Modelling and Evaluating the Deployment of an Autonomous Vehicle Zone in Transportation Networks

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Abstract

The advancement of technologies has fast-tracked the deployment of autonomous vehicles (AVs), however, they necessitate the modernization of the existing road infrastructure to meet the technical and safety requirements of AVs. Due to the high cost and prolonged time required for upgrading road infrastructure, it is expected that AVs will be introduced gradually into existing transport networks of human-driven vehicles (HVs) either jointly or separately in a special area or particular zones within existing networks. However, the deployment of AVs with mixed traffic is challenging and complex due to the different route choice principles of HVs that behave selfishly, and AVs perform cooperatively. Moreover, the deployment of AVs requires proper strategy with the guideline. Thus, the broad aim of this research is to develop a framework for modelling and deployment of AV zones with mixed traffic that maximizes the transportation network's performance in terms of the minimization of the total system travel time. In order to accomplish this broad research goal, first, a mathematical model is developed for the design of a multi-lane AV zone in a mixed-user traffic network that takes into account the different route choice decisions of HVs and AVs. The model is formulated using Wardrobe's first and second equilibrium principles: User equilibrium and System optimum, and then an equivalent convex formulation is derived. The model is used to study two different designs for a multi-lane AV zone, where in design 1, both lanes are dedicated for AVs inside the AV zone; and in design 2, one lane is for AVs, and another lane is for mixed traffic. To demonstrate the result, two lanes per direction are considered in the transportation network, however, this model is applicable for any number of lanes. Secondly, a novel solution algorithm is developed to evaluate and compare the performance of the designs in a small simple and a realistic large-scale complex transportation network. For both networks, design 1 provides the best network performance in terms of the improved total system travel time compared to design 2 at a higher number of AV penetration. On the other hand, design 2 performs better than design 1 at a lower AV penetration. In summary, the proposed framework can be used to assist transportation planners and decision-makers to design and successfully deploy AVs seamlessly within existing transport networks with mixed traffic.

Declaration

This statement is to certify that, to the best of the candidate's knowledge, this thesis contains no material which has been accepted for the award of any other degree or diploma in any university or other institution, and that the thesis contains no material previously published or written by another person, except where due reference is made in the text of the thesis. The length of this thesis is less than 35,000 words, exclusive of figures, tables, and references.

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Publications from this thesis

All contributions in this thesis are under review. The list of publications is as follow:

- Keya Roy, Nam H. Hoang, Hai L. Vu "Modelling autonomous vehicles deployment: A mixed traffic multi-lane zone design" (Under review: Transportation Research Part E: Logistics and Transportation Review)

- Keya Roy, Nam H. Hoang, Hai L. Vu "Deployment of autonomous vehicles in mixed traffic: A mixed user traffic assignment framework" (Under review: 2021 IEEE 24th International Conference on Intelligent Transportation Systems (ITSC))

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LIST OF ABBREVIATIONS

AVs	Autonomous Vehicles
AVTT	Average Travel Time
BPR	Bureau of Public Roads
CSL	Cross Nested Logit
GA	Genetic Algorithm
HVs	Human Driven Vehicles
MRE	Mixed Routing Equilibrium
MSA	Method of Successive Average
MUE	Multiclass User Equilibrium
PSL	Path Size Logit
RSRM	Route Swapping Self Regulated Step Size
SAA	Simulated Annealing Algorithm
SO	System Optimum
TSTT	Total System Travel Time
TT	Travel Time
UE	User Equilibrium
VI	Variational Inequality
NDP	Network Design Problem
GP	Goal Programming
MUCSUC	Multi User Class Stochastic User Equilibrium

PART I:

BACKGROUND, LITERATURE AND RESEARCH METHODOLOGY

CHAPTER 1 INTRODUCTION

1.1 Overview

This thesis focuses on transport network modelling and design, emphasizing the evaluation of network performance involving autonomous vehicles (AVs) and human-driven vehicles (HVs). The thesis seeks to understand the different designs for AVs' deployment as a zone in transportation networks to obtain the best network performance. Moreover, it develops a unified mathematical framework with linear constraints to solve a problem of mixed routing network equilibrium by HVs and AVs. The thesis further investigates the effect of the AV penetration on network performance and defines a new theoretical threshold (or saturation point) beyond which there is no further improvement to the network performance (i.e., minimization of the total system travel time) as the AV penetration increases. This introductory chapter presents the motivation for the research, followed by a statement of the research aims and objectives. The last section outlines the organization of the thesis.

1.2 Background and motivation

The world is changing day by day, including the transportation system, which has been transformed rapidly by technologies. However, we still encounter many of the same transport problems: congestion, pollution, accidents, equal accessibility, and increased transport cost. Gradually we have learned that old issues do not disappear, although strategic transport modelling and decision making can reduce them. On the other hand, old problems appear in new forms and seem more complex and challenging to handle. In the past, the transportation system's major development depended on infrastructure improvement; for instance, increased road capacity was mainly done through roadways expansion. But nowadays, mobility and the advancement of the modes of transport are improving gradually due to technological enhancement. To upgrade the transportation system, automated driverless vehicles, electronic payment (e.g., smartcards, video tolling), information technologies are advancing consecutively.

Technological development has progressed significantly over the past two decades in the area of vehicle automation, with a particular emphasis on autonomous driving systems. The idea of automated highways is yet seen as futurist and science fiction, although it was first introduced in 1939 by Norman Bel Gedde at New York World’s Fair sponsored by General Motors. Since then, many research projects have shown that AVs can significantly improve the transportation system. Over the past two decades, technologies for vehicle automation and automated driving systems have developed. According to a report by KPMG (2019) of AV readiness index, or preparedness for AV, now 30 countries in the world have been grappling with policy and investment decisions needed in order to enable AV deployment, thus indicating a significant revolution in the field of automation technology. In addition, most leading car manufacturers are currently designing and testing their prototype AVs for imminent deployment.

Autonomous vehicles have the potential to transform transportation systems and reshape the future of mobility fundamentally. AVs can significantly enhance traffic safety by reducing human errors associated with manual driving (Assidiq et al., 2008; Das, 2018; Papadoulis et al., 2019; Shladover et al., 2012). They can facilitate and aid the mobility of elderly and disabled people (Harper et al., 2016), save fuel (Mersky and Samaras, 2016), and lower emissions (Greenblatt and Saxena, 2015). Furthermore, AVs can improve both the capacity of transport links and transport networks' efficiency, resulting in less congestion (Friedrich, n.d.; Tientrakool et al., 2011). Additionally, Fagnant and Kockelman (2015) show that greater AV penetration can reduce vehicle ownership, providing more parking and green spaces in overcrowded metropolitan areas.

Despite the potential advantages, the deployment of AVs will not happen overnight and on a large scale; instead, it is incremental and in stages (Farah, 2016; Lee, 2020). One of the significant barriers for the large scale market adoption of AVs is the cost, including the required upgrade and investment to the existing infrastructure to meet stringent requirements for AVs such as quality of the roads and lane markings (Assidiq et al., 2008; Fernandez et al., 2014), or the connectivity to support AV operation (Martínez-Díaz and Soriguera, 2018). Although many studies have focused on advancing the technical features of AVs, there have been few on the deployment of AVs and their impact on network performance (Correia et al., 2016). Since AVs will only be deployed in stages and specific areas, the resulting traffic flows will consist of both HVs and AVs operating in the same network. It is worth noting that during the incremental deployment, AVs are expected to operate autonomously only within a dedicated link or zone while being driven by humans outside these areas (Chen et al., 2017). Such a mixed traffic network's performance will largely depend on the design and strategy for AVs deployment, which still has not been fully explored in the literature.

1.3 Problem statement

One of the major problems in the transportation system is congestion that causes long travel time and loss of productivity. Traditional solutions to combat congestion include expanding road infrastructure. However, the improvement of the existing infrastructure needs to be combined with technological advances, including vehicular automation. AVs are designed in such a way that they are connected with the infrastructure and access to information of other AVs as well as travel conditions. As AVs work autonomously, they choose the best decision to reduce the network's overall travel time, which ultimately results in congestion reduction. However, due to the unavailability of the large number of market adoption of AVs, it is necessary to make strategies so that both HVs and AVs can exist in the same network while the overall network performance is improved. In this context, the problem of this study is the optimal deployment of AVs in the different multi-lane AV zones in a mixed-user network. In particular, the main objective is to model and evaluate different AV zone designs where HVs and AVs can travel together to achieve better network performance and thus AVs' optimal deployment. Note that in this research, only private demand and no shared mobility is considered. The research's primary focus is on identifying which strategic design of AV zone deployment can be effective. To the best knowledge, deploying AVs as a zone with different multi-lane designs considering different routing behaviours of HVs and AVs remains unexplored in the literature.

1.4 Research Aim and Objectives

The broad aim of this research is:

to develop a framework for the deployment of AVs with different multi-lane AV zone designs in a mixed-user traffic network that considers the different route choice decisions of HVs and AVs in order to enhance and evaluate the performance of the network in terms of travel system travel time.

In order to achieve this main aim, the key objectives are as follows:

1) To model a transport network with an AV zone and different multi-lane designs in a mixed-user traffic network.
2) To apply the model to evaluate and compare the network performance for the AV zones with two different multi-lane designs in a realistic city-size network.

1.5 Contributions of this research

The main contributions of this thesis are:

1) A new unified mathematical framework for modelling and deployment of multi-lane AV zone in a mixed-user traffic network. The framework provide means to evaluate different multi-lane AV zone designs.
2) A simpler equivalent convex formulation with linear constraints is transformed from the proposed framework. The simple equivalent convex formulation is suitable for solving the mixed routing network equilibrium.
3) A new solution algorithm: fixed point solution algorithm is developed to solve the Mixed Routing Equilibrium (MRE) model, which is efficient in solving larger networks as it requires only a few iterations to converge.
4) Mathematically shows that there is an AV penetration (referred to as a saturation point) where the network performance is maximized in terms of the minimization of the total system travel time for any design of the AV lanes inside the zone. The number of AVs entering the zone remains the same as AV penetration increases beyond the saturation point.
5) The numerical results show that with the flexible design of AV dedicated lanes inside the zone, the network performance can be improved at a smaller AV

penetration. However, in design 1 where HVs are not allowed to enter the AV zone, the network performance degrades at smaller penetrations.

1.6 Organization of this thesis

Figure 1.1 presents the structure of this thesis. The thesis is divided into four parts and is made up of six chapters.

- Part I (Chapters 1-3) covers background, literature, and methodology,
- Part II (Chapter 4) consists of the model formulation and solution algorithm for two designs for the deployment of AVs,
- Part III (Chapter 5) discusses the numerical results of two designs,
- Part IV (Chapter 6) includes the thesis conclusion and recommendation.

Part I of the thesis is made up of three chapters. Chapter 1 introduces the background, the research aims, the objectives, and the thesis's overall structure. Chapter 2 presents a literature review on AV traffic deployment with mixed traffic with both HVs and AVs. In Chapter 2, the deployment methods and design studies of AVs are reviewed, and the gaps or limitations in the current knowledge are presented. In Chapter 3, there is an overview of the research methodology used in this study.

Part II of the thesis consists of Chapter 4, which focuses on developing a mixed-user mixed routing traffic assignment model with different multi-lane AV zone designs. The model can capture different route choice principles of HVs and AVs. Two multi-lane AV zone designs are developed in order to compare their network performance with mixed traffic. Among the two designs, one design is referred to as extreme design, in which both lanes are for AVs inside the AV zone, and another design is the balanced design where one lane is for AVs, and another lane is for mixed traffic inside the AV zone. The solution algorithm of the model is also presented in Chapter 4.

Part III consists of Chapter 5, in which the numerical results are presented for the small (grid) network and the city-size network (city of Melbourne in Australia). Based on the numerical results, different comparisons: flows, total system travel time, average travel time of the small and Melbourne network have been discussed between two designs for the deployment of the multi-lane AV zone with the mixed traffic.

Part IV, Chapter 6 is the concluding chapter of the thesis and presents key findings and their implications, the limitations of the research, and future research directions.

PART I: BACKGROUND, LITERATURE, AND METHODOLOGY

CHAPTER 1: INTRODUCTION
Background, research aim, objectives, and thesis structure

CHAPTER 2: LITERATURE REVIEW
Review of literature and identification of research gaps

CHAPTER 3: RESEARCH METHODOLOGY
Research methodology used in the research

PART II: DEVELOPMENT OF MIXED-USER MIXED ROUTING TRAFFIC ASSIGNMENT MODEL WITH MULTI-LANE AV ZONE DESIGN

CHAPTER 4:
Model formulation and solution algorithm for design 1 & 2

→ **Original contributions:**

- Model formulation, theoretical analysis of the saturation point for different AV's penetration.
- A new and efficient solution algorithm for solving complex and larger network.

PART III: NUMERICAL RESULTS OF TWO DESIGNS

CHAPTER 5:
Numerical results of two multi-lane AV zone designs

→ **Original contribution:**

- Numerical results showing the comparison of the two different multi-lane AV zone designs.

PART IV: CONCLUSION AND RECOMMENDATIONS

CHAPTER 6: CONCLUSION AND RECOMMENDATIONS
Key findings and contributions, critique, future research

Figure 1.1: Thesis Structure.

CHAPTER 2 LITERATURE REVIEW

2.1 Introduction

This chapter aims to provide a review of the existing literature on modelling and designing transport networks involving the AVs with mixed traffic and their effect on the network's performance. In this thesis, the mixed traffic is considered in terms of mixed routing behaviours where HVs follow the UE routing principle, whereas AVs (in autonomous mode) follow the SO routing using the dedicated AV lanes inside the AV zone. Moreover, outside the dedicated lanes, AVs are driven by humans and behave as HVs, i.e., follow the UE routing.

The objectives of this review are to provide an understanding of:

- Studies of different routing behaviours of HVs and AVs (Section 2.2)
- Deployment of AVs mixed with HVs in traffic (Section 2.3)
- Deployment of AVs in separated a lane/link with or without routing consideration (Section 2.4)
- Deployment AVs in separated zone/subnetwork with or without routing consideration (Section 2.5)
- Deployment of AVs with different penetration (Section 2.6)

This chapter is organized following these objectives. It concludes with a summary of the gaps in knowledge identified through this literature review that provide the motivation for this research.

2.2 Different routing behaviour studies for HVs and AVs

From the literature, it is well known that HVs behave selfishly in the transport network with routing decisions that minimize their own travel time. The resulting route choice in the equilibrium is referred to as user equilibrium (or UE), defined by Wardrop's first principle (Wardrop, 1952). By contrast, AVs in autonomous mode are willing to cooperate to reduce congestion and make more efficient use of the network. AVs' routing thus follows a system optimum principle that minimizes the average travel time in the transport network as a whole. Such different routing principles lead to the mixed routing problem of the network.

The concept of mixed routing equilibrium was first introduced by Haurie and Marcotte (1985) and Harker (1988). In a classic UE problem, users do not cooperate at all, and they competitively use the common network resources, while users in an SO system fully cooperate as a single entity. However, because AVs can only operate in autonomous mode within a dedicated lane/link or zone, while outside these facilities they behave like HVs, the resulting mixed traffic routing of HVs and AVs presents a challenging research problem reviewed below.

2.3 Deployment of AVs mix with HVs in a network

In the literature, various frameworks have been reported for investigating the different options for deploying AVs in mixed traffic. Kala et al. (2013), Bailey et al. (2016), Bose et al. (2003), Olia et al. (2018) simulated AVs in mixed traffic through specific micro-simulation software to analyze the mixed traffic flow. Kala et al. (2013) formulated their problem using Matlab, considering the scenario of an infinite straight road without speed lanes defined. Their proposed planning algorithm within mixed traffic revealed that driver behaviour plays a vital role in overall traffic dynamics, although network performance was not evaluated. Bailey et al. (2016) used two different car-following models for HVs and AVs where a variant of the Gipps car-following model is used for HVs while the enhanced intelligent driver model was considered for AVs. Two different scenarios were studied: the effect of a traffic signal and a lane merge were simulated with mixed traffic and varying penetration rates of AVs. They concluded that high AV penetration leads to an increase in the lane capacity while decreasing the average travel time in traffic signals. Olia et al. (2018) also used the car-following and lane-merging modules to model the behaviour of AVs technologies in mixed traffic. They concluded that if AVs are fully autonomous (i.e., without cooperative systems), road capacity can increase up to 109% for an AV penetration rate of 100%. In contrast, if AVs are connected and driven in a cooperative automated manner, road capacity can improve up to 315%. In 2003, Bose and Ioannou (2003) analyzed the effects on traffic-flow characteristics and the environment when semiautomated and automatic vehicles operate together with manually driven vehicles. Their results demonstrated that semiautomated vehicles help smooth traffic flow by filtering the response of rapidly accelerating lead vehicles. Along with this, their environmental findings suggested that fuel consumption, monoxide pollution levels, and CO_2 emissions were reduced up to 3.6%, 19.2%, and 3.4% respectively with an AV penetration rate of 10%. To find the impact on traffic disturbance, efficiency, and safety of the mixed traffic flow of AVs and HVs, in 2021, Sharma et al (2021), utilized intelligent driver model (IDM) with estimation errors since it incorporates human factors such as estimation errors.

To study the deployment of AVs with HVs, a bi-level network design model has been proposed by several researchers (Chen et al., 2017, 2016; Conceição et al., 2017; Madadi et al., 2018; Movaghar et al., 2020; Ye and Wang, 2018). They have proposed both AV zone or AV link designs and a mixed user equilibrium (MUE) model formulation using a bi-level optimization framework where the upper level is to design the AV zone or links, and the lower level formulates mixed user routing behaviour. Wang et al. (2019) proposed the MUE model for the multi-user class using a multiclass traffic assignment model to analyze the evolution of the mixed traffic flows in transportation networks. In addition, Chen et al. (2017) and Wang et al. (2019) formulated mixed routing equilibrium (MRE) models by proposing a variational-inequality (VI) based traffic assignment model. Using the same approach based on the VI formulation, Chen et al. (2016), Liu and Song (2019), and Movaghar et al. (2020) formulated their MUE model.

