# Elasticity of risk aversion and international trade

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**Abstract**

This note analyzes export production in the presence of exchange rate uncertainty under mean-variance preferences. We present the elasticity of risk aversion, since this elasticity concept permits a distinct investigation of risk and expectation effects on exports. Counterintuitive results are possible, e.g. though the home currency is revaluating (devaluating) exports of the firm increase (decrease). This fact may contribute to the explanation of disturbing empirical results.

*JEL classification:* F21, F31
*Key words:* Exchange rate risk, trade, elasticity of risk aversion, mean-variance model, devaluation

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†This paper was done when I were visiting Department of Economics, Monash University, Australia.

# Elasticity of risk aversion and international trade

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This note analyzes export production in the presence of exchange rate uncertainty under mean-variance preferences. We present the elasticity of risk aversion, since this elasticity concept permits a distinct investigation of risk and expectation effects on exports. Counterintutitive results are possible, e.g. though the home currency is revaluating (devaluating) exports of the firm increase (decrease). This fact may contribute to the explanation of disturbing empirical results.

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## 1. International trade and uncertainty

In the last decade exchange rates of the major industrial countries have shown substantial volatility. Exchange rate uncertainty became a concern of international firms and, therefore, affected and is affecting international trade and foreign investments, although empirical findings are mixed; empirical studies regarding the relationship between exchange rate risk and international trade flows do not necessarily confirm the intuition, that higher exchange rate volatilities lead to a reduction in international trade.[1] The purpose of this note is to give an explanation why a positive link between exchange rate risk and exports is possible from a portfolio theoretical point of view. We apply the mean-standard deviation approach for a scale and location family of probability distributions as to examine an exporting firm that is subjected to revenue risk without hedging opportunities.

[1]See Krugman (1989), Bini-Smaghi (1991), Gagnon (1993), Taylor (1995), McKenzie (1999), Lyons (2001) and others.

In order to study the decision problem of a risk averse competitive exporting firm under exchange rate risk we use a basic model from the literature.[2] The firm produces the quantity $Q$ of a final good at increasing marginal cost: $C'(Q) > 0, C''(Q) > 0$. The foreign exchange rate $\tilde{e}$ is random. The commodity price $P$, denominated in foreign currency is given. The objective is to maximize the expected value of a von Neumann-Morgenstern utility function of profit $U(\Pi)$, with $U' > 0$ and $U'' < 0$. $\tilde{\Pi} = \tilde{e}PQ - C(Q)$ denotes risky profit of the exporting firm. Hence, the export decision problem reads:

$$\max_{Q} \; EU(\tilde{e}PQ - C(Q)),$$

where $E$ denotes the expectation operator. Meyer (1987) and others have shown that under some conditions (section 2) the expected utility decision problem can be transformed into the mean ($\mu$)-standard deviation ($\sigma$) framework.[3] That is to say, there exists a function $V(\mu, \sigma)$ such that (i)

$$V(\mu, \sigma) = EU(\tilde{\Pi}) = \int_{-\infty}^{\infty} U(\Pi)\, f_{\tilde{\Pi}}(\Pi; \mu, \sigma)\, d\Pi,$$

where $\mu$ denotes the mean and $\sigma$ the standard deviation of risky profit for the pdf $f_{\tilde{\Pi}}$; and that (ii) function $V$ satisfies the following properties, where $V_x = \partial V/\partial x$ is the partial derivative: $V_\mu > 0$, $V_{\mu\mu} \leq 0$, $V_\sigma < 0$, $\sigma > 0$ and $V_\sigma(\mu, 0) = 0$; the partial derivatives $V_{\sigma\sigma}$ and $V_{\mu\sigma}$ exist and $V$ is a strictly concave function. The indifference curves are upward sloping and concave in the $(\mu, \sigma)$-space.

