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ABSTRACT

This paper develops a general equilibrium model of impersonal networking decisions and bundling sales, which departures from the other models of bundling and tying by allowing substitution between goods, flexible quantities of goods, resale of any goods, competitive market, and ex ante identical utility function for all individuals. Applying Inframarginal Analysis, this model shows that the function of bundling sales in a competitive market is to avoiding direct pricing of goods with the lowest transaction efficiency, like intangible information goods, meanwhile getting them involved in the division of labour and commercialised production, thereby promoting division of labour and aggregate productivity. According to this theory of bundling, bundling in a competitive market is Pareto efficient and it plays a very important role to utilize positive network effects of division of labour on aggregate productivity. Antitrust prosecution should pay more attention to the intention to block free entry rather than bundling itself.

(JEL D23, D41, D58, L11, L23)

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This purpose of the paper is to investigate the function of a particular type of bundling sales in exploiting network effects of the division of labor and in promoting productivity progress. We motivate this task from the following perspectives. First we compare it with the existing literature of bundling and tying sales. We then consider some common internet phenomena which cannot be predicted by the existing literature. Finally, we motivate the research of effects of bundling sales on the network size of division of labor by comparing our task with the literature of endogenous specialization and network effects of division of labor.

An extensive literature has been developed to investigate the role of bundling and tying sales (Bursten 1960, Adams and Yellen 1976, Schmalensee 1984, McAfee, McMillan, and Whinston 1989, Whinston 1990, Hanson and Martin 1990, Varian 1995, 1997, Stigler 1999, and Bakos and Brynjolfsson 1999a, b). This literature focuses on bundling and tying that is associated with monopoly power. The following assumptions are made in this literature. Each consumer consumes at most one unit of a good and has constant valuation of the one unit of good. Resale of a good is not allowed. In addition, differentiated prices cannot be directly charged for individuals with differentiated valuations of goods because of un-observability of such valuations. The assumptions imply that utility is not specified as a function of amounts of all consumption goods and that no substitution between goods is allowed (so-called independent valuations). Hence, interesting interactions and feedback loops between consumption quantities, prices, income, production decisions, and substitution between goods, which are the focus of a standard general equilibrium analysis, are not investigated in this literature. With the quite specific assumptions, it is easy to see that bundling can impose indirect price discrimination under a uniform price of a bundle of goods. Bakos and Brynjolfsson (1999a, b) have nicely presented the intuition about this function of bundling.

In this literature, research results on welfare effects of bundling are inconclusive. Adams and Yellen (1976) emphasize that adverse effects of bundling on welfare come from monopoly power rather than bundling itself. Blair and Kaserman (1978), Chae (1992), Varian (1995), Grimes (1996), Fishburn, Odlyzko and Siders (1997), Delong (1998), Chuang and Sirbu (1999), and Bakos and Brynjolfsson (1997, 1999) pay more attention to positive welfare effects of bundling. Matutes and Regibeau (1992), Tirole (1989, pp. 146-48), and Martin (1999) pay more attention to adverse welfare effects of tie-in sales. Whinston (1990) shows that welfare effects of tying in an oligopoly regime are ambiguous.

As reviewers of some papers in the literature point out, many internet and e-business phenomena are inconsistent with the particular assumptions made in this literature. For instance, there are more than a thousand email or search engine providers and each of them bundles their services. Some of the services are charged positive prices and others are provided free of charge. Also, resale of such services is possible, quantities of such services can be any integer numbers (for instance each person may get several email accounts from each of several providers), and substitution between services are not trivial (that is, a consumer's valuation of a service is not a constant, or a consumer's utility is a function of quantities of such services and other goods).

Bakos and Brynjolfsson, (1999b, p. 3) defend their position by arguing that bundling sales with zero prices of some services is a phenomenon of disequilibrium. We disagree. Zero price of a good implicitly bundled with goods of positive prices can be a general equilibrium phenomenon. A conventional market for petrol and air pump services may illustrate our point. There are many petrol stations which sell petrol at a competitive price and provide air pump services free of charge. This market structure has been in place for long time. The bundling of petrol and air pump services must be a general equilibrium phenomenon. In this market, all consumers' preferences for petrol and air pump service might be very similar, so that the rationale for the type of bundling in the existing literature is irrelevant.[1] The intuition for this phenomenon is quite straightforward. Pricing of air pump services and collection of related payment involves transaction cost to consumers as well as to petrol stations (waiting time, inconvenience, and tangible resource cost for pricing and payment collection). If the production cost of such services can be added to the price of petrol which is complementary to air pump services, then such transaction cost can be avoided. Bundling sales may incur endogenous transaction costs which are the distortions caused by individuals who use air pump services but do not buy petrol from the same petrol station. But as long as reduction of exogenous transaction costs of pricing process of air pump services outweighs the increase in endogenous transaction cost, a competitive market will generate pressure to compel all petrol stations to implement such a bundling price structure. We call this phenomenon implicit bundling which charges a positive price of a good and zero price of another good without an explicit bundle. Implicit bundling is closer to mixed bundling than pure bundling investigated in the existing literature. Other implicit bundling cases include TV programs (TV shows are free of charge and associated advertisements are paid at positive prices by companies selling goods to viewers of TV programs) and an automobile company's marketing operation with positive prices of cars and free internet purchase services. Here, the key point is that competition pressure and prohibitively high pricing cost of some goods are essential for zero prices of goods bundled with goods of positive prices. Therefore, we need a model without monopoly power and with transaction costs and competitive (implicit) bundling. This paper will formalize this story using a general equilibrium model with well specified ex ante identical utility and production functions for all individuals.

[1] As shown by Bakos and Brynjolfsson (1999a), in the existing models of bundling benefit of bundling disappears as consumers' evaluations converge to the same value.

We shall tell the story by formulating the trade-off between positive network effects of division of labor on aggregate productivity and transaction costs. As suggested by Allyn Young (1928), network effect is a notion of general equilibrium. Not only the network size of division of labor depends on the extent of the market (the number of participants in the network of division of labor), but also the number of participants is determined by all individuals' participation decisions in the network of division of labor, which relate to their decisions of their levels of specialization. This circular causation, noted by Young, is of course an essential feature of general equilibrium, analogous to the circular causation between quantities and prices in the fixed point theorem (each individual's quantities demanded and supplied depend on prices, while the equilibrium prices are determined by all individuals' decisions of quantities). Hence, a partial equilibrium model, such as those in the existing literature of bundling, does not work for our task.

Moreover, since we need an assumption of competitive market for investigating network effects of division of labor, we are not confined to the strategic networking decision which is associated with monopoly power. We need a general equilibrium model of impersonal networking decisions to investigate infinite feedback loops between network size of division of labor, each person's participation decision, prices, quantities, and different markets. Yang (2001) and Sun, Yang, and Yao (1999) have drawn the distinction between the strategic networking decision and the impersonal networking decision. For the latter, each decision maker is not concerned with whom she has a trade connection to. She is concerned with how many goods she will trade and how many she will self-provide. Her decision in choosing the number of types of traded partners determines her trade network size and pattern. Impersonal networking decisions take place in a market where no body can manipulate prices, so that implicit bundling with zero prices of some goods may emerge from competitive pressure and free entry. Such impersonal networking decisions generate network effects of division of labor that are not network externalities since we assume that each individual is capable of conducting inframarginal analysis (total cost-benefit analysis across corner solutions in addition to marginal analysis of each corner solution). Inframarginal analysis means that each individual is capable of not only choosing locally optimum resource allocation for a given trade network pattern using standard marginal analysis, but also choosing a globally optimal trade network pattern by comparing several locally optimum values of objective functions. Formally, inframarginal analysis is non-linear programming. Coase (1946, 1960), Buchanan and Stubblebine (1962), and Yang (2001) have shown that a lot of so-called network externalities can be internalized by individuals' inframarginal decisions. They are considered externalities by many economists since these economists assume, naively, that individuals are incapable of doing inframarginal analysis. Many contributors to the literature of inframarginal analysis of network effects of division of labor and impersonal networking decision (see surveys of this literature by Yang and Ng, 1998 and by Yang, 2001, and references there) have shown that marginal cost pricing does not work when individuals conduct inframarginal analysis. Hence, non-marginal cost pricing is compatible with a competitive market with localized increasing returns and impersonal networking decisions.

In this paper, we will specify a general equilibrium model with a continuum of ex ante identical consumer-producers who prefer diverse consumption and specialized production due to economies of specialization in production of three goods. There is the trade-off between transaction costs and positive network effects of division of labor on aggregate productivity. Hence, if the transaction cost coefficient for a unit of goods traded is very large, the positive network effect is outweighed by transaction costs. Therefore, individuals choose autarky where market, institution of the firm, and bundling sales do not occur. As the transaction cost coefficient decreases, the general equilibrium discontinuously jumps to a higher level of division of labor. Markets emerge from the division of labor. However, if the transaction cost coefficient for labor is smaller than that for goods, the institution of the firm and related labor market emerge from the division of labor. Otherwise, the markets for various goods will be used to organize the division of labor in the absence of the institution of the firm and related labor market. If the transaction cost coefficient for a good is extremely large and the equilibrium level of division of labor is sufficiently high, then this good will be implicitly bundled with other goods to avoid prohibitively high pricing cost, meanwhile getting this good involved in the large network of division of labor and commercialised production.

Intuitively, this story can be told as follows. Suppose that an automobile manufacturer, such as General Motor, sells automobiles and internet services for purchasing cars online. Automobiles are tangible goods which are easy to price, but internet services are intangible, very difficult to price. General Motor can bundle two goods together by providing free internet services and by adding the operation cost of internet services to the price of automobiles. If such bundling can save consumers' transaction costs incurred in a purchase deal in excess of the added cost to the price of automobiles, General Motor will have a competitive edge compared to other automobile manufacturers who do not provide such bundled deal. Then a competitive pressure in the market will force all manufacturers to provide such bundled deal. Here, monopoly power, constant and independent valuations of one unit of good, non-resale, and other peculiar assumptions are not needed. In addition, even if all individuals have ex ante identical utility function which allows substitution between goods, productivity gains from bundling may be generated by network effects of division of labor. Without bundling, involvement of the good with prohibitively high transaction cost coefficient in a high level of division of labor and avoidance of direct pricing cost of such a good cannot coexist. Hence, positive network effects of division of labor on aggregate productivity cannot be fully exploited. With the bundling, both of the tasks can be achieved at the same time. Therefore, the network effects can be fully exploited and aggregate productivity can be promoted by the bundling. It is interesting to see that bundling in a competitive market has very important productivity implications even if all individuals have ex ante identical utility and production functions and substitution between different goods is non-trivial.

This paper proceeds as follows. Section 2 is devoted to describe the model. Section 3 solves equilibrium and its comparative statics and reports main findings. The final section concludes the paper.

