Time, Successive Addition, and *Kalam* Cosmological Arguments

Craig (1979) presents and defends several different *kalam* cosmological arguments. The core of each of these arguments is the following ur–argument:

1. The universe began to exist.
2. Whatever begins to exist has a cause of its existence.
3. (Hence) The universe has a cause of its existence.
4. (Hence) God exists.

What distinguishes between the different *kalam* arguments which Craig defends is the sub–argument which is given on behalf of the first premise in this ur–argument.

One of the sub–arguments appeals to *a posteriori* scientific considerations: there is empirical evidence that the universe has only existed for a finite amount of time, and hence that it began to exist a finite number of years ago. The other two sub–arguments appeal to *a priori* philosophical considerations: there are broadly logical arguments which establish the conclusion that the universe could only have existed for a finite amount of time, and hence that it must have begun to exist no more than some finite number of years ago. According to one of these sub–arguments, it is impossible for there to be completed infinities: since the past would be a completed infinity if it were infinite, it follows that the past must be finite. According to the other of these sub–arguments, it is

impossible for there to be completed infinities formed by successive addition: since the past would be a completed infinity formed by successive addition if it were infinite, it follows that the past must be finite.

In this paper, I wish to look at the second of the two *a priori* arguments. Set out in full, this argument might be represented as follows:

1. It is not possible for a series formed by successive addition to be both infinite and completed.
2. The temporal series of (past) events is formed by successive addition.
3. The temporal series of past events is completed (by the present).
4. (Hence) It is not possible for the temporal series of past events to be infinite.
5. (Hence) The temporal series of past events is finite.
6. (Hence) The universe began to exist.
7. Whatever begins to exist has a cause of its existence.
8. (Hence) The universe has a cause of its existence.
9. (Hence) God exists.

There are many criticisms which a non–theist might choose to make of this argument. However, in this paper, I shall be focussing my attention on the second premise, i.e. on the claim that the temporal series of events is formed by successive addition. I shall argue that non–theists can have good reason to refuse to accept this premise—and hence that non–theists can have good reason to reject arguments which make use of this premise. I

shall not be claiming that this is the strongest—or most important—objection which non–theists can make to the argument; however, towards the end of the paper, I shall—in effect—explore the suggestion that reasons for refusing to accept this premise can be generalised to reasons for refusing to accept the conjunction of the two premises in the ur–argument with which we began.

1. Some Preliminary Observations

Our formulation of the argument makes use of the definite description ‘the temporal series of past events’. Use of this description might be thought to be problematic on two counts: first because it involves commitment to (past) events; and second because it requires that there are no more than countably many past events.

Some philosophers have been loathe to admit events into their ontology. Perhaps we might hope to admit them as supervenient entities; but it seems to me to be implausible to think that we can do without them altogether. Of course, even if this is right, we are still left with the difficult task of saying exactly what events are—but I shall not attempt to pursue this project here. I hope that nothing *I* say depends upon my making controversial assumptions about the nature of events.

Some philosophers have been loathe to admit entirely past entities into their ontology. On the best known versions of this view—presentism—only presently existing things exist. But, if there are no entirely past entities, then, in particular, there are no (entirely) past events. (On the plausible assumption that composition for events is mereological, we can drop the qualification: the only events which exist are present events.) And, if there are no past events, then there is no such thing as ‘the temporal series of past events’. Since it would be tedious to reformulate the argument to make it acceptable to presentists, I shall leave it to affronted presentists to perform this task for themselves.

The claim that past events form a temporal *series* is controversial: I take it that it is analytic that a series has no more than countably many members—the real numbers do not form a series—and that whether some countable collections of entities form series depends upon the order in which they are placed—the rational numbers do not form a series under their standard ordering (though, as Cantor taught us, they can be made to form a series under a non–standard re–ordering). For this reason, I would prefer to talk about ‘the temporal order’—or, in the case of ‘the temporal series of past events’, simply ‘the past’. However, I have already departed enough from the language of ‘actual infinities’ in which Craig originally couched the argument, so I shall make liberal use of scare quotes instead.

2. Time and Succession

The premise on which we are focussing attention says, in effect, that time passes by *successive* addition. That is, time is made up of discrete moments which follow one after the other, like beads on a string. Put another way, time has the structure of the natural numbers, which can be generated from zero (the initial moment) by repeated applications of the successor function.

