Betting Strategies in Horse Races

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Abstract

In this paper we consider the strategies a gambler may employ in situations such as horse races. We assume that the gambler knows which horses have odds which are favourable to him, that he wants to bet in such a way as to have a given positive expectation of win on a race, and that he wishes to minimise the probability of loss of his finite capital. We show that the best strategy is to bet on all the horses whose odds are favourable so as to minimise the probability of loss on a race. We further show that in order to achieve the last objective it is advisable to have a bet on a horse with fair odds, and at times on a horse with unfavourable odds, in addition to a bet on a horse with favourable odds.

1. Introduction

The use of probability theory to investigate gambling strategies is not new; see, Feller (1967), Dubins and Savage (1965), Epstein (1977), Breiman (1961), Rotando and Thorp (1992), to name just the more recent contributions. Feller showed that if a gambler's objective is to increase his initial capital of b by an amount a $(a < b)$, and the game is unfavourable to him, then for even money games, and with a view to minimising the probability of his ruin, his initial bet should be for the amount a, and if he loses that game, his next bet should be for the amount $2a$ etc; if the game is a favourable one, he should bet as small an amount as possible. Rotando and Thorp showed that if the game is for even money and the game is a favourable one, i.e. p the probability of win is greater than $1/2$, then to maximise the exponential rate of growth of the gambler's capital, the gambler should bet the fraction $p - q$ (where $q = 1 - p$) of his capital at every stage of the play. Thus, by and large, the strategies explored so far are the ones the gambler may employ from game to game in a sequence of games. In this paper we consider the strategies a gambler may employ within each game. Horse racing provides the most common example of this situation. We shall discuss the subject matter in the context of horse racing, although the conclusions reached are valid in other contexts also. Within each game (i.e. a horse race), there are a number, (usually about ten to twenty) of mutually exclusive betting propositions (i.e. horse to win). We shall assume that the gambler knows the probability of each horse winning the race, and is offered odds (or prices) about these, so that he can divide the race field into the three categories of favourable bets (i.e. those for whom the expectation is positive), fair bets (i.e. those for whom the expectation is zero) and unfavourable bets (i.e. those for whom the expectation is negative). We assume that there is at least one favourable bet in a game and that the gambler is able to bet on a sequences of such games. We also assume that he has a large but finite capital and is playing against an infinitely rich adversary, and that he wants to bet in such a way as to produce a given (positive) expectation of gain per race. The question is: How should he bet if his objective is to minimise the probability of his ruin, i.e. the exhaustion of his capital?

For example suppose in a race there are, among others, three horses A, B, C.

The odds on offer against them winning the race are 5/1, 3/1 and 3/1 and the probabilities of them winning are 0.2, 0.3 and 0.3 respectively. The question is: What bets should the gambler take? Should he bet on the horse with the most favourable odds? Or, should he hedge his bets, i.e bet on all the horses with favourable odds?

We shall show that the probability of his ruin is a decreasing function of the probability of win on a race (with the same expectation of gain), so that in the situation above, rather than bet only on horse A, he should bet on all the three horses A, B, C. Indeed, if the gambler is in the fortunate position of being offered such bets in a succession of races, then with a capital of 80 dollars, betting 8 dollars on A above (to give an expectation of gain of \$1.60 per race) makes the probability of his ruin equal to 0.4662, whereas betting amounts 2, 3 and 3 dollars on A, B and C respectively (to give the same expectation of \$1.60 of gain on the race) would make the probability of his ruin considerably smaller, namely 0.00013.

On the surface the difference between the two probabilities is striking. However, we need not seek too far to see the reason for this difference. The probability of ruin depends very heavily on the results of initial games, and the probability of getting a succession of losses when the loss probability per game is 0.8 is far greater than what it is when the loss probability per game is only 0.2. So, although there is not much to choose between the two alternatives when the capital is infinite, the difference between the loss probabilities per game has a telling consequence when the capital is finite. In practical terms the latter alternative has another advantage over the former. In practice, a gambler is more like to be able to correctly assess that in a particular race, the probability is 0.8 that the winner would come from one of the three horses A, B and C than correctly apportion probabilities of win to individual horses.

If now, we suppose that the situation is such that there is only one favourable bet in the race, e.g. A as above and he is offered a fair bet on horse D at even money in the same race, then betting 8 dollars each on both A and D would (if such situations were available in a succession of races) be preferable to betting 8 dollars on A alone as the probability of ruin now is 0.37729, somewhat less than 0.4662 for betting on A alone. More surprisingly, the probability of ruin for the combination of A and an unfavourable bet in the same race (albeit only marginally unfavourable) is less than what it would be for the single bet on A. These conclusions go counter to our intuition that the gambler must avoid, at all cost, fair and unfavourable bets.

