An Econometric Analysis of Duality Based Models of Australian Broadacre Production

A thesis submitted for the degree of
Doctor of Philosophy

by

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May 2011

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Declaration

I hereby declare that this thesis contains no material which has been accepted for the award of any other degree or diploma in any university or equivalent institutions, and that, to my best knowledge and belief, the thesis contains no material previously published or written by another person, except where reference is made in the text of the thesis.

Signed: ______________________ On:_____/____/_____

Acknowledgements

I wish to express my deep gratitude to Associate Professor Xueyan Zhao and Professor Keith McLaren for giving me the opportunity to undertake this study and for their patience, encouragement, support and generosity with practical advice and comments. The understanding of Associate Professor Xueyan Zhao at a personal level is deeply appreciated. I am immensely thankful to Professor Keith McLaren for his insightful and witty explanations as well as his sharp eye for accuracy.

I would like to acknowledge with gratitude the ARC and New South Wales Department of Primary Industries (NSW DPI) for the APAI scholarship (Project Number: P561073) which supported this study. I also would like to thank Professor John Mullen and Professor Garry Griffith at the NSW DPI for sharing their far-reaching knowledge of the Australian agricultural sector. This study would not have been possible without assistance from Mr Philip Knopke, Mr Walter Shafron and Mr Colin Mues at the Australian Bureau of Agricultural and Resource Economics in compiling and providing the dataset used for estimation in this study. I would like to acknowledge the help from Dr Lisa Lines from Elite Editing & Tutoring for her professional help in proofreading this thesis. I also wish to extend my thanks to Professor Brett Inder, Ms Lizbeth Moon, Ms Philippa Geurens and Ms Elke Blaedel for their administrative supports throughout the time I was at the Department of Econometrics and Business Statistics at Monash University.

I am deeply indebted to my fiancé, Jolyon, and my daughter, Linh, for their patience, understanding and support throughout this study. There have been many nights Linh has acceptingly gone to bed without a bed-time story. I am grateful for having Mr Rhodri Ellis-Jones and Mrs Iva Ellis-Jones and their daughter Nika as part of my life in Australia.

My utmost gratitude goes to my parents for their sacrifice for their children and their unconditional acceptance of who I am.

List of Abbreviations

AAGIS	Australian Agricultural and Grazing Industries Survey
ABARE	Australian Bureau of Agricultural and Resource Economics
BoM	Bureau of Meteorology
CES	Constant elasticity of substitution
CRESH	Constant ratio of elasticities of substitution homothetic
CRETH	Constant ratio of elasticities of transformation homothetic
GDP	Gross domestic product
FIML	Full information maximum likelihood
Translog	Transcendental logarithmic

Abstract

The first key objective of this thesis is to estimate a set of econometric models of Australian broadacre agricultural production by applying the duality theory in production economics. Models of Australian broadacre agriculture are estimated using alternative formulations of econometric modelling of producer decision-making, in which cost minimisation, revenue maximisation and profit maximisation are assumed. A unique, large quasi-micro pooled cross-sectional farm dataset drawn from the Australian Agricultural and Grazing Industries Survey over a sixteen year period, from 1990 to 2005, is used for model estimation. Key policy-relevant outcomes of this investigation are estimates of price elasticities and elasticities of transformation and substitution between broadacre inputs and outputs in Australia.

Multi-product dual cost, revenue and profit functions are specified for Australian broadacre agriculture. The multi-product functions are specified to accommodate the prevalent multi-enterprise operation on Australian broadacre farms. The restricted versions of the dual functions are chosen to account for quasi-fixity and the lumpiness of some capital used in broadacre production. The translog and normalised quadratic, the two most popular flexible functional forms in empirical duality applications, are used to specify the dual functions. In addition, impacts of climatic conditions, production focuses, production scales and rainfall on production are also allowed for in the estimated models.

Systems of demand and/or supply equations are derived from the specified dual cost, revenue and profit functions and estimated using the quasi-micro dataset available. The estimated systems obtained have reasonable statistical goodness-of-fit with high percentage of statistically significant system coefficients. Price variables are also found to significantly influence input demand and output supply. Importantly, using the normalised quadratic form, the estimated system of input demand derived from the cost function and the estimated system of output supply derived from revenue functions satisfy the theoretical curvature conditions implied by rational economic behaviour. The estimated supply and demand system derived from the profit function violates the curvature condition but the violation is not severe. Input demand and output supply in broadacre farming are found to be fairly inelastic with respect to price changes in the short run. However, demand for fertilisers and crop and pasture chemicals are found to be sensitive to changes in their own prices and in general production costs.

The second key objective of this thesis is to investigate three significant issues concerning the application of the duality theory in empirical agricultural production research using data from Australian broadacre agriculture. First, the results obtained from the dual cost, revenue and profit functions are contrasted according to goodness-of-fit, the satisfaction of theoretical regularity conditions and the sensibility of the generated elasticity estimates. The estimation result obtained under the cost minimisation assumption conforms more to economic theory than results obtained under revenue maximisation or profit maximisation. Second, the estimation results are more in line with expectations based on prior economic reasoning when the normalised quadratic functional form is used than when the translog is used. Third, results using data of Australian broadacre farming at two aggregate levels indicate that data aggregation across firms can have a significant impact on research findings, depending on the assumption made regarding the economic behaviour of producers.

Chapter 1

Introduction

1.1 Motivation

Agricultural production makes significant direct and indirect contributions to the overall Australian economy. This sector has generally accounted for around three per cent of the gross domestic product (GDP) over the last few decades (Australian Bureau of Agricultural and Resource Economics [ABARE] 2008), except in recent years, which had severe drought. In financial year 2008, the total value of farm production was $41.2 billion despite the prevalence of exceptionally dry conditions during this period. The importance of farm production is much greater in terms of its contribution to the national export income per annum. Agricultural commodities accounted for around 20 per cent of the total exports in the 1990s and the early years of the new century, prior to the drought, which reduced the sector's importance as an export earner (ABARE 2008). Agricultural production remained an important source of export income even during periods of booming mining exports and unfavourable weather conditions such as those that existed in 2007 and 2008, when export shares fell significantly to 12.9 per cent and 11.7 per cent respectively (ABARE 2008). Farming production also plays a significant role in the economy in terms of employment opportunities. In 2008, this sector directly employed around 303,000 people or 2.9 per cent of the nation's labour force (ABARE 2008). Additionally, this sector indirectly contributes to the economy by providing major inputs and employment opportunities for other important sectors such as food manufacturing and food services.

Government intervention policies have been introduced in response to market, structural, institutional, climatic and other external changes to ensure the long-term vitality of the Australian agricultural sector and the overall economy. In order to implement effective policies, accurate understanding of agricultural broadacre production technology and the economic behaviour of agricultural broadacre producers is crucial. Up-to-date information on production substitution possibilities and the price responsiveness of inputs and outputs in broadacre agriculture is needed for the evaluation and forecast of policy outcomes. For instance, popular equilibrium displacement models for economic evaluation of research and development policies such as Zhao, Griffiths Griffith and Mullen (2000) use price elasticity estimates as modelling inputs to assess changes in the economic welfare of market participants.

Information on substitution possibilities and price responsiveness is particularly essential for Australian broadacre agriculture, which is dominated by multi-enterprise farming practice. Broadacre farmers usually use a mixture of crop and livestock production to manage financial and natural risks and to better utilise natural and human resources. This practice of running multi-enterprise farming activities ensures the ability to switch between products within a reasonably short period to take advantage of weather and market conditions. Farmers' behaviour is therefore complex and diverse, depending on factors such as weather conditions, physical operation environments and risk management strategies. At the same time, the adjustment behaviour of farmers in input utilisation and output supply has implications for aggregate agricultural production, export income, rural welfare and overall economic performance as well as the effectiveness and inter-industry distributional effects of policy changes.

The importance of quantifying key technical relationships of broadacre production technology and broadacre farmers' behaviour strongly necessitates the estimation of an econometric model of the sector. Econometric studies of Australian broadacre production to date are generally dispersed and outdated. Griffith, I'Anson , Hill and

Vere (2001) find that published econometric studies of Australian agriculture conducted from the 1960s through to the 1990s are rare. They also find a lack of a common consensus regarding the estimates of substitution and transformation elasticities between broadacre demands and supplies, which are the key inputs in economic evaluation and forecasting.

Moreover, the elasticity estimates found in previous econometric studies on Australian agriculture are fairly outdated and may not be relevant to the current state of farming production technology. The earliest econometric studies in which data from across Australia are used for estimation are models proposed by Powell and Gruen (1967, 1968). In these studies, multi-product farming practice is recognised as typical in Australian broadacre agricultural production and is accommodated. The latest Australia-wide multi-product study is the ABARE study by Kokic, Beare, Topp and Tulpule (1993), in which the data used for estimation are cross-sectional, farm-level data from 1981 to 1991. Given the significant structural, institutional and market changes, such as the collapse of the Wool Reserve Price Scheme, that took place after this period, the supply estimates calculated from this model can be considered outdated in the current context of production. More recent multi-product studies of Australian broadacre agriculture such as Coelli (1996) and Ahammad and Islam (2004) estimate broadacre farming in Western Australia using data from 1978 to 1997. Considering the diverse climatic and physical conditions affecting Australian broadacre production, these studies' findings are unlikely to be applicable to broadacre production in other states or in Australia as a whole. An econometric model of Australian broadacre production using data after this period has not been estimated. This lack of up-to-date information on broadacre farming across Australia emphasises the need for estimating an econometric model of the Australian broadacre agriculture using more current data.

1.2 Issues in Estimating an Australian Broadacre Agricultural Production Model

Econometric studies on Australian broadacre production in recent decades have applied duality theory in model estimation. This is also the case in the international literature on production economics. In duality theory-based econometric studies, assumptions are made about the economic behaviour of producers under technological constraints, and a function that represents the assumed economic behaviour is specified using economic information (i.e. cost, revenue, profit and prices). Information about price responses and substitution/transformation relationships of input demand and output supply is then drawn for economic and policy interpretations.

The existence of alternative economic optimisation assumptions has led to the use of different formulations in applying the duality approach to modelling production decision-making. In empirical applications of this approach, producers are commonly assumed to minimise production cost, maximise production revenue or maximise production profit. The researcher chooses one from among these assumptions to specify the dual objective function (i.e. cost, revenue or profit function) to form the basis of model estimation using empirical data. In choosing optimising behaviour for a particular study, no general rules exist. The choice of which optimisation behaviour is applicable to the producers is often *ad hoc*, depending on data available, main research purposes and/or prior knowledge of the technology under study. For example, among Australian studies, Ahammad and Islam (2004), Coelli (1996), Kokic *et al.* (1993), Fisher and Wall (1990) and McKay, Lawrence and Vlastuin (1983) assume that broadacre farmers maximise their production profit, while Mullen and Cox (1996) and McKay, Lawrence and Vlastuin (1980) assume they minimise their production cost. These two assumptions of profit maximisation and cost minimisation are critically different in the sense that relative to the former, the latter considers a conditional optimisation problem in which outputs are assumed to be exogenous. Such assumptions about farmers' behaviour, therefore, have implications for which explanatory variables appear in the econometric models. The measures of the technical and economic relationships in these different models have different interpretations and policy implications.

Another significant issue with regard to empirical applications of the duality approach in production economics is the choice of a flexible functional form. Once the choice of the optimisation assumption has been made, researchers must also choose a functional form to specify the dual objective function representing the producers' optimisation behaviour before they can carry out the econometric estimation. This step is necessary because the types of relationships between variables in the dual function have to be defined, or parameterised, before the model can be estimated using empirical data. A wide array of functional forms have been introduced and applied in empirical duality studies. In previously estimated models of Australian broadacre agriculture, Ahammad and Islam (2004), Fisher and Wall (1990) and McKay *et al.* (1983) employ the normalised quadratic functional form, while Coelli (1996) uses the generalised McFadden form and Mullen and Cox (1996), the transcendental logarithmic (translog) form. However, different functional forms produce different estimates of economic and technological relationships. They also differ in terms of the ease with which parametric restrictions can be imposed and tests for theoretical conditions and technological structure can be performed. There has been no general agreement on the superiority of any particular functional form. Considerable efforts have been directed to analytically, empirically and experimentally evaluate the suitability of various functional forms, but the findings have been mixed. Given the lack of a consensus with regard to the relative performance of available functional forms, the choice of functional form remains an issue in applying the duality approach.

The third important issue in estimating a duality theory-based model of agriculture production is that estimated models frequently fail to meet theoretical regularity conditions required by economic theory. This has been observed in Australian studies by Ahammad and Islam (2004), Coelli (1996) and McKay *et al.* (1983). This is a serious issue since such failure implies that the econometric estimates of the underlying production function or the specified dual cost, revenue or profit function are not consistent with economic theory. This appears to have dampened researchers' enthusiasm in adopting the duality approach (Kohli 1993; Barnett and Hahm 1994; Fox and Kivanda 1994; Shumway 1995; Terrell 1996; Lim and Shumway 1997). More discouragingly, finding what has caused the failure is rather challenging. Possible causes include the use of geographically aggregated data, the ignoring of time-series features of the data and the inadequacy of flexible functional forms. There has also been an expectation that the theoretical regularity conditions are not necessarily to be satisfied at a macro level, even when they are at the micro level (Shumway 1995). Moreover, it has been argued that the tests for these regularity conditions are not 'statistical tests', and the violation of these conditions in a particular study may be not significant (Shumway 1995). Therefore, it appears that the failure of the regularity conditions is expectable in the body of Australian and international duality literature, since in many studies, this outcome is merely stated without an accompanying explanation or caution.

Finally, empirical applications of the duality approach, especially in agricultural production, face the issue of the unavailability of micro-level or farm-level data. Since agricultural production consists of numerous geographically dispersed production units of small production scales, micro-level data of sufficiently large sample size have rarely been available because of the financial costs incurred in data collection and confidentiality concerns. This lack of micro-level data creates several significant empirical issues: The first is the consistency between theory and empirical application. Since micro-level data are not available, previous duality theory-based studies of Australian and international agricultural production have typically used time-series data at the aggregate state, regional or national level for model estimation. This practice requires the assumption that the state, region or nation average is representative of individual farms because the underlying theory is applicable at the microeconomic level. However, the conditions required for legitimate use of geographically aggregated data to study farm-level production technology are too stringent to be realistic (Chambers 1988; Liu and Shumway 2004; Shumway and Davis 2001). The empirical findings in aggregate models, therefore, may not reflect farmers' true economic behaviour and the technological constraints they face.

Another issue related to the unavailability of farm-level data is the limited study coverage in many models of Australian and overseas agricultural production, which can potentially lead to misleading findings. The scope of these studies is often narrowed to a single geographical area, industry or sector, attempting to ensure that the average aggregate data used for model estimation are representative of individual farms, which are the primary decision makers. The findings of these studies are generally not applicable to other areas, industries, sectors, or an entire state or nation because of the diversity in production conditions. Moreover, with such a partial coverage, significant interrelationships between different production areas, industries and sectors are ignored. Given that Australian broadacre agriculture is dominated by multi-enterprise farms, this can have distorting impacts on research findings and policy interpretations.

The use of aggregated data instead of micro-level data is also undesirable in several other modelling respects. In econometric models using aggregate data, prices and demands/supplies can be simultaneously determined, and thus prices are not determined outside the models, violating standard assumptions required for robust estimation results. Further, at aggregate levels, prices are likely to move together, causing a multicollinearity problem in estimation. Moreover, even though the aggregate data used in most studies are time-series data, the time-series features of the data are typically ignored. This can result in biased estimation results and erroneous economic and technological interpretations (Lim and Shumway 1997). Finally, because the sample size of aggregate time-series data are usually small, being restricted by the time dimension, the time-series data used in many previous duality theory-based studies are relatively less given the number of parameters to be estimated. This is the possible cause of the small percentage of significant coefficients found in several studies, such as Villezca-Becerra and Shumway (1992), Fulginiti and Perrin (1990) and Akridge and Hertel (1986), undermining the reliability of the econometric estimation results and the policy-relevant measures subsequently generated.

1.3 Objectives of the Thesis

The purpose of this thesis is to estimate a set of econometric models for Australian broadacre agricultural production by using a unique nationally representative pooled cross-sectional quasi-micro level dataset over the period 1990 to 2005. This dataset is provided by ABARE (2007) and is compiled from the annual Australian Agricultural and Grazing Industries Survey (AAGIS). Owing to confidentiality restrictions, farm-level data collected in this survey were not available for studies on Australian broadacre agriculture conducted outside government agencies. In this study, the detailed farm-level data collected were formatted in a way that preserves the micro-level nature of the data as much as possible while maximising the number of observations available for estimation. In the dataset employed in this thesis, farms located in the same production region among 32 regions across Australia, engaged in similar production activities and having similar production scale are grouped into a cell, and the averages of all variables for each cell are examined. The outcome of this data manipulation is that for each year, up to 30 observations are available for each state, as opposed to just one. A final 'quasi-micro' dataset with 1559 observations is obtained for model estimation. The specific aims of the thesis are as follows:

1. The primary aim of this study is to estimate econometric models of Australian broadacre production under three alternative formulations commonly assumed for farmers' decision-making, using a unique, up-to-date, large, pooled cross-sectional, quasi-micro dataset from a nationally representative farm survey. Models derived under these alternative assumptions, cost minimisation, revenue maximisation and profit maximisation, are estimated, whereby key technical and economic relationships and parameters can be estimated. The dual functions representing these three economic optimisation behaviours are

specified for multi-product production technologies to account for the prevalence of multi-enterprise operations in Australian broadacre production. Restricted versions of these dual functions, i.e. the variable cost, revenue and profit functions, are employed to account for the quasi-fixed nature of some production factors in the short-run and the annual basis of the observed data. Moreover, these dual functions are specified using the translog and normalised quadratic forms, the two most popular flexible functional forms in duality applications. Own- and cross-price elasticities of input demands and output supplies as well as the Allen partial and Morishima elasticities of substitution and transformation between inputs and outputs are calculated.

2. The secondary aim of this study is to investigate three significant methodological and empirical issues present in the application of duality theory to estimate agricultural production models via a comprehensive empirical example of the Australian broadacre agricultural industry:
    a. The first issue to be investigated is the assumption made about farmers' economic behaviour. Three common alternative specifications about the farmers' behaviour are assumed: profit maximisation, cost minimisation for given output quantities and revenue maximisation for given input quantities. Three econometric models based on these assumptions are estimated using the same dataset. Goodness-of-fit measures, satisfaction of economic regularity conditions and various estimated economic summary measures of interest obtained from these models are reported and contrasted.
    b. The second issue investigated in this thesis is the choice of functional form used to specify the dual objective function. The estimation results and useful economic measures, such as elasticities, from the two most popular flexible functional forms in duality theory-based econometric studies of production technology, namely, translog and normalised quadratic, are compared and assessed. Given the unique quasi-micro nature of the dataset as opposed to the aggregate time-series data

typically used in previous studies, the comparison adds useful information to empirical literature on duality theory.

c. The third issue examined is the effect of data aggregation across farms on the estimates of key technical and economic parameters, such as price elasticities and elasticities of input substitution and output transformation. This is achieved by further aggregating the AAGIS quasi-micro dataset across farm sizes and then using the resulting aggregated data to estimate econometric models under the assumptions of cost minimisation, revenue maximisation and profit maximisation for Australian broadacre farmers. The estimation results obtained in these aggregate models are compared with those obtained in their corresponding quasi-micro models. This comparison helps identify the effects of aggregation data across farms on estimation results and on estimates of technical and economic relationships between broadacre inputs and outputs. The findings obtained will add to empirical evidence of aggregation problems that have largely been ignored in empirical literature.

1.4 Organisation of the Thesis

The remainder of the thesis is organised into nine chapters. Chapter 2 reviews the empirical applications of duality theory in Australian agricultural production and worldwide production economics research. The chapter will focus on the alternative formulations of production decision-making typically used in empirical analyses, the choice of functional forms for the specification of the dual function, the frequent finding of the estimated models not satisfying the theoretical regularity conditions and the effects of using cross-farm aggregate data on research results.

Chapter 3 presents the economic framework of the dual approach for producer decisions. The dual cost, revenue and profit functions and their associated regularity conditions are discussed in detail. This chapter covers the definitions and formulas of price elasticities of input demands and output supplies as well as Allen partial and Morishima elasticities of substitution and transformation under each dual specification.

Chapter 4 provides background information on the Australian broadacre industry and describes the data used for model estimation in this study. It comprises a review of Australian broadacre agriculture, a detailed description of the quasi-micro data used for the estimation, detailed documentation of the specification and aggregation of a large number of broadacre inputs and outputs into a small number of aggregate input and output variables to be used in the models and a description of the aggregate inputs and outputs included in all models estimated in this study. Issues and problems associated with the data and their solutions will be discussed.

Chapter 5 reports the empirical research results of this study. In this chapter, the restricted dual translog and normalised quadratic cost functions are specified and estimated for Australian broadacre farmers. The regularity conditions, representations of the models and formulations of the price and substitution elasticities between inputs are presented for each of these two dual cost functions. This chapter also covers in detail all empirical issues and problems arising during model estimation. In particular, it provides a detailed explanation of the potential heteroskedasticity observed in this study because of the quasi-micro nature of the data and the corrective action used to deal with this issue. The estimation results of the dual cost function for the two functional forms and the generated estimates of economic and technical relationships between broadacre inputs are presented, discussed and compared. A critical discussion with practical validations of the models' findings is also included.

Chapters 6 and 7 present the specification and estimation of the restricted dual revenue and profit functions for Australian broadacre farming, respectively. These two chapters follow the same format as Chapter 5. Both the dual revenue and profit function will be specified in the translog and normalised quadratic forms along with their associated regularity conditions, derived estimation models and price and substitution/transformation elasticity formulations. Common empirical issues and problems that arise during the estimation of these two dual functions and the estimation of the dual cost function in Chapter 5 will be referred to but not discussed in detail. For each of the dual revenue and profit functions, the estimation results from the two functional forms and their implied elasticity estimates will again be compared and assessed.

Chapter 8 comprehensively discusses and contrasts the results of the three dual objective functions covered in Chapters 5, 6 and 7. In this chapter the relative statistical significance of the estimation results, the satisfaction of theoretical regularity conditions by the estimated models and the estimates of the derived elasticities are summarised and discussed. The chapter also explores whether the three estimated dual functions conform to the Le Chatelier-Samuelson principle, which applies to the relationships between cost minimising- and profit maximising-derived input demands and the relationships between revenue maximising- and profit maximising-derived output supplies. The comparable *compensated* price elasticities, input substitution elasticities and output transformation elasticities obtained from the estimated profit function are also reported and compared to corresponding net elasticities obtained from the cost and revenue functions. In addition, the estimation results obtained from these alternative model formulations will be assessed against broadacre farming practice in Australia.

Chapter 9 is devoted to the issue of using across-farm aggregated data in duality theory-based studies of agricultural production. The effects of data aggregation are studied in the Australian broadacre production context. The chapter presents results for the three dual functions using a dataset that is drawn from the same survey data but is further geographically aggregated than the quasi-micro dataset used in Chapters 5, 6 and 7. These results are then compared to their corresponding results of the quasi-micro models to draw conclusions about whether the aggregation of data across geographical areas significantly alters the research findings on production technology and economic behaviour.

Chapter 10 summarises and concludes the thesis. The chapter includes a summary of important research findings about Australian broadacre agricultural production, the contributions of this study to empirical literature of duality theory-based econometric models of agricultural production and a discussion of future research and applications. It also discusses the implications of the findings of this thesis for policy development.

Chapter 2

Literature Review

2.1 Introduction

Econometric models of Australian broadacre agriculture have been estimated since the early development of econometric principles to provide information useful to economic forecasting and policy making. These models have been continuously improved to better explain the production technologies employed. They have evolved from direct estimation of the production functions in the primal approach, to the indirect estimation of the dual economic objective functions. However, the number of duality-based econometric models estimated has been limited, mainly due to unavailability of suitable data. This lack of data leads to limited geographical and industry study scopes for most estimated models of Australian broadacre agriculture. The lack of farm-level data also leads to the use of data at an aggregate level for model estimation, despite the fact that duality theory is a microeconomic theory. Broadacre production models estimated for Australia as a whole using farm-level data are rare and outdated. There is a need to estimate a model for Australian broadacre production using more current data that are consistent with the theory to better understand this sector for policy making.

The duality theory of the firm has been developed and often applied in international studies of production economics. In applying this theory, researchers indirectly extract information about technical relationships between inputs and outputs by studying the optimisation behaviour of producers using economic information such as market prices, costs, revenues and profits. The indirect retrieval of information about production technologies from modelling optimal economic behaviour distinguishes this approach from the traditional primal approach, which retrieves technical relationships directly by estimating the production functions. The significant presence of the duality approach in empirical research of production economics is due to several advantages this approach offers, including the ease of application, the robustness of the estimation results and the ready availability of information on price-quantity adjustments in input demands and output supplies. However, a few significant issues have surfaced in empirical applications of the duality approach. Some of these issues have remained largely unchallenged and require further investigation to improve future empirical applications.

This chapter reviews the existing literature on econometric modelling of Australian broadacre agriculture and on international empirical duality-based studies of production technologies. Section 2.2 is devoted to an examination of previous econometric studies applied to Australian agricultural production. Section 2.3 follows with a summary of the development and empirical applications of duality theory in empirical production research worldwide. Section 2.4 then focuses on the choice of the dual economic objective function, one of the major issues encountered in empirical applications of duality theory. Section 2.5 continues with a review of the choice of functional form, another significant empirical issue in applications. Section 2.6 discusses issues related to imposing or testing the economic regularity conditions in international empirical literature. Section 2.7 then follows with a discussion of empirical data-related obstacles and issues. The concluding section summarises the chapter, with remarks on significant gaps and issues in applications of the duality approach to Australian broadacre agriculture and international production economics to which this study aims to contribute.

2.2 Econometric Models of Australian Broadacre Agriculture

Researchers have long employed econometric techniques to estimate models of Australian broadacre agricultural production due to this sector's important role in the overall economy in the past and the need to formulate appropriate economic policies. The estimated econometric models of broadacre production have been continuously conceptually improved to account for many particular characteristics, such as the multi-enterprise operation practice and competitive market conditions of broadacre production. However, there remain several limitations that require further research to ensure an accurate understanding of the sector for economic assessment and policy formation: the restrictive structural assumptions in early direct estimates of the production/transformation functions, the limited number of more recent duality-based studies that have been conducted, the possibly biased estimation results in some of these duality-based studies, the lack of consistency in research findings from these studies, the limited industry and regional focuses of these studies, the lack of empirical data at a micro-level and the outdated data used in model estimation.

Early econometric studies of Australian broadacre agriculture are non-duality applications, directly estimating the production functions representing the production technologies employed by broadacre farmers. The estimation of these models can be straightforward, wherein the production functions are specified and estimated as functions of output prices, as in Fisher and Munro (1983), Kloot and Anderson (1977) and Duncan (1972). Some models can be complex systems of supply functions, such as Adams (1987), Dewbre, Shaw, Corra and Harris (1985), Vincent, Dixon and Powell (1980), Powell and Gruen (1968) and Powell and Gruen (1967). These models have extensively contributed to economic forecasting and policy initiation with the well-known ORANI model (Adams 1987 and Vincent, Dixon and Powell 1980) and EMABA model (Dewbre *et al.* 1985). However, these models are limited because rigid assumptions about technological structure are needed to enable estimation. For instance, Powell and Gruen (1968) assumed that elasticities of transformation between outputs are constant. Meanwhile, Dewbre *et al.* (1985) and Vincent *et al.* (1980) assumed that Australian agricultural production has a constant ratio of elasticities of substitution/transformation, homothetic (CRESH/CRETH) structure.

The modelling of Australian agriculture employing econometric methods has appeared to favour the duality approach in recent decades. In applying this approach, researchers use economic information, which includes cost, revenue, profit, and input and output prices, to indirectly extract information on production technology. This approach offers several theoretical and practical advantages over the direct estimation of the production function. For instance, this approach allows researchers to circumvent rigid structural assumptions about production technology as required in earlier non-duality models. The functional form chosen for model specification, such as linear or quadratic, restricts the economic relationships but does not necessarily impose *a priori* structural restrictions on the underlying physical relationships between inputs and outputs (Appelbaum 1979). Explanatory variables in duality-based econometric models also behave better than those in primal models that directly estimate the production function. This is because in the duality-based models these variables are prices, which are exogenously set in competitive markets. In contrast, in the primal models they are input quantities, which can be controlled by producers and can be simultaneously determined with output quantities. Moreover, measures of technical relationships, especially elasticities of substitution or transformation, can be obtained with considerable computational ease in duality applications in comparison to the primal models, especially in the case of complex technologies involving multiple inputs and outputs (Binswanger 1974a). Finally, measures reflecting economic behaviour, the primary interest of production economists, can be simply calculated using the parameter estimates of the dual objective functions and the corresponding supplies/demands. The duality approach, together with flexible functional forms, permits an arbitrary set of price elasticities to be obtained at any data point (Terrell 1996).

The first applications of the duality approach to Australian agriculture were published in the early 1980s. Significant studies using this approach are McKay *et al.* (1980), McKay, Lawrence and Vlastuin (1982) and McKay *et al.* (1983). In their 1980 study, these authors specified and estimated a dual single-output cost function for the Australian sheep industry. The system of cost-minimising input demands derived from this dual function was estimated using 1953–1977 average farm data from the Australian Sheep Industry Survey conducted by the Bureau of Agricultural Economics. This survey covered farm properties with 200 sheep or more. In their 1982 study, the authors estimated dual variable multi-output profit functions for sheep farms in the Wheat-Sheep zone. They extended the scope of their analysis to cover all sheep farms in their 1983 study using the same survey data. Interestingly, how these authors considered the time frame of the decision-making process varies between their first and two later studies. In their first study, five inputs, namely labour, land, livestock, capital and materials and services, were treated equally as variable inputs in their cost function. However, in their later two studies, land, livestock and capital inputs are treated as fixed inputs while the other two inputs are treated as variable inputs. In doing so, their estimated model is a long-run model in their first study but a short-run model in their later studies.

The conventional definition of the sheep industry may have resulted in estimation biases in some past studies of Australian broadacre agriculture. The threshold of 200 sheep used in studies by McKay *et al.* (1980, 1982, 1983), and later in Fisher and Wall (1990), Coelli (1996) and Mullen and Cox (1996) to define the Australian sheep industry, is potentially spurious. This is because the use of this threshold does not necessarily imply that farms considered in these studies only focus on sheep grazing activities. These farms could have any mix of crop and livestock enterprises and could grow cereals or graze beef as their major enterprises while keeping more than 200 sheep. Examples of 'sheep farms' that have other farming activities accounting for a significant part of their operations are those in the Wheat-Sheep zone in Western Australia (Coelli (1996), Footnote 3). This means that these studies are not only for farms with a focus on sheep grazing as the 'sheep industry' may have initially implied. Farms focusing on cereal cropping or cattle grazing were included in these studies if they had at least 200 sheep, but excluded otherwise. This exclusion means that the data samples in these studies are not representative of either the intended 'sheep industry' or a more broadly defined industry.

Duality-based models of Australian broadacre agriculture after studies by McKay, Lawrence and Vlastuin have been few. The significant among these studies include Fisher and Wall (1990), Kokic *et al.* (1993), O'Donnell and Woodland (1995), Coelli (1996), Mullen and Cox (1996), Griffiths, O'Donnell and Cruz (2000), Ahammad and Islam (2004) and Agbola and Harrison (2005). The research outcomes of these studies, especially their estimates of policy-relevant measures, can deviate significantly from each other. For example, the medium-run own-price elasticity of wheat was estimated to be 2.67 for the Pastoral zone and 0.62 for the Wheat-Sheep zone in Fisher and Wall (1990), using farm data from 1967/68 to 1980/81. These estimates are remarkably higher than the national average estimate of 0.23 in Kokic *et al.* (1993) for the 1981–1991 period. The own-price estimates for beef cattle supply in these two studies also differ considerably from each other. This estimate is 0.43 for the Pastoral zone, 0.11 for the Wheat-Sheep zone and 0.16 for the High Rainfall zone in Fisher and Wall (1990) compared to the corresponding estimates of 0.05, 0.15 and 0.07 in Kokic *et al.* (1993).

The significant divergence in the research findings of duality applications in Australian broadacre agriculture is due to a number of factors. Importantly, these studies differ from each other in terms of their industry and geographical focuses, assumptions made about the economic behaviour of broadacre farmers, functional forms used in specifying the estimated dual function and the time periods studied. For instance, Griffiths *et al.* (2000) and O'Donnell and Woodland (1995) focused on the sheep industry alone. The definition of sheep grazing farms in these two studies is more stringent than in McKay *et al.* (1980, 1982, 1983), Fisher and Wall (1990), Coelli (1996) and Mullen and Cox (1996), in that the receipts from sheep grazing in such a farm must be greater than 80 per cent of its total production revenue. The industry focus is even more rigid in Griffiths *et al.* (2000) where only merino woolgrowers, as opposed to non-merino woolgrowers and prime lamb producers, were studied. Although such a definition ensures a well-defined study scope, the findings from these studies are unlikely to be relevant to any other industry or Australia as a whole.

The scope in previous duality-based studies of Australian broadacre agriculture is not only limited in terms of industry focus but also by geographical coverage. Work done by Coelli (1996) and Ahammad and Islam (2004) focused on Western Australia. The study scope in Coelli (1996) was narrowed to the Wheat-Sheep zone of the state only and the inclusion of farms having at least 200 sheep. This narrow focus helps to reduce biases in this study when compared to earlier studies by McKay *et al.* (1980, 1982, 1983), since sheep grazing is prevalent in the Wheat-Sheep zone in Western Australia. However, the findings of this study are not applicable elsewhere because the production systems employed in this geographical area differ significantly to those in the same zone but different states (as a broadacre zone spreads over several states), as well as to those in other production zones in Australia. Meanwhile, unlike Coelli (1996), the model of Western Australia's broadacre agriculture in Ahammad and Islam (2004) included farms in all three broadacre zones in the state, namely the Wheat-Sheep, High Rainfall and Pastoral zones. As acknowledged by the authors, the output and cost structures of broadacre farming in Western Australia are very different to those in the rest of Australia. This implies that research findings in these two studies are not likely to be relevant for economic assessment at a national level.

Beside the study scope, previous duality applications to Australian broadacre production differ from each other in terms of the dual objective functions specified and the functional forms used to specify these dual functions. Cost minimisation and profit maximisation have been assumed for Australian broadacre farmers. The dual cost function under the cost minimisation assumption was specified in McKay *et al.* (1980), O'Donnell and Woodland (1995), Mullen and Cox (1996) and Griffiths *et al.* (2000), while the dual profit function under the profit maximisation assumption was specified in McKay *et al.* (1982, 1983), Fisher and Wall (1990), Coelli (1996) and Ahammad and Islam (2004). Regarding the functional form, translog was employed in all applications among the above that estimated the dual cost function. This functional form was also used to specify the dual profit function in McKay *et al.* (1982, 1983). Meanwhile, Fisher and Wall (1990) and Ahammad and Islam (2004) used the normalised quadratic form and Coelli (1996) used the generalised McFadden form to specify their profit functions.

A common trend shared in previous duality-based studies of Australian broadacre agriculture is the use of geographically aggregated data for model estimation. The state or national average farm data are most often used in these studies. Similar to McKay *et al.* (1980, 1983), national average time-series farm data over the 1953–1994 period were used in Mullen and Cox (1996). Meanwhile, national pooled time-series cross-sectional data observed annually over the 1953–1976 period were used in O'Donnell and Woodland's (1995) study of lamb- and wool-producing sectors. Compared to datasets used for estimation in other studies this dataset was largest, with 584 observations. The pooled time-series cross-sectional datasets were also used for estimation in Fisher and Wall (1990) and Ahammad and Islam (2004) but they are much smaller in sample size. In Fisher and Wall (1990), farms across Australia were grouped by broadacre zone and by state to form observation units whose annual average data over the 1968–1981 period was used for model estimation. In Ahammad and Islam (2004), farms in Western Australia were grouped by broadacre zone and the annual average data observed for the three zones from 1978 to 1997 were pooled together to form a data sample.

There is a departure from the limited coverage and the use of time-series aggregate data common in past duality-based models of Australian broadacre agriculture. This exception is the ABARE study conducted by Kokic *et al.* (1993). In this study, the authors modelled Australian broadacre agriculture using national time-series crosssection data at the farm-level for the period from 1981 to 1991. The focus of their study was to obtain estimates of price and substitution elasticities for major broadacre products (namely wool, lamb, mutton, beef and wheat) for policy forecast and assessment. Broadacre farmers were assumed here to maximise their net cash income (as a form of profit), subject to a constraint of fixed operating areas. By assuming the cost function as a sum of the exponentials of output quantities and approximating the fixed operating area function by a linear Taylor series, the authors arrived at a fairly simple expression of the production costs and profits, as linearly related to the output quantities, to be estimated using their national farm-level data. Considering the fact that Australian broadacre production has undergone considerable structural change due to the dismantlement of the price support scheme for wool in 1991, the research outcomes of this study would generally not reflect the current production context.

The Australian duality-based models cited above applied the static model of the firm in traditional duality theory, in which the aspect of temporal dynamics in production decisions is ignored. In contrast to these studies, Agbola and Harrison (2005) applied the dynamic model of the firm and formed an optimal inter-temporal profit maximisation problem for broadacre farmers in pastoral areas. This study followed important theoretical work by Lucas (1967), Gould (1968), Treadway (1969), McLaren and Cooper (1980) and Epstein (1981), as well as the empirical applications of Taylor and Monson (1985), Vasavada and Chambers (1986) and Fernandez-Cornejo, Gempesaw II, Elterich and Stefanou (1992). Although the dynamic optimisation framework employed in Agbola and Harrison (2005)'s study is considered more realistic than the static optimisation framework, it has not gained popularity in agricultural production analysis, probably due to its highly restrictive and intractable structures (Shumway 1995).

In summary, the econometric approach to production research has been long applied to Australian broadacre agriculture. However, the constructed econometric models have been limited in number and restricted in several aspects. Early econometric models applied the primal approach to directly estimate the production or transformation functions, often containing stringent assumptions about the structure of production technologies. More recent models have applied the duality approach to indirectly obtain information about underlying production technologies by estimating the dual cost and profit functions using economic information. However, these later models were estimated using out-dated data and suffer some significant limitations that restrict their usefulness in economic evaluation and policy making. Estimation results in some of these studies may have suffered biases due to the selection criteria used to define their study scopes. These models' results also exhibit significant inconsistency regarding their generated information about economic responses of output supply and input demand. Moreover, study coverage in these models has often been limited to an industry or geographical area, limiting their usefulness for economic evaluation at a national level. Finally, the duality studies conducted have used geographically aggregated data for model estimation instead of farm-level data as implied by the theory.

2.3 Applications of the Duality Theory of the Firm in International Literature

The neoclassical duality theory of the firm has a long development history. The duality between the production function and the economic objective functions representing producers' economic optimisation behaviour was first recognised by Hotelling (1932) and Samuelson (1947). However, firm acceptance of the theory only took place after theoretical proofs were published by Shephard (1953), Uzawa (1962), Uzawa (1964), Diewert (1971) and Diewert (1974). Shephard (1953) employed distance function properties to prove the one-to-one relationship between the cost and production functions. Later, Uzawa (1962) demonstrated the duality between the production possibilities set and the cost function in set notations. Diewert (1971) then provided proofs of duality between the production function, the production possibilities set and the cost function. The theory was extended by Diewert (1974) when the duality between the factor requirement function and the revenue function was theoretically established. This work followed the unpublished original work of McFadden in 1966, which was later published in significant book volumes on the duality approach edited by Fuss and McFadden in 1978 (McFadden 1978). Other publications contributing significantly to the establishment and application of the duality theory in empirical research include Christensen, Jorgenson and Lau (1973), Fuss and McFadden (1978), Diewert and Wales (1987), Shumway (1983) and Chambers (1988). Coming from a different path, Theil (1977) and Laitinen and Theil (1978) extended the theory of consumer demand to production decision-making.

The neoclassical duality theory became widely accepted in production economics by the late 1970s and has been often applied since then, especially in agricultural production analysis. In essence, this theory is an application of the Euler-Legendre transformation for solving differential equations in mathematics to economic problems where it is known as the Shephard, Samuelson-McFadden (Chambers 1988) and Hotelling lemma (Paris, 1989 and Jorgenson and Lau, 1974a). Applying this transformation, an economic problem under technological constraints can be solved using at least two equivalent approaches. When producers are assumed to minimise production costs, their cost-minimising behaviour can be represented by a cost function relating production cost to input prices and output quantities. If this cost function is differentiable, Shephard's lemma states that its first partial derivatives with respect to input prices are equal to the cost-minimising input demands. Similarly, when the producers are assumed to maximise their production revenues, a revenue function can be specified as a function of output prices and input quantities to represent their optimisation behaviour. In this case, the Samuelson-McFadden lemma states that the first partial derivatives of this dual revenue function with respect to output prices are equal to the revenue-maximising output supplies. Finally, when profit maximisation behaviour is assumed, according to Hotelling’s lemma, the first partial derivatives of the profit function with respect to input and output prices are equal to the profit-maximising input demands and output supplies.

Early empirical duality applications often specified the cost function to represent producers' optimisation behaviour. This is likely because the duality was first established between the production and cost functions. The study on returns to scale of electricity supply by Nerlove (1963) was probably the first significant published application of the duality theory. The author specified and estimated a logarithmically transformed generalised Cobb-Douglas cost function with a single production output and three inputs. A cross-sectional dataset consisting of 155 observations was used for model estimation. It is interesting that positive serial correlations were detected by the authors but no correction action was taken. In contemporary econometric practice, this problem is termed heteroskedasticity and needs to be corrected for in order to obtain reliable test statistics of the model. The modelling framework in this study was later re-applied in Alcantara and Prato (1973), a study of sugarcane production in the state of Sao Paulo, Brazil.

The adoption of the duality approach was promoted by the introduction of flexible functional forms in the 1970s. The practical convenience offered by the duality approach coupled with the use of flexible functional forms was perhaps first demonstrated in Binswanger (1974a). It was verified in this study that when the translog functional form, introduced by Christensen *et al.* (1973), is used to specify the cost function, the price and substitution elasticities of input demands can be directly related to the derived input cost shares and the parameters of the cost function. Besides the convenience in obtaining substitution elasticities, various possible production technologies, such as those having non-homothetic, non-constant returns to scale or non-constant elasticities of substitution structures, can be accommodated using this functional form. This advantage over the traditional Cobb-Douglas or constant elasticity of substitution (CES) functional forms, where the elasticities of substitution are constrained to be constant over the entire population, also comes with little computational effort since the derived input demands (i.e. the first partial derivatives of the translog cost function with respect to input prices) are linearly related to the logarithmic prices. The specification of the translog cost function since then has been adopted in numerous duality applications such as Berndt and Wood (1975), Christensen and Greene (1976), Halvorsen (1977), Kohli (1978), Kako (1978), Brown, Caves and Christensen (1979), McKay *et al.* (1980), Ball and Chambers (1982) and Ray (1982).

Besides the translog, several other flexible functional forms were introduced and used in empirical applications of the duality theory. The 'flexible' notation was used to refer to this class of functional forms due to their capability to allow input-output relationships to be estimated without many *a priori* restrictions. This capacity places flexible functional forms greatly advantageous to the conventional Cobb-Douglas and CES forms. The generalised Leontief form was introduced by Diewert (1971) and the normalised quadratic form was introduced by Lau and Yotopoulos (1972) (but attributed by Lau (1978b) to Jorgenson and Lau (1974b) and Jorgenson and Lau (1974a)). Denny (1974) developed the generalised quadratic form, a generalisation of the generalised Leontief, CES and Cobb-Douglas. Later, generalised Box-Cox form, which includes translog, square-root quadratic and generalised Leontief forms as special cases, was suggested by Khaled (1978). Although there have been studies of cost functions that used these functional forms, such as the generalised Leontief in Denny, May and Pinto (1978) and Lopez (1980), generalised Box-Cox in Berndt and Khaled (1979) and Appelbaum (1979), and the normalised quadratic in Featherstone and Moss (1994), the translog has remained the most popular form in the estimation of dual cost functions; recent examples including Truett and Truett (2006), Ollinger, MacDonald and Madison (2005), Kwack and Sun (2005), MacDonald and Ollinger (2005), Macdonald and Ollinger (2000) and Gagne and Nappi (2000).

Following the extension of duality to the revenue and profit functions in McFadden (1978) and Diewert (1971, 1974), researchers appear to have became more comfortable with the dual profit function. This is probably because the profit maximisation assumption of economic behaviour is more readily accepted than the cost minimisation assumption (Lee and Chambers 1986). When producers are assumed to maximise their profits, they are assumed to adjust input levels as well as output levels in response to any price change. In contrast, when they are assumed to minimise their production costs, they are assumed to take levels of outputs as given. This implies that if an input price changes, producers only adjust the level of inputs used to minimise the production cost, while keeping the output levels unchanged. The practicality of specifying a dual cost function is reduced in restricting the output levels to remain unchanged.

Early applications assuming profit maximisation behaviour are limited in many aspects. In Lau and Yotopoulos (1972) and Yotopoulos, Lau and Wuu-Long (1976), the profit functions were specified respectively for Indian and Taiwanese agricultural production using a linear normalised functional form. This functional form is simple and restricts the underlying production function to have the Cobb-Douglas functional form. Later, Kohli (1978), Sidhu and Baanante (1981) and Antle (1984) adopted the translog form in their specifications of dual profit functions whereby more general structures of production technology were accommodated. However, all later studies share a common feature of having only a single (aggregate) output in their profit functions.

The development from single-output profit functions to multiple-output ones marks another major improvement in the duality approach. The specification of multi-product profit functions was first raised and dealt with by Lau (1972). The pioneering empirical applications of multi-product technologies are Weaver (1983), Shumway (1983), Lopez (1984), Squires (1987), Ball (1988) and Moschini (1988). Since these studies were published, multi-output profit functions have become the most often specified in empirical duality applications. This is probably due to the fact that in reality producers generally produce two or more distinctive outputs, which require different production input mixes and have different profit margins. Complementary and substitutive relationships between outputs have different implications for producers in allocating their production resources. Information on these relationships is useful for economic evaluation and decision-making but cannot be retrieved from single-output profit functions.

Multi-output profit functions appear to be coupled with the normalised quadratic functional form. Translog has not been often used to specify multi-output profit functions, despite its prevalence in cost function studies. Infrequent examples of translog multi-output profit functions include McKay *et al.* (1982, 1983) and Haughton (1986). The uncommonness of translog in studies of multi-output profit functions may be due to the fact that it is difficult to obtain estimation results of models derived from these functions. This is evidenced in Haughton (1986), in which the author used a large cross-sectional dataset for estimation but obtained a very poor fit for the model.

Another significant development of the duality approach is the establishment of dynamic dual models in addition to the traditional static dual models. Lucas (1967), Gould (1968), Treadway (1969), McLaren and Cooper (1980) and Epstein (1981) contribute significantly to the theoretical foundations of dynamic duality. In a dynamic framework, producers are assumed to optimise the net present value of future profits or costs, subject to technological constraints. Optimal output supplies and input demands are derived in the same manner as in the static duality framework. The dynamic approach has been applied by Taylor and Monson (1985), Vasavada and Chambers (1986) and Leblanc and Hrubovcak (1986) for agricultural production in the United States, by Fernandez-Cornejo *et al.* (1992) for German dairy farming, by Thijssen (1996) for Dutch dairy farming and by Agbola and Harrison (2005) for agricultural production in Australian pastoral areas. Considering that farming typically involves large initial investments whose returns are gained over a long time period, the dynamic framework may be more suitable than the static framework in agricultural production research. When using the dynamic framework, researchers can also test for instantaneous and independent input adjustments and calculate short-run and long-run elasticities of input demands, output supplies and input substitution. Despite these advantages, dynamic dual models have not been estimated as often as the static ones, possibly due to the restrictive structure and low tractability discussed in the previous section.

Since its establishment, the duality approach has been applied to almost every economic problem in which producers optimise their economic objectives under technological constraints. Duality-based models have been estimated in transportation (Caves, Christensen and Tretheway 1980), telecommunication (Bloch, Madden, Coble-Neal and Savage 2001), banking and financial services (Featherstone and Moss 1994 and Khaled, Adams and Pickford 2001), power generation (Christensen and Greene 1976), food processing (Ball and Chambers 1982; Morrison Paul 2001; Marsh 2005 and Ollinger *et al.* 2005), non-agricultural manufacturing (Morrison 1988; Diewert and Wales 1995; Gagne and Nappi 2000 and Truett and Truett 2006) and international trade (Kohli 1993 and Tombazos 1998). This approach has become particularly pervasive in agricultural production research. Noteworthy examples of agricultural research include Binswanger (1974a), Yotopoulos *et al.* (1976), Lopez (1980), Ray (1982), Shumway (1983), Lee and Chambers (1986), Haughton (1986), Moschini (1988), Shumway, Saez and Gottret (1988), Fulginiti and Perrin (1990) and Coelli (1996). The popularity of the duality approach in agricultural research has been partly promoted by the availability of production and price data for the rural sector, due to its economic importance in the past.

In short, the duality approach has been long developed and often applied in production economics. Early applications of this approach often specified and estimated dual cost functions. More recent duality applications appear to have favoured dual profit functions. Studies of profit functions have also moved from single-output cases to multi-product cases, to accommodate for more complex production technologies often encountered in reality. The application of the duality approach in empirical production economics has been encouraged by the introduction of flexible functional forms, among which the translog and normalised quadratic have most often been used. The methodological and empirical advantages this approach offers have led to its frequent applications across production industries and different parts of the world.

Despite vigorous theoretical proof and wide acceptance in empirical research, the application of the duality approach is not without challenge. Fundamental issues have surfaced that require further investigation to reaffirm and improve the practical applicability of this approach. Some of these issues are discussed in the following sections.

2.4 Choice of Dual Objective Function

When applying the duality theory, the first issue researchers have to deal with is the choice of the dual objective function to be specified or, more exactly, the economic behaviour of the producers to be assumed. Researchers have commonly assumed cost minimisation, profit maximisation and, to a lesser extent, revenue maximisation behaviours and estimated the dual cost, profit or revenue function accordingly. These assumptions are fundamentally different, a fact that has been rarely emphasised in empirical applications. Cost minimisation and revenue maximisation are 'conditional optimisations' problems, where outputs (in cost minimisation) and inputs (in revenue maximisation) are assumed to be exogenous in the decision-making process. In contrast, when profit maximisation is assumed, neither inputs nor outputs are assumed to be exogenous. General rules of which among these three dual functions should be estimated in a particular context are not available, despite their consequential implications for the econometric models estimated. The decision on the dual objective function specified is *ad hoc* and largely driven by the researcher’s perceptions of producers’ optimisation behaviour, the purposes of the research and what data are available for estimation. The reasons for choosing a dual function over other alternatives are rarely explicitly stated or explained.

There has been a general tendency to favour the dual profit function in the international literature of empirical duality applications. The popularity of the profit maximisation assumption has probably been furthered by the general acceptance of the capitalist model and the rise of large corporations with which the maximisation of profits based on a system of property rights is most publicly emphasised. From a technical point of view, assuming profit maximisation helps researchers avoid simultaneous equation problems (Lopez 1984; Squires 1987 and Shumway 1995). The input demand and output supply equations derived from dual profit functions do not include input and output quantities as their explanatory variables. Instead, they include input and output prices as their explanatory variables, which are variables given to producers in a competitive market. In contrast, the input demand functions, derived from the cost function, and the output supply functions, derived from the revenue function, include output quantities and input levels are their explanatory variables respectively. The quantities of input demand and output supply can simultaneously determine each other, causing simultaneity biases in estimation.

Despite a general tendency to favour the dual profit function, there have been numerous empirical studies in which cost minimisation is assumed. Empirical applications specifying cost functions are as recent as studies by Truett and Truett (2006), Ollinger *et al.* (2005), Kwack and Sun (2005), MacDonald and Ollinger (2005), Kuroda and Lee (2003), Morrison Paul (2001), Khaled *et al.* (2001), Macdonald and Ollinger (2000) and Gagne and Nappi (2000). This continuing popularity of the dual cost function is partly because the cost function is the first function theoretically proven to be dual to the production function. When the cost function is specified in the translog form, a system of cost shares of input demands is derived and estimated with computational ease, since the shares vary between zero and unity. Moreover, the price and substitution elasticities of input demands are straightforwardly related to the system parameters and cost shares. More readily available information on input costs, alongside the government agencies' strong emphasis on reporting producer-paid price indices, have likely contributed to the practicality of specifying the cost function.

In contrast to the frequent application of the dual cost and profit functions, published studies specifying revenue functions are very rare despite their theoretical duality being forcefully proven by McFadden (1978) and Diewert (1974). This dual function can be considered a restricted profit function, where all production inputs are assumed to be fixed. The assumption of fixity of all production inputs is the likely cause of the scarcity of revenue functions because it is probably feasible only in research of international trade where a nation's production inputs are, to a degree, fixed. Under this assumption, the timeframe under which the economic optimisation behaviour is studied is very short. In contrast, all outputs can plausibly be fixed in the short- to medium-term and thus the cost minimisation assumption is more reasonable. Accordingly, the specification of the dual profit function is the most pragmatic, since in this optimisation process neither inputs nor outputs are assumed to be fixed or given.

In the rare applications of the dual revenue function, the revenue function was often estimated without being the primary purpose of the research. This dual function either formed part of a larger econometric model or a step prior to the major research objective. In Gordon (1989), a translog multi-product revenue function was specified and estimated for the Canadian cattle industry. The assumption of constant returns to scale was made, allowing the author to redefine the revenue function as an aggregate output price index, which was then incorporated into a single-output Cobb-Douglas profit function. A revenue function was specified by Kohli (1994) to model Canadian international trade, in which exports were considered outputs and assumed to be weakly separable from all imports, domestic production factors and domestic outputs to allow a two-stage profit maximisation. The export functions for different destinations were derived from the dual revenue function (the second profit-maximisation stage) and estimated concurrently with the import functions derived from a dual cost function (the first profit-maximisation stage). In a more recent study by Weinberg (2002), a multi-input revenue function was estimated for agricultural production as a step prior to assessing the production surplus for water allocation between agricultural, urban and environmental usages in the United States. The author assumed revenue maximisation because of a lack of information on the costs and values of inputs that did not permit a profit function to be specified. The revenue maximisation assumption was justified by the presence of a quantity-rationed input, the water resource, which was fixed in the short run.

The issue of dual objective function choice is probably most problematic in agriculture, in which duality applications have most often been applied. Agricultural production differs significantly to other industries and sectors, with thousands of small family-run farming businesses. Farmers operate with limited financial and physical resources, without specialised skills in financial and operation management and in various production conditions that they have little control over. Lifestyle choices can be as important as economic drivers in farmers' decision-making. Therefore, farmers' behaviour can depart significantly from the profit maximisation behaviour popularly assumed in production economics research and can differ significantly to producers in other industries and sectors, especially those made up mainly by large, for-profit companies.

Cost minimisation has been popularly assumed in agricultural production research. Significant studies estimating cost functions for agricultural production include Binswanger (1974a), Binswanger (1974b), Kako (1978), McKay *et al.* (1980), Ray (1982), Lopez (1980), Moschini (1990), Griffiths *et al.* (2000) and Kuroda and Lee (2003). It can be argued that cost minimisation may not be suitable for agricultural production since farming's operating environment is competitive, with numerous producers and no regulatory restrictions on output levels. Farmers, therefore, can increase their output levels whenever they see it is profitable to do so.

Applications of the dual profit function are particularly popular in agriculture production research. Early significant studies include Weaver (1983), Shumway (1983), McKay *et al.* (1982, 1983), Lopez (1984), Antle (1984), Haughton (1986), Ball (1988), Shumway *et al.* (1988), Moschini (1988), Fisher and Wall (1990), Coxhead (1992) and Polson and Shumway (1992). This dual objective function has also been specified in recent studies by Coelli (1996), Lim and Shumway (1997), Farooq, Young, Russell and Iqbal (2001), Ahammad and Islam (2004) and Abrar, Morrissey and Rayner (2004). However, there are weaknesses to this problem formation. Under the assumption of profit maximisation, optimisation requires a staged decision-making process that can become exceedingly complex, especially in multiple-output operations with a large number of inputs and outputs. This complex process is impractical where agricultural producers running multiple-enterprise operations lack highly specialised managerial and operational skills. Moreover, farmers would not be able to pursue profit maximisation, especially in the short-term, when they are restricted by the availability of production resources (Lee and Chambers 1986), by the dependency of crops and livestock on seasons in a year or by long-run rotation schemes implemented for controlling crop and livestock diseases and preserving natural resources.

Overall, there exist alternative ways of forming a dual optimisation problem in empirical applications of the duality approach. Cost minimisation and profit maximisation have been commonly applied, while revenue maximisation has been rarely assumed. There are no general rules to guide researchers in deciding what function among the dual cost, revenue and profit functions to specify for a particular production technology. The choice of one dual function over another is not generally explicitly stated, explained or proven. This function choice issue is even more difficult in agricultural production due to its competitive market environment, the presence of thousands of small family-run producers with limited financial, managerial and physical resources and the dependency of agricultural outputs on seasonal cycles in the short-term and on rotation schemes implemented for risk and resource management in the long-term.