The assumption that time has this structure has been made by many Strict Finitists and others. But it can be contested. In particular, it might be insisted that time is dense or continuous, i.e. that it has a structure more like that of the rational or real numbers. If time is dense—i.e. if, between any two distinct times, there is a third time distinct from each—then it seems clear that it will not be accurate to say that time grows by *successive* addition. Rather, we should say that time grows by continuous addition, or accretion, or the like.

Perhaps it might be said that, while it can be conceded that time is dense, it should not be conceded that 'the temporal series of events' is dense. (Perhaps it might be denied that there is any such thing as time, over and above 'the temporal series of events'.) However, if we suppose that there can be continuous processes, then it is hard to see how this insistence can be justified. Suppose—for the sake of argument—that temperature is a continuous quantity and that the temperature of an object O increases continuously from 15°C to 16°C over a period of one minute (from t_1 to t_2). If 'the temporal series of events' is discrete, there will be a first moment M_1 after t_1. Since the temperature of O increases

continuously, the temperature of O at M_1 must still be 15°C; else, there will have been a discontinuous jump in the temperature of O. Repeated application of this argument shows that, at t_2, the temperature of O will still be 15°C, which contradicts our assumption that the temperature of O at t_2 is 16°C. In order to avoid contradiction, a defender of the claim that time is discrete must insist that there are no continuous processes.

Perhaps it might be said that this is right: there are—and can be—no continuous processes. Everything is quantised; any varying quantity of anything varies by making little jumps from moment to moment. However, while I don't see that this metaphysical speculation is ruled out by logical or metaphysical considerations, it seems to me that—putting things as mildly as I can—there is not the slightest reason to suppose that it is correct. It is true that current physics seems to tell us that *some* fundamental quantities are quantised; however, it is also true that current physics is full of continuous quantities. Moreover, attempts to produce fully quantised physical theories have hitherto always ended in disaster: in particular, attempts to produce theories with quantised spacetime have always ended up with non–renormalisable infinities all over the place. If we suppose that physics should be our guide in metaphysics, then we have good reason to think that the world is full of continuous quantities, and hence have good reason to think that time is not discrete. And even if we think that metaphysics is prior to physics, it is hard to see why we should embrace the conclusion that the world is everywhere discrete. It can hardly be said that this claim is obvious, or that it is revealed to intuition.

Perhaps it might be objected that my argument from continuity depends upon absurd Cantorian assumptions about the applicability of real arithmetic to the physical world. Can we really make sense of the idea that there are continuum many times between any two distinct times (and continuum many points between any two distinct points)? I think so. Moreover, it is worth pointing out that the Strict Finitist alternative—the cinematographic model of the universe—is hardly more consonant with untutored intuition: it isn't easy to believe that the history of the universe consists of a finite series of discrete states across which quantities vary by making discontinuous jumps. In any case, we are now moving on to the territory which is covered by the first of the *a priori* sub–argument for the first premise of the ur–argument: for we are now moving to a discussion of the question whether it is possible for there to be completed infinities. We shall return again to this question.

3. Time and Addition

The premise on which we are focussing attention says, in effect, that time passes by successive *addition*.That is, time accumulates in the direction of the future. Suppose we said instead that time passes by successive subtraction—or perhaps, better, that time accumulates in the direction of the past. Then we would have no reason to accept the third premise of the argument: it would be the temporal series of *future* events which is completed by the present.

However, there would then be another argument to confront:

1. It is not possible for a series formed by successive subtraction to be both infinite and completed.
2. The temporal series of (future) events is formed by successive subtraction.
3. The temporal series of future events is completed (by the present).
4. (Hence) It is not possible for the temporal series of future events to be infinite.
5. (Hence) The temporal series of future events is finite.
6. (Hence) The universe ceases to exist.
7. Whatever ceases to exist has a cause of its ceasing to exist
8. (Hence) The universe has a cause of its ceasing to exist.
9. (Hence) God exists.

Since this argument is no more acceptable to non–theists, it seems doubtful that non–theists can object to the original argument on the ground that it gets the objective direction of time the wrong way round. However, it might be possible for non–theists to urge that there is *no* objective direction to time, and hence to claim that ‘the temporal series of events’ is formed neither by addition nor subtraction. Perhaps the past and the future are both infinite, but the present completes neither; and perhaps this is so even if ‘the temporal series of events’ is discrete.