2.4 Deployment of AV dedicated lanes/links with/without routing consideration

To address the problem of the optimal deployment of AVs, different approaches are suggested in studies: dedicated AV lanes (Chen et al., 2016; Conceição et al., 2020; Liu and Song, 2019; Mahmassani, 2016; Movaghar et al., 2020; Razmi Rad et al., 2020; Van Arem et al., 2006; Wang et al., 2019) or dedicated AV links (Ye and Wang, 2018). In particular, dedicated lanes to AVs using existing infrastructure is one of the most flexible and low-cost strategies for deployment due to the high uncertainty in AVs' penetration rate (Movaghar et al., 2020). Several studies have suggested lane allocation as dedicated AV lanes (Levin and Boyles, 2016, 2015), where mixed routing, i.e., the interaction between HVs and AVs in the AV dedicated lanes is not considered.

Liu and Song (2019) developed an AV’s deployment model by considering a new form of managed lanes for AVs, designated as autonomous vehicle/toll (AVT) lanes, which provide free access to AVs while allowing HVs to access the lanes by paying a toll. Wu et al. (2020) have formulated a mathematical framework for a linear traffic corridor with dedicated AV expressways and human-driven local streets using the UE and SO principles to evaluate exit locations' effect on future AV expressway planning. Unlike the deployment of dedicated

AV expressways, Scherr et al. (2019) have modelled service network design with AVs (Society of Automotive Engineers (SAE)level 4) in platoons considering a heterogeneous infrastructure wherein such a platoon may only drive in AV zones but require a HV to guide them elsewhere. Chen et al. (2016), Movaghar et al. (2020); Liu and Song (2019) used metaheuristic algorithms for the deployment of AVs using dedicated AV lanes, whereas Wang et al. (2019) used a route-swapping based algorithm for the deployment of AV dedicated lanes and formulate a Multiclass traffic assignment model to estimate the mixed traffic flows of AVs and HVs.

In the paper of Chen et al. (2016), the objective function of the upper level is the deployment of AV lanes and the forecasting of AV penetration, while the MUE model is formulated at the lower level. They developed their problem using the active set algorithm and AV diffusion mode. Their results show that with the gradual increase of AV penetration, AV lanes can be deployed simultaneously when the AV market penetration rate reaches above 20%. In 2020, Movaghar et al. (2020) also studied the deployment of AVs considering the capacity changes of a roadway shared by AVs and HVs. They also formulate their problem also as a bi-level network design model where the objective function of the upper level is to determine the AV lane locations, and the objective function of the lower level is to develop the MUE model using VI formulation. They use hybrid machine learning and an optimization algorithm that employ a multivariate linear regression and an integer linear program to solve their bilevel problem iteratively. Their results show that with a fixed lane capacity, the total system travel time of the network can be improved up to 3.85% at 60% AV penetration, while the improvement can be up to 9.88% with a variable capacity.

Regarding the deployment of AVs as dedicated lanes, both Liu and Song (2019) and Wang et al. (2019) develop their MUE problem using VI formulation, but Liu and Song (2019) use the convex optimization method while Wang et al. (2019) use the RSRM method to solve their problem. In contrast to the deployment of AVs as dedicated AV lanes, Yipeng et al. (2018) deploy AVs as dedicated AV links (i.e., stretches of roads dedicated to AVs) where pricing is imposed for those links considering the variational AV market penetration. To solve the bi-level problem, they use the relaxation algorithm and the outer approximation algorithm to solve the upper level and lower level problems, respectively. Their results indicate that the bi-level network design model is more effective than congestion pricing at a higher AV market penetration rate and vice versa.

As discussed above, AV lanes are considered in Yipeng et al. (2018), Chen et al. (2016), Movaghar et al. (2020), and Liu and Song (2019), where the authors formulate the multiclass user equilibrium using UE route choice for both HVs and AVs, while Wang et al. (2019) use different route choice principles, i.e., AVs follow the UE routing principle but HVs follow the cross-nested logit (CNL) model to capture the uncertainty of HVs as well as to overcome the route overlap issue whereby the flows on routes with overlapping links are overestimated. It has been shown that deploying AVs as dedicated lanes is an effective way to amplify AVs' capacity-improvement benefit and boost AVs' market penetration. However, the implementation of dedicated lanes for AVs would also worsen the congestion due to the forced detouring of HVs (Conceição et al., 2017). Furthermore, it is also impractical to deploy dedicated AV lanes throughout the network, as the dedicated AV lanes may be underutilized, reducing the network's efficiency when the penetration of AVs is relatively low (Liu and Song, 2019). Note, in this thesis, it is assumed that AVs need a dedicated lane to operate in autonomous mode.

2.5 Deployment of AVs in separated zone/subnetwork with or without routing consideration

In contrast to the AV link deployment, Chen et al. (2017), Conceição et al. (2017), Madadi et al. (2018) formulate the problem of optimal deployment of AVs within a zone. In particular, Chen et al. (2017) developed a theoretical framework for deploying an AV zone and consider mixed routing of HVs and AVs where AVs can travel through the zone, but HVs cannot enter the zone. Their bi-level optimization model consists of two levels where the objective function of the upper level is the set up of the AV zone and in the lower level, they develop the MRE model using VI formulation. The authors then develop SAA (simulated annealing algorithm) to solve their problem. Using mathematical programming, Conceição et al. (2017) developed AV dedicated zones in which a walking penalty is imposed where HVs can not access, and their bi-level problem is solved through the branch-and-bound method. In the paper of Madadi et al. (2018), they have developed three disconnected AV sub-networks/zones by metaheuristic algorithms and used the monte carlo-labeling in combination with path size logit (PSL) approach to capture the mixed traffic, where vehicles are allowed everywhere in the network while AVs are operating autonomously within the zone. While Conceição et al. (2017), Madadi et al. (2018) consider the same UE routing principle for both AVs and HVs, Chen et al. (2017) formulate the mixed traffic of HVs and AVs using two routing principles i.e., UE for HVs and SO for AVs within the zone.

2.6 Deployment of AVs with different penetration

The performance of the transportation network not only depends on the deployment of the AVs but also their penetration rates (Calvert et al., 2017; Chen et al., 2016; Nieuwenhuijsen et al., 2018; Tientrakool et al., 2011; Van Arem et al., 2006). If AVs are deployed in a transportation network with different penetration rates, there will be an effect on the network performance for different penetration rates. For example, Tientrakool et al. (2011) suggested that highway capacity can be increased up to 43% for 100% of AVs and vehicle-to-vehicle connectivity, but it can be increased up to 270% if vehicle-to-infrastructure communication is present. Calvert et al. (2017) suggested and validated through an empirical study and simulation that a low penetration rate of AVs may negatively affect traffic flow and road capacities due to higher gap times in the early stages of deployment. They also concluded that further improvement in traffic performance would only be seen at penetration rates beyond 70%. Focusing on AVs' implementation strategies and operational concerns, Masoud and Jayakrishnan (2017) developed a model introducing shared ownership of AVs, where they have indicated that AVs ridership can reduce the number of vehicles, higher AV penetration may affect the travel time in the network. In 2006, Van Arem et al. (2006) presented a scenario considering a highway merging from four to three lanes where they adapt the MIXIC simulation model to incorporate the CACC system features with different penetration rates of AVs. Their result reveals that there is a slight increase in the traffic-flow efficiency beyond the penetration rate of 40%. Furthermore, Nieuwenhuises et al. (2018) developed a model using systems dynamics under a functional approach where the diffusion of AVs is studied as well as six levels of automation with different fleet sizes, technology maturity, and average purchase price and utility have considered. Their results indicate that the introduction of SAE level-5 AVs will be possible by 2045. AVs level 5 with 90% of the market penetration will be possible between 2060 -2080. After 2100, 100% of AVs of level 5 will only possible.

During this transition period to full automation of AVs, the issue of AV market penetration rate is very important as it creates a major impact on the network performance discussed above. Obtaining the full automation of AVs with 100% penetration will take decades. Thus, for a long time, the heterogeneous traffic flow is inevitable, and it is necessary to deploy

AVs strategically with a lower penetration rate. Proper deployment designs for the deployment of AVs with the consideration of different aspects of AVs are required to leverage the advantages of AVs in the transportation network.

Table 2.1 shows the summary of the relevant literature regarding the deployment of AVs.

2.7 Knowledge gaps

This review of the literature to date regarding the deployment of AVs in the transportation network has identified clear gaps in the knowledge as follows:

1) Although several studies have considered the mixed traffic condition for the deployment of AVs (Chen et al., 2017, 2016; Liu and Song, 2019; Madadi et al., 2018; Movaghar et al., 2020; Ye and Wang, 2018), only a few (Chen et al., 2017; Wang et al., 2019) have considered the mixed routing principle. More importantly, no existing studies have evaluated different designs for the deployment of autonomous vehicles, nor have they investigated a combination of dedicated lanes and zones to cater for different traffic demands and AV penetrations.
2) To formulate the mixed route choice model for HVs and AVs, most researchers use VI formulation. One of the significant drawbacks of this formulation is that it is difficult to solve, and thus previous studies often resort to heuristic methods in order to obtain solutions.
3) When AVs are deployed with different AV penetration rates, there is a point of AV penetration (which is referred to as the saturation point in this thesis) beyond which either there is no change in the network performance, or the network performance is maximized. Note that in this thesis network performance is measured in the total system travel time (TSTT). Although the literature identifies different impacts of different AV penetration rates, there has been limited discussion of the impact of the saturation point on the performance of the whole transportation network.

To address the first gap discussed above, this thesis has developed a unified MUE framework to design and evaluate the AV deployment with mixed routing considering different designs to achieve a better network performance. Furthermore, the proposed framework allows the optimization of lane allocation for AVs inside a zone that tackles the underutilization issue mentioned previously.

Due to the complexity of the mixed route choice model for HVs and AVs and the non-linearity setting of the problem, this thesis has proposed a new solution algorithm by deriving and integrating the problem into a single convex formulation with linear constraints. The advantage of the proposed algorithm is that, in each iteration, it finds the descent direction to circumvent solving the computationally intensive problem. This will address the second gap discussed previously.

To address the third gap, this thesis has mathematically derived an expression for the saturation point where the performance of the network is maximized, and it has shown that the number of AVs utilizing the AV zone remains the same for any AV penetration values beyond that point. This also infers that HVs and AVs have the same end-to-end travel time at the saturation point. In other words, at this point, AVs do not have an incentive to switch to autonomous (or self-driving) mode within its journey from O to D. Note that because the AV zone is within the network, the end-to-end journey will always involve leg(s) where AVs being driven by human (i.e., behave like HVs).

2.8 Summary

In this chapter, a review of the relevant literature has been presented on the development of AVs with mixed traffic in the transportation network. Different aspects for the deployment of AVs, for example, dedicated lanes/links, dedicated zone, different routing principles, and penetration rates of AVs have been discussed in this chapter.

To address the knowledge gaps discussed above, this research project aims to obtain a deeper knowledge about how to deploy in assessing the deployment of AVs with mixed traffic in the transportation network. In the next chapter, the research methodology will be presented in detail.

Table 2. 1: Summary of literature regarding the deployment of AVs.

Reference	AV deployment		Formulation/ Approach			Method/ Algorithm	Routing Principle
	Dedicated lanes/links	Zone/ Sub-network	Bi-level network design problem (NDP)		Other		
Chen et al. (2017)	---	✓	✓	Upper Level- Set up the AV zone Lower Level-MRE model-VI	---	Upper Level-SAA Lower Level-Convex Optimization	HVs-UE AVs-SO
Conceição et al., (2017)	---	✓	✓	Upper Level- Selection of AV dedicated links Lower Level-MUE Model	---	Branch-and-bound (Mathematical method) and Successive linear programming	HVs-UE AVs-UE
Bahman Madadi et al. (2018)	---	✓	✓	Upper Level -GP which entails the choice of AV links Lower Level-MUCSUE Model Monte Carlo-labeling combination and PSL	---	Upper level-GA/SAA/SPSA Lower level-A linear approximation algorithm	HVs-UE AVs-UE
Yipeng Ye et al. (2018)	✓	---	✓	Upper Level – Network Design and Congestion pricing Lower Level-MUE model	---	Upper Level-Relaxation algorithm Lower Level-Outer-approximation algorithm	HVs-UE AVs-UE
Chen et al. (2016)	✓	---	✓	Upper Level – Deployment of AV lanes and forecast	---	Active set algorithm and AV diffusion mode	HVs-UE AVs-UE

Note: '✓' indicates the use of such a factor or technique

Reference	AV deployment		Formulation/ Approach			Method/ Algorithm	Routing Principle
	Dedicated lanes/links	Zone/ Sub-network	Bi-level network design problem (NDP)		Other		
				of AV penetration Lower Level-MUE model			
Sara Movaghar et al. (2020)	✓	---	✓	Upper Level – Location of AV lanes deployment Lower Level-MUE model-VI; considering the capacity variations	---	Hybrid machine learning and optimization: multivariate linear regression and an integer linear program	HVs-UE AVs-UE
Zhaocai Liu et al. (2019)	✓	---	---	---	MUE model-VI	Convex Optimization	HVs-UE AVs-UE
JianWang et al. (2019)	✓	---	---	---	Multiclass traffic assignment model-VI	RSRM	HVs-CNL AVs-UE

Note: '✓' indicates the use of such a factor or technique

CHAPTER 3 RESEARCH METHODOLOGY

3.1 Introduction

The previous chapter provides a literature review of the deployment of AVs and their impacts on network performance based on different deployment strategies. The review identifies research gaps and discusses opportunities to advance knowledge in addressing these gaps.

This chapter describes the overall research approach to address the identified research gaps and achieve the objectives outlined in Section 1.4, including the mixed-user, mixed routing behaviour, and traffic assignment problem based on Wardrop's principle (Wardrop, 1952). Theoretically, there are two approaches for the traffic assignment: User equilibrium (UE) and System optimum approach (Sheffi, 1985). In this study, these two routing principles are considered to solve the mixed routing equilibrium (MRE) problem. User Equilibrium (UE) principle is for HVs and System Optimum (SO) principle is for AVs.

3.2 User Equilibrium (UE) routing principle

UE principle is based on John Glen Wardrop's first principle of route choice. According to this principle, travelers will select a route so as to minimize their personal travel time between their origin and destination. User equilibrium (UE) is said to exist when travelers at the individual level cannot unilaterally improve their travel times by changing routes. Here this principle is used because HVs behave selfishly and the traveler selects the path with the shortest TT (Travel Time), which is the major criterion. The formulation of the Wardrop equilibrium for a general network was first formulated as a mathematical programming problem by Beckmann et al. (1956).

3.3 System Optimum (SO) routing principle

System Optimum Routing Principle is used when there are only AVs in the whole network. SO principle is based on John Glen Wardrop's second principle of route choice. According to this, preferred routes are those, which minimize total system travel time. With System-Optimum (SO) route choices, no traveler can switch to a different route without increasing total system travel time. Travelers can switch to routes decreasing their TTs, but only if System-Optimum flows are maintained. Realistically, travelers will likely switch to non-

System-Optimum routes to improve their own TTs. Here this principle is used because travelers cooperate with each other to provide a better network performance by minimizing the sum of the total system travel time in the network.

3.4 Overall research approach

The overall research approach includes two key research components, which were designed to achieve the two research objectives. The first research component focuses on deploying AVs with HVs in a MRE model with linear constraints, which are then applied with different penetration rates of AVs in the transportation network. The second component investigates the network performance with the consideration of AV's penetration rate and multiple AV zones to identify the best network performance that is achieved by the deployment of AVs. Linkages between research gaps, research component's objectives, subtasks, and outcomes are in Figure 3.2. The following sections present a brief description of each research component.

3.5 Research components

3.5.1 Research component 1: Modelling the transport networks involving the AV zone with mixed traffic for different designs.

Two designs are considered where the mix-routing strategy is applied for the whole network or only in the AV zone, respectively. In the first design, the AV zone is only for AVs. Outside the zone, both AVs and HVs can traverse, and AVs is driven by human and behave like HVs. In the second design, both HVs and AVs can operate in the whole network, but AVs can only in self-driving mode (e.g., freeway platooning) within the AV zone and in AV dedicated lane(s). Outside the zone, AVs are driven by humans and act similarly to HVs as in design 1. In this design, HVs can enter and exit the zone, but they are not allowed to mix with AVs; that is, HVs and AVs share the same road but use different lanes. For each design, UE model is formulated to calculate the flow distribution of HVs. SO model is formulated to calculate the flow distribution of AVs and MRE model is formulated either for the whole network or in the AV zone based on the designs. Figure 3.1 represents the two designs. After the formulation of the model for two designs, i.e., the research component 1, the next step is to evaluate the network performance i.e., the research component 2 to get the best design for the deployment of the AV zone.

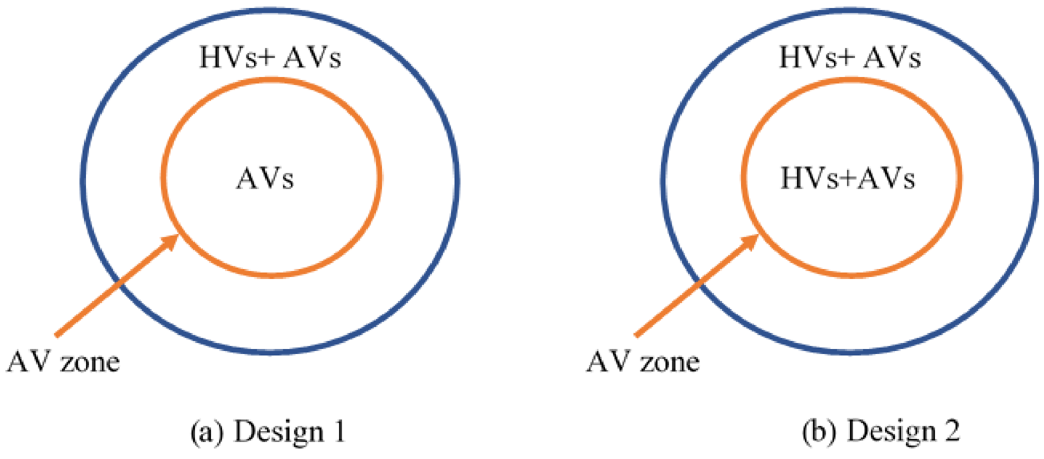

Figure 3. 1: Different multi-lane designs of AV zone

3.5.2 Research component 2: Evaluating the network performance for each design in order to find a better design and zone allocation in terms of the network overall performance.

