Professional learning of primary mathematics teachers: Investigating pathways and processes for changing practice

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Abstract

Changing teaching practice can be challenging, from both the perspective of teachers and those involved in designing, facilitating and supporting teachers with change. The success of an intervention, in relation to changes in teachers' knowledge, disposition or practice, often varies at both an individual teacher level within a context, as well as across contexts. Much research literature has focused primarily on the overall effectiveness of professional learning, with teacher learning viewed predominantly as an indicator of program success. Although there has been a recent shift to gain insights into learning processes within a particular context, it is recognised that teacher professional learning is a complex interconnected process that requires further exploration.

This thesis contributes to the developing empirical and theoretical knowledge base on primary teacher professional learning in mathematics. The study adopted design-based research methodology to investigate processes involved in teachers learning sophisticated instructional practices through onsite collaborative professional learning experiences. The teaching and learning of mental computation with conceptual understanding and fluency, provided the context in which to study the change pathways and processes of ten teachers across three different school settings in Australia. Multiple qualitative data, including lesson observations, were collected.

Each teacher suggested a combination of stimuli instigated and supported their learning, two particular professional learning experiences were critical: modelled lessons and collaborative facilitated planning. Both enhanced opportunities for

professional dialogic interactions and development of shared ideas, or elements of social dynamics, which were consequential for learning and change.

Fundamental to change seemed to be teachers evidencing a learner stance, in particular, characteristics they exhibited associated with a growth mindset (Dweck, 2000; 2006), and if they acted as learners by self-reflecting on their professional learning experiences. For teachers whose thinking or practice was challenged during the intervention, and who displayed attributes of a growth mindset, such experiences created opportunities for them to self-reflect and enhance their learning.

Aspects of the institutional context, both external and internal factors, influenced opportunities for the teachers to learn and change practice. Internal support structures, such as an instructional coach and a culture of professional learning in which accountability was integral, had potential to afford opportunities for teachers to learn and develop practice. However, it for such structures to be effective in supporting learning and change, teachers needed opportunities to be active learners within a supportive, innovative learning community. Moreover, the extent to which the teachers developed a shared vision on effective and equitable teaching of mental computation that was aligned with the goals of the intervention, influenced the changes they made to their practice.

This study highlighted the idiosyncratic nature of teacher professional learning and the complexity with implementing onsite collaborative programs. Yet, the study also found that when teachers view themselves as learners, are situated within an innovative, supportive learning community, and are given opportunities to engage in collaborative professional learning experiences attuned to their developing shared vision for effective and equitable teaching of mathematics, opportunities for teacher learning and change are enhanced.

Declaration

This thesis contains no material which has been accepted for the award of any other degree or diploma at any university or equivalent institution and that, to the best of my knowledge and belief, this thesis contains no material previously published or written by another person, except where due reference is made in the text of the thesis.

Signature: ……………………

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Acronyms and definitions

ACARA	Australian Curriculum, Assessment and Reporting Authority
AP	Assistant Principal
CCK	Common Content Knowledge
CEM	Catholic Education Melbourne
DC	Domain of Consequence
DET	Department of Education and Training (Victoria, Australia)
DP	Domain of Practice
ED	External Domain
ENL	Empty Number Line
IB	International Baccalaureate
IMPG	Interconnected Model of Professional Growth
KCT	Knowledge of Content and Teaching
LFIN	Learning Framework in Number
MDT	Meta-Didactical Transposition model
MKT	Mathematical Knowledge for Teaching
ML	Mathematics Leader
MOE	Ministry of Education (New Zealand)

MUHREC	Monash University Human Research Ethics Committee
NAPLAN	National Assessment Program - Literacy and Numeracy
NC	Numeracy Coach
NCTM	National Council of Teachers of Mathematics
NDP	Numeracy Development Projects (New Zealand)
NNS	National Numeracy Strategy (UK)
PCK	Pedagogical Content Knowledge
PD	Personal Domain
PL	Professional Learning
PNG	Papua New Guinea
PYP	Primary Years Programme (IB curriculum program)
QCA	Qualifications and Curriculum Authority (UK)
QSR	International qualitative research software developer
SCK	Specialised Content Knowledge
SMK	Subject Matter Knowledge
SURF	Strategies Understanding Reading Fast Facts
UK	United Kingdom

1. Learning about teacher professional learning

Just as young learners construct so, too, do teachers. (Fosnot, 1996, p. 216)

Changing teaching practice can be challenging, from both the perspective of teachers and those involved in designing and facilitating professional learning to support teachers with change. The success of an intervention, in relation to teachers constructing new ideas and experiencing changes in knowledge, disposition or practice, often varies at both an individual level within a context, as well as across contexts (Hiebert, 2013; Wilkie, 2019; Wilkie & Clarke, 2015). Much research literature has focused primarily on the overall effectiveness of professional learning, with teacher learning viewed predominantly as an indicator of program success (Goldsmith, Doerr, & Lewis, 2014). Although there has been a recent shift to gain insights into learning processes within a particular context and the variability issue, it is recognised that teacher professional learning is a complex interconnected process that requires further exploration (Arzarello et al., 2014; Clarke & Hollingsworth, 2002; Goldsmith et al., 2014).

This thesis reports on a study that investigated processes involved in teachers learning sophisticated instructional practices through onsite collaborative professional learning experiences. The intent of this study was to gain insights into learning processes; how teachers learn, ways to foster learning and support teachers as they develop their practice within the institutional context in which they work. Design-based research methodology was adopted to explore the experiences of ten primary school teachers from across three different school settings in Australia. The teaching and learning of mental computation with conceptual understanding and fluency, provided the context in which to study the change pathways and processes of the teachers.

The intention of this chapter is to introduce the study, and myself, as the researcher, designer and facilitator of the intervention. In the first section, I set the scene by framing teaching professional learning for the purpose of this study and provide further details about the context – mental computation – in which teacher professional learning was studied. In the second section reasons for the research are discussed, where this study is positioned within the research field, the practical significance and my personal impetus for the research. This is followed by an outline of the main aim and research questions. The final section provides an overview of the thesis structure.

1.1. Context and background

In this section, I set out to define and frame teacher professional learning for the purpose of this study. This is followed by explanation of the context in which teacher learning and change processes were investigated.

1.1.1 Defining and framing teacher professional learning

Over the last three decades there has been much diversity surrounding the idea of 'teacher change' (Clarke & Hollingsworth, 1994), how to theorise professional learning (Lave, 1996; Sfard, 1998; Shulman, 1986;), and what constitutes effective professional development (Desimone, 2009) in relation to the overarching goal of improving student learning in mathematics. For many years, teacher change was predominantly associated with planned 'one-shot' professional development activities such as workshops, conferences and seminars (Clarke & Hollingsworth, 2002, p. 948). The focus of such activities was on improving particular knowledge or skills considered essential for teachers to master: a perspective based on a deficit training-mastery model.

However, perspectives on teacher change have since evolved. Fundamental to more recent views on teacher change is the notion of growth or learning (Clarke & Hollingsworth, 2002). With change being seen as a continuous process, teacher professional learning is now viewed as ongoing development of knowledge and effective teaching practices (Wilkie, 2019). Central to this more recent perspective is the idea of teachers being considered learners and schools as learning communities; this signifies a shift from professional development being viewed as a way to change teachers, to teachers being considered as agents to change their own practice (Clarke & Hollingsworth, 2002). Many of the activities related to new perspectives on professional learning, such as lesson observations, co-teaching, discussing or moderating student work, occur within the school context, usually in teacher's own classrooms with their students (Desimone, 2011), thus highlighting the importance of teacher learning being situated in the context in which they work. The nature of these activities essentially places social interaction as central to teacher professional learning (Desimone, 2009).

For the purpose of this study, I have adopted the term *professional learning* to reflect the current stance on teacher change as learning or growth, with teachers as active learners. This perspective influenced the research focus and the design of the intervention for this study. Essentially the study is based on “the notion that teacher’s learning experiences, in combination with a host of contextual and personal factors, drive changes in what teachers know and do” (Covay Minor, Desimone, Caines Lee, & Hochberg, 2016, p. 4).

1.1.2. The context for studying change processes: Mental computation

The teaching and learning of mental computation with conceptual understanding and fluency, provided the context in which to study the change processes of the teachers. In this subsection, reasons for studying teacher change within this context are discussed. First, a brief outline of the conceptualisation of mental computation adopted for the purpose of this study is given.

Mental computation is interpreted as encompassing mental strategies (mental processes that students use to estimate and solve problems) and knowledge (recall of facts) that students can learn to assist with other aspects of mathematics. Teaching for mental computation is broadly interpreted to involve presenting students with calculations in which they have to work out the answer using known facts, rather than simply recalling facts they have memorised. For this study mental computation is considered a “vehicle for promoting thinking” (McIntosh, Nohda, Reys, & Reys, 1995, p. 238). It is through thinking and working with numbers that students develop computation strategies, knowledge of facts and number sense.

Number sense is a term associated with mental computation; it involves using numbers flexibly by decomposing and recomposing numbers to solve problems efficiently and easily. For example, recognising that 21 – 6 is the same as 20 – 5 is easier than counting back to work out the answer. Mental computation is considered critical to developing number sense in students; when numbers are treated holistically as quantities rather than digits the “methods are more meaningfully and conceptually based” (Threlfall, 2002, p. 31).

Underpinning the approach to mental computation adopted for this study is a perspective on learning based on social constructivist principles. This is interpreted as students being asked to explain ‘how’ they solved the problem and ‘why’ their method works (Caney, 2004). It places emphasis on students building on existing knowledge to develop new mental strategies (Heirdsfield, 2002) and interacting with the teacher and each other to develop mathematical ideas for themselves (Ernest, 1994). The context of mental computation was considered a pertinent topic through which to investigate the processes involved in changing teacher practice through professional learning experiences for two main reasons, which will be explained next.

First, the *Australian Curriculum: Mathematics* identifies four proficiency strands, considered as actions or ways for students to learn mathematics (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2014). Such actions are important in ensuring a balanced curriculum (Sullivan, 2011). The teaching of mental computation with conceptual understanding and fluency, concerns three of the proficiencies: conceptual understanding, reasoning and fluency (the fourth proficiency is problem solving). In a review of national and international research on approaches to teaching mathematics, Sullivan (2011) commented on limited opportunities given to students to build their own understanding of concepts and to reason. With reasoning being considered "central to the discipline of mathematics," it has been posited that students need to be taught to reason, and to critique the reasoning of their peers (Boaler, 2016, p. 28). Learning to compute mentally is considered important in strengthening reasoning, communication and connection of mathematical ideas in students (Heirdsfield, 2011). Adopting a constructivist approach to teaching and learning of mental computation, communicating thinking between the teacher and students and between students, using clear models to represent mental strategies and using interactive explanations, should help students to achieve conceptual understanding (Sullivan, Clarke, & Clarke, 2013a).

Second, the advantages of an emphasis on teaching mental computation have been widely reported by researchers (McIntosh, Reys, & Reys, 1997; Sowder 1990). Heirdsfield and Cooper (2004) commented on the significance of mental computation in promoting number sense when students are encouraged to develop their own computation strategies. Likewise, Boaler (2016) highlighted the value of developing number sense. She suggested that students successful in mathematics are those able to compute with "numbers flexibly and conceptually" (p. 35), in other words, students who have good number sense. Attention has also been drawn to the increasing need for students to develop number sense, estimation skills, problem solving and mental computation skills, due to the prevalence of technology in society (Kamii, Lewis, & Jones, 1991). Changes in mathematics curricula across the world in the last two decades (ACARA, 2014; National Council of Teachers of Mathematics [NCTM], 2000; Qualifications and Curriculum Authority [QCA], 1999) reflect the importance of teaching and learning mental computation.

1.1.2.1. Mental computation in the context of Australia

Although the advantages of a focus on mental computation have been widely reported, a recent study on the development of addition and subtraction strategies in primary schools in the Australian state of New South Wales (Gervasoni, Guimelli, & McHugh, 2017) suggested that large groups of students from Grades 3 to 5 continue to use counting-based strategies, notably 51% of students at the beginning of Grade 3. The study also highlighted that student development of derived strategies did not improve from Grade 3 to Grade 5 (Gervasoni et al., 2017). Recommendations from the study include a focus on basic and derived[1] strategies in addition and subtraction with 2-digit and 3-digit numbers and professional learning opportunities for teachers on "powerful pedagogical actions and tools that will assist all children to learn these strategies" (Gervasoni et al., 2017, p. 274). With many teachers acting against curriculum advice and introducing formal written methods for addition and subtraction with 2-digit and 3-digit numbers in Grades 2 and 3, before students are flexible with mental computation (Gervasoni et al., 2017), designing an

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[1] The terms 'basic' and 'derived' strategies refer to growth points within a framework for addition and subtraction, established by the Early Numeracy Research Project (ENRP). Basic strategies include: doubles, commutativity, adding 10, ten facts, other known facts. Derived strategies include: near doubles, adding 9, building to next ten, intuitive strategies. The ENRP was conducted between 1999-2002 with 35 primary schools working in collaboration with Australian Catholic University, Victorian Department of Employment, Education and Training, Catholic Education Melbourne and the Association of Independent Schools in Victoria.

intervention targeted at Year 3 for the purposes of this study was considered apposite. In essence, a focus on mental computation is arguably an apt context for teachers to develop instructional practices that enhance students' development of the proficiencies of the Australian mathematics curriculum. Such a context

> …provides a rich site for students to develop methods for solving problems and to gain important understandings about the number systems and about operations within number systems. Studying computation serves as a vehicle for building mathematical understandings. (Hiebert et al., 1997, p. 26)

1.2. Reasons for the research

The focus of this section is to position this thesis within the field of research and at the same time highlight the need to conduct the study. The impetus for the research – the personal motivation that has driven it – will also be discussed.

1.2.1. Contribution to the research and literature

Studies of teacher professional learning have raised controversies and complexities ranging from issues associated with "effective professional growth and how it is conceived and measured" (Wilkie & Clarke, 2015, p. 91) to processes of change. Research on change processes concerns investigating the means "through which teachers transform their knowledge and apply new ideas to changes in practice" (Loucks-Horsley, Stiles, Mundry, Love, & Hewson, 2010, p. 23). Literature indicates that transforming new content knowledge "into improved instructional strategies is not automatic; the process works through various mechanisms" (Covay Minor et al., 2016, p. 3). Although there has been a shift in research focus to look at processes involved in professional learning (Goldsmith et al., 2014), until recently it has been "a neglected area. Few studies have addressed the issue directly" (Timperley, Wilson, Barrar, & Fung, 2007, p. xl). Further to this, the number of design studies investigating professional learning is "relatively small compared with classroom design studies" (Cobb, Jackson, & Dunlap, 2015, p. 492). Arguably, further design-based studies on teacher professional learning that, "produce knowledge that will be useful in guiding others as they attempt to support teachers' learning in other settings", are required (p. 495). Thus, conducting a study on professional learning of teachers, which includes a focus on learning processes seems much needed.

An additional challenge for those involved in designing, facilitating and supporting teachers with professional learning is referred to in the literature as the variability issue (Goldsmith et al., 2014). Even when teachers engage in what is considered high-quality professional learning, i.e. professional learning based on the five core features outlined by Desimone (2009), there is considerable variation in what is learnt and how new knowledge is transformed into teaching practice (Covay Minor et al., 2016). While there is substantial evidence of variation in teachers' responses to professional learning, little is known about how and why such variation occurs (Covay Minor et al., 2016). It is therefore important for studies on professional learning to explore and dig deeper into this issue of variation.

In summary, it is recognised that teacher professional learning is a complex, idiosyncratic, interconnected process and that further exploration on how to support teachers with changing their practice is needed (Clarke & Hollingsworth, 2002; Goldsmith et al., 2014).

1.2.2. Practical context: Changing teaching

Since the advent of Trends in International Mathematics and Science Study (TIMSS) in 1995 there has been increased focus on improving student achievement in mathematics, particularly across the US, UK and Australia. It is widely accepted that teaching practice is a major factor influencing student outcomes in mathematics (Askew, Brown, Rhodes, Wiliam, & Johnson, 1997; Hiebert & Grouws, 2007). Thus, in recent years much attention has been given to teachers improving their practice; in particular, there has been an increased focus on learner-centred practices. Such practices essentially involve instigating student thinking, reasoning and problem solving and developing methods which build conceptual understanding (Young-Loveridge, Mills, & Bicknell, 2012). It is recognised that approaches to teaching that involve teachers facilitating purposeful mathematical discussions by representing, explaining and developing the ideas of learners, place considerable demands on teachers' Pedagogical Content Knowledge (Boerst, Sleep, Ball, & Bass, 2011; Hill & Ball, 2009). Furthermore, teaching for conceptual understanding can also place demands on teachers' Subject Matter Knowledge (Thames & Ball, 2010). When students are required to think, reason and problem solve using a range of strategies, teachers need awareness of a diverse range of strategies students might use (Sullivan, 2011). With pressure placed on both pedagogical knowledge and subject knowledge of teachers, it is perhaps not surprising for researchers to posit that, "most teachers in the US" require "professional support for an extended period of time" (Cobb, Jackson, Henrick, & Smith, 2018, p. 2) to adopt such changes in their practice. A review of the national literature on teaching practice in Australia (Sullivan, 2011) suggested that the situation is not dissimilar in Australia.

In summary, the words of Dylan Wiliam express eloquently, the need for a focus on teacher professional learning:

> The reason that teachers need professional development has nothing to do with professional updating. As far as I am aware, there haven't been any real breakthroughs in teaching for the last two thousand years. Teachers need professional development because the job of teaching is so difficult, so complex, that one lifetime is not enough to master it. (Wiliam, 2011, p. 29)

1.2.3. The impetus for this study

My motivation for this study, the impetus driving the research, is connected to my experience as a primary teacher. Shortly after commencing my teaching career in the UK, the National Numeracy Strategy (NNS) was introduced. Arguably the NNS, had the greatest impact on the teaching of primary mathematics in the UK than any intervention for over a hundred years (Askew, Millett, Brown, Rhodes, & Bibby, 2001). Perhaps equally significant was the impact the launch of the NNS had on my teaching career. Although I had achieved well in mathematics as a student and completed mathematics-based courses as an undergraduate studying economics, I felt my own educational experiences had left me with limited understanding of the subject. It was my own professional learning associated with the implementation of the NNS, which provided the catalyst for developing my understanding of the subject and igniting my zeal for teaching and learning mathematics. My enthusiasm and interest quickly earned me the role of numeracy coordinator in a primary school. A key feature of the NNS was an emphasis on calculation, in particular mental calculation, and central to the pedagogy was whole class discourse: aspects of teaching and learning mathematics I am still fervent about two decades later.

While my professional learning experiences in the UK instigated my interest in teaching and learning mathematics, it was my Papua New Guinean (PNG) experience that ignited my interest in teacher professional learning. The skills I learned as an Aid worker facilitating the learning of pre-service primary teachers were invaluable for teaching me firsthand the significance of relationships, trust and context. Within PNG, I transitioned to an international school system where I initially worked in school leadership, a role which included responsibility for professional learning of teachers, then later as a school adviser working predominantly with Papua New Guinean teachers. The majority of the local teachers I worked with embraced the opportunity to learn and develop their teaching of mathematics. Their own educational experiences had been traditional by nature, and their subject and pedagogical knowledge was initially limited (Hughes, 2016). But they embraced opportunities to learn to teach mathematics in a way they had not previously experienced themselves as learners. It was the eagerness and interest of these teachers that was infectious;

they inspired within me a desire and need to learn more about how to support primary teachers with developing their practice in mathematics.

In this section I have highlighted the need for further exploration of professional learning, in particular a focus on learning processes and variation in teacher learning experiences within a school context, as well as across contexts. I have drawn attention to the complexities of teaching mathematics and critical focus areas for improvement. These issues informed the design of the intervention and the formulation of the research questions, which are outlined in the next section.

1.3. Research aim and questions

The main aim of this study was to investigate the processes involved in teachers learning sophisticated instructional practices through onsite professional leaning experiences. In using the term 'sophisticated instructional practices', I refer to skills teachers develop to elicit student thinking, respond to it, and advance mathematical thinking (Lampert & Graziani, 2009). Such practices involve a focus on improving the interaction between the teacher and students to progress learning, essentially facilitating a productive classroom discussion (Stein, Engle, Smith, & Hughes, 2008).

The study adopted design-based research methodology to investigate how teachers learn and develop practice within the institutional setting in which they work. Integral to the study was an intervention; the teaching content of the intervention involved a (potentially) different approach to teaching mental computation. Mental computation provided the context for the study, the vehicle for studying teacher professional learning.

The central research question is:

> How do professional learning experiences, in the context of teaching mental computation, provide opportunities for primary teachers to learn and develop practice?

The following sub-questions were formed to address the aims of the research:

1) What were the different change pathways and learning processes experienced by each teacher?

2) How do aspects of the teachers' institutional context constrain or afford opportunities to learn and change practice?

It is important to clarify some of the terminology within the research questions. The use of the phrase 'opportunities to learn' relates to the notion of teachers as active learners, agents of their own learning. This term draws parallels between student learning, and the notion of students having opportunity to learn rather than being taught, and teacher learning (Wilkie, 2019). In relation to teacher professional learning, it is acknowledged that attention needs to be given to teacher knowledge and their disposition, one aspect of which is attitudes (Cooke, 2015). This study adopted the view that, "If teachers are viewed as active agents in their own learning, then an examination of their opportunity to learn needs to consider not only their access but also their engagement" (Wilkie, 2019, p. 101).

The focus of the first research sub-question is on change pathways and learning processes of individual teacher participants. The term 'change pathways' is linked conceptually to the Interconnected Model of Professional Growth (IMPG) developed by Clarke and Hollingsworth (2002), one of the theoretical models forming the theoretical framework for this study. The model is explained in depth in Chapter 3 (section 3.1.2).

1.4. Overview of the thesis structure

In this section, I provide an outline of the remaining chapters forming this thesis. The chapters have been ordered to reflect the cycles of designing, implementing and refining the intervention integral to this study, over time.

In Chapter 2, I begin by sharing my perspective on learning, which underpins the study and the lens through which I reviewed the literature. Literature on the development of teacher knowledge and how this relates to changing teacher practice, is discussed in relation to the content focus of the intervention. The connection between teacher disposition and how teachers respond to professional learning opportunities is explored. Issues with conceptualising teacher professional learning, designing effective professional learning, and the influence of contextual factors on opportunities for learning are discussed.

In Chapter 3, I outline the theoretical perspectives underpinning the study and explain two models, which form the theoretical framework for examining processes of teacher change in mathematics. Reasons for adopting design-based research methodology and the design of the professional learning program (the intervention) are discussed; qualitative methods and techniques used to collect and analyse data are explained; and ethical matters, approaches to ensure rigor in the research process, and my role as the researcher and external expert, are considered.

In Chapter 4, findings for the first intervention are presented and discussed in connection to relevant literature. The IMPG, one of the models forming the theoretical framework, was used to represent a change sequence for each teacher participant diagrammatically. The theoretical framework was used to guide discussion of the learning processes each teacher experienced. The significant impact of institutional constraints on opportunities for teachers to learn and change practice is discussed. The outcomes and challenges with implementing the first intervention are reflected upon and suggestions for refining the intervention outlined.

Chapter 5 describes the school context of the second intervention and the process of refining the intervention process. Two different change sequences emerged for this group of teachers, and the IMPG is used to represent the learning processes diagrammatically. The learning processes for the teachers sharing the same sequence pattern are compared and contrasted, in response to the first research sub-question. The critical influence of elements of social dynamics and cognitive dissonance on teacher internalisation of new learning is discussed. In the final part of the chapter, affordances and constraints to learning within the institution context are examined.

In the first section of Chapter 6, background information relating to the third school context and participants is given. Following this, details concerning the refining of the intervention for this particular institutional setting are provided. Change sequences and discussion of the learning processes distinguishing this group of teachers from the preceding two iterations, is presented. The interplay of components of the professional learning program and social dynamics, in particular the intensity of these interactions, is emphasised. Lastly, the influence of the school setting (institutional context) on change processes is discussed in relation to the second research sub-question.

In Chapter 7, the key findings from the three interventions are synthesised in relation to the recent literature on teacher professional learning. In comparing the change sequences for teachers across the three schools, commonalities with factors that influenced the learning pathways of the teachers are discussed. The first part of the chapter examines the three change sequence types that were identified across the interventions in this study; the second part discusses how contextual factors impacted on opportunities for teachers to learn and change practice (the sources of variation). The critical importance of teachers being situated in a learning community, and the conditions that influenced the effectiveness of the learning community are highlighted.

Chapter 8 concludes the thesis by considering how findings from the three interventions might inform future design and implementation of onsite professional learning experiences, and guide further research analysing processes involved in developing teaching practice. Practical implications for those involved in planning and implementing professional learning in schools are discussed, and theoretical implications and considerations for researchers studying change processes are addressed. Finally, limitations of the study and possible avenues for future research are considered.

2. Review of the literature

We never think entirely alone: we think in company, in a vast collaboration; we work with the workers of the past and of the present. (Sertillanges, 1978, p. 145)

The purpose of reviewing the literature on teacher professional learning was to draw upon the work of others to provide a backdrop to guide this study. Studying professional learning, in particular, possible processes for teacher learning, highlighted that attention needs to be given to teachers' knowledge and dispositions. This seems important when it has been suggested that prior knowledge and experiences play a role in moderating the transfer of new learning into classroom practice (Covay Minor et al., 2016). Findings of studies on teacher knowledge, and how this relates to changing practice, are discussed in the context of the content focus of the intervention: mental computation. In this study, teachers were perceived as active agents of their own learning, hence literature concerning their propensity to use new knowledge in the classroom, in other words their disposition, is also considered. I begin this chapter by establishing perspectives on learning underpinning the study, which will influence my interpretation of the literature.

2.1. Theoretical perspective on learning

The theoretical perspective on learning assumed for this study is a strand of constructivism, one broadly recognised as social constructivism. I use the term broadly because of the various hues within the parameters of social constructivism. In this subsection, I set out to clarify the interpretation of social constructivism adopted for this research.

The fundamental idea underpinning constructivism is that knowledge is constructed (Fox, 2001); how this is perceived to happen depends on certain assumptions. For example, views on social construction and to what extent every aspect of the world is socially constructed. It is these assumptions that differentiate the various strands of constructivism (Ernest, 1994). Essentially there are two main perspectives: psychological and social. Radical constructivists such as von Glasersfeld, who hold a psychological perspective, place emphasis on internal cognitive processes of individuals constructing knowledge (Cobb & Yackel, 1996). In contrast, those with a social perspective and an interactionist view of collective learning processes, focus on social process and interaction, and are commonly referred to as social constructivists (Schwandt, 2007). However, to present these perspectives as entirely separate would be a rather simplistic interpretation. The literature indicates that ideas of these two groups have evolved and become blurred (Ernest, 1994). Changes have largely been in response to criticism from scholars highlighting inadequacies with either perspective. For example, von Glasersfeld, has shown that mathematical knowledge is 'taken-as-shared' through agreed rules, thus introducing the notion of social interaction (Ernest, 1994). Notwithstanding, fundamental differences between these two main strands of constructivism remain and essentially learning is either individual and cognitive, or a 'combined individual-social' view (Ernest, 1994). The evolving nature of these perspectives is important to recognise because it is reflected in the stance taken for this study, which is explained next.

The perspective adopted for this study reflects the assumptions of social constructivism and is specifically referred to in the literature as an emergent approach (Cobb & Yackel, 1996). This approach develops key aspects of the psychological perspective (von Glasersfeld, 1992) and allows for analysis of constructive activities of both students and teachers in the social context of the classroom (Cobb & Yackel, 1996). The psychological and social perspectives are integrated; analysis of learning processes encompasses both perspectives. The conjecture by Cobb and Yackel is that when individuals participate in a discussion they are “reorganizing their individual beliefs about their own role, other’s roles, and the general nature of the mathematical activity” (Cobb & Yackel, 1996, p. 178). Simultaneously, the reorganisation of individual beliefs is “enabled and constrained” by the evolving social norms; they describe these two constructs as being reflexively related (Cobb & Yackel, 1996, p. 180). From an emergent perspective, the mathematical development of an individual and the classroom community are another example of a reflexive relationship.

In relation to the learning of mathematics, this means knowledge is actively constructed by individuals as they engage in conversation and negotiate new ideas or knowledge with others. Emphasis is placed on the importance of language; it is through conversing with others that individuals build new knowledge. Ernest (1994) used the metaphor of construction work to explain that new knowledge cannot be built by simply being given new blocks of knowledge; it is built using the products of "previously constructed knowledge" (p. 2). In relation to the context in which teacher professional learning is being studied – mental computation – this perspective on knowledge is interpreted as students needing to engage actively in solving problems, explaining strategies and conversing with others to build new knowledge of new strategies. If students only engage in listening to others explain strategies, this will not result in knowledge of new strategies being constructed. Research on teaching mental computation indicates that the underpinning principles are essentially constructivist (Caney, 2004). Importance is placed on students building on their existing knowledge to develop new mental strategies (Heirdsfield, 2002).

Theoretically, the emergent perspective emphasises the importance of analysing learning "as it is situated in the social context" (Cobb, 2000, p. 310). An important assumption of this perspective is that learning is seen as a process of mathematical enculturation (Cobb, 2000); which Cobb recognises as integrating an aspect of a sociocultural perspective to learning. The emergent perspective considers the influence of the institutional context on learning, specifically the pedagogical community, and that this is continually regenerated by the members of that community (rather than being static and universal) (Cobb, 2000).

Essentially this study is based on the notion that personal factors, which includes teachers' prior knowledge, interact with contextual factors and professional learning experiences to influence teacher classroom actions (Covay Minor et al., 2016; Sullivan, Borcek, Walker, & Rennie, 2015). In the next section, the literature on developing teacher knowledge, in particular how this relates to changing teacher practice, is discussed in the context of the mathematics content focus of the intervention (mental computation).

2.2. Mathematical Knowledge for Teaching (MKT)

The literature draws attention to various pitfalls with models that conceptualise teachers' mathematical knowledge (Zhang & Stephens, 2013). For the purpose of this study, the conceptualisation of Mathematical Knowledge for Teaching (MKT) developed by Hill, Ball and Schilling (2008) was adopted. Building on the earlier work of Shulman (1986), they elaborated on the two categories of MKT: Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK) by developing further sub-categories of knowledge. There has been some criticism of the MKT in relation to categorising different aspects of teachers' knowledge (Wilkie, 2016), limited development of the curriculum knowledge component, and failure to integrate teacher beliefs or context (Zhang & Stephens, 2013). With curriculum change not being a focus of this research, and teacher dispositions and context being addressed through a different theoretical model, these criticisms were not considered problematic. Rather, the MKT was considered useful to guide a review of the literature to highlight and discuss the multiple difficulties teachers experience with teaching mental computation. In addition, Italian researchers (Arzarello et al., 2014) highlighted the MKT as a complement to the Meta-didactical Transposition Model (MDT), one of the theoretical models forming the theoretical framework for this study (see section 3.1.3).

The key components of MKT are presented in Figure 2.1; in bold font are the sub-categories considered pertinent for guiding this study. Each of these components will be discussed in the following subsections.

SUBJECT MATTER KNOWLEDGE (SMK)	PEDAGOGICAL CONTENT KNOWLEDGE (PCK)
Common Content Knowledge (CCK)	***Knowledge of Content and Teaching (KCT)***
Specialised Content Knowledge (SCK)	***Knowledge of Content and Students (KCS)***
Knowledge at the mathematical horizon	*Knowledge of Curriculum (KC)*

Figure 2.1. Components of Mathematical Knowledge for Teaching (MKT) conceptualised by Hill et al. (2008).

2.2.1. Subject matter knowledge (SMK)

The importance of SMK for primary teachers to plan and teach mathematics effectively has been widely discussed by researchers (Askew et al., 1997; Hill & Ball, 2004; Thames & Ball, 2010;). Hill et al. (2008) identified three sub-categories of subject matter knowledge important for teaching: Common Content Knowledge (CCK), Specialised Content Knowledge (SCK) and Knowledge at the Mathematical Horizon (see Figure 2.1). CCK is described as knowledge someone who is good at mathematics may have, such as how to use a procedure to perform a calculation accurately or provide a definition for a basic mathematics term. SCK refers to knowledge unique to teachers, such as identifying common errors in student solutions or designing questions to reveal common misconceptions that can occur when students generalise solutions, for example, applying patterns they have noticed calculating with whole numbers to decimals. Knowledge at the Mathematical Horizon is interpreted to mean knowing ways that mathematics connects to topics in subsequent years. The latter two sub-categories were considered relevant for the purpose of this study and will be discussed in the following subsections, specifically, how they relate to teachers learning to use sophisticated instructional practices to teacher mental computation for conceptual understanding and fluency.

2.2.1.1. Specialised Content Knowledge (SCK)

The literature suggests that SCK is particularly important for the teaching of mental computation with conceptual understanding; there is an important distinction between the knowledge an adult requires to perform computation fluently and the knowledge required to support students' conceptual learning. In the context of teaching mental computation to middle primary students, being able to compute mentally with 2-digit numbers might involve a teacher identifying if a student: understands how to partition numbers in canonical and non-canonical form; can apply conceptual place value; comprehends numbers in terms of the multiplicative nature of the place value system, and applies properties of operations, such as associative and commutative (Heirdsfield, 2011). The Specialised Content Knowledge that a teacher requires is clearly distinct from the procedural knowledge required of an adult good at performing mental computations, and an essential foundation for teaching, if the aim is to develop students' conceptual understanding. This seems an important consideration for those responsible for designing professional learning experiences.