Is it plausible to think that there is no objective direction to time—or to ‘the temporal series of events’? Perhaps the elusiveness of ‘the arrow of time’ has lead some people to think so; however, it seems to me that it is reasonable to suppose that there is an objective directed causal order, and that the direction of this causal order is from what we standardly call ‘the past’ to what we standardly call ‘the future’. At any rate, for the purposes of this paper, I am not going to try to push an objection from the claim that there is no objective direction to time, or to events in time. (Note, by the way, that I might think of this as an ace up my sleeve. If it came to a choice between theism and rejection of an objective direction to time, it might well be the rejection of an objective direction to time which gets the nod. However, there are so many other objections to be made that it hardly seems worthwhile pursuing this one at the moment.)

4. Real Time

Suppose we grant that time does have the structure of the real numbers. Then—as I have argued—we have reason to reject the second premise of the argument under consideration: we should not suppose that ‘the temporal series of events’ is formed by successive addition. And, with the rejection of this premise, the argument falls.

However, there are more consequences for *kalam* cosmological arguments to be derived from the assumption that time is at least dense. The aim of the argument, from the

premise that ‘the temporal series of events’ is formed by successive addition, is to show that the past is finite. Now, if time has the structure of the natural numbers, then showing that the past is finite suffices to establish that there is a first moment of time. However, if time is dense, then showing that the past is finite is not sufficient to establish that there is a first moment of time. This point could have important consequences for all of the different kinds of *kalam* cosmological arguments.

Once we have densely ordered entities, we can distinguish between two different kinds of intervals over those entities: **closed** intervals, which have a first member and a last member; and **open** intervals which have neither a first member nor a last member. (There are also half–closed, or half–open intervals, which have exactly one of a first member and a last member.) If we suppose that time is dense, and then seek to argue that there is a first moment of time, then what we wish to establish is that initial temporal intervals are closed. Establishing that certain intervals—initial temporal intervals which overlap with the present—have finite measure is beside the point: this cannot suffice to show that there is a first moment of time, if time is dense. Since *a posteriori* arguments for the finite age of the universe are only arguments about the measure of a temporal interval, those arguments cannot be used in support of the claim that there is a first moment of time, if time is dense.

Perhaps it might be objected that *kalam* cosmological arguments do not depend upon the assumption that there is a first moment of time (or, at any rate, a first moment of time at which the universe exists). That is, it might be said that the first premise of the

argument—that the universe began to exist—should be interpreted to be saying that the age of the universe is finite. Moreover, it might be added—cf. the discussion of the standard puzzle cases in section 5 below—that it is the impossibility of infinite measure which is established by the usual arguments against completed infinities.

However, there are two reasons why this objection seems unsatisfactory. *First*, consistency would now require that the second major premise of the argument—that whatever begins to exist has a cause of its existence—should be interpreted to be saying that whatever has a finite age has a cause of its existence. But this premise is not a standard causal principle, and it is not obvious that it is true. Indeed, it seems obvious that, in the context of debates between theists and non-theists, it is simply grossly question–begging. *Second*, it seems clear that the version of the argument on which we have been focussing cannot be properly understood in this way: the subargument attempts to establish that there cannot have been infinitely many successive past events, not that the measure of the past is finite. Since all of the *kalam* arguments are supposed to share a common core, and since the argument which we are considering cannot have anything to do with the measure of the past, we are entitled to draw the conclusion that none of the *kalam* cosmological arguments has anything to do with the measure of the past. (Or, perhaps more charitably, we might draw the conclusion that proponents of *kalam* cosmological arguments are wrong to say that those arguments have a common core. However, I shall not pursue this line of thought further here.)

Suppose that it is conceded that it is crucial to the success of *kalam* cosmological arguments that they should establish that there is a first moment of time. Even if the above argument is granted, it might be objected that it is surely not possible to take the distinction between open and closed intervals so seriously. However, if I am right, anyone who doesn't find it immediately and compellingly obvious that the world is in every respect discrete or digital takes the distinction between open and closed intervals seriously, simply because they take the notion of continuity seriously. If time is dense, then there is a difference between temporal intervals with endpoints and temporal intervals without endpoints. For almost all purposes, this difference doesn't matter—but it may make an enormous difference when it comes to questions about the beginning and end of the world. Or so it seems to me.