2.5 Choice of Flexible Functional Form

In applying the duality approach, after resolving the objective function choice issue, researchers normally have to face a second choice issue of functional form. In any application employing parametric estimation methods, the algebraic specification of the relationships between variables of the chosen dual function must be decided before econometric estimation can proceed. In essence, this is a choice about the mathematical representations of the dual cost, revenue or profit functions. Since the forms of the underlying production technology and its dual objective function are unknown, an approximating functional form is chosen to represent the relationships between variables of the dual objective function.

The introduction of flexible functional forms encouraged the adoption of the duality approach in early development. These functional forms allow researchers to avoid imposing structural restrictions on the production technologies being modelled. Flexible functional forms available to researchers include translog, normalised quadratic, generalised Leontief, generalised McFadden, generalised Box-Cox, Fourier, full Laurent, square-root quadratic and generalised Barnett. General mathematical representations of these functional forms are presented in Table 1. Other functional forms, including the symmetric generalised McFadden (Diewert and Wales 1987), the symmetric generalised Barnett (Diewert and Wales 1987), Asymptotically Ideal (Barnett, Geweke and Wolfe 1991) and the general exponential (Cooper and McLaren 1996), were introduced more recently and are yet to gain their significance in empirical research.

The availability of many functional forms presents one more challenging step for researchers, despite their crucial contribution to the adoption of the duality approach in empirical research. As claimed by Lopez (1985), the choice of functional form is a 'purely arbitrary decision'. This choice cannot be based on theoretical grounds (Appelbaum 1979) and depends on both model and data (Anderson, Chaisantikulawat, Guan, Kebbeh, Lin and Shumway 1996). At the same time, functional form choice has crucial consequences for the research results. Different flexible functional forms give different results regarding outcomes of hypothesis testing and estimates of economic and technological relationships (Shumway and Lim 1993).

Table 1: Common Flexible Functional Forms

Functional form	Mathematical presentation
Generalised Leontief	$g(h) = a_0 + \sum_{i=1}^{k} a_i h_i^{1/2} + \frac{1}{2}\sum_{i=1}^{k}\sum_{j=1}^{k} a_{ij} h_i^{1/2} h_j^{1/2}$
Translog	$\ln g(h) = a_0 + \sum_{i=1}^{k} a_i \ln h_i + \frac{1}{2}\sum_{i=1}^{k}\sum_{j=1}^{k} a_{ij} \ln h_i \ln h_j$
Normalised quadratic	$\frac{g(h)}{h_k} = a_0 + \sum_{i=1}^{k-1} a_i \left(\frac{h_i}{h_k}\right) + \frac{1}{2}\sum_{i=1}^{k-1}\sum_{j=1}^{k-1} a_{ij} \left(\frac{h_i}{h_k}\right)\left(\frac{h_j}{h_k}\right)$
Generalised McFadden	$g(h) = \sum_{i=1}^{k} a_i h_i + \frac{1}{2}\sum_{i=1}^{k-1}\left(\sum_{j=1}^{k-1} a_{ij} \frac{h_i h_j}{h_k}\right)$
Box-Cox	$g(h) = a_0 + \sum_{i=1}^{k} a_i h_i(l) + \frac{1}{2}\sum_{i=1}^{k}\sum_{j=1}^{k} b_{ij} h_i(l) h_j(l)$ where $h_i(l) = \left(h_i^{l/2} - 1\right)/(l/2)$
Fourier	$g(h) = u_0 + a'h + \frac{1}{2}h'bh + \sum_{c=1}^{C}\left\{u_{0c} + 2\sum_{j=1}^{J}\left[u_{jc}\cos(jk_c'h) - v_{0c}\sin(jk_c'h)\right]\right\}$ where $b = -\sum_{c=1}^{C} u_{0c} k_c k'_c$ and $\{k_a\}$ is a sequence of elementary multi-indexes
Full Laurent	$g(h) = a_0 + 2\sum_{i=1}^{k}\left(a_i h_i^{\frac{1}{2}} - b_i h_i^{-\frac{1}{2}}\right) + \sum_{i=1}^{k}\sum_{j=1}^{k}\left(a_{ij} h_i^{\frac{1}{2}} h_j^{\frac{1}{2}} - b_{ij} h_i^{-\frac{1}{2}} h_j^{-\frac{1}{2}}\right)$

Flexible functional forms have different empirical applicability due to their different approximating characteristics. For instance, the approximation capacity is local for the translog, generalised Leontief and Box-Cox but global for the Fourier. Some forms allow researchers to test for and impose regularity conditions and technological structures on the dual objective functions globally via parametric methods, while others do not. For example, normalised quadratic and generalised Leontief allow the regularity condition of convexity or concavity to be imposed globally (Shumway 1983; Huffman and Evenson 1989; Shumway and Gottret 1991 and Kohli 1993). None of the available forms seems to be convincingly superior to the others in current duality literature.

A large number of criteria have been suggested in researchers' efforts to reduce the arbitrariness of functional form choice. These criteria include parsimony in parameters, ease of interpretation, interpolative robustness, extrapolative robustness, theoretical consistency, computational facility, desired range of approximation and/or convergence desired, degree of variation of the variables in the data sample used for estimation and the *a priori* expectations of the substitution elasticities (Caves and Christensen 1980; Fuss, McFadden and Mundlak 1978; Mountain and Hsiao 1989 and Anderson *et al.* 1996). The preference of a particular form over another depends on the relative weights a researcher gives to each of these criteria (Lim and Shumway 1997). Probably the most important criterion is the parameter parsimony of the functional form. The main motivation of using flexible functional forms instead of the more traditional Cobb-Douglas or CES is to accommodate more complex technologies. However, using these forms can lead to an excessive number of parameters to be estimated, which often leads to multi-collinearity, instability of parameter estimates and reduced precision in estimation. Creating more parameters becomes even more of an issue in modelling multi-product technologies (Fuss *et al.* 1978). The issue of excessive numbers of model parameters is further compounded by the common use of small time-series data samples for estimation. For a functional form that includes linear, quadratic and interaction terms, the number of parameters to be estimated grows rapidly as the number of explanatory variables increases.

Another important criterion used in functional form decision is the capacity of functional forms to meet the theoretical regularity conditions dictated by economic theory. These regularity conditions should be satisfied in order for the dual objective function to appropriately describe rational economic behaviours. Monotonicity and curvature conditions are probably the two most demanding requirements for functional forms and data. While flexible functional forms allow researchers to study sophisticated technologies, their flexibility comes by sacrificing global regularity properties that are intrinsic to the Cobb-Douglas form. These global regular properties have to be artificially enforced, commonly by means of parametric restrictions.

Depending on the functional form used, imposing restrictions on dual objective functions can have important implications on the structure of the underlying production technology (Lopez 1985 and Blackorby, Primont and Russell 1977). Findings on flexible functional forms' capacity to globally meet the regularity curvature condition, either naturally or artificially via parametric imposition, without sacrificing their flexibility are of great value to researchers.

Although flexible functional forms were originally favoured because of their capacity to approximate the unknown true dual function, for econometric convenience in empirical studies the true dual objective function is assumed to take the exact form of the chosen flexible functional form. Few studies actually spell out this assumption explicitly as did Kako (1978) and Kohli (1993). This assumption is made in order for the error terms in the estimation model to represent only optimisation errors and have the required behaviour of being independently and identically distributed with constant variance and zero means. With this assumption, standard econometric estimation methods can be employed for model estimation. However, this assumption means that the flexible functional form chosen is not used as an approximating form anymore. In order for this exact functional form assumption to be legitimate, much higher *a priori* knowledge of the production technology and economic behaviour of production agents is needed. Moreover, when assuming an exact functional form, certain structural restrictions can apply (Blackorby *et al.* 1977).

Much interest has been given to the approximation properties of flexible functional forms. Most of the popular flexible functional forms have local approximation capacities. These functional forms are expansions around a point, often of the second-order Taylor series type, and therefore are local approximations to the unknown true function. They can provide satisfactory approximating accuracy around the point of approximation, but the degree of accuracy decreases with movement away from the point of approximation (Kohli 1993 and Fisher, Fleissig and Serletis 2001). How closely these functions track the unknown true function at points away from the approximation point is not measurable, since the true function is unknown. The capability of these approximating functional forms to retain global properties, such as monotonicity or curvature, of the unknown true function is not guaranteed either (Gallant and Golub 1984). Owing to the local approximation of these flexible functional forms, it is not guaranteed that regularity conditions will be satisfied by the estimated functions outside the expansion ring.

Beside the issue of approximation accuracy outside the expansion ring, locally flexible functional forms have been questioned regarding their compatibility with econometric estimation methods. The parametric estimation methods used to estimate the dual objective functions are global optimising procedures. The whole data range, not a sub-range around a data point, is included in the optimised criterion function, i.e. the sum of least squares or likelihood function. Meanwhile, local flexible functional forms are expected to only approximate the true dual objective function around a local data point. As argued by Chalfant (1984), the incompatibility of locally flexible functional forms with econometric estimation methods can lead to biased and inconsistent model estimates. Empirical applications have shown that unless being constrained by parametric restrictions, there is no guarantee that the estimated dual function will satisfy the regularity conditions, especially regarding the curvature condition. As a result, a robust estimation outcome according to common goodness-of-fit measures is not automatically satisfactory with regard to duality theory.

There have been numerous attempts to resolve the issue of functional form choice. Studies by Wales (1977), Blackorby *et al.* (1977), Caves and Christensen (1980), Chalfant (1984), Diewert and Wales (1987), Thompson and Langworthy (1989), Anderson *et al.* (1996), Ivaldi, Ladoux, Ossard and Simioni (1996), Terrell (1996), Gagne and Ouellette (1998) and Fisher *et al.* (2001) analytically, experimentally and empirically evaluate the suitability of various functional forms. These studies have produced significant insights into the behaviour of the existing flexible functional forms. For instance, Caves and Christensen (1980) showed that the translog and generalised Leontief functional forms, members of the locally flexible functional form family, can have small regions where theoretical (regularity) conditions of monotonicity and curvature, in consumer demand analysis, are satisfied. Moreover, these regularity regions vary significantly depending on the substitutability between commodities. This finding is directly applicable to duality analyses of production technologies. Later, Lopez (1985) analytically demonstrated that generalised Leontief and normalised quadratic functional forms impose quasi-homotheticity, which implies that the production technology has a linear expansion path or that the marginal rate of substitution is independent of output level, input-output separability in multi-output technologies and additive separability in inputs. The author found that, in contrast, translog does not impose such structural restrictions.

Attempts to lessen arbitrariness in flexible functional form choice through empirical evaluation of flexible functional forms' performances have had mixed results. For instance, Thompson and Langworthy (1989) conducted Monte Carlo experiments to evaluate the performance of translog, generalised Leontief, quadratic and Minflex Laurent forms in approximating profit functions and found that no functional form is superior based on estimates of the Allen partial elasticities of substitution. As Lopez (1985) points out, the comparison of the performance of different functional forms becomes very difficult in the case of multi-product technologies. Most studies evaluating the performance of flexible functional forms have been either experimental, i.e. using Monte Carlo techniques, or empirical in nature, i.e. using real datasets. These studies' findings cannot therefore be generalised. Moreover, the empirical evaluations have often used aggregate and time-series data for estimation that undermines their findings. As a result, the question of which functional form suits best in a particular research application remains more or less unanswerable (Anderson *et al.* 1996). Moreover, Blackorby *et al.* (1977) found the generalised Leontief and translog are separability-inflexible, which somewhat dampened researchers' previously optimistic views about the claimed capabilities of flexible functional forms.

None of the introduced flexible functional forms seems to have completely dominated the others, although some have enjoyed greater acceptance in empirical research. The translog and normalised quadratic forms have been most often applied (Anderson *et al.* 1996). This is because some of their properties are empirically attractive. These functional forms are linear in parameters and thus conveniently estimated using standard econometric estimation procedures. Since they are second-order Taylor series expansions, they have enough parameters to allow for arbitrary second-order effects (i.e. substitution and transformation elasticities) to be obtained at any given data point (Gallant and Golub 1984). Their estimation also does not involve an arbitrarily determined choice of parameters as required for the generalised Box-Cox form (see Table 1), which contains translog, generalised Leontief and generalised square-root quadratic as its special cases, as well as for the Fourier form, which was claimed by Chalfant and Gallant (1985) to have unbiased substitution elasticity estimates.

The translog and normalised quadratic functional forms have their own weaknesses despite their great popularity. The translog functional form has been questioned in regard to its approximation capability and flexibility. Wales (1977), using Monte Carlo simulations, found that the ability of the translog functional form to approximate the true function deteriorates as the true substitution elasticity moves away from unity. This finding was reiterated in Monte Carlo studies by Guilkey and Lovell (1980) and Guilkey, Lovell and Sickles (1983). The translog's flexibility has also been questioned following the discovery of its separability-inflexibility, i.e. it becomes inflexible when separability is imposed, by Blackorby *et al.* (1977).

The normalised quadratic functional form has been widely accepted in empirical research due to its many advantages. This functional form is self-dual, which means that the underlying production technology is quadratic when the dual objective function assumes this form (Shumway 1983; Huffman and Evenson 1989; Thompson and Langworthy 1989; Shumway and Gottret 1991 and Lusk, Featherstone, Marsh and Abdulkadri 2002). The Hessian matrix of a dual normalised quadratic objective function is constant, which can be used to directly derive the Hessian of its dual primal function (Lau 1976). Possessing a constant Hessian matrix, the normalised quadratic functional form displays a global curvature attribute, which is considered by Lopez (1985) to be the main reason for its popularity in duality applications. With this functional form, if the curvature condition is satisfied at a data point, it is satisfied at all data points. This allows the curvature condition to be imposed parametrically without destroying its flexibility (Shumway and Gottret 1991).

Despite its popularity in empirical research, the normalised quadratic form has a significant empirical weakness. When it is used to model multi-product production technologies, a normalising factor (the *numeraire*) has to be chosen among the input prices included in the model. The choice of *numeraire* is at the discretion of the researcher. While this decision is arbitrary, it has critical implications for the research findings and policy interpretations. For the same dual objective function, different *numeraires* result in different econometric models with different right-hand variables and error terms (Shumway and Gottret 1991 and McLaren and Zhao 2009). The equation of the derived demand for the *numeraire* is different from that of the remaining inputs. This asymmetric trait of the normalised quadratic functional form and the absence of a systematic method for selecting a *numeraire*, despite efforts such as those by Shumway and Gottret (1991), has motivated the development of the symmetric McFadden and the symmetric generalised Barnett functional forms by Diewert and Wales (1987). However, these forms do not have the advantages of being self-dual and possessing a constant Hessian matrix like the normalised quadratic functional form (Shumway and Gottret 1991 and Gagne and Ouellette 1998).

The above review demonstrates that choice of functional form is a nontrivial matter in empirical applications. Many flexible functional forms have been introduced and applied but none is considered completely superior to the others. Studies of flexible functional forms, mostly employing experimental or empirical research approaches, have produced mixed results, offering only modest help in the selection of functional form. However, there has been an overall preference in recent literature for the translog and normalised quadratic forms, although each has its own weaknesses. There is a need for further research to explore the issues related to the functional form choice in more depth, so as to assist future empirical applications of the duality theory.

2.6 Frequent Failures in Satisfying Regularity Conditions

The increasing popularity of flexible functional forms appears to have come with an increasing incidence of failure to meet theoretical regularity conditions by the estimated dual functions. These regularity conditions, also referred to as the conditions for economic integrability, are homogeneity, symmetry, monotonicity and curvature. They are necessary for the dual objective functions to have meaningful economic interpretations. The frequent failure to meet these conditions has generated scepticism towards the duality approach (Kohli 1993; Barnett and Hahm 1994; Fox and Kivanda 1994; Shumway 1995; Terrell 1996 and Lim and Shumway 1997).

The violation of the regularity conditions has appeared to be most serious for the curvature and monotonicity conditions. This is partly due to the fact that the other two regularity conditions of homogeneity and symmetry have often been imposed through linear parametric restrictions or a normalisation process during estimation. In contrast, it is difficult, if not impossible, to impose the monotonicity and curvature conditions via parametric restrictions. These two conditions can only be checked or tested after unconstrained econometric estimation.

Early studies applying the duality approach often did not formally identify the need to test for or to enforce regularity conditions. At the same time, the failure to meet regularity conditions does not seem to be a serious problem in studies of single-output technologies. For instance, the single-output translog cost functions estimated for the United States electric power generation in Christensen and Greene (1976) and for Japanese rice production in Kako (1978) satisfy the concavity condition. This condition was also satisfied by the generalised Leontief single-output cost function estimated for the Canadian agricultural production in Lopez (1980) and the translog cost function estimated for the Canadian metal mining industry in Halvorsen and Smith (1986).

Failure to meet the curvature condition in duality applications has occurred as early as when duality theory was first applied. This failure was observed in a study by Halvorsen (1977), where a single-output technology was estimated using cross-sectional data of energy demand in United States manufacturing. Such failure has appeared to be more common for multi-product technologies. Interestingly, Lopez (1984) found that the fitted generalised Leontief multi-product profit function for Canadian agricultural production using a cross-sectional dataset failed to meet the convexity condition at 75 per cent of the sample points. This result contradicts the findings in the author's 1980 study of Canadian agricultural production using a time-series dataset. Meanwhile, Featherstone and Moss (1994) yielded different results when estimating models for rural banks in the United States with and without imposing the curvature condition, which implies that the curvature condition was not satisfied. Other studies where the curvature condition was not satisfied include Squires (1987), Fulginiti and Perrin (1990) and O'Donnell and Woodland (1995). The frequency of failure to meet the regularity conditions has been too high for the null hypothesis that these conditions are met to be accepted at a reasonable confidence level, as examined by Shumway (1995). Shumway conducted a survey of 113 duality studies published in major agricultural journals from 1972 to 1993 and found that the curvature condition was rejected in 77 per cent of cases, based on the surveyed studies' published test results and tests reconstructed by the author. Shumway also stated that this condition was seldom satisfied based on his own experience studying multi-product production technologies using 60 datasets. This statement presumably applies to his 1983 study of agricultural production in Texas (Shumway 1983), where a normalised quadratic profit function of six outputs and three variable inputs was estimated using time-series data from 1957 to 1979.

Different solutions have been proposed in response to the frequent failure to meet the curvature condition. A significant development in this respect was the use of Cholesky factorisation to impose the curvature condition on the dual objective functions, which was proposed by Lau (1978a) and followed by Moschini (1988), Featherstone and Moss (1994), Coelli (1996) and Marsh (2005), among others. This procedure has a shortcoming though, in turning the model to be estimated from linear to nonlinear in parameters. The estimation of the resultant nonlinear system is more difficult and computationally demanding, especially when the production technology studied involves several inputs and outputs or when the data sample is small. Moreover, the method chosen to impose the curvature condition can also have certain implications for the structure of the underlying production technology, depending on the functional form employed. Diewert and Wales (1987) showed that imposing global concavity on the dual cost function might cause upward biases in input substitutability measures in the translog functional form's case and exclusion of input complementarity in the generalised Leontief functional form's case. In general, unconstrained dual objective functions have been more commonly estimated with failure to meet the curvature condition frequently reported.

Probably more challenging to researchers than the frequent violations of regularity conditions is the inability to pinpoint their causes. A number of causes have been suggested. They include incorrect model specification due to potentially incorrect assumptions regarding producers' optimisation behaviour, the inadequacy of the flexible functional forms (Wales 1977), the excessive number of variables/parameters to be estimated and the omission of qualitative variables such as demographic characteristics (Shumway 1983; Kohli 1993 and Fisher *et al.* 2001). Researchers have also frequently attributed the violation of the curvature condition to the inappropriate type and the inadequate quality of the data used for model estimation, especially regarding measurement errors, time-series properties, insufficient sample size and high levels of data aggregation (Squires 1987; Kohli 1993; Shumway 1995 and Tombazos 1998).

The frequent failure to meet regularity conditions has appeared to make it acceptable for the regularity conditions not to be met, although this implies that the estimated dual functions are economically meaningless. Farooq *et al.* (2001) assumed the curvature condition was met without formal testing in estimating a translog profit function for rice farmers in Punjab, Pakistan. As Squires (1987) and Wales (1977) emphasised, failure to satisfy the curvature condition does not necessarily mean that the underlying economic behaviour does not exist. This failure can be caused by a number of other factors, such as poor approximation capability of the functional form used or insufficient variation of the data used for estimation. Meanwhile, Kohli (1993) warned that failure to meet regularity conditions, especially in fairly large estimation models, can be expected due to the excessive demand of flexible functional forms on the data. On a different path, Shumway (1995) argued that rejections of the regularity conditions are not based on statistical tests but on whether the unconstrained estimates have the expected signs. The author reasoned that on a statistical basis, the significance of having the wrong sign may be too small to reject the hypotheses that the regularity conditions are met. He also hypothesised that in an absence of detailed validation, it should not be expected that regularity conditions postulated at a farm-level would hold at an aggregate level.

In summary, this section reviews the issue of failure to meet regularity conditions in empirical application of the duality approach. Duality studies of production economics have often found estimation results to be inconsistent with the theoretical regularity conditions. This inconsistency is most serious for the monotonicity and curvature conditions, particularly in studies of multi-product technologies. There has been a tendency to parametrically impose the curvature condition on the estimation model, commonly via the Cholesky factorisation method. This imposition can make model estimation more difficult and imply a certain structure on the underlying production technology, depending on the flexible functional form used. In general, the causes of failure to meet the regularity conditions have not been rectified. They are believed to be, among other things, the inadequacy of flexible functional forms, the mis-specification of estimation models and, most prevalently, the data used for econometric estimation. However, their causal relationships with the violation of regularity conditions have not been concretely demonstrated.

2.7 Data-Related Issues

Data has been considered a cause of the violation of the theoretical regularity conditions in empirical duality applications. As Fox and Kivanda (1994) argued, tests of hypotheses concerning regularity conditions are joint tests of the theory, data, data observation and measurement procedures. The rejection of these hypotheses can therefore be due to the data used for model estimation. The use of data highly aggregated across geographical areas for model estimation has been believed as the cause of the refutation of regularity conditions (Kohli 1993; Fisher *et al.* 2001 and Lusk *et al.* 2002). Empirical duality applications have predominantly used state, regional or national time-series data for estimation. Many benefits of using aggregate data instead of disaggregate data in duality-based studies have been quoted to justify such a substitution. Examples of these benefits are the savings in the cost of collecting individual farm data, the circumvention of potential collinearity existing in disaggregate data, reduction of computational burden and the avoidance of difficulties in drawing aggregate inferences from models using disaggregated data (Polson and Shumway 1990). However, the main reason of substituting aggregate for disaggregate data is the unavailability of micro-level data, an inherent problem in agricultural production research.

There have been a few issues arising from the use of aggregate time-series data in duality applications that are worth further investigation. The first issue is the possible insufficient sample size for robust econometric estimation. The sample size of aggregate time-series data is limited by the period of time over which the data are observed and therefore is often small. Small data samples restrict the number of parameters to be included in the estimation model and necessitate the aggregation of inputs and outputs into a small number of variables. This reduces the potential benefits of using flexible functional forms and the usefulness of information drawn from the estimated model for policy making. Small data samples can also have detrimental effects on the estimation accuracy and hypothesis testing power. These effects can undermine the outcomes of research on issues such as the choice of flexible functional form or the impact of data aggregation on estimation results in duality applications.

The use of geographically aggregated time-series data also means that there is inconsistency between theory and empirical applications regarding the decision-making entity. This is because the duality approach is based on individual firms or farms, not a state, region or nation. This inconsistency between the decision-making entity in the theory and the observational unit in the empirical data can cause discrepancies between theoretical expectations and empirical findings. Theoretically, consistent aggregation of data across production units is possible if certain conditions are met; yet these conditions are too restrictive to be applicable in reality. Theoretically, consistent aggregation across farms requires that production units use identical production technologies (Wolfson 1993 and Polson and Shumway 1990). As stressed by Liu and Shumway (2004), Shumway and Davis (2001), Wolfson (1993) and Chambers (1988), this condition is unlikely to hold in reality. This is particularly true in agricultural production and, therefore, aggregating data across farms in a state, region or country is likely to be inconsistent with duality theory. Nevertheless, most studies of agricultural production technologies have used aggregate data. Assumption of the existence of consistent aggregation conditions, acknowledgement of the caveats of using aggregate data, or a caution about possible effects of the use of aggregate data for research findings have rarely been explicitly stated in studies of agricultural production.

The matter of whether it is valid to use aggregate data in empirical duality research remains one of the most challenging issues for researchers in this field. The questions of how well aggregate data conveys information about individual production units and what possible impacts the aggregation of data across production units has on research findings have been raised recently. In the context of farming, where the technology employed and operational conditions are highly heterogeneous across farms, the use of aggregate data may result in misleading research results and thus unreliable policy interpretations (Liu and Shumway 2004 and Morrison Paul 2001). The use of aggregate data in duality studies may also introduce simultaneity bias. In a competitive market, firms/farms do not have control over input and output prices. Thus when micro-level data are used the explanatory price variables satisfy the exogeneity assumption. At an aggregate level, however, prices and quantities are simultaneously determined via demand-supply principles. Therefore, the price variables are less likely to be exogenous and estimation biases may occur. Meanwhile, Lim and Shumway (1997) suggested that the ignorance of time-series characteristics of aggregate data may have contributed to the frequent failure of past studies to meet one or more regularity conditions required for rational economic behaviour. They tested the stationarity of variables included in their estimation model and discovered that these variables are integrated in different orders. This may often be the case in empirical studies of agricultural production using time-series data. As argued by the authors, when the variables included in static models are non-stationary, spurious regression and erroneous policy interpretation will result unless appropriate diagnosis testing (i.e. cointegration testing) and correction actions (i.e. taking differences of the appropriate order to make the variables stationary) are carried out. Further, Barten (1969) pointed out that when time-series aggregate data are used, the model coefficients are implicitly assumed to be constant over time, which is likely to be unrealistic.

The matter of cross-farm or geographic data aggregation has received considerable theoretical treatment in the past. However, empirical evidence of potentially negative impacts of using aggregate data in duality applications is limited due to the unavailability of farm-level data. In applying the duality theory to agricultural production in the United States and in ten of its agricultural regions, Shumway *et al.* (1988) found that data aggregation has little impact on the elasticity estimates obtained at the regional and national levels. Liu and Shumway (2004) tested for consistent aggregation at the state and regional level using agricultural production data in the United States and found general supports for consistent aggregation across states. However, the findings from these two studies do not shed any light on the impacts of aggregating data across farms on estimation results due to the use of aggregate state and regional data for estimation.

Overall, empirical duality applications have encountered some significant issues due to the unavailability of farm-level data. State, regional or national time series have typically been used instead of unit-level data for model estimation. The use of aggregate time-series data can undermine the potential of flexible functional forms and multi-product technology frameworks due to the often constrained sample size of this data type. The use of aggregate time-series data can also be inconsistent with the theory since the duality theory is one of microeconomics. Such data inconsistency may be the major cause of frequent failures to meet theoretical regularity conditions in past duality applications. Empirical research on the potential impacts of geographic data aggregation on research findings in duality applications has been limited.

2.8 Conclusion

This chapter presents a review of previously published econometric models of Australian agricultural production and international applications of the duality approach to production economics. A limited number of econometric studies have been estimated for Australian agricultural production due to the unavailability of data needed for model estimation. Early studies applying the primal approach to Australian broadacre production required restrictive assumptions about the structure of the prevailing production technology. Later duality-based models have restricted study scopes, which might potentially cause biased estimation results and have limited usefulness in policy making, due to the criteria used to define the study scopes. Almost all of these studies also used aggregate time-series data instead of farm-level data on which the duality theory is based. The data used for estimation in these studies are now quite dated and may not accurately reflect current production technology.

A review of international duality literature indicates that there have remained important empirical obstacles and issues in the application of the duality theory despite its long history of theoretical development and empirical application. Researchers applying duality theory must choose the dual objective function and flexible functional form among different available functions and forms to represent the producers' economic behaviour before an estimation model can be specified. These choices determine the research outcomes, yet no well-founded guiding rules have been developed for guiding them. The research findings of duality studies have also often been found to be inconsistent with theoretical economic expectations regarding the regularity conditions. Moreover, aggregate time-series data have been dominantly used for model estimations in duality applications. The use of time-series data could result in biased estimation results due to simultaneity and/or insufficient sample sizes. Time-series properties in the data used for estimation have also been ignored in almost all studies, which could result in spurious regression and erroneous policy interpretation. Aggregate time-series data have also been used in duality applications despite the fact that consistent aggregation conditions may have not been satisfied. Finally, there has been little empirical evidence concerning the impacts of aggregating data across production units or geographical areas on duality-based estimation results.

Chapter 3

Dual Economic Models for Production Decisions and Measures of Economic Interest

3.1 Introduction

This chapter covers the theoretical framework of the duality approach in production economics. As discussed in Chapter 2, duality studies of production technology commonly follow one of three alternative formulations. Each of the formulations commences with an assumption about the economic behaviour of the producers via the specification of an economic optimisation problem. These optimisation problems are cost minimisation, revenue maximisation or profit maximisation. A dual cost, revenue or profit function is specified accordingly, each accompanied by a set of theoretical regularity conditions to ensure the duality between the primal production function and the dual objective function. Embedded in the assumed optimisation problem is an assumption about the exogenous and endogenous variables included in the model to be estimated. The optimizing output supply and/or input demand equations are then derived by applying the Envelope Theorem to the defined optimisation problem. This derivation operation allows the regularity conditions of the dual objective function to be directly transferred to the derived demand and supply functions.

In the presentation of the duality framework in this chapter, proof of the existence of duality between the primal production function and each of the dual cost, revenue and profit functions is not provided. Proof for the existence of duality has been the subject of many pioneering publications such as Hotelling (1932), Shephard (1953), Uzawa (1962), and Diewert (1971). Only the general representations of the dual objective functions are considered in this chapter. Their representations in specific functional forms will be covered in Chapters 5, 6 and 7 in which empirical models are presented. For each of the three model formations, quantitative measures of economic interest, such as price elasticities and elasticities of input substitution and output transformation, are also presented and discussed.

The remainder of this chapter is divided into eight sections. Section 3.2 contains a description of the conventional production function with common measurements of economic and technical interest, laying a foundation for subsequent dual problem formulations. Sections 3.3 follows with the specification of the dual cost function, including the theoretical regularity conditions this function satisfies and the elasticities of economic and technical relationships between production inputs. Section 3.4 and Section 3.5 respectively deal with the dual revenue and profit functions, following the same structure as Section 3.3. Section 3.6 considers the specification of restricted dual objective functions in dealing with the fixity of some production inputs in the short run commonly encountered in empirical research. Section 3.7 presents concluding remarks of the chapter.

3.2 Production Function

3.2.1 Single-output Production Function

The production function is more easily defined for the single-output case as the output can be represented by a scalar. In this case, production technology can be mathematically characterised as $y = f(X)$ where y is the output level and X is a vector of n input quantities. This function represents the maximum output for a given input vector. To represent real economic problems, $f(X)$ has to satisfy a set of restrictions which are often referred to as regularity conditions. The commonly accepted restrictions are (McFadden 1963; Diewert 1971; Jorgenson and Lau 1974a and Chambers 1988):

Condition A.1: $f(X)$ is monotonic for all $X > 0$;

Condition A.2: The input requirement set (defined as $V(y) = \{X : f(X) \geq y\}$) is a closed, non-empty and convex set (quasi-concavity) for $y > 0$;

Condition A.3: $f(0) = 0$ (weak essentiality);

Condition A.4: $f(x_1, x_2, ..., 0, x_{n-1}, x_n) = 0$ (strict essentiality);

Condition A.5: $f(X)$ is finite, non-negative, real valued and single valued for all non-negative and finite X; and

Condition A.6: $f(X)$ is everywhere continuously differentiable.

Each of these theoretical conditions has an economic interpretation. For instance, the first condition implies that, if the level of any input increases, the output level never decreases. Technically, it is possible that when other things remain fixed, up to a certain level any further addition of an input can cause a reduction in output. For example, over-fertilising a given piece of land can reduce crop yields. Such an occurrence would, however, never be observed if the farmer is economically rational. Meanwhile, Condition A.2 implies a diminishing marginal rate of technical substitution, which lies behind almost every economic problem. Condition A.6 is more for mathematical convenience than for economic necessity. This condition allows for differentiation of the production function to arrive at several technological measures: importantly, the elasticities of substitution between inputs. Other conditions are straightforward in terms of portraying production problems in reality.

3.2.2 Multiple-output Production Function

For the case of multiple-input and multiple-output production, the presentation of the production function is more generally defined as a set of technically efficient output-input combinations defining production possibilities. The multiple output technology can be presented as: $h(X,Y)=0$, where $X=[x_1,\ x_2,\ ...,\ x_n]$ and $Y=\left[y_1,y_2,...y_m\right]$ are respectively the input and output sets. Since the production technology is no longer represented as a function where the left hand side is a scalar, it is not possible to express the regularity conditions in the same manner as in the single-output case above. The production technology and associated regularity conditions are normally described and discussed in terms of set notions, which consist of the production possibility sets, the input requirement sets and the producible-output sets (see Chambers 1988, Chapter 7 or Diewert 1974). Alternatively, the technology can be represented as $-x_i = L\left(X_{-i},Y\right)$ where x_i is an arbitrarily chosen input and X_{-i} is the vector of all other inputs except x_i (Shumway 1983). In this representation, similar to the single-output case, L is assumed to be finite, nonpositive, real-valued, bounded, continuous, smooth, monotonic, convex in inputs and outputs, and twice-differentiable.

3.2.3 Measures of Economic Interest

The main purpose of estimating the production function is to gain information about substitutability and transformability between inputs and outputs. A series of measures of substitutability have been introduced. The Allen partial elasticity (also termed Allen-Uzawa) was formulated by Allen (1938) and Uzawa (1962). The direct elasticity of substitution was then defined by Hicks (1963). The Morishima elasticity was later introduced, independently by Morishima in 1967 (in an untranslated Japanese work) and Blackorby and Russell in their 1975 unpublished discussion paper (Blackorby and Russell 1989). Other measures of substitution elasticities are the shadow elasticity of substitution, which is a weighted average of two Morishima elasticities (McFadden 1963), and the generalised factor ratio elasticity of substitution, which includes the

Morishima as a special case (Davis and Shumway 1996). Among all the measures introduced, the Allen partial elasticities have been most popularly estimated, although this definition has recently been subject to forceful criticisms for its quantitative and qualitative ambiguity (Blackorby and Russell 1989 and Blackorby, Primont and Russell 2007). For the single output case, the substitution relationship between inputs x_i and x_j is defined as:

$$\sigma_{ij} = \frac{\sum_{k=1}^{n} x_k f_k}{x_i x_j} \frac{F_{ji}}{F}, \qquad (3.1)$$

where $f_i = \frac{\partial f(X)}{\partial x_i}$, $F = \begin{vmatrix} 0 & f_1 & f_2 & \cdots & f_n \\ f_1 & f_{11} & f_{12} & \cdots & \\ f_2 & \cdots & \cdots & & \\ \vdots & \vdots & \vdots & & \vdots \\ f_n & f_{1n} & f_{2n} & \cdots & f_{nn} \end{vmatrix}$, and F_{ji} is the cofactor associated with element f_{ji} (Binswanger 1974a and Chambers 1988). The Allen partial elasticity is symmetric, i.e., $\sigma_{ij} = \sigma_{ji}$, due to the assumption of twice-continuously differentiable technology and Young's theorem of symmetry in differentiation. A positive Allen partial elasticity between two inputs means they are substitutes; a negative one signifies that they are complements. In a substitutive relationship, an increase in price of one input, holding fixed the level of output, evokes an increase in the use of the other input; in a complementary relationship a price increase results in a reduction in the utilisation of both inputs.

In comparison to the Allen partial elasticity of substitution, the Morishima measure has been less often used. However, this measure has been reported in several recent studies, including Agbola and Harrison (2005), Sharma (2002), Fisher *et al.* (2001), Huang (1991) and Mountain and Hsiao (1989), for its more meaningful interpretations compared to the Allen partial measure. The Morishima elasticity of substitution is defined as:

$$\sigma_{ij}^{M} = \frac{f_j}{x_i}\frac{F_{ij}}{F} - \frac{f_j}{x_j}\frac{F_{ij}}{F}, \tag{3.2}$$

where f_j, F_{ij} and F are defined as above.

This elasticity can be related to the Allen partial elasticities in the following fashion (Blackorby and Russell 1989 and Chambers 1988):

$$\sigma_{ij}^{M} = \frac{f_j x_j}{f_i x_i}\left(\sigma_{ij} - \sigma_{jj}\right). \tag{3.3}$$

As with the Allen partial measure, a positive Morishima elasticity implies a substitutive relationship and a negative value implies a complementary one. However, as shown in (3.3), Morishima elasticities are not symmetric, i.e., $\sigma_{ij}^{M} \neq \sigma_{ji}^{M}$. Moreover, a relationship can be complementary by the Allen partial measure $\left(\sigma_{ij} < 0\right)$ while being substitutive by the Morishima measure $\left(\sigma_{ij}^{M} > 0\right)$, which occurs if $\left|\sigma_{ij}\right| < \left|\sigma_{jj}\right|$. The relative reasonableness of the Allen partial and Morishima elasticities is more clearly demonstrated in the following discussion of the dual functions.

3.3 Dual Models Using Cost Functions

3.3.1 Optimisation Problem for Multi-product Production Technologies

If, conditional on output levels, producers are assumed to respond to a change in the market price of an input by adjusting all input levels to minimise the production cost, a dual cost function can be defined for the underlying production technology, even in the case of multiple outputs. Consider the multiple output technology $h(X,Y)=0$ described in the preceding section. Define $W=\left[w_1, w_2, \ldots w_n\right]$ as the input price vector and $V(Y)$ as the input requirement set corresponding to this production technology. The cost function dual to this technology is then defined as $C(W,Y)=\min_{X}\left\{W'X : X \in V(Y)\right\}$. This cost function satisfies the following conditions (Shephard 1953; Uzawa 1964; Binswanger 1974b and Chambers 1988):

Condition C.1: $C(W,Y)$ is non-negative for $W > 0$ and $Y > 0$;

Condition C.2: $C(W,Y)$ is non-decreasing in W;

Condition C.3: $C(W,Y)$ is continuous and concave in W;

Condition C.4: $C(W,Y)$ is positively linearly homogeneous in W, i.e., $C(tW,Y) = tC(W,Y)$ for all $t > 0$;

Condition C.5: $C(W,Y)$ is non-decreasing in Y;

Condition C.6: $C(W,0) = 0$, i.e., there are no fixed costs; and

Condition C.7: $C(W,Y)$ is continuously differentiable in W so Shephard's lemma can be applied to derive the cost-minimising input demands as the first derivatives of the cost function with respect to input prices.

These conditions ensure that the cost function, which represents cost minimising behaviour, is capable of portraying a regular technology and are normally referred to as the regularity conditions. Condition C.1 means that the cost function has to be non-negative in the range of positive input prices and cannot be zero for a positive output. Condition C.2 implies that an increase in the price of any input does not decrease the production cost of a given output level, other things being fixed. Condition C.3 postulates that there are possibilities for substitution between inputs so that, if an input's price increases, the production cost increases by a smaller or equal proportion. The next condition of linear homogeneity in input prices means that the cost-minimising input mix does not change when input prices vary by the same proportion. The requirements of being non-decreasing in outputs and having zero fixed costs (Condition C.5 and C.6) signify that it costs more to produce more outputs and there is no cost in producing nothing. Finally, Condition C.7 has a greater empirical implication than theoretical implication because this condition allows for a systematic derivation of input demands that inherit the properties implied by the cost function.

3.3.2 Output-constrained Cost-minimising Input Demand Functions

Important to the duality relationship between the cost function and the production function is Shephard's lemma. In his 1953 monograph, Shephard provided a proof of the one-to-one correspondence between cost-minimising input mix, conditional on output level, and the production possibility surface (the isoquant in the two-input case) for a given output level.

Shephard's lemma

When the cost function satisfies the conditions described above, the unique, cost-minimising input demand vector is determined as:

$$x_i(W,Y) = \frac{\partial C(W,Y)}{\partial w_i},\ i = 1,2,...n. \qquad (3.4)$$

Applying Shephard's lemma, the regularity conditions of the cost function can be transferred to the derived cost-minimising demand equations. Condition C.2 means that the conditional input demand $x_i(W,Y) = \frac{\partial C(W,Y)}{\partial w_i}$ is positive for all $i = 1,2,...n$ while Condition C.3 holds if the matrix $\left[\frac{\partial^2 C(W,Y)}{\partial w_i \partial w_j} = \frac{\partial x_i(W,Y)}{\partial w_j}\right]$, $(i, j = 1,2,...n)$, is negative semidefinite. Condition C.4 is equivalent to the derived demand $x_i(W,Y) = \frac{\partial C(W,Y)}{\partial w_i}$ being homogeneous of degree zero in W and marginal costs $\frac{\partial C(W,Y)}{\partial y_k}$, $k = 1,2,...m$, being linearly homogeneous in W. The condition of being non-decreasing in outputs (Condition C.5) is satisfied if the shadow price of output $p_k\left(W,Y\right) = \frac{\partial C(W,Y)}{\partial y_k}$ is positive for all $k = 1,2,...m$. Finally, if the cost function is further assumed to be twice-continuously differentiable instead of continuously differentiable (Condition C.7), which is for mathematically convenient derivation of elasticities, a condition that

$\left[\dfrac{\partial C^2(W,Y)}{\partial w_i \partial w_j} = \dfrac{\partial x_i(W,Y)}{\partial w_j}\right]$ is symmetric is resulted. This condition arises from Young's theorem in partial differentiation and is usually referred to as the symmetry condition.

3.3.3 Measures of Economic and Technical Interest

In production economics, researchers are predominantly interested in the economic and technical relationships among inputs and outputs. The common unit-free measures of these technical relationships can be retrieved from the second derivatives of the dual cost function. This operation requires the further assumption that the dual cost function is twice-continuously differentiable in input prices. In the specification of the dual cost function, the net (conditional or output-"compensated" or output-constant) price elasticity for input demand is defined as:

$$\eta_{ij} = \frac{\partial x_i(W,Y)}{\partial w_j}\frac{w_j}{x_i(W,Y)}, \qquad (3.5)$$

which satisfies $\sum_{j=1}^{n}\eta_{ij} = 0$ and $\eta_{ij} = \frac{c_j}{c_i}\eta_{ji}$ where $c_i = \frac{x_i(W,Y)w_i}{C(W,Y)}$ and $c_j = \frac{x_j(W,Y)w_j}{C(W,Y)}$ are respectively the cost shares of input i and input j (Chambers 1988; Bertoletti 2005 and Blackorby and Russell 1989).

Applying the definition of Allen partial substitution elasticity in the primal production function's case (expression (3.1)) to the cost function, it has been proven that net (output-"compensated") Allen partial elasticities of substitution can be expressed as the following (Binswanger 1974a and Uzawa 1962):

$$\sigma_{ij} = \frac{C(W,Y)}{\frac{\partial C(W,Y)}{\partial w_j}\frac{\partial C(W,Y)}{\partial w_i}}\frac{\partial^2 C(W,Y)}{\partial w_i \partial w_j} = \frac{C(W,Y)}{x_j(W,Y)x_i(W,Y)}\frac{\partial^2 C(W,Y)}{\partial w_i \partial w_j}. \qquad (3.6)$$

Under the assumption of twice differentiability of the cost function, the net Allen partial elasticities are symmetric, i.e., $\sigma_{ij} = \sigma_{ji}$.

From the expressions of the net price elasticities, the net Allen partial elasticities of substitution and the net Morishima elasticities of substitution, it is straightforward to establish that $\sigma_{ij} = \dfrac{\eta_{ij}}{c_j}$ and $\sigma_{ij}^M = \eta_{ij} - \eta_{jj}$. These two equalities can be used to demonstrate the relative meaningfulness of the Allen partial and Morishima elasticities (σ_{ij} and σ_{ij}^M) in multi-input productions. The expression $\sigma_{ij} = \dfrac{\eta_{ij}}{c_j}$ implies that once the price elasticity between two inputs is known, the Allen partial elasticities provide no further useful information about the pair's substitutability or complementarity since c_j is positive. As explained by Blackorby and Russell (1989), symmetry of pair-wise elasticities of substitution is unlikely to be a "natural property" in multi-input production, which implies that the Allen partial measure, with its symmetry property as discussed in Section 3.2.3, may not be suitable for multi-input technology cases.

In contrast to the Allen partial elasticities, the net Morishima elasticities between two inputs depend on which of the two input prices changes. A relationship can be substitutive in one direction but complementary in the other direction and the substitution effects depend on what price varies. Since η_{ij} and η_{jj} in the expression $\sigma_{ij}^M = \eta_{ij} - \eta_{jj}$ are price elasticities with respect to the same input j, σ_{ij}^M measures the difference in the percentage change in input i and percentage change in input j with respect to a change in the price of input j. Similarly, $\sigma_{ji}^M = \eta_{ji} - \eta_{ii}$ is the difference between the percentage change in input j and the percentage change in input i in response to a change in the price of input i. As established by Blackorby and Russell (1989), σ_{ij}^M and σ_{ji}^M are not equal to each other except for the cases of two-input and CES-Cobb-Douglas multiple-input production technologies. Moreover, as demonstrated by Chambers (1988), in a multiple-input case, σ_{ij}^M and σ_{ji}^M can have opposite signs, which implies that the direction of the Morishima relationship between two inputs depends on which of the two input prices changes. This asymmetry makes Morishima elasticities more intuitive.

3.4 Dual Models Using Revenue Functions

3.4.1 Optimisation Problem for Multi-product Production Technologies

When producers are assumed to adjust output mix to maximise the production revenue for a given input mix, the revenue function is dual to the underlying production function. For a multiple output technology represented by $h(X,Y)=0$, the dual revenue function is defined as $R(P,X)=\max_{Y}\{P'Y : Y\in U(X)\}$, where P is the vector of output prices, and $U(X)$ is the output production possibilities set. The revenue function should satisfy a set of regularity conditions (Diewert 1974 and Chambers 1988):

Condition R.1: $R(P,X)$ is nonnegative for $P>0$ and $X>0$;
Condition R.2: $R(P,X)$ is nondecreasing in P;
Condition R.3: $R(P,X)$ is continuous and convex in P;
Condition R.4: $R(P,X)$ is positively linearly homogeneous in P;
Condition R.5: $R(P,X)$ is nondecreasing in X; and
Condition R.6: $R(P,X)$ is differentiable in P so revenue-maximising output supplies can be derived by applying the Samuelson-McFadden lemma.

The economic interpretation of these conditions is similar to those for the cost function except that they are interpreted with respect to output prices instead of input prices. Also, the revenue function is convex in output prices, conditional on input levels, while the cost function is concave in input prices (conditional on output levels). Again, the last condition (Condition R.6) is not necessary for the revenue function to accurately portray a regular technology but is employed for mathematical and empirical convenience.

3.4.2 Input-constrained Revenue-maximising Output Supply Functions

When the regularity conditions hold, the Samuelson-McFadden lemma (Chambers 1988), a form of the Shephard-Hotelling lemma named due to contributions by Samuelson (1947) and McFadden (1978a) in extending the duality theory to the revenue function, can be applied to derive the revenue-maximising output supplies.

Samuelson-McFadden lemma

When the revenue function is differentiable and satisfies all the regularity conditions, there exists a unique mix of revenue-maximising output supplies which is determined by taking the partial derivatives of the revenue function with respect to the output prices:

$$y_k(P,X) = \frac{\partial R(P,X)}{\partial p_k},\; k = 1,\; 2,\ldots,\; m. \qquad (3.7)$$

Applying the Samuelson-McFadden lemma, the regularity conditions can be carried over to the derived output supplies. Condition R.2 is satisfied when $y_k(P,X) = \dfrac{\partial R(P,X)}{\partial p_k}$ is positive for all k while Condition R.3 means the matrix $\left[\dfrac{\partial y_k(P,X)}{\partial p_l} = \dfrac{\partial^2 R(P,X)}{\partial p_k \partial p_l}\right]$ $(k,\; l = 1,2,\ldots,\; m)$ is positive semidefinite. Condition R.4 is the same as the derived supplies $y_k(P,X)$, $k = 1,\; 2,\ldots,m$, being homogeneous of degree zero in input prices. Finally, Condition R.5 is satisfied if the shadow price of input $w_i(P,X) = \dfrac{\partial R(P,X)}{\partial x_i}$ is positive for all $i = 1,2,\ldots n$. If the revenue function is further assumed to be twice-continuously differentiable, the symmetry condition: $\frac{\partial y_k(P,X)}{\partial p_l} = \frac{\partial y_l(P,X)}{\partial p_k}$, $(k,\ l = 1, 2, \ldots, m)$, is implied.

3.4.3 Measures of Economic and Technical Interest

Again, with a further assumption that the dual revenue function is twice-continuously differentiable, unit-free elasticities similar to those in the dual cost function analysis can also be derived in the analysis of the dual revenue function. The net (input-constrained) price elasticity of output l with respect to price of output k is formulated as

$$\varepsilon_{kl} = \frac{\partial y_k(P,X)}{\partial p_l}\frac{p_l}{y_k(P,X)}. \tag{3.8}$$

It is reasonable to expect that the formulas for the net transformation elasticities will be the same as in the case of the cost function. However, the proof of this for the single-input technology case will be presented here for completeness. The generalisation to the multi-input technology case will be straightforward.

Assuming that we have the single-input production technology characterized by $x = g(Y)$, where $g(Y)$ is the input requirement function that represents the minimum input level required to produce a given output mix, the net Allen partial elasticity of transformation is defined as:

$\tau_{kl} = \frac{\sum_{r=1}^{m} y_r g_r}{y_k y_l}\frac{G_{lk}}{G}$, where G is the bordered Hessian determinant

$$G=\begin{vmatrix} 0 & g_1 & g_2 & \cdots & g_m \\ g_1 & g_{11} & g_{12} & \cdots & \\ g_2 & \cdots & \cdots & & \\ \vdots & \vdots & \vdots & & \vdots \\ g_m & g_{1m} & g_{2m} & \cdots & g_{mm} \end{vmatrix}$$ and G_{lk} is the cofactor associated with element g_{lk}.

The derivation of the formula for this elasticity in terms of the revenue function is analogous to the derivation of the net Allen partial elasticities of substitution for the cost function case presented in Binswanger (1974a) and Uzawa (1962). Given the problem of maximising the revenue $\sum_{k=1}^{m} p_k y_k$, subject to production requirement condition $x = g(Y)$, the first order conditions are:

$$\begin{cases} g(Y) - x = 0 \\ p_k - \ell g_k = 0 \end{cases},$$

where ℓ is the Lagrangian multiplier and $k = 1, 2..., m$.

The matrix representation of the total differential of these conditions is

$$\ell\begin{bmatrix} 0 & g_1 & g_2 & \cdots & g_m \\ g_1 & g_{11} & g_{12} & \cdots & g_{1m} \\ g_2 & \cdots & \cdots & & g_{2m} \\ \vdots & \vdots & \vdots & & \vdots \\ g_m & g_{1m} & g_{2m} & \cdots & g_{mm} \end{bmatrix}\begin{bmatrix} d\ell/\ell \\ dy_1 \\ dy_2 \\ \vdots \\ dy_m \end{bmatrix} = \begin{bmatrix} \ell dx \\ dp_1 \\ dp_2 \\ \vdots \\ dp_m \end{bmatrix}.$$

Solving for $d\ell/\ell$ and dy_k $(k = 1, 2..., m)$, we have:

$$\begin{bmatrix} d\ln\ell \\ dy_1 \\ dy_2 \\ \vdots \\ dy_m \end{bmatrix} = \frac{1}{\ell}\frac{G_{lk}}{G}\begin{bmatrix} \ell dx \\ dp_1 \\ dp_2 \\ \vdots \\ dp_m \end{bmatrix},$$ which means $\frac{\partial y_l}{\partial p_k} = \frac{1}{\ell}\frac{G_{lk}}{G}$. Substituting this last expression into the general formula of the net elasticities we have:

$$\tau_{kl} = \frac{\sum_{r=1}^{m} y_r g_r}{y_k y_l} \frac{G_{lk}}{G} = \frac{\sum_{r=1}^{m} y_r g_r}{y_k y_l} \times \ell \times \frac{\partial y_l}{\partial p_k}$$

As we have $g_l = \frac{p_l}{\ell}$ from the second equation of the first order conditions (1), the previous expression leads to $\tau_{kl} = \frac{\sum_{r=1}^{m} y_r p_r}{y_k y_l} \frac{\partial y_l}{\partial p_k}$.

Applying the Samuelson-McFadden lemma results in $\frac{\partial y_l}{\partial p_k} = \frac{\partial^2 R(P,X)}{\partial p_k \partial p_l}$. Replacing this equality into the preceding expression of τ_{kl}, we obtain

$$\tau_{kl} = \frac{\sum_{r=1}^{m} y_r p_r}{y_k y_l} \frac{\partial^2 R(P,X)}{\partial p_k \partial p_l} = \frac{R(P,X)}{y_k(P,X) y_l(P,X)} \frac{\partial^2 R(P,X)}{\partial p_k \partial p_l}. \tag{3.9}$$

Similar to the cost function, the input-constrained price elasticities and Morishima elasticities of transformation between output k and output l have the following relationship: $\tau_{kl} = \frac{\varepsilon_{kl}}{r_l}$, and $\tau_{kl}^M = \varepsilon_{kl} - \varepsilon_{ll}$, where r_l is the revenue share of output l, and τ_{kl}^M is the net Morishima elasticity of transformation of output k with respect to (the price of) output l [1].

3.5 Dual Models Using Profit Functions

3.5.1 Optimisation Problem for Multi-product Production Technologies

Let Ω be the production possibilities set for a multiple output technology represented by the function $h(X,Y) = 0$ with input and output price vectors W and P. The profit function dual to this production function is defined as:

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[1] Although Morishima elasticity was originally derived for the cost function, this name is also used here for ease of reference.

$$\pi(W,P) = \max_{X,Y}\left\{P'Y - W'X : (Y,X) \in \Omega;\ W,P > 0\right\}.$$

The set of regularity conditions this profit function satisfies consists of (Lau 1972; Jorgenson and Lau 1974b and Chambers 1988):

Condition P.1: $\pi(W,P)$ is nonnegative for $W > 0$ and $P > 0$;

Condition P.2: $\pi(W,P)$ is nonincreasing in W;

Condition P.3: $\pi(W,P)$ is nondecreasing in P;

Condition P.4: $\pi(W,P)$ is convex and continuous in all of its arguments; and

Condition P.5: $\pi(W,P)$ is positively linearly homogeneous in W and P.

In addition to these conditions, $\pi(W,P)$ is normally assumed to be continuously differentiable in W and P so Hotelling's lemma can be applied to derive the profit maximising output supplies and input demands, and price and substitution elasticities can be extracted from the derived output supplies and input demands.

3.5.2 Profit-maximising Input Demand and Output Supply Functions

Similar to the case of the dual cost and revenue functions, the profit-maximising input demands and output supplies can be derived by applying the Hotelling's lemma.

Hotelling's lemma

Under the regularity and twice-continuous differentiability conditions, there exists a unique set of profit-maximising input demands and output supplies, which are determined by taking the partial derivatives of the profit function with respect to input and output prices:

$$-x_i(W,P) = \frac{\partial \pi(W,P)}{\partial w_i},\ i = 1,\ 2,\ \ldots,\ n, \tag{3.9}$$

and

$$y_k(W,P) = \frac{\partial \pi(W,P)}{\partial p_k},\ k = 1,\ 2,\ \ldots,\ m, \tag{3.10}$$

where $x_i(W,P)$ and $y_k(W,P)$ are profit-maximising input and output quantities.