The need for teachers to have “connected mathematical knowledge” (Askew, 2008, p. 20) is echoed in various studies (Askew et al., 1997; Heirdsfield, 2011; Li, Ma, & Pang, 2008). Findings of a study on effective teaching of numeracy conducted in the UK, indicated a strong connection between teachers who could construct concept maps illustrating a range of connections with associated descriptions linking the topics, and average gains in student learning in numeracy over one year (Askew et al., 1997). Research conducted with Grade 3 teachers in Australia found that when teachers were introduced to a concept map showing how mental computation linked with number concepts; it had a significant impact on designing a series of connected lessons (Heirdsfield, 2011). The two studies aforementioned highlight the influence of connected mathematical knowledge on the planning and teaching of numeracy for conceptual understanding.

Teacher subject knowledge seems to influence teacher pedagogy. Goulding, Rowland and Barber (2002) suggested that if teachers are not confident in their subject knowledge of mathematics they avoid situations in which students may ask unplanned questions and find it difficult to plan and teach learning sequences. Such findings highlight potential challenges for teachers where learner-centred pedagogies involve class sharing and discussion of mental strategies. The findings also highlight the interconnected relationship between SMK and PCK.

This review of relevant literature suggests that for teachers to be successful in teaching mental computation with conceptual understanding, they require sound SCK as a basis to inform their planning and guide teaching practice.

2.2.1.2. Knowledge at the mathematical horizon

Knowledge at the mathematical horizon - knowing where current content will be used in future learning (Sullivan et al., 2013b) is important for teachers to plan learning trajectories. D. Clarke (2008) commented on the importance of teacher knowledge of big ideas or growth points in mathematics for effective planning and teaching. Growth points were developed as part of The Early Numeracy Research Project (ENRP) to provide teachers with a research-based framework to guide observations of student learning and inform planning. D. Clarke (2008) reported that the growth points helped develop teacher knowledge of typical learning trajectories and had a positive outcome on the teaching and learning of mathematics. In the context of teaching mental computation, knowledge of the progression for learning addition and subtraction strategies is important in planning a sequence of lessons. For example, it would be helpful if teachers are aware that students need to be secure in basic strategies such as doubles, commutativity, adding ten, tens facts, and other known facts, before learning derived strategies such as near doubles, adding 9, fact families and intuitive strategies. It can be assumed that teachers need some knowledge of how students typically learn to compute mentally to design a sequence of lessons to facilitate student learning (D. Clarke, 2008).

It has been suggested that a constructivist approach to teaching mental computation also requires teachers to have a sound knowledge of a typical learning trajectory to devise questions which progress student thinking (Fosnot & Dolk, 2001). The use of number strings is one approach to teaching mental computation strategies based on constructivist principles, and which is sequenced to promote student progress. A string is a structured series of related computation problems designed to highlight specific number relationships and operations (Fosnot & Dolk, 2001). An example of a number string is shown in Figure 2.2.

	Thinking behind the design of the number string
46 + 10	The first computation in bold is designed to activate prior knowledge to help students develop the strategy (students should be able to do this mentally with ease). It can be referred to as a 'helper' or support question (Fosnot & Dolk, 2001).
46 + 9	The intent is for students to look at the second computation and consider what is the same and what is different; how it is connected to the previous computation i.e. the second addend is one less.
64 + 20	This computation is a support or helper question. The computations that follow are linked and can be solved using the focus strategy – compensation.
64 + 19	Building on the previous computation, a student might reason that adding 19 is adding one less than 20 e.g. 64 + 20 - 1
36 + 19	The final computation is a challenge, designed to see whether students can apply the strategy developed through previous computations.

Figure 2.2. Example of a number string designed to elicit a compensation strategy for addition (Ontario Ministry of Education, 2006, p. 29).

Askew (2016) described the careful planning of a string of questions as "reasoning chains" (p. 61). Number strings are a structured series of computation problems designed to encourage students to look at the relationship between the numbers; to discern similarities and differences; and to guide them in developing a computation strategy. By focusing on number relationships, it is argued that big ideas are constructed and students develop automaticity with facts by thinking about relationships between facts and numbers, for example, thinking of 9 + 6 as the same as 10 + 5. Fosnot and Dolk (2001) commented that in order to design number strings, teachers need to be knowledgeable with a range of strategies and be able to think flexibly themselves. Having knowledge of the addition and subtraction growth points can support teachers in planning trajectories for mental computation. In summary, number strings offer an approach to teaching mental computation that supports teachers with learning sophisticated instructional practices (Kazemi, Franke, & Lampert, 2009; Lampert et al., 2010).

2.2.2. Pedagogical Content Knowledge (PCK)

A second major category of knowledge required by teachers is Pedagogical Content Knowledge (PCK). PCK involves knowing how to help others learn something you know. Rowland (2004) highlighted that the process of doing this is "enormously complex" (p. 12). Based on findings from a teaching experiment, Kinach (2002) argued that developing PCK is not a simple case of converting SMK to PCK if relational understanding is considered important. Kinach (2002) aimed to transform prospective teachers' instructional explanations to relational ones. Initially in the experiment, instrumental subject matter understanding was translated into instrumental pedagogy. Kinach (2002) argued that making a change in teacher practice involves making a "fundamental shift in their notion of what knowing mathematics entails" (p. 69).

The importance of developing PCK and the challenges it presents for teachers is considered in the following subsections. Two of the sub-categories of PCK identified by Hill et al. (2008) – Knowledge of Content and Teaching (KCT) and Knowledge of Content and Students (KCS) – are discussed in the context of teaching mental computation with conceptual understanding and fluency.

2.2.2.1. Knowledge of Content and Teaching (KCT)

KCT entails knowledge such as sequencing of content within lessons and across a series of lessons, and evaluating and using appropriate representations to explain concepts (Hill et al., 2008). Hill and Ball (2009) identified two aspects of KCT that have potential for improving student learning: support for teachers in representing student mathematical thinking visually; eliciting and interpreting student explanations. Each will be discussed in turn.

With regards to representation, effective teaching of mental computation, in which students share and discuss strategies, involves the teacher using models to display student strategies. Doing so provides a visual image to initiate and support class discussion. Selecting models to encourage thinking can present challenges for teachers (Thames & Ball, 2010). For example, using tools such as a hundred chart may support students in seeing patterns of ten and one but they may not instigate student thinking if the teacher simply provides instructions on how to use the tool without linking it to mental jottings for the calculation. Other models such as the empty number line may encourage students to think about landmarks on the number line, make mental leaps and visualise landing points (Fosnot & Uittenbogaard, 2007). Thames and Ball (2010) illustrated through a vignette the complexity of teaching a seemingly simple mental strategy for subtraction with 2-digit numbers that involves compensation. The vignette highlighted the challenges of teachers using a hundred chart as a tool, and the difficulty of aligning the language of direction with addition and subtraction. The students faced difficulties in describing subtraction as *going up a row on the hundred chart* as *up* is usually associated with *increase*. Thames and Ball (2010) also highlighted the importance of recognising key mathematical issues when analysing student explanations. For example, an explanation that describes the movement on the hundred chart independent of the mental strategy does not indicate sufficient mathematical reasoning. In this case an explanation as to why subtracting twenty, then adding one is equivalent to subtracting nineteen (as with the example number string in Figure 2.2) needs to be coordinated with use of the hundred chart.

Boaler (2016) discussed the importance of students explaining their solutions to others, stating that in mathematics sharing explanations is referred to as reasoning, which is central to the discipline of mathematics. She considered mathematics to be a social subject, with mathematicians proving theories through reasoning and producing arguments to convince other mathematicians of their theory. Although facilitating purposeful mathematics discussion – allowing opportunity for students to share and explain mental strategies used to solve a problem – may be considered one of the fundamentals for effective teaching and learning of mathematics (Boaler, 2016) it would seem this is no easy feat. Boerst et al. (2011) commented on the demands this places on teachers' knowledge as they are required to interpret student thinking, represent mathematical ideas visually for the case, and progress student thinking in the intended direction. In the next paragraph, I draw upon the research of Boerst et al. (2011) to elaborate on the aforementioned challenges and highlight reasons for the need to further gain insight into these issues.

Some of the challenges facilitating mathematical discussions present, is reported by Boerst et al. (2011) in their research on preparing prospective elementary teachers to lead discussions. Firstly, they suggested that the problem or task presented needs to be appropriate to sustain student reasoning through interaction between students and the teacher. Secondly, they suggested that students are required to participate through discussion and take up the ideas shared by others. Thirdly, the teacher needs to be an active participant, engaging in collective dialogue by responding to students and progressing their learning. Outlining these stages highlights the complexity of the process and the demands such discussions place on teachers' KCT.

Nonetheless in working with novice teachers, Lampert et al. (2010) found that using number strings in repeated guided rehearsals provided opportunity for teachers to develop skills in facilitating mathematical discussion. Further to this, Lambert, Imm and Williams (2017) suggested that adopting number strings as a practice, allows teachers to develop more sophisticated questioning and "encourage students to critique reasoning of others" (p. 54) – something Boaler (2016) posited as being at the core of learning mathematics. Therefore, gaining further insights into using number strings as an instructional practice with practising teachers could be of value in supporting purposeful class discussion.

Such a suggestion also seems pertinent in relation to challenges to KCT and teaching mental computation. Number strings provide a tool to investigate exposing students to alternative strategies; giving them opportunity to build on prior knowledge, and make connections between mathematical ideas.

2.2.2.2. Knowledge of Content and Students (KCS)

KCS entails knowing about students and how they learn mathematics. It involves anticipating students' cognitive and affective responses and interpreting students' strategies. For example, in thinking about how students may solve 76 + 9, it may be anticipated that some will add 10 and compensate by subtracting 1, some will adjust and add 75 + 10, some will count on and some will use a standard algorithm. Knowing how students typically approach a task and common misconceptions that may arise is considered important in helping teachers plan worthwhile learning experiences.

Bobis (2009) suggested that when teachers have knowledge of growth points in student learning, and can apply this to knowledge to designing their instruction, a significant difference is made to student learning. It can be assumed that the teacher would need to have specific assessment knowledge about each of their student's level of progress to make effective use of knowledge of growth points. Similarly, it can be assumed that challenging students through questioning, using relevant problem-solving tasks, and developing classroom discourse would require the teacher to have an in-depth knowledge of their students. Murphy (2004) emphasised the importance of teachers building on students' prior knowledge so that connections can be made. Murphy suggested that when learning mental strategies, the learner relies on knowledge of "connected use of number facts, number relations and number operations" (p. 15). Likewise, while emphasis has been placed on the value of mental number strings in developing efficiency in computation, the importance of students first exploring and constructing the big ideas underlying mental strategies through investigations and activities that make meaning has also been foregrounded (DiBrienza & Shevell, 1998; Fosnot & Dolk, 2001). This implies that teachers need to know how the ideas are connected but also to have sound knowledge of their students.

In this subsection the components of PCK interpreted as most relevant to this study have been discussed discretely, within the context of teaching and learning mental computation. However, it is important to acknowledge a potential drawback with reviewing the literature in this way. Wongsopawiro, Zwart and van Driel (2017) posited that these components are "mutually related" (p. 192). They argued the better understanding teachers have of how students learn to compute mentally and the potential misconceptions, the greater teachers' knowledge of instructional strategies such as representations, and the more effective the teaching. Although the literature has highlighted the importance of developing strong PCK to support changes in practice, more needs to be known about how new knowledge is transferred to classroom practice; the change processes involved (Covay Minor et al., 2016).

2.2.3. The challenge of transforming knowledge into practice

Although it is recognised that developing MKT is important in relation to improving instructional practice, the translation of this new knowledge into classroom practice is a complex process; simply, focusing on developing a particular aspect of knowledge does not automatically translate to improved practice (Covay Minor et al., 2016). Instead, it is suggested that various support mechanisms are required in combination with a focus on different aspects of knowledge. Desimone and Hill (2017) advocated that critical to the success of an intervention is "the balance of research-based approaches, PD[2] that included both content and pedagogy, and implementation that provided aligned lesson guidance while still allowing for teacher creativity and invention." (p. 529).

In this section, I have drawn upon relevant literature in relation to each of the components comprising MKT (Hill et al., 2008) to highlight what is known about teacher knowledge in supporting change in practice; challenges of using sophisticated instructional practices to teach for conceptual understanding; and possible options for building on current research to improve teaching and learning. Next, I review the literature on teacher dispositions; how this relates to the design and implementation of professional learning experiences.

2.3. Teacher disposition

In this section, the literature on disposition in relation to teacher professional learning will be reviewed. First, it is important to clarify the meaning of the term 'disposition' for this study. Much of the literature has associated disposition with being either positive or negative. However, Cooke (2015) drew upon the definition of the Australian Association of Mathematics Teachers (AAMT) (1997) and the work of Wilkins (2000), to consider disposition as a "continuum or a

[2] The authors used the abbreviation PD for Professional Development.

measure" (p. 2) to describe willingness to engage with and use mathematics. This echoes the views of Ritchhart (2002) who associated disposition with influencing whether a person chooses to use and apply their knowledge and skills in mathematics. Similarly, in their review of studies related to the professional learning of practising teachers of mathematics, Goldsmith et al. (2014) noted that the term disposition was often used to refer to teachers' "propensity to act upon knowledge and beliefs" (p. 10).

Cooke (2015) conceptualised disposition as including four elements that can be considered measurable: attitudes, anxiety, confidence, and how mathematics is conceptualised. She considered attitudes as encompassing "enjoyment, interest and enthusiasm for mathematics" (p. 5), as well as the value teachers attach to mathematics. In relation to how mathematics is conceptualised, Ernest (1989) considered this to include the extent to which mathematics is perceived as a revisable, problem solving subject, a "static interconnecting set of truths," or a "collection of unrelated facts and skills" (p. 5). Cooke's (2015) conceptualisation of mathematics focused on the understanding required, which reflects the three philosophies articulated by Ernest (1989). The interpretation I have adopted for this study is based on the work of Cooke (2015) but incorporates the ideas of Atallah, Bryant and Dada (2010). They associated conceptualisation of mathematics with actions, such as types of classroom activities, purpose of learning, and thoughts on understanding of mathematics. For this study, I considered teachers' conceptualisation to include aspects described by both Atallah et al. (2010) and Ernest (1989).

Disposition towards mathematics is considered important in the sense that it influences the actions teachers take within their classrooms (Katz & Raths, 1985). The importance of disposition having a direct impact on student learning is emphasised by Cooke (2015), who explained that it influences “how the teacher approaches mathematics; how the teacher sees the development of numeracy; and the classroom climate when mathematics is being used” (p. 3). This is substantiated by Boaler (2014), who posited that teachers’ attitudes to mathematics need to change if it is to become a ‘learning subject’. She advocated the importance of teachers changing their attitudes to learning mathematics and valuing mistakes as part of the learning process to help promote changes in students’ attitudes and learning. Essentially the work of Boaler (2014) is connected to Dweck’s (2000) research on mindsets, although a different concept, it seems worthy of attention and is discussed next.

The notion of growth mindset is based on the belief that intelligence and performance can be improved with hard work. In contrast, a fixed mindset is based on the belief that learning can happen, but the basic level of intelligence of a person does not change (Dweck, 2006). “Mindsets are critically important because research has shown that they lead to different learning behaviours, which in turn create different learning outcomes for students” (Boaler, 2016, p. ix). Building on the work of Dweck (2000), Boaler focused her research to look at innovative teaching that promotes a growth mindset in learning and teaching mathematics. It seems pertinent to consider growth mindsets in relation to this study, when it has been suggested that a focus on developing number sense through learning to compute mentally, may support the development of growth mindsets in both teachers and students.

In relation to professional learning and disposition, Goldsmith et al. (2014) reviewed 106 articles focused on the experiences of practising teachers in mathematics; 36 of the studies indicated an impact of professional learning on either beliefs or dispositions. Three main areas of focus were identified: "an inquiry stance toward teaching, attitudes towards colleagues, and sense of efficacy" (p. 12). The changes related to collegial attitudes included, "increased encouragement and support from colleagues that enabled teachers to try new types of teacher; recognition of colleagues as a source of useful feedback and knowledge; and a strengthened sense of accountability to colleagues" (p. 12-13). The studies that reported on the impact of professional learning on teachers' sense of efficacy or confidence in relation to practice, i.e. that changing their instructional practice can positively influence student learning, were connected to a variety of aspects of professional learning. These included study groups, coaching, collaborative study of teaching and learning of mathematics, and lesson study.

It is important to consider these findings to inform the design of professional learning experiences that enhance opportunities for collaborative experiences and consider components that have a positive influence on teachers' efficacy. In the section that follows, I review the literature on teacher professional learning – what is already known about teacher professional learning – and consider what could be explored further, to enhance opportunities for learning and change in practice.

2.4. Teacher professional learning

In this section, I review the literature on: different perspectives on professional learning; evolving models of professional learning; what is known about learning of practising teacher; the core features of effective professional learning and challenges in relation to variation in teachers learning both within a context and across contexts. Although I have purposively adopted 'professional learning' for this study (see section 1.1.1.), the term 'professional development' appears throughout this section. This reflects the language of earlier literature in which 'development' tended to refer to in-service programs with more of a 'one-size-fits-all' approach (Clarke & Hollingsworth, 1994).

2.4.1. Perspectives on professional learning

The perspective on professional learning adopted for this study is of "change as growth or learning" (Clarke & Hollingsworth, 2002, p. 948). In taking this stance, the design and implementation of professional learning for teachers is thus influenced by general theories of learning. There are two main perspectives on professional learning: a cognitive perspective which focuses on individual development of knowledge (Hill et al., 2008; Shulman, 1986) and a participatory or situated perspective which focuses on ways participants and practices change through social interaction (Lave, 1996). These two perspectives, or metaphors for learning, are conceptualised by Sfard (1998) as acquisition (gaining knowledge) or participation (learning by doing).

Studies on teacher professional learning usually focus on either situated or acquisition metaphors for learning but can often include elements from both. Kazemi and Franke (2004) described their study on the use of student learning to develop teachers' understanding of student mathematical thinking, as an approach to professional learning that is situated in practice. Their study involved a workgroup of ten elementary teachers who met regularly across the academic year to analyse and discuss student work samples. The teachers planned common word problems for each class to allow the workgroup to focus on shared meaning; the teachers selected student work to share with the group. The findings of the study showed shifts in teacher participation as a result of teacher collective inquiry into student learning. However, the study also described the development of individual teachers' knowledge about student thinking and learning (KCS) and their own mathematical thinking about mental strategies, thus indicating elements of the acquisition metaphor for learning. Putnam and Borko (2000) also framed their research using a situated perspective. They described three components of their research: members of the research team working alongside participating teachers in classrooms, teachers sharing learning experiences from workshops as staff development activities, and workshops that focus on developing subject matter knowledge. Although framed in a situated perspective the research included individual learning of concepts and development of teacher knowledge.

Some studies are framed in an acquisition perspective but include elements of the participatory metaphor for learning. For example, Zwiep and Benken (2013) researched upper elementary and middle grades teachers' learning of mathematics and science content. Their study focused on changes in content knowledge (building on and providing coherence on teachers' existing knowledge), and was framed in the acquisition paradigm. However, the context for learning involved teachers participating in workshops with colleagues.

Sfard (1998) argued that there are advantages to both the acquisition metaphor and participation metaphor and that the two perspectives are complementary. She posited that "the most powerful research is the one that stands on more than one metaphorical leg" (p. 11). In the next subsection, I briefly discuss how models of professional learning have evolved and the complexity of studying teacher change.

2.4.2. Models of professional learning

Models of professional learning or growth have evolved over the last three decades. Earlier models, such as that developed by Guskey (1986), indicated teacher change as a linear process. His model placed emphasis on teachers experiencing success in the classroom (in relation to students' learning outcomes) before changes in teachers' attitudes.

Desimone (2009, 2011) argued that a conceptual framework is needed to improve the quality of professional development. She drew upon empirical data to identify five core features of effective professional development considered to have a positive impact on improving teaching practice. Her conceptual framework represents "interactive relationships among the core features of professional development, teacher knowledge and beliefs, classroom practice, and student outcomes" (Desimone, 2011, p. 70). The framework allows for evaluation of three outcomes: teacher learning, teacher change in practice, and student outcomes/achievement. Although her conceptual framework is comprised of interactive components that seem universal with other commonly cited models of professional learning, the model is not cyclical and does not appear to allow for multiple pathways that reflect the idiosyncratic nature of teacher professional learning.

2.4.3. Learning about professional learning of practising teachers

With a focus on establishing what is known about professional learning of practising teachers, Goldsmith et al. (2014) conducted a synthesis of the literature (1985-2008). They defined learning as including changes in knowledge, practices, dispositions or beliefs in such a way to influence teacher knowledge or classroom practice. Their review suggested three main findings: learning tends to occur incrementally and iteratively; the impact of an intervention varies across individuals and contexts; existing research on professional learning in mathematics tends to focus on program effectiveness rather than on teachers' learning. In essence, their synthesis of the literature echoed the findings of Clarke and Hollingsworth (2002). Although the findings did not present substantially new information, they were noteworthy in the sense that they were that they were extracted from a much larger review of literature. Each of the main findings will be discussed further in the following subsections.

2.4.3.1. Learning as iterative and incremental

Through analysing studies on teacher learning specifically, Goldsmith et al. (2014) suggested that teacher learning happens through iterative cycles of experiences both inside and outside the classroom and includes learning from colleagues. These learning experiences are incremental and interconnected in the sense that any small changes in knowledge, beliefs, and dispositions are linked to changes in others. For example, "if a teacher made changes in practice to focus on eliciting student thinking this would lead to deeper understanding of student learning, leading to a need to improve their own subject matter knowledge" (Goldsmith et al., 2014, p. 20). Of particular importance, was further evidence to substantiate a shift away from teacher learning being seen as a linear path from a professional learning experience (Guskey, 2002).

2.4.3.2. Variation in learning across individuals and contexts

There appears a general consensus in the literature that the same professional learning experience can have a different impact on individual teachers within the same institution, and that contextual circumstances also influence opportunities for teachers to learn (Borko, 2004; Fishman, Marx, Best, & Tal, 2003; Goldsmith et al., 2014; Timperley et al., 2007; Wongsopawiro et al., 2017). Desimone (2009) emphasised context as an important moderator and mediator. She suggested four elements as comprising the context: student characteristics e.g., achievement and disadvantage; individual teacher characteristics e.g., experience, knowledge, beliefs and attitudes; contextual factors in the classroom, school, and district; and policy conditions at multiple levels. In addition, she argued for the need of such a model to "identify the variables that mediate (explain) and moderate (interact to influence) a professional development's effects." (p. 184).

The importance of the school context on opportunities for teacher learning is emphasised by Loucks-Horsley et al. (2010), who commented on the need for the school culture to have the capacity to be a collaborative learning community for effective professional learning experiences to be designed and implemented. Various studies have highlighted the influence of contextual factors on teacher learning; designers of professional learning have emphasised the importance certain contextual conditions being in place i.e. school culture to support professional learning. This suggests that further investigation into how professional learning programs work within particular contexts is needed (Arzarello et al., 2014; Goldsmith et al., 2014; Hiebert & Morris, 2012).

2.4.3.3. A focus on program effectiveness

Until recently much of the research has focused on documenting effectiveness of professional development programs or curricula, with changes in teachers' knowledge, disposition or practice treated as indicators of success (Desimone, 2009; Goldsmith et al., 2014; Guskey, 2000). In their synthesis of the literature on learning of practising teachers, Goldsmith et al. (2014) commented that few studies have looked at the processes of change; *how* teachers develop aspects of their knowledge, disposition, beliefs and practice. The need for further studies to focus on learning processes is highlighted in the following excerpt:

> Relatively little of the current research literature focuses primarily on understanding teachers' learning; instead, teachers' learning is often treated as a black box, with the main research focus on whether or not a program has an impact on practice and student learning. (Goldsmith et al., 2014, p. 25)

2.4.4. Core features of effective professional learning

There has been much discussion about the core features of professional learning experiences that have a significant impact on knowledge, skills and changes in classroom practice. Desimone (2009) argued that empirical research "reflects a consensus on at least some of the characteristics of professional development that are critical to increasing teaching knowledge and skills and improving their practice, and which hold promise for increasing student achievement" (p. 183). The features of effective professional development she identified include: content focus, active learning, collective participation, coherence, duration and collective participation. With the exception of content focus, which has been addressed in the review of the literature on developing MKT, each of the features identified by Desimone (2009) form the focus of the following subsections.

2.4.4.1. Active learning

Active learning is recognised as occurring in many forms such as, "observing expert teachers or being observed, followed by interactive feedback and discussion; reviewing student work in the topic areas being covered; and leading discussions" (Desimone, 2009, p. 184). Borko (2004) described active learning as engaging teachers as learners. The importance of opportunities for active learning has been emphasised in various studies. Guskey and Yoon (2009) analysed findings from over 1 300 studies in a synthesis of research on effective professional development. They found the use of workshops that "focused on the implementation of research-based instructional practices, involving active-learning experiences for participants, and involved teachers with opportunities to adapt the practices to their unique classroom situations" (p. 496), had a positive impact on student learning outcomes. In a study focused on developing knowledge of science teachers (specifically use of models and modelling), Justi and van Driel (2006) concluded that aspects of the design of the professional learning program they considered most influential on teacher learning were because they supported teachers in becoming "active meaningful learners" (p. 448). The professional learning program included explicitly connecting reflection on previous practice with new practice and high levels of interaction between teachers and researchers throughout the intervention.

Arguably, instructional coaching supports the notion of active learning because it "typically involves face-to-face, one-on-one interactions" that may occur inside or outside the classroom (Desimone & Pak, 2017, p. 6). Many studies report on positive outcomes of instructional coaching in the field of mathematics; it provides opportunities for the teacher to engage with both the subject content and pedagogical approaches (Desimone & Pak, 2017; Sun, Wilhelm, Larson, & Frank, 2014). However, it also seems that there are various caveats in relation to the success of coaches in developing teacher practice. A recent longitudinal study concerning middle school teachers across four large urban districts, indicated considerable variation in school expectations for coaches; in some cases, coaches were required to complete administrative tasks and attend, rather than facilitate collaborative meetings (Cobb et al., 2018). In studies where coaches were given more managerial tasks e.g., data collection, teacher evaluation, or focused on developing general classroom management skills, it was reported they spent less than 50% of their time engaging in content driven professional development (Desimone & Pak, 2017). Variables within the school context in which coaches work, specifically, weak instructional leaders and collaborative culture, have been found to constrain opportunities for improving teaching practice (Hopkins, Ozimek, & Sweet, 2017).

2.4.4.2. Collective participation

The importance of professional learning communities, or some form of collective participation, has been emphasised as affording opportunities for teacher learning (Borko, 2004; Desimone, 2009; Desimone & Hill, 2017; Timperley et al., 2007). Although Timperley et al. (2007) argued that participation in a learning community was imperative, they also emphasised that collaboration per se, does not support change in teachers' learning. They posited that collaborative experiences needed to include opportunities for teachers to build their knowledge or have their thinking about mathematics and teaching challenged. This is substantiated by findings from other studies. For example, Guskey and Yoon (2009) highlighted the inclusion of outside experts, in the form of researchers or program authors presenting ideas, as having a positive impact on student learning outcomes. In a science study focused on the learning of science teachers, Wongsopawiro et al. (2017) found that university staff (facilitators) played a significant role in developing teachers' knowledge. Borko (2004) also emphasised the key influence of the quality of program facilitators on teacher learning.

It has been suggested that collaboration among colleagues in which teachers explain their practices and discuss student learning (scrutinise schoolwork) can influence teacher beliefs about learning (Kazemi & Franke, 2004). Opportunities for professional conversations have been recognised as important in terms of building knowledge of student thinking and pedagogy (more readily than conceptual and connected mathematical knowledge), and also in providing teachers with encouragement and support to experiment with new ideas in their classrooms (Goldsmith et al., 2014).

Loucks-Horsley et al. (2010) emphasised the importance of professional communities in affording opportunities for teachers to engage in "challenging discourse" (p. 145) considered necessary to create cognitive dissonance and instigate learning. They commented that,

> … challenging discourse is not very common among teachers. Often, teachers equate critical reflection on practice with criticism of personal performance. Building professional cultures, however, by the very definition of the word *professional,* carries with it a commitment to effective practice in oneself and in others who share the profession. (Loucks-Horsley et al., 2010, p. 145-46)

In essence, Loucks-Horsley et al. (2010) posited that establishing a professional community within a school provides opportunities for dialogue that can both create and resolve cognitive dissonance necessary for change to occur.

2.4.4.3. Coherence

In an analysis of studies on professional learning in mathematics, in which the focus was on improving SMK and/or PCK, Timperley et al. (2007) highlighted the importance of aligning policy, research and practice. Coherence between all three aspects was required for a positive impact on teacher and student learning.

Some studies have pointed to a lack of coherence among new curricula materials, state initiatives and professional learning programs as having a negative impact on opportunities for teachers to change practice (Goldsmith et al., 2014). However, in circumstances where an intervention has been directly and clearly linked to the curriculum, the ideas are "much more likely to be adopted" by teachers (Desimone & Hill, 2017, p. 528). The importance of coherence with other school initiatives has also been emphasised; it is suggested that school leaders need to be actively involved in the design of professional learning programs to ensure coherence with other school programs so that adequate time and priority is given (Loucks-Horsley et al., 2010).

2.4.4.4. Duration

There has been some debate on the duration of professional learning (in terms of the number of days, hours or school terms) required to have a positive impact on changes in practice and ultimately student learning. It has been indicated that activities that typically span a semester and include 20 hours or more of contact time, are advantageous (Desimone, 2009). However, a study conducted by Yoon, Duncan, Lee, Scarloss and Shapely, as cited in Desimone and Pak (2017), found that in classes where teachers had undergone as least 14 hours of engagement with professional learning, the impact on student achievement was significant. Although Timperley et al. (2007) recognised that generally adequate duration for professional learning is associated with a positive impact on learning (teacher learning and student learning), they raised the contention that the quality of the learning experiences needs to be considered:

> While an extended time frame with frequent ongoing opportunities to learn does seem to be generally associated with professional development that results in positive outcomes for learners, it is not in itself a guarantee of success…What matters is what occurs within the time. (Timperley et al., 2007, p. 75)

To conclude this subsection on core features of effective professional learning, a more recent review of studies by Cobb, Jackson and Dunlap (2015) reported that common to studies indicated as effective for practising teachers, was a focus on issues central to instruction; the inclusion of instructional materials for teacher use in classrooms; and sustained collaboration of the same group of teachers over time. Their findings essentially substantiated, yet also extended earlier conclusions of Desimone (2009) by suggesting that the inclusion of instructional materials and opportunity to continue support with a group of teachers was critical.

2.4.5. Challenge of changing teaching practice

The literature suggests that the process of changing teaching practice can be challenging; personal and contextual factors interact with professional learning experience to influence change in classroom practice (Covay Minor et al., 2016; Hiebert, 2013; Sullivan et al., 2015).

A focus on *how* teacher learning is instigated and supported seems important. Fosnot and Dolk (2001) posited that to successfully change mathematics teaching, teachers need "experiences that involve action, reflection, and conversation with the context of teaching and learning. They need to construct new beliefs, a new vision of what it means to teach and learn mathematics" (p. 173). This stance is supported by Loucks-Horsley et al. (2010), who highlighted the need for a change in approach of those designing and facilitating professional learning experiences, and posited that such experiences should:

> guide teachers to construct knowledge in the same ways as do effective learning experiences for students. Yet it is surprising to note how often the principle of constructivism is conveyed to teachers in the context of how they should help their students to learn, without it being the basis for how they learn themselves i.e. too many lectures (p. 76).

Loucks-Horsley et al. (2010) essentially argued that there are still too many lectures on how to teach, as opposed to experiences in, the principles of constructivism in professional learning programs.

Through the lens of a social constructivist, my contention is that if teaching practice in mathematics is to be transformed, it seems that opportunities for teachers to be active agents of their own learning need to be enhanced; they need to experiment with new pedagogical in the classroom if their dispositions and practices are to change. Experiencing new pedagogy as an active learner may provide a catalyst to change in teaching practice.

2.5. Research on teaching mental computation

In this section, I discuss challenges reported in the literature in relation to the teaching of mental computation and at the same time highlight how this provides an apposite context for teachers to develop sophisticated instructional practices.

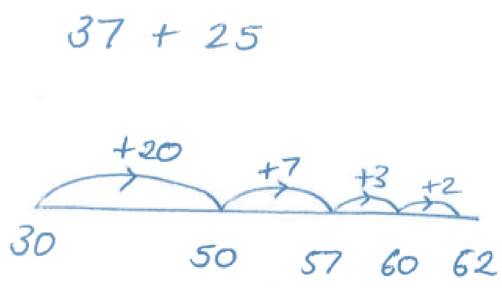
2.5.1. Approaches to teaching mental computation

The literature suggests that direct teaching of holistic mental computation strategies has presented challenges for educators (Caney, 2004; Murphy, 2004; Threlfall, 2002, 2008). The direct teaching of holistic strategies e.g., compensation and near doubles, has proven problematic in the sense that when students are presented with a problem without instruction, their solutions are often not easily aligned to holistic strategies. In addition, this approach has resulted in a focus on choosing a strategy for a calculation, from both a teaching and learning perspective, rather than analysing and thinking about the choice of numbers in a question (Threlfall, 2002). Murphy (2004) emphasised the importance of student prior knowledge and the need for students to have a connected view of mathematics to learn mental computation. She posited that although holistic deductive strategies can be taught to the class, student success depends on prior knowledge to make connections between the mathematical ideas and the strategies. Threlfall (2008) reiterated the views of Murphy (2004) with regards to the importance of student prior knowledge determining strategies used by students and suggests that the focus should be on shaping teaching to "*develop* strategies rather than to acquire them" (Threlfall, 2008, p. 87). Alternative approaches to direct teaching of holistic strategies, which emphasise relational thinking, and the importance of using number knowledge and arithmetical reasoning to compute mentally, are considered to support the learning of mental computation with conceptual understanding (Murphy, 2004; Threlfall, 2008; Wright, Ellemor-Collins & Tabor, 2012). Such approaches would entail using tools such as number talks (Boaler, 2016) and number strings (Askew, 2016; DiBrienza & Shevell, 1998; Fosnot & Dolk, 2001; Lambert et al., 2017).

