MONASH UNIVERSITY

AN ANALYTICAL FRAMEWORK OF PRODUCER-CONSUMERS, ECONOMIES OF SPECIALIZATION AND TRANSACTION COSTS

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An Analytical Framework of Producer-Consumers, Economies of Specialization and Transaction Costs

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Abstract

In this paper, an analytical framework of producer-consumers is presented. It is proved that a producer-consumer does not buy and sell the same good, does not buy and self-provided the same good, and sells one good at most if there exist economies of specialization and transaction costs. Based on this, a lot of classical economic thoughts can be resurrected and many interesting results can be obtained from this kind of analytical framework.

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Abstract

In this paper, an analytical framework of producer-consumers is presented. It is proved that a producer-consumer does not buy and sell the same good, does not buy and self-provided the same good, and sells one good at most if there exist economies of specialization and transaction costs. Based on this, a lot of classical economic thoughts can be resurrected and many interesting results can be obtained from this kind of analytical framework.

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I. Introduction

Samuelson(1967) and Alfred Marshall(1920) consider the essence of economics as the analysis of demand and supply. However, there are two research lines of demand and supply. (1)Marshall's line: Demand and supply are determined by the tradeoff between quantities of different goods consumed in raising utility and the tradeoff between quantities of different factors in raising output. Relative demand in equilibrium is determined by relative taste, relative technology and relative endowments. Aggregate demand is not the focus of the analysis and is given by the dichotomy between pure consumers and pure producers. In other words, resource allocation is the key issue in neoclassical economic theory. Most resource allocation problems are solved in some linear or nonlinear programming models. (2)Allyn Young's line: Demand and supply are two sides of the level of division of labor (or its reciprocal the degree of self-sufficiency). The level of specialization and division of labor determines the extent of the market and aggregate demand and supply(Young, 1928, p539). Hence, we cannot understand what are demand and supply if we do not know the mechanism that determines individuals' level of specialization and the level of division of labor for a society as a whole. As Houthakker (1956, p182) writes:

> Most economists have probably regarded the division of labor, in Schumpeter's words, as an "external commonplace", yet there is hardly any part of economics that would not be advanced by a further analysis of specialization.

studies of specialization is not a subfield of economics, instead, it should be in the central place of economic analyses since individuals' level of specialization determines demand and supply.

Endogenization of individual level of specialization needs a framework of consumer-producers, economies of specialization and transaction costs. I will show that within the framework, an individual's optimum decision cannot be well defined in the absence of the theorem that I will prove. In addition to this, I will prove for a family of utility and production functions that the optimum decision is always corner solution and thereby the marginal analysis for the interior solution is not sufficient for solving the optimum decision. Without the theorem, Young's analysis of demand and supply which are two sides of division of labor cannot be formalized within a mathematical structure.

II. A general framework used to endogenize individual's level of specialization

Dismantling the dichotomy between producers and consumers, we assume each individual is a producer-consumer. Labor is the only initial endowment to everyone and the only variable input in the production of each good. There are m goods which an individual has different capability of producing each of them. For simplicity, we assume that the m goods are all final goods. Labor input in producing good i is denoted as l_i and the quantity consumed of good i is X_i. We further assume:

Assumption 1. Utility function $U(X_1, X_2, ..., X_m)$ is quasi-concave (the bordered Hessian matrix is negative definite) and has positive marginal utility.

Assumption 2. Production function of good i $f_i(\cdot)$ satisfies either: (1) $f_i(0)=0$, $f_i'(\cdot)>0$, $f_i''(\cdot)>0$ and $f_i''(\cdot)$ is continuous or (2) $f_i(l_i)=a_i l_i - b_i$,where $a_i, b_i>0$. Each good produced $f_i(l_i)$ will either be consumed by the producer or sold in the market, the amount of the self-provided consumption is denoted as x_i, the amount of the good sold in the market is denoted as x_i^s, then $x_i + x_i^s = f_i(l_i)$, $x_i \in R_+$, $x_i^s \in R_+$

Assumption 3. The amount the person buys from the market of good i is denoted as x_i^d and assume the transaction cost function $c_i(x_i^d)$ is of: $c_i(0)=0$, $0<c_i'(\cdot)<1$, $c_i''(\cdot)\le 0$, then the corresponding transaction efficiency function $k_i(x_i^d)$ is of: $k_i(0)=0$, $0<k_i'=1-c_i'<1$, $k_i''(\cdot)\ge 0$. Therefore, his consumption of good i is $X_i = x_i + k_i(x_i^d)$. $x_i^d \in R_+$

Assumption 4. Each producer-consumer is a price taker. If market of good i exists, it is under perfect competition. Assume the price of goods i in terms of good 1 is $p_i (p_1 \equiv 1)$.