## 2. Location-Scale parameter condition and elasticity

Let us start by defining the so-called location and scale parameter condition of a probability distribution (Feller (1966), Meyer (1987)). This framework applies for our model of the exporting firm, since the random profit of the firm $\tilde{\Pi}$ is a positive linear transformation of the random foreign exchange rate $\tilde{e}$.

[2] Holthausen (1979), Kawai and Zilcha (1986), Broll, Wahl and Zilcha (1995) and others.

[3] See Schneeweiß (1967), Sinn (1983), Meyer (1987), Ormiston and Schlee (2001) and others.

*Definition 1* (Seed random variable) Let $\tilde{\eta}$ be the seed random variable with zero mean and unit standard deviation. The nondegenerate random foreign exchange rate $\tilde{e}$ is defined to be

$$\tilde{e} = \mu_{\tilde{e}} + \sigma_{\tilde{e}}\,\tilde{\eta}, \text{ with } \sigma_{\tilde{e}} > 0.$$

*Definition 2* (Location and Scale) By using Definition 1 let $\tilde{e}_i = \mu_i + \sigma_i\,\tilde{\eta}$, $\sigma_i > 0$, for $i = 1, 2$.

1. $E(\tilde{e}_i) = \mu_i$ and $\text{Var}(\tilde{e}_i) = \sigma_i^2$ for $i = 1, 2$.
2. The probabilty distributions of $\tilde{e}_1$ and $\tilde{e}_2$ are area preserving under location-scale transformations such that $\text{Prob}(\tilde{e}_1 \leq e_1) = \text{Prob}(\tilde{e}_2 \leq e_2)$ if $e_2 = \mu_2 + \frac{\sigma_2}{\sigma_1}(e_1 - \mu_1)$.
3. We say that $\tilde{e}_2$ is more risky than $\tilde{e}_1$ if $\mu_2 = \mu_1 > 0$ and $\sigma_2 > \sigma_1 > 0$. (Increase in risk.)
4. We say that $\tilde{e}_2$ is more expected than $\tilde{e}_1$ if $\mu_2 > \mu_1 > 0$ and $\sigma_2 = \sigma_1 > 0$. (Increase in mean.)

Note that the pdf of $\tilde{e}$ is a function of $\mu_{\tilde{e}}$, $\sigma_{\tilde{e}}$ and the pdf of $\tilde{\eta}$. A change in expected value and/or standard deviation of the foreign exchange rate can now be introduced within a comparative static analysis of export production.

We now present the so-called elasticity of risk aversion in the mean-standard deviation approach (Lajeri and Nielson (2000), Battermann, Broll, and Wahl (2002)). This concept allows for a distinct investigation of risk and expectation effects on the export decision of the firm.

Let $S = -V_\sigma / V_\mu$, with $\sigma > 0$. $S$ is positive and denotes the marginal rate of substitution between expectation $\mu$ and risk $\sigma$, i.e. the positive slope of the indifference curve. Therefore, $S$ can be interpreted as a measure of risk aversion within the mean-standard deviation framework.

*Definition 3* (Standard deviation elasticity) The elasticity of risk aversion with respect to the standard deviation of the firm's risky profit is stated by

$$\varepsilon_\sigma = -\frac{\partial \ln S}{\partial \ln \sigma}, \ \sigma > 0.$$

$\varepsilon_\sigma$ indicates the percentage change in risk aversion over the percentage change in profit standard deviation, the profit mean being fixed.

*Definition 4* (Mean elasticity) The elasticity of risk aversion with respect to the mean of the firm's risky profit is given by

$$\varepsilon_\mu = \frac{\partial \ln S}{\partial \ln \mu},\ \mu > 0.$$

$\varepsilon_\mu$ indicates the percentage change in risk aversion over the percentage change in profit mean, the profit standard deviation being fixed.

In the following we examine the relationship between trade and a change in the expected value of the foreign exchange rate and its standard deviation, respectively. The relationships are investigated by using the introduced elasticity measures.

