Infinity in Pascal's Wager

The standard formulation of Pascal's Wager casts it in terms of absolute utilities: we use '∞' to represent the absolute utility of wagering for God if God exists. The standard formulation supposes that the decision set-up is represented by the following table:

	God Exists **Pr(God Exists) = p, 0<p<1**	**God does not Exist** **Pr(God does not Exist) = 1-p**
Wager for God	∞	A, where A is finite
Fail to Wager for God	B, where B is finite	C, where C is finite

There are two available courses of action: wager for God, or fail to wager for God. There are two relevant conceivable states of the world: God exists, or God does not exist. The credence that assigned to the claim that God exists is p, whence the credence assigned to the claim that God does not exist is 1-p. The utility of wagering for God, given that God exists, is infinite; all of the other utilities are finite. The expected utility of wager for God is $p.\infty + (1\text{-}p).A = \infty$. The expected utility of failing to wager for God is $p.B + (1\text{-}p).C$ = a finite value. Given that one ought always act so as to maximise expected utility, one ought to wager for God.

One might worry that the mathematics is suspect: '∞' is not a standard number. But there is no difficulty involved in revising the standard numbers N to the extended numbers N' via the addition of '∞', subject to the following rules:

(1) $\forall x \varepsilon N'$: $\infty + x = \infty$
(2) $\forall x \neq 0 \varepsilon N'$: $x.\infty = \infty$
(3) $\forall x \varepsilon N$: $\infty - x = \infty$
(4) $\infty - \infty$ is undefined
(5) ∞/∞ is undefined
(6) $0.\,\infty = 0$

(If you think that there is a rule missing from this list, don't worry: we'll have more to say about the rules governing '∞' later on.)

There are many concerns that have been raised about Pascal's Wager in this standard formulation.

First, there are worries about the assignment of a credence to the claim that God exists. Does it make sense to suppose that we do or can assign credences to that claim? And, if it does make sense to suppose that we do or can assign credences to that claim, are we justified in supposing that the assigned credence can or must be strictly greater than zero and strictly less than 1?

Second, there are worries about the assignment of infinite utility to God's existence given that God exists. Even if we can construct a consistent mathematical formalism, we might think that it makes no sense to suppose that we can have infinite utilities. And, even if we suppose that it does make sense to suppose that we can have infinite utilities, we might think that it is a mistake to suppose that the utility of God's existence if God exists is infinite.

Third, there are worries about exactly what 'wagering for God' amounts to. It cannot be 'believing in God', because believing in God is not an action that you can just choose to perform. But, on Pascal's theology, it is not clear that anything less than believing in God will secure 'an infinity of infinite happiness'. It is simply not obvious that there *is* an action that you can choose to perform that has infinite utility if God exists.

Fourth, there are worries about the suggestion that it can be proper to adjust one's credences in the light of one's utilities. However, exactly, the Wager argument is supposed to work, the overarching idea is that your utilities alone give you a reason to revise up the credence that you give to the claim that God exists. But it seems questionable whether your utilities alone ever can give you an adequate reason to revise your credences.

Fifth, there are worries about the soteriological import of Pascal's wager. The construction of the wager seems to suggest that you may obtain 'an infinity of infinite happiness' merely by acting on a dominant desire to acquire, for yourself, an infinity of infinite happiness. Experience teaches that, in this life, merely acting on a dominant desire to acquire happiness for oneself typically does not lead to happiness. Is it really credible to suppose that things stand differently with the next life?

Beyond these worries, there are three objections to the standard formulation of Pascal's wager that seem particularly formidable.

First, there are many different actions that one might take that are bundled together under 'fail to wager for God'. For example, I might toss a coin in order to decide whether to wager. The expected utility of doing this is ∞. More generally, there is a vast range of 'mixed strategies' that I could pursue, all of which have infinite expected utility. The assumption, in the standard formulation of the wager argument, that there are just two available courses of action that need to be considered is false; and, when we correct this assumption, we no longer get out the conclusion that, in order to maximise expected utility, we must wager for God. (For more detailed development of this objection, see Duff (1986) and Hajek (2003).)