I. A Model with Impersonal Networking Decisions and Bundling Sales

Consider an economy with a continuum of consumer-producers of mass *M*. This assumption implies that population size is very large. It avoids an integer problem of the numbers of different specialists, which may lead to non-existence of equilibrium with the division of labour (see Sun, Yang, and Zhou, 1998). Each consumer-producer has identical, non-satiated, continuous, and rational preference represented by the following utility function:

(1a) $u = f(x^c, y^c)$,

where $x^c \equiv (x + x^d)$ and $y^c \equiv (y + y^d)$ are the amounts of the two final goods that are consumed, x and y are the amounts of the two goods that are self-provided, x^d and y^d are the amounts of the two goods that are purchased from the market, and $f(.)$ is continuously increasing and quasi-concave. For simplicity, it is assumed that $f(.) = (x^c)^\alpha \cdot (y^c)^{1-\alpha}$.

Each consumer-producer's production functions are:

(1b) $x^p = x + x^s = (z + z^d)^\beta \cdot l_x$ and $\beta \in (0,1)$,

$y^p = y + y^s = \text{Max}\{0, l_y - b\}$,

$z^p = z + z^s = \text{Max}\{0, l_z - b\}$, and $b \in (0, 1)$.

where x^p and y^p are the amounts of the two final goods produced, z^p is the amount of the intermediate good produced, z^d is the amount of intermediate good purchased from the market, x^s , y^s and z^s are the amounts of the three goods sold, b is a fixed learning and training cost in producing goods y and z and the parameter β represents the elasticity of output of good x with respect to input level of intermediate good z. $\beta+1>1$ implies that there are increasing returns in producing the final good x. The endowment constraint for each individual endowed with one unit of working time is given as follows:

(1c) $l_x + l_y + l_z = 1$,

where l_i is the amount of labour allocated to the production of good i. This system of production implies that each individual's labour productivity increases as she narrows down her range of production activities. As shown by Yang (2001, chapter 2), the aggregate production schedule for three individuals discontinuously jumps from a low profile to a high profile as each person jumps from producing three goods to a production pattern in which at least one person produces only one good (specialization). The difference between the two aggregate production profiles is considered as positive network effects of division of labour on aggregate productivity. This network effect implies that each person's decision of her level of specialization, or gains from specialization, depends on the number of participants in a large network of division of labour, while this number is determined by all individuals' decisions in choosing their levels of specialization (so-called the Young theorem, see Young, 1928). Since economies of specialization is individual specific (learning by doing must be achieved through individual specific practice and cannot be transferred between individuals), labour endowment constraint is specified for each individual, so that increasing returns are localized.

The budget constraint for an individual is,

(1d) $k_x p_x x^s + k_y p_y y^s + k_z p_z z^s = p_x x^d + p_y y^d + p_z z^d$, and $k_i \in (0,1)$,

where p_i is the price of good i. Fraction 1-k_i of a good sold disappears in transit due to an iceberg transaction cost, or k_i is a trading efficiency coefficient, which represents the conditions governing transactions.[2] k_i relates to transportation conditions and the general institutional environment that affects trading efficiency. We assume that if labour trade occurs, fraction 1-g_i of the amount of labour employed to produce good i disappears in transit from the employee to the employer due to all kinds of transaction costs in labour trade (shirking, measurement cost of quantity and quality of labour, and anticipated moral hazard). Hence, g_i is the trading efficiency coefficient of labour employed to produce good i.

[2] The specification of such iceberg transaction cost is a common practice in the equilibrium models with the trade-off between increasing returns and transaction costs (see Krugman 1995). This specification avoids notoriously formidable index sets of destinations and origins of trade flows.

Due to the continuum number of individuals and the assumption of localized increasing returns in this large economy, a Walrasian regime prevails in this model. The specification of the model generates a trade-off between economies of division of labour and transaction costs. The decision problem for an individual involves deciding on what and how much to produce for self-consumption, to sell and to buy from the market. In other words, the individual chooses nine variables $x_i, x_i^s, x_i^d, y_i, y_i^s, y_i^d, z_i, z_i^s, z_i^d \geq 0$. Hence, there are 2^9 =512 possible corner and interior solutions.

II. Corner Solution in a Configuration and Corner Equilibrium in a Structure

Since corner solutions are allowed in our model, standard marginal analysis of interior solution does not work. We need a three-step inframarginal analysis. In the first step a set of candidates for an individual's optimum decision is identified by ruling out all inefficient interior and corner solutions. Possible network structures of division of labour and related transactions can then be identified as combinations of corner solutions. This first step of inframarginal analysis will be done in subsection 3.1. We then solve for all possible corner solutions and the local equilibrium in each market structure that is a combination of compatible corner solutions, using marginal analysis. The second step will be taken in subsection 3.2. Finally, we will use total cost-benefit analysis to figure out under what condition, which local equilibrium is a general equilibrium. This will be done in subsection 3.3.

A. Configurations and Structures

The set of candidates for each individual's optimum decision includes many corner and interior solutions. In order to narrow down the list of the candidates, Yang and Ng (1993), and Wen (1998) used the Kuhn-Tucker conditions to establish the following lemma:

LEMMA 1: *Each individual sells at most one good, but does not buy and sell the same good, nor buys and self-provides the same good at the same time.*

We define a *configuration* as a combination of zero and positive variables which are compatible with Lemma 1. When labour trade and bundling are allowed, there are 19 configurations from which the individuals can choose. A combination of all individual's configurations constitutes a *market structure*, or *structure* for short. Let us examine all structures that might occur in equilibrium.

1. Structure A: Autarky

Structure A consists of all individuals choosing configuration A (self-sufficiency, or autarky), where an individual produces all the three goods for self-consumption. Configuration A is defined by $x, y, z > 0$ and $x^s = x^d = y^s = y^d = z^s = z^d = 0$.

2. Structures with Partial Division of Labour: P_A, P_B and FP_B

(1) Structure P_A is a division of the population between configurations (xz/y) and (y/x). A person choosing configuration (xz/y) produces goods x and z, buys good y, and sells good x. It is defined by $x, x^s, z, y^d > 0, z^s = z^d = y = y^s = x^d = 0$; A person choosing configuration (y/z) produces good y, buys good x, and sells good y. It is defined by $y, y^s, x^d > 0, x = x^s = z = z^s = z^d = y^d = 0$. Note that structure P_A involves trade of goods x and y, so that trading efficiency coefficients k_x and k_y appear in this structure.

(2) Structure P_B is a division of the population between configuration (zx/y) and (yx/z). A person choosing configuration (zx/y) produces goods x and z, buys good y, and sells good z. It is defined by $x, z, z^s, y^d > 0, x^s = x^d = y = y^s = z^d = 0$; A person choosing configuration (yx/z) produces goods x and y, buys good z, and sells good y. It is defined by $x, y, y^s, z^d > 0, x^s = x^d = y^d = z = z^s = 0$. Note that structure P_B involves trade of goods z and y, so that trading efficiency coefficients k_z and k_y appear in this structure.

(3) Structure FP_B is a division of the population between configuration (l_zx/y) and (yx/l_z). A individual choosing configuration (l_zx/y) produces goods x and z, buys good y, and sells labor for producing intermediate good z. It is defined by $x, z, l_z, y^d > 0, x^s = x^d = y = y^s = z^s = z^d = 0$; A person choosing configuration (yx/l_z) produces goods x and y, sells good y, and employs labor to produce good

z. It is defined by $x, y, y^s, l_z > 0, x^s = x^d = y^d = z^s = z^d = 0$. Note that structure FP_B involves trade of good y and labor l_z, so that trading efficiency coefficients k_y and g_z appear in this structure.

3. Complete Division of Labour

(1) Structure CD with Complete Division of Labour and without the Firm is a division of the population among configurations (x/yz), (z/xy) and (y/x). An individual choosing configuration (x/yz) in structure CD produces and sells good x and buys goods y and z. It is defined by $x, x^s, y^d, z^d > 0, x^d = y = y^s = z = z^s = 0$; An individual choosing configuration (y/x) in structure CD produces and sells good y and buys good x. It is defined by $y, y^s, x^d > 0, x = x^s = y^d = z = z^s = z^d = 0$; An individual choosing configuration (z/xy) in structure CD produces and sells good z and buys goods x and y. It is defined by $z, z^s, x^d, y^d > 0, z^d = x = x^s = y = y^s = 0$. Note that structure CD involves trade of goods x, y, and z, so that trading efficiency coefficients k_x, k_y, and k_z appear in this structure.

(2) Structure FD_A with Complete Division of Labour with the Firm, is a division of the population among configurations ($z/l_x y$), (l_x/xy) and (y/x). An individual choosing ($z/l_x y$) produces and sells good z, hires labour to produce x, and buys good y. It is defined by $z, y^d, l_x, x^s > 0, x^d = y = y^s = z^s = z^d = 0$; An individual choosing (l_x/xy) sells labour for producing x and buys goods x and y. It is defined by $x^d, y^d, l_x > 0, x^s = y = y^s = z = z^s = z^d = 0$; Configuration (y/x) is the

same as in structure CD. Note that structure FD_B involves trade of goods x, y, and labour l_x, so that trading efficiency coefficients k_x, k_y, and g_x appear in this structure.

(3) Complete Division of Labour with Bundling Sales and the Institution of the Firm: Structures FT_A , and FT_B .

(a) Structure FT_A is a division of the population among configurations (x/l_yz), ($l_y/x(y)$) and ($z/x(y)$). An individual choosing (x/l_yz) produces good x, employs labour to produce y, and sells x that is bundled with y. It is defined by $x, x^s, l_y, z^d, y^s > 0, x^d = y^d = z = z^s = 0$; An individual choosing ($l_y/x(y)$) sells labour for producing y, buys good x, and gets the bundled good y. It is defined by $x^d, l_y, y^d > 0, x = x^s = y = y^s = z = z^s = z^d = 0$; An individual choosing ($z/x(y)$) produces and sells z, buys good x, and gets the bundled good y. It is defined by $z, z^s, x^d, y^d > 0, x = x^s = y = y^s = z^d = 0$. Note that structure FT_A involves trade of goods x, z, and labour l_y, so that trading efficiency coefficients k_x, k_z, and g_y appear in this structure. Good y is not directly priced though it is bundled with good x.

(b) Structure FT_B is a division of the population among configurations (x/l_yz), ($l_y/y(x)$) and ($z/y(x)$). Configuration (x/l_yz) in FT_B is symmetric to (x/l_yz) in structure FT_A. An individual choosing this configuration produces good x, hires labour for producing y, sells y, which is bundled with good x. The difference between FT_A and FT_B is that good x is priced and good y is not in the former, while good y is priced and good x is not in the latter; Configuration (l_y/y(x)) is symmetric to (l_y/x(y)) in structure FT_A, but good y is priced and good x is not; Configuration (z/y(x)) is symmetric to (z/x(y)) in structure FT_A, but good y is priced and good x is not. Note that structure FT_B involves trade of goods y, z, and labour l_y, so that trading efficiency coefficients k_y, k_z, and g_y appear in this structure. Good x is not directly priced though it is bundled with good y.