Define $I=\{\ 1,\ 2,\ \ldots,\ m\ \}$, $J=\{i\in I:\ x_i>0\}$, $R=\{i\in I:\ x_i^d>0\}$, $T=\{i\in I:\ x_i^s>0\}$. Later we will show $J\cap R=\varnothing$ and $T\cap R=\varnothing$, $J\cup R=I$ due to quasi-concave utility function. For simplicity, we assume each individual is endowed with one unit of labor. Then we have constraints

$$x_i + x_i^s = f_i(l_i), \qquad i\in \mathrm{I} \tag{1}$$

$$\sum_{i\in I} p_i x_i^d \le \sum_{i\in I} p_i x_i^s \ , \tag{2}$$

which yields
$$\sum_{i\in I} p_i(x_i + x_i^d) \le \sum_{i\in I} p_i f_i(l_i) \tag{3}$$

Labor constraint is
$$\sum_{i\in I} l_i = 1 \tag{4}$$

From (3) and (4), we get

$$\sum_{i=1}^{m} p_i(x_i + x_i^d) \le \sum_{i=1}^{m-1} p_i f_i(l_i) + p_m f_m(1-l_1-l_2-\cdots-l_{m-1}) \tag{5}$$

Let L_i denote a person's labor productivity of good i $(L_i = f_i(l_i)/l_i)$, Level of specialization can be defined as follows:

Definition 1. (an individual's level of specialization in producing a certain good) An individual's labor share in producing a good is defined as the person's level of specialization in producing the good. In our simplified case, his level of specializaton in producing good i is l_i.

Definition 2. (an individual's level of specialization) If a person is involved in some market transactions, the largest labor share he spends on producing one of the goods he sells is defined as his level of specialization denoted by $l_0 = \max_{i \in T}(l_i)$; If he is in autarky, $T = \varnothing$ and his level of specilization is defined as zero.

There exist economies of specialization in the production of good i if his labor productivity of good i increases with his level of specialization in producing the good, ie. if $dL_i / dl_i > 0$.

Lemma. There exist economies of specialization in the production of all goods under Assumption 2.

Proof: (1) When $f_i(0) = 0$, $f_i'(.) > 0$, $f_i''(.) > 0$ and $f_i''(.)$ is continuous , we have

$$0 = f_i(0) = f_i(l_i) - l_i f_i'(l_i) + \frac{l_i^2}{2} f_i''(\xi) > f_i(l_i) - l_i f_i'(l_i) \qquad ,0 < \xi < l_i$$

Thus, $\dfrac{dL_i}{dl_i} = \dfrac{f_i'(l_i)l_i - f_i(l_i)}{l_i^2} > 0, \quad \forall i \in \mathrm{I}$

(2)When $f_i(l_i) = a_i l_i - b_i$ $(a_i, b_i > 0)$, $L_i = a_i - \dfrac{b_i}{l_i}$, $\dfrac{dL_i}{dl_i} = \dfrac{b_i}{l_i^2} > 0$, $i \in \mathrm{I}$

The producer-consumer's decision is to choose x_i, x_i^d, and l_i $(i \in I)$ to maximize his utility under his budget and labor constraints, which is a nonlinear programming problem as follows:

$$\max_{x_i, x_i^d, l_i} U\left(x_1 + k_1(x_1^d), x_2 + k_2(x_2^d), \cdots, x_m + k_m(x_m^d)\right)$$

(6)

$$\text{s.t.} \quad \sum_{i=1}^{m} p_i(x_i + x_i^d) \le \sum_{i=1}^{m-1} p_i f_i(l_i) + p_m f_m(1 - l_1 - l_2 - \cdots - l_{m-1})$$

$$0 \le x_i \le f_i(l_i),\ x_i^d \ge 0,\ l_i \ge 0, \qquad i \in I$$

Since $l_1, l_2, \ldots, l_m$ are not included in the objective function, we need the following theorem to endogenize them in the model. Although Yang and Ng (1993) have proven a similar lemma, it is proved for a very special case: two goods, symmetric Cobb-Douglas utility function, the same constant elasticity production functions for both goods and the same iceberg transaction cost functions in both markets, which is a special case of the problem specified by (1)-(6).