Second, there are many different conceivable ways that the world might be that are bundled together under 'God does not exist'. In particular, there are conceivable sources of infinite utility that we might obtain by performing actions other than wagering for God. Consider, for example:

(a) Very Nice Gods who reward everyone regardless of whether and how they wager;
(b) Nice Gods, each of whom rewards those who wager for it, and all of whom reward wagers on some conceivable Gods while also not rewarding wagers on all of the other conceivable Gods;
(c) Very Perverse Gods who reward everyone except those people who wager for it;
(d) Perverse Gods, each of whom does not reward those who wager for it, and all of whom reward wagers on some conceivable Gods while also not rewarding wagers on all of the other conceivable Gods.

All of these conceivable Gods reward some kinds of wagering on Gods, but many reward different kinds of wagering on Gods. So, there are many different wagers all of which have infinite expected utility. The assumption, in the standard formulation of the wager argument, that there are just two relevant conceivable ways that the world might be is false; and, when we correct this assumption, we no longer get out the conclusion that, in order to maximise expected utility, we must wager for God.

Third, given that we are countenancing infinite utilities—i.e. utilities with value ∞—it seems that we should also countenance infinitesimal credences—i.e., credences with value $1/\infty$. If ∞ is so large that it cannot be increased, then $1/\infty$ is so small that it cannot be properly decreased: if you remove a proper part of something of infinitesimal measure, you are left with something that is also of infinitesimal measure.

Earlier, when we listed the rules for N', you might have expected to see the following rule: $\forall x \varepsilon N$: $N/\infty = 0$. But, if we suppose that both ∞ and $1/\infty$ are added to N to form N', then we actually have the following rules (in addition to those given earlier):

(7) $1/\infty + 1/\infty = 1/\infty$
(8) $\forall x \neq 0 \varepsilon N$: $x.1/\infty = 1/\infty$
(9) $1/\infty - 1/\infty$ is undefined

If, in our standard formulation of Pascal's wager, we set $p=1/\infty$, then it turns out that the expected utility of wagering for God is undefined. And, in that case, the advice that we ought to maximise expected utility will not give us the conclusion that we ought to wager for God.

Perhaps it might be objected that we cannot make sense of infinitesimal credences. But there is a *prima facie* case that we can make sense of infinitesimal credences that seems no worse than the kinds of *prima facie* cases that can be made for the claim that we can make sense of infinite utilities. For each real number r, it is conceivable that there is a God who rewards all and only those who believe that God's favourite number is r. Given that we shouldn't give credence 0 to conceivable hypotheses, we have no option but to give infinitesimal credence to very many of the r-Gods. And, if we can attribute infinitesimal credence to at least some of the r-Gods, then we can attribute infinitesimal credence to Pascal's God. The assumption, in the standard formulation of the wager argument, that the non-zero credence given to the claim that Pascal's God exists is finite, is contestable. But, if this assumption is rejected, we no longer get out the conclusion that, in order to maximise expected utility, we must wager for Pascal's God.

Perhaps it might be objected that, if we countenance infinitesimal credences, we will lose normalisation: in cases in which there is assignment of infinitesimal credence, it will no longer be the case that credences sum to 1. This objection prompts a small adjustment to the theory: rather than requiring the credences sum to 1, we insist that either credences sum to 1, or else they sum to $1+1/\infty$. Rather than having standard normalisation, we have normalisation to within no more than an infinitesimal value.

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In some very interesting publications, Paul Bartha (2007)(2012) has explored ways of rehabilitating Pascal's wager so that it is no longer subject to the 'mixed strategies' and 'many Gods' objections. He makes the following bold conjecture:

> If (1) we can assign positive probability to the existence of deities, (2) we can make sense of infinite utility, (3) we can justifiably revise our beliefs on pragmatic grounds, and (4) we can provide a valid formulation of Pascal's original argument, then the many gods objection poses no additional threat. (Bartha (2012:205).)