The performance of a transportation network not only depends on the topology of the AV zone but also the penetration rate of AVs. So, to evaluate the network performance more precisely, we use different penetration rates of AVs. The average travel time of vehicles (AVTT) and total system travel time (TSTT) of the network are observed with the changing penetration rate of AVs for different designs. Here the traffic pattern after deployment of different penetration rates of AVs in a zone is also observed to understand the proper utilization of AV zone in the network. Multiple AV zones are also considered with different designs in order to find a better design and zone allocation in terms of the network overall performance.

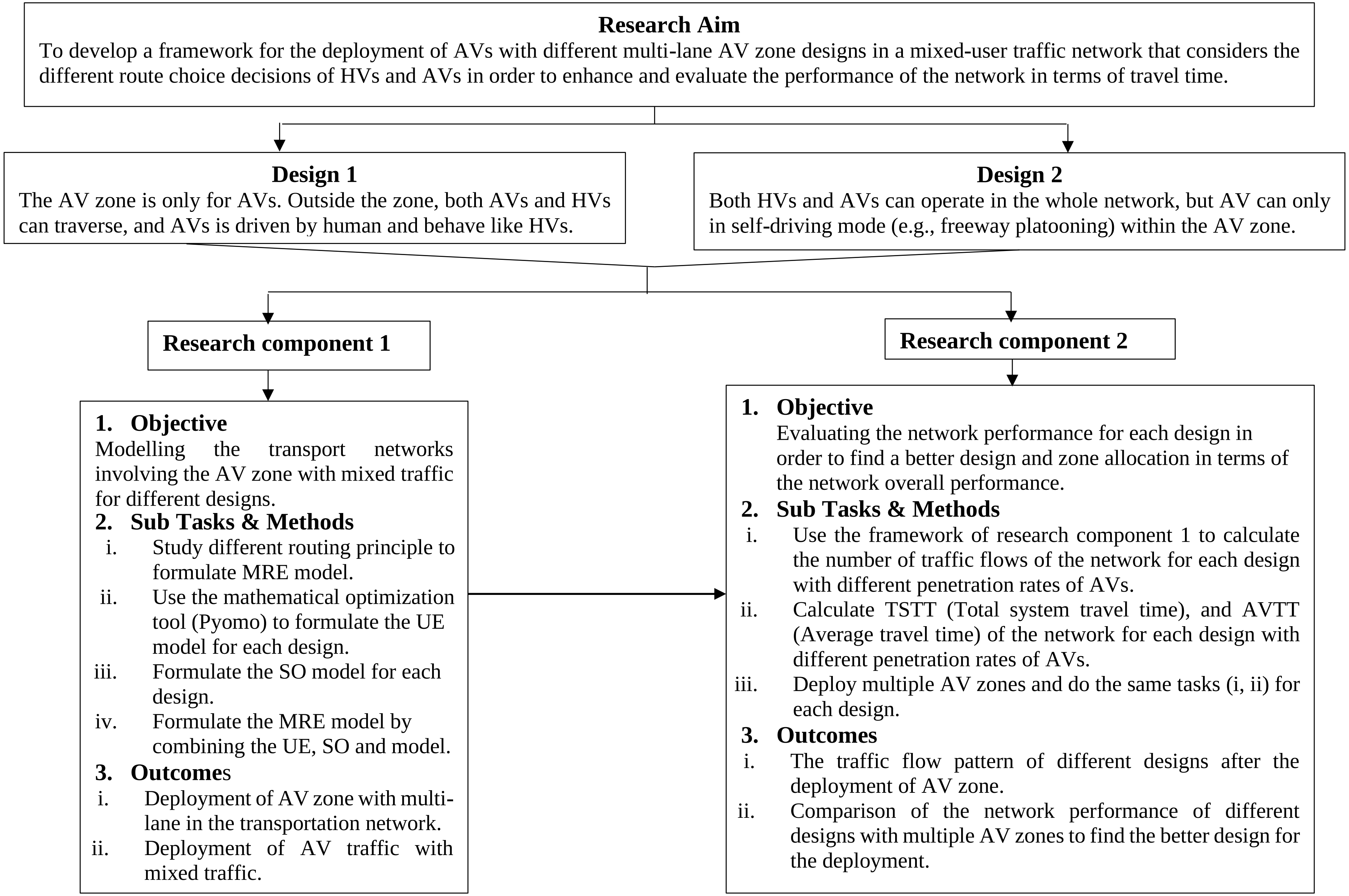

Figure 3. 2: Framework of research methodology.

3.6 Summary

This chapter has presented the overall research approach composed via two research components, in line with the two research objectives. The first research component focuses on modelling the multi-lane AV zone with different designs with mixed traffic considering the mixed routing behaviour that accesses the HV traffic inside the AV zone (design 2). The second research component focuses on the evaluation of the network performance after the deployment of AVs with multiple AV zones, which are then applied to identify the better design and zone allocation for the deployment of AVs.

The next chapter of this thesis presents the model that is developed to formulate mixed-user traffic assignment framework considering MRE model with multi-lane AV zone design and the solution algorithm.

PART II:

DEVELOPMENT OF MIXED-USER TRAFFIC ASSIGNMENT MODEL WITH MULTI-LANE AV ZONE DESIGN.

CHAPTER 4 MODEL FORMULATION AND SOLUTION ALGORITHM FOR DESIGN 1 & 2

4.1 Introduction

This chapter presents the results of research component 1, which focuses on the methodological development of the model for designs 1 and 2. The methodological issues of a general MRE model of mixed HVs and AVs flow with non-linear constraints and its solution algorithm. However, modelling the mixed routing behaviour is not easy as HVs follow the UE principle and AVs follow the SO principle. The aim of this chapter is to therefore propose a new model to formulate this problem where the different multi-lane AV zone designs are considered. A MRE model with non-linear constraint is first proposed, and then it is transformed into a simpler equivalent convex formulation with linear constraints suitable. A new solution algorithm is developed that can solve the problem associate with the simpler equivalent convex problem.

4.2 Problem formulation

In this thesis, the zone-based routing traffic assignment problem in a mixed-user network with AVs is studied. The notations used in this thesis are firstly shown then two possible designs of the AV zone of this work have been introduced and formulate them in a framework of traffic assignment.

4.3 Notations

A transportation network is generally represented by a directed graph $G = (N, A)$, where N is the set of nodes, and A is the set of the directed links connecting certain pairs of nodes. Let W denotes the set of Origin-Destination (O-D) pairs, $\widetilde{W}$ denotes the set of Entrance-Exit (E-E) pairs of the AV network. The Entrance-Exit pairs of the AV network indicate the entrance and exit nodes at the boundary of the AV zone where the AV traffic entering and leaving the zone. M denotes the set of modes or classes of vehicles. There are two possible modes: A and C for vehicles operated in autonomous mode or by human (conventional mode), respectively. Note that, AVs on links in $\tilde{A}$ will be on mode A, otherwise, they are in mode C. The list of sets, variables, and parameters used in this thesis is summarized in Table 4.1, while their definitions will be given in the later corresponding sections.

Let Z be the set of AV zones in the network where each zone $z \in Z$ is defined by a sub-network $G_z = (N_z, E_z)$ with a set of nodes N_z and a set of links E_z.

Table 4. 1: List of notations.

Set/Parameters	Description
$G(N, A)$	Set of nodes (N) and directed links (A) of normal network.
$\widetilde{G}\ (\widetilde{N}, \tilde{A})$	Set of nodes $(\widetilde{N})$ and directed links $(\tilde{A})$ of AV network.
Z	Set of AV zones in the network by $z \in Z$.
$G_z\ (N_z, E_z)$	Set of nodes (N_z) and links (E_z) in the AV zone by $z \in Z$.
Υ_i^-	Set of inflow links to node i.
$\Upsilon_i^{\mp}$	Set of outflow links from node i.
W	Set of Origin-Destination (O-D) pairs.
$\widetilde{W}$	Set of Entrance-Exit (E-E) pairs for the AV zone.
M	Set of modes or classes, including HVs and AVs.
P_m^w	A set of paths between O-D pair $w \epsilon W$ by mode $m \epsilon M$.
$\delta_i{}^{w,m}$	An element in the node-link incidence matrix (defined in Eq (4)).
d_w	Total travel demand between O-D pair including all modes.
d_m^w	Travel demand between O-D pair $w \epsilon W$ by mode $m \epsilon M$.
t_0	Free flow travel time on link $a \epsilon A \cup \tilde{A}$.
$c_a, c_{\tilde{a}}$	Flow capacity (in vehicle/min) on link $a \in A$ (for c_a) or on link $\tilde{a} \in \tilde{A}$ (for $c_{\tilde{a}}$).
$t_{0\tilde{a}}$	Free flow travel time (in minutes) on link $\tilde{a} \in \tilde{A}$.
n_a	A total number of lanes at link a.
$\widetilde{w}(p)$	The E-E pair $\widetilde{w} \in \widetilde{W}$ on path $p \in P$.
$w(p)$	The O-D pair $w \in W$ on path $p \in P$.
α	An element of BPR function.
β	An element of BPR function.
Variables	**Description**
$d_z^{\widetilde{w}}$	Travel demand between E-E pair $\widetilde{w} \in \widetilde{W}$ in AV zone z.
$x_a^{w,m}$	The number of class-m vehicles moving on link a and belonging to O-D pair $w \in W$.
$\widetilde{x}_a{}^{\widetilde{w}}$	Link flow on link $a \in \tilde{A}$ for E-E pair $\widetilde{w} \in \widetilde{W}$.
v_a	Traffic flow on link $a \epsilon A \cup \tilde{A}$.

$t_a(v_a)$	Travel time on link $a \in A \cup \tilde{A}$ given flow v_a.
r_a	The portion of road segment for the dedicated AV traffic
n_a^A	The number of the dedicated lanes for AVs on link *a*.
$y_a^{\tilde{w}}$	is defined as $\frac{\tilde{x_a}^{\tilde{w}}}{d_z^{\tilde{w}}}$ in Eq. (9)

4.3 Multi-lane designs of AV zone

Road segments in the AV zone can have multiple lanes where some lanes are dedicated to AV traffic only, and the other lanes are shared by all vehicles. In this research, the dedicated AV lanes are considered operating at higher flow capacity (more than 3 times than the normal lanes) due to the advantage of AVs, as shown in the literature (Shladover et al., 2012; Tientrakool et al., 2011; Van Arem et al., 2006). Depending on the number of available lanes, in this study different configurations or designs of lanes for AVs have been proposed. For each road a, n_a is defined as the number of lanes and n_a^A be the number of dedicated lanes for AVs only. Let $r_a = \frac{n_a^A}{n_a}$ represent the portion of the road segment for AVs only. Basically, r_a is the ratio of the number of lane(s) within a link dedicated to AVs in autonomous mode over the total number of lane(s) in that link. Let c_a be the flow capacity for each lane in link in a in a mixed traffic or pure HV traffic and $\tilde{c_a}$ be the flow capacity for each dedicated lane in link a ($\tilde{c_a} \gg c_a$). When link a is divided into two parts (one for AVs only and the other for mixed traffic), the dedicated part of link a has the capacity $r_a n_a \tilde{c_a}$ and the mixed traffic part of link a has the capacity $(1 - r_a) n_a c_a$.

The AV zone is the area where it is considered any link in this area can be divided into two parts as above while any link outside of this zone is simply for mixed traffic. One typical setting for AV zone is that $r_a = 1$ for any link in this zone, which indicates that the zone only consists of AV dedicated lanes and it is called the ***design 1 (or the extreme design)***. Another typical design is $r_a = 0.5$ for any link in AV zone, which indicates half of AV dedicated lanes (for example: if a link has two lanes inside the zone then one lane is AV dedicated lane and another lane is normal lane), and it is called the ***design 2 (or the balanced design)***. In terms of routing behaviour, AVs will cooperate within the AV zone and follow a system optimum control while vehicles (either AVs or HVs) outside AV zones will behave selfishly to complete their trips.

In this research, the model formulation of design 1 and design 2 is the same. More precisely, design 2 is the general version of design 1 where between the two lanes, one lane is allocated for AVs, and the other lane is considered for mixed traffic. In this thesis, SAE levels 3 or 4 (but not level 5) is considered for AVs, i.e., AVs can either operate in autonomous mode or driven by human in manual mode.

4.4 The mixed-user mixed-routing traffic assignment model

In this model, the main decision variable $x_a^{w,m}$ is the amount of class- m vehicles moving on link a and belonging to O-D pair w while the main input to our model is d_m^w, the traffic demand of vehicles in class m for O-D pair w. With those inputs and variables, the flow conservation in the traffic assignment is formulated below:

$$\sum_{a\in \Upsilon_i^-} x_a^{w,m} - \sum_{a\in \Upsilon_i^+} x_a^{w,m} = \delta_i{}^{w,m} d_m^w \qquad \forall\, i \in N, m \in M, w \in W \qquad (1)$$

$$x_a^{w,C} = 0 \qquad \forall\, m \in M, w \in W, a \in \tilde{A} \qquad (2)$$

$$x_a^{w,m} \geq 0 \qquad \forall\, a \in A \cup \tilde{A}, w \in W, m \in M \qquad (3)$$

where the parameters $\delta_i{}^{w,m}$ indicates whether node i is a source or destination, specifically,

$$\delta_i{}^{w,m} = \begin{cases} 1 \\ -1 \\ 0 \end{cases} \qquad (4)$$

Here $\delta_i{}^{w,m} = 1$, if node i in the destination in the O- D pair w for the vehicle class m

$\delta_i{}^{w,m} = -1$, if node i in the source in the O- D pair w for the vehicle class m

$\delta_i{}^{w,m} = 0$, otherwise

In Eq. (2), it shows that the HV traffic cannot enter any AV lanes in the AV zone. For evaluating the travel time, the link travel time function $t_{ij}(v_{ij})$ has been derived according to the amount of traffic on each link, that

$$v_a = \sum_{w \in W} \sum_{m \in M} x_a^{w,m} \qquad \forall\, a \in A \cup \widetilde{A} \tag{5}$$

$$t_a(v_a) = t_a^0 \,(1 + \propto \left(\frac{v_a}{c_a}\right)^{\beta}) \qquad \forall\, a \in A \cup \widetilde{A} \tag{6}$$

where t_a^0 is the free-flow travel time and c_a is the flow capacity on link a. For simplicity without losing the generality, the original BPR function (BPR, 1964) is used to estimate the link travel time for any links.

4.4.1 UE routing for HVs

According to Wardrop's principles (Wardrop, 1952), the UE routing happens when no driver (HVs) can unilaterally change its route and improve his or her travel time. Let f_p^C denote the traffic flows on path p for an O-D pair w and t_p be the travel time on path p. In this context, this travel time t_p^C is equal to the accumulation of link cost function as below:

$$t_p^C = \sum_{a \in p} t_a(v_a) \tag{7}$$

This UE condition for the UE travel time π_w is presented as below:

$$f_p^C > 0 \Rightarrow t_p^C = \pi_{w(p)}^C$$

$$t_p^C \geq \pi_{w(p)}^C$$

where the unique O-D pair for each path p is presented as $w(p)$. They can be translated into the following complementarity constraint:

$$0 \leq f_P^C \perp t_p^C - \pi_{w(p)}^C \geq 0 \tag{8}$$

Eq. (8) indicates that both f_P^C and $(t_p^C - \pi_{w(p)}^C)$ should be positive and their product is zero. It means that one of the quantities has to be zero.

4.4.2 Travel time for passing through an AV zone

Let $d_z^{\widetilde{w}}$ be the demand of AV traffic for an E-E pair $\widetilde{w}$ in AV zone G_z. With that, the average travel time of AVs in autonomous mode within the zone is calculated in the following Eq. (9). Here the average travel time is used to calculate the overall travel time for a chosen path p for a single AV vehicle, which is described in Eq. (10) below.

$$t_z^{\widetilde{w}} = \sum_{a \in \widetilde{A}_z} \frac{\widetilde{x}_a{}^{\widetilde{w}} t_a}{d_z^{\widetilde{w}}} = \sum_{a \in \widetilde{A}_z} y_a^{\widetilde{w}} t_a \tag{9}$$

where $y_a^{\widetilde{w}} = \frac{\widetilde{x}_a{}^{\widetilde{w}}}{d_z^{\widetilde{w}}}$. Note that, the travel time through an AV zone is considered as a fixed travel time that equals to the minimum total travel time within the AV zone according to the SO routing principle.

4.4.3 Mixed UE-SO routing for AVs

In this problem, AVs follow the UE routing outside the AV zone while they follow the SO principle in the AV zone. The estimation of travel time for each AV is different from HV where the travel time inside the AV zone is estimated as in Eq. (9); hence the overall travel time for a chosen path p for AV is formulated as below:

$$t_p^A = \sum_{a \in p, a \in A \setminus \tilde{A}} t_a(v_a) + \sum_{z \in Z} t_z^{\widetilde{w}(p)} \tag{10}$$

The end-to-end routing behaviour for AV traffic is based on UE principle, that no AV can unilaterally change its route and improve his or her mixed-routing travel time, mathematically, presented as below:

$$f_p^A > 0 \Rightarrow t_p^A = \pi_{w(p)}^A$$

$$t_p^A \geq \pi_{w(p)}^A$$

These conditions are transformed into the following complementarity constraint:

$$0 \leq f_p^A \perp t_p^A - \pi_{w(p)}^A \geq 0 \tag{11}$$

Eq. (11) represents the same condition as Eq. (8).