III. Specialization

As to problem (6), we can establish

Claim 1. An individual does not buy and sell the same good, ie. $T \cap R = \emptyset$.

Proof: Suppose not. Let x_i^0, x_i^{d0}, l_i^0 $(i \in I)$ denote the solution of the problem (6). Then $\exists$ $j \in T \cap R$, such that $x_j^{d0} > 0$, $x_j^{s0} = f_j(l_j^0) - x_j^0 > 0$, from which there is either $x_j^{s0} > x_j^{d0} > 0$ or $x_j^{d0} \geq x_j^{s0} > 0$.

Step 1. If $x_j^{s0} > x_j^{d0} > 0$, then denote $x_j^* = x_j^0 + x_j^{d0}$, $x_j^{d*} = 0$, $x_j^{s*} = x_j^{s0} - x_j^{d0} > 0$, and $x_i^* = x_i^0$, $x_i^{d*} = x_i^{d0}$, $x_i^{s*} = x_i^{s0}$ for $i \neq j$, $i \in I$. It is easy to verify that labor allocation is the same between the values of the decision variables with asterisks and the optimal ones, ie. $l_i^* = f_i^{-1}(x_i^* + x_i^{s*}) = f_i^{-1}(x_i^0 + x_i^{s0}) = l_i^0$, $\forall i \in I$. Furthermore, x_i^*, x_i^{d*}, $l_i^*(i \in I)$ is a feasible solution of the problem (6) too. However, $X_j^* = x_j^* + k_j(x_j^{d*}) = x_j^0 + x_j^{d0} > X_j^0 = x_j^0 + k_j(x_j^{d0})$ ($0 < k_i' < 1$) while $X_i^* = X_i^0$ for $i \neq j$, $i \in I$. Since $U_i > 0$, we have $U(X_1^*, X_2^*, \cdots, X_n^*) > U(X_1^0, X_2^0, \cdots, X_n^0)$. This contradicts the fact that x_i^0, x_i^{d0}, l_i^0 $(i \in I)$ is the solution of the problem (6). Actually, $\forall i \in I$, if $x_i^{d0} > 0$, $x_i^{s0} > 0$ and $x_i^{d0} < x_i^{s0}$, the person can increase his utility by rearranging his trade plan of the goods without making any production adjustment. So, it is impossible that $x_i^{d0} > 0$, $x_i^{s0} > 0$ hold simultaneously if $x_i^{s0} > x_i^{d0}$ ($\forall i \in I$).

Step 2. If $x_j^{d0} \geq x_j^{s0} > 0$, denote $x_j^* = x_j^0 + x_j^{s0}$, $x_j^{s*} = 0$, $x_j^{d*} = x_j^{d0} - x_j^{s0} \geq 0$, and $x_i^* = x_i^0$, $x_i^{d*} = x_i^{d0}$, $x_i^{s*} = x_i^{s0}$ for $i \neq j$, $i \in I$. It is easy to verify that labor allocation is the same between values of the decision variables with asterisks and the optimal ones, ie. $l_i^* = f_i^{-1}(x_i^* + x_i^{s*}) = f_i^{-1}(x_i^0 + x_i^{s0}) = l_i^0$. Furthermore, x_i^*, x_i^{d*}, l_i^* $(i \in I)$ is a feasible solution of the problem (6) too. However, $X_j^* = x_j^* + k_j(x_j^{d*}) = x_j^0 + x_j^{s0} + k_j(x_j^{d0} - x_j^{s0}) >$ $> X_j^0 = x_j^0 + k_j(x_j^{d0})$ ($k_j(x_j^{d0}) - k_j(x_j^{d0} - x_j^{s0}) = k_j'(\xi) x_j^{s0} < x_j^{s0}$, where $x_j^{d0} - x_j^{s0} < \xi < x_j^{d0}$)

while $X_i^* = X_i^0$ for $i \neq j$. Since $U_j > 0$, we have $U(X_1^*, X_2^*, \cdots, X_n^*) > U(X_1^0, X_2^0, \cdots, X_n^0)$. This contradicts that x_i^0, x_i^{d0}, l_i^0 $(i \in I)$ is the solution of the problem (6). Actually, $\forall$ $i \in I$, if $x_i^{d0} \geq x_i^{s0} > 0$, the person can increase his utility by rearranging his trade plan of the goods without making any production adjustment. So, it is impossible that $x_i^{d0} > 0$, $x_i^{s0} > 0$ hold simultaneously if $x_i^{d0} \geq x_i^{s0}$ ($\forall i \in I$).