Since my list of worries about Pascal's wager is slightly more extensive, I shall interpret this conjecture in the following way: *if we meet all of the other objections to Pascal's wager, then the many Gods objection is thereby already met*.

Bartha argues in detail that, if you think that the field of conceivable Gods is made up of Very Nice Gods, Nice Gods, Very Perverse Gods, and Perverse Gods, along with

(5) Jealous Gods, each of whom rewards all and only those who wager for it (of course, Pascal's God is a Jealous God); and
(6) Indifferent Gods who reward no one, no matter how they wager

then you should apportion all of your credence to the Jealous Gods. If he's right about that, then Very Nice Gods, Nice Gods, Very Perverse Gods, Perverse Gods and Indifferent Gods simply do not make any difficulty for Pascal's wager.

The *first stage* of Bartha's rehabilitation of Pascal's wager is to recast it in terms of *relative utilities*. Let *A«B* mean that B is preferred to A under the weak preference ordering «. If Z«A and Z«B, then U(A,B;Z) is the utility of A relative to B with base point Z. Let [pA, (1-p)B] be a gamble that offers A with probability p, and B with probability 1-p. We have the following three special cases:

(1) U(A,B;Z) = $\infty \leftrightarrow$ B « [pA, (1-p)B] for $0<p\leq 1$
(2) U(A,B;Z) = 0 $\leftrightarrow$ A « [pB, (1-p)A] for $0<p\leq 1$
(3) U(A,B;Z) = 1 $\leftrightarrow$ [pA, (1-p)Z] « B and [pB, (1-p)Z] « A for $0\leq p<1$

A relative decision matrix is a table of relative utilities. We compute the matrix entries relative to the optimal outcome, i.e., in the case of Pascal's Wager, we compute U(A, salvation; Z). According to Bartha, this means that all values in our table are either 0 and 1. '1' indicates the best outcome: salvation; '0' represents all of the other outcomes. When we move to the framework of relative utilities, our initial decision matrix is transformed to look like this:

	God Exists **Pr(God Exists) = p, 0<p<1**	**God does not Exist** **Pr(God does not Exist) = 1-p**
Wager for God	1	0
Fail to Wager for God	0	0

This recasting in terms of relative utilities immediately disposes of the standard 'mixed strategies' objection. If we represent the expanded decision matrix as follows:

	God Exists and Coin Falls Heads **Pr(God Exists and Coin Falls Heads) = p/2, 0<p<1**	**God Exists and Coin Falls Tails** **Pr(God Exists and Coin Falls Tails) =p/2**	**God does not Exist** **Pr(God does not Exist) = 1-p**
Outright Wager for God	1	1	0

Toss Fair Coin to Decide Whether to Wager for God	1	0	0
Neither of the Above	0	0	0

then we get:

EU (Outright Wager for God) = p
EU (Toss Fair Coin to Decide Whether to Wager for God) = p/2
EU (Neither of the Above) = 0.

In order to maximise expected utility, one must wager for God.

However, it is worth noting that, if we countenance infinitesimal credences, then the 'mixed strategies' objection survives in the case in which p is infinitesimal. Given that our representation in terms of relative utilities only does away with the infinite values, in the case in which credence for the existence of God is infinitesimal, we have:

Pr(God exists and coin falls heads) = Pr (God exists and coin falls tails) = p

EU (Outright Wager for God) = p+p+0 = p
EU (Toss fair coin to Decide Whether to Wager for God) = p+0+0 = p
EU (Neither of the above) =0.

The advice to maximise expected utility does not tell us what to do.

