Examining the impact of lesson structure when teaching with cognitively demanding mathematical tasks in the early primary years

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Abstract

Engaging students in a challenging (cognitively demanding) task and launching a mathematics lesson with a task prior to instruction are two characteristics of a reform-oriented approach to mathematics instruction often considered together. The current investigation systematically contrasted teaching with challenging tasks using a task-first lesson structure (Task-First Approach) with that of a teach-first lesson structure (Teach-First Approach) through the delivery of two programs of mathematics instruction to 75 Year 1 and 2 students (7 and 8 year olds). The investigation adopted a quasi-experimental design and included three studies. Study One was quantitative in nature and involved analysing pre- and post- program student outcome data. A series of Mixed Design ANOVAs revealed that both teaching approaches resulted in large gains in student mathematical performance. Moreover, there was no evidence that problem-solving performance differed by lesson structure, although the Teach-First Approach was somewhat more effective in improving mathematical fluency. Study Two was qualitative in nature and involved semi-structured interviews with teacher-participants. Analysis of interview data suggested that there appear to be distinct advantages to both the task-first and teach-first lesson structures. Specifically, teacher-participants perceived that the Teach-First Approach was more focused and efficient, whilst the Task-First Approach was viewed as empowering students, and providing an opportunity to build persistence whilst fostering student mathematical creativity. Despite these differences, there was evidence that the most dramatic shift in teaching practice for teacher-participants would be the incorporation of more cognitively demanding tasks into their mathematics instruction in any capacity. Study Three was also predominantly qualitative and involved semi-structured interviews with student-participants. In line with teacher-perceptions of the student experience (Study Two), analysis of student-participant interviews *indicated that students generally embraced struggle and persisted when engaged in mathematics lessons involving challenging tasks. In addition, many students described enjoying the process of being challenged. Although most students reported preferring the Teach-First Approach when learning with challenging tasks because it provided opportunities for cognitive activation, a substantial minority (41%) of students preferred the Task-First Approach, in part because they relished the higher level of cognitive demand involved. The findings do not support the assumption that for students to learn from cognitively demanding tasks, lessons must begin with these tasks. Given that each approach was revealed to be effective and to possess distinct strengths, it is recommended that early primary teachers give consideration to incorporating both Task-First Approaches and Teach-First Approaches into future mathematics instruction.*

Declaration

This thesis contains no material which has been accepted for the award of any other degree or diploma at any university or equivalent institution and that, to the best of my knowledge and belief, this thesis contains no material previously published or written by another person, except where due reference is made in the text of the thesis.

Signature: …[redacted]………………

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Date: 30th March, 2017.

Publications during enrolment: Peer-reviewed journals and conference proceedings

Thesis Chapter	**Publication Title**	**Status** *(published, in press, accepted or returned for revision, submitted)*	**Nature and % of student contribution**	**Co-author name(s) Nature and % of Co-author's contribution**
7	*Student reflections on learning with Challenging Tasks: "I think the worksheets were just for practice, and the challenges were for maths"*	*Published (Mathematics Education Research Journal)*	*80%. Concept, writing, analysis*	*Sarah Hopkins, Input into manuscript 20%*
6	*How does lesson structure impact teachers' willingness to teach with Challenging Tasks?*	*Accepted 9-2-17 (Mathematics Teacher Education and Development)*	*80%. Concept, writing, analysis*	*Sarah Hopkins, Input into manuscript 20%*
6	*Early primary school teachers' perceptions of students when observing lessons involving cognitively demanding mathematical tasks*	*Submitted (Elementary School Journal)*	*80%. Concept, writing, analysis*	*Sarah Hopkins, Input into manuscript 20%*
5	*Does lesson structure matter for student mathematical performance when teaching with challenging tasks?*	*Submitted (Elementary School Journal)*	*70%. Concept, writing, analysis*	*Sarah Hopkins, Input into manuscript, support with data analysis, re-drafting 30%*
3	*CLASS Challenging Tasks: Using Cognitive Load Theory to inform the design of challenging mathematical tasks*	*Published (Australian Primary Mathematics Classroom)*	*75%. Concept and writing*	*Sarah Hopkins, Support with structure and concept, input into manuscript 25%*
3	*Teaching mathematics in primary schools with challenging tasks: The Big (not so) Friendly Giant*	*Published (Australian Primary Mathematics Classroom)*	*100%*	
3	*How challenging tasks optimise cognitive load*	*Published (Proceedings of 39th Psychology of Mathematics Education conference)*	*100%*	

Note: None of the above articles was co-authored with Monash University students

Publications during enrolment: Editor-reviewed journals (targeted at practitioners)

Thesis Chapter	**Publication Title**	**Status** *(published, in press, accepted or returned for revision, submitted)*	**Nature and % of student contribution**	**Co-author name(s) Nature and % of Co-author's contribution**
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3	*Teaching with challenging tasks: Experiments with counting patterns*	*Accepted 18-10-16 (Primary Maths)*	*100%*	

Note: None of the above articles was co-authored with Monash University students

Student signature: [redacted] Date: 30/03/17

The undersigned hereby certifies that the above declaration correctly reflects the nature and extent of the student's and co-authors' contributions to this work. In instances where I am not the responsible author I have consulted with the responsible author to agree on the respective contributions of the authors.

Main Supervisor signature: [redacted]

Date: 30/03/17

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EXTENDED ABSTRACT

Over the past few decades there have been calls to reform mathematics education in Australia and increase the amount of time students spend engaged in deep problem solving and challenging mathematical tasks (e.g., Hollingsworth, McCrae, & Lokan, 2003). As part of this reform process, it has been argued that traditional lesson structures (i.e., teacher explanation, followed by student practice and correction) are inherently inadequate for meeting contemporary mathematical learning objectives (Sullivan et al., 2014). Instead, reform-oriented teaching approaches have frequently employed a triadic lesson structure: Launch, Explore, Discuss (Stein, Engle, Smith, & Hughes, 2008). Considerable empirical evidence is emerging as to the efficacy of reform-oriented approaches. To summarise, classroom climates perceived by students or expert observers to be more reform-oriented appear to foster students who are more intrinsically motivated to learn mathematics (e.g., Middleton & Midgley, 2002) and perform better mathematically (e.g., Jong, Pedulla, Reagan, Salomon-Fernandez, & Cochran-Smith, 2010).

However, from the perspective of cognitive load theory, launching a lesson with a cognitively demanding activity, which is not explicitly linked to teacher instruction and prior learning, is problematic (Kirschner, Sweller, & Clark, 2006). This argument is based on the idea that working memory has limited capacity to process novel information and is easily overloaded when required to solve an unfamiliar problem (Sweller, Kirschner, & Clark, 2007). Consequently, it may be that an alternative lesson structure which adopted the same reformist content and pedagogy, but began with a less cognitively demanding activity, would result in even larger gains in mathematical performance.

Given these contrasting evaluations, there is a need to disentangle the various elements of a reform-oriented lesson and to empirically investigate the impact that systematically varying one aspect, such as lesson structure, has on subsequent student learning outcomes.

The current investigation contrasted teaching with cognitively demanding tasks (challenging tasks) using a task-first lesson structure (Task-First Approach) with that of a teach-first lesson structure (Teach-First Approach), through the delivery of two programs of mathematics instruction to Year 1 and 2 students (see Figure A.1). The central aim was to investigate how varying lesson structure impacts teaching and learning with challenging tasks.

To address this aim, the investigation included three studies, each of which considered the impact of varying lesson structure from a different perspective. Study One was quantitative in nature, and focused on student outcome data collected pre- and post-program. Study Two involved interviews with participating classroom teachers, and Study Three encompassed interviews with students themselves.

Participants comprised 75 Year 1 and 2 students (7 and 8 years old) and their respective classroom teachers. These students were organised into three composite classes and these pre-existing groups were used in a quasi-experimental, cross-over study design. The cross-over aspect allowed each class to have the opportunity to experience both the Task-First Approach and the Teach-First Approach, which was particularly important for informing the qualitative components of this mixed-methods investigation. Specifically, Class A experienced the patterning unit of work under the task-first lesson structure, and the addition unit of work under the teach-first lesson structure, whereas Class B experienced the inverse. By contrast, Class C experienced an Alternating Approach across both units of work, with each pair of task-first lessons being followed by a pair of teach-first lessons.

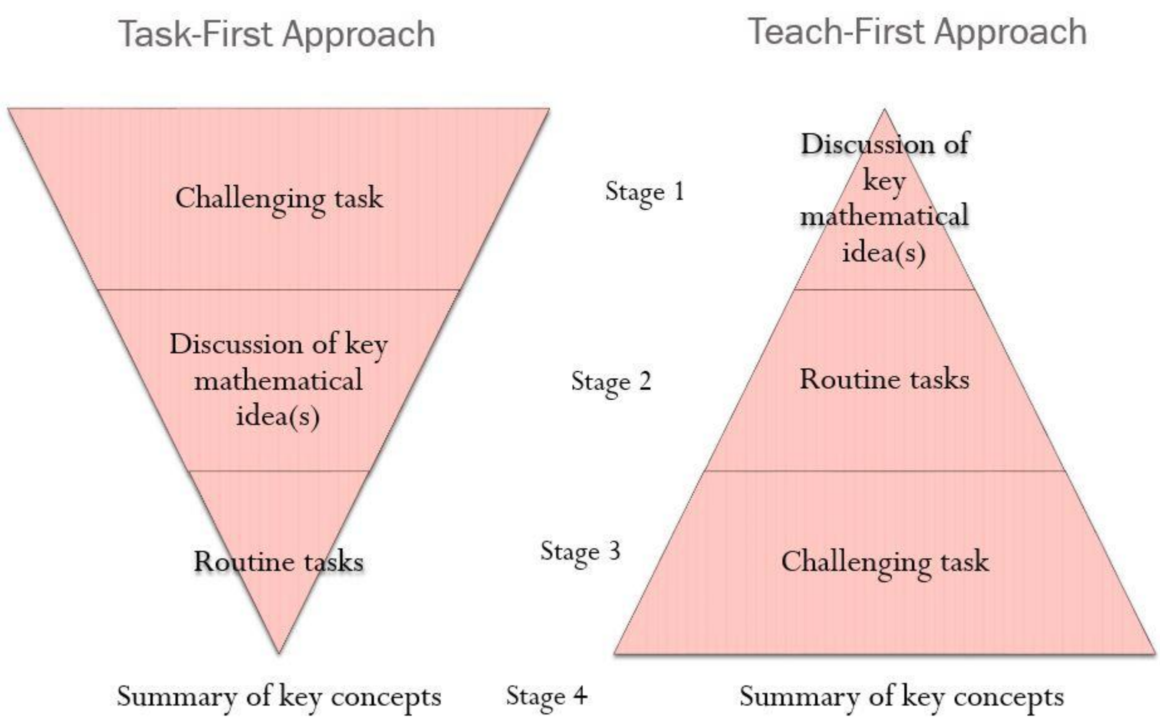


Figure A.1. Deconstructing a Reform-Oriented Approach: Alternative Lesson Structures.

In Study One, a series of mixed randomized-repeated design analyses of variance (Mixed Design ANOVAs) were employed to explore the relationship between participation in the program and student outcomes, including mathematical fluency, problem-solving performance, intrinsic motivation to learn mathematics and task-based student persistence. For each analysis, the within group factor was time (i.e., pre-, post-program) and the between group factor was lesson structure (i.e., Task-First Approach, Teach-First Approach, Alternating Approach). Assessments of mathematical performance revealed that all three instructional approaches resulted in large gains in student performance. Moreover, although the Teach-First Approach was somewhat more effective in improving mathematical fluency, there was no evidence that problem-solving performance differed by lesson structure. By contrast, both intrinsic motivation to learn mathematics and task-based student persistence appeared wholly unrelated to lesson structure, although this may have been in part a consequence of limitations regarding the specific instruments employed to measure these constructs.

Study Two employed interpretative phenomenological analysis to examine teacher-participants interviews following each unit of work. The findings revealed that teacher-participants perceived that students responded positively to learning with challenging tasks. Teacher-participants described students as being autonomous, persistent and highly engaged. Such positive student reactions were attributed by teacher-participants to a variety of factors, including a classroom culture which embraced struggle, high teacher expectations, and consistent classroom routines. However, other previously identified barriers to teaching with challenging tasks, including time and resource constraints, and possessing the relevant mathematical knowledge, remained (to varying degrees) a concern for participants. In addition, participants differed in their views of whether challenging tasks were a suitable means of differentiating instruction, with such evaluations apparently linked to how teacher-participants defined student success. Finally, with regards to lesson structure, teacher-participants perceived both the Task-First Approach and the Teach-First Approach to teaching with challenging tasks to have particular strengths. Specifically, the Task-First Approach was viewed as engaging and empowering students, providing an opportunity to build student persistence, whilst fostering student mathematical creativity. Teachers also placed value on the quality of the mathematical discussion which emerged, and the value of the Task-First Approach for supporting an authentic assessment of student mathematical knowledge. By contrast, the Teach-First Approach was viewed as highly focussed and an efficient approach to learning. It was also perceived as providing an opportunity for lower-achieving, and less confident, students to be successful. Although teacher-participants perceived there to be distinct advantages to both the Task-First and Teach-First Approaches, the study revealed that the most dramatic shift in teaching practice for the teachers involved would be the incorporation of more cognitively demanding tasks into their mathematics instruction in any capacity.

Study Three was divided into two sections. The first section used the Constant Comparative Method to analyse the interview responses of 73 young students regarding the work artefacts they were most proud of creating and why. Five themes emerged which characterised student reflections: Enjoyment, Effort, Learning, Productivity and Meaningful Mathematics. Females were more likely to endorse the Learning theme, and males trended towards disproportionately endorsing the Productivity theme. Overall, there was evidence that students embraced struggle and persisted when engaged in mathematics lessons involving challenging tasks, and moreover that many students enjoyed the process of being challenged. The second section considered the lesson-structure preferences of a subset of participants (Class C; n=23) when learning with challenging tasks. Overall, the majority of students preferred the Teach-First Approach to the Task-First Approach, apparently because it activated their cognition to prepare them for work on the challenging task. However, a substantial minority of students (42% on average) instead endorsed the Task-First Approach, with several students explaining they preferred this structure precisely because it was so cognitively demanding. Other reasons for preferring the task-first lesson structure included that it allowed the focus of the lesson to be on the challenging task and the subsequent discussion of student work.

A key corollary of this suite of findings appears to be that teaching with more cognitively demanding tasks in any capacity constitutes a significant departure from how mathematics is typically experienced in schools, at least for participating students and teachers in the current investigation. In addition, it appears that units of work incorporating challenging tasks can be effective for improving mathematical performance for students in the early years of primary school, regardless of how such lessons are structured. However, given that the Task-First Approach and the Teach-First Approach each possess distinct strengths, it is recommended that early primary teachers give consideration to incorporating both approaches into future mathematics instruction when teaching with challenging tasks.

CHAPTER ONE: INTRODUCTION

A Personal Introduction

I, as the researcher, played many roles in the current investigation. I was responsible for conceptualising the project, developing a process for designing appropriate cognitively demanding mathematical tasks, constructing two units of work incorporating 28 such tasks and hundreds of more routine tasks, teaching these two units of work to student-participants, collecting a range of pre and post data to assess student outcomes, interviewing teacher-participants and student-participants about their perceptions of the units of work, and analysing the resultant data using qualitative and quantitative methods. Given these multifaceted responsibilities, it is necessary for me to share a little of my background (and self), so readers know where I am coming from.

I came to teaching as a mature age student after spending several years working in social policy analysis and evaluation in the private and public sectors. After spending two years full-time in the classroom as a primary school teacher (teaching Year 1 and 2 students), I was given an opportunity to spend some time out of the classroom to develop a framework to better support the use of mental computation strategies. I came up with the idea of SURF Maths. I became so excited about the potential for this framework to improve mathematics instruction that I rushed off to enrol myself in a PhD in mathematics education. My first project outline was essentially to evaluate the implementation of SURF Maths, initially at my school, and then at other schools. However, mainly I was looking forward to implementing the SURF Maths framework at my school, and the opportunity to develop and deliver a program to support the framework. I thought the SURF Maths idea could get students (and teachers) more excited about learning (and teaching) mathematics.

Creating more positive feelings about mathematics in primary schools was, and remains, one of my biggest passions.

The following year my school accepted my proposal to work two days a week exclusively delivering a program of work built around SURF Maths, whilst spending the rest of my time beginning my PhD. I was quickly confronted with the reality of having to develop content to fill my program. I had already decided I wanted all my lessons to fall into one of two categories, i) mathematical games and ii) rich investigations and problem-solving tasks. In my experience, it was these lessons which most engaged students. I held the belief (and still do) that lessons involving games have great capacity to improve procedural fluency, whilst problem-solving tasks can deepen conceptual understanding. When teaching with problem-solving tasks or launching an investigation, I was always confronted by the dilemma of how much to tell students before they began their work. Should I instruct them in key concepts beforehand? Should I conduct a brief review of related prior learning? Should I throw them in the deep-end and hold off with instruction until they had spent some time on the task?

Meanwhile, at university, there were a couple of important developments. Firstly, one of my supervisors, Sarah, had convinced me that evaluating a program is not really about contributing new knowledge and consequently, SURF Maths was perhaps not the best idea for a PhD project. She encouraged me to take a step back, and think more broadly about gaps that I could identify within the mathematics education literature that resonated with me as a teacher as being important and meaningful. Secondly, at this time, I was also introduced to Peter Sullivan’s work on challenging tasks (Peter is one of my supervisors). I began to incorporate some of these tasks into my SURF program, and began experimenting with designing my own challenging tasks. It was out of this background that the current PhD project began to take shape, evolve and (hopefully) finally begin to conclude.

Context for Study

A reform-oriented approach to teaching mathematics

Students in Australian classrooms have tended to complete a high volume of mathematical tasks, but spend relatively little time engaged in deep problem solving (e.g., Hollingsworth et al., 2003). In their analysis of the seminal TIMSS 1999 Video Study, Hollingsworth et al. (2003) argued that this ‘volume’ approach was problematic for several reasons: it denies students opportunities to explore alternative solution pathways and to explain their thinking; it results in an over-emphasis on algorithms and using the ‘correct’ procedure; and it makes it difficult for students to appreciate connections between mathematical ideas and to develop a deeper comprehension of the mathematics behind the problems they are working on. The authors concluded that “it would be beneficial to reduce the time students are expected to spend solving large numbers of short repetitive problems and to use the freed time to work on fewer, more varied, more challenging (but accessible) problems, each for a longer time” (p. xxi).

Over the past decade, Hollingsworth et al.’s (2003) call for change has assisted in further energising attempts to reform mathematics education in Australia. This reform has paralleled similar developments in other countries, particularly the United States. For example, teachers have been encouraged to utilize more challenging, cognitively demanding tasks to better engage students in rich mathematical discussions (Cheeseman, Clarke, Roche, & Wilson, 2013; Stein et al., 2008).

This reform-oriented approach, sometimes referred to as reform teaching (Sherin, 2002), can be viewed as having three distinctive characteristics (see Table 1.1). The first characteristic, which is perhaps the most immediately tangible, relates to the nature of the instructional materials employed to support learning. Specifically, there has been a push towards the use of more complex, multi-faceted, problem solving tasks requiring a higher level of intellectual engagement (Sherin, 2002; Stein et al., 2008). The second characteristic can be described as a more student-centred approach to instruction, involving a greater emphasis on classroom discourse and an adaptive (as opposed to didactic) style of teaching (Sherin, 2002). The third characteristic relates to the employment of a triadic lesson structure. Specifically, reform-oriented approaches have been described as progressing in three stages (Baxter & Williams, 2010; Stein et al., 2008). The lesson begins with the launch phase during which time the teacher introduces students to the task, which generally represents a challenging problem to be solved. During the explore phase, students work on the problem, sometimes collaboratively, while the teacher provides support and guidance. Finally, after students have spent sufficient time engaged with the problem, the lesson enters the discuss and summarize phase, during which time various student-generated approaches to the problem and possible solutions are discussed (Stein et al., 2008). The teacher generally finishes by offering some form of summary comment (Baxter & Williams, 2010).

Table 1.1

Characteristics of a Reform-Oriented Approach

Cognitively demanding tasks	Student centred approach	Triadic lesson structure
Shift towards using more complex tasks to encourage deep mathematical thinking	Characterized by an adaptive style of teaching and a focus on classroom discourse	Launch, Explore, Discuss & summarize

Evidence for a reform-oriented approach

Considerable empirical evidence is emerging as to the efficacy of a reform-oriented approach. To summarize, classroom climates perceived by students as being more reform-oriented appear to foster students who are more intrinsically motivated to learn mathematics (Middleton & Midgley, 2002; Valas & Sovik, 1993). This enhanced intrinsic motivation has in turn been linked to improved mathematical performance (Woolley, Strutchens, Gilbert, & Martin, 2010).

It appears that 'traditional' teaching practices, characterized by a drill and practice emphasis, may impact negatively on the intrinsic motivation of students in relation to learning mathematics. In a study involving 335 Norwegian seventh and eighth graders, Valas and Sovik (1993) found that students who perceived their teacher to be more 'controlling' (i.e., they dictated the pace at which students worked through problems, discouraged cooperation and promoted a single best method or approach) were less intrinsically motivated to learn mathematics, due to their lower levels of self-concept as mathematics learners. Moreover, a longitudinal follow-up investigation by the same authors revealed that current perceptions of a particular teacher's controlling behaviour impacted students' intrinsic motivation to learn mathematics in the subsequent year. The authors concluded that mathematics teachers who were perceived as 'autonomy-supporting' (i.e., they provided student choice, minimised pressure, encouraged student initiative and problem solving) were more likely to motivate students intrinsically to perform well in mathematics.

Adding further weight to this argument, Middleton and Midgley (2002) found that the extent to which the teacher is perceived as pressing students for understanding (through, for example, emphasising student reasoning, asking clarifying questions and probing for further information) was associated with higher levels of personal task goal orientation; a construct which overlaps substantially with intrinsic motivation to learn mathematics. Furthermore, students who were more inclined to perceive their teacher as pressing for understanding tended to demonstrate more positive educational beliefs and behaviours, including higher levels of self-regulation and self-efficacy, and lower levels of help seeking avoidance. Moreover, it was apparent that the positive relationship between perceived push for understanding and these positive outcomes persisted even after controlling for the impact of motivational factors (Middleton & Midgley, 2002). Similarly, Woolley et al. (2010) found that student perceptions of the extent to which their teacher adopted a reform-oriented approach was associated with higher levels of academic achievement, which was partly mediated through higher levels of student intrinsic motivation.

Despite some earlier research suggesting that reform-oriented approaches tend to disproportionately benefit academically stronger students (e.g., Mayer, 1998), more recent research (e.g., Gilbert et al., 2014; Woolley et al., 2010) indicates that reform-oriented practices are particularly critical to the self-efficacy and mathematical performance of low-achieving students. For example, Gilbert et al. (2014), with their sample of 979 middle-school students, found that students with lower mathematical self-efficacy (who were also lower performers) who perceived their classrooms as reform oriented performed significantly better on a standardized assessment task than their counterparts who perceived their classrooms as being more traditional. No interaction between performance and reform-orientation was revealed for high efficacy students. This suggests that students who believe themselves to be less efficacious in mathematics and tend to achieve at a lower level may benefit even more than average and high achieving students from classrooms which emphasize mathematical discussions, problem solving, conceptual understanding, student conjecture and multiple solution pathways.

Finally, it is worth noting that all of the above studies relate to student perceptions of the classroom environment and the orientation of their teacher. By contrast, Jong et al. (2010) employed a teacher observation protocol (RTOP) to determine the extent to which a teacher could be categorized as reform-oriented. The authors found that beginning elementary teachers who utilized reform teaching practices had students who scored significantly higher on a standardized math test. Moreover, the effect size associated with the finding was large ($r = 0.56$), albeit with a small sample of teachers ($n = 22$).

Building on the evidence base: Deconstructing a reform-oriented approach

Despite some initial conjecture that reform-oriented approaches would disproportionately disadvantage students who were already marginalized by the education system (e.g., Hirsch, 1996), the evidence base for the effectiveness of such approaches within mathematics education is now established. However, generally the research into this area has made little attempt to deconstruct the elements which constitute a reform-oriented approach, and to systematically examine its various parts. Are the three previously identified elements of a reformist approach necessarily interdependent? Do they all need to be in place in order for the benefits associated with reform to be realized?

There is a need to disentangle the various elements of a reform-oriented lesson and to empirically investigate the impact that systematically varying one aspect, such as lesson structure, has on subsequent student learning outcomes and on the learning experience of students. To address this issue, the current thesis contrasts two alternative lesson structures, which are presented in Figure 1.1. The central question becomes:

What are the advantages of using cognitively demanding tasks to launch lessons and support subsequent instruction and discussion (task-first lesson structure) compared with using cognitively demanding tasks to extend understanding, following instruction and discussion and the completion of several more routine tasks (teach-first lesson structure)?

This question will be explored from a variety of different perspectives. Study One will consider the question from the point of view of evaluating the impact of lesson structure on student mathematical performance. Study Two will examine teachers' confidence and competence in teaching with cognitively demanding tasks, and whether lesson structure impacts teacher willingness to incorporate such tasks more comprehensively into future instruction. Study Three will explore student perceptions of learning with cognitively demanding tasks and whether students have a preference for a given lesson structure.

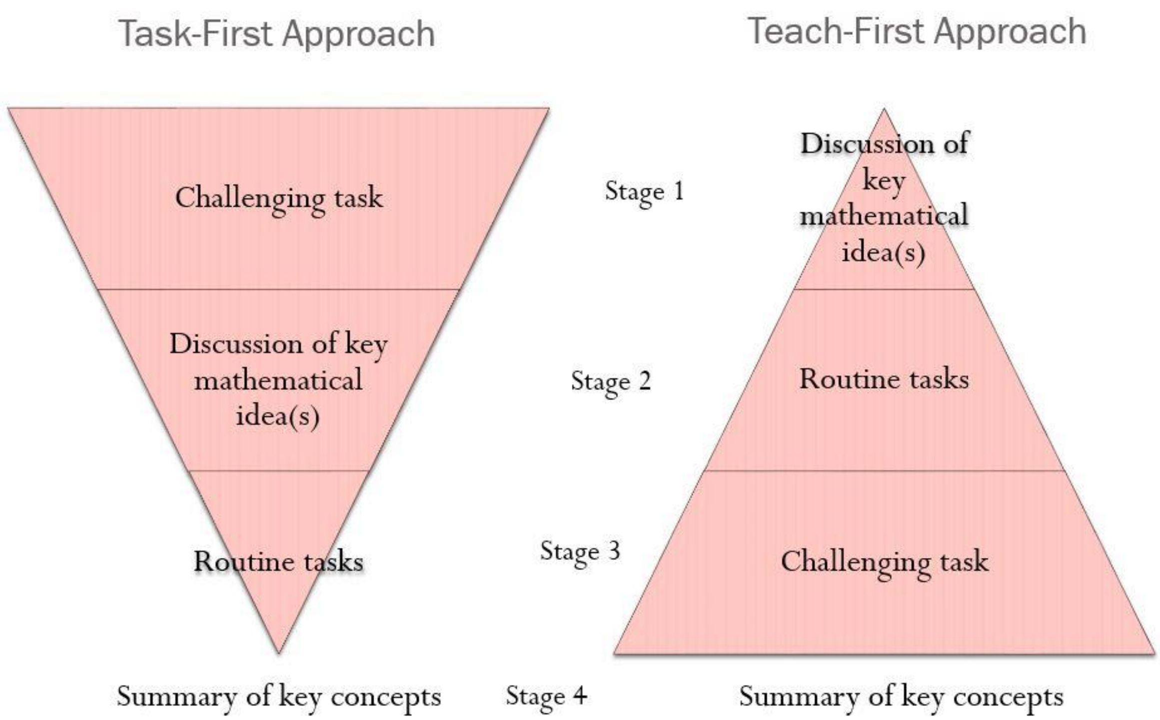


Figure 1.1. Deconstructing a Reform-Oriented Approach: Alternative Lesson Structures.

Defining Key Concepts

The purpose of this section is to define key components of the lesson which are critical to the current study, specifically cognitively demanding tasks and routine tasks. In addition, it will briefly overview, in general terms, the two lesson structures which are to be contrasted; that is, a teach-first lesson structure and a task-first lesson structure.

What are cognitively demanding tasks?

The quality and depth of the mathematical tasks developed and implemented by teachers is thought to be a critical factor in supporting the provision of learning opportunities for students (Anthony & Walshaw, 2010). Indeed, the notion that tasks should be both inherently cognitively demanding and that the teacher should ensure that a high level of challenge is maintained as work on the task unfolds, are frequently highly valued intermediate goals in modern mathematics education (Baxter & Williams, 2010; Henningsen & Stein, 1997). It has been argued that teachers need to be supported to ensure that *all* students are given opportunities to engage with complex mathematical tasks as part of their mathematical learning. Moreover, it is also considered vital that students be given sufficient autonomy to devise their own solution methods to such tasks, at least some of the time (Sullivan et al., 2014).

The term *cognitively demanding tasks* is most readily associated with the early work of Mary Stein and colleagues (e.g., Stein, Grover & Henningsen, 1996), and has also been referred to by the same authors as "high-level tasks" (Henningsen & Stein, 1997, p. 526). Stein et al. (1996) viewed cognitively demanding tasks as existing in juxtaposition to the types of mathematical tasks used in most classrooms, whereby the emphasis is on students absorbing and applying a teacher-introduced algorithm to solve a series of relatively straightforward mathematical problems. Stein et al. instead suggested the need for mathematical tasks that are meaningful, worthwhile and "truly problematic for students" (p. 456) in order to promote high-level mathematical thinking and reasoning. Moreover they argued that cognitively demanding tasks need to possess several important structural characteristics, including being solvable through multiple means, encompassing multiple representations and requiring students to justify their mathematical reasoning. Importantly, the authors emphasised the notion that a high level of cognitive demand needs to be sustained throughout student engagement with the task, which speaks to not only the structure of the task, but the manner in which it is set-up and implemented by the teacher in the classroom. More recently, Stein, Smith, Henningsen and Silver (2009) described cognitively demanding tasks as involving ‘procedures with connections’ and/ or ‘doing mathematics’. The former refers to linking the application of procedures with conceptual understanding, whilst the latter involves pursuing an inquiry where the path forward is not known.

Other authors have also used the term cognitively demanding tasks (e.g., Boston & Smith, 2011), offering insights into the construct that can build on the above discussion. For example, Wilhelm (2014) referred to cognitively demanding tasks as tasks that require students to “make connections to the underlying mathematical ideas” (p. 638), however she also acknowledged that the notions of high and low level of cognitive demand are highly dependent on student prior knowledge. Consequently, given the large variation in student mathematical knowledge within any given classroom, ensuring that all students can access cognitively demanding tasks which they find appropriately challenging is a complex undertaking.

Although the actions of the classroom teacher can open up cognitively demanding tasks to a larger variety of students (e.g., Lambert & Stylianou, 2013), other authors have emphasised the importance of carefully building in greater levels of cognitive differentiation into the structure of the task itself. For example, Sullivan and colleagues highlighted the importance of challenging tasks including enabling and extending prompts, which are developed prior to the delivery of the lesson (e.g., Sullivan & Mornane, 2013; Sullivan et al., 2014). Enabling prompts are designed to reduce the level of challenge through simplifying the problem, changing how the problem is represented, helping the student connect the problem to prior learning and/ or removing a step in the problem (Sullivan, Mousley, & Zevenbergen, 2006). By contrast, extending prompts are designed for students who finish the main task and expose students to an additional task that is more challenging, however that requires them to use similar mathematical reasoning, conceptualisations and representations as the main task (Sullivan, Mousley, & Zevenbergen, 2006[1]).

What are routine tasks?

For the purpose of the current investigation, "routine tasks" is a term used to reflect simpler mathematical tasks which exist in juxtaposition to more cognitively demanding tasks. Generally, routine tasks can be solved by the student following one (or several) established steps, the application of which has been demonstrated during prior instruction and discussion.

Under a task-first lesson structure, the intention of undertaking routine tasks is to consolidate understanding following engagement in a cognitively demanding task (Sullivan & Mornane, 2013). The notion of needing to consolidate learning following knowledge construction has empirical support in the literature (e.g., Dooley, 2012; Tabach, Hershkowitz, & Schwartz, 2006). By contrast, under a teach-first lesson structure, routine tasks likely form the core of the lesson, giving students an opportunity to practise a skill, or explore a concept, discussed during the preceding teacher-facilitated mathematical discussion.

[1] Note that due to Sullivan publishing multiple articles with different authors in 2006, all the names of the authors for these papers are provided throughout (i.e., the "et al." abbreviation is not used), as per APA standards (see http://blog.apastyle.org/apastyle/2011/10/reference-twins.html).

Task-first lesson structure

Task-first lesson structures are defined as encompassing all pedagogical approaches where students begin work on some form of cognitively demanding task prior to any instruction from the teacher regarding the mathematical concepts and/ or procedures embedded in the task.

Such a structure overlaps substantially with notions of discovery-based learning, a construct which has been described as so broadly construed as to be ill-defined (Klar & Nigam, 2004). Alfieri, Brooks, Aldrich, and Tenenbaum (2011) noted that part of the reason that discovery-based learning has been difficult to define is that it is often understood in juxtaposition to an alternative pedagogical approach which involves comparatively more explicit instruction, rather than defined in any absolute sense. In fact, Alfieri et al. (2011) chose to retain this relative construction of discovery-based learning, broadly defining it as involving instances when "the learner is not provided with the target information or conceptual understanding and must find it independently and with only the provided materials" (p. 2). However, it has been noted there is substantial variety regarding the extent to which students are given supports whilst engaged in discovery-based learning (Rosenshine, 2009). In response to this variety, Alfieri et al. proceeded to distinguish between unassisted discovery learning, and enhanced and/or assisted discovery learning, in their meta-analysis into the relative efficacy of discovery-based learning. Enhanced and/or assisted discovery was defined as those discovery-based approaches which had specific mechanisms built into the design of the task in order to guide the discovery process, such as explicit feedback and careful scaffolding.

More specific to the mathematics domain, Stein et al. (2008) proposed a three-stage task-first lesson structure when teaching with cognitively demanding mathematical tasks. The first stage involves the teacher beginning the lesson by *launching* the task. This encompasses presenting the problem to students, ensuring students understand what is expected of them, as well as engaging students in the relevant mathematical mindset. After the task is launched, students *explore* the task, and the teacher encourages students to develop at least one potentially appropriate solution. There is a strong emphasis on students justifying and recording their mathematical reasoning. The final stage of the lesson involves the teacher facilitating a whole-group *discussion*, which provides students with an opportunity to present their particular approach to solving the task. Stein et al. (2008) noted that it is critical that this aspect of the lesson should not devolve into a ‘show and tell’. Instead, the teacher should carefully consider which students are invited to share their work, and emphasise the connections between different student solutions, and between the various student solutions and the underlying mathematical concepts. The teacher may conclude this discussion by offering a summary of the key mathematical ideas. This three-stage approach to teaching with cognitively demanding tasks has been adopted and adapted by several other researchers (e.g., Sullivan et al., 2014).

Teach-first lesson structure

In contrast, teach-first lesson structures encompass pedagogical approaches whereby the lesson begins with some form of explicit instruction by the teacher (or instructor) regarding the mathematical concepts and/ or procedures embedded in the subsequent tasks in which students will be engaged.

The notion of a teach-first lesson structure overlaps with the idea of explicit instruction. As already noted, explicit instruction is a relative term (Alfieri et al., 2011) and consequently, can be defined on a continuum. However, several pedagogical approaches broadly come under the umbrella of explicit instruction. Perhaps most obviously, and again viewing the construct through a fairly broad lens, explicit instruction encompasses direct teaching. Direct teaching is defined by Alfieri et al. (2011) as "the explicit teaching of strategies, procedures, concepts, or rules in the form of formal lectures, models, demonstrations, and so forth and/or structured problem solving" (p. 5). Consequently, direct teaching, and many other approaches which could be loosely described as explicit instruction more generally, do not necessarily preclude students working on cognitively demanding tasks in mathematics. The expectation, however, is that students would have received prior instruction encompassing not only the launching of the task, but in the types of mathematical concepts and structures that would directly support them in solving the task.

By contrast, programs of Direct Instruction can be conceived of as specific embodiments of direct teaching within the paradigm of explicit instruction (e.g., Flores & Kaylor, 2007), which are unlikely to embrace students working on cognitively demanding tasks. With an emphasis on carefully scripted lessons, breaking learning down into manageable chunks, incrementalism in curriculum design, and the gradual and precise process of knowledge and skill acquisition (Ewing, 2011), direct instruction is likely to involve students working exclusively on what have been previously described as routine tasks. Direct Instruction can be viewed as the antithesis of unassisted discovery learning as described by Alfieri et al. (2011).

CHAPTER TWO: LITERATURE REVIEW

This literature review is divided into three sections. The first section examines the literature relevant to a consideration of how lesson structure may impact on student mathematical performance. This is the most comprehensive aspect of the literature review, and includes a discussion of the major substantive arguments in support of task-first lesson structures and teach-first lesson structures. The second section examines the issue of teaching and learning with cognitively demanding tasks from the perspective of teachers and students. Following a discussion of a range of general issues impacting teacher willingness to teach with such tasks, including observed and anticipated student reactions, the potential impact of lesson structure is considered. The third section concludes by outlining the current project and its three studies, including the general research questions arising from this review of the literature.

Lesson Structure and Student Mathematical Performance

This section considers the evidence in support of a task-first lesson structure, including a brief consideration of how such a lesson structure is viewed as being consistent with broader reforms in mathematics education. Next, it discusses how a perfunctory analysis from the perspective of cognitive load theory would suggest that a teach-first lesson structure is likely to be more efficacious. The review then proceeds to unpack the construct of discovery-based learning and considers how different operationalisations and interpretations of discovery-based learning appear to be linked to differences in student performance outcomes. It concludes by offering a framework to describe the link between discovery-based learning and mathematical performance that captures some of these nuances.

Arguments in support of a task-first lesson structure

To some extent, all of the evidence in support of a reform-oriented approach can be viewed as supporting a task-first lesson structure, given that reformist lessons have generally followed the structure: launch, explore, discuss and summarize (Stein et al., 2008). Consequently, it may be expected that a task-first lesson structure produces students who are more intrinsically motivated to learn mathematics (e.g., Middleton & Midgley, 2002; Valas & Sovik, 1993) and who perform better on standardized measures of mathematical performance (e.g., Jong et al., 2010; Woolley et al., 2010). However, from the point of view of the current study, such assertions appear tautological. What specific arguments can be made in support of a task-first lesson structure as distinct from a reform-oriented approach more generally? Three inter-related arguments in support of a task-first lesson structure can be found in the literature.

Firstly, it has been asserted that building a lesson around students tackling a cognitively demanding task may improve student persistence (Sullivan et al., 2013), as students work through the "zone of confusion" (Sullivan et al., 2014, p. 11). The "zone of confusion" captures the idea that being temporarily unsure how to proceed when engaged in a task is part of the process of doing mathematics. The argument is that teachers can facilitate student persistence through normalizing the concept of the 'zone of confusion' in the mathematics classroom. Students should be encouraged to view this state as a prompt for constructive action (e.g., pursuing a trial and error problem solving strategy, accessing the enabling prompt), rather than as a sign of 'failure'. It is through such constructive action and an associated shift in the attitude of the learner that persistence can lead to higher levels of mathematical performance (Sullivan et al., 2014).

Secondly, it has also been postulated that students will be more engaged in the learning material if the task is more challenging (Sullivan, Clarke, Michaels, Mornane, & Roche, 2012), which is likely to be the case when the task is presented first. Indeed, there is some evidence that lessons built around cognitively demanding tasks foster high levels of student engagement (e.g., Roche, Clarke, Sullivan, & Cheeseman, 2013; Sullivan et al., 2014). Given the conceptual and empirical overlap between the constructs of student engagement and intrinsic motivation (e.g., Shernoff, Csikszentmihalyi, Schneider, & Shernoff, 2003), this could be viewed as providing support for the notion that a task-first lesson structure can effectively enhance intrinsic motivation to learn mathematics, in turn improving student mathematical performance.

Thirdly, it can be argued that it is not tackling the task in isolation that is responsible for the positive student outcomes derived from a task-first lesson structure, but rather the fact that launching a lesson with a cognitively demanding task supports and enables the subsequent mathematical discussion. For example, Stein et al. (2008) contended that beginning with a cognitively demanding task is necessary in order to provide the students and teacher with the requisite context to engage in a rich mathematical discussion.

Indeed, it has been well established in the literature for some time that the nature and complexity of tasks introduced in classrooms substantially influence the types of thinking processes in which students subsequently engage, and consequently the student learning that is likely to take place (Stein et al., 1996). Moreover, there are several potential advantages to undertaking a discussion that is grounded in student experience. For instance, such experienced-based discussions offer a mechanism for students to make their thinking public, which provides opportunities for a teacher who is willing to listen carefully and respectfully to students to require students to justify their responses (Woodward & Irwin, 2005). This in turn allows the teacher to restate and reshape student perspectives, so they more appropriately reflect underlying mathematical concepts (Forman, Larreamendy-Joerns, Stein, & Brown, 1998). In addition, as students grow in confidence, experience-based discussions allow for students to engage in a critical evaluation of their own and other students' mathematical ideas (Forman, McCormick, & Donato, 1997).

Therefore, to the extent that task-first lesson structures improve student persistence, intrinsic motivation to learn mathematics and the quality of mathematical discussion in which students engage, they may be expected to lead to higher levels of mathematical performance vis-à-vis teach-first lesson structures.

Criticisms of task-first lesson structures

Perhaps the most significant criticism of beginning a lesson with a cognitively demanding task comes from some cognitive load theorists, and relates to their unequivocal rejection of the efficacy of problem-based approaches to learning (Sweller, 2010). Sweller, a key figure in the development of cognitive load theory, launched a stinging critique of what he and his colleagues termed "unguided or minimally guided instructional approaches" in their provocatively entitled article: *Why minimal guidance during instruction does not work: An analysis of the failure of constructivist, discovery, problem-based, experiential, and inquiry-based teaching* (Kirschner et al., 2006, p. 79). The authors argue that such pedagogical approaches are less effective than those traditional learning approaches that rely more on carefully scaffolded teacher-led instruction. This is suggested to be either a product of an unnecessary level of extraneous cognitive load associated with minimally-guided approaches due to poor instructional design (Kirschner et al., 2006), and/or inappropriately high levels of intrinsic cognitive load, due to the unrealistically ambitious nature of the learning objectives (van Merrienboer & Sweller, 2005). In either case, the authors contend that adopting minimally-guided approaches tend to result in cognitive overload. The suggested mechanism responsible for this cognitive overload is that the number of interacting elements within the set task is high, which increases the number of items the learner has to process simultaneously in working memory to unsustainable levels, in turn impeding learning (Sweller, 2010).

Although this assertion outlining how cognitive load theory establishes the superiority of teacher-led instruction over minimally-guided approaches is not uncontroversial and has attracted a number of critical commentaries (see Hmelo-Silver, Duncan, & Chinn, 2007; Kuhn, 2007; Schmidt, Loyens, van Gog, & Paas, 2007), Sweller and his colleagues have held steadfastly to their position. The extended quote below summarizes their fundamental tenet relating to the perceived contradiction between cognitive load theory and minimally-guided approaches:

> The process of discovery is in conflict with our current knowledge of human cognitive architecture which assumes that working memory is severely limited in capacity when dealing with novel information sourced from the external environment but largely unlimited when dealing with familiar, organized information sourced from long-term memory. If this view of human cognitive architecture is valid, then, by definition, novices should not be presented with material in a manner that unnecessarily requires them to search for a solution with its attendant heavy working

> memory load rather than being presented with a solution [by the teacher] (Sweller et al., 2007, p. 116).

Essentially Sweller and colleagues are claiming that proponents of minimally-guided approaches are choosing to ignore contemporary knowledge of human cognition when designing instruction (Krishner et al., 2006). In Chapter Three, which unpacks the task design principles guiding the current project, this contention of Sweller and colleagues is critiqued, insofar as it is argued that cognitive load theory and problem-based approaches to learning are in fact not incompatible. Instead it is suggested that the former can directly inform the development of the latter through the adoption of an approach to task design that is termed the Cognitive Load Approach to Shaping and Structuring Challenging Tasks (CLASS Challenging Tasks).

Unpacking the broader evidence for discovery-based instruction

Another lens through which to consider the relative efficacy of a task-first lesson structure is to unpack and evaluate the evidence more broadly for discovery-based learning and instruction. Alfieri et al. (2011) attempted to systematically gather and assess the evidence in relation to discovery-based instruction across multiple educational settings and content areas through undertaking two meta-analyses incorporating 164 studies. Their findings made an important distinction between *enhanced* or *assisted* discovered-based learning and *unassisted* discovery-based learning. Specifically, they revealed that *unassisted* discovery-based learning was inferior to explicit instruction in terms of student learning outcomes, whereas *enhanced* discovery-based learning was superior to explicit instruction. Examples of enhanced discovery-based learning included discovery learning that incorporated elements such as feedback, work examples, elicited explanations and/ or scaffolding. The authors conclude that "in line with constructivist aims… participation in guided discovery is better for learners than being provided with an explanation or explicitly taught how to succeed on a task" (p. 11).

The Alfieri et al. (2011) meta-analyses also included a number of moderators within each meta-analysis, including learning domain and age of participants. The relative advantage of explicit instruction over unassisted discovery-based learning was smaller in the domain of mathematics (d= -0.16) compared with other domains (d= -0.39). By contrast, the relative advantage of enhanced discovery-based learning over explicit instruction was similar in mathematics (d= 0.29) compared with other domains (d= 0.30). Moreover, there was some evidence that adults benefited relatively less from explicit instruction compared with non-adults; although, interestingly, neither meta-analysis revealed any notable differences between children (12 years and under) and adolescents. Contrary to postulations put forward by several educational theorists and commentators (e.g., Mayer, 2004; Westwood, 2011), the meta-analysis revealed that children still learnt more through enhanced discovery-based learning than through explicit instruction (d= 0.24).

It could be argued that teaching with cognitively demanding tasks within mathematics education using a teach-first lesson structure could potentially be construed as being either *unassisted* discovery or *enhanced (assisted)* discovery, depending on the specific characteristics of both the task and its delivery. For example, it can be argued that challenging tasks, as described by Sullivan and colleagues (e.g., Sullivan & Mornane, 2013), offer an example of enhanced and/or assisted discovery. This is due to the opportunity for feedback and elicited explanations during the discussion component of the lesson (Stein et al., 2008), as well as the critical role that enabling and extending prompts play in scaffolding and extending student thinking within a challenging task.

A framework linking discovery-based learning and mathematical performance

Combining the findings from the Alfieri et al. (2011) meta-analysis with the arguments and evidence in support of launching a lesson with a cognitively demanding task allows the construction of a framework outlining both how, and under what specific conditions, discovery-based learning can be expected to lead to superior student mathematical performance. This framework is presented in Figure 2.1.

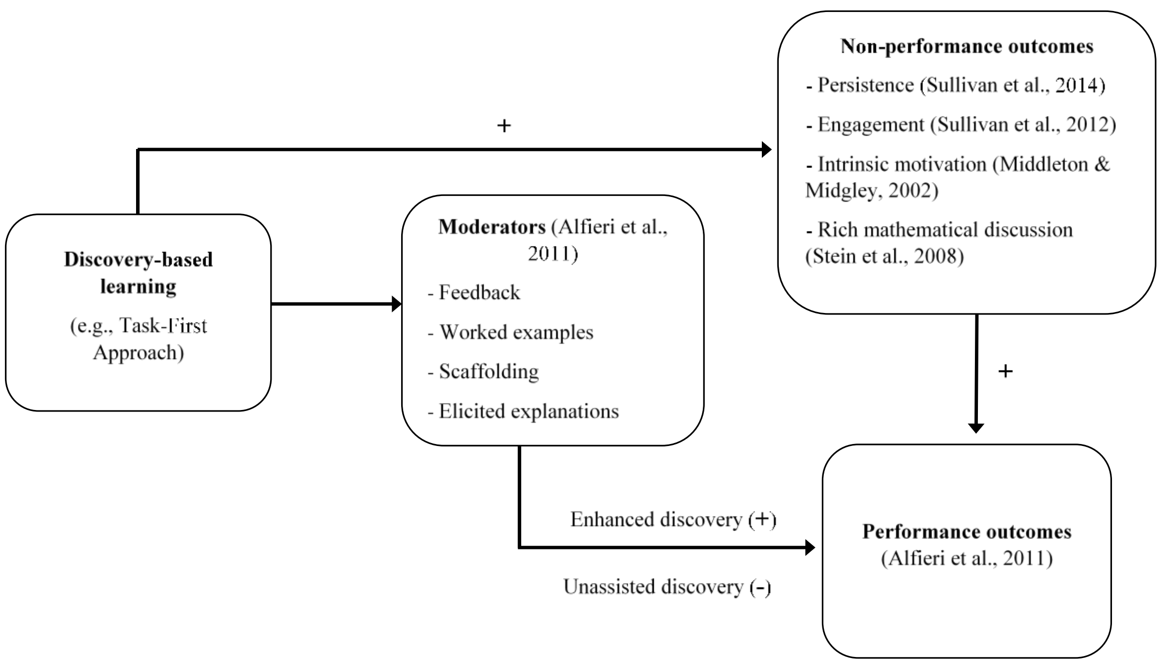


Figure 2.1. Framework Describing the Link between Discovery-Based Learning and Mathematical Performance.

The contention is that discovery-based learning may contribute to improved mathematical performance both indirectly and directly. First, improvements in mathematical performance may be mediated by several non-performance related outcomes.

Specifically, both the process of learning through discovery, as well as the structure of a discovery-based lesson, may generate improvements in non-performance related measures such as student persistence, student engagement, intrinsic motivation to learn mathematics and the quality of mathematical discussions. In addition to these outcomes being valued states in and of themselves, the suggestion in the literature is that generating these non-performance outcomes may in turn help support higher levels of mathematical performance. Second, discovery-based learning can also directly contribute to improved mathematical performance, if some or all of the moderators necessary for ensuring that the discovery-based learning is perceived by students as enhanced discovery (as opposed to unassisted discovery) are present. These moderators include the important aspects outlined in the Alfieri et al. (2011) meta-analysis, such as feedback, worked examples, scaffolding and elicited explanations. As outlined by Alfieri et al., (2011), and implied in Figure 2.1, if these moderators are absent, discovery-based approaches may hamper improvements in mathematical performance, at least relative to more explicit approaches. Note then that one implication of ideas presented in Figure 2.1 is that unassisted discovery-based approaches may have ambivalent impacts on mathematical performance, with meditating variables (e.g., student persistence) supporting higher levels of mathematical performance, but moderating variables (e.g., lack of feedback) supporting lower levels of mathematical performance.

Teaching with Cognitively Demanding Tasks

Despite their apparent efficacy, it has been suggested that teachers are frequently reluctant to pose cognitively demanding tasks to students (Cheeseman et al., 2013; Darragh, 2013; Sullivan, Clarke, & Clarke, 2009; Tzur, 2008). This is linked to the notion that engaging with problem solving tasks involves greater ambiguity and therefore personal risk to students vis-à-vis working on more routine tasks (Henningsen & Stein, 1997). Consequently, the primary concern for these reluctant teachers appears to be that if students perceive the mathematics as ‘too difficult’, they will disengage from the task and withdraw from the lesson (Sullivan et al., 2014). If this happens consistently, teachers worry that students may develop, or continue to hold, a pervasive negative attitude towards mathematics; effectively being “frightened” off the subject (Darragh, 2013, p. 222). From a teacher perspective, such a reaction from students also has clear implications for classroom management and maintaining order (Doyle, 1983). There is in fact some empirical evidence to support the notion that concerns around student reactions is a barrier to teaching with more cognitively demanding tasks.

Leikin, Levav-Waynberg, Gurevich, and Mednikov (2006) found that less than one-quarter of a group of upper high school teachers who attended professional learning on teaching through problem solving agreed to incorporate more cognitively demanding tasks embracing multiple solution methods into their subsequent teaching practice. Teacher concerns centred on whether such an approach could meet the learning needs of lower-achieving students, and doubts regarding student capacity to sufficiently engage in peer-based mathematical discussions, involving both listening to other students and publicly articulating their thinking. Together, these concerns appear to be at least in part linked to teacher anxiety regarding classroom management and a lack of teacher ‘control’.

Sullivan, Clarke, Clarke, and O’Shea (2010) found that two of the three teachers they observed teaching with a particular cognitively demanding task attempted to modify the task through limiting the way students framed the problem and leading them towards a specific procedure for solving it. Following their analysis of reflective interviews with teachers post-task, the authors concluded that it was the teachers' anticipation of unproductive student struggle, in part reflecting the teachers' own personal discomfort with the mathematics involved in the task, which led to them attempting to reduce the potential challenge for students.

Anthony (1996) discussed another side of this classroom control aspect, following observations of a Year 12 mathematics class at a co-educational school. Her analysis revealed that a teacher may be 'rewarded' for reducing the level of cognitive demand, as well as 'punished' for making a lesson or task too difficult. Anthony found at least indirect evidence that students may actively seek to manipulate teachers to reduce the level of challenge, through such actions as articulating a preference for teacher-given summaries and publicly equating what amounts to spoon-feeding with good teaching. In the words of on one particular student: "She's a very good teacher, she writes down the answers for you" (Anthony, 1996, p. 42).

Other studies have demonstrated how teachers, after being exposed to professional learning involving cognitively demanding tasks, can actually come to value student struggle rather than fear it. For example, Wilkie (2015) found that, in contrast to Anthony (1996), the two upper high school teachers in her study were inclined to embrace student discomfort, and not allow a lack of student progress, or a passive student attitude, to pressure them into prematurely sharing potential solution strategies. Furthermore, the third teacher (Ms A) from the Sullivan et al. (2010) study allowed students to continue to pursue their personal solution method despite their substantial difficulty with the corresponding calculations. She took this action even though her decision compromised the timing of her lesson substantially, and meant that, for example, there was no opportunity for students to share their different solution methods.

Despite the reported reluctance of some teachers to tolerate student struggle, there is evidence that students themselves are willing to persevere with extremely challenging mathematical problems, provided that the culture of the classroom, which is in no small part driven by the actions of the teacher, supports such behaviours (e.g., Clarke & Clarke, 2003; Sullivan, Tobias, & McDonough, 2006; Sullivan et al., 2013).

For example, Sullivan, Tobias and McDonough (2006) found that *all* 50 of the Year 8 students who participated in one-on-one assessment interviews as part of their broader study were willing to persist with a series of cognitively demanding tasks (in mathematics and language) for two-hour periods. Students universally persisted with this challenging assessment, irrespective of both their performance on these tasks and their perceptions of the tasks' difficulty. This contrasted substantially with teachers' perceptions of some of these same students, who they labelled as demonstrating "resistance to learning mathematics" (p. 88). Subsequent analyses by the authors led them to attribute this discrepancy to a "classroom culture that censures achievement and effort", which appeared to be primarily driven by the desire for peer acceptance and approval (p. 91).

Although several studies have canvassed the views of teachers in terms of their perceptions of student reactions to working on more cognitively tasks, few studies have attempted to systematically gather data about the student experience directly from students themselves. An exception is a study by Sullivan and Mornane (2013), who surveyed 187 Year 8 students to gauge their reactions to working on a pair of challenging tasks. Specifically, for each task, students were asked whether they liked working on the task, whether they learnt from working on the task, and whether they preferred the task to usual questions. The authors found that students were, on average, somewhat positive about the experience. Most students found the experience of working on the challenging task similar to their regular mathematical instruction in terms of their learning and enjoyment, with the remaining students almost equally divided between rating work on the tasks as inferior or superior to mathematics as “usual” (p. 15). When asked to provide a single word response describing how they felt about the lesson, the most frequently chosen words were: challenged, interested, confused, relaxed, and bored. Interestingly, follow-up interviews with eight students indicated that classroom culture, and in particular student relationships with their classroom teacher, was critical to them being willing to embrace more cognitively demanding tasks. In the words of one particular student: “Because you’d learn more, you’d be more comfortable asking more questions but if I didn’t get on, I wouldn’t ask as many questions because I’d be nervous. But I have a good relationship with my teacher” (p. 18).

Pekrun’s (2006) control-value theory of emotions in an achievement setting can help to make sense of these differing appraisals of student reactions to being confronted with more cognitively demanding tasks. Specifically, Pekrun postulates that students are inclined to experience a range of activity-related emotions when engaged in problem-solving tasks, depending on their perceived control over the task in which they are engaged, and the level of value attached to the task. When students experience both a high level of control and place considerable value on the task, they are likely to experience enjoyment. By contrast, an evaluation of low control and high value is postulated to generate anxiety, whilst an evaluation of high control and low value may lead to frustration and anger. Finally, the pairing of low control and low value suggests boredom (Muis, Psaradellis, Lajoie, Di Leo, & Chevrier, 2015).

It is likely that the creation of a “positive classroom culture”, which has been identified as a precursor to encouraging students to engage and persist with challenging tasks (Sullivan et al., 2013, p. 625), is dependent on supporting students to experience both high control and high value. High control can be fostered through actions such as encouraging students to have autonomy over how they approach their learning whilst allowing opportunities for cognitive activation, ensuring tasks are clearly structured, encouraging self-regulation, allowing students to work cooperatively with peers and promoting a mastery, or cooperative, goal-orientation (Pekrun, 2006). In addition, the valuing of mathematical learning and the task itself (i.e., high value) can be promoted by ensuring that the types of learning materials available, and interactions which occur in the classrooms, are consistent with student needs (Krapp, 2005). Pekrun (2006) suggested that this can be realised through the teacher incorporating “authentic learning tasks and a classroom discourse that engages all students” (p. 334). He also noted that student valuing will be impacted by what Hatfield, Cacioppo, and Rapson (1994) termed “emotional contagion”, as students absorb the reactions of significant others, such as teachers, in terms of how they respond to challenging academic material; essentially highlighting the notion that values can be modelled.

Although the link is far from definitive, it is clear that a myriad of factors that the classroom teacher can directly and indirectly influence impact on the likelihood that a given student will experience both high control and high value, and therefore enjoy mathematics as an activity. Consequently, it may be that studies which have reported positive student responses to cognitively demanding tasks have tended to reflect classroom cultures which support both high control and high value. For example, Clarke and Clarke (2003) worked with a class of Year 2 students and their teacher, introducing two complex mathematical problems. Their narrative account makes prominent the role that the classroom teacher served in supporting an inquisitive culture of enquiry. This was facilitated in part by the classroom teacher's inherently flexible stance, whereby she was willing to delay introducing new topics to take advantage of the momentum created by these two tasks. A clear message was being sent to students that deep thinking, and solving genuine mathematical problems that have captured the class's attention, were valued activities.

To summarise, most studies that considered teacher and student reactions to working on cognitively demanding tasks have been largely limited to students in Years 3 (i.e., age 9) or older, whereas less work has been done with students in the early years of schooling. Moreover, the vast majority of studies in this area have considered teacher perceptions of student reactions, and not considered the voices of students themselves. Directly investigating the reactions of younger students to working on cognitively demanding tasks would seem particularly relevant, given it has been argued by Westwood (2011) that developing a mathematical program that focusses on learning through problem solving is problematic when teaching young students, or those with learning difficulties. He suggested that young students lack the requisite arithmetic knowledge and skills to engage meaningfully in problem-solving, and argued that attempting to do so risks undermining their confidence and cultivating a negative attitude towards mathematics.

Additional barriers to teaching with cognitively demanding tasks

There are other reasons why teachers may be reluctant to pose more cognitively demanding tasks, which go beyond concerns about student reactions. There are at least two additional explanations that have received some attention in the literature.

Firstly, it has been suggested that time and resource constraints are an important factor as to why more teachers do not develop richer problem solving tasks to support mathematics lessons (Sullivan et al., 2014; Sullivan et al., 2012). This is hardly surprising given that a lack of time is an oft-cited obstacle to any significant reform within education (Gandara, 2000). Collinson and Cook (2001), in their qualitative study investigating barriers and motivators to implementing a technologically-oriented reform across three middle schools, identified nine time-related themes which served as barriers to implementing the intended reforms. Some of these themes, such as a lack of common planning time scheduled with other teachers and a lack of discretionary time to comprehend new content, may be particularly relevant to understanding why teachers have not embraced teaching with more challenging mathematical tasks, despite their apparent efficacy.

Secondly, it has also been argued that a lack of pedagogical content knowledge (PCK) is likely to lead to a teacher placing greater focus on routine tasks and procedural fluency, at the expense of deep problem solving and conceptual understanding (Forrester & Chinnappan, 2010). For example, Charalambous (2008) compared the task presentation and enactment of two teachers, one classified as having high-PCK and the other as having low-PCK, and found that the former allocated substantially more time to cognitively demanding tasks than the latter. In addition, the high-PCK teacher was also far more inclined to maintain a high level of cognitive demand as the task unfolded, whereas the low-PCK teacher tended to lead students towards particular procedural approaches and the use of specific algorithms. Therefore, limited PCK, and low levels of mathematical content knowledge more generally (Westwood, 2011), may serve as a further impediment to teaching with more cognitively demanding tasks. This lack of content knowledge could be framed as some teachers feeling they lack the confidence and/ or competence to teach with such tasks.

However, there is a third, related argument which has yet to be put forward. It may be that the recommended task-first lesson structure could also be a barrier to some teachers employing cognitively demanding tasks. Specifically, a lack of confidence and competence in teaching with cognitively demanding tasks may be further amplified when teachers are faced with the expectation that they will be able to successfully develop and deliver an entire lesson around such a task. Although the three-stage launch-explore-discuss (including summary) structure which characterises a typical lesson built around a cognitively demanding task is relatively straightforward to explain, it requires a substantial amount of skill to successfully orchestrate (Stein et al., 2008; Ridlon, 2009; Sullivan et al., 2009; Thomas & Monroe, 2006). In particular, in preparing for the discussion component of the lesson whilst students are engaged in the explore phase, the teacher is required to select particular work samples, sequence these samples in a meaningful way (e.g., least-to-most sophisticated mathematically, least-to-most efficient), connect the relevant mathematical ideas and potentially introduce important alternative solutions and ideas not raised by students (Stein et al., 2008). Although some of this can occur during the lesson planning stage, much of the preparing for the coordinated discussion must happen in-lesson, whilst teachers are simultaneously supporting students struggling to make progress and managing the classroom. This is clearly a demanding undertaking for teachers. Furthermore, as Stein et al. (2008) highlighted, the fact that such lessons are complex and carefully thought through is often masked by the tendency in the literature to present models for these lessons based on the practice of expert facilitators, which may provide little practical guidance for non-experts. Indeed, the lack of expertise and knowledge of generalist primary school teachers has been identified by some commentators as a significant barrier to using problem-based approaches to learning mathematics in primary school environments (e.g., Westwood, 2011).

Current Project

The current project examines the impact of lesson structure when teaching with cognitively demanding tasks from three different perspectives. Study One considers how systematically varying the structure of lessons involving cognitively demanding tasks effects student mathematical performance and important non-performance outcomes (i.e., intrinsic motivation to learn mathematics, student persistence). Study Two examines teacher perceptions of lessons involving cognitively demanding tasks, with particular attention paid to whether lesson structure influences these perceptions. This study considers both teacher perceptions of the impact on the student learning experience, as well as their own confidence and competence in teaching with cognitively demanding tasks. Study Three considers student perceptions when learning with cognitively demanding tasks and, in particular, whether they have a preference for a task-first lesson structure or a teach-first lesson structure. The rationale for each study is briefly summarised below, along with the research questions to be addressed by each study.

Study One: Investigating student outcomes

Reform-oriented approaches have been associated with a number of positive student outcomes, in particular, increased intrinsic motivation to learn mathematics (e.g., Middleton & Midgley, 2002) and mathematical performance (e.g., Jong et al., 2010). One of the key characteristics of a reform-oriented approach is the adoption of a three-stage lesson structure, whereby a cognitively demanding task is launched and subsequently explored by students prior to a teacher-facilitated discussion of the relevant mathematical ideas (Stein et al., 2008).

However, it could be argued that if a reform-oriented approach has been successful, it has been in spite of launching the lesson with the task, rather than because of it. Some interpretations of cognitive load theory might lead to the suggestion that a cognitively demanding task be introduced at a more appropriate time in the lesson; that is, after students have been explicitly exposed to the key mathematical ideas of the lesson and after they have engaged with these ideas through undertaking several more routine tasks. The assumption is that such a structure would prevent students being overwhelmed by the excessive and/or unnecessary cognitive load. Ultimately, however, the decision whether to endorse a task-first lesson structure or a teach-first lesson structure (or both) when teaching with cognitively demanding tasks can be treated as an empirical question.

Given the above tensions, the current study addresses several research questions about the relationship between lesson structure and student mathematical performance and non-performance outcomes. Note that, of the non-performance outcomes discussed in this review (i.e., persistence, intrinsic motivation, engagement and quality of discussion), only the constructs of persistence and intrinsic motivation to learn mathematics are included in the study. Student persistence is included because cultivating this quality is viewed as a particularly critical objective of teaching with cognitively demanding tasks (Sullivan et al., 2013). Intrinsic motivation to learn mathematics has also been included, as it appears to be more clearly specified than the overlapping concept of student engagement in mathematics, and because an appropriate instrument for measuring intrinsic motivation is readily available (Thomson, De Bortoli, & Buckley, 2014).

Finally, it was deemed to be too difficult to also measure the quality of the mathematical discussion. This decision was taken due to the researcher being engaged in the process of actually delivering the program content (see Chapter Four describing the project's methodology) and therefore being unavailable to objectively assess the quality of the mathematical discussion. Although recording and reviewing individual lessons remained a possibility, this course of action would have delayed the project considerably, given the additional ethical considerations associated with recording lessons and the time-intensive nature of the corresponding analysis. However, despite not being measured quantitatively, it is worth noting that Study Two and Study Three will consider perceptions of the quality of the mathematical discussion when relevant.

The *general research questions* for Study One can be stated as follows.

When teaching with cognitively demanding tasks:

1. What is the effect of a task-first lesson structure compared with a teach-first lesson structure on students' mathematical performance?
2. What is the effect of a task-first lesson structure compared with a teach-first lesson structure on students' intrinsic motivation to learn mathematics?

3. What is the effect of a task-first lesson structure compared with a teach-first lesson structure on student persistence?

Study Two: Investigating teacher perceptions

Given the aforementioned issues around launching a lesson with a cognitively demanding task, it may be that teachers would be more comfortable incorporating such tasks to further extend student thinking, rather than having them serve as the core of the lesson. Specifically, teachers may prefer to teach with such tasks using a teach-first lesson structure (e.g., a teacher-facilitated discussion, followed by consolidating work, followed by a cognitively demanding task), rather than using a task-first lesson structure (i.e., the aforementioned launch, explore, discuss structure).

It may be that mastering the use of cognitively demanding tasks to extend student thinking enables the development of the requisite skills, knowledge and confidence for teachers to subsequently use such tasks to launch lessons. The issue of whether teacher confidence and competence with developing and using cognitively demanding tasks is impacted on by lesson structure warrants empirical examination.

Consequently, Study Two was designed to address three research questions.

1. How did teachers perceive students’ response to lessons involving cognitively demanding tasks? What factors did they attribute to these responses?
2. What factors seem likely to influence teachers’ willingness to teach with cognitively demanding tasks in the future (e.g., perceived student reactions, teacher confidence and competence, time constraints)?
3. What differences did teachers perceive between a task-first lesson structure when observing teaching with cognitively demanding tasks and a teach-first lesson

structure? When it came to considering their own teaching practice, did teachers have a preference for one particular type of structure?

Study Three: Investigating student perceptions

The third and final study considers student perceptions of their learning experience when reflecting on instruction involving cognitively demanding tasks. In addition to considering whether students prefer to learn through a task-first lesson structure or a teach-first lesson structure, the current study explored what students' value retrospectively when working on a unit of work involving cognitively demanding tasks. Most previous studies that have considered teacher and student reactions to working on cognitively demanding tasks have been largely limited to students in Years 3 (i.e., age 9) or older (e.g., Leikin et al., 2006; Sullivan, Tobias, & McDonough, 2006), whereas less work has been done with students in the early years of schooling. Moreover, the vast majority of studies in this area have considered teacher perceptions of student reactions, however not considered the voices of students themselves. Directly investigating the reactions of younger students to working on cognitively demanding tasks would seem particularly relevant, given it has been argued by Westwood (2011) that developing a mathematical program that focusses on learning through problem solving is problematic when teaching young students or those with learning difficulties. He suggested that young students lack the requisite arithmetic knowledge and skills to engage meaningfully in problem-solving and argued that attempting to do so risks undermining their confidence and cultivating a negative attitude towards mathematics.

Consequently, this third study addresses the following research questions:

1. What do students value when reflecting on their own learning artefacts following participation in a program involving cognitively demanding tasks?
2. Do students have a preference for a task-first lesson structure or teach-first lesson structure? What factors are reported as influencing this preference?

CHAPTER THREE: TASK DESIGN

The purpose of this chapter is to introduce the cognitively demanding tasks developed for the current project, referred to as challenging tasks, as well as outline the general principles that informed the design of these tasks. The chapter is divided into three sections. In first section, the Cognitive Load Approach to Shaping and Structuring Challenging Tasks (CLASS Challenging Tasks) is described, a process which directly and indirectly guided the development of the challenging tasks included in the current study. The second section overviews the tasks developed for the patterning unit of work, whilst the third section describes the tasks developed for the addition unit of work.

CLASS Challenging Tasks - Using Cognitive Load Theory to Inform the Design of Challenging Mathematical Tasks[2]

In this section, I consider how cognitive load theory can directly inform problem-based task design through outlining a seven-step process for developing challenging mathematical tasks. This process is termed the Cognitive Load Approach to Shaping and Structuring Challenging Tasks (CLASS Challenging Tasks). To demonstrate the practical utility of the framework, I show how it was used to develop a task exploring exponential growth with Year 1 and 2 students.

Context

As discussed in the literature review, there is some evidence suggesting that higher-order mathematical goals, such as the ability to reason and think critically, are more likely

[2] The CLASS Challenging Task approach has been outlined in a peer-reviewed journal article aimed at practicing teachers and teacher-educators: Russo, J., & Hopkins, S. (2017). CLASS Challenging Tasks: Using Cognitive Load Theory to inform the design of challenging mathematical tasks. *Australian Primary Mathematics Classroom*, *22*(1), 21-27.

to be realised when students are given an opportunity to explore concepts prior to instruction (Marshall & Horton, 2011). However, this emphasis on problem-based learning in mathematics is not without its critics. In particular, some cognitive load theorists have argued that launching a lesson with a cognitively demanding activity that is not explicitly linked to teacher instruction and prior learning is problematic (Sweller et al., 2007). The central argument is that our working memory is limited in capacity when required to solve an unfamiliar problem (Kirschner et al., 2006).

However, here it is argued that cognitive load theory and problem-based approaches to learning are not incompatible but rather the former can directly inform the development of the latter. [3] Specifically, a seven-step process for developing a particular type of problem-based approach sensitive to the issue of cognitive load is outlined. This approach is referred to as the Cognitive Load Approach to Shaping and Structuring Challenging Tasks (CLASS Challenging Tasks). Before discussing this particular approach and how it informed task design in the current project, there is a need to consider what is meant by the term challenging task in the first instance, and to explain cognitive load theory in greater detail.

Challenging tasks

Challenging tasks, whether they are used to launch a lesson (i.e., a task-first lesson structure) or extend a lesson (i.e., teach-first lesson structure) are an integral aspect of the

[3] Appendix N includes a copy of the paper I presented at the PME conference in 2015, outlining some preliminary arguments relevant to this contention. The reference for the conference paper is as follows: Russo, J. (2015). How challenging tasks optimise cognitive load. In K. Beswick, Muir, T., & Wells, J. (Ed.), *Proceedings of 39th Psychology of Mathematics Education conference* (Vol. 4, pp. 105-112). Hobart, Australia: PME.

current study. Consequently, it becomes necessary to define what is meant by a challenging task.

Sullivan and Mornane (2013) describe challenging tasks as complex and absorbing problems with multiple solution pathways. Such problems are presented to the entire class, with the teacher encouraging all students to make an attempt at the problem. After a student has spent some time in the ‘zone of confusion’ and remains unsure how to proceed, he or she is given access to ‘just in time’ support through enabling prompts (Sullivan et al., 2011). Enabling prompts are designed to reduce the cognitive demand of the task through changing how the problem is represented, helping the student connect the problem to prior learning and/ or removing a step in the problem (Sullivan, Mousley, & Zevenbergen, 2006). Students who complete the problem early are given access to an extending prompt. This is designed to expose the student to an additional task that is more challenging, however requires them to use similar mathematical reasoning, conceptualizations and representations as the main task.

Consequently, challenging tasks can be viewed as an interpretation of cognitively demanding tasks that meet specific criteria, which are outlined below. These criteria can be differentiated according to whether they are mandatory (i.e., must), or generally expected (i.e., should). Note that these criteria have been adapted from Sullivan et al. (2011), although they have been made more explicit for the purpose of the current investigation.

The challenging task must:

- be solvable through multiple means (i.e., have multiple solution pathways) and may have multiple solutions;

- involve multiple mathematical steps (i.e., as opposed to a single insight facilitating completion of the problem);
- have at least one enabling prompt and one extending prompt developed prior to delivery of the lesson;
- pertain to one (or more than one) of the relevant content stands in mathematics, as set out in the Australian curriculum.

The challenging task should:

- be initially perceived as challenging by the majority of students;
- engage students (i.e., students are motivated to solve the problem);
- involve students spending considerable time working on the task (although the exact length of time will vary substantially, depending on the nature of the task, the age group and the student in question, it is generally expected that in the current study involving students in Years 1 and 2, students will spend at least 10 minutes engaged with the problem);
- involve students having primary control over how they are able to approach the task and when they are able to access enabling and extending prompts, within some constraints established by the teacher.

Cognitive load theory

Cognitive load theory is premised on the well-established idea that our working memory has limited capacity to process novel information from the environment. Our memory can, however, access large amounts of previously processed and organised information from long-term memory (Baddley, 1992). Cognitive load theory essentially explores the instructional consequences of our limited working memory capacity (Sweller, van Merrienbier, & Paas, 1998). It can be used to explain how task design can influence the cognitive load experienced by learners.

Cognitive load refers to the number of interacting elements which need to be processed simultaneously in working memory. There are three types of cognitive load relevant to instruction: intrinsic cognitive load, extraneous cognitive load, and germane cognitive load (Sweller, 2010).

Intrinsic cognitive load is determined by the extent to which the various elements inherent in a particular learning task interact (Sweller, 2010). In instructional terms, this can be thought of as task complexity. Intrinsic cognitive load is also determined by the extent of the learner's expertise with similar tasks and the level of outside support provided to tackle the task (Schnotz & Kurschner, 2007).

By contrast, extraneous cognitive load effectively reflects wasted cognitive load generated by poor instructional design. Importantly, the cognitive mechanism for generating extraneous cognitive load appears to be identical to the mechanism for generating intrinsic cognitive load; both are driven by element interactivity (Sweller, 2010). How then are we expected to distinguish between intrinsic and extraneous cognitive load? It is suggested that if element interactivity can be reduced *without* changing what is learned, the load must be extraneous. Conversely, if element interactivity can only be reduced through changing what is learned, the load must be intrinsic (Beckmann, 2010; Sweller, 2010). Sweller concludes that "the same information may impose an intrinsic or an extraneous cognitive load depending on what needs to be learned" (p. 125).

One possible interpretation of Sweller's conclusion is that distinguishing between extraneous and intrinsic cognitive load requires consideration be given to the learning objectives specified for a particular task. Specifically, does the element of the task under consideration connect to a specific learning objective? If the answer is yes, this element can be considered part of the intrinsic cognitive load of the task. If the answer is no, it is extraneous cognitive load.

The final aspect of cognitive load theory that needs to be considered is germane cognitive load. Within cognitive load theory, germane cognitive load refers to the "working memory resources that the learner devotes to dealing with the intrinsic cognitive load associated with the information". (Sweller, 2010, p. 126). Consequently, germane cognitive load can be considered to be the actual cognitive load the individual is able to dedicate to the material that needs to be learnt.

There is substantial empirical evidence to support cognitive load theory (e.g., Bokosmaty, Sweller, & Kalyuga, 2015; Leppink, Paas, van Gog, van Der Vleuten, & Van Merrienboer, 2014). Moreover, the theory has clear implications for instruction. It implies that teachers should develop instructional tasks that minimise extraneous cognitive load, maximise germane cognitive load and optimise intrinsic cognitive load.

CLASS Challenging Tasks is a process for developing instructional tasks that explicitly attempts to achieve these objectives; that is, minimise extraneous cognitive load, maximise germane cognitive load and optimise intrinsic cognitive load. Each of the seven steps involved in the development of such a task are outlined below. Under each step, a task developed for Year 1 and 2 (ages 7 and 8 years old) student participants for the patterning unit in the current project serves as a running example in order to illustrate this process.

Step 1: Identify your primary learning objective

This step involves being clear about specifying exactly what the instructor intends students to achieve within a particular lesson or series of lessons. It should be a highly valued objective, stated in explicit, student-centred language. This is a critical step in the process, as all subsequent aspects of the lesson are tied back to this primary objective.

In order to reinforce the focus on the most relevant aspect of the lesson, and therefore to maximise germane cognitive load, the primary learning objective should also be included as a summary statement (see Step 7).

Example

A potential primary learning objective relevant to the unit of work on number patterns is: *To understand that doubling is a rule that makes collections (and number patterns) grow very quickly.*

This primary learning objective connects to the Australian Curriculum content descriptions involving ‘describing, continuing and creating number patterns’ outlined for Year 3 and Year 4 students. However, the emphasis on physical objects (i.e., ‘collections’), as well as numbers, implies that this learning objective is also appropriate for younger students. For example, Year 1 students ‘investigate and describe number patterns formed by skip-counting and patterns with objects’ (ACARA, 2015).

Despite these linkages, it needs to be noted that this remains a clearly ambitious learning objective, given that the concept of ‘repeated doubling’, essentially exponential growth, does not appear formally in the Australian Curriculum until Year 9 (ACARA, 2015). However, given that a lack of understanding of exponential growth patterns has wide-ranging implications for higher level mathematics and financial literacy (Connolly &

Nicol, 2015), it can be argued that students should be exposed to such ideas at an earlier stage in their mathematical development.

Step 2: Develop a problem-solving task

The second step involves developing (or sourcing) a problem-solving task that seems useful for meeting this primary learning objective. This problem-solving task should be engaging for students, have multiple solution pathways, involve multiple mathematical steps and take considerable time to solve.

Example

The following problem-solving task was developed, consistent with the primary learning objective: *To understand that doubling is a rule that makes collections (and number patterns) grow very quickly*. The task uses the engaging context of a magical donut tree in order to provide students with an opportunity to explore repeated doubles patterns.