According to Sun, Yang, and Zhou (1998, see also Yang, 2001, chapter 13), a general equilibrium exists for a general class of the models of which the model in this paper is a special case under the assumptions that the set of individuals is a continuum, preferences are strictly increasing and rational; and both local increasing returns and constant returns are allowed in production and transactions. A general equilibrium in this model is defined as a set of relative prices of goods and all individuals' labour allocations and trade plans, such that, (1) Each individual maximizes her utility, that is, the consumption bundle generated by her labour allocation and trade plan maximizes her utility function for given prices; (2) All markets clear.

Since the optimum decision is always a corner solution and the interior solution is never optimal according to Lemma 1, we cannot use standard marginal analysis to solve for a general equilibrium. We adopt a three-step approach to solving for a general equilibrium. The first step is to narrow down the set of candidates for the optimum decision and to identify configurations that have to be considered. We can identify structures from compatible combinations of configurations. In the second step, each individual's utility maximization decision is solved for a given structure. The utility equalization condition between individuals choosing different configurations and the market clearing conditions are used to solve for the relative price of traded goods and numbers (measure) of individuals choosing different configurations. The relative price and numbers, and associated resource allocation are referred to as corner equilibrium for this structure. General equilibrium occurs in a structure where, given corner equilibrium relative prices in the structure, no individuals have an incentive to deviate from their chosen configurations in this structure. In the second step, we can substitute the corner equilibrium relative prices into the utility function for each constituent configuration in the given structure to compare the utility between this configuration and any alternative configurations. This comparison is called a total cost-benefit analysis. The total cost-benefit analysis yields the conditions under which the utility in each constituent configuration of this structure is not smaller than any alternative configuration. With the existence theorem of general equilibrium proved by Sun, Yang, and Zhou (1998), we can completely partition the parameter space into subspaces, within each of which the corner equilibrium in a structure is a general equilibrium. As parameter values shift between the subspaces, the general equilibrium will discontinuously jump between structures. The discontinuous jumps of structure and all endogenous variables are called inframarginal comparative statics of general equilibrium. The three steps constitute an inframarginal analysis.

The corner equilibria in the structures are solved in the following subsection.

B. Corner Solution in a Configuration and Corner Equilibrium in a Structure

In this subsection, we first use two examples to illustrate how marginal analysis can be conducted to solve for the corner solution in each configuration and for the corner equilibrium in each structure. The first example is the corner solution in configuration A that is the corner equilibrium in autarky structure A.

Autarky is a structure where each individual chooses configuration A. An individual's decision problem in A is:

(2a) Max: $u_A = x^\alpha \cdot y^{1-\alpha}$,

subject to the following constraints:

(2b) $x = z^\beta \cdot l_x$, $\quad y = l_y - b$,

$z = l_z - b$, and $l_x + l_y + l_z = 1$.

The solution is:

(2c) $$l_x = \frac{\alpha \cdot (1-2b)}{1+\alpha\beta},$$

$$l_y = \frac{(1-\alpha)\cdot(1-2b) + b\cdot(1+\alpha\beta)}{1+\alpha\beta},$$

$$l_z = \frac{\alpha\beta\cdot(1-b)+b}{1+\alpha\beta}, \text{ and } \quad u_A = \frac{\beta^{\alpha\beta}\cdot(1-\alpha)^{1-\alpha}\cdot(1-2b)^{1+\alpha\beta}\cdot\alpha^{\alpha(1+\beta)}}{(1+\alpha\beta)^{1+\alpha\beta}},$$

where u_A is per capita real income in structure A.

Next, we consider the corner equilibrium is structure FT_A with bundling sales and the institution of the firm. This structure involves the division of the population among configurations (x/l_yz), ($l_y/x(y)$) and ($z/x(y)$). An individual choosing (x/l_yz) is the employer of a firm. She specializes in producing good x, and hires labour to produce final good y. She sells good x, buys intermediate good z and labour, and bundles good y with good x, which means good y is not directly priced, and people can obtain some amount of good y when they buy good x from the market. The ratio of the amounts of the two goods bundled is chosen by the employer under competition pressure in the market.

In structure FT_A, the decision problem for an individual choosing configuration (x/l_yz) is:

(3a) Max: $u_{FTA1} = x^{\alpha} \cdot y^{1-\alpha}$,

subject to the following constraints,

(3b) $x + x^s = (z^d)^{\beta} \cdot l_x$ and $l_x = 1$,

$Y^s = g_y \cdot L_y - b$ and $L_y = 1$,

$y^s = h \cdot x^s$,

$y + y^s = N \cdot Y^s$,

$k_x p_x x^s = p_z z^d + w \cdot N \cdot L_y$,

where g_y is again the transaction efficiency coefficient for labour hired to produce good y. Moreover, N is the number of workers hired by the employer to produce good y, w is the wage rate, and h is the bundling ratio between goods y and x. In order to distinguish inter flow of goods from market trade flow, we use capitalized decision variables to denote internal flow. Hence, Y^s is internal transfer of good y produced by an employee to the employer and y^s is the amount of good y provided free of charge by the firm. Here, x is priced and y is not. We assume $h = e \cdot \frac{p_x}{w}$. This implies that an individual selling x, buying labour, and bundling y with x, must choose the bundling ratio $h = y/x$ according to p_x/w. For a small relative market price p_x/w, she must give away a small amount of y for each unit x sold. Otherwise, a small value of p_x/w may not be enough to cover the production cost of y which is not directly priced. Here, e is as given to the owner of the firm, while later based on the Yao Theorem (see Yang 2001, chapter 6, p.156) , we can role out the real optimal bundling ratio of good x and y. In addition, l_x is the decision variable of the employer, while l_y is an employee's decision variable. u_{FTA1} is the utility for an x specialist-employer choosing (x/l_yz).

The solution to the decision problem yields demand function for labour and good z, supply function of good x, and indirect utility function for configuration (x/l_yz).

Similarly, an employee choosing configuration (l_y/x(y)) has the following decision problem,

(3c) Max: $u_{FTA2} = (x^d)^{\alpha} \cdot (y^d)^{1-\alpha}$,

subject to the following constraints,

(3d) $y^d = h \cdot x^d$, $L_y = 1$, $w \cdot L_y = p_x \cdot x^d$.

The solution of this problem yields demand for goods x and y, supply of labour, and indirect utility function for configuration (l_y/x(y)).

An individual choosing configuration (z/x(y)) has the following decision problem:

(3e) Max: $u_{FTA3} = (x^d)^{\alpha} \cdot (y^d)^{1-\alpha}$,

subject to the production function, endowment constraint, and budget constraint:

(3f) $z^s = l_z - b, \quad l_z = 1, \qquad y^d = h \cdot x^d, \quad k_z p_z z^s = p_x x^d .$

The solution to this problem yields demand for goods x and y, supply of good z, and indirect utility function for configuration (z/x(y)).

The two utility equalization conditions across three configurations yield the corner equilibrium relative prices of goods x and z and labour.

(3g) $\frac{w}{p_z} = k_z(1-b)$, and

$$\frac{w}{p_x} = [\frac{k_x(g_y - b) \cdot k_z \cdot \beta}{(1-\alpha)\cdot(1-\beta)+\alpha\beta}]^{\frac{\beta}{1-\beta}} \cdot \frac{\alpha^{\alpha} \cdot (1-\beta) \cdot [(1-\alpha)\cdot(\alpha-\beta)+\alpha\beta]^{(1-\alpha)+\frac{\beta}{1-\beta}}}{(1-\alpha)^{1-\alpha}} .$$

Based on the Yao Theorem, maximising utility with respect to *e*, yields the optimal value of e :

(3h) $e = \frac{k_x(1-\alpha)^2 \cdot (g_y - b)}{(1-\alpha)\cdot(1-\beta)+\alpha\beta}$.

The two independent market clearing conditions for goods x and z (the other market clearing condition is not independent due to Walras' law) yield the corner equilibrium relative numbers of specialists producing goods x, y, and z.

(3i)

$$\frac{M_x}{M_z}=\{[\frac{(1-\alpha)\cdot(\alpha-\beta)+\alpha\beta}{(1-\alpha)\cdot(1-\beta)+\alpha\beta}]^{(1-\alpha)+\frac{\beta}{1-\beta}}\cdot\frac{\alpha^{\alpha}\cdot(1-\beta)\cdot[(g_y-b)\cdot\beta]^{\frac{\beta}{1-\beta}}\cdot}{(1-\alpha)^{1-\alpha}\cdot k_x^{\frac{1-2\beta}{1-\beta}}}\}^{\frac{1}{1-\beta}}\cdot(\frac{k_z^2}{1-b})^{\frac{\beta}{1-\beta}},$$

and

$$\frac{M_y}{M_x}=[\frac{(1-\alpha)\cdot(\alpha-\beta)+\alpha\beta}{(1-\alpha)\cdot(1-\beta)+\alpha\beta}]\cdot\frac{(1-\alpha+\alpha\beta)}{\beta}-[\frac{k_x(g_y-b)\cdot k_z\cdot\beta}{(1-\alpha)\cdot(1-\beta)+\alpha\beta}]^{\frac{\beta}{1-\beta}}\cdot\frac{\alpha^{\alpha}\cdot(1-\beta)\cdot[(1-\alpha)\cdot(\alpha-\beta)+\alpha\beta]^{(1-\alpha)+\frac{\beta}{1-\beta}}}{(1-\alpha)^{1-\alpha}},$$

where M_x is the number of x specialist-employers choosing (x/l_yz), M_z is the number of specialist producers of good z choosing ($z/x(y)$), and M_y is the number of employees choosing ($l_y/x(y)$). The relative numbers of specialists, together with population size identity $M_x+M_z+M_y=M$, yieldthe corner equilibrium numbers of different specialists. Plugging relative prices into an indirect utility function of any of three configurations yields the per capita real income in this structure:

(3j)

$$u_{FTA}=\alpha^{\alpha}\cdot(1-\beta)\cdot(1-\alpha)^{1-\alpha}\cdot(k_z\cdot\beta)^{\frac{\beta}{1-\beta}}\cdot\{\frac{k_x(g_y-b)\cdot[(1-\alpha)\cdot(\alpha-\beta)+\alpha\beta]}{(1-\alpha)\cdot(1-\beta)+\alpha\beta}\}^{(1-\alpha)+\frac{\beta}{1-\beta}}.$$

In Structure FT_A, a firm produces both good x and y, while selling x with good y bundled. The percentage h of good x and y is dependent on the relative price of good x and labour, and e. Note that good y is bundled through the purchase of good x, therefore transaction costs in directly pricing good y is avoided.