In this study, in order to solve the main problem $\mathcal{P}$ (Eq. 16), the sub-optimization problem $\mathcal{P}_s$ (Eq. 12), for the AV traffic in any AV zone z must be solved first. As the AV traffic in any AV zone z also has to follow the system optimum assignment that the solution of $\widetilde{x}_a{}^{\widetilde{w}}$ is derived from the sub-optimization problem for each zone:

$$[\mathcal{P}_s] \; min \sum_{a \in \tilde{A}} v_a t_a \, (v_a) \tag{12}$$

$$\text{s.t:} \sum_{a \in \Upsilon_i^-} \tilde{x}_a^{\tilde{w}} - \sum_{a \in \Upsilon_i^+} \tilde{x}_a^{\tilde{w}} = \delta_i^{\tilde{w}} d_z^{\tilde{w}} \qquad \forall\, i \in \tilde{N}, \tilde{w} \in \tilde{W} \tag{13}$$

$$\tilde{x}_a^{\tilde{w}} \geq 0 \qquad \forall\, a \in \tilde{A}, \tilde{w} \in \tilde{W} \tag{14}$$

Note that, in $\mathcal{P}_s$, the constraints in Eqs. (13), (14) only apply for AV links or lanes, and the travel time $t_a(v_a)$ is calculated from Eq. (10). In this sub-problem link flows outside the AV zone, $x_a^{w,A}$, are considered as parameters. Furthermore, the relation between the link flow $x_a^{w,A}$ and $\tilde{x}_a^{\tilde{w}}$ is shown below:

$$v_a = \sum_{\tilde{w} \in \tilde{W}} \tilde{x}_a^{\tilde{w}} = \sum_{w \in W} x_a^{w,A} \tag{15}$$

In Eq. (15), the link flow $\tilde{x}_a^{\tilde{w}}$ is for the travel demand $d_z^{\tilde{w}}$ between E-E pair $\tilde{w} \in \tilde{W}$ in any AV zone z where, $x_a^{w,A}$ is for the travel demand d_m^w between O-D pair $w \in W$ by mode $m \in A$.

In summary, the overall problem for given dedicated AV lanes in AV zone is formulated as the following model $\mathcal{P}$:

$$[\mathcal{P}] \min T = \sum_{a \in A} v_a t_a(v_a) \tag{16}$$

$$\text{s.t: Eqs: } (1) - (11), (13) - (15)$$

The problem $\mathcal{P}$ is considered as the network performance evaluation model, where using this model, different performances for different lane designs can be evaluated. So, this model will provide achievable performance (minimized total system travel time) for a given lane design. The main decision variable of this problem is the AV traffic flow, as some AVs can decide whether or not to enter the AV zone in autonomous mode.

In the following section, the analysis of this model is provided to study its insights and properties for developing the solution method in this thesis.

4.5 Analytical studies for a given design of dedicated AV lanes

In this section, to study the relationship between the network performance and the given design of dedicated AV lanes, the proposed model has been analyzed. Firstly, it has shown that the above model with complementarity constraints can be transformed into a simpler convex model with linear constraints for the equilibrium travel time of AVs and AVs in a mixed-traffic network. Then the impact of AV penetration on the network is discussed.

4.5.1 An estimated convex model with linear constraints

The model $\mathcal{P}$ is a complex problem with two non-linear complementarity constraints (Eqs. (8), (11)). Hence, further analysis on model $\mathcal{P}$ is provided to enhance the solution method in the next section. Let's consider the following simpler model $\mathcal{P}_1$. Problem $\mathcal{P}$ is related to the link-based model whereas the problem $\mathcal{P}_1$ is the conversion of problem $\mathcal{P}$, based on the path-based model.

$$[\mathcal{P}_1] \min F = \sum_{a \in A \backslash \tilde{A}} \int_0^{v_a} t_a(x) dx + \sum_{a \in \tilde{A}} \frac{1}{\beta_a + 1} v_a t_a(v_a) + \sum_{z \in Z} \sum_{w \in W} d_z^w \, t_{w,z}^0 \tag{17}$$

$$\text{s.t: Eqs.} (1) - (5), (14) - (16)$$

where

$$t_{\tilde{w},z}^0 = \sum_{a \in \tilde{A}} \frac{\beta_a t_a^0 y_a^{\tilde{w}}}{\beta_a + 1} \tag{18}$$

Proposition 1. *The model $\mathcal{P}_1$ is equivalent to $\mathcal{P}$ if $\beta_a = \beta$ for any $a \in \tilde{A}$ and $t_{w,z}^0$ is constant for any O-D pair w in any zone z.*

Proof. See Appendix A.

Remark 1. *The problem $\mathcal{P}_1$ actually works for any travel time function $t_a(v_a)$ having the following property:*

$$v_a t'(v_a) = \beta(t(v_a) - \xi)$$

that the proof in Appendix A is still correct. Therefore,

$$\begin{aligned} \beta \frac{dv_a}{v_a} &= \frac{d(t(v_a) - \xi}{(t(v_a) - \xi)} \\ \Leftrightarrow \beta d\ln v_a &= d\ln(t(v_a) - \xi) \\ \Leftrightarrow \ln\,(t(v_a) - \xi) &= \beta \ln v_a + \xi_1 \\ \Leftrightarrow t(v_a) &= e^{\xi_1} v_a^{\beta} + \xi \end{aligned}$$

According to the above transformation, problem $\mathcal{P}_1$ can be applied to any travel time function $t_a(v_a) = e^{\xi_1} v_a^{\beta} + \xi$ where β, ξ, ξ_1 are parameters of real values.

Note that, model $\mathcal{P}$ is complex and non-convex due to its non-linear complementarity constraints (Eqs. (8), (11)), whereas model $\mathcal{P}_1$ is simpler than the original model $\mathcal{P}$ due to its linear constraints (Eqs: $(1) - (11), (13) - (15)$) and convex objective function. Based on this result, an algorithm is proposed to efficiently solve $\mathcal{P}$ by repeatedly solving $\mathcal{P}_1$ and updating $t^0_{w,z}$, accordingly in the solution algorithm.

4.5.2 The impact of AV penetration on network performance

In this study, it has been investigated the impact of AV penetration on network performance in terms of the total system travel time of the network for a given AV zone and dedicated AV lanes. Let T^* be the optimal network performance (i.e., TSTT) with 100% AV penetration in the model $\mathcal{P}$. Clearly, it is the best achievable network performance among all possible AV penetrations. But the best network performance can also be achievable with the lower number of AV penetration, which has shown in the numerical result section.

Let us consider the following model $\mathcal{P}_2$

$$[\mathcal{P}_2] \min D = \sum_{w \in W} d_w^A$$

$$\text{s.t: Eqs: } (2) - (11), (13) - (15)$$

$$\sum_{m \in M} \sum_{a \in \Upsilon_i^-} x_a^{w,m} - \sum_{m \in M} \sum_{a \in \Upsilon_i^+} x_a^{w,m} = \delta_i{}^w d_w \qquad \forall\, i \in N, m \in M, w \in W \tag{19}$$

$$\sum_{a \in A} v_a t_a(v_a) = T^* \qquad \forall\, i \in N, m \in M, w \in W \tag{20}$$

$$d_w^A + d_w^C = d_w \qquad \forall\, w \in W \tag{21}$$

Note that, the traffic demand for each mode d_w^m is considered as variable in problem $\mathcal{P}_2$, however, they are considered by a given total demand d_w between the O-D pair w. This $\mathcal{P}_2$ model aims to find the minimum AV traffic (or the minimum AV penetration) to achieve the best performance T^*. Let, d_w^{m*} be the outcome from the solution of this model. This solution is called the saturation point (beyond which there is no further improvement to the network performance i.e., minimization of the total system travel time) of the network.

Proposition 2. *Given the total demand d_w and the saturation point of AV demand d_w^{A*}. Then,*

i. $\pi_w^A \le \pi_w^C$ *for any* $w \in W$.
ii. *If AV chooses a route without passing any AV zone, then this route is also chosen by HV, hence HV and AV have the same equilibrium travel time.*
iii. *If the AV traffic demand exceeds the saturation points ($d_w^{A*} < d_w^A$), then AVs always choose routes passing through AV zones.*

Proof.

i. Due to the domain of feasible routes for AVs includes the domain of feasible routes for HVs, it leads to the conclusion that $\pi_w^A \le \pi_w^C$ for any $w \in W$and for any $\pi_w^A \le d_w^A \le d_w$.
ii. According to the UE condition Eq. (11), the route (without passing any AV zone) has the equilibrium travel time. As it is not passing any AV zone and the feasible routes for AVs include the feasible routes for HVs, this route is also chosen by HV from this O-D pair w (as $d_w^{A*} < d_w^A$). As any chosen route has the same equilibrium travel time for HV and AV, the shared routes between AV and HV lead to the conclusion that they have the same equilibrium travel time.

iii. Assume that AV chooses a route without passing any AV zones. Using the same argument in (ii), the AV traffic on the route (not passing through any AV zone) can be converted to HV traffic while any constraint will not be violated, including the UE conditions for both AVs and HVs. Hence, smaller d_w^A is obtained, which is contradict to the model $\mathcal{P}_2$ for the definition of saturation point. Therefore, at saturation point $d_w^{A*} < d_w^A$, AVs always choose routes passing through AV zones.

Remark 2. *According to the above proposition, this result also infers that if the saturation point is zero, then no AV traffic passing through AV zones. It means that AV zone is not useful for the network in this case. Additionally, if AVs and HVs have the same equilibrium travel time at the saturation point, then AVs have common routes with HVs. In this case, the number of AVs inside AV zones is also saturated as having more traffic in AV zone would increase the overall travel time of AVs while it might reduce the HV travel time. This point is shown via the numerical result.*

4.6 Solution method

In the previous section, this study shows that the original model $\mathcal{P}$ could be simplified to a more efficient convex linear-constraint model $\mathcal{P}_1$. In this section, it is aimed to propose an algorithm for solving the mixed-routing mix-traffic network based on this result. Figure 4.1 describes the flow illustration of the algorithm.

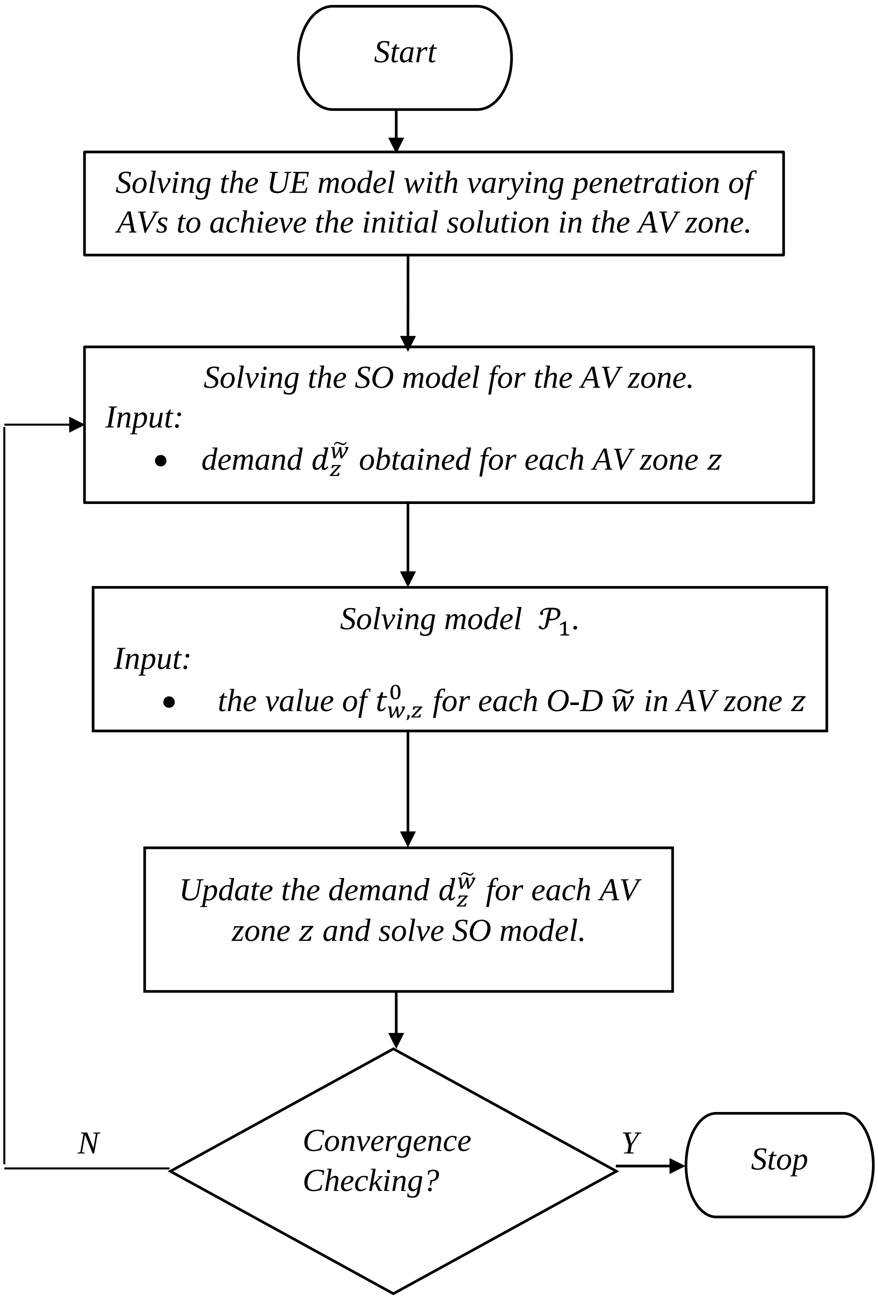

Figure 4. 1: Fixed point algorithm flow illustration.

The overall algorithm is described below.

- **Step 1:** Initialize the solutions. Set $i = 0$. The UE model is solved below to achieve the initial solution in the AV zone

$$[\mathcal{P}_U] \min U = \sum_{a \in A} \int_0^{v_a} t_a(x)dx$$

$$\text{s.t: Eqs. } (1) - (5)$$

- **Step 2:** Evaluate $t^0_{w,z}$. Update $i = i + 1$. From the solution in the previous step, the demand $d_z^{\widetilde{w}}$ is obtained for each AV zone z. Find $t^0_{w,z}$ for each O-D $\widetilde{w}$ in AV zone z by solving the SO model in this zone.

$$[\mathcal{P}_s]\ min \sum_{a \in \tilde{\tilde{A}}} v_a t_a(v_a)$$

$$\text{s.t: } \sum_{a \in \Upsilon_i^-} \widetilde{x}_a{}^{\widetilde{w}} - \sum_{a \in \Upsilon_i^+} \widetilde{x}_a{}^{\widetilde{w}} = \delta_i{}^{\widetilde{w}} d_z^{\widetilde{w}} \qquad \forall\, i \in \widetilde{N}, \widetilde{w} \in \widetilde{W}$$

$$\widetilde{x}_a{}^{\widetilde{w}} \geq 0 \qquad \forall\, a \in \widetilde{A}, \widetilde{w} \in \widetilde{W}$$

- **Step 3:** Calculate the demand $d_z^{\widetilde{w}}$. Solve $\mathcal{P}_1$ with the value of $t^0_{w,z}$ computed in Step 2. The result of demand $d_z^{\widetilde{w}}$ for each AV zone z will be put back to Step2.
- **Step 4:** Check the convergence with the error $\in$. The algorithm stops when the following condition is satisfied:

$$\sum_{\boldsymbol{a} \in \boldsymbol{A}} \left| \boldsymbol{v}_{\boldsymbol{a}}^{(\boldsymbol{i})} - \boldsymbol{v}_{\boldsymbol{a}}^{(\boldsymbol{i}-\mathbf{1})} \right| \leq \in$$

Otherwise, perform Step 4a, then return to Step 2.

- ***Step 4a:*** The following step is optional (before running to Step 2) to avoid the oscillation if any, by updating the demand $d_z^{\widetilde{w}(i)}$ for the iteration i according to the Method of Successive Average (MSA) (Sheffi, 1985) below:

$$d_z^{\widetilde{w}(i+1)} = \frac{d_z^{\widetilde{w}(i)}}{i+1} + \frac{i * d_z^{\widetilde{w}(i-1)}}{i+1}$$

The efficiency of the algorithm. The models $\mathcal{P}_{S,}$ $\mathcal{P}_U$, $\mathcal{P}_1$ used in this algorithm are all convex with linear constraints and they are solved efficiently with the interior point method

(Potra and Wright, 2000). By avoiding the establishment of path flows, they have a complexity scalable to the size of the network. i.e., $O(A)$.

The optimal solution. If the algorithm can obtain a converged solution without using MSA, then we can guarantee the local optimal solution as the algorithm is based on the fixed point method where

$$t^0_{\tilde{w},z} = \mathcal{P}_S(d^{\tilde{w}}_z)$$

$$d^{\tilde{w}}_z = \mathcal{P}_1\left(t^0_{\tilde{w},z}\right)$$

$$\Rightarrow t^0_{\tilde{w},z} = \mathcal{P}_S\left(\mathcal{P}_1\left(t^0_{\tilde{w},z}\right)\right) \text{ and } d^{\tilde{w}}_z = \mathcal{P}_1\left(\mathcal{P}_S\left(t^0_{\tilde{w},z}\right)\right)$$

The need of MSA is because it is not guaranteed that the oscillation in this algorithm can be avoided. However, it is aimed to minimize this occurrence by limiting the number of fixed-point variables which are relied on the convergence of single variable $t^0_{\tilde{w},z}$ for each OD $\tilde{w}$ in AV zones in this algorithm. In the numerical result, even for the large network, there is still have not observed the issue of oscillation, hence avoid the use of MSA in this algorithm.

The model is formulated in Pyomo (Python Optimization Modelling Objects) (Hart et al., n.d.) that is a collection of Python software packages for formulating optimization models and solved using Ipopt (Interior Point Optimizer). Each design is run on a computer with a processor of 3.20 GHz Intel Core i7 and 32GB RAM. In this work, the number of variables and constraint is small, less than 7000, and the solution time is 5 minutes for each iteration, and it often takes no more than 6 iterations to get the convergence of results.