Doubling Donuts task:

Kai and Amaya loved donuts, so their mum decided to plant a donut tree. The tree was magical. Every day, the number of donuts on the tree doubled. Kai was having his birthday party on Friday, so the family decided to not pick any of the donuts off the tree until then. On Monday, there were 3 donuts on the tree. Your job is to work out how many donuts there were on the tree by Friday.

Although in this instance there is no attempt to ensure that the task is authentic from the point of accurately connecting it to students' lived experience, the task is instead endeavouring to resonate with students' imagination. The notion of a donut tree being an impossibility is only a comparatively recent discovery for Year 1 and Year 2 students; young children tend to maintain a wistful curiosity about fantastical possibilities, which can in turn be used to engage them in learning activities (Smith & Mathur, 2009). It is worth noting that providing a context or story to which students can relate has been noted elsewhere within the mathematics education literature as an important aspect identified by teachers for engaging students in challenging tasks (Clarke, Cheeseman, Roche & van der Schans, 2014).

Step 3: Ascertain potential secondary learning objectives

This third step involves identifying additional potential learning objectives that appear to be embedded in the task as it is currently structured. As part of this process, it is useful to ask the question: What other skills and knowledge am I assuming that students either have, or will develop, through engaging in this task as it is currently presented? This step essentially involves making implicit mathematical knowledge as explicit as possible.

Example

Potential secondary learning objectives that may be embedded within the Doubling Donut problem include:

- to identify appropriate ways of mathematically representing worded problems;
- to solve number pattern problems using only abstract representations (i.e., using numerals);
- to independently initiate a number pattern through following a written instruction;
- to explore patterns involving numbers greater than 20 using abstract or quasi-abstract (i.e., pictorial) representations; and

- to make connections between the problem and doubles patterns.

Step 4: Sort secondary learning objectives into intrinsic cognitive load (relevant) and extraneous cognitive load (less relevant)

When viewing these potential secondary learning objectives, it is apparent that some connect to, and build on, the primary learning objective, whilst others could actually detract from it. It might be suggested that one implication of cognitive load theory is that it is not prudent to expect students to simultaneously develop skills and capacities that do not share obvious synergies. Consequently, the next step involves sorting these potential secondary learning objectives into those which appear highly relevant (i.e., those we have determined should be part of the intrinsic cognitive load of the task) and those which are less relevant (i.e., those we have determined are part of the extraneous cognitive load).

Example

Table 3.1.

Doubling Donuts: Sorting the Learning Objectives

Relevant secondary learning objectives (Intrinsic cognitive load)	Less relevant secondary learning objectives (Extraneous cognitive load)
To explore patterns involving numbers greater than 20 using abstract or quasi-abstract (i.e., pictorial) representations.	To identify appropriate ways of mathematically representing worded problems.
To make connections between the problem and doubles patterns.	To solve number pattern problems using only abstract representations (i.e., using numerals)
	To independently initiate a number pattern through following a written instruction.

The three less relevant potential learning objectives, which are determined to be outside the scope of this lesson, can be viewed as generating potential extraneous cognitive load. It is critical to note that there is nothing inherently wrong with these learning objectives. Indeed, in a different context, these could be deemed to be highly relevant. For instance, if the focus of the lesson was on students solving worded problems, clearly finding appropriate ways of representing worded problems would be a highly relevant learning objective – and identified as a source of intrinsic cognitive load, rather than an extraneous source. The classification depends on what one is hoping to achieve through the particular lesson.

Step 5: Redesign the task to remove extraneous cognitive load

Step five involves removing the extraneous cognitive load through redesigning the task. Essentially, it involves acknowledging that certain potential learning objectives embedded in the initial problem solving task were out of scope, and that it is advantageous to redesign the task to take the focus off these less relevant elements.

Example

The redesign of the Doubling Donut task involves presenting some additional information to students. Specifically, students are provided with a table to help them represent the problem mathematically. This table also contains a pictorial representation of the problem. Furthermore, the doubling pattern has been initiated for students, (i.e., students are not expected to generate it themselves). This redesign of the task is intended to remove the three sources of extraneous cognitive load previously identified.

To help you, have a go at completing the following table. Remember, each day the number of donuts doubles.

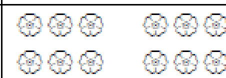
	Donuts on tree	Picture
Monday	3	
Tuesday	6	
Wednesday		
Thursday		
Friday		

Figure 3.1. Doubling Donuts Challenging Task

Step 6: Develop prompts to optimise intrinsic cognitive load

Although obvious sources of extraneous cognitive load have been removed by the information provided to students in Figure 3.1, the task as it currently presented is likely to be too easy for some students, and too challenging for other students. To address this issue, enabling and extending prompts are used (Sullivan et al., 2014). Such prompts are developed prior to the delivery of the lesson as part of the task design process.

As previously discussed earlier in the chapter, enabling prompts are designed to reduce the level of challenge through simplifying the problem, changing how the problem is represented, helping the student connect the problem to prior learning and/ or removing a step in the problem (Sullivan, Mousley, & Zevenbergen, 2006). Extending prompts expose students to an additional task that is more challenging. Generally, this extending prompt, although more challenging, requires students to engage with similar mathematical reasoning, conceptualisations and representations as the main task (Sullivan et al., 2014). A brief rationale for the development of prompts is included below.

According to cognitive load theory, to maximise learning, intrinsic cognitive load needs to be at an appropriate level as determined by the interaction between the complexity of the problem and the expertise of the learner (Sweller, 2010). If intrinsic cognitive load is too high, students will have insufficient germane cognitive load to devote to the task and learning will not occur. However, if intrinsic cognitive load is too low, learning is also undermined. Not only is cognitive capacity underutilised, but more expert learners may choose to disengage and 'tune out' if the level of challenge is inadequate (Schnotz & Kurschner, 2007).

Enabling and extending prompts are accessed by students when required during the lesson. Through introducing enabling prompts and extending prompts on a 'just in time' basis, the level of intrinsic cognitive load (i.e., learner-specific complexity) can be optimised for a given learner. In the first instance, accessing sequenced enabling prompts can reduce the amount of interactivity amongst the elements of the task until the task is at an appropriate level of challenge for a given learner's expertise. Although this process necessarily alters the presentation and/ or nature of the learning task, and therefore impacts on the learning objectives, the enabling prompts need to be designed in such a manner that does not undermine the primary learning objective. Accessing enabling prompts may, however, suspend some or all of the secondary learning objectives. Likewise, the extending prompt should continue to build on the primary learning objective, rather than require students to demonstrate qualitatively different skills and capacities.

Although students who have accessed prompts are in reality working on slightly different problems, critically they have a similar experience in having worked on the same cognitively demanding task. This enables them to actively participate in the discussion component of the lesson, and reflect on the key mathematical concepts explored.

Example

For the Doubling Donut task, the first enabling prompt (Enabling Prompt A in Figure 3.2) offers a simpler problem for students. Instead of there being three donuts on the tree on Monday, there is only one donut. This means that on Friday, there will only be sixteen donuts on the tree. Consequently, accessing this prompt effectively suspends the secondary learning objective: *To explore patterns involving numbers greater than 20 using abstract or quasi-abstract (i.e., pictorial) representations.*

Although the exploration of large numbers is relevant to our primary learning objective, using numbers larger than twenty may inhibit some students from making progress with the task. Consequently, this prompt is designed for students to explore the power of doubles patterns using smaller, more familiar numbers.

To help you, have a go at completing the following table. Remember, each day the number of donuts doubles.

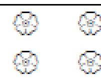
	Donuts on tree	Picture
Monday	1	
Tuesday	2	
Wednesday		
Thursday		
Friday		

Figure 3.2. Enabling Prompt A: An Easier Problem

The second enabling prompt (Enabling Prompt B in Figure 3.3) provides students with access to artefacts used in the classroom presented in relation to doubles patterns. Specifically, students are shown pictures of subitising and doubling cards used during previous instruction to attempt to connect prior understanding of doubles patterns to the current problem. Accessing this prompt effectively suspends the secondary learning objective: *To make connections between the problem and doubles patterns.*

Again, connecting the concept of doubling to the current task was a desired learning objective, because this lesson is attempting to build on this previous understanding through exploring repeated doubling patterns. However, if students cannot bring to mind the concept of ‘doubling’, it is necessary to provide students with some prompt, otherwise they are unlikely to engage productively with the problem at all.

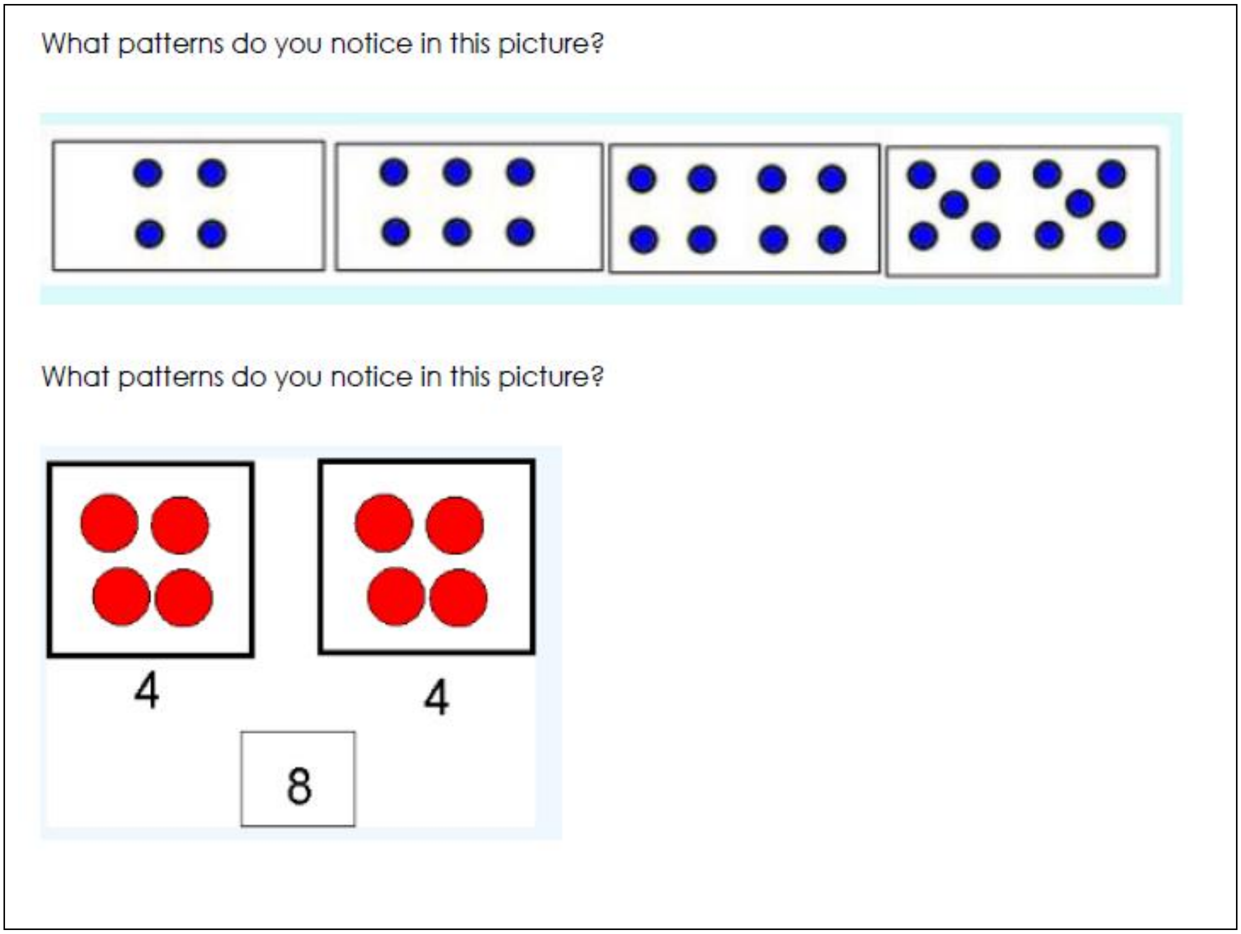


Figure 3.3. Enabling Prompt B: Previous Material about Doubles Facts

Finally, the extending prompt for this problem solving task invites students to continue the double patterning by postponing the day of the party. Specifically, the prompt states:

> How many donuts would be on the tree if Kai decided to have the party on Saturday instead? How about if he had the party on Sunday? Can you keep the pattern going?

This prompt builds on the primary learning objective through emphasising the relevance of exploring large numbers in understanding exponential patterns. It effectively introduces an additional secondary learning objective: *To explore patterns involving numbers greater than 100 using abstract or quasi-abstract (i.e., pictorial) representations.*

Step 7: Include a lesson summary to maximise germane cognitive load

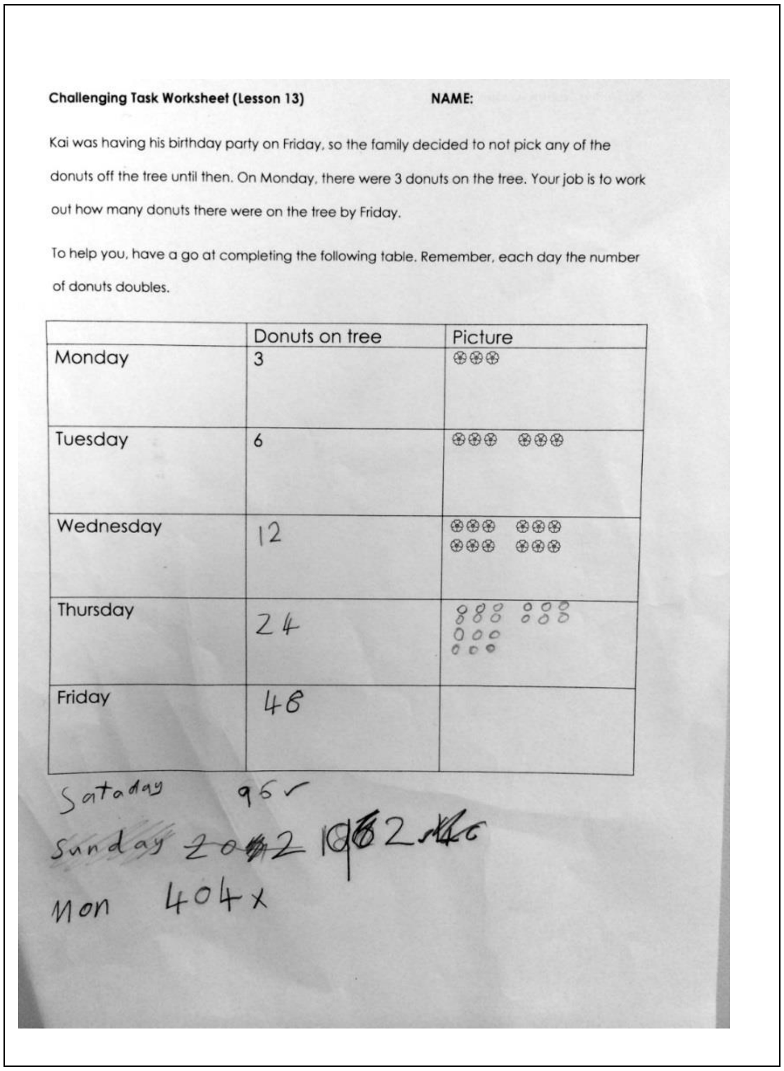
As part of the process, the teacher should aim to conclude the lesson by offering a two minute summary to reinforce the primary learning objective. This focuses students' attention on the most relevant part of the lesson, and therefore helps to maximise germane cognitive load. The process of developing this summary is straightforward. The primary learning objective should be restated to students by the teacher, with a sample of student work which effectively illustrates the primary learning objective presented alongside it. The summary is the only component of this seven-step process in which part of the process (i.e., the student work sample) is developed *in* the lesson, rather than *prior* to the lesson.

As an aside, it is worth noting that, in contrast to the primary learning objective, secondary learning objectives are not shared with students. This is mainly to ensure that students are not overloaded with less relevant information (extraneous cognitive load), enabling them to instead focus on the primary objective. In addition, the specific secondary learning objectives are dependent on the enabling and extending prompts a student accesses and therefore will be different for different students.

Example

The lesson summary for the Doubling Donut task (i.e., restating the primary learning objective) is: *To understand that doubling is a rule that makes collections (and number patterns) grow very quickly*.

Alongside the lesson summary, a student attempt at the task which makes very clear the power of continuously doubling numbers can be presented (see Figure 3.4).



Challenging Task Worksheet (Lesson 13) **NAME:**

Kai was having his birthday party on Friday, so the family decided to not pick any of the donuts off the tree until then. On Monday, there were 3 donuts on the tree. Your job is to work out how many donuts there were on the tree by Friday.

To help you, have a go at completing the following table. Remember, each day the number of donuts doubles.

	Donuts on tree	Picture
Monday	3	
Tuesday	6	
Wednesday	12	
Thursday	24	
Friday	48	

Sataday 96✓

Sunday ~~2042~~ 192

Mon 404x

Figure 3.4. Sample of Student Work

Summarising CLASS Challenging Tasks

This section outlined a seven step process for developing problem solving tasks informed by cognitive load theory. This process has been labelled the Cognitive Load Approach to Shaping and Structuring Challenging Tasks (CLASS Challenging Tasks), and can be used to produce challenging mathematical tasks that aim to optimise cognitive load for a given student. These seven steps are outlined in the figure below.

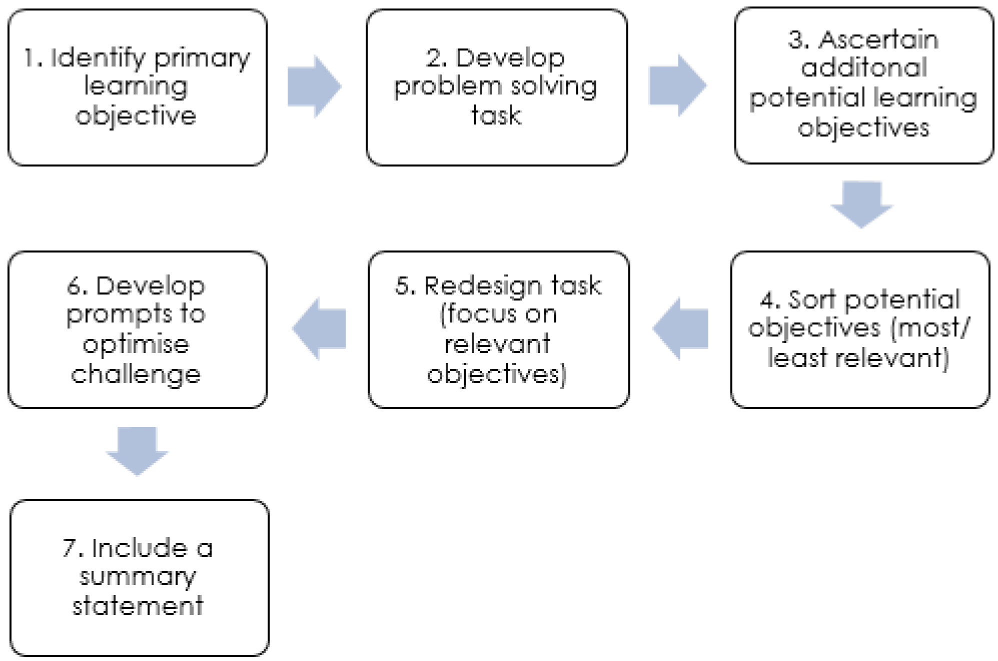


Figure 3.5. Seven Step Process Outlining CLASS Challenging Tasks

CLASS Challenging Tasks is an illustration of how developments in mathematics education can be combined with insights from educational psychology (e.g., cognitive load theory) to generate novel approaches to mathematics instruction and task construction. All of the challenging tasks developed for the current project have been informed, to a greater or lesser extent, by the CLASS Challenging Task process.

Tasks Developed for the Patterning Unit of Work

Including the Doubling Donut task described in the previous section, sixteen challenging tasks were developed for the patterning unit of work. These challenging tasks were organised thematically, with either two or three tasks included under each theme. This thematic organisation was intended to help to minimise extraneous cognitive load *across the sequence of lessons*, rather than simply reducing extraneous load *within an individual lesson* (i.e., as previously described through the CLASS Challenging Task design process).

Specifically, because the context in which the mathematics was situated was consistent across consecutive lessons, students invested fewer valuable working memory resources attempting to ascertain what the problem was asking them to do. However, this attempt to reduce extraneous cognitive load through providing a familiar context needed to be balanced against the desire to ensure that there was sufficient variety in task context across the unit of work to maintain student engagement and prompt learning transfer. The issue of keeping students engaged in learning is often overlooked within cognitive load theory research, and is arguably beyond the scope of what cognitive load theory is useful for investigating (de Jong, 2010). Consequently, the question of optimal contextual variety to both sufficiently reduce extraneous cognitive load and maintain student engagement cannot easily be approached systematically, and reminds us that the process of task design is a very complex undertaking.

In total, the sixteen tasks were organised around seven themes. Within a given theme, the tasks generally required students to work on closely related mathematical concepts. The seven themes included:

- About our room and the people in it – 2 challenging tasks
- Fiona the Frog goes hopping – 3 challenging tasks
- Hundred chart challenges – 3 challenging tasks
- Baskets and Boundaries – 2 challenging tasks
- Creepy Crawlies – 2 challenging tasks

- Food: Eating and being eaten – 2 challenging tasks
- More challenging patterns with Mr Russo – 2 challenging tasks

Each of these themes, and their related tasks, are now considered in turn, including the rationale for the design of each suite of tasks. The tasks, which are designated by a bordered area, replicate the text and images as presented to students. For the sake of brevity, the enabling and extending prompts, as well as the learning objectives, are not included in this discussion. Interested readers should refer to Appendix A for this additional information. Note that several of these tasks have been published as articles in journals aimed at practicing teachers. References to these articles have been included as footnotes when relevant.

About our room and the people in it[4]

Challenging Task 1: How many feet?

Without leaving your seat, or talking to anyone, can you work out how many feet are in the room right now? How did you count them? Can you count them a different way? What do you think is the easiest way to count them?

[4] Russo, J. (2015). Teaching with challenging tasks: Two 'how many' problems. *Prime Number*, *30*(4), 9-11.

Challenging Task 2: How many fingers?

Without leaving your seat, or talking to anyone, can you work out how many fingers are in the room right now? How did you count them? Can you count them a different way? What do you think is the easiest way to count them?

Rationale

It is apparent that both tasks are deliberately similar in structure, however require the student to draw on different counting patterns to find an efficient means of solving the respective problem. Consequently, for the second task, students are able to draw on a nearly identical representation as the first task, allowing the focus of their learning during this second session to be more explicitly on the skip-counting patterns.

The somewhat fluid nature of the information contained in the questions (i.e., relating to the fluidity around how many people might be in a room at any given point in time) serves to reinforce the notion that the quality of student thinking is more important than the precise answer arrived at. Specifically, allowing for variation in whether students choose to count the teacher's feet and hands, and whether any students are temporarily absent from the class (e.g., toilet break) when a particular student begins his or her count, means that more than one answer is potentially acceptable. The intention is that student attention during the post-task discussion can instead be directed to the greater efficiency associated with skip counting compared with counting by ones, rather than which students had the 'correct answer'.

Fiona the frog goes hopping[5]

Context (shared with students before work on each of the three tasks)

Fiona the Frog loved to hop. In fact, she loved to hop so much that, instead of swimming, she even hopped through the water. But as much as Fiona loved hopping, she needs to rest once in a while. She knew she could rest on land, or on a lily pad. Today, we are going to go hopping with Fiona the frog. We are going to pretend that all the numbers ending with zero are lily pads, which means that Fiona can take a little rest if she lands on one of these numbers.

Challenging Task 1: Hopping forwards from zero

Fiona the frog needs to get from one side of the lake (0) to the other side (100). The only way she can do so is by landing on lily pads (numbers which end with a zero, 10 to 90). She must land on at least 5 lily pads, or she won't make it. Once she starts on her journey, Fiona always covers the same distance with each hop. You have to decide how far Fiona should go with each hop.

Should she hop in ones, twos or threes to get to the other side of the lake, as quickly and safely as she can? How did you decide?

[5] Russo, J. (2016). Teaching with challenging tasks: Hopping with Fiona the Frog. *Prime Number*, *31*(2), 10-11.

Challenging Task 2: Hopping forwards from seven

Fiona the frog needs to get from one side of the lake to the other (100). This time a big gust of wind picked her up and dumped her on number 7, so she gets a bit of a head start. Also, Fiona has been exercising a lot, and does not need to take as many rests anymore. She now only has to land on 3 lily pads to arrive safely. Once she starts on her journey, Fiona always covers the same distance with each hop. You have to decide how far Fiona should go with each hop.

Should she hop in ones, twos or threes to get to the other side of the lake, as quickly and safely as she can? How did you decide?

Challenging Task 3: Hopping backwards from 100

Now Fiona has to get back over to the other side of the lake. So this time, she needs to start at 100 and end at 0, so you will have to count backwards! Again, Fiona is fitter now because of all her exercising, so she still only needs to land on 3 lily pads to make it. Again, once she starts on her journey, Fiona always covers the same distance with each hop. You have to decide how far Fiona should go with each hop.

Should she hop back in ones, twos or threes to get back over to the other side of the lake, as quickly and safely as she can? How did you decide?

Rationale

Considered in isolation, any one of the Fiona the Frog challenges at least ostensibly appears to contain some extraneous cognitive load. The non-mathematical knowledge and thinking required to engage in the challenge is substantial, and the notion of multiples of ten representing lily pads may seem convoluted. However, the intention behind developing this set of tasks is to provide a highly engaging context for students to explore counting patterns. Moreover, each of the three challenges use both the same basic narrative and mechanism to attempt to manage the level of potentially extraneous cognitive load. This means that, taken as a whole, this set of tasks is less problematic than any one task considered in isolation. This is because whilst the average level of extraneous cognitive load is reduced with each subsequent task, the value of the narrative context actually increases, as students become somewhat invested in the Fiona the Frog character. However, perhaps more than any other set of tasks, the Fiona the Frog challenges highlight the potential tension between two competing objectives: providing a meaningful problem scenario through which to engage students, and managing cognitive load.

An important additional suggestion provided to students before each of the Fiona the Frog tasks is for them to use counters or whiteboard markers to record the path taken by Fiona on their hundred chart, using a different colour for each counting pattern. Advising students to use a hundred chart to keep track of Fiona's path again assists with problem representation and managing extraneous cognitive load. Moreover, it simultaneously serves to create a compelling visual pattern that effectively draws student attention to the relationship between the different counting sequences, directly supporting the primary learning objective, and thereby increasing germane cognitive load.

Hundred chart challenges[6]

Challenging Task 1: Third time lucky

Starting at 0, I skip counted by twos to 100, placing a counter on all the numbers I landed on. Next, again starting at 0, I skip counted by fives to 100, placing a counter on all the numbers I landed on. Finally, starting at 0, I skip counted by tens to 100, again placing a counter on all the numbers I landed on.

What are the numbers with three counters on them – the numbers I landed on three times?

Challenging Task 2: Fourth time luckier

Starting at 0, I skip counted by twos to 50, placing a counter on all the numbers I landed on. Next, starting at 0, I skip counted by threes to 50, again placing a counter on all the numbers I landed on. After that, I did the same thing counting by fives and then tens.

There is only one number with four counters on it. What is that number?

[6]Russo, J. (Accepted). Teaching with challenging tasks: Experiments with counting patterns. *Primary Mathematics*.

Challenging Task 3: Twos, threes, fours and fives; which number will survive?

Starting at 0, I skipped counted by twos to 40, crossing off the numbers as I went. Then I did the same thing, but instead skip counted by threes. Next, I did it by fours. Finally, I skip counted again, but counted by fives.

Some numbers were crossed off more than once, but some numbers survived – they weren't crossed off at all. Can you guess which 10 numbers survived? Now check if you are right.

Rationale

These three tasks, collectively referred to as the Hundred Chart Challenges, are similar to the Fiona the Frog Challenges such that students have an opportunity to explore different counting sequences and informally examine the interrelationships between different sequences. For this set of tasks, rather than rely on students buying into a specific narrative (e.g., Fiona the Frog, and her adventures), the Hundred Chart Challenges intends to engage students through allowing them to assume the role of a scientist 'experimenting' with numbers. Specifically, the hands-on nature of these challenging tasks, whereby students can make predictions and run a number of 'experiments' with number patterns, is intended to encourage students to both invest in the process of working through these tasks and pay attention to the relevant mathematics. The notion of students playing the role of a scientist with the Hundred Chart Challenges will be reinforced by the manner in which these challenging tasks are launched, and the language used by the teachers.

***Baskets and boundaries (sports with Bugs Bunny)*[7]**

Challenging Task 1: Bugs Bunny and Baskets

Basketball is an awesome sport! Even Bugs Bunny thinks so! In basketball, you can score points in three different ways. Scoring a free throw is worth 1 point, scoring a field goal is worth 2 points and scoring a three-pointer is worth 3 points

(Name of School) played Ferntree Gully Primary School in the big basketball game.

At the end of the basketball game, the (Name of School) team had won 36 points to 27 points. How many free throws, field goals and three pointers might the (Name of School) team have scored? Solve the problem in at least two different ways.

Challenging Task 2: Bugs Bunny and Boundaries

Cricket is also a pretty great sport! Bugs Bunny plays it during summer, when it is too hot for basketball. In cricket, you can score runs in a lot of different ways, but the fastest and most fun way to score runs is to hit boundaries. If a ball hits the rope, it is worth 4 runs. If a ball goes over the rope, it is worth 6 runs.

Will Smith was an excellent cricketer, and scored 36 runs for (Name of School) that day. He scored all of his runs hitting only fours and sixes. How many fours and how many sixes might Will Smith have scored in his innings? Solve the problem in at least two different ways.

[7] Russo, J. (2016). Teaching with challenging tasks: Baskets and boundaries. *Prime Number*, *31*(3), 7.

Rationale

This pair of challenging tasks returns to the idea of attempting to generate an engaging context for students to explore number patterns; in this instance, the notion of a familiar character, such as Bugs Bunny, participating in sports that many students had previously played themselves. The prompt for students to solve the problem in more than one way is an indication to students that there are multiple solutions, and solution pathways, associated with this task. As discussed in more detail in Appendix N, open-ended problem structures may serve to reduce extraneous cognitive load through both the *goal-free effect* put forward by cognitive load theorists (e.g., Bobis, Sweller, & Cooper, 1994) and the related *means-free effect* proposed in the current analysis. Specifically, ensuring that there are multiple viable pathways to a particular solution reduces extraneous cognitive load through several mechanisms. For example, such a task structure increases the probability that learners have some prior knowledge of strategies that can bridge the problem and solution states, and reduces the search time for students to identify an appropriate problem solving strategy.

Creepy crawlies

Challenging Task 1: Ants

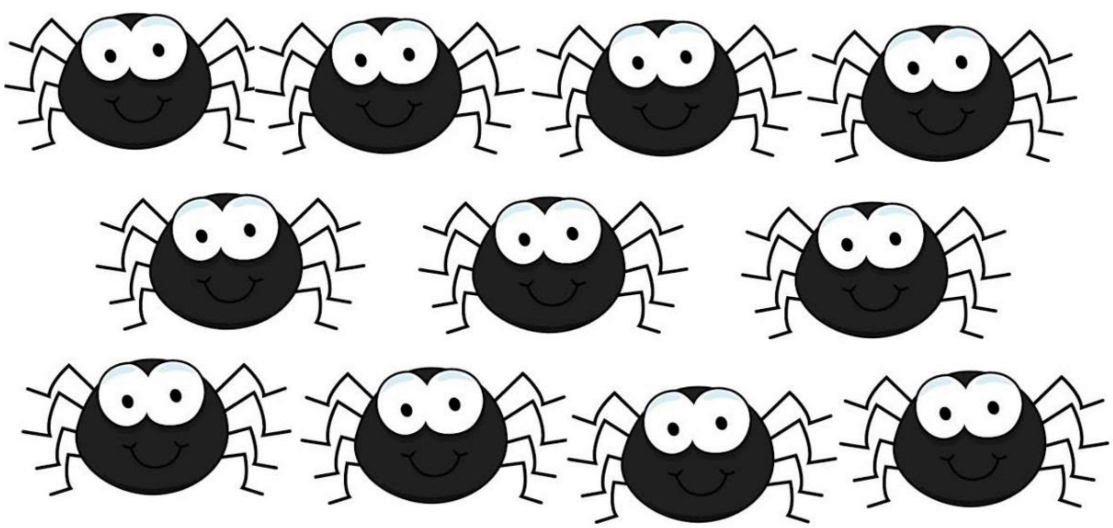
Ants are classified as insects because they have six legs. Ants are busy little insects, who like to live together in large groups. Below is a small army of ants, with their leader, Maximus.

Can you work out how many ant legs are in the picture, without counting by ones?

Can you do it another way? Make sure you show your thinking.

Challenging Task 2: Spiders

Spiders are classified as Arachnids because they have eight legs. Spiders are crafty creatures, who like to spin webs and catch insects in them. Below is a large family of spiders.



Can you work out how many spider legs are in the picture, without counting by ones? Can you do it another way? Make sure you show your thinking.

Rationale

Compared with the tasks considered previously, the Ant Challenge and Spider Challenge provide a highly concrete representation to students, with both these tasks essentially equating to pure counting problems. Removing the need for students to consider problem representation in any capacity allows students to focus on exploring more complex skip-counting sequences (e.g., sixes and eights). This in turn allows students to concentrate on examining the relative efficiency of both previously known, and newly explored, skip-counting sequences for calculating the number of objects in a collection, as well as highlighting the interrelationships between the different sequences. For example, the structure of the Spider Challenge emphasises the fact that counting by fours and counting by eights are intimately connected, with two groups of 4 (i.e., each side of the spiders body) equating to one group of 8 (i.e., the spider taken as a whole). This problem structure also allows students to take a ‘risk’ by pursuing a more challenging counting sequence (e.g., counting by fours or counting by eights), secure in the knowledge that they can verify their answers through drawing on a better known sequence (e.g., counting by twos).

Food: Eating and being eaten

The Doubling Donuts Challenge, used as the exemplar task when overviewing the CLASS Challenging Task approach earlier in this chapter, was one of the challenging tasks contained within this theme. The other task was the Halving Giant Challenge, which is presented below.

Challenging Task 1: Doubling Donuts As previously described.

Challenging Task 2: Halving Giant[8]

A not so friendly giant moved into town. His favourite food was Year 2 children. Actually, he refused to eat anything else! He decided that every night, while the town slept, he was going to stick his ginormous tongue through the windows of houses and eat half of the Year 2 children in the town.

When the not so friendly giant arrived in town on Monday, there were 64 Year 2 children in the town. How long will it take until there is only 1 Year 2 child left?

To help you, have a go at completing the following table. Remember, each night the giant will eat half of the children.

	Year 2 children	Picture	
Monday	64	☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺	☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺
Tuesday		☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺	
Wednesday			
Thursday			
Friday			
Saturday			
Sunday			
Monday			

[8] Russo, J. (2016). Teaching mathematics in primary schools with challenging tasks: The Big (not so) Friendly Giant. *Australian Primary Mathematics Classroom, 21*(3), 8-15.

Rationale

The Halving Giant task is almost identical in structure to the Doubling Donuts challenge, with the key difference being that, rather than informally exploring exponential growth, it involves an informal exploration of exponential decline. Consequently, the rationale previously outlined for the Doubling Donut challenge structure is also wholly applicable to the Halving Giant problem.

More challenging patterns with Mr Russo

Challenging Task 1: Growing Fruit (counting forwards)

This Sunday afternoon, Mr Russo is inviting 50 parents and children to his farm in Belgrave to eat some of the fruit he has grown. He wants each of his guests to have:

1 apple, 1 pear, and 2 oranges.

All the fruit on Mr Russo's trees grow in patterns.

	Mon	Tues	Wed	Thurs	Fri	Sat	Sun
Apples	3	13	23	33	43	?	?
Pears	1	2	4	8	16	?	?
Oranges	10	10	20	30	50	?	?

Will he have enough fruit to feed all his guests on Sunday? Explain your thinking.

Challenging Task 2: Launching rockets (counting backwards)

Mr Russo was planning on launching rockets from the roof of the school. His countdown would begin at 100, and when he got to 0, he would fire the rocket.

He began the countdown for Rocket A:

100, 90, 95, 85, 90, 80, 85…

Continue the counting sequence. Will the rocket fire? Why?

Next, he began the countdown for Rocket B:

100, 90, 80, 90, 100, 90…

Continue the counting sequence. Will the rocket fire? Why?

Finally, he began the countdown for Rocket C:

100, 90, 81, 73, 66, 60…

Continue the counting sequence. Will the rocket fire? Why?

Rationale

Rather than introducing or consolidating a specific counting pattern, these two tasks are designed to get students to consider the overall sequence, and not simply two consecutive numbers, when deciphering countering patterns. Consequently, the notion of needing to identify the rule governing the pattern becomes more central to the process. Given that identifying a rule, in many instances a non-linear rule, is a highly abstract and cognitively demanding task for students of this age group, steps were taken to ensure any extraneous cognitive load is minimised. Most importantly, the sequences are presented to students as completion problems, removing the (extraneous) cognitive load associated with problem representation. This allows students to focus instead their attention on examining the pattern, rather than constructing it.

Tasks Developed for the Addition Unit of Work

The focus of the second unit of work is addition. The overarching learning objective for this unit is for students to look for patterns to help them solve addition problems in efficient ways. The addition unit of work was shorter than the patterning unit, and contained only twelve tasks. Rather than being organised thematically, tasks were paired such that a pair of tasks required students to apply an almost identical mathematical process in order to generate appropriate solutions. Whilst the first task was presented and represented largely in numerical form (e.g., as a number sentence), the second task in the pair was structured as a worded problem. This paired structure was chosen for several reasons. Firstly, its regularity across the unit of work (number sentence with a new concept, followed by worded problem consolidating this concept) helps to reduce the extraneous cognitive load generated by students having to continually devote valuable working memory to deciphering the structure of a task, allowing them to instead focus on the mathematical ideas. Secondly, it enables students to consolidate their understanding of a particular addition strategy across consecutive lessons. Thirdly, regularly including worded problems offers opportunities to engage students in the task by providing a mini-narrative (which sometimes overlapped pairs of lessons) in which students can invest. Finally, this paired structure encourages students to connect the (possibly) more opaque worded problem version of the task to the more transparent number sentence representation, facilitating learning transfer.

This implies that a secondary learning objective embedded in this unit of work is for students to be able to ‘translate’ worded problems into number sentences. It is important to note that this ‘translation’ learning objective is indeed secondary to the primary learning objectives, which are focussed around efficient addition strategies. Specifically, in line with the CLASS Challenging Task approach, for each of the worded problem tasks, this secondary objective was removed by an enabling prompt which effectively ‘translated’ the worded problem into a number sentence for the student. For example, consider the challenging task: “Can you add all of the digits from one to nine together, and explain your approach to a partner?” The first enabling prompt represented the task for the student as a number sentence (1+2+3+4+5+6+7+8+9=), making the problem to be solved less opaque and unfamiliar. Consequently, under this construction, and in contrast to the Doubling Donuts task outlined in Section 1, finding an appropriate means of representing this worded problem is not viewed as extraneous cognitive load, but rather is construed as part of the intrinsic cognitive load of the task. However, as this problem representation aspect is anticipated to impede some students engaging with the core mathematical idea inherent in the task - that is, that changing the order of the addends can help to identify more efficient ways of adding numbers – the first enabling prompt altered the task to remove this secondary objective.

In order to draw attention to the primary learning objective, that is, that students should be solving problems in efficient ways, students are encouraged to solve a given challenging task using more than one approach. Specifically, for the non-worded problems, students were given the following instructions: “Can you work out the answer to this number sentence? Try doing it another way. Which way do you prefer and why?”. The idea is that through requiring students to draw on multiple methods for solving a given task, students are better able to meaningfully compare and contrast problem solving approaches, and therefore determine whether a particular approach is efficient.

A final point concerning the structure of the addition unit relates to another one of the enabling prompts. In order to provide students struggling to begin the main task with support deciphering the relevant pattern for tasks, the character, Maths Man is introduced. Maths Man provides students accessing the enabling prompt with a hint about how the simpler problem, also contained within the enabling prompt, can be solved by applying a strategy that would also be advantageous to the main problem. For example, the picture below (Figure 3.6) relates to a possible strategy for solving the problem 9+9+9+3=. It shows Maths Man suggesting to students that they partition the final addend, three, into three ones, in order to create three groups of tens. This strategy could then be accessible for students when they tackled the main task: 9+9+9+9+9+9+9+7=

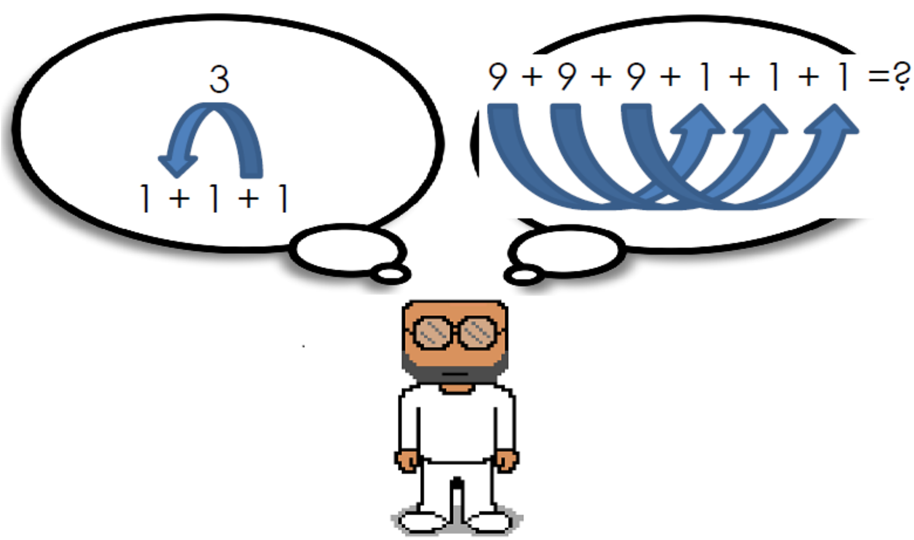


Figure 3.6. Maths Man Suggests a Strategy for Solving the Problem 9+9+9+3=

This section will now outline the challenging tasks included in the addition unit. For the patterning unit, each set of problems under a given theme required its own specific explanation regarding context and structure. This was due to the idiosyncratic scenarios presented in the patterning unit, and the fact that each set of problems contained a somewhat distinctive structure. By contrast, the above discussion can be considered sufficient justification for the structure of the more uniform tasks contained in the addition unit. Instead, for each pair of tasks, a very brief rationale outlining the mathematical value of the particular strategy (or strategies) that serves as the focus of the lesson is offered.

An overview of the six pairs of tasks included in the addition unit is provided below.

- Change the order – Rainbow Facts (Complements to 10);
- Change the order – Doubles Facts;
- Change the order – Multiples of 10;
- Bridging through 10 (and 100);
- Partitioning and regrouping into lots of 10; and
- Complements to 100.

Change the order – Rainbow Facts

Challenging Task 1: Number sentence Can you work out the answer to this number sentence? 0 + 1 + 2 + 3 + 4 + 6 + 7 + 8 + 9 + 10 = Try doing it another way. Which way do you prefer and why?

Challenging Task 2: Worded problem

Your friend challenged you to add all of the digits from 1 to 9 together. Can you work out a quick way of finding the answer?

Rationale

Mental computation can sometimes involve changing the order in which numbers are added in order to arrive at an answer in an efficient manner. The foci of these lessons are on recognising the power of ten facts to aid the addition of a long sequence of numbers.

Change the order – Doubles Facts

Challenging Task 1: Number sentence

Can you work out the answer to this number sentence?

1 + 5 + 2 + 4 + 3 + 2 + 5 + 4 + 1 + 3 =

Try doing it another way. Which way do you prefer and why?

Challenging Task 2: Worded problem

Will Smith was an excellent cricketer. In one over, he hit Mr Russo for:

6, 2, 2, 2, 2, and 6. In the next over, he did exactly the same thing. How many runs did Will score?

Rationale

Again, mental computation can sometimes involve changing the order in which numbers are added to facilitate efficient addition. These two lessons focus on identifying doubles facts (e.g., 4+4, 3+3) as a potentially efficient strategy for adding a long sequence of numbers.

Change the order – Multiples of 10

Challenging Task 1: Number sentence Can you work out the answer to this number sentence? 3 + 30 + 97 = Try doing it another way. Which way do you prefer and why?

Challenging Task 2: Worded problem Shaalev was also an excellent cricketer and played on the same team as Will. In four innings, Shaalev scored 96, 60, 4 and 40. How many runs did he score altogether?

Rationale

Again, it may be worthwhile to consider changing the order in which we add numbers to support efficient addition. These lessons focus on recognising the power of ten facts (e.g., 7+3; 8+2; referred to as rainbow facts), and the related idea of multiples of ten (referred to as friendly numbers), to support addition.

Bridging through 10 (and 100)

Challenging Task 1: Number sentence

Can you work out the answer to this number sentence?

99 + 26 =

Try doing it another way. Which way do you prefer and why?

Challenging Task 2: Worded problem

Johnny did some Christmas shopping for his parents. He got his dad a new vacuum cleaner, which cost $98. For his mum, he bought a new fishing rod for $35. How much money did he spend on his parents altogether?

Rationale

Mental computation can involve the use of a range strategies to make addition easier. One of the key strategies involves partitioning and regrouping numbers. In some instances, this may involve students deciding to 'bridge through 10' in order to complete a number sentence such as 9 + 7 (9 + 1 + 6). Again, this lesson seeks to leverage off student's knowledge of key target numbers (e.g., 10, 100). However, rather than focusing on the order in which they add numbers to find a more efficient approach, the focus is instead on partitioning and regrouping.

Partitioning and regrouping into lots of 10

Challenging Task 1: Number sentence

Can you work out the answer to this number sentence?

9 + 9 + 9 + 9 + 9 + 9 + 9 + 7 =

Try doing it another way. Which way do you prefer and why?

Challenging Task 2: Worded problem

The next day, Johnny was back at the shops doing more Christmas shopping. He decided to get his two sisters tickets to see Katie Perry in concert. The tickets cost $99 each. He also got his dog, Sook, a plastic bone for $2. How much more money did he spend on his Christmas shopping?

Rationale

Another strategy for making addition easier involves partitioning and regrouping numbers. In some instances, this may involve students deciding to partition one number in order to regroup into lots of 10 (e.g., 9 + 9 + 9 + 3 = 9 + 9 + 9 + 1 + 1 + 1).

Complements to 100

Challenging Task 1: Number sentence You did a problem for your maths homework, but can only remember part of it: □5 + □5 = 100 What might the problem have been? Write down as many different possibilities as you can think of.

Challenging Task 2: Worded problem John and Wendy loved baking. John wanted to try out a brand new cupcake recipe that had come to him in a dream. John and Wendy baked and baked all day, and finished the day with 100 cupcakes. They decided to share all the cupcakes between them. They both decided that John should get more cupcakes because he was the one who had dreamed up this new recipe. How many cupcakes might John have got? How many cupcakes might Wendy have got? Write down as many combinations as you can.

Rationale

Building on their understanding of the mathematical patterns involved in rainbow facts (tens facts), it is appropriate to expose students to the more sophisticated concept of complements to 100 (also referred to as 'super rainbow facts'). This is important prerequisite knowledge for the application of the more sophisticated partitioning strategy, bridging through 100. It is important to note that this pair of tasks are structured as open-ended missing addend problems, rather than as ‘pure’ addition problems. Consequently, in contrast to the other pairs of tasks considered, the emphasis was on students identifying more than one possible solution, rather than on them discovering multiple means of solving the problem and determining which method is the most efficient.

Having overviewed the task design process, and introduced the corresponding challenging tasks that supported the current study, it is now prudent to introduce the methodological approach that guided the current research

CHAPTER FOUR: METHODOLOGY

This chapter overviews the research methodology which underpins the current study. It is divided into three sections. The first section outlines the research paradigm informing the study and associated epistemology. The second details the method, including participants, procedures, and measures. The third discusses the analytical approaches employed to organise and interpret the data, as well as the specific research questions investigated.

Research Paradigm and Epistemology

Study design

In one sense, the current study can be described as adopting a *quasi-experimental design*, with naturally occurring groups (i.e., grades) receiving different interventions (i.e., Task-First Approach, Teach-First Approach, Alternating Approach). Generally, such a study design is most frequently associated with positivism or post-positivism and quantitative analytical approaches (Teddie & Tashakkori, 2009). Indeed, quantitative methods are used to analyse student outcome data; a major focus of the current study. However, given that some of the research questions proposed appear better addressed qualitatively, a decision was taken to analyse such questions using qualitative analytical approaches. Consequently, the study can also be described as adopting a *parallel mixed-method design* (Teddie & Tashakkori, 2009). Moreover, in line with other mixed-method research (e.g., Teddlie, Reynolds, & Pol, 2000), the current study should be considered more closely underpinned by a *pragmatist paradigm*, rather than post-positivism.

Pragmatism as a research paradigm

Mixed method researchers advocate the use of "whatever methodological tools are required to answer the questions under study" (Teddlie & Tashakori, 2009, p. 7). As a result, the paradigm of pragmatism frequently underlies studies adopting mixed-method approaches. Pragmatism has been described as:

> A deconstructive paradigm that debunks concepts such as "truth" and "reality" and focuses instead on "what works" as the truth regarding the research questions under investigation. Pragmatism rejects the either/ or choices associated with the paradigm wars, advocates for the use of mixed methods in research, and acknowledges that the values of the researcher play a large role in the interpretation of results (Tashakkori & Teddlie, 2003, p. 713).

Pragmatism concurs with postpositivism in that an external reality exists independent of our minds, however pragmatism is more aligned with constructivism in denying the existence of a universal truth (Teddie & Tashakkori, 2009). The pragmatists choice of which truth to privilege is driven by considering which version is likely to lead to usable, desirable outcomes (Cherryholmes, 1992). Pragmatism's epistemology, axiology and framing of causality will now be briefly described, with consideration given to how these aspects relate to the current study.

Epistemology concerns the relationship between who is knowing (the researcher) and what is known (the participant). Pragmatists assert that the either/ or choice offered by 'objective' positivism and 'subjective' constructivism is in fact a false dichotomy, and instead argue that epistemological issues exist on a continuum (Teddie & Tashakkori, 2009). Pragmatists suggest that different types of research projects, and different stages within projects, will lead themselves to more or less interaction between participants and researchers. In instances where less interaction between participants and researchers is required to address a particular research question (e.g., Who is the preferred Prime Minister? – answered using a large, anonymous, on-line survey with a random sample of the Australian electorate), an objectivist frame is appropriate. By contrast, when the researcher and participants need to interact to make sense of a particular research question (e.g., How can the experience of meditation be described phenomenologically? – answered using semi-structured interviews with Tibetan Buddhist monks), a subjectivist frame needs to be adopted. In the current study, some of the questions regarding student mathematical performance encourage an objectivist frame, whilst other questions more interested in unpacking the experience of students and teachers require a subjectivist frame.

With regards to axiology, pragmatists concur with constructivists that values are central to conducting research, however rather than be overly concerned that their research will be value-bound, they admit to using their values to guide and direct their research (Teddie & Tashakkori, 2009). Pragmatists attempt to study what matters, ask the ‘best’ questions, collect the data they deem most important, and analyse it in a way that sheds the most light on the relevant research questions and the overall topic of interest. As a primary school teacher with a particular interest in how mathematics is taught in primary schools, and a genuine curiosity about how different lesson structures might impact student learning based on my own experience informally experimenting with different approaches in the classroom, it is very apparent that my values informed my choice of research topic. Moreover, my previous experience undertaking two minor theses – one entirely quantitative and the other entirely qualitative –extended my appreciation of the strengths of both analytical methods for addressing a given set of research questions. Such a flexible, context-dependent approach to data analysis is consistent with a pragmatist perspective.

When considering issues of causality, pragmatists share post-positivists concerns with internal validity of constructs (Did the independent variable, and not some other factor, cause the effect on my dependent variable?), as well as constructivists' concerns with issues of credibility (Is my description of reality as the researcher consistent and coherent with that of my participants?) (Teddie & Tashakkori, 2009). The current study shared pragmatists concerns about the need to focus on both internal validity and credibility. For example, the cross-over methodology was adopted in part to boost internal validity; that is, to increase the likelihood that the independent variable (i.e., lesson structure) was responsible for changes in the dependent variable (i.e., mathematical performance), rather than some extraneous factor (e.g., class composition). By contrast, interviewing teachers about their experiences of the program was included in the study design in part to boost credibility concerning the variable student mathematical performance. Specifically, did teacher observations concur with the corresponding quantitative analysis? If not, it might be that the participants and the researcher have a different understanding about what constitutes the construct of 'mathematical performance', or 'improvement'. Without the qualitative semi-structured interviews with teachers, the study would not have had the opportunity to informally test the notion that 'mathematical performance' as construed by the researcher overlaps satisfactorily with the notion of 'mathematical performance' as construed by this important subgroup of study participants.

Method

Participants

Participants included Year 1 and Year 2 students (*n*=75), and their respective teachers (*n*=3), who attended a small-to-medium size primary (elementary) school in the Shire of Yarra Ranges, Victoria, Australia. In Victoria, students typically turn 7 years of age during Year 1, and 8 years of age during Year 2.

The school's 2015 enrolment was 258 students, with an Index of Community Socio-Educational Advantage (ICSEA) of 1042, which is slightly above the national average (1000). This indicates that the educational and occupational backgrounds of these families and students attending the school were somewhat advantaged compared to the average Australian school. In addition, only three per cent of students had a language background other than English (MySchool, 2015).

All Year 1 and Year 2 students (76 students) participated in two units of work developed for the current study as part of their regular mathematics instruction. However, the parents of one student declined to have their child participate in the corresponding data collection and research study. In addition, two study participants were excluded from the subsequent student outcome data analysis. One of these students had a mild intellectual disability, and participated in the program with the support of a teaching aide. Although anecdotally this student and her teaching aide spoke positively of the learning experience provided through the program, it was not possible to include her data in the student outcome analysis, due to her receiving some support during the assessment sessions. The other student was excluded due to the reliability of multiple assessments she completed being undermined (i.e., she repeatedly copied off other students).

The three teacher participants varied in terms of their teaching experience. Whereas Rachel[9] was in her first year of teaching, Sally was in her ninth year and Polly had been teaching for over twenty-five years.

Participants were grouped according to pre-existing class structures. This was done for a variety of different reasons, in particular, to work in with school timetabling and to assist with classroom management. Although it was initially anticipated that four groups of students (i.e., four classes) would participate in the program, subsequent resourcing decisions at the school meant that only three groups were included. Each group was of approximately equal size (i.e., Class A = 26, Class B = 26, Class C = 24).

Researcher relationship to participants

At the time of the study, I had begun my fourth year as a primary school teacher at the study school. The year prior to the study, I moved into a specialist mathematics teacher role in a part-time capacity (2 days per week), and continued in this specialist role during the year of the study (focusing on Grade 3 and 4 students). Although care was taken to separate out my role in the research study from my regular mathematics specialist teaching role, it needs to be acknowledged that pre-existing relationships with students (many of whom I had taught the previous year for two hours per week) may have impacted on the magnitude of overall student outcome effect sizes. However, it was not anticipated that these pre-existing relationships would have generated additional between-group variance, due to the approximately uniform distribution of previously taught students across the groups (i.e., Class A = 15, Class B = 14, Class C = 13).

[9] Note that actual teacher names have been replaced with pseudonyms.

In addition, it should be noted that I had established close working relationships with two of the three teachers (Sally and Polly) prior to the commencement of the study. These pre-existing collegial relationships were not anticipated to impact the study in any meaningful way, particularly considering the largely passive role of the classroom teacher in the implementation of the research program.[10]

Measures

Five measures were developed to support the assessment of student outcomes (see Table 4.1). Each of these measures is elaborated on below.

Table 4.1

Measures: Administered Pre- and Post-Program

Measure	Type	Number of items (per admin.)
Mathematical fluency	Assessment	24
Problem Solving Performance	Assessment	2
Intrinsic Motivation to Learn Mathematics	Questionnaire	4
Persistence D1 (Intent to Continue)	Questionnaire	1
Persistence D2 (Cognitive Demand)	Questionnaire	5

Note: If a student did not complete a particular questionnaire item on a given administration, or incorrectly completed an item, their score on the corresponding scale was not calculated (i.e., their score on the scale was effectively treated as missing data).

[10] It should be noted that, for parsimony, in the remainder of this chapter describing the study methodology, I refer to myself in the third person as "the researcher" when relevant.

Mathematical fluency

Students completed assessments of mathematical fluency developed specifically for the current study: one assessment pre and post the patterning unit, and one assessment pre and post the addition unit. In each assessment, there were 24 questions, and students had 30 minutes to complete the assessment. Students had access to materials to support their response to the task (specifically abacuses, counters and hundred charts).

It is worth noting that each question on the post program assessments was subtly different from the corresponding questions on the pre-program assessments, to prevent students simply recalling answers. For example, 48+98+2 became 47+97+3.

For the patterning unit assessments, students had to complete the number pattern by identifying the missing number, which could appear anywhere in a seven-number sequence. For example:

- ☐ 47 57 67 77 87 97
- 3 6 12 ☐ 48 96 192

In the addition unit assessments, students had to either provide the missing addend or the total. For example:

- 93 + ☐ = 200
- 19 + 19 + 19 + 3 = ☐

Assessments were subsequently scored such that participants were awarded one point for a correct response and no points for an incorrect response. Consequently, the highest score a participant could achieve for mathematical fluency was 24, whilst the lowest score was 0.

Results of data screening suggest that the overall scores on the pre and post program assessments of mathematical fluency approximated a normal distribution. Specifically, although there was a suggestion that the post-program data was somewhat negatively skewed for the patterning unit (skewness = *-.806; SE = 0.285*), the absolute value of the skewness statistic remained less than 1, implying that mathematical fluency could be considered acceptably normally distributed (Bandalos & Finney, 2010; Osborne, 2010).

Problem solving performance

Students completed assessments of problem solving performance developed specifically for the current study: one assessment pre and post the patterning unit of work, and one assessment pre and post the addition unit of work. In each assessment, there were two problems. Students had ten minutes (an initial seven minutes plus a further three minutes) to work on each problem. Again, students had access to materials to support their working out (specifically abacuses, counters and hundred charts).

In total, eight problems for measuring problem solving performance were developed. Each problem was assessed by use of a marking key. Specifically, assessments were scored such that participants were awarded a score between 0 and 3 for each problem depending on their response.

Note that a marking key was preferred to a rubric because it was difficult to make judgments about the quality of a student’s mathematical thinking from examining their written responses in isolation. Anecdotal observations during the assessments revealed that students frequently engaged with the supporting materials (e.g., abacuses, hundred charts) and self-talk to express their mathematical thinking, however most students did not attempt to record this 'working out' process on paper. Clearly, particularly due to the age of the students, some form of structured interview would have been preferable in order to gain insight into the quality of mathematical thinking. However, the time-intensive nature of designing and administering such an interview was prohibitive.

The specific marking keys employed for each of the problem solving assessment tasks are set out below.

Patterning unit

The rubric used to assess work on the Skip Counting problem (pre-program assessment) and Donuts problem (post-program assessment) was as follows.

- 0 – no evidence of mathematical thinking.
- 1 – evidence of mathematical thinking (no solution). *Evidence of attempts to skip-count by twos, threes and/ or fives (Skip counting) or accurately Doubling to at least 16 (Donuts).*
- 2 – solution provided/ partially extended. *Solution of 30 or 60 offered (Skip-counting).Doubles to 64 or 128 (Donuts).*
- 3 – solution extended. *Solution of 30, 60 or 90 offered, or solution further generalised (Skip-counting). Doubles to 256 or beyond (Donuts).*

The rubric used to assess work on the Cricket problem (pre-program assessment) and Basketball problem (post-program assessment) was as follows.

- 0 – no evidence of mathematical thinking

- 1 – evidence of mathematical thinking (no solution). *Evidence of attempts to add relevant scores together (e.g., 4+6 for cricket; 2+3 for basketball) or evidence of attempts to skip-count by fours and/or sixes (cricket), by twos and/or threes (basketball).*
- 2 – solution provided/ two solutions provided
- 3 – more than two solutions provided

Addition unit of work

The rubric used to assess work on the Complements to 100 problems (pre and post program) was as follows.

- 0 – no solutions
- 1 – some solutions (between 1 and 4 solutions provided)
- 2 – most solutions (between 5 and 8 solutions provided)
- 3 – all solutions

Finally, the rubric used to assess work on the Favourite Numbers problems (pre and post program) was as follows.

- 0 – no solutions
- 1 – one solution
- 2 – two solutions
- 3 – three solutions

Participant scores on the two problems were summed. Consequently, the highest score a participant could achieve on each of the assessments was 6, while the lowest score was 0. A small sample of each of the assessments (*n*=8) was cross-marked by one of the teacher-participants using the above marking key. This teacher-participant concurred with the researcher's judgments, following some clarification around the distinction between "no evidence of mathematical thinking" and "some evidence of mathematical thinking" when no correct solution was offered by the student. These clarifications were subsequently included in the above marking key in italics.

Results of data screening suggest that the overall scores on the pre and post program assessments of problem solving performance were normally distributed, with the exception of the pre-program patterning unit scores, which were somewhat positively skewed (*skewness = .860*; *SE = 0.330*). However, again, as the absolute value of skewness still falls within the most conservative of recommendations (less than one), problem solving performance can be considered acceptably normally distributed (Bandalos & Finley, 2010; Osborne, 2010).

Intrinsic motivation to learn mathematics

Students' level of intrinsic motivation can be defined as the amount of interest or enjoyment they feel in relation to mathematics (Thomson et al., 2014). In the current study, intrinsic motivation to learn mathematics was operationalised through adapting the measure included in the Programme for International Student Assessment (PISA) 2012 study (Thomson et al., 2014; originally developed by Pekrun, 1993). The items were measured on 4-point likert scales, as per the PISA study. Students were asked, "Thinking about your views on maths: to what extent do you agree with the following statements: *Strongly Agree (4), Agree (3), Disagree (2), Strongly Disagree (1)*?" The items included:

- I enjoy reading about maths;
- I look forward to my maths lessons;

- I do maths because I enjoy it; and
- I am interested in the things I learn in maths.

The maximum score on this instrument is 16 and the minimum score is 4. Reliability coefficients (Cronbach Alpha) for the four administrations of this measure are provided in Table 4.2. Cronbach Alpha values for the intrinsic motivation to learn mathematics instrument varied from poor (α=0.55) to acceptable (α=0.72). It is worth noting that deleting any of the four items would not have consistently improved Cronbach Alpha values.

Table 4.2

Cronbach Alpha values for the Intrinsic Motivation to Learn Mathematics Measure

Assessment	Patterning unit (α)	Addition unit (α)
Pre-Program	0.55	0.62
Post-Program	0.62	0.72

Further analysis revealed that scores on the intrinsic motivation to learn mathematics measure were not normally distributed. Specifically, scores were consistently negatively skewed on each of its four administration (skewness *= -1.14 to -1.23; SE = 0.285*). This indicates a ceiling effect apparent in the measure. This ceiling effect is driven by study participants overwhelmingly having positive views towards mathematics, particularly in comparison to the older student participants in the PISA study, for whom the measure was originally designed. For example, in the 2012 PISA study (see Thomson et al., 2014), only 36% of 15 year old students in the OECD and 45% in Australia agreed or strongly agreed with the statement “I look forward to my mathematics lessons”. The equivalent percentage for current study participants, prior to the study intervention (i.e., during the patterning unit pre-program assessment), was 92%. Given this large difference, further research examining the relationship between changes in intrinsic motivation to learn mathematics as students move through their early schooling may be warranted. Finally, it is worth noting that the high levels of intrinsic motivation reported by participants in the current study (i.e., the aforementioned ceiling effect) may have undermined the capacity to detect differences between groups, and therefore to address the research question: What is the effect of a task-first lesson structure compared with a teach-first lesson structure on students' intrinsic motivation to learn mathematics? This potential ramification of the ceiling effect has been noted elsewhere (see, for example, Albanese, 2000).

Persistence

Before outlining the specific measure of persistence adopted for the current project, the issues which arose during the process of identifying a potentially appropriate measure of persistence are first considered.

Persistence is an academic and personal quality that is highly valued in a school environment (Vernon & Bernard, 2006). In this context, persistence can be defined as the ability to direct concentrated and sustained effort to a task in order to achieve a particular goal, in the presence of some form of uncertainty and/or adversity. The choice of whether or not to persist with a task depends on the assumptions made, and beliefs held, by the learner. Dweck (2000, 2010) contended that persistence is a direct corollary of a growth mindset, such that an individual who believes that performance is malleable and reflects effort and application, rather than innate ability, is more likely to persist in the face of adversity. Likewise, proponents of attribution theory (e.g., Weiner, 1985) might suggest that the inclination to persist would be consistent with an individual anticipating that success on a particular task is primarily attributable to internal factors (i.e., an internal locus), which are within his or her control (e.g., a product of effort rather than aptitude).

Although at face value persistence appears to be critical to learning, there have been relatively few attempts to systematically measure persistence, or the closely-related construct of perseverance, particularly amongst school children. A notable exception is the OECD through their Program of International Student Assessment (PISA). In their assessment of 510 000 students in 2012 across 65 participating countries and economies, 15 year old students were asked about their perseverance when attempting to solve problems (OECD, 2013). PISA operationalised students' perseverance through examining responses to a series of five statements, which students could respond to on a 5-point scale: "*very much like me*", "*mostly like me*", "*somewhat like me*", "*not much like me*" *or* "*not at all like me*". The questions asked students to consider the extent to which they (the student) feel they resemble someone:

- who gives up easily when confronted with a problem;
- who puts off difficult problems;
- who remains interested in the tasks that he or she starts;
- who continues to work on a task until everything is perfect; and
- who does more than is expected of him or her when confronted with a problem (OECD, 2013).

These data were subsequently used to create an index of perseverance, which was shown to correlate significantly with mathematical performance within a given country.

Specifically, across OECD countries, 5.7% of the variance in mathematical performance can be explained by changes in this index of perseverance. This rate was higher in some countries, such as Australia (8.2%) (OECD, 2013).

Although it is ostensibly encouraging that there is empirical evidence to suggest a relationship between perseverance and mathematical performance, there are a number of issues with the measure employed by the OECD (2013) in their PISA study.

Firstly, the items comprising the index developed from the PISA data represent global measures of perseverance. Such measures, therefore, do not account for the fact that levels of perseverance may vary depending on the specific domain being considered. For example, a student may report low levels of perseverance on a mathematical problem, however high levels of perseverance on a language-related problem, due to, for example, contextually-determined assumptions and beliefs about causal attributions regarding performance. Moreover, a global measure of perseverance cannot be assumed to represent the actual levels of perseverance demonstrated by individuals on a given task.

Secondly, although the OECD does provide extensive information regarding PISA data collection methodologies and provides users with access to extensive summaries of PISA data, information regarding the psychometric properties of the index of perseverance (e.g., internal consistency of the scale items) is not readily available.

This issue leads directly to an additional concern regarding the validity of the items claimed to be measuring the construct of perseverance. Particularly in the absence of relevant psychometric information, one may question the face and content validity of some of the items included in the scale.

Consider, for example, the item concerning whether an individual “puts off difficult problems”. It is unclear that such an item discriminates appropriately between students possessing particular sets of attitudes and behaviours because the researchers are making the potentially problematic assumption that students have some control over the types of problems they pursue. This may or may not be true in an educational context depending on the pedagogical approach within a particular school or classroom. Perhaps more fundamentally, this item may actually be measuring a different construct. It could be argued that a behavioural pattern of putting off difficult problems is more relevant to a scale measuring an avoidance-confrontation dichotomy than a perseverance-‘giving-up’ dichotomy. Indeed, Nezu and Nezu (2001) suggested that the “individual with a strong avoidant style prefers to avoid problems than confront them” and demonstrates a pattern of behaviour “that is characterised by procrastination, passivity, inaction and dependence” (p. 188). An individual with an avoidant problem-solving style who “puts off difficult problems” may also fail to persevere (i.e., give up easily), however it appears that these may not be equivalent constructs.

Similarly, the item asking about a student’s tendency to continue working on a task until everything is perfect may relate more to a potentially unhealthy, even pathological, perfectionism than to perseverance. For example, Rice, Blair, Castro, Cohen, and Hood (2003) found that a sample of professional psychotherapists volunteered statements such as “won’t stop until everything is perfect” to explain what they viewed as the at least partially compulsive desire to perform well amongst perfectionists. Consequently, of the five items included in the perseverance scale developed for PISA, at least two of the items appear to be measuring a different construct.

Finally, even if these above concerns could be placed to one side, the wording of the items used in the PISA study is too sophisticated for most 7 to 8 year old primary school students to adequately comprehend and respond to.

Given the above issues, it would seem to be of importance to develop a new scale for measuring persistence or perseverance. For the current study, the choice was taken to focus on the construct of persistence, rather than the closely-related construct of perseverance, because persistence is a term which has more currency in Australian school contexts, particularly primary schools. For example, the You Can Do It! Education program, which has been widely employed in schools across Australia, cites persistence as one of the five keys to social and emotional learning (Bernard & Walton, 2011; Vernon & Bernard, 2006).

Persistence can be considered to be a multi-dimensional construct. Specifically, most definitions of persistence (e.g., Macquarie Dictionary, Oxford English dictionary) emphasise that it involves two dimensions: (1) continued effort or application, (2) in the face of difficulty or adversity. Consequently, persistence was measured through considering both a student's willingness to continue with a cognitively demanding task (dimension 1) and the extent to which they found the task cognitively demanding in the first instance (dimension 2).

A student's willingness to continue with a cognitively demanding task (dimension 1) was measured using a single item, gauging their *intent to continue*. Specifically, students were asked: Do you want to keep going on the activity for a bit longer, or do you want to do something else? Students were given the response options: *I really, really want to keep* *going (3); I want to keep going (2); I am not sure if I want to keep going (1); I want to stop (0)*. Students were allocated a score between 3 and 0, depending on their response.

By contrast, the extent to which students found the task cognitively demanding (dimension 2) was measured by considering students score on the *cognitive demand* instrument.

For the patterning unit, response to items measuring cognitive demand were recorded on a 3-point likert scale. Students were asked: “How well do these sentences describe how you are feeling about this activity”: *A lot how I feel (3); A little how I feel (2); Not how I feel (1)*. For the addition unit, response to items were measured on a 4-point scale. Students were asked to “Tell us what you think about this activity” in relation to the items below, with the response options including: *Strongly Agree (4), Agree (3), Disagree (2), Strongly Disagree (1).*

The items included (* indicates items to be reverse-scored):

- My brain is working very hard right now;
- I am finding this activity challenging right now;
- I am putting in lots of effort right now;
- I think this is a difficult activity; and
- I can do this activity easily*.

For the patterning unit, the maximum score on the cognitive demand instrument is 15 and the minimum score is 5. For the addition unit, the maximum score is 20 and the minimum score is 5. Cronbach Alpha values for this measure are displayed in Table 4.3.

Table 4.3 shows the reliability of the scale, which varied from ‘unacceptable’ during the patterning unit pre-program assessment (α=0.47; α=0.49), to ‘poor-to-acceptable' during the patterning unit post-program assessment (α=0.72; α=0.62), to 'good' during the addition unit ($0.79 \leq \alpha \leq 0.83$). Deleting any of the items, including the reversed score item, would not have consistently improved Cronbach Alpha values.

Table 4.3

Cronbach Alpha values for the Cognitive Demand Measure

Assessment	Patterning unit (α)	Addition unit (α)
Pre-Program (Problem 1)	0.47	0.83
Pre-Program (Problem 2)	0.49	0.82
Post-Program (Problem 1)	0.72	0.79
Post-Program (Problem 2)	0.62	0.81

Examinations of histograms, skewness and kurtosis statistics in relation to the cognitive demand instrument suggested that scores tended to approximate a normal distribution.

Scores on the two dimensions (i.e., Dimension 1 - intent to continue and Dimension 2 - cognitive demand) were subsequently multiplied together to create a composite measure of persistence for a given problem solving task.

For example, consider a student's responses during the addition unit assessment who, on one of the two problem solving questions, recorded a very high score on Dimension 1 (3) and a very high score on Dimension 2 (20). This student has indicated that they 'really, really want to keep going' on the activity despite finding it extremely cognitively demanding. Such a student was allocated the maximum persistence score of 60 (20×3). This can be contrasted with other examples as follows:

- A student with a very high score on Dimension 1 (3), and a very low score on Dimension 2 (5). This indicates a student who is not particularly challenged by the activity, and who is enthusiastic to continue working on the task. This response profile results in a low persistence score (15).
- A student with a low score on Dimension 1 (1) and a very high score on Dimension 2 (20). This indicates a student who is finding the activity extremely cognitively demanding and is unsure whether they want to continue working on the task. This response profile results in a low-to-moderate persistence score (20).
- A student with a moderate score on Dimension 1 (2) and a moderate score on Dimension 2 (13). This indicates a student who is somewhat challenged by the activity, and who is willing to continue to keep working on the task. This response profile results in a moderate persistence score (26).

It is worth noting that the lowest possible score on the intent to continue dimension was zero (students responding "I want to stop"), whereas the lowest score on the other dimension of persistence, cognitive demand, was five (i.e., a score of one on each of the five items). Although obviously this classification is arbitrary in a vacuum, when creating a composite variable such decisions become important. The rationale is that in order to demonstrate any level of intent to continue, you need to be willing to continue (or at least be willing to consider continuing) working on a task. By definition, choosing to stop working on a task cannot be understood as demonstrating an intent to continue, even at a low level. This can be contrasted with the other dimension of persistence; cognitive demand. A task perceived as having a very low level of cognitive demand cannot be equated with the complete absence of any cognitive demand. Put another way, whilst the construct of cognitive demand likely actually exists on a continuum at all levels, intent to continue contains discrete transitions between at least some levels, each of which represents a qualitatively different intent. [11]

Persistence scores on each of the two problem solving tasks included in a given assessment were subsequently summed to create an overall persistence score. Consequently, for the patterning unit, the maximum score on this measure is 90 (45+45) and the minimum score is 0. For the addition unit, the maximum score is 120 (60+60) and the minimum score is 0.

Examinations of histograms, skewness and kurtosis statistics in relation to the persistence measure suggested that scores tended to approximate a normal distribution on three of the four assessments. The exception was for the addition unit post-program data, whereby persistence was somewhat positively skewed (skewness$= 0.827$; $SE = 0.302$). This appears to be a consequence of the relatively large percentage (17.5%) of participants who indicated that they 'wanted to stop' working on both of the two post-program problem

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[11] This idea can be most easily understood through exploring several scenarios. Consider the following three students:

Student A: A student who is finding the task easy and is enthusiastic to continue working on it.

Student B: A student who is finding the task difficult and wants to stop.

Student C: A student who is finding the task easy and wants to stop.

Clearly all these three students have relatively low persistence; however, should they be considered to have equally low persistence?

Whatever scores we allocate to students intent to continue responses (i.e., 0 to 3 or 1 to 4), two of these three students will have an equal persistence score. If we include a zero on our intent to continue scale (i.e., 0 to 3), Student A's persistence score is 15, whilst Student B and Student C have a persistence score of 0. Conversely, if our intent to continue scale begins at one (i.e., 1 to 4), Student A's persistence score is 20, Student B's persistence score is 20, and Student C's persistence score is 5.

Given this constraint, I felt that it was more appropriate to conclude that Student A is more persistent than Student B and Student C (who are equally persistent), rather than Students A and B being equally persistent and more persistent than Student C. That is, if given the choice, it is more conceptually sound to treat Student B and Student C as having the same level of persistence – because they both want to stop working on the task – than treat Student A and Student B as the same. The contention is that the student who is finding the task easy but is still enthusiastic to keep working on it (Student A) is demonstrating more persistence than the person who is finding the task hard and wants to give up (Student B).

solving tasks. However, as the absolute value for skewness still falls within the most conservative of recommendations, less than one, the persistence measure could be considered acceptably normally distributed (Bandalos & Finney, 2010; Osborne, 2010).

Procedure

The first unit of work was undertaken in the study school over the course of Term 2, 2015. This unit of work related to number patterning (patterning unit). The patterning unit consisted of sixteen 55-minute teaching sessions and four 50-minute assessment sessions. Two of these assessment sessions were undertaken prior to the unit of work beginning, and the other two at the conclusion of the unit of work. A detailed outline of this unit of work is available in Appendix A, with associated student worksheets available in Appendix B (challenging tasks) and Appendix C (consolidating tasks).

The patterning unit was delivered across a 6-week period, comprising between three and four sessions per week. Care was taken to ensure that each of the three groups received the same number (8) of morning teaching sessions (9am or 10am) and afternoon teaching sessions (11:30am or 12:30pm), due to possible interactions between time of day and magnitude of learning effects (e.g., Folkard, Monk, Bradbury, & Rosenthall, 1977). For similar reasons, assessment sessions were structured such that the post-program sessions replicated the pre-program sessions as closely as possible. For example, Group A sat both their pre and post program fluency assessments at 9:10am on a Monday morning, whilst Group B sat both these respective assessments at 11:40am on these same days.

The second unit of work was undertaken in the study school over the course of Term 3, 2015. This unit of work related to addition and missing addend problems (addition unit). It consisted of twelve 55-minute teaching sessions and also comprised four 50-

minute assessment sessions. A detailed outline of this unit of work is available in Appendix D, with associated student worksheets available in Appendix E (challenging tasks) and Appendix F (consolidating tasks).

The addition unit was also delivered across a 6-week period, comprising of three sessions per week. Again, care was taken to ensure that each of the three groups received the same number (6) of morning teaching sessions (9am or 10am) and afternoon teaching sessions (11:30am or 12:30pm).

Within the patterning unit, each of the three groups of students participating in the program participated in either the task-first condition, the teach-first condition or the alternating condition (two task-first sessions, followed by two teach-first sessions, followed by two task-first sessions etc.). Specifically, Class A participated in task-first, Class B in teach-first and Class C in the alternating condition. For the addition unit, Class A and Class B were inverted, such that Class B participated in the task-first condition and Class A the teach-first condition. Class C remained in the alternating condition. This cross-over approach ensured that all participants experienced both the task-first and teach-first conditions. This description of the program structure is summarised in Table 4.4.

Table 4.4

Structure of the Overall Research Program

Unit of Work	Task-First	Teach-First	Alternating
Patterning: Term 2	Class A	Class B	Class C
Addition: Term 3	Class B	Class A	Class C

It should be noted that to minimise the possibility that other unaccounted for instruction may have impacted the integrity of the findings, the researcher was able to ensure that the program constituted the entirety of the number and algebra instruction for students during its duration (i.e., the 6 week period in Term 2, and the 6 week period in Term 3). Other mathematics sessions during the period in which the program was running (generally one to two sessions per week), were run solely by the regular classroom teacher and focussed on applied subjects such as geometry, measurement and probability.