Following this procedure, we can solve for corner equilibria in all structures. Information about such solutions of corner equilibria in 8 structures is summarized in Tables 1 and 2.

C. General Equilibrium and Its Inframarginal Comparative Statics

We now consider the third step of inframarginal analysis. Based on the first two steps of the inframarginal analysis, we will partition the parameter space into subspaces within each of which a particular structure occurs in equilibrium.

For any given structure, each individual can plug the corner equilibrium prices into her indirect utility functions for all configurations including those that are not in this structure. She has no incentive to deviate from a constituent configuration in this structure if this configuration generates a utility value that is not lower than in any alternative configurations under the corner equilibrium values of prices in this structure. Each individual can conduct such total cost-benefit analysis across configurations. Let indirect utility in each constituent configuration not be smaller than in any alternative configurations under the corner equilibrium prices in this structure. We can obtain a system of semi-inequalities that involves only parameters. This system of semi-inequalities defines a parameter subspace within which the corner equilibrium in this structure is the general equilibrium. This total cost-benefit analysis is very tedious and cumbersome. Fortunately, the Yao Theorem (see Yang 2001, chapter 6, p.156) can be used to simplify this total cost-benefit analysis. It states that in an economy with a continuum of ex ante identical consumer-producers having rational and convex preferences and production functions displaying individual specific economies of specialization, a Walrasian general equilibrium exists and it is the

Pareto optimum corner equilibrium. Here the Pareto optimum corner equilibrium is a corner equilibrium that generates the highest per capita real income. Since our model in this paper is a special case of the above mentioned general class of models, the individuals have no incentive to deviate from their chosen constituent configurations in a structure if and only if individuals' corner equilibrium utility value in this structure is not lower than that in any other corner equilibria. With the Yao theorem, we can then compare corner equilibrium per capita real incomes across all structures, and the comparison partitions the five-dimension (α, β, g, b, k) parameter space into several subspaces, within each of which one corner equilibrium is the general equilibrium. As parameter values shift between different subspaces, the general equilibrium discontinuously jumps between corner equilibria. This is referred to as inframarginal comparative statics of general equilibrium.

In order to obtain analytical solution of the inframarginal comparative statics, we consider the economy with $\alpha = \beta = 0.5$. A close examination of per capita real incomes in different structures, given in Table 2, generates the results in the following table, in which trading efficiency coefficients in an entry positively correlate to per capita real income in a structure associated with the column.

From Table 2, we can see that as any trading efficiency coefficient in the second row tends to zero, the per capita real income in the corresponding structure in the first row in Table 3 goes to zero. For instance, per capita real income in structure FD_A positively depends on trading efficiencies of goods x and y, k_x, k_y, and trading efficiency of labour employed to produce x, g_z. The per capita real income converges to zero as any of k_x, k_y, g_z goes to zero. Since per capita real income in autarky (structure A) is independent of the trading efficiency coefficients, if all trading efficiency coefficients are sufficiently close to zero, per capita real income in autarky will be greater than that in any other structures with trade. Also, we can see from Table 3 that a structure with partial division of labour (P_A, P_B, or FP_B) involves trading efficiency coefficients of two types of goods and/or labour, while a structure with the complete division of labour (three goods are involved in commercialised production) involves trading efficiency coefficients of three types of goods and/or labour. Hence, as trading efficiencies of more types of goods and labour are improved, the general equilibrium will discontinuously jump from autarky to partial division of labour, followed by the complete division of labour. Hence, the first conclusion from the total cost-benefit analysis of per capita real income in various structures is that trading efficiency determines the general equilibrium network size of division of labour.

The second conclusion from the third step of inframarginal analysis is that the institution of the firm is a way to replace trade of goods with trade of labour. As we can see from Table 3, all structures with the firm (FP_B, FD_A, FT_A, FT_B) involve trading efficiency coefficient of labour employed to produce good i, g_i. Per capita real incomes in all structures without the firm (P_A, P_B, CD) are independent of trading efficiency of labour. Hence, if the trading efficiency is higher for labour than that for goods, the institution of the firm and related labour market will be used to more efficiently organize the division of labour. Otherwise, the markets for goods will be used to organize the division of labour in the absence of the institution of the firm and related labour market. This formalizes the theory of Coase (1937) and Cheung (1983). This is consistent with the inframarginal analysis of the theory of the firm by Yang and Ng (1995) and the model formalizing the theory of irrelevance of the size of the firm developed by Liu and Yang (2000).

Third conclusion can be obtained by comparing structures with the firm and bundling (FT_i, i = A, B) and those with the firm and without bundling (FP_B, FD_A). A comparison between structures FD_A and FT_A shows that if trading efficiency is prohibitively low for good y (k_y tends to zero), then not only a structure without the firm (such as structure CD) cannot be used to coordinate the complete division of labour with three goods involved in commercialised production, but also structures with the firm (FD_A, FD_B) cannot be used to coordinate the complete division of labour in the absence of bundling. This is because structures CD and FD_A involve marketing and pricing of good y, while structure FT_A with bundling avoids direct pricing of good y, when it gets good y involved in commercialised production.

In order to make results more concrete, we explicitly solve for general equilibrium and its inframarginal comparative statics for some specific ranges of parameter values.

We first assume that the trading efficiency of good y, k_y, is very close to zero. From Tables 2 and 3, we can see that this implies zero per capita real incomes in structures P_A, P_B, CD, FD_A, FT_B, since per capital real incomes in these structures are positively dependent on k_y and they go to zero as k_y tends to zero. Hence, the set of candidates for equilibrium structure consists of structures A and FT_A in which per capita real incomes are independent of k_y. As shown in Tables 2 and 3, per capita real income in structure A is independent of trading efficiency, per capita real income is structure FT_A depends on k_x, k_z, and g_y. Therefore, when k_x, k_z, and g_y are very small, the general equilibrium is the corner equilibrium in structure A. When k_x and k_z are large, the general equilibrium is the corner equilibrium in structure FT_A. The inframarginal comparative statics of general equilibrium are summarized in Table 4.

The inframarginal comparative statics in Table 4 indicate that as trading efficiencies increase from very small to very large values, the general equilibrium discontinuously jumps from autarky to the division of labour. Due to prohibitively low trading efficiency of good y, the division of labour must be organized via the institution of the firm that sells good x and provides good y free of charge. A particular structure with the firm and bundling can be used to avoid trade of a particular type of labour. Structure FT_A can be used to avoid trade of labour employed to produce good x. Suppose that good y is an information good and x is a hardware. Hence, the output and input of producing x are easy to measure, but the output and input of producing y is prohibitively expensive to measure. For instance, labour employed to produce good y is intellectual efforts put in thinking and research. The quantity and quality of such efforts are prohibitively expensive to measure. Under this circumstance, bundling in structure FT_A is to avoid all direct pricing of output and input of the activity producing intangible good y.

As shown in Yang and Ng (1995), the institution of the firm can indirectly price intangible intellectual properties via claims to residual rights of the firm. But, the model in this paper shows that the institution of the firm coupled with bundling can enlarge the scope for such indirect pricing of intellectual properties. In the case of Table 4, the institution of the firm is not enough to indirectly price all input and output of the activity producing good y in the absence of implicit bundling. Hence, without implicit bundling, the division of labour and commercialised production of information goods becomes impossible, so that positive network effects of such commercialised production through specialization cannot be fully exploited.

In order to compare the roles of structures with and without bundling, we consider the case with $k_x \to 0$. The inframarginal comparative statics of general equilibrium within this range of parameter values are summarized in Table 5.

The inframarginal comparative statics in Table 5 indicate that as trading efficiencies increase from very low to very high levels, the general equilibrium evolve from autarky first to the partial division of labour, then to the complete division of labour. The partial division of labour is coordinated by the institution of the firm and related labour market if trading efficiency for labour is high.

Otherwise it is organized by the markets for goods in the absence of the firm and related labour market. The complete division of labour can be organized only via the institution of the firm which sells good y with good x bundled due to prohibitively low trading efficiency of good x. A comparison between Tables 4 and 5 shows that direct pricing of a good (x or y) must be avoided via bundling if the trading efficiency of this good is extremely low.

Following Sun, Yang, and Yao (1999, see also Yang, 2001), it can be shown that a general equilibrium in our model is Pareto optimal. This first welfare theorem in our model with impersonal networking decisions and endogenous network size of division of labour implies that very function of the market is to coordinate impersonal networking decisions and to fully utilize network effects of division of labour on aggregate productivity, net of transaction costs. Bundling in a competitive market is an effective way to promote division of labour and productivity progress. This, together with the inframarginal comparative statics of equilibrium given in Tables 4 and 5, lead to the following proposition.

PROPOSITION 1: *Absolute level of transaction efficiency of goods and labour determines the level of division of labour. As transaction efficiency is improved, the equilibrium level of division of labour increases. Relative level of transaction efficiency for labour to that for goods determines if the division of labour is organised by labour market and the related institution of firm. Bundling sales can be used to avoid direct pricing of output and input of the activity with the lowest* *transaction efficiency, meanwhile getting this activity involved in the division of labour, thereby promoting the division of labour and productivity progress. Bundling sales based on impersonal networking decisions has no adverse effects on welfare.*

Proposition 1 implies that the antitrust prosecution should focus on the existence of intention to block free-entry rather than on bundling sales itself because according to Proposition 1 and the Yao theorem, bundling sales will promote the division of labour and increase the aggregate productivity if it occurs in equilibrium. Bundling does not generate distortions in a competitive market.

Following Yang (2001), it is easy to prove that marginal cost price no longer holds in a structure with the division of labour and that the aggregate production schedule discontinuously jumps to a higher level as the network of division of labour expands. Due to the trade-off between transaction costs and positive network effects of division of labour on aggregate productivity, the equilibrium and Pareto optimum may be different from the PPF. As trading efficiency is improved, the equilibrium network size of division of labour enlarges, and the equilibrium and Pareto optimum become closer to the PPF.

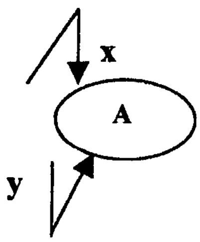
III. Concluding Remarks

This paper develops a Walrasian general equilibrium model based on impersonal networking decisions to investigate the role of bundling sales in a competitive market and e-business. The following features distinguish our model of bundling from other models in the literature. In our model there is no monopoly power, substitution between different goods and resale of goods are allowed. An ex ante identical utility function is specified for all individuals whose valuations of each good are not a constant. Each individual can choose size and pattern of her trade network by choosing her level of specialization subject to impersonal prices. Hence, gains to each person's level of specialization depends on the number of participants in the network of division of labour, while the number of participants depends on each person's participation decision in the network, which is determined by her decision in choosing her level of specialization. Since individuals are capable of doing inframarginal analysis in choosing an utility maximizing trade network from many possible corner solutions, the equilibrium network size and pattern of division of labour is Pareto efficient despite of the existence of network effects of division of labour on aggregate productivity.