The derived input demands and output supplies inherit properties from the properties of the profit function as in the case of the dual cost and revenue function. Condition P.2 implies $-x_i = \frac{\partial \pi(W,P)}{\partial w_i}$ has to be nonpositive (the input demands are nonnegative) and Condition P.3 implies $y_k = \frac{\partial \pi(W,P)}{\partial p_k}$ has to be nonnegative. When $\pi(W,P)$ is twice-continuously differentiable, Hotelling's lemma and Young's theorem imply the symmetry condition: $\frac{\partial x_i(W,P)}{\partial w_j} = \frac{\partial x_j(W,P)}{\partial w_i}$; $\frac{\partial y_k(W,P)}{\partial p_l} = \frac{\partial y_l(W,P)}{\partial p_k}$, and $\frac{\partial x_i(W,P)}{\partial p_k} = \frac{-\partial y_k(W,P)}{\partial w_i}$. With the additional condition of twice-continuously differentiability, Condition P.4 of convexity in prices leads to the restriction that the Hessian matrix, whose elements are the second derivatives of the profit function with respect to input and output prices, must be positive semidefinite. This implies that the matrix of slopes of derived input demands and output supplies

$$\left[\begin{array}{cccc|cccc}
\frac{\partial y_1(W,P)}{\partial p_1} & \frac{\partial y_1(W,P)}{\partial p_2} & \cdots & \frac{\partial y_1(W,P)}{\partial p_m} & \frac{\partial y_1(W,P)}{\partial w_1} & \frac{\partial y_1(W,P)}{\partial w_2} & \cdots & \frac{\partial y_1(W,P)}{\partial w_n} \\
\frac{\partial y_2(W,P)}{\partial p_1} & \frac{\partial y_2(W,P)}{\partial p_2} & \cdots & \frac{\partial y_2(W,P)}{\partial p_m} & \frac{\partial y_2(W,P)}{\partial w_1} & \frac{\partial y_2(W,P)}{\partial w_2} & \cdots & \frac{\partial y_2(W,P)}{\partial w_n} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\frac{\partial y_m(W,P)}{\partial p_1} & \frac{\partial y_m(W,P)}{\partial p_2} & \cdots & \frac{\partial y_m(W,P)}{\partial p_m} & \frac{\partial y_m(W,P)}{\partial w_1} & \frac{\partial y_m(W,P)}{\partial w_2} & \cdots & \frac{\partial y_m(W,P)}{\partial w_n} \\
\hline
\frac{-\partial x_1(W,P)}{\partial p_1} & \frac{-\partial x_1(W,P)}{\partial p_2} & \cdots & \frac{-\partial x_1(W,P)}{\partial p_m} & \frac{-\partial x_1(W,P)}{\partial w_1} & \frac{-\partial x_1(W,P)}{\partial w_2} & \cdots & \frac{-\partial x_1(W,P)}{\partial w_n} \\
\frac{-\partial x_2(W,P)}{\partial p_1} & \frac{-\partial x_2(W,P)}{\partial p_2} & \cdots & \frac{-\partial x_2(W,P)}{\partial p_m} & \frac{-\partial x_2(W,P)}{\partial w_1} & \frac{-\partial x_2(W,P)}{\partial w_2} & \cdots & \frac{-\partial x_2(W,P)}{\partial w_n} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\frac{-\partial x_n(W,P)}{\partial p_1} & \frac{-\partial x_n(W,P)}{\partial p_2} & \cdots & \frac{-\partial x_n(W,P)}{\partial p_m} & \frac{-\partial x_n(W,P)}{\partial w_1} & \frac{-\partial x_n(W,P)}{\partial w_2} & \cdots & \frac{-\partial x_n(W,P)}{\partial w_n}
\end{array}\right]$$

must be positive semidefinite. Finally, Condition P.5 implies that derived output supplies and input demands are positively homogeneous of degree zero in P and W, or stated mathematically $y_k(W,P) = y_k(tW,tP)$ and $x_i(W,P) = x_i(tW,tP)$ for all $t > 0$.

3.5.3 Measures of Economic and Technical Interest

The derivation of gross price elasticities is analogous to the cases of the dual cost and revenue functions. The elasticity of demand for input i $\left(i = 1,2,...n\right)$ with respect to price of input j $\left(j = 1,2,...n\right)$ is:

$$\xi_{ij} = -\frac{\partial x_i(W,P)}{\partial w_j}\frac{w_j}{x_i(W,P)}. \tag{3.11}$$

The elasticity of supply of output k $\left(k = 1,2...,m\right)$ with respect to price of output l $\left(l = 1,2...,m\right)$ is:

$$\xi_{kl} = \frac{\partial y_k(W,P)}{\partial p_l}\frac{p_l}{y_k(W,P)}. \tag{3.12}$$

Finally, the cross-price demand elasticities of input i $\left(i = 1,2,...n\right)$ with respect to price of output k $\left(k = 1,2...,m\right)$ are:

$$\xi_{ik} = -\frac{\partial x_i(W,P)}{\partial p_k}\frac{p_k}{x_i(W,P)}. \tag{3.13}$$

The economic interpretations of the price elasticities here, however, are different to those of the net elasticities derived from the cost and revenue functions. These elasticities represent the gross effects, i.e., they include both substitution and expansion effects, as the quantities of both inputs and outputs are allowed to vary in response to price changes. These elasticities are not conditional on output or input quantity as in the case of the cost and revenue functions where the outputs are exogenous in the former and the inputs are exogenous in the latter.

When a profit function is estimated, the substitution and expansion effects in input and output responses can be separated. Lopez (1984) and Chambers (1988) provide the derivation of the *compensated* price elasticities of input demand and output supply, which are comparable to the net elasticities obtained from the cost and revenue functions, using only estimates of the profit function. The term "compensated" is used here to make a distinction between the *net* elasticities indirectly obtained from the profit function and the comparable net elasticities obtained from the cost and revenue functions. In calculating the *compensated* price elasticities of input demand, all outputs are assumed to be fixed when an input price changes. They are, therefore, measures of substitutability that are net of adjustments in output quantities. Analogously, in the calculation of the *compensated* price elasticities of output supply, all inputs are assumed to be fixed when an output price changes. They are measures of output transformability that are net of input effect. It is in this sense that the *compensated* elasticities are *net* and are comparable to the net elasticities obtained from the cost and revenue functions.

Let $A_1 = \left[\frac{\partial y_k(W,P)}{\partial p_l}\right]_{m\times m}$ be the matrix of responses of *output* supplies to *output* price changes, $A_2 = \left[\frac{\partial y_k(W,P)}{\partial w_i}\right]_{m\times n}$ the matrix of responses of *output* supplies to *input* price changes, $A_3 = \left[\frac{-\partial x_i(W,P)}{\partial p_k}\right]_{n\times m}$ the matrix of responses of *input* demands to *output* price changes and $A_4 = \left[\frac{-\partial x_i(W,P)}{\partial w_j}\right]_{n\times n}$ the matrix of responses of *input* demands to *input* price changes. Then, the *compensated* price responses of input demand and the gross input demand price responses have the following relation:

$$\left[\frac{\partial^2 C(W,Y)}{\partial w_i \partial w_j}\right]_{n\times n} = -A_4 + A_2^T \times A_1^{-1} \times A_2 . \qquad (3.14)$$

Similarly, the *compensated* price responses of output supply are related to the gross output supply price responses as in the following equality:

$$\left[\frac{\partial^2 R(P,X)}{\partial p_k \partial p_l}\right]_{m\times m} = A_1 - A_3^T \times A_4^{-1} \times A_3. \tag{3.15}$$

These *compensated* price response equalities can be expressed in terms of unit-free elasticities using the definition (3.5).

The gross Allen partial elasticities of substitution/transformation between input i and input j ($i, j = 1, 2, ...n$), between output k and output l ($k, l = 1, 2, ...m$), and between input i ($i = 1, 2, ...n$) and output k ($k = 1, 2, ...m$) respectively, are given as:

$$\kappa_{ij}^{I} = -\frac{\pi(W,P)}{x_i(W,P)\, x_j(W,P)} \frac{\partial^2 \pi(W,P)}{\partial w_i \partial w_j}\ , \tag{3.16}$$

$$\kappa_{kl}^{O} = \frac{\pi(W,P)}{y_k(W,P)\, y_l(W,P)} \frac{\partial^2 \pi(W,P)}{\partial p_k \partial p_l}, \tag{3.17}$$

and

$$\kappa_{ik}^{IO} = -\frac{\pi(W,P)}{x_i(W,P)\, y_k(W,P)} \frac{\partial^2 \pi(W,P)}{\partial w_i \partial p_k}. \tag{3.18}$$

The gross Morishima elasticities are expressed similarly to their net measures as: $\kappa_{ij}^{M,I} = \xi_{ji} - \xi_{ii}$, where $i, j = 1, 2, ...n$; $\kappa_{kl}^{M,O} = \xi_{lk} - \xi_{kk}$, where $k, l = 1, 2, ...m$; and $\kappa_{ik}^{M,IO} = \xi_{ki} - \xi_{ii}$, where $i = 1, 2, ...n$ and $k = 1, 2, ...m$ (Blackorby *et al.* 2007). [2]

It is important to note the differences of the elasticities of substitution and transformation derived from different dual objective functions above. The elasticities drawn from the cost function are termed net elasticities in order to explicitly indicate that they represent substitution possibilities net of the output effect (Blackorby *et al.*

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[2] Although Morishima elasticity was originally derived for the cost function, this name is used for the profit function here, in conjunction with "net" and "gross", to differentiate between cost, revenue and profit functions, for ease of reference. Davis and Shumway (1996) named this measure differently as factor ratio elasticity of substitution but, in essence, it is Morishima.

2007 and Bertoletti 2005). They are also categorised as compensated or Hicksian measures (Gordon 1989 and Lopez 1984). Similarly, elasticities of transformation generated from the revenue function are net of input effect. In contrast, the substitution/transformation elasticities obtained in the profit function estimation are gross or uncompensated elasticities (or Marshallian elasticities in Gordon 1989; Lopez 1984 and Squires 1987) since they represent the combined effect of adjustments in both inputs and outputs. However, similar to the *compensated* price elasticities from (3.14) and (3.15), the *compensated* Allen and Morishima elasticities can be computed after a profit function is estimated.

3.6 The Restricted Dual Models

Production operations normally involve different inputs that have different adjustment timeframes. Many of these inputs, typically capital or durable items, can only be adjusted over time periods that are longer than the normal production cycles due to their lumpy nature and heavy investment requirements. Because of these restrictions, producers cannot adjust the quantities of inputs instantaneously in response to market price changes. When data on production are reported at time intervals shorter or equal to production cycles, these inputs become fixed or quasi-fixed in nature.

To account for the fixity of some of the production inputs, the dual objective functions are sometimes specified in their restricted (or variable) forms. The dual functions are restricted, within the timeframe considered, due to there being no adjustments in fixed (and quasi-fixed) input quantities, despite changes in their prices. The dual restricted functions represent short-run optimisation problems as opposed to the long-run problems where all the fixed inputs are optimally adjusted. According to these specifications, producers adjust the levels of variable inputs and outputs conditional on given levels of fixed inputs. Fixed input quantities are considered exogenous in such specifications. Therefore, unlike the treatment of variable inputs, the quantities of the fixed inputs rather than their prices appear on the right hand side as the explanatory variables in dual restricted cost and profit functions. The derived input demands and output supplies derived from these two dual functions are thus conditional on the levels of fixed inputs.

Let Z be the vector of the fixed-input quantities. The general representations of the dual restricted cost function in Section 3.3.1 and of the dual restricted profit function in Section 3.5.1 become

$$C(W,Y,Z) = \min_X \{W'X : X \in V(Y,Z)\} \text{ and}$$

$$\pi(W,P,Z) = \max_{X,Y} \{P'Y - W'X : (Y,X) \in \Omega;\ W,P > 0\},$$

where X represents variable inputs only. Although the general representation of the dual restricted revenue function becomes $R(P,X,Z) = \max_Y \{P'Y : Y \in U(X,Z)\}$, essentially there is no differentiation between the variable and fixed inputs since the levels of outputs supplied are conditional on the variable inputs as well as the fixed inputs. The imputed value of an incremental unit of a fixed input, normally termed the 'shadow price' of that input, can be derived as the first derivative of the dual restricted objective function with respect to the quantity of that input (Paris 1989; Chambers and Just 1989 and McKay *et al.* 1983). The regularity conditions for each of the dual restricted functions are analogous to those described and discussed for the dual unrestricted functions in the preceding sections.

3.7 Summary

This chapter lays out the three common duality-based model formulations in studying multiple-output, multiple-input production technologies. Production problems of cost minimisation, revenue maximisation and profit maximisation are formed. The dual cost, revenue or profit functions representing these optimisation problems are defined in their general forms as functions of prices and/or quantities. A set of theoretical regularity conditions and their economic interpretation are described for each of the three dual objective functions. The optimizing supply and demand functions are then derived by applying the Shephard, Samuelson-McFadden and Hotelling lemmas. The derived input supplies are output-constrained in the case of the cost minimisation problem and the derived output supplies are input-constrained in the case of the revenue maximisation problem.

Price elasticities of supply and demand and elasticities of substitution and transformation between inputs and outputs are defined for each of the three duality-based model formulations described in this chapter. In the dual cost analysis, these elasticities are net measures since they are net of adjustments in output levels. Similarly, the elasticities defined for the dual revenue function are net of adjustments on input levels. The elasticities defined in the case of the dual profit function are gross measures, showing the combined effects of adjustments in both inputs and outputs. Finally, the restricted dual objective functions are defined for cases in which some production factors are fixed during normal production cycles.

Chapter 4

An Overview of Australian Broadacre Agriculture and Data

4.1 Introduction

The objectives of this chapter are threefold. Firstly, this chapter presents an overview of Australian broadacre production, the empirical context to which the duality framework described in Chapter 3 will be applied. This overview covers key characteristics of broadacre production that have critical implications for model specification. Many features specific to Australian broadacre agricultural production, such as the strong reliance of production on rainfall and the operation of multiple enterprises on farms, have to be addressed in the econometric specification of the models to achieve sound estimation results.

The second objective of this chapter is to provide a detailed description of the large unique quasi-micro dataset of Australian broadacre production used in this study. This quasi-micro dataset sets this study apart from previous studies of agricultural production in Australian and overseas. Farm-level data are often unavailable for studies conducted outside government research bodies. The latest and only published study of Australian broadacre agricultural production using farm-level data is the ABARE study by Kokic *et al.* (1993) for the period from 1981 to 1991. In contrast, the quasi-micro dataset used in this thesis covers the period from 1990 to 2005 (ABARE 2007). This quasi-micro dataset was drawn from the AAGIS data and was formed in a way so as to retain the micro-level nature of farm production as much as possible, while still maintaining the confidentiality requirement. In retaining the micro-level nature of the data, the impacts of cross-farm aggregation on research findings that most previous applications are subject to, are circumvented. The quasi-micro nature of this unique dataset also allows the data sampled across time to be pooled together to create a sample that is advantageously larger than most previous duality applications to Australian and international agricultural production. At the same time, the unique quasi-micro nature of this data raises significant econometric implications that have often been encountered in previous agricultural applications. These issues have to be addressed during estimation. Owing to the uniqueness and important implications of this data, the scheme by which it is formed will be described in detail in this chapter.

The third objective of this chapter is to provide a detailed description of the exogenous and endogenous variables to be included in the econometric models of Australian broadacre agriculture estimated in this study. These variables are the aggregate variable inputs, aggregate fixed inputs and aggregate outputs of broadacre agriculture. They are formed after a careful process of aggregating across numerous input and output items reported in the original AAGIS dataset. These aggregate variables will be included in all models estimated in this thesis.

The remainder of this chapter is divided into five sections. Section 4.2 describes Australian broadacre production with a focus on characteristics that have important implications for econometric modelling. Section 4.3 follows with a detailed description of the AAGIS quasi-micro dataset of Australian broadacre agriculture used in this study. Section 4.4 provides definitions of variables included in models estimated in the subsequent chapters. The chapter finishes with a summary in Section 4.5.

4.2 An Overview of Australian Broadacre Agricultural Production

Broadacre agricultural production continues to contribute significantly to the Australian economy despite its diminishing contribution to total economic output over the last few decades. Broadacre agriculture is prevalent across the country and is dominated by multi-product producers. The sector is spread across three distinct broadacre zones with varying rainfall and topographical conditions that determine farm operation types. Broadacre farmers generally have some flexibility in adjusting their output mixes within reasonably short time periods in response to weather and market changes. Compared to other sectors in the Australian economy, broadacre farmers predominantly run small family businesses in competitive input and output markets.

4.2.1 The Role of Australian Broadacre Agriculture

Taking a macro perspective, broadacre production dominates Australian agricultural production and contributes significantly to the Australian economy. Broadacre agriculture accounts for around three per cent of gross domestic product and the Australian labour force. In the 2008 financial year, broadacre farming had a total production value of $41.2 billion and employed 300,000 people (ABARE 2008). Export of broadacre commodities is one of Australia's main sources of foreign income. Major commodities produced in broadacre production are coarse grains, oilseeds, wool, beef and sheep meat. Beef cattle production has been increasingly popular with a total annual production, domestically slaughtered or exported live, of $7.6 billion in 2008 ABARE (2008). The importance of grains and oilseeds has diminished despite continuous production increases. Collectively, the value of these crops summed to $9.0 billion in 2008. The role of wool in broadacre agriculture has diminished significantly over the last two decades but this product remains popular; the value of wool exported in 2008 was $2.8 billion. Lamb production, in contrast, doubled between 2001 and 2008 (ABARE 2005, 2008).

4.2.2 Operational Conditions of Australian Broadacre Agriculture

Broadacre farms are present in most parts of Australia except for the dry central areas, where there is not enough rainfall, and the narrow eastern coasts, where intensive livestock grazing and cropping on smaller scales are feasible. The production technology employed on broadacre farms is strongly conditioned by climatic and topographical conditions. Broadacre farms are normally large farms, running extensive cropping and grazing operations that rely heavily on natural rainfall for soil moisture and require reasonably flat topography to utilise large-scale machinery in cropping activities.

Broadacre farmers have adapted to Australian climatic and topographical conditions by a common practice of producing a mixture of two or more products at the same time. The practice of simultaneously producing different products helps farmers efficiently utilise their different production resources and manage production risks, such as those caused by unexpected weather events or soil degradation. Broadacre farmers have some flexibility in adjusting their output mix within a reasonably short time to respond to market price movements and the unfolding of weather conditions. These farmers also have to make optimal decisions about the mix of different broadacre products, since these products can either complement or substitute for each other in a normal multi-enterprise production operation.

By international standards, Australia suffers from low and highly variable rainfall. Rainfall varies significantly not just through time but also across the continent during a year. The influence of rainfall is profound for broadacre cropping and grazing activities due to their sole reliance on natural rainfall for soil moisture. Moreover, even when rainfall is sufficient, extensive broadacre cropping requires deployment of machinery to be economically viable. As a result, production systems employed by broadacre farmers vary across geographical areas depending on climatic and topographical conditions of the land surface.

Production systems with typical product mixes have been formed within geographical areas having similar climatic and topographical conditions. These conditions vary significantly from north to south and from coastal to central areas of the continent. The northern parts of the country have a summer-dominant rainfall climate, while the southern parts have a winter-dominant rainfall climate. Rainfall, one of the most important determining physical factors in broadacre production, decreases significantly from coastal to central areas. In Australia, typical broadacre production operations have been adopted within the same rainfall belt formed in coastal-inland direction across six states. Three broadacre production zones have been identified by ABARE according to their rainfall conditions and the types of broadacre production regimes adopted. These broadacre zones are the Pastoral zone, Wheat-Sheep zone and High Rainfall zone, as shown in Figure 1 (ABARE 2003).

4.2.2.1 Pastoral Zone

The Pastoral zone covers the majority of Australia's land mass from central areas outward. Due to this zone's arid and semi-arid conditions, the only economically feasible production regime is extensive cattle or sheep grazing on native pastures (ABARE 2003). Farms within some regions of the Pastoral zone only carry out cattle grazing on native pasture.

4.2.2.2 Wheat-Sheep Zone

Adjacent to the Pastoral zone is the Wheat-sheep zone (ABARE 2003). A mixture of wheat (and other coarse grains) and sheep products are typically produced on farms in this zone. This is because the Wheat-Sheep zone is characterised by higher rainfall than the Pastoral zone, sufficient for grain production, and flat topography that allows specialised cropping machines to be profitably utilised. This broadacre zone contributes the largest share of total broadacre production, despite a smaller land area compared to the Pastoral zone.

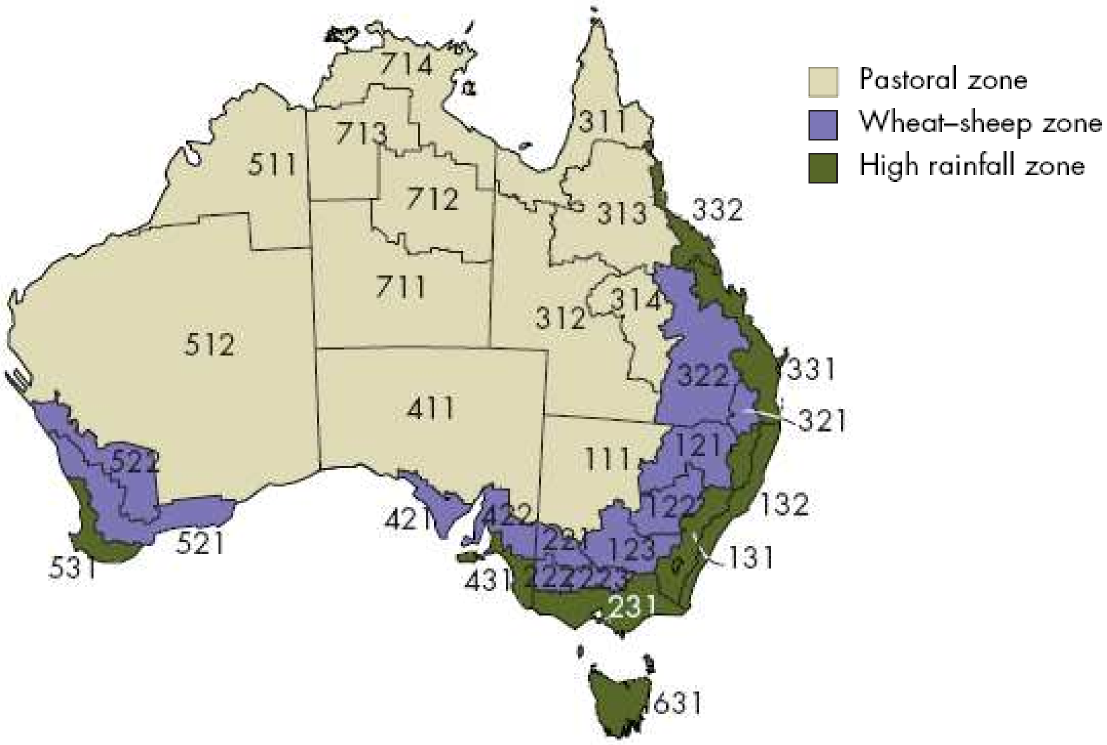

Figure 1: Australian Broadacre Agricultural Production Zones (ABARE 2003)

4.2.2.3 High Rainfall Zone

The third broadacre zone identified by ABARE is the High Rainfall zone (ABARE 2003). This broadacre zone comprises the eastern and south-western coastal seaboards, parts of southern coastal seaboard and Tasmania. This zone receives more rainfall than either the Wheat-Sheep or Pastoral zones, with fairly humid weather conditions. Farms in the High Rainfall zone are smaller than those in the Wheat-Sheep zone. In some regions in the High Rainfall zone, higher levels of rainfall and steeper topography limit large-scale grain production but allow for economically feasible non-broadacre cropping and intensive grazing activities.

4.2.2.4 Broadacre Production Regions

A number of production regions are identified by ABARE for each broadacre zone for survey sample stratification. The three broadacre zones are comprised of 32 broadacre production regions. Farms within each region run similar enterprise mixes. These enterprise mixes can be roughly identified as belonging to one of five broadacre production types defined by ABARE (2003). Farms that mainly grow cereal grains, coarse grains, oil seeds and/or pulses fall into the 'wheat and other crops' category. Farms that produce these crops and simultaneously graze livestock constitute the 'mixed livestock-crops' group. Farms that focus mainly on sheep grazing only and on beef grazing only are correspondingly classified as 'sheep specialists' and 'beef specialists'. Finally, farms that run both sheep and beef grazing enterprises fall into the 'mixed beef-sheep' category.

4.2.3 Competitiveness of Broadacre Agriculture Markets

Australian broadacre farmers generally operate in competitive input and output markets. Being supported by well-developed information and transport infrastructures, broadacre farmers can access input and output markets fairly quickly in response to changes in market and weather conditions to optimise their production objectives. The majority of broadacre products are exported, while a large proportion of broadacre production inputs are imported. However, Australian broadacre farmers are price takers, being small suppliers and consumers in the international commodities markets. The prices of Australian broadacre inputs and outputs are, therefore, heavily influenced by forces in international markets. As a result, Australian broadacre farmers have little or no power in influencing buying prices for production inputs or the selling prices of their products.

In summary, Australian broadacre agriculture is prevalent and diverse across the continent. Broadacre farmers rely solely on natural rainfall for soil moisture required for crop and livestock production. They typically produce different broadacre products on their farms to best manage production risks and utilise their production resources. In general, broadacre farming is spread across three broadacre zones characterised by significantly different climatic and topographic conditions. Farmers in a broadacre zone typically produce a similar mix of broadacre outputs. Broadacre farms are dominated by family-run businesses, having little or no influence over their input and output prices.

4.3 The Australian Agricultural and Grazing Industries Survey Data

A unique, pooled cross-sectional dataset of Australian broadacre agricultural production is used for estimation in this study. This dataset covers broadacre farms across the three broadacre zones in Australia over the period from 1990 to 2005. This period is much more current than previous studies of Australian broadacre agricultural production. This dataset also has a quasi-micro nature, making it crucially different to datasets typically used in previous applications in agricultural production in Australia and overseas. The quasi-micro nature of the data allows the formation of a large, pooled cross-sectional dataset, enhancing the statistical significance of the model estimation.

4.3.1 The Formation of the Quasi-Micro Farm Data

The pooled cross-sectional quasi-micro data are provided by ABARE and are drawn from the AAGIS. The AAGIS survey collects detailed annual information on input costs, output receipts and quantities, as well as values and quantities of invested capital of farm businesses across Australia that have an estimated annual farming value of $22,500 or more. This survey data underlies the AgSurf broadacre farming data published by ABARE on its official website for research and monitoring purposes. The list and definition of variables included in the original dataset is presented in Appendix A.

The quasi-micro data was provided by ABARE through specific request and is uniquely formatted for this study. Farm-level data are more desirable but not available due to confidentiality constraints enforced by ABARE. Therefore, the data are formatted so that the confidentiality of individual farms is maintained while the number of data points available to the study is maximised. This effectively retains the micro-level nature of the data as much as possible. Maintaining a micro-level nature while maximising sample size is achieved by dividing all farms surveyed in a particular year into small homogeneous groups and then computing their average data. The average data of a group (interchangeably data cell, farm cell or observational unit) is made available for this study if the group has at least five farms.

In the final outcome of the investigation of the data, farms in each surveyed year are assigned into data cells using three criteria. These criteria are production region, broadacre industry and production size. For each survey year, farms in the same production region, same broadacre industry and having the same production size form a data cell:

1. Farms are initially assigned to data cells based on the production region they are situated in. Farms in the same production region are grouped together. For the whole data sample, there are thirty-two broadacre regions across the three broadacre zones.
2. Farms in the same region are then categorised as Cropping or Livestock farms, based on the relative share of revenue they receive from these two activities. These two broader broadacre industry categories are formed here instead of the five categories (i.e. Wheat and other crops, Mixed livestock–crops, Sheep specialist, Beef specialist, and Mixed beef-sheep) traditionally defined by ABARE. This is due to the fact that when the five traditional industry categories are used, too many data cells with less than five farm members would occur in the final sample. The average data of these undersized data cells cannot be released by ABARE and the number of data cells in the final sample available

for estimation is substantially reduced. By classifying farms into cropping and grazing industry categories, a larger sample is formed.

3. The final criterion used to form farm data cells is the individual farm size. In this last step, farms in the same industry and same production region are divided into three categories, based on their sizes of operation. The size of a farm is determined by the level of total cash revenues it received during the surveyed year. Farms are classified as large if total annual receipts are at least $400,000, medium if annual receipts are between $200,000 and $400,000 and small if annual receipts are $200,000 or less.

For each farm cell formed using the categories described above, average data are calculated for each of sixteen years from 1990 to 2005. Allowing for the withholding of data due to confidentiality constraints and the late reporting commencement for some regions, a sample of 1559 data points was provided, making up the original quasi-micro dataset. Since this study estimates multi-product production technologies employed by Australian broadacre farmers, observations in the original AAGIS quasi-micro data that appears to be of single-product farms are removed from the sample used for estimation in this study. These single-output observations have been identified through consulting experienced researchers in the broadacre agricultural production as well as examining the mixture of outputs produced by farms observed in the data. The final data sample used in this study consists of 1,343 observations spread over 22 broadacre regions of the three broadacre zones. By operational size, there are 455 large farms, 411 medium farms and 477 small farms. Taking an industry-wide perspective, the sample is made up of 619 observations in the cropping industry and 724 in the livestock industry. Since the identification of a broadacre region implies a broadacre zone, the observations in the sample can also be identified by broadacre zone. The sample contains 833 observations in the Wheat-Sheep zone, 309 in the High Rainfall zone and 201 observations in the Pastoral zone.

4.3.2 Selective Statistics of the Quasi-Micro Farm Data

Some selective statistics calculated from the AAGIS data are presented in Tables 2, 3 and 4. These tables display the average costs and receipts of important broadacre inputs and outputs in different production zones, industries and sizes. The tables show potential differentiation of production technologies by broadacre farmers across broadacre zones, broadacre industries and operation scales. For instance, Table 2 indicates significant differences in patterns of farming costs and revenues across production zones. Farms in the Pastoral zone pay a higher proportion of the total production costs for freighting and hiring labour than those in the Wheat-Sheep and High Rainfall zones. Almost 82 per cent of total farm revenue in the Pastoral zone is generated from livestock grazing activities and 51.4 per cent from beef-cattle alone. In contrast, farms in the Wheat-Sheep zone earn roughly equal amounts from cropping, beef cattle grazing and combined sheep and wool production. Meanwhile, broadacre farms in the High Rainfall zone rely as heavily on grazing activities, albeit more on sheep grazing, as those in the Pastoral zone. These farms receive 73.4 per cent of the total revenue from sheep, beef and wool production together.

The cost and revenue compositions differ markedly between Cropping farms and Livestock grazing farms. As shown in Table 3, there are large differences in cost shares of contracts, fertilisers, crop and pasture chemicals, and fuel, oil and grease between Cropping and Livestock grazing farms. Cropping farms, on average, also pay much higher costs for fertilisers and crop and pasture chemicals than Livestock farms. Interestingly, only minor differences exist between these two farm types in interest payments and handling and marketing expenses. On the output side, Cropping farms receive approximately 27.7 per cent of their total revenue from sheep, beef and wool combined, despite their focus on cropping activities. In contrast, on average Livestock farms generate 84.5 per cent of their total revenue from these three products.

There are some distinctions in input and output compositions between farm operations of different scales. The revenue share of wheat production is positively associated with farm size, increasing from around eleven per cent for small farms to seventeen per cent for large farms. The share of sheep and wool production, in contrast, decreases significantly from 34 per cent for small and medium farms to around 22 per cent for large farms. Interestingly, the share of beef production is smallest for medium farms. On the input side, cost shares of interest payments and fuel, oil and grease expenses decrease as farm size increases although these costs remain significant at all operational scales. Hired labour cost makes up a larger share of total cost for large farms than for smaller farms.

Table 2: Average Values of Selected Inputs and Outputs by Broadacre Zone[*]

	Pastoral zone		Wheat-Sheep zone		High Rainfall zone	
	Costs & receipts ($)	Cost or receipt share	Costs & receipts ($)	Cost or receipt share	Costs & receipts ($)	Cost or receipt share
Contracts for cropping	2,064	0.5%.	7,702	2.5%	5,224	2.0%
Contracts for livestock	5,764	1.4%	912	0.3%	901	0.3%
Fertilisers	3,588	0.8%	25,415	5.9%	22,063	3.3%
Crop and pasture chemicals	3,096	0.9%	18,041	8.4%	8,510	8.6%
Fuel, oil and grease	24,581	6.0%	21,432	7.0%	13,908	5.4%
Interest	39,429	9.6%	27,431	9.0%	25,513	9.9%
Seed	1,324	0.3%	4,274	1.4%	3,174	1.2%
Handling and marketing	28,420	6.9%	20,350	6.7%	17,019	6.6%
Total freight	23,619	5.8%	13,619	4.5%	7,670	3.0%
Hired labour wages	31,092	7.6%	12,240	4.0%	13,581	5.3%
Wheat gross receipts	27,884	5.2%	91,848	23.4%	15,672	4.6%
Sheep gross receipts	28,208	5.3%	32,622	8.3%	41,219	12.0%
Beef cattle gross receipts	273,612	51.4%	100,562	25.6%	134,728	39.3%
Wool gross receipts	125,364	23.5%	51,347	13.1%	75,218	22.0%

Note: [*] Values in 2004–05 dollars

Table 3: Average Values of Selected Inputs and Outputs by Broadacre Industries*

	Cropping		Livestock	
	Costs & receipts ($)	Cost or receipt share	Costs & receipts ($)	Cost or receipt share
Contracts for cropping	9,918	3.4%	3,185	1.0%
Contracts for livestock	479	0.2%	2,624	0.8%
Fertilisers	32,426	8.7%	11,930	1.1%
Crop and pasture chemicals	25,384	11.1%	3,547	3.7%
Fuel, oil and grease	24,995	8.5%	16,049	5.0%
Interest	27,165	9.3%	30,171	9.3%
Seed	5,332	1.8%	2,081	0.6%
Handling and marketing	20,478	7.0%	21,060	6.5%
Total freight	14,865	5.1%	12,791	4.0%
Hired labour wages	10,157	3.5%	19,827	6.1%
Wheat gross receipts	129,444	32.8%	9,434	2.3%
Sheep gross receipts	30,776	7.8%	36,644	9.0%
Beef cattle gross receipts	35,748	9.1%	218,601	53.6%
Wool gross receipts	42,882	10.9%	89,321	21.9%

Note: * Values in 2004–05 dollars

Table 4: Average Values of Selected Inputs and Outputs by Operational Size*

	Large farm		Medium farm		Small farm	
	Costs & receipts ($)	Cost or receipt share	Costs & receipts ($)	Cost or receipt share	Costs & receipts ($)	Cost or receipt share
Contracts for cropping	12,711	2.0%	4,421	2.0%	1,771	2.1%
Contracts for livestock	3,900	0.6%	816	0.4%	182	0.2%
Fertilisers	42,258	4.7%	16,704	4.2%	5,486	3.0%
Crop and pasture chemicals	28,913	6.8%	9,505	7.5%	2,554	6.4%
Fuel, oil and grease	36,538	5.9%	17,076	7.6%	7,229	8.4%
Interest	54,304	8.7%	23,513	10.5%	8,986	10.5%
Seed	6,670	1.1%	2,884	1.3%	1,231	1.4%
Handling and marketing	39,452	6.4%	17,558	7.8%	5,777	6.7%
Total freight	29,721	4.8%	8,949	4.0%	2,643	3.1%
Hired labour wages	36,474	5.9%	7,888	3.5%	1,685	2.0%
Wheat gross receipts	139,198	16.8%	44,917	15.7%	10,819	11.4%
Sheep gross receipts	60,640	7.3%	31,558	11.0%	10,522	11.1%
Beef cattle gross receipts	304,009	36.7%	70,066	24.5%	27,828	29.3%
Wool gross receipts	118,020	14.2%	66,308	23.2%	21,511	22.7%

Note: * Values in 2004–05 dollars

4.3.3 The Pooled Cross-Sectional Nature of the Quasi-Micro Data

The AAGIS quasi-micro farm data initially appears as panel data, since farm cells having the same identification are observed repeatedly across years. By using production region, broadacre farming industry and operational size as criteria to assign farms to data cells, farm data cells with the same identifiers are formed for each of the sixteen years from 1990 to 2005. Data cells are identified by the production region the constituent farms are located in, by the broadacre industry the constituent farms belong to and by the scale the constituent farms operate at. In each year, and for each of the broadacre regions included in the sample, farms are allocated into six observational cells according to their dominant outputs and their production size: (1) cropping farm, large size, (2) cropping farm, medium size, (3) cropping farm, small size, (4) livestock farm, large size, (5) livestock farm, medium size, and (6) livestock farm, small size. The formation of data cells with the same identifiers over time gives the data sample the appearance of panel data. However, this is not the case, for the following reason.

In this study, the quasi-micro farm data provided is more appropriately treated as pooled cross-sectional data. The motivation for this is that the number of farms in a data cell is fairly small and varies significantly from year to year. This significant variation in the number of constituent farms in a farm cell from year to year has two sources. The first cause of this variation is the year-to-year change in the farm sample of AAGIS survey. A farm included in the previous year's survey sample may not be included in the current year's survey sample. Conversely, a farm participating in the survey in the current year may have not participated in the survey in the previous year. The second source of the variation in the sample of a data cell is the variation in individual farms' output mixes and production values. The output mix on a farm can change from year to year due to changing production decisions and failures/successes of different production activities. This means that a farm can move from one size to another, since ABARE uses total revenue thresholds to classify farm size. Even when that farm's size does not change, the relative revenue shares of its outputs can change, resulting in a change of its industry classification. As a result, a particular farm can move from one data cell to another through time, through either a change in its size classification or a change in its industry classification. This means that the data observed for a particular cell through time are of different constituent farms. Therefore, the quasi-micro dataset fails to qualify as of panel data type.

4.3.4 The Availability of Semi-Regional Data

In addition to the quasi-micro data, the original AAGIS dataset provided by ABARE contains average farm data at higher aggregate levels. As explained in Section 4.3.1, when quasi-micro data are formed, the assignment of farms to observational cells is based on three criteria in a sequential order. Farms surveyed across Australia are first divided into 32 production regions. Farms in each of these production regions are then split into two broadacre industry groups according to their dominant product/enterprise. Finally, farms within each of these broadacre industries are divided into three subgroups according to their production size. Average data of the two broadacre industries before farms are further split into three sizes is also provided by ABARE. This industry average data are essentially the quasi-micro data aggregated across farm sizes.

Since the industry average data provided are at a higher aggregate level than the quasi-micro data, and has a sufficiently large number of observations, it is used in this study to assess the impacts of aggregating data across farms on estimation results. This is achieved by comparing the estimation results using this industry average data with those obtained from the quasi-micro data. For ease of reference, this industry average data are termed as semi-regional data. The 'semi-regional' term is to reflect the fact that when this industry average data are aggregated across the two broadacre industries, the regional average data as published on ABARE's Agsurf website is obtained.

In summary, a large, unique quasi-micro AAGIS data sample of Australian broadacre agricultural production is used in this study. The average data of farms in the same production region, in the same broadacre industry and having the same operation size is obtained for each year over the 1990–2005 period. With this data formation, the quasi-micro data obtained describes more closely the farm-level production decision-making than most previous studies of Australian broadacre production. Input and output compositions differ significantly for farms in different broadacre zones, industries and operating scales. The quasi-micro nature also gives the AAGIS dataset the feature of pooled cross-sectional type. A semi-regional dataset containing average data for the two broadacre industries in each production region is also available for estimation. This semi-regional data are used to assess the extent to which aggregation of data across Australian broadacre farms affects the modelling results.

4.4 Aggregation Procedure

Aggregation of observed data items into a smaller number of aggregate inputs and outputs is necessary since there are more than thirty input and output items included in the original AAGIS dataset. This aggregation process is a prerequisite to estimation and often a complicated step in empirical studies of agricultural production. During this process, many significant empirical issues, especially the unobservability of most production inputs, the high frequency of missing data due to the quasi-micro nature of the data and treatment of inputs as being variable or fixed, need to be resolved. More importantly, how individual inputs and outputs are aggregated have consequential implications for research findings and policy relevance. To ensure the best aggregation outcome, this process is carried out with a careful inspection of data, an extensive review of previous Australian and overseas studies of agricultural production, in-depth consultation with experienced researchers in the broadacre agriculture sector and trials of estimations of alternative models with different sets of aggregate variables.

The issue of missing data in this study, due in part to the quasi-micro nature of the AAGIS dataset available, is different to most previous studies. Firstly, in this study the problem of the unavailability of input prices is encountered, which is inherent to empirical research of agricultural production. Although the AAGIS survey collects detailed financial and operating data of broadacre farm businesses, the survey does not have information on the prices of the majority of production inputs in the dataset. The national price indices published in ABARE Australian Commodity Statistics (ABARE 2008) are used for input prices. More detail is provided below in the description of the formation of the aggregate inputs. Secondly, output prices are calculated from reported total output receipts and output quantities. Due to the reporting of individual crop products and the small number of similar farms in each observational cell, many data observations have zero quantities for some crop outputs, as farms do not always produce all crops reported in the survey. The State level averages of observed output prices are used for farm cells with zero output quantities.

The aggregation of inputs and outputs involves an important step in deciding whether to treat an input as variable or fixed. This step is necessary due to the fact that some agricultural production factors are fixed during normal production cycles. For instance, farmers take approximately two and four years to adjust labour and capital, respectively (Agbola and Harrison 2005). In contrast, the quantities of such inputs as fertilisers or fuel can be adjusted almost instantaneously.

The decision to treat an input as variable or fixed is straightforward for most inputs. However, there are two significantly complicated cases regarding interest payments input and service cost of livestock capital. Regarding the first case, interest payments input is considered variable in most previous Australian and international duality studies of agricultural production. For example, interest payments input has been treated in the same way as "other variable materials and services" in Ahammad and Islam (2004), Fisher and Wall (1990) and McKay *et al.* (1982). However, the majority of the interest paid by farmers results from fixed capital acquisitions and is not related to short-run production decisions. Therefore, the part of total interest paid by farmers that is linked to long-term capital decisions should not be placed among variable inputs and should be viewed as a fixed input.

It is noteworthy that total interest paid includes some interest payments related to short-term production decision, such as those paid on harvest loans. With the increasing integration of financial funding and operational management in farm businesses, fostered by the expansion of rural conglomerates such as Elders and Wesfarmers into financial and banking services, it has become popular to fund expenses incurred for variable inputs, especially those related to cropping activities, through short-term loans. In a way, the interest paid for these short-term loans can be considered as the opportunity cost of funds tied up in the inputs funded. It would, therefore, be appropriate to add interest paid for these loans to these inputs' original purchase costs to fully account for the actual economic costs the farmer has incurred.

Information on the amount of interest paid on short-term loans is not available in the original AAGIS farm dataset. This component of interest paid, therefore, must be imputed. Interest paid is approximated as equal to the crop-related expenses that farmers paid multiplied by the prevailing nominal interest rate and divided by two. The nominal interest rate of three-year fixed term deposits in retail banking is used for this purpose. The monthly interest rates for this category of term deposits are published on the Reserve Bank of Australia website and are averaged to generate the yearly rate. The two-year moving average of this yearly interest rate series is the nominal interest rate used for the calculation of short-term loan interest. Short-term loans are expected to be less than a year in duration and interest payments on these loans are calculated for an approximate six-month term.

A second issue worth noting in the discrimination between variable inputs and fixed inputs is the classification of the service cost of livestock capital as a variable input. A variable classification is used in this study despite the fact that livestock inputs, especially beef cattle, are often carried over from one production cycle to the next. A variable classification also contradicts the normal treatment of similar durable inputs, such as buildings and machineries, as fixed in previous Australian studies such as Fisher and Wall (1990) and McKay *et al.* (1982). This departure is purposefully executed in order to recognise that farmers have some freedom in increasing or reducing stock numbers within a one year-time window (Mullen and Cox 1996) in contrast to the much higher degree of fixity of buildings and machineries.

4.4.1 Construction of aggregate indices

After the missing data in the original AAGIS dataset are filled and inputs are classified as variable or fixed inputs, aggregate price and quantity indices are constructed. The price indices are directly calculated using the prices and quantities of the component inputs or outputs. The quantity indices are indirectly derived by dividing the total aggregate value of expenses/receipts by the corresponding constructed price indices. The Fisher formula is chosen over Laspeyres, Paasche or Tornqvist in the construction of price indices because of several desirable properties (see Coelli, Prasada Rao, O'Donnell and Battese 2005, Section 4). For a particular aggregate input/output, the Fisher index is defined as the geometric mean of the Laspeyres and Paasche indices:

$$P_{st}^F = \sqrt{P_{st}^L \times P_{st}^P} \ ,$$

where $P_{st}^L =$ Laspeyres index $= \dfrac{\sum_{m=1}^{M} p_{mt} q_{ms}}{\sum_{m=1}^{M} p_{ms} q_{ms}}$, $P_{st}^P =$ Paasche index $= \dfrac{\sum_{m=1}^{M} p_{mt} q_{mt}}{\sum_{m=1}^{M} p_{ms} q_{mt}}$,

the s subscript denotes price and quantity at the base year, the t subscript denotes the current time period in which the index number is calculated and M is the number of component inputs/outputs of the aggregate input/output. The computed Fisher price index number measures the change in the price of the aggregate input/output from the base period s to period t.

In this study the AAGIS farm dataset is considered as a cross-sectional dataset although the observation period is spread over sixteen years. Therefore, the construction of the aggregate input and output price indices require choosing the base farm cell in addition to choosing the base year. The choice of the base farm among 1334 farm observations reflects two important aspects. Firstly, from an intuitive perspective, the base farm should produce the largest number of broadacre outputs included in the dataset. It should represent the full basket of goods for which the aggregate index is to be constructed. Secondly, from a mathematical perspective, the base farm should have the least zero or missing prices and quantities. Because the base farm's prices appear in the denominator in the Paasche index formula and its quantities appear in the denominator of the Laspeyres index formula, their geometric means, the Fisher index, can be undefined if the base farm does not produce some outputs, resulting in missing output prices and zero output quantities. Most farms do not produce all nine individual crops reported. After a careful inspection of the data, the 2003 observation of the Cropping farm group of the Large size in the Wheat-Sheep zone in Western Australia is selected to be the base farm.

Once the index formula and the base farm observation are chosen, the next step is to determine how many aggregate inputs and outputs are formed and what their component inputs/outputs are. Because the aggregation possibilities are numerous with more than forty individual inputs and outputs in the AAGIS farm dataset, aggregation was carried out without formal testing for consistent aggregations. The manner in which they are aggregated is based on information available in the original dataset, previous empirical studies in Australian broadacre production and international agricultural production, and statistical and theoretical reasonableness of estimated models obtained from alternative aggregations. In many earlier studies of Australian broadacre production, inputs and outputs appeared to be generally grouped into small numbers of aggregate inputs and outputs because of the small data samples available. For instance, in Ahammad and Islam (2004), Coelli (1996), and McKay *et al.* (1983), all materials and services were grouped under one aggregate input. While this aggregation is necessary to conserve the degrees of freedom, the grouping of production factors that are specific to different enterprises into one category implies that some information on production flexibility is lost. For this reason, this study endeavours to leave inputs and outputs as disaggregated as possible to best exploit the large data sample available. As an example, Mullen and Cox (1996) estimated a translog cost function in which variable inputs are grouped into six aggregate inputs, namely contracts, services, materials, labour, livestock purchases, and use of livestock capital. Unlike these authors, aggregation of variable inputs in this study has been performed so as to separate inputs into output-specific groups where possible. For example, grouping inputs specific to cropping into one category and inputs specific to livestock grazing into a separate category can help explain the responsiveness of farmers to these different and potentially competing production activities. Research findings based on such a form of aggregation can assist a detailed assessment of possible economic impacts on different agricultural industries.

From a technical perspective, there is a degree of separation between cropping and livestock grazing in that they can be implemented independently of each other. At the same time, they complement each other, helping farmers to better utilise their production resources, while competing against each other for limited resources such as land or labour. From a data perspective, farms in the sample are identified as cropping or livestock farms, based on their dominant revenue generating activity. From these two perspectives, it is possible and worthwhile to seek out information on any relationships between inputs and outputs specific to these two broad production activities/industries.

With this motive, it seems natural to separate inputs and outputs into cropping and livestock categories wherever possible. This form of aggregation was carried out by Moschini (1988) and is the form followed in this study. In addition, it is important in social and economic policy development to understand the demand for hired labour. Therefore, the hired labour input has been separated from other inputs in this study.

Due to the complete dependence of livestock grazing operation on livestock inputs and the long biological gestation required to produce young animals, the service cost of livestock is kept separate from other inputs. The five aggregate variable inputs below were initially formed.

1. Contracts, services and materials for Livestock;
2. Contracts, services and materials for Cropping;
3. Other Contracts, services and materials;
4. Hired labour; and
5. Service cost of livestock capital.

The results of the dual cost, revenue and profit functions using these five aggregate inputs are unsatisfactory, having low percentage of significant price coefficients, violating the regularity curvature conditions and having unexpected own-price elasticities. As a result, a new aggregation of the variable inputs was sought. This alternative aggregation, from which the aggregate inputs and outputs described below were generated, produced statistically and theoretically reasonable estimation results considered superior to the results presented in many previous Australian and international duality studies. This form of aggregation was therefore chosen for all models estimated in this study.

4.4.2 Aggregate Outputs

The construction of quantity and price indices for aggregate outputs is straightforward since receipts and quantities of individual outputs are in fact observed in the original AAGIS dataset. The actual prices received by each farm cell are derived by dividing the observed receipts by the corresponding observed quantities. When quantity sold and quantity produced are both reported for a crop output, the latter was incorporated in the aggregation to reflect the actual amount produced. The value of total output produced is estimated to be proportional to the gross receipts of the amount sold. Similarly, the number of animals turned off instead of the number of animals sold was used in constructing the aggregate indices of livestock outputs.

Grains

Wheat is grouped with barley, oats, grain legumes, oilseeds, canola, field peas, lupins and sorghum to create the aggregate Grains output. Cotton and rice are excluded from this study since they are produced using intensive production technology and irrigated water. This aggregation is the same as that in Ahammad and Islam (2004), Mullen and Cox (1996) and McKay *et al.* (1983) but differs from Coelli (1996), in which wheat, oats and barley are grouped into an aggregate crop output and all other crops are grouped with a beef cattle output.

The prices of the component outputs comprising the aggregate Grains output are derived by dividing observed gross receipts by quantities. The quantities and derived prices are then used to construct the aggregate Grains price index using the Fisher formula.

Sheep

The sheep output, including lamb, reported in the AAGIS dataset is unaltered. Its selling price is equal to the gross receipts divided by the number of sheep sold. The number of sheep sold is used because there is no information about the number of sheep being transferred out or turned off as in the case of beef cattle. The derived selling price is then used to construct the price index for this output. The quantity index is implicitly derived by dividing the total receipts by the calculated price index number.

Beef

This aggregate output consists of Beef and “Other livestock” outputs. The value of “Other livestock sold” is generally marginal compared to that of Beef. The received beef price equals the gross receipt from beef divided by quantity of beef sold. This derived price is multiplied by the number of beef turned off to generate the total value of beef cattle produced. This total value, not the recorded total receipts, is used to derive the implicit quantity index. Meanwhile, the index of prices received for the livestock sector published by ABARE (ABARE, 2005) is used as the price of "Other livestock sold".

Wool

Wool is kept as a stand-alone output. Similar to the Beef output, the total value of wool produced, instead of the gross receipt of wool sold, is used to derive its implicit quantity index. This total value is calculated as equal to the actual wool price received multiplied by quantity of wool produced. The actual price received is calculated as the gross wool receipt divided by the quantity of wool sold.

4.4.3 Variable Inputs

More than thirty input items are aggregated into five aggregate variable inputs. As previously explained, input prices are not observed in the original AAGIS dataset and are substituted by the ABARE national indices of prices paid in ABARE (2005). Input quantities are then calculated using these price indices and the observed total costs. The derived quantities and the price indices are then used to construct the five aggregate price indices.

Contracts, Services and Materials for Livestock

Production inputs aggregated into the aggregate Contracts, services and materials for livestock (CSM Livestock) input are fodder, livestock materials, livestock purchases, contracts for livestock, AI stud, herd test, vet fees, agistment expense, stores and rations, and shearing and crutching. The ABARE national price indices selected for these inputs are: fodder and feedstuffs, chemicals and medicines, contracts, and shearing rates. The quantities of the component inputs are calculated as their individual costs divided by the corresponding national price indices. These calculated quantities and national price indices are used to construct the price and quantity indices of the aggregate CSM Livestock input.

There is a need to fully account for the livestock that are actually brought into production on each farm during each surveyed year. This is done by summing the livestock purchased by the farm, the livestock transferred onto the farm, and the farm's negative operating gains; an imputed value reported by ABARE as the balance to reconcile the animal stock at the start and at the end of each surveyed year after all animal trading movements have been accounted for. Quantities and values of livestock purchases and inward transfers are observed so their actual purchase prices are observed. For the operating gains, however, only the value is observed. Therefore, the quantity of the operating gains is calculated as equal to its imputed value divided by the livestock price received, as in Alexander and Kokic (2005).

Fertilisers and Crop and Pasture Chemicals

Endowed with low fertile soil, broadacre farms in Australia generally operate at large scales with high usage of chemical inputs. This strong reliance on chemical inputs justifies grouping fertilisers and crop and pasture chemicals into a separate aggregate input. The aggregate Fertilisers, crop and pasture chemicals (FC) input consists of two separately reported inputs: fertilisers, and crop and pasture chemicals. These two inputs account for a significant share of the total production cost, regardless of geographical locations, production focuses and operational sizes. They also share the characteristic of being manufactured inputs. The construction of the price and quantity indices for the aggregate FC input is similar to that for the aggregate CSM Livestock input. The ABARE producer-paid national price indices for fertilisers and chemicals in ABARE (2005) are used as the prices of the component inputs.

Other Contracts, Services and Materials

The aggregate Other contracts, services and materials (Other CSM) input encompasses the remaining variable inputs except for fuel, oil and grease. They are seeds, seedlings and plants, electricity, repairs and maintenance of buildings and structures, contracts for cropping, repairs of machinery and plant, handling and marketing, freight, rates and drain water, insurance, land rent, lease, telephone, accountancy fee, advisory services fee, other administrative expenses, other material expenses and other service expenses. The ABARE producer-paid price indices used are those for electricity, maintenance - structure, maintenance - plant and equipment, selling expenses, freight outwards, rates and taxes, insurance, other overheads, and other materials and services.

Fuel, Oil and Grease

This single input item in the original AAGIS dataset accounts for a significant share of total production cost for farms across broadacre zones, production industries and production sizes. It is, therefore, considered as an aggregate input on its own right. The ABARE producer-paid price index of fuel and lubricants is used as the actual price of this input in the construction of the Fisher price index.

Livestock Trading

The last aggregate variable input is the service cost of livestock capital – briefly referred to as the Livestock trading input. This approach follows the Australian study by Mullen and Cox (1996). Livestock can be thought of as a capital item because only part of the total stock held at the beginning of each year will be consumed during that year. In normal operation only animals that have reached their final production stage are turned off and all others are carried over to the next production cycle. Farmers, however, can adjust the level of livestock held within a fairly short time period in response to market and weather conditions. In this way, the livestock input has the nature of a variable input.

The value of Livestock trading input is calculated as equal to the opening balance of livestock multiplied by the nominal interest rate sourced from the Reserve Bank of Australia, as previously described above in the discussion about interest payments. The data on the opening balance of livestock on hold is not available in the original dataset and, therefore, has to be derived using other available information. In this study, this opening balance is calculated as the estimated number of livestock held at the beginning of year multiplied by the average of livestock purchasing and selling prices. The opening number of livestock is estimated as the number of livestock at the end of the year, plus the number of livestock turned off, minus the total number of livestock purchased and transferred in minus the estimated number of livestock negative operating gains.

4.4.4 Fixed Inputs

Total Fixed Capital

The physical units of capital used in farm production are incorporated as fixed inputs in the models estimated in this study. This is because it usually takes more than one production cycle for farmers to adjust the amounts of land, building and other fixed improvements as well as plant and machinery. The information needed is the opening balances of these capital items. However, the opening balances of the capital items are not included in the original dataset and have to be imputed using other available information such as demonstrated in the following formula:

Opening balance = Closing balance + Depreciation - Net capital additions - Total imputed capital appreciation.

Ideally, the capital items should be accounted for separately as individual items because of differences in how they are utilised in farming and the rate at which they depreciate. Since there is no data on depreciation of individual capital items, it is not possible to categorise capital into two fixed aggregate inputs of (a) Land, building and fixed improvements and (b) Plant and machinery as in Mullen and Cox (1996), Fisher and Wall (1990) and Coelli (1996). Therefore, one aggregate fixed capital input is created by adding land, buildings and other fixed improvements, and plant and machinery inputs together. Since the closing balance of plant and machinery is not reported throughout the study period, the following imputation is used:

Opening balance of total capital = Farm equity closing balance/Equity ratio*100 + Depreciation - Net capital additions - Total imputed capital appreciation - Calculated opening livestock capital.

The service cost of the capital is then estimated as this imputed opening balance multiplied by the real interest rate and added depreciation costs. The resulting service cost of capital and the published ABARE price index for capital in ABARE (2005) are used afterwards to generate the price index and the implicit quantity index of the aggregate fixed capital using the Fisher formula.

Fixed Labour

The quantity and price indices of the operator's and family's labour are calculated using the total worked weeks reported in the AAGIS dataset, adjusted for the estimated number of weeks worked by hired labour. Assuming that hired labour and fixed labour receive the same wage rate, the number of weeks worked by the operator and his/her family equals the imputed fixed labour cost multiplied by the total number of weeks worked by all labour and divided by the sum of imputed fixed labour cost and hired labour cost. This estimated number of weeks worked is then used with the imputed fixed labour cost to derive the price of fixed labour. This derived price and the estimated number of weeks worked by operator and his/her family are then used to construct the Fisher price and implicit quantity indices of Fixed labour.

4.5 Other Data Sources

Since rainfall a plays crucial role in Australian broadacre agriculture, models of this sector should incorporate rainfall information. This information is not available in the ABARE farm data and so is sourced from the Australian Bureau of Meteorology (BoM). The BoM rainfall data obtained is monthly rainfall for 115 rainfall districts defined by BoM over the period from 1890 to 2006 (BoM 2007). To make this rainfall data complementary to the AAGIS farm data, the rainfall districts are matched with the thirty-two broadacre regions identified by ABARE. Once the rainfall districts of a broadacre region are identified, the region's annual calendar-year rainfall is calculated as the average of the component districts' annual rainfall. This annual rainfall is incorporated, transformed or untransformed, as an exogenous variable in the econometric models estimated in this study. Average financial-year rainfall and the pair of average January-to-June rainfall and average July-to-December rainfall (as in Fisher and Wall 1990), are also created. Estimation results suggest that the financial-year rainfall variable does not discernibly improve the model's results compared to the calendar-year rainfall variable. Also, based on the judgement that the production response to the timing of rain is not a primary focus of this study and to conserve the degrees of freedom, the annual rainfall variable is chosen over the pair of the two half-year rainfall variables. The calendar-year rainfall variable is therefore chosen for this study.