2.5.2. Categorisation and organisational framework for strategies

One of the challenges teaching mental computation may present for teachers concerns the categorisation of strategies and knowing which strategies to focus on in their teaching. This can partly be attributed to confusion between materials created for analysing student learning and resources created for *teaching* mental strategies. The intention of much of the work of Wright et al. (2012) and McIntosh (2005) was to equip teachers with tools to analyse and assess student learning and their development of strategies. For example, Wright et al. (2012) explained a strategy referred to as split-jump for adding and subtracting with 2-digit numbers. Split-jump describes a situation in which students separate off the 'tens' of both numbers (referred to as splitting numbers based on place value), then use jump to append the 'ones' sequentially (see example in Figure 2.3.). The identification of such a strategy is to support teachers with analysing and supporting student learning, rather than a strategy for teachers to direct students' attention to in their class teaching.

Informal notation for split-jump strategy



An empty number line to indicate that the tens in both numbers are added first; the ones are added sequentially (referred to as jumping). In this example the student makes use of a 'landmark' or 'friendly number' to make adding the ones easier.

Possible student description of the strategy

30 and 20 is 50. Then the 7 makes 57. Then to add the 5 I went to 60 and then to 62.

Figure 2.3. Example of the split-jump strategy for addition with 2-digit numbers (Wright et al., 2012, p. 100).

Although the literature indicates a consensus on the classification of mental computation strategies, there is some disparity in relation to the naming of strategies. The range of strategies and the concepts underpinning the strategies (see Appendix D) may also present challenges for teachers. The quite recent proposal of an organisational framework to support the teaching of mental computation – Strategies, Understanding, Reading, and Fast Facts framework (SURF) – trialled by a small number of schools in Victoria, Australia, suggested this may be the case (Russo, 2015). The inception of the SURF framework suggested a need for further refinement of the teaching of mental computation strategies to deepen student understanding and develop flexibility in student thinking.

2.6. Summary

The literature was reviewed through the lens of a social constructivist. In the context of learning to compute mentally, this was interpreted as students needing to engage actively in solving problems, explain strategies and converse with others to build new knowledge. A review of the literature highlighted some of the critical issues regarding the influence of mathematical knowledge for teaching, in particular SCK, KCT and KCS, on planning and teaching of computation for conceptual understanding. It seems crucial that teachers have knowledge of the big ideas or growth points concerning mental computation and also the connections between these ideas, to adopt an approach to teaching with conceptual understanding. Decisions about pedagogy seem to be influenced by teacher subject matter knowledge. Supporting teachers with suggestions on instructional practices such as the use of number strings to teach mental computation are likely to provide an effective structure for developing both their subject matter knowledge and their pedagogical content knowledge. Certainly, more needs to be known about how new knowledge is transferred to classroom practice, and the change processes involved. The literature emphasised the influence of teacher disposition and the promotion of a growth mindset on student learning; the importance of providing collaborative learning experiences to enhance efficacy and confidence in teachers was also evident. While there seems general agreement about the key features of effective professional learning, there is much to learn about how personal factors, knowledge and disposition, interact with contextual factors and professional learning experiences, to influence a change in practice. The synthesis of knowledge on teacher professional learning discussed in this chapter was used to inform the research design for this study; this is presented in the next chapter.

3. Research Design

Real change can come when we focus not only on what and how things can be done, but when we also work to understand why. (McKenney & Reeves, 2012, p. 1–2)

The aim of this study was to investigate the processes involved in teachers learning sophisticated instructional practices through onsite professional learning experiences. My intent was to gain insights into how teachers learn and the support mechanisms that foster learning and change, within the different institutional contexts in which the teachers worked.

In this chapter, I explain how design-based research methodology informed the design of the professional learning. I discuss my perspectives on learning, my epistemological assumptions, and the theoretical framework underpinning the study. Participant selection, including ethical considerations, choice of methods of data collection and the analysis process are explained. Issues related to the quality of the research design are addressed; the scope of the study and reflexivity are also considered.

3.1. Theoretical framework

In this section, the theoretical framework underpinning the research is explained. This study connects two theoretical models: Interconnected Model of Professional Growth (IMPG) developed by Clarke and Hollingsworth (2002) and the Meta-Didactical Transposition model (MDT) developed by Italian researchers (Arzarello et al., 2014), to investigate processes of change stimulated by professional learning. First, I discuss the ontological and epistemological assumptions upon which I have based this study.

3.1.1. Theoretical perspective on teacher and student learning

The theoretical perspective on learning assumed for this study is broadly referred to as social constructivism, a version in which the focus is on "how our experience of some particular object or idea" is socially constructed (Schwandt, 2007, p. 40). This view rejects that idea that every aspect of the world is socially constructed, and contrasts that of a strong social constructivism, which seem to deny any ontology of the real (Schwandt, 2007). From an ontological position, this implies that any insights gained in relation to teaching practice would emerge and change through social interactions observed between teachers, between the teacher and students, and between the teachers and myself, as the researcher.

In adopting a social constructivist perspective, I aimed to interpret other people's understanding of the world; this is often referred to as interpretivism (Creswell, 2013). From an epistemological position, this involved interpreting knowledge as being constructed through individual experiences and through interactions with others (Creswell, 2013). In researching changes to teaching practice, this meant I viewed the changes through the individual experiences of the research participants as I co-constructed understandings through interactions with them. My intent was to describe the different perspectives of the participants to gain insights into links among aspects of teacher knowledge, teacher disposition, salient outcomes and teaching practice, as they learned sophisticated instructional practices. It is recognised that these socially negotiated meanings are subjective, they were formed through interaction with others and within the historical and cultural norms of the teachers within the school context (Creswell, 2013). Drawing upon two decades of experience as a primary school practitioner, and my experiences with teachers in an advisory capacity, my aim was to interpret the data reflexively in my role as researcher.

In the following subsection, the two theoretical models that form the theoretical framework for this study are explained. The models were adopted to investigate changes to teaching stimulated by professional learning experiences.

3.1.2. Interconnected Model of Professional Growth (IMPG)

One of the theoretical models informing the research design for this study was the Interconnected Model of Professional Growth (IMPG) developed by Clarke and Hollingsworth (2002). The model is empirically grounded on studies of professional learning in mathematics. It is based on the earlier work of Clarke (1988), who incorporated elements of Guskey's (1986) linear model for teacher development, into a cyclical model (Clarke & Hollingsworth, 2002). The IMPG allows for a focus on analysis of development of teacher knowledge and changes in teacher practice in response to an external stimulus. The model was adopted for this study to provide insights into mechanisms for change that might support the teachers learning sophisticated instructional practices to teach mental computation. The components of the IMPG are displayed in Figure 3.1; the focus of the remainder of this subsection is on explaining this model.

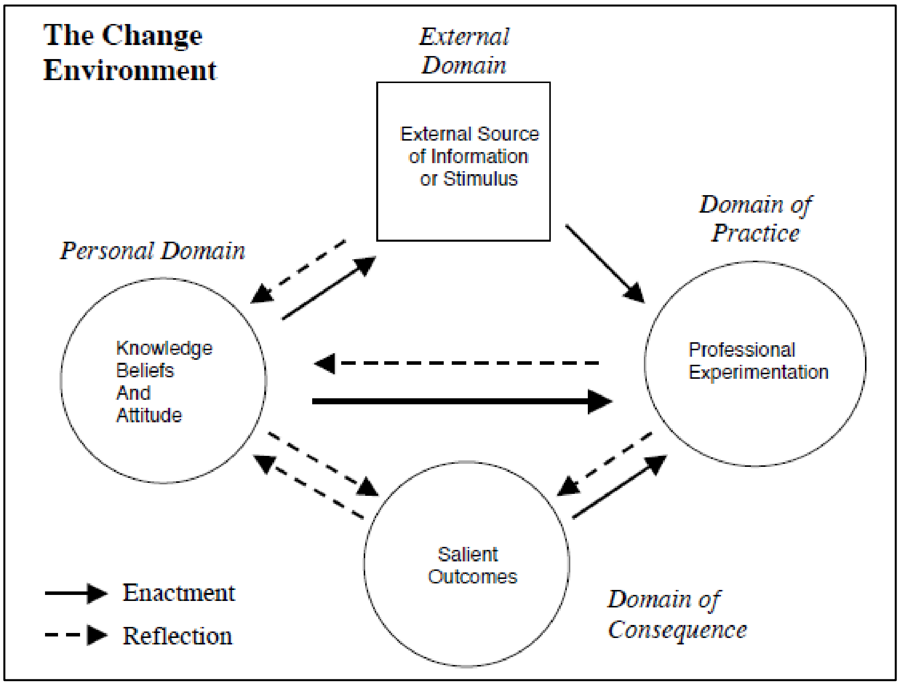


Figure 3.1. The Interconnected Model of Professional Growth (IMPG) (Clarke & Hollingsworth, 2002, p. 951).

The IMPG shown in Figure 3.1 is comprised of four change domains: the Personal Domain (teacher knowledge, beliefs and attitudes), the Domain of Practice (professional experimentation), the Domain of Consequence (outcomes salient to the individual teacher) and the External Domain (sources of professional learning). For the purpose of this study, in relation to the Personal Domain, the conceptualisation of the two main components of Mathematical Knowledge for Teaching (MKT) (Hill et al., 2008) were adopted and a focus on aspects of teacher disposition towards teaching and learning mathematics (Cooke, 2015). The four domains constitute the personal and professional world of the teacher and are situated in the Change Environment, which is the context in which teachers work. The visual representation of the IMPG distinguishes the External Domain from the three other domains by depicting it as a square rather than a circle. For the purpose of this research, it is important to clarify the interpretation of the model through a social constructivist lens. The intent of the visual representation of the model is to covey the concept of the External Domain as “distinguished from the other domains by its location outside the teacher’s personal world” (Clarke & Hollingsworth, 2002, p. 951). However, from a social constructivist perspective this is an inadequate explanation for a far more complex situation. Rather, a bi-directional or reflexive interaction is interpreted to occur between teachers’ knowledge and the institutional context in which they are situated. Thus, reflecting the changing nature of pedagogical community of teachers as it interacts with contextual factors.

When change occurs in one domain, the mediating processes of reflection (represented by the dotted arrow in the model) and enactment (represented by the bold arrow in the model) connect the various change domains. In this model reflection is considered “as active, persistent and careful consideration” (Clarke & Hollingsworth, 2002, p. 953) and is associated with a change in cognition. Enactment is distinct from acting, in that it involves cognitive processes and displays a change in behaviour (Zwart, Wubbels, Bergen, & Bolhuis 2007). The mediating processes of enactment and reflection essentially conceptualise the process of change. A change sequence is said to occur when a change in one domain can be connected to another domain (Clarke & Hollingsworth, 2002). The model does not consider all change to be long term; it may be a case of experimentation. In situations where data indicate a change sequence is associated with long term change it is referred to as a growth network (Clarke & Hollingsworth, 2002, p. 958). In Table 3.1 explanations of how the four change domains provided a framework to investigate possible pathways for development of teacher knowledge

Table 3.1. *Using change domains to frame the professional learning program*

Change domain	Details of the professional learning program
External Domain	Teachers were given opportunities to learn through a variety of external stimuli. The professional learning program included: ▪ professional learning session facilitated by the researcher to explain and discuss the intervention and explore the instructional tools that comprise the intervention; ▪ provision of professional reading on instructional tools: number strings and number talks; ▪ modelling of a learning sequence (three lessons) by the researcher; ▪ provision of a teacher resource book containing outlines for learning sequences on key mental strategies for addition and subtraction (based on research literature) and underlying theoretical principles of the study; ▪ student assessment task and assessment interview both pre- and post-intervention; ▪ iterative discussions with other teacher participants and the researcher to reflect on student learning and collaboratively plan mental computation lessons.
Personal Domain	▪ Participants were asked to reflect on their knowledge and disposition in relation to teaching and learning mathematics in a pre-intervention survey and post-intervention in individual semi-structured interviews. ▪ Lesson observations were conducted pre- and post- intervention to interpret changes in aspects of the Personal Domain. ▪ A researcher's journal was kept to record informal post-teaching reflections shared with the researcher and interpretations of discussions at planning meetings.
Domain of Practice	Enactment of new teacher knowledge through iterative classroom experimentation was captured through: ▪ lesson reflections with the researcher (recorded in a researcher's journal); ▪ discussions at planning meetings (recorded in a researcher's journal); ▪ lesson observations conducted pre- and post-intervention; ▪ post-intervention survey.
Domain of Consequence	▪ Individual semi-structured interviews with participants and post-intervention surveys provided opportunity for the participants to reflect on their own learning and the learning of their students and identify outcomes they considered salient.

and practice, specifically in relation to learning sophisticated practices to teach mental computation, are displayed. For this study, criteria were used to examine relationships between changes in knowledge, disposition and practice and the domains of IMPG, to guide my interpretation of the change sequences for each participant. The criteria used were modified from the work of Justi and van Driel (2006) (see, Table 3.9, in the section on data analysis for further details of the criteria).

The Clarke and Hollingsworth (2002) model recognises multiple pathways and the individuality of teacher growth in response to an external stimulus. The design of the professional learning program in this study aimed to provide teachers with various opportunities for learning. In addition, IMPG pays attention to the importance of individual teacher's interpretations of change, which can sometimes differ from the researcher's observations. For this study, sources of data related to the Personal Domain that focused on teachers' own interpretations of change and perceptions of professional learning included individual semi-structured interviews, post-professional learning surveys and post-intervention surveys. Data on changes I perceived as the researcher, included observations of teaching, recordings of informal post-teaching reflections and discussions at planning meetings.

The IMPG supports either a cognitive or a situative perspective on learning; this resonates with my perspective that theories of learning do not need to be a dichotomous choice (Clarke & Hollingsworth, 2002). The model allows analysis to focus on teachers' individual development of knowledge. In this study teachers were provided with sources of professional reading and asked to select and share examples of student learning at planning meetings. The IMPG also allows for a focus on learning through practice and interactions with others, in other words a situated perspective on learning. The model reflects my personal stance on learning, that knowledge is constructed through individual experiences and through interactions with others.

Various studies on teacher learning and professional growth have adopted the IMPG (e.g., Chan, Roche, Clarke, & Clarke, 2019; Justi & van Driel., 2006; Lebak, 2015; Lomas, 2018; Wilkie, 2019; Wilkie & Clarke, 2015; Wongsopawiro et al., 2017; Zwart et al., 2007) to guide research in the fields of mathematics and science. Some of these researchers identified inadequacies with the model and reinterpreted certain characteristics to support the analysis of their data. A summary of these modifications is presented in Table 3.2.

Most of the reinterpretations of the IMPG concern representing learning and change that occurs *within* a domain, with analysis of change sequences being recognised as the strength of the model (Zwart et al., 2017). Rather than suggest modifications to the IMPG, for the purpose of this study I chose to build on the work of Wilkie (2019), and also adopt a second model to complement the strengths of the IMPG. In her study the IMPG was used as an “analysis tool for exploring different change pathways of teachers” (p. 99); the MDT was used to examine the influence of institutional constraints and affordances on changes in practice. In this study, the intent was to use the MDT to focus on the influence of institutional factors on change sequences and learning processes within the domains of the IMPG; in particular, the influence of social interactions among participants, and with myself as the researcher. Figure 3.3. provides an overview of the research design and indicates the two models being used in conjunction to form the theoretical framework for this study. The features of the MDT are explained in the following subsection.

Table 3.2. *Summary of modifications to the IMPG incorporated by various studies*

Literature reference	Main focus of study	Summary of modifications
Chan et al. (2019)	Investigate construction of knowledge of mathematics teachers	▪ Reconceptualisation of teacher professional growth to include two mechanisms of teacher learning: consolidation of existing knowledge and beliefs and development of new knowledge and beliefs.
Justi & van Driel (2006)	Investigate development of knowledge in beginning science teachers	▪ Redefined change and growth to align with the duration of their study; differentiated change and growth using criterion of complexity.
Lomas (2018)	Analyse of changes in knowledge and beliefs of two practising primary teachers (mathematics)	▪ Proposed structural change to the Personal Domain; arrows of reflection to show interaction among the internal components of the domain (knowledge, beliefs and attitude).
Wongsopawiro et al. (2017)	Investigate development of teacher knowledge (PCK) (science)	▪ Adapted the Personal Domain to identify various elements of PCK and reflect the focus of their study. ▪ Adapted External Domain to identify components of the professional learning program.
Zwart et al. (2007)	Investigate learning processes in relation to a reciprocal peer coaching program (various subjects)	▪ Divided the External Domain to reflect specific aspects of their professional learning program. ▪ Treated the Personal Domain as a whole rather than individual components. ▪ Integrated planning and teaching of lessons into the Domain of Practice. ▪ Extended the notion of reflection to include the intention of participants.

3.1.3. Meta-didactical transposition model (MDT)

The second model comprising the theoretical framework, and informing the research design, is the MDT. It was developed by Italian researchers and is based on Chevallard's (1985, 1992, 1999) Anthropological Theory of Didactics (Arzarello et al., 2014). The MDT is both descriptive and interpretative. The model was principally adopted to inform the analysis of institutional constraints and affordances experienced by the teacher participants. Arzarello et al. (2014) considered the MDT a complement to the Clarke and Hollingsworth (2002) model, in which the influence of constraints within the school context are more implicit. In contrast, MDT identifies various constraints imposed on institutions such as national curriculum requirements and assessments, as well as constraints existing within institutional environments, such as school traditions or culture, teachers' time and space (Arzarello et al., 2014). Essentially, the model considers constraints encountered or perceived when teachers engage in collaborative professional learning experiences.

In addition, the MDT considers the "complex dynamic interplay" (Arzarello et al., 2014, p. 351) between teachers and researchers, when both engage in professional learning. Analysing the data retrospectively highlighted the importance of this feature of the model. Central to the MDT is the notion of praxeology, which consists of "the tasks, techniques, and justifying discourses" (Arzarello et al., 2014, p. 353) that develop during professional learning experiences. Interactions between teachers and researchers can result in the development of new praxeologies; teachers' praxeologies can change from being external to internal through an internalisation process. For example, in this study, teachers experimented with the use of number strings as a new didactic praxeology to elicit student reasoning about mental computation strategies. It is through the brokering process, which essentially describes the "transition of mathematical concepts from one community to another" (Arzarello et al., 2014, p. 357) that a shared praxeology between the teachers and researcher can evolve. In the context of professional learning, broker actions by the researcher support the transfer of mathematical concepts to teachers. The aforementioned features of the MDT are depicted in Figure 3.2.

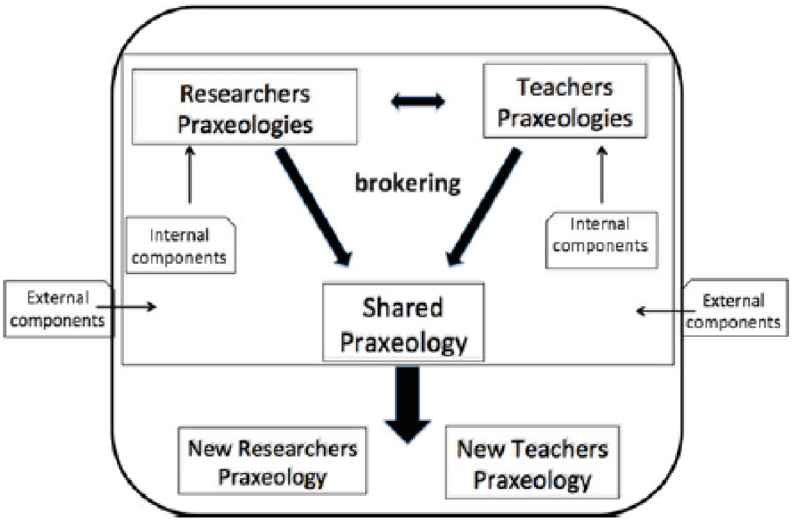


Figure 3.2. The Meta-Didactical Transposition (MDT) model (Arzarello et al., 2014, p. 355).

The five interrelated features, which comprise the MDT and conceptualise teacher change, are summarised in Table 3.3.

The MDT was considered pertinent for this study for three reasons. The model focuses on: the influence of institutional aspects; the internalisation of new practice and knowledge; and the relationship between theoretical knowledge and teaching practice. Each of the aforementioned reasons will be discussed in turn. First, the model focuses on the evolving relationships among teachers, the researcher and role of the institutions in teacher professional learning (Arzarello et al., 2014). In this study, the MDT was used to examine the influence of institutional affordances and constraints on changes to teachers' practices and their disposition towards teaching and learning mathematics. Based on the findings of previous studies examining the challenges of changing teacher practice through professional learning (e.g., Sullivan et al., 2013c; Wilkie, 2019), institutional constraints are an important consideration.

Table 3.3. *Features of the Meta-Didactical Transposition model (Arzarello et al., 2014)*

Feature	Description of the feature
Institutional aspects	Participatory experiences of teachers and researchers involved in a project may be constrained by aspects of the social context in which they are situated i.e., changes to teacher practice may be constrained.
Meta-didactical praxeologies	Interactions between teachers and researchers during professional learning experiences, in which theoretical reflections and justifications are discussed, can result in the development of new teacher praxeologies and/or new researcher praxeologies. The development of a shared praxeology between teachers and researcher is an outcome central to the MDT model.
Internal and external components	During professional learning experiences teachers' praxeologies (techniques and knowledge) can change from being external to internal. This internalisation process is described as a meta-didactical trajectory; the use of this term resonates with the work of Simon (1995) and the concept of a hypothetical learning trajectory. Researcher praxeologies may also change following interaction with the teacher community.
Brokering process	The brokering process describes the "transition of mathematical concepts from one community to another" (Arzarello et al., 2014, p. 357). In the context of professional learning, broker actions by the researcher support the transfer of a mathematical concept to teachers.
Double dialectics	Double dialectic describes dialectic at the meta-didactic level between teachers and researchers, usually concerning a difference in interpretation of student personal meanings in a classroom didactic situation (teaching moment). It is through double dialectic that development of teacher professional competencies occurs, as teacher praxeologies align with those of the researchers.

Second, the MDT model was considered apposite because of the focus on a process of internalisation in "which some of the external components become internal as a result of the process of Meta-Didactical Transposition" (Arzarello et al., 2014, p. 356). Although the Clarke and Hollingsworth (2002) model recognises that changes to teachers' practices may be affected by both external and internal influences within the context of the Change Environment, it treats the External Domain as distinct from the internal domains. For the purpose of this study, the MDT model was considered a complement to Clarke and Hollingsworth (2002) for analysing changes in teacher practices, knowledge and dispositions:

> Our model is similar but not identical to that of Clarke & Hollingsworth, since ours underscores the interdependence of such changes with the institutions (according to the ATD approach), and focuses on the Meta-didactical components of the processes, which remain more implicit in Clarke & Hollingsworth approach. (Arzarello et al., 2014, p. 369)

Finally, the model also complements the MKT model described by Ball, Thames and Phelps (2008), which characterises "MKT through the analysis of the daily practice of teachers" (Arzarello et al., 2014, p. 350). MKT and MDT models both focus on the interconnected relationship of theoretical knowledge and teaching practice; the main difference is that MKT focuses on the structure of knowledge while MDT places more emphasis on how the components evolve (Arzarello et al., 2014, p. 369) i.e., how components once considered external become internal.

> As the MKT model refines Shulman's PCK model (Pedagogical Content Knowledge, 1986), so the Meta-Didactical Transposition model enriches the MKT one. (Arzarello et al., 2014, p. 369)

3.1.4. Theoretical framework: Compatibility of the models

The two models that comprise the theoretical framework for this study were considered complementary in the sense that the strengths of each allowed them to be employed to different ends. The IMPG was used to analyse change sequences and learning processes for teachers from across different school settings, whereas the features of the MDT enhanced the analysis of learning processes within certain domains situated within the change environment. In terms of theoretical compatibility, the extent to which the theoretical positions of both groups of researchers converge and are coherent with a social constructivist perspective will be discussed in this subsection.

Clarke and Hollingsworth (2002) posit that their model of professional growth is coherent with either a situative or cognitive perspective on learning. Although Ernest (1994) does not discuss the position of situated learning in detail, he does suggest that there are aspects of this position "that might be consistently combined with social constructivism" (p. 15). However, as was illustrated in the discussion about the relationship between the teacher, the External Domain and the other domains in the IMPG (see section 3.1.2) there is ambiguity in relation to the extent the learning theory underpinning the IMPG is consistent with the position on social constructivism assumed for this study. Although the arrows of reflection and enactment conceptualise change processes in the IMPG, the existence of interactionist social processes within the change environment are more implicit.

On the other hand, a focus on social interaction and construction of meaning within the school context, lies at the heart of the MDT. The dynamic interplay between researchers (or educators) and teachers in developing a shared praxeology, is congruous with the social constructivist stance adopted for this study. The underpinning theory is based on Chevallard's (1992), as cited in Arzarello et al. (2014), Anthropological Theory of Didactics which emphasises that "the very nature of mathematical objects in school is dependent on the person or institution with which it is related (Arzarello et al., 2014, p. 352). The focus on the internalisation of new practice and knowledge (meta-didactical components of the change process) can be strongly aligned with Cobb's (2000) explanation of the reflexive relationship between the individual and class or teaching community as ideas are constructed through social interaction within the school context.

In summary, the two models indicate different degrees of theoretical alignment with the perspective on learning underpinning this study. While it is recognised there are similarities between the MDT and IMPG (Arzarello et al., 2014) and the two models are theoretically compatible, the MDT clearly underscores the interdependence of changes in teacher practices, knowledge and dispositions with aspects of the institutional context.

In the next section the research methodology used to inform the investigation of processes involved in teachers learning sophisticated instructional practices, stimulated through an onsite professional learning program, is explained.

3.2. Methodology

In this section, I explain the research methodology used to inform the study, and reasons for the methodology and the design of the professional learning program. A diagrammatical representation showing the linking of the theoretical framework and research methodology is displayed in Figure 3.3.

The study adopted a design-based research methodology, which involves “active innovation and intervention in classrooms” (Kelly, 2003, p. 3) and focuses on both student learning and teacher learning (Gravemeijer & van Eerde, 2009). Integral to the study was an intervention to suggest a (potentially) different approach to teaching mental computation. The instructional practices and aspects of the content comprising the intervention presented new learning for the participants. The study involved the teachers and I working collaboratively to achieve change in classroom practice (Baumgartner et al., 2003). In design-based research, “meaningful cooperation” between researchers and participants is considered crucial (McKenney & Reeves, 2012, p. 17). A holistic view of intervention was adopted for the study: the intervention involved “interactions between materials, teachers, and learners” through iterative cycles (Baumgartner et al., 2003, p. 5). The study encompassed the following features considered to define design-based research: theoretically orientated, interventionist, collaborative, iterative and responsively grounded (McKenney &

Reeves, 2012). A description of the five features embodied in this study is summarised in Table 3.4.

Table 3.4. *Features of design-based research embodied in this study (McKenney & Reeves, 2012)*

Feature	**Description**
Theoretically orientated	A central focus was on the development of aspects of instructional practice; sophisticated practices that involved eliciting student reasoning, representing thinking visually and orchestrating productive whole class discussion to teach mental computation with conceptual understanding and fluency. The intent was to construct theory about processes of learning and how to support learning.
Interventionist	The study involved an intervention by the researcher to suggest a (potentially) different approach to teaching mental computation. The intent was to support teachers with new instructional practices and study the processes through which they learned and developed their practice.
Collaborative	The researcher and teachers worked collaboratively to achieve change in classroom practice through professional experimentation, with the teachers learning from the researcher and vice versa.
Iterative	The study involved iterative cycles of refinement in the sense that all subsequent interventions were based on the outcomes of previous ones. It involved ongoing processing of analysing teacher engagement and learning, and modifying the support provided through the professional learning experiences and resources.
Responsively grounded	The study was based on participant expertise, literature and field testing. The intervention evolved over iterations of enactment, reflection and refinement across three schools. The process was responsively grounded in the sense that it involved refining both student learning and teacher learning processes.

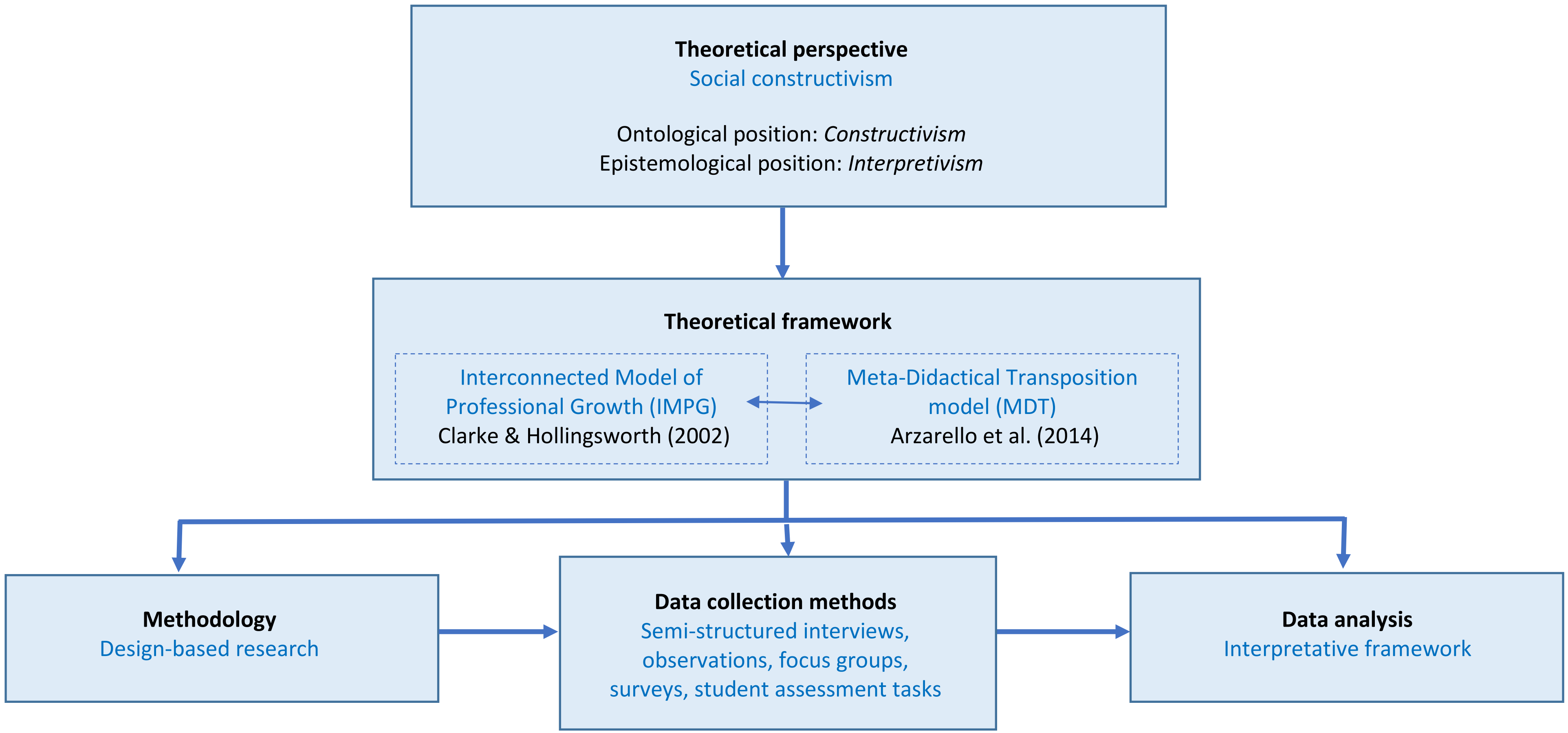


Figure 3.3. Flowchart to depict the research design.

Design-based methodology was considered as analogous with the theoretical models for professional learning employed for this study: IMPG developed by Clarke and Hollingsworth (2002) and MDT model developed by Arzarello et al. (2014). The methodology: connects theory and educational practice; is grounded in interactions of local practice involving researchers and teachers working collaboratively to improve teaching and learning; and aims to gain insights into how theory can be developed to improve teaching practices and learning outcomes through iterative cycles of refinement (Baumgartner et al., 2003). This study involved iterative cycles of refinement with teachers within each school, and across the three intervention cycles implemented at different schools with different groups of teachers. The intent of this study was to contribute to the 'building blocks' of local theory (McKenney & Reeves, 2012).

There are two common types of educational design studies: classroom design studies and professional development design studies (Cobb et al., 2015). The focus of this study is on the latter. Essentially, design-based (or educational design) research is recognised as having a focus on one of two prevailing orientations: research conducted *on* interventions or research conducted *through* interventions (McKenney & Reeves, 2012, p. 7). In this study the teaching component of the intervention provided the context for the study. It was the vehicle for studying another phenomenon: teacher professional learning; specifically, how teachers learn sophisticated instructional practices to teach mental computation. This study therefore adopted the orientation of design-based research conducted *through* an intervention to gain insights into possible pathways for change in teaching practice stimulated by professional learning experiences.