Step 3. From step 1 and 2, we know that $x_i^{d0} > 0$ and $x_i^{s0} > 0$ can't hold simultaneously either in the case $x_i^{d0} < x_i^{s0}$ or $x_i^{d0} \geq x_i^{s0}$. This establishes Claim 1.

Claim 2. An individual sells one good at most.

Proof: From Claim 1, since $x_i^d = 0$ if $x_i^s > 0$ and $X_i = x_i + k_i(x_i^d)$, we have $x_i > 0$ if $x_i^s > 0$ due to the convex preference represented by $U(X_1, X_2, \ldots, X_m)$. Suppose claim 2 is not true, then in the solution of the problem (6), $\exists$ $j, k \in T$, such that $x_j^0 > 0$, $x_j^{s0} > 0$, $x_k^0 > 0$, $x_k^{s0} > 0$. Let $l^0 \equiv l_j^0 + l_k^0$, where $l_j^0 = f_j^{-1}(x_j^0 + x_j^{s0})$ and $l_k^0 = f_k^{-1}(x_k^0 + x_k^{s0})$, $\overline{l_j} \equiv f_j^{-1}(x_j^0)$, $\overline{l_k} \equiv l^0 - \overline{l_j}$, $\overline{\overline{l_k}} \equiv f_k^{-1}(x_k^0)$ and $\overline{\overline{l_j}} \equiv l^0 - \overline{\overline{l_k}}$. It is easy to verify: $\overline{l_j} < l_j^0 < \overline{\overline{l_j}}$, $\overline{\overline{l_k}} < l_k^0 < \overline{l_k}$ while $l_j^0 + l_k^0 = \overline{l_j} + \overline{l_k} = \overline{\overline{l_j}} + \overline{\overline{l_k}} = l^0$. Let $g(l_j) = p_j f_j(l_j) + p_k f_k(l^0 - l_j)$, $\overline{l_j} \leq l_j \leq \overline{\overline{l_j}}$, we have $\max\limits_{\overline{l_j} \leq l_j \leq \overline{\overline{l_j}}} g(l_j) = \max\left(g(\overline{l_j}), g(\overline{\overline{l_j}})\right)$ because (1)$g''(l_j) = p_j f_j''(l_j) + p_k f_k''(l^0 - l_j) > 0$ when $f_i''(\cdot) > 0 (i \in I)$, so any interior extreme point is a minimum point. This implies that the maximum point is at a corner, either $\overline{l_j}$ or $\overline{\overline{l_j}}$; (2) $g(l_j) = p_k a_k l_0 + (p_j a_j - p_k a_k) l_j$ $- p_j b_j - p_k b_k$ either monotonically increases with l_j if $p_j a_j - p_k a_k > 0$ or monotonically decreases with l_j if $p_j a_j - p_k a_k < 0$ when $f_i(l_i) = a_i l_i - b_i$ $(\forall i \in I)$. If $\max\limits_{\overline{l_j} \leq l_j \leq \overline{\overline{l_j}}} g(l_j) = g(\overline{l_j})$ (otherwise $\max\limits_{\overline{l_j} \leq l_j \leq \overline{\overline{l_j}}} g(l_j) = g(\overline{\overline{l_j}})$), then $g(l_j^0) = p_j f_j(l_j^0) + p_k f_k(l_k^0) <$ $g(\overline{l_j}) = p_j f_j(\overline{l_j}) +$ $p_k f_k(\overline{l_k})$ (or $g(l_j^0) = p_j f_j(l_j^0) + p_k f_k(l_k^0) <$ $g(\overline{\overline{l_j}}) = p_j f_j(\overline{\overline{l_j}}) + p_k f_k(\overline{\overline{l_k}})$), which means $p_j(x_j^0 + x_j^{s0}) + p_k(x_k^0 + x_k^{s0}) < p_j f_j(\overline{l_j}) + p_k f_k(\overline{l_k})$, ie. through rearranging production by producing only x_j^0 of good j for self consumption and transferring labor $l_j^0 - \overline{l_j}$ to producing additional amount of good k, the person can get extra receipt $p_k\left(f_k(\overline{l_k}) - f_k(l_k^0)\right) - p_j x_j^{s0}$ (or producing only x_k^0 of good k and transferring labor $l_j^0 - \overline{\overline{l_k}}$ to producing additional amount of good j, the person can get extra receipt

$p_j\left(f_j\left(\overline{l_j}\right)-f_j\left(l_j^0\right)\right)-p_k x_k^{s0}$). Since we have $U_i > 0(\forall i \in I)$, the extra receipt can be used to buy some goods to increase utility. This contradicts the assumption that $x_j^0, l_j^0, x_k^0, l_k^0$ is part of the solution of the problem (6). This argument by negation establishes Claim 2.