4.7 Summary

This chapter has explored different designs for the multi-lane AV zone deployment in the transportation network. Based on the different route choice principles of HVs and AVs, the MRE model is formulated. Analytical studies for the design of dedicated AV lanes have been performed where an estimated convex model with linear constraint has developed. The impact of AV penetration on the network performance has been discovered theoretically where the saturation point of AV penetration indicates the point at which the network performance is maximized and beyond which there is no change of the network performance. A fixed point solution algorithm has been developed to solve the model where the UE and SO problems for HVs and AVs respectively solve repeatedly and converge at a point.

PART III:

NUMERICAL RESULTS FOR DESIGN 1 AND 2

CHAPTER 5 NUMERICAL RESULTS FOR DESIGN 1 AND 2

5.1 Introduction

This section presents the results from two numerical examples, including a small grid network and a city-size network (the network of the city of Melbourne, Australia). For the small network, the AV zone is designed at the centre of the network, and the results are presented for a particular AV zone. For the city-size network, the initial AV zone is placed based on the existing traffic congestion hotspots that have high traffic density in the network, and the results are given for different AV zones in this network. In these examples, each road in the AV zone has two lanes, however, the model can work well also with the different number of lanes, and two different designs for the dedicated AV lanes in the AV zones are considered as follows:

- Design 1: Both lanes are dedicated to AV traffic, and HV cannot enter the AV zone at any time. This design corresponds to $r = 1$.
- Design 2: Only one lane is dedicated to AV traffic, while the other lane can be shared by both AVs and HVs. This design corresponds to $r = 0.5$.

Using these two designs, the impact of dedicated AV lanes on the network performance and the saturation points is investigated numerically.

5.2 Results for the Small network

For the small (grid) network shown in Figure 5.1, the O-D demand is 400 veh/min (with varying AV penetration rate from 0% to 100%) from node 1 to 16. The AV zone in both designs is within the circled area (including nodes 6, 7, 10, 11 in Figures 5.2 and 5.3) where each link in this zone has two lanes. The AV zone for designs 1 and 2 are shown in Figure 5.2 and Figure 5.3, respectively. Inside the AV zone, the flow capacity for the AV lane is triple compared to when it is used by mixed traffic. Note that, both lanes are set as AV dedicated lanes in the design 1, while only one lane of the link is dedicated for AV in the design 2. The link characteristics of the grid network are shown in Table 5.1.

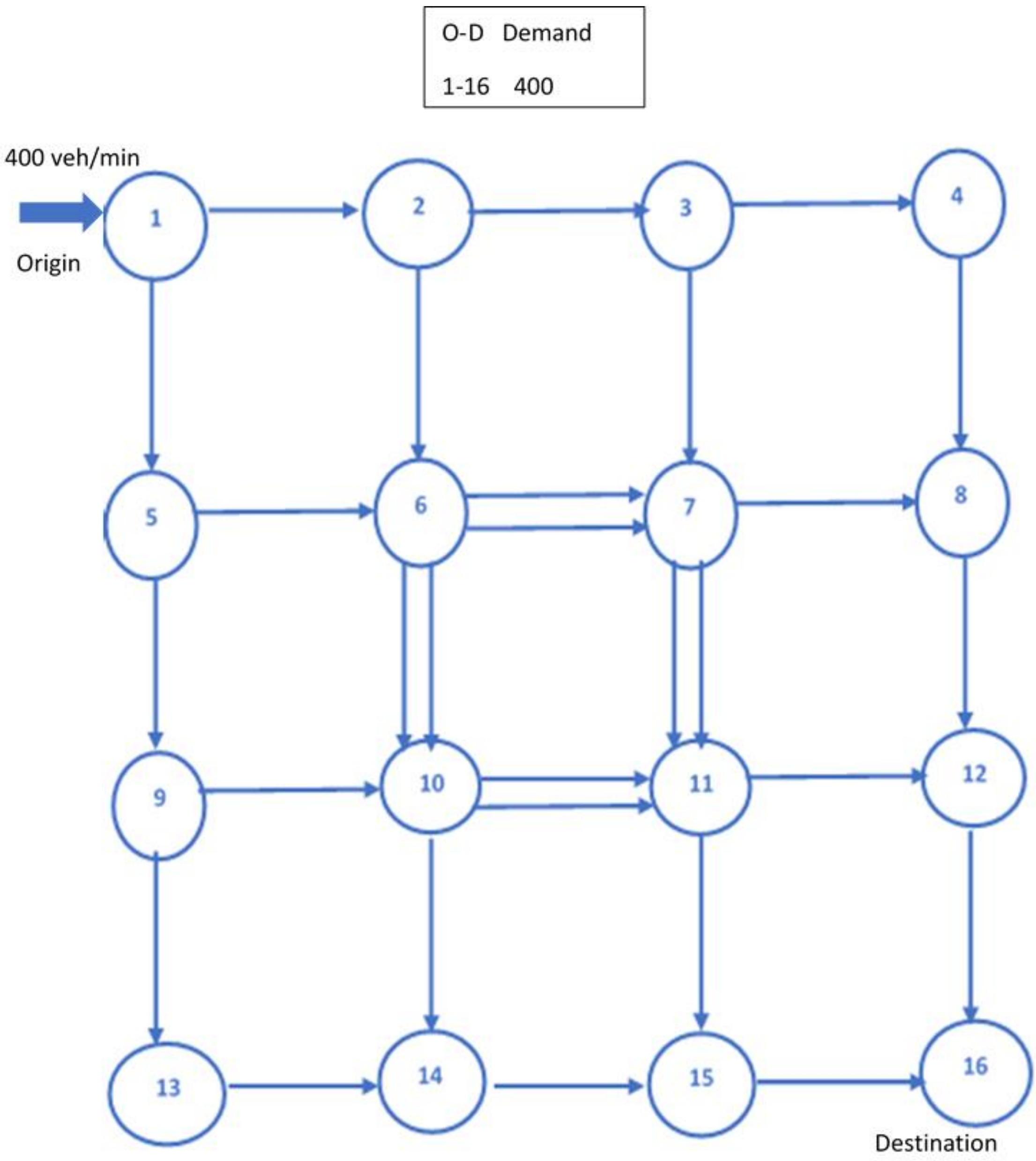

Figure 5. 1: Small Network.

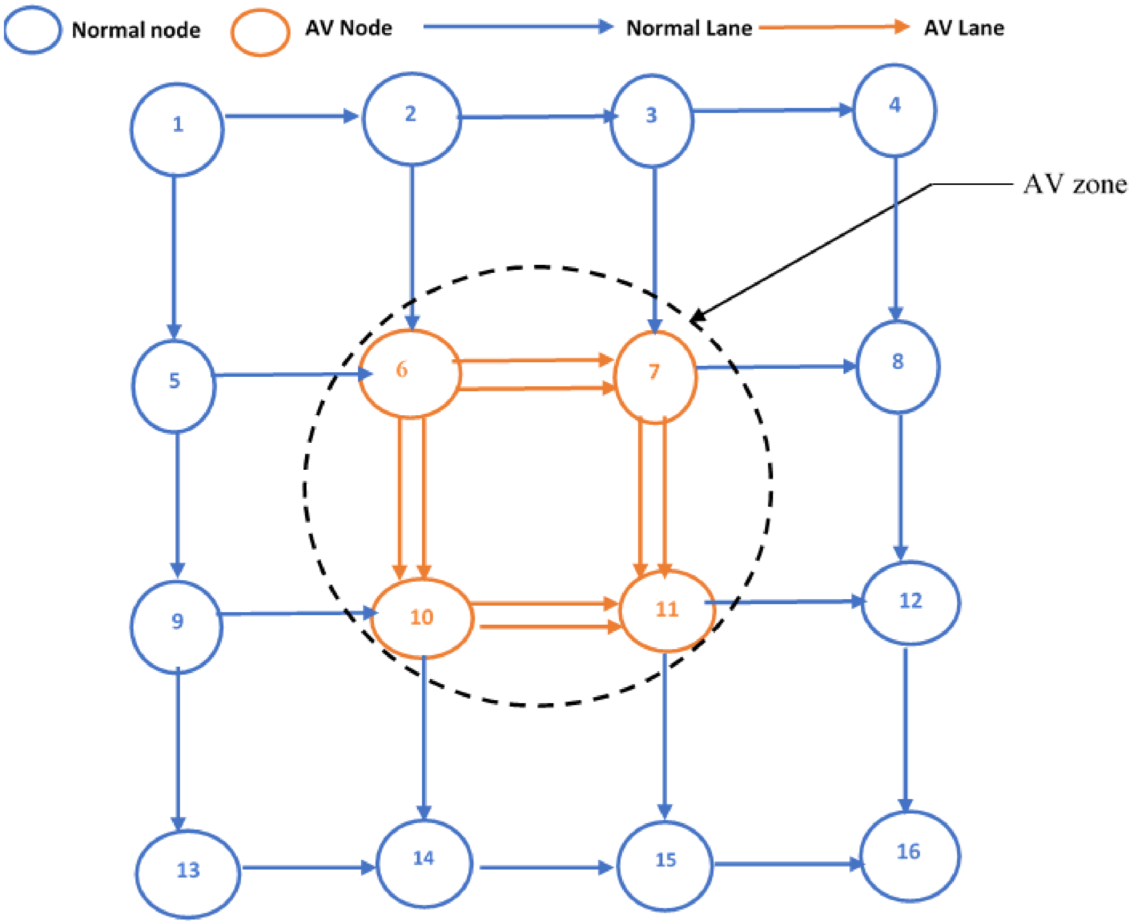

Figure 5. 2: Design 1 of the small network.

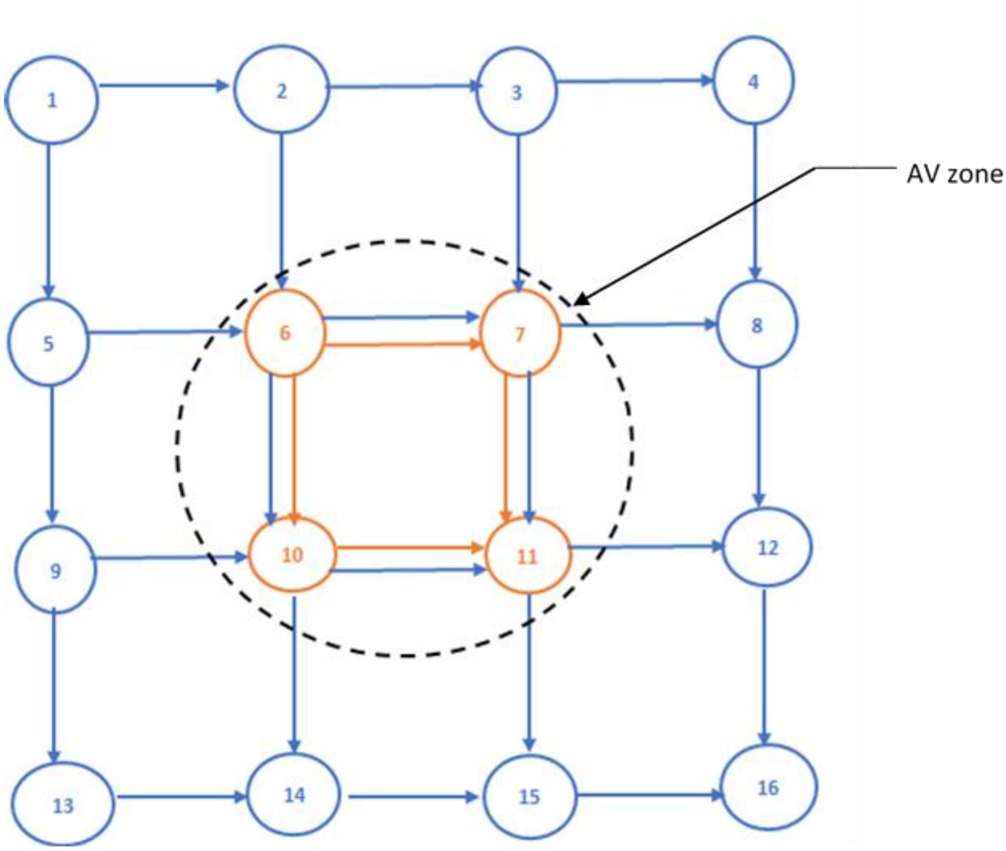

Figure 5. 3: Design 2 of the small network.

Table 5. 1: Network characteristics of small network.

Link	Free flow travel time t_0 (min)	Flow capacity c_a (veh/min)	Link	Free flow travel time t_0 (min)	Flow capacity c_a (veh/min)
1-2	5	105	7-11	17	35
1-5	5	105	8-12	5	35
2-3	5	105	9-10	17	70
2-6	5	70	9-13	5	35
3-4	5	35	10-11	17	35
3-7	17	70	10-14	17	70
4-8	5	35	11-12	5	70
5-6	5	70	11-15	5	70
5-9	5	35	12-16	5	105
6-7	17	35	13-14	5	36
6-10	17	35	14-15	5	35
7-8	17	70	15-16	5	105

5.2.1 Comparison of traffic flows passing and not passing through the AV lanes

Figure 5.4 indicates the change of traffic flows passing and not passing through the AV lanes for different penetration of AVs of both designs. In both designs, as the penetration of AVs increases, traffic volume passing through the AV lanes (inside the zone) increases, and not passing through the AV lanes decreases. Importantly, at high AV penetrations (more than 40%), the number of AV traffic passing through the AV zone in design 1 is significantly greater than in design 2. However, the flows detouring the AV lanes are lower in design 1 at higher AV penetrations than design 2. Furthermore, the number of AVs passing through the AV zone becomes stable after 80% in design 1 and 40% in design 2, respectively. So, design 1 can attract more AV traffic than design 2 at higher AV penetration. These results indicate that it is better to reserve a lane for a mixed traffic when the AV penetration is low, however, dedicating more lanes to AVs with higher penetration will attract more traffic getting through the AV zone and improve the overall network performance up to the saturation point as theorized above.

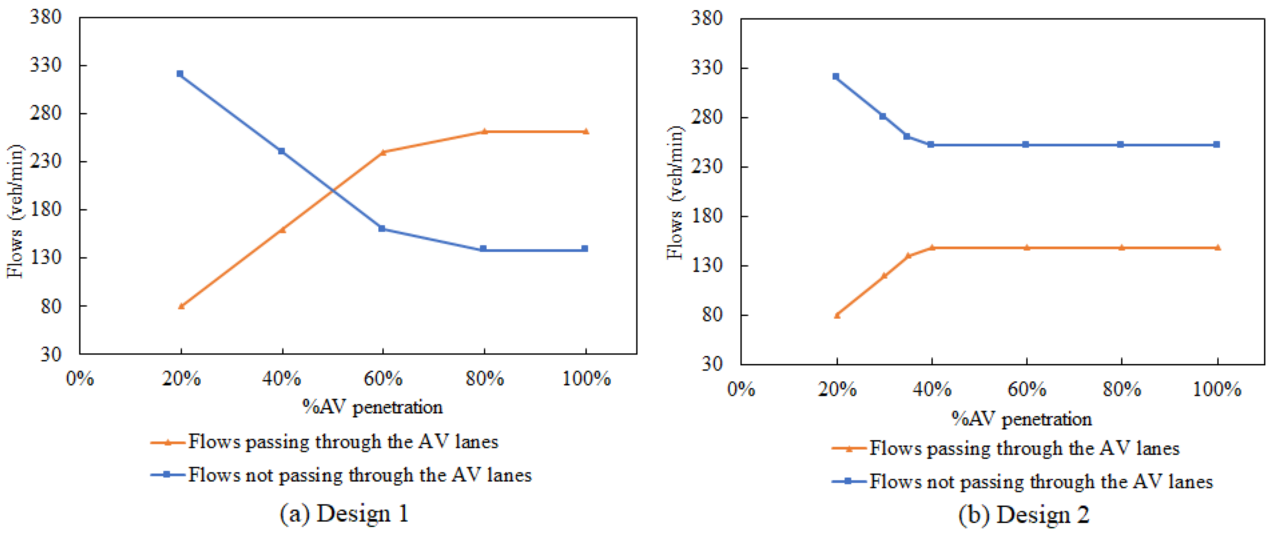

Figure 5. 4: Traffic volume (veh/min) for different penetration of AVs.

5.2.2 Comparison of average travel time of vehicles passing and not passing through the AV lanes.

Figure 5.5 plots the average travel time (AVTT) of vehicles for the two designs with different penetration of AVs. In both designs, the SO solution or minimum AVTT is achieved with 100% AV traffic, and the network performance is improved up to 27.5% compared to no AV vehicles. Figures 5.5 shows as the AV penetration increases, AVTT of vehicles is increasing when passing through the AV lanes and decreasing when not passing through the AV lanes. Observe that in design 1, at 20% of AV, AVTT of vehicles not passing through the AV lanes is very high, in fact, it is much higher than without AV zone/lanes (only HVs). It is because in design 1, as AV zone only allows AVs, and many dedicated AV lanes are idle at the low AV penetration.

Making it worse at lower penetration, where HVs are not allowed to pass through the AV zone/lanes, creating more congestion which increases the AVTT of vehicles in design 1. In design 1, AVTT starts to improve after 60% AV penetration compared to no AV zones/lanes case, and at 80%, the network performance is improved up to 13.3%. Thus, for design 1, mini-mum 60% penetration of AVs is needed. For design 2 (see Figure 5.5 (b)), 30% AV

penetration is required to improve the network performance compared to no AV scenario, and at 40% AV penetration, the network performance is maximized with 5% overall reduction in the AVTT. This result indicates that dedicating more lanes to AVs improves the network performance but also requires higher AV penetration. Moreover, more exclusive AV lanes with lower penetration worsen the network performance.