Lesson structure: Task-First Approach and Teach-First Approach

A prototypical task-first lesson and teach-first lesson are delineated and contrasted in Figure 4.1.

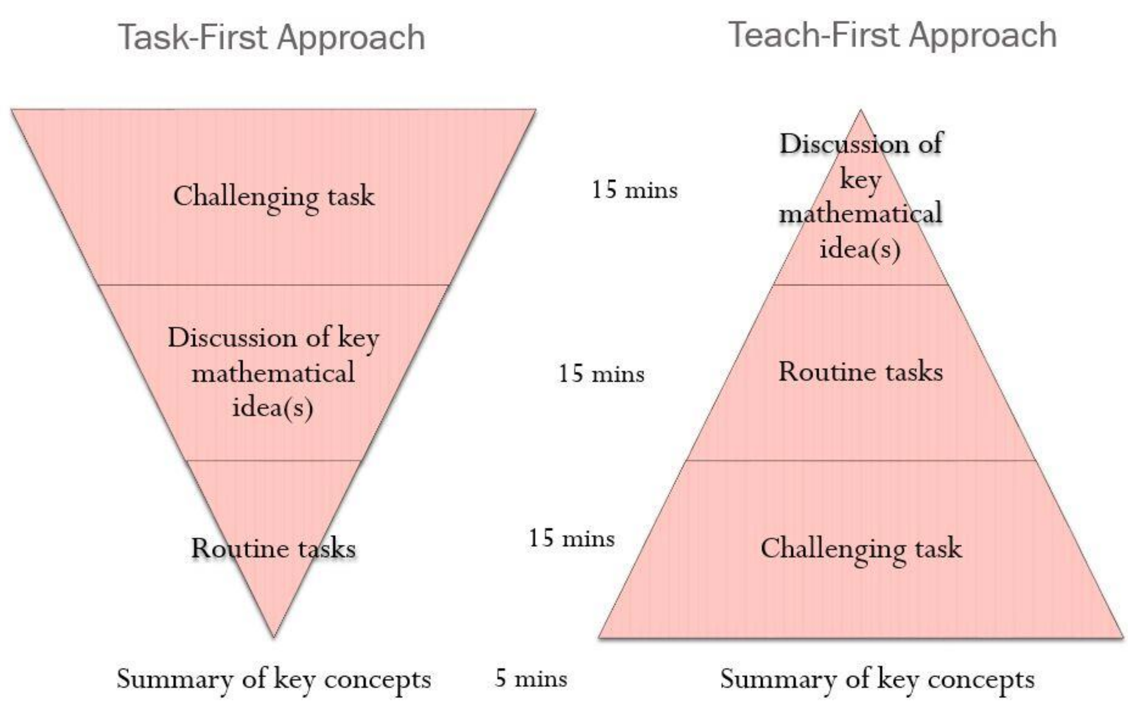

Figure 4.1. Alternative Lesson Structures: Overview

The structure of the Task-First Approach largely mimicked Stein et al.'s (2008) suggested three-stage structure (launch, explore, discuss/ summarize), although an additional fourth stage (consolidate) was introduced. The notion of consolidating student understanding towards the conclusion of a session involving challenging tasks (i.e., after the *discuss* phase) is not novel, and is considered by some authors to be a critical aspect of such task-first lessons (e.g., Sullivan et al., 2014). Consequently, the Task-First Approach began with the *launch* of the challenging task by the researcher. Students were then provided with an opportunity to *explore* the task. Although permitted to work collaboratively, students were expected to develop their own individual solutions and, importantly, be able to explain their own mathematical reasoning. During this phase, students accessed enabling and extending prompts as required. The class then came back together to *discuss* the key mathematical ideas, and examine potential solutions to the challenge. Towards the end of the discussion phase, the teacher would emphasize those solution methods used by students that were most efficient for solving the challenging task (and therefore reflected the lesson's learning objective), as well as, on occasion, highlighting additional efficient solution methods not considered by students. Students were then exposed to several more 'routine' mathematical tasks, designed to *consolidate* their understanding. The researcher then concluded the session with a brief summary, clearly stating the learning objective and inviting two or three students to very briefly present their responses to some of the routine tasks that reinforced this objective.

By contrast, the Teach-First Approach began with what can be considered a student-centered 'mini-lesson', where the teacher introduced the learning objective and proceeded to facilitate a discussion of the key mathematical ideas to be explored in the session. During this initial discussion, the teacher would highlight and unpack with students efficient strategies for solving problems of a particular type, which reflected the specific lesson objectives. Students were then given a series of ‘routine’ mathematical tasks to *establish* their understanding of the concepts introduced. These ‘routine’ tasks became progressively more complex, and students generally worked on these tasks independently. The class then briefly came back together, and the challenging task was launched. Again, the researcher concluded the session with a brief summary. This involved restating the learning objective and calling upon one or two carefully selected students, whose approach to the challenging task aligned with the learning objective, to briefly explain how they approached the challenging task.

Although it would be difficult to argue that the Teach-First Approach precisely equated to any sort of ‘traditional’ lesson structure, there is evidence that it is broadly consistent with some key features of a ‘typical’ Australian mathematics lesson. For example, the TIMSS study of Grade 8 mathematics classes found that the first half of mathematics lessons tend to be dominated by public ‘teacher-talk’, as previously learnt material is reviewed, and/or new material is introduced (Hollingsworth et al., 2003). This period of public-time is roughly synonymous with the ‘discussion of key mathematical ideas’ which characterised the first part of the Teach-First Approach in the current study. By contrast, according to the Trends in International Mathematics and Science Study (TIMSS), the second half of a ‘typical’ Australian mathematics lesson tends to be dominated by students working on a series of concurrent problems at their tables (Hollingsworth et al., 2003). Given that many textbooks tend to structure sets of problems so that they become increasingly complex, pinnacling in multi-step questions focussing on 'application' of the mathematics (e.g., Evans, Lipson, Jones, Blaine, & McCoy, 1996), this second half of the lesson is roughly synonymous with students first working on a series of routine tasks, before shifting their focus to the challenging task. Consequently, it is asserted that the Teach-First Approach had substantially more in common with a 'typical' Australian mathematics lesson than the Task-First Approach.

Figure 4.1 also includes idealised time allocations for a typical mathematics lesson, whereas actual average time allocations for the two units of work are displayed in Tables 4.5 and 4.6. Note that, on average, most sections of the lesson were slightly shorter in duration compared with times shown in Figure 4.1, and less than the 55 minutes allocated to the session; with the average overall length of the lesson being approximately 43 minutes for the Patterning unit and approximately 45 minutes for the Addition unit. These shorter lesson lengths reflect the fact that any non-mathematical activity (e.g., check-in, classroom administration, interruptions, transitioning between activities) was not included in these time allocations (or, when relevant, was retrospectively excluded). The only mathematical activity that was excluded from these time allocations was the two to three minutes it typically took to launch the challenging task. Consequently, the challenging task component in Tables 4.5 and 4.6 only reflects the amount of time students spent *exploring* (Stein et al., 2008) the challenging task.

Including the launching of the challenging task, students spent on average 83% (Patterning) and 86% (Addition) of their lesson time engaged in mathematical work. Although the slightly different methodologies in characterising what constitutes non-mathematical work means that direct comparison is somewhat difficult, results from TIMSS suggest that the equivalent percentage is higher (95%) for Year 8 students. These differences may be attributable to the relative ages of the students, as well as contrasts in the organisational dynamics and cultures of primary schools compared with secondary schools. Supporting this idea, results from the current study compare rather favourably to observational studies of primary (elementary) school environments in which average instructional time data has been collated (e.g., 77%, Smith, 2000), including observations specific to Year 2 classrooms (81%; Rosenshine, 1980).

Table 4.5

Average Time Allocations (minutes): Patterning Unit

	Challenging Task	Discussion	Routine Tasks	Summary	Total
Class A (Task-First)	16	14	10	2	42
Class B (Teach-First)	12	16	12	4	44
Class C (Alternating)	15	14	11	3	43
Average	15	15	11	3	43

Table 4.6

Average Time Allocations (minutes): Addition Unit

	Challenging Task	Discussion	Routine Tasks	Summary	Total
Class A (Teach-First)	14	13	13	4	44
Class B (Task-First)	16	14	11	4	45
Class C (Alternating)	15	13	13	4	44
Average	15	13	12	4	45

The role of the classroom teacher and the researcher

The researcher was responsible for developing the units of work and the respective lesson plans, and for leading the teaching during the sessions. This approach, whereby the researcher took primary teaching responsibilities for all lessons, was chosen principally to ensure that the program was implemented as intended.

By contrast, the regular classroom teacher acted as a relatively passive co-teacher, assisting with classroom management and providing occasional support and guidance to students to assist with the smooth running of the lessons (under the assistance of the researcher). The active ‘teaching’ role of the classroom teacher was deliberately minimised in order to reduce the possibility that some form of confounding ‘teacher effect’ would generate additional variability across the three study groups. Encouraging a more passive role also allowed classroom teachers sufficient space to engage as study participants. The three teachers maintained reflective lesson diaries, which they were invited to review prior to their subsequent semi-structured interviews with the researcher.

Data Collection

Student outcome data: Assessment and questionnaires

Both at the beginning of the patterning unit (pre-program) and at its conclusion (post-program), the administration of ‘pencil and paper’ program assessment and questionnaires to student participants over two 50-minute sessions were carried out by the researcher, with the support of the respective classroom teachers.

During the first session, the intrinsic motivation to learn mathematics questionnaire and the mathematical fluency assessment were administered (see Appendix G). The former was administered to the whole class simultaneously at the beginning of the session, with the researcher reading out each question. The latter was then completed by students individually.

During the second session, the problem solving performance assessment was administered to students. This assessment comprised two problem solving tasks. After being allocated seven minutes to work on the first task individually, the whole class simultaneously completed a questionnaire comprising the cognitive demand items and the intent to continue item, with the researcher reading out each question. The questionnaire required students to reflect on their experience of working on this particular problem solving task. After completing the questionnaire, students were allocated a further three minutes to work on the task. This same procedure was followed for the second problem solving task (see Appendix H).

The addition unit replicated this same process exactly over the corresponding assessment sessions (see Appendix I and Appendix J), with one notable exception. Specifically, the cognitive demand dimension from the persistence measure was altered such that responses were recorded on a 4-point likert scale, rather than a 3-point likert scale. This decision was taken primarily because the notion of agreeing or disagreeing with statements based on the principle of "two thumbs up (strongly agree), one thumb up (agree), one thumb down (disagree), two thumbs down (strongly disagree)", which had already been adopted for the intrinsic motivation to learn mathematics scale, anecdotally appeared more communicable than probing students about the strength of their feelings. As this instrument was still in its development phase, exploring different measurement scales in attempts to improve face validity and reliability seemed appropriate.

It is worth noting that, across both units of work, fewer students completed the problem solving performance assessment and associated questionnaires (*n*=63; *n*=62) than the mathematical fluency assessment and intrinsic motivation to learn mathematics questionnaire (*n*=73). This was mainly due to the decision by the researcher to allow students to complete the mathematical fluency assessment at the next available opportunity (e.g., the next day), if the student was absent from the whole-of-group assessment session for any reason (e.g., illness); a course of action not feasible for the problem solving performance assessment.

For the mathematical fluency assessment, several steps were taken to minimise the possibility that these 'catch-up' assessment sessions would undermine the integrity of the study: 1) the instructions for these 'catch-up' sessions were identical to that of the whole-of-group sessions; 2) in all instances, the pre-assessment sessions occurred before the student began their participation in the program; 3) in all instances, the 'catch-up' post-assessment sessions occurred in the same week as the whole-of-group post-assessment sessions. Primarily due to its more heavily teacher-facilitated nature, it became evident that these three criteria could not be met for the problem solving performance assessment. Consequently, a decision was taken to not run 'catch-up' sessions for this assessment.

Student perspective data: Semi-structured interviews

Between one and two weeks after the completion of the patterning unit, short, semi-structured interviews were undertaken with each of the student participants (*n*=75). Note that semi-structured interviews were chosen over a questionnaire primarily due to the age of the students (i.e., 7 and 8 years old), and the difficulties some students of this age have in expressing themselves fluently and coherently in written form (Hill, 1997). The main purpose of these interviews was to ascertain what in particular students valued about the work they completed during the unit. In addition, students in Class C (*n*=24) were asked to compare the Task-First Approach and Teach-First Approach, and convey which lesson-type they preferred and why. Student interviews were short in duration, with a typical interview taking around 10 to 15 minutes to complete. Moreover, the majority of this interview time was occupied by students selecting their relevant work samples for discussion, rather than actually conversing with the researcher. During the interview, student responses were copied down verbatim by the researcher, and then read back to the student (referred to formally as "reflection"; Hill, 1997, p. 180). This approach allowed students to clarify, modify and add to their comments when necessary.

Similarly, one week after the completion of the addition unit, semi-structured interviews were undertaken only with those participants from Class C (*n*=23). The purpose of these interviews was to invite this sub-group of participants to again compare the Task-First Approach and Teach-First Approach, and convey which lesson-type they preferred and why. It was decided to limit this comparison question to Class C participants, as they were the only group who were in a position to meaningfully compare the two approaches, having experienced both approaches regularly throughout both units of work. These interviews, which were essentially built around a single question, were brief in duration, taking less than 5 minutes to complete.

It is worth noting that during the program, at the beginning of a Class C lesson, students were shown one of the two diagrams below (see Figure 4.2), corresponding to the lesson structure for that particular session. The primary purpose of exposing students to this diagram was to allow students to prepare themselves for how they would be expected to engage mathematically within the session.

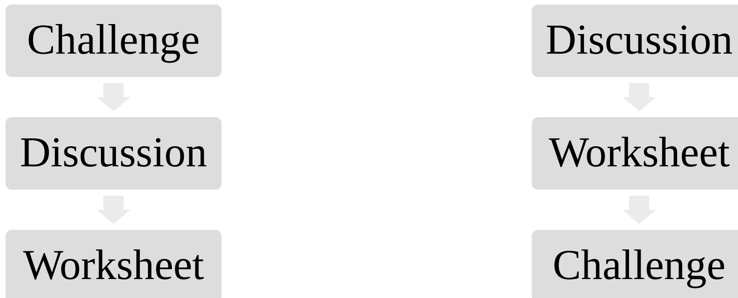

Figure 4.2. Diagram for Participants Summarising the Task-First Approach (left) and the Teach-First Approach (right)

During follow-up interviews with students in Class C, both diagrams in Figure 4.2 were shown to students. The intention was to provide students with a visual aid to remind them about the two contrasting lesson structures experienced during the program.

The student interview schedule is included in Appendix K.

Teacher perspective data: Semi-structured interviews

One week after the completion of both the patterning unit and the addition unit, semi-structured interviews were undertaken by the researcher with each of the classroom teachers individually. Classroom teachers had been encouraged to informally record their reflections and observations (in the form of a diary) following each of the lessons. They were then invited to spend some time reflecting on these experiences prior to their interviews with the researcher.

The purpose of the interviews was to glean teacher perspectives on both the program, and teaching with challenging tasks generally. During the second interview, teachers were also given an opportunity to contrast the Task-First approach with the Teach-First approach.

In total, six interviews were completed (3 teacher participants x 2 units of work). Each interview was audio-recorded and subsequently transcribed by the researcher. These interviews were all between 22 minutes and 31 minutes in length, with the second round of interviews (mean = 30 minutes) being slightly longer than the first round of interviews (mean = 23 minutes). Interview transcripts were shared with teachers, who were given an opportunity to add or amend material. Upon reflecting on her interview, one teacher-participant (Polly) indicated she wanted to add some additional comments to conclude her second interview. These comments were transcribed and incorporated as an addendum.

The teacher interview schedule is included in Appendix L.

Approaches to Data Analysis

Student outcome data: Mixed Design ANOVA

Student outcome data was analysed quantitatively, using SPSS Statistics, Version 22. As there were no significant differences between the three groups of students (i.e., the three classes) in terms of pre-program scores on any of the key measures, it was decided to analyse student outcome data using a mixed randomised-repeated design analysis of variance (Mixed Design ANOVA). Although the analysis could have reasonably been undertaken using Analysis of Covariance (ANCOVA), with the pre-program data included as a co-variate, the advantage of a mixed repeated measures design is the (usually) increased statistical power, due to the smaller error terms (Tabachnick & Fidell, 2007).

A separate Mixed Design ANOVA was undertaken for each dependent variable, and each unit of work was analysed separately.[12] Dependent variables analysed included: mathematical fluency, problem solving performance, intrinsic motivation to learn mathematics and persistence. For each analysis, the within group factor was time (i.e., pre-, post-program) and the between group factor was lesson structure (i.e., task-first, teach-first, alternating).

Before proceeding further, it is worth mentioning that, despite some administrations of the measures employed in the current study producing distributions that deviated somewhat from normal, generally no attempt was made to transform the data. As Howell (1997) noted, analysis of variance procedures are very robust to violations of normality, particularly in instances where the homogeneity of regression slope assumption has not been violated and samples sizes are relatively equal. Moreover, Tabachnick and Fidell (2001) noted that when undertaking ANOVA with relatively equal sample sizes in groups, no outliers, two-tailed tests, and approximately 20 degrees of freedom for the randomised error term, normality of the sampling distribution can generally be assumed. Consequently, given that these conditions for robustness of ANOVA outlined above were met in the current study, data transformations were only considered for the intrinsic motivation to learn mathematics measure, which was both significantly negatively skewed (z values for skewness were greater than +2.58; $p<0.01$) and had absolute values for skewness greater than 1 (Bandalos & Finney, 2010; Khine, 2013; Osborne, 2010).

[12] Note that the cross-over design inhibits the researcher from combining the two units of work into a MANOVA design.

Student perspective data: Constant Comparative Method

By contrast, student perspective data was analysed qualitatively. Due to the large number of cases (n=75) and the fact that the student interviews were short in duration, the decision was taken to choose an analytical technique that emphasised comparisons across cases and categorical classifications, rather than individual case studies and rich narrative accounts (Teddie & Tashakkori, 2009). Specifically, the Constant Comparative Method (Glaser, 1965, 1969) was adopted to support categorical organisation of the qualitative student perspective data. The Constant Comparative Method is a four stage process and involves: "(1) comparing incidents applicable to each category, (2) integrating categories and their properties, (3) delimiting the theory, and (4) writing the theory" (Glaser, 1969, p. 220). It has links to Grounded Theory approaches (Strauss & Corbin, 1994). Each of these four steps is elaborated in Table 4.7.

After the Constant Comparative Method yielded a more parsimonious set of categories (i.e., Step 3 in Table 4.7), some attempt was made to quantify the frequency in which participants appeared in various categories. Consequently, the analysis of student perspective data truly involves a ‘mixed method’ approach, with some attempt to describe the data quantitatively alongside the qualitative analysis.

Table 4.7

Constant Comparative Method (Adapted from Glaser, 1965)

Step	Description
1) Comparing incidents applicable to each category	After the data are read and reread to identify relevant themes and categories, the incident (e.g., the section of the relevant transcript) is coded to as many categories of analysis as possible. The advice is "while coding an incident for a category, compare it with the previous incidents coded in the same category" (p. 439).
2) Integrating categories and their properties	Shift in focus from comparing incidents (i.e., Step 1) to developing more general rules that define a given category arising from analysis of these incidents. This process is iterative; each incident within that category is compared back to this set of tentative rules to refine the rules.
3) Delimiting the theory	The original list of categories is reduced, with the goal of developing a smaller set of categories operating at a higher level of abstraction. The goal is for categories to become "theoretically saturated", whereby coding an additional incident to that category does not add to the explanatory power of that category (p. 441).
4) Writing the theory	Through working back through this process, the analyst is able to explicate their theory.

Teacher perspective data: Interpretative Phenomenological Analysis (IPA)

Interpretive Phenomenological Analysis (IPA) was selected as the analytic method for revealing themes and patterns embedded in the semi-structured interviews conducted with teachers. Interpretive Phenomenological Analysis is a contemporary approach to qualitative data analysis, developed specifically to capture subjective experiences (Smith, Harre, & Van Lagenhove, 1995). According to Jonathon Smith (2004), who developed IPA, the method has three defining characteristics: it is idiographic, in that each case should be analysed until some form of gestalt is achieved before various cases are compared and cross-case themes developed; it is inductive, in that it is flexible enough that unanticipated themes can emerge during analyses; and it is interrogative, in that it attempts to illuminate and build on existing research.

Interpretative Phenomenological Analysis was selected as the analytical tool for the teacher interviews, due to the small number of cases (n=3) and the related emphasis on context and privileging the narrative, rather than attempting to immediately distil responses into categories. More specifically, part of the semi-structured interviews with teachers attempted to unpack participants' own teaching practice; their lived experience of being a teacher. This highly personal, subjective world can only really be understood through dialogue with participants constructed in participants' own terms. This focus resonates with a phenomenological approach, such as IPA.

However, participants were not only required to play the role of 'reflective practitioner' during the interview; they also had to step into the role of detached 'expert' observer as they made sense of another practitioner's teaching practice (i.e., the researcher's), alternative pedagogies (e.g., teaching with challenging tasks, different lesson structures), and evaluated the overall impact of the program on their students (both individually and collectively). Given that some of the strengths of IPA include both its flexibility and its capacity to handle complexity (Smith & Osborn, 2008), this dual role aspect provides further support for the notion that IPA is an appropriate analytical tool for interpreting the teacher interviews.

Smith and Osborn (2008) described four broad steps that a researcher should consider when employing IPA to analyse qualitative data. These four steps guided the analysis of teacher interview data in the current study, and have been distilled in Table 4.8. In addition to the four steps outlined in Table 4.8, the researcher will endeavour to enhance the reliability of the superordinate themes which emerge in the current study through discussing each theme with his supervisor.

Table 4.8

Stages in the Analysis of Interview Transcripts (Adapted from Smith & Osborn, 2008)

Step	Description
1) Looking for themes in the first case	A transcript is read several times, and annotated in an unstructured, fluid manner upon each reading, with significant elements of the transcript highlighted. Initial notes are transformed into themes, represented by precise phases which capture the meaning of the text, perhaps with reference to the relevant pedagogical terminology.
2) Connecting the themes	Themes emerging from the transcript are listed, and links and connections between them examined. The process is iterative and the original transcript frequently referred back to so as to ensure

	the emerging clusters of themes resonate with the participants' actual words and meanings. Superordinate themes, and relevant examples in the text, are then developed into a summary table, with themes less relevant to the emerging structure omitted.
3) Continuing the analysis with other cases	Steps 1 and 2 are repeated for subsequent interviews, with the analyst constantly endeavouring to balance discerning repeating patterns with highlighting emergent issues. Final superordinate themes are developed, and earlier cases are reviewed in light of any new superordinate themes that have emerged.
4) Writing up	The final step is concerned with translating the themes into narrative accounts. The analysis again becomes expansive, as themes are explained, elaborated and examined. The analyst should be careful to distinguish between what a respondent has stated, and their account of what the respondent has stated.

Specific research questions addressed by the current project

Although the research questions were presented in general terms at the conclusion of the literature review, it is necessary to restate the research questions in more specific terms following the Task Design and Methodology chapters. Consequently, specific research questions for each of the three studies are presented below.

Study One

When teaching with challenging tasks:

1. What is the effect of a Task-First Approach compared with a Teach-First Approach on students' mathematical fluency?
2. What is the effect of a Task-First Approach compared with a Teach-First Approach on students' mathematical problem solving performance?
3. What is the effect of a Task-First Approach compared with a Teach-First Approach on students' intrinsic motivation to learn mathematics?
4. What is the effect of a Task-First Approach compared with a Teach-First Approach on task-based student persistence?

Study Two

1. How did teachers perceive students' response to lessons involving challenging tasks? What factors did they attribute to these responses?
2. What factors seem likely to influence teachers'willingness to teach with challenging tasks in the future (e.g., perceived student reactions, teacher confidence and competence, time constraints)?
3. What differences did teachers perceive between a Task-First Approach when observing teaching with challenging tasks and a Teach-First Approach? When it came to considering their own teaching practice, did teachers have a preference for one particular type of approach?

Study Three

1. What do students value when reflecting on their own learning artefacts following participation in a program involving challenging tasks?

2. Do students have a preference for a Task-First Approach or a Teach-First Approach? What factors are reported as influencing this preference?

CHAPTER FIVE: STUDY ONE – STUDENT OUTCOMES

This chapter comprises findings and discussion from Study One, which involved the quantitative analysis of student outcomes. It begins by briefly summarising key descriptive statistics, before systematically considering what the student outcome data revealed about each of the four research questions, specifically:

1. What is the effect of a Task-First Approach compared with a Teach-First Approach on students' mathematical fluency?
2. What is the effect of a Task-First Approach compared with a Teach-First Approach on students' mathematical problem solving performance?
3. What is the effect of a Task-First Approach compared with a Teach-First Approach on students' intrinsic motivation to learn mathematics?
4. What is the effect of a Task-First Approach compared with a Teach-First Approach on task-based student persistence?

The second part of the chapter is devoted to discussing the implication of these findings, particularly as they relate to the literature concerning student outcomes associated with cognitively demanding tasks and discovery-based learning.

Descriptive Statistics

Descriptive statistics for the student outcome measures are reported below. Note that there is no attempt to undertake any inferential analysis of data (e.g., statistical significant testing) in this section. Instead, pre and post comparisons will be more robustly investigated in subsequent sections, as part of the analysis of variance techniques employed to address the research questions. Descriptive data for the two instruments which comprise the persistence measure are included in Appendix M.

Mathematical fluency

Means and standard deviations for the four mathematical fluency assessments for all students are presented in Table 5.1. The mean score on these assessments varied from 10.96 for the patterning unit pre-test to 16.34 for the addition unit post-test. Moreover, it is apparent that mean scores on these measures increased from the pre-program assessment to the post-program assessment across both units of work.

Table 5.1

Means and Standard Deviations for Mathematical Fluency

Assessment	N	Mean	SD
Patterning unit (pre)	73	10.96	5.81
Patterning unit (post)	73	14.47	4.56
Addition unit (pre)	73	12.62	5.86
Addition unit (post)	73	16.34	6.18

*Recall that the maximum score on this measure is 24, whilst the minimum score is 0.

Problem solving performance

Means and standard deviations for the four problem solving performance assessments for all students are presented in Table 5.2. The mean score on these assessments varied from 1.57 for the patterning unit pre-test to 3.33 for the patterning unit post-test. Again, for both the patterning unit and the addition unit, scores on this measure were higher during the post-program assessments than during the pre-program assessments.

Table 5.2

Means and Standard Deviations for Problem Solving Performance

Assessment	N	Mean	SD
Patterning unit (pre)	63	1.57	1.62
Patterning unit (post)	63	3.33	1.62
Addition unit (pre)	62	1.97	1.76
Addition unit (post)	62	3.08	2.12

*Recall that the maximum score on this measure is 6, whilst the minimum score is 0.

Intrinsic motivation to learn mathematics

Intrinsic motivation to learn mathematics was measured on four occasions during the current study. Mean scores and standard deviations are provided in Table 5.3.

Table 5.3

Means and Standard Deviations for Intrinsic Motivation to Learn Mathematics.

Assessment	N	Mean	SD
Patterning unit (pre)	73	13.38	2.22
Patterning unit (post)	73	14.26	1.79
Addition unit (pre)	72	14.19	1.81
Addition unit (post)	73	13.64	2.47

*Recall that the maximum score on this measure is 16, whilst the minimum score is 4.

In contrast to measures of mathematical performance, although intrinsic motivation to learn mathematics scores were, on average, higher during the post-program assessment compared with the pre-program assessment for the patterning unit, this was not the case for the addition unit. Specifically, the mean score of intrinsic motivation to learn mathematics actually declined somewhat between the pre-program and post-program assessments.

Persistence

Means and standard deviations for student persistence scores are displayed in Table 5.4. Again, in a similar manner to intrinsic motivation to learn mathematics, whilst comparisons of pre and post data indicate that the mean score on this measure of persistence increased during the patterning unit, it decreased during the addition unit.

Table 5.4

Means and Standard Deviations for Persistence

Assessment	N	Mean	SD
Patterning unit (pre)	57	32.49	23.07
Patterning unit (post)	64	41.34	21.54
Addition unit (pre)	63	57.96	31.48
Addition unit (post)	63	37.00	29.87

*Recall that for the patterning unit, the possible range of scores on this measure was 0 to 90 (45+45), whereas for the addition unit, it was 0 to 120 (60+60).

Task-First vs Teach-First: Comparing Mathematical Fluency

A Mixed Design ANOVA was performed on mathematical fluency scores for each unit of work. The within group factor was time (i.e., pre-program, post-program) and the between group factor was lesson structure (i.e., task-first, teach-first, alternating). Results of the evaluation of the assumptions of independence, linearity, homogeneity of variance, sphericity and homogeneity of intercorrelations were satisfactory for both analyses.

Patterning unit

With regards to the patterning unit, the interaction effect between time and lesson structure was non-significant, with $F(2, 70) = 0.288$, $p > 0.05$. This suggests that allocation to a particular lesson structure did not impact on student mathematical fluency within this particular unit of work. By contrast, the main effect for time was significant and large, with $F(1, 70) = 78.748$, $p < 0.05$, partial $\eta^2 = 0.529$. This suggests that, irrespective of how the lesson was structured, participation in the research program had a substantial impact on mathematical fluency within the patterning unit. Means and standard deviations are presented in Table 5.5, whilst Figure 5.1 provides a visual representation of mean scores.

Table 5.5

Means and Standard Deviations for Mathematical Fluency by Time and Lesson Structure: Patterning Unit

Time	Lesson Structure	N	Mean	SD
Pre-program	All	73	10.96	5.81
	Task-First (Class A)	25	10.96	5.47
	Teach-First (Class B)	24	11.00	5.73
	Alternating (Class C)	24	10.92	6.46
Post-program	All	73	14.47	4.56
	Task-First (Class A)	25	14.16	4.51
	Teach-First (Class B)	24	14.42	4.47
	Alternating (Class C)	24	14.83	4.87

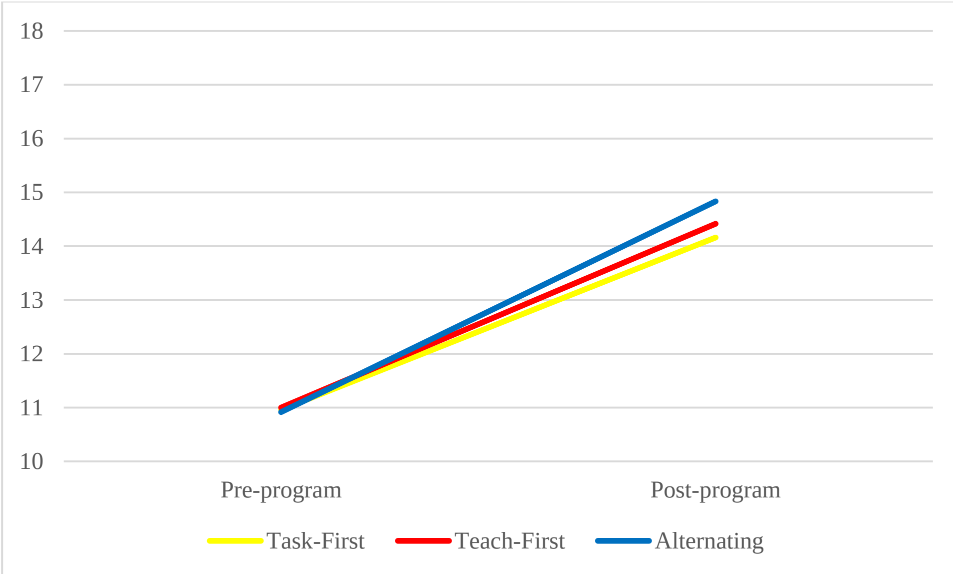

Figure 5.1. Mean Scores for Mathematical Fluency by Time and Lesson Structure (Patterning Unit)

Addition unit

In contrast with the patterning unit, for the addition unit, the interaction effect between time and lesson structure was significant and moderate, with $F(2, 70) = 3.913$, $p < 0.05$, $\eta^2 = 0.101$. This suggests that allocation to a particular lesson structure impacted on changes in student mathematical fluency. Interrogating Table 5.6 and Figure 5.2 indicates that participating in the Teach-First Approach on average led to greater improvements in mathematical fluency scores compared with the other two approaches. Moreover, the main effect for time was also significant and large, with $F(1, 70) = 116.761$, $p < 0.05$, partial $\eta^2 = 0.625$. Consequently, participation in the research program also had a very notable influence on mathematical fluency within the addition unit, independent of the particular lesson structure to which students were allocated.

Table 5.6

Means and Standard Deviations for Mathematical Fluency by Time and Lesson Structure: Addition Unit

Time	Lesson Structure	N	Mean	SD
Pre-program	All	73	12.62	5.86
	Task-First (Class B)	26	11.46	5.39
	Teach-First (Class A)	24	12.42	6.22
	Alternating (Class C)	23	14.13	5.92
Post-program	All	73	16.34	6.18
	Task-First (Class B)	26	14.31	6.54
	Teach-First (Class A)	24	17.50	5.91
	Alternating (Class C)	23	17.43	5.67

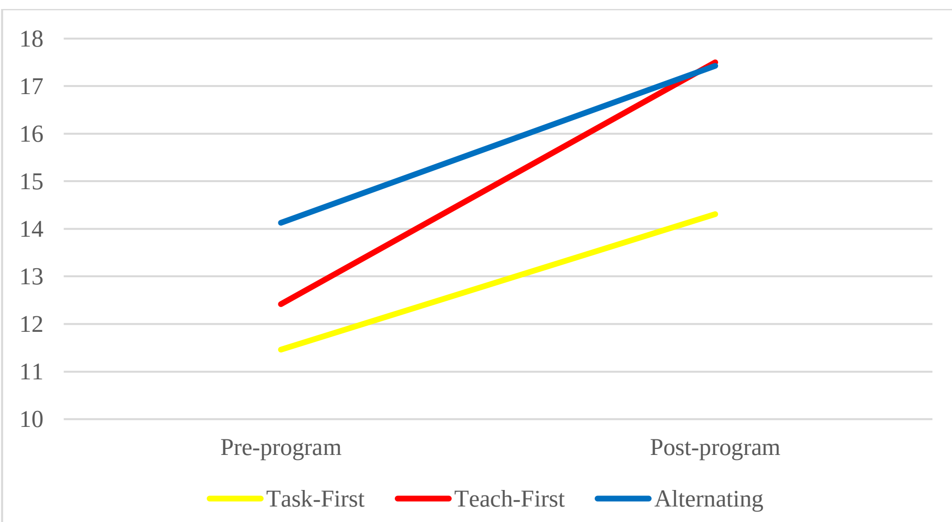

Figure 5.2. Mean Scores for Mathematical Fluency by Time and Lesson Structure (Addition Unit)

Combined analysis

Given the inconsistencies in the above analyses, in an attempt to ascertain the presence of an overall effect for lesson structure, data from the two units of work were combined into a single analysis. This involved the separate calculation of difference scores (post-program test scores minus pre-program test scores) for the task-first condition and the teach-first condition. The variable representing the task-first difference score comprised participants from Class A's results from the patterning unit and participants from Class B's results from the addition unit. By contrast, the variable representing the teach-first difference score comprised participants from the Class B's results from the patterning unit and participants from Class A's results from the addition unit.

A two-tailed paired-samples t-test was undertaken to compare difference scores in mathematical fluency in the combined task-first condition with difference scores in the combined teach-first condition. There was a significant difference between the task-first (M=2.96, SD=2.97) and the teach-first (M=4.25, SD=3.45) difference scores; $t(47)= -2.05$, $p<0.05$. The effect size for this analysis ($d = 0.30$) was small-to-medium following Cohen's (1992) convention. Specifically, students on average improved their mathematical fluency scores more when the lesson was structured such that the teaching component was first compared with when the task component was first.

Summary

There is evidence that a Teach-First Approach improved mathematical fluency more than a Task-First Approach. Specifically, mathematical fluency on the addition unit was highest for the teach-first group, whilst a combined analysis which pooled data from both units of work suggested that participants improved their mathematical fluency more in the teach-first condition than in the task-first condition. It is worth noting that the main effect for time was notably larger than the effect of lesson structure. This suggests that participation in any form of the program when challenging tasks are used was more important than the manner in which the respective lessons were structured for improving mathematical fluency.

Task-First vs Teach-First: Comparing Problem Solving Performance

A Mixed Design ANOVA was performed on problem solving performance scores for each unit of work. The within group factor was time (i.e., pre-program, post-program) and the between group factor was lesson structure (i.e., task-first, teach-first, alternating). Evaluation of the assumptions of independence, linearity, homogeneity of variance, sphericity and homogeneity of intercorrelations were satisfactory for both analyses.

Patterning unit

Allocation to a particular lesson structure did not impact on student problem solving performance within the patterning unit, with $F(2, 60) = 2.510$, $p > 0.05$. By contrast, there was evidence that, irrespective of how the lesson was structured, participation in the research program had a very substantial impact on problem solving performance, with $F(1, 60) = 104.932$, $p < 0.05$, partial $\eta^2 = 0.636$. Means and standard deviations are presented in Table 5.7, whilst Figure 5.3 provides a visual representation of mean scores. Although consideration of Figure 5.3 suggests that students experiencing the Alternating Approach may have had smaller gains in problem solving performance scores compared with the other two approaches, it needs to be noted that these differences were not large enough to be statistically significant.

Table 5.7

Means and Standard Deviations for Problem Solving Performance by Time and Lesson Structure: Patterning Unit

Time	Lesson Structure	N	Mean	SD
Pre-program	All	63	1.57	1.62
	Task-First (Class A)	23	1.30	1.40
	Teach-First (Class B)	19	1.47	1.74
	Alternating (Class C)	21	1.95	1.75
Post-program	All	63	3.33	1.62
	Task-First (Class A)	23	3.44	1.44
	Teach-First (Class B)	19	3.37	1.67
	Alternating (Class C)	21	3.19	1.81

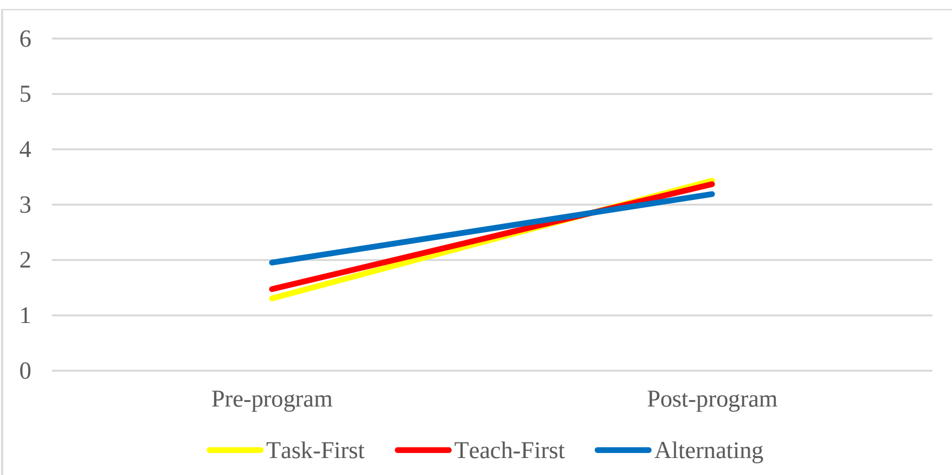

Figure 5.3. Mean Scores for Problem Solving Performance by Time and Lesson Structure (Patterning Unit)

Addition unit

Similarly, allocation to a particular lesson structure did not impact on student problem solving performance within the addition unit, with $F(2, 59) = 0.509$, $p > 0.05$; however, overall participation in the program was again related to large performance gains, with $F(1, 59) = 28.154$, $p < 0.05$, partial $\eta^2 = 0.323$. Means and standard deviations are presented in Table 5.8, whilst Figure 4.4 provides a visual representation of mean scores.

Table 5.8

Means and Standard Deviations for Problem Solving Performance by Time and Lesson Structure: Addition Unit

Time	Lesson Structure	N	Mean	SD
Pre-program	All	62	1.97	1.76
	Task-First (Class B)	22	1.55	1.63
	Teach-First (Class A)	22	1.82	1.65
	Alternating (Class C)	18	2.67	1.91
Post-program	All	62	3.33	1.62
	Task-First (Class B)	22	2.41	2.02
	Teach-First (Class A)	22	3.18	2.02
	Alternating (Class C)	18	3.78	2.24

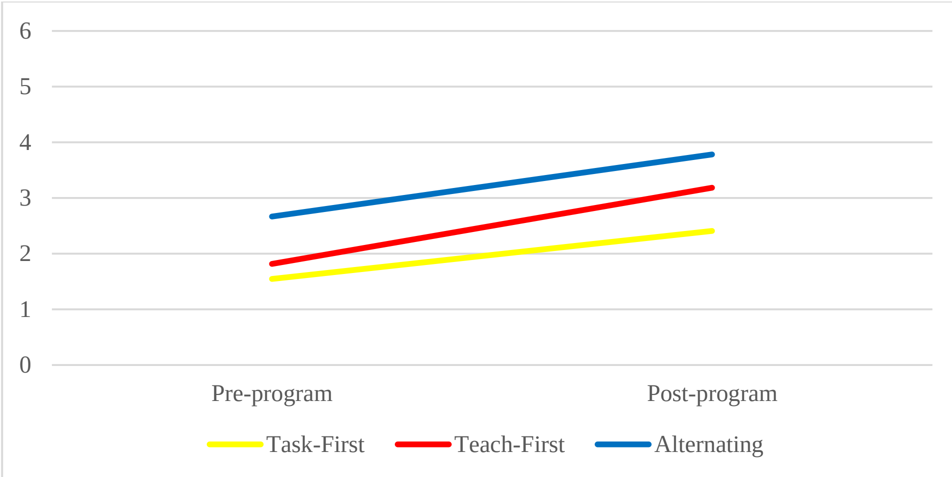

Figure 5.4. Mean Scores for Problem Solving Performance by Time and Lesson Structure (Addition Unit)

Summary

Taking both the above analyses together, it can be concluded that, across both units of works, there was no evidence that lesson structure resulted in differential improvement in students' problem solving performance. By contrast, it was apparent that participation in the program overall had a large impact on problem solving performance.

Task-First vs Teach-First: Comparing Intrinsic Motivation to Learn Mathematics

A Mixed Design ANOVA was performed on intrinsic motivation to learn mathematics scores for each unit of work. The within group factor was time (i.e., pre-program, post-program) and the between group factor was lesson structure (i.e., task-first, teach-first, alternating). Results of the evaluation of the assumptions of independence, linearity, homogeneity of variance, sphericity and homogeneity of intercorrelations were satisfactory for both analyses. However, given the fact that all four assessments of intrinsic motivation to learn mathematics were substantially negatively skewed, indicating a ceiling effect, an attempt was made to transform the data. Data were reflected, and a logarithmic transformation was undertaken, after which the intrinsic motivation to learn mathematic scores appeared to approximate a normal distribution.

Patterning unit

With regards to the patterning unit, the interaction effect between time and intrinsic motivation was non-significant, with $F(2, 70) = 1.508$, $p > 0.05$. This suggests that allocation to a particular lesson structure did not impact on students' intrinsic motivation to learn mathematics within this unit of work. By contrast, the main effect for time was significant and large, with $F(1, 70) = 14.016$, $p < 0.05$, partial $\eta^2 = 0.167$. This suggests that, irrespective of how the lesson was structured, participation in the research program had a positive impact on student intrinsic motivation to learn mathematics across the patterning unit. Means and standard deviations are presented in Table 5.9, whilst Figure 5.5 provides a visual representation of mean scores.

Table 5.9

Means and Standard Deviations for Intrinsic Motivation to Learn Mathematics by Time and Lesson Type: Patterning Unit

Time	Lesson Type	N	Mean	SD
Pre-program	All	73	.48	.27
	Task-First (Class A)	25	.51	.29
	Teach-First (Class B)	24	.44	.28
	Alternating (Class C)	24	.48	.27
Post-program	All	73	.35	.28
	Task-First (Class A)	25	.31	.26
	Teach-First (Class B)	24	.30	.28
	Alternating (Class C)	24	.44	.28

Note: Due to the data being reflected prior to the logarithmic transformation, a lower mean score indicates a higher level of intrinsic motivation to learn mathematics.

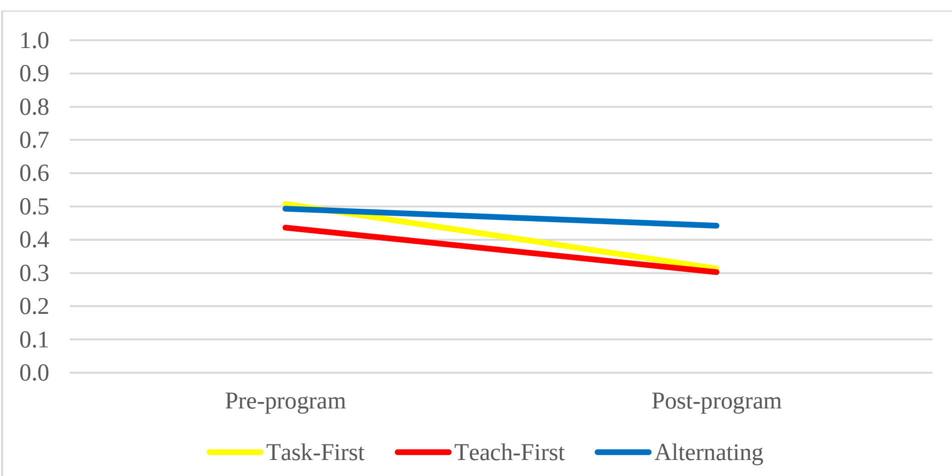

Figure 5.5. Mean Scores for Intrinsic Motivation to Learn Mathematics by Time and Lesson Type (Patterning Unit)

Addition unit

With regards to the addition unit, the interaction effect between time and intrinsic motivation was non-significant, with $F(2, 69) = 0.341$, $p > 0.05$. This suggests that allocation to a particular lesson structure did not impact on student intrinsic motivation to learn mathematics within this unit of work. Moreover, the main effect for time was non-significant, with $F(1, 69) = 0.925$, $p > 0.05$. This suggests that participation in the research program did not impact on student intrinsic motivation to learn mathematics across the addition unit. Means and standard deviations are presented in Table 5.10, whilst Figure 5.6 provides a visual representation of mean scores.

Table 5.10

Means and Standard Deviations for Intrinsic Motivation to Learn Mathematics by Time and Lesson Type: Addition Unit

Time	Lesson Type	N	Mean	SD
Pre-program	All	72	.36	.28
	Task-First (Class B)	25	.30	.29
	Teach-First (Class A)	24	.34	.28
	Alternating (Class C)	23	.45	.24
Post-program	All	72	.40	.33
	Task-First (Class B)	25	.36	.31
	Teach-First (Class A)	24	.34	.33
	Alternating (Class C)	23	.52	.32

Note: Due to the data being reflected prior to the logarithmic transformation, a lower mean score indicates a higher level of intrinsic motivation to learn mathematics.

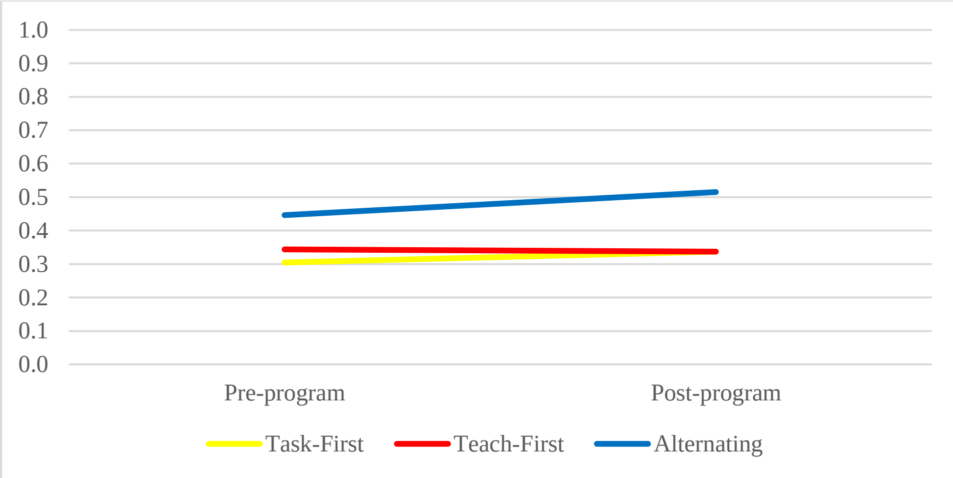

Figure 5.6. Mean Scores for Intrinsic Motivation to Learn Mathematics by Time and Lesson Type (Addition Unit)

Summary

Overall, there was no evidence to suggest that a particular lesson structure leads to higher (or lower) levels of intrinsic motivation to learn mathematics. Although overall intrinsic motivation to learn mathematics for program participants improved during the patterning unit, there was no change in intrinsic motivation across the addition unit.

Task-First vs Teach-First: Comparing Persistence

A Mixed Design ANOVA was performed on persistence scores for each unit of work. The within group factor was time (i.e., pre-program, post-program) and the between group factor was lesson structure (i.e., task-first, teach-first, alternating). Results of the evaluation of the assumptions of normality, independence, linearity, homogeneity of variance, sphericity and homogeneity of intercorrelations were satisfactory for both analyses.

Patterning unit

With regards to the patterning unit, the interaction effect between time and student persistence was non-significant, with $F(2, 47) = 0.535$, $p > 0.05$. This suggests that allocation to a particular lesson structure did not impact on student persistence scores within the patterning unit. By contrast, the main effect for time was significant and large, with $F(1, 47) = 12.002$, $p < 0.05$, partial $\eta^2 = 0.203$. This suggests that, irrespective of how the lesson was structured, participation in the research program had a positive impact on student persistence across the patterning unit. Means and standard deviations are presented in Table 5.11, whilst Figure 5.7 provides a visual representation of mean scores.

Table 5.11

Means and Standard Deviations for Persistence by Time and Lesson Type: Patterning Unit

Time	Lesson Type	N	Mean	SD
Pre-program	All	50	30.28	22.52
	Task-First (Class A)	18	22.56	20.16
	Teach-First (Class B)	15	35.13	26.47
	Alternating (Class C)	17	34.18	20.06
Post-program	All	50	41.66	21.19
	Task-First (Class A)	18	37.89	20.92
	Teach-First (Class B)	15	46.40	17.61
	Alternating (Class C)	17	41.47	24.53

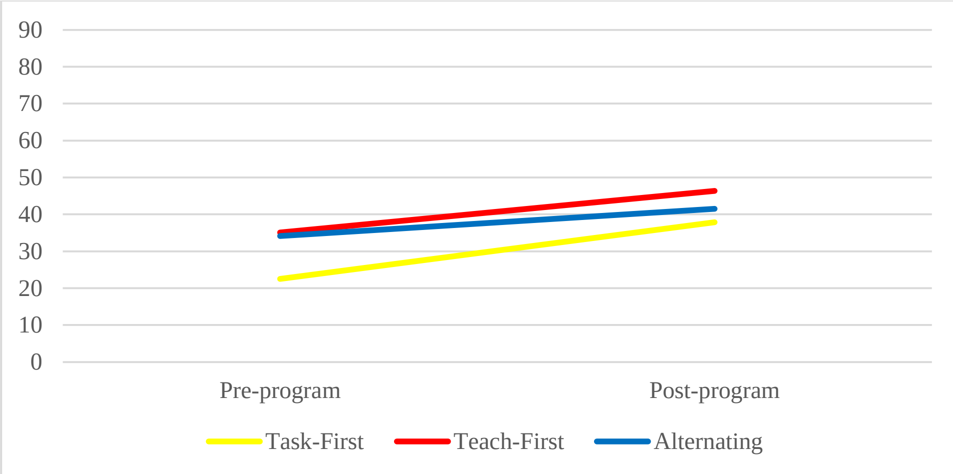

Figure 5.7. Mean Scores for Persistence by Time and Lesson Type (Patterning Unit)

Addition unit

With regards to the addition unit, the interaction effect between time and persistence was non-significant, with $F(2, 54) = 1.510$, $p > 0.05$. This suggests that allocation to a particular lesson structure did not impact on student persistence scores within this unit of work. By contrast, the main effect for time was significant and large, with $F(1, 54) = 24.529$, $p < 0.05$, partial $\eta^2 = 0.312$. This implies that, in juxtaposition to the patterning unit, participation in the research program overall had a negative impact on student persistence scores across the addition unit. Again, means and standard deviations are presented in Table 5.12, and Figure 5.8 provides a visual representation of mean scores.

Table 5.12

Means and Standard Deviations for Persistence by Time and Lesson Type: Addition Unit.

Time	Lesson Type	N	Mean	SD
Pre-program	All	57	57.21	32.51
	Task-First (Class B)	18	52.67	27.33
	Teach-First (Class A)	21	59.76	34.55
	Alternating (Class C)	18	58.78	36.02
Post-program	All	57	37.86	29.83
	Task-First (Class B)	18	43.11	34.81
	Teach-First (Class A)	21	36.76	33.61
	Alternating (Class C)	18	33.89	18.52

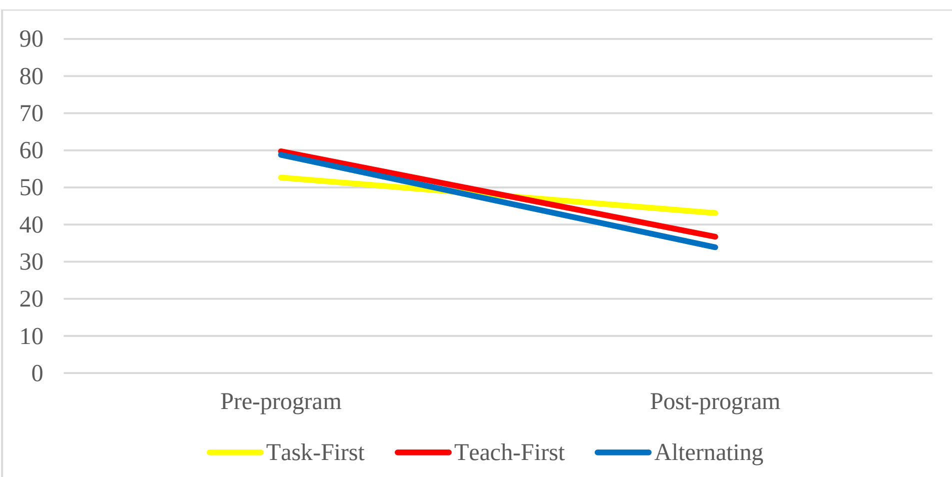

Figure 5.8. Mean Scores for Persistence by Time and Lesson Type (Addition Unit)

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Summary

There was no evidence that student persistence levels differed by lesson structure in either the patterning unit or the addition unit. Furthermore, although persistence scores increased during the patterning unit, they actually decreased during the addition unit, making the overall impact of participation in the program on persistence scores ambivalent.

In addition to the primary analysis outlined above, in order to assess whether multiplying the two dimensions of persistence was suppressing important relationships between the independent variables and the individual dimensions of persistence, intent to continue and cognitive demand were also analysed separately. Regarding intent to continue, the pattern of results was very similar to the original persistence analysis outlined previously. Specifically, the lesson structure interaction was non-significant, while persistence increased during the patterning unit and decreased during the addition unit. Regarding cognitive demand, the lesson structure interaction was again not significant; however, in contrast to intent to continue, the level of cognitive demand decreased across both units of work.

This latter observation regarding cognitive demand is consistent with what one would expect. Specifically, given that the post-program assessments contained similar problem solving tasks to the pre-program assessments, both in terms of structure and complexity, the post-program assessments should be perceived by students as less cognitively demanding. This is because students may have become more familiar with such problems, having been repeatedly exposed to similar problems whilst completing the respective units of work.

Discussion of Findings (Study One)

Intrinsic motivation to learn mathematics and persistence: Measurement issues

Intrinsic motivation to learn mathematics increased significantly during the patterning unit of work, suggesting that participation in the program impacted positively on this dimension, at least initially. Beyond this finding, it is difficult to draw any other definitive conclusions relating to intrinsic motivation to learn mathematics in the current study. However, rather than concluding that the construct has little relevance to the study, the lack of notable findings may be driven by limitations with the way the construct was operationalised.

Specifically, the intrinsic motivation to learn mathematics measure was consistently negatively skewed, indicating a ceiling effect. This ceiling effect was driven by study participants' overwhelmingly positive views towards mathematics, particularly in comparison to the older student participants in the PISA study, for whom the measure was originally designed. For example, in the 2012 PISA study (see Thomson et al., 2014), only 36% of 15 year old students in the OECD and 45% in Australia agreed or strongly agreed with the statement "I look forward to my mathematics lessons". The equivalent percentage for current study participants, *prior to* the study intervention (i.e., during the patterning unit pre-program assessment), was 92%. Given this alarming difference, further research examining the relationship between changes in intrinsic motivation to learn mathematics as students move through their primary schooling may be warranted. The very high levels of intrinsic motivation reported by participants in the current study (i.e., the aforementioned ceiling effect) may have undermined the capacity to detect differences between groups, and therefore to address the associated research question. This potential ramification of the ceiling effect has been noted elsewhere (see, for example, Albanese, 2000).

In contrast to the contention that the Task-First Approach may build student persistence relative to a Teach-First Approach, there was no evidence that persistence was related to the manner in which the lesson was structured. Moreover, the relationship between persistence and overall participation in the program was inconsistent; despite increasing substantially during the patterning unit, persistence decreased substantially during the addition unit. This appears to challenge the assumption that tackling a challenging task may improve student persistence and help to build a mastery orientation (Sullivan et al., 2013; Sullivan et al., 2014).

However, it needs to be noted that the task-based measure of persistence employed was challenging to administer to students of this age group. It required students to complete a questionnaire *after* they were already substantially fatigued from having worked on the associated problem solving task. Moreover, the design of the questionnaire itself was cognitively taxing on students, despite attempts to make it less so through changing the scale associated with the measure during the addition unit. Each of the six items associated with the persistence measure had to be read aloud to the students as a group, and ensuring students accurately completed the questionnaire was in itself very time consuming. Consequently, it is possible that some students (despite instructions to the contrary) answered the persistence items at least partly in relation to the experience of completing the questionnaire, rather than of working on the problem solving task, potentially undermining the validity of the measure.

Mathematical performance: Mixed findings

While there is considerable evidence that reformist-based pedagogical approaches are associated with higher levels of mathematical performance (e.g., Jong et al., 2010), prior research in this area has not sufficiently unpacked reformist pedagogy and examined its component parts to see whether all these aspects are necessary for generating superior performance. The current study found that when teaching with challenging tasks, it was not necessary to adopt a task-first lesson structure to achieve substantial learning gains in mathematical performance. On the contrary, there was some evidence to suggest that a teach-first structure was particularly important for fluency performance. Specifically, the Teach-First Approach resulted in greater gains in mathematical fluency performance than the Task-First Approach in the addition unit but not the patterning unit. Before discussing the implications of these findings, it is necessary to first try to explain why lesson structure was found to impact fluency but not problem solving outcomes, and student performance with addition but not patterning.

Problem solving and patterning as (relatively) ill-structured domains

Spiro and DeSchryver (2009) have argued that more explicit approaches to teaching may be more effective for learning in well-structured domains, and that inquiry-based methods more appropriate for learning in ill-structured domains. Although this position is contentious (e.g., Clark, 2009), it may serve to explain the differential findings in the current study. Perhaps most obviously, the skills and knowledge that facilitate fluency performance are almost by definition more clearly structured than the equivalent skills and knowledge that facilitate problem solving performance. Whilst developing mathematical fluency involves acquiring and applying algorithmic-type knowledge, problem solving ability necessarily involves contexts where the individual is assumed *not to know* how to solve the problem a priori. Consequently if we adopt the position of Spiro and DeSchryver (2009), it is perhaps not surprising that whilst the Teach-First Approach resulted in greater relative improvements in fluency performance, the Task-First Approach was equally effective when it came to problem solving performance.

A similar argument can be made in relation to the units of work themselves; that is, that the addition unit more closely resembled a well-structured domain and the patterning unit an ill-structured domain, and therefore, according to Spiro and DeSchryver (2009), that a more explicit teaching approach is likely to be more effective in the former but not the latter. For the addition unit, the learning objectives generally revolved around exposing students to, and encouraging them to apply, efficient partitioning and (re)combination strategies when confronted with particular problems.

For example, during the addition unit fluency assessments, students were confronted with the number sentence 19+19+19+3. Through drawing on the strategies they were exposed to during the unit, they would be expected to assess the problem and recognize that nine is one away from ten. Then, through partitioning the last term, three, into three ones and distributing these ones onto the nines, the student could effectively transform the number sentence into 20+20+20, and solve this less challenging problem. Although this is undoubtedly a complex learning sequence for many seven and eight year old students, it is also very clearly defined. The addition unit also had the benefit of students having access to clearly defined back-up strategies (e.g., adding numbers left to right and using place value partitioning, counting-on from the larger number, and counting-on from left to right), which, although not always efficient, theoretically always led to a correct answer.

By contrast, in the patterning unit, students were generally required to identify and continue number patterns, some of which involved non-arithmetic sequences. The complex and varied nature of the sequences meant that students were encouraged to decipher the pattern through drawing on a range of strategies (e.g., visually creating the pattern on a hundreds chart, counting how many numbers there were between each of the terms in the sequence, and using blocks to physically create and model the pattern). However, in clear contrast to the addition unit, these strategies were often exploratory and did not definitively lead the student to the solution.

For example, as part of the patterning unit fluency assessment, students were asked to find the next term in this series of numbers: 5, 6, 8, 11, 15, 20. To work out that the skip-counting pattern is increasing by one each time and that the next term is 26, the student had to consider multiple terms in the sequence, and their relationship to adjacent terms. Many students instead noticed the first term in the sequence, and the last two terms in the sequence, and assumed that the next term was 25, thinking that the pattern involved counting by fives. Notions such as distinguishing between arithmetic and non-arithmetic sequences are not only complex but effectively rely on navigating knowledge of imperfect heuristics, rather than clearly defined algorithms. Moreover, unlike with addition, there is an absence of reliable (albeit less efficient) back-up strategies, which will always lead to a correct answer. Thus, in many ways, even the fluency aspects of the patterning unit more closely approximated an ill-structured domain, which may explain why the Task-First

Approach was as effective as the Teach-First Approach in improving fluency performance for this unit of work.

Examining the superior mathematical fluency associated with the Teach-First Approach

The finding that the Teach-First Approach was somewhat more effective than the Task-First Approach when it came to improving mathematical fluency, although consistent with the position of some cognitive load theorists very critical of discovery-based approaches to instruction (e.g., Mayer 2004; Sweller et al., 2007), seems to contradict the results of Alfieri et al.'s (2011) meta-analysis. Alfieri et al. concluded that, whilst more explicit instructional approaches were superior to unassisted discovery-based learning, enhanced discovery-based learning was generally superior to all other teaching approaches considered. As the Task-First Approach was constructed, at least ostensibly, as an iteration of enhanced discovery-learning (i.e., it involved feedback, elicited explanations during the discussion component of the lesson, and provided enabling prompts to scaffold student thinking) it might have been expected to result in superior learning outcomes compared with the Teach-First Approach. One possible explanation for this somewhat surprising result is that the Task-First Approach may have actually more closely resembled unassisted discovery-learning than enhanced discovery-learning in the current study, at least for some students some of the time.

Task-First Approach: Enhanced or unassisted discovery?

As articulated previously, there are at least three reasons why it can be contended that the Task-First Approach would be experienced by participants as enhanced, rather than unassisted, discovery: opportunities for feedback whilst working on the task, elicited explanations during the discussion component of the lesson, and, perhaps most importantly, use of enabling prompts.

One of the key scaffolds built into the learning experience when accessing a challenging task are enabling prompts (Sullivan, Mousley, & Zevenbergen, 2006). As outlined in Chapter Three when discussing the CLASS Challenging Task design process, the information contained in the enabling prompts used in the current study was intended to reduce the level of cognitive demand (i.e., intrinsic cognitive load) inherent in the associated tasks. This was generally achieved through removing a step in the main task (e.g., representing a worded problem as a number sentence), providing students with an easier problem intended to serve as a gateway to the main task, and connecting the task to relevant prior learning (e.g., linking the bridging through 10 strategy to knowledge of rainbow facts). Importantly, in line with the literature, it was generally expected that students accessing enabling prompts did so at their own initiative, rather than being directed to by the researcher. Although at times the researcher would suggest to a student that they consider getting the ‘hint sheet’ (the student-friendly term adopted in the current study to refer to enabling prompts), it was regularly emphasised prior to the lesson that students themselves were in the best position to determine if and when they needed support.

However, it has been argued that such adaptive, learner-controlled, instructional environments may be less effective when used with novice learners, who may struggle to effectively embrace the greater control they are provided with (Kalyuga, Renkl, & Pass, 2010). An alternative way of framing this same problem is that young children, such as the participants in the current study, may lack the meta-cognitive skills to effectively self-regulate their learning (Flavell, 2000; Kuhn, Black, Keselman, & Kaplan, 2000). Consequently, it may have been that some participants either did not access the enabling prompt when it would have been of benefit to their learning, waited too long into the lesson to access the prompt, or could not make sense of the prompt once they accessed it.

In addition to these potential issues around accessing and effectively using enabling prompts, there may have been limitations to the other built-in enhanced discovery components. Although the majority of students were provided with feedback during the sessions and many given opportunities to elucidate their reasoning during subsequent class discussions, these activities occurred within the natural flow of the lesson and no attempt was made to systematically ensure that all students received such opportunities in every lesson. Taken together, these factors may have resulted in some students unintentionally experiencing a given task-first lesson almost exclusively as unassisted discovery-based learning, rather than enhanced discovery-based learning.

However, given that the Task-First Approach still resulted in large improvements in both measures of mathematical performance across both units of work, it is important to not over-emphasize problems or limitations with this teaching approach. Indeed, based on the findings of the current study, it cannot be concluded, as some of its more severe critiques have done (e.g., Mayer, 2004), that inquiry-based approaches, even if they are experienced by some participants as ‘unassisted discovery-based learning’, are not effective in improving performance.

Teach-First Approach: Best of both worlds?

Consequently, an alternative (but compatible) account for explaining the current study findings would be to (rather than highlight the limitations of the Task-First

Approach) focus on the strengths of the Teach-First Approach. In fact, it could be argued that Teach-First Approach in the current study was as an exemplary model of mathematics instruction. For example, the 'mini-lesson' which began the lesson embraced a student-centred stance, through maximizing active student participation and encouraging students to reconstruct teacher explanations in their own language. Moreover, the 'routine tasks' students were confronted with to reinforce their understanding of the mini-lesson were carefully scaffolded to become progressively more challenging, with the challenging task itself then becoming a natural, more cognitively demanding extension of the previously worked-through material. Importantly, therefore, the Teach-First Approach still endeavoured to adopt the other two pillars of a reform-oriented approach to mathematics instruction that have been outlined elsewhere; a student-centred approach to instruction and the inclusion of more cognitively demanding tasks (Sherin, 2002). In a sense, the Teach-First Approach empowered students with the requisite mathematical knowledge to engage deeply with the material, whilst still providing them with the opportunity to pursue a meaningful mathematical inquiry.

Concluding thoughts

The findings of the current study support the efficacy of teaching with challenging tasks, but call into question the assumption that it is necessary to begin a lesson with a cognitively demanding task in order for the associated learning benefits of using challenging tasks to be fully realized. Taken as a whole, participation in the program generated large improvements in both fluency and problem-solving performance across both units of work. Moreover, effect sizes across the time dimension were more consistent and far more substantial in magnitude than any between-group differences attributable to lesson structure.

It is a commonly argued that teachers, at least in countries such as Australia and the US, need to utilize more cognitively demanding tasks in classrooms to maximize learning opportunities for students; however, that many teachers continue to prefer to use more routine or procedurally-focused tasks, at the expense of deep problem solving and conceptual understanding (e.g., Hollingsworth et al., 2003; Stein et al., 2008). Reluctance to teach with cognitively demanding tasks may be exacerbated if teachers are faced with the expectation that they begin the lesson with a challenging task, allowing children to work on it with only minimal guidance before discussing student-generated approaches and solutions as a class. Lessons based on a launch-explore-discuss structure, which characterize more typical reform-orientated lessons that make use of cognitively demanding tasks, are complex and require a substantial amount of skill to successfully orchestrate (Stein et al., 2008; Ridlon, 2009; Sullivan, Clarke, Clarke & O'Shea, 2010; Thomas & Monroe, 2006). It is possible that reluctant teachers may be more comfortable, and gain confidence with incorporating cognitively demanding tasks into their practice, if these tasks were instead used to extend student thinking towards the end of a lesson. Importantly, findings from Study One suggest that adopting this alternative lesson structure would not undermine the capacity of such tasks to support improvements in student mathematical performance. Study Two will examine whether the above assertion that classroom teachers may be more willing to teach with challenging tasks if allowed to pursue either a Task-First Approach or a Teach-First Approach is in fact supported by evidence. In addition, it will also evaluate some of the strengths and limitations associated with these respective approaches to mathematics instruction from the perspective of the classroom teachers observing the program.

CHAPTER SIX: STUDY TWO – TEACHER PERSPECTIVES[13]

This chapter encompasses the analysis of the six (three interviews by two units of work) semi-structured interviews with teacher-participants. As noted in the methodology section, this qualitative interview data will be reconstructed through the lens of interpretative phenomenological analysis. The presentation of this chapter has been organised around the corresponding research questions put forward in the introduction:

1. How did teachers perceive students' response to lessons involving challenging tasks? What factors did they attribute to these responses? (i.e., Perceptions of Student Reactions)
2. What factors seem likely to influence teachers' willingness to teach with challenging tasks in the future (e.g., perceived student reactions, teacher confidence and competence, time constraints)? (i.e., Future Teaching Intentions)
3. What differences did teachers perceive between a Task-First Approach when observing teaching with challenging tasks and a Teach-First Approach? When it came to considering their own teaching practice, did teachers have a preference for one particular type of approach? (i.e., Observations about Lesson Structure)

The sub-headings under each section reflect the specific themes as they emerged from the synthesis of the teacher-participant narratives. These themes have been presented as study findings. Specific extracts from interviews with teacher-participants have been included under the summary of the relevant theme, in order to provide an illustration of the

[13] Findings from this chapter has been outlined in a peer-reviewed journal article accepted for publication: Russo, J., & Hopkins, S. (Accepted). How does lesson structure shape teacher perceptions of teaching with challenging tasks?. *Mathematics Teacher Education and Development.*

corresponding analysis. The second part of the chapter is devoted to discussing the implications of these findings.

Perceptions of Student Reactions

In general, teacher-participants noted students reacted positively to lessons involving challenging tasks. Specifically, three themes emerged from the analysis of teacher interviews. First, teachers perceived students to embrace the struggle and persist with challenging tasks, even when students were failing to make significant progress. Second, it appeared that the program supported a classroom culture which embraced struggle, which teachers attributed to high expectations, a high level of student autonomy, and consistent routines. Third, teachers perceived that teaching with challenging tasks may have made learning mathematics more purposeful, which in turn led to higher levels of student engagement. Each of these themes is elaborated on below.

Teachers perceived students to embrace struggle and persist with challenge

None of the three teachers identified lack of student persistence as a barrier to teaching with challenging tasks in the future, nor did they identify lack of persistence of students as a shortcoming of the current program. On the contrary, Polly marvelled at students' willingness to keep working on tasks, even in instances when they were failing to make significant progress or it was not clear if they fully understood the task.

> They keep trying. They never give up… And I think that was one of the great things – that they kept trying. And, in fact, in the beginning, they did give up, because they didn't know what to do. But by the end they were trying and trying and trying. And that was a really great thing that I think

> the program developed. This whole thing of persistence that… you may not get something right straight away – that you might not solve something straight away. And you keep on going regardless of that… Maybe in our normal maths program we give them so much, what do you call it – scaffolding – that they go back, they complete their task, it is easy for them, they are successful with that, so that when something hard comes along, they think "Oh, can't do that". So this is definitely something that drew out of them - the ability to just keep thinking and trying. *Polly, Addition Unit.*

Rachel concurred with Polly's perspective, again emphasising how a persistent disposition took time to develop, as students adjusted to tackling more challenging mathematical tasks.

> To start off with, I was a bit nervous because as I said, some of my kids would sit there and not do anything, but as time went on, this changed... I guess when a lot of them get the challenge, they're thinking, "Hmmm… what is this?" Yet they're still, you know, going off to their tables very quickly and starting. *Patterning Unit, Rachel*.

Sally highlighted how one struggling student's continued application and persistence assisted him in developing greater self-efficacy as a mathematician.

> I feel like the last six weeks with you has changed his attitude about maths. I feel like it empowered him, that he became capable of thinking "I can do this". And he didn't come in here like that. He didn't come in here with that attitude. And I think that there were other kids like him who just… who really grew. Sally, *Patterning Unit.*

Teachers perceived that a classroom culture which embraced struggle was developed through high teacher expectations, student autonomy and a consistent routine

Teachers discussed the interrelated themes of high expectations, student autonomy and consistency in routine as critical to student growth through the program. During both post-program interviews, Polly emphasized how high teacher expectations supported the development of greater student autonomy in their role as learners.

> I think the fact that there was an expectation that they were going to use their brains, and they were going to be stretched… And I think that is what threw them at first, because they were so completely disoriented and confused by the fact that “oh, you mean we can actually think for ourselves”. But once they kind of cottoned-on to the fact that “ok this is my responsibility”, I think that was a little bit of a process for them… I think that is probably one of the good things about the program – that it was starting to make them feel responsible for their own learning. If they were going to get somewhere, then they would work it out... Even if they didn’t maybe come to the right conclusions, they were still encouraged to use their brains in ways they hadn’t done before and I just think that bit by bit, that encouraged them to keep going. *Polly, Patterning Unit.*

> Because normally what we would have taught is very much step-by-step “go and do this” and concrete materials, and a lot of all that sort of thing... Oh yes, they did have their abacus, but they had to know how to use it. It was much more dependent on them being able to apply their knowledge… But I think they made progress with that, most of them. Your real weak ones struggled with it, but most of them made a lot of progress. *Polly, Addition Unit.*

As Rachel and Sally highlight, these high expectations in relation to how students approached the lessons were supported by strong classroom routines.

> I have quite a few students who, they lack confidence, and they don't want to do anything without extra assistance from me. And I noticed quite a few of them changed as the time went on; they were excited about going off and starting the challenge. They started to get the tools to be able to tackle the challenge without asking for help… I think to start off with, and you probably would have noticed too, a lot of them didn't use the hint sheet. And they would go "oh I don't know what to do". But they hadn't taken the hint sheet. And I think as time went on they started to realise… "oh, hang on, there is a hint sheet there. I can go off and use that"… I think the routine of coming in and realising that they should get a hundred chart to start them off. But also as they became more confident, it was a belief that "I can at least start this. I know what to do". *Patterning Unit, Rachel.*

> I think that the consistency in the way the lessons were approached - even though we were the group that swapped and changed - always having that same structure – that there will be a worksheet, that there will be a challenge, that the things that you need will be in this location. That they had to use the same resources each week to help them, that the hint sheet was in the same place each week. I think some of that helped them. But also that there was a consistent expectation that they get a hundred charts, that they get a beads frame, that you put those things out for them every time. So there was no doubt that they were meant to be using those things. Maybe that got it into their heads that actually, that is what good mathematicians do. *Sally, Patterning Unit*.

As all three teachers highlighted, an important part of fostering student autonomy is empowering students with the necessary resources to help themselves. Sally highlighted how this tendency had apparently been internalized by students and transferred to other learning situations.

> It is interesting. I gave mine a maths test yesterday. And you know when you do a maths test, you often, not that it is something that I do often, but when I do do it – I will put out resources and kids won't use them. You know you will put out beads frames, you will put out hundred charts, you will put out counters and you will say "Use these?". Well I would say that 70% of my grade yesterday had hundred charts and beads frames, and they were using them effectively, which I thought was really exciting. If nothing else, I thought that was a big change – that they saw those resources as useful and they knew how to use them. That was the thing that I saw that really changed. *Sally, Patterning Unit.*

Teachers perceived that learning was more purposeful for students, which was reflected by a high level of student engagement

In different ways, all three of the teachers noted how teaching with challenging tasks seemed to engage students, in part through situating the mathematical learning as purposeful and relevant.

For Sally, this was reflected in students grasping a greater purpose in computational knowledge.

> And I feel like some of my year ones took a bit more pride in learning their rainbow facts and knowing them. It was like beforehand they didn't really see the point and I felt like some of them were really starting to (see it)… I don't know. You teach them, you teach them, you teach them and you think they've got it, and they just don't seem to see the relevance. But I felt like throughout this, they began to see it. *Sally, Addition Unit.*

For Rachel, it meant students were highly motivated and focussed to solve the challenging task.

> The kids loved it. I don't know about the other grades, but my kids were so engaged. And I think having the challenge first. They came in, and they were ready – "What's our challenge today?". Yeah I think that it was a really good group of lessons. I think that the kids were so engaged that they were learning. That is why they went so far, because they were really interested and they were determined to take on that challenge. *Rachel, Patterning Unit.*

For Polly, it meant students were able to relate more to the mathematics, because the problems they were solving were contextualised.

> I think the little problems that you gave, the challenge, really got them in. I think they liked that… It wasn't just a straight worksheet of work, it was this little interesting scenario. It sort of appealed to their imagination and their child-likeness. So I think that was really good. It sort of drew them in. *Polly, Addition Unit.*

Future Teaching Intentions

Despite concurring that students reacted positively to the program, there was substantial variability in teacher-participants' enthusiasm to teach with challenging tasks in the future, due to their different perspectives as to who such tasks are suitable for and their confidence teaching with such tasks. In particular, whether challenging tasks can be used to appropriately differentiate instruction for all students or whether they should be reserved for 'high performers' was a point of contention. In addition, a substantial obstacle to teaching with challenging tasks, even amongst teachers more confident in their mathematical knowledge, was the time and expertise involved in planning such a unit of work. Finally, the time-intensive nature of using challenging tasks in instruction was also of concern, particularly following the patterning unit of work. Each of these issues is considered in more detail below.

Teachers differed in their belief about whether challenging tasks can be used to appropriately differentiate instruction

One of the issues which the three teacher-participants differed most markedly on was whether challenging tasks can be used to effectively teach all students in the early years of schooling.

Sally was enthusiastic about using challenging tasks to differentiate instruction.