The preceding discussion was by necessity a hypothetical one; in practice a gambler intending to bet on a succession of races in a season or a life-time would meet a large variety of betting propositions. What the above tells us is that, a gambler must consider only races where there is at least one bet which is favourable, should bet such that the probability of loss on a race is minimum, and to satisfy this objective, even take fair and unfavourable bets.

2. Preliminaries

Let X be the net gain made by the gambler on a game, be it with one bet or more than one bet and let $E(X) = \mu > 0$. Let us assume we have a sequence of games, for each of which the net gain has a distribution, the same as that of X. Then we have a sequence $\{X_i\}$ of independent and identically distributed random variables, and $S_n = \sum_1^n X_i$ is the net gain after n games. If the gambler's capital is $b > 0$, $\{S_n\}$, $(n = 0, 1, 2, 3, \cdots)$ is a random walk process with an absorbing barrier at $-b$, the gambler's ruin corresponding to the absorption of the random walk at $-b$. A convenient tool used to derive the probability of absorption in random walks is Wald's Identity (Wald (1947)) which is as follows:

Wald's Identity: Let N be the first time the random walk is absorbed, and let $M(\theta)$ be the moment generating function $(m.g.f.)$ of X. Then,

$$E\left[e^{\theta S_N} \quad M(\theta)^{-N}\right] = 1 \tag{1}$$

for all θ such that $|M(\theta)| \geq 1$.

We shall also need the following lemma, also due to Wald (1947).

Wald's Lemma: Let X be a $r.v.$ assuming both positive and negative values with $E(X) \neq 0$, and let $M(\theta)$ exist for θ in an interval (c, d) around zero. Then there exists one and only one real $\theta_0 \neq 0$ such that $M(\theta_0) = 1$.

For our case, with $E(X) > 0$, we have $\theta_0 < 0$.

Putting $\theta = \theta_0$ in (1), we obtain

$$E\left[e^{\theta_0 S_N}\right] = 1. \qquad (2)$$

Suppose now our $r.v.$ X is integer-valued with the minimum value -1, and let b be an integer. Then, at absorption, the barrier $-b$ is reached exactly, i.e. $S_N = -b$, and we have, from (2), the probability of the gamblers ruin, $P(R)$ given by

$$P(R) = e^{b\theta_0}. \qquad (3)$$

We shall find it convenient to consider the probability generating function $(p.g.f.)$ rather than the $m.g.f.$ Putting $e^{\theta_0} = z_0$, we have

$$P(R) = z_0^b \qquad (4)$$

where z_0 is the unique solution $(0 < z_0 < 1)$ of $P(z) = 1$. Suppose now the $r.v.$ X is integer-valued with the minimum value $-M$, $(M > 1)$, then at absorption, we have $S_N = -b, -b-1, -b-2, \cdots, -b-M+1$. For this case, it can be shown, using Rouche's theorem, that $P(z) = 1$ has M roots

z_j, $(j = 1, 2, \cdots, M)$ $\left(|z_j| < 1\right)$. One of the roots z_1, say is the same as z_0 above, lying between 0 and 1. Using the identity (1) with *p.g.f.* $P(z)$ of X instead of the *m.g.f.* $M(\theta)$, and putting $z = z_j$ $(j = 1, 2, \cdots, M)$ and denoting $\Pr(S_N = -b - i)$ by P_i, we have the M equations.

$$\sum_{i=0}^{M-1} P_i z_j^{-b-i} = 1 \qquad (j = 1, 2, \cdots, M). \tag{5}$$

These can be solved for $\{P_i\}$. $P(R)$ is then given by

$$P(R) = \sum_{i=0}^{M-1} P_i .$$

For the case $M = 2$, the root z_2 is also real, $(-1 < z_2 < 0)$, and solving (5) we obtain

$$P(R) = \frac{(1 - z_2)z_1^{b+1} - (1 - z_1)\, z_2^{b+1}}{z_1 - z_2}. \tag{6}$$

For our application, b is large compared to the negative values assumed by X, and hence we may neglect the overshoot over the barrier $-b$, and we have the approximate result,

$$P(R) \approx z_1^b \tag{7}$$

Obviously the smaller $|z_2|$ is compared to z_1, the better is the approximation. The closeness of the approximation may be seen by taking the example where X assumes values 2, 1, 0, −2 with probabilities 0.3, 0.1, 0.4, and 0.2 respectively. We obtain $z_1 = 0.7483$ and $z_2 = -0.6022$, and we notice that $|z_2|$ is fairly close to z_1. For $b = 10$, 15 and 20, the exact and approximate values of $P(R)$ obtained from (6) and (7) are as follows:

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b	10	15	20
Exact	0.0496	0.0114	0.0027
Approximate	0.0551	0.0129	0.0030

Let's now consider the scenario described in the introduction, where we have three horses A, B and C with odds 5/1, 3/1 and 3/1 and probabilities of win 0.2, 0.3 and 0.3 respectively. Suppose, we bet the amounts 2, 3 and 3 dollars on A, B, and C. X now takes the values 4 and -8 with probabilities 0.8 and 0.2 respectively, so that we have $P(z) = 0.8z^4 + 0.2z^{-8}$. Now, the probability of ruin for this case with $b = 80$, is the same as when we have $b = 20$ and X takes the values 1 and -2 with probabilities 0.8 and 0.2. The resulting $p.g.f.$ is $P(z) = 0.8z + 0.2z^{-2}$, and taking $P(z) = 1$, yields $z_1 = 0.6404$, $z_2 = -0.3904$. The approximate and exact values of $P(R)$ obtained from (6) and (7) are 0.00013 and 0.00012 respectively. Table 1 gives the approximate values of $P(R)$ obtained from (7) with $b = 80$, for various possible combinations of bets for the game (all with the same expected gain on the game). We notice that $P(R)$ decreases as q the probability of loss in the game decreases.

<u>*Table 1*</u>

$P(R)$, the probability, of the gambler's ruin with a capital of 80 units

Betting Combination	8 units on A only	8 units on B (or C) only	3.2 units on A, 4.8 units on B (or C)	4 units each on B and C	2 units on A, 3 units each on B, C
Prob. of loss on a game, q	0.8	0.7	0.5	0.4	0.2
P(R)	0.4662	0.2769	0.0591	0.0173	0.00013
Var(X)	368.64	215.04	92.16	61.44	23.04

For large values of b, we may consider another relationship. The number of steps before absorption, N, is now large and since we are dealing with sums of random variables, we may use the Central Limit Theorem; see Bartlett (1955). This effectively means that we can assume that the net gain per game has a normal distribution. The relationship $M(\theta) = 1$ gives us $\theta_0 = -2\mu / \sigma^2$, where σ^2 is the variance of X, and from (3) we note that $P(R)$ is an increasing function of σ^2, for a constant μ. Table 1 brings out this relationship between $P(R)$ and σ^2 fairly strongly.

In the general theory given in the rest of the paper, we shall assume that we have either a unit bet on the game (Section 3), or else the primary bet is of unit amount (Sections 4 and 5). In this case, the net gain is unlikely to assume integer values; however, with b large and the loss per game restricted to under 2, the overshoot is small and may be neglected, and $P(R)$ is approximately given by (7). The problem therefore reduces to comparing the values of z_1 obtained for various betting strategies. The z_1 is the unique positive solution $(0 < z_1 < 1)$ of $P(z) = 1$, where the definition of $p.g.f.$ is extended to include the case of non-integer valued random variables.

3. Betting on Favourable Bets

For betting only on favourable bets, we can bet amounts on each horse, so that the net gain is the same. Let p be the combined probability of a win and $q(= 1 - p)$ the probability of a loss. We shall assume the total bet is one unit; thus c the overall odds we are getting is such that $cp - q = \mu > 0$. The net gain is the random variable X assuming values c and -1 with probabilities p and q respectively.

Here $P(z) = pz^{(\mu + 1)/p - 1} + q z^{-1}$.

It is fairly easy to show that (for constant μ) the root $z_1 (0 < z_1 < 1)$ of $P(z) = 1$ decreases as p increases from 0 to 1.

First, we note that $P(1) = 1$, $P'(1) = \mu > 0$, $P(z) \to \infty$ as $z \to 0$, and that $P(z)$ is a convex function of z, $(0 < z \leq 1)$. This means, there is a value z', $(0 < z' < 1)$ at which $P(z)$ attains its minimum value. Solving $P'(z) = 0$, we have $z' = \left[\dfrac{1 - p}{1 + \mu - p}\right]^{p/(\mu + 1)}$, and we know $z_1 < z'$. Now, z_1 is a function of p, implicity defined by

$$p z_1^{(\mu + 1)/p - 1} + q z_1^{-1} = 1. \tag{8}$$

Differentiating (8) with respect to p, we find

$$\frac{1}{z_1} \frac{d z_1}{d p} = \frac{z_1^{(\mu + 1)/p}\left[1 - (\mu + 1) p^{-1} \log z_1\right] - 1}{1 - p - (\mu + 1 - p) z_1^{(\mu + 1)/p}}.$$

The numerator above is of the form $x(1 - \log x) - 1$ where $0 < x < 1$. This is always negative. From the value of z' above, it is easy to see that the denominator above is positive for $z_1 < z'$. Thus dz_1/dp is negative, i.e. z_1 decreases as p increases.