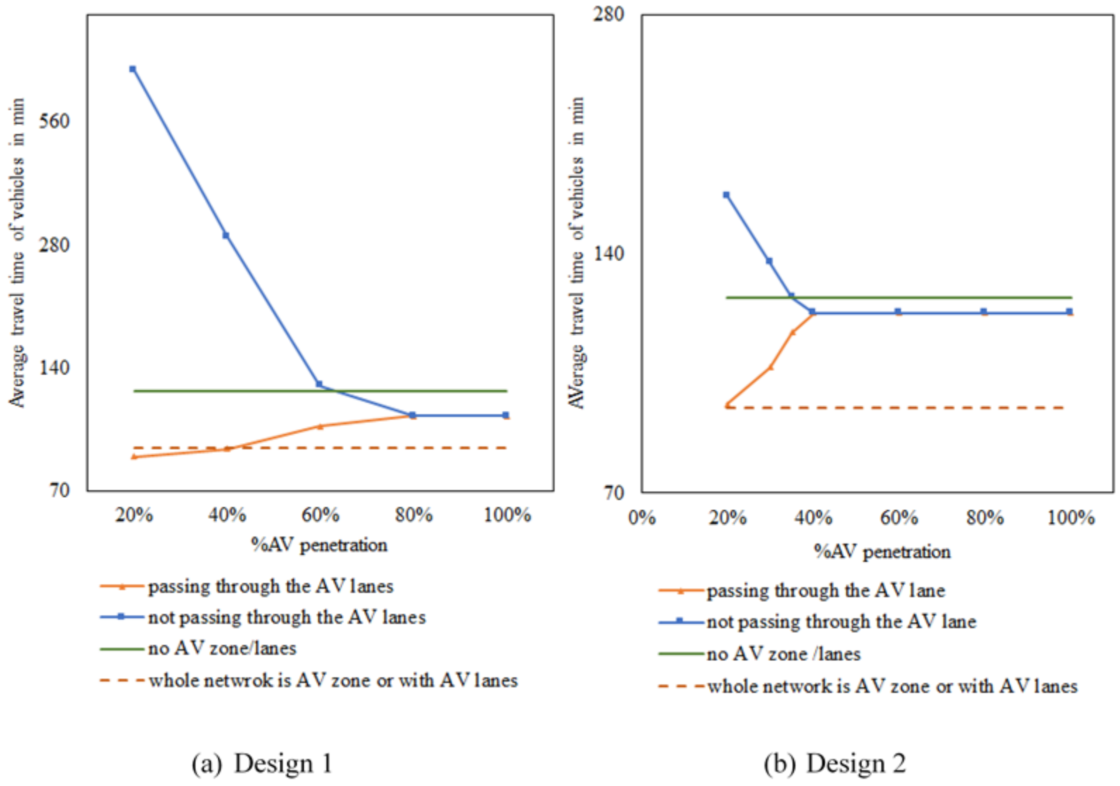

Figure 5. 5: Average travel time of vehicles (min) for different penetration of AVs.

5.2.3 Saturation point of AV penetration for two different designs

Observe from Figures. 5.4 (a, b), 5.5 (a, b) that, in both designs, there is a value of AV penetration at which the number of flows as well as the AVTT of vehicles passing or not passing through the AV lanes plateau. For design 1, the saturation point happens at 80%; on the other hand, for design 2 it is 40%. These results indicate that dedicating more lanes to AV delays the saturation point of AV penetration. Also, note that although high AV penetration is required for the better utilization of design 1, it is more effective than design 2 as it can improve the network performance up to 13.3%.

5.2.4 Comparison of vehicle routing for different designs

To illustrate how the AV zone has been utilized in design 1, Figures 5.6 and 5.7 demonstrate the flow pattern of vehicles for 20% and 80% AV penetrations, respectively. Beyond 80%, the flow patterns remain the same. At 20% AV penetration, only small AV flows pass through the zone (Figure 5.6), while most AVs pass through the zone at 80% AV penetration (Figure 5.7). Moreover, one new path 1→5→6→10→11→15→16 has formed at 80% AV penetration which contains 27% AV traffic of the total demand.

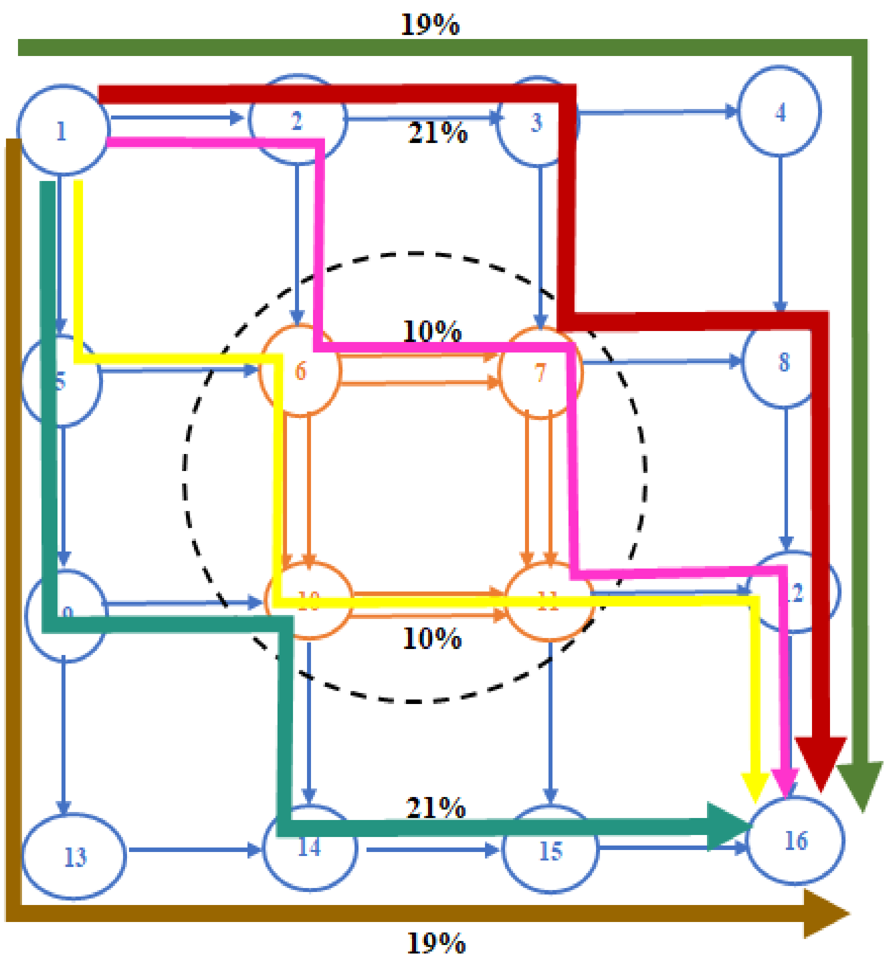

Figure 5. 6: Flow pattern of design 1 at 20% AV.

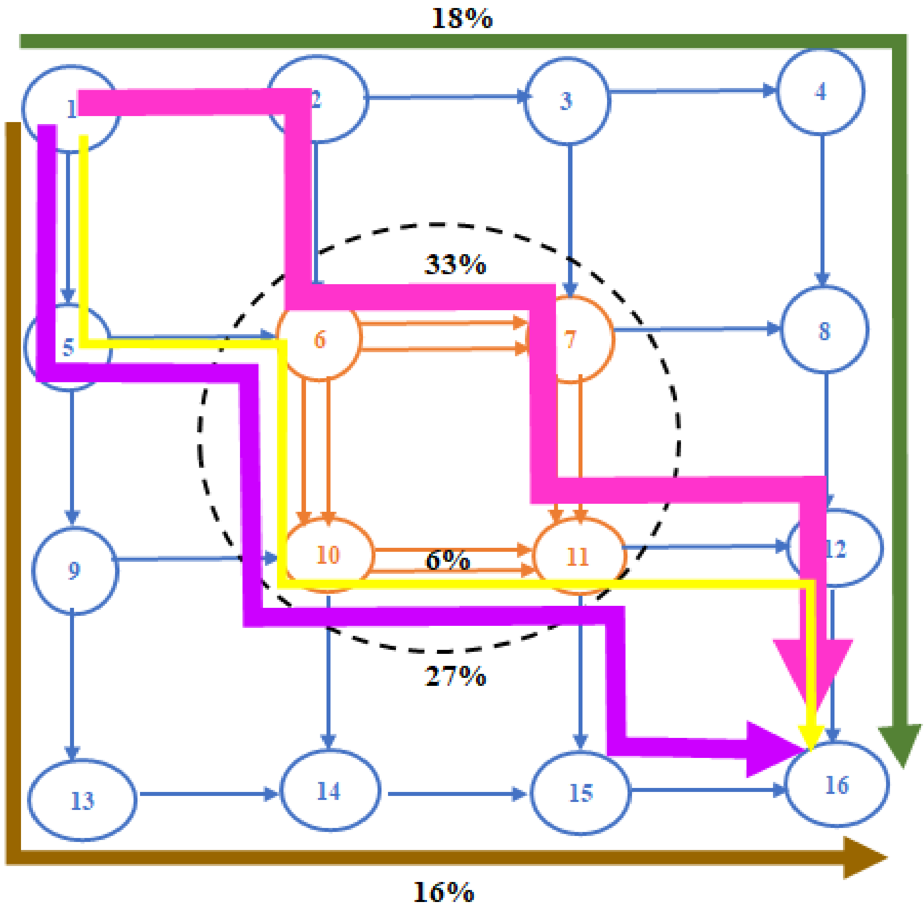

Figure 5. 7: Flow pattern of design 1 at 80%-100% AV

Figure 5.8 describes the flow patterns of 20%, while Figure 5.9 illustrates the flow patterns of 40% AV penetration for design 2. In design 2 at 20% AV penetration, as HVs can enter into the AV zone, there are much more flows passing through the AV zone compared to the design 1. On the other hand, after reaching the 40% penetration of AV, there is no significant increase of flows passing through the AV zone in the design 2. We observe that in both designs, at higher penetration of AVs, more than 60% flows of the total AV demand pass through the AV zone. So, these results indicate that the high penetration of AVs attracts more vehicles to the AV zone in both designs. However, more exclusive AV lanes require higher penetration to have more flows passing through the AV zone.

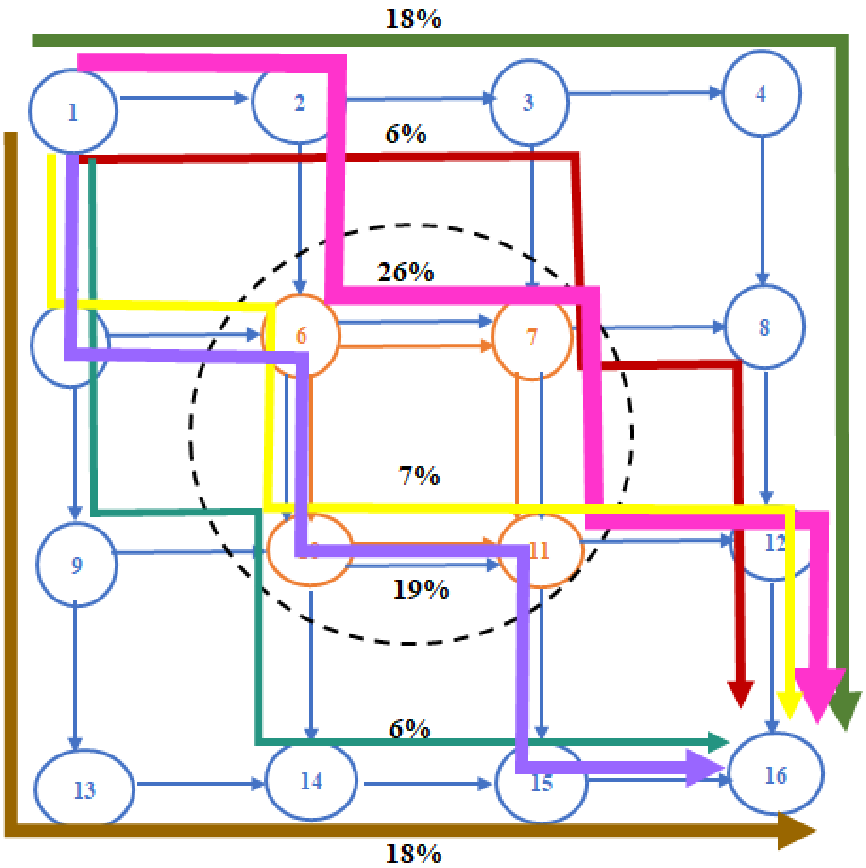

Figure 5. 8: Flow pattern of design 2 at 20% AV.

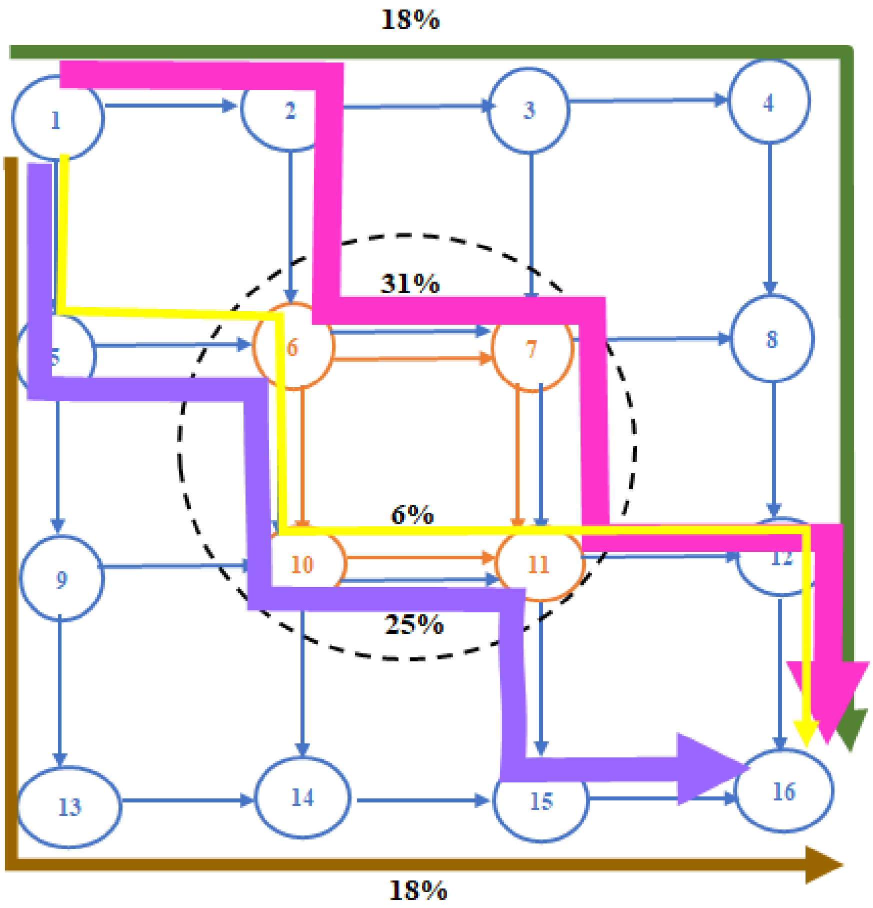

Figure 5. 9: Flow pattern of design 2 at 40%-100% AV.

5.3 Results for the city size network (Melbourne Network)

In this section, the performance of the simplified Melbourne network in Figure 5.10 has been studied, consisting of 80 nodes, 384 links, and 30 O-D pairs covering the Melbourne CBD. In the Figure 5.10, the virtual links indicate the connections from the centroid to the road network. A combination of two consecutive AV zones with two designs of AV lanes for the Melbourne network has been evaluated. Similar to the small network case, the flow capacity for the AV lanes is triple compared to that used by mixed traffic. Table 5.6 represents the network characteristics of the Melbourne Network.

The topology was taken from a Melbourne network modelled in AimSun together with the OD matrices and calibrated using SCATS (Sydney Coordinated Adaptive Traffic System) data from the Melbourne network.

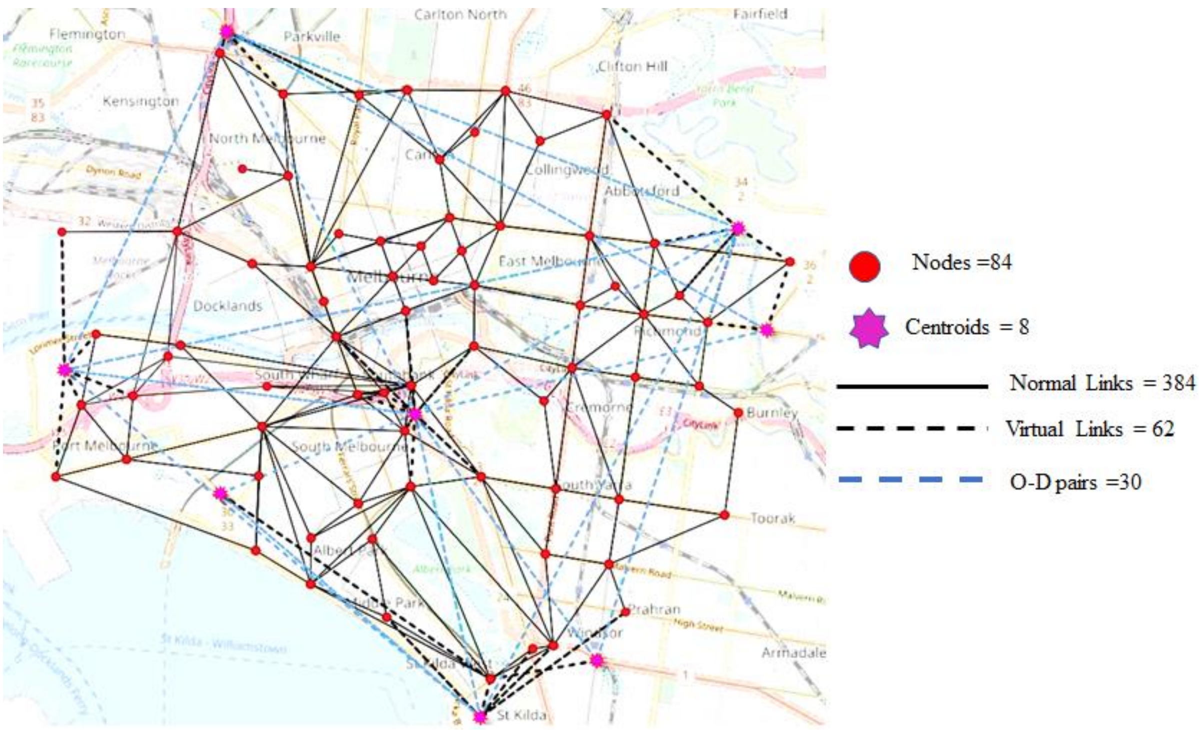

Figure 5. 10: Melbourne network.