> I really like the concept. Because I think one of the biggest challenges in a classroom environment is differentiation… And what I really liked about this is that you didn't need to group the kids. All kids had something. All kids could find an in at some level in every lesson. But also that there was massive room for growth and extension in all areas. Rarely did a kid finish everything. In fact, I don't think we had a lesson where a kid was sitting there twiddling their thumbs. And I think that that was fantastic. So I really liked it from that perspective. *Sally, Patterning Unit.*

In fact, by the end of the second unit of work, it was clear that Sally had internalized the challenging task model as being central to her teaching philosophy.

> A really good open-ended challenging task is I think one of the purest and simplest and best ways of differentiating an activity. And I think more and

> more, the more I go through my teaching career, the more I think that when we group kids, particularly in maths, when we have say four small groups, and they're all doing different things; I think that is about baby-sitting the other three groups while you work with one. And so the quality time you get with your kids is really one lesson a week. And I think that when you use an open-ended task and you use it well, it gives you an opportunity to get a bigger range and a bigger gauge of where your kids are at. *Sally, Addition Unit.*

Rachel, by contrast, was somewhat ambivalent about whether a single challenging task could effectively meet the learning needs of a heterogeneous classroom of students.

> I think it has been really beneficial for the upper kids. That extension sometimes doesn't get to them. I think the thing I would change is, towards the end, as it got more difficult, I think some kids just fell away. And it is probably a waste of their time at that point, when they could be doing something that is more at their level. I think that is how I felt towards the end. That is was still really good for the high achievers, they were still getting something out of it, but those middle and lower, I think it was just way past them. *Rachel, Patterning Unit.*

> I think it would be worthwhile for certain groups of students (to develop future units of work built around challenging tasks). I don't know how you feel about it, but some of them who have that mathematical thinking, I think they find it really challenging and it is good for them to broaden their thinking about maths. Whereas I think others are kind of struggling a bit as it is, and challenging tasks are perhaps a little overwhelming for them… But then I guess it is also good for them to start thinking that way, and getting the practice to start thinking that way – that it is not just a number sentence

> every time. It is worded problems, and its different sorts of tasks. *Rachel, Addition Unit*

Of the three teachers, Polly was the most concerned that challenging tasks were mainly only appropriate for high-achieving students. This clearly impacted her willingness to consider teaching with challenging tasks in the future.

> You've got that tension again between the kids you are wanting to fly with it, and the kids in the middle who are starting to get it – and you want to give them the opportunity to sort of grasp it – they're on the verge of it all. And then you've got the kids who have got no idea, and you think "is this good for them… is it wasting their time?" Should it be something that has a specialist area where your high flyers can go off and… Yes it is going to be great for those kids that are capable of deep thinking. But there are some kids who are never going to be capable of deep thinking. And if they can just get the basics, and they can write a number the right way, they can know that five is five no matter if it is there or there or what it looks like you would be happy with that. So I guess your expectations of different kids are there. And I think the more I even speak about it now, the more I really think it should just be for those kids that are more highly able… *Polly, Patterning Unit*

Indeed, it may be that at least some of the difference in opinion between Polly and Sally reflects their different understanding as to what might constitute a 'successful' learning experience for some of the lower-achieving students. Sally clearly did not expect that all her students would be able to solve the challenging task.

> I just think that is something that has really changed – they can find 60 on a hundred's chart. They might not be able to manipulate that number, they

> might not necessarily know how many groups of tens that is, but I feel like that is something that has really changed with my bottom ended kids is that can access information more easily. They can find what they need. They particularly can find their way around a hundred chart, which I actually think is something that would be easy to discount and say "big deal". But I think it is huge. I think it is big. *Sally, Patterning Unit.*

In juxtaposition, consider Polly's own opinion about how much the only student (Kate) with a diagnosed (mild) intellectual disability gained from participating in the second unit of work, which, in fact, involved experiencing the lesson with the challenging task first. One can imagine that had Sally observed Kate's personal and mathematical growth in her classroom, she would have interpreted this as a strong endorsement for using challenging tasks to differentiate instruction. However, Polly clearly remained very focussed on whether students were making progress with the tasks themselves and using this to gauge whether the unit of work was appropriate or not, rather than focussing more on the relative growth in a given student's level of mathematical understanding.

> Students were certainly better able to show initiative. Even the likes of Kate, on the days that she didn't have Jo (her aide) – and I didn't always work one-on-one with her. She was able to use some strategies – you know it was quite amazing really – and draw on some number facts that she knew to try and work things out. So I think there was definitely progress there in all of it… But I still think, it is a pretty big ask, what we are asking them to do, little Grade oners and twoers. And we were asking them to do some pretty challenging tasks. And they certainly made progress. It is not like they arrived at the end of this, and they were completely confident using everything, and you could say "ok, they started with nothing, now they have got it under their belt" – you can't say that… Which makes me wonder, how

> much of it is developmental that they are not actually able to because of their age, to be able to get to that point. That is looking at the average child – not looking at your high achievers. *Polly, Addition Unit.* [14]

Teacher confidence to teach with challenging tasks differed, however none of the teachers were confident planning challenging tasks.

The most experienced teacher interviewed, Polly, was quite upfront in acknowledging that her lack of a deep, interconnected understanding of mathematical knowledge was a barrier to her incorporating challenging tasks in her classroom in the future.

> I wonder too if teaching something like that requires someone who has the passion and the knowledge of where they are going a lot more than what I do. Maybe I would be able to replicate what you have done, but if I strike problems or, just the unknown that can happen in the lesson, I might feel a little bit like I am floundering because I might not have the knowledge of where I am wanting to go so much as what you do. You know I relate that to like me with music. I could maybe teach you how to teach a unit of work, but I know where I am going down the track because I have got all that knowledge. But if something happened in a lesson, you might think "Why has that happened? Is that important, or not important?" you know… All those sorts of questions. *Polly, Patterning Unit.*

Polly was particularly concerned about her capacity to plan a unit of work built around challenging tasks.

[14] Incidentally, post-hoc analysis of student outcome data revealed that, despite Polly's perception (and to a lesser extent Rachel's), the program did not appear to disproportionately benefit 'high flyers' (see Appendix O for further information).

> Nup, I just don't have the know-how (to plan a unit of work involving challenging tasks)… You know, if someone else planned it, and I had to administer it, I would be more inclined to do that. But no, there is no way in the world… I wouldn't know what to do... It would be like me saying to you "Ok, James, watch me take some music lessons. Ok, you feel equipped. Ok, now I want you to do… I have done some lessons on rhythm, but now I want you to do some lessons on melody. Ok. Ah… You think you know what to do?". *Polly, Addition Unit.*

Even after her re-consideration of the value of teaching with challenging tasks, as Polly began contemplating the next unit of work following the completion of the program, planning the units remained a significant obstacle.

> After looking at the textbooks, like the Nelson resource we usually teach from, and comparing it to the work we did with addition this term, it seems way too basic. It's not challenging enough. It makes me think I'd be willing to teach a unit of work that included more challenging tasks because it seems more interesting and stimulating for students... But I still have my reservations about planning it. I'd need help with that. *Polly, addendum to the addition unit interview.*

By contrast, Rachel, a first year teacher, tended not to refer to her own pedagogy, even when prompted to discuss if and how she would incorporate challenging tasks into her own teaching practice in the future. However, she did note that observing the program was beneficial for extending her pedagogical knowledge-base in mathematics instruction.

> I think it has been good for me to see how you are teaching some of those strategies, and that I can take some of those lessons on for my kids next year as well. *Rachel, Addition Unit.*

Finally, as is apparent in her discussion of using challenging tasks to differentiate instruction, Sally did not indicate any reservations regarding her capacity to competently teach with such tasks. Having said this, Sally's comments do indicate that she views the sourcing and sequencing of a high-quality unit of work built around open-ended tasks as a demanding undertaking for a generalist teacher such as herself.

> I think the thing that happens in teaching is you are expected to be a jack of all trades. But sometimes I feel like you are a jack of all trades but an expert in none. And what was really nice about both of those units of work for me was that I learnt things too… I felt like this broke things down nice and simply, and it gave some really tangible things to focus on, and really tangible strategies to hone in on. And it meant that I didn't have to sift through all of the possible options to work out what was most relevant, what is most helpful, what is going to take the kids from point A to point B. *Sally, Addition Unit.*

In addition, Sally indicated that, as comfortable as she was teaching with challenging tasks, she would likely require external assistance to plan a unit of work based around such tasks in the future.

> I can't see how I would have the capacity to organise a unit of work based around challenging tasks. But certainly, if it was something I was doing in conjunction with someone who knew what they were doing, I would definitely be interested. *Sally, Patterning Unit.*

Teachers perceived challenging tasks as very time intensive to teach, particularly within the patterning unit, and voiced concerns about the 'crowded curriculum'

Despite acknowledging the value of the program to students, the notion of spending twenty lessons teaching and assessing one important aspect of number and algebra (i.e., number patterning) was raised by all three teacher participants. As Rachel noted, teaching with challenging tasks to interrogate deeply one particular topic can mean forgoing opportunities to explore other areas within mathematics.

> Having five weeks out of this term has sort of cut out other topics that we needed to have covered. So that would be one barrier I guess, how long you would spend on that topic. Yeah I think that would be one of the biggest barriers. *Rachel, Patterning Unit.*

Polly contemplated how much discretion teachers have within a government school environment in determining how to allocate the scarce resource of time.

> I mean obviously if you are working at a deeper level you need more time. And even within that hour's maths lesson, you could see that there was frustration there for kids who really wanted to pursue it, there wasn't the time there either. That's life at the moment, isn't it? We're confined by the constraints of time in everything. But when you look at what we have to teach in the curriculum, that's the way it has been and that's the way it still is. And you do think "Well ok, we have done a week on this topic", and you can see that the kids haven't got it, but you have to move on. And do we have the flexibility working in a state school that is governed by an education department, do we have that flexibility to say what we are going to teach in the maths curriculum… what we are going to teach and what we are going to leave out? *Polly, Patterning Unit.*

Although Sally also voiced concerns about the time-intensive nature of the unit, she acknowledged that the interconnections and overlap between different curriculum areas requires a more nuanced evaluation of spending five weeks on a single topic.

> And I guess it is worthwhile, because when you're teaching division, when you're teaching multiplication, when you're teaching fractions, the stuff that you've covered will all connect. But my concern is how do we fit everything in – and will we fit everything in?... If I wasn't thinking about the structure of the school environment, and what I need to get through, then I would definitely teach this unit… because I think that the kids really grew. *Sally, Patterning Unit.*

Interestingly, this was not the case with the addition unit of work, with none of the teachers indicating that they were particularly concerned about the length of the unit. This seemed to be both a product of the unit of work being somewhat shorter (sixteen lessons) and addition being a topic viewed as being particularly valuable to students of this age. As Sally states:

> I just think kids like adding numbers. Addition is inherently interesting… Doing the three days with you and the two days on applied maths in the classroom, that actually worked quite well – I didn't have any complaints about that. I felt like I was ticking all the boxes I needed to tick. *Sally, Addition Unit.*

Observations about Lesson Structure

Teacher-participants perceived both the Task-First Approach and the Teach-First Approach to teaching with challenging tasks to have distinct strengths. The Task-First

Approach was viewed as engaging and empowering for students, providing an opportunity to build student persistence whilst fostering student mathematical creativity. Teachers also placed value on the quality of the mathematical discussion that emerged, and the value of the Task-First Approach for an authentic assessment of student mathematical knowledge. By contrast, the Teach-First Approach was viewed as highly focussed and an efficient approach to learning. It was also perceived as providing an opportunity for lower-achieving and less confident students to be successful. Each of these ideas is elaborated on below. The section finishes by considering the expressed opinions of the three study participants in terms of how they might teach with challenging tasks in the future.

Task-First Approach: Encourages creativity and empowerment

There seemed to be a consensus amongst the three teachers that beginning a lesson with a challenging task appears to foster more creative forms of mathematical thinking. Sally clearly placed considerable value on the freedom given to students in the Task-First Approach.

> You know sometimes we can stifle kids creativity, if we are teaching this strategy, and this way of thinking… When you are saying this is the challenge, go and tackle it, you are not stifling that thinking that they already have going on. So I loved that element of it. I think that was probably my favourite part of the lesson –hearing them explain their thinking. *Sally, Patterning Unit.*

Similarly, Rachel emphasises the importance of students being empowered to develop their own solutions to problems.

> And they had to think of it themselves. And I think that is a really important skill to learn – that you find your own way to solve it. I mean, I always found you get taught a certain way and you're thinking "that doesn't make any sense to me". Whereas if you had of had a different strategy, then it could have made a lot more sense… Yeah I think there was some pride. That they had achieved it. They all wanted to share their ideas. *Rachel, Patterning Unit.*

However, this greater creativity, empowerment and freedom was also accompanied by greater uncertainty and some discomfort, particularly when students were first exposed to the previously unfamiliar experience of beginning a lesson with a challenging task. As Rachel noted:

> I think to start off with, as I said, some of them didn't know what to do. And they are not used to that style of teaching. They are used to "We talk about this first, then we get an idea, and then we go off and do our work". *Rachel, Patterning Unit.*

By contrast, the perception was that beginning with the teacher-facilitated mini-lesson first directed students in a specific direction, and led to many students attempting to apply teacher-demonstrated strategies. Rachel emphasised how the different lesson structure completely changed the tone of the lesson, and the type of mathematical thinking demonstrated by students.

> Yeah. It was a big difference for me I think. The challenge first, you were getting to see how their brains ticked a lot of the time, because they were sent off and they had to find their own way to work out the challenge, based on previous lessons, and what they thought might be the best strategy to use.

> This time I guess I found it different because they were getting the lesson first, and they had to go off and do what was taught in the lesson, and apply that strategy. Some of them still did try and use a different strategy, even though they had the lesson first, but the majority of them were just using that strategy that you had taught to them in that lesson, which was a huge difference I think. *Rachel, Addition Unit.*

Task-First Approach: Engages students

Similarly, there also seemed to be a consensus amongst teachers that having the challenge first was an engaging way in which to begin the lesson. As Rachel and Sally outline many students seemed to eagerly await the challenge.

> I think there was a certain buzz when they came in and had to do the challenge first. I think I noticed that in my class. They were a bit excited: "I wonder what the challenge is going to be today?". *Rachel, Addition Unit.*

> I think having that challenge first seems to draw them in. *Rachel, Patterning Unit.*

> You know, being set a challenge each week and there was some kids like… You know Josh and Neo, and Liam… they were looking at the challenge when you put it up on the board and they wanted to be first, and they wanted to work it out. You know, they were doing that in their head. *Sally, Patterning Unit.*

Task-First Approach: Builds persistence

There was also a perception amongst teachers that the Task-First Approach helped to build student persistence. According to Rachel, this opportunity to persist was directly a result of the greater uncertainty students experienced with regards to how to proceed with the task. By contrast, because students had a clearer idea of what they were required to do under the Teach-First Approach, there was less of a need for students to demonstrate persistence.

> I think a lot of them showed a lot of persistence in the challenge first. Even though a lot of them, sometimes, didn't know what was going on, I think they showed persistence in wanting to work it out… And I guess it was different with the lesson first. I had a few that weren't catching on, but a lot of them were catching on to what was going on, so they were able to get the work done. *Rachel, Addition Unit.*

Polly placed more emphasis on the psychological aspect of having to undertake the challenge first in building persistence. She essentially concluded that if students could get through the challenge, they knew that the rest of the lesson would be manageable, providing them with an incentive to persist through the challenging task. In juxtaposition, the prospect of having to work through the challenging task towards the end of the lesson could become overwhelming for students in her view, a consequence of both fatigue and perhaps anxiety brought about by anticipating difficulties with the task.

> I think they persisted more with the challenge first… I don't know whether it was the structure of having the challenge first, or because they come in and they are not as tired. They come in, ready to learn, and they hit the task. They've done the hardest bit first, and then it's kind of downhill from there. And then when they get to the (consolidating) task at the end, well that was easy-pezi… Whereas what happened last term was we did the worksheet, and that was very hard for them, and then it just got harder again… I don't know whether psychologically, subconsciously, whether "ok the hard bit is done now, now I have got the easy bit" and you sort of think "oh good, I can

> do this". I mean as humans, when we know something is easier we sort of relax a little bit more, whereas when we know something hard is coming up, we kind of brace ourselves for it. *Polly, Addition Unit.*

Task-First Approach: Meaningful discussion and reflection

Both Sally and Polly acknowledged the value in having students work on a task prior to a teacher-led discussion. Polly perceived that having the mini-lesson after engaging with the challenging task meant that students were more invested in the class discussion, and receptive to listening to peers describe different ways of approaching the task. She also emphasised the importance of students then being able to put these learnings into practice through undertaking the consolidating task.

> And I think it made coming back and discussing the teaching – or the reflection on what just happened – more meaningful because they tried it out first. And they came back and they thought "Ah ok, so I could have done it like this. Oh gee, I didn't do it like that, but I could have". I think that was actually more beneficial in many ways than giving them everything that they needed to do a task, and then sending them off... I think it was great to have that reflection after. It gave them the opportunity to then apply that knowledge to the easier work. And it made a lot more sense with the easier work. *Polly, Addition Unit.*

Sally also placed considerable value on the post-task discussion with students.

> That is the thing that I said to you that I miss when you flip-it. Where there is no time for that really in-depth discussion. *Sally, Addition Unit.*

However, Sally also noted one of the shortcomings of a lesson being reliant on an effective, teacher facilitated discussion; that is, the physical and psychological availability of the classroom teacher. It is clear that Sally perceives the Task-First Approach as quite demanding on a teacher as the lesson unfolds in real-time.

> In the day-to-day of the classroom as well, you need to consider the fact that you can't always be that available during a maths class either because… I mean we have been talking about down here how much one-on-one testing sucks up of your time. So, when we are trying to collect data about our kids, you have got to do that during a maths lesson because when else are you going to do it… And sometimes, when you are on your own, they come in from lunch and this argument has happened, or that argument has happened… So you have to be really engaged and really involved to be able to then do that (coordinate the discussion) effectively. And I know for me in my classroom this year, with all the behavioural issues I have, that I would really struggle with that. *Sally, Addition Unit.*

Task-First Approach: Authentic assessment opportunity

Related to notion that the Task-First Approach prompts students to demonstrate creative mathematical thinking is the idea that beginning a lesson with a challenging task provides an authentic assessment opportunity. Sally notes how working within this structure allowed her to observe students in their capacity to apply their knowledge without direct scaffolding from a teacher.

> I got a chance to see what they knew before you had planted anything in their brain. And I thought that was really fantastic. *Sally, Patterning Unit.*

Elaborating on this idea in more detail following the second unit of work, Sally discussed her recent experience of teaching an open-ended lesson on data.

> It was fascinating, fascinating. I really just said to them “Here is a paint pot. How can we decide what is the most popular colour in our classroom”. And the maths thinking that came out of that was about 50 times greater than if you said to them “Here’s graph paper. We’re making a graph on our favourite colour”. Because all of a sudden you get all this information that you didn’t know you would. Kids who could tally and kids who would change their mind when others were collecting the data, and told this person blue and this person yellow and this person red – who clearly don’t know very much about data collection (laughs). *Sally, Addition Unit.*

Rachel concurred with Sally that the Task-First Approach provided valuable insight into student mathematical thinking, relative to more conventional teaching approaches.

> But I really liked that I got to see how they could solve the problem, what strategies they already knew, what strategies they could learn to do themselves... they could show their own thinking, without having to have that lesson first. *Rachel, Patterning Unit.*

Teach-First Approach: Greater focus

Rachel noted that students appeared more focussed and on task when the lesson began with the teacher facilitated discussion. She linked this more focussed behaviour to the contrasting sense of ambiguity and discomfort students experienced when they began the lesson with the challenging task.

> With the structured lesson first, it seems more pulled together. They are more focussed on the floor: they come in, sit down and get their lesson, and then they go off and get their work done, because they are not sort of distracted from not knowing what to do… I think they were more on task (in the teach-first scenario). I think some of them, with the challenge first, when they didn't know what to do, they just got distracted, and just started mucking around a little bit. *Rachel, Addition Unit.*

Teach-First Approach: More efficient approach to learning

Rachel also suggested that, in part because students were more on-task under the Teach-First Approach, it appeared the more efficient approach to instruction in terms of student learning. By contrast, the Task-First Approach seemed more time intensive, as students had to work through the aforementioned ambiguity before making progress with the task.

> I think that was one of my concerns the first time is that some of them had just started to work out a way to solve the problem, and the time was up. And they had to come to the floor, and they didn't find any success because they didn't have the time. Whereas with the lesson first, they had that time to sit there and go "Ok I know how I have to do this". And they used it (time) quite well I think… They went off, and knew what they had to do, and decided to try it straight away. *Rachel, Addition Unit.*

Teach-First Approach: Allows less confident and lower performing students to be successful

Rachel and Sally also offered the perspective that beginning with the lesson tended to better support less confident and lower achieving students, who benefitted from greater teacher scaffolding and support. By contrast, many of the more confident and higher achieving students appeared to thrive when the lesson was launched with the challenging task.

> I think generally the weaker, less confident kids responded to the lesson first, and then building to the challenge. And then I think that the kids who had the confidence, and the willingness to have a go, loved having the challenge first. *Sally, Patterning Unit.*

> I think opening with the challenging tasks for those top-end bright thinkers encourages them to be a little bit more autonomous and a bit more creative, and gives them an opportunity to come to the solution before you've shoved it down their throat so to speak. *Sally, Addition Unit.*

> I had more concerns with kids in the first unit of work, when the challenge was first, than I did this term… I think with the lesson first, it made the challenges much easier… because they had had the lesson, then they'd practiced it, and then they can apply it. So I think a lot of them found it much easier, which I guess is good, because then they felt like they could do it – they felt successful. *Rachel, Addition Unit.*

How to best teach with challenging tasks: Teacher conclusions

The three teacher-participants differed considerably in their conclusions as to how they would consider teaching with challenging tasks in the future.

Sally: Conceptualises lesson structure as the 'best fit' for a given student, rather than 'best practice'.

Sally perceived that different students responded differently to the different lesson structures. According to Sally, whereas some students thrived when engaging with the

Task-First Approach, other students responded better to, and seemed to have a preference for, the Teach-First Approach. In her view, such diversity makes the emergence of any form of 'best practice' in relation to lesson structure somewhat unlikely.

> Look I really struggle with this one, because I actually think there is a percentage of the grade who respond best to each… I feel like having seen the difference in the way the kids tackled those problems and how they coped differently, it actually really opens my eyes to the fact that, as teachers, it is really important to mix things up. *Sally, Addition Unit.*

Moreover, Sally concludes that it becomes the responsibility of the teacher to look past her individual preferences for a particular learning approach and ensure she provides a variety of different learning experiences for students.

> It is easy to do what comes naturally to you, but you might not be aware of the impact that's having on your students. And I think that was what was really powerful about this for me, was that sometimes, in the teaching of it, one way may have felt more unnatural – I don't know if one felt easier or more normal for you? But typically speaking we would go lesson-first, then challenge. But it really opened my eyes to the fact that maybe when we take that closed off view, we're actually kind of disabling some of those kids, and, you know, holding them back. *Sally, Addition Unit.*

Although Sally suggests that generally the students she perceives as more mathematically capable demonstrated a relative preference for the Task-First Approach, in her view this was not exclusively the case.

> I am just looking at my notes here. “Surprised by the fact that those students who most enjoyed being challenged weren’t necessarily the strongest students”. So Liam, who finds maths really challenging, approached the (challenging) task in a really focussed way. And Jake quit when things got difficult, even though he is more than capable. And you look at those two kids. Jake is probably more capable than Liam. And he just threw in the towel. *Sally, Patterning Unit.*

Rachel: Preferred teach-first for supporting lower-achieving students and managing the classroom

Despite acknowledging the strengths of the Task-First Approach, particularly with regards to engaging students, building persistence and the opportunity for authentic assessment, overall Rachel indicated a preference for the Teach-First structure. In part this was related to her perception that the Teach-First Approach was more efficient.

> It is a tricky one. I think they learn more when the lesson’s first, but I like to see what they can do when they don’t have the lesson first, because it is challenging them a lot. But I guess that wasn’t good for all of them, for all those students... I think the students got more out of it (with the lesson first)… I would probably feel more comfortable having the lesson first myself as a teacher. *Rachel, Addition Unit.*

Additional reasons for Rachel preference for the Teach-First Approach related to her capacity as a teacher to provide sufficient support to those students who required it, and to manage the classroom accordingly.

> I find then, especially when you are on your own… because in here we’ve got the two of us and we can roam around. But when it is just you in the

> classroom, if you are giving out a challenge first and there are a lot of kids who don't know what they are doing and need that support, I think it can be quite tricky to get around to all of them, to manage that… Whereas, when you have the lesson first, the majority of them will catch on, and be able to do the work independently, and you just have to have a small group who you just work with. *Rachel, Addition Unit.*

Polly: Apparent preference for the Task-First Approach, however difficult to analyse the impact of lesson structure independently of a 'practice effect'

As was apparent during the teacher interviews, teaching with challenging tasks is powerfully different from how the study teachers typically approached a mathematics lesson. Consequently, there is a risk of overstating the impact of lesson structure, as compared to a 'practice' effect, whereby students became accustomed to the higher expectations and greater autonomy typically encountered during a lesson involving challenging tasks. Polly indicated that, although she perceived the second unit of work, which was structured such that the task was first, to be more effective, she was uncertain to the structure of the lesson or prolonged exposure to challenging tasks.

> Yeah, I thought it was really interesting seeing both ways of doing it… I don't know if it was just the fact that the first time they had been exposed to everything in the first term there that they floundered a lot and… I don't know what it was. I think this term worked better than last term… I think they were more settled with the challenge first… But once again, it is hard to know isn't, because of them having had the term before. They were more used to the whole format of everything. But I felt like they got into it more… What I think I noticed this time is that they knew where they were going. They knew what it was all about this time, so it wasn't this whirl of overwhelming stuff that was happening, and they had no idea what was

> going on… They already came to it with a better frame of mind because of understanding the expectations. And they were ready to work right from the beginning, instead of floundering from the beginning. They understood what the hint sheet was all about more – and even I understood more what the hint sheet was all about. And… I could see that most of them were working really hard on trying to solve the problems and use the strategies that had been talked about. Some of them needed some guidance or reminders of strategies – “Remember you can use this”. And of course that is where the hint sheet comes in. But I felt that more of them were on task and they seemed to get a grasp of what was happening more. They might not have come to the right answers, but I think one of the good things was that they were grappling with the problem. *Polly, Addition Unit*

Discussion of Findings (Study Two)

Teacher perceptions of student reactions to challenging tasks

Generally, teachers concluded that students reacted very positively to what was expected of them in the program. Perhaps most critically, all three teachers concurred that students generally demonstrated a high level of persistence and a willingness to embrace struggle. Although it has been argued that excessive persistence without the requisite understanding can be problematic as students may engage unproductively in problems (Star, 2015), the results from the current study are encouraging given that persistence is a difficult quality to cultivate.

Interestingly, this finding contrasts with some prior research suggesting that teachers’ reluctance to embrace challenging tasks reflects their concern that students would be unwilling to persist with a problem which they could not immediately solve (e.g., Leikin et al., 2006; Sullivan et al., 2010). There are at least three potential explanations for this difference.

Firstly, It is possible that the young age of the children in the current study (i.e., 7 and 8 years old) was a factor in their general enthusiasm for embracing the challenging task, even when they were struggling to make progress and comprehend what was required. By contrast, many other studies examining teacher perceptions of student reactions to challenging tasks, such as Sullivan et al. (2010) have focussed on teachers of children nine years of age and older. Ridlon (2009) contended that young children may be particularly willing to persist with challenging mathematical problems, and have a positive disposition towards learning mathematics, compared with older students.

Secondly, it may have been that observing the practice of another teacher (in this case, the primary researcher), rather than attempting to teach with such tasks themselves, allowed the teacher the relevant time and space to more objectively consider how most students were actually responding to such tasks, rather than becoming overly fixated on the reactions of a small subset of students, such as those struggling with the mathematics. One of the participants observed that, within a classroom, students perceived as struggling tend to occupy a substantial proportion of a teacher's time and energy. It is conceivable, therefore, that teachers may incorrectly generalise their perceptions of these 'strugglers', with whom they spend a disproportionate amount of time with, to students in a general sense. Consequently, the observational methodology employed in the current study may partially explain the positive perceptions of teacher-participants, given that several other studies have instead gathered data about teacher perceptions of challenging tasks using other means, such as self-reflections (e.g., Sullivan et al., 2013; Sullivan et al., 2010).

Thirdly, it may be that the prolonged exposure to challenging tasks across two school terms helped to develop a propensity in students to persist with the challenge. Indeed, it was apparent, particularly from Polly's and Rachel's accounts, that the early stages of the first unit of work were associated with high levels of teacher anxiety, as students began to navigate, some perhaps for the first time, what has been termed the "zone of confusion" (Clarke et al., 2014, p. 58; Sullivan et al., 2014, p. 11). Indeed, the current study provides some support for the contention that student persistence can be facilitated through normalising the concept of the 'zone of confusion' in the mathematics classroom, and providing students with avenues for taking constructive action when confronted with this state. Specifically, teachers perceived a willingness to persist evolved in part through consistently high teacher expectations with regards to students' capacity to help themselves, and a commitment to providing students with the requisite resources to do so (e.g., enabling prompts, bead frames, hundred charts).

This emphasis on 'normalising' student autonomy is consistent with other research, which suggests that developing a classroom climate which supports student effort (Sullivan et al., 2013), and empowers students to be responsible for how they choose to solve problems (Sullivan et al., 2010), are critical components to successfully teaching with challenging tasks. However, although a classroom climate which promotes autonomy is necessary for providing students with the capacity to persist with a challenging task, it is unlikely to be sufficient. Willingness to persist with such tasks also reflects how engaged students are in the lesson.

There was strong evidence in the current study that teachers perceived students as highly engaged in both units of work. This engagement was seen to be both a product of the challenging nature of the learning material, and the mathematics being situated in a context to which students could relate. Providing a context or story to which students can connect has been discussed elsewhere as an important aspect identified by teachers for engaging students in challenging tasks (Clarke et al., 2014).

Interestingly, one teacher (Sally) also perceived that being engaged with a challenging task supported the learning of more rudimentary material. She observed that the proposition of tackling a challenging task appeared to provide students with a more compelling purpose to master basic number facts and patterns. This notion that active engagement by students in purposeful learning supports the development of basic arithmetic has been well established within the mathematics education literature for some time (e.g., Baroody & Hume, 1991; Carpenter, Fennema, Peterson, Chiang, & Loef, 1989).

Using challenging tasks to differentiate instruction

Despite general agreement between the three teachers that students engaged and persisted with challenging tasks, teachers differed in their views as to whether the tasks were appropriate for all students. Polly, and to a lesser extent Rachel, noted that many of her lower-achieving students struggled significantly with many of the challenges. This led Polly to conclude, at least before she had an opportunity to further reflect on the value of the program, that perhaps challenging tasks should be reserved for higher-achieving students who are more confident with mathematics. She also contemplated whether learning through challenging tasks is actually more appropriate for older students, and wondered whether the concepts discussed were too abstract for seven and eight year old children. Although Polly's concern that the program disproportionately benefitted higher achieving students was not borne out in the analysis of student outcome data (which actually found some support for the contrary claim; that is, that lower achieving students benefitted most – see Appendix O), the fact that she retained this perception is in itself important. Indeed, her assumption that a curriculum which is focused on learning through problem solving fails to best serve students with learning difficulties in mathematics, and perhaps younger students more generally, does resonate with arguments put forward by some educational researchers (e.g., Westwood, 2011). Polly would likely align herself with the views of Star (2015), who concluded that “perseverance is only useful when students have sufficient prior knowledge and metacognitive abilities to make their struggles potentially productive” (p. 9-10).

However, others within mathematics education may take issue with Polly’s conclusion. For example, Pogrow (1988) argued that not exposing lower-achieving students to more challenging work in an effort to protect their esteem might paradoxically prevent them from developing real self-confidence in mathematics. He instead encouraged teachers to allow all students, not just high performers, to experience what he termed “controlled floundering” (p. 83), or what others have referred to as “productive failure” (Kapur, 2008, p. 379), or the “zone of confusion” (Clarke et al., 2014, p. 58). Moreover, given that challenging tasks are designed with the intention of students being able to augment the level of challenge they are confronted with through accessing enabling and extending prompts, it has been argued that students at different ability levels can all potentially benefit from working on challenging tasks (Sullivan, Mousley & Zevenbergen, 2006).

In support of this position, and in strong contrast to the other two participants, Sally concurred that challenging tasks are an appropriate means of differentiating instruction.

Her views in many ways embodied how many mathematics educators and researchers hope that classroom teachers will come to view challenging tasks (e.g., Sullivan, Mousley & Zevenbergen, 2006). Sally perceived that such tasks enabled almost all students to engage in meaningful mathematics throughout the lesson, and described her commitment to further integrating challenging tasks into her personal pedagogy.

Part of the explanation for the contrary conclusions reached by Polly and Sally concerning the capacity of challenging tasks to differentiate instruction may relate to their different points of reference. Whereas Polly seemed concerned with whether students had success with the task and understood the underlying mathematical concepts in relation to the stated learning objectives, Sally was focussed on overall student mathematical engagement, as well as the relative growth in a student's level of mathematical knowledge. These contrasting criteria for defining success allowed Sally to celebrate modest demonstrations of learning, such as a student identifying a two-digit number on a hundred chart. By contrast, Polly remained sceptical about the value of the program for low achieving students, even while acknowledging the "amazing" progress of Kate, a student with a mild intellectual disability, in her ability to independently apply her mathematical knowledge to a challenging task.

Sullivan et al., (2014) state that, once a challenging task has been posed, there "is no expectation that all students can complete this learning task successfully, although the intention is to facilitate engagement of all students with the learning tasks" (p. 3). Consequently, it appears that Sally's understanding of what constitutes success aligns more closely with how experts (e.g., Sullivan et al.) intended work on such tasks to be evaluated. However, the very different perspectives of these two teachers when observing similar student behaviours reminds us that how we choose to define success in the learning of mathematics is somewhat subjective, potentially contested and, at least in part, socially, culturally and historically determined (Ellis & Berry, 2005).

Evaluating other barriers to teaching with challenging tasks

Prior research has demonstrated that the willingness of teachers to teach with challenging tasks may be constrained by their lack of mathematical content knowledge and/or mathematical pedagogical content knowledge (Charalambous, 2008). There was some support for this idea in the current study. Although Sally, and less explicitly Rachel, indicated they would be confident to teach with challenging tasks in the future, Polly anticipated that her limited expertise in mathematics may be a barrier. Polly repeatedly contrasted her lack of deep mathematical understanding with her considerable expertise in teaching music. She suggested that teaching in a more student-led manner incorporating more open-ended, cognitively-demanding tasks required deeper content mastery (by the teacher), in part because of the difficulties anticipating where precisely such a lesson may be heading.

Indeed, Polly's position is potentially justifiable when you consider the research. It is well documented that a problem-based approach to teaching mathematics requires considerable skill and expertise for teachers to execute effectively (Ridlon, 2009; Stein et al., 2008). Moreover, Roh (2003) argued that attempts to pursue a problem-based learning agenda when the teacher lacks the necessary mathematical knowledge is likely to result in them selecting inappropriate tasks and/or not implementing said tasks appropriately. Consequently, it may be that a teacher who has sufficient insight to realise they lack a deep understanding of the relevant mathematics actually serves their students best by focussing on more explicit instruction and routine tasks.

Interestingly, although Polly was the only teacher who indicated she lacked confidence teaching with challenging tasks, all three teacher-participants described themselves lacking the capacity to plan a unit of work based around challenging tasks. To some extent, the views of the teachers in the current study reflect Sullivan et al's (2014) assumption that planning such tasks is a time-intensive undertaking. However, it is also evident from interview responses that teachers perceive the skills and knowledge involved in developing, sourcing and sequencing challenging tasks as distinguishable from the skills and knowledge associated with executing a lesson that incorporates such tasks.

The last noted barrier to teaching with challenging tasks within the current program was the idea that the curriculum may be too crowded to accommodate in-depth consideration of one particular topic. Although, in the current research project, this concern was raised by teacher-participants exclusively in relation to the patterning unit of work, it is consistent with prior research suggesting that teachers may perceive the curriculum as 'too crowded' to warrant investigating single topics or ideas more comprehensively (Siemon, Bleckly, & Neal, 2012). It is worth noting that these concerns are in spite of attempts to ensure that the revised Australian Curriculum covered fewer topics in more depth (Westwood, 2011).

This viewpoint may reflect the fact that teacher-participants tended to implicitly emphasise the content description aspects of the curriculum at the expense of the four proficiency strands: understanding, reasoning, fluency and problem solving (ACARA, 2015). Specifically, it appears that all teacher-participants felt a responsibility to cover each ‘topic’ in the curriculum, and that this perceived obligation outweighed any corresponding responsibility to teach for depth in order to develop these proficiencies. Teachers fixating on content whilst failing to sufficiently prioritise mathematical process (as embodied by the reasoning and problem solving proficiency strands) has been highlighted previously as a potential obstacle to ensuring that the Australian Curriculum in mathematics is enacted by teachers in ways intended by its developers (Stacey, 2010).

The impact of lesson structure on teacher perceptions of challenging tasks

Analysis of the teacher interviews revealed that teachers perceived each of the two lesson structures to offer distinct benefits. Although no specific hypotheses were put forward with regards to how teacher-participants may perceive the two different lesson structures to impact on student learning, it is worth noting that the teacher perceptions uncovered in the current study were highly consistent with the arguments and evidence contained within prior research. Specifically, it was found that the Task-First Approach was perceived by teachers as better able to: i) foster mathematical creativity as students had the opportunity to ‘discover’ idiosyncratic, and often more than one, solution methods (e.g., Leikin, 2009; Sullivan & Davidson, 2014); ii) promote meaningful discourse amongst students (e.g., Forman et al., 1998; Woodward & Irwin, 2005); iii) build student persistence (Sullivan et al., 2014); and iv) effectively engage students through challenge (e.g., Sullivan et al., 2012). Conversely, there was also some support for the postulation put forward by some cognitive load theorists that a lesson which begins with some form of explicit teaching, such as the Teach-First Approach, constitutes a more focussed, efficient approach to instruction (e.g., Kirschner, et al., 2006; Sweller et al., 2007). Indeed, even the belief that a Teach-First Approach may be more appropriate for lower-achieving students, not mentioned in our brief literature review, has precedence (e.g., Westwood, 2011). Overall, this implies that framing the Task-First Approach or Teach-First Approach as an either/or proposition is perhaps overly simplistic, as both approaches appear to have distinct strengths in terms of student learning outcomes, at least as construed by observing teachers. This conclusion is perhaps contrary to conventional wisdom. For example, Sullivan et al., (2014) argued that traditional lesson structures, characterised by some form of teacher explanation preceding student practice (equating to the Teach-First Approach in the current study), can inhibit the possibility of even well-designed tasks facilitating opportunities for students to engage in higher level mathematical thinking.

The central tension identified by teacher-participants in the current study between wanting students to discover and subsequently own their personalised solution method, and teachers leading students towards the most efficient (or mathematically important) solution method, is not novel and has been revealed in previous research. For example, Star and Rittle-Johnson (2008) found that encouraging year six students to discover their own methods for solving linear equations led to them demonstrating a broader variety of problem solving strategies, however directed teaching in how to solve such equations resulted in students incorporating more efficient strategies. This tension has been described elsewhere by Baxter and Williams (2010) as “managing the dilemma of telling”, and is the central theme of their paper which observes the classroom practice of two teachers who are attempting to employ problem-based approaches to learning mathematics (Baxter & Williams, 2010, p. 7).

A corollary of the finding that the Task-First Approach and the Teach-First Approach have distinct strengths is that a particular teacher’s preference for one approach over another will likely depend in part on what student learning outcomes she prioritises as a teacher. For example, a teacher who is strongly focussed on meeting the needs of the three or four students in her classroom who have severe difficulties with mathematics, may be inclined to embrace the Teach-First Approach. By contrast, a teacher who views mathematics learning as being principally about struggle and discovery will likely embrace a Task-First Approach. The notion that the idiosyncratic values that teachers hold regarding what they believe should be the primary learning objective impacts on their subsequent approach to instruction, has been raised in a variety of other primary education contexts, including foreign-language learning (e.g., Pichon, 2014) and the use of technology in classrooms (e.g., Warwick & Kershner, 2008).

Interestingly, Sally put forward an equity-based argument as to why teachers need to suspend their own preferences for a particular lesson structure and consider incorporating a mixture of approaches in their classrooms. Sally noted that, anecdotally, it appeared some students responded better to, and had a preference for, the Task-First Approach, whilst other students responded better to, and had a preference for, the Teach-First Approach. She implied that such diversity in student reactions makes the emergence of any single-form of best practice somewhat unlikely. Sally's position is consistent with the proposition that one size is unlikely to fit all within the context of mathematics education (Ridlon, 2009), although others may argue that preferring the Teach-First Approach reflects such students having not yet developed a mastery orientation towards mathematics. For example, Dweck (2000) contended that students who are more willing to take risks, and are less concerned about social affirmation, are more inclined to both embrace, and persist with, a challenge. Despite this tension, the idea that students have a distinct preference for a Task-First Approach or a Teach-First Approach warrants more direct empirical investigation, and is explored in Study Three (see Chapter Seven).

Finally, there was some support for the notion that the Task-First Approach may support less confident and experienced teachers in experimenting with incorporating more challenging tasks into their mathematics lessons. Specifically, Rachel, a first-year graduate teacher, noted that part of her reason for preferring the Task-First Approach was the less demanding classroom management aspect. However, it is noteworthy that, at least in Polly's case, a teacher who self-identified as having (relatively) limited content knowledge in the area of mathematics was still reluctant to incorporate challenging tasks even when such tasks were used to extend student thinking, following a more traditional lesson structure (i.e., Teach-First Approach).

Notwithstanding the above discussion, overall it appears that the type of tasks incorporated into lessons and the overall pedagogical approach adopted was perceived by teachers as being at least as important as the specific lesson structure adopted. For participating teachers, it is likely that incorporating challenging tasks regularly into lessons in any capacity represents a significant shift in their teaching practice. Consequently, whether the task is used to launch a lesson or extend student thinking may be secondary to the presence of challenging tasks themselves. As a corollary of this, it appeared that, when comparing the two units of work, practice with challenging tasks likely played a role, as the class adapted to learning with challenging tasks. For example, Polly was more reluctant to teach with challenging tasks in part because she perceived them as too challenging for many young students, particularly lower-achieving students; yet, paradoxically, Polly was more enthusiastic and endorsing of the Task-First Approach. Rather than reflecting the fact that such a structure offers less challenge, it is likely that her observation reflects a ‘practice’ effect, whereby she perceived her students as being more familiar with the expectations and routines associated with the program during the second unit of work, which coincided with her class participating in the task-first structure.

Concluding thoughts

This study revealed that teachers perceived their students to react very positively to the use of more challenging tasks during mathematics instruction, at least when observing instruction facilitated by a teacher with some expertise in teaching mathematics to young students. Students were seen to demonstrate substantial persistence and engagement, whilst making considerable progress mathematically. Although there were some concerns raised about the challenges of a crowded curriculum, it appeared that two of the three teacher-participants enthusiastically endorsed the idea of incorporating challenging tasks into future instruction, provided they were given some support with the planning of such tasks.

One study participant, Polly, remained somewhat uncertain about teaching with challenging tasks, at least for the majority of the research project. Her candidness should be appreciated. It was telling that despite Polly observing the program to be highly effective (at least for many of her students), she was still reluctant to consider incorporating challenging tasks in future mathematics instruction, at least during her initial interviews. This reluctance appeared to reflect both her concerns about the learning outcomes for lower-achieving students as well as anxiety in relation to whether she had sufficient mathematical knowledge to teach using such tasks. Her subsequent, rather dramatic, reconsideration once she began planning for the next unit of work is promising and does speak to the power of such tasks vis-à-vis what was currently available through standard teacher resources at the school, particularly in the early years.

Finally, with regards to lesson structure, teacher-participants perceived both the Task-First Approach and the Teach-First Approach to teaching with challenging tasks to have particular strengths. Specifically, the Task-First Approach was viewed as providing an opportunity to engage and extend students, whilst fostering student mathematical creativity, whereas the Teach-First Approach was viewed as highly supportive, focussed, and efficient. However, although there appear to be distinct advantages to both the Task-First and Teach-First Approaches, the study revealed that a significant shift in teaching practice for many teachers may be the incorporation of more cognitively demanding tasks into their mathematics instruction in any capacity. In order to build on this discussion, the third and final study (Chapter Seven) will explore student reactions to learning through challenging tasks, including their personal preferences for how such lessons should be structured.

CHAPTER SEVEN: STUDY THREE – STUDENT PERSPECTIVES[15]

This chapter analyses student perceptions of the program. It is divided into two sections. The first section considers student responses to the brief semi-structured interview (10 to 15 minutes) following the patterning unit of work.[16] During this interview, students were asked to review their portfolios, select the two pieces of work that they were most proud of creating and explain why they had chosen these two pieces of work. The second section relates to student interview responses following both units of work from students in Class C (i.e., the 'Alternating' group) as to their preferred structure of the lesson (i.e., Teach-First Approach or Task-First Approach) and their rationale for their preference.[17] Consequently, these two sections address the following research questions respectively:

1. What do students value when reflecting on their own learning artefacts following participation in the program? (Student Reflections on their Work)
2. Do students have a preference for a task-first or teach-first lesson structure? What factors are reported as influencing this preference? (Student Reflection on the Structure of their Lessons)

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[15] Findings from this chapter has been outlined in a peer-reviewed journal article accepted for publication: Russo, J., & Hopkins, S. (2017). Student reflections on learning with challenging tasks:'I think the worksheets were just for practice, and the challenges were for maths'. *Mathematics Education Research Journal, Online First*, 1-29.

[16] Note that the primary reason that these student interviews were not undertaken for the second unit of work (i.e., the addition unit) was due to time constraints. In particular, the interviews would have taken around six school days to administer, and the second unit of work concluded a week before school holidays. The second unit of work began a fortnight later than initially intended, due to the researcher having to work around the school's scheduling.

[17] Note that, for the patterning unit, students in Class C were asked an additional question at the end of the standard student interview to glean their lesson structure preference. For the addition unit, students in Class C were very briefly interviewed (less than 5 minutes) with the sole purpose of establishing their lesson structure preference.

The analytical approach to address both these questions is the Constant Comparative Method (Glaser, 1965, 1969), as described in Chapter Four. After the Constant Comparative Method yielded a more parsimonious set of categories and themes, the frequency in which participants appeared in various themes was quantified, and tests of associations were performed. Specifically, this involved employing tests of statistical significance suitable for analysing categorical data (i.e., chi-square tests of independence; Fisher's Exact Test) to investigate whether gender, year level and performance were associated with how students responded to challenging tasks.[18]

Student Reflections on Their Work

Following analysis of the verbatim responses provided by 73 students participating in the program with regards to what work they were most proud of creating and why, 11 categories emerged indicative of what students valued about their work in these particular lessons (see the bullet points in Figure 7.1).[19] These categories were subsequently refined and organised into five themes: Enjoyment, Effort, Learning, Productivity and Meaningful Mathematics, which were in turn clustered under three meta-themes: Process, Outcome and Contents. It is important to emphasise that the development of categories and themes was an iterative process; categories and themes were augmented, merged, and refined as more interview transcripts were read.

[18] This quantitative analysis was intended to be exploratory. Consequently, the analysis is only included in the text of the results section when deemed noteworthy; in particular, when it was statistically significant or approached statistical significance. The absence of any commentary indicates that the association between gender, year level or ability and the relevant theme was not statistically significant ($p < 0.05$) nor approaching statistical significance ($p < 0.10$).

[19] Although 75 interviews were attempted, two responses were not considered due to the student being non-responsive.

For example, during the initial analysis, a sixth theme, ‘Level of Challenge’, was included under the Contents meta-theme. This theme appeared to capture student reflections indicating they valued a particular artefact because they had made progress with the corresponding mathematical task, despite it being experienced as challenging or difficult. However, as the analysis progressed, it was revealed that this notion of ‘a good job on a hard task’ was so ubiquitous (describing almost all participants) as to make the explanatory power of this theme in isolation largely redundant. Instead, to orchestrate a more meaningful enquiry, it was decided to comment on the level of challenge inherent in a task when the idea arose in connection to other themes such as, enjoyment (i.e., It was fun because it was hard), effort (it was hard and yet I didn’t give up), and productivity (i.e., it was hard but I did so much work).

Following this categorisation process, five (6.8%) participant responses remained unallocated to any one of the eleven final categories and corresponding five themes. In one case, the student provided insufficient detail for their response to be categorised. In the remaining four cases, and despite prompting to attempt to ascertain why specifically the student was proud of the work, the student’s response was largely limited to describing the task. For example, after being asked to explain why they had chosen a challenging task involving a fruit-themed counting pattern, the student replied:

> Because on Saturday they ate bananas, and Sunday they ate fourteen.
> Because everyone didn’t get a banana. The next day, they did plums.
> Because it was 18 plums.

A summary of the analysis, with the number of students indicating a particular theme, is provided in Figure 7.1. The median number of themes endorsed by students was 2 (*M*=1.70; *SD*=0.95; min 0, max 4).

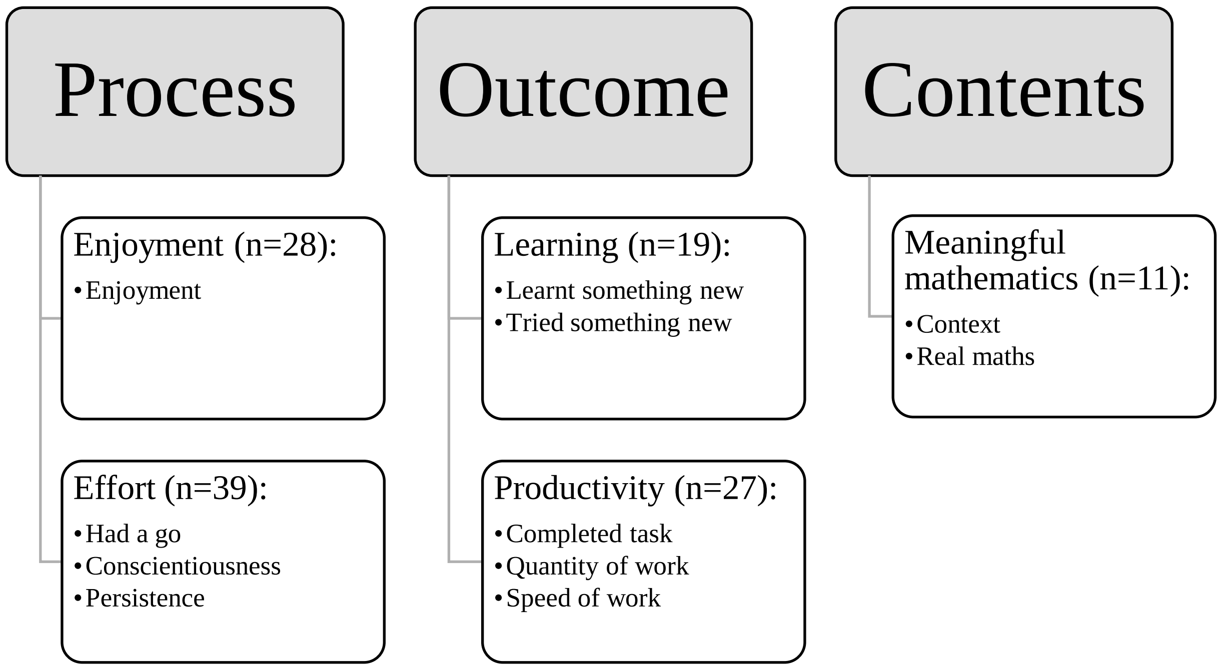

Figure 7.1. What Students' Valued When Reflecting on their Work Artefacts

Note: n represents the number of students whose responses were categorised to this theme during the interview.

An attempt will now be made to elaborate on each of the five themes that emerged from the student interview data. A key aspect of this elaboration is the inclusion of quotations from students. In general, those quotations which most compellingly captured a given aspect of a particular theme were included. At times, however, additional, less compelling quotations have also been included. The intention is to compensate for the lack of detailed articulation provided by many of these young participants when discussing particular themes through demonstrating the frequency with which an issue was raised.

Enjoyment

This theme reflects a single category; the notion that the student enjoyed working on a particular task. Over one-third (38%) of participants indicated that they enjoyed working on their selected artefact, suggesting that many young students value an activity in part because they derive pleasure or satisfaction from working on it.

The theme of Enjoyment emphasises the process of working on the task, rather than attempting to capture the participant 'liking' the work they produced when reflecting on the task retrospectively during the interview. Distinguishing a student's affective judgment on the process itself from their feelings about the output created hinges on a careful consideration of the language used by students. For example, whilst the comment "it was fun to do" would have been included under this theme, a comment such as "it looked like a good task" without further elaboration would not be included, given that the latter comment might imply that the student is simply taking pride in having produced this piece of work, rather than attempting to comment on the actual process of working on it.

The most frequent reason put forward by participants as to why they enjoyed working on a particular task was because they found it difficult. This indicates that students derived satisfaction from the process of being challenged mathematically.

> Because I like the challenge and I like those patterns. And some of the patterns were really hard, but I think I figured some of them out. *Female student, Year 2.*

> This one is cool. It is quite challenging to do. I like to do challenges. *Male student, Year 2.*

> Because I like challenging. It is fun to do challenging stuff. Because it is quite hard for you - and it is really fun because it is hard for you. *Female student, Year 1.*

> It was fun as well. Because it was sort of hard for you. I like challenges because I need to use my brain more. *Female student, Year 2.*

In addition, there were several examples of students commenting on their enjoyment of applying a particular mathematical process in a general sense, with the role of the task primarily to facilitate their engagement in the specific mathematics. In particular, students appeared to enjoy the process of doubling, halving, as well as specific counting patterns.

> Because I like counting patterns… Because I like doing doubling; it is fun! I want to do doubling all day and just keep going and going and going.
> *Female student, Year 2.*

> Because I really like counting backwards. I like counting backwards by twos and threes and stuff. *Male student, Year 1.*

> Because I love to count by twos. I really like the number 2. *Female student, Year 1.*

> Because I really like halving. It was fun. *Female student, Year 1.*

Another notable reason as to why a given task was described as enjoyable to work on was because of the problem solving context in which the task was situated. Although the issue of context is considered in more detail when discussing the theme ‘Meaningful

Mathematics', it is worth noting that both the doubling and halving challenges, as well as the tasks involving sport, were described as enjoyable by multiple participants.

> I liked this one the most. I like halving numbers and I like it because of the giant eating people. *Male student, Year 2.*

> Because I like basketball. Because I like doing maths that has sports in it - like maths sports… It is nearly the same as that. I used to do cricket when I was like five. And I really like cricket, because when I am the batter, I am always so good at it. It is funny that it (the task) has Will in it. *Male student, Year 2.*

Finally, several students made broad statements about the task being enjoyable to work on, without linking this enjoyment to any other particular aspects of the task, or their own learning preferences.

> I chose it because it was really fun. *Female student, Year 2.*

Effort

The theme of effort captures the three interrelated categories of 'having a go', 'persistence' and 'conscientiousness'. Over half (53%) of participants had their interview responses coded to at least one of these three categories, meaning that 'effort' was the most prevalent theme to emerge from analysis of the interview data.

Given the apparent conceptual overlap between 'having a go', 'persistence' and 'conscientiousness', it is necessary to elaborate somewhat on the precise meaning of these three categories. 'Having a go' implies that a student was willing to attempt a task that they found challenging. More specifically, it means the student indicated that simply 'having a go' at a difficult task was a reason he or she took pride in a particular piece of work. By contrast, 'persistence' refers to the notion that, through sustained effort, a student managed to make progress with (or complete) a task. Persistence also implies that the student overcame some form of adversity (e.g., finding the work difficult, not knowing where to begin) in order to complete the task. Finally, 'conscientiousness' captures the related idea that a student worked hard and/or demonstrated a high level of concentration or effort when working on the task.

The notion that 'having a go' at a cognitively demanding task is reason enough for taking pride in a piece of work is interesting, particularly because students providing this explanation often had made minimal progress with the actual task they selected. This rationale can be contrasted with those students who indicated that they selected the work sample because they had completed or finished the task (see the theme Productivity). It is also noteworthy that several students used the specific phrase "had a go" or "gave it a go", suggesting that these idioms, used by both the researcher and classroom teachers continually throughout the program, had been internalised by students.

> Because it was really difficult and I gave it a go. *Female student, Year 2.*

> Because it was challenging and I had a go at it. *Male student, Year 1*.

> I had a go at the extra challenge. Because most of the challenges are hard. Basically most of the time I don't get up to the extra challenge, because they are usually quite hard. *Male student, Year 2.*

There was substantial emphasis placed during the program on persevering with a task even if you were initially unsure how to begin; that is, in students confronting what

Sullivan, Clarke and colleagues have referred to as the “zone of confusion” (Clarke et al., 2014, p. 58; Sullivan et al., 2014, p. 11). Consequently, it was perhaps not surprising that many students framed their capacity to persist with the task in these terms.

> It was also kind of hard and I couldn’t really get started at the start. But I found my way to getting started. (How do you think you found your way to getting started?). Because I was thinking a lot. *Male student, Year 1.*

> Because at the start when you were explaining it, I thought I couldn’t do it… Then I did it. *Male student, Year 2.*

Other students discussed persistence more in terms of experimenting with previously unknown mathematical processes.

> Because first I thought it was 78. Then I counted it again and it was 77. Then I counted again and it was 78. It was a bit tricky for me but I figured it out. I counted by sixes. I got the 120s chart, and I counted by sixes. I circled every sixth number: 1,2,3,4,5,6. I have never done this before. But then I did it, and I was proud of myself. *Male student, Year 1.*

> Because even though it was really really hard for my brain to figure out, I got it anyway. Before I did this challenge I didn’t know how to split things in half with numbers - but now I do. *Female student, Year 1.*

Finally, the role of the ‘hint sheet’ (i.e., the enabling prompt) in students’ construction of the persistence concept was complex and somewhat contradictory. Some students associated using the hint sheet with persistence, while other students implied accessing the hint sheet lessened the value of the challenge, and actually indicated a lack of persistence. It is noteworthy that this latter view was constructed despite the researcher and teachers attempting to destigmatise the hint sheet through, for example, regularly praising students who used it effectively.

> Because when I thought it was too hard, I needed a bit of help. I couldn't figure it out so I got the hint sheet. And because I know how to count by threes and twos I just made the pattern. *Female student, Year 1.*

> I worked hard, and I didn't even use a hint sheet, on either of them. I only used two hint sheets. I did not need a hint sheet on these. That is why I am proud of them. *Female student, Year 1.*

The final category placed under this theme was conscientiousness. Although ostensibly equivalent to persistence, conscientiousness captures those comments indicating that the student believed that they had worked hard and concentrated, without making any reference to finding the task challenging, or overcoming any obstacles. In a sense, conscientiousness can be considered a necessary, but not sufficient, condition for persistence. A conscientious approach was frequently linked by students to the high volume of work produced during the session (see the theme: Productivity).

> I did a lot of work and was concentrating a lot. *Female student, Year 1.*

> Because it is the same as the other one, I got past 100. I think I did really well on it. Because I was just putting my head down and working on that day. *Male student, Year 2.*

> Because I counted on all the way to 200. Because it is quite hard to count to 200 with twos. It takes quite a long time… I put a lot of effort into it. *Male student, Year 2.*

Learning

This theme contains two categories, both of which link the task chosen by the student to the notion of learning. 'Tried something new' refers to the idea that the student noted that they chose the particular task because they either tried a new strategy, approach, or experimented with a previously unknown mathematical process. By contrast, 'learnt something new' reflects the fact that the student identified a specific process or concept that was better understood as a result of engaging in a task.

It is important to note that in order to be categorised under this theme, the student needed to make a specific reference to a strategy attempted, or a process learnt, rather than a generic reference to learning. For example, if the student stated, without elaborating, "I like doing challenges, because I get to learn new stuff" or "I didn't used to know anything, but now I know a lot", these comments were not included under this theme. The rationale was that, in such instances, it was not clear what the link was between the work undertaken on a specific task and the learning that occurred.

Overall, the theme of Learning arose in approximately one-quarter (26%) of participant interviews. Female participants were more likely to have their response coded to the Learning theme than male participants. Specifically, whereas over one-third (38%) of female participant responses were coded to the Learning theme, the equivalent rate for male participants was around one-sixth (17%). Moreover, a chi-square test of independence indicated that this trend whereby female students were more likely to highlight the theme of Learning was large enough to be statistically significant [X^2 (1, 73) = 3.895, p <.05].

Many students indicated that they were proud of the piece of work they had chosen because they had engaged with new mathematical ideas or processes. At times, this category overlapped with the theme of Effort, and specifically the notion of ‘Having a go’.

> Because I liked… Because I had never counted by eights before and I wanted to try it. *Female student, Year 1.*

> Because I knew those two (3 and 6) equalled, but I didn’t know what double 12 was, but I used a hundred chart to figure it out. *Female student, Year 2.*

> Because I used more than one way with it. Because I thought that would be good - to work it out more than one way. And I thought it would be good to use the sixes more than fours, because it was good to get to knowing the sixes better. *Female student, Year 2.*

> Well I worked it out a quick way. I counted the legs by crossing them out and writing the number so I didn’t lose track. (So you were proud of your strategy?) Yep. *Male student, Year 2.*

By contrast, other students emphasised that they had actually acquired a new mathematical skill or piece of mathematical knowledge as a consequence of undertaking the task. Although it is possible that students appropriately credited engagement with a single mathematical task as being responsible for supporting the development of a particular skill, it may be that the task was illustrative of the skills developed across the unit of work as a whole.

> Because it taught me how to count by threes. Because I didn't know how to count by threes before, like when I was in prep. *Female student, Year 1.*

> I counted by fours, eights and twos. And I am really impressed by that. And I counted in different ways that I have never knew how to count by before. And I think that I learnt a lot more maths stuff at this school. We counted by twos and we did doubles (at our old school), but we never understood it. And now I understand it. *Female student, Year 2.*

> I didn't know that you had to skip two numbers to count by threes (You learnt that and it made you proud?). Yep... The threes were kind of easy, but before I didn't know you had to skip two numbers to count by threes. *Female student, Year 2.*

Productivity

The theme of Productivity comprises three interrelated categories, each emphasising a specific aspect of how a student perceived themselves as being highly productive while working on the chosen task. Student responses were categorised under this theme if they made reference to completing or finishing the task (implying they got 'their work done'), the quantity of work they produced whilst engaged in the task, or the speed at which they produced this work.

Overall, over one-third (37%) of transcripts were coded to the theme of productivity. Although males appeared somewhat more likely to raise the issue of Productivity when reflecting on their work than females (44% vs 28%), this difference was not large enough to be statistically significant [X^2 (1, 73) = 1.920, p >.05].

Many students highlighted the fact that they had chosen a task because they had managed to complete it or finish it. This statement was generally made in the context of the student completing the task or worksheet either despite it being experienced as very challenging (i.e., demonstrating persistence), or in contrast to other pieces of work, which they were not able to finish.

> Because most of the Fiona the Frog challenges were quite challenging - like this one. Because I completed the challenge. *Female student, Year 1.*

> Because I finished it… I put one down, and it got bigger and bigger and harder. It got trickier and trickier - but I still finished it. *Male student, Year 2.*

> Because I finished it. It was easy. Easier than all the others. (How come that made you proud?) Because I could finish it. *Male student, Year 1.*

> I finished it all, and I got to do another piece, but I didn't finish that piece… I chose it because I like the math's sheet and I finished it in time. *Female student, Year 1.*

There were also frequent references to the quantity of work students had produced as a rationale for them being proud of the task..

> Because I was proud of me, and getting so much done… I got up to the eleventh one. *Male student, Year 1.*

> I am proud of it because I wrote the most things down I have ever done. *Female student, Year 2.*

> I was pretty proud of it because of all my counting. All that work using my brain to count by tens all the way up to 630. *Male student, Year 2*.

> Because I went on to another sheet and into the hundreds. It is hard for me to count in the hundreds. Because I need to go onto two sheets, onto two pieces of paper. *Female student, Year 2.*

Finally, the issue of the speed at which they managed to undertake the task was raised by several participants. Participant comments about speed often appeared in a specific context; for example, in connection to another identifiable characteristic of the task, such as its high level of challenge, or in connection to another category within the Productivity theme, such as the large quantity of work completed.

> Because I did it so quick and it was a challenging task. *Male student, Year 1*.

> I didn’t think I could do this many numbers in the time. Because I did this many numbers, and I felt proud of myself for doing them. *Male student, Year 2.*

> Well because I did it fast and I did it right. I got heaps of it done. Well I got to the super challenge and I think I did that. *Male student, Year 2.*

Meaningful Mathematics

This theme, termed Meaningful Mathematics, captures two interrelated aspects of student perceptions of the contents of the mathematical work they undertook within the program. The category of ‘real maths’ refers to comments by students implying that a particular task was more purposeful or meaningful in a mathematical sense, either in comparison to other tasks within the program, or maths at school more generally. By contrast, the category of ‘context’ emphasises that the student valued a particular task because of the context in which the mathematics was situated. The assumption is that the student chose this contextualised task in part because it was perceived as being more relevant and/or meaningful.

Although only 15% of participant transcripts were coded to this theme, making it the least prevalent theme overall, the notion of students valuing mathematical work because it is meaningful is of particular interest. The notion of ‘real maths’ in particular captures some very powerful insights from students about why they valued working on challenging tasks.

> I chose these because I think the worksheets were just for practice. And the challenges and the hint sheets were for maths. *Male student, Year 1.*

> Because it shows them how I do challenges my way. *Female student, Year 2.*

> Because the challenges stretch your brain further into maths... Because maths gets you very far. Like if you want to be a doctor. Like if your blood pressure is 600 degrees the patient could die. Same if it was lower. It needs to be exactly in the middle... I just chose these because it stretched your brain a lot. Maths is the main thing of work you need to do at school. It is more important than writing and art. Maths helps you do anything. It helps you do marine biology. Or if you want to be a marine - like you need to know how far away a bomb is... Because when you do challenges at maths, they help you do all sorts of things. Because those challenges are very important. *Male student, Year 2.*

> The challenges make me think about counting and stuff. And the help sheets only warm up my brain. They don't do what the challenges do for me... They (the challenges) make me think a lot. I am really really proud of them. I love counting. The challenges make me think hard and count really good. *Female student, Year 1.*

In addition, several students highlighted how the context in which the task was situated enhanced their motivation to engage with the task, and the pleasure they derived from working on it. Although some relevant examples emphasising the importance of context have already been considered previously under the theme of Enjoyment, some additional extracts from interviews in relation to this category are provided below.

> Because these (pointing to the consolidating worksheets) were just like counting by tens, and counting by fives, and counting by fours and counting by threes. And the challenges were like seeing if Fiona the Frog could make it to the other side of the river. *Male student, Year 1.*

> Because I like ants. They are the smallest animals. *Male student, Year 1.*

> I liked doing it. Because with the people you have to like chop them (in half) - and then you had to count how many there were each day. *Female student, Year 1.*

To summarise, student responses as to why they were proud of a particular work artefact were described by five themes: Enjoyment, Effort, Learning, Productivity and Meaningful Mathematics. Whereas Enjoyment reflected a single category, Effort encompassed the categories of having a go, conscientiousness and persistence. Learning described students either learning something new or trying something new, and disproportionately reflected the views of female participants. Productivity captured three interrelated categories, specifically the notion of taking pride in a work artefact because the task was completed, because a large quantity of work was produced or because the work was produced quickly. Meaningful Mathematics was the final theme discussed, and reflected students valuing work because it was presented in a rich context, or because the challenging task involved doing ‘real maths’, as opposed to more routine mathematical work. The implications of these findings are discussed later in the chapter. However, before proceeding to this discussion, it is necessary to first consider student reflections on the different lesson structures they experienced across the program.

Student Reflections on the Structure of the Lessons

The main thrust of the current thesis is to explore and contrast the impact of teaching with challenging tasks using two different lesson structures. Previous chapters compared the impact these different structures had on student outcomes (Chapter Five), as well as teacher perceptions of teaching with challenging tasks (Chapter Six). The purpose of this section is to examine student perceptions of the two lesson structures. In particular, it attempts to ascertain whether students had a preference for a given structure, and the rationale provided for that preference.

It was determined that only students in Class C (the “Alternating Group”) be asked to consider this particular issue. The rationale is that Class C students would likely be the group who were in a position to meaningfully compare the two approaches, having experienced both approaches regularly throughout the two units of work.

During the program, at the beginning of a Class C lesson, students were shown one of the two diagrams below (see Figure 7.2), corresponding to the lesson structure for that particular session. The primary purpose of exposing students to this diagram was to make the lesson routine for the day clear to them.

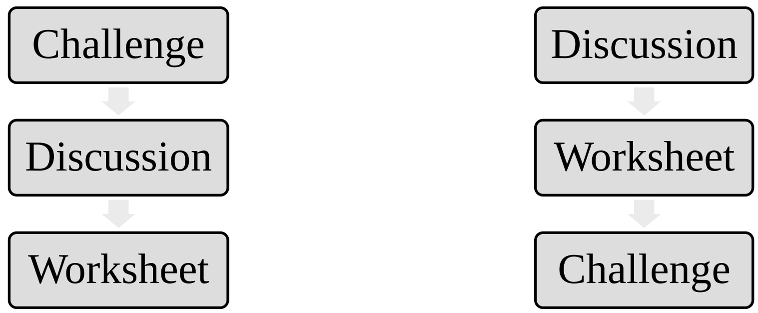

Figure 7.2. Diagram for Participants Summarising the Task-First Approach (left) and the Teach-First Approach (right)

During follow-up interviews with students in Class C, both diagrams in Figure 7.2 were shown to students. The intention was to provide students with a visual aid to remind them about the two contrasting lesson structures experienced during the program. Students were then asked the following question:

> Some of our classes began with the challenge first, then the discussion on the floor and then the worksheet, and some of our classes began with the discussion on the floor first, then the worksheet and then the challenge. Which did you like more? Why?

Following verbatim transcription of the responses provided by 23 student participants from Class C regarding their lesson structure preferences, and their rationale for such preferences, the analysis proceeded in two phases.[20] First, the nature of lesson structure preferences indicated by students is summarised and described. Following on

[20] Although 24 interviews were attempted, one response was not considered due to the student being non-responsive. In addition, one respondent was not available for the second interview (which was to be undertaken after the addition unit), and consequently his data has only been included from the first interview (after the patterning unit).

from this, the specific rationale for a given preference provided by students was subjected to Constant Comparative Analysis in order to distil a parsimonious set of themes that captured the explanations students provided for their preferences.

Describing student preferences

The first important finding is that individual students had clear preferences for either the Task-First Approach or the Teach-First Approach. During both interviews, all students put forward a definite preference for a particular lesson structure. In addition, almost all students were able to answer this question immediately. This suggests that, in general, student preferences were not constructed retrospectively during the interview, but rather reflected their actual lesson structure preferences experienced while they were engaged in the program.

Following the patterning unit of work, approximately half of participants indicated they preferred the Teach-First Approach (n=12; 52%) and approximately half of participants indicated they preferred the Task-First Approach (n=11; 48%). By contrast, after the addition unit of work, almost two-thirds of students specified they preferred the Teach-First Approach (n=14; 64%) over the Task-First Approach (n=8; 36%). Consequently, it is apparent that at least some students who preferred the Task-First Approach for the first unit of work undertaken preferred the Teach-First Approach for the second unit. Indeed, although most participants demonstrated a consistent lesson structure preference across the two units of work (9 Teach-First, 6 Task-First, n=15; 68%), around one-third of participants changed their preference when interviewed following the addition unit (n=7; 32%). This may suggest that lesson structure preference is in part driven by the nature of the material to be learnt.

In order to establish whether particular participant characteristics were related to a preference for a particular lesson structure, several cross-tabulations were undertaken for each of the units of work. Note that, due to the small sample size, two-sided Fisher's Exact Tests were employed instead of the chi-square statistic to assess the statistical significance of these relationships. This analysis revealed that, for the patterning unit, high performers (above median score) on the post-program assessment of mathematical fluency were more likely than low performers to prefer the Task-First Approach (69% vs 20%; $p = 0.036$, Fisher's Exact Test, 2-sided). Although this trend was also evident in the addition unit, it was not statistically significant (50% vs 20%; $p = 0.204$, Fisher's Exact Test, 2-sided). Similarly, although Year 2 students were ostensibly more likely to prefer the Task-First Approach than Year 1 students, particularly for the addition unit (47% vs 14%), this trend again was not substantial enough to be statistically significant ($p = 0.193$, Fisher's Exact Test, 2-sided). By contrast, there were no notable difference between males and females in their preference for the Task-First Approach across either the patterning unit (males = 46%, females = 50%; $p = 1.000$, Fisher's Exact Test, 2-sided) or the addition unit (males = 30%, females = 42%; $p = 0.675$, Fisher's Exact Test, 2-sided).

Exploring the rationale for student preferences

In contrast to the analysis of student reflections on their work samples, each student response was allocated to one specific theme. This was because students typically provided a single reason for their stated lesson structure preference. Data from the two interviews were analysed together because, although students frequently provided a similar rationale across both interviews for their lesson structure preference, this was not always the case.

Following the Constant Comparative Analysis of preference data, three themes (encapsulating seven categories) emerged describing why some students preferred the Task-First Approach. By contrast, only two themes (encapsulating three categories) emerged describing why some students preferred the Teach-First Approach (see Figure 7.3).

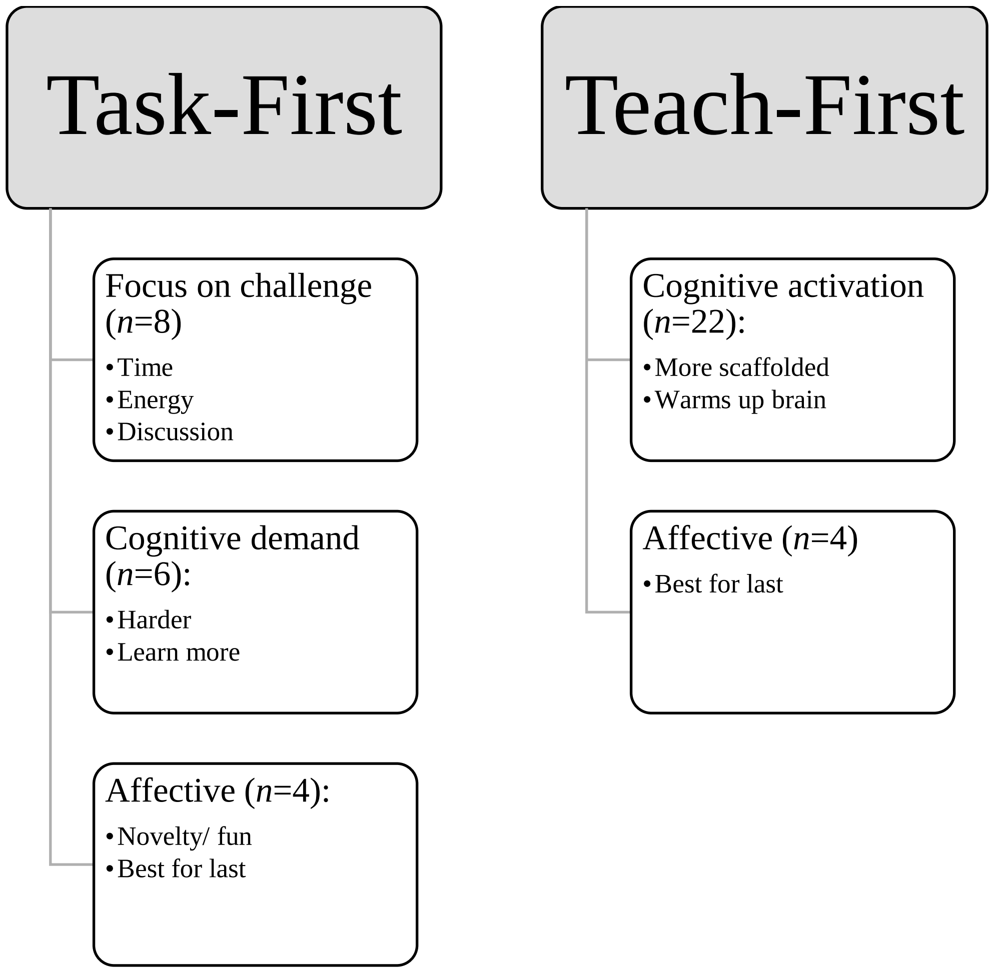

Figure 7.3. Rationales Provided by Students for their Lesson Structure Preferences.

Note: *n* represents the number of students who endorsed this theme during one or other of the two interviews. Two participants provided inadequate detail in their responses, and their responses could not be coded.

Rationale for preferring the Task-First Approach

There was a relatively broad variety of reasons provided by students as to why they preferred the Task-First Approach. Specifically, the three themes describing this preference were, Focus on challenge, Cognitive demand, and Affective.

Focus on challenge

The most frequent rationale for students preferring the Task-First Approach is that it allowed the focus of the lesson to be more explicitly on the challenging task. This demonstrates that a certain subset of students placed substantial value on being able to invest significant time, energy and effort in exploring, and subsequently discussing, the various challenging tasks.

Some students indicated that they appreciated beginning the lesson with the challenging task because they tended to have more energy earlier in the lesson, and they perceived that less time was available for the challenge if it occurred last in the session.

> Because if I had it the other way, it would be harder for me to do the challenge? (Why is that?). Because I would lose my energy from the worksheet. *Male student, Year 2, Patterning Unit.*

> Because I like doing the challenge first, because I like making my brain work good (hard) – and then it doesn't have to work too hard at the end. *Male student, Year 2, Addition Unit.*

One of the most interesting aspects of students valuing the focus on the challenge was the notion that this allowed the discussion component of the lesson to be built around work on the challenging task, which in turn provided opportunities for students to learn directly from their peers (as opposed to the teacher).

> I like the lesson (discussion) after the challenge, then you can have the lesson to actually work it out. Instead when the challenge is last, you only have a couple of minutes to work it out together. (Why?) So then I can find out how the other kids have done it. I like seeing the other ways they have done it, not just the one way I did it. *Female student, Year 2, Addition Unit.*

> Because you get more time to talk about what you have worked out, and you get more time on the challenge to think. Because you can explain – you can learn stuff. And that is how I got really good at maths. I listened to people and they told me strategies. And then you know I might be able to do it that way next time. *Male student, Year 2, Addition Unit.*

> Because you can talk about the challenge. Because then you know what the answer is, because other people talk about it. *Female student, Year 1, Addition Unit.*

Cognitive demand

Other students emphasised that beginning the lesson with the challenge was more cognitively demanding, and consequently preferable. This stance is particularly interesting because these students appeared to be effectively stating that they did not want their cognition to be activated prior to working on the task.