An alternative approach to design-based research, in which the research is conducted *on* an intervention, involves a focus on the design of the intervention and how it works. McKenney and Reeves (2012) posited that although design research usually focuses on one orientation, there are often points during a study when both orientations are reflected. In this study three main cycles of the intervention were implemented sequentially at three different schools, and each time the structure of the intervention was refined and adapted with the purpose of increasing opportunities for learning. In these circumstances it could be said that the research was at times being conducted *on* the intervention itself. The following subsections outline the study's design process and describe how design-based methodology informed the design of the professional learning program.

3.2.1. Intervention design

In this subsection, I outline the design process and explain the principles underpinning the design of the professional learning program (the intervention). The purpose of the intervention was to gain insights into processes involved in changing teacher practice through a school-based collaborative professional learning program; specifically, how any changes in teachers' knowledge, disposition and practice might relate to their interactions with the researcher and other participating teachers. The study also considered how aspects of the institutional context either constrained or afforded opportunities for teachers to learn and change their practice.

3.2.1.1 The design process

The intervention design was informed by design-based methodology since it focused on instigating and supporting a change in practice, and was theoretically orientated. The design process was guided by a generic model developed by McKenney and Reeves (2012) which identifies three main stages: analysis and exploration; design and construction; evaluation and reflection. Each of these key stages will be discussed respectively, in relation to the first intervention cycle.

The initial stage of the design process, analysis and exploration, involved reviewing the literature on the teaching content of the intervention (mental computation) and refining the problem in the context of teaching in Australian primary schools with a nationally prescribed curriculum (McKenney & Reeves, 2012). It was apparent that mental computation would provide a pertinent context for enhancing opportunities for teachers to develop instructional practices that focus on reasoning and conceptual understanding; proficiencies in the Australian mathematics curriculum recognised as requiring greater attention (Sullivan, 2011). Literature on designing professional learning was also explored with the aim of identifying elements considered essential for instigating and supporting professional learning of teachers (Desimone, 2009). In addition, this stage of the design process involved conducting a pilot of the teaching resources for the intervention. The pilot involved classroom experimentation with sequence of lessons, which I modelled for the class teacher. Lesson debriefs focused on the lesson structure, pedagogical approach and student learning. Lesson plans were shared, as well as a sample of the teacher resource book; the teacher provided useful verbal feedback on the resources.

The second stage, design and construction, drew on the data from the initial exploration stage to design the professional learning experiences and resource materials for the teachers. The third stage involved implementation of the intervention, a stage of the design process McKenney and Reeves (2012) referred as *evaluation and reflection*. This stage entailed refining aspects of the program to support teachers. For example, increasing the modelled lessons from one lesson to a sequence of three lessons, as well as refining the teaching resources. In relation to the teaching component, the participants were involved in designing and experimenting with the suggested learning sequence of instructional activities to collaboratively develop hypothetical learning trajectories[3] for mental computation (Gravemeijer & van Eerde, 2009), throughout the implementation of the intervention. Specific refinements to the intervention design are discussed in each of the chapters reporting on the three interventions (see sections 4.4, 5.2, 5.5, 6.2, 6.5). A model depicting the design process is displayed in Figure 3.4.

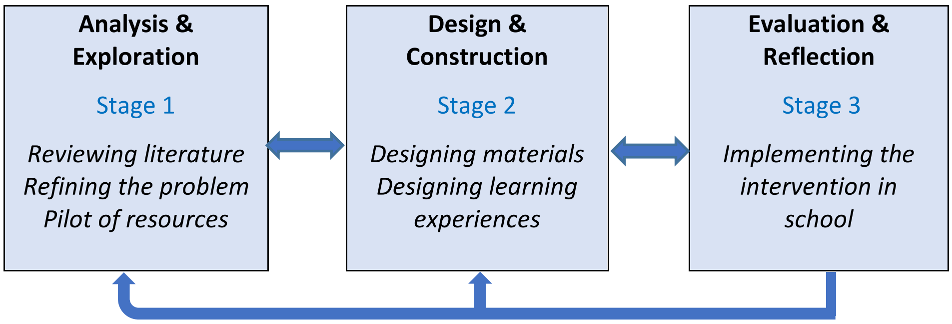


Figure 3.4. Model to display the design process followed for this study (adapted from McKenney & Reeves, 2012).

[3] The notion of a hypothetical learning trajectory was introduced by Simon (1995), who explained it as being "consideration of the learning goal, the learning activities, and the thinking and learning in which the students might engage" (p. 133).

The design process displayed in Figure 3.4 was repeated for the design and implementation of the second intervention cycle; it involved returning to the initial ideas and literature (stage 1) and making refinements to the design (stage 2) and implementation (stage 3). The third intervention cycle involved a shorter process: just the second and third stages of the design process. The arrows underneath the model (Figure 3.4) indicate the cyclical nature of the process and revisiting various stages. The arrows between the stages indicate movement between stages during an intervention cycle (note an intervention cycle refers to a main cycle of implementation at a school site).

3.2.1.2 Supporting the learning process: Intervention design principles

In designing a professional learning program to support teachers with learning sophisticated instructional practices to teach mental computation, I predominantly drew upon the work of Desimone (2009). Her work suggested a research consensus on common features of effective professional learning (effective in the sense of maximising learning opportunities for both students and teachers). These features include attention to: content focus, active learning, coherence, collective participation, and duration. Table 3.5 provides an overview of the core features on which the intervention design was based, with details of the program components designed to support teacher learning. Each of the components presented in the table, including the theory underpinning the design, are discussed next.

Table 3.5. *Overview of the design of the professional learning program based on core features highlighted by Desimone (2009)*

Program components to support learning	**Core design feature** *Based on Desimone (2009)*
Provision of a framework for teaching mental computation: Teacher resource book	Content focus Coherence
Professional learning session	Active learning Content focus
Modelled lessons (including lesson debriefs)	Active learning
Facilitated collaborative planning	Collective participation Active learning
Provision of professional reading (research papers) on instructional practices	Content focus

The teacher participants in this study were provided with a researcher-developed framework for teaching sequences of lessons on mental computation strategies for addition and subtraction (Gravemeijer & van Eerde, 2009), which was presented as a hard-copy teacher resource book. The framework included a suggested trajectory of mental computation strategies and an outline of a learning sequence for teaching mental computation. A sample of the teacher resource book is presented in Appendix B. Gravemeijer and van Eerde (2009) highlighted similarities between the design-based research approach and the iterative nature of the mathematics teaching cycle developed by Simon (1995). They suggested that teachers require support in designing hypothetical learning trajectories and that local instruction theory offers a framework of reference for teachers constructing such trajectories. In the context of this study, local instruction theory concerns a theory about a possible learning process for teaching mental computation. In line with design-based methodology, the teacher resource book was refined and adapted for each school context, based on outcomes of the intervention in the previous school context.

An initial professional learning session focusing on the instructional tools – the theory underlying the approach and pedagogies to teach the learning sequences – was facilitated at each school. This session was crucial in providing an opportunity to interact with the participants so that they developed “a taken-as-shared understanding of the rationale for and intent of the agreed-upon innovation before the teaching experiment” began (Cobb, 2000, p. 331). Sharing the rationale for the study with the participants was important for not only defining and clarifying the problem, but also in terms of engaging the participants in the study (McKenney & Reeves, 2012). The session provided opportunity for teacher participants to work collaboratively with colleagues and to experiment with some of the suggested instructional activities provided in the teacher resource book. A lesson structure designed to support facilitation of purposeful discussion, a key component of the approach, was unpacked with the teachers in this session. This lesson structure was used for each lesson in the sequence, and had been purposively designed to foster a sense of classroom community to which all students contribute and learn from each other (Sullivan et al., 2016). The professional learning session also included discussion relating to “classroom social norms” (Gravemeijer & van Eerde, 2009, p. 514) and the importance of a growth mindset culture (Dweck, 2006) to foster and support a classroom learning environment that involves students taking risks.

The professional learning included opportunity for the teachers to observe a researcher-modelled lesson or sequence of three lessons, referred to in the teacher resource book as a learning sequence. Although I assumed a leading role in facilitating student learning through the use of instructional activities and whole class discussion, I was conscious of maintaining an ongoing dialogue with teachers throughout the modelled lessons and whenever possible involving them in the decision-making process. The process of planning and teaching the learning sequences is depicted in Figure 3.5.

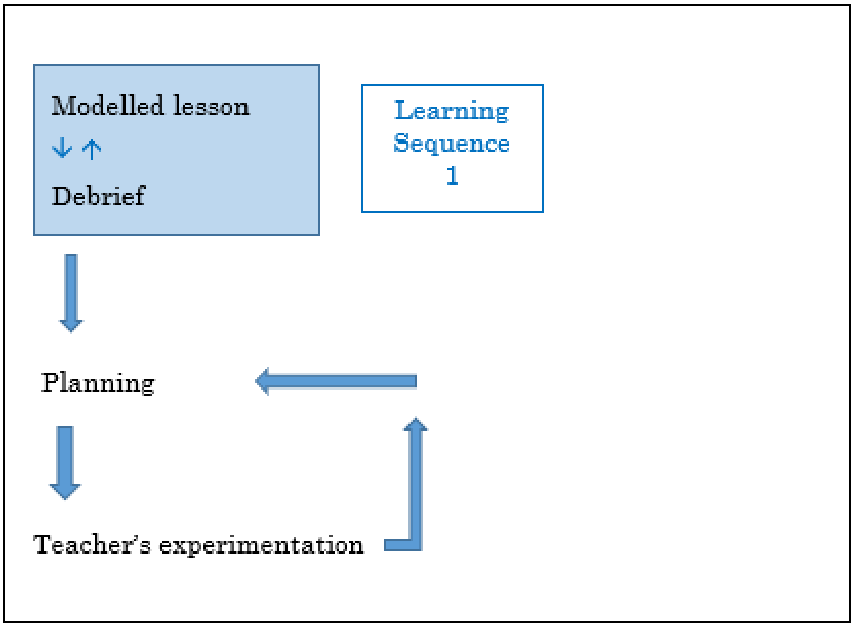


Figure 3.5. The planning and teaching of the learning sequences.

An integral part of the professional learning was facilitated collaborative planning. These sessions provided opportunities for the teachers and I to work collaboratively on achieving a change in classroom practice (Baumgartner et al., 2003). Planning meetings at each school differed slightly due to the influence of contextual factors in each institutional environment. Generally, however, the focus was on sharing student work samples, which were either tasks recorded in student workbooks, photographs of student mini-whiteboards, or poster paper with recordings of student strategies shared in whole class discussion. Within these meetings, the role I assumed was to: pose questions to stimulate discussion about the complexity of the student mental strategies; draw out the foundational knowledge (Wright et al., 2012) applied in student development of mental strategies; and highlight mathematical principles underlying strategies and potential student misconceptions (Kazemi & Franke, 2004). The collective discussion and inferred learning from analysis of student work on mental strategies was used to inform the planning and teaching of subsequent mental computation lessons. Essentially these sessions involved the teachers and myself collectively revising and refining local instruction theory. The teacher participants had an active role in the research process throughout the intervention. Analysing data and planning subsequent teaching actions were collaboratively undertaken by the teachers and myself (Gravemeijer & van Eerde, 2009). In keeping with the nature of design-based methodology, the teacher participants were encouraged to adapt and refine the resources initially provided at the outset of the intervention, to meet the learning needs of their own students.

3.2.2. Participants

The participants were Year 3 primary school teachers and their respective students from three schools located in the south-eastern suburbs of Melbourne. Of central importance was the need to study learning in context, in real learning environments (Collins, Joseph & Bielaczyc, 2004). Year 3 was purposively selected for the focus of the study because this year level has traditionally been recognised as the stage in primary schooling at which students are introduced to more formal written calculation methods for addition and subtraction. I was interested to look at how teachers might teach mental computation at this year level.

The intervention was conducted separately at each school site; the schools did not attend cross-campus professional learning sessions together as a whole group. The schools were purposively invited to participate in the research project; they were selected because there were distinct differences between each school context and the learning environments. The intent was to gain insights into learning processes within different contexts, an aspect of design-based research considered important (Cobb et al., 2015). The schools that participated in the study included a Victorian state primary school (3 teachers), a Catholic Education primary school (4 teachers) and an independent K–12 school (3 teachers). Detailed descriptions of each school context are provided in the chapters reporting the intervention findings for each school (Chapters 4 – 6). An overview of the key features of the participants and school contexts is presented in Table 3.6.

Table 3.6. *Overview of the teacher participants and contextual features of the participating schools*

Features	**School A**	**School B**	**School C**
No. of participating teachers	3	4	3
School system	State (government)	Catholic (private)	Independent
School internal support with the program	Year 3 learning specialist (team leader) actively involved	Mathematics leader (ML) actively involved	Numeracy coach (NC) actively involved
Reasons for school interest in participating in the program	Integrate more opportunities for student reasoning into lessons School mathematics program has limited focus on mental computation	School transitioning from ability grouping intervention program to whole class approach ML interested in approaches to embed mental computation into the mathematics program	Support with embedding fluency in mental strategies prior to introducing formal algorithms Interested in number strings as way to enhance student reasoning

3.2.3. Implementation of the intervention

There was some variation in the implementation of the intervention across the three different schools, in relation to duration and iterative cycles, which will be explain in this subsection. An overview of the implementation of the intervention is presented in Figure 3.6.

PILOT	**INTERVENTION CYCLE 1**	**INTERVENTION CYCLE 2**		**INTERVENTION CYCLE 3**
Term 4, 2017	Term 1, 2018 Weeks 7 - 9	Term 2, 2018 Weeks 1 - 4	Term 3, 2018 Weeks 1 - 2	Term 3, 2018 Weeks 2-4
School A	**School A**	**School B**		**School C**

Figure 3.6. Overview of the intervention across three different schools.

As indicated in Figure 3.6, each time the intervention was implemented at a school site this was considered an intervention cycle (main cycle). Iterative cycles existed within in main intervention cycle; these iterative cycles reflected the planning and teaching cycle for a sequence of three lessons (usually one cycle occurred in a school week). Variations in relation to duration and iterative cycles were predominantly the influence of constraints within the institutional contexts. At school B, high levels of student engagement in the first week of the project influenced a team decision to implement a second phase of the intervention at the beginning of the following school term.

3.2.4. Ethical considerations

Ethical considerations are usually focused around four key issues: potential of harm to participants, consent, privacy, and deception (Bryman, 2012). Ethics was considered: before the research was conducted, at the beginning of the research, during the data collection, at the data analysis stage, and when reporting and publishing the research (Creswell, 2013).

Prior to commencement of the research, ethics approval was sought from the Monash University Human Ethics Committee (MUHREC). Once approval was granted by MUHREC, ethics clearance was also sought both from the Victorian Department of Education and Catholic Education Melbourne, since the three interventions were conducted in schools from each sector (see Appendix C for Explanatory Statements).

Central to the research process is a need to show respect for the participants. Pseudonyms and codes were used in all documentation to protect the privacy of the individual participants. In this study, publication of the research will be shared with participants and stakeholders; they will be emailed a report summary. Transparency was integral to the research process from the outset. Teachers were clearly informed about the purpose of the research; information was emailed to schools prior to an initial meeting, in which I explained the purpose of the study. Each participant was provided with detailed information regarding the goals of the research (explanatory statements); consent forms clearly stipulated requirements from participants with an 'opt out' option for each aspect of the data collection. In addition, transcripts were emailed to participants to check accuracy of data and give them an opportunity to make further comments in relation to the study.

The participants made a commitment to the research with regard to time; they engaged with the study outside direct contact they had with me, as the researcher, during the research process. Creswell (2013) discussed the need to show "reciprocity" for the participants, meaning the intent is for participants to benefit in some way from their commitment. In this study, participant benefits included: professional learning sessions on number strings; one-on-one classroom support and feedback; facilitated planning; teacher resource book; an opportunity to co-construct assessment tools; resources to support classroom teaching e.g., PowerPoint slides, research-based professional reading materials.

3.3. Methods of data collection

In this section, I describe the various methods of data collection, the contribution they made to address the research questions, and the data collection process. A range of qualitative data was considered most apt for studying in-depth the process of teachers learning sophisticated instructional practices to teach mental computation (Cobb et al., 2015). The study involved multiple data collection methods to gain multiple perspectives of the participants and provide thick, rich descriptive data to allow for rigorous retrospective analysis (Creswell, 2013). The data methods were purposively chosen to consider both the participatory paradigm of learning i.e., data related to teachers' interactions with each other and the researcher, and the acquisition paradigm i.e., data related to each teacher's individual development of knowledge. A summary of the data collection methods, the purpose of each method and the research sub-questions (see section 1.3) addressed by each method are presented in Table 3.7.

Multiple sources of data were collected from each intervention at three different stages: *pre-*, *during* and *post*-intervention. Table 3.8 provides an overview of the planned data collection process, specifically, the various research methods planned for each stage of the research process.

Table 3.7. *Summary of data collection methods, purposes and research sub-questions addressed*

Data collection methods	Purposes	Research sub-questions[4] Q1	Q2
Pre-intervention teacher survey	▪ Insights into teachers' perceived knowledge of mental computation, current classroom practice and perspective on teaching and learning mental computation.	✓	
Pre- & post- intervention lesson observations	▪ Insights into changes in teachers' practice. ▪ Triangulation of teacher self-reported data.	✓	✓
Post-professional learning session questionnaire	▪ Insights into teacher responses to the instructional approaches including any dissonance created, new learning and how the session supported this learning.	✓	✓
Focus group: weekly planning meetings	▪ Insights into classroom practice (teacher experimentation with new ideas in their classrooms) and student learning.	✓	✓
Post-intervention semi-structured interview	▪ Insights into teachers' self-perceived changes in knowledge and practice; affordances and constraints to these changes.	✓	✓
Focus group: post-intervention reflection	▪ Insights into teachers' self-perceived changes in knowledge and practice; affordances and constraints to these changes.	✓	✓
Individual semi-structured interviews with ML or NC (who were actively involved in the intervention)	▪ Insights into perceived changes in teacher knowledge and practice; affordances and constraints to change.	✓	✓
Student assessment tasks	▪ Triangulation of teacher self-reported data on student learning and changes in practice.	✓	

[4] The research sub-questions: 1) What were the different change pathways and learning processes experienced by each teacher? 2) How do aspects of the teachers' institutional context constrain or afford opportunities to learn and change practice?

Table 3.8. *Overview of the planned data collection process for the research*

Stage	**Research methods**	**Sources of data**
PRE- intervention	Teacher written survey Lesson observation – snapshot Student assessment task (written)	Questionnaires Researcher's journal Student assessment samples
DURING intervention	Professional learning session Post-professional learning session survey Modelled lessons and lesson debriefs Focus groups: Weekly collaborative planning meetings	Researcher's journal Questionnaires Student work samples Photographs of student mini-whiteboards (student strategies)
POST- intervention	Student assessment task (written) Student assessment interviews Lesson observations Individual teacher semi-structured interviews Individual teacher surveys Focus group: reflection on the intervention Individual semi-structured interviews with ML or NC within who were actively involved in the intervention	Student assessment samples Researcher's journal Transcriptions of audio recordings – individual and focus group Questionnaires

The intent was to follow the data collection process outlined in Table 3.8 each time the intervention was implemented at the three different schools (the schools in this study are referred to as schools A, B and C). However, there were slight modifications to the data collection process at each school due to various constraints within the institutional contexts. The constraints and consequent adaptations to the process are discussed in detail in each chapter reporting the findings for each intervention (see sections 4.4, 5.2, 5.5, 6.2, 6.5). See Appendix E for a summary of the data collected for each intervention.

In the following subsections, the rationale and limitations of each data method of data collection are discussed.

3.3.1. Surveys

Data were collected from the teachers using three different surveys at various stages within each intervention. Although the focus of all survey data collection was on aspects of the Personal Domain in the Change Environment (Clarke & Hollingsworth, 2002) each survey had a different purpose, as explained below.

Prior to the commencement of the study, the participating teachers completed an initial survey (see Appendix D) to gain the following information:

- their knowledge of mental computation strategies;
- their use of resources to support and inform planning and teaching of mental computation;
- current approaches to teaching mental computation; and
- perspectives on teaching and learning of mental computation.

The data were used to inform the design of the professional learning session for that group of teachers.

Following the professional learning session, a further survey was conducted to gather information on participant responses to the new approach to teaching mental computation, in particular insights into any dissonance the session may have created and any self-perceived learning. The survey included statements for participants to respond to using Likert-style scale response items and free-format responses (refer to Appendix D).

A final survey was conducted post-intervention to gather information about teachers' perceived changes to aspects of their knowledge, their disposition towards teaching and learning mental computation, and changes to their teaching practice (refer to Appendix D).

3.3.2. Focus group: Collaborative planning

Facilitated, collaborative weekly planning meetings were effectively focus groups with the year level teams of teachers in each school. The focus of the data collection was on aspects of the Domain of Practice and its connection with other domains (Clarke & Hollingsworth, 2002). The weekly planning meetings were important in terms of offering participants support, but also provided me with an opportunity to learn from the participants (Cobb, 2000). The focus was on discussing students' learning and work samples, evaluating the sequence of mental computation sessions for the week (including constraints encountered), and planning the following week. The sessions provided an opportunity for the teachers and I to share and discuss reflections on classroom experiences, as well as to interpret student learning, which at times resulted in teachers and the researcher developing new praxeologies (Arzarello et al., 2014). The meetings were crucial in terms of revising and adapting learning trajectories (Cobb, 2000).

3.3.3. Observations

Lesson observations were conducted (by myself as researcher) pre- and post-intervention to gain insights into changes in teachers' instructional practices (Cobb et al., 2015). An initial lesson observation was conducted pre-intervention for a snapshot of classroom culture and pedagogical approaches in mathematics lessons. A second observation of a mental computation lesson was scheduled with each research participant towards the end of the intervention. The purpose of the second observation was to gather data about changes in aspects of knowledge and practice. This was considered particularly important, considering many design-based research studies focus on teacher participation in professional learning rather than on documenting changes in teacher instructional practices. During the observations I assumed the role of a non-participant observer and recorded descriptive and reflective notes on teacher instruction, and interactions between teacher and students, as well as between students. The observations were important not only to gain insights into any changes in practice, but as a means of triangulating data from the focus groups (planning meetings) and semi-structured interviews. Desimone (2009) posits that observation data is particularly valuable in removing self-report bias of surveys and interviews.

3.3.4. Semi-structured interviews

A semi-structured interview was conducted with each research participant following the intervention to gather information about their self-perceived changes in knowledge and practice (see Appendix D for the interview schedule). The focus of the data collection was on the Personal Domain. Clarke and Hollingsworth (2002) emphasise that participants can interpret the professional learning in different ways that are salient to them, and also view these salient outcomes differently depending on their existing value system. It was therefore important to collect data on each teacher's self-perceptions and use this to corroborate any changes I observed during the intervention. Open-ended interview questions were used to 'guide' the interview; the research sub-questions were used to devise the interview questions (Bryman, 2012). This method of data collection provided some flexibility to listen and respond to individual responses of the research participants, adapting the questions during the interview as appropriate (Creswell, 2013). The semi-structured interviews also gave the research participants opportunity to raise further questions or queries about the goals of the intervention and support mechanisms (Bryman, 2012). The interviews were audio-recorded for transcription purposes.

3.3.5. Student assessment

Although the study was primarily concerned with researching changes to teaching practice, data on student learning of mental computation were also collected pre- and post-intervention for two main purposes: first, as a way to develop the professional learning of the participants by analysing student mental strategies and second, to evaluate the intervention strategy. Student assessment data provided another data source to triangulate teacher self-reported data on their teaching experiences. Student learning outcomes are also often a salient outcome for teachers with regards to their being willing to make changes to their professional practice (Clarke & Hollingsworth, 2002).

Two types of student assessments were planned: written assessment tasks, which were completed pre- and post-intervention, and individual student assessment interviews, which were conducted post-intervention. The written task was conducted with all students; they were asked to record their thinking using mental jottings. Once the results had been processed for the written task, teachers were asked to select three students, representative of a higher, middle and lower achiever, and to conduct a short one-on-one assessment interview with each. Clinical assessment interviews are commonly used by teachers in Australia to assess student mathematical knowledge and are reported to be beneficial in terms of gaining insights into student learning and misconceptions (e.g., Gervasoni et al., 2017). The researcher initially designed both assessment tasks using the literature on mental computation, and then made some revisions in collaboration with the teacher participants (see Appendix D for samples).

3.3.6. Researcher's journal

Throughout the three interventions I had many informal conversations and discussions with teachers. I documented these incidences in a Researcher's journal by taking handwritten notes immediately after the conversations, sometimes during the conversations. On occasions, I used voice memos on my phone to record the key details and my immediate reflections. The handwritten notes and voice memos were later entered as fieldnotes on QSR International NVivo qualitative analysis software.

3.4. Methods of data analysis

The study was concerned with primary data analysis (Bryman, 2012). An interpretative approach to data analysis was adopted (Cobb et al., 2015) since the intent was to interpret data reflexively from both the perspective of an experienced primary school practitioner and of a researcher. The data analysis was conducted on two different levels: ongoing analysis concerning the teaching and learning of mental computation from the onset of the first intervention; and retrospective analysis of the data on completion of each intervention and after completion of all three interventions (Cobb et al., 2015). The ongoing analysis was essential to support the teachers' learning and inform the planning and teaching in classrooms. The retrospective analysis allowed for "a more thorough and systematic analysis of the same data" (Gravemeijer & van Eerde, 2009, p. 512). The retrospective analysis mainly concerned theoretical development (Cobb, 2000), as informed by the study's theoretical framework. In the following subsection, the process of retrospective analysis will be described.

3.4.1. Retrospective data analysis process

"The data analysis stage is fundamentally about data reduction" in order to comprehend the data to answer the research questions (Bryman, 2012, p. 13). The data analysis process involved moving in "analytic circles" rather than following a sequential process (Creswell, 2013, p. 182). Creswell (2013) identified key stages involved in the qualitative data analysis process: managing or organising; reading and memoing; describing, classifying and interpreting data; and representing the data. These stages were used to guide the data analysis process for each intervention (note that Stage 2 was modified to 'reading and initial coding'), each of which are outlined below.

Stage 1: Managing the data

Managing the data involved transcribing the planning meetings, semi-structured interviews, and focus group and voice memos from informal conversations. Survey responses that teachers had handwritten were typed and saved as digital files. The files were clearly labelled using pseudonyms for participants and/or school references i.e., School A, B, C. The files were uploaded onto NVivo.

Stage 2: Reading and initial coding

Each data source was coded using NVivo. Using the software, five thematic nodes[5] were created based on the framework of one of the theoretical models: the IMPG (Clarke & Hollingsworth, 2002) (see Figure 3.2). The names of the five nodes were derived from the components of the IMPG: the four main domains and the Change Environment. A node was also created to code memorable quotes considered critical in addressing the research questions. Initial broad codes (referred to as sub-nodes) were created within the five top-level nodes and informed by features of the IMPG. Initially, broad codes or sub-nodes were only formed for two of the top-level nodes: Personal Domain and the Change Environment. Within the node for the Personal Domain the two initial broad codes (sub-nodes) were formed: knowledge and disposition. For the Change Environment node, the initial broad codes (sub-nodes) formed were: affordances and constraints.

Stage 3: Describing, classifying and interpreting data

The data were re-read and the initial codes refined to describe the data in more detail. For example, for the top-level node Personal Domain, the coding was later refined to include sub-codes for relevant aspects of MKT conceptualised by Hill et al. (2008). The broad code 'disposition' was refined to include elements

[5] Nodes are essentially containers for the coding from multiple sources of data.

identified in the work of Cooke (2015) (see Figure 3.7 for initial coding). Re-reading was repeated due to the iterative nature of the process and further sub-nodes created for the coding of the data. The final coding hierarchy that evolved through the process of re-reading along with descriptions of each node is presented in Appendix E.

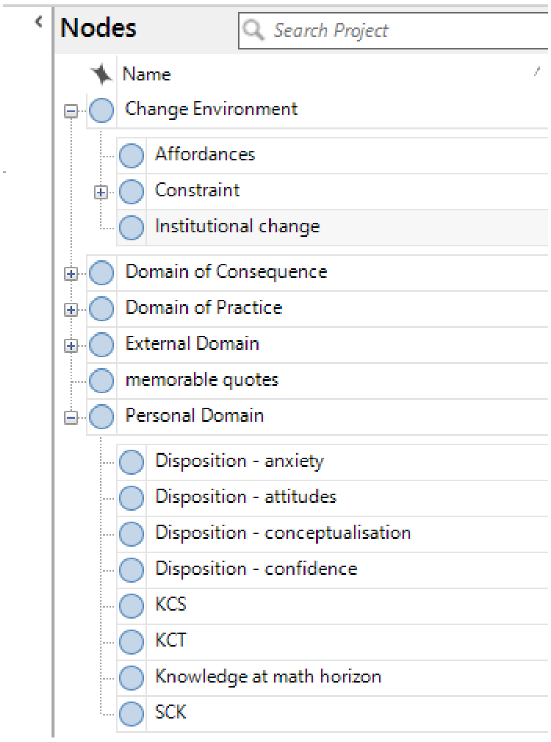


Figure 3.7. Thematic nodes created in QSR NVivo for initial coding of data sources.

The IMPG was used to analyse how teacher participation in the professional learning program led to changes in the four domains. Once the data were coded, criteria originally developed by Justi and van Driel (2006), were modified and used to examine relationships between changes in knowledge, disposition and practice and the domains of IMPG. The criteria used to establish relations between the domains in the IMPG are displayed in Table 3.9. The criteria guided the selection of coded data from NVivo, indicating learning processes of teachers within each of the domains of the IMPG. Examples of data highlighting learning processes within domains were recorded in a table to establish the learning pathway (the change sequence) for each teacher.

Table 3.9. *Criteria to establish relations between domains in the IMPG, adapted from Justi and van Driel (2006)*

Relation[6]	**Mediating process**	**Criteria**
PD to ED	Enactment	When a specific aspect of teachers' initial knowledge (SCK or aspects of PCK) or disposition influenced what they did or said about teaching mental computation during their participation in one of the learning experiences i.e. PL session, modelled lessons.
ED to PD	Reflection	When something that was done or discussed during one of the learning experiences modified teachers' initial knowledge or disposition on teaching mental computation.
ED to DP	Enactment	When something that was done or discussed during one of the learning experiences influenced something that occurred in their teaching practices.
PD to DP	Enactment	When a specific aspect of teachers' knowledge or disposition influenced something that occurred in their teaching practices.
DP to PD	Reflection	When something that teachers did in their teaching practice modified aspects of their knowledge or disposition on teaching mental computation (without reflection on classroom outcomes first).
DP to DC	Reflection	When the teacher noticed and/or reflected on something that they or their students did in their teaching practice that caused specific outcomes e.g. student learning, teacher use of visual representations, student motivation.
DC to DP	Enactment	When a specific outcome made teachers state how they would modify the associated teaching practice in the future.
		When a specific outcome made the teacher change their practice at that moment (reflection-in-action).
DC to PD	Reflection	When teachers reflected on a specific outcome, thus changing a specific aspect of their knowledge or disposition on teaching.
		When a teacher's evaluative reflection on salient outcomes led to a change in knowledge or disposition.
PD to DC	Reflection	When a specific aspect of teachers' knowledge helped them in reflecting on/analysing a specific outcome of their teaching practice.

[6] The codes representing the domain relations in Table 3.9 are: ED - External Domain; PD - Personal Domain; DP - Domain of Practice; and DC - Domain of Consequence.

Stage 4: Representing the data

The IMPG was used to depict change sequences for each teacher diagrammatically. Arrows indicating reflection and enactment were numbered to show the change process based on teachers' describing their experiences alongside my interpretation of the change process based on observations, contributions to planning meetings, and interview responses. Examples of change sequence diagrams for each teacher are presented throughout each chapter reporting the findings from each intervention: Chapters 4, 5 and 6 (see section 4.2.2, Figure 4.1, for the first teacher change sequence presented). Diagrams depicting the change sequence for each teacher were compared and analysed for similarities and differences.

Next, the MDT (Arzarello et al., 2014) (see Figure 3.2) was used to further analyse and describe influences on the learning processes within the domains of the change environment. The tables created for each participant to show the learning processes within domains (see Stage 3) were revisited; examples of any institutional factors and social dynamics influencing learning processes were coded within the tables. An excerpt showing the analysis of the learning processes of teachers is displayed in Figure 3.8. The sample presented in Figure 3.8 clearly shows an example of an influence of social dynamics (use of the MDT) on a learning process and the coding used to identify this. Further samples of this stage of the data analysis process are presented in Appendix E.

This indicates data coded at the External Domain (ED) in relation to modelled lessons

Teacher: Ian

Arrow	Domain Link	Mediating process	Criteria	Summary	Examples of learning processes
1	ED to PD	Reflection	When something that was done or discussed during one of the learning experiences modified teachers' initial knowledge or disposition on teaching mental computation.	Reflects on external stimuli: modelled lessons, teacher resource book and professional dialogue during collective planning. He has new PCK: how to use new instructional tools to facilitate student learning of mental computation strategies.	*...I think the modelling was really good, so watching you do a number string lesson. It's very different to reading it, and seeing it. I think that was very valuable.* **ED-MODELLED** *...The modelling was really, is, really valuable. And even with the prescriptive lesson I liked the teacher background before as well, yeah that was really useful, you know all the understanding clearly exactly what the strategy was, the use for it and.....and how to go forward with it. That was really valuable.* **ED-MODELLED** *...I loved how the lessons were just there, the equations were there, everything was there, it made our lives a lot easier than having to come up with new equations.* **ED-TEACHER RESOURCE BK** *...I think they need to have the lessons very prescriptive, at first, so that the teachers have a really clear idea exactly how it should be run so that the data is consistent, that was really good. Because we would be able to create our own now, based off all those three. So I think we need that.* **ED-TEACHER BK** **PLANNING MEETING 2**: conversation highlighted that Ian had been reading the teacher resource book. He commented on students progressing from using an ENL to record their thinking (field notes). *...And then also you coming in for the meeting was really valuable as well, just to clarify after we tried it for a week, to be able to clarify. Like I was struggling with the number talk, thinking it was exactly the same as the number string, that from you coming in, I think it clarified that difference.* **ED-PROF CONVERSATIONS** **Planning meeting 2:** Importance of bi-directional illustrated. Ian was confused about the lesson based on idea/concept of a number talk – how it would evolve into a lesson (he had only seen the number string lesson modelled). Jenna supported by backing up with an example of using a student strategy to drive forward the lesson and next question (Planning meeting 2 – field notes). Ian reflected on this in his interview and the value of this conversation.