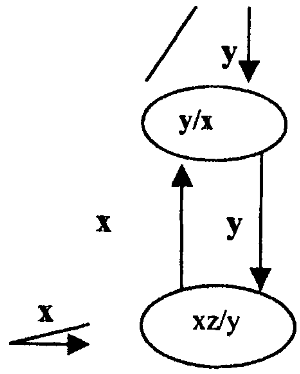
The function of the institution of the firm and bundling is to get the activity with the lowest trading efficiency involved in the division of labour and commercialised production, meanwhile avoiding direct pricing of the outputs and inputs of this activity. Implicit bundling coupled with the institution of the firm can provide a greater scope for indirectly pricing goods with the lowest trading efficiency than the institution of the firm alone can do. In our model, the complete division of labour can be organized by trade of three types of goods and labour. But there are six types of goods and labour: x, y, z, l_x, l_y, l_z. Hence, a competitive market will find a three element combination from six elements to fully exploit total positive network effects of division of labour on aggregate productivity net of total transaction costs. Note that total equilibrium value of transaction cost may increase as a consequence of evolution of division of labour caused by improvements in trading efficiency. For instance, as trading efficiency is improved, the general equilibrium jumps from autarky, where transaction cost is zero and aggregate productivity is lower than the PPF, to the division of labour where total transaction cost is positive and aggregate productivity is higher.

Since the general equilibrium in our model is always Pareto optimal as long as nobody can block free entry into any sector and nobody can manipulate relative prices and numbers of specialists, policy implications of our model is straightforward. Bundling in a competitive market is efficient and it ensures that network effects of division of labour can be fully exploited when goods involved in the network of division of labour are associated with prohibitively high transaction costs. Hence, bundling in a competitive market can promote aggregate productivity by enlarging the scope for trading off network effects of the division of labour on aggregate productivity against transaction costs. Bundling itself cannot be a source of distortions in a competitive market. Bundling may generate distortions only if it is used in connection with monopoly power. Hence, in antitrust cases, such as in the case of Microsoft vs. the United States, attention should be placed on the existence of intention to block free entry in an attempt of gaining monopoly power rather than on alleged adverse effects of bundling itself on welfare. To business practitioners, our model suggests that successful bundling of intangible e-business with some tangible 'mortar-brick' business is a key for commercial viability of e-business companies.

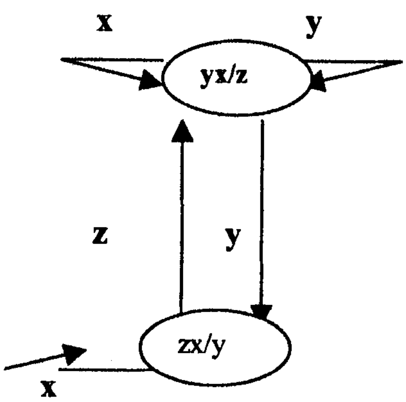
A promising extension of our model is to assume that the seller of a bundle of goods cannot choose bundling ratio. We may assume that each buyer of implicitly bundled goods must allocate resource to use those goods that are free of charge. Hence, it is the buyer rather than the seller who chooses bundling ratio subject to her resource endowment constraint. When a firm sells information goods via website, she usually cannot choose bundling ratio of goods with positive prices and goods free of charge. We speculate that the extended model will confirm results in the current paper with this assumption that is more relevant to real e-business.



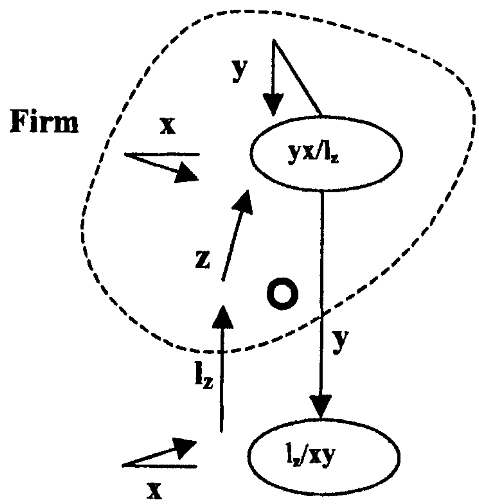
Autarky



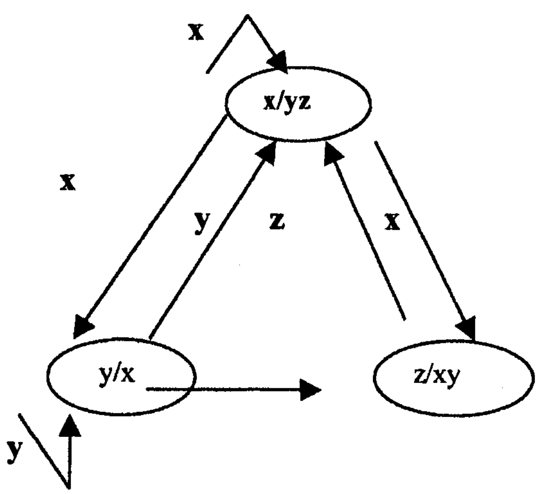
Structure P_A



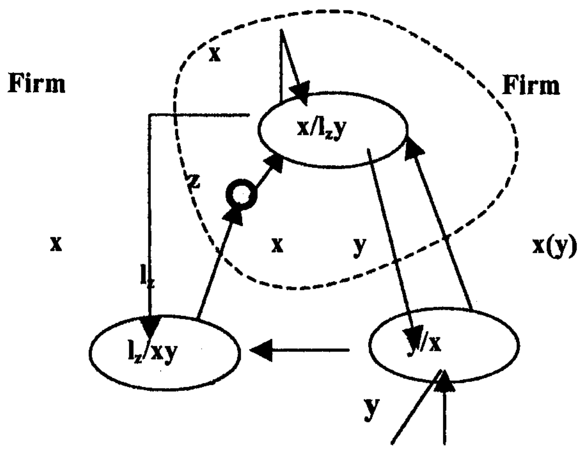
Structure P_B



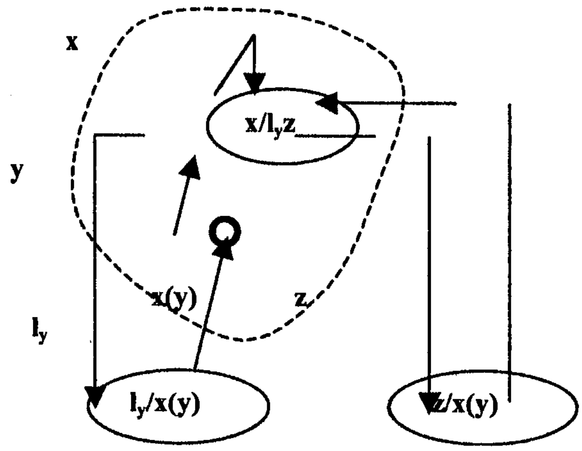
Structure FP_B



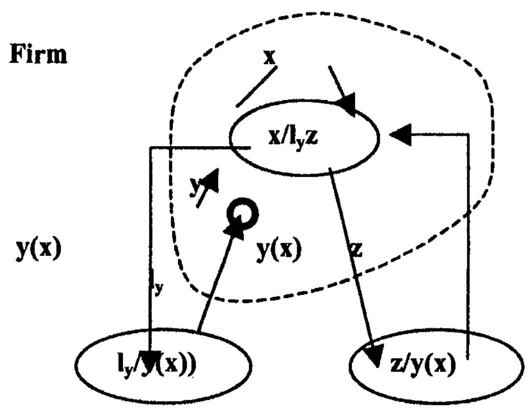
Structure CD



Structure FD$_A$



Structure FT$_A$



Structure FT$_B$

FIGURE 1. CONFIGURATIONS AND STRUCTURES

TABLE 1 – RELATIVE PRICE AND NUMBER OF SEPECIALISTS

Structure	Relative Prices	Relative Number of Specialists
A	N/A	N/A
P_A	$\frac{p_x}{p_y}=\frac{k_y^{\alpha}\cdot(1+\beta)^{1+\beta}}{k_x^{1-\alpha}\cdot\beta^{\beta}\cdot(1-b)^{\beta}}$	$\frac{M_x}{M_y}=\frac{\alpha\cdot k_y^{1-\alpha}}{(1-\alpha)\cdot k_x^{\alpha}}$
P_B	$\frac{p_y}{p_z}=(\frac{k_z^{1-\alpha}}{k_y^{\alpha\beta}})^{\frac{1}{1-\alpha+\alpha\beta}}$	$\frac{M_z}{M_y}=\frac{\alpha\beta\cdot k_y^{\frac{1-\alpha}{1-\alpha+\alpha\beta}}}{(1-\alpha)\cdot k_z^{\frac{\alpha\beta}{1-\alpha+\alpha\beta}}}$
FP_B	$\frac{p_y}{w}=P_{FP}$	$\frac{M_z}{M_y}=M_{FPB}$
CD	$\frac{p_x}{p_z}=(\frac{1-b}{1-\beta})^{1-\beta}\cdot\frac{k_z^{1-\beta}}{\beta^{\beta}\cdot k_x^{1-\alpha+\alpha\beta}}$ $\frac{p_z}{p_y}=\frac{k_y^{\alpha}}{k_z}$	$\frac{M_y}{M_x}=\frac{(1-\alpha)+\alpha\beta(1-k_x\cdot k_z\cdot k_y^{1-\alpha})}{\alpha\cdot(1-\beta)\cdot k_x^{1-\alpha}}$ $\frac{M_x}{M_z}=\frac{(1-\beta)}{\beta\cdot k_z\cdot k_x^{\alpha}}$
FD_A	$\frac{w}{p_y}=k_y^{\alpha}\cdot(g_z-b)$ $\frac{w}{p_x}=k_x^{1-\alpha+\alpha\beta}\cdot(1-\beta)^{1-\beta}\cdot\beta^{\beta}\cdot(g_z-b)^{\beta}$	$\frac{M_x}{M_z}=\frac{g_z\cdot(1-\beta)}{\beta\cdot k_x^{\alpha}}$ $\frac{M_y}{M_x}=\frac{k_x^{\alpha}\cdot(1-\alpha)}{k_y^{1-\alpha}\cdot\alpha\cdot(1-\beta)}$
FT_A	$\frac{w}{p_z}=k_z(1-b)$ $\frac{w}{p_x}=P_{FTA1}$	$\frac{M_x}{M_z}=M_{FTA1}$ $\frac{M_y}{M_x}=M_{FTA2}$
FT_B	$\frac{w}{p_z}=k_z(1-b)$ $\frac{w}{p_y}=P_{FTB}$	$\frac{M_x}{M_z}=M_{FTB1}$ $\frac{M_y}{M_x}=M_{FTB2}$