5. Time and Infinity Machines

Many recent defenders of *kalam* cosmological arguments have followed Craig in supposing that certain familiar puzzles about infinity—the Tristram Shandy paradox, Hilbert's Hotel, Benardette's serrated continuum, and so forth—lend support to the impossibility premises in those arguments. However, it seems to me that there are several reasons for being sceptical about this claim. While these puzzles are very interesting in their own right—and deserving of considerable philosophical reflection—they do not

support arguments for the conclusion that ‘the temporal series of past events’ must be finite.

Let me begin with a kind of concession. It seems to me to be plausible to think that one cannot have a serial collection of things which are related by ‘successive addition’, in which there is a first member correctly labelled ‘1’ and a last member correctly labelled ‘ω’, and in which each member of the collection has the same ‘size’ and is at the same ‘distance from its neighbours’. So there are versions—perhaps in some sense ‘the standard versions’—of well–known puzzles about infinity which, in my view, are impossible. Moreover, if there were a ‘temporal series of events formed by successive addition’, then it might be very tempting to claim that it could not be infinite: for one might think that, if it were, one would have a serial collection of things (moments) which are related by ‘successive addition’, in which there is a first member correctly labelled ‘1’ and a last member correctly labelled ‘ω’, and in which each member of the collection has the same ‘size’ and is the same ‘distance from its neighbours’. (I assume that, strictly speaking, moments have neither ‘size’ not ‘distance from their immediate neighbours’, but—for now—I also assume that this somehow suffices for satisfaction of the relevant conditions. Moreover, I assume that the relation of ‘successive addition’ forces the requirement that later times are assigned higher ordinal numbers. Finally, I assume that the claim that the temporal series if infinite brings with it the requirement that the present be assigned the label ‘ω’.) However, it seems to me that to think in this way would be mistaken: there is nothing in a ‘temporal series of events formed by successive addition’ which requires that there be a first member which is correctly labelled ‘1’; rather, there

must be earlier members of the series of the form ω–n, for all natural numbers n. (And, if it is said that this is formation by 'subtraction' rather than 'addition', then the point is that there is no reason why the passage of time cannot involve this kind of 'subtraction'. There is no denial here that the direction of the passage of time is from the past to the future, i.e. this objection is not related to the worries which were raised in section 3. Rather, the point here is that the past might be 'turtles all the way down': each moment is supported by the one which comes immediately before it.)

If I am right about this last point, then it is clear that Hilbert's Hotel, the Tristram Shandy paradoxes, Craig's Library, and so forth are simply irrelevant for *kalam* cosmological arguments: the versions of these stories which are problematic are ones in which all of the conditions for impossibility which I listed above are satisfied; but, in the case of a 'temporal series of events formed by successive addition', it may be that one of these conditions is not satisfied. Suppose, though, that I am wrong on this last point: suppose that a 'temporal series of events formed by successive addition' must have members which are 'infinitely distant' from the present. No matter; there is another way in which 'the temporal series of events' differs from the standard puzzle cases and which may suffice to undermine the support which they are sometimes supposed to lend to *kalam* cosmological arguments.

As a preliminary to stating this further difference, let me begin by noting at this point that one standard way to retell the stories about Hilbert's Hotel etc. in which the air of paradox is apparently removed is to introduce something like 'density' or 'points of

accumulation'. The model here is Grunbaum's discussion of infinity machines. Consider, for example, the Thomson lamp. If we suppose that the amplitude of switching is constant, then we get kinematical inconsistency: but if we suppose that the amplitude of switching can decrease to zero, then we can restore kinematical consistency (and perhaps dynamical consistency as well); and, moreover, we can control the end–state into which the system settles by controlling the initial description of the system.

If the world is continuous, then this kind of infinite behaviour could be everywhere. Consider, for example, a simple pendulum. Suppose that, if it is displaced .5 cm from its midpoint, it will take .5 seconds to return to the midpoint. Suppose, further, that if its maximal distance of displacement is x cm, then, due to damping, its maximal displacement on the other side will be 0.5x cm. Suppose, finally, that the pendulum is initially displaced 1 cm. Then, we can calculate that it will come to rest after 2 second, and that it will have successively been displaced by amounts +1cm, –0.5cm, +0.25cm, –0.125 cm, and so on. There is an infinite series here, with a first member, and a last member. But there is no inconsistency, because the requirements of constancy of 'size' and 'distance from immediate neighbours' are not satisfied. Of course, in practice, this behaviour will not be realised: thermodynamic effects will soon swamp the momentum due to the initial displacement. But it also seems that the behaviour of the pendulum is not logically impossible: there is nothing logically impossible about the scenario which I have described.