> (Why is it better to start with the challenge?). Because the challenges are harder first. Because when you do the worksheet that is easier, you get your brain a little bit started – and this makes the challenge easier. *Male student, Year 2, Addition Unit.*

> I like the challenges. (Prompt). Because I like the hard stuff. (Is it harder when it is first). Yeah (Why?). I don't know, I just think it is hard first. *Male student, Year 2, Addition Unit.*

In addition to what appeared to be an innate preference for working on a task they perceived as 'harder', two students also noted that they felt they could learn more when working on tasks they found difficult. A quotation from one of these students is included below.

> I like the challenge first because it was meant to be a challenge. You were actually supposed to do the challenge first and not cheat. I like them both, but if I had to pick one, I would pick the challenge first… I actually learn better with the challenge first. Because if you do the challenge, I start to get it in mind a lot - so I learn more quicker. *Female student, Year 2, Patterning Unit.*

Affective

Finally, some students indicated they preferred the Task-First Approach predominantly for more affective reasons. Specifically, two students indicated that beginning a lesson with the challenging task was more enjoyable, perhaps because such an approach was different from what they would regularly experience at school.

> Starting with the challenge was my favourite.(Why?) I don't know, it seems different. (Different from what you would normally do in class?) Yeah that's right, different. *Female student, Year 2, Patterning Unit.*

> Because like I was doing my best. The challenge was fun. It was really good for me. I like challenges. *Male student, Year 1, Patterning Unit.*

Two other students indicated they preferred the Task-First Approach because it allowed them to tackle the worksheet last, which was their favourite aspect of the lesson. This notion of 'saving the best for last' is a reminder that students provide complex and varied reasons for preferences, and that a preference for the Task-First Approach does not means that these students preferred the challenging task to the consolidating worksheet task.

> Because you can get it [the challenging task] over and done with and then you can do the lesson on the floor and then the worksheet. (What was the favourite part of your lesson?). The worksheets. *Female student, Year 1, Patterning Unit.*

> Worksheet last because then you get to do the fun things last. And then you can go back to the challenge if you had time. (What was the favourite part of your lesson?). The worksheets. *Female student, Year 1, Patterning Unit.*

Rationale for preferring the Teach-First Approach

Overall, the majority of students preferred the Teach-First Approach to the Task-First Approach. Moreover, and notwithstanding a small number of students who enjoyed saving the challenging task to the end of the lesson, students who indicated this preference justified their choice by making some form of reference to the notion of Cognitive Activation.

Cognitive Activation

Many students made an explicit reference to the notion that the initial teacher-facilitated discussion (i.e., the mini-lesson) and the opportunities to engage with the worksheet ‘warmed up their brain’, so they had the confidence and requisite understanding to tackle the challenging task.

> Because when I was getting ready for the challenge my brain was warmed up. *Female student, Year 2, Patterning Unit.*

> I just liked the challenge being last. Because with the worksheet it gets my brain working for the challenge. *Male student, Year 2, Patterning Unit.*

> Because the lesson could just warm up my brain a little. *Female student, Year 1, Addition Unit.*

> Challenge-first can be more challenging. The lesson first might be a bit easier. Because then if the challenge is first, your brain might not be ready, and you might get an answer wrong. *Male student, Year 1, Addition Unit.*

> Because the lesson is normally always easier than the challenge – so then my brain is warmed up for a harder challenge. *Male student, Year 2, Addition Unit.*

This notion of a student preferring to be ‘warmed up’ for the challenging task can be linked to the idea that many students found the challenging task difficult, and consequently placed a high level of value on the teacher-facilitated discussion and consolidating task to scaffold the challenge.

> Because the challenge was the hardest and I could do that at the end. The lesson on the floor and the worksheet, they helped a bit with the challenge. *Male student, Year 2, Patterning Unit.*

> So we could get some clues before we did the worksheet and the challenge. *Male student, Year 1, Addition Unit.*

> Because the challenge is a little bit hard. Because then you can explain it. *Female student, Year 1, Addition Unit.*

> Because the challenge is easier to work out if you have had something else to help you before it. Because with the lesson, say you show us a new strategy to make it easier, then that would help us with the worksheet, and that would help us with the challenge too. *Female student, Year 2, Addition Unit.*

Affective

Students also indicated that they preferred the Teach-First Approach because the Challenging Task was their favourite part of the lesson, and they enjoyed ‘saving the best for last’.

> I liked the challenge last. It’s my favourite. *Male student, Year 2, Patterning Unit.*

> Because whenever I am doing the challenge I really like it, and I sometimes like to have the things I really like last. *Female student, Year 2, Patterning Unit.*

> Because I like having challenges last, because it makes it more exciting. *Male student, Year 2, Addition Unit.*

To summarise, most students interviewed preferred the Teach-First Approach, although a substantial minority (43%) of disproportionately higher-performing students instead preferred the Task-First Approach. The general rationale for this preference amongst those students favouring a teach-first structure was that beginning with the discussion component effectively ‘warmed up their brain’. By contrast, a greater variety of reasons were provided for favouring the task-first structure, with the two most frequently mentioned being an opportunity to focus more on the challenging task, and the higher level of cognitive demand associated with this structure.

Discussion of Findings (Study 3)

Student reflections on their work

The current study provides evidence that students as young as seven and eight years of age embrace a high level of challenge when learning mathematics. The implication is that instructional approaches that rely heavily on problem-solving are in fact appropriate for primary-school age children, contrary to the views of some commentators (e.g., Westwood, 2011).

Many of the participants in the current study reported enjoying the process of working on a particular task precisely because they found the task difficult. This may in part be because a classroom culture supporting students to experience high control and high value in relation to their mathematical learning was fostered, enabling them to embrace the process of being intellectually challenged (Pekrun, 2006). For example, when working on the challenging task, students were provided with autonomy over how they approached the task and whether or not to access the enabling prompt. Moreover, they were encouraged to learn from and work cooperatively with peers, and had a clear understanding of how the overall lesson was structured. All of these characteristics were likely to promote high control. In addition, ensuring that mathematical discussions were student centred, and that the mathematical content was presented in rich, engaging contexts, may have supported students in experiencing what Pekrun (2006) termed 'high value'.

Although this notion that a problem is enjoyable to work on because it is hard or challenging is not particularly prevalent in the mathematics education literature, parallels can be drawn with research into student attitudes whilst engaged in playing and designing educational computer games focused around mathematical learning (Ke, 2014). In the current study, many students valued their work because it gave them a sense of being challenged mathematically, and were willing to 'have a go' at a task even if they initially perceived it to be beyond their ability level. Moreover, a substantial number of these students noted that they were proud of their work because they were able to persist, overcome obstacles and eventually make significant progress with the task. The notion that challenging tasks improve student persistence has been discussed elsewhere (Sullivan et al., 2013). This study adds to this literature by capturing how students themselves recognize and value persistence as an important quality when engaged in mathematical work.

The notion of enjoyment as a rationale for valuing a particular task was also linked to students deriving satisfaction from applying a specific mathematical process, such as doubling numbers or pursuing skip-counting patterns. This highlights the inherent joy many students took from engaging in mathematics within the current program of work. The notion that young students are naturally curious and enthusiastic about mathematics has been well recognized for several decades (e.g., NRC, 1989). In terms of the control-value theory postulated by Pekrun (2006), cultivating a love for mathematics and its application is likely to lead to students attaching high value to a given mathematical task.

In other instances, enjoyment of working on the task was attributed to the rich context in which the task was perceived to be situated. It is important to note that comments by students relating to context straddled both ‘real world’ scenarios to which students could relate (e.g., problems involving sport such as basketball and cricket), as well as highly contrived narratives which were perceived as meaningful to students (e.g., Fiona the Frog having to cross to the other side of the lake). Providing a story or context that students can relate to in some manner has been revealed elsewhere as an important component for engaging students in mathematical tasks (Clarke et al., 2014).

The two outcome-related themes to emerge from the analysis were described as Learning and Productivity. Specifically, when reflecting on their work, students valued both learning and experimenting with new mathematical concepts and processes, as well as being productive in terms of completing a task, and/or producing a substantial quantity of work in a limited time period. There was some evidence that females were more inclined to emphasize learning.

Finally, students’ valued engaging in meaningful mathematics. Although to some extent this reflects a desire to work on mathematical problems that are meaningfully contextualized, a particularly interesting notion to emerge from the current study was the idea that some students viewed working on challenging tasks as ‘real maths’, and distinguished it from more routine mathematical work. The emergence of this theme suggests that some students were beginning to connect mathematics to a purpose beyond its consideration as a mandatory classroom activity, and view it as relevant to both their individual identity as learners and the external world beyond the classroom. This finding is particularly promising for research in this area, as the notion of making mathematical work more meaningful and connected is consistent with the objectives of teaching with more cognitively demanding tasks (e.g., Sullivan & Mornane, 2013).

Student reflections on the structure of their lessons

Most students in the study preferred the Teach-First Approach when learning mathematics in lessons involving challenging tasks. According to these students, this was primarily because the teacher-facilitated mathematical discussion and the consolidating worksheets served as cognitive activators, effectively 'warming up their brains' so students were ready to work through the challenging task. Lower-performing students were disproportionately inclined to indicate that they preferred the Teach-First Approach, which provides further validity for the cognitive activation explanation offered by students. These observations are consistent with Pekrun's (2006) control-value theory of emotions in an achievement setting. When students are simultaneously given substantial autonomy over how they approach a task and provided with opportunities for cognitive activation, they are likely to experience a high level of control. When accompanied with a high level of value, this is theorized to generate academic enjoyment. By contrast, the comparative expertise of higher-achieving students implies that their sense of control is less likely to be undermined when confronted with a challenging task prior to any instruction (i.e., Task-First Approach). For these students, the activation of knowledge held in long-term memory may effectively substitute for knowledge provided from an external source (e.g., a teacher), which may explain their greater relative preference for the Task-First Approach.

Although fewer students indicated that the Task-First Approach was their preferred lesson structure, the explanations they provided for these preferences were of particular interest. Several students with this preference indicated that they valued the fact that the focus of the lesson was very much around the challenging task under this structure. This appeared to partially relate to students acknowledging that they have finite mental resources available, and would rather use their available ‘energy’ to work on the challenge. However, most compellingly, three students specifically indicated that they found discussing the mathematics after exploring the challenging task particularly important, as it provided them with an opportunity to learn from other students.

The potential power of the discussion component of the lesson following work on a cognitively demanding task to build students’ mathematical understanding has been noted elsewhere (e.g., Jackson, Garrison, Wilson, Gibons, Shahan, 2013; McClain, 2002; Stein et al., 2008). Indeed, ensuring that teachers possess both the pedagogical and mathematical knowledge to, in the first instance, value, and, in the second instance, facilitate, such a discussion has been viewed as a critical aspect of converting a task into a “worthwhile learning experience” (Sullivan et al., 2009, p. 103). However, the notion that students as young as seven or eight years old can identify that they themselves benefit directly from participating in such discourse is noteworthy, and contrasts with teacher concerns that even much older students struggle to meaningfully engage in whole-of-class discussions around mathematics (e.g., Leikin et al., 2006).

The other theme to emerge, Cognitive Demand, can be considered the antithesis of students preferring the Teach-First Approach because it supported Cognitive Activation. Specifically, it appears that that some students preferred the Task-First Approach precisely because the lack of discussion beforehand made it more challenging. This notion that 'hard is good', which exists in juxtaposition to the idiom 'help is good', reminds us that one size is unlikely to fit all within the context of mathematics education (Ridlon, 2009), and perhaps indicates that teachers should contemplate varying the structure of lessons on equity grounds. It is noteworthy that Sally, the teacher-participant of Class C whose students were interviewed for this aspect of the study, anticipated that her students had both distinct lesson structure preferences and different tolerances for ambiguity. In fact, she independently proposed that it is the responsibility of the teacher to vary the lesson structure to cater for the needs of a diverse range of students.

Finally, consideration of the Affective theme indicates that concluding that a student necessarily preferred the challenging task component of the lesson if they endorsed the Task-First Approach (and preferred the consolidating worksheet if they endorsed the Teach-First Approach) is too simplistic and inaccurate. Whereas some students indicated that they preferred the Task-First Approach because it was novel and exciting (perhaps implying that the challenging task was their favourite aspect of the lesson), other students indicated they preferred the Task-First Approach because it allowed them to postpone their favourite part of the lesson to the end; that is, the worksheets. A similar phenomenon of 'saving the best for last' was observed when analysing Teach-First preferences, although in this instance it involved students wanting to delay working on the favoured challenging task until all other work was completed.

Concluding thoughts

Ostensibly, there appears to be some tension between the two central findings of the current study. On the one hand, there is evidence that students value effort, embrace struggle, and persist when engaged in mathematics lessons involving challenging tasks, and moreover that many students enjoy the process of being challenged. On the other hand, the majority of students preferred the Teach-First Approach because it served to activate their cognition, thereby making the level of challenge more manageable. Although to some extent these findings reveal the views of different groups of students, this was certainly not exclusively the case, with many students desiring cognitive activation to reduce challenge and still valuing the process of struggling with a challenging task.

However, this apparent tension resolves if we consider that, for many students, it is likely that work on challenging tasks remained cognitively demanding irrespective of the structure of the lesson. Beginning with the teacher-facilitated discussion and some less challenging work, as is the case with the Teach-First Approach, appears to have allowed these students to approach the challenging task with more confidence and greater expectations of success. In terms of Pekrun's (2006) control-value theory, the opportunity for cognitive activation supports these students to gain more control over their learning. It may be postulated that if students continue to build their self-efficacy and self-belief in the domain of mathematics, they would increasingly be inclined to take risks and confront the unknown, and thereby be more embracing of the Task-First Approach.

CHAPTER EIGHT: SYNTHESIS OF FINDINGS AND CONCLUSIONS

The purpose of this chapter is to synthesise the findings from the three studies and propose directions for future research into how to teach with cognitively demanding tasks. The chapter is divided into two sections. The first section offers a synthesis of the findings contained within the three studies, in order to examine the broad question put forward in the introduction, specifically: What are the advantages of using cognitively demanding tasks to launch lessons and support subsequent instruction and discussion (task-first lesson structure) compared with using cognitively demanding tasks to extend understanding, following instruction and discussion, and the completion of several more routine tasks (teach-first lesson structure)? The second section will outline several potential directions for future research into this area, before offering some brief concluding thoughts.

Synthesising the Findings of the Three Studies

What have we learnt about the impact (and role) of lesson structure when teaching with cognitively demanding tasks? It is apparent that each of the three studies can shed its own particular light on this question.

Study One revealed that, when it comes to student mathematical performance, there is some evidence that a teach-first lesson structure improves mathematical fluency more substantially than a task-first lesson structure. Although this effect was only apparent across the addition unit and not the patterning unit, pooling data across both units of work revealed that the aggregate net impact of lesson structure on improvement in mathematical fluency was statistically significant, although the effect size was relatively small. By contrast, improvement in student problem-solving performance was unrelated to lesson structure. The key finding when examining the impact of the program on student mathematical performance was that differences over time were far larger than differences between groups, suggesting that participation in the program in *any* capacity (i.e., using an Alternating, Teach-First, or Task-First Approach) was substantially more important than participation in a *particular* lesson structure. It was concluded that teaching with cognitively demanding tasks is effective, regardless of how the lessons are structured.

By contrast, both intrinsic motivation to learn mathematics and task-based student persistence appeared unrelated to lesson structure, although this may have been in part a consequence of limitations regarding the specific measures employed to assess these constructs. With regard to intrinsic motivation to learn mathematics, the key observation was that the young students in the current study were highly motivated to learn mathematics *before* they even began participation in the program. The associated ceiling effect may have undermined the capacity for this measure to detect differences in motivation as it related to lesson structure. By contrast, the two-dimensional student persistence measure proved difficult to administer, limiting any conclusions that can be drawn from this data. Specifically, it may have inadvertently measured (at least partly) self-reported student persistence with the process of engaging with the questionnaire, rather than their work on the associated problem-solving task.

Study Two revealed that teachers perceive learning through cognitively demanding tasks to foster student persistence, irrespective of how the lesson was structured. This in part reflects teachers' perceptions that the two units of work were experienced by most students as highly challenging, and that students were viewed as responding very positively to this high level of challenge. Having said this, there was some indication that the task-first structure was perhaps superior for building persistence, and that, more generally, both structures appeared to possess particular strengths. Specifically, in addition to enhanced persistence, the task-first structure was perceived by teachers as supporting mathematical creativity and empowerment, engaging students, promoting meaningful discussion and reflection, and providing an opportunity for rich, authentic assessment. By contrast, the teach-first structure was viewed as more efficient, focussed and offering greater opportunities for less confident students to be mathematically successful. The three teachers held conflicting views in terms of how they would teach with cognitively demanding tasks in the future. Whereas Rachel preferred the greater efficiency and opportunities for lower achieving students associated with the teach-first lesson structure, Polly indicated that she viewed her students as responding better to the task-first lesson structure, although noted that distinguishing lesson structure from a ‘practice effect’ across the two units of work was difficult. Sally, whose class experienced the alternating structure across both units of work, remained ambivalent about which structure to endorse. She concluded that the onus is on the teacher to vary the manner in which learning is structured, having observed that some of her students appeared to thrive when the task was first, whilst other students responded better to instruction which began with a teacher-led discussion.

Indeed, this observation that students may possess idiosyncratic preferences for a particular lesson structure was borne out in Study Three. Whereas most students interviewed preferred the teach-first lesson structure, a substantial minority of disproportionately higher-performing students instead preferred the task-first lesson structure. The general rationale for this preference amongst those students favouring a teach-first structure was that beginning with the discussion component effectively ‘warmed up their brain'. By contrast, a greater variety of reasons were provided for favouring the task-first structure, with the two most frequently mentioned being an opportunity to focus more on the challenging task, and the higher level of cognitive demand associated with this structure.

A key implication to emerge from this suite of studies is that the findings do not support the assumption that for students to learn from cognitively demanding tasks, lessons must begin with these tasks. They instead suggest there is more than one way of incorporating challenge tasks into mathematics lessons to produce sizeable learning gains. However, this does not imply that the Task-First Approach and the Teach-First Approach generate equivalent learning experiences for students. Specifically, taken together, the three studies provide distinctive, contrasting portrayals of the two approaches. The teach-first lesson structure can be described as a highly focussed, efficient approach to learning that effectively activates prior knowledge and provides opportunities for students to be successful and to feel suitably supported. On the other hand, the task-first lesson structure can be described as a highly dynamic, explorative approach to learning that effectively maintains a high level of cognitive demand and provides opportunities for students to be mathematically creative and to feel suitably challenged. Consequently, given that each approach has apparent advantages over the other, it is recommended that early primary teachers give consideration to incorporating both approaches into their future mathematics instruction when teaching with challenging tasks.

Strengths, Limitations, Conclusions and Future Directions

There were several strengths to the current investigation that warrant highlighting. Firstly, the investigation made every effort to ensure that both the teach-first and task-first learning experiences were of a similar, high quality. All lessons were delivered by the same teacher-researcher who had specific experience in specialised primary mathematics instruction, and all learning materials were developed in direct consultation with experts in curriculum development (i.e., the author's two PhD supervisors). This is important given that choice of comparison group, and the manner in which the comparison group is constructed, can have a substantial impact on resultant study findings within evaluations of discovery-based instruction (Alfieri et al., 2011). Secondly, the experimental design ensured that all students experienced both the Task-First Approach and the Teach-First Approach, and provided confidence that differences in student outcomes and experiences across groups were driven by lesson structure and not an unaccounted for, extraneous variable. This is arguably particularly noteworthy given the relative lack of experimental research within mathematics education, with as few as three percent of studies adopting experimental designs (Alcock, Gimore, & Inglis, 2013). Thirdly, the investigation took place in actual classrooms, with all the complexities and challenges associated with attempting to conduct a mathematics program within the reality of a school, thereby adding to the ecological validity of the investigation. This can be contrasted with much of the experimental research in mathematics education, which tends to remove students from classrooms so they can receive some form of one-on-one, or sometimes small-group, intervention in a controlled, yet necessarily contrived, setting (e.g., DeCaro & Rittle-Johnson, 2012). Fourthly, the mixed method design allowed the issue of teaching with cognitively demanding tasks to be examined from a number of different perspectives, including teachers, participating students and student outcome data. Most other studies which have explored this topic have tended to consider it through a single lens (e.g., Sullivan, Borcek, Walker, & Rennie, 2016) or, at most, from two different perspectives (e.g., Sullivan & Mornane, 2013).

Conversely, the investigation did also have a number of limitations, some of which were directly linked to its strengths. The single-school, single-teacher investigation design, whilst enabling the implementation of the program to be carefully controlled by the researcher, limits the generalisability of the findings. In addition, the relatively small sample size (n=75 across three groups) limited the statistical power of the quantitative aspect of the investigation (Study One), meaning that it was likely only medium and large effect sizes could have been detected, potentially masking subtler differences in performances between the three groups. Finally, as the program was implemented in a real-world school setting, it was not possible to control for all factors that may have differentially impacted the three groups over time (e.g., evolving classroom dynamics).

Beyond addressing the limitations of the current investigation (e.g., larger sample, incorporating multiple teachers/ schools), there are several directions for future research suggested by the investigation findings. One interpretation of the analysis is that young students may experience what was intended to be enhanced discovery-based learning as unassisted discovery, in part due to their lack of metacognitive skills and capacity to effectively self-regulate their learning (Flavell, 2000; Kuhn et al., 2000). This provides a possible caveat to the conclusion arising from the Alfieri et al. (2011) meta-analysis that enhanced discovery-based learning is superior to more explicit approaches to instruction, which is in turn superior to unassisted discovery. It might be that determining what constitutes enhanced discovery vis-a-vis unassisted discovery depends on the interaction between the learning material, structural characteristics of the learning environment and the learner; what is constructed as an enhanced discovery-based experience from one learner's perspective may be interpreted as an unassisted discovery-based experience from a different learner's point of view. Consequently, future research might endeavour to more explicitly account for the extent to which discovery learning is experienced as enhanced or unassisted from the perspective of learners themselves.

In addition, it has been contended that the relative advantage of more explicit teaching approaches compared with discovery- or inquiry-based learning depends on how clearly structured a particular learning domain is (Spiro & DeSchryver, 2009). Indeed, this may have been one of the motives for Alfieri et al. (2011) including subject-matter areas as a moderator in their meta-analysis. The current investigation adds to this discussion by emphasizing that it is perhaps not sufficient to consider domains as aggregate subject-matter areas, as other research has done, but that the structure of the learning materials within a given subject-matter area is also critical. Future research could endeavour to build on this hypothesis by comparing task-first performance with teach-first performance in a single area of mathematics instruction, such as multiplication, and create two otherwise identical conditions, one which was clearly-structured (e.g., instruction focusing on number sentences) and another which was somewhat ill-structured (e.g., instruction focusing on worded problems).

With regards to student perceptions of cognitively demanding tasks, the current investigation suggested that beginning with the teacher-facilitated discussion and some less challenging work, as is the case with a teach-first lesson structure, may allow students to approach a cognitively demanding task with more confidence, and thereby have more success with their learning. It was speculated that if students continue to build their self-efficacy and self-belief in the domain of mathematics, they may be increasingly inclined to take risks and confront the unknown, and thereby be more embracing of a task-first lesson structure. Future research could examine this issue longitudinally. Specifically, it could explore whether students whose regular mathematics instruction frequently incorporates challenging tasks have converging lesson structure preferences, such that they prefer to tackle the challenging task first as their confidence and competence as mathematicians increases.

To conclude, although there seems little doubt that reformist-based pedagogical approaches are associated with higher levels of mathematical performance (e.g., Jong et al., 2010), prior research in this area has not sufficiently unpacked reformist pedagogy and examined its component parts to see whether all these aspects are necessary for generating superior performance. Study One found that the specific lesson structure adopted was not particularly important for student performance outcomes. However, to the extent that differences in performance levels could be explained by lesson structure, the teach-first structure, encompassing a more explicit teaching approach, resulted in greater gains in mathematical fluency than the task-first structure. The key finding, however, was that teaching with cognitively demanding tasks appears to be effective for improving student mathematical performance, regardless of whether such tasks are used to launch lessons, or to extend student understanding.

Study Two revealed that teachers perceived their students to react positively to the use of more cognitively demanding tasks during mathematics instruction, irrespective of how the lesson was structured. Students were seen to demonstrate substantial persistence and engagement whilst making considerable progress mathematically. However, although both the task-first lesson structure and the teach-first lesson structure were perceived by teachers as efficacious, they were described as having distinct strengths; with the former offering greater challenge and creativity, and the latter offering greater support and learning efficiency.

Study Three, which considered the students' perspective, added to this complex picture. On the one hand, there was evidence that students valued effort, embraced struggle, and persisted when engaged in mathematics lessons involving cognitively demanding tasks, and moreover that many students enjoy the process of being challenged. On the other hand, the majority of students preferred the teach-first lesson structure because it served to activate their cognition, thereby making the level of challenge more manageable. Consequently, it is likely that work on such tasks remained cognitively demanding for many of these young students irrespective of how their learning was structured.

A key corollary of this suite of findings appears to be that teaching with more cognitively demanding tasks in any capacity constitutes a significant departure from how mathematics is typically experienced in schools, at least for participating students and teachers in the current investigation. Moreover, teaching with more cognitively demanding tasks improves both mathematical fluency and problem solving performance, regardless of how the corresponding lesson is structured. Consequently, teacher-educators should continue to encourage and support teachers to incorporate such tasks into their mathematics instruction, even in the early years of primary school. Part of the role of outside expertise, such as teacher-educators, would appear to be to continue to supply teachers with suitable cognitively demanding tasks, whilst perhaps initially allowing teachers to structure lessons around these tasks in a manner in which they are most comfortable. Although there seems little doubt that the task-first and teach-first lesson structures have distinctive strengths and generate different learning experiences, it is difficult to prescribe one particular structure over another based on the results of the current investigation. Such a determination likely depends on the skill, personality and knowledge of the teacher, the nature of the mathematical material to be learnt, the specific learning objectives emphasised during the particular lesson (or suite of lessons), and the preferences, personalities and mathematical ability of students. Ideally, teachers would strive to provide students with opportunities to experience both types of lesson structures when planning lessons incorporating challenging tasks.

CHAPTER NINE: TEACHER-RESEARCHER REFLECTIONS

The purpose of this chapter is to reflect on my role as a teacher-researcher in the current project. It emphasises what I have learnt through the experience of teaching these two units of work built around challenging tasks to young students.

This chapter is divided into two sections. In the first section, I reflect on a number of facets of my experience delivering the two units of work, paying some particular attention to whether and how the different lesson structures impacted my experience in my role as teacher. The second section outlines some potential modifications I would consider making to the units of work, and the specific challenging tasks developed, if I was to teach this program again in the future.

The Teaching Experience in Retrospect

During the delivery of the two units of work, I kept a daily journal, capturing both my observations in my capacity as researcher, and my teaching reflections in my capacity as an educator. Given that entries into this journal were undertaken prior to interviews with teachers and students, and post program student outcome assessments, these journal entries can be considered largely independent of other sources of data collected throughout this project. As such, they offer a unique perspective into the program, making this section recommended reading for any prospective researchers (or teachers) interested in delivering either or both of these two units of work in the future. Rather than focus on student learning outcomes (e.g., mathematical performance, student persistence), and the perceived student experience of the program, which have been covered fairly comprehensively in Chapters Five through Seven, my analysis of the journal entries revolves around the first-hand experience of designing and delivering the program.

My reflections are organised around four particularly salient themes which emerged following review of my journal entries. The issue of lesson structure is not addressed separately, but rather is discussed insofar as it is embedded within each of these themes. These themes include: classroom management, maintaining and managing cognitive demand, time management, and tensions between competing discussion objectives. In order to illustrate and elaborate on some of the more complex themes, extended quotations from my journal are included in the chapter when deemed particularly relevant.

Classroom management

In general, having well established routines and a clear, consistent lesson structure appeared to aid with classroom management, even in instances when students were engaged with challenging work. The advantage of clearly established routines and structures is that they enable students to psychologically and practically prepare themselves for what will be demanded of them in the lesson. Consequently, it was perhaps not surprising that, within each unit of work, classroom management issues, such as the time students took to transition between activities and general off-task behaviour, were, for the most part, less of a concern as the unit progressed, despite the content of the lessons becoming progressively more challenging.

Key routines and structures that were consistent across all lessons included: a) ensuring that all potential resources (e.g., bead-frames, hundred charts, enabling prompts) were located in the same place each lesson, b) at the beginning of each lesson, directing student attention to the electronic whiteboard for the launch of the task (Task-First Approach) or for the teacher-led discussion (Teach-First Approach), c) displaying and stating the primary learning objective on a piece of A3 paper both at the beginning of the lesson and during the summary phase, d) explaining to students that each lesson would contain three essential phases, that is, a whole-group discussion, a worksheet containing more routine tasks and work on a challenging task, and outlining the order in which these phases would occur.

With regard to classroom management and lesson structure, lessons involving the Teach-First Approach generally felt calmer and more controlled than equivalent lessons involving the Task-First Approach. The Teach-First Approach also tended to feel ‘safer’ from a teacher’s perspective as the lessons generally unfolded in a more predictable manner. A key differentiating aspect was that, under the Teach-First Approach, the material included in the teacher-led discussion was largely developed prior to the delivery of the lesson (see Appendix A and D). By contrast, under the Task-First Approach, although student responses could be partly anticipated in advance, both the structure of the discussion and the specific student work samples which supported the discussion were generally determined during the lesson.

However, there were also benefits to teaching with the Task-First Approach with regards to classroom management, provided that classroom management objectives are not conceived of narrowly as being limited to pursuing control, order and predictability. Specifically, it was my experience that the Task-First Approach tended to generate a classroom environment that was more chaotic and unsettled but simultaneously more dynamic; with greater energy, spontaneity, participation and engagement.

Having drawn these distinctions, it is also worth noting that classroom management issues differed notably by cohort, independent of lesson structure. As I noted in a journal entry during the addition unit:

> Across both units of work, Class B are the quietest group on the floor (as in, the less likely to contribute and engage in the discussion) and the loudest group when working, which seems a particularly unsatisfying combination from a teaching point of view. Within Class B, there are perhaps seven students – around one-third of the class – who regularly, voluntarily contribute to discussion time. It would be no exaggeration to say that this number is at least double in the other two classes.

Maintaining and managing cognitive demand

Strongly related to the theme of classroom management was the notion of maintaining and managing cognitive demand, particularly when the lesson began with the challenging task (Task-First Approach). I was prepared for this issue, having read literature prior to delivering the program outlining teacher discomfort with student struggle, and, in particular, perceived and actual pressure from students to reduce the level of cognitive demand (e.g., Darragh, 2013; Henningsen & Stein, 1997; Tzur, 2008). Despite this knowledge, there were certainly occasions, particularly early in the program, where emergent classroom management issues, such as those arising from multiple students verbalising their confusion and frustration with a task, tested my resolve and commitment to maintaining a high level of cognitive demand. For example, I had two particular students in two different groups, who were more than capable mathematically, but would frequently act (e.g., throw objects around the room, have tantrums) when they perceived a task or concept as too difficult. There were occasions when I was severely tested not to over-explain a novel task during its launch, particularly in the patterning unit of work. As I noted in my journal:

> This lesson (i.e., first Hundred Chart Challenge) provided another example about the anxiety I, in my role as teacher, can encounter when I take students outside their comfort zone, and some yell out things like ‘I can’t do it’. It probably resulted in me providing some further ‘priming’ to each group, essentially carefully reading the task again to emphasise its various component parts, and suggesting they tackle the task one step at a time. This is support/ advice which I wasn’t initially intending on providing. I will hopefully stay strong in the next session, given that students should now be familiar with the workings of these types of problems.

This commitment to maintaining a high level of cognitive demand, and the associated challenge of doing so, provides further justification for the decision to have the program implemented by the researcher, rather than by the respective classroom teachers. As I noted in another journal reflection:

> With the first Fiona the Frog challenge, I ‘primed’ all the groups by letting them know that if they could count by ones, twos or threes it might be a help in solving this problem. In this manner, I connected the problem somewhat to prior learning, although within very definite constraints (e.g., I did not outline how to begin counting by threes). This is obviously a tough line to walk, and I think walking this line would be even more challenging if I was their regular classroom teacher and not approaching the program as a researcher (where I have an a priori commitment to maintaining cognitive demand). This is perhaps another reason why this study wouldn’t work as well if I got the teachers to implement the lessons; there would just be far too much variability in terms of how they were implemented (i.e., variability amongst teachers in their level of comfort with student discomfort, and how much they ‘told’ students about how to approach the task).

Time management

Although there are benefits to teaching within a highly structured program (as noted under Classroom Management), one of the corresponding challenges to such an approach is the issue of time management. The current program was no exception. My aim was to spend between 10 and 15 minutes 'on task' for each of the three major components of the lesson, and around 2 to 4 minutes on the lesson summary. Although my records demonstrate that this objective was, on average, achieved (see Tables 4.5 and 4.6 in Chapter Four), the need to carefully 'watch the clock' to ensure that sufficient time was allocated to each aspect of the lesson added significant strain to the teaching experience, particularly in the early stages.

More specifically, during several lessons within both teaching structures (i.e., Task-First and Teach-First), I felt that insufficient time was allocated to the challenging task segment of the lesson in particular. By contrast, it was often difficult to maintain the flow of the discussion component of the lesson beyond the initial 7 to 10 minutes. Indeed, many students appeared to 'tune out' for the last 5 minutes or so of the discussion, regardless of whether the discussion occurred at the beginning of the lesson (i.e., Teach-First Approach) or after work on the challenging task (i.e., Task-First Approach).

In addition, the highly structured nature of the program and the corresponding time constraints were at times significant barriers to spontaneously exploring important mathematical ideas as they arose, and building on momentum generated within a lesson. Consider this example recorded in my journal during the patterning unit when counting by fours was first introduced in Class B (recall that Class B experienced the patterning unit through the Teach-First Approach).

> Despite the lesson feeling somewhat overwhelming, the students did a good job of engaging with the challenging task and picking up the counting by fours patterns. I was very impressed by the quality of the student contributions during the Class B discussion which began the lesson. The students independently highlighted many of the patterns associated with counting by fours, as we generated and discussed this counting sequence on a hundred chart. For example:
>
> - "It counts every second number, so it is counting by 20s up and down" (Lochie)
> - "You have to skip three numbers" (Penelope)
> - "None of the numbers have any neighbours; they're smelly numbers" (Abby).
> - "You only count even numbers… you count every second even number" (Ash)
> - "There are two numbers on one row, and three numbers on the next row" (Teddy).
>
> Ordinarily we would have run with this momentum and created some form of anchor chart pointing out what we notice when we count by fours. This would have allowed student ownership over many of the ideas highlighted (e.g., they could have put their names next to a particular idea with their contributions). This is one of the frustrations of running such a tight program in a research context – it can sometimes go against my teaching instincts. The mini-lesson had already run to 20 minutes, which was longer than desired, and it was very necessary for the students to move on to the worksheet (i.e., 'routine tasks').

Time management was in many ways a particular challenge during the teach-first lesson structure. This was the case for several related reasons, many of which revolve around not being able to discuss the challenging task in any depth. Firstly, during such lessons, there was not sufficient space to pursue interesting threads that emerged after students had grappled with the challenging task. Secondly, it was certainly limiting (and somewhat dissatisfying, from both a teacher and student perspective), to have the teacher summarise how one may have gone about solving the challenging task. Even though samples of student work were used during these summaries, it was difficult to incorporate meaningful student input, given the tight time frames during this final phase (i.e., two to four minutes). Thirdly, as this final part of the lesson is intended to function as a summary, it was not appropriate to explore student misconceptions that only became apparent once students had engaged with the challenging task. As is evident from the following journal entry, this inability to address misconceptions sometimes felt like a missed learning opportunity.

> With the Task-First Approach, there is more of a chance to address student misconceptions, because you can use student responses to explicitly discuss these when they are most relevant (i.e., when they are tackling the task). For example, during a task-first lesson, Alannah counted by threes by generating the following pattern 3, 6, 9, 13, 16, 19 etc when working on the challenge. She concluded that Fiona the Frog wouldn't land on any lily pads if she hopped by threes, and therefore she should hop by twos. Although she was 'correct' in her answer (i.e., hopping by threes does not allow Fiona to land on enough lily pads), it was for the wrong reasons. However, because of the discussion after the task, we could examine whether this approach made sense, why Alannah might have thought it did in the first instance, and a strategy for getting around this (i.e., looking for diagonal and not vertical patterns when counting by threes; making sure we skip over two numbers, no less or no more). By contrast, when this same misconception emerged during a teach-first structure (i.e., when Jill was working on the challenging

task), there was no time to address it in a meaningful manner. This was amplified by the fact that, like Alannah, Jill got the ‘right’ answer – and therefore may have even had her misconception validated during the lesson summary.

Tensions between competing discussion objectives

When delivering the current program, I experienced a clear tension between, on the one hand, leading students towards important mathematical ideas, and ensuring such ideas were appropriately articulated, and, on the other hand, letting students’ authentic engagement with the task, and the students themselves, drive the discussion. A similar tension also existed between emphasising the most efficient student solution and celebrating the diversity of student responses, many of which were novel, idiosyncratic in terms of process and thus very much ‘owned’ by a particular student. In broad terms, such tensions can be framed as the need to ensure that discussion is instructionally-optimised coming into conflict with an attempt to ensure that the discussion is appropriately student-centred.

Although such tensions are not novel to this program, with similar dialectics being highlighted in several other studies and commentaries (e.g., Baxter & Williams, 2010; Lobato, Clarke, & Burns, 2005; Stein et al., 2008), they remain considerably challenging to manage. Attempting to identify and synthesise these tensions whilst in the act of teaching is extremely cognitively demanding, and I think is a powerful reminder that teaching as a profession is so complex that it will always be considered as much an art as a science.

It is noteworthy that these tensions existed, to some extent, regardless of how the lesson was structured. In either instance, there was a constant need to balance the desire to achieve a specific learning objective with allowing students to take ownership of the discussion component. As I noted in my journal after teaching a task-first lesson early in the patterning unit:

> Today we tackled the second Fiona the Frog problem. At the moment, with the Fiona the Frog problems, the discussion of the solutions has been more teacher-led than I intended, albeit with substantial student input. This is for a few reasons. Partly it is the complex structure of the problem (the lily pads, the lake) which seems a barrier to granting full control to student explanations (in contrast to more simply structured tasks like 'how many fingers in the room'). Also, there is essentially only one right answer to these problems, and many students seem to be approaching these problems in similar ways. Consequently, in order to 'build the explanation', I feel compelled to take charge and explore why counting by ones is too slow and why twos doesn't work (at least not for this task). It is not that I am stating the solution, but rather that I am leading students down a very particular pathway by asking specific questions of them which do not necessarily reflect how they went about solving the problem. I was conscious today that if I did not set the direction, the notion that counting by twos beginning on an odd number results in becoming 'stuck' on the odd number (which is an important mathematical idea, and one of the key learning objectives of the lesson) would not have emerged. This is largely because many of the students chose to count by threes first, either because they intuited that it was likely to be the correct answer, or because counting by the higher number first (and then ruling it out if necessary) is the most efficient strategy. In more general terms, sequencing the work for the task-first condition so the mini-lesson flows can be quite challenging, and is very dependent on the particular student being able to articulate their thinking clearly. I can make the mistake of over-estimating some students' capacity to explain their strategy use (e.g., Ashleigh).

Not surprisingly, synthesising these tensions and effectively coordinating the discussion component of the lesson was more demanding when I was unwell. As I noted during one of my reflective entries during the addition unit.

> Another lesson teaching sick. This is definitely impacting my energy and my ability to think on my feet and make connections between student responses and to the underlying mathematical concepts. Possibly the most challenging aspect is thinking about the various student approaches, and selecting the three or four students with the most noteworthy approaches to share. This is partly because I have less energy to get around to all the students, and partly because actually reflecting on the appropriate sequencing is already the most challenging aspect of the lesson.

Task Design in Retrospect

I now reflect on two aspects of the task design process following the delivery of the program. First, I consider some general observations about the suite of tasks delivered within each unit of work, and put forward some potential modifications in relation to the overall structure of the units of work. Second, I retrospectively examine how some of the specific tasks could potentially be modified to better realise their learning objectives. These suggested modifications are consistent with the CLASS Challenging Task approach outlined in Chapter Three.

General observations about the overall structure of the program

Tensions between new concepts and consolidation

Across both units of work, there was an ongoing tension between attempting to introduce new concepts and providing sufficient opportunity for students to consolidate their understanding. Although I was careful to build in what I believed to be adequate repetition and revision, with the benefit of hindsight, both units of work could have been potentially improved.

With regards to the patterning unit, in retrospect, it was determined that it would have been prudent to introduce counting by threes and fours using more tangible demonstrations (e.g., counting eyes and noses; counting arms and legs), whilst developing corresponding challenging tasks that embodied concrete representations. The premise is that students should be given at least one opportunity to explore a given counting pattern focused exclusively on counting a collection of objects, before any less concrete representations are introduced. As the unit was currently structured, some patterns (e.g., threes and fours) were first encountered by students in the somewhat more abstract context of a hundred chart. This change would involve introducing two additional lessons into the program; however, the overall length of the program could remain at sixteen lessons if the two mixed counting pattern lessons were not retained (see *Modification to specific tasks*).

Similarly, with the addition unit, even though the strategies introduced and the associated learning objectives were all interrelated, the level of abstraction involved in partitioning and recombining numbers was substantial. Consequently it may have been prudent to include additional challenging tasks (with associated lessons), which provided more concrete representations of these strategies to students. For example, a problem similar to 9+9+9+9+9+9+9+7= could have been presented to students as a series of incomplete ten frames containing counters (see Figure 9.1). The enabling prompt could then have reorganised the final addends, that is, the seven counters, so that each counter was directly under each of the ten frames. Perhaps an additional two lessons to begin the unit of work could have been developed drawing on these more concrete representations. Moreover, these more concrete representations could have also been built into the enabling prompts for the remaining lessons (i.e., rather than relying solely on the more abstract partitioning strategies presented by Maths Man).

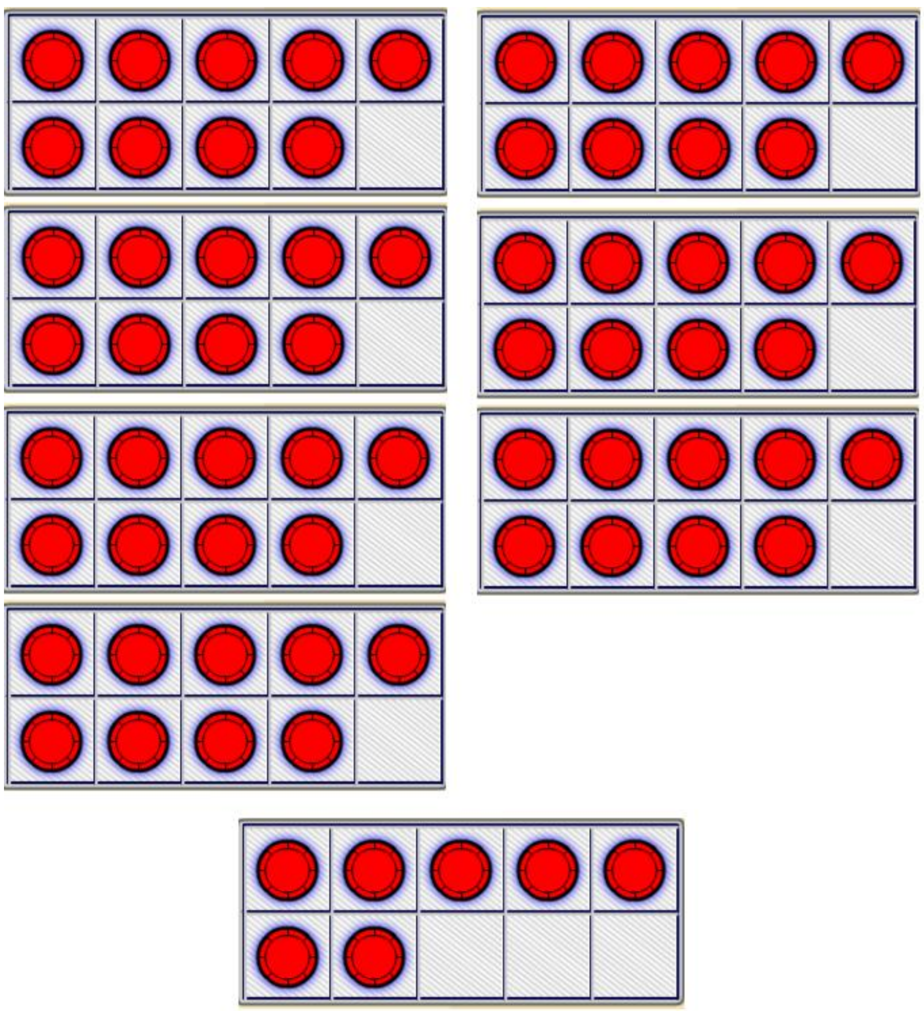

Figure 9.1. A More Concrete Representation of the Equation: 9+9+9+9+9+9+9+7=

Tensions between extraneous cognitive load and engagement

An issue which was anticipated in the Task Design chapter, and which played out in practical terms, was the tension between introducing (largely) extraneous cognitive load when students first encounter new problem contexts and structures, and the need to engage students through offering interesting and varied problem contexts. Although this was less of an issue during the more consistently structured addition unit, it arose frequently during the more novel patterning unit. Consistently across the patterning unit, the first lesson adopting a particular theme was generally experienced as difficult and demanding *to teach and learn*, as many students appeared to struggle to understand what the challenging task entailed, before they even considered what was required of them mathematically. However, subsequent tasks with a similar structure were often experienced as relatively straightforward. Two of the more difficult to comprehend themes were the Fiona the Frog tasks and the Hundred Chart Challenges, which arguably justifies the fact that each of these themes comprised three lessons, whilst the other themes involved only two lessons. Consider three related journal entries recorded during the corresponding Fiona the Frog challenging tasks.

> Challenging Task 1: As a group, students were frustrated with the Fiona the Frog problem. Comprehending the structure of the problem was a clear challenge.
>
> Challenging Task 2: Overall the lessons ran more smoothly today, which reflected the fact that students were more familiar with the context of the Fiona the Frog challenge, and also the fact that the routines are more established.
>
> Challenging Task 3: Most of the students had success with the challenging task activity today – they've certainly come a long way since Monday! Again, it shows the importance of routine, and building on similar problem

structures. Students adapt and absorb – they learn the rules of the game (if these are clearly stated), and internalise them.

Modifications to specific tasks

Although I would anticipate not making any significant alterations to the specific tasks included in the addition unit if I were to deliver the unit again, there were two sets of tasks within the patterning unit that I would likely adapt. The first set of tasks I would consider modifying are the two 'how many' tasks which began the unit of work, and the second set, the two mixed counting pattern tasks that concluded the unit.

Potential modifications to the 'how many' tasks

The first set of tasks to be considered are the two 'how many tasks'. Recall that the tasks asked students to "Without leaving your seat, or talking to anyone, can you work out how many feet (fingers) are in the room right now? How did you count them? Can you count them a different way? What do you think is the easiest way to count them?

I originally anticipated that the fluid nature of what was being counted would be a strength of the task, as students coming and going from the room (e.g., bathroom breaks) would redirect attention away from finding a 'correct' answer and towards exploring skip-counting patterns. Indeed, in general, this was what I observed during the session. However, in retrospect, I may have considered providing students with more support around problem representation in order to further reduce extraneous cognitive load in line with the CLASS Challenging Task process. Although appropriately representing the problem was not the central focus of the task, it remained a challenging aspect for some students, and may have undermined their capacity to focus on the primary learning objective (i.e., skip-counting). One means of removing this source of extraneous cognitive load would have been to instead provide a photograph of a large group of students (more than 15, less than 30), and ask students to calculate the number of feet (Task 1) and hands (Task 2) in the photograph.

However, some may argue that this modification makes the problem somewhat less engaging to students and perhaps less mathematically interesting. Although this position is certainly defensible, adopting the CLASS Challenging Task process would suggest that it is still necessary to change aspects of the lesson, specifically the learning objectives. The tasks, therefore, could have been retained in their original form and reframed, with the problem representation aspect now being considered central to the tasks. However, to signify this change, the primary learning objective would need to be reimagined, and restated as something like: ‘To find a quasi-abstract or abstract means of representing a concrete counting-based mathematical problem’ (presented to students as: ‘To use pencil and paper to find a way of solving a real-life counting problem’). The notion of skip-counting sequences supporting efficient counting would then become secondary learning objectives, with the enabling prompts altered accordingly to ensure that these secondary learning objectives were now subservient to the new primary learning objective.

For example, under this new focus, the enabling prompt may outline a simpler problem requiring students to draw a quick sketch of all the children at their tables to calculate how many people are in the classroom at this very moment. This simpler problem draws the focus to the primary learning objective (problem representation), by essentially removing the secondary learning objective (skip-counting patterns). Arguably this reimagining generates a more compelling mathematical problem, and indeed was how I structured these tasks when I published them in the teaching journal *Prime Number*.

However, given that the focus of the first unit of work in the current thesis was explicitly around number patterns, the first proposal of providing a photograph of children to shift the focus away from problem representation would still seem the more appropriate modification.

Potential modifications to the mixed counting pattern tasks

The second set of tasks to be considered are the mixed counting pattern tasks. Recall that, for both tasks, students had to consider three counting patterns. For the first task, students had to determine whether Mr Russo would have enough fruit on his farm to feed his guests, whilst for the second task, students had to decide whether a rocket would launch by examining whether a given counting pattern would reach zero.

Again, in retrospect, feedback from teachers participating in the program suggested that these final two tasks were too challenging for most students. This is despite the fact that the number patterns were scaffolded, and presented as completion problems. It appears that the intrinsic cognitive load of this task was likely too high for some students, and that the enabling prompts were insufficient at providing an ‘in’ to the main task through reducing the cognitive demand to a more manageable level. In particular, the non-linear sequence which described the growth of the oranges was difficult for students to recognise, which was in turn further amplified by the fact that each guest was to consume two oranges instead of one orange.

Furthermore, despite the fact that they were framed as completion problems, there was arguably unnecessary extraneous cognitive load embedded in the tasks as well. For example, the fruit task required students to link the completed number patterns back to both information contained in the original question and to the fruit ‘key’ provided, in order to decide whether Mr Russo would have enough fruit to feed his 50 guests. The requirement to coordinate information from multiple sources is known to generate extraneous cognitive load (e.g., Sweller, Chandler, Tierney, & Cooper, 1990; Ward & Sweller, 1990).

Given that feedback from teachers was that the patterning unit was time intensive, I would likely remove these two tasks in future iterations of this unit of work. However, if I were to retain the tasks and instead modify them, I would consider making the completion problems the entire task, rather than linking it to any broader problem solving context. This restructuring would reiterate the focus on the primary learning objective, that is, to get students to consider an entire number pattern, and not just consecutive terms, when attempting to identify the rule governing the pattern.

Concluding Thoughts

Overall, I felt privileged to have the opportunity to plan and deliver these two units of work built around challenging tasks. As a teacher, I found teaching with such tasks simultaneously stimulating, engaging and enjoyable. Although in a day-to-day sense, delivering the lessons and collecting data was exhausting, on a deeper level, the process was extremely energising. I look forward to continuing to incorporate challenging tasks into my mathematics instruction, and to future opportunities to design additional units of work built around the CLASS Challenging Task approach.

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APPENDIX A

Teaching with tasks that make children think: Patterning unit of work

This unit of work was developed by James Russo (Belgrave South Primary School and Monash University), in consultation with his PhD supervisors Sarah Hopkins and Peter Sullivan, as part of his PhD project.

TEACHER'S GUIDE FOR THE UNIT OF WORK

TABLE OF CONTENTS

CURRENT UNIT OF WORK

The focus on the current unit of work is on number patterns, particularly skip-counting. A central aspect of the unit is the notion that skip-counting is often a more efficient approach to calculating how many things are in a collection. Additionally, the unit of work will focus on the specific patterns generated through skip-counting, and how different skip-counting patterns are related to each other. Finally, some attention will be given to exponential growth (doubling and halving), and more complex number patterns.

LESSON SPECIFIC LEARNING OBJECTIVES

The specific learning objectives for each of the 16 lessons are outlined below.

Lesson 1

- **Counting by 2's** can be a more efficient way of working out how many things there are in a collection

Lesson 2

- **Counting by 5'sor 10's** can be a more efficient way of working out how many things there are in a collection

Lesson 3

- **Counting by 3's** can be a more efficient way of working out how many things there are in a collection
- When we count forwards by different amounts, we notice patterns. Starting at 0:
  - **Counting by 1's** means we land on all the numbers
  - Skip **counting by 2's** means we land on all the even numbers (numbers ending with 0, 2, 4, 6, 8)
  - Skip **counting by 3's** means we land on an odd number (1, 3, 5, 7, 9), then an even number (0, 2, 4, 6, 8) an odd number, then an even number…

Lesson 4

- When we count forwards by different amounts, we notice patterns. **Starting at a different number** (e.g. 7, or another odd number) changes these patterns:
  - Counting by 1's means we land on all the numbers
  - Skip counting by 2's means we land on all the odd numbers
  - Skip counting by 3's means we land on an even number , then an odd number, an odd number, then an even number…

Lesson 5

- When we count **backwards** by different amounts, we notice patterns. Starting at 100:
  - Counting by 1's means we land on all the numbers (numbers ending with 0, 2, 4, 6, 8)
  - Skip counting by 2's means we land on all the even numbers (numbers ending with 0, 2, 4, 6, 8)
  - Skip counting by 3's means we land on an odd number (1, 3, 5, 7, 9), then an even number (0, 2, 4, 6, 8) an odd number, then an even number…

Lesson 6

- When we count by different amounts, we notice patterns. Starting at 0:
  - Skip counting by 2's means we land on all the even numbers (numbers ending with 0, 2, 4, 6, 8)
  - Skip **counting by 5's** means we land on all numbers ending with 0 or 5
  - Skip **counting by 10's** means we land on all numbers ending with 0

Lesson 7

- When we count by different amounts, we notice patterns. Starting at 0:
  - Skip counting by 2's means we land on all the even numbers (numbers ending with 0, 2, 4, 6, 8)
  - Skip counting by 3's means we land on an odd number (1, 3, 5, 7, 9), then an even number (0, 2, 4, 6, 8) an odd number, then an even number…
  - Skip counting by 5's means we land on all numbers ending with 0 or 5

- Skip counting by 10's means we land on all numbers ending with 0

Lesson 8

- **Counting by 4's** can be a more efficient way of working out how many things there are in a collection
- When we count forwards by different amounts, we notice patterns. Starting at 0:
  - Skip counting by 2's means we land on all the even numbers (numbers ending with 0, 2, 4, 6, 8)
  - Skip counting by 3's means we land on an odd number (1, 3, 5, 7, 9), then an even number (0, 2, 4, 6, 8) an odd number, then an even number…
  - Skip **counting by 4's** means we land on every second even number
  - Skip counting by 5's means we land on all numbers ending with 0 or 5

Lesson 9

- We can combine different counting patterns to show there is more than one way of reaching a target number (**counting by 1's, 2's and 3's**)

Lesson 10

- We can combine different counting patterns to show there is more than one way of reaching a target number (**counting by 4's, and 6's**)

Lesson 11

- We can count how many things there are in a collection in **many different ways**.
- **Counting by 6's** can be a more efficient way of working out how many things there are in a collection. Counting by 6's is related to counting by 2's and counting by 3's.

Lesson 12

- We can count how many things there are in a collection in many different ways.
- **Counting by 8's** can be a more efficient way of working out how many things there are in a collection. Counting by 8's is related to counting by 2's and counting by 4's.

Lesson 13

- **Doubling** is a rule that makes collections (and number patterns) grow very quickly.

Lesson 14

- **Halving** is a rule that makes collections (and number patterns) shrink very quickly.

Lessons 15 and 16

- We need to look at the **whole number pattern**, and not just two consecutive numbers, to work out what the rule is.

STANDARDS FROM THE AUSTRALIAN CURRICULUM

The unit of work related to a number of content descriptions in the Australian Curriculum. These have been separated into those content descriptions which are central to the unit of work (Primary content descriptions) and those that are more peripheral (Auxiliary content descriptions).

Primary Content Descriptions

- Develop confidence with number sequences to and from 100 by ones from any starting point. Skip count by twos, fives and tens starting from zero (grade 1)
- Investigate number sequences, initially those increasing and decreasing by twos, threes, fives and ten from any starting point, then moving to other sequences (grade 2)

Auxiliary Content Descriptions

- Describe, continue, and create number patterns resulting from performing addition or subtraction (Grade 3)
- Investigate number sequences involving multiples of 3, 4, 6, 7, 8, and 9 (Grade 4)

ABOUT OUR ROOM (LESSONS 1 AND 2)

All of the maths we are going to do today is going to be about things in our classroom.

LESSON 1: FEET IN OUR ROOM (FORWARDS BY 2'S)

MINI-LESSON CONCEPTS (TASK FIRST):

- Demonstration of counting by 2's using an abacus/ counting frame
- How many shoes do your two teachers' have altogether? How did you count them? Can you count them a different way? What do you think is the easiest way to count them? Physical demonstration
- How many legs are on both the teachers' chairs? How did you count them? Can you count them a different way? What do you think is the easiest way to count them? Physical demonstration and photograph

- How many eyes do the Grade 1 boys in the classroom have altogether? How did you count them? Can you count them a different way? What do you think is the easiest way to count them? Physical demonstration
- How many arms do the Grade 1 boys in the classroom have altogether? How did you count them? Can you count them a different way? What do you think is the easiest way to count them? Physical demonstration

ROUTINE TASKS

Worksheet(s):

- Count by 2 objects
- Count by 2 balloons

CHALLENGING TASK

Without leaving your seat, or talking to anyone, can you work out how many feet are in the room right now? How did you count them? Can you count them a different way? What do you think is the easiest way to count them?

Extending Prompt:

For an extra challenge, I want you to work out how many toes there are in the room.

Enabling Prompt 1 (augmented problem):

How many feet are there in this picture? How did you count them? Can you count them a different way? What do you think is the easiest way to count them?

PICTURE OF 10 FEET

Enabling Prompt 2 (100's chart):

What pattern do you notice in this picture?

PICTURE OF COUNTING BY 2'S ON A HUNDREDS CHART

LESSON 2: FINGERS IN OUR ROOM (5'S AND 10'S FORWARDS)

MINI-LESSON CONCEPTS (TASK FIRST):

- Demonstration of counting by 5's and 10's using an abacus/ counting frame.
- How many toes do your two teachers have altogether? How did you count them? Can you count them a different way? What do you think is the easiest way to count them? Physical demonstration
- How many fingers do all the Grade 1 girls have in the classroom? How do you know? Can you count them a different way? What do you think is the easiest way to count them? Physical demonstration

ROUTINE TASKS

Worksheet(s):

- Count by 5 objects
- Count by 10 number line

CHALLENGING TASK

Without leaving your seat, or talking to anyone, can you work out how many fingers are in the room right now? How did you count them? Can you count them a different way? What do you think is the easiest way to count them?

Extending Prompt:

For an extra challenge, I forgot to tell you – thumbs don't count as fingers! How many fingers are there in the classroom now?

Enabling Prompt 1 (augmented problem):

How many fingers are there in this picture? How did you count them? Can you count them a different way? What do you think is the easiest way to count them?

PICTURE OF 6 HANDS

Enabling Prompt 2 (100's chart):

What pattern do you notice in this picture?

PICTURE OF COUNTING BY 5'S ON A HUNDREDS CHART

What pattern do you notice in this picture?

PICTURE OF COUNTING BY 10'S ON A HUNDREDS CHART

FROGGIE'S ADVENTURE (LESSONS 3, 4 AND 5)

Lesson Context:

Fiona the Frog loved to hop. In fact, she loved to hop so much that, instead of swimming, she even hopped through the water. But as much as Fiona loved hopping, she need to rest once in a while. She knew she could rest on land, or on a Lilly pad.

Today, we are going to go hopping with Fiona the frog. We are going to pretend that all the numbers ending with zero are Lilly pads, which means that Fiona can take a little rest if she lands on one of these numbers.

LESSON 3: FORWARDS BY 1'S, 2'S, 3'S

MINI-LESSON CONCEPTS (TASK FIRST):

- Brief review of counting by 1's, 2's and 3's (number chart focus).
- Do we land on odd numbers? Do we land on even numbers? What numbers do we land on more than once?
- What patterns do you notice? Do you land on all the numbers ending with 2? What about all of the numbers ending with 5? What about all of the numbers ending in 0?

ROUTINE TASKS

Worksheet(s):

- Count by 3 objects
- Count by 3 number line

CHALLENGING TASK

Fiona the frog needs to get from one side of the lake (0) to the other (100). The only way she can do so is by landing on Lilly pads (numbers which end with a zero, 10 to 90). She must land on at least 5 Lilly pads, or she won't make it. Once she starts on her journey, Fiona always covers the same distance with each hop. You have to decide how far Fiona should go with each hop.

Should she hop in 1's, 2's or 3's to get to the other side of the lake, as quickly and safely as she can? How did you decide?

Suggestion: Use counters or a whiteboard marker to record the path taken by Fiona on your 100's chart.

Extending Prompt:

Fiona the frog has been eating her vitamins, and she can now hop by 4's if she wants. Should she hop in 1's, 2's, 3's or 4's to get over to the other side of the lake, as quickly and safely as she can? How did you decide?

Enabling Prompt 1 (Augmented Task):

Fiona the frog needs to get from one side of the lake (0) to the other (20). The only way she can do so is by landing on a Lilly pad, the number 10.

Should she hop in 1's, 2's or 3's to get to the other side of the lake, as quickly and safely as she can? How did you decide?

Enabling Prompt 2 (100's chart):

What pattern do you notice in this picture?

PICTURE OF COUNTING BY 1'S ON A HUNDREDS CHART TO 10

What pattern do you notice in this picture?

PICTURE OF COUNTING BY 2'S ON A HUNDREDS CHART TO 20

What pattern do you notice in this picture?

PICTURE OF COUNTING BY 3'S ON A HUNDREDS CHART TO 30

LESSON 4: FORWARDS BY 1'S, 2'S, 3'S (ODD BEGINNINGS)

MINI-LESSON CONCEPTS (TASK FIRST):

- Brief review of counting by 1's, 2's and 3's (number chart focus).
  - Do we land on odd numbers? Do we land on even numbers? What numbers do we land on more than once?
  - What patterns do you notice? Do you land on all the numbers ending with 2? What about all of the numbers ending with 5? What about all of the numbers ending in 0?
- How do all these patterns change if we begin from an odd number (like 1 or 7), rather than an even number (like 0 or 6)

ROUTINE TASKS

Worksheet(s):

- Count by 2's – odd numbers (Custom)
- Count by 3's – different starting point (Custom)

CHALLENGING TASK

Fiona the frog needs to get from one side of the lake to the other (100). This time a big gust of wind picked her up and dumped her on number 7, so she gets a bit of a head start. Also, Fiona has been exercising a lot, and does not need to take as many rests anymore. She now only has to land on 3 Lilly pads to arrive safely.

Once she starts on her journey, Fiona always covers the same distance with each hop. You have to decide how far Fiona should go with each hop. Should she hop in 1's, 2's or 3's to get to the other side of the lake, as quickly and safely as she can? How did you decide?

Suggestion: Use counters or a whiteboard marker to record the path taken by Fiona on your 100's chart.

Extending Prompt:

Fiona the frog has been eating her vitamins, and she can now hop by 10's if she wants!!! Should she hop in 1's, 2's, 3's or 10's to get over to the other side of the lake, as quickly and safely as she can? How did you decide?

Enabling Prompt 1 (Augmented Task):

Fiona the frog needs to get from one side of the lake (7) to the other (20). The only way she can do so is by landing on a Lilly pad, the number 10.

Should she hop in 1's, 2's or 3's to get to the other side of the lake, as quickly and safely as she can? How did you decide?

Enabling Prompt 2 (100's chart):

What pattern do you notice in this picture?

PICTURE OF COUNTING BY 1'S ON A HUNDREDS CHART FROM 7 TO 20

What pattern do you notice in this picture?

PICTURE OF COUNTING BY 2'S ON A HUNDREDS CHART FROM 7 TO 19

What pattern do you notice in this picture?

PICTURE OF COUNTING BY 3'S ON A HUNDREDS CHART FROM 7 TO 22

LESSON 5: BACKWARDS BY 1'S, 2'S, 3'S

MINI-LESSON CONCEPTS (TASK FIRST):

- Count back by 1's from 100, leaving a trail of counters along the way

  100, 99, 98, 97, 96, 95….

  Stop when you get back to 70. What patterns do you notice? How many of the numbers ending with zero (90, 80, 70) are covered?

- Skip count back by 2's from 100, leaving a trail of counters along the way

  100, 98, 96, 94, 92…

  Stop when you get back to 70. What patterns do you notice? How many of the numbers ending with zero are covered?

- Skip count back by 3's from 100, leaving a trail of counters along the way

  100, 97, 94, 91, 88…

  Stop when you get back to 70. What patterns do you notice? How many of the numbers ending with zero are covered?

  Remember, when you are skip counting by 3's, you will always skip over two numbers and land on the third number. The pattern will always go 'odd', 'even', 'odd', 'even'. You can use count back 3 to help you. For example 100, 99, 98, 97.

ROUTINE TASKS

Worksheet(s):

- Count by 2's karate
- Count by 3's penguins

CHALLENGING TASK

Now Fiona has to get back over to the other side of the lake. So this time, she needs to start at 100 and end at 0, so you will have to count backwards! Again, Fiona is fitter now because of all her exercising, so she only needs to land on 3 Lilly pads to make it. Once she starts on her journey, Fiona always covers the same distance with each hop. You have to decide how far Fiona should go with each hop.

Should she hop back in 1's, 2's or 3'sto get back over to the other side of the lake, as quickly and safely as she can? How did you decide?

Suggestion: Use counters or a whiteboard marker to record the path taken by Fiona on your 100's chart.

Extending Prompt:

Fiona the frog has been eating her vitamins, and she can now hop by 4's if she wants. Should she hop back in 1's, 2's, 3's or 4's to get back over to the other side of the lake, as quickly and safely as she can? How did you decide?

Enabling Prompt 1 (Augmented Task):

Fiona the frog needs to get back over to the other side of the lake. She has to hop backwards, beginning at 20 and ending at 0. The only way she can do so is by landing on a Lilly pad, the number 10.

Should she hop in 1's, 2's or 3's to get back to the other side of the lake, as quickly and safely as she can? How did you decide?

Enabling Prompt 2 (100's chart):

What pattern do you notice in this picture?

PICTURE OF COUNTING BACK BY 1'S ON A HUNDREDS CHART FROM 100 TO 91

What pattern do you notice in this picture?

PICTURE OF COUNTING BACK BY 2'S ON A HUNDREDS CHART FROM 100 TO 82

What pattern do you notice in this picture?

PICTURE OF COUNTING BACK BY 3'S ON A HUNDREDS CHART FROM 100 TO 73

What pattern do you notice in this picture?

PICTURE OF COUNTING BACK BY 4'S ON A HUNDREDS CHART FROM 100 TO 64

HOW MANY TIMES AND WHICH NUMBERS WILL SURVIVE? (LESSONS 6, 7 & 8)

Lesson Context:

Exploring counting patterns on a 100's chart.


Hundred Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

LESSON 6: FORWARDS BY 2'S, 5'S, 10'S

MINI-LESSON CONCEPTS (TASK FIRST):

The following teaching points will largely focus around demonstrations using a hundreds chart.

- Not sure how to skip count by 2's from 0. The beginning of the pattern is

    - 0, 2, 4, 6, 8…
    - You will always land on a number that ends with a 0, 2, 4, 6, 8. These are the even numbers.
- Not sure how to skip count by 5's from 0. The beginning of the pattern is
    - 0, 5, 10, 15, 20…
    - You will always land on a number that ends with a 0 or a 5.
- Not sure how to skip count by 10's from 0. The beginning of the pattern is
    - 0, 10, 20, 30, 40…
    - You will always land on a number that ends with a 0.

ROUTINE TASKS

Worksheet(s):

- Count by 2 sheet
- Count by 10 balloons

CHALLENGING TASK

Starting at 0, I skip counted by 2's to 100, placing a counter on all the numbers I landed on. Next, I skip counted by 5's to 100, again placing a counter on all the numbers I landed on. Finally, I skip counted by 10's to 100, again placing a counter on all the numbers I landed on? What are the numbers with three counters on them – the numbers I landed on three times?

Extending Prompt:

Tackle the task again, but instead of skip counting from 0, start skip counting from 6. How does starting from 6 change the counting patterns? What numbers do you land on three times?

Enabling Prompt 1 (Augmented Problem)

Starting at 0, I skip counted by 2's to 20, placing a counter on all the numbers I landed on.

Next, I skip counted by 5's to 20, again placing a counter on all the numbers I landed on.

Finally, I skip counted by 10's to 20, again placing a counter on all the numbers I landed on.

What is the number with three counters on it – the number I landed on three times?

Enabling Prompt 2 (100's chart):

What pattern do you notice in this picture?

PICTURE OF COUNTING BY 5'S ON A HUNDREDS CHART

What pattern do you notice in this picture?

PICTURE OF COUNTING BY 10'S ON A HUNDREDS CHART

What pattern do you notice in this picture?

PICTURE OF COUNTING BY 2'S ON A HUNDREDS CHART

LESSON 7: FORWARDS BY 2'S, 3'S, 5'S, 10'S

MINI-LESSON CONCEPTS (TASK FIRST):

- Brief review of counting by 2's, 5's and 10's (number chart focus).
- Mini-lesson focus on skip-counting by 3's (object and number chart focus).
    - How many eyes and noses do your teachers have altogether? How did you count them? Can you count them a different way?
    - How many eyes and noses do the Grade 1 girls in this class have? How did you count them? Can you count them a different way?
    - Not sure how to skip count by 3's from 0. The beginning of the pattern is 3, 6, 9, 12, 15…

      You will always skip over two numbers and land on the third number. The pattern will always go 'odd', 'even', 'odd', 'even'. You can use count on 3 to help you. For example 3, 4, 5, 6.

ROUTINE TASKS

Worksheet(s):

- Count by 5 golf
- Count by 3 balloons

CHALLENGING TASK

Starting at 0, I skip counted by 2's to 50, placing a counter on all the numbers I landed on. Next, I skip counted by 3's to 50, again placing a counter on all the numbers I landed on. I did the same thing counting by 5's and 10's. There is only one number with four counters on it. What is the number?

Extending Prompt:

What if I continued skip counting to 100 instead of 50? How many numbers would I have landed on four times? What are these numbers? Do you notice any interesting patterns with these numbers?

Enabling Prompt 1 (Augmented Problem)

Starting at 0, I skip counted by 3's to 30, placing a counter on all the numbers I landed on.

Next, I skip counted by 5's to 30, again placing a counter on all the numbers I landed on.

What are the numbers with two counters on them – the numbers I landed on twice?

Enabling Prompt 2 (100's chart):

What pattern do you notice in this picture?

PICTURE OF COUNTING BY 5'S ON A HUNDREDS CHART

What pattern do you notice in this picture?

PICTURE OF COUNTING BY 3'S ON A HUNDREDS CHART

What pattern do you notice in this picture?

PICTURE OF COUNTING BY 10'S ON A HUNDREDS CHART

What pattern do you notice in this picture?

PICTURE OF COUNTING BY 2'S ON A HUNDREDS CHART

LESSON 8: FORWARDS BY 2'S, 3'S, 4'S & 5'S

MINI-LESSON CONCEPTS (TASK FIRST):

The following teaching points will largely focus around demonstrations using a hundreds chart.

- Not sure how to skip count by 2's from 0. The beginning of the pattern is

  0, 2, 4, 6, 8…

  You will always land on a number that ends with a 0, 2, 4, 6, 8. These are the even numbers.

- Not sure how to skip count by 3's from 0. The beginning of the pattern is

  3, 6, 9, 12, 15…

  You will always skip over two numbers and land on the third number. The pattern will always go 'odd', 'even', 'odd', 'even'. You can use count on 3 to help you. For example **3**, 4, 5, **6**.

- Not sure how to skip count by 5's from 0. The beginning of the pattern is

  0, 5, 10, 15, 20…

  You will always land on a number that ends with a 0 or a 5.

- Not sure how to skip count by 10's from 0. The beginning of the pattern is

  0, 10, 20, 30, 40…

  You will always land on a number that ends with a 0.

- Not sure how to skip count by 4's from 0. The beginning of the pattern is

  0, 4, 8, 12, 16, 20.

  Stop when you get back to 40. What patterns do you notice? How many of the numbers ending with zero are covered?

  When you are skip counting by 4's, you will always land on even numbers (if you start on an even number). A good way of skip counting by 4's is to skip count by 2's twice. For example, **0**, 2, **4**, 6, **8**. You can use count on 4 to help you. For example **0**, 1, 2, 3, **4**.

ROUTINE TASKS

Worksheet(s):

- Count By 4 (Objects)
- Count By 4 (Skiing)

CHALLENGING TASK

Starting at 0, I skipped counted by 2's to 40, crossing off the numbers as I went. Then I did the same thing, but instead skip counted by 3's. Then by 4's. Then I did it by 5's. Some numbers were crossed off more than once, but some numbers survived – they weren't crossed off at all. Can you guess which 10 numbers survived? Now check if you are right.

Extending Prompt:

What if I also skip counted by 6's, 7's, 8's, 9's and 10's? Would all 10 numbers still survive? How many more numbers would get crossed off?

Enabling Prompt 1 (Augmented Problem)

Starting at 0, I skip counted by 2's to 18, crossing off the numbers as I went.

Next, starting at 0, I skip counted by 3's to 18, crossing off the numbers as I went.

Some numbers were crossed off more than once, but some numbers survived – they weren't crossed off at all. Can you guess which 6 numbers survived? Now check if you are right.

Enabling Prompt 2 (100's chart):

What pattern do you notice in this picture?

PICTURE OF COUNTING BY 5'S ON A HUNDREDS CHART

What pattern do you notice in this picture?

PICTURE OF COUNTING BY 3'S ON A HUNDREDS CHART

What pattern do you notice in this picture?

PICTURE OF COUNTING BY 4'S ON A HUNDREDS CHART

What pattern do you notice in this picture?

PICTURE OF COUNTING BY 2'S ON A HUNDREDS CHART

SPORTS (LESSONS 9 AND 10)

LESSON 9: BASKETBALL GAME (FORWARDS BY 1'S, 2'S, 3'S)

Lesson Context:

Basketball is an awesome sport! Even Bugs Bunny thinks so!

In basketball, you can score points in three different ways.

Scoring a free throw is worth 1 point

Scoring a field goal is worth 2 points

Scoring a three-pointer is worth 3 points

MINI-LESSON CONCEPTS (TASK FIRST):

- Introduce the basketball terminology, and go through several examples: I scored 6 points, how might I have scored them? I scored 10 points, how might I have scored them? I scored 2 three-pointers, and 1 field goal. How many points did I score? I scored 2 field goals, 2 free throws and 1 three pointer, how many points did I score?
- Use the number chart to continue explore mixed counting patterns. For example 1's and 3's (1, 4, 5, 8 etc), 2's and 3's (2, 5, 7, 10 etc) and 1's and 2's (1, 2, 4, 5, 7 etc).

ROUTINE TASKS

Worksheet(s):

- Custom Basketball worksheet

CHALLENGING TASK

Belgrave South played Ferntree Gully in the big basketball game.

VS

At the end of the basketball game, the Belgrave South team had won 36 points to 27 points. How many free throws, field goals and three pointers might the Belgrave South team have scored?

Solve the problem in at least two different ways.

Extending Prompt:

What is the highest number of three-pointers both teams could have scored altogether?

Enabling Prompt 1 (Augmented Task):

Cooper Brooks scored 11 points for Belgrave South. How might he have scored his 11 points?

Enabling Prompt 2 (100's chart):

What pattern do you notice in this picture?

PICTURE OF COUNTING BY 3'S ON A HUNDREDS CHART

What pattern do you notice in this picture?

PICTURE OF COUNTING BY 2'S ON A HUNDREDS CHART

What pattern do you notice in this picture?

PICTURE OF COUNTING BY 2'S, THEN 3'S, THEN 2'S ETC ON A HUNDREDS CHART

LESSON 10: CRICKET GAME (FORWARDS BY 4'S, 6'S)

Challenging Task

Cricket is also a pretty great sport!

Bugs Bunny plays it during summer, when it is too hot for basketball.

In cricket, you can score runs in a lot of different ways, but the fastest and most fun way to score runs is to hit boundaries.

If a ball hits the rope, it is worth 4 runs.

If a ball goes over the rope, it is worth 6 runs.

MINI-LESSON CONCEPTS (TASK FIRST):

- Introduce the cricket terminology, and go through several examples: I scored 10 runs, how might I have scored them? I scored 16 runs, how might I have scored them? I scored 2 fours, and 1 six. How many runs did I score? I scored 2 sixes and 3 fours, how many points did I score?
- Use the number chart to continue explore mixed counting patterns. For example 4's and 6's (4, 10, 14, 20, 24, 40); 6's and 4's (6, 10, 16, 20).

ROUTINE TASKS

Worksheet(s):

- Custom Cricket worksheet

CHALLENGING TASK:

Belgrave South played Ferntree Gully in the big cricket game.

VS

Will Sargood was an excellent cricketer, and scored 36 runs for Belgrave South that day. He scored all of his runs hitting only 4's and 6's.

How many 4's and how many 6's might Will Sargood have scored in his innings?

Solve the problem in at least two different ways.

Extending Prompt:

Could Will have scored 37 runs hitting only 4's and 6's? Explain your thinking.

What if Will Sargood had of scored 50 runs? How many 4's and how many 6's might he have scored?

Enabling Prompt 1 (Augmented Task):

Shaalev Ryan also batted well and made 18 runs hitting only boundaries without getting out. How might he have scored his 18 runs?.

Enabling Prompt 2 (100's chart):

What pattern do you notice in this picture?

PICTURE OF COUNTING BY 6'S ON A HUNDREDS CHART

What pattern do you notice in this picture?

PICTURE OF COUNTING BY 4'S ON A HUNDREDS CHART

What pattern do you notice in this picture?

PICTURE OF COUNTING BY 6'S, THEN 4'S, THEN 6'S ETC ON A HUNDREDS CHART

CREEPY CRAWLIES (LESSONS 11 AND 12)

LESSON 11: ANTS (COUNTING FORWARDS BY 2'S, 3'S, 6'S)

Ants are classified as insects because they have six legs. Ants are busy little insects, who like to live together in large groups.

Can you think of any other animals with 6 legs?

MINI-LESSON CONCEPTS (TASK FIRST):

- Revise skip counting objects by 2, 3 and 6. Highlight how sometimes we can choose how we want to count because the problem can be solved in more than one way.
- Present picture of 1 ant, then 2 ants, then 3 ants and discuss various skip counting approaches.
- Revise/ introduce skip counting by 2, 3 and 6 using the 100's chart, noting the relationship between them.