4.6 Summary

This chapter sets out the empirical context of this thesis. Broadacre agricultural production is carried out in diverse climatic and physical conditions across Australia. Broadacre farming relies on natural rainfall for water input. Broadacre farmers typically produce multiple products for risk management and resource utilisation. They implement typical production regimes within each of the three broadacre zones, which are comprised of more than thirty production regions.

In its second objective, this chapter describes in detail the unique large quasi-micro AAGIS farm data used for estimation in this study. This quasi-micro data hold information on broadacre farming production across Australia from 1990 to 2005. The farm-level production decision is better preserved in this AAGIS quasi-micro data compared to most previous Australian studies of broadacre agricultural production. Aggregate data at a semi-regional level is also available for estimation, which is for assessing the aggregation issue in duality applications.

Chapter 5

Estimating Restricted Multi-Product Cost Functions for Australian Broadacre Production

5.1 Introduction

In this chapter, restricted multi-product cost functions are specified and estimated using the AAGIS quasi-micro data described in Chapter 4. As discussed in Chapter 3, in this model formulation of production decision-making, broadacre farmers are assumed to take output levels as given and adjust input levels, whose prices are exogenously determined, to minimise production cost. The dual restricted cost function is specified with four aggregate outputs, five aggregate variable inputs and two aggregate fixed inputs as described at the end of Chapter 4. Both translog and normalised quadratic functional forms are used for the specification of this dual function. For each functional form, output-constrained cost-minimising input demand functions are derived and net price elasticities for input demand and elasticities of input substitution are calculated after model estimation. The estimation results and the elasticity estimates obtained from the two functional forms are then compared and contrasted.

This chapter is organised into six sections. Section 5.2 commences with the general specification of the dual restricted translog multi-product cost function and the empirical implementation of estimating this function for Australian broadacre farming using the AAGIS quasi-micro data. The general specification component encompasses a mathematical expression of the translog multi-product cost function, a description and discussion of parametric restrictions for the theoretical regularity conditions, a derivation of net price elasticities and elasticities of substitution between inputs and a formation of the system of equations for the econometric model consistent with a translog cost function. The empirical implementation component covers all steps in estimating the econometric model derived from the translog cost function when applied to the set of four outputs, five variable inputs and two fixed inputs of Australian broadacre production. The chapter continues with Section 5.3, which deals with the general specification and empirical estimation of the restricted normalised quadratic multi-product cost function for Australian broadacre agriculture. This section is organised in the same manner as Section 5.2. Section 5.4 is solely devoted to the issue of heteroskedasticity encountered throughout this study due to a unique feature of the quasi-micro data used for model estimation. It includes an explanation of the potential heteroskedasticity and actions taken to mitigate this problem for both translog and normalised quadratic functional forms. In Section 5.5, empirical results for the two models derived from the translog and normalised quadratic cost functions are presented and described. In this section, estimates of net price and substitution elasticities of input demands obtained from these two functional forms are also presented. A discussion and comparison of the two sets of results from the two functional forms constitute Section 5.6. The chapter then concludes with summaries and remarks.

5.2 Specification of a Restricted Translog Multi-product Cost Function

In this section, the theoretical framework for modelling production technology via specifying a restricted translog multi-product cost function is described. Following the theoretical framework is a detailed account of the econometric model. The estimation of the econometric model for Australian broadacre agriculture, with a set of four aggregate outputs, five aggregate variable inputs and two aggregate fixed inputs as described in Chapter 4, is finally described.

5.2.1 The Translog Multi-Product Cost Function

The restricted translog multi-product cost function of the technology producing outputs $Y=[y_1, y_2, \ldots y_m]$ using variable inputs $X=[x_1,\ x_2,\ \ldots,\ x_n]$ can be represented as follows:

$$\begin{aligned}\ln C(W,Y,Z,T) &= \alpha_0 + \sum_{i=1}^{n}\alpha_i \ln w_i + \sum_{k=1}^{m}\beta_k \ln y_k + \sum_{g=1}^{v}\lambda_g \ln z_g + \tfrac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_{ij}\ln w_i \ln w_j \\ &+ \tfrac{1}{2}\sum_{k=1}^{m}\sum_{l=1}^{m}\beta_{kl}\ln y_k \ln y_l + \tfrac{1}{2}\sum_{g=1}^{v}\sum_{h=1}^{v}\lambda_{gh}\ln z_g \ln z_h + \sum_{i=1}^{n}\sum_{k=1}^{m}\delta_{ik}\ln w_i \ln y_k + \sum_{i=1}^{n}\sum_{g=1}^{v}\gamma_{ig}\ln w_i \ln z_g \\ &+ \sum_{k=1}^{m}\sum_{g=1}^{v}\varphi_{kg}\ln y_k \ln z_g + \sum_{i=1}^{n}\rho_{ti}T\ln w_i + \sum_{k=1}^{m}\phi_{tk}T\ln y_k + \sum_{g=1}^{v}\psi_{tg}T\ln z_g + \theta_t T + \tfrac{1}{2}\theta_{tt}T^2\end{aligned}$$

where $W=[w_1, w_2, \ldots w_n]$ is the input price vector and $Z=[z_1, z_2, \ldots z_v]$ is a vector of fixed inputs and other non-price, non-quantity exogenous variables that can affect the production and T is a technological index. Cost, price and quantity variables C, W, Y, X and Z are non-negative.

5.2.2 Regularity Conditions

As discussed in Chapter 3, the translog cost function defined above alone is not sufficient to describe a producer's decision-making process. The set of regularity conditions for this cost function can be translated into equality and non-equality restrictions on the derivatives of the cost function. For this cost function, the cost function is monotonic (see Condition C.2 in Chapter 3) if the first partial derivatives of the cost function with respect to prices (in their logarithmic forms)

$$\frac{\partial \ln C(W,Y,Z,T)}{\partial \ln w_i} = \alpha_i + \sum_{j=1}^{n}\alpha_{ij}\ln w_j + \sum_{k=1}^{m}\delta_{ik}\ln y_k + \sum_{g=1}^{v}\gamma_{ig}\ln z_g + \rho_{ti}T$$

are positive (Sickles and Streitwieser 1998; Gagne and Ouellette 1998 and Binswanger 1974b).

Whether the regularity condition of concavity (Condition C.3) is satisfied is related to the matrix $A-\hat{c}+c'c$, where $A=\left[\alpha_{ij}\right]_{n\times n}$, $\hat{c}$ is an $n\times n$ diagonal matrix with its diagonal elements being the input cost shares and c is a vector of input cost shares. If this matrix is negative semi-definite at each observation point, the concavity condition is met (Diewert and Wales 1987). In contrast to the monotonicity and concavity conditions, the condition of linear homogeneity in prices (Condition C.4) and the condition of symmetry (the extra condition of twice-continuous differentiability) have global implications for the cost function's parameters. The former requires that

$\sum_{i=1}^{n}\alpha_i=1$, and $\sum_{i=1}^{n}\alpha_{ij}=\sum_{i=1}^{n}\delta_{ik}=\sum_{i=1}^{n}\gamma_{ig}=\sum_{i=1}^{n}\rho_{ti}=0$,

and the latter requires $\alpha_{ij}=\alpha_{ji}$, $\beta_{kl}=\beta_{lk}$ and $\lambda_{gh}=\lambda_{hg}$.

5.2.3 The Output-Constrained Input Demands

When the translog cost function specified above satisfies all the regularity conditions, applying Shephard's lemma, obtaining (3.4), and the Chain Rule leads to a system of output-constrained input demand cost share equations as follows:

$$\frac{\partial \ln C(W,Y,Z,T)}{\partial \ln w_i}=\frac{\partial C(W,Y,Z,T)}{\partial w_i}\frac{w_i}{C(W,Y,Z,T)}=$$

$$c_i=\alpha_i+\sum_{j=1}^{n}\alpha_{ij}\ln w_j+\sum_{k=1}^{m}\delta_{ik}\ln y_k+\sum_{g=1}^{v}\gamma_{ig}\ln z_g+\rho_{ti}T,$$

where $i = 1,2,...n$ and c_i denotes the share of ith variable input in the total variable cost. This means the monotonicity condition of the cost function implies that the derived cost-minimising shares c_i, $i = 1,2,...n$, are positive. Besides, the parametric restrictions for the homogeneity and the symmetry conditions of the cost function are the same as the restrictions required for the adding-up condition, a condition results from the fact that cost shares sum to unity at each observation (Halvorsen 1977).

5.2.4 The Net Price and Substitution Elasticities of Input Demands

The net price and substitution elasticities for the cost function case are defined as (3.5) and (3.6) in Section 3.3. Applying these definitions to the translog cost function specified above, the net own-price elasticity of demand for input i (η_{ii}) and the net cross-price elasticity of demand for input i with respect to price of input j (η_{ij}) have the following expressions: $\eta_{ij} = \frac{\alpha_{ij}}{c_i} + c_j$ and $\eta_{ii} = \frac{\alpha_{ii}}{c_i} + c_i - 1$ (McKay *et al.* 1980 and Binswanger 1974a). Regarding the elasticities of input substitution, the net Allen partial measure σ_{ij} is expressed as: $\sigma_{ij} = \frac{\alpha_{ij}}{c_i c_j} + 1,\ i \neq j$ (McKay *et al,.* 1980 and Binswanger 1974a). The net Morishima elasticity of substitution σ_{ij}^M is derived straightforwardly through the relationship $\sigma_{ij}^M = \eta_{ij} - \eta_{jj}$ as established in Subsection 3.3.3.

It can be shown from the expression of the net own-price elasticities that the own-price parameter α_{ii} does not need to be negative in order for the own-price elasticity η_{ii} to be negative as expected by economic theory. The inequality $\eta_{ii} = \frac{\alpha_{ii}}{c_i} + c_i - 1 < 0$ is equivalent to $\eta_{ii} = \frac{\alpha_{ii} + c_i^2 - c_i}{c_i} < 0$, which in turns is equivalent to the inequality $\alpha_{ii} + c_i^2 - c_i < 0$. Rearranging the left hand side of the last expression we have the corresponding inequality:

$$\left(c_i - \frac{1}{2}\right)^2 + \alpha_{ii} - \frac{1}{4} < 0 \text{ or}$$

$$\alpha_{ii} < \frac{1}{4} - \left(c_i - \frac{1}{2}\right)^2. \tag{5.1}$$

At the same time, since $0 < c_i < 1$ we always have $-\frac{1}{2} < \left(c_i - \frac{1}{2}\right) < \frac{1}{2}$, which implies that $\frac{1}{4} > \frac{1}{4} - \left(c_i - \frac{1}{2}\right)^2 > 0$. These last inequalities together with result (5.1) imply that the own-price parameter α_{ii} can be positive but, as a necessary condition, must be smaller than $\frac{1}{4}$, in order to satisfy the rational economic expectation that the own-price elasticity η_{ii} is negative. This result has been supported by findings in Sickles and Streitwieser (1998), Halvorsen and Smith (1986), McKay *et al.* (1980) and Kako (1978), in which some estimates of the own-price parameters are found to be positive but smaller than 0.25. The own-price demand elasticities obtained for the corresponding inputs are negative, as expected for rational economic behaviour. The derived necessary condition above also implies that enforcing the curvature condition via parametrically imposing the price matrix $A = \left[\alpha_{ij}\right]_{n \times n}$ to be negative semi-definite, which requires α_{ii} to be nonnegative, such as in Jorgenson and Fraumeni (1981) is too restrictive.

5.2.5 Empirical Implementation

Empirical studies specifying translog cost functions conventionally estimate the system of the derived cost share functions. When the estimation is carried out, error terms are first added to these n system equations. These error terms are assumed to be linearly additive to the share equations and normally distributed with mean zero and nonzero constant contemporaneous variance-covariance matrix. Since the left-hand side variables of the equations in the system being estimated are cost shares that always add up to unity, the errors of the system share equations sum up to zero at all data points. The system is, therefore, singular and cannot be estimated. To overcome this problem, one share equation is dropped from the system and the system of the remaining share equations is estimated. The estimates of this new system of $(n-1)$ share equations are then used to calculate the parameters of the dropped share equation using the adding-up condition. Barten (1969) proves that when using the Full Information Maximum Likelihood (FIML) method for system estimation, the estimates of this new system of $(n-1)$ share equations are invariant to the equation deleted. Therefore, the choice of share equation to be deleted from the original derived share system does not have any consequences on the estimation results and on the elasticity estimates when FIML is used for estimation.

Two regularity conditions are imposed during the estimation of the derived cost share system. As shown in Section 5.2.2, the linear homogeneity and symmetry conditions can be enforced on the cost function using restrictions on the parameters of the derived share system. It is, however, not possible to impose the monotonicity and concavity conditions on the translog cost function. These two conditions can only be checked locally after the estimation. The monotonicity condition is met if the predicted shares of the input costs are positive for the whole data sample. The concavity condition is satisfied if the estimated $A-\hat{c}+c'c$ matrix, as defined in Section 5.2.2, is negative semi-definite. There have been attempts (for example Jorgenson and Fraumeni 1981) to impose the global concavity condition on the translog cost function by restricting the price coefficient matrix $\left[\alpha_{ij}\right]$ to be negative semi-definite. Section 5.2.4, however, shows that α_{ii}s can be positive without sacrificing the negativity of the own-price elasticities of input demand, which implies that the price coefficient matrix $\left[\alpha_{ij}\right]$ needs not to be negative semi-definite. Moreover, it has been acknowledged in Terrell (1996), Diewert and Wales (1987), Gagne and Ouellette (1998) and Gagne and Nappi (2000) that the imposition of negative semi-definiteness on the matrix $\left[\alpha_{ij}\right]$ is stronger than required by the concavity condition and can destroy the flexibility of the functional form through imposing *a priori* restrictions on own- and cross-price elasticities.

In specifying the restricted translog multi-product cost function for Australian broadacre production, the restricted (variable) cost is defined as the sum of expenses spent on variable inputs. The cost shares of variable inputs are calculated as expenses divided by total variable cost. As described at the end of Chapter 4, the restricted cost function has five aggregate variable inputs, four aggregate outputs and two fixed input quantities. The five variable inputs $X = [x_1,\ x_2,\ ...,\ x_5]$ are: (1) Contracts, services and materials for livestock, (2) Fertilisers and chemicals, (3) Other contracts, services and materials, (4) Fuel, oil and grease and (5) Livestock trading. The four outputs $Y = \left[y_1, y_2, ...y_4\right]$ are: (1) Grains, (2) Sheep, (3) Beef and (4) Wool. The two fixed inputs z_1 and z_2 are Total capital and Fixed labour.

The translog cost function defined for Australian broadacre agriculture also includes six other non-economic exogenous variables, which are grouped with fixed inputs in $Z = \left[z_1, z_2, ...z_8\right]$. Among these exogenous variables are qualitative dummy variables accounting for possible effects of agro-climatic conditions (two zone dummies), of enterprise focuses (one industry dummy) and of production scales (two size dummies) on production technologies employed by broadacre farmers across Australia. Beside these qualitative variables, an annual rainfall variable, created from the monthly rainfall data by rainfall-districts provided by the Bureau of Meteorology (BoM 2007), is included since Australian broadacre agriculture depends on natural rainfall for soil moisture.

Five cost share equations are derived from the translog cost function for the variable inputs:

$$c_i = \alpha_i + \sum_{j=1}^{5} \alpha_{ij} \ln w_j + \sum_{k=1}^{4} \delta_{ik} \ln y_k + \sum_{g=1}^{8} \gamma_{ig} \ln z_g + \rho_{ti} T\ ,\ i = 1,\ 2, \ldots,\ 5.$$

After the parametric restrictions for the homogeneity and symmetry conditions are imposed and the share equation of the Livestock trading input is arbitrarily deleted from the system, a cost share system with 70 coefficients is estimated using the AAGIS quasi-micro data sample[3].

For the translog functional form, an empirical problem encountered in this study is the presence of zeros in a significant number of observations for some output quantities. This is due to the quasi-micro nature of the data. In the original AAGIS data, some farms do not produce certain broadacre crops reported. Despite the fact that all crop outputs are aggregated into a single aggregate output, the resulting aggregate Grains quantity is zero for a significant number of the observations. The logarithmic Grains quantity appearing on the right hand-side of share equations is not defined for these observations. This also happens to some Sheep output observations. To overcome this issue, all observations for all output quantities are increased by one. The addition of one unit to quantity levels does not significantly affect non-zero quantities since outputs are normally produced in large quantities on farms.

After the share system is estimated using FIML, a system-wide McElroy R^2 is calculated to assess explanatory power (see Appendix B for the steps in computing this system-wide R^2). This goodness-of-fit measure is more meaningful than the R^2 or adjusted-R^2 popularly computed for individual equations. The coefficient estimates and fitted shares are then used to assess whether the monotonicity and concavity conditions are met and to calculate the net price elasticities of demands, Allen partial elasticities of substitution and Morishima elasticities of substitution. Since these elasticities are nonlinear functions of the system parameter estimates, their standard errors are estimated using a bootstrapping method as in studies by Marsh (2005), Sharma (2002), Eakin, McMillen and Buono (1990), Green, Hahn and Rocke (1987), Krinsky and Robb (1986), Freedman and Peters (1984) and Gallant and Golub (1984). The bootstrapping procedure is described in detail in Appendix C.

[3] When the cost function is included, the derived equation system cannot be estimated using the AAGIS quasi-micro data. The cost share system is, therefore, estimated without the cost function.

5.3 Specification of a Restricted Normalised Quadratic Multi-Product Cost Function

This section describes the general framework and empirical estimation of production technology when the dual restricted multi-product cost function is specified using the normalised quadratic functional form.

5.3.1 The Normalised Quadratic Cost Function

Define $C'(W',Y,Z,T)$ and $W'=[w'_1,w'_2,...w'_{n-1}]$ as the total variable cost and variable input prices normalised by the nth variable input price. Normalised prices w'_i, $i = 1,2,...n-1$, are obtained as $w'_i = \frac{w_i}{w_n}$, where $W=[w_1,w_2,...w_n]$ are variable input prices defined in Section 5.2. The restricted multi-product cost function in normalised quadratic form has the following specification:

$$C'(W',Y,Z,T) = \alpha_0 + \sum_{i=1}^{n-1}\alpha_i w'_i + \sum_{k=1}^{m}\beta_k y_k + \sum_{g=1}^{v}\lambda_g z_g + \tfrac{1}{2}\sum_{i=1}^{n-1}\sum_{j=1}^{n-1}\alpha_{ij} w'_i w'_j + \tfrac{1}{2}\sum_{k=1}^{m}\sum_{l=1}^{m}\beta_{kl} y_k y_l$$
$$+ \tfrac{1}{2}\sum_{g=1}^{v}\sum_{h=1}^{v}\lambda_{gh} z_g z_h + \sum_{i=1}^{n-1}\sum_{k=1}^{m}\delta_{ik} w'_i y_k + \sum_{i=1}^{n-1}\sum_{g=1}^{v}\gamma_{ig} w'_i z_g + \sum_{k=1}^{m}\sum_{g=1}^{v}\varphi_{kg} y_k z_g + \sum_{i=1}^{n-1}\rho_{ti} w'_i T$$
$$+ \sum_{k=1}^{m}\phi_{tk} y_k T + \sum_{g=1}^{v}\psi_{tg} z_g T + \theta_t T + \tfrac{1}{2}\theta_{tt} T^2.$$

where all variables beside $C'(W',Y,Z,T)$ and $W'=[w'_1,w'_2,...w'_{n-1}]$ are defined as in Section 5.2.

5.3.2 Regularity Conditions

The implications of the regularity conditions on the dual restricted multi-product cost function are more straightforward for the normalised quadratic form than for the translog. The condition of monotonicity in prices (Condition C.2) requires the first partial derivatives of the normalised cost function with respect to normalised input prices $\frac{\partial C'(W,Y,Z,T)}{w'_i} = \alpha_i + \sum_{j=1}^{n-1} \alpha_{ij} w'_j + \sum_{k=1}^{m} \delta_{ik} y_k + \sum_{g=1}^{v} \gamma_{ig} z_g + \rho_{ti} T$, $i = 1,2,...n-1$, to be positive. This condition places no restrictions on the parameters of the cost function. The global concavity (Condition C.3) requires the price coefficient matrix $\left[\alpha_{ij}\right]_{(n-1)\times(n-1)}$ to be negative semi-definite. The cost function in this functional form automatically satisfies the condition of homogeneity in prices (Condition C.4) due to the normalisation process. Finally, the symmetry condition requires $\alpha_{ij} = \alpha_{ji}$ $(i, j = 1,2,...n-1)$, $\beta_{kl} = \beta_{lk}$ $(k, l = 1,2,...m)$ and $\lambda_{gh} = \lambda_{hg}$ $(g, h = 1,2,...v)$.

5.3.3 The Output-Constrained Input Demands

When the restricted normalised quadratic cost function specified above satisfies the regularity conditions, the output-constrained cost-minimising demand equations of variable inputs are derived by applying Shephard's lemma as follows:

$$x_i = \frac{\partial C(W,Y,Z,T)}{\partial w_i} = \frac{\partial C'(W,Y,Z,T)}{\partial w'_i} = \alpha_i + \sum_{j=1}^{n-1} \alpha_{ij} w'_j + \sum_{k=1}^{m} \delta_{ik} y_k + \sum_{g=1}^{v} \gamma_{ig} z_g + \rho_{ti} T,$$

$i = 1,2,...n-1$.

The demand equation for the *numeraire*, input n, can be derived as the first derivative of the un-normalised cost function with respect to the *numeraire* price (Polson and Shumway 1992 and Shumway and Alexander 1988):

$$x_n = \alpha_0 + \sum_{k=1}^{m} \beta_k y_k + \sum_{g=1}^{v} \lambda_g z_g + \tfrac{1}{2} \sum_{i=1}^{n-1} \sum_{j=1}^{n-1} \alpha_{ij} w'_i w'_j + \tfrac{1}{2} \sum_{k=1}^{m} \sum_{l=1}^{m} \beta_{kl} y_k y_l$$
$$+ \tfrac{1}{2} \sum_{g=1}^{v} \sum_{h=1}^{v} \lambda_{gh} z_g z_h + \sum_{k=1}^{m} \sum_{g=1}^{v} \varphi_{kg} y_k z_g + \sum_{k=1}^{m} \phi_{tk} y_k T + \sum_{g=1}^{v} \psi_{tg} z_g T + \theta_t T + \tfrac{1}{2} \theta_{tt} T^2.$$

These n derived demand equations contain all the coefficients of the specified dual cost function. The demand equation of the *numeraire* has a different functional form compared to the other $n-1$ equations, being quadratic in all exogenous variables.

5.3.4 The Net Price and Substitution Elasticities of Input Demands

Regarding the measures of economic interest, the own- and cross-price elasticities of the conditional demands for input i, where $i = 1,2,...n-1$, have the following expressions: $\eta_{ij} = \alpha_{ij} \times \frac{w'_j}{x_i}$ with $j = 1,2,...n-1$. The price elasticities related to the normalising input (the *numeraire*) are expressed as (Coxhead 1992): $\eta_{in} = -\sum_{j=1}^{n-1} \eta_{ij}$, $\eta_{ni} = -\eta_{in} \times \frac{w'_i \times x_i}{x_n}$, and $\eta_{nn} = -\sum_{j=1}^{n-1} \eta_{nj}$.

5.3.5 Empirical Implementation

When production technology is modelled using the normalised quadratic cost function, the system of derived demand equations is estimated. Linearly additive error terms from a normal distribution, with mean zero and nonzero constant contemporaneous variance-covariance matrix, are added to the system equations. The system of these equations is then estimated using the FIML method. Unlike the translog form case, there is no singularity problem in the estimation of this system. For this functional form, FIML estimates are not invariant to the choice of the *numeraire*. The choice of *numeraire* also has a substantial impact on the robustness of estimation results. Methods for choosing the *numeraire* have been proposed, such as by Shumway and Gottret (1991), but they are yet to gain significant acceptance in empirical research.

The imposition of the regularity conditions on the normalised quadratic demand system is fairly straightforward. The condition of linear homogeneity in prices is automatically enforced by the normalising process. The monotonicity condition cannot be imposed by means of parametric restrictions and therefore can only be checked after estimation. Unlike the translog's case, the concavity condition of the normalised quadratic cost function can be globally enforced via parametric restrictions. This global imposition is usually achieved by means of the Cholesky decomposition. In the Cholesky decomposition, the price coefficient matrix $\left[\alpha_{ij}\right]_{(n-1)\times(n-1)}$ is replaced by the negative of the product of a lower triangle matrix and its transposed as follows:

$$\left[\alpha_{ij}\right]_{(n-1)\times(n-1)} = -\begin{bmatrix} a_{11} & 0 & 0 & 0 \\ a_{12} & a_{22} & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{1(n-1)} & a_{2(n-1)} & \dots & a_{(n-1)(n-1)} \end{bmatrix} \times \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1(n-1)} \\ 0 & a_{22} & \cdots & a_{2(n-1)} \\ 0 & 0 & \ddots & \vdots \\ 0 & 0 & \dots & a_{(n-1)(n-1)} \end{bmatrix}$$

$$= -\begin{bmatrix} a_{11}a_{11} & a_{11}a_{12} & \cdots & a_{11}a_{1(n-1)} \\ a_{12}a_{11} & a_{12}a_{12}+a_{22}a_{22} & \cdots & a_{12}a_{2(n-1)}+a_{22}a_{2(n-1)} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1(n-1)}a_{11} & a_{1(n-1)}a_{12}+a_{2(n-1)}a_{22} & \dots & a_{1(n-1)}a_{1(n-1)}+\dots+a_{(n-1)(n-1)}a_{(n-1)(n-1)} \end{bmatrix}.$$

The imposition of negative semi-definiteness on the price coefficient matrix $\left[\alpha_{ij}\right]_{(n-1)\times(n-1)}$ does not place unwanted restrictions on price elasticities of input demands as the translog form does. However, this transformation has the drawback of turning linear relationships in system equations into nonlinear relationships that can make estimation more difficult. Despite this, this decomposition method has been popularly applied in empirical studies where the normalised quadratic functional form is employed. When the concavity condition is imposed by Cholesky decomposition, the symmetry condition is implicitly imposed as well, since this decomposition implies $\alpha_{ij} = \alpha_{ji}$. When the concavity condition is not enforced, the symmetry condition can be separately imposed during the estimation via restrictions $\alpha_{ij} = \alpha_{ji}$.

With five variable inputs defined for Australian broadacre agriculture, all five four-equation demand systems derived from the cost function using alternative *numeraires* are estimated using the FIML method[4]. For example, when x_5 is used as the *numeraire*, the estimated demand system is:

$$x_i = \alpha_i + \sum_{j=1}^{4} \alpha_{ij} w'_j + \sum_{k=1}^{4} \delta_{ik} y_k + \sum_{g=1}^{8} \gamma_{ig} z_g + \rho_{ti} T \text{ with } i = 1,2,...4 .$$

The estimation results of these five alternative demand systems are then compared against each other using the percentage of significant own-price coefficients, the percentage of own-price coefficients with expected signs, the percentage of significant price coefficients and the percentage of significant coefficients in the whole system. Based on these criteria, the system with the aggregate Fertilisers and chemicals (FC) input as the *numeraire* is superior among the five alternative systems. The estimation results of this system are presented and discussed later in Section 5.5.

5.4 Adjusting for Heteroskedasticity for Quasi-Micro Farm Data

The quasi-micro nature of the dataset raises some unique econometric issues in model estimation. Due to temporal discontinuity of observations for the majority of the distinctive farm cells in the dataset, as explained in section 4.3.3, the dataset is more like a pooled cross-sectional dataset than a panel dataset. As a result, no action is taken to correct autocorrelation in this study. In contrast, the unique quasi-micro nature of the dataset means that the issue of heteroskedasticity is more serious than usual. Each data point used for model estimation in this study is the average data observed for all surveyed farms in that cell, where the number of constituent farms varies significantly across cells. For instance, an average input is observed for farm cell A with a large number of farm members and for farm cell B with a small number of members. It is then expected that the input variance is much larger for cell A than for cell B. This particular form of heteroskedasticity is explained and discussed in Wooldridge (2006,

[4] The numeraire demand equation is excluded from the derived demand systems due to the excessive number of parameters it contains.

Chapter 8). The potential heteroskedasticity in the model is demonstrated here for the normalised quadratic cost function.

Suppose that error term u_i with a constant variance ($Var\left(u_i\right) = \sigma_i^2$) is added to the farm-level model derived from the normalised quadratic cost function specified for Australian broadacre agriculture:

$$x_i = \alpha_i + \sum_{j=1}^{4} \alpha_{ij} w'_j + \sum_{k=1}^{4} \delta_{ik} y_k + \sum_{g=1}^{8} \gamma_{ig} z_g + \rho_{ti} T + u_i ,\ i\ =\ 1,2,...4 .$$

Each observation in the quasi-micro data used for model estimation in this study is, however, the average data of sampled farms in a data cell. Let d_e be the number of farms constituting data cell e and $\tilde{x}_{i,e}$ $\tilde{w}'_{j,e}$, $\tilde{y}_{k,e}$ and $\tilde{z}_{g,e}$ be the observed average prices, quantities and other non-economic exogenous variables for this data cell. The econometric model at the quasi-micro level is:

$$\tilde{x}_{i,e} = \alpha_i + \sum_{j=1}^{4} \alpha_{ij} \tilde{w}'_{j,e} + \sum_{k=1}^{4} \delta_{ik} \tilde{y}_{k,e} + \sum_{g=1}^{8} \gamma_{ig} \tilde{z}_{g,e} + \rho_{ti} T + \tilde{u}_{i,e} , \text{ where } \tilde{u}_{i,e} = \frac{1}{d} \sum_{e=1}^{d_e} u_{ie} .$$

The equality concerning the error term means that if $Var\left(u_i\right) = \sigma_i^2$ for all individual farms as assumed then the variance of the error term for data cell e is $Var\left(\tilde{u}_{i,e}\right) = \dfrac{1}{d_e} \sigma_i^2$. If all data cells have the same number of constituent farms, i.e. $d_e \equiv d$ is constant, then the quasi-micro model also satisfies the homoskedasticity condition. However, in the quasi-micro data available in this study, d_e varies significantly across data cells, being as small as 5 and as large as 73. As a result, the variance of the error term in the quasi-micro model is not constant and depends on the number of constituent farms in the cells.

As seen above, it is fairly straightforward to determine whether heteroskedasticity exists for normalised quadratic functional form because for this form the left-hand variables of the model are input quantities. It is less clear whether a heteroskedasticity issue arises for the translog. For this functional form, the left-hand variables of the model are cost shares of inputs while the average data observed in the AAGIS data are input quantities. Appendix D demonstrates that heteroskedasticity also exists in a model based on the translog cost function.

A robust econometric estimation result requires adjustments to correct for heteroskedasticity or to alleviate its consequences. In a simple linear regression, the conventional means for overcoming this problem is to weight variables by the square root of the cells' sample size (Wooldridge 2006, Chapter 8). This weighting is employed in Lopez (1984) and is also used in this study for both translog cost share and normalised quadratic demand quantity systems.

5.5 Empirical Results

5.5.1 Estimated Coefficients

5.5.1.1 Translog Cost Share System

The estimated coefficients of the translog share system are presented in Table 5. The number of significant system coefficients is 67 (or 70.5 per cent of total number of system coefficients) at the 5% level. At the 10% level, 70 coefficients are statistically significant. At the 5% level, 64 per cent of the price coefficients α_{ij}s are statistically significant. The system's McElroy R^2 is 0.63 and the adjusted R^2 of the system equations ranges from 0.71 to 0.86. The own-price coefficient α_{ii} is significant at the 5% level for Contracts, services and materials for livestock (CSM livestock) and Livestock trading demand equation. The own-price coefficients of the CSM livestock, FC and Other Contracts, services and materials (Other CSM) equations are negative, which indicates that their own-price elasticities will be negative as expected. The own-price coefficients for Fuel, oil and grease (FOG) and Livestock trading equations are positive. The absolute values of these own-price coefficients are well below the threshold of 0.25, meeting the necessary condition for the resulting own-price elasticities to be negative, as discussed in Subsection 5.2.4. Overall, the cost share system derived from the restricted translog multi-product cost function is fairly well fitted using the AAGIS quasi-micro data of Australian broadacre farming.

The estimation results of the cost share system show that the theoretical regularity conditions of monotonicity and concavity in input prices are not satisfied by the translog cost function. Negative shares are predicted at 18.4 per cent and 7.4 per cent of observations for FC and Livestock trading inputs respectively. The proportion of negative predicted shares in the total data sample is fairly low for CSM livestock, Other CSM and FOG inputs, being 3.4 per cent, 0.4 per cent and 2.5 per cent of observations respectively. Regarding the concavity condition, the matrix $A-\hat{c}+c'c$, as calculated by Diewert and Wales's (1987), is negative semi-definite at 566 out of the 1,343 data points in the sample.

5.5.1.2 Normalised Quadratic Quantity Demand System

Table 6 displays coefficient estimates of the normalised quadratic demand quantity system. At the 5% level, the percentage of significant system coefficients is 61.1 per cent, which is considerably lower than in the translog cost share system. At the 10% level, however, this percentage is 70.8 per cent, which is comparable to that of the translog share system. Regarding the input prices, 62.5 per cent of the coefficients of these variables are significant at the 5% level, slightly lower than that obtained in the translog share system. Notably, this percentage increases sharply to 75 per cent at the 10% level. Moreover, all own-price coefficients are statistically significant and negative at the 5% level as expected. In addition, the McElroy R^2 is 0.85 and the equations' adjusted R^2 ranges between 0.78 and 0.91. Overall, all these measures suggest that the goodness-of-fit of the normalised quadratic quantity system is reasonably good.

Regarding the regularity conditions unconstrained during the estimation, the normalised quadratic cost function achieves a better result than the translog cost function. Similar to the translog case, the estimated normalised quadratic demand quantity system does not satisfy the monotonicity condition with significant frequency of violations across the observations in the sample. The percentage of negative predicted quantities of input demands are respectively 13.0 per cent, 5.7 per cent, 6.1 per cent, and 11.1 per cent for CSM livestock, Other CSM, FOG, and Livestock trading. However, the derived normalised quadratic quantity demand system automatically satisfies the concavity condition. All four eigenvalues of the price coefficient matrix are negative without parametric restrictions.

The estimation results of the translog share system and the normalised quadratic quantity system show that both functional forms give reasonable goodness-of-fit in modelling Australian broadacre agriculture. However, from an economic point of view the normalised quadratic form appears to perform better than the translog. The estimated system of demand quantities derived from the former satisfies the concavity condition while the estimated system of cost shares derived from the latter does not. Considering the frequent failure of this condition in previous duality-based applications as discussed in Chapter 2, the satisfaction of the concavity condition of the estimated normalised quadratic demand system here is remarkable. Given this better performance of the normalised quadratic form, the estimation result of the normalised quadratic demand quantity system is chosen to be discussed in detail as follows.

Table 5: Translog Cost Function—Estimated Parameters of System of Derived Input Demand Share Equations

	Input demand share equation									
	CSM livestock		FC		Other CSM		FOG		Livestock trading	
	Coefficient	z-Statistic	Coefficient	z-Statistic	Coefficient	z-Statistic	Coefficient	z-Statistic	Coefficient	z-Statistic
Constant	0.518**	5.690	0.123**	2.051	0.519**	7.067	0.023	0.948	−0.187**	−3.578
CSM livestock price	−0.100**	−7.514	0.023**	2.872	0.037**	3.721	−0.004	−1.086	0.044**	8.803
FC price	0.023**	2.872	−0.007	−0.379	0.019	0.860	−0.026**	−3.347	−0.009	−1.644
Other CSM price	0.037**	3.721	0.019	0.860	−0.042	−1.526	0.029**	2.756	−0.04**	−5.797
FOG price	−0.004	−1.086	−0.026**	−3.347	0.029**	2.756	0.008	1.617	−0.007**	−2.260
Livestock trading price	0.044**	8.803	−0.009	−1.644	−0.04**	−5.797	−0.007**	−2.260	0.012**	3.496
Crops quantity	−0.006**	−3.398	0.004**	3.465	0.002**	2.072	0.001**	2.803	−0.003**	−3.765
Sheep quantity	0.002	0.491	0.004*	1.826	0.002	0.778	0.001	0.944	−0.009**	−5.727
Beef quantity	0.027**	8.860	−0.028**	−21.542	−0.008**	−3.747	−0.002**	−3.242	0.012**	7.365
Wool quantity	0.005	1.454	−0.001	−0.309	−0.000	−0.292	−0.004**	−4.220	−0.0001	−0.077
Capital quantity	−0.024**	−3.279	−0.003	−0.608	0.0076	1.279	−0.004**	−2.365	0.024**	5.967
Fixed labour quantity	−0.056**	−4.835	0.009	1.165	0.001	0.149	0.024**	8.078	0.021**	3.220
Dummy variable D	−0.118**	−12.854	0.068**	11.491	0.075**	11.317	0.03**	14.354	−0.058**	−11.189
Dummy variable Z1	0.025**	2.342	0.04**	5.104	−0.027**	−3.647	−0.007**	−2.557	−0.033**	−7.772
Dummy variable Z2	0.019*	1.737	0.062**	7.737	−0.04**	−5.157	−0.016**	−5.616	−0.028**	−5.884
Dummy variable S1	0.045**	3.307	0.079**	9.846	−0.058**	−5.534	−0.025**	−8.102	−0.044**	−6.245
Dummy variable S2	0.031**	3.094	0.035**	6.044	−0.031**	−4.252	−0.011**	−5.188	−0.025**	−5.590
Rainfall	−0.041**	−3.551	0.010	1.333	0.0201**	2.353	0.0021	0.861	0.009*	1.849
Time	0.008**	8.565	0.001	1.042	−0.006**	−5.607	−0.002**	−5.612	−0.001	−1.006
Adjusted R^2	0.71		0.84		0.86		0.76		0.77	

Note: D = 1 when the farm is in the cropping industry, Z1 = 1 when the farm is in the Wheat-Sheep zone, Z2 = 1 when the farm is in the High Rainfall zone, S1 = 1 when the farm size is greater than $400,000, and S2 = 1 when the farm size is between $200,000 and $400,000

** Significant at the 5% level

* Significant at the 10% level

Table 6: Normalised Quadratic Cost Function—Estimated Parameters of System of Derived Input Demand Quantity Equations

	Input quantity equation							
	CSM livestock		Other CSM		FOG		Livestock trading	
	Coefficient	z-Statistic	Coefficient	z-Statistic	Coefficient	z-Statistic	Coefficient	z-Statistic
Intercept	86.88	0.14	1,762.35**	8.02	112.67**	2.35	−62.19	−1.22
CSM livestock price	−795.77**	−2.02	−185.11	−1.32	−72.77**	−3.27	37.38*	1.76
Other CSM price	−185.11	−1.32	−1,670.6**	−5.24	181.06**	2.57	1.23	0.16
FOG price	−72.77**	−3.27	181.06**	2.57	−197.94**	−9.89	−5.12**	−2.64
Livestock trading price	37.38*	1.76	1.23	0.16	−5.12**	−2.64	−8.26**	−3.34
Grains quantity	0.015	0.22	0.146**	18.23	0.028**	19.28	−0.008*	−1.78
Sheep quantity	0.415**	2.75	−0.344**	−6.11	−0.007	−0.57	0.036**	2.64
Beef quantity	1.083**	58.18	0.175**	18.27	0.018**	9.17	0.099**	63.44
Wool quantity	0.261**	2.56	0.544**	16.44	0.006	0.81	−0.016*	−1.95
Capital	0.004	0.87	0.033**	34.71	0.002**	10.42	0.0002	−0.55
Fixed labour	−1.637**	−2.99	−0.232	−1.31	0.154**	4.33	0.238**	4.84
Dummy variable D	53.43	0.14	327.67**	5.10	80.19**	6.08	−35.55	−1.33
Dummy variable Z1	1,715.13**	7.96	8.99	0.11	−27.92*	−1.72	−144.72**	−9.01
Dummy variable Z2	1,325.06**	4.26	−307.69**	−3.38	−84.51**	−4.65	−123.73**	−5.93
Dummy variable S1	74.18	0.21	748.87**	7.38	136.82**	7.16	67.84*	1.89
Dummy variable S2	114.86	0.31	124.11	1.33	41.79**	2.59	2.34	0.07
Rainfall	−375.84	−1.12	−256.41**	−2.57	29.369*	1.76	95.093**	3.77
Time	−16.48	−0.68	−21.11**	−2.00	−6.41**	−3.32	5.24**	2.47
Adjusted R^2	0.78		0.91		0.84		0.85	

Note: D = 1 when the farm is in the cropping industry, Z1 = 1 when the farm is in the Wheat-Sheep zone, Z2 = 1 when the farm is in the High Rainfall zone, S1 = 1 when the farm size is greater than $400,000, and S2 = 1 when the farm size is between $200,000 and $400,000

** Significant at the 5% level

* Significant at the 10% level

5.5.2.1 Quantity Demand for Contracts, Services and Materials for Livestock

As shown in Table 6, the demand for CSM livestock has negative significant relationships to its own price and the price of the aggregate FOG input. This implies that the demand for this aggregate livestock input decreases if these two prices increase. In contrast, CSM livestock demand has a significantly positive relationship with Livestock trading price. With respect to the outputs, the demand for CSM livestock input is significantly and positively related to the quantities demanded for all three livestock outputs, i.e. Sheep, Beef and Wool, which appears logical. Based on statistical evidence, the level of Grains produced does not have an effect on the quantity of CSM livestock demanded, which is expected.

The relationships between the quantity of CSM livestock demanded and the remaining exogenous factors included in the model are fairly sensible. Fixed capital does not have a statistically significant influence on demand for CSM livestock, other things being equal. This insignificant relationship likely follows from the fact that livestock grazing does not require heavy investments in specialised machinery or storage facilities. The exclusion of livestock capital from the total capital is also likely to contribute to the insignificance in this pair-wise relationship. In contrast, higher devotion of the operator and family to on-farm activities (represented by weeks worked by fixed labour) appears to be associated with lower utilisation of CSM livestock input. The statistical insignificance of the industry dummy variable suggests that demand for CSM livestock input is not influenced by the choice of key products in farming operations. As Cropping farms receive nearly half of their total revenue from combined Beef, Sheep and Wool production, as described in Chapter 4, this result is acceptable. In contrast, both the zone dummy variables are significantly positive in CSM livestock demand equation. This implies that there is higher use of the CSM livestock input in the Wheat-Sheep and High Rainfall zones than in the Pastoral zone, *ceteris paribus*. Finally, all remaining exogenous variables are statistically insignificant, suggesting that production size, rainfall and technical progress have no significant effects on the level of CSM livestock input demanded over the study period.

5.5.2.2 Quantity Demand for Other Contracts, Services and Materials

Demand for the aggregate Other CSM input is negatively influenced by its own price and the aggregate FOG price input, based on the statistical evidence. No statistical support is found for significant relationships between this input's demand and the prices of CSM livestock and Livestock trading inputs. The usage of this input, however, significantly relates to the levels of all broadacre outputs produced. An increase in Grains, Beef and Wool output levels are associated with an increase in demand for the Other CSM input. An increase in Sheep output, in contrast, appears to have a negative impact on the demand for the Other CSM input. With regard to fixed capital and labour inputs, only physical capital significantly and positively affect demand for Other CSM. This relationship is possibly because fixed capital usually incurs ongoing running costs so the higher the capital level is, the higher the demand for the aggregate Other CSM input.

Demand for the Other CSM input is influenced by more non-quantity, non-price factors than demand for the CSM livestock input. There is strong statistical evidence indicating that cropping and livestock grazing farms utilise Other CSM input differently. The estimation result also shows that usage of Other CSM input differs between the High Rainfall zone and the other two broadacre zones (between which no significant difference is found). At the same time, large farms are found to have appreciably higher demand for the Other CSM input than medium and small farms, *ceteris paribus*. Finally, unlike the demand for CSM livestock, statistical evidence indicates that higher rainfall and technical progress help reduce use of the Other CSM input.

5.5.2.3 Quantity Demand for Fuel, Oil and Grease

In the estimated demand equation of the aggregate FOG input, the coefficients of all input prices are statistically significant. A high level of significance suggests that demand for FOG is fairly responsive to market signals. The demand for this input is negatively affected by its own price as well as CSM livestock and Livestock trading prices. In contrast, the relationship between demand for FOG and the price of Other CSM is positive. Moreover, the level of FOG demand significantly increases with the levels of Grains and Beef produced, suggesting a more intensive use of machinery in the production of these two outputs. In contrast, the estimated system indicates that the production of Sheep and Wool does not significantly affect the demand for fuel, oil and grease.

Positive relationships between the levels of FOG input demand and the two fixed inputs, physical capital and fixed labour, is strongly supported by the statistical evidence. All other qualitative factors, i.e. predominant production activities, agro-climatic zone and production scale, also have significant effects on the level of FOG used. Given other characteristics fixed, Cropping farms appear to use more FOG than Livestock farms, farms in the High Rainfall zone use less of this input than those in the Wheat-Sheep zone, farms in the Wheat-Sheep zone use less of this input than those in the Pastoral zone and the larger the farm is, the higher level of this input demanded. Additionally, there is weak statistical evidence that an increase in rainfall is associated with an increase in demand for FOG. Finally, the estimated system suggests that technical advances lead to a decreased use of this input as in the case of the Other CSM input.

5.5.2.4 Quantity Demand for Livestock Trading

The Livestock trading demand equation in the estimated quantity system is also statistically reasonable. In this equation, the coefficients of own price and FOG price are highly significant. The signs of these coefficients suggest that Livestock trading demand is negatively affected by increases in these two input prices. Conversely, the coefficient of CSM livestock price is weakly significant and positive in this demand equation. The relationship between demand for Livestock trading input and Other CSM price is also positive but insignificant. Regarding the outputs, demand for the Livestock trading input appears to be influenced by the levels of all outputs. The production levels of Beef and Sheep have highly significant and positive effects on demand for Livestock trading input, in line with expectations. There is weak statistical evidence that an increase in Grains or Wool production level reduces demand for Livestock trading input.

The interactions between the demand for Livestock trading input and the remaining exogenous variables are fairly reasonable. Demand for the Livestock trading input is significantly and positively related to the fixed labour input, rainfall and time trend. The positive relationship between Livestock trading and rainfall is sensible. However, the increase of the Livestock trading input with the passage of time is unexpected given the time trend is often considered a proxy for disembodied technological progress. This finding may be related to an increasing trend of cattle production in Australia over the study period (ABARE 2008). Regarding the influences of climatic conditions on production, it is strongly evident from the estimation result that demand for Livestock trading is higher for farmers in the Pastoral zone than those in the High Rainfall zone, who in turn have higher demand of this input than those in the Wheat-Sheep zone. Moreover, the estimation result weakly suggests that large farms have higher demand for Livestock trading than medium and small farms. There is no statistical evidence supporting the existence of significant differences between Cropping and Livestock farms in utilising the Livestock trading input.

5.5.2 Net Price and Substitution Elasticities of Input Demands

Table 7 and Table 8[5] respectively report estimates of net own- and cross-price elasticities (with corresponding bootstrapping standard errors) calculated from the estimated translog and normalised quadratic derived demand systems. For the translog

[5] Because variables in this study are generally highly skewed to the right due to the micro-quasi nature of the data, we have chosen to evaluate these measures at all sample points and report their median values instead of reporting them at mean values as is conventional.

form, eighteen out of the 25 elasticities are statistically significant at the 5% level, compared to seventeen for the normalised quadratic form. Elasticity results from both functional forms satisfactorily have statistically significant and negative own-price elasticities as expected by economic theory. However, the magnitude of own-price elasticity differs significantly between the two result sets. For instance, the translog result shows that the demand for CSM livestock is elastic to its own price with an elasticity estimate of −1.20. The estimate of the same elasticity from the normalised quadratic is much lower in magnitude, being −0.431. Further, the demand for Livestock trading input is fairly responsive to its own price by the translog form (with an elasticity estimate of −0.744) but very inelastic by the normalised quadratic (with an elasticity estimate of −0.078). Similarly, the magnitude of the own-price elasticity for FOG from the translog form is significantly higher than that from the normalised quadratic form, 0.81 compared to 0.531. In contrast, the own-price elasticity of FC is much smaller (in magnitude) by the translog than by the normalised quadratic form, −0.905 compared to −1.756. The estimates from the two functional forms are closest for the own-price elasticity of Other CSM, −0.623 in the translog compared to −0.796 in the normalised quadratic.

The two price elasticity sets commonly show that demand for most inputs is inelastic to changes in alternative inputs' prices. However, half of the cross-price elasticity estimates change sign between these two elasticity sets. The largest discrepancies are the elasticity estimates of FC demand with respect to Other CSM price, Other CSM demand with respect to FC price, FOG demand with respect to Other CSM price and Livestock trading demand with respect to CSM livestock price. Notably, an increase of one per cent in Other CSM price results in an increase of 0.61 per cent in demand for FC according to the translog, compared to an increase of 1.03 per cent according to the normalised quadratic. Similarly, the demand elasticity of Other CSM with respect to FC is much lower from the translog than from the normalised quadratic. In contrast, the elasticity of FOG demand with respect to Other CSM price is estimated to be 0.906 by the translog, compared to 0.525 by the normalised quadratic. Finally, the price elasticity estimate of Livestock trading with respect to CSM livestock is 0.80 in the translog's result, much higher than the estimate of 0.152 in the normalised quadratic's result.

Table 7: Own- and Cross-Price Elasticities of Input Demand—Translog Form [a, b]

	With respect to price of				
Demand for	Contracts, materials & services for livestock	Fertilisers and chemicals	Other contracts, services & materials	Fuel, oil & grease	Livestock trading
Contracts, materials & services for livestock	-1.197**	0.188**	0.640**	0.046**	0.306
	(0.048)	(0.026)	(0.033)	(0.015)	(0.018)
Fertilisers and chemicals	0.324**	-0.905**	0.605**	-0.085**	0.008
	(0.035)	(0.102)	(0.125)	(0.039)	(0.039)
Other contracts, services & materials	0.297**	0.137**	-0.623**	0.136**	-0.021
	(0.021)	(0.05)	(0.057)	(0.021)	(0.015)
Fuel, oil & grease	0.149**	-0.271**	0.906**	-0.810**	-0.022
	(0.051)	(0.09)	(0.118)	(0.055)	(0.038)
Livestock trading	0.802**	-0.016	-0.042	-0.012	-0.744**
	(0.038)	(0.054)	(0.048)	(0.022)	(0.03)

Note: [a] Medians of elasticities evaluated at all observation points
[b] Bootstrapping standard errors (500 trials) are in parentheses
** Significant at the 5% level
* Significant at the 10% level

Table 8: Own- and Cross-Price Elasticities of Input Demand—Normalised Quadratic Form [a, b]

Demand for	With respect to price of: Contracts, materials & services for livestock	Fertilisers and chemicals	Other contracts, services & materials	Fuel, oil & grease	Livestock trading
Contracts, materials & services for livestock	−0.431**	0.491**	−0.13*	−0.051**	0.06**
	(0.123)	(0.124)	(0.076)	(0.014)	(0.023)
Fertilisers and chemicals	0.391**	−1.756**	1.03**	0.037	−0.009
	(0.108)	(0.198)	(0.167)	(0.034)	(0.011)
Other contracts, services & materials	−0.068*	0.782**	−0.796**	0.083**	0.001
	(0.041)	(0.118)	(0.143)	(0.031)	(0.009)
Fuel, oil & grease	−0.157**	0.197	0.525**	−0.531**	−0.031**
	(0.046)	(0.166)	(0.197)	(0.048)	(0.009)
Livestock trading	0.152**	−0.055	0.006	−0.026**	−0.078**
	(0.061)	(0.071)	(0.04)	(0.008)	(0.017)

Note: [a] Medians of elasticities evaluated at all observation points
[b] Bootstrapping standard errors (500 trials) are in parentheses
** Significant at the 5% level
* Significant at the 10% level

Table 9: Allen Partial Elasticities of Substitution—Translog Cost Function [a, b]

	Contracts, materials & services for livestock	Fertilisers and chemicals	Other contracts, services & materials	Fuel, oil & grease	Livestock trading
Contracts, materials & services for livestock					
Fertilisers and chemicals	1.857**				
	(0.211)				
Other contracts, services & materials	1.352**	1.240**			
	(0.079)	(0.245)			
Fuel, oil & grease	0.747**	−1.207**	1.857**		
	(0.172)	(0.478)	(0.261)		
Livestock trading	2.871**	0.223	−0.098	−0.167	
	(0.132)	(0.286)	(0.118)	(0.316)	

Note: [a] Medians of elasticities evaluated at all observation points
[b] Bootstrapping standard errors (500 trials) are in parentheses
** Significant at the 5% level
* Significant at the 10% level

Table 10: Allen Partial Elasticities of Substitution—Normalised Quadratic Cost Function [a, b]

	Contracts, materials & services for livestock	Fertilisers and chemicals	Other contracts, services & materials	Fuel, oil & grease	Livestock trading
Contracts, materials & services for livestock					
Fertilisers and chemicals	0.551**				
	(0.25)				
Other contracts, services & materials	−0.242*	1.694**			
	(0.142)	(0.318)			
Fuel, oil & grease	−0.598**	0.332	0.975**		
	(0.165)	(0.353)	(0.371)		
Livestock trading	0.384**	−0.163	0.012	−0.306**	
	(0.153)	(0.153)	(0.153)	(0.153)	

Note: [a] Medians of elasticities evaluated at all observation points
[b] Bootstrapping standard errors (500 trials) are in parentheses
** Significant at the 5% level
* Significant at the 10% level

The estimates of Allen partial elasticities of substitution obtained from the translog and normalised quadratic cost functions are respectively shown in Tables 9 and 10. These two sets of results differ in portraying substitutive and complementary relationships among broadacre production inputs. Half of the Allen partial elasticity estimates change signs between the two functional forms' results. The CMS livestock-other CMS, CMS livestock-FOG, and FC-livestock trading relationships are found to be substitutive by the translog form but complementary by the normalised quadratic. The translog suggests that FC is complementary to FOG, and other CMS is complementary to livestock trading but the normalised quadratic suggests the contrary. The two functional forms find a substitutive relationship between CMS livestock and FC, CMS livestock and livestock trading, FC and other CMS, and other CMS and FOG but a complementary relationship between FOG and livestock trading. Out of ten Allen partial elasticities, seven are statistically significant at the five per cent level in the translog's result, compared to six in the normalised quadratic's result. The livestock trading-FC and livestock trading-other CMS elasticities are statistically insignificant in both functional forms' results.

Table 11: Morishima Elasticities of Substitution—Translog Cost Function [a, b]

	Contracts, materials & services for livestock	Fertilisers and chemicals	Other contracts, services & materials	Fuel, oil & grease	Livestock trading
Contracts, materials & services for livestock	-	1.193**	1.260**	0.861**	1.052**
		(0.128)	(0.084)	(0.059)	(0.036)
Fertilisers and chemicals	1.743**	-	1.209**	0.723**	0.743**
	(0.103)		(0.173)	(0.042)	(0.05)
Other contracts, services & materials	1.514**	1.085**	-	0.951**	0.749**
	(0.065)	(0.142)		(0.069)	(0.032)
Fuel, oil & grease	1.386**	0.685**	1.496**	-	0.723**
	(0.074)	(0.093)	(0.167)		(0.063)
Livestock trading	1.952**	0.934**	0.618**	0.782**	-
	(0.074)	(0.127)	(0.064)	(0.085)	

Note: [a] Medians of elasticities evaluated at all observation points
[b] Bootstrapping standard errors (500 trials) are in parentheses
** Significant at the 5% level
* Significant at the 10% level

The estimates of the Morishima elasticities of substitution calculated from the two functional forms conform to each other. As shown in Tables 11 and 12, all pair-wise elasticities are positive in both result sets, indicating the substitutive nature of pair-wise relationships among broadacre inputs. Moreover, almost all elasticities obtained are highly significant in both functional forms' results. These findings contrast strongly to the significant divergence between the Allen partial elasticity estimates from the two functional forms. From the relationship $\sigma_{ij}^{M} = \eta_{ij} - \eta_{jj}$ (see Subsection 3.3.3.), the positive Morishima elasticity estimates obtained imply that an increase in the price of a broadacre input will cause either an increase in the use of alternative inputs (i.e. a substitutive response) or a decrease in the use of alternative inputs (i.e. a complementary response) accompanied by a larger percentage-wise decrease in its own demand.

