

Efficiency and Cross Efficiency Measures: A Validation Using OECD Data

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Abstract

This paper makes use of cross efficiency measures, a means of validating Data Envelopment Analysis (DEA) scores using different weighting schemes. It is demonstrated using data from 30 OECD countries. We recommend this as a means of bringing non stochastic DEA and stochastic modelling closer together, especially useful in modelling of health (and health care) production functions, given their multiple output nature.

Keywords

Cross efficiency, data envelopment analysis, international comparisons, production of health, efficiency measurement.

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Introduction

The use of data envelopment analysis in estimating the efficiency of health care provision is growing rapidly [1] as it is the only technique available to estimate multiple input – multiple output models without resorting to some level of aggregation. Validation of the DEA models has been looked at elsewhere [2] in terms of model specification, here we extend validation further in terms of post analysis validation of efficiency scores, using the computationally intensive method of cross efficiency analysis, which brings together elements of stochastic and nonstochastic modelling in terms of subjecting DEA to some measure of uncertainty.

Methods

Data Envelopment Analysis

Detailed descriptions of DEA, and the economic theory of production and efficiency measurement which underlie it, can be found in a number of sources [1,3], only a brief description is given here. DEA is an indicator of efficiency, defined as the ratio of a weighted sum of the outputs of a productive unit to a weighted sum of its inputs, constrained to lie between unity (efficient) and zero. Thus, for a country with n outputs and m inputs the ratio is:

$$Efficiency = \frac{\sum_{r=1}^n u_r * y_r}{\sum_{i=1}^m v_i * x_i}$$

where: y_r = quantity of output r , u_r = weight attached to output r , x_i = quantity of input i , v_i = weight attached to input i ; and a value equal to unity implies complete efficiency. The weights are specific to each unit so that: $0 < Efficiency \leq 1$.

The method assumes that each country attaches its own weights to both inputs and outputs and that the efficiency of a particular country is assessed by comparing it against similar countries, ie it is based on self appraisal. Of all of the possible sets of weights, the set is chosen which gives the highest efficiency score.

From this problem, a linear programme can be defined by constraining the numerator or denominator of the efficiency ratio to be equal to unity. The problem then becomes either maximise weighted output with weighted input equal to unity or minimise weighted input with weighted output equal to unity. The solution to the output maximisation problem is directly analogous to a radial measure of technical efficiency [1]. The method we use is output maximisation, as, conceptually, countries will wish to maximise the health of their populations given their resource inputs (this contrasts with health care production models where outputs are taken as exogenous). DEA can be undertaken for constant or varying returns to scale models, we use the latter as this is conceptually analogous to the theory of the production of health; estimating the relationship between country level average health and gross national product suggests decreasing marginal returns [4] and Culyer and Wagstaff [5] assume that the relationship between health care expenditure and health is concave. DEA is used to measure the efficiency of each year of data on a static cross sectional basis.

Cross Efficiency

Cross efficiency is a two stage process [6]. First the basic DEA model is run. Cross efficiency then compares every country with all other countries, applying the weights of the other countries, from the original DEA estimation, to the country under consideration to ascertain the effect this has on the original country's efficiency rating. An average cross efficiency score is arrived at from the re estimation. A matrix is then used to estimate cross efficiencies, see Table 1. It would be expected that average cross efficiency scores would be lower than the original scores, as a country cannot have a cross efficiency score higher than the original DEA score, as this shows each country in its best possible light.

Table 1 Matrix of cross efficiencies, basic DEA efficiencies are in the leading diagonal, E35 is the cross efficiency accorded Country 5 using country 3's weights. Ak and ek can be averaged with or without the leading diagonal (in practice given a large enough data set it makes little difference)¹.

Rating country	Rated country					Averaged appraisal of peers
	1	2	3	4	5	
1	E ₁₁	E ₁₂	E ₁₃	E ₁₄	E ₁₅	A ₁
2	E ₂₁	E ₂₂	E ₂₃	E ₂₄	E ₂₅	A ₂
3	E ₃₁	E ₃₂	E ₃₃	E ₃₄	E ₃₅	A ₃
4	E ₄₁	E ₄₂	E ₄₃	E ₄₄	E ₄₅	A ₄
5	E ₅₁	E ₅₂	E ₅₃	E ₅₄	E ₅₅	A ₅
	e ₁	e ₂	e ₃	e ₄	e ₅	
Averaged appraisal by peers						

A comparison can then be made; averaging down each column gives the countries average cross efficiency using its own inputs and outputs and other countries weights (average appraisal by peers). This analysis allows a ranking of units, including those which are on the production frontier. Cross efficiency helps eliminate one problem with DEA, where most of the weight in a ratio can be placed on a single variable, with the rest being given near zero weights. Several methods of restricting weights have been tried but are arbitrary [7]. Cross efficiency allows analysis based on peer appraisal with weights which are internally derived rather than externally imposed [6].