Perhaps you might think that, in the spirit of the move to relative utilities, we should set infinitesimal credences to zero. But if we set p=0 in the above calculation, then all of the expected utilities are 0, and the advice to maximise expected utility fails to tell us what to do. (Bartha explicitly refuses to allow for infinitesimal utilities: 'every other entry in the table is 0 because it represents the ratio of a finite to an infinite utility'. Given that he rules out infinitesimal utilities on these grounds, he surely rules out infinitesimal credences on exactly the same grounds.)

The *second stage* in Bartha's rehabilitation of Pascal's wager is to introduce some constraints on acceptable credences with respect to the many Gods. Given that the thrust of Pascal's Wager is that, in certain cases, you ought to update your credences in the light of your utilities, it is plausible that, for those cases, your credences ought to be stable equilibrium points under the updating in question.

The specific updating rule that Bartha proposes is as follows:

> Suppose that there are finitely many possibilities S_1, …, S_n with corresponding wagers W_1, … W_n, a relative decision matrix A, and an initial subjective probability vector $\mathbf{p} = (p_1, \ldots, p_n)$, where p_i is the initial subjective probability for S_i. Let $U(W_i) = A(\mathbf{p})$ represent the expected relative utility of wager W_i. Let $\hat{U} = p_1U(W_1) + \ldots + p_nU(W_n)$ represent the average (relative) expected utility. Then the updated subjective probability for S_i is $p_i' = p[U(W_i)/\hat{U}]$.

The consequent constraint the Bartha imposes on acceptable credences with respect to the many Gods is that a viable probability distribution for a Pascalian decision problem should be a stable equilibrium under this updating rule. (A probability distribution $[p_i]$ over possibilities $S_1 \ldots S_n$ is an *equilibrium distribution* if $p_i' = p_i$ for all i. An equilibrium distribution $[p_i]$ over possibilities $S_1 \ldots S_n$ is *stable* if for any small (and mathematically admissible) set of changes Δp_i, application of the updating rule to the distribution $[p_i + \Delta p_i]$ leads to convergence to $[p_i]$.)

Bartha observes that the precise details of the rule are not important; the rule is but one of a large family of evolutionarily robust updating rules that would deliver similar results across the kinds of cases in which we are interested. He also notes that, while insistence that credences should be stable equilibrium points under a suitable updating rule does demand only giving credence to Jealous Gods in some decision situations involving the range of Gods under consideration, it does not do so in all cases.

Consider, first, the following decision scenario (in which it is assumed that each of the initial credences is some positive, finite value):

	Jealous God Exists	**Indifferent God Exists**	**Very Nice God Exists**	**Very Perverse God Exists**	**Nice God Exists**	**Perverse God Exists**
Wager on Jealous God	1	0	1	1	1	1
Wager on Indifferent God	0	0	1	1	0	0
Wager on Very Nice God	0	0	1	1	0	0
Wager on Very Perverse God	0	0	1	0	0	0
Wager on Nice God	0	0	1	1	1	0
Wager on Perverse God	0	0	1	1	0	0
Do not Wager on any of the Above	0	0	1	1	0	0

In this case, under the updating rule, the only stable equilibrium point is to give all of your credence to the Jealous God.

Consider, second, the following decision scenario (with the same assumption as before):

	Jealous God Exists	**Indifferent God Exists**	**Very Nice God Exists**	**Very Perverse God Exists**	**Nice God Exists**	**Perverse God Exists**	**No God Exists**
Wager on Jealous God	1	0	1	1	0	0	0

Wager on Indifferent God	0	0	1	1	1	0	0
Wager on Very Nice God	0	0	1	1	0	0	0
Wager on Very Perverse God	0	0	1	0	0	0	0
Wager on Nice God	0	0	1	1	1	1	0
Wager on Perverse God	0	0	1	1	0	0	0
Do not Wager on any of the Above	0	0	1	1	0	0	0

In this case, under the updating rule, there is a stable equilibrium point in which you give all of your credence to the Nice God.