Table 12: Morishima Elasticities of Substitution—Normalised Quadratic Cost Function [a, b]

	Contracts, materials & services for livestock	Fertilisers and chemicals	Other contracts, services & materials	Fuel, oil & grease	Livestock trading
Contracts, materials & services for livestock	-	1.448**	0.583**	0.459**	0.116**
		(0.203)	(0.195)	(0.053)	(0.029)
Fertilisers and chemicals	0.394**	-	1.247**	0.525**	0.061**
	(0.126)		(0.227)	(0.055)	(0.015)
Other contracts, services & materials	0.320**	1.953**	-	0.628**	0.079**
	(0.134)	(0.282)		(0.08)	(0.019)
Fuel, oil & grease	0.193	1.676**	1.314**	-	0.045**
	(0.133)	(0.224)	(0.31)		(0.021)
Livestock trading	0.471**	1.481**	0.798**	0.503**	-
	(0.134)	(0.203)	(0.139)	(0.055)	

Note: [a] Medians of elasticities evaluated at all observation points
[b] Bootstrapping standard errors (500 trials) are in parentheses
** Significant at the 5% level
* Significant at the 10% level

5.5.3 Estimation Using Subsamples by Zone, by Size and by Industry

In the models estimated, qualitative dummy variables are included in the translog and normalised quadratic cost functions to account for climatic, industry and size effects in broadacre production. An alternative to the use of dummy variables is to estimate the models using subsamples segregated by climate, industry or size. In other words, econometric models are estimated separately for three broadacre zones, two production industries and three production sizes. This alternative approach is promising since it permits differences of any form between farms in different zones, in different industries or having different sizes. In contrast, when dummy qualitative variables are used, only vertical shifts of estimation equations (i.e. different intercepts) between zones, sizes and industries are allowed. However, when separate models for zone, size and industry subsamples are estimated, their estimation results are too poor for both functional forms. The estimated systems for these subsamples have a low percentage of significant system coefficients, a low percentage of significant price coefficients and a large proportion of insignificant own-price coefficients. For instance, among the estimated normalised quadratic quantity systems for these eight subsamples, only the systems for the Wheat-Sheep zone (having 833 observations) and small size (having 477 observations) have more than half of the system coefficients being significant at the 5% level.

The poor estimation results for zone, size and industry subsamples are unexpected considering the high percentage of significant coefficients of the zone, size and industry dummy variables when using the whole data sample in both the translog share and normalised quadratic quantity systems estimated. The poor results for data subsamples are likely due to insufficient sample size relative to the number of system parameters to be estimated. For instance, the Pastoral zone subsample has 201 observations whilst the normalised quadratics quantity system has 58 parameters to be estimated. However, in the case of the normalised quadratic form, the results are unsatisfactory for cropping and livestock subsamples despite their large sample sizes of 619 and 724 observations, respectively. This suggests that broadacre farms focusing on different products interact with each other and that they belong to the same population. This interaction between farms having different production focuses is probably due to the fact that Cropping farms, on average, collectively receive about 46 per cent of total revenue from livestock activities, as described in Chapter 4. The interaction between farms in different industries implies that research findings in studies of Australian broadacre agricultural production whose research scope is limited to a single industry may be significantly distorted.

5.6 Discussion of Estimation Results

The estimation results are reasonable for both translog and normalised quadratic cost functions. This contrasts with previous international and Australian duality-based studies that estimated cost functions. The percentage of significant system coefficients at the 5% level, 71.6 per cent for the translog and 61.1 per cent for the normalised quadratic, are much higher than that in many studies such as Kuroda and Lee (2003), Gagne and Nappi (2000), Bloch *et al.* (2001), Akridge and Hertel (1986) and Halvorsen and Smith (1986). System-wide McElroy R^2 for the two estimated models are also fairly high, being 0.85 for the normalised quadratic model. The normalised quadratic model also explains individual input demands well and evenly with adjusted-R^2 ranging between 0.78 and 0.91. Moreover, input prices are found to significantly influence input demands, with 64 per cent and 62.5 per cent of price coefficients being significant at the 5% level in the translog share and the normalised quadratic quantity systems respectively. In the normalised quadratic quantity system, all own-price coefficients are statistically significant and negative as expected.

The statistical significance of many cross-price coefficients in the two estimated demand systems suggests that the demand for a broadacre input is jointly determined by its price and prices of alternative inputs. This implies that Australian broadacre farmers make production decisions concerning different inputs simultaneously. This simultaneous decision-making process highlights the importance of accommodating interrelationships between broadacre inputs when intervention policies concerning the rural sector and the wider economy are made. This implies that results from single-output studies, such as Griffiths *et al.* (2000) or O'Donnell and Woodland (1995) and McKay *et al.* (1980) should be used with caution.

The estimation of the translog and normalised quadratic cost forms generates mixed results regarding regularity conditions that cannot be parametrically imposed during the estimation. The estimated translog cost function is not concave at more than half of the data points. This failure to meet the concavity condition by the translog cost is similar to the findings in McKay *et al.* (1980) although these authors based their conclusion on the non-negative semi-definiteness of the (assumingly Allen) 'partial elasticities of substitution'. If using Allen partial elasticities, the estimation result of the translog cost function using quasi-micro data in this thesis fairs better since the eigenvalues of the Allen partial elasticity matrix obtained here are all negative.

In contrast to the translog form's result, the estimated normalised quadratic quantity system meets the concavity condition without artificial parametric restrictions. This is an unexpectedly positive result considering the frequent violations of this condition in previous duality applications, especially in applications to agricultural production. The satisfaction of the concavity condition and the statistical significance of all own-price coefficients strongly suggest that farmers do substitute inputs for each other in response to price changes for given output levels and that the demand curves of broadacre inputs are downward sloping. These outcomes also indicate that the normalised quadratic form is more suitable than the translog in specifying restricted multi-product cost functions for Australian broadacre agricultural production.

The estimation outcomes regarding the monotonicity condition are not as encouraging as for the concavity condition, for both functional forms used. This condition is violated at numerous data points for both functional forms. This finding is contradictory to findings in McKay *et al.* (1980), in which the predicted shares of all inputs are positive at all data points, and in Mullen and Cox (1996), where the estimated cost function is not concave at only three data points for one of the six variable inputs included in their model. These two studies used highly-aggregated time-series data for model estimation, in contrast to the quasi-micro data used in this study. In the quasi-micro dataset used in this study, some input quantities vary extensively and are close to zero in a considerable number of observations. Therefore, the cost shares and demand quantities predicted in this study are expected to be close to zero or negative. The violation of the concavity condition in this study is similar to the research findings in Ollinger *et al.* (2005), where pooled cross-sectional data are used to estimate a translog cost function for the United States poultry industry.

The findings on price responsiveness in this study generally conform to economic theory. All the own-price elasticities of broadacre inputs have the expected signs and are statistically significant for both functional forms used. More than two-thirds of all cross-price elasticities are statistically significant in the two functional forms' results.

Results from the normalised quadratic functional form, which are better than the translog's results regarding the concavity condition, indicate that demand for most broadacre inputs in Australian agriculture are not responsive to price change in the short-term. This implies that government interventions or market movements affecting input prices will not cause significant adjustments in demand for these inputs. The demand for the aggregate FC is an exception to this general trend. Demand for FC is very responsive to its own price and price of Other CSM. Its own-price elasticity estimate is −1.8, indicating that a one per cent increase in its price leads to a 1.8 per cent decrease in its demand, given output levels are fixed. An increase in general production costs (price of Other CSM) causes a decrease of a slightly larger percentage in FC demand.

Similar to most previous studies of agricultural production, this study also suffers from the unavailability of data on farm-specific input price. National annual price indices constructed by ABARE were used in place of unobserved actual input prices for model estimation. As a result, variations in input prices across geographical areas are not present in the data used for estimation. In contrast, input prices are commonly expected to vary across geographical areas, being higher in remote areas such as those in the pastoral and Wheat-Sheep zones due to higher transportation costs. Moreover, input costs in the data, and thus the implicit input quantity indices derived from the costs and national price indices (see Chapter 4), are at a quasi-micro level. Therefore, it would be judged that the price elasticity estimates obtained in this study may have been overestimated. However, this cannot be proven to be the case. By way of construction, the national price indices, like the aggregate price indices constructed in this study (see Chapter 4), should reflect the relative variations in input prices and quantities demanded across geographical areas in a way that is consistent with the price-demand relationships being estimated in this study. Moreover, if the price variation across geographical areas is similar for all production inputs the normalisation of prices in estimating the normalised quadratic cost function may have lessened any inflating impacts that the use of national input price indices may have on price elasticity estimates. This can be demonstrated using the derived quantity demand equations and the formulas of price elasticities for the normalised quadratic functional form.

Regarding input substitutability, findings from the two functional forms differ significantly in Allen partial elasticity estimates but are strongly consistent in Morishima estimates. A large proportion of Allen partial estimates change sign between the two functional forms. In contrast, Morishima elasticity estimates from the two functional forms consistently suggest that all pair-wise relationships among broadacre inputs, conditional on output levels, are substitutive. Almost all Morishima elasticities are also statistically significant in both functional forms' results.

The consistency of Morishima elasticity estimates obtained from the two functional forms, and their strong statistical robustness, suggests that the Morishima measure is more robust to the choice of functional form in estimation of dual cost functions. These findings further strengthen the increasing preference for the Morishima measure over the more traditional Allen partial measure in recent literature, recognising that the Morishima measure has a less ambiguous interpretation of substitution than the Allen partial measure (Agbola and Harrison, 2005; Sharma, 2002; Fisher *et al.*, 2001; Huang, 1991; and Mountain and Hsiao, 1989). Morishima estimates obtained from the two estimated cost functions of Australian broadacre production commonly suggest that all broadacre inputs are substitutes. This result is similar to a study of the Australian pastoral region by Agbola and Harrison (2005) in which twenty out of the twenty-five reported short-run Morishima elasticities are positive.

Due to differences in geographical coverage, nature of data used, data aggregation methods, dual objective functions estimated and functional forms used, it is not possible to validate this study's findings by a direct comparison of estimation results obtained to those in previous studies of Australian broadacre farming. Nevertheless, the strong statistical significance of the estimation results obtained in this study, via using a much larger data sample, strongly supports that elasticity estimates generated by the models here are reliable and valuable for policy valuation.

5.7 Conclusion

In this chapter, Australian broadacre production technology is modelled under the assumption that farmers aim to minimise their production costs, conditional on output levels. Restricted multi-product translog and normalised quadratic cost functions are specified for a set of five variable inputs, four outputs, two fixed inputs and six other exogenous variables. The derived translog cost share system and the derived normalised quadratic demand quantity systems are estimated using AAGIS quasi-micro data. Heteroskedasticity caused by the quasi-micro nature of AAGIS data is corrected during the estimation. The symmetry and homogeneity conditions are imposed during the estimation, while the monotonicity and concavity conditions are checked after estimation. The estimation results are statistically robust with high percentages of statistically significant system coefficients and price coefficients. The own-price coefficient estimates have expected signs. The monotonicity condition is violated in both functional forms due to the quasi-micro nature of the data used for model estimation. The estimated translog cost share system does not satisfy the concavity condition but the normalised quadratic cost function satisfies this condition without being enforced by parametric restrictions.

The results of price-quantity interactions are statistically significant and economically meaningful for broadacre inputs. All own-price elasticities obtained from both functional forms have the expected signs and are highly significant using bootstrapping standard errors. A majority of cross-price elasticities are statistically significant regardless of what functional form is used. The results suggest that, in the short-term, demand for broadacre inputs is generally inelastic with respect to input prices. This implies that policies designed to influence input demands through market prices will not be very effective. An exception is demand for fertilisers and chemicals for crop and pasture, which is found to be elastic to their own prices and prices of contracts, non-petroleum materials and services in the normalised quadratic cost function's results.

The substitution elasticity estimates obtained from the two cost functions provide an important insight into relative reliability of the Allen partial and Morishima measures. By the Morishima measure, the estimated translog and normalised quadratic cost functions commonly suggest that all broadacre production inputs are substitutes. Meanwhile, significant divergence is found in Allen partial elasticity estimates obtained from these two functional forms. Moreover, almost all Morishima elasticities are statistically significant while a number of Allen partial elasticities are statistically insignificant in the two functional forms' results. The high stability and strong statistical significance of Morishima elasticities in both cost functions estimated in this study suggest that this measure is more reliable than the Allen partial measure in measuring technical relationships between production inputs.

Chapter 6

Estimating Restricted Multi-Product Revenue Functions for Australian Broadacre Production

6.1 Introduction

In this chapter, the investigation of Australian broadacre production continues through the specification and estimation of restricted multi-product revenue functions using AAGIS quasi-micro data. Unlike in Chapter 5, the formulation of the model in this chapter assumes farmers take input levels as given and adjust output levels to maximise total production revenue, given that output prices are exogenously determined. Under this assumption, a dual revenue function is specified, and revenue-maximising input demand equations are derived and estimated.

Revenue maximisation is rarely assumed in empirical applications of duality, in contrast to the popularity of profit maximisation and cost minimisation. Assuming fixity of all production inputs is too restrictive to be realistic. However, there are several reasons for assuming revenue-maximisation behaviour to investigate Australian broadacre agricultural production. Firstly, broadacre farms are predominantly family-run businesses with limited natural, financial and managerial resources. Farmers have limited flexibility in adjusting these resources in the short term. Despite farmers having some flexibility to switch between different products within a year's time

Interval, limitations in resource endowments may not allow them to maximise production profits in the short term since this requires adjusting input levels and costs.

The second reason for assuming revenue maximisation in Australian broadacre farming is the complex, staged decision-making process needed for technical and economic efficiency in profit maximisation. To maximise profits, farmers have to adjust and allocate the quantities of both inputs and outputs in response to relative price changes. With numerous inputs and outputs, profit maximisation is a mathematically complicated optimisation problem that can only be solved with computer aid, and is impractical for normal farming operation with limited natural, financial and managerial resources. In contrast, production revenue is a good proxy, guiding production decisions to ensure financial survival and profit making. The revenue maximisation also involves fewer entities (i.e. only outputs but not inputs) that allocation decision is feasibly implemented by farmers.

The third reason supporting the specification of a dual revenue function for Australian broadacre farming concerns the information retrieved about the underlying production technology. Specifying a revenue function reveals price-quantity responses and transformation possibilities between outputs when input levels are fixed. As Gordon (1989) points out, these net (or conditional or input-compensated or input-constant) elasticities do not encompass the adjustments in input quantities caused by price changes while the gross (or uncompensated or Marshallian) elasticities drawn from a profit function do.

With the above motivations, dual restricted multi-product revenue functions are specified in this chapter for Australian broadacre agriculture. These functions will have the same four aggregate outputs, five aggregate variable inputs, two aggregate fixed inputs, five dummy variables, rainfall variable and time trend as in the cost functions estimated in Chapter 5. As in Chapter 5, both translog and normalised quadratic revenue functions are specified and estimated. Empirical issues common in the estimation of dual cost and revenue functions are discussed briefly in this chapter and referred to Chapter 5. For the translog form, the system of input-constrained output revenue share functions is derived and estimated. For the normalised quadratic form, the system of input-constrained output supply quantities is derived and estimated. After the estimation of each of these derived systems, the net price elasticities and elasticities of transformation are estimated for broadacre outputs. The results from the two functional forms are then compared.

The structure of this chapter is similar to Chapter 5. Section 6.2 covers the general framework of specifying a dual restricted translog multi-product revenue function and its empirical application to Australian broadacre agricultural production. This section includes a description of the theoretical regularity conditions, the derivation of input-constrained output supplies, and the derivation of net price and transformation elasticities for output supplies. Section 6.3 follows the same structure as Section 6.2 for modelling Australian broadacre technology via specifying a dual restricted normalised quadratic multi-product revenue function. The estimation results for Australian broadacre agriculture from restricted translog and normalised quadratic multi-product revenue functions are described in Section 6.4. These results are then discussed in Section 6.5. Section 6.6 concludes the chapter, summarising the chapter's empirical findings.

6.2 Specification of a Restricted Translog Multi-Product Revenue Function

This section presents the general framework of estimating a restricted multi-product revenue function in the translog functional form. The empirical application of this framework to Australian broadacre agriculture for the same set of four outputs, five variable inputs[6], two fixed inputs and six other exogenous variables as in Chapter 5 is then described.

6.2.1 The Restricted Translog Multi-Product Revenue Function

The restricted translog multi-product revenue function describing a technology that uses variable inputs $[x_1,\ x_2,\ ...,\ x_n]$ to produce outputs $[y_1, y_2, ...y_m]$ has the following representation:

$$\ln R(P,X,Z,T) = \alpha_0 + \sum_{i=1}^{n}\alpha_i \ln x_i + \sum_{k=1}^{m}\beta_k \ln p_k + \sum_{g=1}^{v}\lambda_g \ln z_g + \tfrac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_{ij}\ln x_i \ln x_j$$
$$+\tfrac{1}{2}\sum_{k=1}^{m}\sum_{l=1}^{m}\beta_{kl}\ln p_k \ln p_l + \tfrac{1}{2}\sum_{g=1}^{v}\sum_{h=1}^{v}\lambda_{gh}\ln z_g \ln z_h + \sum_{i=1}^{n}\sum_{k=1}^{m}\delta_{ik}\ln x_i \ln p_k + \sum_{i=1}^{n}\sum_{g=1}^{v}\gamma_{ig}\ln x_i \ln z_g$$
$$+\sum_{k=1}^{m}\sum_{g=1}^{v}\varphi_{kg}\ln p_k \ln z_g + \sum_{i=1}^{n}\rho_{ti}T\ln x_i + \sum_{k=1}^{m}\phi_{tk}T\ln p_k + \sum_{g=1}^{v}\psi_{tg}T\ln z_g + \theta_t T + \tfrac{1}{2}\theta_{tt}T^2$$

where R is the production revenue, p_k is the price of output k, z_g is the gth fixed input quantity or a non-quantity, non-price exogenous variable and T is a time trend.

6.2.2 Regularity Conditions

As described in Chapter 3, the restricted translog product revenue function specified above satisfies a set of theoretical regularity conditions to describe the rational economic behaviour of producers. The condition of monotonicity in output prices (Condition R.2) requires that partial derivatives of the logarithmic revenue function with respect to logarithmic prices are positive. Whether the revenue function satisfies the condition of convexity in output prices (Condition R.3) is related to the sign of definiteness of matrix $[\beta_{kl}]_{m\times m} - \hat{r}_{m\times m} + r_{1\times m}{'}r_{1\times m}$, where $[\beta_{kl}]_{m\times m}$ is the matrix of price coefficient estimates, $\hat{r}_{m\times m}$ is the diagonal matrix with its diagonal elements being the

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[6] In the specification of a revenue function, there is no real distinction between variable inputs and fixed inputs since their quantities are all assumed to be given. However, for ease of referencing when comparing the results from the three dual functions in later part of the thesis, this distinction is maintained in this chapter.

shares and $r_{1\times m}$ is the vector of the shares. Following the demonstration by Diewert and Wales (1987), it is straightforward to verify that the translog revenue function specified above satisfies the convexity condition if the matrix $[\beta_{kl}]_{m\times m} - \hat{r}_{m\times m} + r_{1\times m}{}' r_{1\times m}$ is positive semi-definite. The conditions of linear homogeneity (Condition R.4) and symmetry (or twice-continuous differentiability) both imply global restrictions on the parameters of the revenue function. The specified revenue function satisfies the homogeneity condition if $\sum_{k=1}^{m}\beta_k = 1$ and $\sum_{k=1}^{m}\beta_{kl} = \sum_{k=1}^{m}\delta_{ik} = \sum_{k=1}^{m}\varphi_{kg} = \sum_{k=1}^{m}\phi_{tk} = 0$, and the symmetry condition if $\alpha_{ij} = \alpha_{ji}$, $\beta_{kl} = \beta_{lk}$, and $\lambda_{gh} = \lambda_{hg}$. Similar to the translog cost function, the restrictions for these two regularity conditions are also those needed for the adding-up condition.

6.2.3 The Input-Constrained Output Supplies

When the revenue function specified above satisfies the regularity conditions, a system of input-constrained revenue share equations is derived by applying the Chain Rule and the Samuelson-McFadden lemma:

$r_k = \beta_k + \sum_{l=1}^{m}\beta_{kl}\ln p_l + \sum_{i=1}^{n}\delta_{ik}\ln x_i + \sum_{g=1}^{v}\varphi_{kg}\ln z_g + \phi_{tk}T$, $k = 1, 2\ldots, m$, where r_k is the revenue share of output k in the total revenue.

6.2.4 The Net Transformation and Price Elasticities of Output Supplies

The Allen partial elasticities of transformation between output supplies can be derived for the translog revenue function following the same derivation of elasticities of substitution between inputs for the cost function case by Binswanger (1974). Firstly, the coefficients of the interaction terms between output prices in the translog revenue function specified above can be expressed as:

$$\beta_{kl} = \frac{\partial^2 \ln R}{\partial \ln p_k \partial \ln p_l} = p_l \frac{\partial}{\partial p_l}\left(\frac{\partial R}{\partial p_k}\frac{p_k}{R}\right) = p_l \left(\frac{\partial^2 R}{\partial p_k \partial p_l}\frac{p_k}{R} - \frac{p_k}{R^2}\frac{\partial R}{\partial p_k}\frac{\partial R}{\partial p_l}\right).$$

Substituting $\frac{\partial R}{\partial p_k} = y_k$ (from the Samuelson-McFadden lemma) into the expression above we have $\beta_{kl} = \frac{p_k p_l}{R}\frac{\partial^2 R}{\partial p_k \partial p_l} - \frac{p_k p_l}{R^2} y_k y_l$, which leads to

$$\frac{\partial^2 R}{\partial p_k \partial p_l} = \frac{R}{p_k p_l}\left(\beta_{kl} + \frac{p_k y_k p_l y_l}{R^2}\right) = \frac{R}{p_k p_l}\left(\beta_{kl} + r_k r_l\right).$$

Substituting the last expression into the definition of Allen partial elasticity $\tau_{kl} = \frac{R}{y_k y_l}\frac{\partial^2 R}{\partial p_k \partial p_l}$ as derived in Chapter 3 (Subsection 3.4.3), we arrive at $\tau_{kl} = \frac{\beta_{kl}}{r_k r_l} + 1$.

It is straightforward to verify that the net own- and cross-price elasticities between output supplies are related to the coefficients and shares as: $\varepsilon_{kk} = \frac{\beta_{kk}}{r_k} + r_k - 1$ for $k = 1, 2 \ldots, m$, and $\varepsilon_{kl} = \frac{\beta_{kl}}{r_k} + r_l$ for all $k \neq l$.

A 'rule of thumb' notion about values of the parameters of the translog revenue function can be assessed using the expression of the own-price elasticities of output supplies. As derived above, for this functional form, the relationship between the own-price coefficient β_{kk} and r_k is: $\varepsilon_{kk} = \frac{\beta_{kk}}{r_k} + r_k - 1$. It is expected that own-price elasticity ε_{kk} is positive for all output supplies. In economic interpretation, positive own-price elasticity means that an increase in an output’s price leads to an increase in its supply. This implies that $\beta_{kk} \geq 0$ because $0 \leq r_k \leq 1$. At the same time, we have $\lim_{\beta_{kk} \to 0} \frac{\beta_{kk}}{r_k} + r_k - 1 = r_k - 1$, the right hand side of which is negative since $r_k \leq 1$. This means that when β_{kk} is arbitrarily close to zero, the own-price elasticity ε_{kk} is very likely to be negative, contradicting economic theory. For example, if $\beta_{kk} = 0.05$, the own-price elasticity ε_{kk} is negative whenever the revenue share r_k falls in the range 0.053–0.947, which is extremely wide given $0 \le r_k \le 1$. Therefore, β_{kk} is expected to be not too close to zero.

6.2.5 Empirical Implementation

When the general framework described above is applied to the same set of variables as the cost function in Chapter 5, a system of revenue share equations is derived for output supplies as follows:

$$r_k = \beta_k + \sum_{l=1}^{4} \beta_{kl} \ln p_l + \sum_{i=1}^{5} \delta_{ik} \ln x_i + \sum_{g=1}^{8} \varphi_{kg} \ln z_g + \phi_{tk} T$$

where r_k is the revenue shares of Grains, Sheep, Beef and Wool outputs; p_l is the prices of Grains, Sheep, Beef and Wool; x_i is the quantities of CSM livestock, *FC*, Other CSM, FOG and Livestock trading inputs; z_g is two aggregate fixed inputs (total capital and operator's labour), five dummy variables (two zone dummy variables, two size dummy variables and one industry dummy variable) and an annual rainfall variable; and T is the time trend. Additive normally distributed errors with constant covariance are added to these system equations. As in the estimation of the cost share system in Chapter 5, a revenue share equation is deleted from this system to overcome the issue of singularity of the variance-covariance matrix caused by the adding-up condition. The system of the remaining share equations is estimated using the FIML estimation method. The conditions of homogeneity and symmetry in output prices are imposed during estimation while the monotonicity and convexity conditions are checked after estimation. Again, for each observational cell, all system variables are multiplied by the square root of the cell size (the number of constituent farms) to correct for heteroskedasticity as discussed in detail in Chapter 5 (Section 5.4).

An issue arises in the estimation of the revenue share system that does not occur in the estimation of the cost share system in Chapter 5. Since output prices are not realised when production decisions are made, the expected prices of outputs, instead of their actual prices, are included of the revenue share system. In the estimation here, broadacre farmers are assumed to have naïve price expectations as in Ahammad and Islam (2004), Lim and Shumway (1997), Coelli (1996), Lim and Shumway (1992) and Shumway and Alexander (1988). That is, these farmers are assumed to expect that the output prices received in the current year will be received in the following year. The output prices appearing on the right-hand-side of the revenue share equations are therefore lagged by one period to represent this price expectation. In the quasi-micro data used in this study, some lagged output prices are not available due to the discontinuance of data cells, which resulted from the confidentiality issue discussed in Chapter 4 (Section 4.3). These missing prices are replaced by the average of prices observed in the same year for all farm cells in the same region. Due to the loss of observations in 1990, since lagged prices are not available for this year, the final data sample used for estimation of the revenue share system has 1272 data observations.

6.3 Specification of a Restricted Normalised Quadratic Multi-Product Revenue Function

This section consists of two broad components. The first component describes the general framework for specifying a restricted multi-product revenue function in the normalised quadratic functional form. The second component of this section describes the empirical implementation of specifying the restricted normalised quadratic multi-product revenue function for Australian broadacre agriculture using the same set of outputs, inputs and other exogenous variables as in the specification of the restricted cost functions in Chapter 5.

6.3.1 The Restricted Normalised Quadratic Multi-Product Revenue Function

Consider the multi-product production technology using variable inputs $[x_1,\ x_2,\ ...,\ x_n]$ to produce outputs $[y_1, y_2, ...y_m]$[7]. Let output m be arbitrarily chosen as the normalising factor. The normalised production revenue and the normalised prices of the remaining outputs are as follows: $R'(P', X, Z, T) = \dfrac{R(P, X, Z, T)}{p_m}$ and $p'_k = \dfrac{p_k}{p_m}$, $k = 1, 2 \ldots, m-1$. The restricted normalised quadratic multi-product revenue function for this production technology has the following specification:

$$\begin{aligned}
R'(P', X, Z, T) &= \alpha_0 + \sum_{i=1}^{n} \alpha_i x_i + \sum_{k=1}^{m-1} \beta_k p'_k + \sum_{g=1}^{v} \lambda_g z_g + \tfrac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_{ij} x_i x_j \\
&+ \tfrac{1}{2} \sum_{k=1}^{m-1} \sum_{l=1}^{m-1} \beta_{kl} p'_k\, p'_l + \tfrac{1}{2} \sum_{g=1}^{v} \sum_{h=1}^{v} \lambda_{gh} z_g z_h + \sum_{i=1}^{n} \sum_{k=1}^{m-1} \delta_{ik} x_i p'_k + \sum_{i=1}^{n} \sum_{g=1}^{v} \gamma_{ig} x_i z_g \\
&+ \sum_{k=1}^{m-1} \sum_{g=1}^{v} \varphi_{kg} p'_k\, z_g + \sum_{i=1}^{n} \rho_{ti} T x_i + \sum_{k=1}^{m-1} \phi_{tk} T p'_k + \sum_{g=1}^{v} \psi_{tg} T z_g + \theta_t T + \tfrac{1}{2} \theta_{tt} T^2
\end{aligned}$$

where $Z = [z_1,\ \mathrm{z}_2,\ ...,\ \mathrm{z}_v]$ are fixed input quantities and other non-price, non-quantity factors that exogenously influence production revenue, and T is the time trend.

6.3.2 Regularity Conditions

When specified in the normalised quadratic form, the dual revenue function satisfies the condition of monotonicity in output prices (Condition R.2) if the first partial derivatives of the normalised revenue function with respect to the normalised output prices are positive: $\dfrac{\partial R'(P', W, Z, T)}{\partial p'_k} = \beta_k + \sum_{l=1}^{m-1} \beta_{kl} p'_l + \sum_{i=1}^{n} \delta_{ik} x_i + \sum_{g=1}^{v} \varphi_{kg} z_g + \phi_{tk} T > 0$.

The revenue function is convex in output prices (Condition R.3) if the Hessian matrix of price derivatives $[\beta_{kl}]_{(m-1)\times(m-1)}$ is positive semi-definite. This dual revenue function

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[7] Similar to the translog revenue function, the distinction between variable inputs and fixed inputs is maintained here for the ease of referencing despite there being no real distinction between them.

automatically satisfies the regularity condition of linear homogeneity in output prices (Condition R.4) due to the normalisation process. Finally, the specified revenue function meets the symmetry condition if $\alpha_{ij} = \alpha_{ji}$, $\beta_{kl} = \beta_{lk}$ and $\lambda_{gh} = \lambda_{hg}$.

6.3.3 The Input-Constrained Output Supplies

When the revenue function specified above satisfies the regularity conditions, a system of input-constrained output supply equations is derived by applying the Chain Rule and the Samuelson-McFadden lemma as:

$$y_k = \beta_k + \sum_{l=1}^{m-1} \beta_{kl} p'_l + \sum_{i=1}^{n} \delta_{ik} x_i + \sum_{g=1}^{v} \varphi_{kg} z_g + \phi_{tk} T \ , \ k = 1, 2 \ldots, m-1.$$

The supply equation for the *numeraire* output is derived by taking the first derivative of the un-normalised revenue function with respect to the *numeraire* price:

$$y_m = \alpha_0 + \sum_{i=1}^{n} \alpha_i x_i + \sum_{g=1}^{v} \lambda_g z_g + \tfrac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_{ij} x_i x_j + \tfrac{1}{2} \sum_{k=1}^{m-1} \sum_{l=1}^{m-1} \beta_{kl} p'_k \, p'_l$$
$$+ \tfrac{1}{2} \sum_{g=1}^{v} \sum_{h=1}^{v} \lambda_{gh} z_g z_h + \sum_{i=1}^{n} \sum_{g=1}^{v} \gamma_{ig} x_i z_g + \sum_{i=1}^{n} \rho_{ti} T x_i + \sum_{g=1}^{v} \psi_{tg} T z_g + \theta_t T + \tfrac{1}{2} \theta_{tt} T^2.$$

6.3.4 The Net Price and Transformation Elasticities of Output Supplies

Applying the definition of the net (input-constrained) price elasticities in Chapter 3, it is straightforward to establish the relationships between the own- and cross-price supply elasticities and the function parameters, output prices and output quantities as:

$\varepsilon_{kl} = \beta_{kl} \times \dfrac{p'_l}{y_k}$ for $k, \ l = 1, \ 2, ..., \ m-1$;

$\varepsilon_{lm} = -\sum_{k=1}^{m-1} \varepsilon_{lk}$ for $l = 1, \ 2, ..., \ m-1$;

$\varepsilon_{mk} = \varepsilon_{km} \times \dfrac{p'_k \times y_k}{y_m}$ for $k = 1, \ 2, ..., \ m-1$; and

$$\varepsilon_{mm} = -\sum_{k=1}^{m-1} \varepsilon_{mk} \text{ for } k = 1, 2, ..., m-1.$$

As shown in Chapter 3, the net Allen partial and Morishima elasticities of transformation (respectively denoted as τ_{kl} and τ_{kl}^M, with $k, l = 1, 2, ..., m$) have the following relationships with the price elasticities: $\tau_{kl} = \dfrac{\varepsilon_{kl}}{r_l}$ and $\tau_{kl}^M = \varepsilon_{kl} - \varepsilon_{ll}$.

6.3.5 Empirical Implementation

When estimating a production technology presented by a normalised quadratic revenue function, the choice of the *numeraire* needs to be addressed before the estimation can be carried out. Once the decision of the *numeraire* is made, the system of derived output supplies, excluding the revenue function and the *numeraire* supply equation due to their excessive number of parameters, is then estimated using the FIML estimation method. There is no need to parametrically impose the homogeneity condition since it is automatically enforced through the normalisation of prices. The symmetry condition can be imposed on the system during estimation using parametric restrictions $\beta_{kl} = \beta_{lk}$. The condition of convexity can be enforced using the Cholesky factorisation technique, which implicitly imposes $\beta_{kl} = \beta_{lk}$ for the symmetry condition as well. In this decomposition, the price coefficient matrix $\left[\beta_{kl}\right]_{(m-1)\times(m-1)}$ is replaced by a product of a lower triangle matrix and its transpose is as below:

$$\left[\beta_{kl}\right]_{(m-1)\times(m-1)} = \begin{bmatrix} a_{11} & 0 & 0 & 0 \\ a_{12} & a_{22} & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{1(m-1)} & a_{2(m-1)} & \dots & a_{(m-1)(m-1)} \end{bmatrix} \times \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1(m-1)} \\ 0 & a_{22} & \cdots & a_{2(m-1)} \\ 0 & 0 & \ddots & \vdots \\ 0 & 0 & \dots & a_{(m-1)(m-1)} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}a_{11} & a_{11}a_{12} & \cdots & a_{11}a_{1(m-1)} \\ a_{12}a_{11} & a_{12}a_{12} + a_{22}a_{22} & \cdots & a_{12}a_{2(m-1)} + a_{22}a_{2(m-1)} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1(m-1)}a_{11} & a_{1(m-1)}a_{12} + a_{2(m-1)}a_{22} & \dots & a_{1(m-1)}a_{1(m-1)} + \dots + a_{(m-1)(m-1)}a_{(m-1)(m-1)} \end{bmatrix}.$$

The monotonicity condition is checked after estimation. This condition is satisfied if the estimated quantities of outputs are positive for all observations.

Since each of the four outputs defined for Australian broadacre agriculture (as described in Chapter 4) can act as the normalising factor, four alternative output supply systems can be derived and estimated. All four systems are estimated using FIML, without the revenue function and the *numeraire's* supply equation[8], and with the symmetry condition imposed. The best system is chosen based on the percentage of statistically significant price coefficients, the percentage of positive own-price coefficients, the percentage of significant system coefficients and the relative practicality of calculated elasticity estimates. The estimation results show that Beef price is the best choice as the *numeraire* and thus the system of the derived supply equations for Grains, Sheep and Wool is chosen for the entire analysis in this chapter.

6.4 Empirical Results

6.4.1 Translog Revenue Share System

The estimation result of the translog revenue share system (including the deleted share equation of the Wool output) in Table 13 shows a very good fit for the system. It has 77.6 per cent of the parameters (59 out of 76) being statistically significant at the 5% level. At the 10% level, 82.9 per cent of system parameters are significant. Prices are found to be important determinants of the revenue shares of outputs, with fourteen among sixteen price coefficients being significant at the 5% level. Moreover, all own-price coefficients are positive and highly significant, implying that revenue share of a product increases if there is an increase in its own price. In addition, in each share equation, the coefficients of alternative prices are negative, which indicates that revenue share of a particular output diminishes if the price of any alternative output increases. The system-wide McElroy R^2 is 0.77, suggesting that the estimated system

[8] When the *numeraire's* demand equation is included, FIML method fails to obtain an estimation result.

explains the observed output supplies quite well. However, the adjusted R^2 of individual equations varies broadly, from 0.48 in the Sheep supply equation to 0.92 in the Grains supply equation.

In contrast to the high statistical goodness-of-fit, the findings regarding the regularity conditions are unsatisfactory for the estimated translog share system. Using the price coefficient estimates and predicted shares, the calculated matrix $[\beta_{kl}]_{m\times m} - \hat{r}_{m\times m} + r_{1\times m}{}' r_{1\times m}$ has at least one negative eigenvalue at almost every data observation. This result means that the convexity condition is seriously violated. The estimated share system also violates the monotonicity condition with the percentage of predicted negative shares being 14.9 per cent for Grains, 4.4 per cent for Sheep and 2.0 per cent for Wool.

An important finding from the estimation of the translog revenue share system is that all own-price coefficient estimates are significantly smaller than 0.25. These estimates range from 0.062 (in the Grains equation) to 0.172 (in the Wool equation). From the discussion in Section 6.2.4, these results imply that there is a fairly wide range of revenue shares where the own-price supply elasticities are negative, in contradiction to economic theory. Using the predicted shares and estimates of own-price coefficient β_{kk}, the own-price elasticities are found to be negative in 1044 observations for Grains, 757 observations for Sheep, 1015 observations for Beef and 739 observations for Wool (out of the total 1272 observations).

There are two causes of negative own-price elasticities for output supplies. Some negative own-price elasticities result from the negative predicted shares used to calculate them. However, this contribution of negative predicted shares to the prevalence of negative own-price elasticities of output supplies is fairly small. As presented above, the percentage of negative predicted shares is less than five per cent for Sheep and Wool, and less than fifteen per cent for Grains. This result implies that the estimated own-price coefficients β_{kk} are the major cause of the negativity of the own-price supply elasticities.

An examination of own-price elasticity estimates at individual observations shows that there is a wide range of positive revenue shares where own-price elasticities of output supplies are negative. For the Grains output, which has the smallest estimate of own-price coefficient in its supply equation, the generated own-price elasticity is negative when the predicted revenue share is greater than 6.7 per cent and less than 95 per cent. In contrast, the own-price elasticity of Wool supply, which has the largest own-price coefficient estimate, is negative when its revenue share is greater than 22.1 per cent. The largest predicted share for this output is 66 per cent. For Sheep supply, which has an own-price coefficient estimate of 0.084, the own-price elasticity is negative when its revenue share greater than 9.2 per cent. The largest predicted share for this output is 33.9 per cent. Finally, the own-price elasticity of Beef supply, which has the estimated own-price coefficient of 1.44, is negative when its predicted revenue share falls between 17.5 per cent and 81.8 per cent.

The empirical outcome regarding the own-price supply elasticities discussed above challenges the suitability of the translog functional form in the specification of the revenue function. This empirical finding can be added to evidence of the translog's poor performance found in previous studies such as Monte Carlo experiments by Wales (1977) and Guilkey *et al.* (1983). This poor result and the acceptable result of the translog cost function found in Chapter 5 imply that the suitability of the translog functional form may depend on which among the cost, revenue and profit functions is estimated. Based on the unreasonable finding above, the estimation results of the translog revenue share system are not discussed further.

6.4.2 Normalised Quadratic Supply Quantity System

The estimation result of the normalised quadratic quantity system is reasonable based on the statistical significance of system coefficients and the system-wide McElroy R^2. As shown in Table 14, there are 42, out of a total of 54, statistically significant system coefficients at the 5% level. This number is equivalent to 77.8 per cent of the total number of system coefficients, which increases to 83.3 per cent at the 10% level. Among nine price coefficients, four are significant at the 5% level and six are significant at the 10% level. The McElroy system-wide R^2 is 0.65 and the adjusted R^2 is 0.87, 0.49 and 0.58 for Grains, Sheep and Wool equations, respectively.

The result obtained for the normalised quadratic quantity system is economically meaningful. The own-price coefficient is positive in all equations and statistically significant in Grains and Sheep equations. Importantly, the estimated price matrix $\left[\beta_{kl}\right]_{(m-1)\times(m-1)}$ is positive semi-definite without being parametrically imposed by the Cholesky decomposition. This positive semi-definiteness means that the condition of convexity in output prices is naturally satisfied by the estimated supply quantity system. The monotonicity condition is violated with negative quantities being predicted at 22.6 per cent of observations for Grains, 8.5 per cent for Sheep and 11.6 per cent for Wool. This result is expected, considering the quasi-micro nature of the data used for estimation. The individual estimated derived supply equation is now described and discussed.

Table 13: Translog Revenue Function—Estimated Parameters of Derived Revenue Share System

	Revenue share equation							
	Grains		Sheep		Beef		Wool	
	Coefficient	z-Statistic	Coefficient	z-Statistic	Coefficient	z-Statistic	Coefficient	z-Statistic
Constant	0.352**	2.662	0.564**	6.947	-0.722**	-3.726	0.806**	4.590
Grains price	0.063**	24.729	-0.019**	-7.285	-0.007	-1.075	-0.036**	-6.922
Sheep price	-0.019**	-7.285	0.084**	22.287	-0.034**	-5.371	-0.031**	-6.190
Beef price	-0.007	-1.075	-0.034**	-5.371	0.144**	8.350	-0.103**	-7.828
Wool price	-0.036**	-6.922	-0.031**	-6.190	-0.103**	-7.828	0.171**	13.220
CSM livestock quantity	-0.072**	-10.950	0.007*	1.843	0.078**	9.267	-0.013	-1.613
FC quantity	0.016**	3.223	0.018**	6.133	-0.049**	-9.333	0.015**	3.518
Other CSM quantity	0.063**	4.396	0.001	0.105	-0.139**	-5.693	0.075**	3.426
FOG quantity	0.066**	6.334	-0.029**	-4.861	0.011	0.797	-0.048**	-3.725
Livestock trading quantity	-0.074**	-10.212	0.007	1.591	0.1204**	12.772	-0.053**	-6.210
Capital	-0.021*	-1.750	-0.038**	-4.991	0.072**	4.295	-0.014	-0.899
Fixed labour	-0.035**	-2.046	0.005	0.498	0.066**	2.779	-0.036*	-1.715
Dummy variable D	0.276**	22.174	-0.033**	-4.047	-0.063**	-3.155	-0.179**	-10.517
Dummy variable Z1	0.046**	2.495	-0.042**	-3.801	0.168**	8.034	-0.172**	-9.750
Dummy variable Z2	0.021	0.980	-0.047**	-3.693	0.166**	7.176	-0.139**	-6.998
Dummy variable S1	0.166**	7.411	0.002	0.121	-0.148**	-4.193	-0.02	-0.610
Dummy variable S2	0.077**	5.780	0.018**	2.075	-0.126**	-5.948	0.031*	1.665
Rainfall	0.1084**	7.109	-0.05**	-5.305	-0.110**	-5.003	0.0522**	2.914
Time	0.01**	7.900	-0.005**	-5.422	-0.003	-1.445	-0.003	-1.568
Adjusted R^2	0.92		0.48		0.76		0.58	

Note: D = 1 when the farm is in the Cropping industry, Z1 = 1 when the farm is in the Wheat-Sheep zone, Z2 = 1 when the farm is in the High Rainfall zone, S1 = 1 when the farm size is greater than $400,000 and S2 = 1 when the farm size is between $200,000 and $400,000

** Significant at the 5% level

* Significant at the 10% level

Table 14: Normalised Quadratic Revenue Function—Estimated Parameters of Derived Output Supply Quantity System

	Output quantity equation					
	Grains		Sheep		Wool	
	Coefficient	z-Statistic	Coefficient	z-Statistic	Coefficient	z-Statistic
Constant	−1,770.8**	−4.75	569.79**	4.65	1,083.05**	6.78
Grains price	242.76**	4.26	−75.67**	−2.93	−62.62*	−1.66
Sheep price	−75.67**	−2.93	202.38**	4.90	−23.47	−0.49
Wool price	−62.62*	−1.66	−23.47	−0.49	59.41	0.91
CSM livestock quantity	−0.06**	−2.91	−0.01**	−2.30	−0.03**	−4.09
FC quantity	3.425**	51.22	0.178**	5.79	0.132**	2.75
Other CSM quantity	0.333**	6.22	0.088**	4.27	0.353**	13.12
FOG quantity	0.468**	2.27	−0.461**	−5.21	−1.242**	−9.66
Livestock trading quantity	−0.872**	−3.52	−0.254**	−4.51	−0.511**	−7.24
Capital quantity	−0.015**	−5.13	0.001	1.30	0.0003	−0.17
Fixed labour quantity	0.215	0.59	−0.508**	−4.61	−0.609**	−3.80
Dummy variable D	767.68**	4.41	−272.24**	−5.83	−432.93**	−6.02
Dummy variable Z1	−73.84	−0.35	−234.7**	−4.19	−942.24**	−12.86
Dummy variable Z2	−397.51*	−1.76	−145.55**	−2.50	−821.26**	−10.72
Dummy variable S1	492.14**	2.31	785.37**	10.79	895.77**	8.74
Dummy variable S2	−102.81	−0.50	443.14**	7.07	561.96**	6.45
Rainfall	1,138.30**	6.38	164.41**	2.45	407.46**	4.80
Time	50.74**	4.06	−11.83**	−2.55	−3.91	−0.63
Adjusted R^2	0.87		0.49		0.58	

Note: D = 1 when the farm is in the Cropping industry, Z1 = 1 when the farm is in the Wheat-Sheep zone, Z2 = 1 when the farm is in the High Rainfall zone, S1 = 1 when the farm size is greater than $400,000 and S2 = 1 when the farm size is between $200,000 and $400,000

** Significant at the 5% level

* Significant at the 10% level

6.4.2.1 Supply of Grains

The estimated Grains supply equation indicates strongly that the level of Grains supplied is significantly influenced by its own price and Sheep price. Grains supply increases in response to an increase in the expected Grains price but decreases in response to an increase in the expected Sheep price. There is also weak evidence of the negative relationship between Grains supply and Wool price. Notably, the level of Grains produced is found to be significantly linked to all variable input levels. This output supply is negatively related to CSM livestock and Livestock trading quantities and positively related FC, Other CSM and FOG quantities. Regarding the two fixed inputs, physical capital and operator's labour, only capital has a statistically significant, negative relationship with the supply of Grains.

Among the non-price, non-quantity exogenous variables, the industry dummy, the Large farm dummy, the rainfall and the time trend variables are highly significantly in the Grains supply equation. The estimated coefficients of these variables suggest that Cropping farms produce more Grains than Livestock farms, Large farms produce more of this output than Small and Medium farms, higher rainfall leads to higher Grains production, and technical advance increases Grains production, *ceteris paribus*. No significant differences are detected between Medium and Small farms in Grains production. Finally, there is weak evidence that farms located in the High Rainfall zone produce less Grains than those located in the Pastoral and Wheat-Sheep zones, but no significant difference is found between the two drier zones.

6.4.2.2 Supply of Sheep

Interestingly, the estimated Sheep supply equation has more statistically significant coefficients than the Grains supply equation despite the fact that it has a much lower adjusted R^2. Based on statistical evidence, the supply of Sheep is positively influenced by its expected price and negatively influenced by the expected Grains price. The expected Wool price does not appear to have a significant effect on the supply of Sheep output. Moreover, all variable inputs are significant in determining the level of Sheep supplied. According to the coefficient estimates, Sheep supply is negatively related to CSM livestock, FOG and Livestock trading input levels while being positively related to FC and Other CSM levels.

Sheep supply has fairly significant relationships with the other exogenous variables in the system, based on statistical evidence. Sheep supply is significantly and negatively related to the labour contributed by operator and family members, but not significantly related to fixed capital. All qualitative dummy variables, rainfall variable and time trend are significant at the 5% level in the Sheep supply equation. The coefficient estimates suggest that Cropping farms produce less Sheep than Livestock farms, and so do farms in the Wheat-Sheep and High Rainfall zones compared to farms in the Pastoral zone, *ceteris paribus*. The larger a farm is and the higher the rainfall is, the more Sheep is produced. Interestingly, the estimated Sheep supply equation indicates that the passage of time has a negative impact on the Sheep supply over the study period.

6.4.2.3 Supply of Wool

Unlike the Grains and Sheep supplies, Wool supply is not significantly determined by prices of outputs, including its own, based on statistical evidence. Grains price has a weakly significant and negative relationship with the supply of Wool. The coefficients of Wool and Sheep prices are not significant in this supply equation, even at the 10% level. In contrast, all variable inputs are found to be statistically significant in the estimated Wool supply equation, as they are in the Grains and Sheep supply equations. Similar to the Sheep supply response, a decrease in Wool supply is associated with an increase in CSM livestock, FOG and Livestock trading inputs.

The statistical significance and the direction of the relationships of Wool supply with the remaining system exogenous variables are similar to those of the relationships of Sheep supply with them. All of these non-price exogenous variables, except for the fixed capital and time trend, are highly significant in the Wool supply equation. The coefficient estimates are negative for the industry and zone dummy variables and positive for the size dummy and rainfall variables. It is noteworthy that the time trend is found to be insignificant in this supply equation, in contrast to its strong significance in the Sheep supply equation.

6.4.3 Net Price and Transformation Elasticities of Output Supplies

Table 15 reports the net own- and cross-price elasticities of output supplies computed from the estimated normalised quadratic supply system. The price elasticities of all outputs are generally very low and less than half of them are statistically significant at the 5% level, based on their bootstrapped standard errors. The own-price elasticity is positive for all outputs as expected but is statistically significant only for Grains and Sheep. The own-price elasticity estimate indicates that a one per cent increase in Sheep price triggers a 0.159 per cent increase in its supply. For Grains, a one per cent increase in price leads to a 0.073 per cent increase in its supply, which is economically insignificant. The own-price elasticities of Beef and Wool supplies are not only statistically insignificant but also small in magnitude.

The cross-price elasticity estimates are negative for all output pairs except for the Wool-Beef pair. For outputs, a negative cross-price elasticity means a substitutive relationship. However, the statistical insignificance of the majority of the cross-price elasticities suggests that the supply of a broadacre output is not responsive to the prices of alternative outputs. Based on statistical evidence, the Grains supply responds to changes in prices of Sheep and Wool but not Beef. Meanwhile, Sheep and Wool supplies only respond to changes in Grains price, beside their own prices. Finally, over the short term, Beef supply is not significantly affected by changes in the expected prices of all outputs.

As shown in Table 16 and Table 17, Allen partial and Morishima elasticity estimates of output transformation are in accord with the price elasticity estimates obtained. The pair-wise substitutive relationships between outputs are verified by both Allen partial and Morishima measures. However, in each of these two sets of transformation elasticities, only around half of the elasticities are statistically significant. This low significance suggests that there may be little scope for transformation between broadacre outputs.

There are significant differences between the Allen partial and Morishima elasticities of output transformation. For instance, the substitutive relationship between Wool and Sheep is not statistically significant by the Allen partial measure. In contrast, the transformation relationship between these two outputs is highly significant by the Morishima measure, regardless of whether the transformation response is measured with respect to Wool or Sheep price change.

According to the Morishima elasticity measure, when Grains and Sheep prices change, there are adjustments in the supply of all alternative outputs. However, when Wool price changes, there is only a significant substitutive adjustment in Sheep supply, but not in Grains supply or Beef supply. Finally, changes in Beef price generally do not lead to substitutive adjustments in other output supplies.

Table 15: Own- and Cross-Price Elasticities of Output Supply [a, b]

	With respect to price of			
Supply for	Grains	Sheep	Beef	Wool
Grains	0.073**	−0.016**	−0.029	−0.023**
	(0.019)	(0.005)	(0.018)	(0.009)
Sheep	−0.086**	0.159**	−0.033	−0.035
	(0.025)	(0.034)	(0.06)	(0.048)
Beef	−0.024*	−0.003	0.065	−0.0002
	(0.013)	(0.007)	(0.045)	(0.026)
Wool	−0.047**	−0.012	0.004	0.058
	(0.021)	(0.019)	(0.06)	(0.05)

Note: [a] Medians of elasticities evaluated at all observation points
[b] Bootstrapping standard errors (500 trials) are in parentheses
** Significant at the 5% level
* Significant at the 10% level

Table 16: Allen Partial Elasticities of Output Transformation [a, b]

	Grains	Sheep	Beef	Wool
Grains	.			
Sheep	−0.158** (0.045)	.		
Beef	−0.047** (0.024)	−0.045 (0.087)	.	
Wool	−0.094** (0.04)	−0.165 (0.217)	−0.001 (0.1)	.

Note: [a] Medians of elasticities evaluated at all observation points
[b] Bootstrapping standard errors (500 trials) are in parentheses
** Significant at the 5% level
* Significant at the 10% level

Table 17: Morishima Elasticities of Output Transformation [a, b]

	Grains	Sheep	Beef	Wool
Grains	-	−0.169** (0.035)	−0.074* (0.039)	−0.073 (0.051)
Sheep	−0.133** (0.032)	-	−0.083 (0.09)	−0.094** (0.043)
Beef	−0.082** (0.017)	−0.152** (0.036)	-	−0.056 (0.069)
Wool	−0.100** (0.028)	−0.174** (0.036)	−0.058 (0.094)	-

Note: [a] Medians of elasticities evaluated at all observation points
[b] Bootstrapping standard errors (500 trials) are in parentheses
** Significant at the 5% level
* Significant at the 10% level

6.5 Discussion of Estimation Results

Estimation results obtained from restricted translog and normalised quadratic revenue functions for Australian broadacre agriculture reveal some important insights into the empirical applicability of the duality approach. Firstly, the strong statistical significance of the results for both functional forms suggests that, in the short term, broadacre farmers appear to maximise production revenue for given production resources. Using the large AAGIS quasi-micro dataset for estimation, the estimated systems of derived revenue shares and quantity supplies have a much higher percentage of significant coefficients than many previous studies, including Villezca-Becerra and

Shumway (1992), Fulginiti and Perrin (1990) and Akridge and Hertel (1986). Output prices are found to be important determinants of the supply of broadacre products with strong statistical significance and correct signs. More importantly, the regularity condition of convexity is satisfied naturally by the estimated supply system derived from the normalised quadratic revenue function. This outcome is very encouraging given this regularity condition has often been violated in previous international and Australian empirical applications.

The estimated translog revenue share system displays a lack of correspondence between statistical fit and economic meaningfulness. This implies that a good statistical fit in econometric estimation does not necessarily mean that the estimated model meaningfully portrays rational economic behaviour. The estimated translog revenue share system initially appears strongly attractive, as indicated by the high percentage of significant system coefficients, the high statistical significance of almost all price variables and having positive own-price coefficients. However, the estimated model does not satisfy the convexity and the monotonicity condition over much of the data sample. Moreover, the own-price elasticity estimates are negative for all output supplies in a disproportionately high percentage of observations. This result implies that the supply curves of broadacre products are downward sloping instead of upward sloping as predicted by economic theory. As discussed in Section 6.4.1, the small revenue shares for a considerable number of observations, a result of the quasi-micro nature of the data used for estimation, contribute to this discouraging outcome, but the main cause of this occurrence is the small magnitude of the positive own-price coefficient estimates.

The results obtained from the normalised quadratic revenue function do not accord with those from the translog revenue function. Compared to the translog share system, the statistical goodness-of-fit of the normalised quadratic supply quantity system is lower, but still reasonable. More than three-quarters of this system's coefficients are statistically significant at the 5% level. Using this functional form, the outcomes concerning the theoretical regularity conditions and elasticity estimates are very satisfactory. The estimated supply system satisfies the convexity condition without artificial parametric restrictions. All estimates of own-price elasticities are positive as expected. The monotonicity condition is not satisfied by the estimated normalised quadratic quantity system, but this is expected considering the quasi-micro nature of the data sample used for estimation.

The estimates of price-quantity responses obtained from the normalised quadratic revenue function suggest that the supplies of broadacre products are fairly rigid toward price movements in the short term. All price elasticities of output supplies are found to be small in magnitude. A half of own-price elasticities and two-thirds of cross-price elasticities are statistically insignificant, suggesting that the supply of broadacre products is generally not influenced by prices, including own prices. The lack of responsiveness in Wool supply with respect to its own price may be an indirect consequence of the collapse of the Wool Reserve Price Scheme in 1991, which led to substantial Wool stockpiles and severe price depression, motivating farmers to move away from Wool production. Meanwhile, the inelastic supply of Beef with respect to its own price may be due to the long production cycle of this product. Moreover, the lack of responsiveness in Beef supply toward all alternative output prices implies that the production of Beef is, to a large extent, separate from cropping and other grazing activities.

The estimates of price elasticities, Allen partial elasticities of transformation and Morishima elasticities of transformation generated from the normalised quadratic revenue function are consistent with each other. Estimates of all cross-price, Allen partial and Morishima elasticities are negative, suggesting that broadacre outputs generally compete against each other for production resources. All three of these elasticity sets have a high proportion of statistically insignificant elasticities, indicating that there is little possibility for transformation among broadacre outputs in Australia in the short term.

It is not possible to verify the results obtained from the normalised quadratic revenue function by means of comparison, since there has been no study of Australian agricultural production that estimates a revenue function. Nevertheless, the sign of all generated elasticity estimates are the same as those obtained in Vincent *et al.* (1980), in which a profit function in the CRESH/CRETH form is estimated. The price elasticities obtained in this study are discernibly higher than those obtained in this chapter. However, a much smaller sample of time series data, observed over the period from 1952/53 to 1973/74, was used for estimation in this study. Moreover, the output price and quantity data used in this study were aggregated over vast geographical areas. The functional form used in this study is also less flexible than the normalised quadratic and the goodness-of-fit of its estimated derived supply system is less statistically significant than the system estimated in this chapter. These suggest that elasticity estimates obtained in this chapter would reflect more accurately the production technology and economic behaviour of Australian broadacre producers than those obtained in Vincent *et al*. (1980).