Claim 3. An individual does not buy and self-provide the same good, ie. $J \cap R = \varnothing$.

Proof: Suppose not. Then in the solution of the problem (6), $\exists j \in J \cap R$, ie. $x_j > 0$, $x_j^d > 0$. From Claim 2, we know T contains only one element if he is not in autarky. Let $T = \{i_0\}$, then $x_{i_0} > 0$ and $x_{i_0}^s > 0$. Let $l_j \equiv f_j^{-1}(x_j)$, $l_{i_0} \equiv f_{i_0}^{-1}(x_{i_0}+x_{i_0}^s)$, $\bar{l} \equiv l_j + l_{i_0}$, $x_{i_0}^{s'} \equiv \dfrac{p_j}{p_{i_0}} x_j^d$, $\overline{l_{i_0}} = f_{i_0}^{-1}\left(x_{i_0}+x_{i_0}^s - x_{i_0}^{s'}\right)$, then $0 < l_j < \overline{l_j} = \bar{l} - \overline{l_{i_0}}$. Now, consider changing l_j: let $X_j(l) = x_j(l) + k_j\left(x_j^d(l)\right) = f_j(l) + \; k_j\left(\dfrac{p_{i_0}}{p_j}\left(f_{i_0}(\bar{l}-l) - f_{i_0}\left(\overline{l_{i_0}}\right)\right)\right)$, $0 \le l \le \overline{l_j}$. It is easy to verify that $X_j''(l) = f_j''(l) + \left(\dfrac{p_{i_0}}{p_j} f_{i_0}'(\bar{l}-l)\right)^2 k_j'' + \dfrac{p_{i_0}}{p_j} k_j' f_{i_0}''(\bar{l}-l) > 0$, which means $\max\limits_{0 \le l \le \overline{l_j}} X_j(l) = \max\left(X_j(0), X_j\left(\overline{l_j}\right)\right)$. So, $X_j(l_j) = x_j + k_j\left(x_j^d\right) < \max\limits_{0 \le l \le \overline{l_j}} X_j(l)$ ($0 < l_j < \overline{l_j}$). In other words, the consumption of good j can be increased through the adjustment of labor allocation between good j and the good he sells without affecting the production and consumption of any other goods. This contradicts $x_j > 0$, $x_j^d > 0$, $x_{i_0} > 0$ and $x_{i_0}^s > 0$ is a part of the solution of the problem (6). This argument by negation establishes Claim 3.

Theorem 1. *For problem* (6), *the optimal decision of the individual does not involve buying and selling the same good, does not involve self-providing and buying the same good, and does not involve selling more than one good.*

Proof: Claims 1, 2, and 3 are enough to establish this theorem.

From this theorem, we know that optimal solution of problem (6) is always a corner solution. The producer-consumer has to compare his utility levels of those corner solutions that are compatible with the theorem. Autarky(a profile in which $x_i > 0$,

$x_i^d = x_i^s = 0$, $\forall i \in I$) is one possible solution. If the person is in autarky, the problem (6) becomes

$$\max_{l_i} U(x_1, x_2, \cdots, x_m) = U(f_1(l_1), f_2(l_2), \cdots, f_m(l_m))$$

$$\text{s.t.} \quad l_i > 0,\ (i \in I),\ \sum_{i \in I} l_i = 1$$

After his labor share in producing each good has been solved out, we can calculate the utility level. If he is not in autarky, then he sells and only sells one good since he has no initial wealth and we have proved he sells one good at most with economies of specialization. We denote the good he sells as good i_0 ($T=\{i_0\}$) without loss of generality. Suppose he self-provides n goods besides the one he sells, so J contains $n+1$ elements ($0 \le n \le m-2$), then R contains m-n-1 elements, ie. he buys m-n-1 goods. In these cases, the problem (6) becomes