Here,

$$P_{FPB}=\frac{p_y}{w}=\{\frac{(1+\alpha\beta)^{\frac{1-\alpha}{\alpha}}\cdot k_y\cdot[g_z(1-b)\cdot(1-\alpha)-b(1+\alpha\beta)]}{(\alpha\beta)^{\frac{1-\alpha}{\alpha}}\cdot(1-\alpha)\cdot(1-b)^{\frac{1}{\alpha}}}\}^{\frac{\alpha\beta}{(1-\alpha)\cdot(1-\alpha+\alpha\beta)}},$$

$$M_{FPB}=\frac{M_z}{M_y}=\{\frac{(1+\alpha\beta)^{\frac{1-\alpha}{\alpha}}\cdot k_y\cdot[g_z(1-b)\cdot(1-\alpha)-b(1+\alpha\beta)]}{(\alpha\beta)^{\frac{1-\alpha}{\alpha}}\cdot(1-\alpha)\cdot(1-b)^{\frac{1}{\alpha}}}\}^{\frac{\alpha\beta}{(1-\alpha)\cdot(1-\alpha+\alpha\beta)}}\cdot[\frac{(1+\alpha\beta)^2}{(1-\alpha)\cdot(1-b)^2}],$$

$$P_{FTA1}=\frac{w}{p_x}=[\frac{k_x(g_y-b)\cdot k_z\cdot\beta}{(1-\alpha)\cdot(1-\beta)+\alpha\beta}]^{\frac{\beta}{1-\beta}}\cdot\frac{\alpha^\alpha\cdot(1-\beta)\cdot[(1-\alpha)\cdot(\alpha-\beta)+\alpha\beta]^{(1-\alpha)+\frac{\beta}{1-\beta}}}{(1-\alpha)^{1-\alpha}},$$

$$M_{FTA1}=\frac{M_x}{M_z}=\{[\frac{(1-\alpha)\cdot(\alpha-\beta)+\alpha\beta}{(1-\alpha)\cdot(1-\beta)+\alpha\beta}]^{(1-\alpha)+\frac{\beta}{1-\beta}}\cdot\frac{\alpha^\alpha\cdot(1-\beta)\cdot[(g_y-b)\cdot\beta]^{\frac{\beta}{1-\beta}}}{(1-\alpha)^{1-\alpha}\cdot k_x^{\frac{1-2\beta}{1-\beta}}}\}^{\frac{1}{1-\beta}}\cdot(\frac{k_z^2}{1-b})^{\frac{\beta}{1-\beta}},$$

$$M_{FTA2}=\frac{M_y}{M_x}=[\frac{(1-\alpha)\cdot(\alpha-\beta)+\alpha\beta}{(1-\alpha)\cdot(1-\beta)+\alpha\beta}]\cdot\frac{(1-\alpha+\alpha\beta)}{\beta}-[\frac{k_x(g_y-b)\cdot k_z\cdot\beta}{(1-\alpha)\cdot(1-\beta)+\alpha\beta}]^{\frac{\beta}{1-\beta}}\cdot\frac{\alpha^\alpha\cdot(1-\beta)\cdot[(1-\alpha)\cdot(\alpha-\beta)+\alpha\beta]^{(1-\alpha)+\frac{\beta}{1-\beta}}}{(1-\alpha)^{1-\alpha}},$$

$$P_{FTB1}\frac{w}{p_y}=[\frac{k_y(g_y-b)\cdot k_z\cdot\beta}{(1-\alpha)\cdot(1-\beta)+\alpha\beta}]^{\frac{\beta}{1-\beta}}\cdot\frac{\alpha^\alpha\cdot(1-\beta)\cdot[(1-\alpha)\cdot(\alpha-\beta)+\alpha\beta]^{(1-\alpha)+\frac{\beta}{1-\beta}}}{(1-\alpha)^{1-\alpha}},$$

$$M_{FTB1}=\frac{M_x}{M_z}=\{[\frac{(1-\alpha)\cdot(\alpha-\beta)+\alpha\beta}{(1-\alpha)\cdot(1-\beta)+\alpha\beta}]^{(1-\alpha)+\frac{\beta}{1-\beta}}\cdot\frac{\alpha^\alpha\cdot(1-\beta)\cdot[(g_y-b)\cdot\beta]^{\frac{\beta}{1-\beta}}}{(1-\alpha)^{1-\alpha}\cdot k_y^{\frac{1-2\beta}{1-\beta}}}\}^{\frac{1}{1-\beta}}\cdot(\frac{k_z^2}{1-b})^{\frac{\beta}{1-\beta}},$$

$$M_{FTB2}=\frac{M_y}{M_x}=[\frac{(1-\alpha)\cdot(\alpha-\beta)+\alpha\beta}{(1-\alpha)\cdot(1-\beta)+\alpha\beta}]\cdot\frac{(1-\alpha+\alpha\beta)}{\beta}-[\frac{k_y(g_y-b)\cdot k_x\cdot\beta}{(1-\alpha)\cdot(1-\beta)+\alpha\beta}]^{\frac{\beta}{1-\beta}}\cdot\frac{\alpha^\alpha\cdot(1-\beta)\cdot[(1-\alpha)\cdot(\alpha-\beta)+\alpha\beta]^{(1-\alpha)+\frac{\beta}{1-\beta}}}{(1-\alpha)^{1-\alpha}}.$$

TABLE 2 – PER-CAPITA REAL INCOME IN DIFFERENT STRUCTURES

Structure	Per-Capita Real Income, u
A	$\frac{\beta^{\alpha\beta}\cdot(1-\alpha)^{1-\alpha}\cdot(1-2b)^{1+\alpha\beta}\cdot\alpha^{\alpha(1+\beta)}}{(1+\alpha\beta)^{1+\alpha\beta}}$
P_A	$\frac{\alpha^{\alpha}\cdot(1-\alpha)^{1-\alpha}\cdot(1-b)^{1+\alpha\beta}\cdot\beta^{\alpha\beta}\cdot(k_x\cdot k_y)^{\alpha(1-\alpha)}}{(1+\beta)^{\alpha(1+\beta)}}$
P_B	$u_{PB}=\frac{\alpha^{\alpha}\cdot(1-\alpha)^{1-\alpha}\cdot(1-b)^{1+\alpha\beta}\cdot(\alpha\beta)^{\alpha\beta}\cdot(k_z\cdot k_y)^{\frac{\alpha\beta(1-\alpha)}{1-\alpha+\alpha\beta}}}{(1+\alpha\beta)^{1+\alpha\beta}}$
FP_B	$u_{FPB}=\{\frac{(1+\alpha\beta)^{\frac{1-\alpha}{\alpha}}\cdot k_y\cdot[g_z(1-b)\cdot(1-\alpha)-b(1+\alpha\beta)]}{(\alpha\beta)^{\frac{1-\alpha}{\alpha}}\cdot(1-\alpha)\cdot(1-b)^{\frac{1}{\alpha}}}\}^{\frac{\alpha\beta}{(1-\alpha)\cdot(1-\alpha+\alpha\beta)}}\cdot[\frac{(1-\alpha)^{1-\alpha}\cdot\alpha^{\alpha+\beta}\cdot(1-b)^{1+\beta}\cdot\beta^{\beta}}{(1+\alpha\beta)^{1+\beta}}]$
CD	$\alpha^{\alpha}\cdot(1-\alpha)^{1-\alpha}\cdot(1-b)^{1-\alpha+\alpha\beta}\cdot\beta^{\alpha\beta}\cdot(1-\beta)^{\alpha(1-\beta)}\cdot k_z^{\alpha\beta}\cdot k_x^{\alpha(1-\alpha+\alpha\beta)}\cdot k_y^{\alpha(1-\alpha)}$
FD_A	$\alpha^{\alpha}\cdot(1-\alpha)^{1-\alpha}\cdot(g_z-b)^{1-\alpha+\alpha\beta}\cdot\beta^{\alpha\beta}\cdot(1-\beta)^{\alpha(1-\beta)}\cdot k_y^{\alpha(1-\alpha)}\cdot k_x^{\alpha(1-\alpha+\alpha\beta)}$
FT_A	$u_{FTA}=\alpha^{\alpha}\cdot(1-\beta)\cdot(1-\alpha)^{1-\alpha}\cdot(k_z\cdot\beta)^{\frac{\beta}{1-\beta}}\cdot\{\frac{k_x(g_y-b)\cdot[(1-\alpha)\cdot(\alpha-\beta)+\alpha\beta]}{(1-\alpha)\cdot(1-\beta)+\alpha\beta}\}^{(1-\alpha)+\frac{\beta}{1-\beta}}$
FT_B	$u_{FTB}=\alpha^{\alpha}\cdot(1-\beta)\cdot(1-\alpha)^{1-\alpha}\cdot(k_z\cdot\beta)^{\frac{\beta}{1-\beta}}\cdot\{\frac{k_y(g_y-b)\cdot[(1-\alpha)\cdot(\alpha-\beta)+\alpha\beta]}{(1-\alpha)\cdot(1-\beta)+\alpha\beta}\}^{(1-\alpha)+\frac{\beta}{1-\beta}}$

TABLE3 – TRADING EFFICINECY COEFFICIENTS THAT POSITIVELY AFFECT PER-CAPITA REAL INCOME IN A STRUCTURE

A	P_A	P_B	FP_B	CD	FD_A	FT_A	FT_B
n/a	k_x, k_y	k_z, k_y	g_z, k_y	k_x, k_y, k_z	k_x, k_y, g_z	k_x, k_z, g_y	k_z, k_y, g_y

TABLE 4 – GENERAL EQUILIBRIUM AND ITS INFRAMARGIANL COMPARATIVE STATICS WHEN $k_y \to 0$

Trading efficiency of goods	k_x and k_z are small	k_x and k_z are large
Equilibrium structure	A	FT_A

APPENDIX

THE CORNER EQUILIBRIA OF DIFFERENT MARKET STRUCTURES:

1. Partial Division of Labour: P_A

Structure P_A consists of two configurations, (xz/y) and (y/x). In the structure P_A, given that $x, x^s, z, y^d > 0, z^s = z^d = y = y^s = x^d = 0$, an individual in configuration (xz/y) has the following decision problems,

(A1a) Max: $u_{PA1} = x^{\alpha} \cdot (y^d)^{1-\alpha}$,

subject to the following constraints:

(A1b) $x + x^s = z^{\beta} \cdot l_x$ and $\beta \in (0,1)$,

$z = l_z - b$ and $b \in (0,1)$,

$l_x + l_z = 1$, $k_x \cdot p_x \cdot x^s = p_y \cdot y^d$,

where u_{PA1} is the utility for an individual in configuration (xz/y). The equations of constraints state an individual's budget constraint, endowment constraint, and the production function. Similarly, an individual in configuration (y/x) has the following decision problem:

(A1c) Max: $u_{PA2} = (x^d)^{\alpha} \cdot y^{1-\alpha}$,

subject to the following constraints:

(A1d) $y + y^s = l_y - b$ and $b \in (0,1)$,

$l_y = 1$, $k_y \cdot p_y \cdot y^s = p_x \cdot x^d$,

where u_{PA2} is the utility for an individual in configuration (y/x).