ROUTINE TASKS

Worksheet(s):

- Custom insects legs sheet

CHALLENGING TASK

Below is a small army of ants, with their leader, Maximus.

Can you work out how many ant legs are in the picture, without counting by 1's?

Can you do it another way?

Make sure you show your thinking.

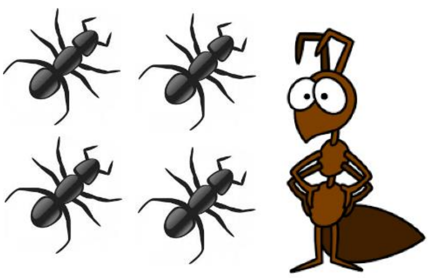
Extending Prompt

What if there had of been 20 ants and Maximus in the ant army? How many legs would they have had altogether? Can you do it another way?

Make sure you show your thinking.

Enabling Prompt 1 (Augmented problem)

Below is a very small army of ants, with their leader, Maximus.



Can you work out how many ant legs are in the picture, without counting by 1's?

Can you do it another way?

Make sure you show your thinking.

Enabling Prompt 2 (100's chart):

What pattern do you notice in this picture?

PICTURE OF COUNTING BY 6'S ON A HUNDREDS CHART

What pattern do you notice in this picture?

PICTURE OF COUNTING BY 43S ON A HUNDREDS CHART

What pattern do you notice in this picture?

PICTURE OF COUNTING BY 2'S ON A HUNDREDS CHART

LESSON 12: SPIDERS (COUNTING FORWARDS BY 2'S, 4'S, 8'S)

Spiders are classified as Arachnids because they have eight legs. Spiders are crafty creatures, who like to spin webs and catch insects in them.

Can you think of any other animals with 8 legs?

MINI-LESSON CONCEPTS (TASK FIRST):

- Revise skip counting objects by 2, 4 and 8. Highlight how sometimes we can choose how we want to count because the problem can be solved in more than one way.
- Present picture of 1 spider, then 2 spiders, then 3 spiders and discuss various skip counting approaches.
- Revise/ introduce skip counting by 2, 4 and 8 using the 100's chart, noting the relationship between them.

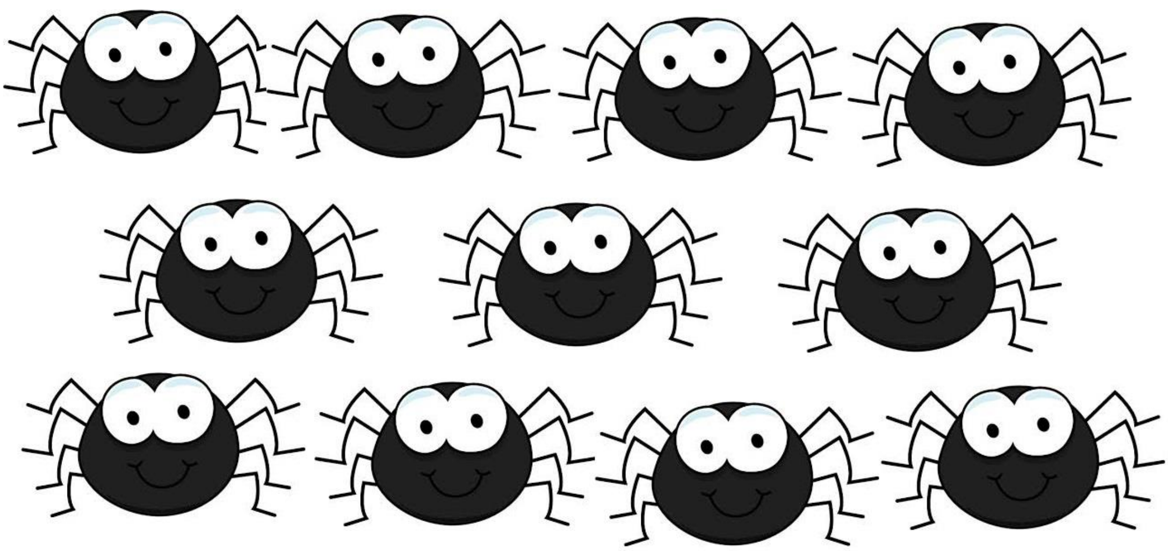
ROUTINE TASKS

Worksheet(s):

- Custom octopus legs sheet

CHALLENGING TASK

Below is a large family of spiders.



Can you work out how many spider legs are in the picture, without counting by 1's?

Can you do it another way?

Make sure you show your thinking.

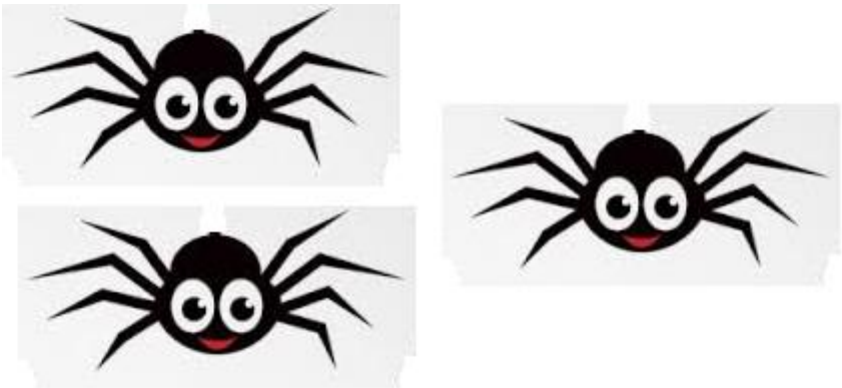
Extending Prompt

How many spider legs in a family of 16 spiders? Can you do it another way?

Make sure you show your thinking.

Enabling Prompt 1 (Augmented problem)

Below is a small family of spiders.



Can you work out how many spider legs are in the picture, without counting by 1's?

Can you do it another way?

Make sure you show your thinking.

Enabling Prompt 2 (100's chart):

What pattern do you notice in this picture?

PICTURE OF COUNTING BY 4'S ON A HUNDREDS CHART

What pattern do you notice in this picture?

PICTURE OF COUNTING BY 8'S ON A HUNDREDS CHART

What pattern do you notice in this picture?

PICTURE OF COUNTING BY 2'S ON A HUNDREDS CHART

FOOD (LESSONS 13 AND 14)

LESSON 13: DOUBLES

Lesson Context:

Kai and Amaya loved donuts, so their mum decided to plant a donut tree.

The tree was magical. Every day, the number of donuts on the tree doubled.

If there were 2 donuts on the tree today, there would be 4 donuts on the tree tomorrow (2+2=4). If there were 5 today, there would be 10 tomorrow (5+5=10).

MINI-LESSON CONCEPTS (TASK FIRST):

- Present a range of examples of doubling donuts. Focus on first order doubling (i.e., today and tomorrow) and second order doubling (today, tomorrow and the next day). Demonstrate using pictures of donuts, as well as the 100's chart.

ROUTINE TASKS

Worksheet(s):

- Custom doubling sheet

CHALLENGING TASK

Kai was having his birthday party on Friday, so the family decided to not pick any of the donuts off the tree until then. On Monday, there were 3 donuts on the tree. Your job is to work out how many donuts there were on the tree by Friday. To help you, have a go at completing the following table. Remember, each day the number of donuts doubles.

	Donuts on tree	Picture
Monday	3	❀❀❀
Tuesday	6	❀❀❀ ❀❀❀
Wednesday		❀❀❀ ❀❀❀ ❀❀❀ ❀❀❀
Thursday		
Friday		

Extending Prompt:

How many donuts would be on the tree if Kai decided to have the party on Saturday instead? How about if he had the party on Sunday?

Enabling Prompt 1 (Augmented problem)

Kai was having his birthday party on Friday, so the family decided to not pick any of the donuts off the tree until then. On Monday, there was 1 donut on the tree. Your job is to work out how many donuts there were on the tree by Friday.

To help you, have a go at completing the following table. Remember, each day the number of donuts doubles.

	Donuts on tree	Picture
Monday	1	❁
Tuesday	2	❁ ❁
Wednesday		❁ ❁ ❁ ❁
Thursday		
Friday		

Enabling Prompt 2 (Images):

What patterns do you notice in this picture?

PICTURES OF DOUBLING PATTERNS

LESSON 14: HALVES

Lesson Context:

A not so friendly giant moved into Belgrave South. His favourite food was Grade 2 children. Actually, he refused to eat anything else!

He decided that every night, while the town slept, he was going to stick his ginormous tongue through the windows of houses and eat half of the Grade 2 children in the town.

MINI-LESSON CONCEPTS (TASK FIRST):

- Present a range of examples of halving children. Focus on first order doubling (i.e., today and tomorrow) and second order doubling (today, tomorrow and the next day). Demonstrate using pictures of children, as well as the 100's chart.

ROUTINE TASKS

Worksheet(s):

- Custom halving sheet

CHALLENGING TASK

When the not so friendly giant arrived in Belgrave South on Monday, there were 64 Grade 2 children in the town. How long will it take until there is only 1 Grade 2 child left?

To help you, have a go at completing the following table. Remember, each night the giant will eat half of the children.

	Grade 2 Children	Picture
Monday	64	☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺
Tuesday		☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺
Wednesday		
Thursday		
Friday		
Saturday		
Sunday		
Monday		

Extending Prompt

Next stop for the not so friendly giant was Ferntree Gully. When the not so friendly giant arrived in Ferntree Gully on Monday, there were 640 Grade 2 children in the town. How long will it take until there are only 10 Grade 2 children left? Do you notice any patterns when you compare this table to Belgrave South?

	Grade 2 Children
Monday	640
Tuesday	
Wednesday	
Thursday	
Friday	
Saturday	
Sunday	
Monday	

Enabling Prompt 1 (Augmented task)

When the not so friendly giant arrived in Belgrave South on Monday, there were 16 Grade 2 children in the town. How long will it take until there is only 1 Grade 2 child left?

To help you, have a go at completing the following table. Remember, each night the giant will eat half of the children.

	Grade 2 Children	Picture
Monday	16	☺☺☺☺ ☺☺☺☺ ☺☺☺☺ ☺☺☺☺
Tuesday		☺☺☺☺ ☺☺☺☺
Wednesday		
Thursday		
Friday		
Saturday		
Sunday		
Monday		

Enabling Prompt 2 (Images):

What patterns do you notice in this picture?

PICTURES OF HALVING PATTERNS

What patterns do you notice in this picture?

MORE CHALLENGING PATTERNS WITH MR RUSSO (LESSONS 15 AND 16)

Lesson Context

Mr Russo has two hobbies: growing fruit and launching rockets!






LESSON 15: COUNTING FORWARDS

MINI-LESSON CONCEPTS (TASK FIRST):

- This is a review lesson, looking at some of the counting sequences (forward counting) we have explored so far. The focus will be on writing a sequence of numerals, asking students to guess the next number in the sequence, and also to identify what the pattern/ rule is. This will be linked with exploring what the pattern looks like on the 100's chart, and how we can use the 100's chart to identify and continue the pattern.
- Review 2's (odd and even), 3's, 4's (0), 5's (0 and 6 as starting points), 6's, 8's and 10's (0 and other numbers as starting points). Also very briefly look at a non-standard pattern (e.g., 4's and 6's, doubling).

ROUTINE TASKS

Worksheet(s):

- Number patterns basic
- Number line patterns

CHALLENGING TASK

This Sunday afternoon, Mr Russo is inviting 50 parents and children to his farm in Belgrave to eat some of the fruit he has grown. He wants each of his guests to have:

One apple 

One pear 

Two oranges 

All the fruit on Mr Russo's trees grow in patterns.

	Mon	Tues	Wed	Thurs	Fri	Sat	Sun
Apples	3	13	23	33	43	?	?

Pears	1	2	4	8	16	?	?

Oranges	10	10	20	30	50	?	?

Will he have enough fruit to feed all his guests on Sunday? Explain your thinking.

Extending Prompt

Mr Russo also wondered if his 50 guests would each be able to each have:

Three bananas 

Two cherries 

All the fruit on Mr Russo's trees grow in patterns.

	Mon	Tues	Wed	Thurs	Fri	Sat	Sun
Bananas	15	30	45	60	75		
Cherries	1	3	7	15	31		

Will he have enough fruit to feed all his guests on Sunday? Explain your thinking.

Enabling Prompt 1 (Augmented problem)

This Sunday afternoon, Mr Russo is inviting 20 parents and children to his farm in Belgrave to eat some of the fruit he has grown. He wants each of his guests to have:

One strawberry 

One plum 

All the fruit on Mr Russo's trees grow in patterns.

	Mon	Tues	Wed	Thurs	Fri	Sat	Sun
Strawberry	2	4	6	8	10		
							
Plum	3	6	9	12	15		
							

Will he have enough fruit to feed all his guests on Sunday? Explain your thinking.

LESSON 16: COUNTING BACKWARDS

MINI-LESSON CONCEPTS (TASK FIRST):

- This is a review lesson, looking at some of the counting sequences (backward counting) we have explored so far. The focus will be on writing a sequence of numerals, asking students to guess the next number in the sequence, and also to identify what the pattern/ rule is. This will be linked with exploring what the pattern looks like on the 100's chart, and how we can use the 100's chart to identify and continue the pattern.
- Review 2's (odd and even), 3's, 4's, 5's, 6's, 8's and 10's. Also very briefly look at a non-standard pattern (e.g., halving). Focus on counting down from 100, and other starting points.

ROUTINE TASKS

Worksheet(s):

- Launching rockets

CHALLENGING TASK

Mr Russo was planning on launching rockets from the roof of Belgrave South Primary School. His countdown would begin at 100, and when he got to 0, he would fire the rocket.

He began the countdown for Rocket A:

100, 90, 95, 85, 90, 80, 85…

Continue the counting sequence. Will the rocket fire? Why?

Next, he began the countdown for Rocket B:

100, 90, 80, 90, 100, 90…

Continue the counting sequence. Will the rocket fire? Why?

Finally, he began the countdown for Rocket C:

100, 90, 81, 73, 66, 60…

Continue the counting sequence. Will the rocket fire? Why?

Extending Prompt:

He began the countdown for Rocket D:

128, 64, 32, 16, 8…

Continue the counting sequence. Will the rocket fire? Why?

Enabling Prompt 1 (Augmented problem)

Mr Russo was planning on launching rockets from the roof of Belgrave South Primary School.

His countdown would begin at 20, and when he got to 0, he would fire the rocket. 

He began the countdown for Rocket X: 20, 19, 18, 17, 16…

Continue the counting sequence. Will the rocket fire? Why?

He began the countdown for Rocket Y: 20, 18, 19, 17, 18, 16, 17…

Continue the counting sequence. Will the rocket fire? Why?

He began the countdown for Rocket Y: 20, 19, 18, 19, 20, 19, 18…

Continue the counting sequence. Will the rocket fire? Why?

Enabling Prompt 2 (100's chart):

What pattern do you notice in this picture?

VARIOUS SKIP-COUNTING PATTERNS ON A HUNDREDS CHART COUNTING BACKWARDS

APPENDIX B

Teaching with tasks that make children think: Patterning unit of work

This unit of work was developed by James Russo (Belgrave South Primary School and Monash University), in consultation with his PhD supervisors Sarah Hopkins and Peter Sullivan, as part of his PhD project.

CHALLENGING TASK WORKSHEETS

Challenging Task Worksheet (Lesson 1) **NAME:**

Without leaving your seat, or talking to anyone, can you work out how many feet are in the room right now? How did you count them? Can you count them a different way? What do you think is the easiest way to count them?

Extending Prompt (Lesson 1):

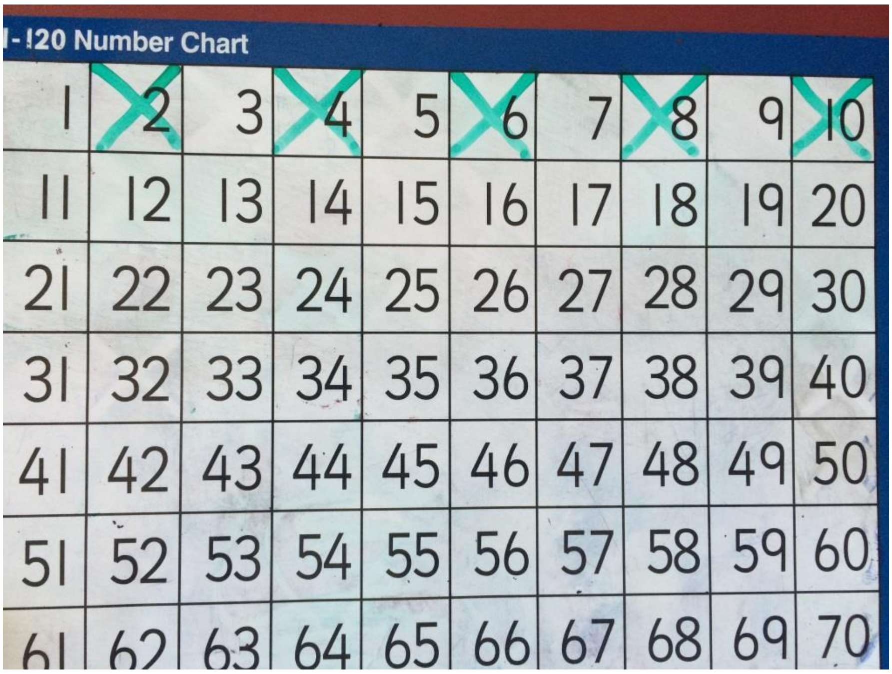
For an extra challenge, I want you to work out how many toes there are in the room.

Enabling Prompt A (Lesson 1): **NAME:**

How many feet are there in this picture? How did you count them? Can you count them a different way? What do you think is the easiest way to count them?

Enabling Prompt B (Lesson 1):

What pattern do you notice in this picture?



Challenging Task Worksheet (Lesson 2) **NAME:**

Without leaving your seat, or talking to anyone, can you work out how many fingers are in the room right now? How did you count them? Can you count them a different way? What do you think is the easiest way to count them?

Extending Prompt (Lesson 2):

For an extra challenge, I forgot to tell you – thumbs don't count as fingers! How many fingers are there in the classroom now?

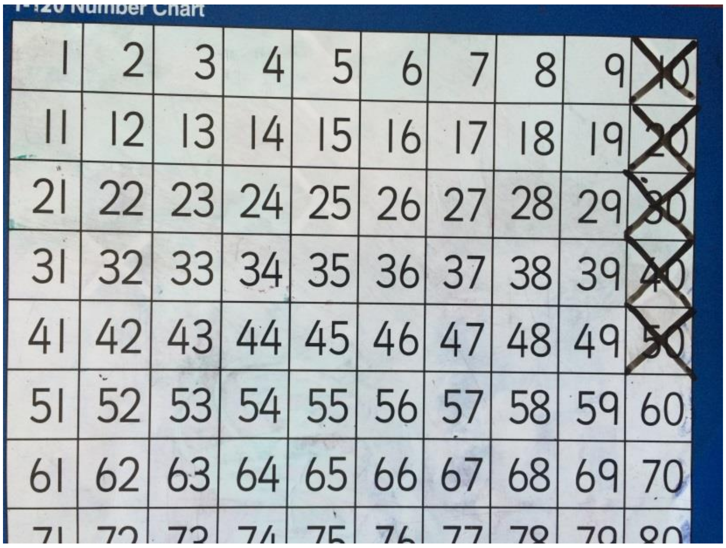
Enabling Prompt A (Lesson 2): **NAME:**

How many fingers are there in this picture? How did you count them? Can you count them a different way? What do you think is the easiest way to count them?

Enabling Prompt B (Lesson 2):

What pattern do you notice in this picture?

What pattern do you notice in this picture?



Challenging Task Worksheet (Lesson 3) **NAME:**

Fiona the frog needs to get from one side of the lake (0) to the other (100). The only way she can do so is by landing on Lilly pads (numbers which end with a zero, 10 to 90). She must land on at least 5 Lilly pads, or she won't make it. Once she starts on her journey, Fiona always covers the same distance with each hop. You have to decide how far Fiona should go with each hop.

Should she hop in 1's, 2's or 3's to get to the other side of the lake, as quickly and safely as she can? How did you decide?

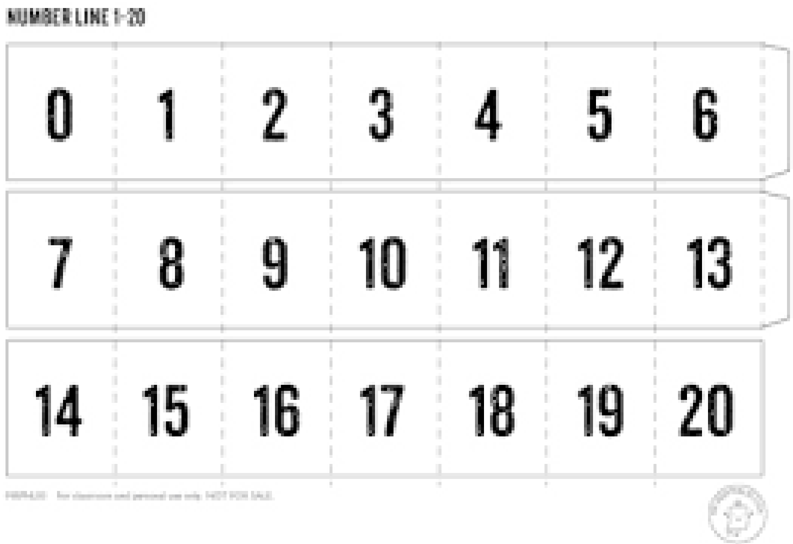
Suggestion: Use counters or a whiteboard marker to record the path taken by Fiona on your 100's chart.

Extending Prompt (Lesson 3):

Fiona the frog has been eating her vitamins, and she can now hop by 4's if she wants. Should she hop in 1's, 2's, 3's or 4's to get over to the other side of the lake, as quickly and safely as she can? How did you decide?

Enabling Prompt A (Lesson 3): **NAME:**

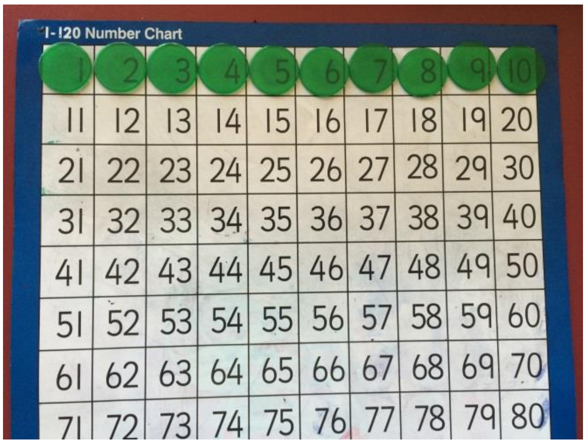
Fiona the frog needs to get from one side of the lake (0) to the other (20). The only way she can do so is by landing on a Lilly pad, the number 10.



Should she hop in 1's, 2's or 3's to get to the other side of the lake, as quickly and safely as she can? How did you decide?

Enabling Prompt B (Lesson 3):

What pattern do you notice in this picture?



What pattern do you notice in this picture?

What pattern do you notice in this picture?

Challenging Task Worksheet (Lesson 4) **NAME:**

Fiona the frog needs to get from one side of the lake to the other (100). This time a big gust of wind picked her up and dumped her on number 7, so she gets a bit of a head start. Also, Fiona has been exercising a lot, and does not need to take as many rests anymore. She now only has to land on 3 Lilly pads to arrive safely.

Once she starts on her journey, Fiona always covers the same distance with each hop. You have to decide how far Fiona should go with each hop. Should she hop in 1's, 2's or 3's to get to the other side of the lake, as quickly and safely as she can? How did you decide?

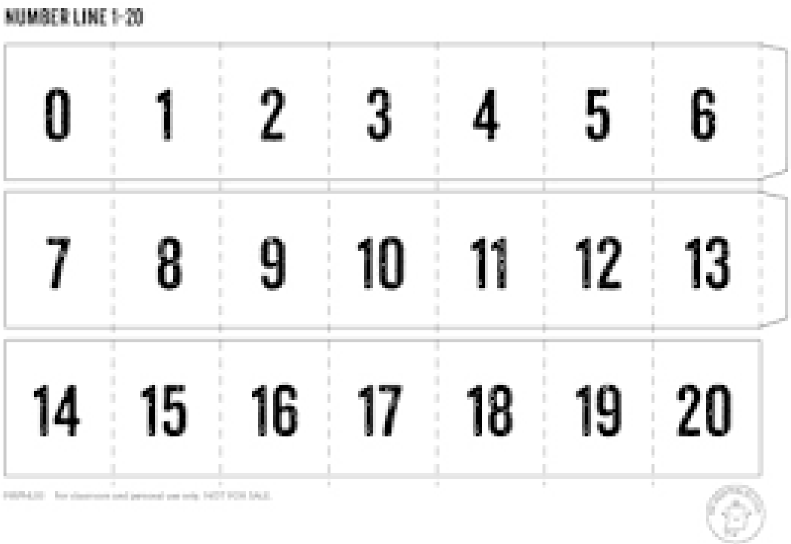
Suggestion: Use counters or a whiteboard marker to record the path taken by Fiona on your 100's chart.

Extending Prompt:

Fiona the frog has been eating her vitamins, and she can now hop by 10's if she wants!!! Should she hop in 1's, 2's, 3's or 10's to get over to the other side of the lake, as quickly and safely as she can? How did you decide?

Enabling Prompt A (Lesson 4): **NAME:**

Fiona the frog needs to get from one side of the lake (7) to the other (20). The only way she can do so is by landing on a Lilly pad, the number 10.

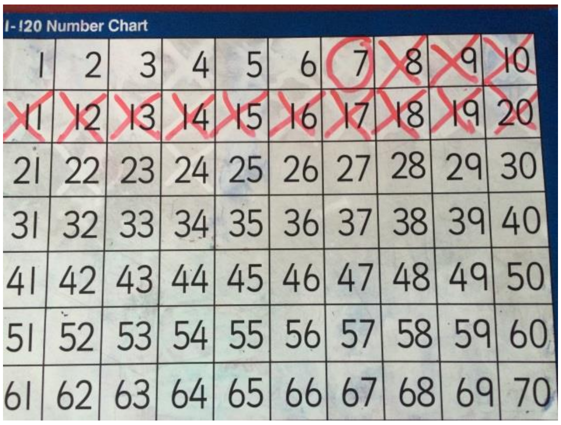


Should she hop in 1's, 2's or 3's to get to the other side of the lake, as quickly and safely as she can? How did you decide?

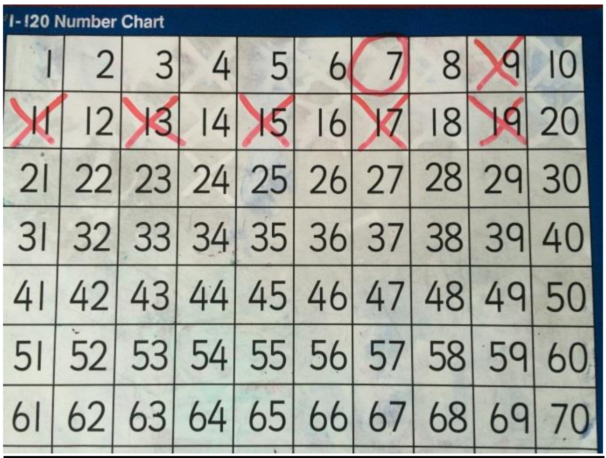
Appendix B: Challenging Task Worksheets - Patterning Unit of Work

Enabling Prompt B (Lesson 4):

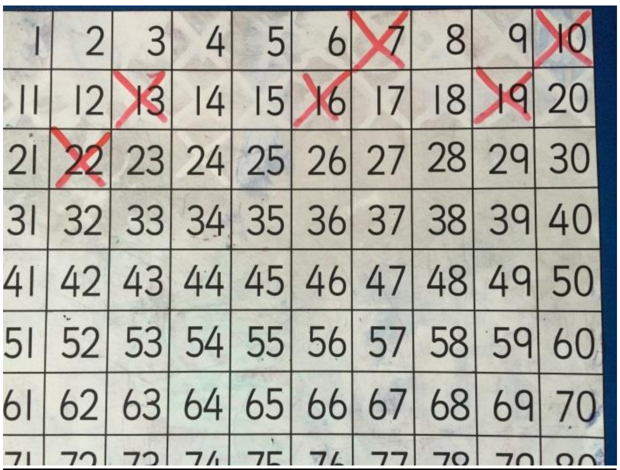
What pattern do you notice in this picture?



What pattern do you notice in this picture?



What pattern do you notice in this picture?



Challenging Task Worksheet (Lesson 5) **NAME:**

Now Fiona has to get back over to the other side of the lake. So this time, she needs to start at 100 and end at 0, so you will have to count backwards! Again, Fiona is fitter now because of all her exercising, so she only needs to land on 3 Lilly pads to make it. Once she starts on her journey, Fiona always covers the same distance with each hop. You have to decide how far Fiona should go with each hop.

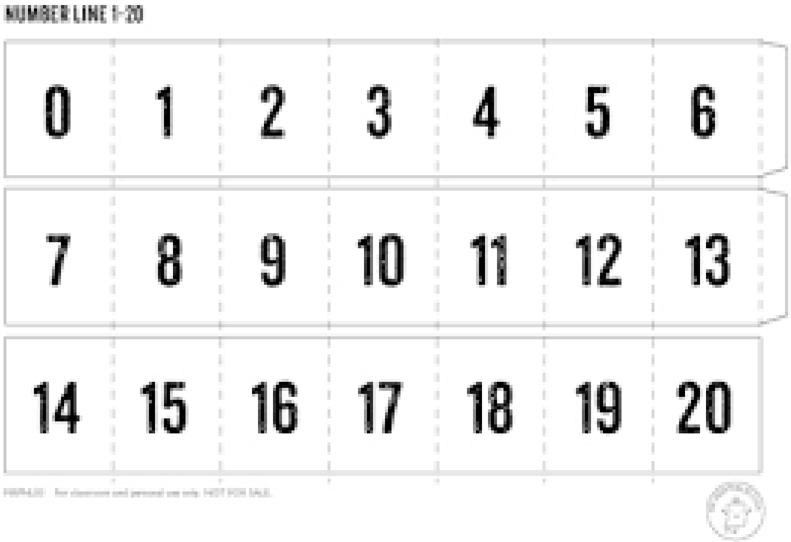
Should she hop back in 1's, 2's or 3'sto get back over to the other side of the lake, as quickly and safely as she can? How did you decide?

Extending Prompt (Lesson 5):

Fiona the frog has been eating her vitamins, and she can now hop by 4's if she wants. Should she hop back in 1's, 2's, 3's or 4's to get back over to the other side of the lake, as quickly and safely as she can? How did you decide?

Enabling Prompt A (Lesson 5): **NAME:**

Fiona the frog needs to get back over to the other side of the lake. She has to hop backwards, beginning at 20 and ending at 0. The only way she can do so is by landing on a Lilly pad, the number 10.



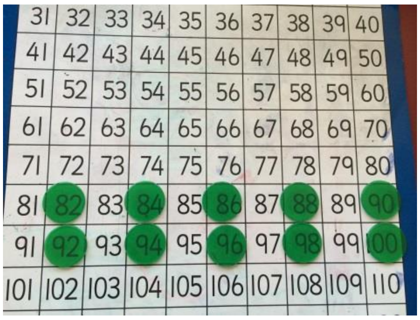
Should she hop in 1's, 2's or 3's to get back to the other side of the lake, as quickly and safely as she can? How did you decide?

Appendix B: Challenging Task Worksheets - Patterning Unit of Work

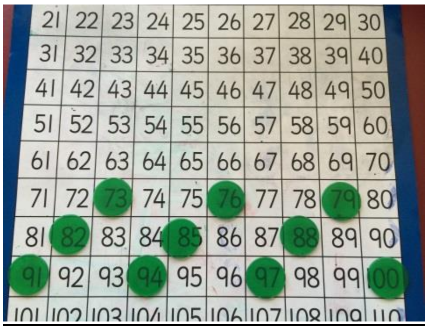
Enabling Prompt B (Lesson 5):

What pattern do you notice in this picture?

What pattern do you notice in this picture?



What pattern do you notice in this picture?



What pattern do you notice in this picture?

Challenging Task Worksheet (Lesson 6) **NAME:**

Starting at 0, I skip counted by 2's to 100, placing a counter on all the numbers I landed on.

Next, I skip counted by 5's to 100, again placing a counter on all the numbers I landed on.

Finally, I skip counted by 10's to 100, again placing a counter on all the numbers I landed on.

What are the numbers with three counters on them – the numbers I landed on three times?

Extending Prompt (Lesson 6):

Tackle the task again, but instead of skip counting from 0, start skip counting from 6. How does starting from 6 change the counting patterns? What numbers do you land on three times?

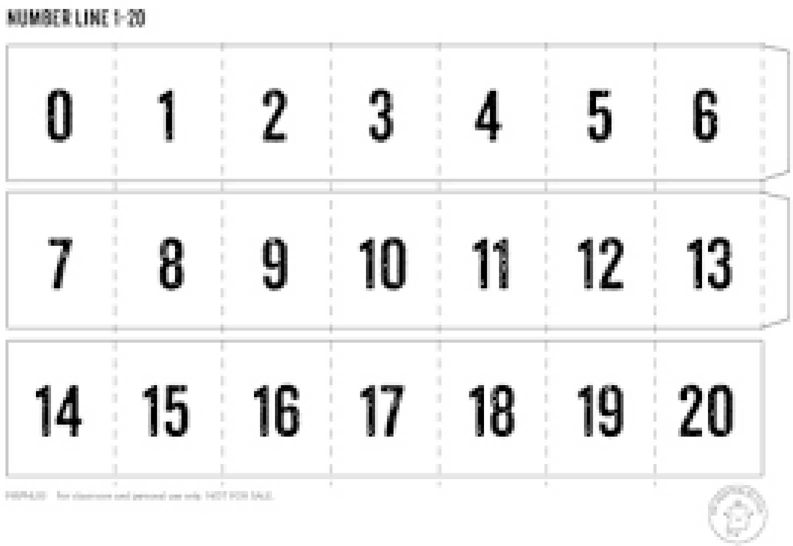
Enabling Prompt A (Lesson 6)

Starting at 0, I skip counted by 2's to 20, placing a counter on all the numbers I landed on.

Next, I skip counted by 5's to 20, again placing a counter on all the numbers I landed on.

Finally, I skip counted by 10's to 20, again placing a counter on all the numbers I landed on.

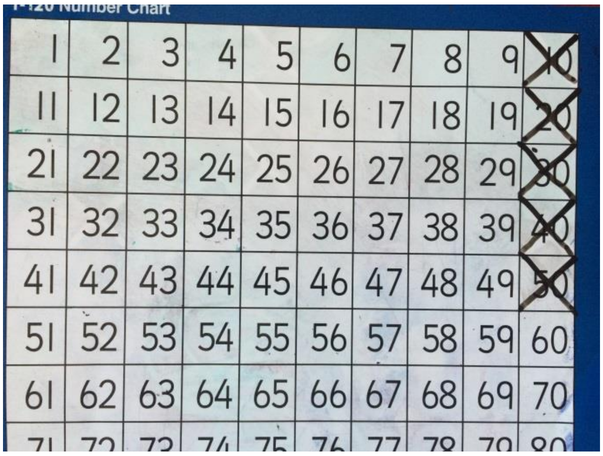
What is the number with three counters on it – the number I landed on three times?



Enabling Prompt B (Lesson 6):

What pattern do you notice in this picture?

What pattern do you notice in this picture?



What pattern do you notice in this picture?

Challenging Task Worksheet (Lesson 7)

Starting at 0, I skip counted by 2's to 50, placing a counter on all the numbers I landed on. Next, I skip counted by 3's to 50, again placing a counter on all the numbers I landed on. I did the same thing counting by 5's and 10's. There is only one number with four counters on it. What is the number?

Extending Prompt (Lesson 7):

What if I continued skip counting to 100 instead of 50? How many numbers would I have landed on four times? What are these numbers? Do you notice any interesting patterns with these numbers?

Enabling Prompt A (Lesson 7) **NAME:**

Starting at 0, I skip counted by 3's to 30, placing a counter on all the numbers I landed on.

Next, I skip counted by 5's to 30, again placing a counter on all the numbers I landed on.

What are the numbers with two counters on them – the numbers I landed on twice?

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30									

Appendix B: Challenging Task Worksheets - Patterning Unit of Work

Enabling Prompt B (Lesson 7):

What pattern do you notice in this picture?

What pattern do you notice in this picture?

What pattern do you notice in this picture?

What pattern do you notice in this picture?

Challenging Task Worksheet (Lesson 8) **NAME:**

Starting at 0, I skipped counted by 2's to 40, crossing off the numbers as I went. Then I did the same thing, but instead skip counted by 3's. Then by 4's. Then I did it by 5's. Some numbers were crossed off more than once, but some numbers survived – they weren't crossed off at all. Can you guess which 10 numbers survived? Now check if you are right.

Extending Prompt (Lesson 8):

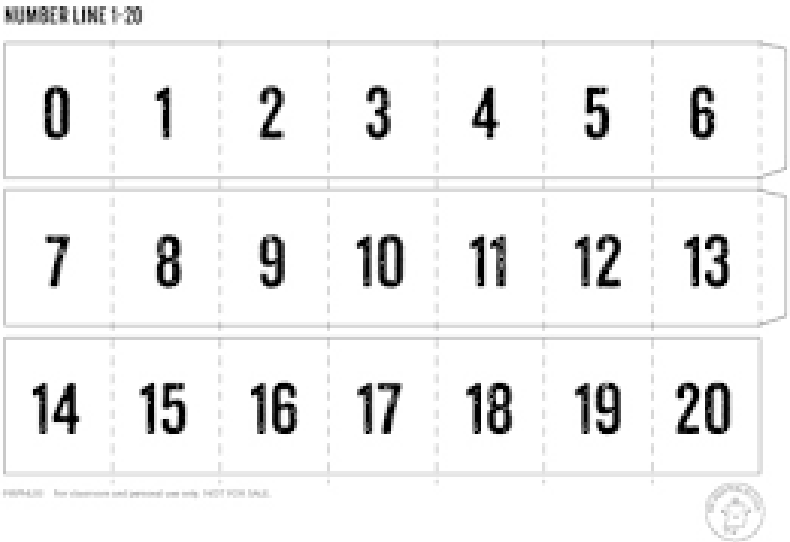
What if I also skip counted by 6’s, 7’s, 8’s, 9’s and 10’s? Would all 10 numbers still survive? How many more numbers would get crossed off?

Enabling Prompt A (Lesson 8) **NAME:**

Starting at 0, I skip counted by 2's to 18, crossing off the numbers as I went.

Next, starting at 0, I skip counted by 3's to 18, crossing off the numbers as I went.

Some numbers were crossed off more than once, but some numbers survived – they weren't crossed off at all. Can you guess which 6 numbers survived? Now check if you are right.

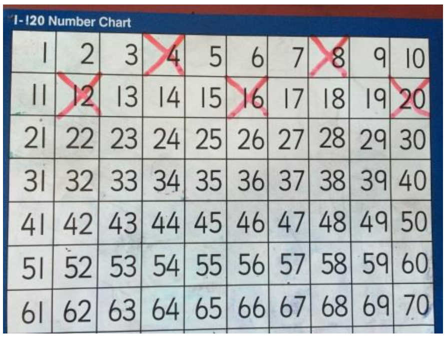


Enabling Prompt B (Lesson 8):

What pattern do you notice in this picture?

What pattern do you notice in this picture?

What pattern do you notice in this picture?



What pattern do you notice in this picture?

Challenging Task Worksheet (Lesson 9) **NAME:**

Belgrave South played Selby in the big basketball game.

VS

At the end of the basketball game, the Belgrave South team had won 36 points to 27 points.

How many free throws, field goals and three pointers might the Belgrave South team have scored?

Solve the problem in at least two different ways.

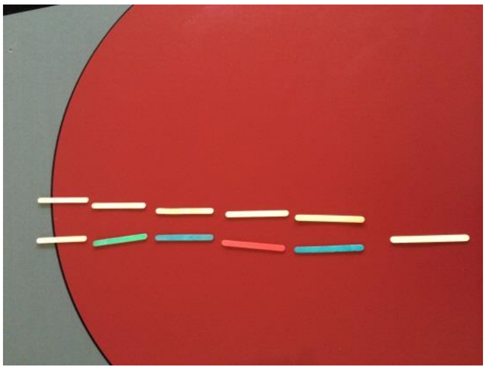
Extending Prompt (Lesson 9):

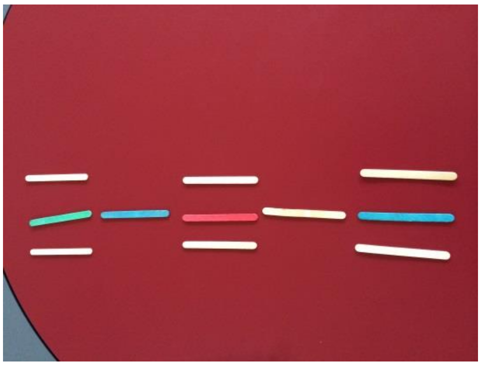
What is the highest number of three-pointers both teams could have scored altogether?

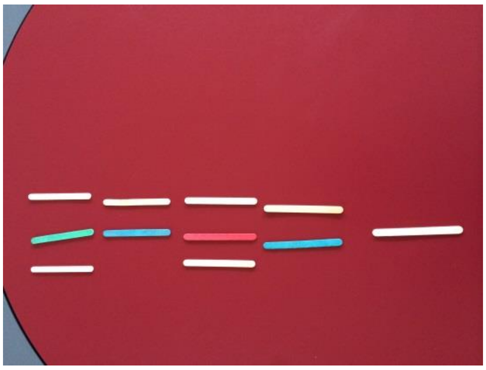
Enabling Prompt A (Lesson 9): **NAME:**

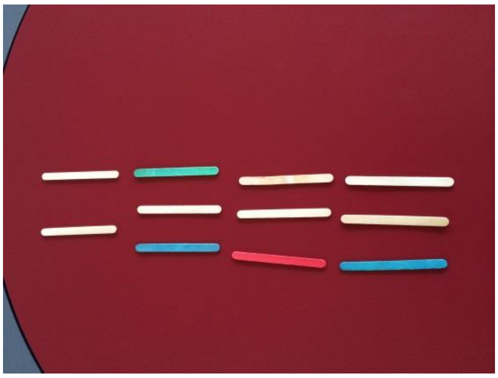
Cooper Brooks scored 11 points for Belgrave South. How might he have scored his 11 points?

These pictures might help you.







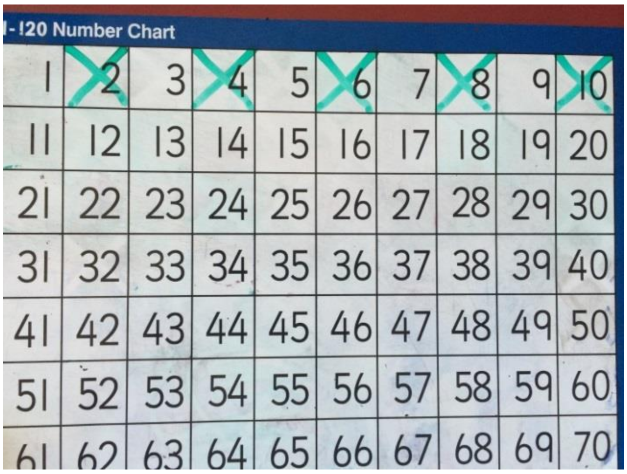


Appendix B: Challenging Task Worksheets - Patterning Unit of Work

Enabling Prompt B (Lesson 9):

What pattern do you notice in this picture?

What pattern do you notice in this picture?



What pattern do you notice in this picture?

Challenging Task Worksheet (Lesson 10) **NAME:**

Belgrave South played Ferntree Gully in the big cricket game.

VS

Will Sargood was an excellent cricketer, and scored 36 runs for Belgrave South that day. He scored all of his runs hitting only 4's and 6's.

How many 4's and how many 6's might Will Sargood have scored in his innings?

Solve the problem in at least two different ways.

Extending Prompt (Lesson 10):

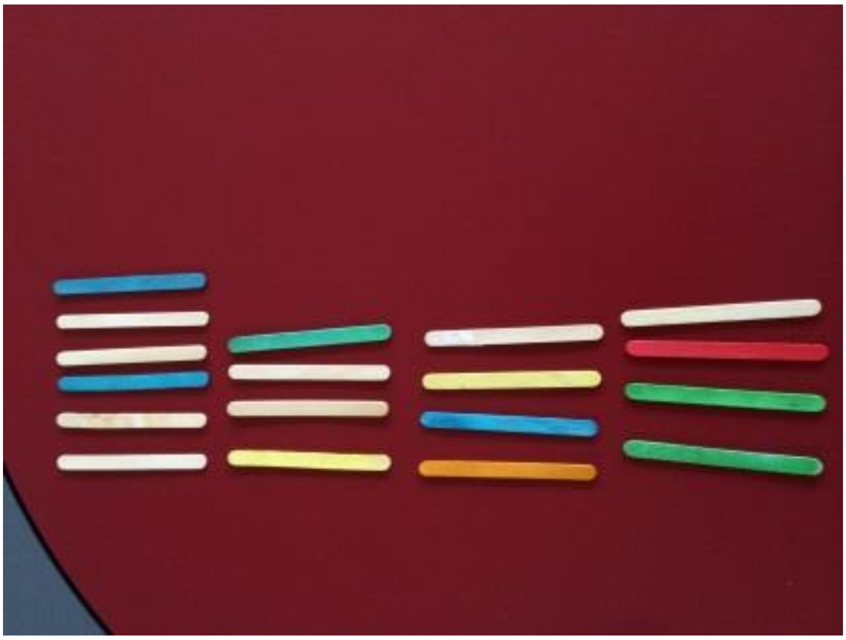
Could Will have scored 37 runs hitting only 4's and 6's? Explain your thinking.

What if Will Sargood had of scored 50 runs? How many 4's and how many 6's might he have scored?

Enabling Prompt A (Lesson 10): **NAME:**

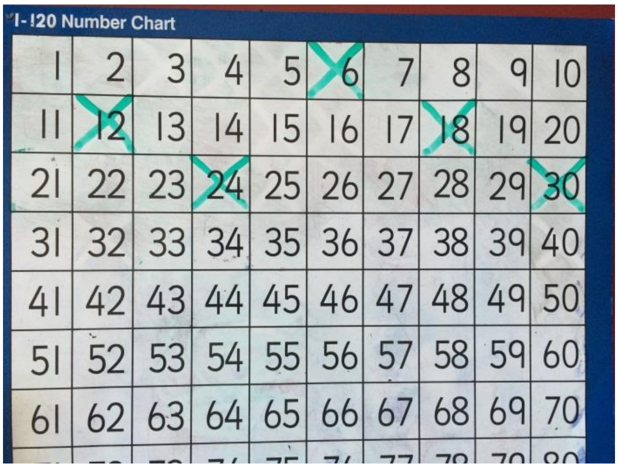
Shaalev Ryan also batted well and made 18 runs hitting only boundaries without getting out.

How might he have scored his 18 runs? These pictures might help you.

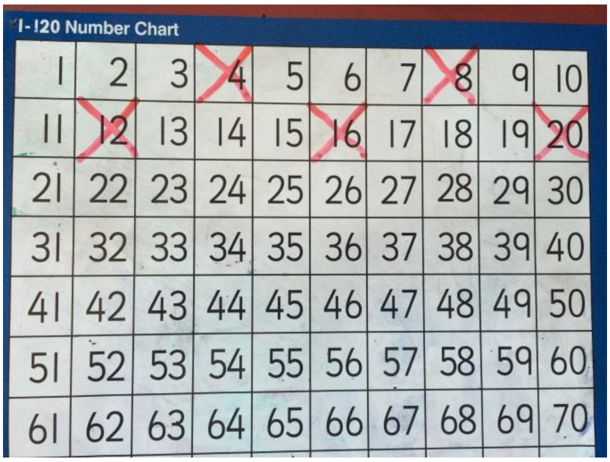


Enabling Prompt B (Lesson 10):

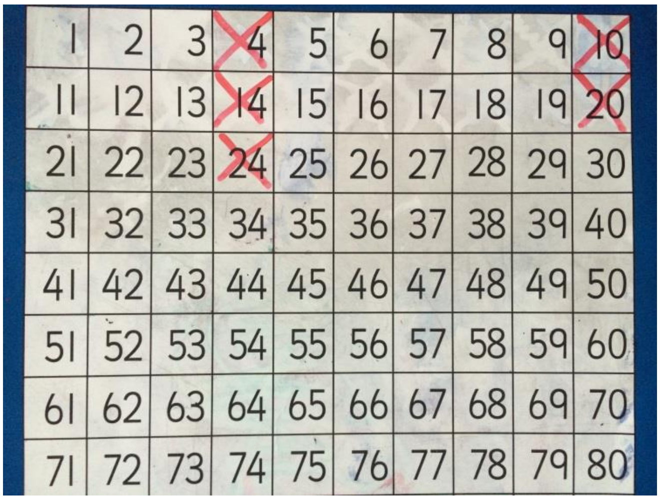
What pattern do you notice in this picture?



What pattern do you notice in this picture?



What pattern do you notice in this picture?



Challenging Task Worksheet (Lesson 11) **NAME:**

Below is a small army of ants, with their leader, Maximus.

Can you work out how many ant legs are in the picture, without counting by 1's?

Can you do it another way?

Make sure you show your thinking.

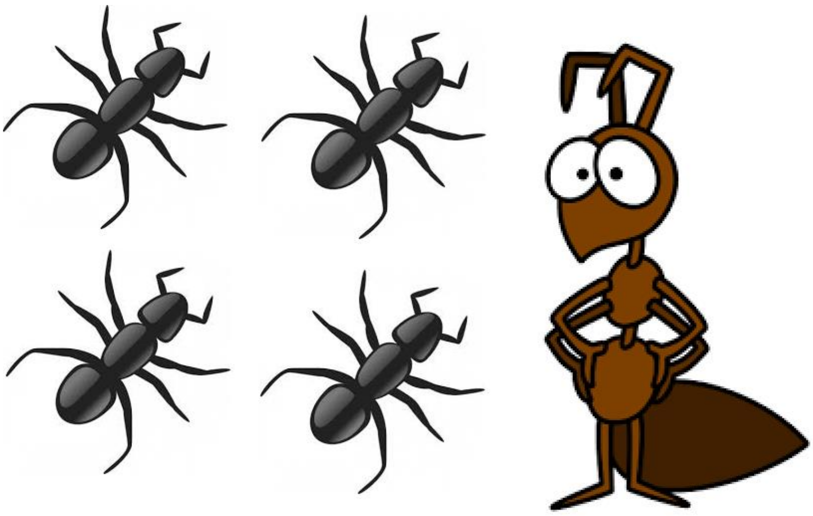
Extending Prompt (Lesson 11)

What if there had of been 20 ants and Maximus in the ant army? How many legs would they have had altogether? Can you do it another way?

Make sure you show your thinking.

Enabling Prompt A (Lesson 11) **NAME:**

Below is a very small army of ants, with their leader, Maximus.



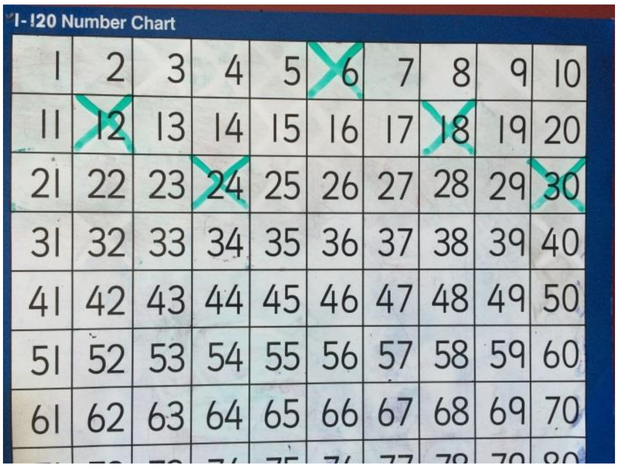
Can you work out how many ant legs are in the picture, without counting by 1's?

Can you do it another way?

Make sure you show your thinking.

Enabling Prompt B (Lesson 11):

What pattern do you notice in this picture?

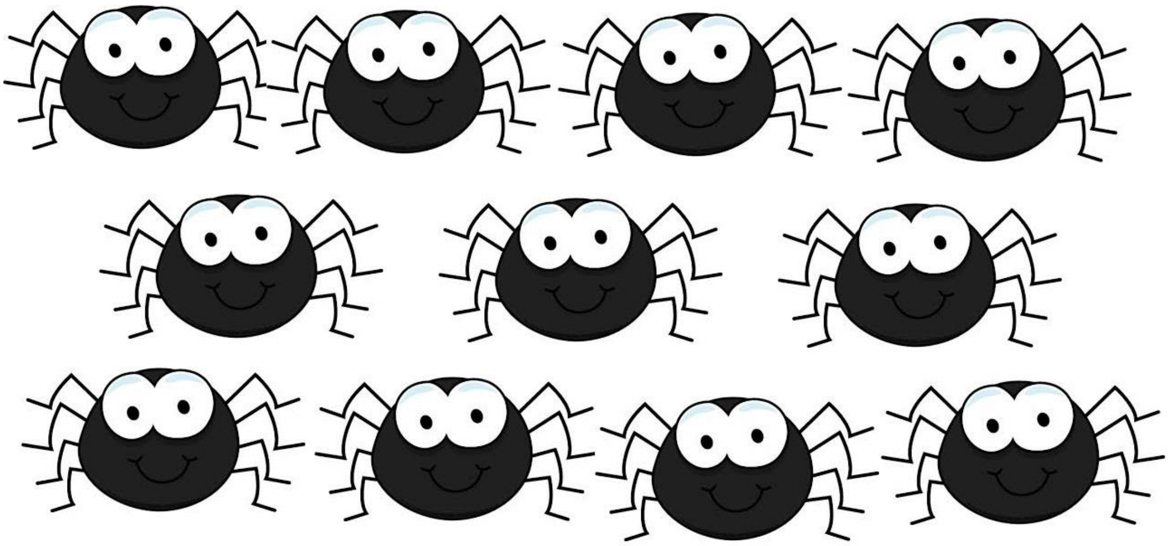


What pattern do you notice in this picture?

What pattern do you notice in this picture?

Challenging Task Worksheet (Lesson 12) **NAME:**

Below is a large family of spiders.



Can you work out how many spider legs are in the picture, without counting by 1's?

Can you do it another way?

Make sure you show your thinking.

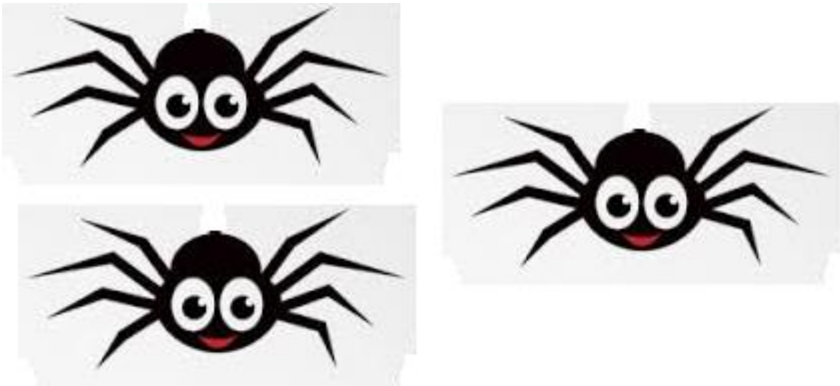
Extending Prompt (Lesson 12)

How many spider legs in a family of 16 spiders? Can you do it another way?

Make sure you show your thinking.

Enabling Prompt A (Lesson 12) **NAME:**

Below is a small family of spiders.



Can you work out how many spider legs are in the picture, without counting by 1's?

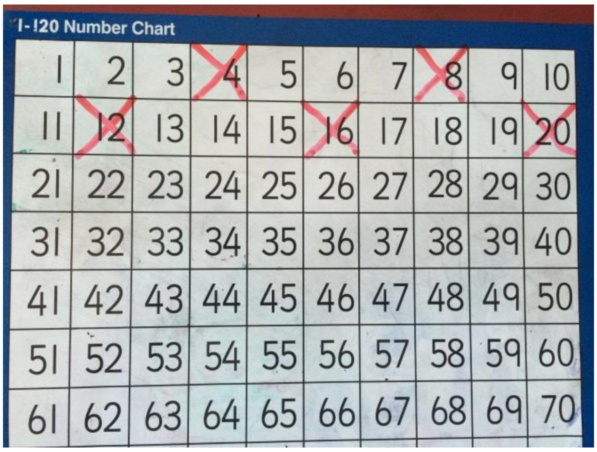
Can you do it another way?

Make sure you show your thinking.

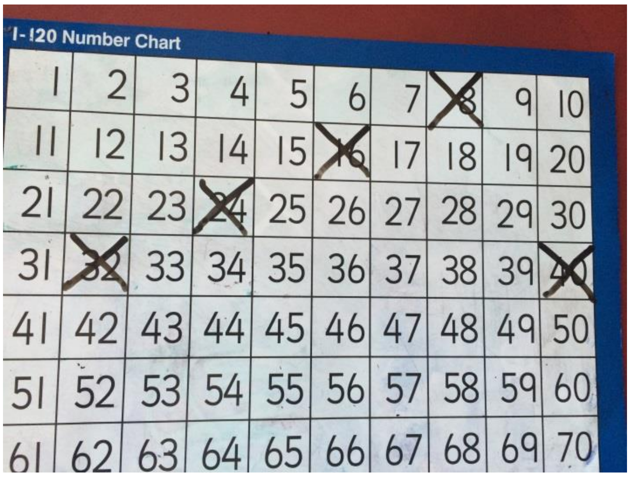
Appendix B: Challenging Task Worksheets - Patterning Unit of Work

Enabling Prompt B (Lesson 12):

What pattern do you notice in this picture?



What pattern do you notice in this picture?

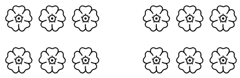


What pattern do you notice in this picture?

Challenging Task Worksheet (Lesson 13) **NAME:**

Kai was having his birthday party on Friday, so the family decided to not pick any of the donuts off the tree until then. On Monday, there were 3 donuts on the tree. Your job is to work out how many donuts there were on the tree by Friday.

To help you, have a go at completing the following table. Remember, each day the number of donuts doubles.

	Donuts on tree	Picture
Monday	3	
Tuesday	6	
Wednesday		
Thursday		
Friday		

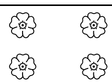
Extending Prompt (Lesson 13):

How many donuts would be on the tree if Kai decided to have the party on Saturday instead? How about if he had the party on Sunday? Can you keep the pattern going?

Enabling Prompt A (Lesson 13) **NAME:**

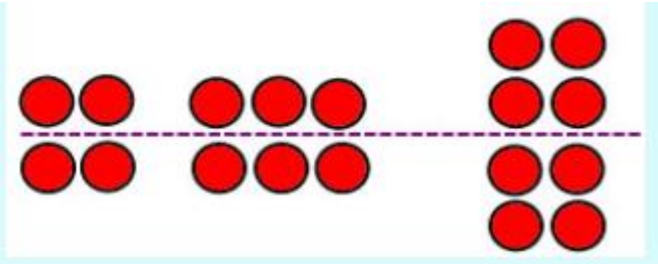
Kai was having his birthday party on Friday, so the family decided to not pick any of the donuts off the tree until then. On Monday, there was 1 donut on the tree. Your job is to work out how many donuts there were on the tree by Friday.

To help you, have a go at completing the following table. Remember, each day the number of donuts doubles.

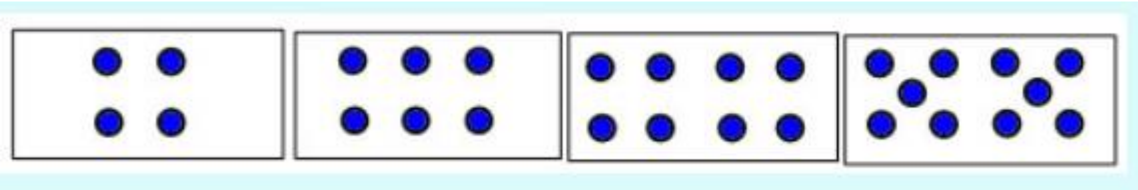
	Donuts on tree	Picture
Monday	1	
Tuesday	2	
Wednesday		
Thursday		
Friday		

Enabling Prompt B (Lesson 13):

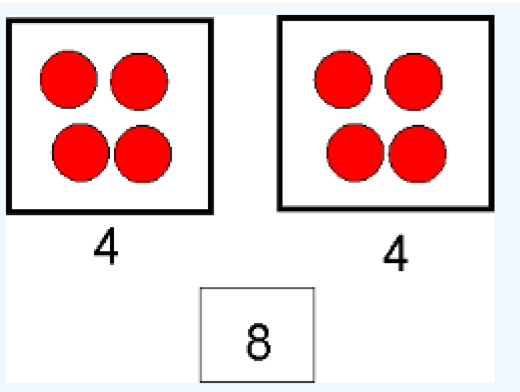
What patterns do you notice in this picture?



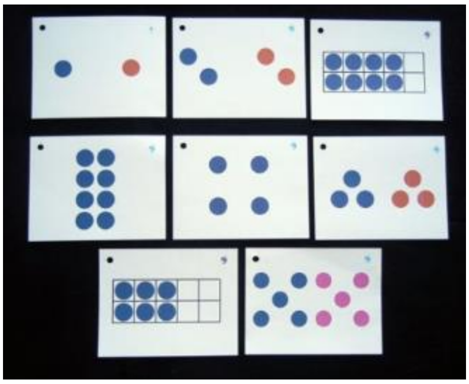
What patterns do you notice in this picture?



What patterns do you notice in this picture?



What patterns do you notice in this picture?



Challenging Task Worksheet (Lesson 14) **NAME:**

When the not so friendly giant arrived in Belgrave South on Monday, there were 64 Grade 2 children in the town. How long will it take until there is only 1 Grade 2 child left?

To help you, have a go at completing the following table. Remember, each night the giant will eat half of the children.

	Grade 2 Children	Picture
Monday	64	☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺
Tuesday		☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺
Wednesday		
Thursday		
Friday		
Saturday		
Sunday		
Monday		

Extending Prompt (Lesson 14)

Next stop for the not so friendly giant was Ferntree Gully. When the not so friendly giant arrived in Ferntree Gully on Monday, there were 640 Grade 2 children in the town. How long will it take until there are only 10 Grade 2 children left? Do you notice any patterns when you compare this table to Belgrave South?

	Grade 2 Children
Monday	640
Tuesday	
Wednesday	
Thursday	
Friday	
Saturday	
Sunday	
Monday	

Enabling Prompt A (Lesson 14) **NAME:**

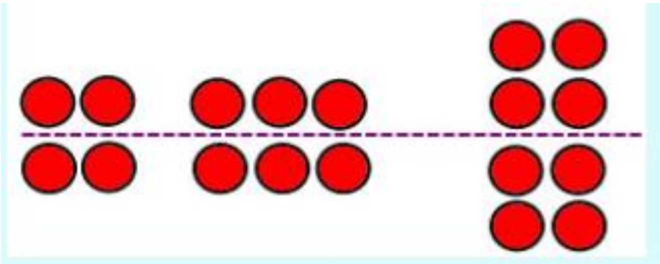
When the not so friendly giant arrived in Belgrave South on Monday, there were 16 Grade 2 children in the town. How long will it take until there is only 1 Grade 2 child left?

To help you, have a go at completing the following table. Remember, each night the giant will eat half of the children.

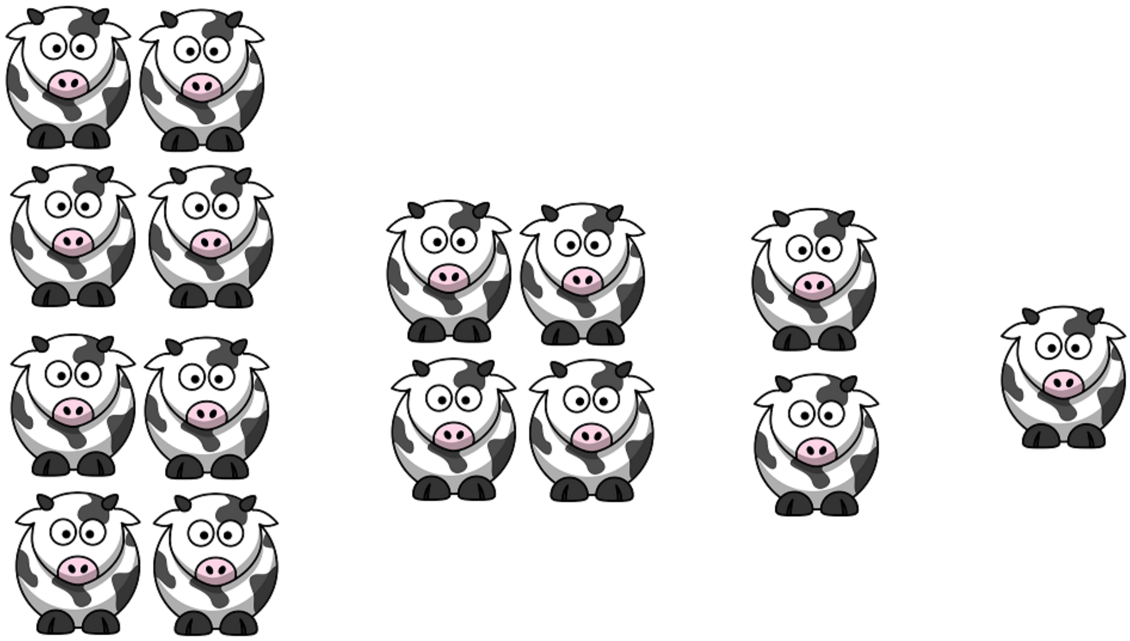
	Grade 2 Children	Picture
Monday	16	☺☺☺☺ ☺☺☺☺ ☺☺☺☺ ☺☺☺☺
Tuesday		☺☺☺☺ ☺☺☺☺
Wednesday		
Thursday		
Friday		
Saturday		
Sunday		
Monday		

Enabling Prompt B (Lesson 14):

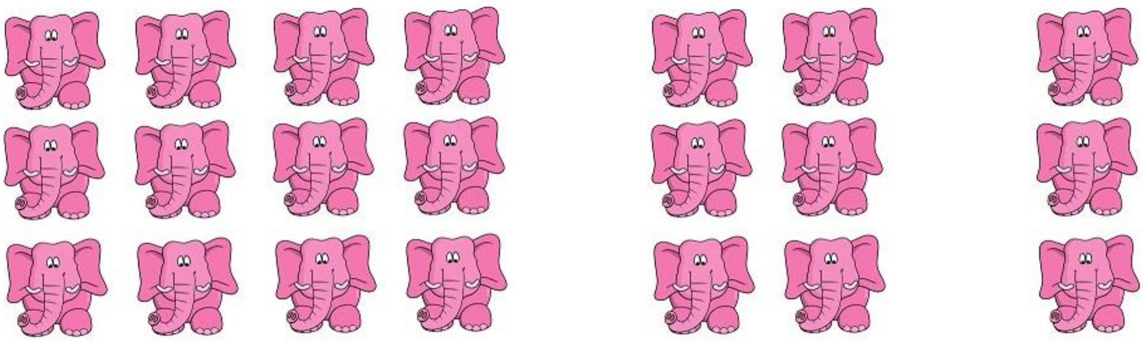
What patterns do you notice in this picture?



What patterns do you notice in this picture?



What patterns do you notice in this picture?



Challenging Task Worksheet (Lesson 15) **NAME:**

This Sunday afternoon, Mr Russo is inviting 50 parents and children to his farm in Belgrave to eat some of the fruit he has grown. He wants each of his guests to have:

One apple 

One pear 

Two oranges 

All the fruit on Mr Russo's trees grow in patterns.

	Mon	Tues	Wed	Thurs	Fri	Sat	Sun
Apples	13	18	23	28	33		
Pears	1	2	4	8	16		
Oranges	10	10	20	30	50		

Will he have enough fruit to feed all his guests on Sunday? Explain your thinking.

Extending Prompt (Lesson 15)

Mr Russo also wondered if his 50 guests would each be able to each have:

Three bananas 

Two cherries 

All the fruit on Mr Russo's trees grow in patterns.

	Mon	Tues	Wed	Thurs	Fri	Sat	Sun
Bananas	15	30	45	60	75		
Cherries	1	3	7	15	31		

Will he have enough fruit to feed all his guests on Sunday? Explain your thinking.

Enabling Prompt A (Lesson 15) **NAME:**

This Sunday afternoon, Mr Russo is inviting 20 parents and children to his farm in Belgrave to eat some of the fruit he has grown. He wants each of his guests to have:

One strawberry 

One plum 

All the fruit on Mr Russo's trees grow in patterns.

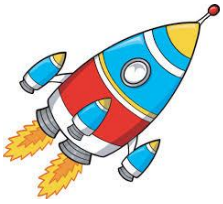
	Mon	Tues	Wed	Thurs	Fri	Sat	Sun
Banana	2	4	6	8	10		
Plum	3	6	9	12	15		

Will he have enough fruit to feed all his guests on Sunday? Explain your thinking.

Challenging Task Worksheet (Lesson 16) **NAME:**

Mr Russo was planning on launching rockets from the roof of Belgrave South Primary School.

His countdown would begin at 100, and when he got to 0, he would fire the rocket.



He began the countdown for Rocket A:

100, 90, 95, 85, 90, 80, 85…

Continue the counting sequence. Will the rocket fire? Why?

Next, he began the countdown for Rocket B:

100, 90, 80, 90, 100, 90…

Continue the counting sequence. Will the rocket fire? Why?

Finally, he began the countdown for Rocket C:

100, 90, 81, 73, 66, 60…

Continue the counting sequence. Will the rocket fire? Why?

Extending Prompt (Lesson 16:

He began the countdown for Rocket D:

128, 64, 32, 16, 8…

Continue the counting sequence. Will the rocket fire? Why?

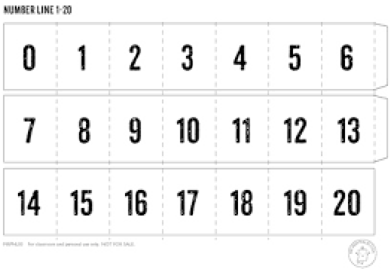
Enabling Prompt A (Lesson 16) **NAME:**

Mr Russo was planning on launching rockets from the roof of Belgrave South Primary School.

His countdown would begin at 20, and when he got to 0, he would fire the rocket. 

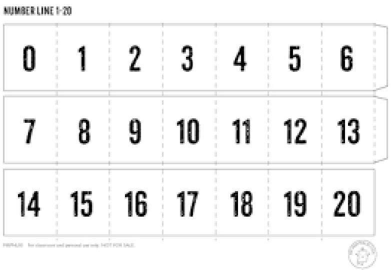
He began the countdown for Rocket X: 20, 19, 18, 17, 16…

Continue the counting sequence. Will the rocket fire? Why?



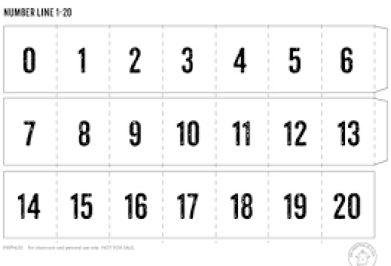
He began the countdown for Rocket Y: 20, 18, 19, 17, 18, 16, 17…

Continue the counting sequence. Will the rocket fire? Why?



He began the countdown for Rocket Y: 20, 19, 18, 19, 20, 19, 18…

Continue the counting sequence. Will the rocket fire? Why?



APPENDIX C

Teaching with tasks that make children think: Patterning Unit of Work

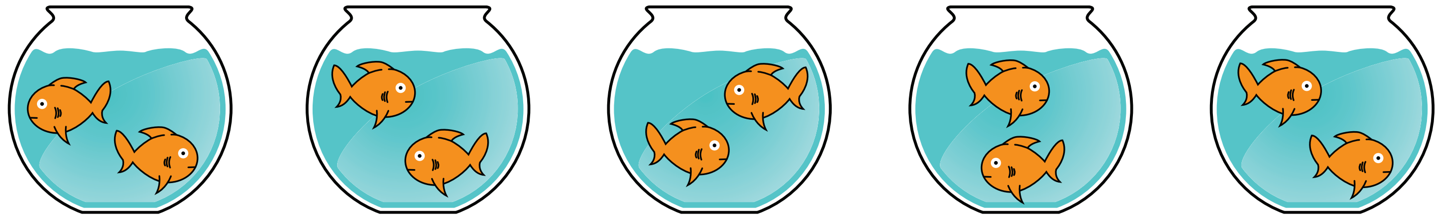
This unit of work was developed by James Russo (Belgrave South Primary School and Monash University), in consultation with his PhD supervisors Sarah Hopkins and Peter Sullivan, as part of his PhD project.

CONSOLIDATING WORKSHEETS

Name: ______________________________

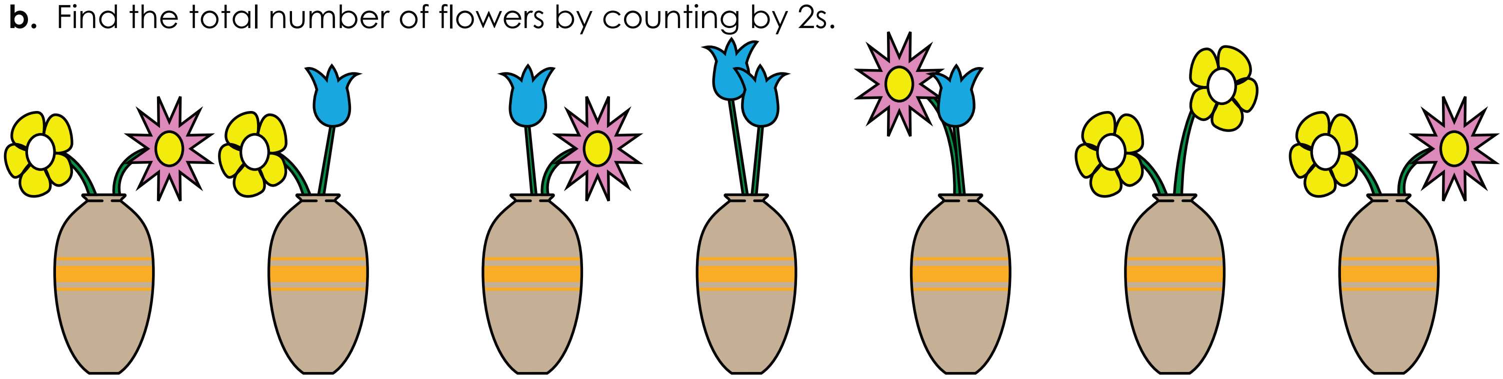
Counting by 2s

a. Find the total number of fish by counting by 2s.



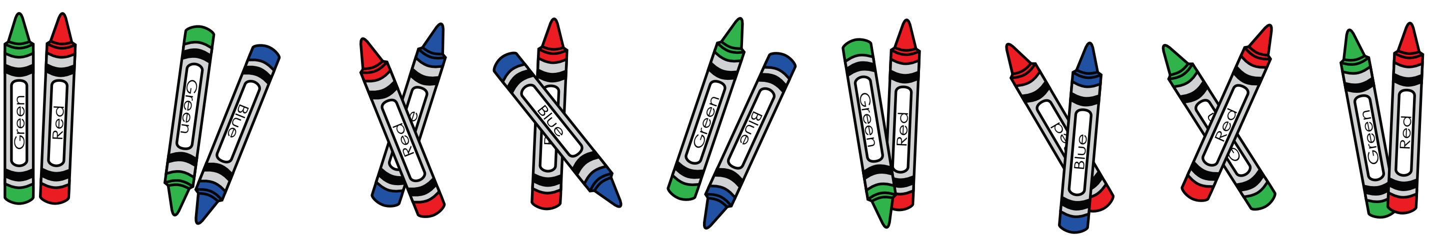
2 ______ 4 ______ ______ ______ ______

b. Find the total number of flowers by counting by 2s.



______ ______ ______ ______ ______ ______ ______

c. Find the total number of crayons by counting by 2s.



______ ______ ______ ______ ______ ______ ______ ______ ______

d. Find the total number of snails by counting by 2s.

______ ______ ______ ______

Name: ____________________

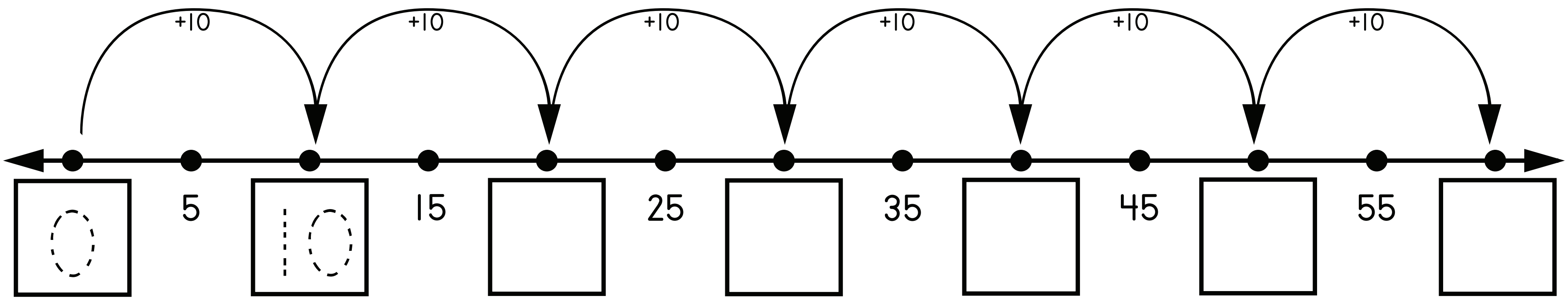
Count by 2s

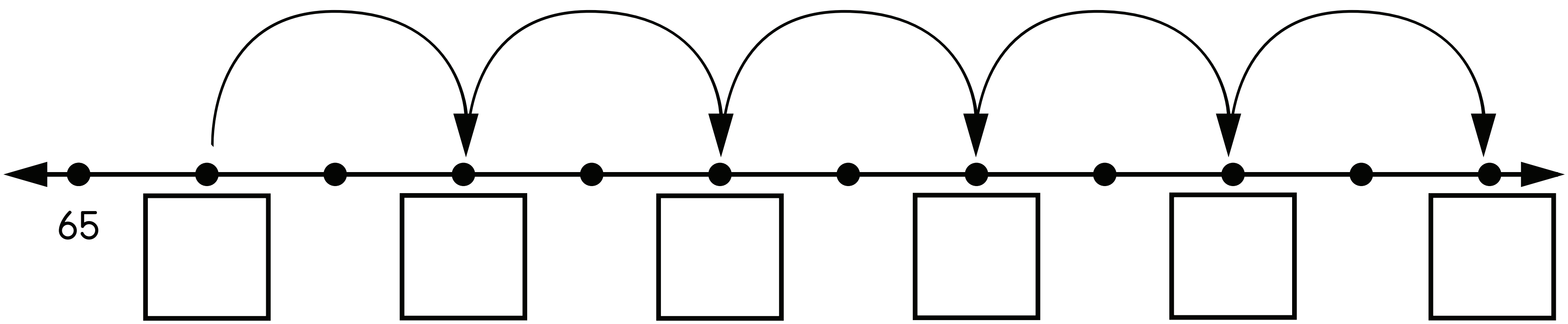
Count by 2s and fill in the missing numbers in the balloons.