In order to get the result that the Pascalian wants, we need a further condition. Bartha opts for the following:

> A viable probability distribution for a Pascalian decision problem must be a strongly stable equilibrium. (An equilibrium distribution $[p_i]$, $1 \leq i \leq n$, over possibilities $S_1, \ldots, S_n$ is *strongly stable* if for any new possibility S_{n+1}, and any (mathematically admissible) set of changes Δp_i, $1 \leq i \leq n+1$, application of the updating rule to the distribution $[p_i+\Delta p_i]$ leads to convergence to $[p_i]$, and, in particular to $p_{n+1}=0$.

The motivation for this proposal is the observation that there is a sense in which both of the results that we get in the two cases discussed above depend upon arbitrary features of the Gods that figure in the decision problem. In the first example, the Nice God and the Perverse God reward only followers of the Jealous God (except, of course, that the Nice God rewards its own followers); in the second example, the Perverse God rewards only followers of the Nice God, and the Nice God rewards only followers of itself and the Indifferent God.

It may be that a slightly different condition is mandated. When deciding what to do, you really should take all of the relevant possibilities into account. Given our account of the various Gods that are up for consideration, a *serious* decision scenario is one in which there are no asymmetries introduced in connection with the Nice Gods and the Perverse Gods. Consider, for example, the following, somewhat more complex, decision scenario (where wagering actions are specified in the left column, states of the world are specified in the top row, J is a Jealous, I is indifferent, VN is Very Nice, VP is Very Perverse, the Ni are Nice, the Pi are Perverse, and 0 is the state in which there are no Gods):

	J	**I**	**VN**	**VP**	**N1**	**N2**	**N3**	**N4**	**P1**	**P2**	**P3**	**P4**	**0**
J	1	0	1	1	1	0	0	0	1	0	0	0	0
I	0	0	1	1	0	1	0	0	0	1	0	0	0
VN	0	0	1	1	0	0	1	0	0	0	1	0	0
VP	0	0	1	0	0	0	0	1	0	0	0	1	0

N1	0	0	1	1	1	0	0	0	1	0	0	0	0
N2	0	0	1	1	0	1	0	0	0	1	0	0	0
N3	0	0	1	1	0	0	1	0	0	0	1	0	0
N4	0	0	1	1	0	0	0	1	0	0	0	1	0
P1	0	0	1	1	1	0	0	0	0	0	0	0	0
P2	0	0	1	1	0	1	0	0	0	0	0	0	0
P3	0	0	1	1	0	0	1	0	0	0	0	0	0
P4	0	0	1	1	0	0	0	1	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0	1	0	0	0

In this scenario, the only stable equilibrium position (given our assumption that each of the initial credences is some positive, finite value) is to give all of your credence to the Jealous God. Since this scenario has the kind of symmetry that is plausibly the target of the strong stability condition, it is plausible to conclude that the assessment of this scenario establishes that, if there is a choice between a Jealous God, an Indifferent God, a Very Nice God, a Very Perverse God, the full range of Nice Gods, the full range of Perverse Gods, and no God, you should wager on the Jealous God.

Does this mean that Bartha's conjecture is vindicated? Bartha himself urges caution:

> At the moment, I'm unsure whether or not other types of deity can participate in a strongly stable equilibrium. That leaves room for a remnant of the many-gods objection, and for doubts about the sufficiency of the requirement of strong stability.

I think that this caution does not go far enough. It is clear that we haven't yet considered all conceivable Gods. Consider the possibility of a jealous cartel: a group of Gods, each of whom rewards all and only those people who wager on one among the group of Gods. Let J be a regular Jealous God, and let J1, J2, J3, and J4 be a jealous cartel. If we are deciding just between these five, then the table that represents our decision problem is as follows:

	J	**J1**	**J2**	**J3**	**J4**
J	1	0	0	0	0
J1	0	1	1	1	1
J2	0	1	1	1	1
J3	0	1	1	1	1
J4	0	1	1	1	1

It is obvious that any stable equilibrium point gives zero credence to J, and distributes all of the credence over J1-J4. Moreover, in a wider, properly symmetric contest in which we add an Indifferent God, a Very Nice God, a very Perverse God, an appropriate bunch of Nice Gods, an appropriate bunch of Perverse Gods, and the option of wagering for no God, the jealous cartel will emerge triumphant.