The findings in this chapter regarding price and transformation elasticities have important implications within Australian broadacre production for policy development and farm management. The small magnitude of own- and cross-price elasticities of output supplies and the insignificance of the majority of price and transformation elasticities imply that there is little possibility of influencing supplies of broadacre products over the short term through price interventions. This means that farmers generally cannot take advantage of favourable price movements or alleviate losses induced by unfavourable price movements in the short term. Volatility in output prices, especially due to fluctuations in exchange rates, can have substantial effects on year-to-year survival and/or profitability of farming operations. Therefore, management of risks created by exchange rates or market price fluctuations through strategies such as hedging and forward contracting is important in improving the viability of broadacre farming in Australia.

Chapter 7

Estimating a Restricted Multi-Product Profit Function for Australian Broadacre Production

7.1 Introduction

Estimation results for Australian broadacre production from restricted multi-product cost and revenue functions were presented in the two previous chapters. In this chapter, the results from a restricted multi-product profit function for Australian broadacre production are presented. In this formulation of the optimisation problem, broadacre farmers are assumed to respond to price changes by adjusting both inputs and outputs, not just inputs alone or outputs alone as in the conditional optimisation specification of cost and revenue functions. This makes the profit maximisation assumption more realistic than the cost minimisation and revenue maximisation assumptions made in the two previous chapters. This behavioural assumption also appears to have been preferred over cost minimisation and revenue maximisation in empirical applications of the duality approach.

In this chapter, a restricted normalised quadratic multi-product profit function is specified for Australian broadacre agriculture with the same four outputs, five variable inputs, two fixed inputs and six other exogenous variables included in the models estimated in the two previous chapters. The system of unconstrained supply and demand equations derived from this profit function is estimated using the available AAGIS quasi-micro data. After the system is estimated, gross price elasticities and elasticities of input substitution and output transformation are calculated for all outputs and variable inputs. Unlike Chapters 5 and 6, in this chapter only the estimation results of the profit function in the normalised quadratic form are presented and discussed. The translog functional form is unsuitable for specifying the profit function in this study because the observed profit is small or negative for a substantial number of observations in the AAGIS quasi-micro data. The unsuitability of the translog form in specifying a multi-product profit function has also been evidenced by its unpopularity and its discouraging estimation results in past studies of profit functions, such as in McKay *et al.* (1982, 1983), Haughton (1986) and Squires (1987).

The structure of this chapter is similar to that of Chapters 5 and 6. A general representation of the restricted normalised quadratic multi-product profit function is presented in Section 7.2. This section comprises descriptions of the regularity conditions, the derived input demand and output supply equations and the gross price elasticities and elasticities of substitution and transformation between inputs and outputs. In Section 7.3 the estimation results are presented. Section 7.4 contains a discussion of the estimation results and the elasticity estimates obtained. Section 7.5 then concludes the chapter with remarks and comments on the estimation results.

7.2 Specification of a Restricted Normalised Quadratic Multi-Product Profit Function

This section presents a general theoretical framework for investigating production technology through specifying a restricted multi-product profit function in the normalised quadratic functional form. A general mathematical representation of this profit function is first presented, which is followed by a description of the regularity conditions. The section continues with the derivation of the input demand and output supply equations. Finally, the gross price elasticities and elasticities of input substitution and output transformation for inputs and outputs are derived.

7.2.1 The Restricted Normalised Quadratic Multi-Product Profit Function

Define $\pi'(W',P',Z,T)$, w'_i and p'_k as the variable profit, the price of input i ($i = 1,\ 2,\ \ldots,\ n-1$) and the price of output k ($k = 1,\ 2,\ \ldots, m$) normalised by the price of input n: $\pi'(W',P',Z,T) = \dfrac{\pi(W,P,Z,T)}{w_n}$, $w'_i = \dfrac{w_i}{w_n}$ and $p'_k = \dfrac{p_k}{w_n}$. The restricted normalised quadratic multi-product profit function can have the following representation:

$$\pi'(W',P',Z,T) = \alpha_0 + \sum_{i=1}^{n-1}\alpha_i w'_i + \sum_{k=1}^{m}\beta_k p'_k + \sum_{g=1}^{v}\lambda_g z_g + \tfrac{1}{2}\sum_{i=1}^{n-1}\sum_{j=1}^{n-1}\alpha_{ij} w'_i\, w'_j +$$
$$\tfrac{1}{2}\sum_{k=1}^{m}\sum_{l=1}^{m}\beta_{kl} p'_k\, p'_l + \tfrac{1}{2}\sum_{g=1}^{v}\sum_{h=1}^{v}\lambda_{gh} z_g z_h + \sum_{i=1}^{n-1}\sum_{k=1}^{m}\delta_{ik} w'_i\, p'_k + \sum_{i=1}^{n-1}\sum_{g=1}^{v}\gamma_{ig} w'_i\, z_g$$
$$+\sum_{k=1}^{m}\sum_{g=1}^{v}\varphi_{kg} p'_k\, z_g + \sum_{i=1}^{n-1}\rho_{ti} w'_i\, T + \sum_{k=1}^{m}\phi_{tk} p'_k\, T + \sum_{g=1}^{v}\psi_{tg} z_g T + \theta_t T + \tfrac{1}{2}\theta_{tt} T^2$$

where z_g consists of the fixed capital level, fixed labour level, industry dummy variable, two zone dummy variables, two size dummy variables and rainfall variable, and T is time trend, as defined in Chapters 5 and 6.

7.2.2 Regularity Conditions

As described in Chapter 3, the regularity conditions for profit functions consist of monotonicity in prices (Conditions P.2 and P.3), convexity in prices (Condition P.4), linear homogeneity in prices (Condition P.5) and the symmetry condition (twice-continuously-differentiable assumption). For the profit function presented above, the condition of monotonicity is met if the quantities of variable input demands and output supplies are positive. The convexity condition requires that the Hessian matrix of price coefficients $\begin{bmatrix}\alpha_{ij} & \beta_{kl}\end{bmatrix}$ is positive semi-definite. The condition of linear homogeneity is automatically satisfied due to the normalisation process. Finally, the symmetry condition is met if $\alpha_{ij} = \alpha_{ji}$, $\beta_{kl} = \beta_{lk}$ and $\lambda_{gh} = \lambda_{hg}$.

7.2.3 The Input Demands and Output Supplies

When the normalised quadratic profit function specified above satisfies the regularity conditions, applying Hotelling's lemma leads to a system of input demand and output supply equations, as follows:

$$-x_i = \alpha_i + \sum_{j=1}^{n-1} \alpha_{ij} w'_j + \sum_{k=1}^{m} \delta_{ik} p'_k + \sum_{g=1}^{v} \gamma_{ig} z_g + \rho_{ti} T \text{ with } i = 1,\ 2,\ \dots,\ n-1$$

$$y_k = \beta_k + \sum_{l=1}^{m} \beta_{kl} p'_l + \sum_{i=1}^{n-1} \delta_{ik} w'_i + \sum_{g=1}^{v} \varphi_{k\mathrm{g}} z_g + \phi_{tk} T \text{ with } k = 1,\ 2,\ \dots, m.$$

7.2.4 The Gross Price and Substitution/Transformation Elasticities

Applying the definition of gross elasticities in Chapter 3 to the normalised quadratic profit function, the gross price elasticity of demand for variable input i ($i = 1,\ 2, ..., n-1$) is $\xi_{ij} = \alpha_{ij} \times \frac{w'_j}{x_i}$ with respect to price of variable input j ($j = 1,\ 2, ..., n-1$), and $\xi_{ik} = \delta_{ik} \times \frac{p'_k}{x_i}$ with respect to price of output k ($k = 1,\ 2, ..., m$). Similarly, the gross price elasticities of supply of output k ($k = 1,\ 2, ..., m$) with respect to output l ($l = 1,\ 2, ..., m$) and input i ($i = 1,\ 2, ..., n-1$) are $\xi_{kl} = \beta_{kl} \times \frac{p'_l}{y_k}$ and $\xi_{ki} = \delta_{ki} \times \frac{w'_i}{y_k}$, respectively (Marsh 2005).

With $\xi_{in} = -\sum_{j=1}^{n-1} \xi_{ij} - \sum_{k=1}^{m} \xi_{ik}$ and $\xi_{kn} = -\sum_{i=1}^{n-1} \xi_{ki} - \sum_{l=1}^{m} \xi_{kl}$, $i = 1,\ 2, ..., n-1$ and $k = 1,\ 2, ..., m$, other gross price elasticities related to the *numeraire* input n are: $\xi_{ni} = \xi_{in} \times \frac{w'_i \times x_i}{x_n}$;

$\xi_{nk} = \xi_{kn} \times \frac{p'_k \times y_k}{x_n}$; and $\xi_{nn} = -\sum_{i=1}^{n-1} \xi_{ni} - \sum_{k=1}^{m} \xi_{nk}$ (Huffman and Evenson 1989 and Coxhead 1992).

7.3 Empirical Estimation

The restricted normalised quadratic multi-product profit function is specified for Australian broadacre production with four outputs, five variable inputs and two fixed inputs as before. With five variable inputs, there are five alternative *numeraires* and, therefore, five alternative systems of supply and demand equations derived from the normalised quadratic profit functions using different *numeraires*. Once a system of input demand and output supply equations is derived, additive multivariate normal errors are added to the equations. The system is then estimated using the FIML estimation method. All variables are again weighted by the square root of the sample size of observational cells before estimation to correct for heteroskedasticity (as explained in Chapter 4). As in the case of the revenue function estimated in Chapter 6, broadacre farmers are assumed to have naïve price expectations and the output prices are lagged by one period to reflect these expectations. The estimated supply and demand system excludes the profit function since the FIML method fails to obtain an estimation result when the profit function is included. This supply and demand system contains 144 parameters and is estimated with 1272 data observations.

Regarding the regularity conditions, the system of derived demand and supply equations is constrained to satisfy the symmetry condition by parametrically restricting $\alpha_{ij} = \alpha_{ji}$ and $\beta_{kl} = \beta_{lk}$. For the convexity condition, when positive semi-definiteness is imposed on the price coefficient matrix by Cholesky decomposition, the estimation of the system fails to yield a result. Therefore, the system is estimated without the imposition of this convexity condition. Meanwhile, the monotonicity condition is not parametrically enforceable and is checked after estimation by examining whether the predicted input and output quantities are non-negative for all observations in the data sample.

The choice of the *numeraire* among the five variable inputs is made by comparing the estimation results of the five alternative derived supply and demand systems. The selection criteria are the percentage of significant system coefficients, the percentage of own-price coefficients having the expected sign and the percentage of significant price coefficients. Based on these criteria, the results obtained for the supply and demand system when the FC input is the *numeraire* are better than those obtained for alternative supply and demand systems. The results of the quantity system using FC as the *numeraire* are therefore presented and discussed in this chapter. With this *numeraire*, the derived demand and supply system is:

$$-x_i = \alpha_i + \sum_{j=1}^{4} \alpha_{ij} w'_j + \sum_{k=1}^{4} \delta_{ik} p'_k + \sum_{g=1}^{8} \gamma_{ig} z_g + \rho_{ti} T \text{ , } i = 1,2,...4 \text{ and}$$

$$y_k = \beta_k + \sum_{l=1}^{4} \beta_{kl} p'_l + \sum_{i=1}^{4} \delta_{ik} w'_i + \sum_{g=1}^{8} \varphi_{kg} z_k + \phi_{tk} T \text{ , } k = 1,2,...4,$$

where x_i and w'_j are respectively quantities of CSM livestock, Other CSM, FOG and Livestock trading inputs and their prices divided by FC price; y_k and p'_l are respectively quantities of Grains, Sheep, Beef and Wool outputs and their prices divided by FC price.

7.4 Estimation Results

7.4.1 Estimated Coefficients

The estimated parameters of the derived supply and demand system for Australian broadacre agriculture are displayed in Table 18. 63.2 per cent of system parameters are significant at the 5% level and 66.7 per cent of the parameters are significant at the 10% level. Of the estimated own-price coefficients, all eight are positive as expected by economic theory and six are statistically significant at the 5% level. Less than half of the price coefficients (28 out of 64) are statistically significant at the 5% level, compared to more than three-quarters of the non-price variables (i.e. fixed inputs, qualitative dummy, rainfall and time trend)[9]. The adjusted R^2 of individual supply and demand equations varies between 0.42 and 0.8710.

Of the regularity conditions, the estimated supply and demand system does not satisfy the monotonicity and convexity conditions. The percentage of negative predicted quantities, at which the profit function is not monotonic, ranges from 5.4 per cent for Sheep output to 30.2 per cent for Grains output. The frequency of this violation is fairly high for Beef (27.8 per cent), CSM livestock (23.5 per cent) and Livestock trading (24.9 per cent). With respect to the convexity condition, the matrix of the price coefficient estimates is not positive semi-definite; two out of eight eigenvalues of this price coefficient matrix are negative.

Supply of Grains

From a statistical viewpoint, the estimated Grains supply equation is reasonable. Ten out of eighteen equation coefficients are significant at the 5% level. Surprisingly, the estimated supply equation suggests that Grains supply is not determined by its expected own price. Beef price is not found to have a significant effect on Grains supply, suggesting that production decisions concerning Grains cropping and Beef cattle grazing may be separate from each other. Grains supply is, however, significantly influenced by Sheep and Wool prices. Increases in these two output prices would

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[9] The high percentage of significant qualitative dummy variables in the estimated system suggests that it is worthwhile to estimate a separate supply and demand system for each broadacre industry, each broadacre zone and each operation size. However, the estimation results obtained are very poor for these supply and demand systems, which is likely due to insufficient sample sizes given the large number of coefficients to be estimated.

[10] Due to the large number of equations and observations in the estimated supply and demand system, the McElroy system-wide R^2 cannot be computed for this system. This is because of the inability of the Eviews software to carry out the inversion of a large matrix, whose dimensions are the number of system equations and the number of data observations, involved in the computation.

reduce Grains supply. With respect to broadacre inputs, there is weak evidence that an increase in Livestock trading price is associated with an increase in Grains production. The prices of all other variable inputs, i.e. CPM Livestock, Other CSM and FOG, do not influence Grains supply, based on statistical evidence.

The estimated Grains supply equation shows that Grains supply is more significantly related to non-price factors than to the prices of broadacre inputs and outputs. The relationship between Grains supply and total fixed capital is highly significant. Greater supply of Grains is associated with increased capital investment, *ceteris paribus*. No statistical evidence was found to support a significant relationship between Grains supply and fixed labour. The equation coefficients of all other exogenous variables, except the time trend, are highly significant. This strong significance suggests that production specialisation, geographical location and production scale all influence Grains supply. The estimated results also suggest that higher rainfall enhances Grains production considerably, other things being fixed. Finally, the insignificance of the time trend in this supply equation suggests that technological progress has no significant influence on Grains production over the 1990–2005 period.

Supply of Sheep

The goodness-of-fit of the estimated Sheep supply equation is comparable to that of the Grains supply equation. The Sheep supply equation has the same number of significant coefficients as the Grains supply equation. However, unlike Grains supply, Sheep supply is found to be significantly and positively impacted by its expected price, which is in accordance with economic theory. Moreover, Sheep supply is also strongly determined by the expected prices of Grains and Beef. An increase in these expected prices would reduce Sheep production. Meanwhile, the estimation outcome suggests that Sheep production is not significantly related to Wool price.

Table 18: Estimated Parameters of System of Supply and Demand Quantity Equations Derived from Normalised Quadratic Profit Function

	Output supply and input demand equations							
	Grains		Sheep		Beef		Wool	
	Coefficient	z-Statistic	Coefficient	z-Statistic	Coefficient	z-Statistic	Coefficient	z-Statistic
Constant	−2,851.3**	−4.66	684.13**	4.53	2,568.6**	3.46	1,483.8**	6.77
Grains price	179.92	1.44	−84.9**	−3.34	−145.95	−1.39	−91.84**	−2.32
Sheep price	−84.9**	−3.34	272.41**	4.73	−157.52**	−2.71	−42.93	−0.74
Beef price	−145.95	−1.39	−157.52**	−2.71	943.78**	2.98	−256.14**	−2.69
Wool price	−91.84**	−2.32	−42.93	−0.74	−256.14**	−2.69	33.64	0.42
CMS livestock price	204.12	1.51	242.72**	2.50	−1414.3**	−3.68	619.99**	4.81
Other CSM price	63.29	1.36	−59.25	−0.73	−187.55*	−1.76	−138.55	−1.59
FOG price	9.73	1.27	−26.57	−1.47	9.71	0.45	−0.92	−0.05
Livestock trading price	25.43*	1.70	6.94	0.36	−112.74**	−2.73	19.12	0.78
Capital	−0.02**	−4.04	0.001	1.20	0.06**	10.13	0.01**	3.15
Fixed labour	−0.69	−0.98	−0.59**	−4.40	−0.95	−1.09	−0.92**	−4.17
Dummy variable D	2,633.6**	10.37	−173.21**	−3.56	−1,432.4**	−4.21	−283.71**	−3.56
Dummy variable Z1	1,354.3**	4.12	−72.11	−1.20	−2,514.1**	−7.92	−782.51**	−9.51
Dummy variable Z2	934.3**	2.56	26.48	0.44	−3,109.0**	−8.06	−684.15**	−7.90
Dummy variable S1	4,112.8**	13.54	900.39**	10.62	534.21	1.09	1,175.5**	8.85
Dummy variable S2	880.3**	2.68	466.7**	5.94	−113.28	−0.26	617.96**	5.07
Rainfall	1,673.0**	5.58	95.45	1.26	75.94	0.21	242.84**	2.30
Time	−5.71	−0.28	−28.53**	−3.91	126.43**	4.50	−43.81**	−4.71
Adjusted R^2	0.69		0.46		0.54		0.52	

Note: D = 1 when the farm is in the Cropping industry, Z1 = 1 when the farm is in the Wheat-Sheep zone, Z2 = 1 when the farm is in the High Rainfall zone, S1 = 1 when the farm size is greater than $400,000 and S2 = 1 when the farm size is between $200,000 and $400,000

** Significant at the 5% level

* Significant at the 10% level

Table 18 (continued): Estimated Parameters of System of Supply and Demand Quantity Equations Derived from Normalised Quadratic Profit Function

	Output supply and input demand equations							
	CMS-Livestock		Other CSM		Fuel, oil and grease		Livestock trading	
	Coefficient	z-Statistic	Coefficient	z-Statistic	Coefficient	z-Statistic	Coefficient	z-Statistic
Constant	−3,176.8**	−2.59	−2,058.0**	−6.35	−217.2**	−3.04	−223.8**	−2.52
Grains price	204.12	1.51	63.29	1.36	9.73	1.27	25.43*	1.70
Sheep price	242.72**	2.50	−59.25	−0.73	−26.57	−1.47	6.94	0.36
Beef price	−1,414.3**	−3.68	−187.55*	−1.76	9.71	0.45	−112.74**	−2.73
Wool price	619.99**	4.81	−138.55	−1.59	−0.92	−0.05	19.12	0.78
CMS livestock price	2,133.7**	3.31	225.58	1.36	84.05**	3.21	57.56	1.16
Other CSM price	225.58	1.36	1,018.27**	2.66	−63.2	−0.59	82.41**	2.56
FOG price	84.05**	3.21	−63.2	−0.59	136.41**	4.04	27.57**	3.30
Livestock trading price	57.56	1.16	82.41**	2.56	27.57**	3.30	25.48**	2.35
Capital	−0.08**	−9.08	−0.04**	−30.14	−0.004**	−11.80	−0.01**	−8.86
Fixed labour	3.14**	2.82	0.77**	3.06	−0.13**	−3.21	−0.15	−1.39
Dummy variable D	1,591.1**	3.63	−362.55**	−4.27	−129.14**	−9.22	199.13**	4.90
Dummy variable Z1	1,192.8**	2.52	705.29**	6.53	44.23**	2.44	407.48**	11.48
Dummy variable Z2	2,130.2**	3.65	1,166.68**	10.62	127.56**	6.74	443.97**	10.32
Dummy variable S1	−1,268.7*	−1.93	−1,757.4**	−15.18	−245.72**	−12.40	−85.61	−1.45
Dummy variable S2	−340.37	−0.47	−424.27**	−3.55	−57.67**	−3.20	17.45	0.41
Rainfall	121.72	0.19	−74.8	−0.56	−64.1**	−3.16	−94.86**	−2.39
Time	−94.7**	−2.07	42.74**	3.11	9.25**	3.56	−19.78**	−4.57
Adjusted R^2	0.42		0.87		0.81		0.60	

Note: D = 1 when the farm is in the Cropping industry, Z1 = 1 when the farm is in the Wheat-Sheep zone, Z2 = 1 when the farm is in the High Rainfall zone, S1 = 1 when the farm size is greater than $400,000 and S2 = 1 when the farm size is between $200,000 and $400,000

** Significant at the 5% level

* Significant at the 10% level

Regarding the variable inputs, there is strong statistical evidence that CSM livestock price has a significant and positive relationship with Sheep supply. The prices of all other variable inputs are not found to be significantly related to Sheep supply. The level of Sheep production also does not appear to have a significant link with fixed capital. However, sheep output supply is found to be significantly and negatively related to the labour input of the operator and family members, *ceteris paribus*.

The estimation results show that Sheep supply has significant relationships with several other exogenous variables. The negativity and statistical significance of the industry dummy variable suggests that, other things being fixed, farms that focus on cropping generally produce less Sheep than those focusing on livestock grazing. No evidence was detected to support the hypothesis that there are significant differences among farms located in different production zones. Rainfall also appears to have no significant impact on Sheep supply. Meanwhile, production scale is found to be an important factor influencing Sheep production; the larger a farm is, the greater the Sheep supply is. Finally, the estimated supply equation indicates that Sheep supply decreases with the passage of time.

Supply of Beef

Compared to the estimated Grains and Sheep supply equations, more variables are found to be statistically significant in the estimated Beef supply equation. The number of significant price coefficients indicates Beef supply is more responsive to price changes than all other output supply and input demand. Beef supply seems to respond well and positively to changes in its expected price. This output supply is negatively affected by an increase in the expected prices of Sheep and Wool as expected. Grains price appears to have no significant influence on Beef supply.

Both livestock-related variable inputs CSM livestock and Livestock trading appear to play a significant role in Beef production. An increase in these input prices is associated with a reduction in Beef supply. The impact of changes in Other CSM price on Beef supply is found to be weakly significant. Meanwhile, FOG input price is not found to have a significant influence on Beef supply. In relation to fixed inputs, Beef supply is positively associated with the level of physical capital but not with the level of fixed labour input.

Statistical evidence suggests that Beef supply is significantly affected by several factors that are unrelated to production inputs and outputs. Farms that concentrate on cropping activities produce less Beef output than farms that concentrate on grazing activities, other things being equal. Further, farms in the High Rainfall zone generally produce less Beef than farms in the Wheat-Sheep zone, which in turn produce less Beef than farms in the Pastoral zone. In addition, Beef supply increases with the passage of time over the period from 1990 to 2005. In contrast, no significant effects on Beef supply of production scale and rainfall level are found in the estimated Beef supply equation.

Supply of Wool

The estimated Wool supply equation has the highest number of statistically significant coefficients when compared to other output supply equations. However, this estimated supply equation indicates that Wool production has a 'default' nature. The own-price coefficient is not statistically significant in this supply equation, suggesting that Wool supply is not responsive to its expected price during the period studied. In contrast, Wool supply appears to be significantly influenced by the prices of Grains and Beef outputs. Higher price of these alternative outputs would lead to lower Wool supply. Moreover, Wool supply is not significantly affected by changes in the price of any variable input except CSM livestock. The relationship between Wool supply and CSM livestock price is found to be positive. In contrast to variable inputs, both fixed inputs are statistically significant in the Wool supply equation. The level of Wool produced is positively associated with physical capital level but negatively associated with fixed labour quantity.

All non-price, non-quantity exogenous variables in the system are found to have significant effects on Wool supply. When other things are fixed, Cropping farms produce less Wool than Livestock farms; farms in the Wheat-Sheep zone produce less Wool than those in the High Rainfall zone, which in turn produce less Wool than those in the Pastoral zone. Also, the larger the farm, the more Wool produced; the higher the rainfall, the more Wool produced; and Wool supply decreases with the passage of time.

Demand for Contracts, Services and Materials for Livestock

The demand for the CSM livestock input is found to be more responsive to price signals than all other output supplies and input demands. The own-price coefficient of this input demand equation is highly significant and positive as expected (note that in Table 18, the left-hand side of each of the input demand equations is the negative of the quantity). The demand for CSM livestock is consistently significantly related to the prices of all livestock outputs (i.e. Sheep, Beef and Wool). The higher the Beef price, the higher the demand for the CSM livestock input. However, an increase in the expected price of Sheep or Wool generally causes a reduction in demand for CSM livestock. The coefficient of Grains price is statistically insignificant in this demand equation, which is reasonable.

Regarding the variable inputs, the demand for CSM livestock is found to be significantly related to FOG price, but not to Other CSM or Livestock trading prices. An increase in FOG price would generally lead to a decrease in demand for CSM livestock. Meanwhile, both fixed inputs (physical capital, and operator and family members' labour) are significantly related to the demand for the CSM livestock input. An increase in capital investment is associated with an increase in demand for CSM livestock demand. In contrast, the more devoted the operator and family are in their farming operation, the lower the demand for CSM livestock is.

The demand for CSM livestock is influenced by many non-price and non-quantity exogenous factors, based on statistical evidence. Among the qualitative variables, the coefficient of the industry dummy variable is significantly positive, implying that Cropping farms have lower demand for CSM livestock than Livestock farms, other things being equal. Both zone dummy variables are positive and statistically significant. The magnitudes of the coefficients of these two variables suggest that the demand for CSM livestock input is highest for farms in the Pastoral zone and lowest for farms in the High Rainfall zone. Regarding operational scale, there is weak evidence supporting the existence of differences between large farms and farms of other sizes in the way CSM livestock is utilised. The larger a farm is, the higher its demand for CSM livestock is, other things being fixed. Statistical evidence also indicates that rainfall does not have a significant effect on CSM livestock demand. Finally, demand for CSM livestock is found to increase with the passage of time over the 1990–2005 period.

Demand for Other Contracts, Services and Materials

The estimated demand equation for Other CSM input indicates that this input demand is responsive to its own price. An increase in this input price would lead to a decrease in its own demand as expected. The estimation result, however, shows that Other CSM demand is fairly unresponsive to movements in the prices of alternative variable inputs and all outputs. In this input demand equation, only Livestock trading price is significant at the 5% level and Beef price at the 10% level. According to the estimated coefficients, Other CSM demand decreases when the price of Livestock trading increases, but increases when expected Beef price increases.

In contrast to the low statistical significance of the price variables, all non-price exogenous variables, except rainfall, are highly significant in the estimated equation of Other CSM demand. Growth in capital investment is associated with increased demand for the Other CSM input. However, the increased involvement of the operator and family members in production is linked with decreased demand for the Other CSM input. The estimation results also suggest that Cropping farms generally have higher demand for Other CSM than Livestock farms with the same characteristics in all other aspects. The demand for this input is lower for farms in the Pastoral zone compared to farms in the Wheat-Sheep and High Rainfall zones. The estimated demand equation also indicates that the larger a farm is, the higher is its demand for the Other CSM input, *ceteris paribus*. Finally, the passage of time appears to have a negative impact on demand for Other CSM.

Demand for Fuel, Oil and Grease

The number of significant coefficients in the demand equation for the aggregate FOG input ranks among the highest across all supply and demand equations in the estimated system. The quantity of FOG is found to be significantly related to its own price in the expected direction. However, the estimated equation indicates that the demand for FOG is not significantly influenced by the price of any broadacre output. In contrast, the demand for FOG is significantly affected by changes in CSM livestock and Livestock trading prices. An increase in these two input prices would be associated with a decrease in demand for FOG.

In contrast to the low significance of price variables, demand for FOG is significantly influenced by all non-price exogenous variables included in the estimated system. Both fixed input quantities are positively related to the quantity of FOG demanded. Cropping farms have higher demand for this input than Livestock farms, other things being equal. The demand for FOG input also increases with operation scale. Considering geographical location, farms in the High Rainfall zone demand less of FOG input than farms in the Wheat-Sheep zone, which demand less of this input than farms in the Pastoral zone. Furthermore, the higher rainfall appears to imply higher demand for FOG. Lastly, it is indicated that FOG demand is negatively related to the passage of time.

Demand for Livestock Trading

The estimated supply and demand system signifies that prices are significant determinants of demand for the Livestock trading input. The quantity of Livestock trading demanded is negatively influenced by its own price as expected. This input demand also responds positively to changes in expected Beef price. There is weak evidence supporting a negative relationship between the demand for Livestock trading and the expected Grains price. Regarding the variable inputs, demand for the Livestock trading input is found to be strongly and negatively related to the prices of Other CSM and FOG inputs.

Many of the non-price exogenous variables included in the system are found to significantly influence demand for the Livestock trading input. Based on statistical evidence, fixed capital has a positive relationship with Livestock trading demand. There are also significant differences in demand for this input between farms having different production focuses and operation scales. Other things being equal, Cropping farms demand less of the Livestock trading input than Livestock farms while farms in the Pastoral zone demand more of this input than farms in the Wheat-Sheep and High Rainfall zones. An increase in rainfall appears to have a positive influence on the demand for Livestock trading, as does the passage of time.

7.4.2 Gross Own- and Cross-Price Elasticities of Input Demands and Output Supplies

The gross price elasticities of output supply and input demand generated from the estimated quantity system are shown in Table 19. The elasticity estimates obtained are generally small in magnitude, suggesting that supply of broadacre outputs and demand for broadacre inputs are not elastic to price changes. Despite having small absolute values, approximately 60 per cent of all price elasticities are significant at the 5% level.

All nine own-price elasticities have the correct sign, being positive for outputs and negative for inputs. The own-price elasticity is highly significant for seven (out of nine) inputs and outputs, being Sheep, Beef, CSM livestock, FC, Other CSM, FOG and Livestock trading. In addition, the own-price elasticity of Grains supply has a p-value of 0.051, implying that Grains supply is probably influenced by its own price. The own-price elasticity of Wool supply is statistically insignificant, in accordance with the insignificance of the own-price coefficient in the estimated supply equation of this output.

Regarding the cross-price adjustments of broadacre outputs and inputs, pair-wise relationships between outputs are found to be acceptable. All these pair-wise elasticities of output supply are negative. This result implies that broadacre outputs are gross substitutes. Moreover, two-thirds of these cross-price supply elasticities are highly significant. At the 10% level, cross-price supply elasticities between Grains and Beef are also significant. Regarding the magnitude of the cross-price responses between outputs, the supply elasticities of Sheep and Wool with respect to Beef price are highest, being around 0.30, and are highly statistically significant. These elasticities mean that if expected Beef price increases by ten per cent, Sheep and Wool production decreases by approximately three per cent. Sheep and Wool production, however, respond very differently to changes in the expected price of Grains. A ten per cent rise in Grains price will only induce a one per cent or less reduction in the supply of these two outputs.

Similar to the pair-wise elasticities between outputs, the adjustment in an input demand with respect to the price of another input is generally small in magnitude. However, the cross-price quantity adjustments of input pairs are more statistically significant and vary more greatly in terms of direction than those of output pairs. Sixteen of the twenty cross-price elasticities between inputs are statistically significant at the 5% level. Half of these input-input pair-wise elasticities are negative (implying complementary relationships) while the other half are positive (implying substitutive relationships).

Table 19 also displays the elasticities of output supplies with respect to input prices (right upper corner) and the elasticities of input demands with respect to output prices (left lower corner). These cross-price elasticities are less statistically significant than the cross-price output-output and input-input elasticities discussed above. Less than half of these input-output elasticities (seventeen out of 40) are significant at the 5% level. It is noteworthy that when the price of the CSM livestock input increases, the supply of Sheep and Wool increases but the supply of Beef decreases. Demand for the CSM livestock and Livestock trading inputs increases while demand for FC decreases if the expected price of Beef increases.

7.4.3 Gross Elasticities of Substitution and Transformation

The estimates of gross Allen partial elasticities of transformation and substitution are reported in Table 20. All six output-output transformation relationships are negative, suggesting that they are gross substitutes. The transformation relationships of Sheep - Grains, Sheep - Beef, Wool - Grains and Wool - Beef pairs are statistically significant at the 5% level. On the input side, most inputs are also substitutes. Among the input-input substitution relationships, those of the FOG - MSC Livestock, Livestock trading - MSC Livestock, Livestock trading - FOG, Livestock trading - Other MSC, MSC Livestock - FC and FC - Livestock trading pairs are statistically significant. The FOG - Other MSC and FC - Other CSM relationships are complementary but statistically insignificant.

The estimates obtained for Allen partial elasticities reveal some interesting input-output relationships in broadacre farming. Only ten out of these twenty relationships are statistically significant. An increase in Beef production is found to be associated with an increase in the usage of all inputs except for Livestock trading. Meanwhile, Grains production has negative relationships with the amounts of CSM livestock, FOG and FC inputs but no significant relationships with Other CSM and Livestock trading inputs.

Increases in Wool production appear to require extra CSM livestock and FO inputs but an increase in Sheep production appears to have no significant impact on the quantities of all variable inputs except CSM livestock.

The estimates of the gross Morishima elasticities of transformation and substitution are displayed in Table 21. Compared to Allen partial elasticities, Morishima elasticities are more often statistically significant. Forty-six of the seventy-two (equivalent to 63.9 per cent) pair-wise relationships are significant at the 5% level. The estimates of Morishima elasticities are generally in line with Allen partial estimates regarding the direction of the relationships. In terms of output transformability, eight out of twelve Morishima elasticity estimates are statistically significant and all Morishima estimates except one are negative. These results suggest that broadacre outputs compete against each other for resources regardless of which output price changes. The substitutive responses between Grains and Sheep and between Beef and Sheep are strongly significant, regardless of which output price changes. The transformation elasticity between Beef and Grains is found to be positive but also statistically insignificant.

The Morishima elasticity estimates for pairs of variable inputs have the expected signs. Two-thirds of these twenty elasticities are statistically significant. All except one of these elasticities are negative, indicating that broadacre inputs are substitutes as suggested by the Allen partial measure. The complementary relationship found for FOG input with respect to Livestock trading input is statistically insignificant. Interestingly, the substitution adjustments in response to changes in FOG price are significant for all other variable inputs. In contrast, changes in Livestock trading price do not appear to cause any significant substitution among inputs.

The Morishima elasticity for an output with respect to an input is positive for all outputs. This implies that an increase in an input price causes a larger percentage-wise decrease in that input's demand than the increases it causes in output supplies (see discussion in Chapter 3, Section 3.3.3). Moreover, three quarters of these elasticity estimates are highly significant. It is notable that Morishima estimates of Beef supply with respect to CSM livestock, Livestock trading and FC are not statistically significant.

Most Morishima elasticity estimates of input demand with respect to output supply are negative. This outcome implies that when there is an increase in an output price, the increase in that output's supply is generally larger, in percentage, than the resulted increases in demand for all inputs. Notably, a change in the expected price of Sheep significantly influences the demand of all variable inputs while a change in the expected price of Wool has no significant effect on the demand for most variable inputs.

Table 19: Own- and Cross-Price Elasticities of Supply and Demand [a, b]

Supply of and demand for	With respect to price of								
	Grains	Sheep	Beef	Wool	Contracts, services & materials for livestock	Fertilisers and chemicals	Other contracts, services & materials	Fuel, oil & grease	Livestock trading
Grains	0.037*	−0.011**	−0.040*	−0.021**	0.035*	−0.02	0.015*	0.002**	0.009**
	(0.019)	(0.003)	(0.022)	(0.008)	(0.019)	(0.019)	(0.009)	(0.001)	(0.004)
Sheep	−0.107**	0.246**	−0.282**	−0.070	0.265**	0.064	−0.081	−0.036*	0.018
	(0.027)	(0.043)	(0.082)	(0.067)	(0.071)	(0.122)	(0.091)	(0.019)	(0.038)
Beef	−0.036*	−0.026**	0.350**	−0.084**	−0.301**	0.217**	−0.052**	0.003	−0.062**
	(0.019)	(0.008)	(0.071)	(0.022)	(0.051)	(0.057)	(0.022)	(0.004)	(0.014)
Wool	−0.074**	−0.024	−0.296**	0.036	0.416**	0.025	−0.122**	−0.001	0.035
	(0.025)	(0.023)	(0.078)	(0.066)	(0.063)	(0.09)	(0.06)	(0.012)	(0.031)
Contracts, services & materials for livestock	−0.054*	−0.042**	0.579**	−0.228**	−0.509**	0.37**	−0.071**	−0.025**	−0.032*
	(0.029)	(0.012)	(0.099)	(0.035)	(0.093)	(0.088)	(0.034)	(0.007)	(0.018)
Fertilisers and chemicals	0.023	−0.001	−0.321**	−0.015	0.286**	−0.512**	0.335**	0.062**	0.064**
	(0.024)	(0.013)	(0.093)	(0.025)	(0.072)	(0.171)	(0.094)	(0.023)	(0.026)
Other contracts, services & materials	−0.028*	0.016	0.117**	0.079**	−0.082**	0.449**	−0.487**	0.029	−0.084**
	(0.017)	(0.018)	(0.049)	(0.039)	(0.039)	(0.13)	(0.153)	(0.035)	(0.025)
Fuel, oil & grease	−0.024	0.041*	−0.033	0.003	−0.170**	0.527**	0.171	−0.331**	−0.138**
	(0.016)	(0.023)	(0.048)	(0.043)	(0.046)	(0.186)	(0.214)	(0.059)	(0.028)
Livestock trading	−0.046**	−0.009	0.323**	−0.047	−0.094*	0.230**	−0.179**	−0.057**	−0.099**
	(0.021)	(0.019)	(0.076)	(0.042)	(0.051)	(0.091)	(0.056)	(0.012)	(0.028)

Note: [a] Medians of elasticities evaluated at all observation points
[b] Bootstrapping standard errors (500 trials) are in parentheses
** Significant at the 5% level
* Significant at the 10% level

Table 20: Allen Partial Elasticities of Substitution and Transformation [a, b]

	Grains	Sheep	Beef	Wool	Contracts, services & materials for livestock	Other contracts, services & materials	Fuel, oil & grease	Livestock trading	Fertilisers and chemicals
Grains									
Sheep	−0.026**								
	(0.006)								
Beef	−0.003	−0.027**							
	(0.002)	(0.008)							
Wool	−0.020**	−0.038	−0.027**						
	(0.007)	(0.036)	(0.007)						
Contracts, services & materials for livestock	−0.006*	−0.048**	0.127**	−0.081**					
	(0.003)	(0.014)	(0.02)	(0.014)					
Other contracts, services & materials	0.003	0.004	0.074**	0.002	0.086**				
	(0.003)	(0.003)	(0.021)	(0.003)	(0.025)				
Fuel, oil & grease	−0.006**	0.03	0.01**	0.043**	0.013*	−0.005			
	(0.003)	(0.032)	(0.005)	(0.021)	(0.007)	(0.013)			
Livestock trading	−0.005	0.077*	−0.004	0.002	0.034**	0.001	−0.054		
	(0.003)	(0.041)	(0.005)	(0.026)	(0.01)	(0.017)	(0.067)		
Fertilisers and chemicals	−0.009**	−0.011	0.066**	−0.019	0.034**	0.050**	0.042**	0.094**	
	(0.004)	(0.021)	(0.017)	(0.016)	(0.017)	(0.023)	(0.013)	(0.021)	

Note: [a] Medians of elasticities evaluated at all observation points
[b] Bootstrapping standard errors (500 trials) are in parentheses
** Significant at the 5% level
* Significant at the 10% level

7.5 Discussion of Estimation Results

The estimated system of supply and demand equations derived from the restricted normalised quadratic multi-product profit function for Australian broadacre agriculture is reasonable, from both statistical and economic points of view. Compared to previous studies of profit functions, the estimated system is sensible with 63.2 per cent of system coefficients being significant at the 5% level. This percentage is higher than that achieved in similar studies such as Shumway and Alexander (1988), Shumway *et al.* (1988), Moschini (1988) and Fulginiti and Perrin (1990). Using annual data from 1951 to 1982, Shumway and Alexander estimated restricted normalised quadratic profit functions for ten production regions in the United States. The percentage of significant coefficients of the estimated derived supply and demand systems ranges from 27 per cent to 48 per cent for these ten regions. The percentage of significant system coefficients is 42 per cent in Shumway *et al.* (1988) and 57.8 per cent in Moschini (1988), despite the fact that aggregate data are used for model estimation in these studies.

The failure to meet the monotonicity and convexity conditions for the estimated profit function in this chapter is a result common to many studies of normalised quadratic profit functions. Examples of these studies are Shumway (1983), Huffman and Evenson (1989), Fisher and Wall (1990), Coxhead (1992) and Ahammad and Islam (2004). In particular, when Fisher and Wall (1990) estimated normalised quadratic profit functions for three broadacre zones in Australia, the monotonicity condition was violated for the Pastoral zone despite their use of aggregate zone-state data for estimation. Meanwhile, in the quasi-micro dataset used for estimation in this study, there are a large number of observations where the observed output supplies, especially Grains supply, are zero or small. This data feature likely contributes to the significant

Table 21: Morishima Elasticities of Substitution and Transformation [a, b]

	Grains	Sheep	Beef	Wool	Contracts, services & materials for livestock	Other contracts, services & materials	Fuel, oil & grease	Livestock trading	Fertilisers and chemicals
Grains	-	-0.245^{**}	-0.353^{**}	-0.046	0.425^{**}	0.465^{**}	0.49^{**}	0.333^{**}	0.088^{**}
		(0.042)	(0.075)	(0.062)	(0.092)	(0.158)	(0.154)	(0.06)	(0.028)
Sheep	-0.118^{**}	-	-0.689^{**}	-0.110^{*}	0.786^{**}	0.551^{**}	0.347^{*}	0.268^{**}	0.116^{**}
	(0.028)		(0.126)	(0.064)	(0.114)	(0.175)	(0.184)	(0.066)	(0.035)
Beef	0.005	-0.252^{**}	-	-0.113^{*}	0.049	0.136	0.409^{**}	0.331^{**}	0.02
	(0.021)	(0.04)		(0.062)	(0.046)	(0.107)	(0.162)	(0.06)	(0.019)
Wool	-0.092^{**}	-0.270^{**}	-0.644^{**}	-	0.906^{**}	0.514^{**}	0.3^{*}	0.329^{**}	0.125^{**}
	(0.027)	(0.038)	(0.113)		(0.105)	(0.143)	(0.163)	(0.062)	(0.025)
Contracts, materials & services for livestock	-0.046^{*}	-0.284^{**}	0.061	-0.259^{**}	-	0.100	0.370^{**}	0.347^{**}	0.060^{*}
	(0.027)	(0.042)	(0.063)	(0.067)		(0.086)	(0.162)	(0.063)	(0.033)
Other contracts, services & materials	0.005	-0.244^{**}	-0.09	-0.049	0.176^{*}	-	0.605^{**}	0.359^{**}	0.076^{**}
	(0.022)	(0.038)	(0.079)	(0.062)	(0.096)		(0.179)	(0.054)	(0.027)
Fuel, oil & grease	-0.057^{**}	-0.214^{**}	-0.248^{**}	0.027	0.42^{**}	0.713^{**}	-	0.363^{**}	0.028
	(0.018)	(0.051)	(0.07)	(0.067)	(0.092)	(0.201)		(0.095)	(0.029)
Livestock trading	-0.056^{**}	-0.172^{**}	-0.382^{**}	-0.032	0.268^{**}	0.850^{**}	0.657^{*}	-	-0.025
	(0.019)	(0.05)	(0.081)	(0.081)	(0.102)	(0.178)	(0.341)		(0.043)
Fertilisers and chemicals	-0.017	-0.242^{**}	-0.073^{*}	-0.081	0.375^{**}	0.245^{**}	0.199	0.341^{**}	-
	(0.024)	(0.038)	(0.041)	(0.053)	(0.112)	(0.114)	(0.144)	(0.087)	

Note: [a] Medians of elasticities evaluated at all observation points
[b] Bootstrapping standard errors (500 trials) are in parentheses
** Significant at the 5% level
* Significant at the 10% level

proportion of negative supply and demand quantities predicted from the estimated supply and demand system.

The result that the convexity condition is not satisfied by the estimated profit function is surprising given the satisfaction of the curvature conditions by the estimated normalised quadratic cost function in Chapter 5 and by the estimated normalised quadratic revenue function in Chapter 6. This outcome contradicts the finding in Fisher and Wall's (1990) that the convexity condition holds. However, only two out of eight eigenvalues of the matrix of the estimated price coefficients are negative. This suggests that the violation of the convexity condition may not be so severe. Moreover, all eight own-price coefficients have the correct signs, indicating that the derived supply curves are upward sloping and the derived demand curves are downward sloping as predicted by economic theory. Six of these own-price coefficients are also statistically significant at the 5% level. This result is substantially more encouraging compared to that in Ahammad and Islam (2004). In their study of broadacre agricultural production in the state of Western Australian, the estimated own-price coefficient is negative in two estimated supply and demand equations. Moreover, almost all of the own-price coefficients of their estimated supply and demand system are insignificant at the 5% level.

The failure to impose the convexity condition on the profit function using the Cholesky decomposition in this chapter contrasts with many previous duality applications. Shumway *et al.* (1988), Shumway and Alexander (1988), Dupont (1991), Polson and Shumway (1992) and Marsh (2005) successfully imposed this condition by means of parametric restrictions. The failure to impose the convexity condition in this chapter is surprising when considering the fact that only two out of the eight eigenvalues of the matrix of the estimated unrestricted price coefficients are negative. It is possible that the convexity condition demands too much on the quasi-micro dataset used for estimation in this study. In contrast, as pointed out by Coxhead (1992), the imposition of this regularity condition by parametric restrictions is only practically possible by forcing some cross-price coefficients to be zero, which has no economic justification.

Moreover, the estimated unconstrained system of the derived supply and demand equations portrays reasonably well rational supply and demand behaviour, as shown by the correct sign and statistical significance of the own-price coefficients as well as by the positiveness of the majority of the eigenvalues of the price matrix. Therefore, the failure to impose the convexity condition may be considered as not seriously violating the assumption of profit maximisation for broadacre farmers in Australia.

In contrast to the less conclusive findings regarding the regularity conditions, the estimated model of Australian broadacre producers generates sensible measures of price responsiveness for output supply and input demand. Six out of the eight own-price supply and demand elasticities are statistically significant at the 5% level. Another own-price elasticity (of Grains supply) is also significant at the 5.5 per cent level. All the own-price elasticity estimates obtained are in line with economic expectations, being positive for output supplies and negative for input demands. They are also within the range of elasticity estimates obtained in previous studies of Australian agricultural production, supporting the view that broadacre production in Australia is fairly rigid in the short term. For instance, the estimate of the own-price elasticity for Wool supply is 0.037 in this study. This value is almost identical to the value of 0.04 obtained by Fisher and Wall (1990) for the Wheat-Sheep zone. The own-price elasticity estimate is 0.35 for Beef supply in this chapter, compared to the 0.11–0.43 range for the three broadacre zones in Fisher and Wall’s study. While the own-price elasticity estimate of 0.25 for Sheep supply in this chapter is lower than those obtained in Fisher and Wall (1990) and much higher than the long-run estimate of 0.041 in Coelli's (1996) study, it is very close to the estimate obtained in Vincent *et al.* (1980) for the Wheat-Sheep zone. An exception to the conformity of the elasticity estimates obtained in this chapter to the previous studies’ estimates is the own-price elasticity estimate of Grains supply. This estimate is significantly lower than in previous studies, regardless of the geographical coverage and whether the profit function is specified for the short run or the long run.

The estimated system of derived supply and demand equations suggests that there is little scope for substitution among broadacre inputs and outputs in Australia. More than half of all pair-wise relationships are statistically significant, for both Allen partial and Morishima measures of transformation and substitution. All output-output and input-input relationships are found to be substitutive in the short term. This finding is consistent with findings from previous studies of Australian broadacre production such as Vincent *et al.* (1980) and Agbola and Harrison (2005).

The relatively higher significance of non-price, non-quantity exogenous variables in the estimated supply and demand system implies that there are significant differences in the operation of farms across broadacre industries, broadacre zones and operation sizes. Rainfall is found to have a significant positive effect on Grains and Wool production but not on Sheep and Beef production. Meanwhile, the time trend is found to have significant relationships with all broadacre output supplies and input demands except Grains supply. With time, the supply of Beef and the demand for livestock-related inputs increase, while the supply of Sheep and Wool decreases. The common interpretation of the time trend as representing the disembodied effects of technological progress may not be appropriate in this study. It is probably more appropriate to interpret this finding as a result of the general trend over the study period in favour of beef grazing in Australia to meet increasing demand from overseas markets. The decline in the supply of Wool and Sheep supply may result from the dismantlement of government support for the wool industry via the Wool Price Scheme, which collapsed in 1991.

7.6 Summary

In this chapter, the results of estimating a restricted normalised quadratic multi-product profit function for Australian broadacre production are presented. The restricted normalised quadratic profit function is specified for the same set of five variable inputs, four outputs, two fixed inputs and six other exogenous variables as in Chapters 5 and 6. The system of the supply and demand equations derived from this profit function is estimated with the symmetry and homogeneity conditions imposed using the quasi-micro AAGIS data available. The estimated supply and demand system has a reasonable statistical goodness-of-fit. This system, however, does not satisfy the regularity conditions of monotonicity and convexity in prices. It is also not possible to impose the convexity condition on the system using parametric restrictions.

Despite the violation of the convexity condition, the profit maximisation assumption may still be appropriate in Australian broadacre production. The violation of the convexity condition is not serious and may be due to the quasi-micro nature of the data used for estimation. All own-price coefficients in the estimated supply and demand system have correct signs, implying that output supplies are upward-sloping and input demands are downward-sloping as expected by economic theory. Most of these own-price coefficients are also statistically significant. Moreover, all gross own-price elasticities of supply and demand are found to be in line with rational economic behaviour.

The estimates of gross price elasticities obtained from the estimated supply and demand system suggest that broadacre production in Australia is fairly unresponsive to price signals in the short term. In this chapter, none of the output supplies or input demands are found to be responsive to price changes. This means that the effectiveness of government policies designed to influence the supply of and demand for broadacre outputs and inputs through intervening market prices will be low. Many of the elasticities are statistically significant, which implies that over the short term, broadacre farmers try to take advantage of positive price movements or eliminate any adverse effects of negative price movements.

The findings regarding the Allen partial and Morishima elasticities of substitution and transformation obtained from the restricted normalised quadratic profit function are encouraging. More than half of these elasticities are statistically significant. The estimates of these two elasticity measures suggest that all broadacre outputs are gross substitutes of each other as are all broadacre inputs.

The results indicate that production operations differ significantly between broadacre zones, sizes and industries. This finding implies that it is worthwhile investigating production technology separately for each broadacre zone, production size and industry. Given the large systems of derived supply and demand equations, obtaining a reliable research result in such an investigation requires a much larger dataset than the quasi-micro dataset available for this study.

Chapter 8

The Estimated Cost, Revenue and Profit Functions of Australian Broadacre Production Contrasted

8.1 Introduction

In the three preceding chapters, estimation results for Australian broadacre production under cost minimisation, revenue maximisation and profit maximisation assumptions were presented and discussed. Profit maximisation assumption appears to be more readily accepted by researchers than cost minimisation and revenue maximisation assumptions. However, the failure of the estimated system of supply and demand equations derived from the profit function to meet the curvature condition, as discussed in Chapter 7, raises doubts about the applicability of the profit maximisation assumption to Australian broadacre production. In contrast, the curvature condition is automatically satisfied in the estimation results of the normalised quadratic cost function in Chapter 5 and the normalised quadratic revenue function in Chapter 6. These findings suggest that assuming cost minimisation and revenue maximisation may be more appropriate for Australian broadacre production than the profit maximisation assumption.

In this chapter, the estimation results from the normalised quadratic cost, revenue and profit functions obtained in Chapters 5, 6 and 7 are contrasted. Only the results from the normalised quadratic functional form are discussed since the results from the translog revenue and profit functions are either unavailable or unreasonable. Contrasting results is only possible between cost and profit functions and between revenue and profit functions. Estimation results from the cost and revenue functions cannot be contrasted since the former concerns only the inputs and the latter concerns only the outputs.

The contrast in this chapter covers three aspects of the estimation results. The three result sets from the three dual objective functions are first assessed with regard to their statistical goodness-of-fit. Although these results are not strictly comparable to one another, since the estimated systems have different endogenous variables, their explanatory power may provide insights into the appropriateness of the assumption made about the economic behaviour of Australian broadacre farmers. The degree to which each of the three result sets conforms to economic theory is then contrasted. Conformation is based on satisfaction of the theoretical regularity conditions by the estimated system of derived demand and/or supply. The net elasticity estimates obtained from the cost and revenue functions and the gross elasticity estimates obtained from the profit function, while not directly comparable, should appropriately portray rational economic behaviour as demonstrated by the Le Chatelier-Samuelson principle explained later in the chapter. Finally, estimates of the comparable *compensated* elasticities of input demand and output supply obtained from the estimated profit function are compared to the corresponding net estimates obtained from the cost and revenue functions.

The main body of this chapter is organised into four sections. In Section 8.2, the goodness-of-fit of the estimated systems of the demand and/or supply equations derived from the cost, revenue and profit functions for Australian broadacre production, as presented in Chapters 5, 6 and 7, are contrasted. Section 8.3 follows with a discussion on whether and to what degree to which the results from the three dual functions appropriately describe the rational economic behaviour assumed for Australian broadacre farmers. In Section 8.4, the estimates of price elasticities and elasticities of substitution and/or transformation obtained from the three dual functions are contrasted. This section includes a discussion on whether the estimation results in Chapters 5, 6 and 7 follow the Le Chatelier-Samuelson principle. Section 8.5 concludes this chapter with findings about economic behaviour of Australian broadacre farmers.

8.2 Goodness-of-fit of Estimation Results Contrasted

The goodness-of-fit measures of the estimated demand or/and supply systems derived from the normalised quadratic cost, revenue and profit functions in the three previous chapters are displayed in Table 22. The six measures presented are more in line with each other for the estimated cost function than for the revenue and profit functions. The cost function also has the highest percentage of significant price coefficients at the 5% level. Moreover, the cost function model explains consistently well the demand for all included variable inputs (excluding the *numeraire*), with the adjusted R^2 ranging from 0.78 to 0.91 for the individual derived demand equations. In contrast, the adjusted R^2 fluctuates over a large range for equations in the quantity systems derived from the revenue and profit functions. The adjusted R^2 varies between 0.49 and 0.87 in the system of supply equations derived from the revenue function and between 0.42 and 0.87 in the system of supply and demand equations derived from the profit function.

Regarding input demand, the estimation results from the cost and profit functions are fairly consistent with each other. The own-price coefficient is statistically significant and has the correct sign in all estimated equations of input demand derived from these two dual functions. At the 5% level, the number of significant input prices is the same in the two systems. Out of sixteen price coefficients, only four change sign between the two systems.

With respect to output supply, modelling results from the revenue and profit functions agree well with each other. All coefficients of (normalised) output prices in the estimated supply equations derived from these two dual functions are positive as expected. Both estimated systems consistently suggest that Sheep supply responds significantly and positively to its own price while Wool supply does not. Both systems also indicate that Sheep price does not have any significant impact on Wool supply and vice versa. One noteworthy difference between the two system results is the influence of Grains price on its own supply. Grains price significantly influences its supply in the revenue function's result but not in the profit function's result. The likely cause of this difference is that these two systems have different *numeraires*, being Beef price in the revenue function case and FC price in the profit function case.

Table 22: The Goodness-of-fit of Three Restricted Normalised Quadratic Dual Functions

Goodness-of-fit measure	Cost function	Revenue function	Profit function
Percentage of significant system coefficients at 5% level	61.1%	77.8%	63.2%
Percentage of significant system coefficients at 10% level	70.8%	83.3%	66.7%
Percentage of significant price coefficients at 5% level	62.5%	44.4%	43.8%
Percentage of significant price coefficients at 10% level	75.0%	66.7%	50.0%
Adjusted R^2 for individual equations	0.78 - 0.91	0.49 - 0.87	0.42 - 0.87
System McElroy R^2	0.85	0.65	Not available

8.3 Theoretical Regularity Conditions Contrasted

For the underlying economic behavioural assumption to hold, a set of regularity conditions should be satisfied by each of the estimated systems of supply and/or demand equations derived from the cost, revenue and profit functions. The key regularity conditions consist of homogeneity, symmetry, monotonicity and curvature in input and output prices. Homogeneity and symmetry conditions were imposed during estimation for all three dual functions while monotonicity and curvature conditions were checked after estimation.

The monotonicity condition is violated in all three result sets and is most seriously violated in the case of the profit function. As shown in Table 23, the percentage of data points at which the monotonicity condition is violated (i.e. when a predicted input or output quantity is negative) is considerable for all three dual functions. The pattern of violation of this regularity condition is similar among the three models. CSM livestock and Livestock trading inputs have high percentage of negative predicted quantities for both cost and profit functions. Similarly, Grains supply has the highest percentage of negative predicted quantities, followed by Wool supply and then Sheep supply, in results from the revenue and profit functions.