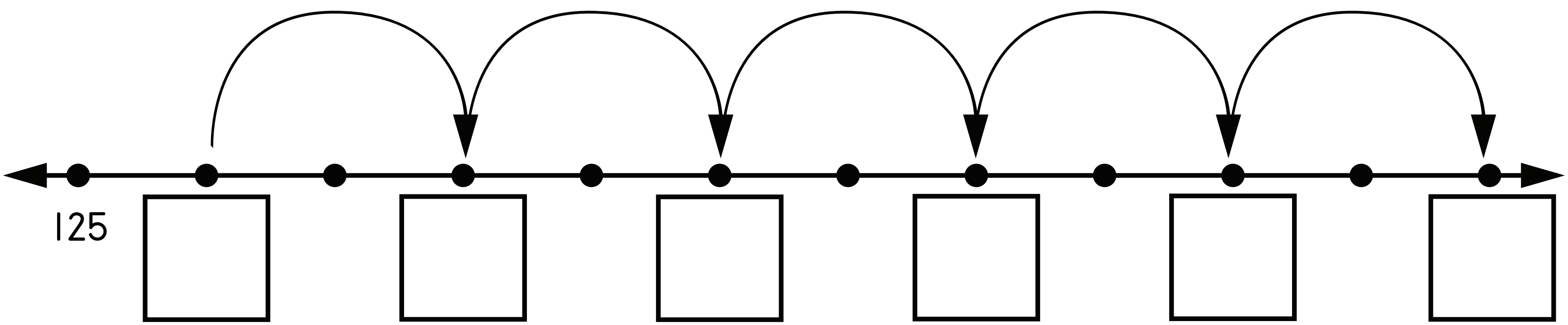
Name:______________________

Count by 10s

Count by 10s and fill in the missing numbers on the number lines.



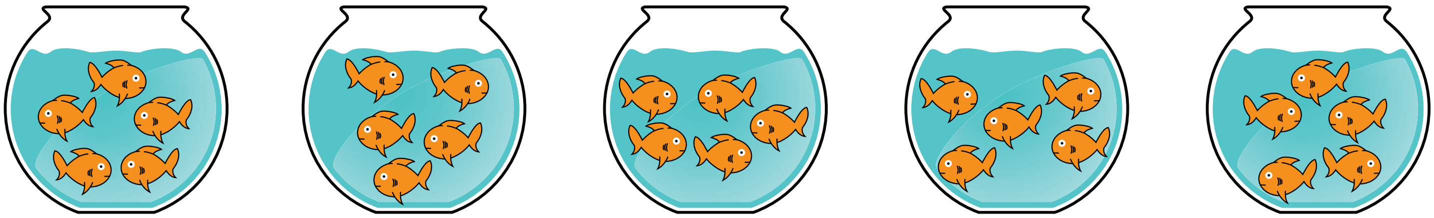




Name: ______________________________

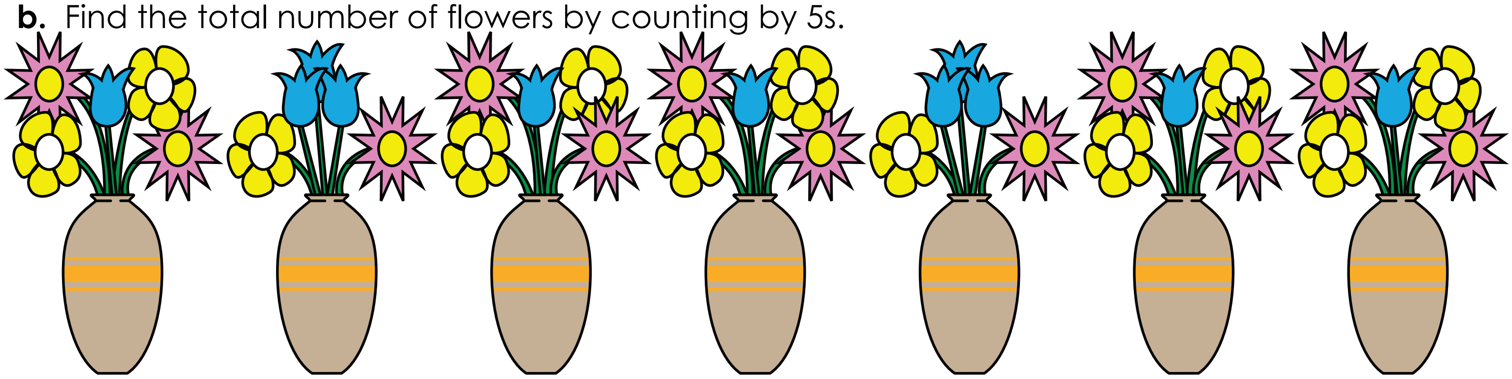
Counting by 5s

a. Find the total number of fish by counting by 5s.



5 ______ 10 ______ ______ ______ ______

b. Find the total number of flowers by counting by 5s.

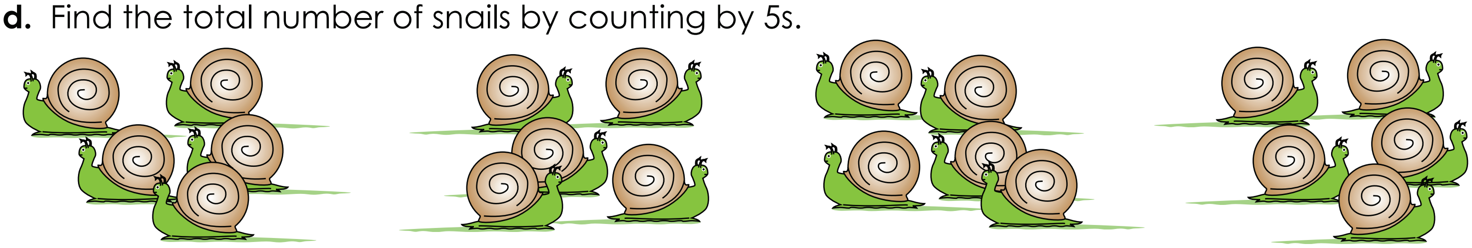


______ ______ ______ ______ ______ ______ ______

c. Find the total number of crayons by counting by 5s.

______ ______ ______ ______ ______ ______ ______ ______ ______

d. Find the total number of snails by counting by 5s.

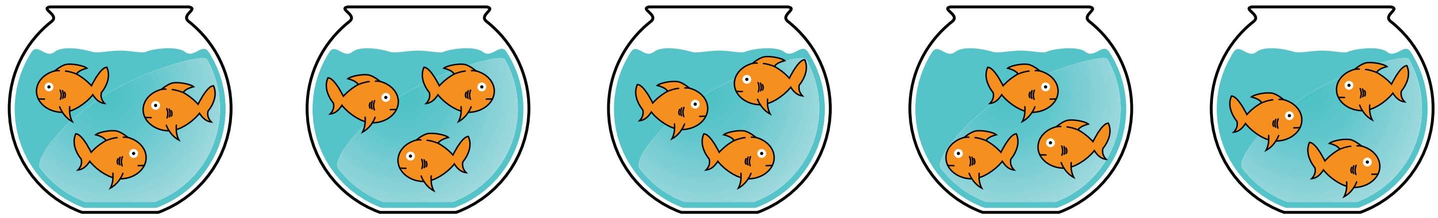


______ ______ ______ ______

Name: ___________________________________

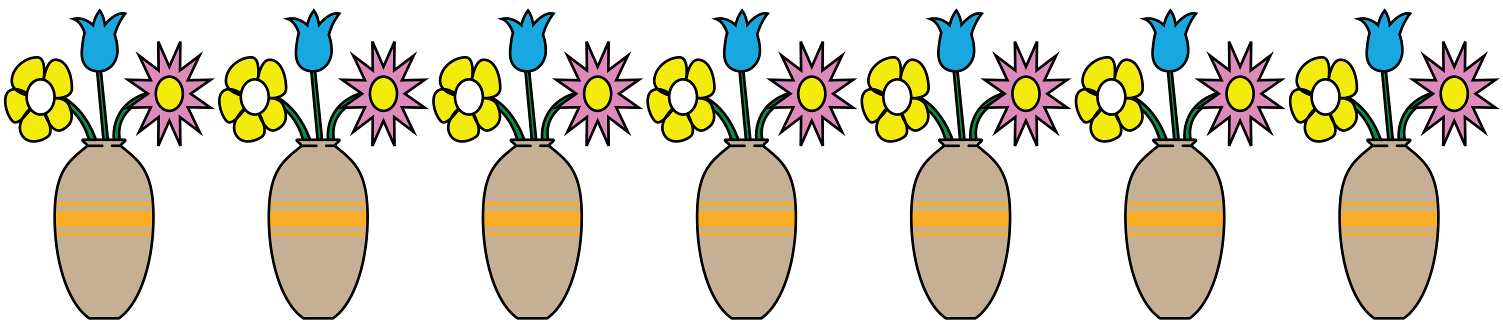
Counting by 3s

a. Find the total number of fish by counting by 3s.



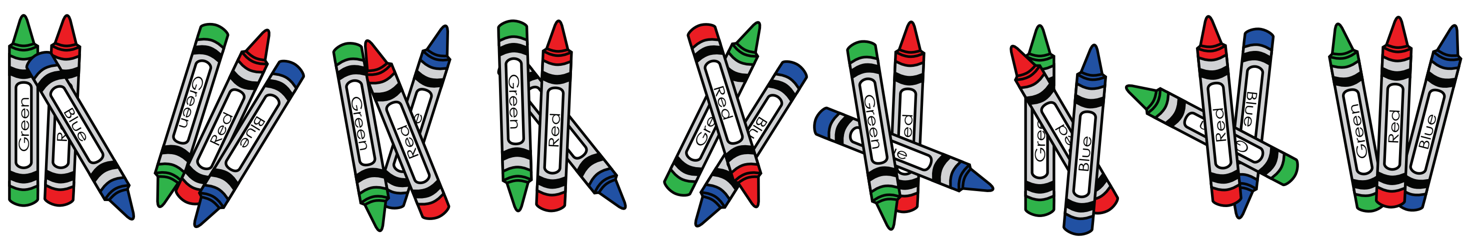
3 ______ 6 ______ ______ ______ ______

b. Find the total number of flowers by counting by 3s.



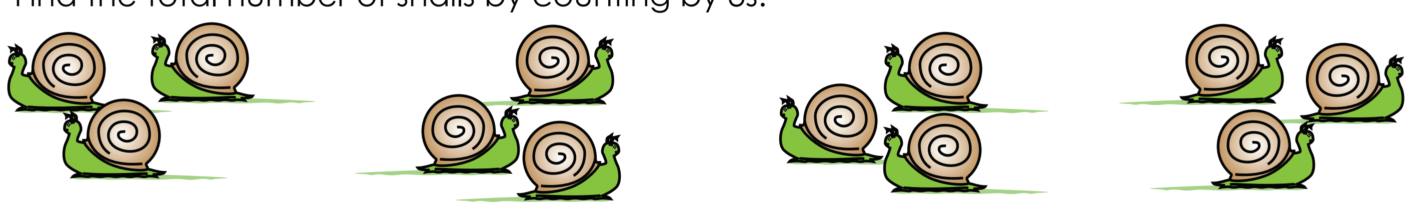
______ ______ ______ ______ ______ ______ ______

c. Find the total number of crayons by counting by 3s.



______ ______ ______ ______ ______ ______ ______ ______ ______

d. Find the total number of snails by counting by 3s.

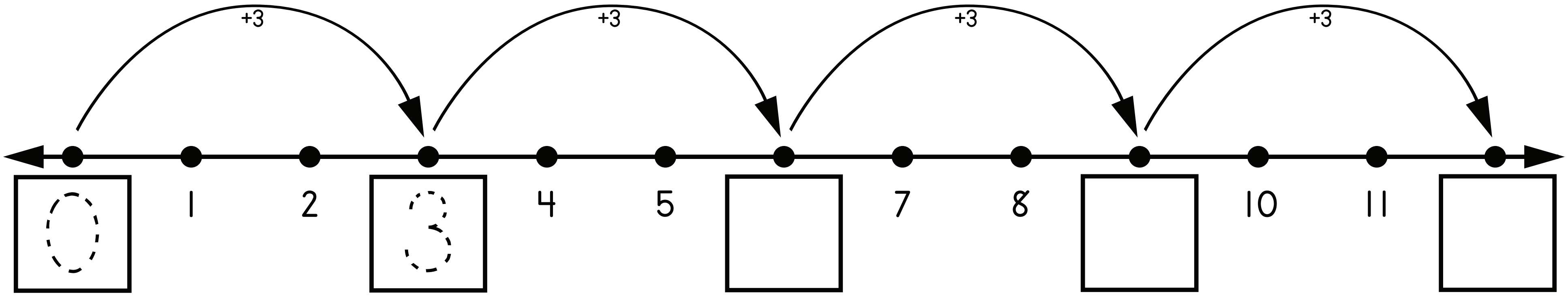


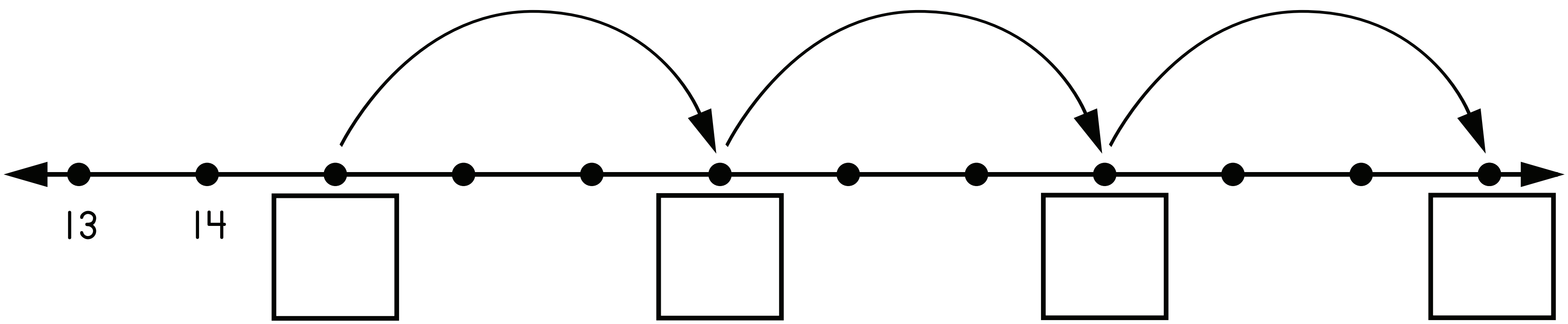
______ ______ ______ ______

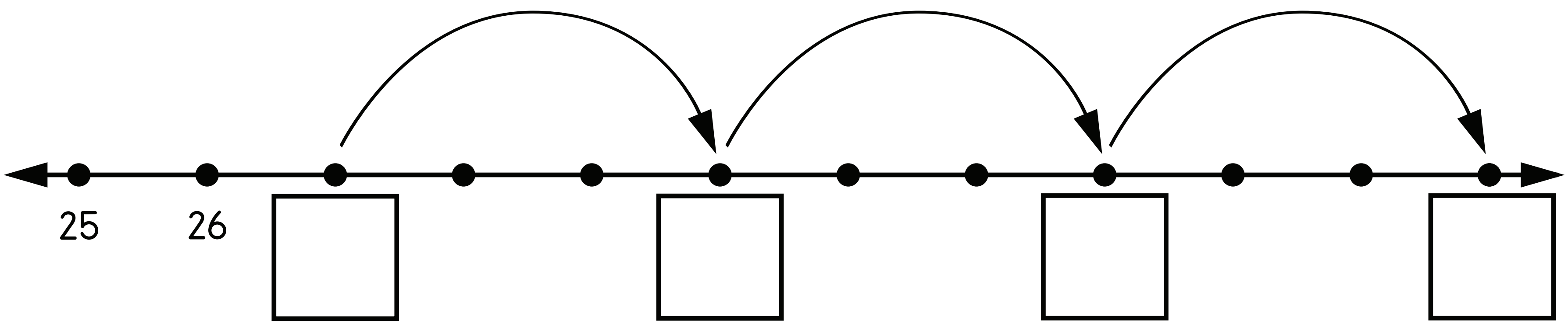
Name: ____________________

Count by 3s

Count by 3s and fill in the missing numbers on the number lines.







Name: ______________________________ Date: ______________

Counting patterns

Fill in the blanks and finish the counting pattern

1. 1, 3, 5, 7, 9, ______, ______, 15, 17, 19

2. 19, 17, 15, 13, 11, ______, ______, 5, 3, 1

3. 3, 6, 9, 12, 15, 18, ______, ______, 27

4. 4, 7, 10, 13, 16, 19, ______, ______, 28

5. 33, 35, 37, 39, 41, 43, ______, ______, ______

6. 1, 4, 7, 10, ______, ______, ______, 22, ______

7. ______, 3, 5, 7, ,9, 11, 13, 15, ______, 19

8. 50, 53, 56, 59, 62, 65, 68, ______, ______

9. 54, 56, 58, 60 ______, ______, ______, 68

10. 7, 9, ______, 13, 15, ______, 19, 21, ______

11. ______, ______, ______, ______, 23, 25, 27, 29

12. 5, 8, 11, 14, 17, ______, ______, ______ Name: ______________________________ Date: ______________

Counting patterns

13. 80, 83, ______, 89, 92, ______, 98, 101, ______

14. 110, 113, 116, ______, ______, ______, ______

15. 100, 97, 94, 91, ______, ______, ______, ______

16. ______, ______, 107, 105, 103, 101, 99, 97

17. ______, ______, ______, ______, ______, ______, ______, ______, ______

18. ______, ______, ______, ______, ______, ______, ______, ______, ______

ANSWER KEY

Counting patterns

1. 1, 3, 5, 7, 9, (11), (13), 15, 17, 19
2. 19, 17, 15, 13, 11, (9), (7), 5, 3, 1
3. 3, 6, 9, 12, 15, 18, (21), (24), 27
4. 4, 7, 10, 13, 16, 19, (22), (25), 28
5. 33, 35, 37, 39, 41, 43, (45), (47), (49)
6. 1, 4, 7, 10, (13), (16), (19), 22, (25)
7. (1), 3, 5, 7, ,9, 11, 13, 15, (17), 19
8. 50, 53, 56, 59, 62, 65, 68, (), ()
9. 54, 56, 58, 60 (62), (64), (66), 68
10. 7, 9, (11), 13, 15, (), 19, 21, (23)
11. (15), (17), (19), (21), 23, 25, 27, 29
12. 5, 8, 11, 14, 17, (20), (23), (26)
13. 80, 83, (86), 89, 92, (95), 98, 101, (104)
14. 110, 113, 116, (119), (122), (125), (128)
15. 100, 97, 94, 91, (88), (85), (82), (79)
16. (111), (109), 107, 105, 103, 101, 99, 97
17. (), (), (), (), (), (), (), (), ()
18. (), (), (), (), (), (), (), (), ()

Counting patterns

This worksheet was created with the Fill-in-the-Blank Worksheet Generator on Super Teacher Worksheets (www.superteacherworksheets.com).

Worksheet Title: Counting patterns

Created By: Mr Russo

Date Created: Mar 29, 2015

Filename: counting patterns- 2\\\\\\\'s, 3\\\\\\\'s

Direct Link:
http://www.superteacherworksheets.com/custom/?fi=qQmrg

Name: ______________________________

Count by 2s

a.

2	4			10			

b.

64		68		72						84	

c.

30						42					

d.

				84				92			

f.

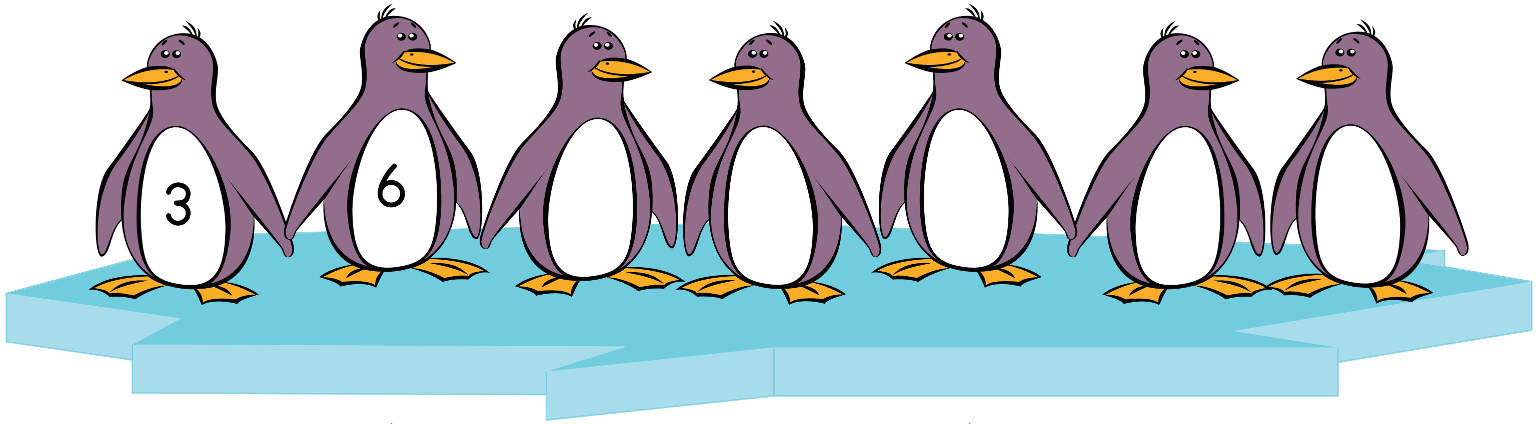
					26						

Name:___________________

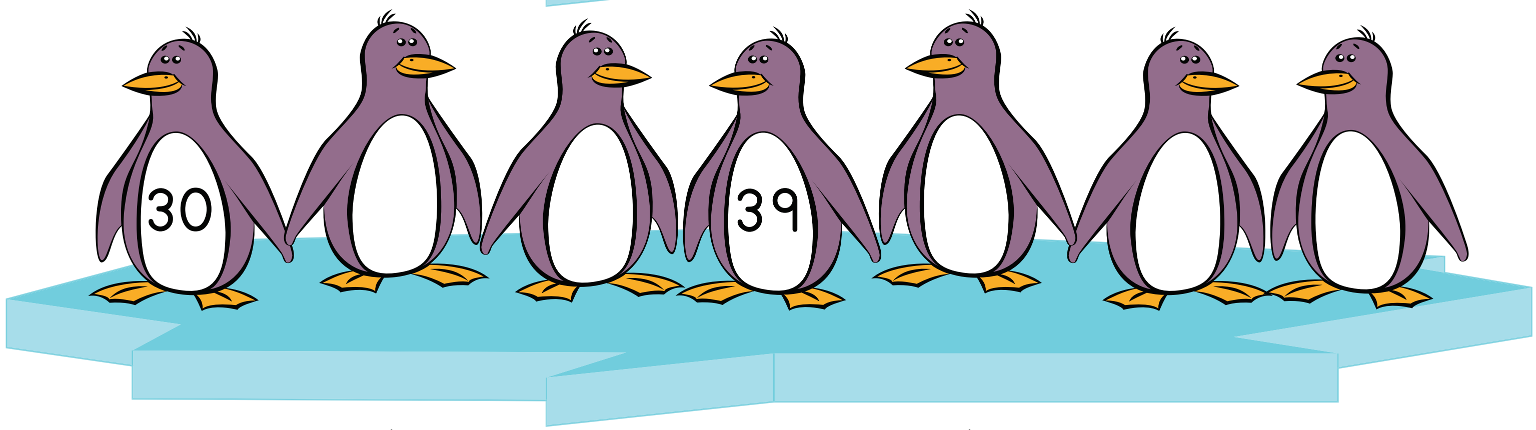
Count by 3s

Count by 3s and fill in the missing numbers in each group of penguins.

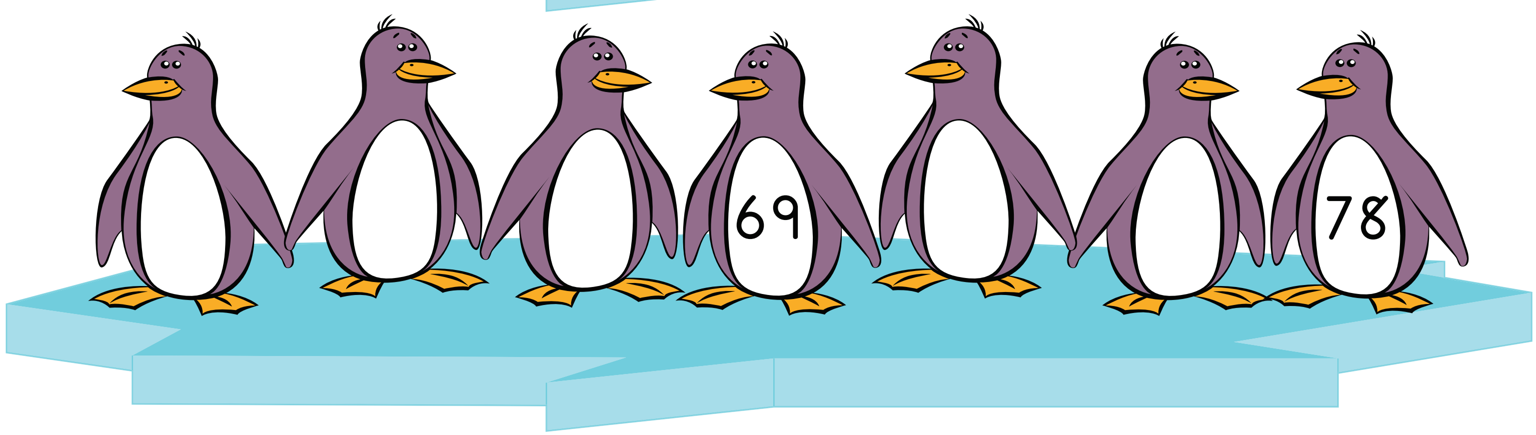
a.



b.



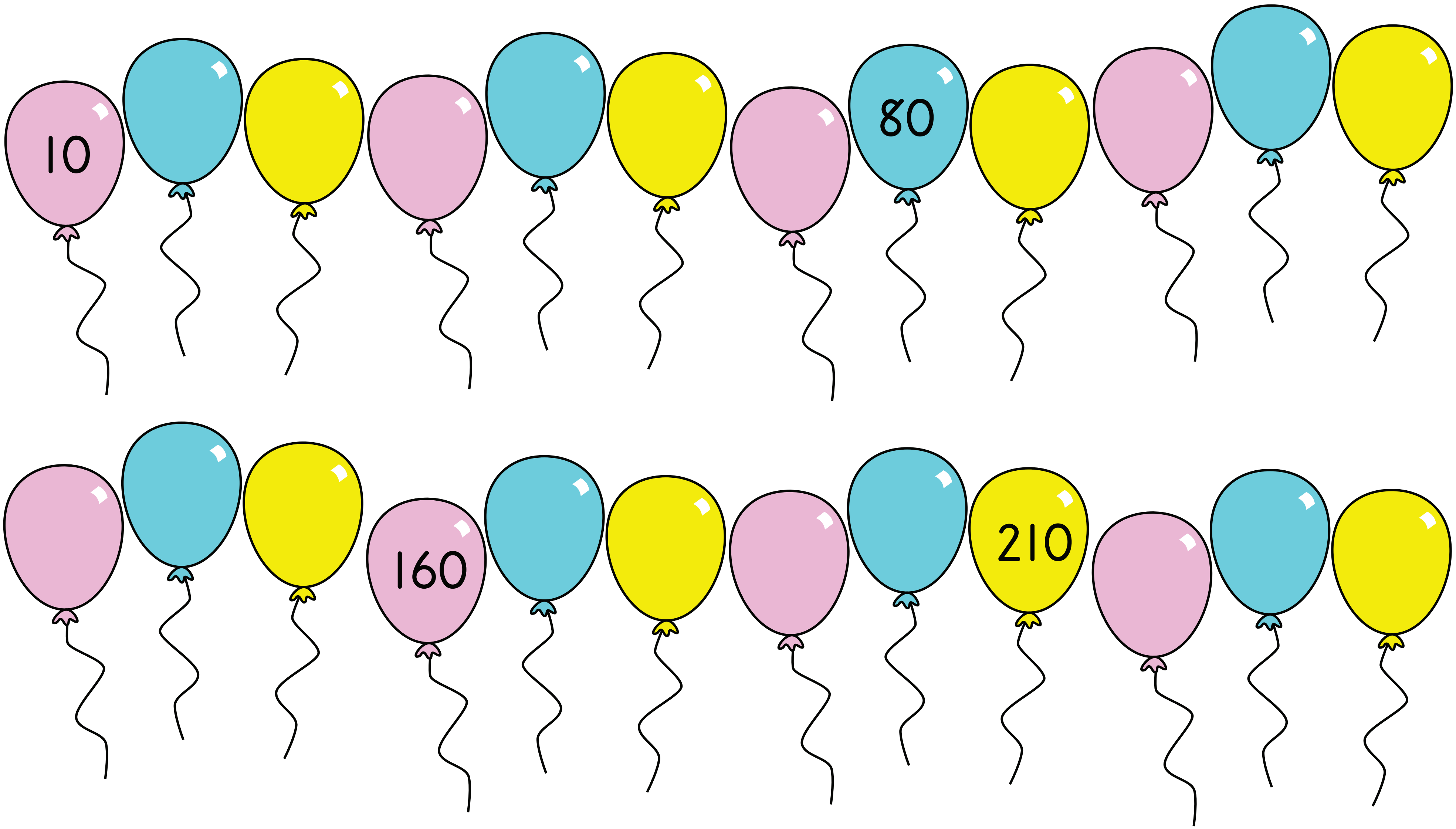
c.



Name:______________________________

Count by 10s

Count by 10s and fill in the missing numbers in the balloons.



Name:______________________________

Skip Count by 2s

Write the missing numbers.

2	4			
		16		
	24			30
			38	

Name: ______________________________

Count by 5s

a.

0	5	10			

b.

70			85		

c.

					55			

d.

			115	

Name: ____________________

Count by 3s

Count by 3s and fill in the missing numbers in the balloons.

Name: ______________________________

Count by 4s

a.

4	8					28	

b.

36						60				76	

c.

				32						56	

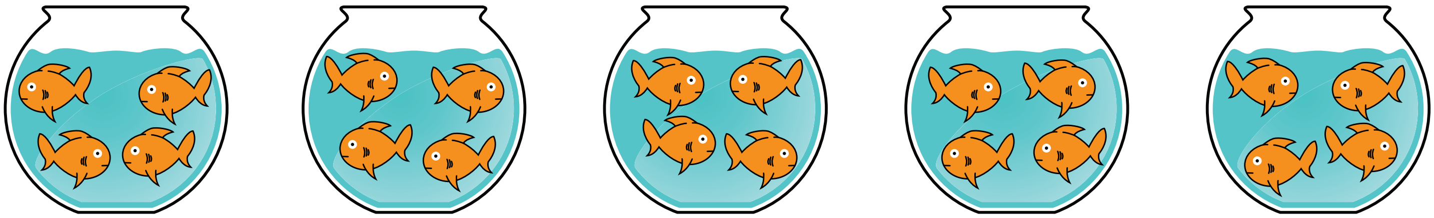
d.

						96	

Name: ______________________________

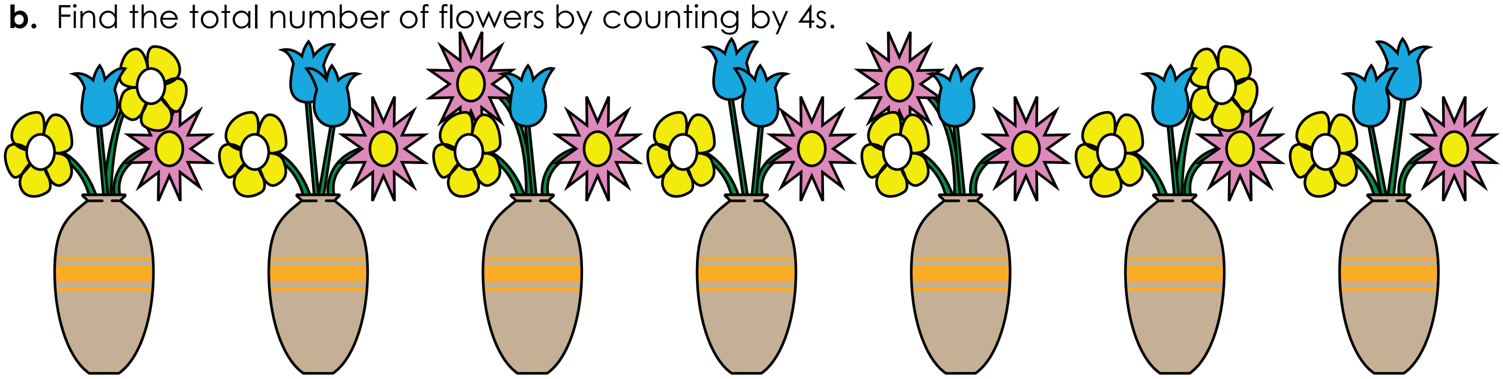
Counting by 4s

a. Find the total number of fish by counting by 4s.



4 ______ 8 ______ ______ ______ ______

b. Find the total number of flowers by counting by 4s.

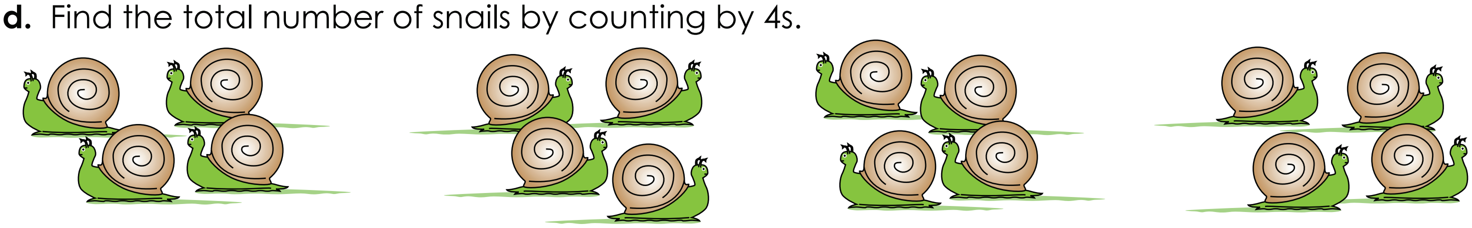


______ ______ ______ ______ ______ ______ ______

c. Find the total number of crayons by counting by 4s.

______ ______ ______ ______ ______ ______ ______ ______ ______

d. Find the total number of snails by counting by 4s.



______ ______ ______ ______

Name: ____

Bugs Bunny Patterns

Choose the best answer for each question.

1. Bugs Bunny scored 8 points. He might have scored?
- **a.** 1 three-pointer, 1 field goal, 1 free throw
- **b.** 2 three-pointers, 2 field goals, 1 free throw
- **c.** 2 three-pointers, 1 field goal
- **d.** 3 field goals

2. Bugs Bunny scored 10 points. He might have scored?
- **a.** 4 Field Goals
- **b.** 2 Three Pointers
- **c.** 3 Three Pointers and 1 Free Throw
- **d.** 7 Free Throws

3. Bugs Bunny scored 12 points. He might have scored?
- **a.** 6 Field Goals
- **b.** 3 Three Pointers and 3 Field Goals
- **c.** 10 Free Throws
- **d.** 4 Three Pointers and 1 Field Goal

4. Bugs made 4 Field Goals and 4 Three Pointers. He scored
- **a.** 18 points
- **b.** 20 points
- **c.** 22 points
- **d.** 24 points

5. Bugs made 4 Free Throws, 2 Field Goals and 3 Three Pointers. He scored
- **a.** 5 points
- **b.** 50 points
- **c.** 17 points
- **d.** 19 points

Name: ____

6. What number is next in the pattern: 2, 5, 7, 10, 12, 15
- **a.** 16
- **b.** 17
- **c.** 18
- **d.** 19

7. What number is next in the pattern: 2, 3, 5, 6, 8, 9
- **a.** 10
- **b.** 11
- **c.** 12
- **d.** 13

8. What number is next in the pattern: 1, 4, 5, 8, 9, 12
- **a.** 13
- **b.** 14
- **c.** 15
- **d.** 16

9. What number is next in the pattern: 2, 4, 6, 8, 10, 12
- **a.** 13
- **b.** 14
- **c.** 15
- **d.** 16

10. What number is next in the pattern: 3, 4, 7, 8, 11, 12
- **a.** 13
- **b.** 14
- **c.** 15
- **d.** 16

Name: ________________________________ Date: ________________

More patterns with Bugs Bunny

Choose the best answer for each question.

1. Bugs Bunny scored 10 runs. He might have scored?
- **a.** 1 six, 1 four
- **b.** 2 fours
- **c.** 2 sixes
- **d.** 3 fours

2. Bugs Bunny scored 12 runs. He might have scored?
- **a.** 2 fours
- **b.** 3 fours
- **c.** 2 sixes and 1 four
- **d.** 2 fours and 1 six

3. Bugs Bunny scored 16 runs. He might have scored?
- **a.** 2 sixes and 2 fours
- **b.** 3 fours
- **c.** 1 four and 2 sixes
- **d.** 4 sixes

4. Bugs hit 4 fours and 2 sixes. He might have scored?
- **a.** 29 runs
- **b.** 20 runs
- **c.** 32 runs
- **d.** 28 runs

5. Bugs made 3 fours and 3 sixes. He might have scored?
- **a.** 10 runs
- **b.** 20 runs
- **c.** 24 runs
- **d.** 30 runs

Name: ________________________________ Date: ________________

More patterns with Bugs Bunny

6. What number is next in the pattern: 2, 8, 12, 18, 22, 28, 32

- **a.** 34
- **b.** 26
- **c.** 36
- **d.** 38

7. What number is next in the pattern: 12, 16, 20, 24, 28, 32

- **a.** 34
- **b.** 36
- **c.** 38
- **d.** 40

8. What number is next in the pattern: 6, 12, 18, 24, 30, 36

- **a.** 40
- **b.** 41
- **c.** 42
- **d.** 43

9. What number is next in the pattern: 2, 6, 10, 14, 18, 22

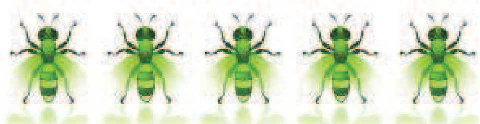
- **a.** 24
- **b.** 25
- **c.** 26
- **d.** 27

10. What number is next in the pattern: 6, 10, 14, 20, 24, 30

- **a.** 32
- **b.** 34
- **c.** 26
- **d.** 36

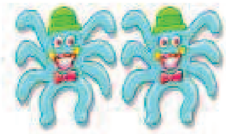
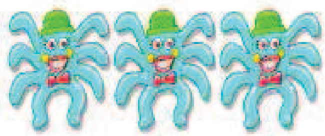
How many legs?

Name:
Grade:

6 	
	24
	36
	
54	

How many legs?

Name:
Grade:

8 	
	32
	
56	

Name: ____

Doubling Donuts

Fill in the best answer for each question. Using some counters or an abacus to help you.

1. I have 5 donuts today, so I will have ______ donuts tomorrow

2. I have 3 donuts today, so I will have ______ donuts tomorrow

3. I have 7 donuts today, so I will have ______ donuts tomorrow

4. I have 10 donuts today, so I will have ______ donuts tomorrow

5. I have 50 donuts today, so I will have ______ donuts tomorrow

6. I have 1 donut on Monday, so I will have 2 donuts on Tuesday and ______ donuts on Wednesday

7. I have 2 donuts on Monday, so I will have 4 donuts on Tuesday and ______ donuts on Wednesday

8. I have 5 donuts on Monday, so I will have 10 donuts on Tuesday and ______ donuts on Wednesday

Name: ____

Doubling Donuts

9. I have 4 donuts on Monday, so I will have ______ donuts on Tuesday and ______ donuts on Wednesday

10. I have 6 donuts on Monday, so I will have ______ donuts on Tuesday and ______ donuts on Wednesday

11. I have 8 donuts on Monday, so I will have ______ donuts on Tuesday and ______ donuts on Wednesday

12. I have 11 donuts on Monday, so I will have ______ donuts on Tuesday and ______ donuts on Wednesday

13. I have ______ donuts on Monday, so I will have 20 donuts on Tuesday and ______ donuts on Wednesday

14. I have ______ donuts on Monday, so I will have ______ donuts on Tuesday and 28 donuts on Wednesday

15. I have 9 donuts on Monday, so I will have ______ donuts on Tuesday and ______ donuts on Wednesday

16. I have 20 donuts on Monday, so I will have ______ donuts on Tuesday, ______ donuts on Wednesday and ______ donuts on Thursday

Name: ____

Halving Children

Fill in the best answer for each question. Using some counters or an abacus to help you.

1. There are 6 children today, tomorrow there will be ______ children

2. There are 10 children today, tomorrow there will be ______ children

3. There are 16 children today, tomorrow there will be ______ children

4. There are 20 children today, tomorrow there will be ______ children

5. There are 100 children today, tomorrow there will be ______ children

6. There are 24 children on Monday, so there will be 12 children on Tuesday and ______ children on Wednesday

7. There are 16 children on Monday, so there will be 8 children on Tuesday and ______ children on Wednesday

8. There are 36 children on Monday, so there will be 18 children on Tuesday and ______ children on Wednesday

Name: ____

Halving Children

9. There are 24 children on Monday, so there will be 12 children on Tuesday and ______ children on Wednesday

10. There are 20 children on Monday, so there will be 10 children on Tuesday and ______ children on Wednesday

11. There are 40 children on Monday, so there will be ______ children on Tuesday and ______ children on Wednesday

12. There are 32 children on Monday, so there will be ______ children on Tuesday and ______ children on Wednesday

13. There are 100 children on Monday, so there will be ______ children on Tuesday and ______ children on Wednesday

14. There are ______ children on Monday, so there will be ______ children on Tuesday and 7 children on Wednesday

15. There are ______ children on Monday, so there will be ______ children on Tuesday and 12 children on Wednesday

16. There are ______ children on Monday, so there will be 28 children on Tuesday and ______ children on Wednesday

Name: ______________________________

Number Line Patterns

Write the missing numbers for each number line pattern.

a.

57	______	61	63	65	______	______

b.

87	92	97	______	______	______	117

c.

24	______	36	______	______	54	60

d.

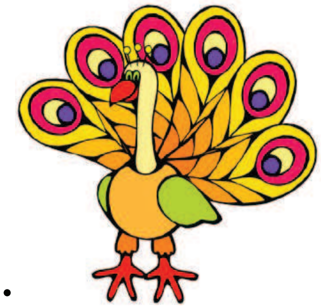
______	______	46	55	______	73	82

e.

______	______	8	12	16	20	______

Name: ______________________________

Number Patterns



Write the numbers that come next.

1. 1, 3, 5, 7, 9, ______, ______, ______ ...

2. 2, 4, 6, 8, 10, ______, ______, ______ ...

3. 24, 34, 44, 54, ______, ______, ______ ...

4. 3, 6, 9, 12, 15, ______, ______, ______ ...

5. 35, 40, 45, 50, ______, ______, ______ ...

6. 11, 22, 33, 44, ______, ______, ______ ...

7. 9, 19, 29, 39, ______, ______, ______ ...

Name: ____

Launching Rockets

Fill in the blanks and count down to zero to help launch Mr Russo's rockets.

1. 100, 90, 80, 70, 60, 50, 40, ______, ______, ______, ______

2. 50, 45, 40, 35, 30, 25, 20, ______, ______, ______, ______

3. 22, 20, 18, 16, 14, ______, ______, ______, ______, ______, ______, ______

4. 24, 21, 18, 15, ______, ______, ______, ______, ______

5. 32, 28, 24, 20, 16, ______, ______, ______, ______

6. 120, 100, 80, 60, ______, ______, ______

7. ______, ______, ______, ______, 12, 10, 8, 6, 4, 2, ______

8. 90, ______, 70, 60, 50, ______, ______, 20, ______, ______

9. 60, 54, 48, 42, 36, ______, ______, ______, ______, ______, ______

10. 80, 72, 64, 56, 48, ______, ______, 24, 16, ______, ______

11. ______, ______, ______, 18, 15, 12, 9, ______, ______, ______

Name: ____

Launching Rockets

12. ______, ______, ______, 100, 80, 60, ______, ______, ______

13. 77, ______, 63, 56, 49, ______, 35, 28, ______, 14, ______, ______

14. 99, 88, 77, ______, ______, ______, 33, ______, ______, ______

15. 1000, ______, 800, ______, 600, 500, 400, ______, ______, ______, ______

16. 200, 175, 150, 125, 100, ______, ______, ______, ______

17. ______, ______, ______, ______, ______, ______, ______, ______, ______

18. ______, ______, ______, ______, ______, ______, ______, ______, ______

APPENDIX D

Teaching with tasks that make children think: Addition and missing addends

This unit of work was developed by James Russo (Belgrave South Primary School and Monash University), in consultation with his PhD supervisors Sarah Hopkins and Peter Sullivan, as part of his PhD project.

TEACHER'S GUIDE FOR THE UNIT OF WORK

Challenging Tasks: Addition Unit of Work

Current unit of work

The focus of the current unit of work is on addition. In particular, the overarching learning objective for the unit is for students to look for ***patterns*** to help us solve maths problems in ***efficient*** ways. The unit of work has been designed for Grade 1 and 2 students. The challenging tasks presented in the unit contain a mixture of number sentences and worded problems.

After completing a lesson built around a challenging task, it is suggested that students be given an opportunity to consolidate their understanding of a particular concept through undertaking a series of more routine tasks. Worksheets containing these routine tasks have been included with the current unit of work. Each challenging task has a corresponding consolidating worksheet included. These worksheets could either be completed straight after the challenging task (as part of a 90-minute maths session), as a homework activity, or during a short (approx. 20 minute) follow-up session the following day.

Lesson specific learning objectives

The specific learning objectives for each of the 12 lessons (6 by 2) are outlined below.

- Lesson 1A and 1B: To look for ***patterns*** to help us solve maths problems in ***efficient*** ways. Today we looked to make **rainbow facts (complements to 10)**.
- Lesson 2A and 2B: To look for ***patterns*** to help us solve maths problems in ***efficient*** ways. Today we looked to make **doubles facts**.
- Lesson 3A and 3B: To look for ***patterns*** to help us solve maths problems in ***efficient*** ways. Today we looked to make **tens numbers (friendly numbers)**.
- Lesson 4A and 4B: To look for ***patterns*** to help us solve maths problems in ***efficient*** ways. Today we looked to **bridge through 10.**
- Lesson 5A and 5B: To look for ***patterns*** to help us solve maths problems in ***efficient*** ways. Today we looked to **break up numbers and regroup into lots of 10.**
- Lesson 6A and 6B: To look for ***patterns*** to help us solve maths problems in ***efficient*** ways. Today we looked to make **super rainbow facts (complements to 100).**

Standards from the Australian Curriculum

This unit of work connects with a number of key ideas in primary school mathematical curricula. The most relevant content descriptions connecting the current unit of work to the Australian curriculum are included below.

Key content descriptions:

- Represent and solve simple addition and subtraction problems using a range of strategies including counting on, partitioning and rearranging parts (grade 1)
- Solve simple addition and subtraction problems using a range of efficient mental and written strategies (grade 2)

Other relevant content descriptions:

- Develop confidence with number sequences to and from 100 by ones from any starting point. Skip count by twos, fives and tens starting from zero (grade 1)
- Count collections to 100/ 1000 by partitioning numbers using place value (grade 1/2)

- Recognise, model, read, write and order numbers to at least 100/ 100 (grade 1/2)
- Explore the connection between addition and subtraction (grade 2)
- Use equivalent number sentences involving addition and subtraction to find unknown quantities (Grade 4)

Lesson 1A and 1B: Change the order – Rainbow Facts

Rationale for the lesson:

For students to be able to make addition easier when adding more than two numbers through looking for patterns and changing the order in which the numbers are added.

Mental computation can sometimes involve changing the order in which we add numbers in order to arrive at an answer in an efficient manner. The foci of these lessons are on recognising the power of ten facts (e.g., 7+3; 8+2) and doubles facts (e.g., 4+4, 3+3) to aid the addition of a long sequence of numbers. Formally students will be partitioning and regrouping numbers through applying the associative properties of addition.

Prior knowledge:

- Assumes that students are familiar with tens facts (rainbow facts) and have some appreciation of these patterns.
- Assumes that students are able to both add groups of 10, and add a multiple of 10 to a single digit number with some fluency.

Teacher-facilitated discussion and instruction of key mathematical ideas:

- There is always more than one way of adding more than two numbers together.
- We can add numbers in *any order* we like, depending on what is most helpful for solving a problem.
- Looking for particular *patterns* can help decide which order we should add numbers in.

Challenging Task A (More than 1 way):

Task:

Can you work out the answer to this number sentence?

0 + 1 + 2 + 3 + 4 + 6 + 7 + 8 + 9 + 10 =

Try doing it another way. Which way do you prefer and why?

Enabling prompt:

b. Simpler problem: 1 + 2 + 8 + 9 = ?
c. Hint about pattern: Can you see any rainbow facts?
d. Help from a friend

Extending prompt:

Can you work out the answer to this number sentence?

1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 =

Can you solve it another way? Which way did you think was easier? Why?

Possible solutions:

The intention is for students to see that the problem can be more easily solved if the numbers are rearranged. For instance, students may choose to add up the numbers as a series of rainbow facts:

=1+9+2+8+3+7+4+6+ 10 + 0

=10+10+10+10+10

=50

Challenging Task B (Worded problem):

Task:

Your friend challenged you to add all of the digits from 1 to 9 together. Can you work out a quick way of finding the answer?

Enabling prompt:

a. Number sentence: 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 =
b. Simpler problem: 1 + 5 + 5 + 9 = ?
c. Hint about pattern: Can you see any rainbow facts?
d. Help from a friend

Extending prompt:

Can you work out a quick way of adding all the numbers from 1 to 20 together?

Possible solutions:

The intention is for students to see that the problem can be more easily solved if the numbers are rearranged. For instance, students may choose to add up the numbers as a series of rainbow facts:

=1+9+2+8+3+7+4+6+5

=10+10+10+10+5

=45

Lesson 2A and 2B: Change the order – Doubles Facts

Rationale for the lesson:

For students to be able to make addition easier when adding more than two numbers through looking for patterns and changing the order in which the numbers are added.

Mental computation can sometimes involve changing the order in which we add numbers in order to arrive at an answer in an efficient manner. The foci of these lessons are on recognising the power of ten facts (e.g., 7+3; 8+2) and doubles facts (e.g., 4+4, 3+3) to aid the addition of a long sequence of numbers. Formally students will be partitioning and regrouping numbers through applying the associative properties of addition.

Prior knowledge:

- Assumes that students are familiar with doubles facts and have some appreciation of these patterns.
- Assumes that students are able to add groups of 10 with some fluency.

Teacher-facilitated discussion and instruction of key mathematical ideas:

- There is always more than one way of adding more than two numbers together.
- We can add numbers in *any order* we like, depending on what is most helpful for solving a problem.
- Looking for particular *patterns* can help decide which order we should add numbers in.
- Sometimes there is more than one helpful (efficient) approach.

For this lesson:

- Focus on identifying doubles facts, tens facts and making 10 (out of 3 or more numbers) as relatively efficient approaches to solve a particular problem.

Challenging Task A (More than 1 way):

Task:

Can you work out the answer to this number sentence?

1 + 5 + 2 + 4 + 3 + 2 + 5 + 4 + 1 + 3 =

Try doing it another way. Which way do you prefer and why?

Enabling prompt:

b. Simpler problem: 3 + 5 + 5 + 2 + 2 + 3 =
c. Hint about pattern: Can you see any doubles facts? Can you see any ways of making groups of 10?
d. Help from a friend

Extending prompt:

Can you work out the answer to this number sentence?

117 + 25 + 25 + 50 =

Can you solve it another way? Which way did you think was easier? Why?

Possible solutions:

To solve the problem efficiently, students will likely have to rearrange the numbers. One option is to identify doubles facts, and then rainbow facts from these:

= 1+1+2+2+3+3+4+4+5+5

= 2+4+6+8+10

= 2+8+4+6+10

= 10+10+10

= 30

Another possible solution is to regroup numbers into lots of 10 at the outset:

= 1+5+4+1+5+4+2+3+2+3

= 10+10+10

= 30

Although this involves at least one less step, it also requires more complex pattern recognition (e.g., that the four numbers 2+3+2+3 can add to make 10).

Challenging Task B (Worded problem):

Task:

Will Sargood was an excellent cricketer. In one over, he hit Mr Russo for:

6, 2, 2, 2, 2, and 6.

In the next over, he did exactly the same thing.

How many runs did Will score?

Enabling prompt:

a. Number sentence: 6 + 2 + 2 + 2 + 2 + 6 + 6 + 2 + 2 + 2 + 2 + 6 =
b. Simpler problem: 6 + 2 + 2 + 6 =
c. Hint about pattern: Can you see any doubles facts? Can you see any ways of making groups of 10?
d. Help from a friend

Extending prompt:

In five innings, Will Sargood scored: 60, 25, 25, 60 and 50. How many runs did he score altogether?

Possible solutions:

To solve the problem efficiently, students will likely have to rearrange the numbers. One option is to work out how many runs he scored off the first over by identifying doubles facts, and then double your answer to account for the second over:

= 6 + 6 + 2 + 2 + 2 + 2

= 12+4+4

= 12+8

= 20

= Double 20

= 40

Another possible solution is to write down all 12 terms and look for groups of 10.

= 6 + 2 + 2 + 2 + 2 + 6 + 6 + 2 + 2 + 2 + 2 + 6

= 6 + 4 + 4 + 6 + 6 + 4 + 4 + 6

= 10 + 10 + 10 + 10

= 40

Lesson 3A and 3B: Change the order – Multiples of 10

Rationale for the lesson:

For students to be able to make addition easier when adding more than two numbers through looking for patterns and changing the order in which the numbers are added.

Mental computation can sometimes involve changing the order in which we add numbers in order to arrive at an answer in an efficient manner. The foci of these lessons are on recognising the power of ten facts (e.g., 7+3; 8+2) and doubles facts (e.g., 4+4, 3+3) to aid the addition of a long sequence of numbers. Formally students will be partitioning and regrouping numbers through applying the associative properties of addition.

Prior knowledge:

- Assumes that students are familiar with tens facts (rainbow facts) and have some appreciation of these patterns.
- Assumes that students are able to count to 100, and can recognise that 100 + 30 will equal 130. Even if students do not have a full appreciation for the place value concept, it is anticipated that this response can be generated though some understanding in language of how we construct numbers beyond 100 (i.e., we state 100 and 30 to represent 130).

Teacher-facilitated discussion and instruction of key mathematical ideas:

- There is always more than one way of adding more than two numbers together.
- We can add numbers in *any order* we like, depending on what is most helpful for solving a problem.
- Looking for particular *patterns* can help decide which order we should add numbers in.

For this lesson:

- Focus on identifying making multiples of 10 as relatively efficient approaches to solve a particular problem.

Challenging Task A (More than 1 way):

Task:

Can you work out the answer to this number sentence?

3 + 30 + 97 =

Try doing it another way. Which way do you prefer and why?

Enabling prompt:

b. Simpler problem: 1 + 49 + 10 =?
c. Hint about pattern: How could we change this number sentence into 50 + 10 =?
d. Help from a friend

Extending prompt:

A. Can you work out the answer to this number sentence? 393 + 94 + 7 + 6 =

Can you solve it another way? Which way did you think was easier? Why?

Possible solutions:

The intention is for students to see that the problem can be more easily solved if the numbers are rearranged. In particular, if students can identify the fact that 97 + 3 = 100 and solve the problem by simply adding the 30 last they would have identified an efficient solution. Some students may choose to count on by 10's when adding 30 to 100.

Another possible solution involves rearranging the numbers so that they can be added in order from largest to smallest. This has the potential to result in some confusion as students count by 10's (to 30) from 97, however it may be experienced as relatively efficient for students with strong place value concepts.

Challenging Task B (Worded problem):

Task:

Shaalev was also an excellent cricketer and played on the same team as Will. In four innings, Shaalev scored 96, 60, 4 and 40. How many runs did he score altogether?

Enabling prompt:

a. Number sentence: 96 + 60 + 4 + 40 =
b. Simpler problem: 2 + 98 + 10 =?
c. Hint about pattern: How could we change this number sentence into 100 + 10 =?
d. Help from a friend

Extending prompt:

The Belgrave South team was on 296 runs. Then Shaalev came into bat and made 75 runs. After Shaalev got out, Wll came into bat, but only made 4 runs. How many runs is the Belgrave South team on now?

Possible solutions:

The intention is for students to see that the problem can be more easily solved if the numbers are rearranged. In particular, if students can identify the fact that 96 + 4 = 100 and solve the problem by adding another 100 (60+40), they would have identified an efficient solution.

Another possible solution involves rearranging the numbers so that they can be added in order from largest to smallest. This has the potential to result in some confusion as students count by 10's from 96, however it may be experienced as relatively efficient for students with strong place value concepts.

Lesson 4A and 4B: Bridge Through 10 (and 100)

Rationale for the lesson:

Mental computation can involve the use of strategies to make the addition easier. One of the key strategies involves partitioning and regrouping numbers. In some instances, this may involve students deciding to 'bridge through 10' in order to complete a number sentence such as 9 + 7 (9+1+6). Again, this lesson seeks to leverage off student's knowledge of key target numbers (e.g., 10, 100). However, rather than focus on the order in which we add numbers to find a more efficient approach, we can instead focus on partitioning and regrouping.

Prior knowledge:

- Assumes that students are familiar with tens facts
- Assumes that students are able to add a multiple of 10 (including 100) to a single digit number with some fluency.
- Assumes that students have some capacity to add 10 to a 2-digit number.

Teacher-facilitated discussion and instruction of key mathematical ideas:

- There is almost always more than one strategy when adding any two numbers together.
- Some strategies are more helpful (efficient) than others.
- We can look for patterns and break up numbers into parts in order to make addition easier.
- The larger the numbers, the more important it is to look for clever shortcuts.

For this lesson:

- Focus on partitioning numbers in order to bridge through 10 (and 100) to make addition easier.

Challenging Task A (More than 1 way):

Task:

Can you work out the answer to this number sentence?

99 + 26 =

Try doing it another way. Which way do you prefer and why?

Enabling prompt:

b. Simpler problem: 49 + 6 =
c. Hint about pattern: If we break up the 6 into 1 and 5, we can change this number sentence into 49 + 1 + 5 = ? Can you make a friendly number?
d. Help from a friend

Extending prompt:

Can you work out the answer to this number sentence?: 99 + 99 + 99 = ?

Can you solve it another way? Which way did you think was easier? Why?

Possible solutions:

One efficient solution is to partition the 26 into 25 and 1, and bridge through 100:

= 99 + 1 + 25

= 100 + 25

= 125

Another solution would involve partitioning the 99 into 90 and 9, and partitioning the 26 into 20 and 6. The problem can then be solved by adding the 10's and then adding the 1's.

= 90 + 20 + 9 + 6

= 110 + 15

= 110 + 10 + 5

= 125

This approach is obviously less efficient.

Some students may attempt to count on from 99. Although obviously not efficient, this may be attractive, particularly for those students very fluent at counting on. At this point, it becomes necessary to expose these students to the extending prompt.

Challenging Task B (Worded problems):

Task:

Johnny did some Christmas shopping for his parents. He got his dad a new vacuum cleaner, which cost $98. For his mum, he bought a new fishing rod for $35. How much money did he spend on his parents altogether?

Enabling prompt:

a. Number sentence: 98 + 35 =
b. Simpler problem: 48 + 5 =
c. Hint about pattern: If we break up the 5 into 2 and 3, we can change this number sentence into 48 + 2 + 3 = ? Can you make a friendly number?
d. Help from a friend

Extending prompt:

Johnny decided to get his parents some more presents. He got his dad a lovely shirt for $97, and his mum a new pair of cross-trainers for $199? How much extra money did Johnny spend on his parents?

Possible solutions:

One efficient solution is to partition the 35 into 2 and 33, and bridge through 100:

= 98 + 2 + 33

= 100 + 33

= 133

Another (less efficient) solution would involve partitioning the 98 into 90 and 8, and partitioning the 35 into 30 and 5. The problem can then be solved by adding the 10's and then adding the 1's.

= 90 + 30 + 9 + 5

= 120 + 13

= 133

Some students may attempt to count on from 98. Although obviously not efficient, this may be attractive, particularly for those students very fluent at counting on. At this point, it becomes necessary to expose these students to the extending prompt.

Lesson 5A and 5B: Partitioning and regrouping into lots of 10

Rationale for the lesson:

Mental computation can involve the use of strategies to make the addition easier. One of the key strategies involves partitioning and regrouping numbers. In some instances, this may involve students deciding to partition one number in order to regroup into lots of 10 (e.g., 9 + 9 + 9 + 3 = 9 + 9 + 9 + 1 + 1 + 1).

Prior knowledge:

- Assumes that students are familiar with tens facts
- Assumes that students are able to both add groups of 10 and add a multiple of 10 (including 100) to a single digit number with some fluency.
- Assumes that students have some capacity to add 10 to a 2-digit number.

Teacher-facilitated discussion and instruction of key mathematical ideas:

- There is almost always more than one strategy when adding any two numbers together.
- Some strategies are more helpful (efficient) than others.
- We can look for patterns and break up numbers into parts in order to make addition easier.
- The larger the numbers, the more important it is to look for clever shortcuts.

For this lesson:

- Focus on partitioning numbers in order to bridge through 10 (and 100) to make addition easier. Rather than manipulating two digits, the focus in this lesson is on more than two digits (essentially, in these examples, a single digit number can be drawn down in order to create several multiples of 10, which can then be easily added).

Challenging Task A (More than 1 way):

Task:

Can you work out the answer to this number sentence?

9 + 9 + 9 + 9 + 9 + 9 + 9 + 7 =

Try doing it another way. Which way do you prefer and why?

Enabling prompt:

b. Simpler problem: 9 + 9 + 9 + 3 =
c. Hint about pattern: If we break the 3 into 1, 1 and 1, we can rewrite the number sentence as 9 + 9 + 9 + 1 + 1 + 1 = ? Can you see any rainbow facts?
d. Help from a friend

Extending prompt:

99 + 98 + 97 + 7 =

Can you solve the number sentence another way? Which way did you think was easier? Why?

Possible solutions:

One of the most efficient solutions to this problem (particularly for this age group) involves partitioning the 7 into lots of 1, and matching these ones with the nines.

7 = 1 + 1 + 1 + 1 + 1 + 1 + 1

= 9 + 9 + 9 + 9 + 9 + 9 + 9 + 1 + 1 + 1 + 1 + 1 + 1 + 1

= 10 + 10 + 10 + 10 + 10 + 10 + 10

= 70

Another (probably less efficient) solution involves beginning the problem by identifying doubles facts (9+9=18), and bridging through 10 (9+7=16). The issue with this approach is that it leaves the students with four 2-digit numbers to add together. Students can complete the problem either by splitting the numbers in 10's and 1's, or by leveraging off their knowledge of doubles facts.

= 18 + 18 + 18 + 16

= 10 + 10 + 10 + 10 and 8 + 8 + 8 + 6

= 40 + 30

= 70

As a final obviously efficient alternative, some students may be capable of counting by 9's and decide on this strategy.

Challenging Task B (Worded problem):

Task:

The next day, Johnny was back at the shops doing more Christmas shopping. He decided to get his two sisters tickets to see Katie Perry in concert. The tickets cost $99 each. He also got his dog, Sook, a plastic bone for $2. How much more money did he spend on his Christmas shopping?

Enabling prompt:

a. Number sentence: 99 + 99 + 2 =
b. Simpler problem: 19 + 19 + 2 =
c. Hint about pattern: If we break the 2 into 1 and 1, we can rewrite the number sentence as 19 + 19 + 1 + 1= ? Can you see how we could make 20 + 20?
d. Help from a friend

Extending prompt:

John forgot that his 4 cousins and their dog Fletcher were also going to be at his house on Christmas day. He decided to buy his cousins entrance to bounce, which cost $49 each. For Fletcher, he bought a new collar for $5. How much more money did poor Johnny have to spend?

Possible solutions:

The most efficient solution to this problem involves partitioning the 2 into lots of 1, and matching these ones with the nines.

= 99 + 1 + 99 + 1

= 100 + 100

= 200

Another (probably less efficient) solution involves partitioning the 99's into 10's and 1's:

= 90 + 90 + 9 + 9 + 2

= 180 + 18 + 2

= 180 + 20

= 200

Lesson 6A and 6B: Complements to 100

Rationale for the lesson:

Building on their understanding of the mathematical patterns involved in rainbow facts (tens facts), it is appropriate to expose students to the more sophisticated concept of complements to 100 (also referred to as 'super rainbow facts'). This is important prerequisite knowledge for the application of the more sophisticated partitioning strategy, bridging through 100.

Prior knowledge:

- Assumes that students are familiar with tens facts
- Assumes that students have an understanding of the meaning of the equality sign.
- Assumes that students have enough knowledge of addition to conceptualise a missing addends problem.

Teacher-facilitated discussion and instruction of key mathematical ideas:

For this lesson:

- Focus on identifying complements to 100. Demonstrate to students how a 100's chart can be used to work out a given number's complement to 100 by first counting how many 1's to get to the next 10 (i.e., the end of the row), and then how many 10's to get to 100.

Challenging Task A (Non-worded problem):

Task:

You did a problem for your maths homework, but can only remember part of it:

◯5 + ◯5 = 100

What might the problem have been? Write down as many different possibilities as you can think of.

Enabling prompt:

b. Simpler problem: 85 + ◯5 = 100
c. Hint about pattern: 85 + 5 =90; so 85 + 15 = ?
d. Help from a friend

Extending prompt:

You did a problem for your maths homework, but can only remember only part of it:

◯◯5 + ◯5 = 200

What might the problem have been? Write down as many different possibilities as you can think of.

Possible solutions:

Any solution where the two missing numbers add to 9 (i.e., 9 tens) is correct.

Challenging Task B (Worded problem):

Task:

John and Wendy loved baking. John wanted to try out a brand new cupcake recipe that had come to him in a dream. John and Wendy baked and baked all day, and finished the day with 100 cupcakes. They decided to share all the cupcakes between them. They both decided that John should get more cupcakes because he was the one who had dreamed up this new recipe. How many cupcakes might John have got? How many cupcakes might Wendy have got? Write down as many combinations as you can.

Enabling prompt:

a. Number sentence: John's cupcakes + Wendy's cupcakes = 100 cupcakes. Remember, John has to have more cupcakes than Wendy.
b. Simpler problem: If John got 90 cupcakes, how many cupcakes did Wendy get?
c. Hint about pattern: This question is all about knowing your super rainbow facts.
d. Help from a friend

Extending prompt:

Because the cupcakes were so delicious, the next week Wendy and her sister Polly decided to bake another 100 of these special new cupcakes. Wendy and Polly decided to share the cupcakes between them equally, so they would both have the same amount. They also thought that they should give some cupcakes to John because it was still his recipe. They decided that john deserved at least 10 cupcakes for his wonderful recipe, but that John should definitely have less cupcakes than Polly and Wendy, because they did all the baking. How many cupcakes might John, Wendy and Polly ended up with? Write down as many combinations as you can.

Possible solutions:

All solutions in which John has 51 or more cupcakes, and Wendy has enough cupcakes to bring the total to 100 are appropriate (i.e., 51, 49; 52, 48; 53, 47). Students who recognise the pattern should be encouraged to provide their solutions in a systematic manner.

APPENDIX E

Teaching with tasks that make children think: Addition and missing addends

This unit of work was developed by James Russo (Belgrave South Primary School and Monash University), in consultation with his PhD supervisors Sarah Hopkins and Peter Sullivan, as part of his PhD project.

CHALLENGING TASK WORKSHEETS

Challenging Task Worksheet (Lesson 1A) NAME:

Can you work out the answer to this number sentence?

0 + 1 + 2 + 3 + 4 + 6 + 7 + 8 + 9 + 10 =

Try doing it another way. Which way do you prefer and why?

0 + 1 + 2 + 3 + 4 + 6 + 7 + 8 + 9 + 10 =
0 + 1 + 2 + 3 + 4 + 6 + 7 + 8 + 9 + 10 =

Extending Prompt (Lesson 1A): **NAME:**

Can you work out the answer to this number sentence?

1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 =

Try doing it another way. Which way do you prefer and why?

1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 =
1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 =

Enabling Prompt (Lesson 1A): **NAME:**

<u>Simpler problem:</u> 1 + 2 + 8 + 9 =

1 + 2 + 8 + 9 =

<u>Hint about pattern:</u>

Can you see any rainbow facts?

Challenging Task Worksheet (Lesson 1B) NAME:

Your friend challenged you to add all of the digits from 1 to 9 together. Can you work out a quick way of finding the answer?

Extending Prompt (Lesson 1B): **NAME:**

Can you work out a quick way of adding all the numbers from

1 to 20 together?

Enabling Prompt (Lesson 1B): **NAME:**

Number sentence: 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 =

1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 =

Enabling Prompt (Lesson 1B): **NAME:**

Simpler problem: 1 + 5 + 5 + 9 =

1 + 5 + 5 + 9 =

Hint about pattern:

Can you see any rainbow facts?

Challenging Task Worksheet (Lesson 2A) **NAME:**

Can you work out the answer to this number sentence?

$1 + 5 + 2 + 4 + 3 + 2 + 5 + 4 + 1 + 3 =$

Try doing it another way. Which way do you prefer and why?

$1 + 5 + 2 + 4 + 3 + 2 + 5 + 4 + 1 + 3 =$
$1 + 5 + 2 + 4 + 3 + 2 + 5 + 4 + 1 + 3 =$

Extending Prompt (Lesson 2A): **NAME:**

Can you work out the answer to this number sentence?

117 + 25 + 25 + 50 =

Try doing it another way. Which way do you prefer and why?

117 + 25 + 25 + 50 =
117 + 25 + 25 + 50 =

Enabling Prompt (Lesson 2A): **NAME:**

Simpler problem: 3 + 5 + 5 + 2 + 2 + 3 =

3 + 5 + 5 + 2 + 2 + 3 =

Hint about pattern:

Can you see any doubles facts? Can you see any ways of making groups of 10?

Challenging Task Worksheet (Lesson 2B)   NAME:

Will Sargood was an excellent cricketer.

In one over, he hit Mr Russo for: 6, 2, 2, 2, 2, and 6.

In the next over, he did exactly the same thing.

How many runs did Will score?

Extending Prompt (Lesson 2B): NAME:

In five innings, Will Sargood scored: 60, 25, 25, 60 and 50. How many runs did he score altogether?

Enabling Prompt (Lesson 2B): **NAME:**

<u>Number sentence:</u> 6 + 2 + 2 + 2 + 2 + 6 + 6 + 2 + 2 + 2 + 2 + 6 =

6 + 2 + 2 + 2 + 2 + 6 + 6 + 2 + 2 + 2 + 2 + 6 =

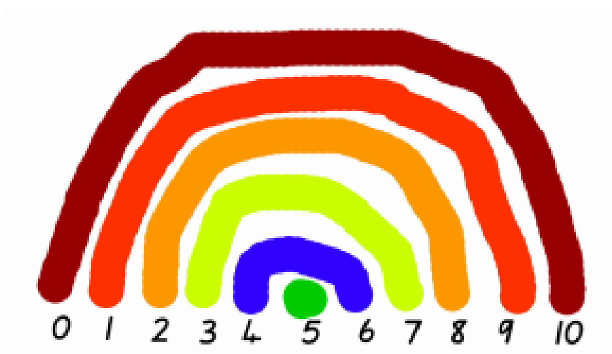
Enabling Prompt (Lesson 2B): **NAME:**

<u>Simpler problem:</u> 6 + 2 + 2 + 6 =

6 + 2 + 2 + 6 =

<u>Hint about pattern:</u>

Can you see any doubles facts? Can you see any ways of making groups of 10?



Challenging Task Worksheet (Lesson 3A)　　NAME:

Can you work out the answer to this number sentence?

3 + 30 + 97 =

Try doing it another way. Which way do you prefer and why?

3 + 30 + 97 =
3 + 30 + 97 =

Extending Prompt (Lesson 3A): NAME:

Can you work out the answer to this number sentence?

393 + 94 + 7 + 6 =

Try doing it another way. Which way do you prefer and why?

393 + 94 + 7 + 6 =
393 + 94 + 7 + 6 =

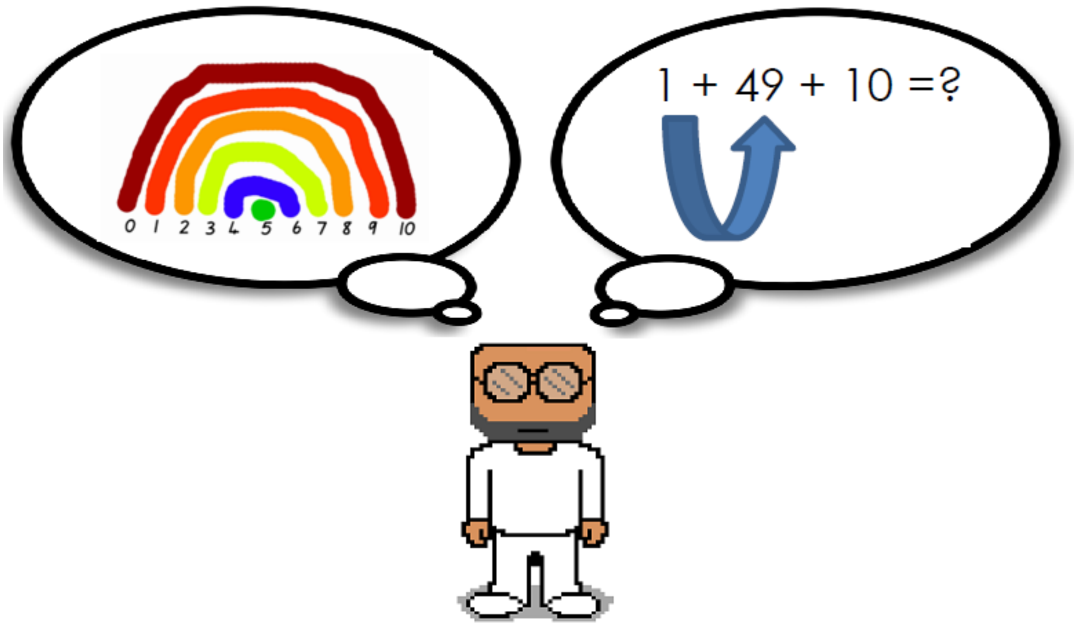
Enabling Prompt (Lesson 3A): **NAME:**

<u>Simpler problem:</u> 1 + 49 + 10 =

1 + 49 + 10 =

<u>Hint about pattern:</u>

How could we change this number sentence into 50 + 10 =?



Challenging Task Worksheet (Lesson 3B)   NAME:

Shaalev was also an excellent cricketer and played on the same team as Will.

In four innings, Shaalev scored 96, 60, 4 and 40. How many runs did he score altogether?

Extending Prompt (Lesson 3B): **NAME:**

Belgrave South was playing Ferntree Gully in the cricket final.

VS

The Belgrave South team was on 296 runs. Then Shaalev came into bat and made 75 runs. After Shaalev got out, WIl came into bat, but only made 4 runs. How many runs is the Belgrave South team on now?

Enabling Prompt (Lesson 3B): **NAME:**

Number sentence: 96 + 60 + 4 + 40 =

96 + 60 + 4 + 40 =

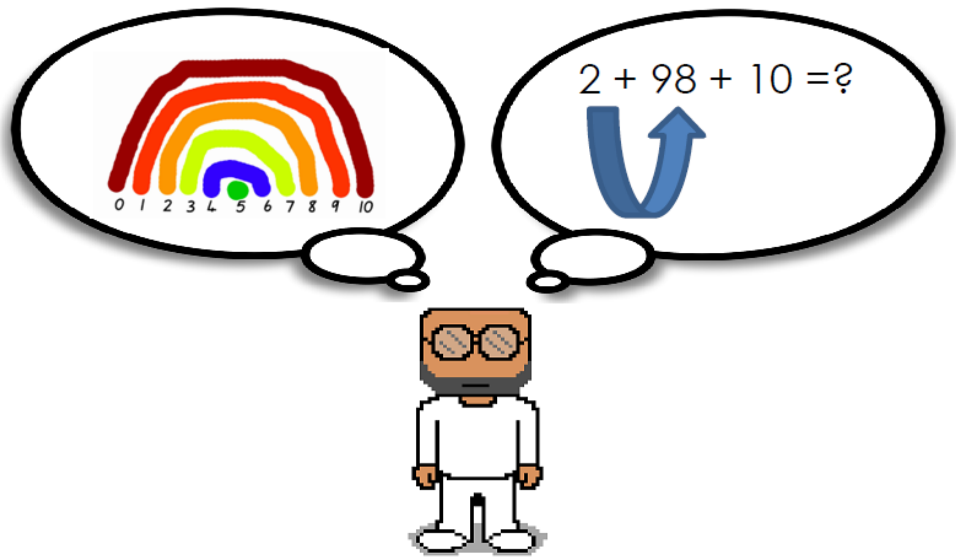
Enabling Prompt (Lesson 3B): **NAME:**

Simpler problem: 2 + 98 + 10 =?

2 + 98 + 10 =?

Hint about pattern:

How could we change this number sentence into 100 + 10 =



Challenging Task Worksheet (Lesson 4A)   NAME:

Can you work out the answer to this number sentence?

99 + 26 =

Try doing it another way. Which way do you prefer and why?

99 + 26 =
99 + 26 =

Extending Prompt (Lesson 4A): **NAME:**

Can you work out the answer to this number sentence?

99 + 99 + 99 =

Try doing it another way. Which way do you prefer and why?