Data

The data are from 30 OECD countries, on health, health expenditure and schooling. The health variable is calculated from burden of disease work [8] and is a morbidity adjusted measure of life expectancy; DALEs. Health care expenditure data is taken from Pouillier and Hernandez [9] and measures total health expenditure per capita (public and private) in 1997 international dollars.

¹ This table is a simple adaptation from Doyle and Green [12].

Schooling is measured as the average years of schooling in the adult population. The output, health (measured in DALEs), is a function of the physical inputs into the health system, health expenditure per capita, and non-health service inputs, schooling.

Results

Table 2 Efficiency, cross efficiency, and variance (ranked by cross efficiency).

	<i>Original score</i>	<i>Average cross efficiency</i>	<i>Variance (cross efficiency)</i>
<i>Spain</i>	100.00	98.17	5.79
<i>Greece</i>	100.00	98.12	4.52
<i>Japan</i>	100.00	96.75	24.94
<i>Italy</i>	100.00	95.83	32.12
<i>United Kingdom</i>	98.02	95.41	7.59
<i>Portugal</i>	99.40	95.09	6.01
<i>France</i>	100.00	95.06	48.25
<i>Norway</i>	97.01	93.80	20.10
<i>Sweden</i>	97.87	93.66	33.04
<i>Austria</i>	97.46	93.19	34.93
<i>Ireland</i>	95.25	93.09	5.38
<i>Belgium</i>	96.14	93.01	22.62
<i>Netherlands</i>	96.74	92.99	29.52
<i>Australia</i>	96.63	92.74	22.35
<i>Czech Republic</i>	98.73	92.29	23.87
<i>Canada</i>	96.62	92.29	30.58
<i>Iceland</i>	95.60	92.25	22.42
<i>Finland</i>	95.17	91.83	16.83
<i>Turkey</i>	100.00	91.72	35.19
<i>Poland</i>	100.00	91.35	33.50
<i>Mexico</i>	98.76	91.20	17.04
<i>Luxembourg</i>	95.37	91.20	33.64
<i>Switzerland</i>	97.18	90.80	71.68
<i>Slovakia</i>	97.11	90.46	24.50
<i>New Zealand</i>	93.97	89.83	21.36
<i>Germany</i>	94.41	89.20	51.29
<i>Hungary</i>	97.55	88.97	29.26
<i>Denmark</i>	92.97	88.27	32.98
<i>Rep Korea</i>	91.36	87.15	17.63
<i>United States</i>	93.91	83.40	130.01

Table 2 shows the original DEA scores, the average cross efficiency score and the variance. The average of the cross efficiencies (overall mean 92.3, st dev 3.2), are lower overall than the original DEA scores (mean 97.1, st dev 2.37) as the countries which are efficient in the original DEA analysis do not come out as efficient in every cross efficiency analysis, thereby reducing the overall average efficiency, and, as stated earlier, a country cannot have a cross efficiency score higher than its original DEA score.

The cross efficiency average scores are seen as representing a true peer assessment as each country is assessed as how it performs using all other countries weights, thereby reflecting a countries all round performance. In other words, no matter which combination of weights are used on a countries inputs and outputs, if a country has a high cross efficiency score on average we can assume they are actually using their inputs and outputs efficiently. This is a type of sensitivity analysis, 29 different sets of weights are applied to each unit, undertaking the DEA analysis again each time. Obviously the true number of potential weight combinations is enormous, but as stated previously here the intuitively appealing process of using the weights from within the analysis process rather than some arbitrary external imposition of weights is used.

A simple means of observing which countries vary the least from their highest efficiency score (ie have the lowest level of uncertainty) is to calculate the variance, see Table 2. This shows countries efficient under the original DEA run generally have the lowest level of uncertainty, implying they are more likely to be actually using their inputs and outputs efficiently. It also highlights uncertainty concerning some countries efficiency scores, as some have a large variance. Finally, cross efficiency does allow us to rank those countries which originally scored 100, see Table 2.

Discussion

We have demonstrated how a non stochastic method (DEA) can be drawn closer to stochastic modelling, in terms of introducing an element of uncertainty. DEA is being used increasingly to measure the efficiency of health services [1], as it is the only method which can deal with multiple input – multiple output models without resorting to some means of aggregation. Methods are available for validating model specification [2], but cross efficiency takes the validation process a stage further. Once the model has been specified, cross efficiency allows validation of the actual results using peer appraisal.

Here we have demonstrated that cross efficiency is a useful method of post DEA analysis validation of results. We have also demonstrated how variance can be used to highlight uncertainty around efficiency scores, and how cross efficiency can be used as a further means of ranking efficient units. A further interpretation of the variance may be that it is indicative of variable selection problems in the original model, a large variance may point to variables being omitted from the original model, impacting upon the weighting structure.

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