How should we think about a jealous cartel? Where a Jealous God says 'You must believe in me (in order to obtain salvation)!' a God in a jealous cartel says 'You must believe in someone who is enough like me (in order to obtain salvation): it needn't *be* me; near enough is good enough!'

If this result stands, it is bad news for Pascal and Bartha. Their God is a Jealous God; but Pascal's wager—on Bartha's reconstruction—quite clearly tells you not to wager on a Jealous God. So Pascal's wager does not give the result that Pascal wants.

That's not to say that Pascal's wager tells you to wager on a God who belongs to a jealous cartel. There are several problems here.

First, it is clear that a bigger jealous cartel trumps a smaller jealous cartel. How big should be the jealous cartel to which the God on which you wager belongs? It looks as though the only acceptable answer to this question is: infinitely large! But, if that's right, then we are now looking at a decision problem involving at least infinitely many Gods that belong to jealous cartels, infinitely many Perverse Gods and infinitely many Nice Gods. That means that we'll have an infinite number of occurrences of '1' in many of the rows in our table. If—perhaps *per impossible*—all of the candidate Gods are getting infinitesimal probability, there will be lots of Gods with the maximal expected utility: the sum of infinitely many infinitesimals—whatever that is! (I assume that it is simply undefined.) At the very least, our attempt to extricate ourselves from entanglement with infinities appears to have foundered …

Second, I see no good reason to suppose that the list of kinds of conceivable Gods that have been considering is complete. In particular, the Gods that we have considered so far distribute their rewards according to the Gods that are believed in by those who would like to have the rewards. But there are lots of conceivable Gods who, while represented as Indifferent on the table—because they do not distribute rewards according to the God-beliefs of those who would like to have the rewards—nonetheless do differentially bestow rewards. Consider, for example, a God who rewards only those who do not allow their credences to be affected by their utilities. More generally, consider the—plausibly infinite—class of conceivable Gods who will not reward you for anything that depends upon your engagement in Pascalian wagering.

Perhaps it might be objected that, while it was fine to countenance a Very Nice God, a Very Perverse God, an Indifferent God, and a range of Nice Gods and Perverse Gods, it is not fine to countenance the greatly expanded range of Gods that have crept into my discussion. Bartha says the following, in the context of justifying the requirement of strong stability:

> It is important that [a] new theological possibility is 'in the neighbourhood'. From the bare fact that a deity appears to be logically possible, one need not—indeed, cannot always—infer positive probability. The thought here is that the many-gods objection rests on the view that it is not reasonable to assign positive probability only to one deity. I am generalising this point to the 'pantheon of possibilities' [mentioned above]: anyone who assigns one of these gods a positive probability should be willing to entertain a tiny positive probability for the other types …. These are relevant possibilities for anyone who takes Pascal's argument and the many gods objection seriously. (202)

This might seem to open up the prospect of admitting into consideration Jealous, Very Nice, Very Perverse, Nice, Perverse, and Indifferent Gods, while not admitting into consideration jealous cartels and Gods who do not reward anything that is dependent upon engagement in Pascalian wagering. However, it seems to me that it would be very odd to admit Jealous, Very Nice, Very Perverse, Nice, Perverse, and Indifferent Gods for consideration while not admitting jealous cartels for consideration; and it seems to me that, if anything, it would be even odder to admit Jealous, Very

Nice, Very Perverse, Nice, Perverse, and Indifferent Gods for consideration while not also admitting for consideration Gods who do not reward anything that is dependent upon engagement in Pascalian wagering. If we are prepared to countenance the rather hard to motivate behaviour of the Very Perverse God and at least some of the Nice Gods and Perverse Gods, surely we ought to be prepared to countenance members of a jealous cartel whose behaviour is plausibly motivated in much the same way that the behaviour of Jealous Gods is motivated. And surely, too, we ought to be prepared to countenance Gods who do not reward anything that is dependent upon engagement in Pascalian wagering, since it seems readily intelligible that one might suppose that the behaviour of such Gods is motivated by their respect for rationality and integrity.