To see how z_1 depends on μ and p, we let $z_1 = 1 - \delta$, and use Taylor series expansion for the left hand side of (8) and solve for δ. We obtain

$$\delta = \frac{2 \mu p}{(1 + \mu)(q + \mu)}.$$

Thus, (for a constant value of μ) as p increases, δ increases, and since $P(R) = z_1^b$, $P(R)$ decreases. Table 2 gives the variation of the values of z_1 obtained from (8) with respect to values of q for $\mu = 0.05$, 0.1, 0.2, and it can be seen that as q decreases, z_1 decreases. Indeed, since the ruin probability is an exponential function of z_1, the effect on the ruin probability is very much greater.

Table 2

z_1 values for values of q and μ for favourable games.

μ \ q	0.2	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.05	0.5145	0.7163	0.8156	0.8744	0.9132	0.9407	0.9612	0.9770	0.9897
0.10	0.3458	0.5640	0.6966	0.7846	0.8469	0.8932	0.9291	0.9576	0.9808
0.20	0.2183	0.4101	0.5556	0.6667	0.7535	0.8229	0.8795	0.9265	0.9662

4. Betting on an additional Fair Bet

Suppose now there is only one favourable bet in a race with probability of win p and expectation $\mu > 0$, so that the odds offered are given by $c = (1 + \mu) / p - 1$. Let us assume we have an additional fair bet in the race with probability of win p' i.e. the odds are q' / p', where $q' = 1 - p'$. Whatever amount x we have on the fair bet leaves our expectation of the net gain unchanged. Betting a unit amount on the favourable bet and an amount x units on the fair bet, the net gain X has the distribution

X	$c - x$	$x q' / p' - 1$	$-1 - x$
Pr	p	p'	$1 - p - p'$

To minimise the probability of ruin by betting on this game (and successive such games), we should take that value of x such that the root z_1 $(0 < z_1 < 1)$ of the resultant equation $P(z) = 1$, i.e.

$$p z^{c+1} + p' z^{x/p'} - z^{1+x} + 1 - p - p' = 0 \tag{9}$$

takes its minimal value.

Putting $z = 1 - \delta$ and using Taylor series expansion for the left hand side of (9), we obtain

$$\delta = \frac{2\mu}{c(c+1)p - 2x + x^2 q' / p'}$$

For given c and p, the maximum value of δ is obtained when $x = p'/q'$. Substituting the value of x in (9) we solve for z. Table 3 gives the z_1 values obtained for $\mu = 0.1, 0.2,$ $p = 0.1, 0.2, 0.3, 0.4, 0.5$ and $p' = 0.1, 0.2, 0.3, 0.4$. The required z_1 values are those which correspond to the case $u = 0$. The corresponding z_1 values for the same p with only favourable bets is given in each case. From the table we note, for example, that for $\mu = 0.2$, the single favourable bet at $p = 0.2$ gives $z_1 = 0.9265$. However, with the additional fair bet at $p' = 0.5$, yields $z_1 = 0.9071$, so that with $b = 40$, $P(R)$ decreases from 0.04723 to 0.02026.

We note that as p' increases the value of z_1 decreases. So if an additional fair bet is to be taken, the shorter the odds the better it is. Ideally, of course, we should take a fair bet at $p' = 1 - p$, but quite obviously, in practice this would be unattainable, as it would mean there is no loss, only a possibility of a gain.

5. Betting on an additional unfavourable bet

Suppose now we have, as before, a favourable bet of one unit with probability of win p, and expectation $\mu > 0$. Let us assume we have in addition an unfavourable bet with probability of win p', and let us assume the odds offered are $q' / p' - u$ to 1, so that the unfavourability factor is $u > 0$. It is obvious that to maintain the expectation of the total transaction of the two bets to μ, we need to increase the amount on the favourable bet. Let $x > 0$ be the increase of the amount on the favourable bet and y the amount on the unfavourable bet. Since the loss on the unfavourable bet (due to its unfavourability) has to be compensated by the extra gain on the favourable bet, we have the relation $p'uy = \mu x$. The net gain X on the game has the probability distribution