$$\max U(X_1, X_2, \cdots, X_m), \qquad \text{where } X_i = x_i = f_i(l_i)\ (i \in J-T),\ X_{i_0} = x_{i_0}$$

$$X_i = k_i(x_i^d), \quad i \in R$$

$$\text{s.t.} \quad \sum_{i \in R} p_i x_i^d \le p_{i_0}\left(f_{i_0}\left(1 - \sum_{i \in J-T} l_i\right) - x_{i_0}\right)$$

$$0 < \sum_{i \in J-T} l_i < 1$$

$$x_{i_0} > 0,\ l_i > 0 (i \in J-T),\ x_i^d > 0 (i \in R)$$

Solving this problem for all possible i_0 and n yields the optimal labor share in producing each good in each case: l_i^* ($i \in J-T$) and $l_{i_0}^* = 1 - \sum_{i \in J-T} l_i^*$, optimal self-provided amount of the good he sells $x_{i_0}^*$, optimal trade plan: $x_i^{d*} (i \in R)$, $x_{i_0}^{s*} = f_{i_0}(l_{i_0}^*) - x_{i_0}^*$, optimal consumption bundle $(X_1^*, X_2^*, \cdots, X_m^*)$, and the corresponding utility level $U(X_1^*, X_2^*, \cdots, X_m^*)$. Since $1 \le m-n-1 \le m-1$, the above decision problem includes $C_m^1 (C_{m-1}^1 + C_{m-1}^2 + \cdots + C_{m-1}^{m-1})$ profiles of variables. Each of these profiles and the case of autarky represents his different specialization levels in producing each and every good as well as his different levels of specialization. Comparing the utility levels in these profiles and in autarky, we can identify the optimal production plan (thus his optimal specialization level), the optimal trade plan, and the optimal consumption plan that the optimal decisions yield the maximum utility which are dependent on his utility function, production functions, relative prices of the goods, transaction efficiency of each good market.

From a simple example, we can establish the following proposition:

Proposition. The separation between production decision and consumption decision of each consumer-producer will lead to non-optimal decisions when transaction costs in some good markets outweight the economies of specialization.

Example: $m=3$, $U=XYZ$; $f_x(l_x)=l_x^2$, $f_y(l_y)=l_y^3$, $f_z(l_z)=l_z^2$; $p_x=1$, $p_y=0.5$, $p_z=2$

$$k_x(x^d)=\frac{1}{20}x^d,\ k_y(y^d)=y^d-\frac{1}{2},\ k_z(z^d)=\frac{1}{2}z^d,\ l_x+l_y+l_z=1$$

It is easy to verify that this example satisfies assumption 1-4. Since the consumer-producer has no initial wealth and does not sell his labor directly, he will make production decision first under neoclassical dichotomy between consumption decisions and production decisions. Because total cost of his production is the one unit of labor, so he will arrange his production plan to maximize his total revenue as follows:

$$\max_{l_x,l_z}\left(l_x^2+2l_z^2+0.5(1-l_x-l_z)^3\right)$$

the solution is $l_x=0,\ l_y=0,\ l_z=1$, ie. he will totally specialize in producing good z and his level of specialization is 1. So he will be in a configuration of selling good z and buying both goods x and y. Then, as a consumer in neoclassical framework, he will choose his trade plan to maximize his utility as follows:

$$\max_{x^d,y^d,z} U=\frac{1}{20}x^d z\left(y^d-\frac{1}{2}\right)$$

$$\text{s.t.}\quad x^d+0.5y^d\le 2(1-z)$$

$$0<z<1,\ x^d>0,\ y^d>0$$

the solution is $x^d=0.583$, $y^d=1.67$, $z=0.291$, and $U^0=0.0099$

However, in the configuration of self-providing good x, selling good z and buying good y, his maximization problem is: $\max_{l_x,y^d,z} U=l_x^2\left(y^d-\frac{1}{2}\right)z$

$$\text{s.t.}\quad 0.5y^d\le 2\left((1-l_x)^2-z\right)$$

$$0<l_x<1,\ y^d>0,\ z>0$$

the solution is $l_x = 0.276$, $y^d = 1.297$, $z = 0.2$, $U^* = 0.012$

Obviously, he can reach higher utility level in the configuration of self-providing good x, selling good z and buying good y than totally specializing in producing good z which is the optimal configuration under the dichotomy between production decisions and consumption decisions. Furthermore, the optimal production plan (thus the optimal level of specialization), the optimal trade plan, the optimal consumption plan and the maximum utility level comes from the best configuration which leads to the highest utility level while the configuration of totally specializing in producing good z and the configuration of self-providing good x, selling good z and buying good y are only two possible candidates. In other words, the maximum utility level U^* that our consumer-producer can reach will be higher than or equal to 0.012. Certainly, $U^* > U^0$. Therefore, the neoclassical dichotomy between consumption decisions and production decisions will lead to non-optimal decisions when the transaction costs in some good markets outweight the economies of specialization.