Criteria used to establish relations between domains in the IMPG.

Sources of data on Ian indicating learning processes within the External Domain.
Text in *italics* are quotes from his semi-structured interview.

Use of the MDT to identify social dynamics occurring within the External Domain.

Figure 3.8. A sample of data analysis at Stage 4 of the process for one teacher (Ian): Using criteria to establish relations between domains in the IMPG.

The MDT was also used for analysis of institutional constraints on changes to professional practice within change sequences. The stages outlined were conducted cyclically for each school intervention. The findings and analysis for each school site are presented in Chapters 4-6.

3.5. Quality of the research design

There are many different perspectives regarding appropriate strategies for assessing the quality of the research design of qualitative studies. It has been argued that use of positivist terminology such as 'validation' and 'reliability' suggests acceptance of qualitative research in the quantitative world and that using terms associated with quantitative studies appears defensive (Creswell, 2013). This study adopted the stance taken by Creswell (2013) who emphasises adopting multiple strategies to assess the "*accuracy* of the findings, as best described by the researcher and the participants" (p. 249-50). Although Creswell (2009, 2013) uses the term 'validation strategies' he underscores the difference between this approach and that adopted in quantitative studies. He posits that adopting such strategies refers to a process of checking accuracy and consistency of findings, whereas in quantitative studies it involves examining consistency of responses and validity (Creswell, 2009). Creswell (2013) recommends engaging in a minimum of two strategies for any research. For the purpose of this study, I employed three strategies identified by Creswell (2013): triangulation, clarifying researcher bias and rich, thick description. In the remainder of this section, the strategies adopted to assess the accuracy of the findings, will be discussed.

Triangulation provides a way of checking accuracy of findings through corroborating multiple uses of sources of data, methods and theories (Creswell, 2013). Data from lesson observations were used to triangulate self-reported data collected from semi-structured interviews and surveys; student data was also used as a source of triangulation in relation to teacher self-reported data on student learning. The process of coding data from the interviews and cross-referencing with data collected from planning meetings and surveys also constitutes triangulation. Baumgartner et al. (2003) emphasised the importance of triangulating data to maintain objectivity. They described how the researcher is placed in the position of promoting the intervention whilst at the same time remaining a critic; therefore, it is crucial to triangulate multiple sources of data.

Clarification of researcher bias is an important process, it involves the researcher clarifying their position, biases or assumptions that may impact the study. Throughout the reporting of this study, I was conscious of stating my personal perspectives on learning (social constructivism) and describing how it may have influenced the research process and interpretation of the data (Creswell, 2013). I also provided the reader with a brief history of my professional experiences in connection with the impetus for this research.

Providing thick, rich description is a strategy in the sense that it allows “readers to make decisions regarding transferability” (Creswell, 2013, p. 252). My intent was to provide detailed description of each institutional context to clarify how specific aspects of the context mediated teachers’ learning and changes to classroom practice (Cobb et al., 2015). This approach also allows the reader to decide for themselves if the information presented might be transferable to other settings. Audio recording of interviews and focus groups were used to allow for detailed transcriptions. Detailed field notes were also recorded in a researcher journal for each school site. I used the software program, NVivo as a process to ensure the coding of data was reliable.

3.6. Consideration of reflexivity and the scope of the study

The intent was to interpret the data reflexively from both the perspective of an experienced primary school practitioner and facilitator of professional learning, and as a researcher. Throughout this study I was conscious to report on changes in knowledge, disposition and practice, perceived by the teachers. I was also aware that I was explaining "their perspectives from my perspective as the researcher" (Simon & Tzur, 1999, p. 254). To this end, I drew on direct quotes from the teachers but explicated when I was interpreting their reflections or actions. A recognised strength of being in the position of both researcher and facilitator of professional learning is being able to identify and conceptualise key issues that are not necessarily shared by the teacher (Simon & Tzur, 1999). Interpreting the data from my perspective as the researcher is subtly different from reporting on just the perspectives of the teachers; the theoretical framework guided the interpretation of the data and allowed a focus on issues considered important to the field of professional learning in primary mathematics.

It is important to consider some of the constraints associated with design-based methodology. One of the criticisms with design-based research is the involvement of the researcher in the research process, which could result in the bias of the researcher becoming part of the research process, compromising the credibility and trustworthiness of statements made by the researcher (Anderson & Shattuck, 2012). However, studies on professional learning have emphasised the positive impact an external expert can have on enhancing teacher knowledge and challenging current teacher thinking about mathematics (Timperley et al., 2007; Wongsopawiro et al., 2017). It has been contended that, “inside knowledge adds as much as it detracts from the research validity” (Anderson & Shattuck, 2012, p. 18). Arguably, the skills I developed teaching in primary classrooms for two decades, and my focus on developing teaching of early number as a school adviser, in effect positioned me as both an external expert and researcher for this study.

Design based research presents the challenge of bounding the scope of the study due to the multiple iterations it can involve. The study involved three main cycles of intervention at different schools. In terms of outputs, it is intended that this study will contribute to the “building blocks” of local theory (McKenney & Reeves, 2012). The study was conducted in practice (which is associated with local level theory), with three main cycles of the intervention studied in ten classrooms across three school settings. Iterative cycles of refinement occurred within the implementation of the intervention at each school. The insights on learning gained from local theory are within specific contexts “and not across a wide range of settings” (McKenney & Reeves, 2012, p. 35), to allow for in-depth analysis.

Design-based research can often involve small-scale interventions, such as individual teachers or schools, which it has been argued limits the impact of the research findings (Anderson & Shattuck, 2012). Similarly, Baumgartner et al. (2003) described one of the challenges of design-based research as developing research trajectories that prove valuable in terms of achieving goals (improving teaching practice and subsequently student learning outcomes), and developing knowledge that can contribute to the field on a global level. Yet the methodology also provides an opportunity to study a particular aspect of mathematics teaching in depth and with practising teachers in their own context, with the potential for useful insights into both teaching and learning (Collins et al., 2004). Design-based research connects theory and educational practice and is grounded in the “needs, constraints, and interactions of local practice” (Baumgartner et al., 2003, p. 8), which can be useful in gaining insights into how theory can be applied in classroom contexts to achieve desired learning outcomes. In terms of this study, design-based methodology provided an opportunity to study the teaching of mental computation in depth, paying attention to constraints presented by three different school contexts and how the intervention could be applied to each context to improve teaching practice and student learning outcomes.

3.7. Summary

The aim of this study was to investigate the processes involved in teachers learning sophisticated instructional practices through onsite professional learning experiences. The theoretical perspective on learning underpinning the study was social constructivism. Two theoretical models were used to investigate changes to teaching stimulated by professional learning: IMPG developed by Clarke and Hollingsworth (2002) and the MDT developed by Italian researchers (Arzarello et al., 2014). The study adopted a design-based methodology, which was considered analogous with the theoretical models employed for the study. Integral to the study was an intervention to suggest a (potentially) different approach to teaching mental computation. The design of the intervention involved the teachers and I working collaboratively to achieve a change in practice and improve student learning in three different school contexts. An interpretive approach to data analysis was adopted, with the intent to interpret the data reflexively from both the perspective of an experienced primary school practitioner and as a researcher. Multiple sources of data were collected to allow for thick, descriptive analysis (Creswell, 2013). Data were analysed on two different levels: ongoing analysis concerning the teaching and learning of mental computation from the onset of the first intervention; and retrospective analysis of the data on completion of each intervention and after completion of all three interventions (Cobb et al., 2015). Attention was given to describing and analysing connections between the institutional contexts, professional learning experiences and changes in classroom practice. The findings and data analysis for each school intervention are presented in Chapters 4 – 6.

4. Learning and the challenge of contextual constraints

Being puzzled, being unsure, being mistaken, and changing tack through trial and error, seem to be both integral and conducive to creative research. (Minkin, 1997, p. 15)

This chapter reports on the findings of the first intervention cycle. Notably, this was my first experience assuming the combined role of researcher, designer and facilitator of the intervention; one that confirmed the complexities and challenges of studying teacher professional learning. Much valuable learning about the ongoing process of refining and adapting to the social context of the school setting was highlighted, thus emphasising the importance of studying the influence of the institutional context on opportunities for learning.

Findings for this intervention cycle are presented in relation to the two research sub-questions (see Chapter 1, section 3). In response to the first sub-question, change sequences are depicted diagrammatically and learning processes discussed. The influence of institutional factors on opportunities for teachers to learn and change practice are considered, in relation to the second sub-question. In the last section, attention is given to the process of refining the intervention. First, given the nature of this study being design-based research, it is important to describe the context in which the intervention was implemented.

4.1. School context and background

I begin this chapter by setting the scene with a description of the institutional (school) context in which the first cycle of the intervention was implemented.

This is followed by a report on the early negotiation process that evolved in preparing for the intervention.

The first intervention cycle was conducted at a co-educational government primary school catering for Years P-6. The school was located in the Melbourne eastern suburbs region. It had a diverse cultural representation and embraced an international perspective on education as an International Baccalaureate World School (IB school), offering students the Primary Years Programme (PYP). In addition to being recognised as driving high academic achievement, the school prided itself on the extensive range of extra curricula and specialist learning experiences, which included Physical Education, Visual Arts, Performing Arts (which was supplemented by the option of instrumental lessons during school hours) and Languages. Students with additional learning needs were also supported through the provision of Learning Support and English as an Additional Language (EAL). From a parental perspective the school appeared a vibrant, inclusive and supportive learning environment with high academic standards.

From my perspective as an experienced educator in primary education, at the outset the school appealed as a setting in which to conduct this study. Central to the intervention was the facilitation of purposeful mathematics discussion. Teachers seemed receptive to whole class discussion and students generally appeared confident to communicate their ideas within a class community. It appeared that the foundations on which to build mathematical discussion were in place. This school (School A) had also been involved in the pilot of the teaching resources for the intervention, towards the end of the previous school year. The Year 3 teachers, and the acting Assistant Principal (AP) at that time, expressed an interest in participating in the study once the Department of Education and Training (DET) granted ethics approval. This suggested that the involvement of the school in the piloting had fostered an interest and possible sense of ownership with the study (McKenney & Reeves, 2012), thus providing a setting conducive for the research.

4.1.1. Setting up the intervention

When negotiations regarding the intervention recommenced at the beginning of the new academic year, there had been some changes in the leadership at the school, which included a newly appointed Year 3 team leader (Adele) and acting AP. Nonetheless, the new Year 3 team leader initially expressed an interest in the study, noting that a focus on developing reasoning skills would be particularly relevant for the students. However, early indications of potential challenges at this site soon emerged. In the email correspondence that followed, Adele expressed concerns about participating in the research (personal correspondence 23 Feb, 2018). She explained that the transition with the new Year 3 team had not been smooth and she was concerned that participation in the project may contribute to stress levels of some team members. Despite her early concerns, an introductory meeting with the Year 3 team to explain the purpose of the study proceeded, after which team participation in the project was confirmed. It seemed that an opportunity for professional learning from an external source, on an aspect of mathematics the teachers indicated they were not familiar with during the initial meeting, and the provision of teaching resources, was appealing for the team.

An initial challenge was negotiating time to run a professional learning session for the teachers. Adele expressed concern at running a session after school hours due to the leadership focus on managing workload; the year had started with professional learning sessions on personal wellbeing and work-life balance facilitated by a growth coach. In agreement with this goal, it was negotiated that the session would be facilitated during time allocated for planning. As a result of time limitations, I agreed to model the first *sequence* of lessons (initially I had intended to model the first lesson only) with the intent this might provide opportunities for the teachers to engage with the study through brokering and dialogic interactions with me in their classrooms.

A second potential and notable challenge I perceived in the early stages, was the withdrawal of four part-time teachers (from two Year 3 classes) and the unexpected movement of an additional teacher to another area of the school. This concerned me because of the possible impact on opportunities for collective planning and discussion. Although the model of professional growth adopted for this study (Clarke & Hollingsworth, 2002) supports either the cognitive (Hill et al. 2008; Shulman, 1986) or situative (Lave, 1996) perspective on learning, it seemed important to integrate both perspectives in this study, especially as collective participation was recognised as an important component of professional learning (Desimone, 2009). Interestingly, during the two weeks I had been at the school it was evident that year level weekly meetings were not being used for planning purposes as intended, but for administration. In an informal conversation with one of the teachers (Belinda) I learned this situation had occurred because the new team leader needed to familiarise herself with the school systems and review processes. This was corroborated with an informal conversation with Adele (team leader) who explained that she had been purposively moved to Year 3 to improve planning and teaching. Reflecting on this information, collective planning opportunities may have been a challenge with this team regardless of whether or not the whole team was involved in the project. However, being presented with this situation in the early stages of the project was concerning from a research perspective, particularly when the intention was to initiate collaborative professional learning experiences. It later transpired that the situation did present some challenges, which will become apparent in the discussion of the change sequences in the next section.

Details relating to the teachers, who agreed to participate in the study, are presented in Table 4.1.

Table 4.1. *Teacher participants and years of teaching experience*

Teacher name (pseudonym)	Number of years teaching	Number of years in Year 3
Adele (team leader)	10 years	0
Belinda	4 years	3
Clare	Graduate teacher	0

The table above provides a summary of the years of teaching experience for each of the participants. Contextual background information relating to each teacher is discussed in the following section, in which the change sequences for each participant are explored.

4.2. Change sequences

In this section the change sequences I interpreted for each teacher, are presented and explained in response to the first research sub-question. The Interconnected Model of Professional Growth (IMPG) developed by Clarke and Hollingsworth (2002) was used to guide the data analysis and represent the change processes diagrammatically for each teacher. In response to the first sub-question, the changes perceived by the teachers, reported through individual semi-structured interviews, post-professional learning and post-intervention surveys are described. Alongside this data, changes interpreted by the researcher based on observations[7], debriefs, conversations and meetings are discussed. The MDT model (Arzarello et al., 2014) (see Figure 3.2) was used in conjunction with the IMPG (see Figure 3.1) to describe and analyse the development of the teachers' praxeologies within the domains of the change environment. In the following subsections, contextual background information on each teacher is described, then followed by a diagrammatical representation of the change sequence and corresponding explanation for each teacher in turn.

4.2.1. Contextual background: Adele

Adele was an experienced classroom teacher and had senior leadership responsibilities within the school. She had previous experience as a team leader

[7] Only initial lesson observations at the beginning of the intervention were conducted, it was not possible to conduct post-intervention observations at this school.

in the upper primary area of the school, moving to Year 3 at the beginning of the school year in a new role as a Learning Specialist[8]. Adele had started her teaching career in secondary education before changing to primary teaching; her experience was predominantly in upper primary classes. Her lack of prior experience at Year 3 level presented her with some challenges, which she described as a positive professional experience:

> It's been a challenge for me, which is great. It got me to look at the curriculum more and I found that professionally it helped me a lot. (Adele, interview)

This response could be interpreted as indicating that Adele had a growth mindset approach to learning (Dweck, 2000; 2006). This is further substantiated by her behaviour at our initial introductory meeting; she critically reflected on her own practice, sharing that instigating opportunities for students to reason was something she did not do enough. Her reflection suggested that she had a positive attitude towards opportunities for new learning and appeared intrinsically driven to improve her own practice.

In a pre-intervention lesson observation, Adele was observed consistently reinforcing the importance of perseverance and hard work with her class. She openly praised students for "working hard," noting that she was pleased to see some students showing "perseverance" with the challenging parts of the task

[8] The Learning Specialist role had been recently created by Department of Education and Training (DET); each teacher promoted to this position had the responsibility for leading their year level teaching teams. The learning specialists at this school were each given one day per fortnight to work alongside teachers in their year level team; the intention was for them to develop the professional capacity of their team.

(Researcher's journal). During the lesson debrief, Adele shared that her focus for Term 1 had been to build resilience in students. The general feedback from Parent Teacher interviews earlier in the school year, was their children were easily bored and quick to give up when presented with challenges. These observations suggest that Adele recognised the importance of instilling a growth mindset approach (Dweck, 2006) in her students, and could be interpreted as indicating that she personally held these values.

Further evidence that Adele positioned herself as a learner was her interaction during the professional learning session. This was her first exposure to number strings, yet she was willing to take a risk in sharing her ideas and approaches to mental computation. Adele did not consider herself a specialist in mathematics but she had assumed a shared role as a mathematics and science specialist teacher across Years 4 – 6 earlier in her career; this experience could possibly explain her confidence and engagement in the session.

In the following subsection, the changes Adele perceived in her practice, and my interpretation of the changes, in response to various components of the professional learning program, are discussed.

4.2.2. Adele's change sequence: Internalising new learning

Adele's change sequence, as I interpreted it from the data, is presented in Figure 4.1. Each stage of her change sequence; how external stimuli led to change in her Personal Domain (development of new PCK) and subsequent changes in the

Domain of Practice and the Domain of Consequence, is discussed in the following subsections.

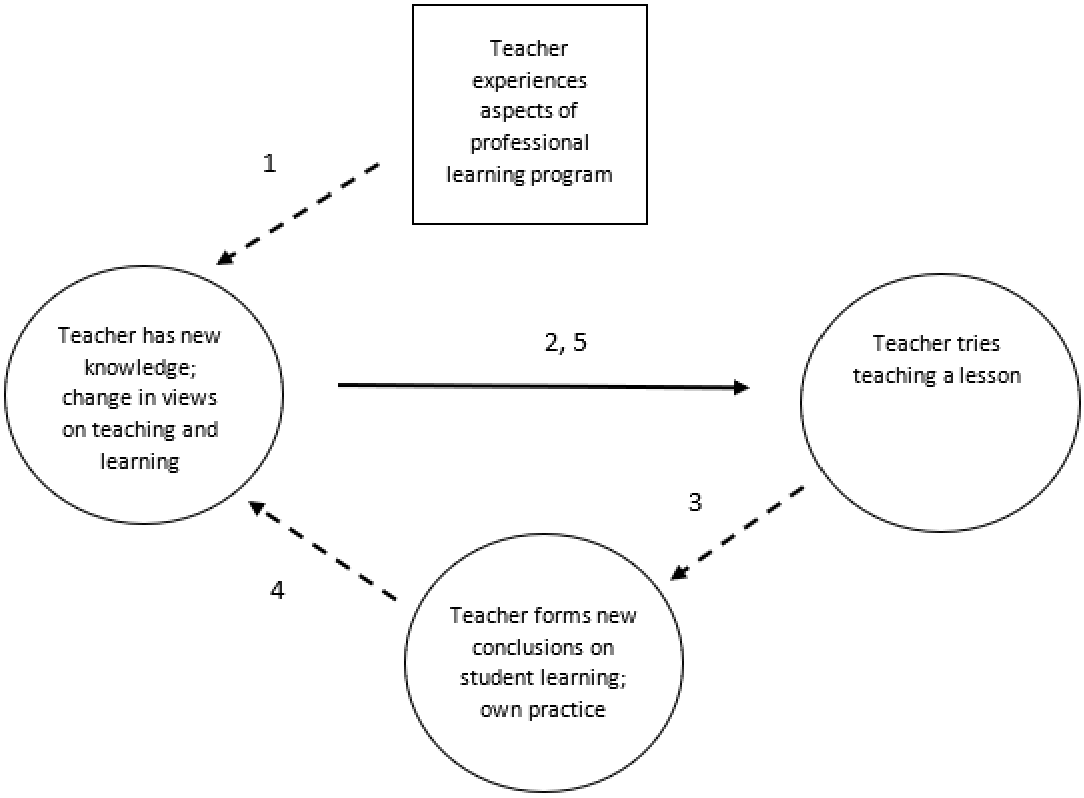


Figure 4.1. Adele's change sequence.

4.2.2.1. Arrow 1: External stimuli and change in the Personal Domain

The introductory meeting about the intervention provided an external stimulus for Adele. During our initial discussion about number strings as "reasoning chains" (Askew, 2016, p. 61), she shared that many of her students struggled to explain their thinking and to reason. Our discussion on the purpose of number strings also led her to reflect on and share a recent teaching experience in which she had introduced mental strategies to her class. She described starting the lesson by modelling some examples of strategies for the students to practise later in the lesson. The idea of using number strings appealed to her as a way to provide opportunity for her students to develop their reasoning skills and as a tool to support a move away from explicit teaching. She further reflected on this in her semi-structured interview:

> I guess that's the one thing that kids struggle with here, is that they are so good at following a method but it's that reasoning that they don't have because they are so used to… vertical subtraction and addition and I think that's what we probably need to focus on a lot more, is that they have that time to reason and that you help build that skill in them. (Adele, interview)

Adele had made an initial connection with number strings as a potential tool to support student development of reasoning skills, a learning focus she suggested would be of benefit to her students. This indicated Adele had reason to engage with the intervention.

Following the introductory meeting, I facilitated a professional learning session in which the teachers were given the opportunity to work through a number string in much the same way that it was intended they would use the instructional tool with their classes. This session appeared to provide a further external stimulus for Adele. She was an active participant and was keen to share her strategies as the group worked through a number string. She was particularly interested in the use of visual representations (and mental jottings) to record student thinking; she took photos of the visual representations I recorded on the board during the session. Her enthusiasm during this part of the session suggested that using visual representations to support conceptual learning might be new pedagogical knowledge, specifically Knowledge of Content and Teaching (KCT) for Adele. Her reflections in her semi-structured interview and final survey corroborated this interpretation. She repeatedly reflected on the use of visual representations in relation to either a change in her practice or a positive impact on student learning, making seven references to either of the aforementioned. When asked specifically about ways in which her participation in the project had contributed to her professional learning, in her interview she commented "I learnt a lot more about number talks and the importance of sharing their thinking on the board visually rather than just orally." This observation reflects findings from Hill and Ball (2009) who suggest that mathematical representation is an area which has potential for improving student learning but one in which teachers need support.

Through the brokering process of experiencing number strings herself as a learner – an approach viewed as important through the lens of a social constructivist and advocated by Fosnot (1996) – Adele was interested in enacting this new knowledge by experimenting with the approach in her classroom. This observation also concurs with general agreement amongst empirical research findings that active learning is an important feature of teacher professional learning (Desimone, 2009).

The opportunity to observe a sequence of modelled lessons provided Adele with an additional external stimulus. Through brokering, it seemed that observing me teach helped her internalise new knowledge about using visual representations to stimulate student thinking and reasoning. In the final survey, Adele identified components of the program that were most helpful; she referred to the modelled lessons twice:

> Professional reading to give context and then *seeing it in action*. It helps me to develop my understanding and confidence so I can implement my new learning.
>
> The modelling sessions so that I could observe the students. (Adele, final survey)

From my perspective as a researcher, I found it interesting that she placed emphasis on the modelled sessions as an important component. During the lessons we had minimal time to converse or to debrief afterwards because her leadership responsibilities (urgent meetings for which she had little advance warning) drew her away part-way through the lessons. Although she managed to observe the instructional tools in action and the whole class discussion, the professional conversations and meta-didactical opportunities for praxeologies to evolve were constrained. The following subsection describes how the aforementioned changes in the Personal Domain, as perceived by Adele, were enacted and appeared to create change in the Domain of Practice.

4.2.2.2. Arrow 2: Change in the Domain of Practice

Adele described how she enacted her new knowledge in the classroom, teaching lessons with a focus on using bridging as a strategy to add mentally. Her enactment of new knowledge is represented diagrammatically as Arrow 2 (Figure 4.1) in her change sequence. A significant change in practice Adele described was allowing time for students to engage with a task first, before asking them to contribute strategies to whole class discussion:

> Get them to have a go and get them to bring their strategies. I didn't give enough time for that…and actually visually put them on the board so all the kids could see them. (Adele, interview)

> So, I've seen the impact it's had on my teaching as well as the students. (Adele, interview)

In the reflection above, Adele mentioned three changes to her classroom practice: allowing thinking time for students to work on a computation without explicit instruction; allowing time for students to explain their strategies to the class; and using visual representations to display student thinking and stimulate further learning. The first change in practice she described is synonymous with allowing student' experimentation before practice: an approach developed by Sullivan et al. (2016) in exploring a structure for mathematics lessons that activate cognition. This change suggests that she internalised the lesson structure exemplified in the modelled sessions. Her reference to using visual representations to share and further stimulate student thinking gives the impression that Adele's praxeology was in the process of change; she described experimenting with a new approach to teaching mathematics that involved a different lesson structure and instructional tools that allowed for more student driven learning.

Through classroom experimentation, Adele started to reflect on her current practice and the school approach to teaching mathematics:

> I find that we do a lot of explicit here [sic] and not a lot of inquiry based, let them explore and find out. (Adele, interview)

In this reflection she refers to two common and contrasting approaches to teaching: teacher-centred i.e., direct instruction, explicit teaching, and student-centred (i.e., inquiry-based), whole class discussions (Prodromou, Robutti, & Panero, 2018). The use of number strings entails a more nuanced approach (an instructional tool proportionally more student-centred). It could be inferred that her classroom experimentation was supporting a transformation of this praxeology to an internal component for Adele. She seemed aware of the differences between this approach to teaching and the style she described as being predominant in her classroom pre-intervention. In the final survey she described the challenges the new approach presented for her students:

> Many of the students initially found it challenging to find other methods other [sic] than algorithms. (Adele, final survey)

Although initially challenging, it would appear, however, that allowing more opportunities for students to think and reason in this way resulted in some positive outcomes for students. In the following subsection, Adele's reflections on her perception of changes in her classroom practice and the subsequent outcomes she considered salient are discussed.

4.2.2.3. Arrow 3: Salient outcomes - change in the Domain of Consequence

Adele reflected on her classroom experimentation and the outcomes she considered salient; her reflection is represented diagrammatically by Arrow 3 (Figure 4.1) in her change sequence. Her reflections largely focused on the positive impact of her classroom experimentation on student learning (seven out of eight references to salient outcomes were coded as being related to aspects of student learning). In her interview, she described an improvement in student engagement, confidence and attitude towards learning as a positive change:

> And then I found the kids, because they like to have their stuff put on the board, they're all wanting to… I've got another one, I've got another one… so that's impacted their engagement, and then to see the impact that's had on what they're transferring and I think sometimes they get stuck in their way with one particular method of doing something. By putting those strategies on the board they start to become more confident trying different things, which is really good.
>
> …The students learned to take more risks with their learning and to look for alternative ways other than using an algorithm. (Adele, interview)

The students enjoyed the opportunity to share their thinking and approaches to computation; having their ideas displayed visually was a pivotal part of the learning process. In describing the impact on student confidence and their willingness to take risks, Adele's reflection could be interpreted as suggesting that students were becoming more flexible in their thinking. This reflects the views of Boaler (2014; 2016) who posited that a change in teacher attitudes to learning can lead to subsequent changes in students' attitudes and learning. It also echoes findings of Heirdsfield and Cooper (2004) in relation to student flexibility with computation strategies. They reported that student beliefs in self and teaching, distinguished between students who were flexible thinkers, confident in developing and using their own mental strategies, and those who were inflexible in thinking and were dependent on teacher-taught procedures.

Student progress in developing skills to explain strategies and use visual representations to show thinking were particularly salient for Adele. This could be connected to an earlier interpretation, that the use of visual representations to show student thinking was new learning for Adele. When asked specifically about the impact of using visual representations in her teaching, Adele replied "hugely, because so many of those kids you could talk about maths for a long time and they cannot connect with it" (interview). It seemed that visually representing student thinking acted as a stimulus for other students. While the use of visual representations had a positive impact on student learning generally, it was most significant for her lower achieving students:

> By drawing a number line it's… opened the door for them to see how you would do it. As I said, with the recent assessment they've done the fact that they actually, some of my lowers in particular, drew a number line how we had been doing it, to do the jump strategy or bridging, they would

> never normally do that so it's fantastic that they've picked that up and continued to go with it which is good. It's had an impact, which is great. (Adele, interview)

In describing the outcomes of an assessment conducted towards the end of the term, some weeks after the teaching of mental computation, Adele's reflection implied that students had retained their learning – an indication that they had developed some conceptual understanding of the strategies learned.

Adele was particularly excited about the students showing that they could transfer their learning; she made various references to this in her interview:

> Normally, with a lot of units, they'll forget and we'll do a revision at the end of the term… 'Ahh…yes we did this but I can't remember' but they're actually independently using that strategy in other areas of maths that isn't just simply addition and subtraction. We were looking at time and they were trying to work out the difference between a time and another time so they used a number line to help them.
>
> … If you look at my lower students and their confidence and the fact that they are transferring that knowledge, we haven't covered that concept for a little while. (Adele, interview)

Her comments also suggested that students had applied their learning to new mathematical situations and that this had been evidenced by visual representations of their thinking.

In addition to the salient outcomes Adele perceived in relation to student learning, she also recognised changes her own professional learning as being substantial. When asked what advice she would give to another school considering participation in the research project, she responded with "take it on because professionally what I got out of it [sic]." She went on to elaborate that what she had learnt would be useful in supporting students learning in the future for…

> … building the proficiencies, and trying to give kids independence to explore mathematics as opposed to doing a lot of explicit teaching. (Adele, interview)

It is interesting that Adele again referred to explicit teaching and her intention to move away from this approach in the future. This could be interpreted as highlighting changes in her pedagogical knowledge and practice she considered salient. In the following subsection, Adele's reflection on the outcomes she considered salient and my interpretation of this connecting to a change in her disposition, will be discussed.

4.2.2.4. Arrow 4: The Personal Domain and a change in disposition

Similar to Guskey's description of teacher change (2002), following Adele's perception of some improvement in student learning outcomes as a result of classroom experimentation, she appeared to display a change in her disposition. This change is represented diagrammatically by Arrow 4 (Figure 4.1) in her change sequence.

Adele reflected on her approach to teaching. Her interview responses suggest that she considered it important to revise her thinking on lesson structure to allow students adequate time to construct understanding of new ideas:

> I guess, analysing how I might structure a lesson.
>
> It's like if we slow down and they get a good grasp it has impact on every other area of mathematics so it's better off to really do it well than to keep pushing along because you want to get through everything. (Adele, interview)

Her comments indicate that she recognised the benefits of adopting this approach to teaching in relation to student learning. This is substantiated further in her semi-structured interview, in which she reflected on procedural approaches to learning, stating that "it's just so much more than that, they're not learning strategies that are going to help them build their mathematics and to be able to do things independently." Her reflection implied further recognition of the importance of students being required to think, explain and reason to facilitate learning. This interpretation of changes in Adele's views on approaches to teaching, namely time for students to think and explain ideas, suggests that this new approach was becoming an internal component of her praxeologies. This interpreted change in her views teaching and learning appeared to drive a desire to change her pedagogical approach in the future. In the next subsection, Adele's reflections on future directions and my interpretation of these are explored.

4.2.2.5. Arrow 5: Change in future practice

Adele's interview responses suggest her intent to continue experimentation with this approach to teaching. This interpretation has been represented as an arrow of enactment from the Personal Domain to the Domain of Practice and is depicted as Arrow 5 (Figure 4.1) on her change sequence diagram.

In her interview, Adele reflected on her experiences with the project and considered how she could apply her learning to future teaching…

> … how can I use the stuff that I've read now in the role that I'm going into and because their focus really is on building the proficiencies.
>
> … You gave us a couple of booklets on number talks and I thought 'I reckon I can use this'. (Adele, interview)

At the time of the interview, Adele had just accepted a teaching position at a different school and was contemplating how she could her could apply her new learning in that role. She described the position at the new school as requiring a focus on developing mathematical proficiencies: understanding, reasoning, problem solving, and fluency (ACARA, 2014).

4.2.2.6. Adele's change sequence: A summary

Adele's change sequence suggested that reflection on various external stimuli – professional learning session; observation of researcher modelling lessons; teacher resource book, and professional reading materials – initiated a change in the Personal Domain. The initial changes Adele described were interpreted as being predominantly in aspects of PCK. Learning about using visual representations to show student thinking and adapting lesson structure so that students were engaged in thinking before explicit instruction, were most consequential. Adele internalised her new knowledge through classroom experimentation. She reflected on this experience and described various salient outcomes: student use of computation methods other than written algorithms; development of student skills to reason and explain thinking, improved student engagement in lessons; and a growth mindset approach to new learning. There was a sense of passion and enthusiasm in the way she expressed these changes. This is reflected in the number of references she made to her perceived changes in practice and the subsequent impact on students throughout her interview. The salient outcomes seemed to result in a change in Adele's disposition: she appeared to internalise a new approach to teaching. Her change sequence reflected actions of a teacher who evidenced a growth mindset approach to learning and positioned herself as a learner from the outset of the project. A summary of Adele's change sequence is presented in Table 4.2.