X	$c(1+x)-y$	$(q'/p'-u)y-(1+x)$	$-1-x-y$
Pr.	p	p'	$1-p-p'$

so that the equation $P(z) = 1$ becomes

$$p z^{(1+x)(1+c)} + p' z^{(1/p'-u)y} - z^{1+x+y} + 1 - p - p' = 0 \tag{10}$$

As before, to find the values of x and y so that the root z_1 $(0 < z_1 < 1)$ of (10) takes its minimum value, we let $z = 1 - \delta$, expand the left hand side of (10) by a Taylor series expansion and solve for δ in terms of μ, p, p', c, x, y and u. The maximum value of δ is obtained when

$$y = \frac{-(c+1)(2c+1)\,p\,p'u/\mu + 2 + p'u(1-\mu)/\mu}{2\left[pp'^2u^2(1+c)^2/\mu^2 + (1-p'u)^2/p' - (\mu+p'u)^2/\mu^2\right]}$$

Using the value of y and x given by $x = p'uy/\mu$ in (10) we solve for z. The values of z_1 for $\mu = 0.1$, 0.2, $p = 0.1$, 0.2, 0.3, 0.4, $p' = 0.1$, 0.2, 0.3, 0.4, 0.5, and $u = .01$, $.05$ are given in Table 3. The asterisks * in the table correspond to the cases where the values of x and y are negative, and therefore not admissible. The symbols # correspond to the cases where $p + p' = 1$; these cases are obviously not realizable in practice.

From Table 3 we note for example that for $\mu = 0.1$, the single favourable bet at $p = 0.4$ yields $z_1 = 0.8932$, and with an additional fair bet at $p' = 0.5$ (the case $u = 0$) yields $z_1 = 0.7510$. However, if the additional bet is an unfavourable bet with $p' = 0.5$ and $u = 0.05$, we have $z_1 = 0.8410$, so that with $b = 20$, the values of $P(R)$ for the three cases above are 0.1045, 0.0033, and 0.0313 respectively.

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Table 3

z_1, *values for values of* p, p', μ, u for additional fair and unfavourable bets.

	p = 0.1 $\mu = 0.1$ $(z_1 = 0.9807)$					$\mu = 0.2$ $(z_1 = 0.9662)$				
u p'	0.1	0.2	0.3	0.4	0.5	0.1	0.2	0.3	0.4	0.5
0	.9805	.9802	.9799	.9793	.9785	.9658	.9653	.9647	9638	.9626
0.01	.9806	.9804	.9803	.9802	.9802	.9658	.9655	.9651	.9647	.9643
0.05	.9807	.9808	*	*	*	.9660	*	*	*	*
	p = 0.2 $\mu = 0.1$ $(z_1 = 0.9575)$					$\mu = 0.2$ $(z_1 = 0.9265)$				
u p'	.1	.2	.3	.4	.5	.1	.2	.3	0.4	0.5
0	.9565	.9550	.9530	.9500	.9451	.9248	.9225	.9193	.9146	.9071
0.01	.9565	.9554	.9541	.9524	.9500	.9249	.9229	.9203	.9168	.9115
0.05	.9569	.9568	.9571	*	*	.9252	.9242	.9237	.9236	.9240
	p = 0.3 $\mu = 0.1$ $(z_1 = 0.9291)$					$\mu = 0.2$ $(z_1 = 0.8795)$				
u p'	.1	.2	.3	.4	.5	.1	.2	.3	0.4	0.5
0	.9259	.9216	.9153	.9051	.8858	.8747	.8682	.8588	.8438	.8165
0.01	.9261	.9224	.9173	.9097	.8959	.8749	.8689	.8605	.8476	.8246
0.05	.9267	.9250	.9240	.9236	.9241	.8754	.8712	.8667	.8612	.8530
	p = 0.4 $\mu = 0.1$ $(z_1 = 0.8932)$					$\mu = 0.2$ $(z_1 = 0.8229)$				
u p'	.1	.2	.3	.4	.5	.1	.2	.3	0.4	0.5
0	.8860	.8755	.8587	.8277	.7510	.8124	.7974	.7741	.7330	.6404
0.01	.8863	.8766	.8620	.8357	.7709	.8126	.7983	.7766	.7389	.6537
0.05	.8872	.8809	.8738	.8636	.8410	.8133	.8018	.7863	.7615	.7057
	p = 0.5 $\mu = 0.1$ $(z_1 = 0.8469)$					$\mu = 0.2$ $(z_1 = 0.7535)$				
u p'	.1	.2	.3	.4	.5	.1	.2	.3	0.4	0.5
0	.8316	.8075	.7642	.6627	#	.7327	.7011	.6473	.5343	#
0.01	.8863	.8766	.8620	.8357	#	.7329	.7023	.6508	.5425	#
0.05	.8872	.8809	.8738	.8636	#	.7339	.7071	.6645	.5754	#

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