## 3. Risk and mean effects on international trade

We model a increase in exchange rate risk by augmenting the standard deviation $\sigma_{\tilde{e}}$, holding the mean $\mu_{\tilde{e}}$ constant (Definition 2 (3.)).

**Proposition 1** (Trade and risk) Suppose the exchange rate becomes more risky, i.e. the standard deviation of the foreign exchange rate increases. Then the firm's export decreases (remains constant, increases) if and only if the standard deviation elasticity of risk aversion is less than (equal to, greater than) unity.

**Proof.** The first order condition with respect to export volume $Q$ implies

$$(\mu_{\tilde{e}} - S\sigma_{\tilde{e}})P = C'(Q).$$

By applying the implicit function theorem we get

$$\begin{aligned} \operatorname{sign}\frac{\partial Q}{\partial \sigma_{\tilde{e}}} &= \operatorname{sign}\left(-\frac{\partial S}{\partial \sigma_{\tilde{e}}}\frac{\sigma_{\tilde{e}}}{S} - 1\right) \\ &= \operatorname{sign}(\varepsilon_\sigma - 1). \quad \text{q.e.d.} \end{aligned}$$

Now we model an increase in exchange rate expectation by augmenting the mean $\mu_{\tilde{e}}$, holding the standard deviation $\sigma_{\tilde{e}}$ constant (Definition 2 (4.)).

**Proposition 2** (Trade and expectation) Suppose a higher exchange rate becomes more expected, i.e. the mean of the foreign exchange rate increases. If the mean elasticity of risk aversion is less than or equal to unity, then the firm's export increases.

**Proof.** From the first order condition with respect to export volume $Q$ (see proof of Proposition 1), applying the implicit function theorem we obtain

$$\begin{aligned} \operatorname{sign}\frac{\partial Q}{\partial \mu_{\tilde{e}}} &= \operatorname{sign}\left(1-\frac{\partial S}{\partial \mu_{\tilde{e}}}\sigma_{\tilde{e}}\right) \\ &= \operatorname{sign}\left(1-\varepsilon_{\mu}R\right), \end{aligned}$$

with $0 < R = S\sigma/\mu < 1$. q.e.d.

Whether or not there is a risk effect on international trade depends upon the magnitude of the profit standard deviation elasticity of risk aversion. With unit elastic risk aversion there is no risk effect. That is, the firm's optimum export production remains unchanged although the exchange rate becomes more risky. If risk aversion is (in)elastic than the firm will (diminish) extend its export production. Hence, the elasticity measure provides a distinct answer to the question how a change in exchange rate risk affects international trade. With this result in mind contradicting empirical findings are not unlikely when the elasticity is highly unstable over time.

The intuition that a devaluation of the home currency stimulates export production of the firm is not true in general. A sufficient condition which supports this intuition is that we have unit elastic or inelastic risk aversion with repect to the profit mean. Furthermore, there exist a critical elasticity level, $1/R$ (see proof of Proposition 2), which implies that there is no mean effect at all on international trade. Sufficient elastic risk aversion, i.e. $\varepsilon_{\mu} > 1/R$ induces the firm to lessen export production although the expected value of the foreign exchange rate increases. Again, contradicting empirical results that build on different samples should not be that surprising.

Note, that $R$ ($S$) can be interpreted as a measure of relative (absolute) risk aversion within the mean-standard deviation approach (Lajeri and Nielsen (2000)).

## 4. Summary

The paper studies optimum production decision of an international firm using the mean-standard deviation model. It is shown that an increase in the exchange rate risk (or, expectation) may have a negative, positive or no impact on trade. The direction depends upon the elasticity of risk aversion with respect to the standard deviation (or, the mean) of the firm's random

profit. The classic elasticity concept is a straightforward instrument to show when so-called nonintuitive risk and mean effects on trade occur. Contradicting empirical findings on the subject may be the result of highly unstable elasticities over time.

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