Table 23: Monotonicity Condition — Percentage of Negative Predicted Supply and Demand Quantities

Supply and demand quantity	Dual function		
	Cost	Revenue	Profit
Grains		22.6%	30.2%
Sheep		8.5%	5.4%
Wool		11.6%	9.7%
Contracts, services & materials for livestock	13.0%		23.5%
Other contracts, services & materials	5.7%		6.4%
Fuel, oil & grease	6.1%		7.2%
Livestock trading	11.1%		24.9%

All three dual functions achieve better results for the curvature condition than for the monotonicity condition. The matrix of the estimated price coefficients of the demand system derived from the cost function is negative definite, implying that the cost function specified for Australian broadacre agriculture satisfies the concavity condition. The revenue function specified for Australian broadacre agriculture also satisfies the convexity condition, implied by the positive definiteness of the estimated price matrix of the system of supply equations derived from this dual function. Unlike the cost and revenue functions, the convexity condition is not satisfied by the estimated system of supply and demand equations derived from the profit function. This estimated system does not have a positive semidefinite price coefficient matrix, with two out of eight eigenvalues being negative. An attempt to impose positive semidefiniteness on this price coefficient matrix via parametric restrictions also failed to yield an estimation result. These findings may suggest that the profit maximisation assumption may be less suitable in describing Australian broadacre production than cost minimisation or revenue maximisation assumptions.

Despite failing to meet the convexity condition, results from the profit function indicate that Australian broadacre farmers' economic behaviour is generally rational. All supply and demand curves derived from this dual function have the expected slopes with respect to their own prices, being upward sloping for output supply and downward sloping for input demand. Moreover, only two out of the eight eigenvalues of the estimated price matrix of the supply and demand system derived from the profit function are negative. These two outcomes suggest that the violation of the convexity condition is not too severe and that the violation may be due to the use of quasi-micro farm data and national input price indices for estimation as discussed in Chapter 7 (Section 7.5). These outcomes, and the satisfactory estimation results from the cost and revenue functions regarding the curvature condition, indicate that Australian broadacre farmers generally are rational economic agents.

8.4 The Estimated Elasticities Contrasted and Compared

The elasticity estimates obtained in Chapters 5, 6 and 7 are generated under different assumptions regarding the optimisation behaviour of Australian broadacre farmers and regarding the set of variables being exogenous or endogenous. The elasticity estimates in these three result sets are therefore of different notions, as explained in Chapter 3, and cannot be directly compared meaningfully. The elasticities obtained from the cost and revenue functions are net (conditional or compensated) measures while those obtained from the profit function are gross (unconditional or uncompensated) measures. It is possible for a pair of inputs and outputs to be classified as substitutes by the net measure but as complements by the gross measure (Bertoletti 2005).

8.4.1 The Le Chatelier-Samuelson Principle

The own- and cross-price elasticities of input demand obtained from the cost and profit functions, despite not being comparable with each other, are fairly similar to each other. As shown in Table 24, all net and gross own-price elasticities of input demand are negative. Only six out of twenty cross-price elasticity estimates change sign between the two result sets (Table 25). The net elasticity estimates from the cost function are generally higher than the corresponding gross estimates from the profit function. For instance, demand for the Fertilisers and Chemicals input is elastic with respect to own price and to the price of Other CSM input by the net measure but not by the gross measure.

The net and gross price elasticities of output supply obtained from the revenue and profit functions exhibit high degree of similarity. As shown in Table 24, own-price elasticities of output supply from these two dual functions are positive. Table 26 shows that all cross-price elasticity estimates but one do not change sign between the net and gross elasticity sets. In both result sets, the elasticity estimates are smaller than 0.5, indicating that output supply is not responsive to price change in the short run. The gross price elasticity estimates obtained from the profit function are generally higher than the corresponding net estimates obtained from the revenue function. For instance, the elasticities of Beef supply with respect to its own price and with respect to Sheep and Wool prices are less than 0.1 when generated from the revenue function but are approximately 0.3 when generated from the profit function.

Table 24: Own-price Elasticity Estimates from Three Dual Functions [a]

Own-price elasticity of supply and demand	Dual function		
	Cost	Revenue	Profit
Grains		0.073**	0.037
Sheep		0.159**	0.246**
Beef		0.065	0.350**
Wool		0.058	0.036
Contracts, services & materials for livestock	−0.431**		−0.509**
Fertilisers and chemicals	−1.756**		−0.512**
Other contracts, services & materials	−0.796**		−0.487**
Fuel, oil & grease	−0.531**		−0.331**
Livestock trading	−0.078**		−0.099**

Note: [a] Medians of elasticities evaluated at all observation points
** Significant at 5% level

Table 25: Cross-price Elasticities of Input Demand: Net Estimates from Cost Function and Gross Estimates from Profit Function [a]

Demand for	With respect to price of: Contracts, services & materials for livestock – Cost function	Contracts, services & materials for livestock – Profit function	Fertilisers and chemicals – Cost function	Fertilisers and chemicals – Profit function	Other contracts, services & materials – Cost function	Other contracts, services & materials – Profit function	Fuel, oil & grease – Cost function	Fuel, oil & grease – Profit function	Livestock trading – Cost function	Livestock trading – Profit function
Contracts, services & materials for livestock			0.491**	0.370**	−0.130	−0.071**	−0.051**	−0.025**	0.060**	−0.032
Fertilisers and chemicals	0.391**	0.286**			1.030**	0.335**	0.037	0.062**	−0.009	0.064**
Other contracts, services & materials	−0.068	−0.082**	0.782**	0.449**			0.083**	0.029	0.001	−0.084**
Fuel, oil & grease	−0.157**	−0.170**	0.197	0.527**	0.525**	0.171			−0.031**	−0.138**
Livestock trading	0.152**	−0.094	−0.055	0.230**	0.006	−0.179**	−0.026**	−0.057**		

Note: [a] Medians of elasticities evaluated at all observation points
** Significant at 5% level

Table 26: Cross-price Elasticities of Output Supply: Net Estimates from Revenue Function and Gross Estimates from Profit Function [a]

Supply of	With respect to price of: Grains – Revenue function	Grains – Profit function	Sheep – Revenue function	Sheep – Profit function	Beef – Revenue function	Beef – Profit function	Wool – Revenue function	Wool – Profit function
Grains			−0.016**	−0.011**	−0.029	−0.040	−0.023**	−0.021**
Sheep	−0.086**	−0.107**			−0.033	−0.282**	−0.035	−0.070
Beef	−0.024	−0.036	−0.003	−0.026**			−0.0002	−0.084**
Wool	−0.047**	−0.074**	−0.012	−0.024	0.004	−0.296**		

Note: [a] Medians of elasticities evaluated at all observation points
** Significant at 5% level

Although the net and gross elasticity estimates obtained from the three dual functions specified in Chapters 5, 6 and 7 cannot be directly compared with each other in a meaningful manner, there are some theoretical expectations regarding their relative magnitudes. The net elasticities of demand and supply obtained from the cost and revenue functions can be related to the gross elasticities obtained from the profit function by decomposing the impact of a price change into substitution and expansion effects (Lopez 1984 and Chambers 1988). The relationships between the net and gross elasticities demonstrate a form of the Le Chatelier-Samuelson principle. According to this principle, for the multi-product case, the own-price elasticities of derived input demand obtained from the cost function, i.e. under output-constrained profit maximisation, should be smaller, in absolute value, than those obtained from the profit function, i.e. under output-unconstrained profit maximisation (Chambers 1988, pages 105 and 275-276). Similarly, the own-price elasticities of output supply derived from the profit function should be larger than the corresponding elasticities of output supply derived from the revenue function. In other words, once the constraint of output or variable input quantities being fixed is removed, the curves of input demand and output supply derived from the profit function are expected to be steeper than the corresponding curves derived from the cost and revenue functions.

The net and gross price elasticities obtained for Australian broadacre production from the cost and profit functions are consistent with the described Le Chatelier-Samuelson principle only for CSM livestock and Livestock trading inputs. The absolute values of own-price elasticities of FC, Other CSM and FOG inputs are higher from the cost function than from the profit function (see Table 24). This inconsistency is also exhibited in results obtained by McKay *et al.* (1983) and McKay *et al.* (1980) when the own-price elasticities of the variable inputs included in their models (namely, labour, and materials and services) are compared. The own-price elasticity estimates of these two inputs from the profit function (McKay *et al.* 1983) have much smaller absolute values than those from the cost function (McKay *et al.* 1980).

Since reported elasticities are the medians of elasticities calculated at all data sample points, the comparison above may not be accurate. An inspection of the elasticity estimates at each data observation point shows that the Le Chatelier-Samuelson principle holds at 894 (out of 1272) data points for CSM livestock input and at 988 data points for Livestock trading input. For the FC input, the Le Chatelier-Samuelson principle holds at 501 data points, which contrasts with a serious violation when the medians of the net and gross own-price elasticity estimates of this input are compared. Finally, the number of data points at which the Le Chatelier-Samuelson principle holds is 95 and 112 for Other CSM and FOG inputs, respectively.

Similar to input demand, the net and gross own-price elasticity estimates obtained for output supply do not conform to the Le Chatelier-Samuelson principle. As shown in Table 24, the gross own-price elasticity is higher than the corresponding net elasticity only for Beef supply and Sheep supply. The opposite is found for the supply of Grains and Wool. An inspection of elasticity estimates calculated at each sample point reveals that the Le Chatelier-Samuelson principle is conformed to at 995 data points for Sheep supply and 893 data points for Beef supply. For the Grains and Wool supply, this theoretical expectation is met at 423 and 239 data points, respectively.

The failure of the estimation results from the cost, revenue and profit functions to conform to the Le Chatelier-Samuelson principle can be attributed to the failure of the profit function to satisfy the convexity condition described in Chapter 7. This assertion is based on the fact that the Le Chatelier-Samuelson principle is a consequence of the concavity of the cost function and the convexity of the revenue and profit functions (Chambers 1988). Therefore, the violation of the convexity condition by the estimated profit function appears to be serious, despite the low degree of severity of the violation as explained in Section 8.3.

The failure of the three dual functions to conform to the Le Chatelier-Samuelson principle may suggest the proposition that the profit maximisation assumption is less suitable for Australian broadacre production than the cost minimisation and revenue maximisation assumptions. Among the three result sets, the profit function has the lowest percentage of significant price coefficients (Table 22) and violates the monotonicity condition most frequently (Table 23) while failing to satisfy the convexity condition. This poorer result from the profit function is potentially due to the tendency of producers to make separate production choices for crops and livestock production due to popular deployment of the ley rotation practice, as suggested by Reynolds and Gardiner (1980). Furthermore, according to Fisher and Munro (1983), it takes wool growers in New South Wales three years to significantly restructure their enterprise combinations. This implies that adjustments in output mixture required for revenue or profit maximisation are long-run. An examination of the quasi-micro dataset in this study shows that the revenue shares of Grains and Beef outputs have a correlation coefficient of −0.69, sensibly suggesting that if a broadacre farm is significantly engaged in cropping activities, it is not likely to carry out significant Beef production. Moreover, these two outputs do not influence the supply of the other, as indicated by the insignificance of their prices in each other's supply equation derived from the profit function (Chapter 7). The revenue shares of Grains and Wool outputs have a correlation coefficient of −0.5, suggesting that production decisions concerning these two outputs are possibly made independently. This possible separation in production decisions is also supported by the finding that Wool price is weakly significant in the Grains supply equation derived from the revenue function.

The independent decision-making in cropping and livestock grazing activities discussed above implies that the restricted revenue function specified in Chapter 6 is not appropriate for Australian broadacre production. Land and other capital inputs have lumpy nature and are normally fixed during typical production cycles. For an individual farm, the fixity of land and the deployment of a fixed rotation regime together imply that, in the short run, output levels are, to a certain extent, predetermined. The allocation of available land resource to different outputs over the long-run is determined by the rotation systems adopted by farmers. This means that, in the short run, farmers do not maximise production revenue received from crops and livestock. This is possibly the reason why the percentage of significant price coefficients is much lower for the revenue function than for the cost function. Furthermore, this probably explains why the majority of price elasticities obtained from the estimated revenue function are statistically insignificant. Finally, as a consequence of this decision-making process, all price elasticities of output supply are significantly smaller than those of input demand, regardless of whether they are obtained from the revenue or profit functions.

The results discussed above may imply that Australian broadacre farmers focus on minimising production costs in the short run. If this is the case, demand for production factors could be expected to be more responsive to price change than output supply, an outcome which is supported by the results from the cost and profit functions. In the short run, the cost minimisation assumption appears to be more consistent with features peculiar to Australian broadacre agriculture than the revenue maximisation and profit maximisation assumptions. Broadacre production is subject to the irreversibility of large capital investments as well as uncertainty due to exogenous changes in weather conditions and unpredicted international commodity prices. Other aspects of production operation such as income stability, disease control, financial constraints, risks preferences, and the maintenance and improvement of the long-term productivity of natural resources, can cause farmers to deviate from revenue and profit maximisation in the short run. Broadacre agricultural production entails long-term capital investment in different broadacre enterprises. Once investment decisions are made, the mixture of products for subsequent production cycles is in effect predetermined, thereby reducing short-run production flexibility. Farmers are likely to be risk averse (Beal 1996), which implies that farmers, although being opportunistic, are inclined to follow their long-run risk management strategies, instead of short-run adjustments for revenue or profit maximisation. Furthermore, Agbola and Harrison (2005) found that farmers require a period of more than two years to adjust sheep and cattle numbers. This implies that farmers cannot maximise their profit or revenue within a time period shorter than two years. Such a long-run decision-making process in multi-product broadacre farming is probably the reason why the cost minimisation assumption has the best estimation results in this study and why short-run price elasticities are generally very small for the supply of broadacre products.

8.4.2 The Net and *Compensated* Price Elasticities Compared

After the system of input demand and output supply equations derived from the profit function is estimated, the system parameter estimates and the predicted demand and supply quantities are used to calculate the *compensated* price elasticities of input demand and supply using the response decomposition in Lopez (1984), (see equations 3.14 and 3.15 in Chapter 3). The *compensated* elasticity estimates obtained from the estimated system of input demand and output supply derived from the normalized quadratic profit function are presented in Tables 27 and 28, along with the corresponding net elasticities obtained from the cost and revenue functions in Chapter 5 and 6. As shown in Table 27, the *compensated* elasticity estimates of input demand obtained the profit function correspond fairly well with the net elasticity estimates obtained from the cost function. Only four out of sixteen *compensated* price-demand elasticities have different signs to corresponding net elasticities. Interestingly, the net measures of these four price-demand relationships, obtained from the cost function, are statistically insignificant. Additionally, the *compensated* own-price elasticity of CSM livestock and the *compensated* elasticity of FG with respect to Other CMS price are significantly lower than the corresponding net elasticities obtained from the cost function.

In contrast to the similarity the *compensated* price elasticities display, in terms of the direction of substitution relationships, the statistical significance of the derived *compensated* elasticities are significantly lower compared to the net price elasticities. Only five of the twenty-five *compensated* elasticities are statistically significant at the 5% level while seventeen of the net elasticities are statistically significant at the 5% level.

With regard to output supply, the comparable *compensated* price elasticities obtained from the profit function describe fairly different price responses compared to the net elasticities obtained from the revenue function. As shown in Table 28, six out of sixteen *compensated* elasticities derived from the profit function differ in sign compared to the net elasticities from the revenue function. Among these six *compensated* elasticities are the own-price elasticities of Beef and Wool, both of which are found to be negative. Two other *compensated* elasticities that have different signs compared to corresponding net elasticities are cross-price elasticities with respect to Beef price. Similar to input demand, net measures of these price-supply relationships, obtained from the revenue function, are statistically insignificant. Furthermore, the magnitudes of the *compensated* output supply elasticities are generally higher than their corresponding net elasticities.

A notable feature of the *compensated* elasticity estimates is their much higher standard errors in comparison to those for the net elasticity estimates from the cost and revenue function settings. All *compensated* elasticities obtained from the profit function are statistically insignificant and many of the standard errors of these elasticities are very high. There appears to be some correspondence between the *compensated* and net elasticities in terms of the magnitude of standard errors. Among the *compensated* elasticities that have the largest standard errors are those with respect to Beef price. Separately, the net price elasticities with respect to Beef price estimated from the revenue function are all statistically insignificant.

The relatively larger standard errors of *compensated* price elasticities derived from the profit function setting are somewhat intuitive. In the case of input demands, they are specific to output mix produced, thus conditioning them on farm specific output quantities as in the case of cost function setting is likely to achieve better statistical fit and hence lower standard errors. In contrast, whilst the exogeneity of output supply is relaxed in the profit function setting, the non farm specific input prices as explanatory variables are likely to have much less explanatory power. Another way of looking at the likely causes is the inversion of the estimated price matrix required in calculating these *compensated* price elasticities (see equations 3.14 and 3.15 in Chapter 3). Because inversion of a matrix is a nonlinear transformation, small changes in the estimated price coefficients can result in very large changes in elasticities, and hence large standard errors. The consequences of inverting a matrix on standard errors was highlighted by Binswanger (1974a) as an advantage offered by the duality approach over the primal approach since when the primal approach is used, calculating the substitution/transformation elasticities requires inverting the matrix of the estimated production function coefficients.

Table 27: Net and *Compensated* Price Elasticities of Input Demand from Cost and Profit Functions [a, b]

	With respect to price of									
	Contracts, services & materials for livestock		Fertilisers & chemicals		Other contracts, services & materials		Fuel, oil & grease		Livestock trading	
Demand for	Cost function	Profit function	Cost function	Profit function	Cost function	Profit function	Cost function	Profit function	Cost function	Profit function
Contracts, materials & services for livestock	−0.431**	−0.038	0.491**	−0.065	−0.13*	0.066	−0.051**	−0.028	0.06**	0.067
	(0.123)	(0.712)	(0.124)	(0.501)	(0.076)	(0.419)	(0.014)	(0.037)	(0.023)	(0.092)
Fertilisers and chemicals	0.391**	−0.046	−1.756**	−0.402	1.03**	0.370	0.037	0.062	−0.009	0.009
	(0.108)	(0.36)	(0.198)	(0.45)	(0.167)	(0.256)	(0.034)	(0.03)	(0.011)	(0.032)
Other contracts, services & materials	−0.068*	0.075	0.782**	0.506	−0.796**	−0.529	0.083**	0.030	0.001	−0.065
	(0.041)	(0.506)	(0.118)	(0.388)	(0.143)	(0.503)	(0.031)	(0.052)	(0.009)	(0.106)
Fuel, oil & grease	−0.157**	−0.192	0.197	0.522	0.525**	0.178	−0.531**	−0.325	−0.031**	−0.139
	(0.046)	(0.259)	(0.166)	(0.261)	(0.197)	(0.316)	(0.048)	(0.064)	(0.009)	(0.054)
Livestock trading	0.152**	0.194	−0.055	0.040	0.006	−0.139	−0.026**	−0.057	−0.078**	−0.046
	(0.061)	(0.269)	(0.071)	(0.135)	(0.04)	(0.213)	(0.008)	(0.022)	(0.017)	(0.064)

Note: [a] Medians of elasticities evaluated at all observation points
[b] Bootstrapping standard errors (500 trials) are in parentheses
** Significant at 5% level

Table 28: Net and *Compensated* Price Elasticities of Output Supply from Revenue and Profit Functions [a, b]

	With respect to price of							
	Grains		Sheep		Beef		Wool	
Supply of	Revenue function	Profit function	Revenue function	Profit function	Revenue function	Profit function	Revenue function	Profit function
Grains	0.073**	0.028	−0.016**	−0.017	−0.029	0.032	−0.023**	−0.043
	(0.019)	(0.199)	(0.005)	(0.232)	(0.018)	(0.766)	(0.009)	(0.294)
Sheep	−0.086**	−0.161	0.159**	0.177	−0.033	0.287	−0.035	−0.306
	(0.025)	(2.33)	(0.034)	(3.28)	(0.06)	(9.904)	(0.048)	(3.962)
Beef	−0.024*	0.028	−0.003	0.026	0.065	−0.272	−0.0002	0.129
	(0.013)	(0.671)	(0.007)	(0.825)	(0.045)	(4.271)	(0.026)	(1.981)
Wool	−0.047**	−0.147	−0.012	−0.107	0.004	0.454	0.058	−0.293
	(0.021)	(1.03)	(0.019)	(1.358)	(0.06)	(6.924)	(0.05)	(3.652)

Note: [a] Medians of elasticities evaluated at all observation points

[b] Bootstrapping standard errors (500 trials) are in parentheses

** Significant at 5% level

8.4.3 Allen Partial Elasticities of Substitution and Transformation Contrasted and Compared

A contrast of result sets from Chapters 5, 6 and 7 reveals that the input substitution relationships described by Allen partial elasticities differ significantly between net and gross measures obtained from the cost and profit functions. As shown in Tables 29 and 30, six out of ten Allen partial elasticity estimates for inputs change sign between the net and gross result sets from these two dual functions. In contrast, estimates of net and gross Allen partial elasticities of output transformation obtained from the revenue and profit functions are negative, a result which strongly suggests that broadacre outputs are substitutes of one another.

The *compensated* Allen partial elasticities of input substitution and output transformation are presented in Table 31 and 32, along with the net Allen partial elasticities obtained from the cost and revenue functions. As shown, the *compensated* elasticity estimates are significantly smaller in magnitudes compared to the net elasticities. A majority of these *compensated* elasticities are smaller than 0.05. Almost all *compensated* Allen partial elasticities of substitution and transformation are statistically insignificant.

Table 29: Allen Partial Elasticities of Substitution: Net Estimates from Cost Function and Gross Estimates from Profit Function [a]

Input	Contracts, services & materials for livestock		Fertilisers and chemicals		Other contracts, services & materials		Fuel, oil & grease		Livestock trading	
	Cost function	Profit function	Cost function	Profit function	Cost function	Profit function	Cost function	Profit function	Cost function	Profit function
Contracts, services & materials for livestock										
Fertilisers and chemicals	0.551**	0.086**								
Other contracts, services & materials	−0.242	0.013	1.694**	−0.005						
Fuel, oil & grease	−0.598**	0.034**	0.332	0.001	0.975**	−0.054				
Livestock trading	0.384**	0.034**	−0.163	0.050**	0.012	0.042**	−0.306**	0.094**		

Note [a] Medians of elasticities evaluated at all observation points
** Significant at 5% level

Table 30: Allen Partial Elasticities of Transformation: Net Estimates from Revenue Function and Gross Estimates from Profit Function [a]

Output	Grains		Sheep		Beef		Wool	
	Revenue function	Profit function	Revenue function	Profit function	Revenue function	Profit function	Revenue function	Profit function
Grains								
Sheep	−0.158**	−0.026**						
Beef	−0.047**	−0.003	−0.045	−0.027**				
Wool	−0.094**	−0.020**	−0.165	−0.038	−0.001	−0.027**	.	.

Note: [a] Medians of elasticities evaluated at all observation points
** Significant at 5% level

Table 31: Net and *Compensated* Allen Partial Elasticities of Input Substitution from Cost and Profit Functions [a, b]

Input	Contracts, services & materials for livestock		Fertilisers & chemicals		Other contracts, services & materials		Fuel, oil & grease		Livestock trading	
	Cost function	Profit function	Cost function	Profit function	Cost function	Profit function	Cost function	Profit function	Cost function	Profit function
Contracts, materials & services for livestock										
Fertilisers and chemicals	0.551**	−0.011								
	(0.25)	(0.122)								
Other contracts, services & materials	−0.242*	−0.012	1.694**	−0.006						
	(0.142)	(0.083)	(0.318)	(0.02)						
Fuel, oil & grease	−0.598**	0.038	0.332	0.001	0.975**	−0.056				
	(0.165)	(0.052)	(0.353)	(0.018)	(0.371)	(0.1)				
Livestock trading	0.384**	−0.069	−0.163	0.004	0.012	0.032	−0.306**	0.095		
	(0.153)	(0.094)	(0.153)	(0.03)	(0.153)	(0.051)	(0.153)	(0.037)		

Note: [a] Medians of elasticities evaluated at all observation points
[b] Bootstrapping standard errors (500 trials) are in parentheses
** Significant at 5% level

Table 32: Net and *Compensated* Allen Partial Elasticities of Output Transformation from Revenue and Profit Functions [a, b]

	Grains		Sheep		Beef		Wool	
Output	Revenue function	Profit function	Revenue function	Profit function	Revenue function	Profit function	Revenue function	Profit function
Grains								
Sheep	−0.158**	−0.039						
	(0.045)	(0.504)						
Beef	−0.047**	0.002	−0.045	0.027				
	(0.024)	(0.043)	(0.087)	(0.807)				
Wool	−0.094**	−0.039	−0.165	−0.168	−0.001	0.041		
	(0.04)	(0.271)	(0.217)	(2.022)	(0.1)	(0.635)		

Note: [a] Medians of elasticities evaluated at all observation points
[b] Bootstrapping standard errors (500 trials) are in parentheses
** Significant at 5% level

8.4.4 Morishima Elasticities of Substitution and Transformation Contrasted and Compared

The gross and net Morishima elasticities exhibit greater consistency than the Allen partial elasticities in classifying substitutive and complementary relationships when the underlying economic behavioural assumption changes. As shown in Tables 33 and 34, the gross Morishima elasticities from the profit function and the net Morishima elasticities from the cost and revenue functions are positive for input demands and negative for output supplies. Both the net and gross measures are also highly statistically significant. These results suggest that all broadacre inputs and outputs are gross and net substitutes.

Similar to the price elasticities, the *compensated* Morishima elasticities from the profit function differ significantly from the net Morishima elasticities from the cost and revenue functions. The magnitudes of the *compensated* elasticities are significantly smaller than the net elasticities. The standard errors of the *compensated* elasticities are also much higher compared to the corresponding net elasticities, especially for output transformation. The likely reasons for this are discussed in Section 8.4.2.

Table 33: Morishima Elasticities of Input Substitution: Net Estimates from Cost Function and Gross Estimates from Profit Function [a]

Input	Contracts, services & materials for livestock		Fertilisers and chemicals		Other contracts, services & materials		Fuel, oil & grease		Livestock trading	
	Cost function	Profit function	Cost function	Profit function	Cost function	Profit function	Cost function	Profit function	Cost function	Profit function
Contracts, services & materials for livestock	-	-	1.448**	0.100	0.583**	0.370**	0.459**	0.347**	0.116**	0.060
Fertilisers and chemicals	0.394**	0.176	-	-	1.247**	0.605**	0.525**	0.359**	0.061**	0.076**
Other contracts, services & materials	0.320**	0.420**	1.953**	0.713**	-	-	0.628**	0.363**	0.079**	0.028
Fuel, oil & grease	0.193	0.268**	1.676**	0.850**	1.314**	0.657	-	-	0.045**	−0.025
Livestock trading	0.471**	0.375**	1.481**	0.245**	0.798**	0.199	0.503**	0.341**	-	-

Note: [a] Medians of elasticities evaluated at all observation points
** Significant at 5% level

Table 34: Morishima Elasticities of Output Transformation: Net Estimates from Revenue Function and Gross Estimates from Profit Function [a]

Output	Grains		Sheep		Beef		Wool	
	Revenue function	Profit function	Revenue function	Profit function	Revenue function	Profit function	Revenue function	Profit function
Grains	-	-	−0.169**	−0.245**	−0.074	−0.353**	−0.073	−0.046
Sheep	−0.133**	−0.118**	-	-	−0.083	−0.689**	−0.094**	−0.110
Beef	−0.082**	0.005	−0.152**	−0.252**	-	-	−0.056	−0.113
Wool	−0.100**	−0.092**	−0.174**	−0.270**	−0.058	−0.644**	-	-

Note: [a] Medians of elasticities evaluated at all observation points
** Significant at 5% level

Table 35: Net and *Compensated* Morishima Elasticities of Input Substitution from Cost and Profit Functions [a, b]

Demand for	Contracts, services & materials for livestock		Fertilisers & chemicals		Other contracts, services & materials		Fuel, oil & grease		Livestock trading	
	Cost function	Profit function	Cost function	Profit function	Cost function	Profit function	Cost function	Profit function	Cost function	Profit function
Contracts, materials & services for livestock	-		1.193**	0.312	1.260**	0.535	0.861**	0.341	1.052**	0.115
			(0.128)	(0.318)	(0.084)	(0.783)	(0.059)	(0.075)	(0.036)	(0.149)
Fertilisers and chemicals	1.743**	−0.006	-		1.209**	0.688	0.723**	0.353	0.743**	0.040
	(0.103)	(0.488)			(0.173)	(0.585)	(0.042)	(0.062)	(0.05)	(0.064)
Other contracts, services & materials	1.514**	0.102	1.085**	0.711	-		0.951**	0.358	0.749**	0.001
	(0.065)	(1.038)	(0.142)	(0.576)			(0.069)	(0.106)	(0.032)	(0.037)
Fuel, oil & grease	1.386**	−0.176	0.685**	0.765	1.496**	0.706	-		0.723**	−0.092
	(0.074)	(0.725)	(0.093)	(0.514)	(0.167)	(0.478)			(0.063)	(0.072)
Livestock trading	1.952**	0.240	0.934**	0.354	0.618**	0.300	0.782**	0.331	-	
	(0.074)	(0.868)	(0.127)	(0.341)	(0.064)	(0.309)	(0.085)	(0.091)		

Note: [a] Medians of elasticities evaluated at all observation points
[b] Bootstrapping standard errors (500 trials) are in parentheses
** Significant at 5% level

Table 36: Net and *Compensated* Morishima Elasticities of Output Transformation from Revenue and Profit Functions [a, b]

Output	Grains		Sheep		Beef		Wool	
	Revenue function	Profit function	Revenue function	Profit function	Revenue function	Profit function	Revenue function	Profit function
Grains	-		−0.169**	−0.177	−0.074*	0.274	−0.073	0.276
			(0.035)	(3.334)	(0.039)	(4.346)	(0.051)	(3.489)
Sheep	−0.133**	−0.161	-		−0.083	0.629	−0.094**	0.020
	(0.032)	(2.303)			(0.09)	(14.376)	(0.043)	(2.369)
Beef	−0.082**	−0.002	−0.152**	−0.108	-		−0.056	0.401
	(0.017)	(0.536)	(0.036)	(4.129)			(0.069)	(5.35)
Wool	−0.100**	−0.156	−0.174**	−0.287	−0.058	0.729	-	
	(0.028)	(1.026)	(0.036)	(1.741)	(0.094)	(11.041)		0.276

Note: [a] Medians of elasticities evaluated at all observation points
[b] Bootstrapping standard errors (500 trials) are in parentheses
[**] Significant at 5% level

8.5 Conclusion

In this chapter, estimation results obtained for Australian broadacre farming under assumptions of cost minimisation, revenue maximisation and profit maximisation in Chapters 5, 6, and 7 are contrasted. Overall, all three result sets display reasonable goodness-of-fit. With respect to the theoretical regularity conditions, all three estimation results fail to satisfy the monotonicity condition. Violation of this condition is most serious for the profit function. The curvature conditions are met by the estimated cost and revenue functions but not by the profit function. The net own-price elasticities obtained from the cost and revenue functions and the gross own-price elasticities obtained from the profit function do not always conform to the Le Chatelier-Samuelson principle. The *compensated* price, Allen partial and Morishima elasticities from the profit function are significantly different to, and less statistically significant than, the corresponding net elasticities from the cost function and revenue functions.

Based on goodness-of-fit measures and the degrees to which the three models appropriately describe rational economic behaviour, cost minimisation may be more appropriate for Australian broadacre farmers than the alternative assumptions of revenue or profit maximisation over the short-run. A potential explanation for Australian broadacre farmers' focus on minimising production costs over the short run is the popular deployment of the ley rotation practice. Australian broadacre farmers may also seek to minimise production costs to cope with uncertain weather conditions, international price setting for broadacre outputs and the irreversibility of capital investments.

Regarding measures of substitutability and transformability, results from the three dual functions suggest that the Morishima elasticity measure appears to be more stable than the Allen partial elasticity measure. The Allen partial measure is found to classify input and output pairs differently under the three different assumptions regarding economic behaviour of Australian broadacre farmers. In contrast, the Morishima elasticity measure consistently indicates substitutive relationships for input pairs and output pairs across all three dual functions. Also, the Morishima elasticity measure is more statistically significant than the Allen partial measure for all three estimation result sets.

Chapter 9

Estimating Australian Broadacre Production Models Using Semi-Regional Data—An Empirical Investigation into Aggregation Issues

9.1 Introduction

The unavailability of farm-level data leads to the common use of aggregate time-series data for model estimation in duality applications. Commonly used aggregate data are state or national average time series data. Theoretically, estimation results obtained from aggregate data reflect the underlying economic behaviour of individual producers if the conditions for consistent aggregation are satisfied. However, the theoretical conditions for consistent aggregation across farms are too restrictive to be applicable in reality, especially to agricultural production (Chambers 1988; Wolfson 1993; Shumway and Davis 2001; Liu and Shumway 2004). Agricultural production consists of thousands of farms operating under such diverse physical and climatic conditions that the production technology employed is not identical across farms. Therefore, the bias introduced by data aggregation across farms is likely to be nontrivial in the agricultural sector and empirical findings that rely on geographically aggregated data may not accurately portray production behaviour at the farm level.

The effects of cross-farm data aggregation on research findings in duality applications have been scarcely investigated in empirical literature. Many benefits, such as interpretation ease and policy relevance, have been used as rationales for using aggregate data in place of farm-level data. Biases introduced by the aggregation of data across farms are implicitly assumed to be trivial. Empirical studies on the impacts of data aggregation across farms or geographical areas on research findings are few and have produced mixed results (Shumway and Davis 2001). Some of these studies, including Reed and Riggins (1981) and Shumway *et al.* (1988), use aggregate data at two different levels, such as at the state- and region-level, for model estimation in their evaluation of aggregation bias. Since farm-level data are not used for model estimation at any stage in these studies, the aggregation bias is not truly dealt with.

The aim of this chapter is to provide some empirical evidence of the effects of data aggregation across farms on the estimated technical and economic relationships between Australian broadacre inputs and outputs. This is achieved by estimating normalised quadratic cost, revenue and profit functions using the available semi-regional data, described in Section 4.3.4, that is drawn from the same survey data but is more aggregated than the quasi-micro data used in previous chapters. The three dual objective functions are all estimated, since the impact of data aggregation on estimation results may depend on what economic behaviour (cost minimisation, revenue maximisation or profit maximisation) is assumed and on how the nature and behaviour of production inputs and outputs differ between micro- and macro-level. The results from the cost, revenue and profit functions using the semi-regional data are then compared to their corresponding results using the quasi-micro data in Chapters 5, 6 and 7. It is hoped that the best among the results of the three dual functions will be least affected by data aggregation.

The organisation of this chapter is as follows. Section 9.2 provides a brief description of the AAGIS semi-regional dataset and empirical application in this chapter. Section 9.3 presents results from the normalised quadratic cost function using the semi-regional dataset and compares these results with those obtained from the quasi-micro dataset in Chapter 5. Sections 9.4 and 9.5 follow the same structure of Section 9.3 in dealing with the revenue and profit functions. The overall findings about the effects of farm-wise data aggregation will be discussed in Section 9.6. Finally, Section 9.7 offers a summary of this chapter.

9.2 Semi-Regional Data and Empirical Implementation

The data used for model estimation in this chapter is at a higher aggregate level than the quasi-micro data used in Chapters 5, 6 and 7. This AAGIS aggregate data are referred to as semi-regional data for the reasons explained in Chapter 4 (Section 4.3.4). Essentially, the semi-regional data are the quasi-micro data aggregated across three farm sizes.

Similar to the quasi-micro dataset, observational cells in the AAGIS semi-regional dataset are formed by broadacre region and broadacre industry with the average data of these cells observed. Essentially, one semi-regional data cell is made up of three quasi-micro data cells of large, medium and small farm sizes that are in the same broadacre region and industry. This cell's observed data are the averages of the component quasi-miro data cells (of three sizes), weighted by the estimated populations represented by these quasi-micro cells, instead of the actual number of farms surveyed. In doing so, the obtained average data are representative for the farm population represented by that semi-regional data cell. With 32 production regions and two broadacre industries, 795 semi-regional observations are available over the 1990–2005 period. To be consistent with the quasi-micro models, observations of farm cells that do not produce multiple broadacre outputs are excluded and the final dataset used for estimation in this chapter has 621 observations.

The empirical implementation in estimating semi-regional models is the same as in estimating quasi-micro models in previous chapters except for one or two aspects. Importantly, the variables are not weighted by the cell sample size to correct for heteroskedasticity, as required for the quasi-micro models. This is because, in the semi-regional dataset, the number of constituent farms in a data cell is generally large. Moreover, the way the data are weighted to generate semi-regional averages, as explained above, makes it less clear if heteroskedasticity, if present, is related to the sample size of individual cells.

9.3 The Restricted Multi-Product Normalised Quadratic Cost Function

With variable inputs $[x_1, x_2, ..., x_5]$, outputs $[y_1, y_2, ... y_4]$, fixed inputs z_1 and z_2, industry dummy variable z_3, zone dummy variables z_4 and z_5, rainfall variable z_6 and time trend T, the restricted normalised quadratic cost function has the following representation:

$$C'(W',Y,Z,T) = \alpha_0 + \sum_{i=1}^{4}\alpha_i w'_i + \sum_{k=1}^{4}\beta_k y_k + \sum_{g=1}^{6}\lambda_g z_g + \tfrac{1}{2}\sum_{i=1}^{4}\sum_{j=1}^{4}\alpha_{ij} w'_i w'_j + \tfrac{1}{2}\sum_{k=1}^{4}\sum_{l=1}^{4}\beta_{kl} y_k y_l$$
$$+ \tfrac{1}{2}\sum_{g=1}^{6}\sum_{h=1}^{6}\lambda_{gh} z_g z_h + \sum_{i=1}^{4}\sum_{k=1}^{4}\delta_{ik} w'_i y_k + \sum_{i=1}^{4}\sum_{g=1}^{6}\gamma_{ig} w'_i z_g + \sum_{k=1}^{4}\sum_{g=1}^{6}\varphi_{kg} y_k z_g + \sum_{i=1}^{4}\rho_{ti} w'_i T$$
$$+ \sum_{k=1}^{4}\phi_{tk} y_k T + \sum_{g=1}^{6}\psi_{tg} z_g T + \theta_t T + \tfrac{1}{2}\theta_{tt} T^2,$$

where $C'(W',Y,Z,T)$ and $[w'_1, w'_2, ... w'_4]$ are total variable cost and prices of CSM livestock, Other CSM, FOG and Livestock trading inputs normalised by the price of the aggregate FC input w_5. FC price is chosen to be the *numeraire* to be consistent with the quasi-micro model. Applying Shephard's lemma, the system of derived input demand equations is obtained as:

$$x_i = \alpha_i + \sum_{j=1}^{4}\alpha_{ij} w'_j + \sum_{k=1}^{4}\delta_{ik} y_k + \sum_{g=1}^{6}\gamma_{ig} z_g + \rho_{ti} T \text{ with } i = 1, 2, 3 \text{ and } 4.$$

The FIML estimates of the derived demand system using the semi-regional data are presented in Table 37. The results here are better than those obtained using the quasi-micro data (see Chapter 5, Table 6) in terms of statistical significance and directions of price-quantity relationships. The proportion of significant system coefficients is 68.8 per cent in this aggregate model compared to 61.1 per cent in the quasi-micro model. All own-price coefficients are negative and statistically significant at the 5% level as in the quasi-micro model. More price coefficients are statistically significant in this aggregate model than in the quasi-micro model. Thirteen out of the 64 system coefficient estimates change their signs between the two models. There are more price coefficients than non-price coefficients among these thirteen coefficients. In particular, all the coefficients of alternative inputs in the CSM livestock demand equation change signs between the semi-regional and quasi-micro models. The relationships the rainfall variable and time trend have with all input demands remain unchanged with the data aggregation.

The overall goodness-of-fit of the estimated semi-regional model is close to that of the estimated quasi-micro model. This aggregate model has a system McElroy R^2 of 0.84, close to the quasi-micro model's measure of 0.85. Similarly, its individual equation adjusted R^2 of 0.78–0.87 range is comparable to the quasi-micro model's range of 0.78–0.91.

Regarding theoretical regularity conditions, the semi-regional model's results are better than the quasi-micro model's results. At the aggregate semi-regional level, the convexity condition is satisfied by the estimated derived demand system, with all eigenvalues of the price matrix being negative. The violation of the monotonicity condition is also less frequent at the semi-regional level than at the quasi-micro level. In the semi-regional model, the percentage of negative predicted quantities are 1.4 per cent for CSM livestock input, 0.6 per cent for Other CSM input, 2.7 per cent for FOG input and 5.6 per cent for Livestock trading input. The corresponding percentages in the quasi-micro model are 13.0 per cent, 5.7 per cent, 6.1 per cent and 11.1 per cent, respectively. The better result of the semi-regional model is expected because there are considerably fewer observations that have very small input quantities in the semi-regional data than in the quasi-micro data.

The estimates of net price elasticities, Allen partial elasticities of substitution and Morishima elasticities of substitution obtained from the normalised quadratic cost function using semi-regional data are shown in Tables 38, 39 and 40, respectively. These elasticity estimates, when compared to those obtained in the quasi-micro model

(Tables 8, 10 and 12), reveal some notable results. All own-price elasticity estimates here are negative, as in the quasi-micro model, and highly statistically significant. Out of twenty cross-price elasticities, ten are statistically significant at the semi-regional level, compared to twelve at the quasi-micro level. Half of the cross-price elasticities obtained change signs between the two levels of aggregation, some of which are statistically significant in one or both result sets. For example, the elasticity of FOG with respect to CSM livestock price is significantly negative in the micro-quasi model but significantly positive in the semi-regional model. Moreover, it appears that the larger the magnitude of a price elasticity estimate at the quasi-micro level is, the more stable it is when data are further aggregated. For instance, an own-price elasticity of −1.785 for FC input in the semi-regional model is almost identical to an estimate of −1.756 obtained in the quasi-micro model. It is also found that the demand of FC input is elastic with respect to the price of Other CSM input in both semi-regional and quasi-micro models.

Concerning input substitutability, the stability of elasticity estimates with respect to data aggregation depends on the measure employed. The Allen partial elasticity estimates obtained from the semi-regional model differ significantly to the corresponding estimates from the quasi-micro model. Half of the Allen partial elasticity estimates change sign between the semi-regional and quasi-micro models. For example, according to this measure, the FOG-CSM livestock relationship is significant and substitutive in the semi-regional model but significant and complementary in the quasi-micro model. In contrast, none of the Morishima elasticity estimates changes sign between the quasi-micro and semi-regional models. The Morishima elasticity estimates obtained at the semi-regional level here suggest that all broadacre inputs are substitutes of one another.

Table 37: Parameter Estimates of Demand System Derived from Cost Function Using Semi-Regional Data

	Input quantity equation							
	Contracts, services & materials for livestock		Other contracts, services & materials		Fuel, oil & grease		Livestock trading	
	Coefficient	z-Statistic	Coefficient	z-Statistic	Coefficient	z-Statistic	Coefficient	z-Statistic
Constant	366.12**	3.81	851.99**	3.92	21.37	0.45	−80.32**	−2.57
Contracts, materials & services for livestock	−195.71**	−5.56	95.51**	2.27	39.24**	3.28	−3.67	−1.04
Other contracts, services & materials	95.51**	2.27	−1351.3**	−4.89	150.34**	2.44	9.82*	1.68
Fuel, oil & grease	39.24**	3.28	150.34**	2.44	−166.87**	−9.12	−4.42**	−2.49
Livestock trading	−3.67	−1.04	9.82*	1.68	−4.42**	−2.49	−3.24**	−2.21
Crops	−0.011	−0.96	0.186**	21.03	0.031**	17.82	0.002	0.53
Sheep	0.361**	9.69	−0.069	−0.68	0.033	1.55	0.055**	3.14
Beef	0.28**	26.05	0.265**	14.08	0.028**	7.62	0.128**	52.04
Wool	0.112**	4.62	0.476**	9.43	0.004	0.38	−0.009	−1.08
Capital	5.635**	3.52	22.58**	12.09	2.668**	5.88	−0.07	−0.13
Fixed labour	0.171	1.21	0.725**	3.74	0.207**	4.53	0.135**	2.79
Dummy variable D	−86.23**	−2.47	209.57**	4.13	81.32**	7.91	−12.91	−1.03
Dummy variable Z1	48.06	1.24	163.62**	2.60	−22.3**	−2.13	−37.91**	−3.31
Dummy variable Z2	−22.17	−0.50	−71.98	−1.08	−70.54**	−5.74	−18.78	−1.32
Relative rainfall	−148.56**	−3.52	−269.54**	−3.23	11.67	0.83	28.91**	2.15
Time	−17.61**	−3.86	−13.56	−1.50	−7.77**	−4.60	4.44**	3.52
Adjusted R^2	0.80		0.85		0.78		0.87	

Note: D = 1 when the farm is in the Cropping industry, Z1 = 1 when the farm is in the Wheat-Sheep zone and Z2 = 1 when the farm is in the High Rainfall zone

** Significant at the 5% level

* Significant at the 10% level

Table 38: Own- and Cross-Price Elasticities of Input Demand—The Semi-Regional Cost Function Model [a, b]

	With respect to price of				
Demand of	Contracts, services & materials for livestock	Fertilisers and chemicals	Other contracts, services & materials	Fuel, oil & grease	Livestock trading
Contracts, services & materials for livestock	−0.324**	0.144*	0.144**	0.057**	−0.012
	(−0.055)	(−0.083)	(−0.07)	(−0.014)	(−0.011)
Fertilisers and chemicals	0.121	−1.785**	1.486**	−0.03	0.006
	(−0.084)	(−0.272)	(−0.274)	(−0.047)	(−0.015)
Other contracts, services & materials	0.067**	0.697**	−0.859**	0.093**	0.015*
	(−0.032)	(−0.12)	(−0.144)	(−0.036)	(−0.009)
Fuel, oil & grease	0.167**	−0.114	0.552**	−0.603**	−0.034**
	(−0.042)	(−0.155)	(−0.213)	(−0.059)	(−0.01)
Livestock trading	−0.036	0.026	0.086*	−0.035**	−0.044**
	(−0.033)	(−0.063)	(−0.051)	(−0.01)	(−0.014)

Note: [a] Medians of elasticities evaluated at all observation points
[b] Bootstrapping standard errors (500 trials) are in parentheses
** Significant at the 5% level
* Significant at the 10% level

Table 39: Allen Partial Elasticities of Substitution between Inputs—The Semi-Regional Cost Function Model [a, b]

	Contracts, services & materials for livestock	Fertilisers and chemicals	Other contracts, services & materials	Fuel, oil & grease	Livestock trading
Contracts, services & materials for livestock	.				
Fertilisers and chemicals	0.62				
	(−0.426)				
Other contracts, services & materials	0.301**	3.364**			
	(−0.146)	(−0.627)			
Fuel, oil & grease	0.709**	−0.457	1.151**		
	(−0.179)	(−0.576)	(−0.446)		
Livestock trading	−0.134	0.069	0.197	−0.48**	.
	(−0.121)	(−0.121)	(−0.121)	(−0.121)	

Note: [a] Medians of elasticities evaluated at all observation points
[b] Bootstrapping standard errors (500 trials) are in parentheses
** Significant at the 5% level
* Significant at the 10% level

Table 40: Morishima Elasticities of Substitution between Inputs—The Semi-Regional Cost Function Model [a, b]

	Contracts, services & materials for livestock	Fertilisers and chemicals	Other contracts, services & materials	Fuel, oil & grease	Livestock trading
Contracts, services & materials for livestock	-	1.978**	1.047**	0.66**	0.031
		(−0.34)	(−0.186)	(−0.062)	(−0.019)
Fertilisers and chemicals	0.475**	-	2.469**	0.566**	0.045**
	(−0.125)		(−0.448)	(−0.043)	(−0.017)
Other contracts, services & materials	0.399**	2.617**	-	0.692**	0.057**
	(−0.069)	(−0.405)		(−0.091)	(−0.018)
Fuel, oil & grease	0.518**	1.679**	1.457**	-	0.011
	(−0.086)	(−0.287)	(−0.342)		(−0.017)
Livestock trading	0.292**	1.83**	1.028**	0.574**	-
	(−0.071)	(−0.292)	(−0.156)	(−0.065)	

Note: [a] Medians of elasticities evaluated at all observation points
[b] Bootstrapping standard errors (500 trials) are in parentheses
** Significant at the 5% level
* Significant at the 10% level

9.4 The Restricted Multi-Product Normalised Quadratic Revenue Function

With the same set of model variables as in Section 9.3, the restricted normalised quadratic revenue function has the following representation:

$$R'(X,P',Z,T)=\alpha_0+\sum_{i=1}^{5}\alpha_i x_i+\sum_{k=1}^{3}\beta_k p'_k+\sum_{g=1}^{6}\lambda_g z_g+\tfrac{1}{2}\sum_{i=1}^{5}\sum_{j=1}^{5}\alpha_{ij}x_i x_j+\tfrac{1}{2}\sum_{k=1}^{3}\sum_{l=1}^{3}\beta_{kl}p'_k\, p'_l$$
$$+\tfrac{1}{2}\sum_{g=1}^{6}\sum_{h=1}^{6}\lambda_{gh}z_g z_h+\sum_{i=1}^{5}\sum_{k=1}^{3}\delta_{ik}x_i p'_k+\sum_{i=1}^{5}\sum_{g=1}^{6}\gamma_{ig}x_i z_g+\sum_{k=1}^{3}\sum_{g=1}^{6}\varphi_{kg}p'_k\, z_g+\sum_{i=1}^{5}\rho_{ti}Tx_i+\sum_{k=1}^{3}\phi_{tk}Tp'_k$$
$$+\sum_{g=1}^{6}\psi_{tg}Tz_g+\theta_t T+\tfrac{1}{2}\theta_{tt}T^2.$$

where $R'(X,P',Z,T)$, p'_1, p'_2 and p'_3 are the variable production revenue, Grains price, Sheep price and Wool price normalised by Beef price. Beef price is chosen as the *numeraire* to be consistent with the quasi-micro model in Chapter 6. Applying the Samuelson-McFadden lemma to this revenue function, the system of supply equations is derived as follows:

$$y_k=\beta_k+\sum_{l=1}^{3}\beta_{kl}p'_l+\sum_{i=1}^{5}\delta_{ik}x_i+\sum_{g=1}^{6}\varphi_{kg}z_g+\phi_{tk}T\text{ , with } k=1,2\text{ and }3.$$

Coefficient estimates of the derived supply system using semi-regional data are presented in Table 41. The results here are less positive than that of the quasi-micro model in Chapter 6 (Table 14). The proportion of significant system coefficients at the 5% level is 62.5 per cent here compared to 77.8 per cent in the quasi-micro model. Further, the semi-regional model has only one significant own-price coefficient, compared to two in the quasi-micro model. Compared to the cost function, a smaller proportion of system coefficient estimates change sign between the quasi-micro and semi-regional models derived from the revenue function. Only eight of the 48 system coefficient estimates change sign, and most are statistically insignificant. It is notable that the coefficient of time trend is positive in all supply equations in the semi-regional model while being negative in the Sheep and Wool equations in the quasi-micro model.

The overall fit of the estimated output supply system using semi-regional data is comparable to the overall fit obtained using the quasi-micro data. The adjusted R^2 s of the individual supply equations in this aggregate model are fairly close to those in the quasi-micro model. However, the system McElroy R^2 is higher for this aggregate model than for the quasi-micro model, being 0.71 compared to 0.65.

The revenue function's results concerning the regularity conditions are improved when the data used for estimation is aggregated from the quasi-micro level to the semi-regional level. The convexity condition is satisfied in the semi-regional model, as in the quasi-micro model. Similar to the cost function case, the violation of the monotonicity condition is considerably less severe in the semi-regional model than in the quasi-micro model. The percentage of negative predicted quantities for Grains, Sheep and Wool outputs is respectively 18.5 per cent, 0.3 per cent and 18.1 per cent in this model, compared to 22.6 per cent, 8.5 per cent and 21.5 per cent in the quasi-micro model.

The estimates of price elasticities and elasticities of transformation for four broadacre outputs generated from the estimated semi-regional model are presented in Tables 38, 39 and 40. These elasticity estimates obtained, when compared to their corresponding estimates obtained in the quasi-micro model of the revenue function (Tables 15, 16 and 17—Chapter 6), appear to be more stable under data aggregation than those obtained from the cost function. All own-price supply elasticities are positive as expected in the semi-regional model. The direction of the majority of the cross-price relationships also remain unchanged between the semi-regional and quasi-micro models. Moreover, the direction of all pair-wise transformation relationships do not change when data are aggregated using both Allen partial or Morishima measures. However, fewer price and transformation elasticities are statistically significant in the semi-regional model than in the quasi-micro model.

Table 41: Parameter Estimates of Supply System Derived from Normalised Quadratic Revenue Function Using Semi-Regional Data

	Output quantity equation					
	Grains		Sheep		Wool	
	Coefficient	z-Statistic	Coefficient	z-Statistic	Coefficient	z-Statistic
Constant	−834.8*	−1.93	288.26**	2.43	751.35**	4.02
Grains price	99.05	1.35	−49.79**	−2.16	−50.28	−1.61
Sheep price	−49.79**	−2.16	186.03**	5.37	3.63	0.09
Wool price	−50.28	−1.61	3.63	0.09	44.51	0.71
CSM livestock quantity	−0.5**	−2.39	0.42**	13.46	0.43**	7.81
FC quantity	3.712**	25.02	0.378**	6.94	0.338**	3.99
Other CSM quantity	0.716**	6.38	0.047	1.20	0.436**	7.77
FOG quantity	0.342	0.76	−0.13	−0.81	−1.225**	−4.79
Livestock trading quantity	−0.981	−1.62	−0.633**	−6.30	−1.163**	−7.63
Capital quantity	−1.894	−0.32	−2.271	−1.08	−5.065	−1.58
Fixed labour quantity	−1.002*	−1.66	−0.32**	−2.07	−0.378	−1.51
Dummy variable D	368.61**	2.53	−73.06**	−2.33	−180.19**	−3.22
Dummy variable Z1	−242.38*	−1.92	−214.78**	−5.42	−865.29**	−15.13
Dummy variable Z2	−480.71**	−2.87	−163.31**	−3.62	−703.93**	−10.52
Relative rainfall	680.75**	3.21	137.47**	2.74	349.31**	4.54
Time	54.18**	3.55	2.01	0.48	13.32**	2.06
Adjusted R^2	0.83		0.42		0.59	

Note: D = 1 when the farm is in the Cropping industry, Z1 = 1 when the farm is in the Wheat-Sheep zone and Z2 = 1 when the farm is in the High Rainfall zone

** Significant at the 5% level

* Significant at ten per cent level

Table 42: Own- and Cross-Price Elasticities of Output Supply—The Semi-Regional Revenue Function Model [a, b]

	Grains	Sheep	Beef	Wool
Grains	0.044*	−0.016**	−0.001	−0.024*
	(−0.026)	(−0.005)	(−0.026)	(−0.013)
Sheep	−0.093**	0.276**	−0.164*	0.009
	(−0.03)	(−0.053)	(−0.099)	(−0.071)
Beef	0	−0.025*	0.055	−0.004
	(−0.026)	(−0.015)	(−0.067)	(−0.042)
Wool	−0.064*	0.004	−0.005	0.08
	(−0.036)	(−0.033)	(−0.106)	(−0.088)

Note: [a] Medians of elasticities evaluated at all observation points
[b] Bootstrapping standard errors (500 trials) are in parentheses
** Significant at the 5% level
* Significant at the 10% level

Table 43: Allen Partial Elasticities of Transformation of Output Supply—The Semi-Regional Revenue Function Model [a, b]

	Grains	Sheep	Beef	Wool
Grains				
Sheep	−0.209**			
	(−0.065)			
Beef	−0.006	−0.365*		
	(−0.062)	(−0.212)		
Wool	−0.165*	0.038	−0.019	
	(−0.085)	(−0.309)	(−0.216)	

Note: [a] Medians of elasticities evaluated at all observation points
[b] Bootstrapping standard errors (500 trials) are in parentheses
** Significant at the 5% level
* Significant at ten per cent level

Table 44: Morishima Elasticities of Transformation of Output Supply—The Semi-Regional Revenue Function Analysis [a, b]

	Grains	Sheep	Beef	Wool
Grains	-	−0.3**	−0.051	−0.116
		(−0.053)	(−0.077)	(−0.093)
Sheep	−0.153**	-	−0.203	−0.07
	(−0.047)		(−0.164)	(−0.075)
Beef	−0.047	−0.297**	-	−0.089
	(−0.044)	(−0.065)		(−0.141)
Wool	−0.133**	−0.271**	−0.065	-
	(−0.055)	(−0.06)	(−0.179)	

Note: [a] Medians of elasticities evaluated at all observation points
[b] Bootstrapping standard errors (500 trials) are in parentheses
** Significant at the 5% level
* Significant at the 10% level

9.5 The Restricted Multi-Product Normalised Quadratic Profit Function

Following the estimation of the normalised quadratic profit function using quasi-micro data in Chapter 7, the price of FC input is chosen as the *numeraire* to specify the normalised quadratic profit function in this chapter. Using the same set of variables as in sections 9.3 and 9.4, this normalised quadratic profit function has the following representation:

$$\pi'(W',P',Z,T) = \alpha_0 + \sum_{i=1}^{4}\alpha_i w'_i + \sum_{k=1}^{4}\beta_k p'_k + \sum_{g=1}^{6}\lambda_g z_g + \tfrac{1}{2}\sum_{i=1}^{4}\sum_{j=1}^{4}\alpha_{ij} w'_i\, w'_j +$$
$$\tfrac{1}{2}\sum_{k=1}^{4}\sum_{l=1}^{4}\beta_{kl} p'_k\, p'_l + \tfrac{1}{2}\sum_{g=1}^{6}\sum_{h=1}^{6}\lambda_{gh} z_g z_h + \sum_{i=1}^{4}\sum_{k=1}^{4}\delta_{ik} w'_i\, p'_k + \sum_{i=1}^{4}\sum_{g=1}^{6}\gamma_{ig} w'_i\, z_g \quad .$$
$$+\sum_{k=1}^{4}\sum_{g=1}^{6}\varphi_{kg} p'_k\, z_g + \sum_{i=1}^{4}\rho_{ti} w'_i\, T + \sum_{k=1}^{4}\phi_{tk} p'_k\, T + \sum_{g=1}^{6}\psi_{tg} z_g T + \theta_t T + \tfrac{1}{2}\theta_{tt} T^2$$

where $\pi'(W',P',Z,T)$, w' and p' are profit, input prices and output prices normalised by FC price.