Bartha justifies the claim that one cannot always 'infer positive probability from logical possibility' by appeal to 'Gale's denumerable infinity of sidewalk crack deities'. Here is what Gale (1991: 350) says:

> From the fact that it is logically possible that God exists, it does not follow that the product of the probability of his existence and an infinite number is infinite. In a fair lottery with a denumerable infinity of tickets, for each ticket it is true that it is logically possible that it will win, but the probability of its doing so in infinitesimal, and the product of an infinite number and an infinitesimal is itself infinitesimal. …. There is at least a denumerable infinity of logically possible deities who reward and punish believers …. For instance, there is the logically possible deity who rewards with infinite felicity all and only those who believe in him and step on only one sidewalk crack in the course of their life, as well as the two-crack deity, the three-crack deity, and so on, *ad infinitum*.

I do not think that we should be persuaded by Gale's *argument*. In N', it is not true that 'the product of an infinite number and an infinitesimal is itself infinitesimal'; more generally, there is no coherent theory of infinities and infinitesimals on which the product of an infinite number and an infinitesimal is always an infinitesimal. If N' is our background mathematical theory, then it is not true that it is logically possible for there to be a fair lottery with a denumerable infinity of tickets. The requirement that the lottery is fair means that each ticket has an equal chance of winning. But, in N', there is no number x which satisfies $x.\infty=1$. Moreover—though I admit that this controversial—I do not think that there is any coherent theory of infinities and infinitesimals against which we can establish that it is logically possible that there is a fair lottery with a denumerable infinity of tickets. (See Oppy (2006: 188).)

Earlier, I gave a *prima facie* case for admitting infinitesimal credences: but that *prima facie* case involved uncountably many Gods. While, in a case involving denumerably many Gods, it is possible to give positive credence to all of them—for example, if, for all n, one gives probability $1/2^n$ to the n-crack deity, then one gives positive credence to all of Gale's sidewalk crack deities—there is no way that one can give positive credence to all of uncountably many Gods. If one is prepared to allow that there are uncountably many possible Gods, one can rightly insist that there is no legitimate inference of positive probability from logical possibility.

But, as I have already insisted, whether or not one is prepared to countenance uncountably many possible Gods, there is another option: one can allow that some of the Gods admitted for consideration in Pascal's wager are given only infinitesimal credence. If one takes this option, then one will say that the many-gods objection rests on the view that it is not reasonable to assign *non-zero* probability only to one deity. When we 'generalise to our pantheon of possibilities', what we say is that anyone who assigns one of these gods a positive probability should be willing to entertain a tiny—i.e. non-zero—probability for the other types.

(*)

Time to take stock.

Bartha conjectured that, if we meet all of the other objections to Pascal's wager, then the many-Gods objection is already met. Moreover, he showed that, if all other objections to Pascal's wager are already met, then, in a choice between a Jealous God, an Indifferent God, a Very Nice God, a Very Perverse God, the full range of Nice Gods, the full range of Perverse Gods, and no God, you should wager on the Jealous God. However, he worried that there might be other types of Gods that can participate in strongly stable equilibria—and, if that were so, then it would remain the case that, even if all other objections to Pascal's wager were met, the many-Gods objection would still be a significant objection to Pascal's wager.

I have argued that Bartha's worry is well-founded. There are other types of Gods, no less worthy of consideration that those that figure in Bartha's deliberations, that are better wagers than the Jealous God. In particular, I have suggested that one does better to wager on a God that is a member of a jealous cartel than one does to wager on a Jealous God.