Table 5. 2: Network characteristics of Melbourne network.

Link	Free flow travel time t_0 (min)	Flow capacity c_a (veh/min)	Link	Free flow travel time t_0 (min)	Flow capacity c_a (veh/min)
64465-64480	2.324	14.167	64528-64513	2.368	10.000
64465-77936	1.681	24.390	64528-64514	2.310	10.833
64465-79782	1.260	28.333	64528-64522	0.977	21.667
64470-64480	0.800	21.667	64528-69289	1.830	26.667
64480-64465	2.575	12.500	64528-72257	3.167	10.000
64480-64470	0.936	12.500	64528-77944	3.026	10.000
64480-70642	1.732	21.667	64528-79120	1.969	10.833
64480-77936	0.810	21.667	64528-79128	1.974	26.667
64480-77979	0.787	25.000	64528-79643	1.439	13.333
64502-64510	1.410	10.000	64528-79668	0.946	26.667
64502-69203	1.933	10.000	64738-65155	0.983	10.000
64502-79101	2.784	10.000	64738-65158	1.083	10.000
64510-64502	1.374	10.000	64738-77815	2.587	10.000
64510-64513	0.945	10.000	64744-65155	2.264	10.000
64510-64528	3.058	10.000	64744-65156	1.068	10.833
64510-69203	1.693	10.000	64744-65161	0.880	45.941
64510-72277	1.765	26.667	64744-72179	1.372	28.333
64510-79101	2.981	10.833	64744-77936	2.179	21.667
64510-79120	0.825	45.941	64744-79810	0.970	26.667
64513-64510	1.036	10.000	64744-79967	1.515	28.333
64513-64514	0.897	10.000	65094-70642	1.245	12.500
64513-64528	2.431	10.000	65094-79101	2.000	10.717
64513-72277	2.016	10.000	65155-64738	0.821	10.000
64514-64513	0.897	10.000	65155-64744	2.200	10.000
64514-64528	2.215	10.833	65155-65156	0.782	21.667
64514-72277	1.155	26.667	65155-65158	1.377	12.500

64514-77944	2.267	10.000	65155-65159	2.894	10.000
64522-64528	0.894	21.667	65155-65161	0.881	25.000
64522-79120	1.528	10.833	65155-70664	2.238	10.000
64522-79128	2.344	10.833	65155-77815	0.886	21.667
64528-64510	3.334	10.000	65155-79548	0.822	10.000
65156-65155	0.782	21.667	70642-65094	1.594	28.333
65156-65159	1.103	10.833	70642-72277	4.911	11.667
65156-77936	1.510	21.667	70642-76228	3.629	28.333
65158-64738	0.978	10.000	70642-77979	2.455	13.333
65158-65155	0.886	25.000	70642-79101	2.065	21.667
65158-65159	1.014	10.833	70664-65155	2.066	10.000
65158-66935	1.994	12.500	70664-77440	1.559	63.333
65159-65155	2.002	10.000	70664-77815	1.965	9.901
65159-65156	0.787	21.667	71788-71794	1.184	25.000
65159-65158	0.975	10.833	71788-77574	2.002	21.207
65159-79782	1.138	13.333	71788-79133	2.581	28.333
65161-64744	0.848	45.941	71794-71788	1.143	12.500
65161-65155	0.881	12.500	71794-72390	2.865	12.500
66935-65158	1.792	25.000	71794-77303	1.324	26.667
66935-77815	2.126	12.500	71794-77574	1.570	20.000
69203-64502	1.675	10.000	71794-79133	5.380	10.833
69203-64510	1.471	10.000	72179-64744	1.446	28.333
69203-72277	0.984	26.667	72179-79535	1.884	16.667
69203-79101	2.278	26.667	72179-79668	1.606	31.667
69237-70614	1.144	10.833	72179-79810	1.163	26.667
69237-79643	1.173	26.667	72254-72257	2.157	11.667
69289-64528	1.734	26.667	72254-77798	1.152	3.286
69289-72257	1.118	16.667	72254-79133	0.930	13.333
69289-77944	1.248	10.000	72257-64528	2.829	10.000
69289-79668	1.083	31.667	72257-69289	0.960	16.667
69289-79711	1.081	10.833	72257-72254	2.454	11.667
70614-69237	1.654	13.333	72257-76292	1.481	8.333
70614-79126	1.214	12.500	72257-77908	1.806	16.667

70614-79128	1.715	13.333	72257-79133	1.873	8.264
70614-79656	1.399	23.333	72257-79150	2.138	8.333
70614-79754	1.566	23.333	72257-79668	0.891	21.667
70642-64480	1.689	21.667	72257-79711	1.956	16.667
72277-64510	1.978	26.667	77323-72390	1.009	25.000
72277-64513	2.134	10.000	77323-77633	2.881	10.000
72277-64514	1.128	26.667	77323-77798	2.259	3.292
72277-69203	1.083	26.667	77440-70664	1.575	47.500
72277-70642	4.553	11.667	77440-72431	1.351	26.667
72277-76228	1.124	26.667	77440-77303	1.165	15.833
72277-77944	1.040	13.158	77440-77815	2.681	9.901
72277-79101	4.611	11.532	77574-71788	2.038	21.207
72390-71794	2.949	14.167	77574-71794	1.646	10.833
72390-77323	1.018	25.000	77574-76410	0.994	20.000
72390-77633	1.734	10.000	77574-77303	2.174	25.974
72390-79133	2.911	28.333	77574-77739	1.166	25.974
72431-77303	1.239	21.667	77574-77752	2.009	20.000
72431-77440	1.844	13.333	77633-72390	1.472	10.000
72431-77752	1.753	21.667	77633-76366	1.120	20.000
76228-70642	3.394	12.500	77633-77323	2.835	10.000
76228-72277	1.060	26.667	77633-77798	3.365	3.268
76228-77944	2.315	5.417	77633-79133	1.488	28.333
76228-77979	1.652	12.500	77739-77574	1.045	25.974
76228-79668	3.001	21.667	77739-77759	1.684	8.333
76228-79967	0.908	25.000	77739-79133	3.320	28.333
76292-72257	1.399	8.333	77752-72431	1.775	21.667
76292-79133	0.652	16.667	77752-77303	3.020	21.207
76366-77633	1.321	20.000	77752-77574	2.042	20.000
76410-77303	0.896	31.667	77759-77739	1.857	8.333
76410-77574	0.903	20.000	77759-77794	1.043	8.333
77303-71794	1.234	26.667	77759-77848	0.909	16.667
77303-72431	1.274	21.667	77759-79133	1.210	8.264
77303-76410	0.879	31.667	77759-79150	1.256	8.333

77303-77440	1.441	23.333	77769-77798	1.817	10.000
77303-77574	1.986	25.974	77794-77759	0.997	8.333
77303-77752	3.002	21.207	77794-79133	2.069	15.587
77798-72254	1.164	3.286	77995-79150	1.100	8.333
77798-77323	3.494	3.279	78936-79754	1.262	29.630
77798-77633	2.031	9.901	79101-64502	2.701	10.000
77798-77769	1.722	10.000	79101-64510	2.806	10.833
77798-79656	1.332	3.292	79101-65094	1.792	10.717
77815-64738	2.819	10.000	79101-69203	2.329	26.667
77815-65155	0.847	21.667	79101-70642	1.974	12.500
77815-66935	2.380	12.500	79101-72277	4.434	11.532
77815-70664	1.674	9.901	79120-64510	0.849	30.966
77815-77440	3.258	9.901	79120-64522	1.517	10.833
77848-77759	1.110	8.333	79120-64528	1.815	10.833
77848-79150	0.870	16.667	79120-79126	4.509	11.667
77908-72257	1.917	16.667	79126-70614	1.214	12.500
77908-79150	0.601	16.667	79126-79120	4.952	11.667
77908-79535	1.292	16.667	79126-79128	0.894	26.667
77908-79668	1.788	16.393	79128-64522	2.340	10.833
77936-64465	1.951	12.346	79128-64528	1.921	26.667
77936-64480	0.810	21.667	79128-70614	1.280	10.833
77936-64744	2.193	21.667	79128-79126	0.894	26.667
77936-65156	1.565	21.667	79128-79754	1.313	13.158
77936-79967	0.784	25.000	79133-71788	2.577	15.833
77944-64514	1.902	10.000	79133-71794	5.080	7.222
77944-64528	2.904	10.000	79133-72254	0.890	13.333
77944-69289	1.252	10.000	79133-72257	1.650	16.393
77944-72277	1.221	10.000	79133-72390	2.898	15.833
77944-76228	2.414	10.833	79133-76292	0.652	16.667
77944-79668	2.099	13.333	79133-77633	1.449	16.667
77944-79711	1.041	10.000	79133-77739	3.504	15.833
77979-64480	0.938	12.500	79133-77759	1.220	8.333
77979-70642	2.076	12.500	79133-77794	1.958	15.587

77979-76228	1.395	12.500	79133-79150	1.793	8.333
77979-79967	0.874	28.333	79150-72257	2.193	8.333
79150-77759	1.238	8.333	79810-64744	1.132	26.667
79150-77848	0.910	16.667	79810-72179	1.163	26.667
79150-77908	0.563	16.667	79810-79967	1.849	10.000
79150-77995	0.964	8.333	79967-64744	1.577	28.333
79150-79133	1.974	8.264	79967-76228	1.000	25.000
79535-72179	1.890	16.667	79967-77936	0.998	12.500
79535-77908	1.140	16.667	79967-77979	0.869	28.333
79548-65155	0.904	10.000	79967-79810	2.058	10.000
79643-64528	1.591	13.333	64470-7701112	0.000	100000.000
79643-69237	1.112	26.667	64502-7701112	0.000	100000.000
79643-79668	1.430	26.667	64738-7701133	0.000	100000.000
79656-79668	1.399	3.286	65094-7701112	0.000	100000.000
79656-79754	0.807	30.000	65158-7701133	0.000	100000.000
79668-64528	2.576	3.279	66935-7701133	0.000	100000.000
79668-69289	1.023	15.833	69237-7700778	0.000	100000.000
79668-72179	1.596	15.833	69289-7701239	0.000	100000.000
79668-72257	2.115	15.415	70614-7700778	0.000	100000.000
79668-76228	3.062	15.833	70642-7701112	0.000	100000.000
79668-77908	1.869	15.587	71788-7701336	0.000	100000.000
79668-77944	2.096	15.587	72179-7701239	0.000	100000.000
79668-78936	1.362	3.292	72257-7701239	0.000	100000.000
79668-79643	3.107	3.279	72277-7701239	0.000	100000.000
79668-79711	0.845	13.333	72390-7701336	0.000	100000.000
79711-69289	1.132	10.833	76228-7701239	0.000	100000.000
79711-72257	1.924	16.667	77323-7701336	0.000	100000.000
79711-77944	1.084	10.000	77440-7701133	0.000	100000.000
79711-78936	2.112	29.525	77769-7700778	0.000	100000.000
79711-79668	0.987	13.333	77815-7701133	0.000	100000.000
79754-77798	1.537	3.300	77908-7701239	0.000	100000.000
79754-79128	2.118	14.167	77944-7701239	0.000	100000.000
79782-64465	1.363	14.167	79101-7701112	0.000	100000.000

79782-65159	1.138	13.333	79126-7700778	0.000	100000.000
79668-7701239	0.000	100000.000	7701120-65158	0.000	100000.000
79711-7701239	0.000	100000.000	7701120-66935	0.000	100000.000
79754-7700778	0.000	100000.000	79101-7701113	0.000	100000.000
7700778-69237	0.000	100000.000	79120-7700890	0.000	100000.000
7700778-70614	0.000	100000.000	65158-7701120	0.000	100000.000
7700778-77769	0.000	100000.000	66935-7701120	0.000	100000.000
7700778-79126	0.000	100000.000	77848-77739	0.831	49.180
7700778-79754	0.000	100000.000	77848-77995	0.503	75.000
7701112-64470	0.000	100000.000	77739-77848	0.833	49.180
7701112-64502	0.000	100000.000	77739-77752	0.644	142.500
7701112-65094	0.000	100000.000	77739-77841	1.341	25.000
7701112-70642	0.000	100000.000	77995-77848	0.501	75.000
7701112-79101	0.000	100000.000	77995-77841	1.024	25.000
7701133-64738	0.000	100000.000	77995-79535	0.878	50.000
7701133-65158	0.000	100000.000	77841-77739	1.381	25.000
7701133-66935	0.000	100000.000	77841-77752	1.110	25.000
7701133-77440	0.000	100000.000	77841-77995	1.024	25.000
7701133-77815	0.000	100000.000	77841-79535	0.675	50.000
7701239-69289	0.000	100000.000	77752-77739	0.666	142.500
7701239-72179	0.000	100000.000	77752-77841	1.089	25.000
7701239-72257	0.000	100000.000	77752-70664	2.012	80.000
7701239-72277	0.000	100000.000	79535-77995	0.815	50.000
7701239-76228	0.000	100000.000	79535-77841	0.740	25.000
7701239-77908	0.000	100000.000	79535-65161	2.427	37.037
7701239-77944	0.000	100000.000	70664-79548	1.247	30.000
7701239-79668	0.000	100000.000	70664-77752	2.037	80.000
7701239-79711	0.000	100000.000	70664-65161	0.741	142.500
7701336-71788	0.000	100000.000	65161-70664	0.774	95.000
7701336-72390	0.000	100000.000	65161-79535	1.968	65.000
7701336-77323	0.000	100000.000	65161-79548	0.929	30.000
7701113-79101	0.000	100000.000	79548-65161	0.867	30.000
7700890-79120	0.000	100000.000	79535-77752	1.847	65.000

77752-79535	1.755	65.000

5.3.1 Comparison of traffic density between different designs

In the following, two AV zones have been selected subsequently based on the traffic density of the links. To obtain the initial link densities, the UE routing principle is performed without considering any AV zone (see Figure 5.11). Then zone 1 for designs 1 and 2 is selected based on one of the most congested areas in the network (see Figures. 5.12, 5.13). Another AV zone (or zone 2) in the CBD is also selected to test with both designs (see Figures. 5.14, 5.15).

Figure 5.12 illustrates the traffic density of the Melbourne network after the deployment of AVs in zone 1 for design 1 with 80% AV penetration. Observe that although the total system travel time (TSTT) significantly improves (4.52%), some other areas of the network have become more congested. A similar pattern can be observed using zone 2 in design 1 (see Figure. 5.13), where the performance improvement is 2.06%. However, if we deploy AVs as design 2 with zone 1or 2 (see Figures 5.14, 5.15), there are no significant changes in the spatial density; nevertheless, the network performance is improved up to 6.5% and 2.33%, respectively.

Figure 5. 11: Traffic density of Melbourne network without considering any AV zone.

Figure 5. 12: Traffic density of Melbourne network with AV zone 1 for design 1.

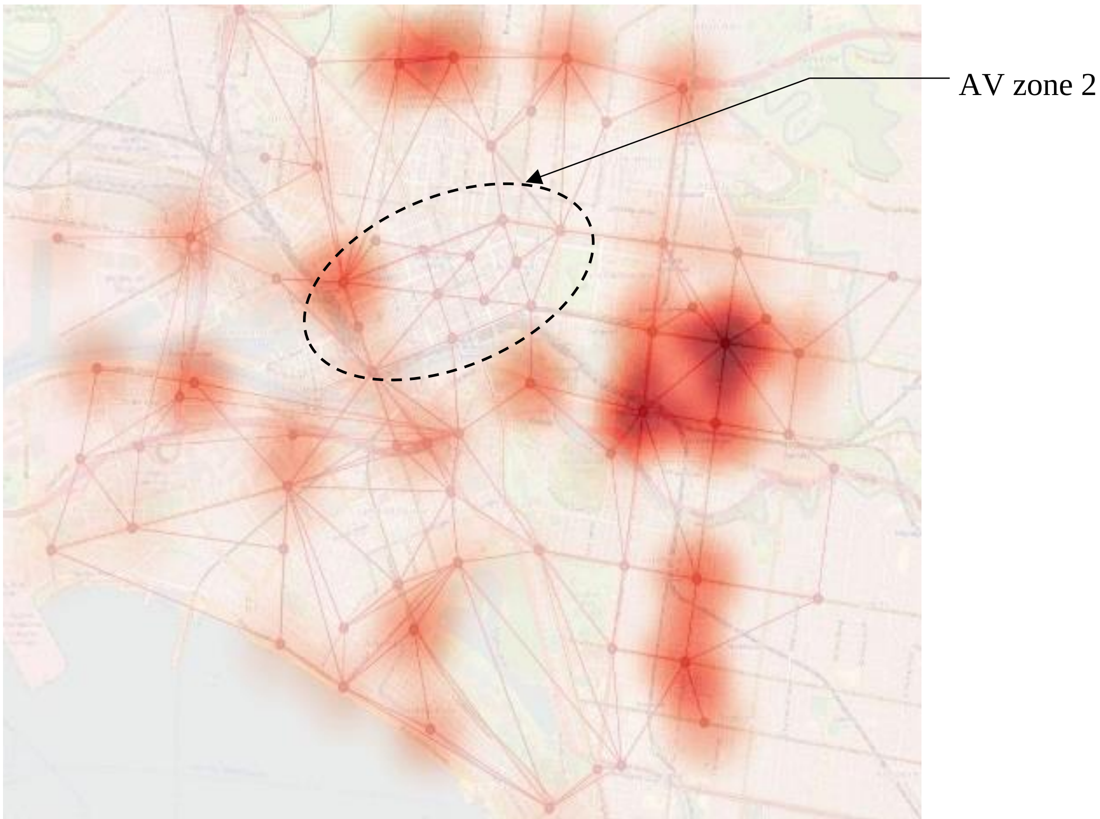

Figure 5. 13: Traffic density of Melbourne network with AV zone 2 for design 1.