Of course, we might not want to say that ‘the temporal series of events formed by successive addition’ is like the pendulum: we might not be able to make sense of the idea that moments vary in ‘size’ or ‘proximity to their immediate neighbours’. But, once we make this concession then—as I in effect noted above—we can no longer suppose that the alleged absurdities of the puzzle cases mentioned earlier give us good reason to suppose that a ‘temporal series of events formed by successive addition’ can not be infinite: for the puzzlement generated by those puzzles depends upon the fact that the analogues of moments are kinds of things which *can* vary in ‘size’ or ‘proximity to their immediate neighbours’, but which do not.

Suppose that I am wrong about this as well: suppose that the alleged disanalogy between the puzzle cases and ‘the temporal series of events formed by successive addition’ fails to obtain. That is, suppose that if the past were a ‘temporal series of events formed by successive addition’, then the absurdities of the puzzle cases would give us reason to hold that the past is finite. No matter; there is still another objection. As I have argued in the preceding sections of this paper, there is no good reason to think that the past is a ‘temporal series of events formed by successive addition’; there is at least as much reason to think that the past is continuous—and, if it is, then it is quite clear that the familiar puzzle cases about countable infinities are simply irrelevant. Craig’s libary has countably many books; Hilbert’s Hotel has countably many guests; Tristram Shandy lives for countably many days; and so on. But the past is continuous; if it is infinite, it simply should not be thought of on the model of a countable number of beads on a string. Where there is genuine continuity, there is—in at least one good sense—completed infinity; but

the familiar puzzle cases pose no kind of challenge to this kind of completed infinity. Perhaps—disregarding the two criticisms made above—if 'the temporal series of events' were a series, then these puzzles would be relevant; but it isn't, and they aren't.

Perhaps it might be objected that there *are* puzzles about continua which will support the claim that the past cannot be infinite. However, as we saw in section 4, once we allow that time is continuous, there is no longer any reason to think that a finite past requires an initial moment of time. Perhaps it might be said that there are puzzles about continua which will support the claim that, if the past is finite, then there is an initial moment of time. However, at the very least, that it is a case which no defenders of *kalam* cosmological arguments have yet attempted to prosecute. Given that time grows by continuous accretion, why shouldn't there be open temporal intervals—and why can't the beginning of the universe involve them? (The main point here can be put a different way: When it is said that the past is finite, this is usually taken to be the claim that the past has finite measure. However, the familiar puzzle cases are not puzzles about measure; rather, they are puzzles about cardinality, or counting. This explains why they are of no use to proponents of *kalam* cosmological arguments. Of course, as I mentioned above, there are puzzles about infinite measure; and there are philosophical puzzles about measure theory—but it is hard to believe that those puzzles give us reason to give up on the idea that time is continuous.)

6. Some Concluding Remarks

In this paper, I have argued, in effect, that proponents of *kalam* cosmological arguments presuppose Strict Finitist metaphysics: they presuppose that the world is fundamentally discrete in all respects. Speaking for myself, I think that there is a pretty good inference from the success of current physics to the conclusion that the world is not fundamentally discrete in all respects. However, since I recognise that this instance of inference to the best explanation is a little contentious, I have only argued for the weaker claim that the controversial nature of this Strict Finitist presupposition creates substantial problems for proponents of *kalam* cosmological arguments. I do not find it plausible to think that there are many theists for whom the Strict Finitist presupposition is more doxastically secure than their belief in God. Nor do I find it plausible to think that there are many people at all who are very securely persuaded of the Strict Finitist presupposition. Consequently, it seems to me that *kalam* cosmological arguments are bound to be pretty useless things.

(Note: This paper was presented at the Gifford Centenary Conference in Aberdeen, in May 2000. I would like to thank: Neil Manson for his sterling work in organising the conference; David Oderberg for his fine commentary on my paper at the conference (reprinted below); Bill Craig for his kind offer to help publish both my paper and David's comments on it; and the other participants who contributed to the discussion of my paper at the conference. There is much which I might have added to the paper in response to David's comments and those of other participants at the conference (though almost

nothing that I would change or retract); however, all of that must wait for some other occasion.)

References

Craig (1979) The Kalam Cosmological Argument London: Macmillan