Based on the utility equalization condition and market clearing conditions, the price of good x in terms of good y, and the relative number of individual selling good x to individuals selling good y are given by:

(A1e) $$\frac{p_x}{p_y} = \frac{k_y^{\alpha} \cdot (1+\beta)^{1+\beta}}{k_x^{1-\alpha} \cdot \beta^{\beta} \cdot (1-b)^{\beta}}, \quad \text{and} \quad \frac{M_x}{M_y} = \frac{\alpha \cdot k_y^{1-\alpha}}{(1-\alpha) \cdot k_x^{\alpha}} .$$

The real per capita income in this structure is,

(A1f) $$u_{PA} = \frac{\alpha^{\alpha} \cdot (1-\alpha)^{1-\alpha} \cdot (1-b)^{1+\alpha\beta} \cdot \beta^{\alpha\beta} \cdot (k_x \cdot k_y)^{\alpha(1-\alpha)}}{(1+\beta)^{\alpha(1+\beta)}} .$$

2. Partial Division of Labour: P_B

Similarly, in structure P_B the decision problem for an individual with configuration (zx/y) is:

(A2a) Max: $u_{PB1} = x^{\alpha} \cdot (y^d)^{1-\alpha}$,

subject to the following constraints:

(A2b) $x = z^{\beta} \cdot l_x$ and $\beta \in (0,1)$,

$z + z^s = l_z - b$ and $b \in (0,1)$,

$l_x + l_z = 1$, $k_z \cdot p_z \cdot z^s = p_y \cdot y^d$,

where u_{PB1} is the utility for an individual in configuration (zx/y). The equations of constraints state an individual's budget constraint, endowment constraint, and the production function.

An individual in configuration (yx/z) has the following decision problem:

(A2c) Max: $u_{PB2} = x^{\alpha} \cdot y^{1-\alpha}$

subject to the following constraints:

(A2d) $x=(z^{d})^{\beta}\cdot l_{x}$ and $\beta\in(0,1)$,

$y+y^{s}=l_{y}-b$ and $b\in(0,1)$,

$l_{x}+l_{y}=1$, $k_{y}\cdot p_{y}\cdot y^{s}=p_{z}\cdot z^{d}$.

The utility equalization and market clearing conditions, yield a set of relative prices and relative number of specialists, and the per capita real income in this structure.

(A2e) $\dfrac{p_{y}}{p_{z}}=\left(\dfrac{k_{z}^{1-\alpha}}{k_{y}^{\alpha\beta}}\right)^{\frac{1}{1-\alpha+\alpha\beta}}$, $\dfrac{M_{z}}{M_{y}}=\dfrac{\alpha\beta\cdot k_{y}^{\frac{1-\alpha}{1-\alpha+\alpha\beta}}}{(1-\alpha)\cdot k_{z}^{\frac{\alpha\beta}{1-\alpha+\alpha\beta}}}$,

and $u_{PB}=\dfrac{\alpha^{\alpha}\cdot(1-\alpha)^{1-\alpha}\cdot(1-b)^{1+\alpha\beta}\cdot(\alpha\beta)^{\alpha\beta}\cdot(k_{z}\cdot k_{y})^{\frac{\alpha\beta(1-\alpha)}{1-\alpha+\alpha\beta}}}{(1+\alpha\beta)^{1+\alpha\beta}}$.

3. Partial Division of Labor with the institution of Firms: Structure FP_B

Structure FP_B is a division of the population between configuration (l_zx/y) and (yx/l_z). Given that $x,z,l_{z},y^{d}>0,x^{s}=x^{d}=y=y^{s}=z^{s}=z^{d}=0$, an individual in configuration (l_zx/y) has the following decision problems,

(A3a) Max: $u_{FPB1}=x^{\alpha}\cdot(y^{d})^{1-\alpha}$.

subject to the following constraints:

(A3b) $x=z^{\beta}\cdot l_{x}$ and $\beta\in(0,1)$,

$z=l_{z}-b$ and $b\in(0,1)$,

$l_{x}+l_{z}+L_{z}=1$, and $w\cdot L_{z}=p_{y}\cdot y^{d}$.

Similarly, A person choosing configuration (yx/l_z) produces goods x and y, sells good y, and employs labor to produce good z. It is defined by $x, y, y^s, l_z > 0, x^s = x^d = y^d = z^s = z^d = 0$. An individual in configuration (yx/l_z) has the following decision problems,

(A3c) Max: $u_{FPB2} = x^\alpha \cdot y^{1-\alpha}$,

subject to the following constraints:

(A3d) $y + y^s = l_y - b$,

$x = (z^d)^\beta \cdot l_x$ and $\beta \in (0,1)$,

$l_x + l_z = 1$, $z^d = N \cdot z^s$,

$z^s = g_z \cdot L_z - b$, and $k_y \cdot p_y \cdot y^s = w \cdot N \cdot L_z$.

The utility equalization and market clearing conditions, yield the price of good y in term of labour to produce good z, and the number of individuals selling good y relative to that of individuals selling labour to produce good z, which are given by:

(A3e) $$\frac{p_y}{w} = \left\{\frac{(1+\alpha\beta)^{\frac{1-\alpha}{\alpha}} \cdot k_y \cdot [g_z(1-b)\cdot(1-\alpha) - b(1+\alpha\beta)]}{(\alpha\beta)^{\frac{1-\alpha}{\alpha}} \cdot (1-\alpha)\cdot(1-b)^{\frac{1}{\alpha}}}\right\}^{\frac{\alpha\beta}{(1-\alpha)\cdot(1-\alpha+\alpha\beta)}}, \text{ and}$$

$$\frac{M_z}{M_y} = \left\{\frac{(1+\alpha\beta)^{\frac{1-\alpha}{\alpha}} \cdot k_y \cdot [g_z(1-b)\cdot(1-\alpha) - b(1+\alpha\beta)]}{(\alpha\beta)^{\frac{1-\alpha}{\alpha}} \cdot (1-\alpha)\cdot(1-b)^{\frac{1}{\alpha}}}\right\}^{\frac{\alpha\beta}{(1-\alpha)(1-\alpha+\alpha\beta)}} \cdot \left[\frac{(1+\alpha\beta)^2}{(1-\alpha)\cdot(1-b)^2}\right].$$

The per capita real income in this structure is,

(A3f)

$$u_{FPB} = \left\{\frac{(1+\alpha\beta)^{\frac{1-\alpha}{\alpha}} \cdot k_y \cdot [g_z(1-b)\cdot(1-\alpha) - b(1+\alpha\beta)]}{(\alpha\beta)^{\frac{1-\alpha}{\alpha}} \cdot (1-\alpha)\cdot(1-b)^{\frac{1}{\alpha}}}\right\}^{\frac{\alpha\beta}{(1-\alpha)\cdot(1-\alpha+\alpha\beta)}} \cdot \left[\frac{(1-\alpha)^{1-\alpha} \cdot \alpha^{\alpha+\beta} \cdot (1-b)^{1+\beta} \cdot \beta^\beta}{(1+\alpha\beta)^{1+\beta}}\right].$$

4. Complete Division of Labor without Firms: Structure CD

There are three Configurations (x/yz), (z/xy) and (y/x) in this structure, where an individual produces only one of good x, y or z, and sells them in exchange for others. The decision problems for the individuals under different configurations are given as below respectively,

In configuration (x/yz):

(A4a) Max: $u_{CD1} = x^{\alpha} \cdot (y^d)^{1-\alpha}$

(A4b) s.t. $x + x^s = (z^d)^{\beta} \cdot l_x$ and $\beta \in (0,1)$,

$l_x = 1$, $k_x \cdot p_x \cdot x^s = p_y \cdot y^d + p_z \cdot z^d$.

In configuration (z/xy):

(A4c) Max: $u_{CD2} = (x^d)^{\alpha} \cdot (y^d)^{1-\alpha}$

(A4d) s.t. $z^s = l_z - b$ and $b \in (0,1)$,

$l_z = 1$, $k_z \cdot p_z \cdot z^s = p_x \cdot x^d + p_y \cdot y^d$.

In configuration (y/x):

(A4e) Max: $u_{CD3} = (x^d)^{\alpha} \cdot y^{1-\alpha}$

(A4f) s.t. $y + y^s = l_y - b$ and $b \in (0,1)$,

$l_y = 1$, $k_y \cdot p_y \cdot y^s = p_x \cdot x^d$.

The utility equalization condition and market clearing conditions, yield a set of relative prices and relative number of specialists,

(A4g) $$\frac{p_z}{p_y} = \frac{k_y^{\alpha}}{k_z} , \qquad \frac{p_x}{p_z} = \left(\frac{1-b}{1-\beta}\right)^{1-\beta} \cdot \frac{k_z^{1-\beta}}{\beta^{\beta} \cdot k_x^{1-\alpha+\alpha\beta}} ,$$

$$\frac{M_y}{M_x} = \frac{(1-\alpha)+\alpha\beta(1-k_x \cdot k_z \cdot k_y^{1-\alpha})}{\alpha \cdot (1-\beta) \cdot k_x^{1-\alpha}}, \qquad \frac{M_x}{M_z} = \frac{(1-\beta)}{\beta \cdot k_z \cdot k_x^{\alpha}}.$$

The per capita real income in this structure is

(A4h) $$u_{CD} = \alpha^{\alpha} \cdot (1-\alpha)^{1-\alpha} \cdot (1-b)^{1-\alpha+\alpha\beta} \cdot \beta^{\alpha\beta} \cdot (1-\beta)^{\alpha(1-\beta)} \cdot k_z^{\alpha\beta} \cdot k_x^{\alpha(1-\alpha+\alpha\beta)} \cdot k_y^{\alpha(1-\alpha)}.$$

5. Complete Division of Labor with the institution of Firms: Structure FD_A

Structure FD_A consists of three individual configurations $(x/l_z y)$, (l_z/xy) and (y/x). Given that $x, x^s, y^d, l_z > 0, x^d = y = y^s = z = z^s = 0$, an individual in configuration $(x/l_z y)$ has the following decision problems,

(A5a) Max: $u_{FDA1} = x^{\alpha} \cdot (y^d)^{1-\alpha}$.