Figure 5. 14: Traffic density of Melbourne network with AV zone 1 for design 2.

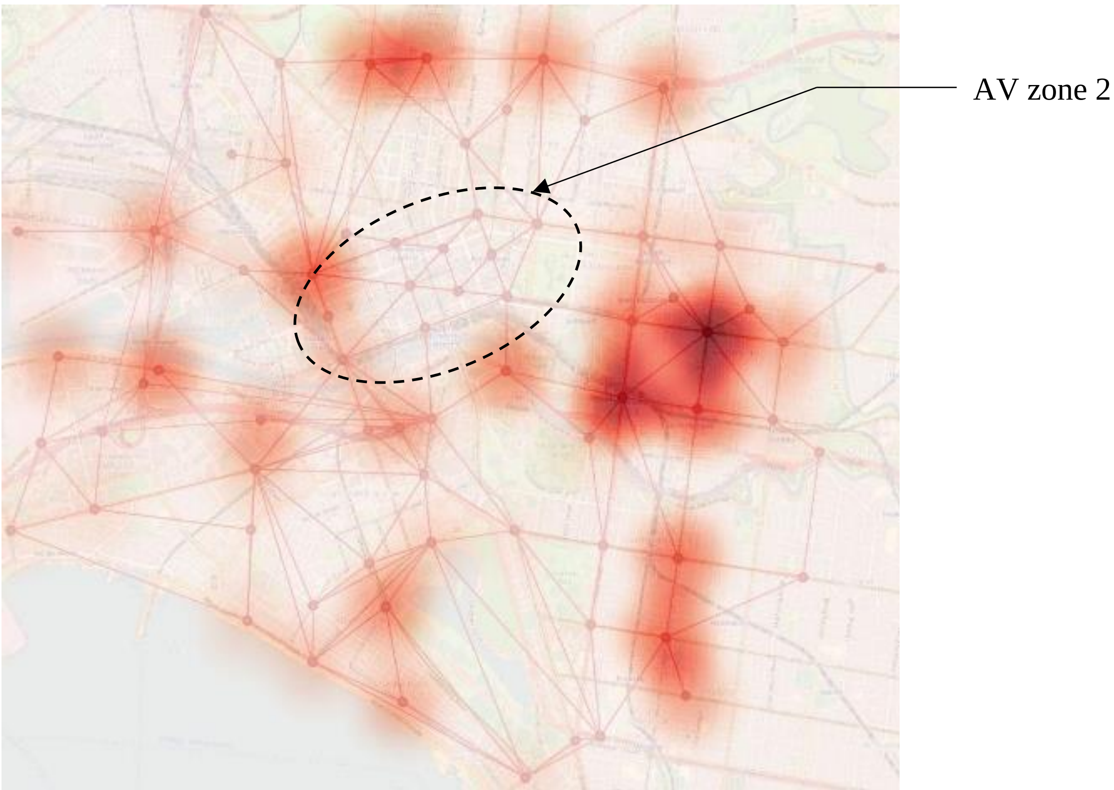

Figure 5. 15: Traffic density of Melbourne network with AV zone 2 for design 2.

5.3.2 Comparison of network performance between different designs

Figure 5.16 illustrates the comparison of network performance for different penetration of AVs between the two designs for the Melbourne network. If the AV penetration is 100%, then the network performance is improved up to 10.6% compared to no AVs in the network. The TSTT of design 1 using zone 1 (see Figure 5.16) is smaller than that of HVs only network at 60% AV penetration, and at 80% penetration, the network performance is improved up to 4.52%. Deployment of zone 1 in design 2, improves the network performance up to 2.05% at 80% AV penetration while it is up to 6.5% and 2.33% for zone 2 with designs 1 and 2, respectively.

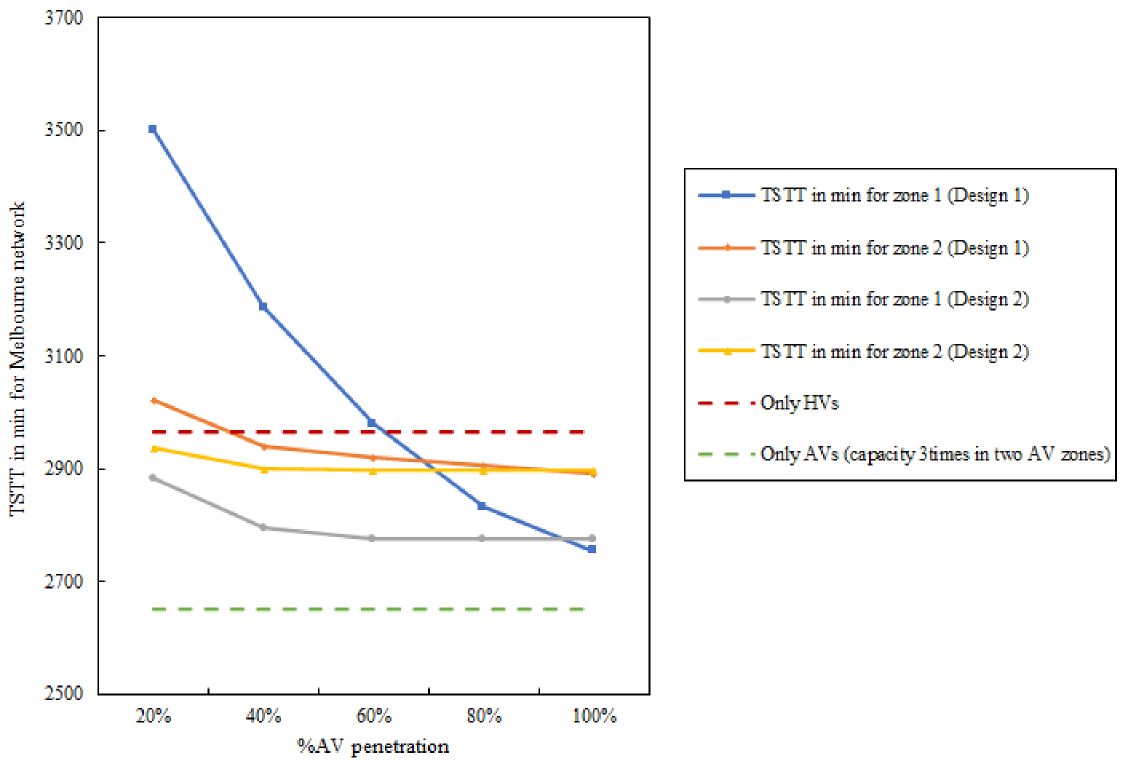

Figure 5. 16: Comparison of network performance between different designs.

5.3.3 Saturation point of AV penetration for two different designs

From Figure 5.16, it is observed that the saturation point of the AV penetration for design 2 is at 60%. Note that there is no saturation point for the Melbourne network in design 1. It is because design 1 has more dedicated AV lanes compared to design 2 and can improve the network performance even at 100% AV penetration. It is also observed that, although high AV penetration is required for better utilization in design 1, it is more effective than design for 80% AV penetration and beyond.

5.4 Summary

This chapter represents the numerical results for two different multi-lane AV zone designs. For the small network, when there is only small penetration of AVs in the transportation network, design 2 with mixed traffic can be used, and design 1 can then be introduced later when there is a sufficient number of AVs in the network. For the Melbourne network, design 1 with AV zone 1 provides the best network performance as it improves the network performance up to 7.18% at 100% AV penetration. However, design 2 with AV zone 1 is better compared to the other designs as it requires a much smaller penetration at 20% of AV to improve the network performance up to 2.83%. Overall, higher AV penetration with a lower number of AV dedicated lanes decreases the network performance. Therefore, governments need to think holistically when planning transportation infrastructure. The models and the results reported in this study could be used in the future strategy for the deployment of AV with mixed traffic in the transportation network.

PART IV:

CONCLUSION AND RECOMMENDATIONS

CHAPTER 6 CONCLUSION AND RECOMMENDATIONS

6.1 Introduction

The aim of this research has been to gain a better understanding of the optimal deployment of autonomous vehicles with mixed traffic in the transportation network. In this thesis, a framework has been developed for the deployment of AVs with different multi-lane AV zone designs in order to maximize the performance of the network in terms of travel time. The research work undertaken has provided a number of original contributions to knowledge in this field, as presented in Chapters 4 and 5. This chapter concludes the thesis by providing a summary of the major contributions, followed by a discussion of the limitations and future directions of this research.

6.2 Summary of key contributions

This research has made contributions in four major areas relevant to the optimization of travel time of the network involving AVs with mixed traffic. These include formulating a unified framework with different multi-lane AV zone designs (Chapter 4); deriving the complex mathematical problem into a simple equivalent convex problem with linear constraints and developing a fixed point solution algorithm (Chapter 4); deriving the saturation point of AV penetration mathematically (Chapter 4); and showing the optimal deployment of AVs via numerical results (Chapter 5). The major contributions in each of those areas are outlined as follows:

1) In this research, a novel mathematical framework has been developed to optimally deploy AVs in a zone with multi-lane using different AV lane allocation designs as well as considering different penetration of AVs. This study considers a mixed routing network where AVs outside of the AV zones follow the user equilibrium routing principle, but AVs within the AV zones follow the system-optimum routing principle while HVs follow the user-equilibrium routing principle both outside and inside the AV zones. These two different routing principles lead to a complex mathematical problem.
2) In this research, the complex problem has been transformed into a simple convex problem with linear constraints. To solve this problem using VI formulation is cumbersome. Instead, a fixed point solution algorithm has been developed where the UE and SO problems are solved simultaneously and converged at a point. The methodology is generic and can be applied to any network configuration and link flow.

3) In addition, a saturation point of AV penetration has been mathematically presented, which indicates an important theoretical result proven in this thesis that at saturation point, the network performance is maximized.
4) Extensive numerical results are also presented based on the small and large-scale network using the Melbourne network, showing the impact of different AV penetrations.

The proposed framework and theoretical insights presented in this thesis provide valuable insights for the optimal future deployment of AVs and the design of AV zones in order to achieve better network performance with mixed traffic.

6.3 Limitations of the research

While this thesis research has provided a number of original contributions to knowledge, it is also subject to a number of limitations.

In Chapter 4, a static deterministic framework has been considered for the design of multi-lane AV zones. While capacity constraint has not been considered in the model, this constraint may influence the UE routing principle and requires further investigation.

In Chapter 5, two AV zones have been selected and based on that the results have been discussed. So, the comparison was performed between these two AV zones.

6.4 Future research

This research has found travel time to be the most influential factor to evaluate the network performance of the transportation network. Future research can explore other factors such as different types of travel costs, vehicular speed, and elastic demand. A pricing scheme for HVs could be included in future work.

To model the framework, this research used a static deterministic approach. However, using a stochastic framework would enable real-time results to be obtained for any network with better flexibility in the solution.

As the AV zones were selected based on the traffic density of the Melbourne network, more AV zones can be selected where traffic is more congested. However, if other congested areas are selected as AV zones, then there will be more scope to compare the result and maybe the result will vary that time.

APPENDIX A.

Proof of Proposition 1

Let's transform the above model $\mathcal{P}_1$ into path-based model as below:

$$[\mathcal{P}_{1P}]\min F = \sum_{a\in A\backslash\tilde{A}} \int_0^{v_a} t_a(x)dx + \frac{1}{\beta+1}\sum_{a\in\tilde{\tilde{A}}} v_a t_a(v_a) + \sum_{\tilde{w},z} d_z^{\tilde{w}}\, t_{\tilde{w},z}^0$$

$$\text{s.t:} \qquad d_m^w = \sum_{p\in P_w} f_p^m \qquad \forall m\in M, w\in W_z$$

$$t_{\tilde{w},z}^0 d_z^w = \sum_{a\in\tilde{\tilde{A}}_z} \frac{\beta t_a^0 \tilde{x}_a{}^{\tilde{w}}}{\beta+1} \qquad \forall m\in M, w\in W_z$$

$$v_a = \sum_{m\in M}\sum_{p\in P: a\in p} f_p^m \qquad \forall a\in A$$

$$d_z^{\tilde{w}} = \sum_{p\in P:\tilde{w}=\tilde{w}(p)} f_p^A \qquad \forall \tilde{w}\in\widetilde{W}$$

Let $\mathcal{L}$ be the Lagrangian function for the above model, it is equal to:

$$\mathcal{L} = F + \sum_{m\in M}\sum_{w\in W} \pi_w^m \left(\sum_{p\in P_w} f_p^m - d_m^w\right) + \sum_{z\in Z}\sum_{\tilde{w}\in\tilde{\widetilde{W}}_z} \mu_z^{\tilde{w}} \left(\sum_{a\in\tilde{\tilde{A}}_z} \frac{\beta t_a^0 \tilde{x}_a{}^{\tilde{w}}}{\beta+1} - t_{\tilde{w},z}^0 d_z^w\right)$$

Assume that we obtain f_p^{m*} as the solution of $\mathcal{P}_2$, hence for HV traffic we have the following Karush-Kuhn-Tucker (KKT) condition:

$$\frac{\partial \mathcal{L}}{\partial f_p^C} = \sum_{a\in p} t_a(v_a) - \pi_w^C \geq 0$$

$$f_p^C \frac{\partial \mathcal{L}}{\partial f_p^C} = 0$$

This above property satisfies the UE condition for HV traffic described in Eq. (8).

For AV traffic, the KKT condition give:

$$\frac{\partial \mathcal{L}}{\partial f_p^A} = \sum_{a\in p\backslash \tilde{A}_z} t_a(v_a) + \frac{1}{\beta+1} \sum_{a\in \tilde{A}_z : a\in p} \left(v_a t_a'(v_a) + t_a(v_a)\right) + t_{\widetilde{w},z}^0 - \pi_w^A - \mu_z^w \left(\sum_{a\in \widetilde{A_z}} \frac{\beta_a t_a^0}{\beta_a + 1} - t_{w,z}^0 \right) \geq 0 \qquad (A.1)$$

$$f_p^A \frac{\partial \mathcal{L}}{\partial f_p^A} = 0 \qquad (A.2)$$

By fixing the traffic outside the AV zone z and the demand $d_z^{\widetilde{w}}$ for any OD $\widetilde{w}$ in this zone, the model $\mathcal{P}_{1P}$ formulates the SO problem in zone z. In fact, we can simply put $C_{w,z} = \sum_{a\in p\backslash \tilde{A}_z} t_a(v_a)$ for any path from the same OD w and having the same pattern outside this zone. Therefore,

$$\frac{\partial \mathcal{L}}{\partial f_p^A} = C_{w,z} + \frac{1}{\beta+1} \sum_{a\in \tilde{A}_z : a\in p} \left(v_a t_a'(v_a) + t_a(v_a)\right) + t_{\widetilde{w},z}^0 - \pi_w^C - \mu_z^{\widetilde{w}} \left(\sum_{a\in \widetilde{A_z}} \frac{\beta_a t_a^0}{\beta_a + 1} - t_{\widetilde{w},z}^0 \right) \geq 0$$

As a result, for any path flow p passing through the AV zone z and sharing the same pattern outside the z, we have

$$\sum_{a\in\tilde{A}_z:a\in p}(v_a t'_a(v_a)+t_a(v_a))\geq G_{w,\tilde{w},z}$$

$$f_p^A\ \sum_{a\in\tilde{A}_z:a\in p}\big(v_a t'_a(v_a)+t_a(v_a)\big)=0$$

where

$$G_{w,\tilde{w},z}=(\beta+1)\left(\pi_w^C-t_{\tilde{w},z}^0-C_{w,z}+\mu_z^{\tilde{w}}\left(\sum_{a\in\tilde{A}_z}\frac{\beta_a t_a^0}{\beta_a+1}-t_{w,z}^0\ \right)\right)$$

Due to linear constraints in $\mathcal{P}_{1P}$, the solution satisfying this condition is the SO solution in zone z. Furthermore, by using BPR function of $t_a(v_a)$ we have

$$v_a t'_a(v_a)+t_a(v_a)=(\beta+1)t_a(v_a)-\beta t_a^0$$

$$\Rightarrow\sum_p\sum_{a\in\tilde{A}_z:a\in p}f_p^A\left(v_a t'_a(v_a)+t_a(v_a)\right)=\sum_p\sum_{a\in\tilde{A}_z:a\in p}(\beta+1)f_p^A\,t_a(v_a)-\sum_p\sum_{a\in\tilde{A}_z:a\in p}\beta f_p^A\,t_a^0$$

$$\Rightarrow\qquad d_z^{\tilde{w}}G_{w,\tilde{w},z}=\ \sum_{a\in\tilde{A}_z}(\beta+1)\tilde{x}^{\tilde{w}}{}_a t_a(v_a)-\sum_{a\in\tilde{A}_z}\beta x_a^{\tilde{w}}\,t_a^0$$

$$\Rightarrow\qquad \frac{G_{w,\tilde{w},z}}{\beta+1}=t_z^{\tilde{w}}-t_{\tilde{w}}^0$$

$$\Rightarrow t_z^{\tilde{w}}\qquad =\frac{1}{\beta+1}\sum_{a\in\tilde{A}_z:a\in p}\left(v_a\big(t'_a(v_a)+t_a(v_a)\big)+t_{\tilde{w},z}^0\right.$$

Therefore, by substituting this into Eq. (A.2), we infer that

$$f_p^A > 0 \Rightarrow t_p^A = \pi_w^A - \mu_z^w(\sum_{a\in\tilde{A}_z} \frac{\beta_a t_a^0}{\beta_a + 1} - t_{\tilde{w},z}^0)$$

$$t_p^A \geq \pi_w^A - \mu_z^w(\sum_{a\in\tilde{A}_z} \frac{\beta_a t_a^0}{\beta_a + 1} - t_{\tilde{w},z}^0)$$

for any path p, this condition satisfies the UE condition for AV traffic defined in Eq. (11). Therefore, the model $\mathcal{P}_1$ is equivalent to $\mathcal{P}$ if $\beta_a = \beta$ for any $a \in \tilde{A}$ and $t_{\tilde{w},z}^0$ is (or becomes) constant for any OD pair w in any zone z.

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