99 + 99 + 99 =
99 + 99 + 99 =

Enabling Prompt (Lesson 4A): **NAME:**

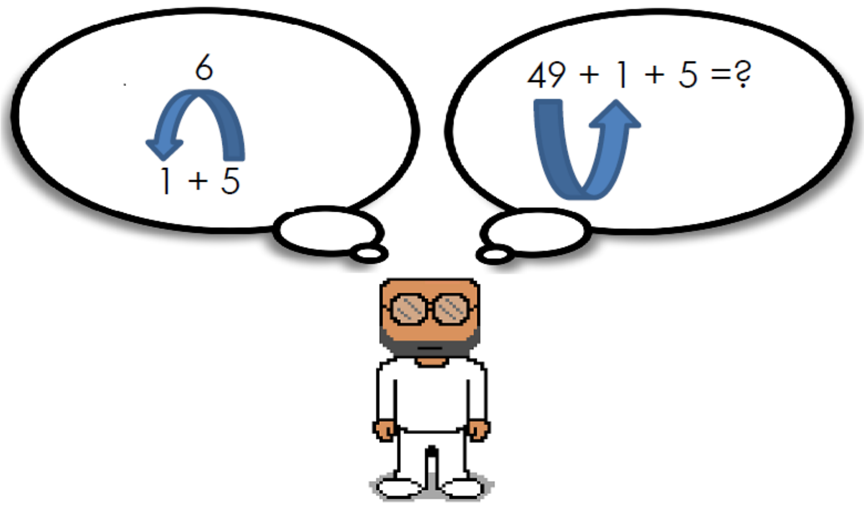
Simpler problem: 49 + 6 =

49 + 6 =

Hint about pattern:

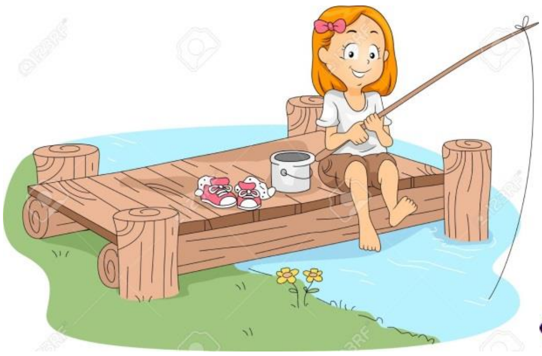
If we break up the 6 into 1 and 5, we can change this number sentence into 49 + 1 + 5 = ?

Can you make a friendly number?



Challenging Task Worksheet (Lesson 4B) NAME:

Johnny did some Christmas shopping for his parents. He got his dad a new vacuum cleaner, which cost $98. For his mum, he bought a new fishing rod for $35. How much money did he spend on his parents altogether?



$35 $98

Extending Prompt (Lesson 4B): **NAME:**

Johnny decided to get his parents some more presents. He got his dad a lovely shirt for $97, and his mum a new pair of cross-trainers for $199. How much extra money did Johnny spend on his parents?

$97 $199

Enabling Prompt (Lesson 4B): **NAME:**

Number sentence: 98 + 35 =

98 + 35 =

Enabling Prompt (Lesson 4B): **NAME:**

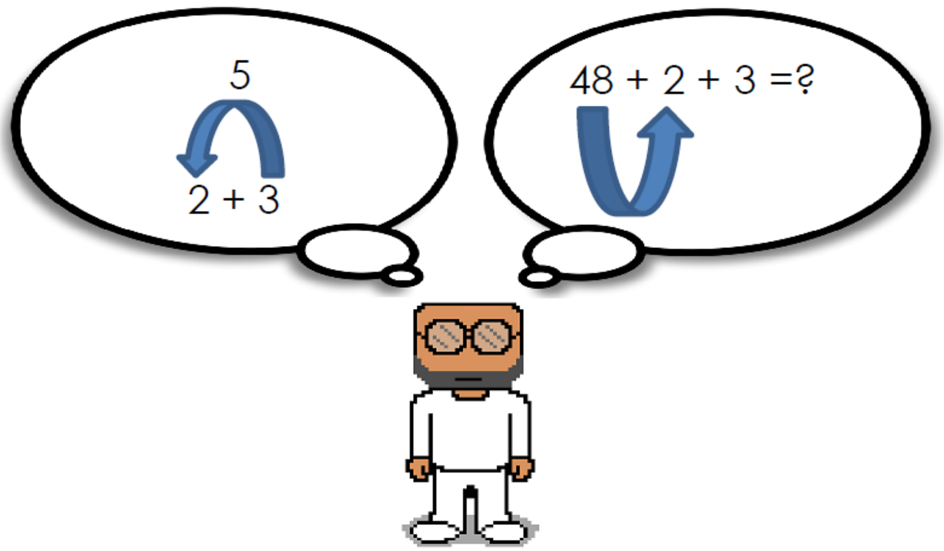
Simpler problem: 48 + 5 =

48 + 5 =

Hint about pattern:

If we break up the 5 into 2 and 3, we can change this number sentence into 48 + 2 + 3 = ?

Can you make a friendly number?



Challenging Task Worksheet (Lesson 5A)   NAME:

Can you work out the answer to this number sentence?

9 + 9 + 9 + 9 + 9 + 9 + 9 + 7 =

Try doing it another way. Which way do you prefer and why?

9 + 9 + 9 + 9 + 9 + 9 + 9 + 7 =
9 + 9 + 9 + 9 + 9 + 9 + 9 + 7 =

Extending Prompt (Lesson 5A): NAME:

Can you work out the answer to this number sentence?

99 + 98 + 97 + 7 =

Try doing it another way. Which way do you prefer and why?

99 + 98 + 97 + 7 =
99 + 98 + 97 + 7 =

Enabling Prompt (Lesson 5A): **NAME:**

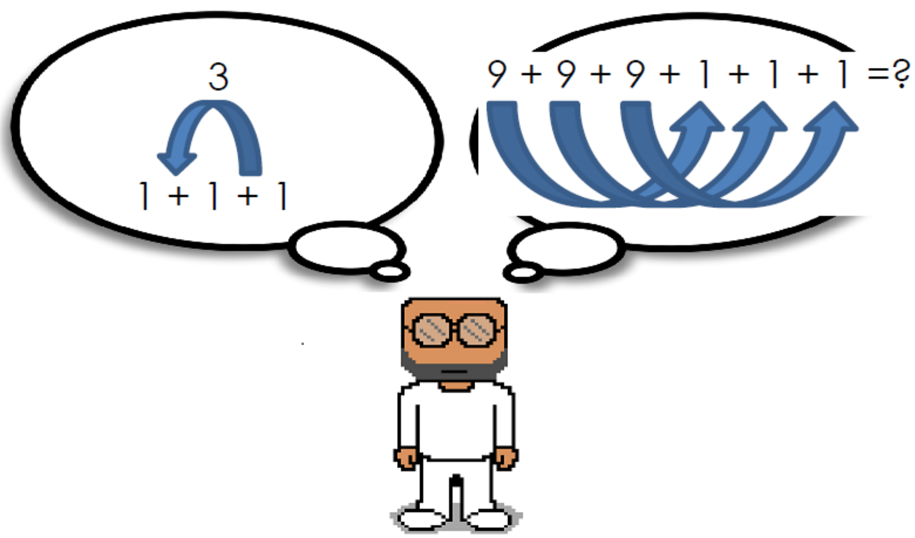
Simpler problem: 9 + 9 + 9 + 3 =

9 + 9 + 9 + 3 =

Hint about pattern:

If we break the 3 into 1, 1, 1, we can rewrite the number sentence as 9 + 9 + 9 + 1 + 1 + 1 =

Can you see any rainbow facts?



Challenging Task Worksheet (Lesson 5B) NAME:

The next day, Johnny was back at the shops doing more Christmas shopping. He decided to get his two sisters tickets to see Katie Perry in concert. The tickets cost $99 each. He also got his dog, Sook, a plastic bone for $2. How much more money did he spend on his Christmas shopping?

$99 $99 $2

Extending Prompt (Lesson 5B): **NAME:**

John forgot that his 4 cousins and their dog Fletcher were also going to be at his house on Christmas day. He decided to buy his cousins entrance to bounce, which cost $49 each. For Fletcher, he bought a new collar for $5. How much more money did poor Johnny have to spend?

$49 $49 $49 $49 $5

Enabling Prompt (Lesson 5B): **NAME:**

Number sentence: 99 + 99 + 2 =

99 + 99 + 2 =

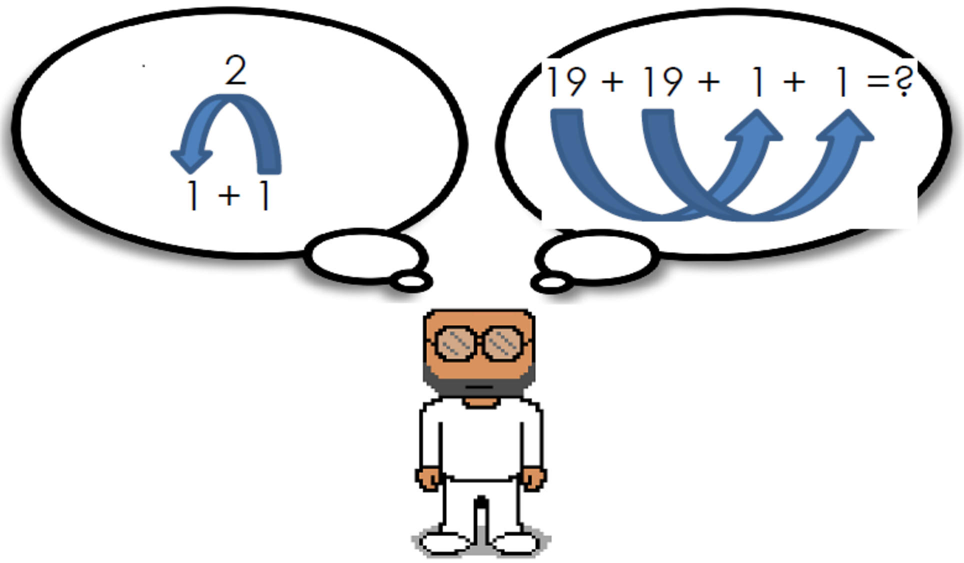
Enabling Prompt (Lesson 5B): **NAME:**

<u>Simpler problem:</u> 19 + 19 + 2 =

19 + 19 + 2 =

<u>Hint about pattern:</u>

If we break the 2 into 1 and 1, we can rewrite the number sentence as 19 + 19 + 1 + 1= ? Can you see how we could make 20 + 20?



Challenging Task Worksheet (Lesson 6A) NAME:

You did a problem for your maths homework, but can only remember part of it:

◯5 + ◯5 = 100

What might the problem have been? Write down as many different possibilities as you can think of.

◯5 + ◯5 = 100	◯5 + ◯5 = 100
◯5 + ◯5 = 100	◯5 + ◯5 = 100
◯5 + ◯5 = 100	◯5 + ◯5 = 100
◯5 + ◯5 = 100	◯5 + ◯5 = 100
◯5 + ◯5 = 100	◯5 + ◯5 = 100

Extending Prompt (Lesson 6A): **NAME:**

You did a problem for your maths homework, but can only remember part of it:

◯◯5 + ◯5 = 200

What might the problem have been? Write down as many different possibilities as you can think of.

◯◯5 + ◯5 = 200	◯◯5 + ◯5 = 200
◯◯5 + ◯5 = 200	◯◯5 + ◯5 = 200
◯◯5 + ◯5 = 200	◯◯5 + ◯5 = 200
◯◯5 + ◯5 = 200	◯◯5 + ◯5 = 200
◯◯5 + ◯5 = 200	◯◯5 + ◯5 = 200

Enabling Prompt (Lesson 6A): **NAME:**

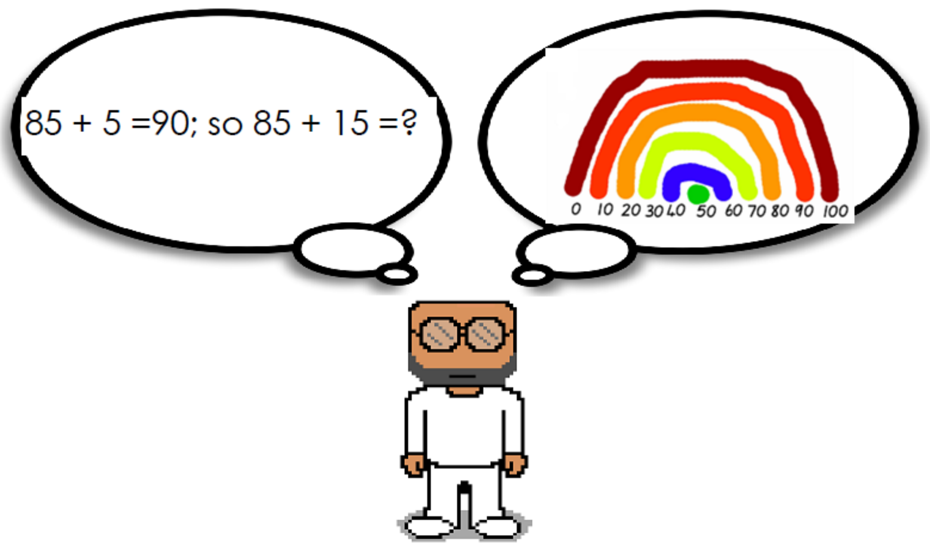
Simpler problem: 85 + ◯5 = 100

85 + ◯5 = 100

Hint about pattern:

85 + 5 = 90 because we know our rainbow facts, so we can work out what 85 + 15 =

Do you know your super rainbow facts?



Challenging Task Worksheet (Lesson 6B)   NAME:

John and Wendy loved baking. John wanted to try out a brand new cupcake recipe that had come to him in a dream. John and Wendy baked and baked all day, and finished the day with 100 cupcakes. They decided to share all the cupcakes between them. They both decided that John should get more cupcakes because he was the one who had dreamed up this new recipe. How many cupcakes might John have got? How many cupcakes might Wendy have got? Write down as many combinations as you can.

Extending Prompt (Lesson 6B): NAME:

Because the cupcakes were so delicious, the next week Wendy and her sister Polly decided to bake another 100 of these special new cupcakes. Wendy and Polly decided to share the cupcakes between them equally, so they would both have the same amount. They also thought that they should give some cupcakes to John because it was still his recipe. They decided that john deserved at least 10 cupcakes for his wonderful recipe, but that John should definitely have less cupcakes than Polly and Wendy, because they did all the baking. How many cupcakes might John, Wendy and Polly ended up with? Write down as many combinations as you can.

Enabling Prompt (Lesson 6B): **NAME:**

<u>Number sentence:</u> : John's + Wendy's = 100

Remember, John has to have more cupcakes than Wendy.

John's + Wendy's = 100

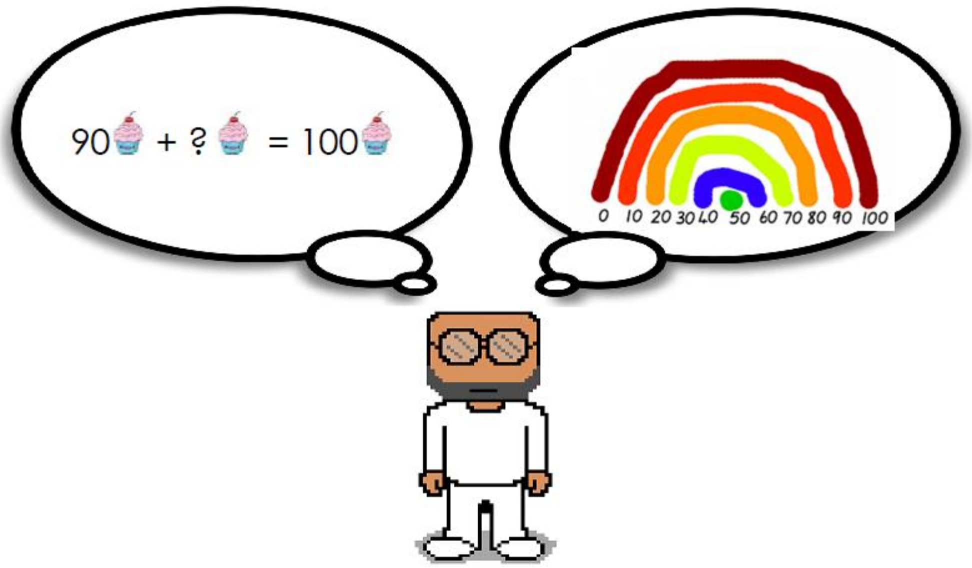
Enabling Prompt (Lesson 6B): **NAME:**

<u>Simpler problem:</u> If John got 90 cupcakes, how many cupcakes did Wendy get?

90 + Wendy's = 100

<u>Hint about pattern:</u>

This question is all about knowing your super rainbow facts.



APPENDIX F

Teaching with tasks that make children think: Addition and missing addends

This unit of work was developed by James Russo (Belgrave South Primary School and Monash University), in consultation with his PhD supervisors Sarah Hopkins and Peter Sullivan, as part of his PhD project.

CONSOLIDATING WORKSHEETS

Name ________________

Change the Order: Rainbow Facts

Olaf and Ana are trying to find Elsa, can you help them by completing these sums to move to the next spot.

Start

4 + 5 + 6 = $\boxed{}$

5 + 2 + 5 = $\boxed{}$

3 + 8 + 2 = $\boxed{}$

1 + 9 + 1 + 9 = $\boxed{}$

2 + 4 + 6 + 8 = $\boxed{}$

Finish

Name: ______________________________________

1A-Change the order: rainbow fact

6. b) Calculate 7 + 2 + 8 + 5 + 3 =

7. b) Calculate 3 + 6 + 9 + 1 + 4 =

8. c) Calculate 5 + 5 + 5 + 95 + 95 + 95 =

9. c) Calculate 3 + 4 + 36 + 37 =

10. c) Calculate 1 + 98 + 99 + 2 =

Name ______________

Change the Order: Rainbow Facts

Rafael and Donatello are trying to find Michelangelo and Leonardo can you help them by completing these sums to move to the next spot.

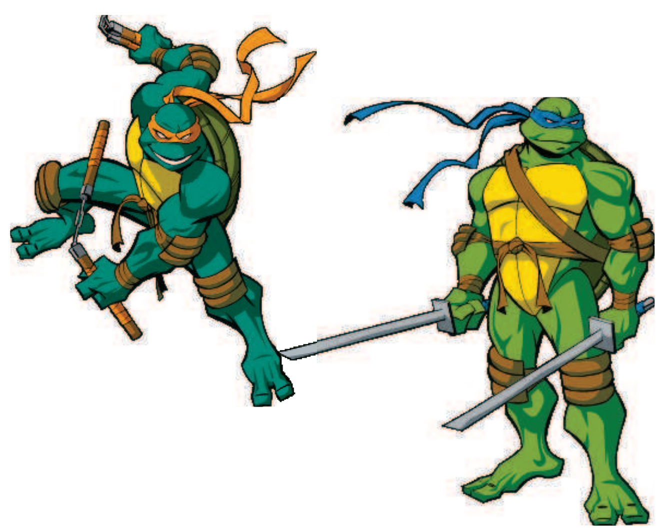
3 + 6 + 7 =

5 + 3 + 5 =

4 + 9 + 1 =

2 + 8 + 2 + 8 =

1 + 4 + 6 + 9 =



Name: ____

1B-Change the order: rainbow fact

6. b) Calculate 7 + 2 + 8 + 6 + 3 =

7. b) Calculate 8 + 6 + 9 + 1 + 4 =

8. c) Calculate 10 + 10 + 10 + 90 + 90 + 90 =

9. c) Calculate 6 + 7 + 34 + 33 =

10. c) Calculate 7 + 96 + 93 + 4 =

Name ________________

Change the Order: Doubles Facts

Olaf and Ana are trying to find Elsa, can you help them by completing these sums to move to the next spot.

Start

3 + 6 + 3 =

5 + 2 + 5 =

2 + 2 + 5 + 5 =

3 + 3 + 3 + 3 =

7 + 5 + 5 + 7 =

Finish

Name: ____

2A-Change the order: doubles fact

6. b) Calculate 9 + 15 + 15 =

7. b) Calculate 10 + 20 + 20 + 10 + ? = 63

8. c) Calculate 3 + 35 + 35 + 3 =

9. c) Calculate 15 + 15 + 35 + ? = 100

10. c) Calculate 2 + 1 + 48 + 48 =

Name ______________

Change the Order: Doubles Facts

Rafael and Donatello are trying to find Michelangelo and Leonardo can you help them by completing these sums to move to the next spot.

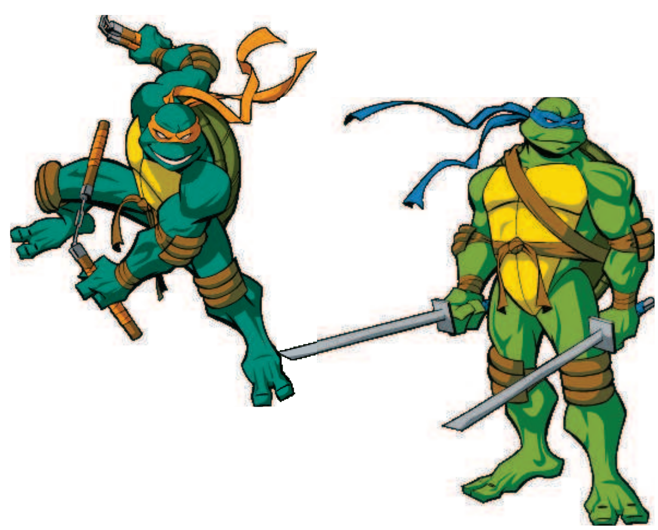
4 + 2 + 4 =

6 + 2 + 6 =

4 + 4 + 5 + 5 =

1 + 10 + 10 + 1 =

6 + 5 + 5 + 6 =



Name: ____

2B-Change the order: doubles fact

6. b) Calculate 7 + 15 + 15 =

7. b) Calculate 5 + 20 + 20 + 5 + ? = 53

8. c) Calculate 3 + 45 + 45 + 10 =

9. c) Calculate 25 + 25 + 25 + ? = 100

10. c) Calculate 2 + 3 + 48 + 48 =

Name _______________

Change the Order: 10s Numbers

Olaf and Ana are trying to find Elsa, can you help them by completing these sums to move to the next spot.

Start

2 + 9 + 8 = ☐

9 + 4 + 1 = ☐

7 + 9 + 1 = ☐

19 + 7 + 1 = ☐

39 + 6 + 1 = ☐

Finish

Name: ______________________________

3A-Change the order: 10s numbers

6. b) Calculate 58 + 7 + 2 =

7. b) Calculate 77 + 9 + 3 =

8. c) Calculate 8 + 96 + 4 =

9. c) Calculate 37 + 97 + 3 =

10. c) Calculate 101 + 1 + 99 + 99 =

Name ______________

Change the Order: 10s Numbers

Rafael and Donatello are trying to find Michelangelo and Leonardo can you help them by completing these sums to move to the next spot.

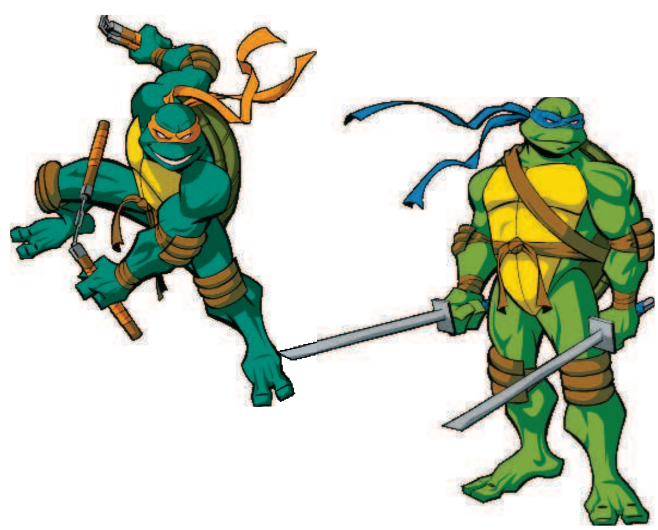
4 + 9 + 6 =

9 + 10 + 1 =

5 + 7 + 3 =

18 + 7 + 2 =

49 + 7 + 1 =



Name: ____

3B-Change the order: 10s numbers

6. b) Calculate 27 + 9 + 3 =

7. b) Calculate 75 + 9 + 5 =

8. c) Calculate 8 + 93 + 7 =

9. c) Calculate 47 + 96 + 4 =

10. c) Calculate 102 + 1 + 98 + 99 =

Name ________________

Bridge Through 10 (and 100)

Olaf and Ana are trying to find Elsa, can you help them by completing these sums to move to the next spot.

Start

9 + 7 =

9 + 6 =

8 + 5 =

5 + 9 =

19 + 7 =

Finish

Name: ______________________________

4A-Bridge through 10 (and 100)

6. b) Calculate 9 + 28 =

7. b) Calculate 64 + 9 =

8. c) Calculate 8 + 88 =

9. c) Calculate 176 + 9 =

10. c) Calculate 8 + 863 =

Name ______________

Bridge Through 10 (and 100)

Rafael and Donatello are trying to find Michelangelo and Leonardo can you help them by completing these sums to move to the next spot.

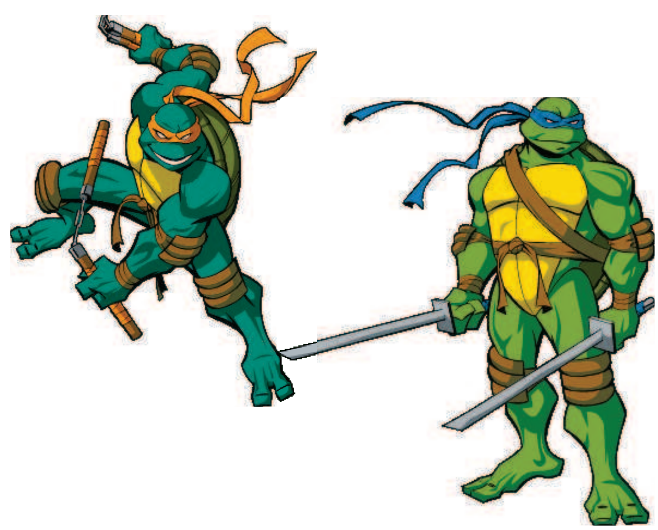
9 + 6 = $\boxed{}$

9 + 4 = $\boxed{}$

8 + 6 = $\boxed{}$

5 + 7 = $\boxed{}$

19 + 8 = $\boxed{}$



Name: ____

4B-Bridge through 10 (and 100)

6. b) Calculate 9 + 38 =

7. b) Calculate 65 + 8 =

8. c) Calculate 7 + 88 =

9. c) Calculate 166 + 9 =

10. c) Calculate 8 + 769 =

Name ________________

Making groups of 10

Olaf and Ana are trying to find Elsa, can you help them by completing these sums to move to the next spot.

Start

9 + 5 = ☐

9 + 8 = ☐

4 + 9 = ☐

19 + 7 = ☐

6 + 28 = ☐

Finish

Name: ____

5A-Making groups of 10

6. b) Calculate 19 + 19 + 2 =

7. b) Calculate 9 + 9 + 9 + 3 =

8. c) Calculate 39 + 29 + 2 =

9. c) Calculate 4 + 19 + 18 + 9 =

10. c) Calculate 5 + 19 + 19 + 19 =

Name ______________

Making groups of 10

Rafael and Donatello are trying to find Michelangelo and Leonardo can you help them by completing these sums to move to the next spot.

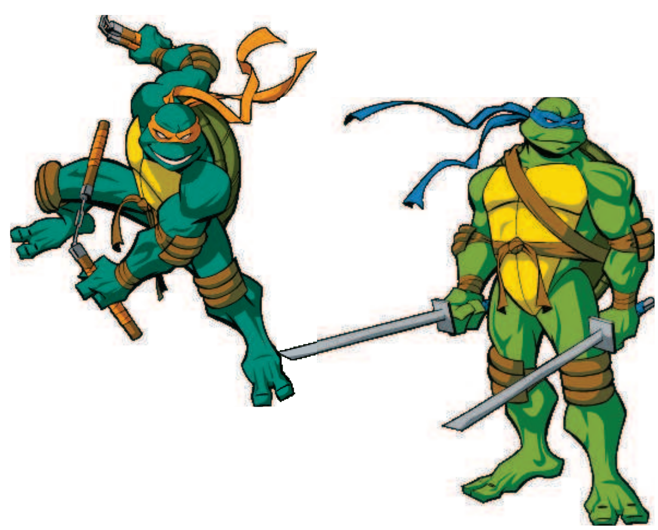
9 + 4 = ☐

9 + 7 = ☐

19 + 6 = ☐

3 + 9 = ☐

5 + 28 = ☐



Name: ____

5B-Making groups of 10

6. b) Calculate 29 + 29 + 2 =

7. b) Calculate 19 + 9 + 19 + 3 =

8. c) Calculate 49 + 39 + 2 =

9. c) Calculate 4 + 18 + 19 + 9 =

10. c) Calculate 5 + 19 + 18 + 19 =

Name _______________

Super Rainbow Facts

Olaf and Ana are trying to find Elsa, can you help them by completing these sums to move to the next spot.

Start

70 + ? = 100

99 + ? = 100

90 + ? = 100

50 + ? = 100

55 + ? = 100

Finish

Name: ______________________________

6A-Super Rainbow Facts

6. b) Calculate 85 + ? = 100

7. b) Calculate 67 + ? = 100

8. c) Calculate 13 + ? = 100

9. c) Calculate 92 + ? = 200

10. c) Calculate 79 + ? = 200

Name ______________

Super Rainbow Facts

Rafael and Donatello are trying to find Michelangelo and Leonardo can you help them by completing these sums to move to the next spot.

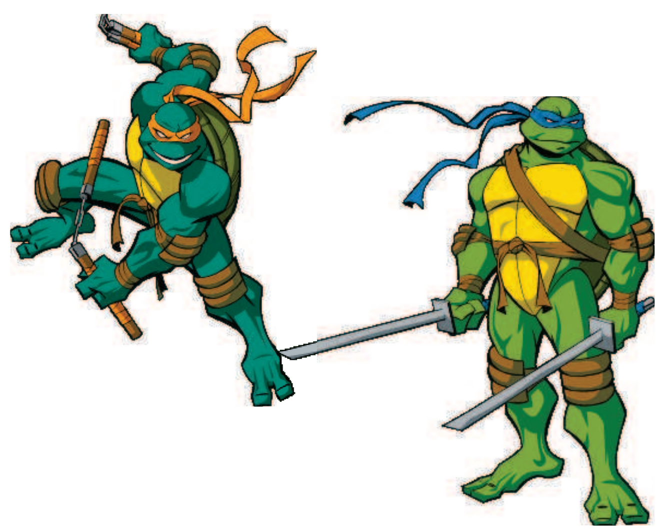
10 + ? = 100

98 + ? = 100

60 + ? = 100

80 + ? = 100

45 + ? = 100



Name: ____

6B-Super Rainbow Facts

6. b) Calculate 15 + ? = 100

7. b) Calculate 33 + ? = 100

8. c) Calculate 73 + ? = 100

9. c) Calculate 62 + ? = 200

10. c) Calculate 99 + ? = 200

Appendix G: Patterning Unit – Intrinsic motivation to learn mathematics questionnaire and Fluency Performance assessments

Intrinsic motivation to learn mathematics questionnaire (pre and post program)

Name: Grade: Date:

Thinking about your views on maths: to what extent do you agree with the following statements?

	Strongly Agree	Agree	Disagree	Strongly Disagree
I enjoy reading about maths	👍👍	👍	👎	👎👎
I look forward to my maths lessons	👍👍	👍	👎	👎👎
I do maths because I enjoy it	👍👍	👍	👎	👎👎
I am interested in the things I learn in maths	👍👍	👍	👎	👎👎

Patterning Unit: Fluency Performance (pre-program)

Name:

Grade:

Date:

Instructions

You will have 30 minutes for this assessment, and there are 24 questions altogether. You will do this assessment at your own pace. Try and do as many questions as you can, and try and show your thinking as you go. Remember, you can skip a question if you get stuck. I don't expect you to finish all the questions.

For each of the questions below, identify the missing number in the sequence by deciding which number belongs in the circle. Write the answer below the question. We will do two practice questions together.

Practice questions

For each of the questions below, identify the missing number in the sequence by deciding which number belongs in the circle.

Practice Question 1.

2 3 4 5 6 7 ◯

Practice Question 2.

2 4 ◯ 8 10 12 14

Appendix G: Fluency assessment and Intrinsic Motivation questionnaire - Patterning Unit of Work

Question 1.

23 24 25 ○ 27 28 29

Question 2.

70 ○ 90 100 110 120 130

Question 3.

11 13 15 17 ○ 21 23

Question 4.

10 13 16 19 22 25 ○

Question 5.

44 43 42 41 ◯ 39 38

Question 6.

5 10 15 20 ◯ 30 35

Question 7.

62 64 66 68 70 72 ◯

Question 8.

◯ 47 57 67 77 87 97

Question 9.

4 8 12 16 20 24 ◯

Question 10.

3 6 9 12 ◯ 18 21

Question 11.

47 50 51 54 55 58 ◯

Question 12.

◯ 97 94 91 88 85 82

Question 13.

81 86 91 96 ◯ 106 111

Question 14.

80 70 ◯ 50 40 30 20

Question 15.

10 14 16 ◯ 22 26 28

Question 16.

106 ◯ 102 100 98 96 94

Question 17.

36 42 48 ◯ 60 66 72

Question 18.

56 64 ◯ 80 88 96 104

Question 19.

42 46 52 56 62 66 ◯

Question 20.

128 ◯ 32 16 8 4 2

Question 21.

5 6 8 11 15 20 ◯

Question 22.

3 6 12 ◯ 48 96 192

Question 23.

90 80 71 63 56 50 ◯

Question 24.

320 160 80 ◯ 20 10 5

Patterning Unit: Fluency Performance (post-program)

Name:

Grade:

Date:

Instructions

You will have 30 minutes for this assessment, and there are 24 questions altogether. You will do this assessment at your own pace. Try and do as many questions as you can, and try and show your thinking as you go. Remember, you can skip a question if you get stuck. I don't expect you to finish all the questions.

For each of the questions below, identify the missing number in the sequence by deciding which number belongs in the circle. Write the answer below the question. We will do two practice questions together.

Practice questions

For each of the questions below, identify the missing number in the sequence by deciding which number belongs in the circle.

Practice Question 1.

2 3 4 ◯ 6 7 8

Practice Question 2.

2 4 6 8 10 12 ◯

Question 1.

23 ◯ 25 26 27 28 29

Question 2.

70 80 90 ◯ 110 120 130

Question 3.

11 13 15 17 19 21 ◯

Question 4.

10 13 16 19 ◯ 25 28

Question 5.

44 43 42 41 40 39 ◯

Question 6.

◯ 10 15 20 25 30 35

Question 7.

62 64 66 68 ◯ 72 74

Question 8.

37 ◯ 57 67 77 87 97

Question 9.

4 8 ◯ 16 20 24 28

Question 10.

3 6 9 12 15 18 ◯

Question 11.

◯ 50 51 54 55 58 59

Question 12.

100 97 94 91 88 85 ◯

Question 13.

81 86 91 96 101 ◯ 111

Question 14.

80 70 60 50 ◯ 30 20

Question 15.

10 14 ◯ 20 22 26 28

Question 16.

106 104 ◯ 100 98 96 94

Question 17.

36 42 48 54 ◯ 66 72

Question 18.

56 64 72 80 88 96 ◯

Question 19.

42 46 52 ◯ 62 66 72

Question 20.

128 64 ◯ 16 8 4 2

Question 21.

5 6 8 ◯ 15 20 26

Question 22.

3 6 12 24 ◯ 96 192

Question 23.

90 80 ◯ 63 56 50 45

Question 24.

320 160 80 40 20 10 ◯

Appendix H: Patterning Unit – Problem Solving Performance assessments and associated questionnaires (intent to continue and cognitive demand)

Patterning Unit: Problem Solving Performance (Pre-program)

Name:

Grade:

Date:

Instructions

You will have 20 minutes for this assessment, and there are 2 maths problems altogether. We will do this assessment together at the same time. I will read each problem to you, and you will have some time to work on each problem (7 minutes). We will then have a little break to answer some questions about how you feel about the problem. You will then have some more time to keep working on the problem (3 minutes).

Don't worry if you don't finish working on the problem or if you get stuck. All I want you to do is to try your best. If you think you have finished the problem, have a look at the extra challenge at the bottom of the page.

Problem 1 (Pre-program)

Starting at 0, I skip counted by **2's** to 30, placing a counter on all the numbers I landed on. Next, I skip counted by **3's** to 30, again placing a counter on all the numbers I landed on. Finally, I skip counted by **5's** to 30, again placing a counter on all the numbers I landed on.

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30									

What are the numbers with three counters on them – the numbers I landed on three times?

What happens if I keep going (past 30) with my skip counting patterns? What other numbers might I land on three times?

Questionnaire for Problem 1 (Pre-program)

Would you like to keep going on the activity for a bit longer, or would you like to do something else?

I really, really want to keep going	👍👍
I want to keep going	👍
I don’t really want to keep going	👎
I want to stop	👎👎

How well do these sentences describe how you are feeling about this activity?

	A lot how I feel	A little how I feel	Not how I feel
1. My brain is working very hard right now	☆☆	☆	
2. I am finding this activity challenging right now	☆☆	☆	
3. I am putting in lots of effort right now	☆☆	☆	
4. I think this is a difficult activity	☆☆	☆	
5. I can do this activity easily	☆☆	☆	

Problem 2 (Pre-program)

Belgrave South played Ferntree Gully in the big cricket game.

VS

Felix was an excellent cricketer, and scored 30 runs for Belgrave South that day. He scored all of his runs hitting only 4's and 6's.

How many 4's and how many 6's might Felix have scored in his innings?

Try and solve the problem as many different ways as you can

Questionnaire for Problem 2 (Pre-program)

Would you like to keep going on the activity for a bit longer, or would you like to do something else?

I really, really want to keep going	👍👍
I want to keep going	👍
I don’t really want to keep going	👎
I want to stop	👎👎

How well do these sentences describe how you are feeling about this activity?

	A lot how I feel	A little how I feel	Not how I feel
1. My brain is working very hard right now	☆☆	☆	
2. I am finding this activity challenging right now	☆☆	☆	
3. I am putting in lots of effort right now	☆☆	☆	
4. I think this is a difficult activity	☆☆	☆	
5. I can do this activity easily	☆☆	☆	

Patterning Unit: Problem Solving Performance (Post-program)

Name:

Grade:

Date:

Instructions

You will have 20 minutes for this assessment, and there are 2 maths problems altogether. We will do this assessment together at the same time. I will read each problem to you, and you will have some time to work on each problem (7 minutes). We will then have a little break to answer some questions about how you feel about the problem. You will then have some more time to keep working on the problem (3 minutes).

Don't worry if you don't finish working on the problem or if you get stuck. All I want you to do is to try your best. If you think you have finished the problem, have a look at the extra challenge at the bottom of the page.

Problem 1 (Post-program)

Belgrave South played Selby in the big basketball game. There were no fouls during the game, so none of the players scored any free throws (1-pointers).

VS

At the end of the basketball game, the Belgrave South team had won 21 points to 20 points – what an exciting victory!

How many field goals (2-pointers) and three pointers (3-points) might the Belgrave South team have scored?

Try and solve the problem as many different ways as you can

Questionnaire for Problem 1 (Post-program)

Would you like to keep going on the activity for a bit longer, or would you like to do something else?

I really, really want to keep going	👍👍
I want to keep going	👍
I don't really want to keep going	👎
I want to stop	👎👎

How well do these sentences describe how you are feeling about this activity?

	A lot how I feel	A little how I feel	Not how I feel
1. My brain is working very hard right now	☆☆	☆	
2. I am finding this activity challenging right now	☆☆	☆	
3. I am putting in lots of effort right now	☆☆	☆	
4. I think this is a difficult activity	☆☆	☆	
5. I can do this activity easily	☆☆	☆	

Problem 2 (Post-program)

Kai and Amaya loved donuts, so their mum decided to plant a donut tree. The tree was magical. Every day, the number of donuts on the tree doubled.

On Monday, there was 1 donut on the tree. Your job is to work out how many donuts there were on the tree by Sunday. To help you, have a go at completing the following table. Remember, each day the number of donuts doubles.

	Donuts on tree	Picture
Monday	1	❀
Tuesday	2	❀❀
Wednesday		❀❀ ❀❀
Thursday		
Friday		
Saturday		
Sunday		

What happens if you keep going (past Sunday) with this pattern? How many donuts might be on the tree? Keep the pattern going for as long as you can.

Questionnaire for Problem 2 (Post-program)

Would you like to keep going on the activity for a bit longer, or would you like to do something else?

I really, really want to keep going 👍👍

I want to keep going 👍

I don't really want to keep going 👎

I want to stop 👎👎

How well do these sentences describe how you are feeling about this activity?

	A lot how I feel	A little how I feel	Not how I feel
1. My brain is working very hard right now	☆☆	☆	
2. I am finding this activity challenging right now	☆☆	☆	
3. I am putting in lots of effort right now	☆☆	☆	
4. I think this is a difficult activity	☆☆	☆	
5. I can do this activity easily	☆☆	☆	

Appendix I: Addition Unit – Intrinsic motivation to learn mathematics questionnaire and Fluency Performance assessments

Intrinsic motivation to learn mathematics questionnaire (pre and post program)

Name: Grade: Date:

Thinking about your views on maths: to what extent do you agree with the following statements?

	Strongly Agree	Agree	Disagree	Strongly Disagree
I enjoy reading about maths	👍👍	👍	👎	👎👎
I look forward to my maths lessons	👍👍	👍	👎	👎👎
I do maths because I enjoy it	👍👍	👍	👎	👎👎
I am interested in the things I learn in maths	👍👍	👍	👎	👎👎

Addition Unit: Fluency Performance (pre-program)

Name:

Grade:

Date:

Instructions

You will have 30 minutes for this assessment, and there are 24 questions altogether. You will do this assessment at your own pace. Try and do as many questions as you can, and try and show your thinking as you go. Remember, you can skip a question if you get stuck. I don't expect you to finish all the questions.

Each question has a missing number, represented by a square ☐. Your job is to work out which number goes in the square to make the number sentence true.

Remember to use any clever short-cuts you know to make figuring out the answer easier.

Practice questions

Before the assessment starts, you will do the two practice questions together with your teacher.

These practice questions should be done together as a class.

Practice Question 1.

$3 + 3 = \square$

Practice Question 2.

$4 + \square = 5$

a)

7 + 3 = ☐

b)

6 + ☐ = 10

c)

$4 + 4 + 2 = \square$

d)

$5 + 8 + 2 = \square$

e)

3 + 10 + 10 + 3 = ☐

f)

9 + 5 + 1 = ☐

g)

9 + 7 = ☐

h)

9 + 9 + 2 = ☐

i)

97 + ☐ = 100

j)

1 + 3 + 7 + 9 = ☐

k)

7 + 15 + 15 = ☐

l)

77 + 8 + 3 = ☐

m)

$38 + 9 = \square$

n)

$19 + 19 + 19 + 3 = \square$

o)

$75 + \square = 100$

p)

$4 + 95 + 96 + 5 = \square$

q)

7 + 45 + 45 + 10 = ☐

r)

48 + 98 + 2 = ☐

s)

$263 + 9 = \square$

t)

$28 + 28 + 28 + 6 = \square$

u)

93 + ☐ = 200

v)

397 + 24 + 3 = ☐

w)

18 + 18 + 18 + 18 + 8 = ☐

x)

10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = ☐

Addition Unit: Fluency Performance (post-program)

Name:

Grade:

Date:

Instructions

You will have 30 minutes for this assessment, and there are 24 questions altogether. You will do this assessment at your own pace. Try and do as many questions as you can, and try and show your thinking as you go. Remember, you can skip a question if you get stuck. I don't expect you to finish all the questions.

Each question has a missing number, represented by a square ☐. Your job is to work out which number goes in the square to make the number sentence true.

Remember to use any clever short-cuts you know to make figuring out the answer easier.

Practice questions

Before the assessment starts, you will do the two practice questions together with your teacher.

These practice questions should be done together as a class.

Practice Question 1.

3 + 3 = ☐

Practice Question 2.

4 + ☐ = 5

a)

6 + 4 = ☐

b)

7 + ☐ = 10

c)

$4 + 4 + 1 + 1 = \square$

d)

$6 + 8 + 2 = \square$

e)

2 + 10 + 10 + 2 = ☐

f)

9 + 6 + 1 = ☐

g)

$9 + 6 = \square$

h)

$8 + 8 + 4 = \square$

i)

95 + ☐ = 100

j)

2 + 4 + 6 + 8 = ☐

k)

8 + 15 + 15 = ☐

l)

78 + 9 + 2 = ☐

m)

48 + 9 = ☐

n)

29 + 29 + 29 + 3 = ☐

o)

$65 + \square = 100$

p)

$3 + 98 + 97 + 2 = \square$

q)

$6 + 45 + 45 + 10 = \square$

r)

$47 + 97 + 3 = \square$

s)

253 + 9 = □

t)

18 + 18 + 18 + 6 = ☐

u)

91 + ☐ = 200

v)

397 + 26 + 3 = ☐

w)

19 + 19 + 19 + 19 + 19 + 5 = ☐

x)

10 + 9 + 8 + 7 + 6 + 4 + 3 + 2 + 1 = ☐

Appendix J: Addition Unit – Problem Solving Performance assessments and associated questionnaires (intent to continue and cognitive demand)

Addition Unit: Problem Solving Performance (Pre-program)

Name:

Grade:

Date:

Instructions

You will have 20 minutes for this assessment, and there are 2 questions altogether. We will do this assessment together at the same time. I will read each question to you, and you will have some time to try and answer each question (7 minutes). We will then have a little break to fill in a short survey about how you feel about the question. You will then have some more time to keep working on the question (3 minutes).

Don’t worry if you don’t finish working on the question or if you get stuck. All I want you to do is to try your best. If you think you have finished the question, have a look at the extra challenge at the bottom of the page.

Before the assessment starts, you will do one practice question together with your teacher.

Practice question 1

You did a problem for your maths homework, but can only remember part of it:

■ + ■ = 8

What might the problem have been?

Practice question 2

Mrs Ould's favourite numbers are:

7	3	2
10	5	1

Choose three numbers from the list to add together and cross them off:

□+□+□=

Problem 1 (pre-program)

You did a problem for your maths homework, but can only remember part of it:

$6\blacksquare + 3\blacksquare = 100$

What might the problem have been? Give as many different answers as you can.

Questionnaire for Problem 1 (pre-program)

Would you like to keep going on the activity for a bit longer, or would you like to do something else?

I really, really want to keep going	👍👍
I want to keep going	👍
I don’t really want to keep going	👎
I want to stop	👎👎

Tell us what you think about this activity

	Strongly Agree	Agree	Disagree	Strongly Disagree
1. My brain is working very hard right now	👍👍	👍	👎	👎👎
2. I am finding this activity challenging right now	👍👍	👍	👎	👎👎
3. I am putting in lots of effort right now	👍👍	👍	👎	👎👎
4. I think this is a difficult activity	👍👍	👍	👎	👎👎
5. I can do this activity easily	👍👍	👍	👎	👎👎

Problem 2 (pre-program)

Mr Russo's favourite numbers are:

97	201	3	45	30
29	2	28	299	55

Choose three numbers from the list to add together and cross them off:

□+□+□=

Choose three different numbers from the list to add together and cross them off:

□+□+□=

Choose three different numbers from the list to add together and cross them off:

□+□+□=

Appendix J: Problem Solving Assessments and Associated Questionnaires - Addition Unit of Work

Questionnaire for Problem 2 (pre-program)

Would you like to keep going on the activity for a bit longer, or would you like to do something else?

I really, really want to keep going	👍👍
I want to keep going	👍
I don't really want to keep going	👎
I want to stop	👎👎

Tell us what you think about this activity

	Strongly Agree	Agree	Disagree	Strongly Disagree
1. My brain is working very hard right now	👍👍	👍	👎	👎👎
2. I am finding this activity challenging right now	👍👍	👍	👎	👎👎
3. I am putting in lots of effort right now	👍👍	👍	👎	👎👎
4. I think this is a difficult activity	👍👍	👍	👎	👎👎
5. I can do this activity easily	👍👍	👍	👎	👎👎

Addition Unit: Problem Solving Performance (Post-program)

Name:

Grade:

Date:

Instructions

You will have 20 minutes for this assessment, and there are 2 questions altogether. We will do this assessment together at the same time. I will read each question to you, and you will have some time to try and answer each question (7 minutes). We will then have a little break to fill in a short survey about how you feel about the question. You will then have some more time to keep working on the question (3 minutes).

Don’t worry if you don’t finish working on the question or if you get stuck. All I want you to do is to try your best. If you think you have finished the question, have a look at the extra challenge at the bottom of the page.

Before the assessment starts, you will do one practice question together with your teacher.

Practice question 1

You did a problem for your maths homework, but can only remember part of it:

■ + ■ = 8

What might the problem have been?

Practice question 2

Mrs Ould's favourite numbers are:

7	3	2
10	5	1

Choose three numbers from the list to add together and cross them off:

□+□+□=

Appendix J: Problem Solving Assessments and Associated Questionnaires - Addition Unit of Work

Problem 1 (post-program)

You did a problem for your maths homework, but can only remember part of it:

$7\blacksquare + 2\blacksquare = 100$

What might the problem have been? Give as many different answers as you can.

Questionnaire for Problem 1 (post-program)

Would you like to keep going on the activity for a bit longer, or would you like to do something else?

I really, really want to keep going 👍👍

I want to keep going 👍

I don't really want to keep going 👎

I want to stop 👎👎

Tell us what you think about this activity

	Strongly Agree	Agree	Disagree	Strongly Disagree
1. My brain is working very hard right now	👍👍	👍	👎	👎👎
2. I am finding this activity challenging right now	👍👍	👍	👎	👎👎
3. I am putting in lots of effort right now	👍👍	👍	👎	👎👎
4. I think this is a difficult activity	👍👍	👍	👎	👎👎
5. I can do this activity easily	👍👍	👍	👎	👎👎

Problem 2 (post-program)

Mrs Bok's favourite numbers are:

98	301	3	75	50
49	2	27	199	25

Choose three numbers from the list to add together and cross them off:

$$\square + \square + \square =$$

Choose three different numbers from the list to add together and cross them off:

□+□+□=

Choose three different numbers from the list to add together and cross them off:

□+□+□=

Questionnaire for Problem 2 (post-program)

Would you like to keep going on the activity for a bit longer, or would you like to do something else?

I really, really want to keep going	👍👍
I want to keep going	👍
I don't really want to keep going	👎
I want to stop	👎👎

Tell us what you think about this activity

	Strongly Agree	Agree	Disagree	Strongly Disagree
1. My brain is working very hard right now	👍👍	👍	👎	👎👎
2. I am finding this activity challenging right now	👍👍	👍	👎	👎👎
3. I am putting in lots of effort right now	👍👍	👍	👎	👎👎
4. I think this is a difficult activity	👍👍	👍	👎	👎👎
5. I can do this activity easily	👍👍	👍	👎	👎👎

Appendix J: Problem Solving Assessments and Associated Questionnaires - Addition Unit of Work

Appendix K: Student perspectives – Semi-structured interview schedule

Patterning Unit Semi-Structured Interview Schedule

All students

(HELP STUDENT SPREADOUT THEIR PORTFOLIO OF WORK AND THEN READ TO STUDENT) Take some time to look through all the great work you have done in maths this term (give students five minutes or so to browse through their work; allow students up to around ten minutes). I want you to pick the two pieces of work you are most proud of creating (encourage students to take their time).

1) Why did you choose this piece of work? (Work Sample A)
2) Why did you choose this piece of work? (Work Sample B)

Further prompts:

- Are there any other reasons why you are proud of this piece of work?
- (After the two pieces of work have been chosen and explained) Why did you choose two worksheets (if relevant)? Why did you choose two challenges (if relevant)?

Students in alternating group only (Class C)

(READ TO STUDENT) Some of your lessons began with the Challenge first, then the lesson on the floor and then the worksheet and some began with the lesson on the floor first, then the worksheet and then the challenge (POINT TO DIAGRAM DEMONSTRATING VISUAL BREAKDOWN OF THE TWO DIFFERENT LESSON STRUCTURES, WHICH HAD BEEN REFERRED BACK TO THROUGHOUT THE UNIT OF WORK). Do you remember these two different types of lessons? (WAIT FOR CONFIRMATION BEFORE CONTINUING; IF THE STUDENT CANNOT REMEMBER OR CONFIRM THERE WERE DIFFERENT TYPES OF LESSONS, DISCONTINUE THE INTERVIEW)

3) Which lesson type did you like more?
4) Why do you think you liked this type of lesson more?

Further prompts:

- Any other reasons?

Addition Unit Semi-Structured Interview Schedule

Students in alternating group only (Class C)

(READ TO STUDENT) Some of your lessons began with the Challenge first, then the lesson on the floor and then the worksheet and some began with the lesson on the floor first, then the worksheet and then the challenge (POINT TO DIAGRAM DEMONSTRATING VISUAL BREAKDOWN OF THE TWO DIFFERENT LESSON STRUCTURES, WHICH HAD BEEN REFERRED BACK TO THROUGHOUT THE UNIT OF

WORK). Do you remember these two different types of lessons? (WAIT FOR CONFIRMATION BEFORE CONTINUING; IF THE STUDENT CANNOT REMEMBER OR CONFIRM THERE WERE DIFFERENT TYPES OF LESSONS, DISCONTINUE THE INTERVIEW)

1) Which lesson type did you like more?
2) Why do you think you liked this type of lesson more?

Further prompts:

- Any other reasons?

Appendix L: Teacher perspectives – Semi-structured interview schedule

Patterning Unit Semi-Structured Interview Schedule

1) Did you notice any changes in your students across the course of the program?

Additional prompts:

- Can you describe these changes?
- What were the most important changes in your view?
- What do you think were the key drivers of this change?

2) Would you be interested in teaching this unit of work to a group of students next year?

Additional prompts:

- Are there any barriers to you teaching this unit of work?
- How could these barriers be overcome?
- Are there any enablers?
- How could they be realised?
- Would you make any changes to the way the unit of work, or the lessons within the unit of work, were structured?

3) Would you be interested in developing, and then teaching, perhaps next term, an additional unit of work using Challenging Tasks in another learning area in mathematics, for example, in place value, geometry or statistics and probability?

Additional prompts:

- What are the barriers to you developing and teaching this unit of work?
- How could these barriers be overcome?
- Are there any enablers?
- How could they be realised?
- Tell me about how you might structure this unit of work, and the specific lessons themselves.

Addition Unit Semi-Structured Interview Schedule

1) Did you notice any changes in your students across the course of the program?

Additional prompts:

- Can you describe these changes?
- What were the most important changes in your view?
- What do you think were the key drivers of this change?

2) Would you be interested in teaching this unit of work to a group of students next year?

Additional prompts:

- Are there any barriers to you teaching this unit of work?
- How could these barriers be overcome?
- Are there any enablers?
- How could they be realised?
- Would you make any changes to the way the unit of work, or the lessons within the unit of work, were structured?

3) Would you be interested in developing, and then teaching, perhaps next term, an additional unit of work using Challenging Tasks in another learning area in mathematics, for example, in place value, geometry or statistics and probability?

Additional prompts:

- What are the barriers to you developing and teaching this unit of work?
- How could these barriers be overcome?
- Are there any enablers?
- How could they be realised?
- Tell me about how you might structure this unit of work, and the specific lessons themselves.

4) You have now seen Challenging Tasks used to both launch lessons and extend understanding. How do you think you would utilise challenging tasks in the future? Why?

Additional prompts (encourage the interviewee to consider a range of issues, including):

- Student learning outcomes; classroom management issues; encouraging student persistence; building intrinsic motivation to learn mathematics; the skills and knowledge required by the teacher; the quality of the mathematical discussion.

Appendix M. Additional data analysis: Persistence measure

This appendix includes a variety of additional data analyses.

Student Outcome Data: Descriptive Statistics

Intent to continue

The single item measuring intent to continue was administered on a total of eight occasions [two units of work (patterning and addition) by two assessments (pre and post) by two problems]. Means and standard deviations are provided in Table M.1.

Table M.1

Means and Standard Deviations for the intent to continue item.

Assessment	N	Mean	SD
Patterning Unit: Pre (Problem 1)	66	1.78	2.02
Patterning Unit: Pre (Problem 2)	64	1.27	2.07
Patterning Unit: Post (Problem 1)	67	2.06	2.69
Patterning Unit: Post (Problem 2)	66	2.00	2.31
Addition Unit: Pre (Problem 1)	66	2.09	4.28
Addition Unit: Pre (Problem 2)	65	2.03	4.03
Addition Unit: Post (Problem 1)	67	1.51	4.19
Addition Unit: Post (Problem 2)	68	1.40	4.45

*Recall that the maximum score on this item is 3, whilst the minimum score is 0.

Cognitive Demand

The cognitive demand instrument was also administered on a total of eight occasions [two units of work (patterning and addition) by two assessments (pre and post) by two problems]. Means and standard deviations for this instrument are provided in Table M.2, which also includes the correlation between cognitive demand scores and performance on the associated mathematical task.

Table M.2

Appendix M: Additional data analysis - Persistence measure

Means, Standard Deviations, and Correlations with associated problem solving task for the Cognitive Demand instrument.

Assessment	N	Mean	SD	r (with task)
Patterning Unit: Pre (Problem 1)	65	11.28	2.02	-.05
Patterning Unit: Pre (Problem 2)	65	12.03	2.07	-.11
Patterning Unit: Post (Problem 1)	66	10.00	2.69	-.41
Patterning Unit: Post (Problem 2)	66	10.71	2.31	-.26
Addition Unit: Pre (Problem 1)	63	14.30	4.28	-.64
Addition Unit: Pre (Problem 2)	64	14.41	4.03	-.47
Addition Unit: Post (Problem 1)	65	12.77	4.19	-.54
Addition Unit: Post (Problem 2)	67	14.22	4.45	-.61

*Recall that for the Patterning Unit, the possible range of scores on this instrument was 5 to 15, whereas for the Addition Unit, it was 5 to 20.

Pooling cognitive demand data for Problem 1 and Problem 2 for each assessment revealed that the level of cognitive demand reported by students decreased across both the Patterning Unit [$t(55)= 4.211$, $p <0.01$, $d=1.14$] and the Addition Unit [$t(54)= 2.402$, $p <0.05$, $d=0.654$]. This observation is consistent with what one would expect. Specifically, given that the post-program assessments contained very similar problem solving tasks to the pre-program assessments, both in terms of structure and complexity, the post-program assessments should be perceived by students as less cognitively demanding. This is because students had become more familiar with such problems, having been repeatedly exposed to similar problems whilst completing the respective units of work.

In order to further examine the characteristics of the cognitive demand instrument, scores on the instrument were correlated with corresponding problem solving performance scores on the associated problem solving task. Given that cognitive demand is intended to measure subjective task complexity, we might expect a negative correlation between scores on this instrument and associated problem solving performance. Correlations varied from negligible during the pre-program assessment for the patterning unit of work ($r = -0.05$ and -0.11), to weak-to-moderate during the post-program assessment for the patterning unit of work ($r = -0.41$ and -0.26), to strongly negative across the addition unit of work ($-0.47 \leq r \leq -0.64$). This perhaps suggests that students were more realistic in appraising how cognitively demanding a particular task was for them, as they became more experienced in completing the cognitive demand questionnaire.

APPENDIX N: HOW CHALLENGING TASKS OPTIMISE COGNITIVE LOAD

James Russo

Monash University

This theoretical paper argues that the reform in mathematics towards more problem-based learning can be made consistent with cognitive load theory through the use of carefully designed challenging tasks. It is argued that such tasks can provide the benefits of problem-based approaches whilst being cognisant of the issue of cognitive overload. Possible directions for future research are suggested.

Context: A reform in mathematics towards problem-based approaches

Over the past several years, there have been calls to reform mathematics education in Australia to increase the amount of time students spend engaged in deep problem solving (e.g., Hollingsworth, Holden, & McCrae, 2003). This reform has paralleled similar developments in other countries, particularly the United States. For example, teachers have been encouraged to utilise more cognitively demanding tasks to better engage students in rich mathematical discussions (e.g., Stein, Engle, Smith, & Hughes, 2008).

As part of this reform process, it has been argued that traditional lesson structures (i.e., teacher explanation, followed by student practice and correction) are inherently inadequate for meeting contemporary mathematical learning objectives (Sullivan et al., 2014). Instead, reform-oriented teaching approaches have frequently employed a triadic lesson structure: Launch, Explore, Discuss (Stein et al., 2008). The lesson begins with the launch phase during which the teacher introduces students to the task, which generally represents a challenging problem to be solved. During the explore phase, students work on the problem, sometimes collaboratively, while the teacher provides support and guidance. Finally, after students have spent sufficient time engaged with the problem, the lesson enters the discuss phase, during which time various student-generated approaches to the problem and possible solutions are discussed. The teacher generally finishes by offering some form of summary comment (Stein et al., 2008).

There is some support for the notion that this Launch-Explore-Discuss lesson structure, which can be characterised as a form of problem-based learning, more effectively meets the contemporary aims of mathematics education. For example, there is empirical evidence to suggest that higher-order mathematical goals, such as the *ability to reason* and *think critically*, are more likely to be realised when students are given an opportunity to explore concepts prior to direct instruction (Marshall & Horton, 2011). Furthermore, building a lesson around students first tackling a cognitively demanding task may improve *student persistence*, as students work through the "zone of confusion" (Sullivan et al., 2014, p. 11).

However, this recent emphasis on problem-based learning in mathematics is not without its critics, particularly within certain branches of educational psychology. In particular, some cognitive load theorists have argued that launching a lesson with a cognitively demanding activity, which is not explicitly linked to teacher instruction and prior learning, is problematic (Sweller, Kirschner, & Clark, 2007). This argument, which is briefly elaborated below, is based on the idea that our working memory has limited capacity to process novel information, and is therefore easily overloaded when required to solve an unfamiliar problem (Sweller, 2010).

Critique of problem-based approaches

It has been asserted that an understanding of human cognitive architecture should lead to the unequivocal rejection of problem-based, and other "minimally-guided", approaches to learning (Kirschner, Sweller and Clark, 2006, p. 75). Specifically, Sweller, a key figure in the development of cognitive load theory, and his colleagues argue that such pedagogical approaches are less effective than traditional learning approaches that rely more on carefully scaffolded direct instruction. The relative ineffectiveness of minimally-guided approaches is thought to be due to unnecessary and irrelevant cognitive load (extraneous cognitive load) brought about by poor instructional design, and/or the overly ambitious nature of the learning objectives resulting in the cognitive load inherent in the learning task (intrinsic cognitive load) being too high (van Merrienboer & Sweller, 2005). In either case, it is contended that adopting minimally-guided approaches tends to result in cognitive overload. The suggested mechanism responsible for this overload is the very high number of interacting elements within the set task, which the learner has to process simultaneously in working memory. This unsustainably high load in turn impedes the formation of new schemas, thus undermining learning (Sweller, 2010).

This assertion outlining how cognitive load theory establishes the superiority of direct instruction over minimally-guided approaches is not uncontroversial and has attracted a number of critical commentaries (e.g., Schmidt, Loyens, van Gog, & Paas, 2007). Sweller and colleagues, however, maintain that proponents of minimally-guided approaches are choosing to ignore contemporary knowledge of human cognition when designing instruction (Sweller et al., 2007):

> The process of discovery is in conflict with our current knowledge of human cognitive architecture which assumes that working memory is severely limited in capacity when dealing with novel information sourced from the external environment but largely unlimited when dealing with familiar, organized information sourced from long-term memory. If this view of human cognitive architecture is valid, then by definition novices should not be presented with material in a manner that unnecessarily requires them to search for a solution with its attendant heavy working memory load rather than being presented with a solution (Sweller et al. 2007, p. 116).

However, the current paper will contend that Sweller and his colleagues' critique of minimally-guided approaches is an overreach, as it *does not apply* to some of the more nuanced approaches to problem-based learning that have evolved in mathematics education. Specifically, the current paper will advance several arguments in support of the notion that launching a lesson with a challenging problem is in fact consistent with our knowledge of human cognitive architecture, provided that the tasks themselves meet particular criteria. Moreover, this analysis will be couched in language and ideas central to cognitive load theory.

What are challenging tasks?

Sullivan and Mornane (2013) describe challenging tasks as complex and absorbing problems with multiple solution pathways. Such problems are presented to the entire class, with the teacher encouraging all students to make an attempt at the problem. After a student has spent some time in the 'zone of confusion' and remains unsure how to proceed, he or she is given access to 'just in time' support through 'enabling prompts' (Sullivan, Mousley, & Zevenbergen, 2006). Enabling prompts reduce the intrinsic cognitive load of the task through changing how the problem is represented, helping the student connect the problem to prior learning and/ or removing a step in the problem (Sullivan et al., 2006). Students who complete the problem early are given access to an 'extending prompt'. This is designed to expose the student to an additional task that is more challenging, however requires them to use similar mathematical reasoning, conceptualisations and representations as the main task.

Consequently, challenging tasks can be viewed as a subset of problem-solving tasks that meets specific criteria. Adapted from the work of Sullivan and his colleagues (e.g., Sullivan & Mornane, 2013), criteria relevant to the issue of optimising cognitive load are presented below.

The task must:

- be solvable through multiple means (i.e., have multiple solution pathways) and may have multiple solutions;
- involve multiple mathematical steps (i.e., as opposed to a single insight facilitating completion of the problem);
- have at least one enabling prompt and one extending prompt developed prior to delivery of the lesson;
- involve students having primary control over how they are able to approach the task and when they are able to access enabling and extending prompts, within some constraints established by the teacher.

How can challenging tasks reduce extraneous cognitive load?

This section introduces two effects discussed in the cognitive load literature which have been linked empirically with extraneous cognitive load. It is argued that challenging tasks possess particular structural characteristics that allow them to leverage off these effects, reducing extraneous cognitive load relative to more teacher-directed learning approaches.

Goal-free (and means-free) effect

One of the earliest ideas within cognitive load theory to gain empirical support was the notion that goal-free tasks can reduce extraneous cognitive load through reducing reliance on a cognitively taxing means-end analysis (Sweller, 1988). Sweller argued that the absence of established schema require individuals to problem solve through adopting a means-end analysis. Although he acknowledges this may be an efficient means of solving a problem, he argues that it places a substantial strain on working memory. Specifically, he suggests that using a means-end analysis requires the problem solver to continually hold in mind several elements simultaneously, including the original problem state, the end goal state, how the two states relate to one another, strategies and operators that could bridge the two states and any subgoals that the problem solver needs to reach as he or she works through a problem. He suggested that this substantial extraneous load inhibits learning, because building an appropriate schema to understand how the relevant concepts interrelate and solving the problem are not compatible goals. Sweller suggested that to circumvent this issue, instructors should provide students with goal-free problems, which allow them to more comprehensively explore and comprehend a concept. Empirical support for the goal-free effect is well established within the literature (e.g., Bobis, Sweller, & Cooper, 1994).

Challenging tasks are open-ended in the sense that they may have multiple solutions. This may result in lower extraneous cognitive load, as described by the goal-free effect. Perhaps more importantly, the fact that challenging tasks have multiple solution-pathways means that they may have a lower (extraneous) cognitive load compared with traditional learning approaches, which emphasise algorithms and ‘one-best method’. This may be termed a ‘means-free effect’. The rationale is similar to the goal-free effect. Essentially, through ensuring that there are multiple viable pathways to a particular solution, instructors are increasing the probability that learners have some prior knowledge of strategies that can bridge the problem and solution states. Moreover, it is likely that the search time for locating an appropriate strategy is reduced, as learners only have to recall one of the multiple means of solving the problem to proceed. Similarly, there is likely to be less emphasis on reaching a specific subgoal, and even when a particular subgoal is still vital to solving the problem, there are almost certainly multiple pathways for reaching that subgoal. To summarise, this enhanced connectivity between the problem and solution states reduces the cognitive load required to productively engage in the problem, and, therefore, enhances the likelihood of learning occurring.

Reducing/ removing the expertise reversal effect

Many of the mechanisms and techniques that have been associated with lower extraneous cognitive load when learners are novices have a paradoxical effect when learners are more expert (Kalyuga, 2007). For example, Renkl and Atkinson (2003) argued that as learner experience with a particular problem type increased, they should progress from worked examples, to completion problems, and finally to fully intact problems. They demonstrated empirically that attempting to provide experts with more scaffolding than they required actually inhibited their learning. This perhaps counter-intuitive finding within the cognitive load literature has been termed “the expertise reversal effect” (Kalyuga, 2007, p. 509).

The expertise-reversal effect has been attributed to another phenomenon within cognitive load theory, the redundancy effect (Kalyuga, 2007). Specifically, requiring experts to process additional information intended to support learning but irrelevant to their learning needs unnecessarily burdens their working memory, resulting in an extraneous cognitive load. It has been suggested that, in order to reduce the expertise-reversal effect and optimise how much support is provided to learners, learning environments need to be tailored so they can adapt to learner expertise (Kalyuga, 2007). Challenging tasks include enabling prompts to provide scaffolding for a problem for those students who require it. Students are primarily responsible for determining if and when they should access these prompts. Structuring support in this manner can reduce the likelihood of the expertise reversal effect inhibiting learning.

While it can be argued that all problem-based approaches by definition reduce the expertise-reversal effect because their low-support approach fundamentally caters to the needs of experts, challenging tasks appear to do so without compromising the level of support offered to non-expert learners. Through the withholding of information, which would otherwise simplify or breakdown the problem (i.e., not automatically providing all students with the enabling prompts), experts are not provided with potentially redundant information.

A further strength of the challenging task approach is that no initial judgements need be made by the teacher in relation to the expertise of the student, and therefore the level of scaffolding and support they will require. Instead, students self-select based on their perceptions of the difficulty of the task. Although teachers clearly have a role in encouraging students who are struggling unproductively with a task to access an enabling prompt, this self-determination increases the accuracy with which expertise is identified. This in turn should serve to further reduce the expertise-reversal effect, in comparison to less precise ways of determining expertise with a given task (e.g., relying on past test scores).

In a more general sense, the use of prompts potentially optimises the level of challenge inherent in the task (i.e., the intrinsic cognitive load). This issue is discussed next.

How can challenging tasks optimise intrinsic cognitive load?

Intrinsic cognitive load is determined by the extent to which the various elements inherent in a particular learning task interact (Sweller, 2010); in other words, task complexity (Schnotz & Kurschner, 2007). A large number of interacting elements requiring simultaneous information processing suggests a high intrinsic cognitive load. In addition to task complexity, intrinsic cognitive load is also determined by the extent of the learner's expertise with similar tasks (which will impact on subjective task complexity) and the level of outside support provided to tackle the task (Schnotz & Kurschner, 2007). In contrast to extraneous cognitive load, the level of intrinsic cognitive load is considered fixed for an individual with a given level of expertise. Changing the level of intrinsic cognitive load can only be achieved through altering the task, which in turn would imply different learning objectives (Sweller, 2010).

To maximise learning, intrinsic cognitive load needs to be at an appropriate level as determined by the interaction between the complexity of the problem and the expertise of the learner (Sweller, 2010). If intrinsic cognitive load is too high, students will become overloaded and learning will not occur. However, if intrinsic cognitive load is too low, learning is also undermined. Not only is cognitive capacity underutilised, but as Schnotz and Kurschner (2007) argue, more expert learners may choose to disengage and 'tune out' if the challenge inherent in the task is inadequate. Essentially this last point is an alternative interpretation of the expertise-reversal effect discussed earlier.

It is proposed that challenging tasks can optimise the level of intrinsic cognitive load through learners utilisng enabling prompts and extending prompts on a 'just in time' basis. In the first instance, accessing sequenced enabling prompts can reduce the amount of interactivity amongst the elements of the task until the task is at an appropriate level of challenge for a given learner's expertise.

For example, consider a challenging task for a Grade 2 student: "Can you add all of the digits from one to nine together, and explain your approach to a partner?" The first enabling prompt may represent the task for the student as a number sentence (1+2+3+4+5+6+7+8+9=), making the problem to be solved far less opaque and unfamiliar. The second enabling prompt may remind students that they do not need to add numbers in the order they are first presented in. The third enabling prompt may ask students to consider if they can see any number bonds equalling ten, and the fourth enabling prompt may provide students with some examples of number bonds equally 10 taken from the problem (i.e., 1+9; 2+8).

By contrast, an extending prompt essentially attempts to increase the number of interacting elements to make the problem more challenging. For example, modifying the above challenging task so that multi-digit numbers need to be added (e.g., "Can you add all of the numbers from eleven to twenty together?") introduces additional place-value elements to the task (i.e., adding multi-digit numbers; understanding place-value to 3-digits).

It needs to be noted that in using prompts to enable the activation of requisite knowledge and facilitate the creation of new ‘intermediate’ knowledge, the nature of the problem has been changed and therefore the learning objectives of the task have been somewhat altered. However, although students are in reality working on slightly different problems, critically they have a similar experience in having worked on the same challenging task. This enables them to actively participate in the discussion component of the lesson, and reflect on the key mathematical concepts explored. Indeed, systematically modifying intrinsic cognitive load by reducing or increasing the number of elements and/or the interactions between elements *without undermining the primary learning objective* has some precedent within cognitive load theory (see the part-whole approach; van Merrienboer, Kester & Paas, 2006).

Consequently, if a particular learning objective is a central focus of a lesson, it should not be compromised by any of the enabling prompts. For example, in the task outlined above, if the primary learning objective was for students to be able to translate worded problems into number sentences, then the first enabling prompt, which effectively does this for the student, is clearly not appropriate.

Summary and Future Research Directions

Whilst there is some evidence that the reform in mathematics education towards problem-based learning has been efficacious (e.g., Marshall & Horton, 2011), other authors cite evidence that problem-based approaches impose too high a cognitive load, and therefore undermine learning (e.g., Sweller et al., 2007). This paper has argued that teaching with challenging tasks can provide the benefits of problem-based approaches (e.g., higher order thinking, persistence) whilst being cognisant of the issue of cognitive overload. There are at least two lines of future research suggested by the arguments put forward in this paper.

Firstly, the contention that enabling and extending prompts effectively modify the intrinsic cognitive load of a task so that it is optimised for a given learner could be examined in a classroom context. This would require multiple measurements of cognitive load to be taken during a particular lesson, as well as data around whether students perceive the level of challenge on offer as optimal. Changes with respect to learners’ perceptions of cognitive load and challenge optimality could then be examined in relation to time.

Secondly, student learning outcomes achieved in classrooms adopting the Launch-Explore-Discuss lesson structure could be contrasted with student learning outcomes achieved by classrooms adopting more traditional lesson structures (i.e., lessons beginning with a period of teacher-facilitated instruction). This would get to the heart of the debate by addressing concerns about whether problem-based learning contexts generate extraneous cognitive load, therefore undermining student learning. Any such study would need to ensure that both classroom types essentially contained the same content and pedagogy, with lesson structure being the only factor allowed to vary.

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Appendix O. Investigating whether participation in the program disproportionately benefitted ‘high flyers’

This analysis revisits the student outcome data in order to briefly address an issue which emerged from the teacher-participant interviews; that is, the notion that the program disproportionately benefitted whom Polly termed ‘high flyers’. In order to address the issue, the magnitude of changes in student mathematical performance scores between pre-program and post-program assessments are compared with absolute student pre-program mathematical performance scores for each unit of work. If the program disproportionality benefited ‘high-flyers’, we would expect a positive correlation between change in performance scores and pre-program performance scores.

Student Outcome Data: Patterning Unit

A Pearson correlation coefficient matrix was created to compare pre-program fluency and mathematical problem solving scores and the respective *changes* in these scores between the pre and post program for the patterning unit of work. This post-hoc analysis reveals that pre-program scores were negatively correlated with improvement for both mathematical fluency ($p < 0.01$) and problem solving performance ($p < 0.01$). This suggests that, for the patterning unit of work at least, ‘high flyers’ actually benefitted *least* from the program (or, perhaps more accurately, low to moderate performers tended to benefit most from participation in the program). According to Cohen’s (1992) convention, the size of these negative correlations varied from moderate for problem solving performance (r = -.43) to strong for mathematical fluency (r = -.62). Given the large improvements noted for participants in the program as a whole, it is likely that these negative correlations reflect the fact that higher achievers already had a relative mastery of the relevant curriculum regarding number patterning, and had less scope to improve compared with lower achievers.

As an additional aside, it is worth noting that the above patterns regarding these correlations remained similar when the analysis was undertaken at the level of individual classes (i.e., Class A, Class B, Class C), rather than simply considering the data set in aggregate (see Table O.1). Consequently, it can also be stated that lesson structure (i.e., task-first, teach-first, alternating) did not moderate the relationship between prior mathematical performance, and gains in mathematical fluency and problem solving.

Table O.1

Lesson structure and the correlations between pre-program scores and improvement: Patterning unit

Lesson Structure	Fluency *r*	Problem Solving *r*
All	-.62	-.43
Task-First (Class A)	-.57	-.54
Teach-First (Class B)	-.63	-.41
Alternating (Class C)	-.67	-.29

Student Outcome Data: Addition Unit

The above analysis was replicated for the addition unit of work. Again, there was no evidence for the notion that ‘high flyers’ disproportionately benefitted more from participation in the program. Although there remained the suggestion that lower achieving students actually improved more, in contrast to the patterning unit, the corresponding negative correlations between performance and improvement were weak and not statistically significant (mathematical fluency, $r = -.16$, $p > 0.05$; problem solving performance $r = -.22$, $p > 0.05$). Consequently, for the addition unit, it should be concluded that pre-program performance did not correlate with the magnitude of student improvement scores.

Moreover, these correlations remained consistently weak and statistically non-significant when the analysis was undertaken at the level of individual classes (see Table O.2). Therefore, it can again be stated that lesson structure (i.e., task-first, teach-first, alternating) did not moderate the relationship between prior mathematical performance, and gains in mathematical fluency and problem solving.

Table O.2

Lesson structure and the correlations between pre-program scores and improvement: Addition unit

Lesson Structure	Fluency *r*	Problem Solving *r*
All	-.16	-.22
Task-First (Class B)	.20	-.25
Teach-First (Class A)	-.35	-.29
Alternating (Class C)	-.33	-.15

It is interesting to note that the direction of the correlation between fluency scores and fluency improvement was positive ($r = .20$) for Class B in the addition unit of work, which was in fact Polly’s class. Ostensibly, this might be interpreted to lend validity to Polly’s claim that participation in the program actually did better support ‘high flyers’. However, it is important to consider that the other three correlations between performance and improvement were negative for Polly’s students and moreover, that the magnitude of these negative correlations were larger

(r = -.25; r = -.41; r = -.63). Consequently, overall, even for Polly’s class, there is substantially more evidence for the contention that the program disproportionately benefitted lower achieving students than the contrary claim put forward by Polly.