Table 4.2. *Summary of Adele's change sequence*

Arrow	Domain Link[9]	Mediating Process	Description of learning process
1	ED to PD	Reflection	Reflects on various external stimuli: introduction to number strings, professional learning session, observing students in modelled lessons, professional reading and teacher resource book. She has new PCK to facilitate student centred learning and support a transition from explicit teaching.
2	PD to DP	Enactment	Enacts new knowledge about teaching mental computation with conceptual understanding. She teaches a lesson on using the bridging strategy to add mentally. She focuses on allowing time for students to engage with the task first and use of visual representations to share their thinking with the class.
3	DP to DC	Reflection	Reflects on student learning outcomes: positive impact on engagement, confidence, attitude towards learning (more flexible in thinking); student development of skills to articulate strategies and represent thinking visually; students retaining and transferring learning. Reflects on outcomes in terms of own practice: transition from explicit thinking; allowing students to think and reason.
4	DC to PD	Reflection	Change in her views on approaches to teaching and learning: recognises importance of students being asked to think, explain and reason to facilitate learning.
5	PD to DP	Enactment	Intends to use this approach for future teaching of computation strategies.

4.2.3. Contextual background: Belinda

Belinda had commenced her teaching career at the current school four years prior; this was her third year of teaching Year 3. She had been involved in the pilot of the teaching resources for this study; at that time, she expressed an interest as a future research participant.

[9] The codes representing the Domain links are: ED (External Domain), PD (Personal Domain), DP (Domain of Practice) and DC (Domain of Consequence).

Belinda appeared to position herself as a learner in the class community; this was something she was observed expressing openly with the class. She was enthused when she explained to the class that they would be working with someone from Monash University so that everyone, including herself, could learn some new strategies to solve problems mentally (Researcher's journal). This indication that she had a growth mindset approach to learning was reinforced during the pre-intervention observation and throughout the modelled lessons; she consistently praised students for "having a go" and encouraged students by affirming that it was okay "to make mistakes" when learning (Researcher's journal).

In the pre-intervention observation, the majority of students in her class were seen to be struggling with adding 2-digit and 3-digit numbers. The students were insistent on using a written algorithm but due to fundamental misconceptions with the procedure and place value, their answers were mostly inaccurate. In an informal conversation following the pre-intervention observation, Belinda expressed concern about huge gaps in student knowledge; she hoped that this approach would help bridge some of those gaps by allowing students time to develop and experiment with new strategies. This awareness of the learning needs of the students in her class provided her with a reason for participating in the project.

As noted earlier, Belinda was involved in the piloting of the teaching resources for this study, and I was consciously aware this may have influenced her responses to some of interview questions. In the pilot she had already observed my teaching of her students and we had engaged in discussions about the development of the teaching resources for the intervention. In the following subsection, the changes Belinda perceived in each domain and my interpretation of the changes, in response to aspects of the professional learning program are presented.

4.2.4. Belinda's change sequence: Developing knowledge and confidence

In this section, each stage of Belinda's change sequence will be discussed: my interpretation of her change sequence is depicted in Figure 4.2. While there were similarities between the two change sequences for Belinda and Adele, the key difference between the changes experienced by these two teachers seemed to be the types of changes within the Personal Domain and the emphasis each participant placed on the importance of certain external stimuli. First, the various external stimuli, which instigated initial changes for Belinda, will be discussed.

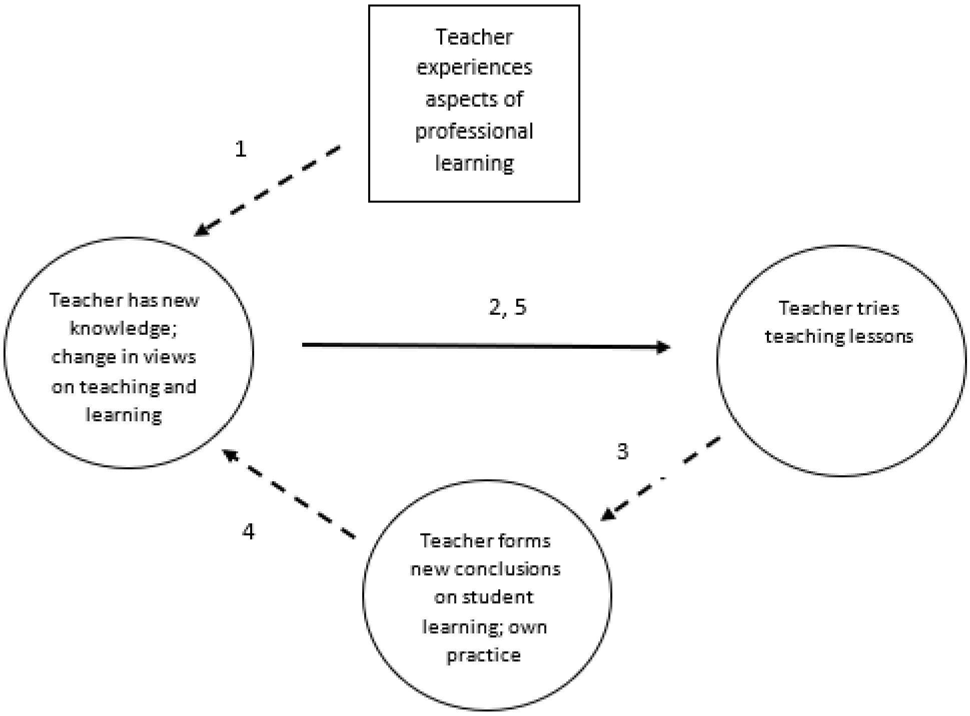


Figure 4.2. Belinda's change sequence.

4.2.4.1. Arrow 1: Change in the Personal Domain

Access to a teacher resource book was an important external stimulus for Belinda. As part of the professional learning program, teachers were each given a copy of the (researcher-developed) resource book. In addition to providing background information specific to the design of the intervention, the book also included brief explanations of various mental strategies, key pedagogical considerations, possible student responses to the tasks, and examples of corresponding visual representations. In her interview, Belinda described how she referred to the book to develop her awareness of the different ways students may respond to a task. She explained that she "… read through it because I was, what happens if there's a student who has a different idea? A different answer?"

Belinda's reflection suggests she considered it important to anticipate student responses in advance, to have some insight into how the whole class discussion may unfold. The section of the book providing examples of possible student solutions, and how these could be represented visually, was particularly useful. Belinda's thought processes, in relation to the importance of anticipating, correlates with the first of five key practices described by Stein et al. (2008) for helping teachers facilitate productive class discussion.

The professional learning session provided a further external stimulus for Belinda. In her survey feedback she rated it as being 'very useful' (the highest rating on the scale). Her given reason for the rating was "I think mental computation is lacking in the curriculum." It could be inferred that she found working through examples of tasks and discussing various strategies useful because she wasn't familiar, or maybe lacked confidence, with this content. Belinda made a further comment relating to the curriculum content in her semi-structured interview, which seemed to corroborate this surmise saying that "in the curriculum it just says to solve addition and subtraction using various strategies. Well, what are the strategies? I think it needs to be more explicit."

Although Belinda did not explicitly identify the modelled sessions in the interview as an external stimulus, she did reflect on her observations of the students and their initial reactions to the approach:

> They weren't open minded, they weren't saying well, 'I am willing to have ago at doing it with a different method.' I just think they are so used to

> one way it was, well 'why do I have to do it that way?' They were just not wanting to do it at all. (Belinda, interview)

Her reflections on the modelled lessons implied she had noticed some initial student resistance to learning new strategies; the new approach challenged their thinking. As aforementioned, the students had all been exposed to formal written algorithms and considered this as the only way in which to approach a task. This observation was discussed informally in conversations both within and following the lessons (Researcher's journal). This noticing seemed to activate a desire for her to think about how she could open students' minds to learn alternative strategies and explain their thinking. In her interview she commented that it "… changed my perspective because I thought 'okay there are different ways to solve problems, how can we, how can I show them that?'" Her reflection suggests an interaction taking place between her new knowledge about different mental strategies (gained through the PL session and reading the resource book) and her observation that students were rigid in their thinking and challenged to justify their thinking. This instigated an aspiration to use her new learning to bring about change in student learning.

It is possible that Belinda did not consider the modelled sessions as notable because she had previously observed my teaching when I trialled the teaching resources. However, from a researcher perspective, it was notable that there was greater interaction between Belinda and myself in these sessions, in comparison with the other two teachers. For example, during the sessions we had a brief discussion about whether we needed to repeat the 3-phase lesson cycle (launch-explore-summary) to allow opportunity for more students to further explore and test strategies. Belinda became involved in some of the decision-making and steering the direction of the lessons; this difference may simply be explained by the pre-existing relationship between us. This level of dialogic interaction and brokering allowed enhanced opportunities for transformation of praxeologies (Arzarello et al., 2014).

The data suggested that the professional dialogue regarding student learning during and immediately after modelled lessons; the opportunity to engage with mental computation tasks in the professional learning session; and access to further information in the teacher resource book, acted as a catalyst for Belinda to experiment with enacting her new knowledge in classroom.

4.2.4.2. Arrow 2: Change in the Domain of Practice

Belinda experimented with enacting her new knowledge in the classroom; the interpretation of this is represented diagrammatically as Arrow 2 (Figure 4.2) in her change sequence. She reflected on changes to her classroom practice; in the final survey she described her biggest change as "using different strategies to teach my students." Experimenting with the new approach seemed to create further opportunities for her to develop her content knowledge on how students learn (KCS). She noticed how students tended to develop their own variations of strategies:

> With some of them they might, for example, when they did their jump strategy they skip counted by different numbers but then they ended up

> getting the right answer instead of the most simple way [sic]. (Belinda, interview)

This reflection suggests that through experimentation Belinda was forming new ideas about how students learn mathematics; that when students construct strategies they build on their existing knowledge (Heirdsfield, 2002). For some students this meant working with smaller numbers, which involved more steps to work out the answer. There was almost some element of surprise that students could achieve success with the tasks by developing their own strategies, even if not necessarily the most efficient approach.

Through experimentation Belinda began to internalise her new knowledge about mental strategies. She reflected on the difference between this way of teaching and her personal experiences learning mathematics in her interview, explaining that "… she was always taught vertical addition… and it's rote learning. But this way is showing and explaining what you are actually doing as well." Asking students to explain their thinking and justify their strategy was a new approach, which contrasted with the way she had been taught. She seemed to recognise that the rote learning approach that she experienced as a student was ineffective if the goal was for students to think and reason. Through the eyes of a social constructivist, it is important to have an experience in a social context to allow personal knowledge to be constructed. Fosnot (1996) posited that teachers need to be engaged in new approaches "in experiences where they can study children and their meaning-making, and in field experiences where they can experiment collaboratively" (p. 216), if they are to adopt reform pedagogy in their classrooms.

4.2.4.3. Arrow 3: Salient outcomes

Belinda reflected on her classroom experimentation and the outcomes she considered salient; her reflection is represented diagrammatically by Arrow 3 (Figure 4.2) in her change sequence. A particularly salient outcome for Belinda was student progress with explaining their thinking; an aspect of learning she highlighted as an area of concern in the early stages of the intervention:

> Confidence with using different strategies… (Belinda, final survey)
>
> … They can explain explicitly what answers they got and how they got there… are able to explain it, confidently. (Belinda, interview)

She described development of student skills to articulate thinking and a subsequent growth in student confidence. In reflecting, she also commented on the benefits of the new approach in relation to students' retaining learning and developing understanding:

> It was really good and they actually remember it.
>
> … And it's not just about vertical addition and subtraction, it's different methods of actually doing it and they actually understand the importance of it.
>
> … I was really happy with the kids and the progress in their understanding. (Belinda, interview)

Based on the second quote above, it could be inferred that Belinda noticed a change in students' disposition; they seemed to recognise the value of what they were learning. This was a significant shift for her students, who at the outset conveyed some resistance to a different approach to learning mathematics.

Belinda identified various outcomes related to student learning which she considered salient: progress in articulating thinking, and subsequent growth in confidence; signs of productive disposition (students' recognising the value and purpose of what they were learning); and students retaining learning. Her recognition of salient outcomes was interpreted as connecting to further changes in her Personal Domain, which will be discussed in the next subsection.

4.2.4.4. Arrow 4: The Personal Domain

Belinda's perception of positive outcomes in relation to student learning indicated subsequent changes in her Personal Domain. This change is represented diagrammatically by Arrow 4 (Figure 4.2) in her change sequence.

In reflecting on the positive outcomes perceived in students' progress with learning new mental strategies and developing skills to articulate their thinking, Belinda seemed to form a new view on student learning of mathematics:

> I think something sequentially is very important for them to build on what they know. To build on that lesson and then they can go build on it a bit further. It's repetition, it takes time. (Belinda, interview)

This reflection suggests that Belinda's praxeology is moving towards a shared praxeology; it seems she was beginning to recognise that given time, students can build on existing knowledge to form new ideas.

In addition, her interview comments indicated she had formed the view that students need to be able to articulate their thinking clearly; "like trying to explain their thinking, and initially they weren't able to do it at all. I think it's something that's really, really important." Her reflection suggests that she has started to internalise the view that student explanations are an important part of learning mathematics. She expressed a viewpoint on the importance of students learning to compute mentally:

> I think there has to be a continuous focus, especially for those children in Grade 3[10]. They need to know everything, to be able to say, 'How will I work this out and how can you actually demonstrate that?' I think it needs to be across the board and not just this one focus. (Belinda, interview)

In suggesting that a focus on student thinking and explaining ideas needs to be integrated across all aspects of mathematics, Belinda indicated that this new approach to teaching was being transformed from an external to an internal component of her praxeologies.

Belinda also recognised important outcomes in terms of her own professional learning. In her interview she reflected on her own learning, "How can we actually organise it to show their thinking? I think that's what I took away." The

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[10] The participant, Belinda, used this terminology to refer to the Year level of the students.

changes that Belinda described in this excerpt relate to her KCT, how to use visual representations to communicate thinking and stimulate discussion and learning for the class. In her response on her post-professional learning survey, she acknowledged that she learned to manage different ways that students may approach a task, "how to approach different student's learning during the lessons." It could be interpreted that this is what Stein et al. (2008) referred to as "monitoring student responses" (p. 326). It involves interacting with the students, listening to their ideas and trying to make sense of what they are thinking. Stein et al. (2008) pointed out, that this is usually easier for teachers who have focused on anticipating student responses during the planning stage, a characteristic that differentiated Belinda from the other teacher participants.

Further changes in the Personal Domain, on which Belinda commented in her interview, concerned her disposition. She shared that she "… became more confident in my ability… I really enjoyed it because it showed me different ways that I can teach the kids how to do it with addition and subtraction." In this excerpt she referred to growth in her own confidence, an element of teacher efficacy (Cooke, 2015), to think mentally and interpret student thinking. She conveyed a sense of satisfaction in her perceived professional growth.

4.2.4.5. Arrow 5: Change in future practice

Belinda indicated an intention for the Year 3 team to incorporate mental strategies into their teaching program in the future; this reflection has been represented as an arrow of enactment from the Personal Domain to the Domain of Practice and is depicted as Arrow 5 (Figure 4.2) in her change sequence.

She shared her intent to continue to focus on exposing students to various strategies to compute mentally in her interview, commenting that "we actually put it in our future learning for next year, for doing different strategies." This suggests she has internalised the importance of giving students opportunities to think, reason and justify different computation strategies.

4.2.4.6. Belinda's change sequence: A summary

Belinda reflected on her experiences in the professional learning session and observation of her students in the modelled sessions. Through the brokering process and dialogic interactions, a change was initiated in her Personal Domain. She recognised a need to experiment with a new approach to teaching computation with her students. She also described how she referred to the teacher resource book to further develop her content knowledge and support a change in her practice. The initial changes Belinda described were interpreted as being in aspects of knowledge: Specialised Content Knowledge (SCK) and Pedagogical Content Knowledge (PCK). Her self-reflections predominantly focused on her development of content knowledge to support a change in practice. Belinda, placed emphasis on perceived change in student confidence and positive attitude to learning. In addition, she reflected on growth in her own skills and confidence with teaching. Belinda certainly appeared to welcome an opportunity to learn and improve both her practice and learning outcomes for students in her class. A summary of Belinda's change sequence is presented in Table 4.3.

Table 4.3. *Summary of Belinda's change sequence*

Arrow	**Domain Link**[11]	**Mediating Process**	**Description of learning process**
1	ED to PD	Reflection	Reflects on opportunity to develop content knowledge in the professional learning session; extends this by accessing the teacher resource book. Reflects on student response to new learning in the modelled lessons; their rigidity and challenge to explain their thinking. She aspires to improve student learning by using her new knowledge and experimenting with this new approach to teaching computation.
2	PD to DP	Enactment	Internalises new knowledge through enactment in her classroom; she teaches lessons involving the jump strategy.
3	DP to DC	Reflection	Reflects on salient outcomes related to classroom practice: improvement in student articulation of thinking and confidence, a shift in student attitudes to learning, students retaining learning.
4	DC to PD	Reflection	Forms a new view on how students learn: she recognises the importance of allowing students time to develop ideas sequentially; articulating thinking. Forms new view on approach to teaching: recognises the value of asking questions to eliciting thinking and use visual representations to facilitate learning. Identifies growth in own confidence to thinking mentally and display student thinking visually.
5	PD to DP	Enactment	Intends to continue teaching mental computation strategies in the future, as part of the Year 3 mathematics program.

4.2.5. Contextual background: Clare

Clare was a graduate teacher who had completed her final placement at this school in a Year 2 class and had subsequently been offered a teaching position. This was her first experience of teaching Year 3 students. She was perceived as a young, energetic and enthusiastic teacher who initially expressed a keen interest

[11] The codes representing the Domain links in Table 1 are: ED (External Domain), PD (Personal Domain), DP (Domain of Practice) and DC (Domain of Consequence).

in participating in the project. In the post-professional learning survey, she indicated that the sequence of learning activities would be "very effective" in relation to improving student learning of mental computation in her class.

Clare was observed teaching her class pre-intervention. At that time, the students were completing a mathematics investigation to demonstrate their learning of addition and subtraction, which had been the focus over previous weeks. It was interesting to note that when students approached her to check sections of the task, she reached for a calculator to quickly confirm if the answer was correct (Researcher's journal). This seemed to be due to the pressure of time, based on the short exchange of conversation we had during the lesson, but it was interesting that the focus was not on checking the process students had used. This could, perhaps, provide an indication of her level of confidence with her own mental computation.

4.2.6. Clare's change sequence: Reflecting on learning experiences

Clare's change sequence, as I interpreted it from the data, is depicted in Figure 4.3. Despite her apparent interest in participating in the study, it appeared that Clare did not experiment herself with the approach in her classroom. In the following subsections her change sequence will be unpacked and discussed.

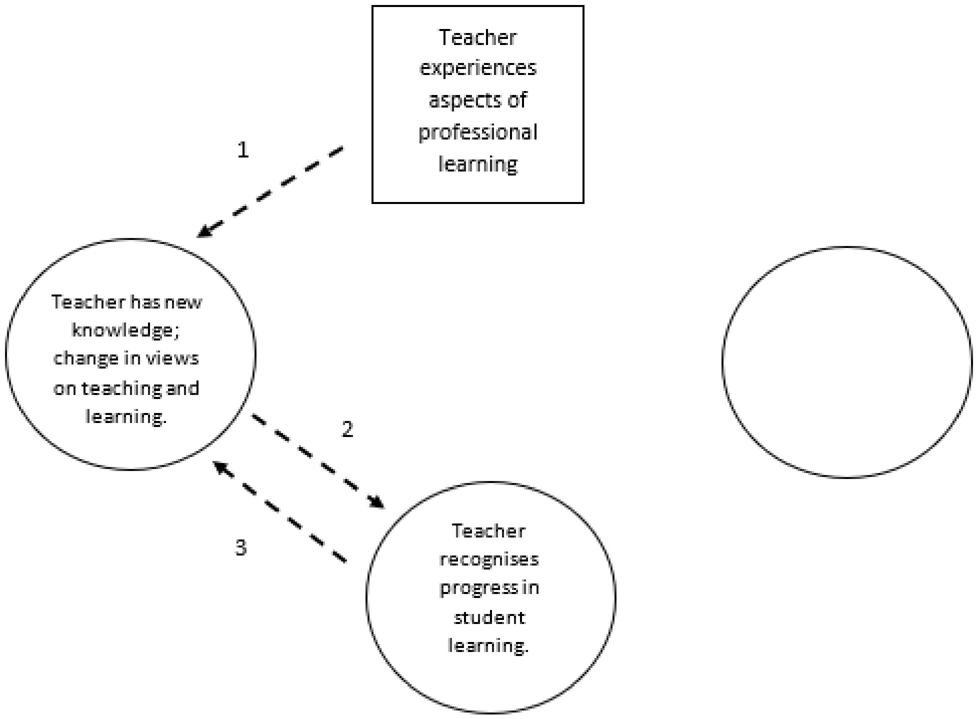

Figure 4.3. Clare's change sequence.

4.2.6.1. Arrow 1: Change in the Personal Domain

The professional learning session provided Clare with an external stimulus. Although very quiet in the session (she did not contribute to discussions), in the survey she completed at the end of the session she indicated the project would be worthwhile for her class:

> Teaching the students the mental strategies behind addition/subtraction is extremely effective since most students are unable to work out the answer without doing a vertical algorithm. (Clare, post-PL survey)

From her comment it could be inferred that her students were familiar with, and perhaps reliant on, written algorithms to perform calculations (triangulation with the pre-intervention lesson observation confirmed this interpretation). The project certainly had potential to support the learning needs of most students in her class, thus providing her with a reason to participate.

The modelling of a sequence of lessons seemed to provide an additional, possibly more valuable, external stimulus for Clare. In completing the post-professional learning survey, she indicated an interest in observing the approach in action by stating it was "useful information – interested in seeing it implemented in the classroom."

The modelled lessons provided some opportunities for brief professional conversations in which the pedagogical approach could be clarified and observations of student learning exchanged. During the first lesson, Clare asked for clarification on the design of the number string; she was unsure how she would know what equations to include in future strings. Although she had just observed a number string being used, she also asked for verification of the questions to ask students when presenting the string. Clare was confused by the example we had discussed in the professional learning session, which was different and therefore involved different discussion points (Researcher's journal). Her questions conveyed the impression that she was interested in learning more about the approach, but also a sense that she was lacking in confidence to implement it immediately in her classroom.

Clare reflected on her observation of the sequence of lessons in her interview, indicating that the approach was different to how she would usually teach because "there would have been more on the actual vertical and carrying and things like that. More of a focus on that." Clare's reflection suggests that her experience of teaching and learning was predominantly traditional in approach, with a focus on written algorithms. It seemed that observing a sequence of modelled lessons provided an opportunity for Clare to develop aspects of KCT.

Through observing students in her class, Clare became aware not only of some gaps in their learning but also of the complex, interconnected nature of learning mathematics:

> I have learned that a lot of them really struggle with place value; to be honest I didn't think that was such a big thing until you came in... Just little things like that and I was 'wow' we need to go back to what we did in Grade 2[12]. (Clare, interview)

Through observing and supporting students in the lessons, she noticed that her students had some misconceptions with place value. It is possible that this gap had previously gone unnoticed because the focus had been on teaching written methods and conventional place value (Wright et al., 2012). Many students were challenged to add a multiple of ten to a number off the decuple (Wright et al., 2012):

[12] Clare used this terminology to refer to the Year level of the students.

> Just 16 plus 10 and it was like, just change to 2 but they really struggled with that. Or plus 20 and they didn't understand that you just have to add 2 to the tens column. (Clare, interview)

Clare's reflections suggest some element of surprise that students struggled to add multiples of ten. Her comments also indicate that her understanding of place value was nested within a conventional approach (Wright et al., 2012), more appropriate for learning written methods of computation. There were a few opportunities during the lessons for Clare and I to share and briefly discuss examples of tasks students were finding challenging. Through the brokering process, Clare was exposed to the importance of discussing computation in terms of quantity value of numbers (Researcher's journal). It would be reasonable to expect a graduate teacher to have limited knowledge of content and how students learn (KCS), and possibly also Specialised Content Knowledge (SCK), at this stage in her career. However, it appeared that an opportunity to observe her students and engage in professional dialogue had stimulated learning in aspects of her mathematical knowledge for teaching.

In observing her students, Clare also noticed some positive learning outcomes. Her reflections on the outcomes she perceived as salient are discussed in the following subsection.

4.2.6.2. Arrow 2: Reflecting on salient outcomes

Clare described some changes she perceived in student learning; these changes were interpreted as salient outcomes for her. The examples given all connected to the sequence of modelled lessons observed, rather than examples of lessons she had taught. In her final survey, she identified students' learning, "other ways of working out addition and subtraction problems that isn't a vertical algorithm [sic]," as the most important outcome for her. When asked to describe further the impact of the project in terms of student learning she identified the following:

> Explaining thinking… Development of strategies – students used this in their school tests which was great to see. (Clare, final survey)

In the comment above, Clare referred to student completion of an assessment task some weeks after the intervention, in which most applied their learning of the jump strategy.

In comparison with the other two teacher participants, Clare was interpreted as making the least number of references to salient outcomes from participating in the project. Through the lens of a social constructivist, this could be partially explained by the evidenced lack of experimentation with the approach herself.

4.2.6.3. Arrow 3: Changes in views on teaching and learning

In reflecting on her own learning, Clare described what she considered most valuable:

> The most valuable thing I learned, I think it's just taking it down to basics. I think we always try and build them up, this is a harder sum, but I think with just stripping it down to the basic thing and if they

> understand that then they will be able to use it for bigger numbers. (Clare, interview)

Clare's reflection on the importance of focusing on the 'basics' suggested a change in her views on how students learn to calculate. When initially asked if the approach was different to how she would have taught addition and subtraction, she commented that she "… would probably have gone back to those basics as well," but then further elaborated and described how the focus would have been on the vertical written algorithm. She indicated that she regarded use of the number line as being appropriate for the lower year levels…

> … That's the younger years. Or I would normally use a number line if the kids were really weak in Grade 3[13], that's when I would use it as opposed to using it for the whole class. (Clare, interview)

As a first-year teacher, Clare seemed to be grappling with the ideas of how children learn to add and subtract and what is considered developmentally appropriate. Her initial thoughts and comments suggested that she viewed student learning as a linear process; she considered certain content appropriate for certain year levels i.e. learning of vertical algorithms for addition and subtraction in Year 3, rather than first ascertaining what students know and using that as a springboard for future learning. However, in her final interview, Clare's reflections indicated a change in her perspective on teaching computation; that mental computation and starting "with the basics" would be

[13] Clare used this terminology to refer to the Year level of the students.

more beneficial for students and that she could "… see the benefit of teaching mental computation – to build the foundations for high school and upper primary years." Participation in the project had started to instigate some changes in Clare's views on teaching and learning addition and subtraction.

4.2.6.4. Clare's change sequence: A summary

Clare's change sequence suggested her reflection on external stimuli (professional learning session and observation of modelled lessons) initiated a change in the Personal Domain. The changes interpreted were in aspects of knowledge (KCS and SCK). Clare developed awareness of gaps in student learning with place value and of strategies other than standard algorithm for teaching addition and subtraction. Through observation of modelled lessons and professional dialogue, Clare recognised positive outcomes in student learning. Her reflection on these outcomes and formation of new ideas on how students can learn addition and subtraction indicated early changes in her perspective on teaching and learning computation.

The interpretation of Clare's change sequence suggested that she was yet to internalise her new knowledge through classroom experimentation. Based on the discussion in the modelled sessions, it seemed that Clare was not yet confident in implementing the teaching approach herself. Perhaps if she had the opportunity to participate in collective planning, or co-teach with either the Year 3 leader or myself, both her learning and perceived salient outcomes would have been enhanced. During the classroom interactions with the researcher, Clare had been interpreted as exhibiting a desire and interest to learn more about the approach and improve student learning. The focus of the intervention was on developing pedagogies to enact ambitious instruction, which has been reported as being challenging for novice teachers and noted an area in which requires further focus for teacher educators (Kazemi et al., 2009).

It is reasonable to expect that an approach to teaching mental computation that requires students to think and reason to develop conceptual understanding may present some challenges to teachers in terms of their subject and pedagogical content knowledge. Although Clare was interpreted as experiencing some development in her knowledge, this required further internalisation. Clare (final survey), was asked to reflect on the most valuable thing she learned from participating in the project, she wrote…

> … How to teach students addition / subtraction using a number line.
>
> 52 + 73 =
>
> 50 + 70 + 2 + 3 =
>
> Thinking like this!

The example she provided in the survey indicated a misconception with the goals of the project and also the subject content. The focus of the project had not been on teaching students to use an empty number line (ENL); rather, the ENL was used as one tool to represent student thinking alongside alternative mental jottings. The mental jotting shared in her response is an example of the split strategy (or partitioning strategy), which was also not a focus of the intervention.

The split strategy is also not appropriate for representing on a number line. It seemed that Clare needed further opportunities for support, or exposure to external stimuli, to enact and internalise learning about mental computation strategies. A summary of Clare's change sequence is displayed in Table 4.4.

Table 4.4. *Summary of Clare's change sequence*

Arrow	**Domain Link**[14]	**Mediating Process**	**Description of learning process**
1	ED to PD	Reflection	Reflects on learning about mental strategies and new pedagogies experienced in professional learning session and observed in modelled lessons.
2	PD to DC	Enactment	Reflects on salient outcomes related to student learning: progress with learning mental strategies and skills to articulate their thinking.
3	DC to PD	Reflection	Reflects on progress in student learning and begins to revise her stance on approaches to teaching computation and the importance of students learning to compute mentally.

4.2.7. Change sequences: A summary for school A

Data on teachers' perspectives on the intervention, the outcomes they considered salient, and their perceived changes in knowledge, disposition and practice, were analysed alongside observations by the researcher to gain insights into possible pathways for change (Clarke & Hollingsworth, 2002). All three teachers appeared to experience a change from a stimulus in the External Domain to a change in the Personal Domain. The data indicated two different change sequences for the three teachers. For two of the teachers, Adele and Belinda, an external stimulus seemed to lead to initial development of new knowledge; the teachers evidenced developing their PCK by learning how to use visual

[14] The codes representing the Domain links are: ED (External Domain), PD (Personal Domain) and DC (Domain of Consequence).

representations to display student thinking and move forward the learning of their class. This new learning was enacted through classroom experimentation. The third teacher, Clare, also experienced an initial change in the Personal Domain but the data suggested that Clare was yet to enact this new knowledge in the classroom. Instead, through the mediating process of reflection, a change in her disposition regarding approaches to teaching computation was perceived.

The data were analysed to explore how aspects of the External Domain might have stimulated a change in aspects of each teacher's Personal Domain and changes in their teaching practice. Modelling the approach, essentially the brokering process, was perceived as most significant for two of the teachers (Adele and Clare). From my perspective, the opportunities for dialogic interactions with the teachers during these sessions seemed particularly useful for clarifying questions or misunderstandings. Access to the teacher resource book was considered pivotal in instigating change for the other teacher (Belinda).

A salient outcome for all teachers seemed to be their noticing of the development of students' skills to explain their thinking and reasoning. Additional student outcomes considered salient were the retaining and transfer of learning, and change in student engagement and attitudes to learning. The outcomes seemed to provide a catalyst for future learning and change in practice; two of the teachers indicated their intent to continue with experimentation of this approach to teaching following the end of the project.

Although all three teachers perceived some gains in terms of knowledge and/or practice from participation in the learning experiences, these changes were both afforded and hindered by some aspects of the institutional context. The influence of these institutional aspects on the professional learning of teachers will be explored in the following section.

4.3. The change environment: The institutional context

In this section, analysis of data concerning the second research sub-question, the influence of institutional aspects on opportunities to learn and change practice is discussed. The Meta-Didactical Transposition (MDT) model (Arzarello, 2014) was used to examine the influence of institutional affordances and constraints on changes to teachers' knowledge, practices and disposition (refer to Figure 3.3).

The school environment had certain structures in place to afford opportunities for teachers to develop their professional practice. For example, weekly team meetings were scheduled for planning purposes. In addition, each term, Staff Professional Practice Days allowed for year level teams to meet for the day to work collaboratively on developing teaching programs. The collaborative approach within the school potentially provided the setting to support the approach of the intervention. The ethos of the IB Primary Years Programme (PYP) also endorsed the pedagogies the intervention aimed to develop in the teachers i.e., students as confident communicators who can articulate and justify strategies. The teachers' perceptions of changes in their knowledge and practice, discussed in the previous section, suggest that there were affordances from implementing the intervention within this school context. However, there were various factors, both external and internal, seen to constrain changes in teaching practice on both an individual and team level. Analysis of multiple data suggested these constraints can be categorised under two main themes: the challenging of finding time to implement the program; and issues with communication constraining developments in practice. Analysis of the data and how it relates to each theme will be discussed in the following subsections.

4.3.1. The challenge of finding time to implement the program

Analysis of the data indicated two common foci related to time as a constraint on the implementation of the program: the timing of the intervention itself, and time available to teachers to implement the intervention. A summary of data highlighting teachers' perceptions of time as a constraint is presented in Table 4.5.

All three teachers commented on the impact of the timing of the implementation of the intervention. The comments suggest that the content of the intervention and the Year 3 mathematics program were misaligned. The focus of the intervention was on developing mental computation strategies for addition and subtraction.