I have also argued that there are other types of Gods, no less worthy of consideration that those that figure in Bartha's deliberations, that make trouble for Pascal's wager, but not because one would do better to wager on them rather than on a Jealous God. In particular, I have suggested that a 'Virtuous' God, who does not reward behaviour that is dependent upon engagement in Pascalian wagering but does reward properly motivated virtue, clearly makes trouble for Pascalian wagering. Consider the following decision scenario:

	Jealous God Exists	**Virtuous God Exists**	**No God Exists**
Wager on Jealous God	1	0	0
Wager on Virtuous God	0	0	0
Don't Wager / Act Virtuously	0	1	0
Don't Wager / Don't Act Virtuously	0	0	0

In this case, Bartha's updating rule does not apply, and there are no relevant notions of stable equilibrium and strongly stable equilibrium. Once we introduce Gods who reward non-wager actions, we introduce a barrier to the updating of credences in the light of utilities. And, once we've done that, we definitely do not get out the conclusion that one ought to wager on the Jealous God in preference to merely acting virtuously.

Finally, I have argued that there is an objection to Pascal's wager that Bartha does not consider, but that interacts in interesting ways with Bartha's treatment of the many-Gods objection. If we are prepared to countenance infinitesimal credences, then we should baulk at the move that recasts Pascal's wager in terms of relative utilities. In the original formulation of Pascal's wager, when infinite utility meets infinitesimal credence, we do not get well-defined results (and quite properly so). But, if we suppose that infinitesimal credences are in no worse standing the infinite utilities, then we cannot accept the assumption—built into Bartha's formulation of relative utilities—that

$\forall x \varepsilon N$: $x/\infty = 0$. Rather, what we should have is: $\forall x \varepsilon N$: $x/\infty = 1/\infty$. But, if that is what we have, then we need to replace almost all of the 0s in our decision tables with $1/\infty$'s.

Consider, for example, this revised decision table:

	Jealous God Exists	Indifferent God Exists	Very Nice God Exists	Very Perverse God Exists	Nice God Exists	Perverse God Exists
Wager on Jealous God	1	$1/\infty$	1	1	1	1
Wager on Indifferent God	$1/\infty$	$1/\infty$	1	1	$1/\infty$	$1/\infty$
Wager on Very Nice God	$1/\infty$	$1/\infty$	1	1	$1/\infty$	$1/\infty$
Wager on Very Perverse God	$1/\infty$	$1/\infty$	1	$1/\infty$	$1/\infty$	$1/\infty$
Wager on Nice God	$1/\infty$	$1/\infty$	1	1	1	$1/\infty$
Wager on Perverse God	$1/\infty$	$1/\infty$	1	1	$1/\infty$	$1/\infty$
Do not Wager on any of the Above	$1/\infty$	$1/\infty$	1	1	$1/\infty$	$1/\infty$

If I initially assign credence of $1/\infty$ to one or more of the conceivable states of the world, then one or more of the expected utilities will be undefined. And, in that case, there will be no way to apply the updating rule to adjust the credences in the light of the utilities. So, if I initially assign credence of $1/\infty$ to one or more of the conceivable states of the world, then I do not get out the conclusion that I ought to wager on the Jealous God.

Furthermore, if I take seriously the idea that there are uncountably many possible Gods, and I understand the requirement on strongly stable equilibrium to require that all possible Gods are taken into account, then it will certainly be the case that there are undefined expected utilities in my wagering calculations. If I take seriously the idea that there are uncountably many possible Gods, and I understand the requirement on strongly stable equilibrium to require that all possible Gods are taken into account, then I simply will not get out the conclusion that I ought to wager on the Jealous God. While Bartha's treatment of some cases involving finitely many Gods is quite compelling, it seems that the uncountably-many-Gods objection remain a serious, independent objection to Pascal's wager.

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