Applying Hotelling's lemma, the system of derived demand and supply equations is:

$$-x_i = \alpha_i + \sum_{j=1}^{4}\alpha_{ij} w'_j + \sum_{k=1}^{4}\delta_{ik} p'_k + \sum_{g=1}^{6}\gamma_{ig} z_g + \rho_{ti} T \quad \text{with } i = 1,2,3 \text{ and } 4 \text{ and}$$

$$y_k = \beta_k + \sum_{l=1}^{4}\beta_{kl} p'_l + \sum_{i=1}^{4}\delta_{ik} w'_i + \sum_{g=1}^{6}\varphi_{kg} z_g + \phi_{tk} T \quad \text{with } k = 1,2,3 \text{ and } 4 .$$

The FIML estimates of this demand and supply system using the semi-regional data are shown in Table 45. The result obtained here is considerably less statistically significant and differ significantly to that from the quasi-micro data (Chapter 7, Table 18). Only 46.9 per cent of the system coefficients in this aggregate model are significant at the 5% level, compared to 63.2 per cent in the quasi-micro model. The percentage of significant price coefficients is also much lower for the semi-regional model than for the quasi-micro model, being 32.8 per cent compared to 43.8 per cent. In addition, the own-price coefficient of the Grains supply equation is negative in this aggregate model while being positive in the quasi-micro model. The own-price coefficient of the Beef supply equation also becomes insignificant in this model. Moreover, a large proportion of system coefficients, including many of Sheep, Wool and CSM livestock prices, change sign between the two models.

The weaker goodness-of-fit of the semi-regional model compared to the quasi-micro model under the profit maximisation assumption is also shown in the adjusted R^2 obtained for individual system equations. The adjusted R^2 is significantly lower in this model than in the quasi-micro model for almost all equations, especially for Sheep supply and Other CSM demand equations. In particular, the Sheep supply equation has an adjusted R^2 of 0.19 in this aggregate model, compared to 0.46 in the quasi-micro model.

With respect to the regularity conditions, the outcome is mixed when the results of the semi-regional and quasi-micro models are compared. The monotonicity condition is violated less frequently in the semi-regional model than in the quasi-micro model, which is expected. Notably, the proportions of negative predicted quantities are 20.7 per cent, 12.0 per cent, 1.7 per cent and 11.5 per cent for Grains, Beef, CSM livestock and Livestock trading, respectively. These figures are considerably lower than those obtained for the quasi-micro model, which are 30.2 per cent, 27.8 per cent, 23.5 per cent and 24.9 per cent. The convexity condition, however, is more seriously violated in the semi-regional model than in the quasi-micro model. The price matrix of the estimated supply and demand system has three negative eigenvalues when using semi-regional data, compared to two negative eigenvalues when using quasi-micro data.

A comparison of the elasticity estimates obtained from the semi-regional and quasi-micro models reveals that geographical aggregation of data has a notable impact on the estimated economic and technical relationships between inputs and outputs under the profit maximisation assumption. Fewer of the price elasticities and elasticities of substitution and transformation are statistically significant in this aggregate model than in the quasi-micro model. A considerable number of these elasticities also change sign between the two models. Importantly, as shown in Table 46, the own-price elasticity of Grains supply is negative in the semi-regional model but positive in the quasi-micro model (Chapter 7, Table 19). The own-price elasticities of Wool supply and FC demand also become insignificant in the semi-regional model. In addition, the magnitude of the own-price elasticity estimates of Beef supply, CSM livestock demand and FC demand are significantly lower in the semi-regional model than in the quasi-micro model. These elasticity estimates are 0.218, −0.365 and −0.2 respectively in the semi-regional model, compared to their corresponding estimates of 0.35, −0.509 and −0.512 in the quasi-micro model. Moreover, 27 out of 72 cross-price elasticities change sign between the two models. When transformation and substitution elasticity estimates obtained in semi-regional model (Table 47 and Table 48) are compared to those obtained in quasi-micro model (Chapter 7, Table 20 and Table 21), 14 out of 36 Allen partial elasticities and 11 out of 72 Morishima elasticities change sign between the two models.

Table 45: Estimated Parameters of Supply and Demand System Derived from Profit Function Using Semi-Regional Data

	Output supply quantity equation							
	Grains		Sheep		Beef		Wool	
	Coefficient	z-Statistic	Coefficient	z-Statistic	Coefficient	z-Statistic	Coefficient	z-Statistic
Constant	−3462.7**	−3.289	680.44**	4.323	18.92	0.039	1210.09**	4.914
Grains price	−200.94	−1.018	−87.64**	−2.648	222.03**	2.007	−130.9**	−2.595
Sheep price	−87.64**	−2.648	214.94**	4.243	−107.32	−1.442	−71.56	−1.187
Beef price	222.03**	2.007	−107.32	−1.442	192.39	0.669	−191.03**	−2.052
Wool price	−130.9**	−2.595	−71.56	−1.187	−191.03**	−2.052	40.9	0.418
CSM livestock price	−22.2	−0.501	−121.62**	−2.893	−40.8	−0.424	−46.79	−0.833
Other CSM price	40.9	0.823	−49.11	−0.668	−62.86	−0.474	108.54	1.210
FOG price	9.81	0.950	9.3	0.533	−1.72	−0.070	44.38**	2.204
Livestock trading price	−22.98	−1.513	11.56	0.636	−58.37	−1.390	12.21	0.569
Capital	63.65**	5.137	8.513**	3.752	45.46**	6.070	15.95**	3.629
Fixed labour	0.87	0.654	−0.1	−0.465	1.76**	2.618	−0.06	−0.172
Cropping industry	2274.09**	9.231	−29.24	−0.804	−1043.8**	−7.019	−71.71	−1.001
Wheat Sheep Zone	835.84**	2.252	−160.4**	−3.116	−891.15**	−5.818	−790.18**	−8.549
High Rainfall Zone	−278.7	−0.631	−216.4**	−4.037	−925.42**	−4.922	−824.61**	−8.707
Rainfall	1093.46**	2.497	43.8	0.604	58.87	0.306	128.86	1.146
Time	90.52**	2.797	0.5	0.072	42.63**	2.463	−6.06	−0.612
Adjusted R^2	0.51		0.19		0.51		0.41	

Note: D = 1 when the farm is in the Cropping industry, Z1 = 1 when the farm is in the Wheat-Sheep zone and Z2 = 1 when the farm is in the High Rainfall zone

** Significant at the 5% level

* Significant at the 10% level

Table 45 (continued): Estimated Parameters of Supply and Demand System Derived from Profit Function Using Semi-Regional Data

	Input demand quantity equation							
	Contracts, services & materials for livestock		Other contracts, services & materials		Fuel, oil & grease		Livestock trading	
	Coefficient	z-Statistic	Coefficient	z-Statistic	Coefficient	z-Statistic	Coefficient	z-Statistic
Constant	−743.62**	−3.817	−500.91	−1.456	−58.98	−0.843	76	1.045
Grains price	−22.2	−0.501	40.9	0.823	9.81	0.950	−22.98	−1.513
Sheep price	−121.62**	−2.893	−49.11	−0.668	9.3	0.533	11.56	0.636
Beef price	−40.8	−0.424	−62.86	−0.474	−1.72	−0.070	−58.37	−1.390
Wool price	−46.79	−0.833	108.54	1.210	44.38**	2.204	12.21	0.569
CSM livestock price	226.94**	4.128	−108.27	−1.434	−38.11**	−2.616	62.18**	3.733
Other CSM price	−108.27	−1.434	846.47**	2.608	−3.42	−0.040	−13.93	−0.467
FOG price	−38.11**	−2.616	−3.42	−0.040	105.4**	3.978	11.21	1.421
Livestock trading price	62.18**	3.733	−13.93	−0.467	11.21	1.421	18.87**	2.334
Capital	−20.81**	−6.748	−51.63**	−14.162	−6.91**	−10.423	−6.91**	−6.514
Fixed labour	−0.65**	−2.484	−1.31**	−3.780	−0.28**	−4.263	−0.33**	−3.624
Cropping industry	397.22**	7.671	−313.86**	−4.335	−117.71**	−10.391	147.47**	6.729
Wheat Sheep Zone	383.32**	5.963	287.94**	2.854	20.39	1.367	160.73**	7.025
High Rainfall Zone	475.42**	7.078	721.44**	6.427	104.53**	5.967	152.94**	5.234
Rainfall	145.43*	1.811	−3.58	−0.030	−43.54**	−2.140	−48.7*	−1.783
Time	15.45*	1.832	−0.29	−0.021	4.47*	1.813	−12.84**	−4.015
Adjusted R^2	0.51		0.64		0.70		0.56	

Note: D = 1 when the farm is in the Cropping industry, Z1 = 1 when the farm is in the Wheat-Sheep zone and Z2 = 1 when the farm is in the High Rainfall zone

** Significant at the 5% level

* Significant at the 10% level

Table 46: Own- and Cross-Price Elasticities of Demands and Supplies—The Semi-Regional Profit Function Model [a, b]

	Grains	Sheep	Beef	Wool	Contracts, services & materials for livestock	Fertilisers and chemicals	Other contracts, services & materials	Fuel, oil & grease	Livestock trading
Grains	−0.065*	−0.02**	0.088**	−0.048**	−0.008	0.052	0.013	0.003	−0.014**
	(−0.037)	(−0.005)	(−0.028)	(−0.014)	(−0.01)	(−0.033)	(−0.012)	(−0.002)	(−0.007)
Sheep	−0.15**	0.272**	−0.254**	−0.151*	−0.26**	0.588**	−0.094	0.017	0.036
	(−0.038)	(−0.051)	(−0.119)	(−0.078)	(−0.06)	(−0.128)	(−0.101)	(−0.024)	(−0.035)
Beef	0.177**	−0.063**	0.218	−0.198**	−0.045	0.053	−0.06	−0.002	−0.091**
	(−0.056)	(−0.029)	(−0.185)	(−0.075)	(−0.069)	(−0.123)	(−0.082)	(−0.015)	(−0.037)
Wool	−0.172**	−0.064*	−0.34**	0.068	−0.072	0.312**	0.156*	0.057**	0.031
	(−0.049)	(−0.033)	(−0.132)	(−0.105)	(−0.06)	(−0.125)	(−0.089)	(−0.018)	(−0.036)
Contracts, services & materials for livestock	0.031	0.119**	0.074	0.082	−0.365**	0.046	0.161**	0.052**	−0.174**
	(−0.038)	(−0.028)	(−0.115)	(−0.068)	(−0.065)	(−0.096)	(−0.075)	(−0.014)	(−0.03)
Fertilisers and chemicals	−0.064	−0.124**	−0.033	−0.163**	0.014	−0.2	0.532**	0.092**	0.031
	(−0.049)	(−0.031)	(−0.102)	(−0.065)	(−0.041)	(−0.152)	(−0.142)	(−0.036)	(−0.031)
Other contracts, services & materials	−0.024	0.02	0.05	−0.082*	0.079**	0.494**	−0.551**	0.002	0.019
	(−0.022)	(−0.021)	(−0.069)	(−0.047)	(−0.037)	(−0.124)	(−0.151)	(−0.037)	(−0.03)
Fuel, oil & grease	−0.033	−0.02	0.008	−0.185**	0.157**	0.508**	0.012	−0.355**	−0.08**
	(−0.022)	(−0.029)	(−0.075)	(−0.056)	(−0.042)	(−0.194)	(−0.224)	(−0.065)	(−0.037)
Livestock trading	0.109**	−0.043	0.394**	−0.073	−0.417**	0.177	0.082	−0.061**	−0.167**
	(−0.052)	(−0.041)	(−0.159)	(−0.085)	(−0.07)	(−0.156)	(−0.133)	(−0.029)	(−0.046)

Note: [a] Medians of elasticities evaluated at all observation points
[b] Bootstrapping standard errors (500 trials) are in parentheses
** Significant at the 5% level
* Significant at the 10% level

Table 47: Allen Partial Elasticities of Substitution and Transformation—The Semi-Regional Profit Function Model [a, b]

	Grains	Sheep	Beef	Wool	Contracts, services & materials for livestock	Fertilisers and chemicals	Other contracts, services & materials	Fuel, oil & grease	Livestock trading
Grains									
Sheep	−0.039**								
	(−0.01)								
Beef	0.039**	−0.063**							
	(−0.014)	(−0.03)							
Wool	−0.039**	−0.079*	−0.069**						
	(−0.011)	(−0.042)	(−0.027)						
Contracts, services & materials for livestock	0.009	0.153**	0.014	0.034					
	(−0.011)	(−0.037)	(−0.023)	(−0.029)					
Fertilisers and chemicals	−0.006	0.031	0.005	0.012	0.006				
	(−0.005)	(−0.033)	(−0.011)	(−0.019)	(−0.005)				
Other contracts, services & materials	−0.006	0.032	0.015	−0.045*	−0.054**	0.026			
	(−0.005)	(−0.035)	(−0.022)	(−0.026)	(−0.025)	(−0.034)			
Fuel, oil & grease	−0.008	−0.037	0.003	−0.11**	−0.116**	0.034	−0.005		
	(−0.005)	(−0.052)	(−0.027)	(−0.033)	(−0.031)	(−0.034)	(−0.085)		
Livestock trading	0.025**	−0.048	0.102**	−0.03	0.164**	0.022	−0.024	0.119**	
	(−0.012)	(−0.045)	(−0.042)	(−0.034)	(−0.029)	(−0.018)	(−0.038)	(−0.055)	

Note: [a] Medians of elasticities evaluated at all observation points
[b] Bootstrapping standard errors (500 trials) are in parentheses
** Significant at the 5% level
* Significant at the 10% level

Table 48: Morishima Elasticities of Substitution and Transformation—The Semi-Regional Profit Function Model [a, b]

	Grains	Sheep	Beef	Wool	Contracts, services & materials for livestock	Fertilisers and chemicals	Other contracts, services & materials	Fuel, oil & grease	Livestock trading
Grains	-	−0.287**	−0.154	−0.12	0.374**	0.265*	0.57**	0.359**	0.161**
		(−0.05)	(−0.216)	(−0.103)	(−0.065)	(−0.15)	(−0.148)	(−0.065)	(−0.047)
Sheep	−0.093**	-	−0.493**	−0.221**	0.105	0.848**	0.454**	0.374**	0.2**
	(−0.045)		(−0.232)	(−0.091)	(−0.068)	(−0.192)	(−0.177)	(−0.067)	(−0.05)
Beef	0.263**	−0.327**	-	−0.264**	0.3**	0.318*	0.528**	0.35**	0.055*
	(−0.085)	(−0.06)		(−0.125)	(−0.073)	(−0.163)	(−0.168)	(−0.066)	(−0.033)
Wool	−0.119**	−0.337**	−0.56**	-	0.295**	0.593**	0.719**	0.413**	0.195**
	(−0.058)	(−0.052)	(−0.236)		(−0.081)	(−0.19)	(−0.167)	(−0.07)	(−0.043)
Contracts, services & materials for livestock	0.108	−0.125**	−0.141	0.014	-	0.27	0.763**	0.412**	0.013
	(−0.066)	(−0.048)	(−0.141)	(−0.106)		(−0.197)	(−0.189)	(−0.069)	(−0.041)
Fertilisers and chemicals	−0.004	−0.419**	−0.277	−0.235**	0.418**	-	1.094**	0.42**	0.199**
	(−0.024)	(−0.053)	(−0.17)	(−0.1)	(−0.084)		(−0.278)	(−0.051)	(−0.041)
Other contracts, services & materials	0.04	−0.242**	−0.141	−0.156	0.465**	0.732**	-	0.357**	0.187**
	(−0.032)	(−0.058)	(−0.155)	(−0.097)	(−0.072)	(−0.25)		(−0.095)	(−0.052)
Fuel, oil & grease	0.035	−0.293**	−0.204	−0.253**	0.56**	0.778**	0.563	-	0.077
	(−0.033)	(−0.059)	(−0.179)	(−0.114)	(−0.087)	(−0.221)	(−0.349)		(−0.066)
Livestock trading	0.198**	−0.304**	0.115	−0.139	−0.001	0.486**	0.702**	0.349**	-
	(−0.079)	(−0.057)	(−0.096)	(−0.108)	(−0.062)	(−0.202)	(−0.206)	(−0.087)	

Note: [a] Medians of elasticities evaluated at all observation points
[b] Bootstrapping standard errors (500 trials) are in parentheses
** Significant at the 5% level
* Significant at the 10% level

9.6 Discussion and Summary

Data aggregation is often expected to have negative impacts on estimation results, since it reduces the sample size. Their potential impacts in duality applications include reduced statistical significance as well as violation of regularity conditions (Squires 1987; Kohli 1993; Shumway 1995; Tombazos 1998). Estimation results obtained in this chapter using the AAGIS semi-regional data, and in previous chapters using the AAGIS quasi-micro models, suggest that the impacts of data aggregation on modelling outcomes may depend on the assumption made about the economic behaviour of producers.

Among the semi-regional models, estimation results from the cost function display higher statistical significance and are more consistent with economic theory than those from the revenue or profit functions. The estimated system of input demand equations derived from the cost function has the highest percentage of significant system coefficients. All own-price coefficients of this estimated system are statistically significant and negative as expected. This estimated demand system also satisfies the theoretical convexity condition without parametric restrictions on system coefficients. Moreover, results from the cost function have a higher percentage of significant price and substitution elasticities than estimation results from the revenue and profit functions. This is in line with the estimation outcomes using quasi-micro data as discussed in Chapter 8.

Data aggregation appears to have no unfavourable impacts on estimation results obtained from the cost function but to have discernibly negative impacts on the revenue and profit function results. As presented in Section 9.3, for the cost function, the semi-regional model has higher fractions of significant price coefficients and significant system coefficients than the quasi-micro model. In contrast, the statistical significance of the estimated systems derived from the revenue and profit functions diminishes when data are aggregated from the quasi-micro level to the semi-regional level. Given the supply system derived from the revenue function has fewer parameters than the demand system derived from the cost function, and that the cost function results are not negatively affected by this data aggregation from the quasi-micro level to the semi-regional level, the deterioration of the revenue function results is unexpected. This implies that poorer results from the revenue function using the semi-regional data compared to those using the quasi-micro data are not solely caused by the smaller sample size of the aggregated semi-regional dataset. This surprising result may be due to the inappropriateness of the assumed revenue maximisation behaviour.

Across the three dual functions, the impact of data aggregation on estimation results is most serious for the profit function. The statistical significance and the degree of conformity to economic theory of the estimated derived demand and supply system are reduced when data are aggregated. The price elasticities obtained from this dual function change sign and become statistically insignificant or vary significantly in magnitude between the quasi-micro and semi-regional levels. A significant proportion of Allen partial elasticity estimates and some Morishima elasticity estimates also reverse their sign between the semi-regional and quasi-micro models. Despite reduced sample size being the likely cause, the unsuitability of the profit maximisation assumption for Australian broadacre farmers cannot be ruled out as a contributing factor of this negative finding. This is discouraging since the dual profit function has been most commonly specified, and often estimated using data aggregated across farms, in applications of the duality approach to Australian and international agricultural production.

Regarding measures of substitution and transformation, the estimates of Morishima elasticities are most stable when data are aggregated across farms. The Morishima elasticity estimates generated from the cost and revenue functions do not change sign between the semi-regional and quasi-micro models. In contrast, the Allen partial elasticity estimates frequently reverse their directions when data are aggregated. For instance, half of the Allen partial elasticities of substitution obtained from the cost function, including some statistically significant ones, change sign between the two models.

Chapter 10

Summary and Conclusions

10.1 Summary of the Thesis

This thesis is motivated by the significant contribution broadacre agricultural production makes to the Australian economy and the lack of an updated national econometric model for this sector in production economics literature. In this thesis, a set of models were estimated for Australian broadacre agricultural production following common model formulations conducted in the empirical literature. In Chapters 5, 6 and 7, the three alternative assumptions of cost minimisation, revenue maximisation and profit maximisation were respectively assumed for Australian broadacre farmers. Econometric models were derived under these alternative assumptions and estimated using a nationally representative AAGIS quasi-micro dataset for the period 1990–2005. The most important outcome of this thesis is a set of current national econometric models and key measures of economic and technical relationships between inputs and outputs for Australian broadacre agriculture that are useful for economic assessment and policy making.

The modelling of Australian broadacre agriculture in this thesis accounted for many of the special and important features of the sector. Multi-product dual cost, revenue and profit functions were specified to accommodate the prevalent practice of producing a mixture of different products in broadacre farming. These dual functions were also specified in their restricted forms to allow for the quasi-fixity and lumpiness of production capital over the short run. The exogenous impacts of weather conditions, production focuses, production scales and technological progress on broadacre farming were also incorporated into econometric models by including rainfall information, qualitative dummy variables and time trend. After restricted multi-product cost, revenue and profit functions were specified for Australian broadacre agriculture, systems of input demand and/or output supply equations were derived by applying the Shephard, Samuelson-McFadden and Hotelling lemmas. These systems were estimated using the available AAGIS quasi-micro dataset.

Besides estimating new econometric models and generating elasticity estimates for Australian broadacre agriculture, this thesis also contributes to production economics literature by providing empirical evidence on some significant issues in application of the duality theory to agricultural production. These issues include: the choice among different formulations of econometric models assuming different economic behaviour for producers, the relative performance of the two most popular translog and normalised quadratic functional forms in the duality-based production literature and the effects of data aggregation across production units on estimation results and key economic and technical measures.

10.2 Key Estimation Results

As presented in Chapters 5, 6 and 7, the estimation of the dual normalised quadratic cost, revenue and profit functions using the AAGIS quasi-micro data of Australian broadacre agriculture yielded reasonable results. Estimated systems of the demand and/or supply equations derived from these dual functions generally have adequate statistical significance. At the 5% level, the percentage of significant system coefficients is above 60 per cent for the three dual functions. In particular, the estimated demand system derived from the normalised quadratic cost function and the estimated supply system derived from the normalised quadratic revenue function have reasonable McElroy system-wide R^2. Considering the especially large sample size and the quasi-micro nature of the dataset used for estimation in this study, in contrast to small datasets of aggregate time-series data commonly used in past agricultural production studies, the obtained estimation results are strongly reliable.

The results obtained from the normalised quadratic cost, revenue and profit functions in this study are significantly better than many previous duality applications. The regularity condition of curvature is satisfied by the estimated system of input demands derived from the cost function and the estimated system of output supplies derived from the revenue function. These findings contrast strongly with the frequent violations of this regularity condition in the existing duality literature. Although the estimated system of output supplies and input demands derived from the profit function violates the convexity condition, the violation is not severe. In this supply and demand system, all estimated own-price coefficients have the correct sign and the matrix of the estimated price coefficients is close to being positive semi-definite. Overall, all derived supply and demand curves estimated in this study have the expected slopes with respect to their own prices.

The estimates of price elasticities and elasticities of substitution and transformation obtained from the estimated models of Australian broadacre agriculture in this study are mostly sensible. Own-price elasticity estimates obtained from the three dual functions are negative for input demands and positive for output supplies as expected. These elasticity estimates suggest that, in the short run, input demand and output supply in Australian broadacre agriculture are generally inelastic with respect to market price changes. In other words, broadacre agricultural production is fairly rigid in the short run. Therefore, the development of exchange markets for agricultural commodities and the deployment of financial risk management strategies using products traded on exchange markets, such as forward and futures contracts, can improve farming profitability and resilience.

Two exceptions to the general price-inelasticity of input demand and output supply in Australian broadacre farming were identified in this study. According to the results obtained from the normalised quadratic cost function, demand for fertilisers and crop-pasture chemicals are elastic with respect to their own prices and prices of other variable inputs, except for petroleum-based and livestock-related inputs. From a policy-making perspective, this implies that in the short run, Australian broadacre farmers will cut back purchases of fertilisers and chemicals for crops and pasture, tolerating lower yields or even risking crop failure, in response to increases in these input prices and general production costs.

With respect to production substitutability and transformability, the Allen partial and Morishima elasticity estimates obtained from the three dual normalised quadratic functions suggest that inputs and outputs in Australian broadacre agriculture are substitutes for one another. The statistical significance of these elasticities also implies that there is some scope for broadacre farmers to substitute inputs and outputs in the short run. Moreover, the elasticities obtained suggest that the Morishima elasticity is more reliable than the more popular Allen partial elasticity. Compared to the Allen partial measure, the Morishima measure was found to be less influenced by the choice of the dual objective function, the choice of flexible functional form used and the level of data aggregation across production units. The direction and statistical significance of Morishima elasticities were found to be quite consistent across the cost, revenue and profit functions, across the translog and normalised quadratic functional forms and across quasi-micro data and semi-regional data.

10.3 Three Alternative Formulations of Duality-based Econometric Models of Production

Previous studies of Australian broadacre production assume that farmers minimise production costs or maximise production profits. In this thesis, models of Australian broadacre farm production were estimated under alternative assumptions of cost minimisation, revenue maximisation and profit maximisation using the same farm dataset. Many aspects of the estimation results presented in Chapters 5, 6 and 7, and the prevalence of ley rotation as a farm management practice, suggest that Australian broadacre farmers may seek to minimise production costs in the short run. Of the three models of farm production, the estimated input demand system derived from the normalised quadratic cost function has more satisfactory statistical fit. This equation system has more price variables that are statistically significant than the estimated systems derived from the revenue or profit functions. Moreover, in the cost function result, all input prices are found to be significant determinants of their own demand. The system of demand equations derived from the cost function also meets the regularity condition of curvature without imposing parameter restrictions. In contrast, wool price was not found to significantly influence its own supply in the revenue function result despite the fact that the estimated system of output supply equations derived from this function satisfies the curvature condition. Meanwhile, grains and wool prices were not found to significantly influence their own supplies in the profit function result. Moreover, the estimated demand system derived from the cost function explains the demand of individual inputs fairly evenly while the estimated systems derived from the revenue and profit functions do not. In addition, when the quasi-micro data are aggregated further to semi-regional data, the results from the cost function remain sensible while those obtained from the revenue and profit functions deteriorate.

The conformity of data with short-run cost minimisation assumption for Australian broadacre farmers discussed above is supported by several characteristics specific to this sector. The popular practice of crop and livestock rotation, in which cropping and livestock grazing activities are operated separately, implies that farmers are unlikely to maximise production revenue and profit in the short run. Moreover, broadacre farming is subject to great uncertainty, caused by variable weather conditions and stochastic changes in international commodity prices, and simultaneously requires large initial capital investment, which is partly or largely irreversible. In dealing with uncertainties and capital irreversibility, farmers would incorporate other objectives such as short-term income stability and long-run investment returns into their decision-making. This means that they do not seek to maximise production revenues or profits in the short run. Cost minimisation, in contrast, does not conflict with these objectives.

10.4 Choice of Functional Forms

Another significant contribution of this thesis is to provide further empirical evidence regarding the relative performance of the translog and normalised quadratic functional forms, the two most popular flexible functional forms in duality literature. In this thesis, these two functional forms were used to specify the dual cost and revenue functions in Chapter 5 and Chapter 6. For each of these dual functions, the results obtained from the two functional forms using the large quasi-micro dataset available show significant differences. For the cost function, the two functional forms display similar statistical fit. However, the estimated system of translog cost share equations does not satisfy the regularity condition of concavity. In contrast, the estimated normalised quadratic input demand system automatically satisfies this condition. Similarly, the estimated output supply system derived from the translog revenue function fails to meet the convexity condition, despite having reasonable statistical fit, and the own-price elasticities generated from this system do not have the expected sign. On the contrary, the results obtained from the normalised quadratic revenue function are as expected by economic theory regarding the convexity condition and own-price elasticities.

10.5 Impacts of Data Aggregation

The final empirical evidence sought in this thesis is to assess the impacts of data aggregation across production units on estimation results and policy-relevant economic elasticities. Due to the unavailability of farm-level data, past duality studies of agricultural production have often used aggregate average state, regional and national data for estimation, despite the fact that the duality theory is concerned with microeconomic behaviour. In this thesis, the models of Australian broadacre farming derived from the dual cost, revenue and profit functions were estimated at two data levels using the available AAGIS quasi-micro and semi-regional datasets. For each dual objective function, the results obtained using these two datasets were compared to assess the impacts of data aggregation on research findings. The results obtained suggest that data aggregation across production farms may have serious impacts on research findings, depending on what economic optimisation behaviour is assumed for the producers. When the quasi-micro data was aggregated to semi-regional data, the statistical significance and consistency with economic theory of the cost function results did not appear to be adversely affected, while those of the revenue and profit function results were worsened.

Regarding key price and substitution/transformation elasticities, research findings at the two data levels in this study were found to differ notably. The cross-price and Allen partial elasticities, including statistically significant elasticities, frequently change sign between the two data levels, even for the well-behaved cost function. However, it was found that the Morishima elasticity measure is robust in classifying complementary and substitutive relationships with respect to data aggregation across farms.

10.6 Limitations and Future Research

In this thesis, the estimated system of input demand and output supply derived from the normalised quadratic profit function does not satisfy the regularity convexity condition. When this condition was imposed using the Cholesky decomposition, the FIML method failed to yield an estimation result. This failure may be due to insufficient data sample size, limitations of the software used for estimation or an inappropriate assumption of economic behaviour for Australian broadacre farmers. Therefore, further investigation into the well-accepted perception of Australian broadacre farmers as short-run profit maximisers is required. Future investigation could be an attempt to impose this condition using a larger dataset or different estimation software. Another potential approach to investigate this issue is to assess the impact of imposing regularity conditions on elasticity estimates using Bayesian methods such as in Griffiths *et al.* (2000), O'Donnell *et al.* (1999) and Terrell (1996).

Regarding the performance of the translog and normalized quadratic functional forms in this thesis, it should be noted that no attempt was made to impose the curvature condition on the translog functional form in estimation. The decision to not impose the curvature condition using the Cholesky decomposition for this functional form was taken due to the potential for *a priori* restrictions on price elasticities resulting from such an action (Terrell 1996 and Diewert and Wales 1987). Use of the Cholesky decomposition at a representative data point in future research, such as in the procedure proposed by Moschini (1999), may guarantee a more satisfactory empirical assessment. Bayesian methods may also be employed to assess the effects that the imposition of the curvature condition has on elasticity estimates for this functional form.

In this thesis, empirical evidence of the choice of flexible functional form is limited to the two most popular functional forms, being translog and normalised quadratic. The approach used to assess the performance of these two functional forms here can be employed to investigate the suitability of other flexible functional forms such as the Generalised Leontief, Fourier, Box-Cox, Asymptotically Ideal and Full Laurent. Like models derived from the translog and normalised quadratic forms, models derived from the other flexible functional forms commonly have a large number of parameters to be estimated. Therefore, a large dataset, ideally at the micro or quasi-micro level, would be valuable for the assessment of the performance of these functional forms in duality applications.

The positive outcomes regarding the curvature conditions in this thesis can be attributed to the large AAGIS quasi-micro dataset used for estimation. This large dataset allows for the inclusion of many qualitative dummy variables to accommodate potential differences across geographical locations, production focuses and operation scales. These dummy variables were found to be statistically significant in all models estimated in this study, indicating that broadacre farming operation differs significantly across zones, industries or scales. The sample in this study is not large enough to allow the estimation of separate models for individual zones, industries and scales. However, information regarding differences in broadacre farming across zones, industries and production sizes is valuable for policy making. Therefore, in the future, a larger dataset that allows estimation of separate models for different broadacre zones, industries and sizes will help in broadening the knowledge of Australian broadacre agriculture.

Risks and uncertainties caused by weather conditions and market price changes were not explicitly dealt with in this thesis and in duality applications in general. This has been considered a significant limitation of the duality theory. Future research can explicitly incorporate risks and uncertainties by specifying error-components models such as in O'Donnell and Woodland (1995) or state-contingent models proposed by Quiggin and Chambers (2006) and Chambers and Quiggin (2004).

There are structural characteristics of Australian broadacre agricultural production technology that were not in the scope of this study but are of significant economic interest. These characteristics include returns to scale, homotheticity, separability and non-jointness in production. The formulation of models and tests described in Morrison Paul (2001), Featherstone and Moss (1994), Fulginiti and Perrin (1990) and Chambers (1988) for multi-product production technologies can be applied to investigate further into these structural characteristics of Australian broadacre agriculture.

Broadacre farming in Australia requires large initial investment in capital that is partially or completely irreversible. This would necessitate long-run production decision-making that would discourage farmers from maximising short-run profits. The findings in this thesis are not consistent with short-run profit maximisation behaviour. This implies that future applications of duality theory in agricultural production should entail careful consideration of the assumption chosen regarding the economic behaviour of farmers. Models of intertemporal dynamics of production decisions, such as Fernandez-Cornejo *et al*. (1992), Vasavada and Chambers (1986) and McLaren and Cooper (1980), can be applied to further understand farmers' adjustment behaviour. Real-options models, traditionally used for pricing risky financial products such as options and futures, can also be applied to model farmers' investment decisions concerning different enterprises simultaneously operated on their farms.

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Appendix A: Definition of Variables in AAGIS Data

Variables	Definition
Accounting services ($)	Accounting expense for the survey year.
Advisory services ($)	Total advisory fees paid during the survey year including to farm consultants.
AI stud fees and herd testing ($)	Total expense during the survey year of artificial insemination, herd testing and stud fees.
Contracts - cropping ($)	Cost of cropping contracts during the survey year including spraying and harvesting.
Contracts - livestock ($)	Cost of livestock contracts during the survey year including mustering.
Crop and pasture chemicals ($)	Expenditure on crop and pasture chemicals during the survey year.
Electricity ($)	Expenditure on electricity during the survey year.
Fertiliser ($)	Expenditure on fertilisers and soil conditioners during the survey year.
Fodder ($)	Expenditure on fodder during the survey year.
Fuel oil and grease ($)	Expenditure on fuel, oil and grease during the survey year.
Insurance ($)	Insurance expense during the survey year.
Interest paid ($)	Total interest paid during the survey year.
Land rent ($)	Total rent paid on land rented or leased during the survey year.
Leasing charges ($)	Total payment made on leased plant and/or livestock during the survey year.
Livestock materials ($)	Expenditure on livestock materials during the survey year.
Water charges ($)	Water expense for the survey year.
Repairs and maintenance ($)	Expenditure on repairs and maintenance of plant, machinery, buildings and structures during the survey year.
Seed ($)	Expenditure on seed chemicals during the survey year.
Shearing crutching ($)	Total amount paid to shearing and crutching contractors during the survey year.
Stores and rations ($)	Cost of stores and rations provided to workers during the survey year.
Telephone ($)	Telephone charges during the survey year.
Vet fees ($)	Total veterinarian expense during the survey year.
Age of owner manager (yrs)	Age of the primary decision maker in the farm business.
Age of spouse (yrs)	Age of the spouse of the primary decision maker in the farm business.
Off farm contracts ($)	Gross receipts from contract work where part of the farm capital is involved.
Average micron of wool sold (micron)	Average micron of the main fleece line.
Beef bulls at 30 June (no.)	Number of beef bulls on hand at 30 June.
Beef calves at 30 June (no.)	Number of branded beef calves less than 12 months of age on hand at 30 June.

Variables	Definition
Beef cows at 30 June (no.)	Number of beef cows on hand at 30 June.
Beef cattle transferred onto farm (no.)	The number of beef cattle transferred onto the surveyed property during the financial year from other properties owned or leased.
Beef cattle transferred off farm (no.)	The number of beef cattle transferred off the surveyed property during the financial year to other properties that are owned or leased.
Steers and other beef cattle at 30 June (no.)	Number of other beef cattle on hand 30 June. The other cattle category includes steers, bullocks, speyed cows and other heifers not included elsewhere.
Beef heifers at 30 June (no.)	Number of beef replacement heifers on hand at 30 June.
Branding rate (%)	Number of calves marked/branded as a percentage of the number of cows mated.
Capital appreciation ($)	Change in the value of land and improvements, plant, livestock and other tradeable stocks such as wool and grain, arising from changes in their process during the financial year.
Canola area sown (ha)	Area of canola sown for harvest during the survey year.
Canola receipts ($)	Gross receipts for canola sold during the survey year.
Canola produced (t)	Total quantity of canola produced.
Field peas area sown (ha)	Area of field peas sown for harvest during the survey year.
Field peas receipts ($)	Gross receipts for field peas sold during the survey year.
Field peas produced (t)	Total quantity of field peas produced.
Lupins area sown (ha)	Area of lupins sown for harvest during the survey year.
Lupins receipts ($)	Gross receipts for lupins sold during the survey year.
Lupins produced (t)	Total quantity of lupins produced.
Cotton receipts ($)	Gross receipts for cotton sold during the survey year.
Rice sold (t)	Total quantity of rice sold.
Barley receipts ($)	Gross receipts for barley sold during the survey year.
Grain legumes receipts ($)	Gross receipts for grain legumes sold during the survey year. Grain legumes includes lupins, field peas, chick peas, cow peas, pigeon peas, mung beans, faba beans, navy beans and other grain legumes.
Oats receipts ($)	Gross receipts for oats (grain) sold during the survey year.
Off farm sharefarming ($)	Receipts from sharefarming livestock or crops on land that is owned by someone else.
Oilseeds receipts ($)	Gross receipts for oilseeds sold during the survey year. Oilseeds include linseed, sunflowers, safflower, canola, soybean, linola and other oilseeds.
Payments to sharefarmers ($)	Payments made to sharefarmers who use land on the surveyed property to produce crops or livestock.
Rice receipts ($)	Gross receipts for rice sold during the survey year.
Sorghum receipts ($)	Gross receipts for sorghum sold during the survey year.
Total crop gross receipts ($)	Total gross receipts from sale of crops and hay during the survey year.
Wheat receipts ($)	Gross receipts for wheat sold during the survey year.
Value of land and fixed improvements ($)	Estimate of the market value of all land operated and fixed improvements as of the end of the financial year. Estimated by the owner-manager or co-operator in the survey.
Total area cropped (ha)	Total farm area cropped (total area of crops sown or planted less areas double counted or interplanted) including areas cut for hay.
Barley area sown (ha)	Area of barley sown for harvest during the survey year.
Barley produced (t)	Total quantity of barley produced.

Variables	Definition
Grain legumes area sown (ha)	Area of grain legumes sown for harvest during the survey year. Grain legumes include lupins, field peas, chick peas, cow peas, pigeon peas, mung beans, faba beans, and navy beans.
Grain legumes produced (t)	Total production of grain legumes during the survey year. Grain legumes includes lupins, field peas, chick peas, cow peas, pigeon peas, mung beans, faba beans, navy beans and other grain legumes.
Oats area sown (ha)	Area of oats sown for harvest during the survey year.
Oats produced (t)	Total quantity of oats produced (grain).
Oilseeds area sown (ha)	Area of winter oilseeds sown for harvest during the survey year and area of summer oilseeds sown during the survey year. Oilseeds include linseed, sunflower, safflower, canola, soybean and linola.
Oilseeds produced (t)	Total area of oilseeds sown during the survey year. Oilseeds include linseed, sunflowers, safflower, canola, soybean, linola and other oilseeds.
Rice area sown (ha)	Area of rice sown for harvest during the survey year.
Sorghum area sown (ha)	Area of sorghum sown during the survey year.
Sorghum produced (t)	Total quantity of sorghum produced.
Wheat area sown (ha)	Area of wheat sown for harvest during the survey year.
Wheat produced (t)	Total quantity of wheat produced.
Wheat sold (t)	Total quantity of wheat sold during the survey year.
Total closing capital ($)	Total capital at June 30 is the closing value of all assets used on the farm including leased equipment but excluding machinery and equipment either hired or used by contractors. ABARE uses market value of land and fixed improvements and livestock/crop inventories and replacement value less depreciation for plant and machinery.
Capital at 1 July ($)	Total capital at 1st of July is the opening value of all assets used on the farm including leased equipment but excluding machinery and equipment either hired or used by contractors. ABARE uses market value of land and fixed improvements and livestock/crop inventories and replacement value less depreciation for plant and machinery.
Change in farm debt ($) (1)	Increase or decrease in farm debt during the survey year.
Farm business debt at 30 June ($) (1)	Total farm business debt at the 30th June.
Equity ratio at 30 June (%)	Value of owned capital, less farm business debt at June 30 expressed as a percentage of owned capital.
Area other crops fertilised (ha)	Total area of other crops (excluding wheat and pasture) that fertiliser was applied to during the survey year.
Area pasture fertilised (ha)	Total area of pasture that fertiliser was applied to during the survey year.
Area wheat fertilised (ha)	Total area of wheat that fertiliser was applied to during the survey year.
Total quantity of gypsum applied (t)	Total quantity of gypsum applied to crops and pasture during the survey year.
Total potassium applied (t)	Total quantity of potassium applied to crops and pasture during the survey year.
Total quantity of lime applied (t)	Total quantity of lime applied to crops and pasture during the survey year.
Total nitrogen applied (t)	Total quantity of nitrogen applied to crops and pasture during the survey year.

Variables	Definition
Total NPK fertilisers applied (t)	Total quantity of NPK fertilisers applied to crops and pastures during the survey year.
Total phosphorus applied (t)	Total quantity of phosophurus applied to crops and pasture during the survey year.
Total area fertilised (ha)	Total area that fertiliser was applied to during the survey year.
Total area gypsum applied (ha)	Total area of crops and pasture that gypsum was applied to during the survey year.
Total area lime applied (ha)	Total area of crops and pasture that lime was applied to during the survey year.
Total area NPK applied (ha)	Total area of crops and pasture that NPK fertilisers were applied to during the survey year.
Buildup in trading stocks ($)	The imputed value of all changes in the inventories of trading stocks during the financial year. It includes the value of any change in herd or flock size or in the stocks of wool, fruit and grains held on farm. It is negative if stocks are run down.
Depreciation ($)	Estimated by the diminishing value method, based on the replacement cost and age of each item. The rates applied are the standard rates allowed by the Commissioner of Taxation. For items purchased or sold during the financial year, depreciation is assessed as if the transaction had taken place at the midpoint of the year. Calculation of farm business profit does not account for depreciation on items subject to a finance lease because cash costs already include finance lease payments.
Farm cash income ($)	Farm cash income is the difference between total cash receipts and total cash costs.
Farm business equity June 30 ($) (1)	Value of owned capital, less farm business debt at June 30.
Produce purchased for resale ($)	Produce purchased for resale.
Profit at full equity ($)	Profit at full equity equals farm business profit, plus rent, interest and finance lease payments, less depreciation on leased items. It is the return produced by all the resources used in the farm business.
Profit at full equity including capital appreciation ($)	Profit at full equity plus capital appreciation.
Farm business profit ($)	Farm business profit equals farm cash income plus buildup in trading stocks, less depreciation expense, less the imputed value of the owner manager, partner(s) and family labour.
Farm liquid assets at 30 June ($)	Liquid assets (readily convertible to cash) owned by or available to the farm business at June 30
Total cash costs ($)	Sum of payments made by the farm business for permanent and casual hired labour (excluding operator or manager, partner and family labour), materials, services, produce purchased for resale, livestock purchases and transfers onto the property, interest and payments to sharefarmers. Capital and household expenditures are excluded from total cash costs.
Total cash receipts ($)	Total of revenues received by the farm business during the survey year, including revenues from the sale of livestock, livestock products and crops, plus the value of livestock transfers off a property. It includes revenue received from agistment, royalties, rebates, refunds, plant hire, contracts, sharefarming, insurance claims and compensation, and government assistance payments.

Variables	Definition
Hours worked on farm by owner manager	Hours worked by the primary decision maker in the farm business. Labour as measured here in hours a week is averaged over the whole year and includes all hours worked in excess of 40 hours in any one week. This variable is available from 1994-1995 onwards.
Handling and marketing ($)	Total handling and marketing expense during the survey year on all commodities.
Harvest loan at 30 June ($)	Balance outstanding at June 30 on advances made for grain sold through the pools system. Repayment of harvest loans is underwritten.
Hours worked on farm by spouse	Hours worked by the spouse of the primary decision maker in the farm business. Labour as measured here in hours a week is averaged over the whole year and includes all hours worked in excess of 40 hours in any one week. This variable is available from 1994-1995 onwards.
Hours worked on farm - total	Total hours worked by farm business managers, partners, family, hired permanent and casual workers and sharefarmers but excluding work done by contractors. Labour as measured here in hours a week is averaged over the whole year and includes all hours worked in excess of 40 hours in any one week. This variable is available from 1994-1995 onwards.
Area grazing land irrigated (ha)	Total area of land primarily used for grazing that was irrigated during the survey year.
Total area irrigated (ha)	Total area of land that was irrigated during the survey year.
Imputed labour cost ($)	Payments for owner/manager and family labour may bear little relationship to the actual work input. An estimate of the labour input of the owner/manager, partners and their families is calculated in work weeks and a value is imputed at the relevant Federal Pastoral Industry Award rates.
Total labour used (weeks)	Labour used is the total number of full time weeks worked by all farm workers including hired labour. If an individual works less than 40 hours in an average week, the estimate is converted into a full time week equivalent.
Wages for hired labour ($)	Wages paid to casual and permanent labour. Excludes amounts paid to contractors such as shearers and wool classers.
Agistment ($)	Amount paid on livestock agisted off farm (excludes lot feeding costs off farm).
Beef cattle sold ($)	Gross receipts for beef cattle sold during the survey year.
Beef cattle purchased ($)	Cost of beef cattle purchases (including beef bulls) during the survey year.
Sheep sold ($)	Gross receipts for sheep sold during the survey year.
Sheep purchased ($)	Cost of sheep purchases (including rams) during the survey year.
Livestock transfers - inwards ($)	The value of livestock transferred onto the surveyed property during the financial year from other properties that are owned or leased.
Livestock transfers - outward ($)	The value of livestock transferred off the surveyed property during the financial year onto other properties that are owned or leased.
Beef herd at 30 June (no.)	Number of beef cattle on hand at 30 June.
Beef cattle purchased (no.)	Number of beef cattle purchased during the survey year.
Beef cattle sold (no.)	Number of beef cattle sold during the survey year.
Beef cattle sold or transferred off farm (no.)	The number of beef cattle turned off equals the number of beef cattle sold plus any cattle transferred to other properties.

Variables	Definition
Dairy cattle at June 30 (no.)	Number of dairy cattle on hand 30 June.
Sheep and lambs shorn (no.)	Number of sheep and lambs shorn during the survey year.
Sheep flock at 30 June (no.)	Number of sheep on hand 30 June.
Sheep purchased (no.)	Number of sheep and lambs purchased during the survey year.
Sheep sold (no.)	Number of sheep and lambs sold during the survey year.
Area operated at 30 June (ha)	Includes all land operated by the farm business at the end of June whether owned or rented by the business. Land sharefarmed on another farm is excluded.
Net capital additions ($)	Total additions/purchases of land, buildings, structures, plant and equipment (excluding leased items) less total amount received from the sale of land, buildings, structures, plant and equipment.
Total family income ($)	Family share of farm cash income less family share of depreciation plus all off- farm income of owner manager and spouse.
Family share of farm income ($) (4)	Ownership share of farm income of owner manager, spouse and dependant children.
Total non-farm income ($)	Total off farm income of the owner manager and spouse in the survey year including rent, dividends and interest.
Total off farm wages ($)	Total off farm wages and salaries earned by the owner manager and spouse during the survey year.
Off-farm work for owner manager	Hours the primary decision maker in the farm business works off farm for wages and salaries. Labour as measured here in hours a week is averaged over the whole year and includes all hours worked in excess of 40 hours in any one week.
Off-farm work for spouse	Hours the spouse of the primary decision maker in the farm business works off farm for wages and salaries. Labour as measured here in hours a week is averaged over the whole year and includes all hours worked in excess of 40 hours in any one week.
Other administration expenses ($)	Includes bank fees, legal fees, postage, printing, stationary, subscriptions and other administrative expenses not listed as separate expense items.
Other farm income ($)	Other farm related receipts that are not listed as a separate revenue items.
Other livestock purchased ($)	Other livestock purchases, excluding sheep, beef cattle and dairy cattle purchased during the survey year.
Other materials ($)	Other material expenses includes wool packs, tree and vine replacements, packing materials, water for livestock, electricity and other material expenses not listed as separate variables.
Other services ($)	Includes business related motor vehicle expenses, plant hire, packing charges, travel and entertainment and other service related expenses not listed as separate cost items. Note that services exclude leasing charges.
Other livestock sold ($)	Gross receipts for other livestock sold. Other livestock excludes sheep, beef cattle and dairy cattle sold during the survey year.
Population	Estimated number of farms in the selected categories.
Rate of return including capital appreciation (%)	Profit at full equity expressed as a percentage of total opening capital (including capital appreciation).
Rate of return excluding capital appreciation (%)	Profit at full equity expressed as a percentage of total opening capital (excluding capital appreciation).
Shire and PPB rates ($)	Includes shire rates, pastoral protection board rates and other rates appearing in financial accounts.
Sample Contributing	Number of sampled farms in the selected categories.

Variables	Definition
Ewes at 30 June (no.)	Number of ewes on hand 30 June.
Lambs at 30 June (no.)	Number of marked lambs less than 12 months of age on hand 30 June.
Rams at 30 June (no.)	Number of rams on hand 30 June.
Wethers at 30 June (no.)	Number of wethers on hand 30 June.
Total freight ($)	Total freight paid during the survey year on all commodities.
Total wool sold (kg)	Total quantity of wool sold during the survey year.
Total wool gross receipts ($)	Gross receipts for wool during the survey year.
Total wool produced (kg)	Total quantity of wool produced during the survey year.
Wool cut per head (kg)	Estimated quantity of wool produced per sheep shorn (including lambs).

Appendix B: The McElroy System-Wide R^2

The adjusted R^2 for individual equations in the derived share/quantity system are not useful in judging how well the estimated systems explain the variation in the shares or quantities. Rewrite the derived supply/demand system by stacking M columns of shares/quantities as $\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{bmatrix} = \begin{bmatrix} X & 0 & \cdots & 0 \\ 0 & X & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & X \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_M \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_M \end{bmatrix} = \mathrm{XB} + \varepsilon$.

McElroy (1977) presents a goodness-of-fit measure for a system of seemingly unrelated regressions as $R^2 = 1 - \frac{\tilde{\varepsilon}'(\hat{\Sigma}^{-1} \otimes I)\tilde{\varepsilon}}{y'(\hat{\Sigma}^{-1} \otimes A)y}$ where $\hat{\Sigma}$ is the estimated $M \times M$ covariance matrix of the disturbances; I is a $T \times T$ identity matrix; T is the number of observations, i is a $T \times 1$ column vector of ones, y is the column vector of stacked dependent variables; and $A = I - \frac{1}{T} ii'$. This system-wide R^2 can be used in conjunction with the adjusted R^2 of individual equations to evaluate the performance of the share or quantity systems.

Appendix C: Bootstrapping Standard Errors for Elasticities

Elasticity reports encompass their point estimates and standard errors. Because elasticities are nonlinear functions of the parameter estimates and the predicted shares/quantities, it is not appropriate to use linear Taylor series approximation (the delta method) to calculate the standard errors. In this study, bootstrapping is used to generate the standard errors of the elasticities. This technique uses the sample data itself to obtain sampling properties of elasticities. Previous applications of the bootstrapping method for reporting elasticities include Marsh (2005), Eakin *et al.* (1990), Green *et al.* (1987), Krinsky and Robb (1986), Freedman and Peters (1984) and Gallant and Golub (1984). This study follows the procedure for calculating standard errors described in Eakin *et al.* (1990), which has the following steps:

1. Estimate the share/quantity system using FIML method.
2. Create and save the residuals from the estimated system.
3. Create a new sample of residuals by drawing randomly, with replacement, from the estimated residuals.
4. Create an artificial sample of the dependent variables by adding the newly drawn sample of residuals to the fitted values of dependent variables from Step 1.
5. Run the estimation of the share/quantity system using FIML and the artificial sample of the dependent variables formed in Step 4 and the actual sample of the independent variables.
6. Calculate and save Allen partial, own-price and cross-price elasticities using the estimated parameters and predicted shares/quantities obtained in Step 5.
7. Save the medians of the calculated elasticities.
8. Repeat the procedure described in Step 3 to step 7 (a trial) for 500 times.

9. Calculate the standard deviations of the distribution of the elasticity medians repeatedly generated from the 500 trials. These standard deviations are the bootstrapped standard errors of the elasticities.

Appendix D: Heteroskedasticity in the System of Share Equations Derived from the Translog Cost Function

If farm-level prices and quantities are observed, the cost share for input i is:

$$c_i = \frac{w_i x_i}{\sum_{j=1}^{n} w_j x_j} = f\left(W, Y, Z, T\right) + u_i, \qquad \text{(E.1)}$$

where n is the number of inputs used in production and error term u_i is independently and identically distributed with a normal distribution of constant variance and mean zero. In this study, the input prices observed for a year are the same for all cells since the ABARE national price indices are used in place of the unobserved actual prices. On the other hand, the cells' observed quantities are the average of the constituent farms' quantities. Consider a cell with e farms over which the average is taken. Let d denote the index of farms in this cell. The observed cost share for input i becomes:

$$\tilde{c}_i = \frac{\bar{w}_i \bar{x}_i}{\sum_{j=1}^{n} \bar{w}_j \bar{x}_j} = \frac{\left(\frac{\sum_{d=1}^{e} w_{id}}{e}\right)\left(\frac{\sum_{d=1}^{e} x_{id}}{e}\right)}{\sum_{j=1}^{n}\left(\frac{\sum_{d=1}^{e} w_{jd}}{e}\right)\left(\frac{\sum_{d=1}^{e} x_{jd}}{e}\right)} = \frac{\sum_{d=1}^{e} w_{id} \sum_{d=1}^{e} x_{id}}{\sum_{j=1}^{n} \sum_{d=1}^{e} w_{jd} \sum_{d=1}^{e} x_{jd}}$$

Because $w_{id} = w_i$ with $d = 1,\ 2,\ \ldots, e$, we have:

$$\tilde{c}_i = \frac{e \sum_{d=1}^{e} w_i x_{id}}{\sum_{j=1}^{n} e \sum_{d=1}^{e} w_j x_{jd}} = \frac{\sum_{d=1}^{e} w_i x_{id}}{\sum_{j=1}^{n} \sum_{d=1}^{e} w_j x_{jd}} \qquad \text{(E.2)}$$

From the definition of c_i in (E.1), we can write $c_{id}\sum_{j=1}^{n} w_j x_{jd} = w_i x_{id}$. Summing the share over all farms in the cell it follows that $\sum_{d=1}^{e} c_{id}\sum_{j=1}^{n} w_j x_{jd} = \sum_{d=1}^{e} w_i x_{id}$. Substituting this into (E.2) results in $\tilde{c}_i = \dfrac{\sum_{d=1}^{e} c_{il}\sum_{j=1}^{n} w_j x_{jd}}{\sum_{d=1}^{e}\sum_{j=1}^{n} w_j x_{jd}} = \dfrac{\sum_{d=1}^{e} c_{id} C_d}{\sum_{d=1}^{e} C_d}$, where C_d is the total cost incurred by farm d. In a special case, if all farms in the cell have the same cost C, this observed share reduces to

$$\tilde{c}_i = \frac{\sum_{d=1}^{e} c_{id} C}{\sum_{d=1}^{e} C} = \frac{\sum_{d=1}^{e} c_{id}}{e} = \bar{c}_i \qquad \text{(E.3)}$$

where $\bar{c}_i$ is the observed average of cost shares for input i for the cell. Although it is unrealistic to assume that all farms within a cell have the same production cost, the derivation above shows that there would be some correlation between the cell sample size and the variance of the error terms, and that weighting is likely to be more appropriate than to ignore the effects of nonconstant variance. (Note that in the AAGIS dataset, the surveyed farms are classified into three farm sizes and therefore the production cost for farms within a cell can be fairly similar to each other, which reinforces (E.3)). In the case where $\bar{c}_i$ instead of c_i are used for econometric estimation, weighting by the square root of the sample size e would be appropriate to account for heteroskedasticity (Wooldridge, 2006). In addition, simple linear regressions show that at least some linear relationships exist between the cell sample size and the squared estimated residuals obtained from the share system estimated without weighting. So, weighting the variables included in the derived cost share system is appropriate to correct for heteroskedasticity caused by the quasi-micro nature of the data used for estimation in this study.