Table 4.5. *Summary of data indicating two common foci related to time as a constraint to change, as perceived by the teachers*

Illustrative quotes indicating timing as a constraint	**Data indicating limited time to implement the intervention as a constraint**
I think timing is really important. I think timing is a really important thing. (Belinda, interview) Obviously when you came we weren't doing addition and subtraction. It would have been great if it had been at the start of the year because that's when we were focusing on addition and subtraction. (Clare, interview) I would have loved to have started it when we first started our addition and subtraction unit (Adele, interview) I really want to tease something out but it was so hard, especially because we had done subtraction and addition and then we did more of it, so we spent all of Term 1 doing addition and subtraction. So it was a long term just to do that. So that would be my only criticism time. (Clare, interview) I would say that if they are concentrating on addition and subtraction that this is a really valuable program. (Clare, interview) Due to the timing of the project and how busy the school is, there was lack of consistency and I feel this may have impacted the success of the project. (Adele, final survey). So I guess the timing was challenging when it was implemented and I think maybe if we had started earlier in the term we would probably have had more growth and been able to come back to it again (Adele, interview)	Yes, I just think allowing more time for it. (Adele, interview) I think making the time to actually have the lessons to do. (Belinda, interview) I think sometimes the curriculum is so dense that we do often rush through things, we don't give them enough time and that's a huge injustice to students (Adele, interview) there's so much in the curriculum that you push to get everything done in the short time frame that you probably don't feel like you give enough time for different things. (Adele, interview) The time. Time to be honest. I wish we could have teased it out a lot more, yeah it was just…the curriculum is so packed and I really find that we are always switching and doing a different thing each week (Clare, interview) Time – not having enough time in the term to implement this. (Clare, final survey)

However, at the time the intervention commenced, students had just completed a unit on addition and subtraction. The team leader suggested that the intervention would be a good opportunity to build on and develop learning in this area. She was aware that there had been minimal focus on mental computation, and that assessment tasks indicated gaps in student learning related to computation. However, in reality it seemed that the time allocated for this topic in the school mathematics program was no longer available.

An issue which Adele described as a significant constraint on the implementation of the program, which was related the timing of the intervention, and also constrained teaching time were assessment requirements (both external and internal):

> I felt like then NAPLAN[15] came in and that impacted it because we had certain things we had to cover for that and then on top of that it's an assessment term, so we had set assessment we had to get done that were dictated by leadership. So constantly those things that come in take away your time and you start to get freaked out about getting everything done and I felt that throughout my team, that they were feeling that. (Adele, interview)

Assessment requirements were identified as impacting the availability of adequate teaching time to implement the intervention. The completion of student portfolios was an internal factor, which had a significant impact on the teaching program. In the week prior to implementation of the intervention, the three hundred minutes allocated to mathematics lessons for the week was

[15] The National Assessment Program for Literacy and Numeracy (NAPLAN) is standardised testing conducting annually in Australian schools to assess basic skills in Literacy and Numeracy. The assessments are administered by the Australian Curriculum, Assessment and Reporting Authority (ACARA). The assessment data is used to compare schools' performance across the country and is published on the Government My School website.

consumed by an investigation to demonstrate learning on addition and subtraction for student portfolios. Preparation for national testing, NAPLAN, created an additional pressure in terms of teaching time. The team leader suggested allocating half of the weekly time for mathematics to the intervention, so that the remainder of the time could be used to teach curriculum content predicted to appear in NAPLAN. This notion of 'teaching to the test' is reported as a common issue in Australian schools (Polesel, Rice, & Dulfer, 2014; Thompson & Harbaugh, 2013). These two assessment requirements impacted considerably on time available to implement the intervention. Fundamentally, teachers did not have time to commit to the research project requirements as outlined in the explanatory statements and consent forms.

The second main focus related to the challenge of finding time to implement the intervention, there was limited time due to an extensive range of extra curricula and specialist learning experiences. One example that highlighted this issue was the team struggling to complete a pre-intervention assessment task within the agreed time frame. It was explained that Diversity Day[16] had limited teaching time available for mathematics that week. The extensive range of specialist programs on offer to students made it difficult to find time to complete the modelled sequence of lessons in each class. Belinda explained that the range of programs individual students attended during time allocated for class teaching meant that she only had five periods per week when all students were present in her class.

[16] Diversity Day was a celebration of the multicultural nature of the school. All students were encouraged to wear traditional costumes related to their cultural background.

The various constraints on finding time to implement the intervention seemed to impact on the depth of reflection by the teachers. They had limited time to contemplate and reflect on questions and actions to develop mathematical praxeology i.e., the thinking about the task, how to organise student thinking, and to understand the reasons for organising it in this way (Prodromou et al., 2017).

In addition to the challenge of finding time to implement the program due to either issues related to timing of the implementation, or time to implement the program, there were also constraints in connection with communication issues.

4.3.2. Communication issues acting as a constraint to change

From the analysis of multiple data, several references relating to both issues of communication within the Year 3 team, and between the team leader and the team, were evident. The analysis was based on teachers' comments in interviews, informal conversations, personal correspondence (email) and my observations during team meetings.

The presence of some pre-existing interpersonal issues within the Year 3 team were seen to constrain opportunities to change practice for individual participants and for the whole team. The Year 3 team was particularly large, consisting of a total of nine teachers, so the potential for interpersonal issues was not unexpected. The issues were first observed at the introductory meeting for the project, where I was greeted by the notably sullen presence of some team members, who, as I later discovered, were the four part-time staff. During the professional learning session, the interaction among team members was minimal and seemed strained. My observation was corroborated by personal communication (email) with the team leader, Adele, when negotiating the implementation of the intervention. She referred to some difficulties with the transition process for the new Year 3 team, and was concerned about stress levels of some teachers. Following the modelled lessons, the four part-time teachers all decided to withdraw from the project. The reason was given that they needed mathematics teaching time to prepare for NAPLAN; this situation seemed to contribute to tension within the team. The stress NAPLAN seemed to place on these teachers is a common issue; it is suggested that the way NAPLAN data is reported by government can be interpreted as an instrument for judging teacher performance (Polesel et al., 2014). In an informal conversation following a modelled session, Belinda commented on the need for the whole team to participate in the project. She also referred to this issue twice in her interview:

> And I think it has to be across the board, not a couple of teachers, it has to be the whole cohort. Everyone needed to be on board in order to do it, to be more successful, I think. (Belinda, interview)

> I think this needs to be implemented across the curriculum/cohort in grade 3[17]. (Belinda, interview)

Belinda's comments suggest she believed the success of the program had been compromised by the lack of a whole team approach. Certainly, from a research perspective, this decision was seen as a constraint on the collaborative and

[17] Belinda used this terminology to refer to the Year level of the students.

participatory approach of the project; it restricted the potential use of scheduled planning time for professional learning in the intervention.

There also appeared some underlying tension in terms of communication between the team leader and some team members. The tension could partially be attributed to internal pressures to revise and improve planning and teaching practice within the team. Through informal conversations during the pre-intervention classroom visits (Researcher's journal), Adele explained that she had been purposively moved to Year 3 to focus on improving planning and teaching practices within that area of the school. However, some of the tension could equally be attributed to a need for greater transparency in communication. In personal correspondence (email) between Adele and myself, in which we arranged the introductory meeting regarding the project, she mentioned that she had not yet discussed the research project with the team. The team knew very little about project prior to my arrival at the school. I surmise that this may have contributed to the tense atmosphere in the room. The following interview excerpt in which Clare commented, "we also didn't know that this was being put in place either, so I wish it had been communicated. And I wish we could have continued it," further corroborates this interpretation. Clare's comment indicates a level of frustration at the lack of prior warning about participation in the research project.

Through a social constructivist lens, it is questionable to what extent such communication issues can be considered an internal institutional constraint. Rather, it seems that personal factors i.e., dispositions, and actions of this team of teachers were also shaping the context in which they worked. In other words, a bi-directional interaction was occurring.

Although there was a need to improve student learning in line with the goals of the project, various factors seemed to constrain possible changes to teaching practice. Some of the constraints were presented by internal and external factors and in relation to limitations of time; some were the result of communication issues within the Year 3 team. The constraints presented by the institutional context resulted in inconsistencies between the intended and the actual intervention implementation. The absence of collective planning time limited opportunities for professional conversations and meta-didactical transformations and the brokering process. Two planned sources of data were also not collected due to time constraints: lessons observations were not conducted at the end of the intervention and the planned post-intervention assessment was replaced with an in-school task. Whilst individual teachers perceived some changes in their knowledge, disposition and practice as a result of participation in the project, this could not be corroborated by researcher observation of teaching practice. The data suggest that constraints within the institutional context hampered opportunities for the teachers to learn and develop through a collaborative approach.

4.4. Refining the intervention process

In reflecting on the outcomes and challenges with implementing the first intervention, it was important to consider both the components and the processes that required refinement to improve future cycles of the intervention.

Since the intention was to investigate *collaborative* professional learning processes for primary teachers, it was important for me to emphasise this requirement with future participating schools. Essentially whole team participation was required for the project to be implemented as intended. The minimum expected timeframe for the project also needed to be communicated clearly from the outset, with some discussion on how and when the intervention would fit within that school's Year 3 mathematics program.

The first intervention highlighted that developing pedagogies to enact it presented some challenges for teachers. The design of the sequence of lessons and the linking of the instructional tools seemed to require further clarification based on the informal conversations in modelled lessons. I revised the layout of the teacher resource book and included more visual representations to explain the teaching process involved in the intervention, and the components of the lesson structure. Time limitations on the professional learning session meant that teachers needed to read the resource book to gain understanding of this, or observe a modelled sequence. On reflection, the PowerPoint slideshow presented to teachers did not include a clear diagram depicting the teaching process or highlight the linking of the lesson sequence. I therefore revised this presentation and integrated clearer visual representations.

For the teachers at School A, this was their first introduction to number strings as an instructional activity. I decided to provide future participants with additional information to support the intricacies involved in using this tool effectively, particularly the key questions that are useful to pose for progressing student discussion.

4.5. Summary

In this chapter the findings of the first intervention at School A were presented and discussed in relation to the two research sub-questions. To address the first question on changes perceived during the professional learning experience, the Interconnected Model of Professional Growth (IMPG) developed by Clarke and Hollingsworth (2002) was used to guide the analysis and represent the change processes diagrammatically for each teacher. Although all three teachers participated in the same professional learning program, the data suggested two different diagrammatical representations of change sequences. For two of the teachers (Adele and Belinda) initial change in the Personal Domain was interpreted as leading to change teaching practice. The third change sequence (Clare) was interpreted as external stimuli initiating change in the Personal Domain, connecting to subsequent change in the Domain of Consequence.

The data were analysed to explore how aspects of the External Domain and Personal Domain appeared to influence opportunities to learn and change practice. For two of the teachers (Adele and Clare) the opportunity to develop knowledge by observing modelled lessons was perceived as most significant in stimulating learning. This iteration suggested the importance of multiple external stimulating changes in the Personal Domain. The key salient outcomes – student progress with explaining thinking and reasoning, student retaining and transferring learning, and changes in student engagement and attitudes – were seen to provide a catalyst for future learning and change in practice.

The focus of second sub-question was on exploring the influence of institutional aspects on opportunities to learn and change practice. The MDT model (Arzarello et al., 2014) was used to guide the analysis of the data. The following factors were interpreted as having a significant impact on opportunities for learning and change to practice: the absence of collective participation; the challenges of limited time to implement the intervention; and issues with timing for the intervention conflicting with other school activities.

After reflecting on the outcomes and challenges with implementing the first intervention, components of the professional learning program were refined, namely the communication of project requirements, and provision of resources to explain and support classroom experimentation. These changes will be discussed more specifically in the next chapter, in the context of the second intervention cycle implemented at a different school in the Melbourne region.

5. Interplay of external and internal influences on learning

Much of what happens in the classroom is determined by a cultural code that functions, in some ways, like the DNA of teaching. (Stigler & Hiebert, 2009, p. xii)

The findings of the second cycle of the intervention form the focus of this chapter. Considering the methodological approach adopted for this study, the first cycle informed the refining and implementation of the second intervention. Factors previously interpreted as constraints to change became key points of discussion in the early negotiation process. Of primary importance, was an emphasis on collective participation, minimum time requirements and the timing of the intervention. With attention given to pre-empting prior challenges, this chapter reports on a different research experience, one that highlights the complex interplay of external and internal influences on teacher learning. While some teachers experienced cognitive dissonance, which led to a change in practice, others were challenged to integrate new ideas within the culture of their classrooms.

In this chapter, change sequences and a comparison of learning processes experienced by a team of Year 3 teachers at a different primary school (School B) in Melbourne are presented. Aspects of the school setting, specifically affordances and constraints to change, are explored. I begin by providing background information relating to the institutional context (school setting), to situate the implementation of this second intervention cycle.

5.1. School context and background

In this section contextual information relating to the institutional (school) setting is presented; this is followed by details relating to the teacher participants.

5.1.1. The institutional (school) context

The second intervention was conducted at a Catholic Parish Primary School located in a south eastern suburb of Melbourne. The co-educational school catered for students in Years P - 6 and prided itself on offering a nurturing environment to support the development of the whole child. During the intervention period, the focus on students' spiritual growth and positive learning environment was evident with the whole school approach to daily class meditation, at the beginning of each day. The feeling of a calm, caring and supportive atmosphere was conveyed. Students were offered an extensive range of co-curricular opportunities. In addition, specialist teachers were employed in Visual Art, Music, Physical Education and Mandarin. Although responsibility for teaching mathematics was with classroom teachers, the school employed a Mathematics Leader (ML), whose role involved supporting teachers in planning, teaching and assessing mathematics across the school. The ML (given the pseudonym Hannah) was not assigned a class and worked four days a week to fulfil this role.

5.1.2. Participant background

Details relating to teachers participating in this intervention, specifically years of teaching experience in middle primary, are presented in Table 5.1. Further background information on each teacher is presented in Section 5.3.

Table 5.1. *Teacher participants at School B and years of teaching experience*

Teacher name (pseudonym)	**Number of years teaching**	**Number of years in middle primary (Years 3 & 4)**
Deryn (team leader)	16	4
Ethan	25	2 (one year in a composite)
Fiona[18]	2	2
Giselle	Graduate teacher	0

In addition to the teacher participants, the school ML assumed a supporting role in the implementation of the intervention. The ML, Hannah, was well-informed about student learning needs in Year 3[19] and thought the subject content of the teaching component of the intervention (mental computation for addition and subtraction) would be helpful in addressing some of these needs. It appeared that Hannah's disposition on students learning mathematics, specifically the importance of learning mental computation before formal written strategies, aligned with my view as the researcher. Bearing in mind general consensus that coherence is an important feature of professional learning that seeks to achieve

[18] Teaching was a second career for Fiona as she had previously worked as a teacher aide for seven years, predominantly in Prep and Year 1 classes.
[19] The ML provided student assessment data at the beginning of the intervention. Data related to the percentage of Year 3 students achieving various stages of the Learning Framework in Number (LFIN). The LFIN is a research-based framework for assessment, instruction and intervention in whole number arithmetic across Grades K-5. The framework was developed as part of Mathematics Recovery® program developed by Robert Wright and David Ellemor-Collins and was an integral part of the school numeracy program.

changes in knowledge and practice of teachers (Desimone, 2009), it was important that the teaching intervention aligned with the school goals on teaching computation[20].

Details relating to the setting up of the second intervention cycle at this school, specifically how findings from the first intervention informed the negotiation process are discussed in the following section.

5.2. The second iterative cycle: The negotiation process

The early negotiation process for the second intervention was informed by findings from the first cycle. In an effort to preempt some of the challenges encountered with the first intervention, it seemed crucial to focus attention on collective participation, minimum time requirements, and the timing of the intervention. In addition, external professional learning resources were modified and the assessment tool co-constructed with the teachers. Modification of teaching resources was based on School B's current student assessment data provided by the ML. Each of the aforementioned were key considerations in setting up the second intervention and will be further discussed in the following subsection.

5.2.1. Setting up the intervention

In learning from the experience of the first intervention cycle and the challenges related to collective participation, I placed emphasis on this component of the

[20] Note that for the purpose of this study the meaning of coherence is taken from the work of Desimone (2009) and includes alignment with both teachers' disposition on teaching and learning mathematics and the school's approach.

professional learning (PL) program in the early stages of negotiation. This was of particular importance because establishing a meaningful cooperative relationship between the participants and myself, is considered a crucial element of the design-based research methodology adopted for this study (Baumgartner et al., 2003; McKenney & Reeves, 2012). The focus on collective participation was raised in initial communication with the ML. Subsequently, the ML shared information about the intervention with the Year 3 team, and general agreement was reached that the project would also be useful in supporting teachers with the school's own professional learning requirements[21]. Although the school's action research program was separate to this study, the existence of this school requirement gave the teachers an additional reason to engage in the intervention. Such a requirement supported a cohesive team approach and could be interpreted as an affordance in the change environment. The situation provided opportunity for a symbiotic relationship between researcher and teachers to develop: the teachers had access to resources and support from an external source to meet their own school goals, and I could study learning processes involved in changing practice.

This symbiotic relationship, between the teachers and myself, evolved through co-construction of the pre- and post-intervention student assessment tasks. Although a sample assessment had been prepared, when this was initially shared the ML (Hannah) suggested that there would be a number of students

[21] Teachers at the school were required to conduct an action research project each year within their planning and teaching teams; this involved reading current research literature on an aspect of teaching and learning, experimenting with the approach in classrooms and reporting the findings to the Principal.

who would not be able to access the tasks (mathematically). The bidirectional interactions between the teachers and myself were important in adjusting the assessment tool to optimise data collected on student learning (Arzarello et al., 2014). Useful student data were needed to inform planning and teaching, as well as measure student progress. In addition, this was an opportunity for me to develop a relationship with the teachers in the early stages of the project, which seemed to further support a shared sense of ownership of the intervention.

Inappropriate timing and misalignment with the school mathematics program had been major challenges with the implementation of the first intervention. For this reason, I considered discussion about the timing of the project as a key part of the early negotiation process with the school. The ML, Hannah, suggested implementation of the project at the beginning of Term 2 to coincide with a unit on addition and subtraction in the Year 3 mathematics program. This decision was favourable in that it also addressed the need for adequate time for the intervention. It was negotiated that a sequence of three lessons would be taught each week over a three-week period (three weeks had been allocated for a unit on addition and subtraction in the Year 3 school program). The remaining two lessons per week were to be allocated to preparation for NAPLAN[22].

Another important adjustment, in preparation for the second cycle, was modification of the professional learning resources. In the first intervention, the

[22] The National Assessment Program for Literacy and Numeracy (NAPLAN) is standardised testing conducting annually in Australian schools to assess basic skills in Literacy and Numeracy. The assessments are administered by the Australian Curriculum, Assessment and Reporting Authority (ACARA). The assessment data is used to compare schools' performance across the country and is published on the Government My School website.

participants experienced some confusion with the instructional tools and use of terminology such as sequence, strategies, and strings. For this reason, it was important to communicate with the ML regarding teachers' prior experience with the instructional tools and content of the intervention, so that the resources could be modified accordingly. Professional conversations with Hannah suggested it would not be unreasonable to expect that this team of teachers might also experience similar challenges, unless the resources were modified. Hannah indicated that the approach and content would be mostly new learning for this group of teachers.

5.2.2. Negotiating the implementation of the intervention

The initial plan, as agreed between the school and myself, was to implement the intervention for a three-week period at the beginning of Term 2. However, during the weekly Year 3 planning meeting (first week of the intervention), further discussion resulted in this plan being modified. The Year 3 teachers, and in particular the team leader Deryn, had observed the students during modelled sessions and concluded that this new approach to teaching would be beneficial, based on the learning needs of their students. The teachers consulted the Year 3 mathematics plan for the year and proposed that a second unit on addition and subtraction, which was planned for the beginning of Term 4, be moved to the beginning of Term 3 to accommodate a second (additional) phase of the intervention.

It is noteworthy that the decision to adjust the plan and introduce a second phase of the intervention was predominantly based on teacher observation of students during the three modelled lessons. The opportunity to observe students engaging in the tasks and reasoning about strategies was sufficient to instigate this decision and change teacher attitudes towards the intervention (assessment task data were not required). This concurs with findings of Guskey (2002) who suggested that evidence of changes in students' learning outcomes is first necessary for changes in teachers' attitudes and beliefs.

Early negotiations about setting up the research project suggested the school environment would be a conducive setting in which to conduct the research. Bidirectional interactions with the team to modify the assessment tool seemed to foster interest and a shared ownership of the intervention. From my perspective, there appeared cohesion within the team, and between the team and myself. The evolving social dynamics (Arzarello et al., 2014) were a positive influence on the early implementation of the project.

5.3. Change sequences

Two different change sequences emerged for the four teachers. It appeared that Deryn and Giselle experienced the same sequence; Ethan and Fiona shared a different change sequence pattern. Although pairs of teachers shared similar sequence patterns, it was apparent that changes within domains and how these changes related to other domains were nonetheless individualised in each teacher's learning experience. For this reason, teacher change within each of the sequences will be compared and contrasted in response to the first research sub-question (refer to section 1.3). First, contextual background information on the teachers sharing a similar change sequence is presented. This will be followed by a diagrammatical representation of the change sequence and supporting explanation of the changes perceived both by teachers and by myself as the researcher (Clarke & Hollingsworth, 2002; Arzarello, 2014).

5.3.1. Contextual background: Deryn and Giselle

Deryn and Giselle were at very different stages in their teaching careers. Deryn was an experienced teacher; she had been teaching for 16 years and assigned responsibility for coordinating the Year 3 team. In contrast, Giselle had only just commenced her second term of teaching a Year 3 class at the time the first phase of the intervention was implemented. As a graduate teacher, this was her first experience of teaching at middle primary level. Despite this difference, there existed similarities in relation to their disposition and learning goals. In this section, what follows is a brief description of each of their contextual backgrounds.

The distinct difference in years of teaching experience did not seem to distinguish learning goals for Deryn and Giselle; both indicated a focus on developing aspects of their Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK). Although Deryn was a senior teacher and had spent four years teaching at middle primary level, her experience was extensively with the junior years. Her reflection on the usefulness of the information in the teacher resource book suggests that a focus on teaching mental strategies involved her developing new knowledge:

> It would be a great resource for me now if I go back and teach the bridging strategy, for example, let's go back and refresh my memory on the problems that relate to it and how to actually teach it, what are the steps before teaching it. (Deryn, interview)

The use of the resource book to look at problems that relate to, or that could elicit a particular strategy from students, suggests she was developing both new Specialised Content Knowledge (SCK) and Knowledge of Content and Teaching (KCT). As a new teacher, Giselle indicated she was learning and experimenting with new knowledge about mental computation strategies:

> I think that because I am a new teacher, everything is new for me so all these strategies are new. (Giselle, interview)

With regards to developing new knowledge, it seemed that both teachers viewed participation in the intervention as an opportunity for new learning.

Deryn and Giselle displayed a similar disposition in relation to their role as teachers in the classroom. During the modelled lessons, Deryn's tendency to explicitly direct students to examples, rather than elicit their thinking and ideas through questioning, was suggestive of a teacher-centred approach in the classroom. Likewise, Giselle's usual practice was to explicitly model a strategy rather than allow opportunity for students to share and discuss their ideas first. She also evidenced a teacher-directed approach to her teaching (Researcher's journal).

Similarities in disposition and shared goals in relation to developing aspects of Mathematical Knowledge for Teaching (MKT), appeared to influence the external stimuli initiating change for both teachers. Their change sequence is presented in the following section.

5.3.2. Change sequence 1: The influence of social dynamics within domains

The change sequence for Deryn and Giselle is represented diagrammatically in Figure 5.1. In the following subsection the learning processes and changes experienced by both teachers are compared and discussed.

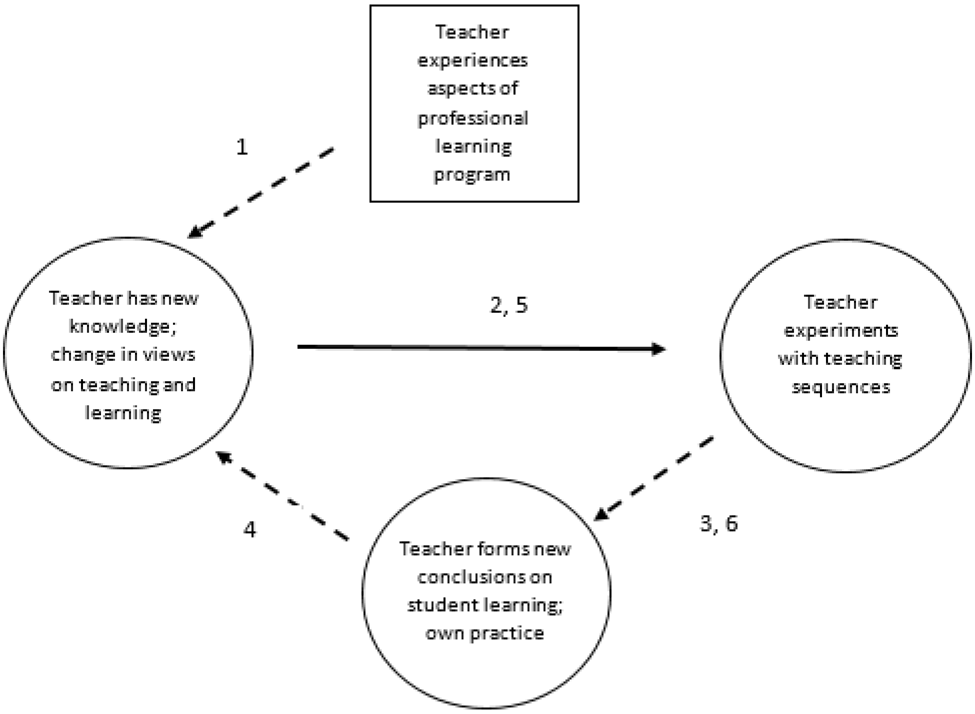

Figure 5.1. Change sequence for Deryn and Giselle.

5.3.2.1. Change in the Personal Domain (Arrow 1)

Deryn and Giselle reflected on various external stimuli, which instigated initial changes in each of their Personal Domains. The data suggest that it was the opportunity to observe a sequence of modelled lessons and participate in collective planning sessions, which were most consequential in igniting change for these two teachers. Each of these influences will be discussed in turn.

For Deryn, it was the brokering of using number strings as an instructional tool to elicit student reasoning that appeared critical to her developing new pedagogical knowledge. Although she had experienced number strings as a learner herself during the professional learning session, brokering in a classroom situation was necessary for her to internalise the purpose and design of the number string (internalise new pedagogical knowledge, specifically KCT). During the debrief at the end of the first modelled lesson she commented how observing this in action had clarified her initial confusion with the terms *number string* and *sequence*. She also noticed how the linking of the questions in a string supported student thinking (Researcher's journal). The importance of observing the modelled lessons was corroborated when she reflected in her semi-structured interview:

> I really like other people coming in and modelling for me. I like listening to others. I like the way, as you said, listening to someone else's questioning, listening to someone else's way of doing things. I really enjoy that and I get a lot out of that... things like that I have found very beneficial. (Deryn, interview)

The benefits that Deryn described were substantiated by her eagerness to organise a program to allow the team to observe a sequence of three lessons. This was a deviation from the original plan, where it had been agreed that I would only model the first lesson in the sequence (because number strings were a new instructional tool).

Observing the modelled lessons was also valuable in initiating a change in Deryn's disposition. In the subsequent planning session, she displayed a level of enthusiasm and interest in number strings as an instructional tool that was not evident in the professional learning session. At the meeting she described changes in student engagement she had noticed during the modelled sessions (Researcher's journal).

It was not simply the opportunity to observe and reflect on the modelled teaching that acted as a stimulus for Deryn's new learning. There appeared influences on learning other than the mediating process of reflection within the Personal Domain. Although I led the instructional teaching, Deryn was an active participant in the lessons. She was involved in orchestrating class discussions by helping to select examples of student thinking (recorded on mini-whiteboards) to address misconceptions and highlight more efficient approaches. This bidirectional interaction between Deryn and myself, was an important part of her internalising the process of facilitating productive classroom discussion (Stein et al., 2008). The evolving social dynamics, namely brokering and bidirectional interactions, appeared a critical influence on her learning.

The opportunity to observe her students in the modelled lessons was also an important influence on Giselle's learning. In her post-professional learning survey she commented that "getting students to verbalise their thinking before documenting their answer," and noticing "how important the discussion of their mental computation is" were key aspects of learning. It was through brokering the facilitation of productive class discussion, that Giselle recognised value in students thinking first and then articulating their thinking verbally. She reflected on the importance of students having opportunities to share and discuss their ideas as part of the learning process. Observation of the modelled lessons seemed to support development of Giselle's pedagogical knowledge.

Similar to Deryn, it was not simply the opportunity to observe a sequence of modelled lessons that stimulated learning for Giselle:

> I think that's why having you in the classroom was nice, because we could sort of say, well you know 'we need a pull out group' or 'we need the lower kids on the floor' so to have you in there, I felt that was useful as a grad, because you know sometimes you need to adapt your lessons or whatever so to be able to have you in there and talk us through what you're doing and then we can talk about what we think, just the discussion was useful. (Giselle, focus group)

It is interesting that Giselle predominantly focused on the professional conversations that occurred during the lessons. Whilst the instructional teaching was modelled, I was conscious of articulating and inviting Giselle into the decision-making process throughout the lesson. Being aware that she was a graduate teacher, my intention was to provide her with insights into the pedagogical approach, namely how to use student thinking and ideas to drive the direction of the lesson and instigate further learning. This brokering process seemed to support her internalisation of new pedagogical knowledge.

Participation in collective planning meetings appeared a significant, additional external stimulus. Deryn emphasised the value of this component of the program:

> You listened to us say how kids aren't ready for that or those numbers are too high and things like that. It was great seeing that you could actually modify it. So there was a basic skeleton of it that you could go higher or lower with it… Yeah, having those discussions in between was good. (Deryn, interview)

> It was a great resource book but it wasn't set in stone either. You know, you listened as we talked in planning meetings about the needs of our kids and where our kids were at, to be able to modify it. (Deryn, focus group)

Her reflections highlight the importance of the bidirectional interaction between Deryn and myself. Listening and responding flexibly to suggestions from the teachers was considered a valuable element of the learning process. During the planning sessions, the teachers and I drew on each other's knowledge to co-construct resources and lesson outlines. This resonates with findings of Desimone and Hill (2017), who posited that adaptation of an intervention is an important element to integrate into professional learning programs. In a study of middle school science teachers, they found that opportunities for teachers to engage in professional dialogue about adapting the intervention to meet the learning needs of lower achieving students was a critical component of teacher learning. It was perhaps the existence of collective planning within the school culture that afforded this level of bidirectional interaction between the team members and myself.

The significance of collective planning was also highlighted by Giselle. In her pre-intervention survey (which it is noted was completed part-way through the intervention) she commented "sharing lesson plans with other Year 3s and designing lesson PowerPoints" was important in supporting her professional learning. Giselle did not refer explicitly to the researcher's role as being influential; instead she alluded to the importance of the bidirectional interaction with her colleagues during meetings. She was developing her planning skills and learning to modify resources; interaction with her colleagues seemed to afford this learning. This influence resonates with findings of Wilkie (2019) who highlighted bidirectional interactions among teachers as an influence that deserved greater attention. By providing teachers with the outline of a learning sequence and resources to modify, it afforded an opportunity to focus conversations on the *how* of teaching (the pedagogical considerations) rather than the *what* of teaching. Although collective planning was embedded within this school culture, Hannah the ML explained that sometimes it was only possible to outline a learning sequence in planners within the timeframe. Giselle's reflection alluded to this affordance.

5.3.2.2. Change in the Domain of Practice (Arrow 2)

Both teachers experimented with enacting new knowledge; each teacher reflected predominantly on the influence of the new pedagogical approach on student learning. Deryn described a significant change in her teaching practice, allowing time for students to think mentally and explain their strategies rather than model a process for students to follow:

> I like the fact that we're actually trying to make them figure out the answer first before the working out, whereas mathematically processing usually we say, 'this is how you work it out'. Whereas we're trying to get them to transfer that into their head; I like that. (Deryn, interview)

Her comments indicate that she was experimenting with a new pedagogical approach, one that gave opportunity for students to develop agency. The classroom actions she described suggest early changes in her praxeology. In contrast, Giselle's initial reflections on classroom experimentation with new pedagogical knowledge highlighted some challenges:

> I think for a lot of them, they really struggled with their ability to be flexible in thinking about different strategies. They have one strategy in their head for how to solve it and when you show them a new strategy they can't adjust to it because they say, 'no that's not how I work it out. I work it this way'… so I think they are a bit rigid in their thinking. (Giselle, interview)

> They were asking, 'why are we learning a different strategy?' And I said, 'well they can be applied to different questions… you are never just going to use that one strategy'. (Giselle, interview)

Giselle's comments suggest the change in pedagogical approach contrasted with the students' usual learning experiences; they were used to receiving direct instruction from the teacher. Initially the students displayed some resistance to reasoning with multiple strategies to solve a problem: they were more comfortable with the expectation to master just one strategy. Despite experiencing some challenges in the early stages of experimentation, Giselle persevered with the new approach in her classroom. The excerpt below shows how she perceived the learning experiences of the students evolved:

> I guess it's just a different approach for the kids in mathematics and that would probably be the biggest thing for them. They were unsure about how the lessons worked, especially at the start, and what they are expected to do and the different strategies they are expected to show. (Giselle, interview)

It was not until Giselle experimented with teaching the second sequence of lessons, that she began to feel at ease with the approach and recognise some benefits from the change in practice. This concurs with the notion of relentless consistency (Brown & Coles, 2013) and the need to establish a learning culture that fosters student agency and allows for learning through teaching practices that have been described as ambitious (Kazemi et al., 2009).

Although Deryn and Giselle focused on enacting almost identical aspects of new knowledge, there were some differences in their classroom experimentation. Some of the challenges Giselle described in relation to this new approach to teaching could be interpreted as a reflection of her limited classroom experience as a graduate teacher. For example, at the planning meeting following her experimentation with a new sequence, Giselle shared how she found it difficult to ensure active participation by all students in the fluency tasks at the beginning of the lesson. She also expressed concern with how to assess progress of individual students within a whole class situation. It would not be unreasonable to assume that these challenges were a reflection of her early development of general pedagogical skills as a graduate teacher rather than issues with teaching mathematics per se. It could also be inferred that establishment of her classroom learning culture was in the early stages of development, in comparison to Deryn.