As shown in the above proposition, dismantling the dichotomy between consumers and producers is essential for us to endogenize the level of specialization. Adam Smith's thought that the specialization and division of labor are limited by the markets have not been reflected in neoclassical microeconomics. Anyway, without the theorem given in this paper, problem (6) is not well defined. With the relationship $x_i + x_i^s = f_i(l_i)$, problem (6) can be written as:

$$\max_{x_i, x_i^d, l_i} U\left(x_1 + k_1\left(x_1^d\right), x_2 + k_2\left(x_2^d\right), \cdots, x_m + k_m\left(x_m^d\right)\right) \tag{6'}$$

$$\text{s.t.} \quad \sum_{i=1}^{m} p_i x_i^d \le \sum_{i=1}^{m} p_i x_i^s$$

$$\sum_{i \in I} f_i^{-1}\left(x_i + x_i^s\right) = 1$$

$$x_i \ge 0,\ x_i^s \ge 0,\ x_i^d \ge 0, \qquad i \in I$$

now, we treat $U(X_1, X_2, \ldots, X_m)$ as $\overline{U}\left(x_1, x_2, \cdots, x_n, x_1^d, x_2^d, \cdots, x_n^d, x_1^s, x_2^s, \cdots, x_n^s\right)$, which is a function of 3m variables x_i, x_i^s, x_i^d ($i \in I$), it is easy to see the Hessian matrix of $\overline{U}$ is semi-negative definite while $\frac{\partial \overline{U}}{\partial x_i^t} = 0$ for all feasible $x_i^t \geq 0$. This means internal stable point may or may not be maximum point. We have to compare the value of $\overline{U}$ of all feasible internal stable points and boundary points to get the optimal solution. But since $\frac{\partial \overline{U}}{\partial x_i^t} = 0$ for all feasible $x_i^t \geq 0$, so internal stable points consist a continuum if there exists one internal stable point which makes the comparison impossible. The Theorem 1 has excluded all internal stable point. It states that optimal solution of problem (6) is always a corner solution and the set of candidates for the optimal solution is narrowed down to a limited number of configurations.

IV. Further Research

In this paper, we haven't considered the case of complementarity between different activities: an increase of specialization level in one production will increase the productivity of another. If the complementarity has been considered, how will a consumer-producer decide their specialization level in producing each good as well as his level of specialization will become more complicated. If we measure the labor input with time and consider consumption time, assuming there are two kinds of household production: one is consuming production--a process of consumption which can increase utility through adjusting the ratio of time to good in the consumption; one is good production, then the specialization problem is closer to reality but hard to manipulate.

Allowing prices to change and considering a economy with people in different possible configurations, we can get general equilibrium conditions and have comparative static analysis. A division of population among different configurations represents a certain pattern and level of division of labor, relevant studies refer to Yang and Ng (1993).

By combining the tradeoff between economies of specialization and transaction costs with the tradeoffs among economies of specialization, economies of complementarity among goods consumption and transaction costs, concurrent increases in specialization and consumption variety has been expounded in Yang and Shi (1992); through adding intermediate goods and introducing a differential in transaction efficiency between intermediate goods market and labor market, why and how firms emerge from the division of labor has been explained in a similar framework (Yang and Ng 1993); and by introducing a differential in transaction efficiency between agricultural and manufacturing sectors, the emergence of a dual structure between the urban and rural sectors has been endogenized in a similar way (Yang and Rice 1994). We believe more classical economic thoughts can be resurrected in this kind of framework. Yet existent equilibrium models are very specific in utility functions, production functions and transaction cost (or the reciprocal transaction efficiency) functions partly because of the absence of theorem 1 and some others similar to the theorem 1. Hence, the theorem 1 has laid the basis for generalizing the equilibrium analyses based on the framework introduced in this paper.

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