Her budget constraint and the production functions are,

(A5b) $x + x^s = (Z^d)^{\beta} \cdot l_x$ and $\beta \in (0,1)$, $l_x = 1$,

$z^s = g_z \cdot L_z - b$, $g_z \in (0,1)$ and $b \in (0,1)$,

$L_z = 1$, $Z^d = N \cdot z^s$,

$k_x \cdot p_x \cdot x^s = p_y \cdot y^d + w \cdot N \cdot L_z$,

where g_z is the transaction efficiency coefficient for labour hired to produce the intermediate good z. It encompasses all costs that relate to the measurement of the effects of efforts exerted for producing the intermediate good z in terms of quantity and quality. In essence, the measurement costs can be explained as pricing costs. N is the number of workers hired by the employer. In this configuration, l_x is the decision variable to the employer, while L_z is as given because it is bought from the labour market. u_{FDA1} is the utility for an individual in configuration (x/l$_z$y), and she is the employer in this structure FD$_A$.

Similarly, an individual in configuration ((l$_z$/xy) has the following decision problems,

(A5c) Max: $u_{FDA2} = (x^d)^\alpha \cdot (y^d)^{1-\alpha}$.

The budget constraint and the production functions are,

(A5d) $L_z = 1$, $w \cdot L_z = p_x \cdot x^d + p_y \cdot y^d$.

The individual who chooses this configuration is the employee of this structure. Moreover, an individual in configuration (y/x) has the decision problem of,

(A5e) Max: $u_{FDA3} = (x^d)^\alpha \cdot y^{1-\alpha}$.

The budget constraint and the production functions are,

(A5f) $y + y^s = l_y - b$ and $b \in (0,1)$, $l_y = 1$,

$k_y \cdot p_y \cdot y^s = p_x \cdot x^d$.

The utility equalization and market clearing conditions, yield the set of prices of good x and y in terms of labour to produce good z; and the number of individuals selling good x, y relative to that of individuals selling labour to produce good z, are given by:

(A5g) $\frac{w}{p_y} = k_y^\alpha \cdot (g_z - b)$, $\frac{w}{p_x} = k_x^{1-\alpha+\alpha\beta} \cdot (1-\beta)^{1-\beta} \cdot \beta^\beta \cdot (g_z - b)^\beta$,

$\frac{M_x}{M_z} = \frac{g_z \cdot (1-\beta)}{\beta \cdot k_x^\alpha}$ and $\frac{M_y}{M_x} = \frac{k_x^\alpha \cdot (1-\alpha)}{k_y^{1-\alpha} \cdot \alpha \cdot (1-\beta)}$.

The per capita real income in this structure is,

(A5h) $u_{FDA} = \alpha^{\alpha} \cdot (1-\alpha)^{1-\alpha} \cdot (g_z - b)^{1-\alpha+\alpha\beta} \cdot \beta^{\alpha\beta} \cdot (1-\beta)^{\alpha(1-\beta)} \cdot k_y^{\alpha(1-\alpha)} \cdot k_x^{\alpha(1-\alpha+\alpha\beta)}$.

6. With Bundling Sales and the institution of the Firms: Structure FT_B

Structure FT_B is with bundling sales and the institution of the firm, and involves the division of population among configurations (x/l_yz), ($l_y/y(x)$) and ($z/y(x)$). In Structure FT_B, a firm specializes in producing good x, and also hires labour to produce another final good y. However, an owner of the firm only sells good y in exchange for intermediate good z and labour employed to produced good y; she bundles good x with good y, which means good x is not directly priced, and people can obtain some amount of good x when they buy good y from the market. The ratio of the amounts of the two goods is set up in a bundling sales.

In structure FT_B, the decision problem for an individual in configuration (x/l_yz) is as follow,

(A6a) Max: $u_{FTB1} = x^{\alpha} \cdot y^{1-\alpha}$,

subject to the following constraints,

(A6b) $x + x^s = (z^d)^{\beta} \cdot l_x$ and $\beta \in (0,1)$, $l_x = 1$,

$Y^s = g_y \cdot L_y - b$, $g \in (0,1)$ and $b \in (0,1)$, $L_y = 1$,

$x^s = h \cdot y^s$, $y + y^s = N \cdot Y^s$,

$k_y \cdot p_y \cdot y^s = p_z \cdot z^d + w \cdot N \cdot L_y$, where g_y is again the transaction efficiency coefficient for labour hired to produce good y. Moreover, N is the number of workers hired by the employer to produce good y . In order to distinguish inter flow of goods from market trade flow, we use capitalized decision variables to denote internal flow. Hence, Y^s is internal transfer of good y produced by an employee to the employer and y^s is the amount of good y sold by the firm. h is the bundling ratio between the bundled good y and the final good x which is for sale. Here, we assume $h = e \cdot \frac{p_y}{w}$. This implies that an individual selling y, buying labour, and bundling x with y, must choose the bundling ratio $h = \frac{x}{y}$ according to $\frac{p_y}{w}$. For a small market value of $\frac{p_y}{w}$, she must give away a small amount of y for each unit x sold. Otherwise, a small value of $\frac{p_y}{w}$ may not be enough to cover the production cost of x which is bundled to the sale of good y. Here, e is as given to the owner of the firm, while later based on the Yao Theorem (see Yang 2001, chapter 6, p.156), we can role out the real optimal bundling ratio of good y and x. In addition, l_x is the decision variable to the employer, while L_y is as given because it is bought from the labour market. u_{FTB1} is the utility for an individual in configuration (x/l_yz), and she is the employer in this structure FT_B.

The solution to the decision problem yields demand function for labour and good z, supply function of good x, and indirect utility function for configuration ($x/l_y z$).

Similarly, an employee choosing configuration ($l_y/y(x)$) has the following decision problem,

(A6c) Max: $u_{FTB2} = (x^d)^a \cdot (y^d)^{1-a}$,

subject to the following constraints,

(A6d) $x^d = h \cdot y^d, \quad L_y = 1 , \qquad w \cdot L_y = p_y \cdot y^d$.

The solution of this problem yields demand for goods x and y, supply of labour, and indirect utility function for configuration ($l_y/y(x)$).

An individual choosing configuration ($z/y(x)$) has the following decision problem:

(A6e) Max: $u_{FTB3} = (x^d)^a \cdot (y^d)^{1-a}$,

subject to the production function, endowment constraint, and budget constraint:

(A6f) $z^s = l_z - b, \quad l_z = 1, \qquad x^d = h \cdot y^d, \quad k_z p_z z^s = p_y y^d$.

The solution to this problem yields demand for goods x and y, supply of good z, and indirect utility function for configuration ($z/y(x)$).

The utility equalization conditions across three configurations yield the corner equilibrium relative prices of goods x and z and labour.

(A6g) $\frac{w}{p_z} = k_z(1-b)$, and

$$\frac{w}{p_y} = [\frac{k_y(g_y - b)\cdot k_z \cdot \beta}{(1-\alpha)\cdot(1-\beta)+\alpha\beta}]^{\frac{\beta}{1-\beta}} \cdot \frac{\alpha^\alpha \cdot (1-\beta)\cdot[(1-\alpha)\cdot(\alpha-\beta)+\alpha\beta]^{(1-\alpha)+\frac{\beta}{1-\beta}}}{(1-\alpha)^{1-\alpha}}.$$

Based on the Yao Theorem, maximising utility with respect to e, yields the optimal value of e:

(A6h) $$e = \frac{k_y(1-\alpha)^2 \cdot (g_y - b)}{(1-\alpha)\cdot(1-\beta)+\alpha\beta}.$$

The two independent market clearing conditions for goods x and z (the other market clearing condition is not independent due to Walras' law) yield the corner equilibrium relative numbers of specialists producing goods x, y, and z.

(A6i)

$$\frac{M_x}{M_z} = \{[\frac{(1-\alpha)\cdot(\alpha-\beta)+\alpha\beta}{(1-\alpha)\cdot(1-\beta)+\alpha\beta}]^{(1-\alpha)+\frac{\beta}{1-\beta}} \cdot \frac{\alpha^\alpha \cdot (1-\beta)\cdot[(g_y - b)\cdot\beta]^{\frac{\beta}{1-\beta}} \cdot}{(1-\alpha)^{1-\alpha}\cdot k_y^{\frac{1-2\beta}{1-\beta}}}\}^{\frac{1}{1-\beta}} \cdot (\frac{k_z^2}{1-b})^{\frac{\beta}{1-\beta}},$$

and

$$\frac{M_y}{M_x} = [\frac{(1-\alpha)\cdot(\alpha-\beta)+\alpha\beta}{(1-\alpha)\cdot(1-\beta)+\alpha\beta}]\cdot\frac{(1-\alpha+\alpha\beta)}{\beta} - [\frac{k_y(g_y-b)\cdot k_z\cdot\beta}{(1-\alpha)\cdot(1-\beta)+\alpha\beta}]^{\frac{\beta}{1-\beta}} \cdot \frac{\alpha^\alpha\cdot(1-\beta)\cdot[(1-\alpha)\cdot(\alpha-\beta)+\alpha\beta]^{(1-\alpha)+\frac{\beta}{1-\beta}}}{(1-\alpha)^{1-\alpha}},$$

where M_x is the number of x specialist-employers choosing (x/l_yz), M_z is the number of specialist producers of good z choosing ($z/y(x)$), and M_y is the number of employees choosing ($l_y/y(x)$). The relative numbers of specialists, together with population size identity $M_x+M_z+M_y=M$, yield the corner equilibrium numbers of different specialists. Plugging relative prices into an indirect utility function of any of three configurations yields the per capita real income in this structure:

(A6j)

$$u_{FTB} = \alpha^{\alpha} \cdot (1-\beta)\cdot(1-\alpha)^{1-\alpha} \cdot (k_z \cdot \beta)^{\frac{\beta}{1-\beta}} \cdot \{\frac{k_y(g_y - b)\cdot[(1-\alpha)\cdot(\alpha-\beta)+\alpha\beta]}{(1-\alpha)\cdot(1-\beta)+\alpha\beta}\}^{(1-\alpha)+\frac{\beta}{1-\beta}}.$$

In Structure FT_B, a firm produces both good x and y, while selling y with good x bundled. The percentage *h* of good x and y is dependent on the relative price of good y and labour, and the wage rate *w* of labour hired to produce good y, and *e*. Note that obtaining good x is bundled through the purchase of good y, therefore we need not take the transaction costs of good x into account separately from good y. In other words, we suppose there is no extra transaction cost to obtain good x when good x is bundled with good y.

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