5.3.2.3. Change in the Domain of Consequence (Arrow 3)

Deryn and Giselle reflected on changes in student affect as a salient outcome of their classroom experimentation but to varying levels. They also described additional consequences specific to each of their classrooms. These are discussed respectively.

Both teachers emphasised changes in student affect, highlighting student engagement as a salient outcome. At the planning meeting in preparation for Term 3 teaching, Deryn shared that it was opportunities for whole class discussion, in particular the comparing of different strategies, which students seemed to enjoy participating in most (Researcher's journal). Similarly, Giselle commented on student engagement during whole class discussion:

> I think they thrive when they're on the floor and using their whiteboards… I feel that for them that's the strongest part of the lesson, when they are actively engaged in it… I think they get the most out of it. (Giselle, interview)

Interestingly, after experimenting with teaching two sequences, Giselle perceived opportunities for whole class discussion as the strength of the lesson in terms of student thinking and engagement. In the early stages of the intervention, she expressed concerns with managing whole class discussion effectively.

In comparison with Deryn, Giselle placed greater emphasis on changes in student affect. In addition to recognising changes in student engagement, she also noticed the positive influence on student confidence:

> I think the confidence goes up, especially when they are on the floor… I feel we are getting responses from students who wouldn't normally say anything… So the whole 'put your thumb up if you've got it', or 'write it down on your board', it's still sort of anonymous for them as in they don't have to speak in front of the class. So I think that for those learners who are a little bit less confident, something like that just gets them in with everyone else and helps them join the group. (Giselle, interview)

Giselle described changes in student participation and confidence as a result of changes in her pedagogical approach. Following initial concerns with effective management of whole class discussion, she had focused on experimenting with strategies such as ‘thumbs up’ (instead of hands up) and use of mini-whiteboards to encourage active participation. She was exploring tools for engineering effective discussions that elicit evidence of student learning (Wiliam, 2011). She recognised the positive impact this had for students who had previously lacked confidence in whole class situations.

Each teacher also identified salient outcomes specific to their individual classroom experimentation experiences; these outcomes and related sources of evidence are presented in Table 5.2.

The importance Deryn attached to student learning outcomes was substantiated by her reflection on student assessment data. The students demonstrated notable improvement in accuracy on mental computation assessment tasks, as well as showing evidence of relational thinking (Wright et al., 2012). This concurs with findings in the literature that student achievement is a powerful agent in terms of teacher professional learning (Goldsmith et al., 2014; Guskey, 2002). It is possible that Deryn’s emphasis on this outcome, which differentiated her changes in this domain from those of Giselle, could be attributed to her coordinator role within the school. Responsibilities attached to her position would have included accountability for student learning outcomes. In contrast, Giselle’s focus on positive consequences in terms of her own professional learning could be interpreted as a reflection of her position as a graduate teacher.

Table 5.2. *Summary of idiosyncratic changes for each teacher within the Domain of Consequence*

	Summary of idiosyncratic changes within the Domain	**Related comments by each teacher**
DERYN	Deryn contemplated the significance of changes in student learning. She considered student development of skills to justify strategy choices and reason as salient. She described student progress with learning mathematical vocabulary to articulate their thinking as significant. She reflected on student assessment task data and improvements reported in student accuracy with computation.	*And figuring out an answer first…and then reasoning how you got to that answer in your head. Not just, 'I just did it'. So getting kids to actually explain how they got the answer. (Deryn, interview)* *I think it's great that the kids can now define words like efficient and mental, things like that. I think it's great that they're using some of that technical vocab. (Deryn, interview)* *Yeah, and just seeing your kids succeed. Like when you said, this many percent answered accurately whereas before…that's great reinforcement that you are on the right track and doing the right thing and that the kids are learning. (Deryn, interview)*
GISELLE	Giselle reflected on outcomes in relation to her own professional learning. She considered development of aspects of her knowledge as most valuable. She reflected on developing understanding about how students learn to compute mentally (KCS). She seemed to recognise development in her pedagogical skills, namely how to use student thinking to facilitate productive discussion and move forward student learning.	*I guess being flexible in understanding how the kids learn and the different ways that they think, and taking on their suggestions and their approaches…So yeah, making a bit more of a conversation about it, rather than just saying that's the answer and that's how we get there. (Giselle, interview)*

5.3.2.4. Change in the Personal Domain (Arrow 4)

Changes occurring in the Personal Domain were similar for each teacher; both described development in aspects of PCK, as well as changes in their disposition towards teaching mathematics. The changes are presented in Table 5.3.

Table 5.3. *Summary of changes for each teacher within the Personal Domain*

	Deryn	**Giselle**
Summary of changes in PCK	Deryn largely attributed changes in PCK to learning to use number strings as an instructional tool. She recognised the value in building on existing student knowledge to foster connections with new strategies (Murphy, 2004). ***Supporting evidence*** *But I suppose the number strings is the bit I've taken out of it, as how to build on their knowledge if that makes sense. (Deryn, interview)*	Giselle described significant development in aspects of her knowledge, mainly aspects of PCK. She was exposed to new learning of mental strategies and ways to represent strategies visually. ***Supporting evidence*** *But I think even just the ways to represent addition and subtraction for me I think about how I did it in school, which was probably just partitioning and vertical addition. But to look at these different strategies that I do in my head but I'd never thought about it explaining ideas in that way to the kids. (Giselle, interview)*
Summary of changes in disposition	Deryn's reflection suggests a change in her attitude regarding mathematics in the classroom and everyday life. Deryn appeared to reconsider the purpose and value in teaching mental computation, suggesting that there had been too much focus on written processes in her classroom. ***Supporting evidence*** *We get bogged down by the processes too much sometimes that we forget about the mental part, when you go into a shop and you give a kid a dollar and twenty cents and they can't add that on….does that make sense? I think that's when you realise the real world is why you teach the mental. But I think sometimes it gets lost or it's just a Tools session. (Deryn, interview)*	Giselle appeared to be changing her perception on how mathematics could be taught; she was starting to recognise that there are often multiple ways to solve a task. She described adopting a more open minded, flexible attitude towards teaching mathematics. ***Supporting evidence*** *I think it's made me think about maths a little bit differently in the ways that I approach it…I think that the way I think about showing them possible solutions to get to the answer is different for me. So I've become a bit more flexible because for me if I am thinking about mental computation I think of it as how I get there, rather than how they get there. So I feel that because there are so many different ways for these kids to get there, it's sort of opens it up as a different way for us to explore one question. So yeah….I guess I'll approach tasks slightly differently, maybe with a bit more of an open minded approach, rather than being so narrow and specific and this is just how we get there and that's, that. (Giselle, interview)*

Deryn's comments indicate that she was internalising a new approach to teaching mental computation and that her praxeology on how students learn to compute mentally was beginning to align with that of the researcher. Giselle's reflection suggests that the way she had been teaching was based on the way she had learned during her school years. It is probable that she had been exposed to new learning as an undergraduate, but had not yet applied it in her own classroom at the time of the intervention. This would align with findings of Sfard (2005) who argued that prospective teachers do not seem to transfer new learning about teaching to their own classrooms

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5.3.2.5. Change in the Domain of Practice (Arrow 5)

Deryn and Giselle were observed teaching the second lesson in a sequence, in the second phase of the intervention at the beginning of Term 3. The purpose was to gain insights into any aspects of the approach they may have internalised. When Deryn was observed facilitating the main part of the lesson, her actions indicated that she had internalised the idea of allowing students thinking time without explicit instruction (this corroborated her self-reported changes in practice described in her interview). This new approach was in contrast to her usual practice, which predominantly involved explicit teaching. This pedagogical practice also involved students reasoning to justify their strategy. During the second iteration, Deryn focused on using student work to create opportunities for reasoning and to stimulate discussion:

> You are exposing them to the different strategies which makes them go 'aha'; that's a more efficient way of doing it than my strategy[sic]. (Deryn, interview)

She was observed using mini-whiteboards with students as a tool to support the engineering of whole class discussion. Following individual thinking time, she asked students to use the boards to explain their thinking visually to a partner. Deryn used this as an opportunity to select examples of student thinking to activate discussion. Encouraging students to develop computation strategies is considered critical in developing number sense (Heirdsfield & Cooper, 2004). She displayed her selected examples of student thinking visually to instigate student reasoning about the efficiency of various mental strategies; it was apparent she was using the Five Practices to guide orchestration of the class discussion (Stein et al., 2008). Likewise, Giselle also focused on facilitating productive whole class discussion. She invited students who she anticipated would have the least efficient strategies to contribute their ideas first, ending the discussion with the more efficient approaches. This was the approach I had modelled in her classroom, and one of the suggestions articulated in the work of Stein et al. (2008) on orchestrating productive discussions. Observing her orchestration of whole class discussion it was evident that allowing students individual thinking time, and using visual representations of student thinking to initiate discussion, was moving from an external to an internal component of her praxeologies.

Noteworthy, was the way both teachers had adapted the lesson structure to include experimentation with focus groups; use of these groups was not originally incorporated into the intervention design. The integration of focus groups was the result of bidirectional interactions between the teachers and myself throughout the first phase of the intervention. Initially, the teachers' perception and my understanding of the purpose of a focus group were very different. The school had been implementing a numeracy intervention program that involved explicit teaching in small ability groups; this was the concept of a focus group the teachers wanted to integrate into lessons. Whilst I recognised there could be some benefit to running a small group session, I was keen for this to be an opportunity for students to articulate their thinking and develop reasoning skills. As a result of our discussions concerning differentiation, the idea of a focus group being a small-scale version of the whole class discussion rather than an opportunity for small group direct teaching moment evolved. Both Giselle and Deryn were observed experimenting with our shared concept of a focus group. It seemed there was potential for a shared praxeology between the teachers and myself to evolve (Arzarello et al., 2014).

Observation of both teachers suggested they were beginning to internalise aspects of this new approach. However, it was also evident that their enacting of new knowledge was in the early stages of development. For example, when observing Deryn, I considered that aspects of her knowledge inhibited her capacity to respond flexibly to student responses she had not anticipated[23]. The subsequent whole class discussion was refocused onto the first two examples shared by students, which were aligned with what she had planned to elicit during the lesson. Similarly, Giselle's enactment of learner-centred techniques appeared in the early stages of development. For example, whole class discussion seemed very teacher directed. There were not opportunities for students to ask their peers questions about their mental methods or moments for students to make decisions to contribute to the discussion (opt into the discussion). Throughout the lesson Giselle tended to summarise key points herself, rather than invite students to do this. At the end of the whole class discussion, Giselle posed a question, explained the strategy she would use to work out the answer mentally, and justified why. There was not any interaction with students, or between students. It is possible that she was conscious of allowing enough time for students to engage with tasks independently and that summarising herself was quicker. However, her actions also suggest she thought it was her role to teach explicitly at some point in the lesson; allowing the students to do the explaining might have signified that the lesson was deficient somehow.

5.3.2.6. A Change in the Domain of Consequence (Arrow 6)

In reflecting on their classroom experimentation, both teachers considered student learning a salient outcome:

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[23] In response to the question, 102 – 97, the student explained that they had counted back from 102 (minuend) to the subtrahend (97). Deryn represented the student thinking visually; she briefly comment that the strategy was counting back but she did not seem convinced this was correct.

> When they are doing it in their heads, when they are at the floor at the start, it just forces those lower students to actually think. (Giselle, focus group)

> … And make kids more aware, or more verbal in actually being able to explain it. (Deryn, focus group)

Giselle reflected on the importance of students learning to compute mentally. Her comment suggests that this approach to teaching mental computation provided all students with an opportunity to engage in the lesson: for active learning across different achievement levels. Deryn considered the positive impact on students recognising when it was appropriate to use mental strategies, and improvement in their capacity to explain their thinking as noteworthy.

In addition, Deryn considered her professional learning as an important consequence of her classroom experimentation:

> I think it was really explicit for me. I've taught the strategies before but have probably got a lot more used to when to use those strategies, that's what I got out of it the most. I've taught jump and all that but being explicit about the type of problems you can use this strategy for. (Deryn, interview)

> And I suppose I like the fact that, I've learned a bit more about some of the strategies too as to when you would use them and things like that, that's been really good. (Deryn, interview)

Deryn reflected on deepening her knowledge (SCK and KCT)[24] about when to use certain mental strategies and the design of questions to elicit a particular strategy.

5.3.2.7. Summary

Deryn and Giselle appeared to experience the same change sequence; the domains were linked by the same type of mediating processes. Despite being teachers at very different stages of their careers, they shared similarities in relation to disposition; they were both open to new learning. The data suggest the sequence of modelled lessons and participation in facilitated collective planning sessions were crucial in initiating change; both enhanced opportunities for brokering and bidirectional interactions (between each teacher and myself and among teachers). These elements of social dynamics (Arzarello et al., 2014) were a critical influence initiating change for both teachers.

Opportunity to observe the teachers corroborated some of the self-reported changes in knowledge and pedagogical approach; it provided insights into the components of the program they were beginning to internalise. As a result of participating in the program, Deryn appeared to develop KCT and SCK. She demonstrated a using student thinking to stimulate new learning. Similarly, Giselle developed skills in facilitating productive discussion alongside developing aspects of knowledge (SCK, KCT and KCS). Although the teachers were accurate in describing changes in their own practice, my observation indicated that these

[24] Specialised Content Knowledge (SCK) and Knowledge of Content and Teaching (KCT).

changes were in the early stages of development. Summaries of the learning processes for each teacher are presented in Table 5.4.

Table 5.4. *Summary of change sequences for Deryn and Giselle*

Arrow	Domain Link	Mediating process	Summary of Deryn's learning processes	Summary of Giselle's learning processes
1	ED to PD	Reflection	Reflects on external stimuli: professional learning session, modelled lessons and related professional conversations, teacher resource book. She has new PCK; how to use number strings as a tool to facilitate student centred learning of mental computation strategies. She experiences a change in her disposition; she is enthused about using new instructional tools.	Reflects on external stimuli: modelled lessons, professional conversations, teaching resources and collaborative planning. She has new pedagogical content knowledge to facilitate student centred learning of mental computation strategies.
2	PD to DP	Enactment	Enacts new knowledge about teaching mental computation with conceptual understanding. She begins teaching a new sequence: using the jump strategy to subtract mentally. She focuses on allowing time for students to think mentally first and explain their strategies. Highlights on the importance of the professional conversations during lessons and in planning meetings.	Enacts new knowledge, predominantly PCK. She teaches two different sequences. She focuses on developing her skills to facilitate whole class discussion and opportunities for students to think and reason.
3	DP to DC	Reflection	Reflects on changes in student affect: engagement in whole class discussion. Reflects on student learning outcomes: reasoning skills, development of mathematical language to articulate thinking; improvement in accuracy in mental computation.	Reflects on salient outcomes in relation to student learning and affect: improvement in engagement, confidence, attitudes to learning and flexibility in thinking. Reflects on her own professional learning, namely developing knowledge about how students learn (KCS) and development of pedagogical skills to facilitate productive discussion.
4	DC to PD	Reflection	Reflects on changes in her PCK, specifically changes in KCT, as a result of experimentation with number strings to build on student knowledge. Reflects on changes in her disposition; her conceptualisation of computation.	Reflects on development of aspects of PCK; learning about mental strategies and ways to present thinking using visual representations. Reflects on her conceptualisation of mathematics and changes in her approach to thinking and teaching.
5	PD to DP	Enactment	Continues with classroom experimentation of new aspects of KCT to teach a new sequence. Focuses on facilitating productive discussion; displaying student strategies visually to instigate learning.	Enacts new knowledge about mental computation strategies (KCT) to teach a new sequence of lessons on indirect addition.
6	DP to DC	Reflection	Reflects on student learning; improvement in their capacity to explain their thinking. Reflects on her own professional learning; development of SCK and KCT in relation to mental computation strategies.	Reflects on impact of classroom experimentation on student learning, highlights the positive impact on lower achieving students.

In Table 5.4. codes relating to the domains are: External Domain (ED), Personal Domain (PD), Domain of Practice (DP), Domain of Consequence (DC)

In the following section background information on the second pair of teachers sharing a different change sequence is presented.

5.3.3. Contextual background: Ethan and Fiona

In terms of disposition on teaching and learning, classroom management techniques and career background, Ethan and Fiona seemed distinctly different teachers. Yet, they evidenced a similar change sequence pattern. This section provides a brief comparison of their contextual backgrounds.

Ethan was a very experienced classroom practitioner; his teaching career spanned twenty-five years. Although he had previously taught a Year 3/4 composite class, this was his first experience of teaching solely Year 3 students. In contrast, Fiona was a relatively new teacher with two and a half years of classroom experience. Her teaching experience was with the middle primary years; she had taught Year 4 for two years before moving to Year 3 at the beginning of the current school year.

Ethan's approach to teaching appeared comparatively different to that observed in the other Year 3 classes. There was a sense of free-flowing student-driven discussion in his class. Generally, the students were not dependent on teacher authorisation when presented with a problem (Researcher's journal). He appeared to view learning as complex and interconnected rather than a linear process; this was reflected in his approach to teaching:

> Because I do believe with some kids I've seen, some kids just don't quite get it and I think let's just leave it a while and move on and they do get

> those 'aha' moments later whether it's 6 months later, a year later. (Ethan, interview)

He was less stringent in his expectations of student behaviour. For example, on occasions when a small number of students became distracted, his strategy was simply to focus attention on the students who were interested and engaged. In comparison, Fiona conveyed a strong presence and authoritative role in the classroom. She mandated clear expectations for student behavior and was active in directing whole class discussions (Researcher's journal).

Despite apparent differences between the teachers, both shared a similar disposition in relation to the teaching and learning of mathematics. Fluency in basic number facts appeared important in Ethan's conceptualisation of mathematics. He described how he incorporated such learning into his class program:

> I've got this little program called Blitz Master which is not so much about teaching; it's just about repetition of basic number facts and it's great. They all start on one level, if they get two in a row perfect score… It's timed. It has to be timed because it has to be the same conditions each time. Once they get two in a row, they get to move up a level, and I've found that the kids just love to be challenged themselves. It's really non-threatening so I use that religiously once a week. I think it's a small thing but I think mental maths needs more of a priority. (Ethan, interview)

Ethan seemed proud of his strategy to promote fluency in number facts; he shared the teaching resources the first time I visited his classroom. I was intrigued to learn more about his conceptualisation of 'mental maths' and if it was limited to fluency with basic facts. Interestingly, his use of timed tasks for learning facts was not aligned with the approach advocated by the ML; she was very clear than use of such resources not in the school planners "nor endorsed" (Hannah, semi-structured interview).

Fiona expressed her disposition on teaching and learning of mathematics in the pre-intervention survey:

> I enjoy Number, especially working mentally as this has high 'life' relevance and underpins other crucial aspects. I also believe being nimble in the mind underpins confidence. Relatively low fluency/speed and simple accuracy is often misinterpreted by students as lack of ability (poor mindset creates self-fulfilling prophecy) (Fiona, pre-intervention survey)

Her comment suggests she perceived speed and accuracy working with basic facts as synonymous with achieving success in mathematics. It was evident that Fiona enjoyed teaching mathematics. Her enjoyment was possibly linked to self-confidence in her subject knowledge; she communicated that mathematics was a "personal strength at school" and that she was the highest performing student in this subject in her Graduate Diploma in Education (Fiona, pre-intervention survey). One of her main learning goals for her class was aligned with those of the project, to "enable clear articulation of what and why." (Fiona, pre-intervention survey). She valued a focus on mental computation, particularly its connection to other aspects of number and real life; her comments suggest she considered the focus of the intervention worthwhile.

Ethan and Fiona were distinctly different in their approach to teaching but shared similar goals for one aspect of student learning: fluency with basic facts. Both placed emphasis on the importance of students being able to compute mentally. Their change sequence and accompanying discussion of the similarities and differences with their learning processes, is presented in the following section.

5.3.4. Change sequence 2: Changes in practice and the influence of cognitive dissonance

Participation in the professional learning program appeared to stimulate elements of cognitive dissonance and influence the learning processes for both Ethan and Fiona. The change sequence for Ethan and Fiona is represented diagrammatically in Figure 5.2. An overview of this change sequence, including a summary of the learning processes, is presented for each teacher (Ethan and Fiona) in Table 5.5

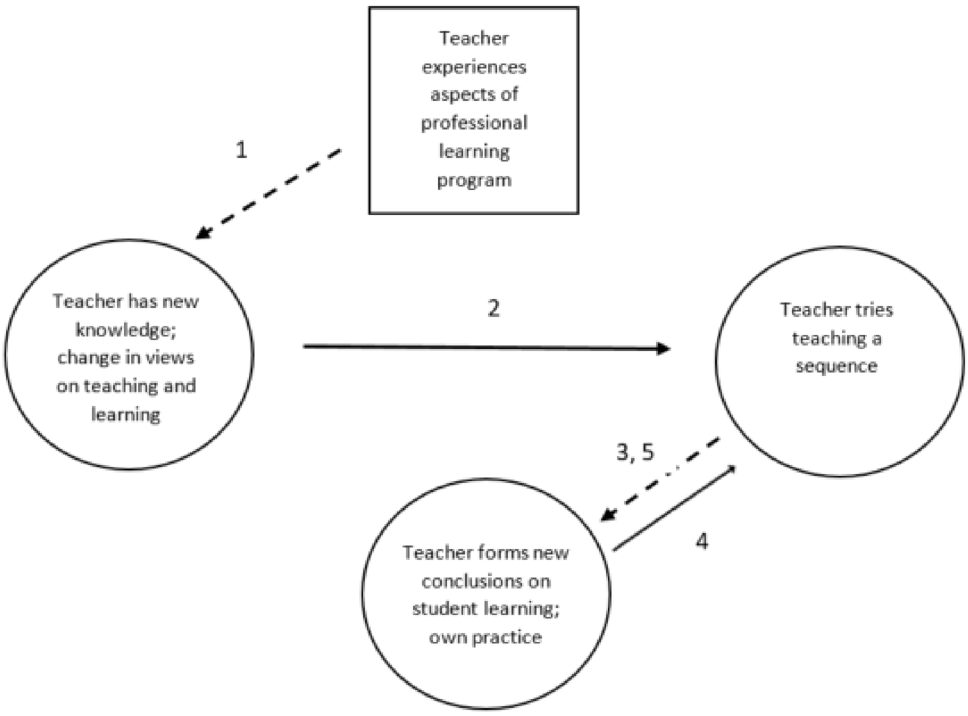

Figure 5.2. Ethan and Fiona's change sequence.

In the following subsections the focus of the discussion is specifically on the influence of cognitive dissonance on changes occurring within certain domains. The changes experienced by each teacher will be compared and discussed. Detailed analysis of the whole change sequence for these teachers is presented in Appendix E.

.

Table 5.5. *Summary of Ethan's and Fiona's change sequences*

Arrow	Domain Link	Mediating process	Summary of Ethan's learning processes	Summary of Fiona's learning processes
1	ED to PD	Reflection	Reflects on external stimuli: observing the researcher model a sequence of lessons and the opportunity to engage in professional conversations. He has new Pedagogical Content Knowledge (PCK) to facilitate productive discussion.	Reflects on external stimuli: professional learning session, modelled lessons and teacher resource book. She has new PCK to facilitate student centred learning of mental computation strategies.
2	PD to DP	Enactment	Enacts aspects of new PCK in the classroom. He experiments with teaching lessons focused on the jump strategy to subtract and bridging to add mentally. He focuses on facilitating whole class purposeful discussion, specifically students articulating thinking verbally and using visual representations to stimulate learning of peers. He expresses some negativity towards change, his classroom management skills seemed to interfere with implementation of the new approach.	Enacts new knowledge, predominantly PCK. She teaches two differences sequences. She focuses on using visual representations of thinking to deepen student understanding of computation processes and move forward student learning for the whole class. Expresses concerns with a whole class approach to teaching and differentiation.
3	DP to DC	Reflection	Reflects on salient outcomes in relation to student learning, namely their development of a range of computation strategies and improved efficiency. Reflects on outcomes in relation to his own professional learning, specifically extending aspects of his pedagogical knowledge (KCS). Positive student learning outcomes appear to create internal conflict with how he could best meet student learning needs and perceived curriculum responsibilities within a given time frame.	Reflects on salient outcomes in relation to student learning, specifically development of reasoning and improvement in articulating thinking using visual representations. She reflects on positive changes in student affect towards learning, as well as benefits in terms of her own professional learning.
4	DC to DP	Enactment	Enacts new knowledge of pedagogical approaches to teach a new sequence of lessons on indirect addition.	Enacts new knowledge of pedagogical approaches to teach a new sequence of lessons on indirect addition.
5	DP to DC	Reflection	Reflects on salient outcomes, he recognises the benefits of a change in practice in terms of both student learning and his own learning.	Reflects on salient outcomes and recognises the benefits of a change in practice in terms of both student learning, and her own professional learning.

In Table 5.5 codes relating to the domains are: External Domain (ED), Personal Domain (PD), Domain of Practice (DP), Domain of Consequence (DC)

5.3.4.1. Arrow 1: Change in the Personal Domain

Ethan and Fiona both reflected on the modelled lessons and associated brokering as critical stimuli to new learning. First, learning processes instigated by this external stimulus will be compared and contrasted for each teacher. This will be followed by discussion of elements of cognitive dissonance that appeared to emerge for both teachers.

Ethan reflected on the modelled lessons, placing emphasis on the importance of observing his students. He explained that with over twenty years of experience as a classroom teacher, it was not observing another practitioner that was of value but the opportunity to observe students (Researcher's journal). Ethan commented that the most useful thing from participating in the sessions was "seeing the misconceptions of some students" (post-professional learning survey). The modelled lessons provided him with the opportunity to notice, contemplate and discuss student misconceptions. Observing his students appeared to support development of his knowledge about how students learn to compute mentally, conceptualised by Hill et al. (2008) as Knowledge of Content and Students (KCS).

Although Ethan was keen to emphasise that it was the opportunity to observe his students that was most important, his comments suggest that observing an external person teach also instigated new learning. In his post-professional learning survey, Ethan reflected on the lessons and commented that they "made [him] think more about utilising students' knowledge/strategies." He recognised that sharing and discussing student thinking was a powerful tool to instigate the learning of others – an observation he discussed fervently in his semi-structured interview and reiterated in his post-professional learning survey…

> … definitely seeing their peers because often as teachers you don't do that. You might work in small groups and sometimes we might show them, whereas you drew it out of them a lot more. (Ethan, interview)
>
> Students were able to see other students' methods and this enabled them to understand that there are efficient strategies. (Ethan, post-professional learning survey).

The first excerpt above suggests that he usually assumed responsibility for providing explanations or modelling of ideas. His usual practice reflected the approach of the school numeracy intervention program (based on ability grouping of students) implemented in previous years. In the second excerpt, he reflected on the new approach supporting students in refining their thinking and developing more efficient strategies. Ethan appeared to be developing knowledge about new pedagogical approaches. Similarly, Fiona considered the modelled lessons important in stimulating new learning:

> The modelled teaching for me and number strings, was an element I hadn't much exposure with, so watching you do that... (Fiona, focus group)

She reflected on the benefit of seeing the pedagogical approach in action, in particular how to use number strings as an instructional tool. The brokering process was an important influence on her learning, perhaps because the new approach was a significant contrast to her usual practice:

> 'I do, we do, you do' would be more the model. So you would demonstrate a strategy, you would engage the children with the strategy. (Fiona, interview)

Allowing students time to think and engage mentally with a task before being invited to contribute ideas to whole class discussion, contrasted with her usual teacher-directed approach.

Although both teachers described opportunities for new learning, exposure to the new approach activated some uncertainty in relation to their disposition. During the modelled lessons Ethan asked for clarification on the project goals for student learning. He seemed slightly disconcerted with students being exposed to a range of strategies and expressed this concern through an analogy of a sports person learning to play golf. He explained that a golfer may learn a new technique but it does not necessarily improve their game. When placed under pressure, the player often reverts to familiar, less effective strategies. It seemed that Ethan was yet to notice the subtle difference between the classroom situation and his analogy. In the classroom students were not being shown how to perform a new technique or strategy, they were constructing the ideas themselves by thinking and reasoning about the ideas of their peers. They were essentially learning based on social constructivist principles (Fosnot, 1996).

Likewise, when implementing a whole class approach to teaching Fiona anticipated some potential challenges with…

> … extended sequence that is substantially whole class teaching; differentiated questions but 'whole' teacher led conversation. (Fiona, professional learning survey)

It seemed that the culture of teaching students in ability groups was embedded in Fiona's classroom. Separate but connected to the concern with a whole class approach, was the range of student learning needs in her class. In her post-professional learning survey she predicted that "… students will vary quite widely as they have entered with highly variable prior knowledge and maths confidence."

Fiona's concerns were not limited to the challenge of differentiated learning needs. In addition, she raised the issue of attitudes to learning in her post-professional learning survey noting a "range of student learning needs and attitudes." She thought there was potential for the higher achievers to disengage if they were seen to be doing similar tasks to the rest of the class (Researcher's journal). The issue seemed to be as much to do with the classroom learning culture as it was about providing tasks that would challenge the thinking of these students.

Despite expressing some concerns with the approach to teaching, both teachers decided to enact their new pedagogical knowledge in their classrooms. The changes they enacted are compared and discussed in the following subsection.

5.3.4.2. Arrow 2: Change in the Domain of Practice

Both Ethan and Fiona experienced challenges with experimentation of new knowledge in their classrooms and expressed some negativity towards change. Ethan expressed difficulties with the transition from small group teaching to a whole class approach:

> I liked the sequence. I guess, again it's another managerial thing, it would be a lot easier if it was done with half the class even… or in groups. I've got them in four groups, which varies a bit each time we test; I group my kids. (Ethan, interview)

His comment suggests that the culture of teaching in ability groups – an approach adopted by the school as a result of their previous involvement in a numeracy intervention program in unison with Catholic Education Melbourne (CEM) – seemed embedded in his classroom. One of the key challenges was in relation to classroom management. Ethan reflected on this extensively in his interview:

> I know this is just a management thing in my class but using the boards can be a bit distracting at times. I use the boards with a small group on the floor where it is easy to control. It's good, I like the idea of it but sometimes some kids can fiddle around so it can be a bit distracting. (Ethan, interview)… because if you've got 26 kids holding up their boards and waiting you are going to lose half of them. It's just a managerial side

> of it… I think it would be best to target a smaller group. (Ethan, interview)

Although Ethan reflected positively when he observed me implement the approach with his class, his own experimentation evoked a negative attitude towards change. The approach I modelled involved students using mini-whiteboards to explain their thinking visually to a partner (after they had individually engaged with the problem mentally). The mini-whiteboards were useful in allowing the teacher to circulate and select appropriate examples, including misconceptions, to orchestrate whole class discussion (Stein et al., 2008). His reflection suggests classroom management issues interfered with his using mini-whiteboards as a tool to support the pedagogical approach. He was yet to establish use of techniques such as learning partners or other systems to help manage whole class discussion more effectively. Experimenting with such systems might have supported him with more effective time management of whole class discussion (Clarke, 2014; Wiliam, 2011).

The challenges Fiona faced were comparable to Ethan's: essentially, she also argued for the need to incorporate ability group teaching into the approach. The modelled approach of providing differentiated learning opportunities was a significant change in practice for her…

> … so certainly, in the sense of this being a broadly whole group, with some tiering within it, but the teaching being whole group was quite different. (Fiona, interview)

Her comments suggest that she perceived differentiation as students receiving different learning tasks and different teacher instruction. Use of enabling and extending prompts, alongside students making decisions on differentiated tasks through which to investigate strategies, did not fit within her conceptualisation of differentiation. Fiona and I clearly shared different perspectives on differentiation; she perceived it as differentiated *teaching* whilst I was focused on differentiated *learning* opportunities. It seems reasonable to suggest that the pre-existence of a school culture advocating a pedagogical approach based on ability grouping, continued to mould her praxeology and acted as a constraint to change. Her initial concerns regarding differentiated teaching appeared to infiltrate this first phase of the intervention.

5.3.4.3. Arrow 3: Salient Outcomes and a Change in the Domain of Consequence

Ethan and Fiona reflected on their classroom experimentation and recognised similar positive outcomes from a change in their practice. The outcomes were related to student learning and their own professional learning (refer to Appendix E). Although Ethan recognised the benefits for students learning to compute mentally, the enactment of the new approach appeared to create internal conflict in relation to his disposition:

> Obviously we'll have to do more on written forms, which we do have to teach. (Ethan, interview)

Ethan perceived the teaching of written calculation methods as his responsibility as a teacher. In his interview response he placed emphasis on the words "we do have to teach". However, his comment reflect his own conceptualisation of mathematics, rather than the perspective shared by the ML (Hannah). It is possible that his disposition concerning his conceptualisation of mathematics was influenced by institutional aspects. This concurs with the argument presented by Hiebert (2013) that mathematics teaching of mathematics can be viewed as a cultural activity. It seemed that this conflict was intensified by a limited timeframe in which to teach addition and subtraction. When asked to reflect explicitly on constraints with the approach, he responded with:

> The number one reason would be time constraint. I don't know what it's like in England but they always use the word the 'crowded curriculum.' (Ethan, interview)

Although he recognised the benefits for students of learning to compute mentally, it seemed that time constraints meant it was difficult to comprehend how mental computation could be integrated into the mathematics program when there was a perceived requirement to teach written methods. He appeared in a quandary in terms of meeting student learning needs and fulfilling his perceived teacher responsibilities in regards to curriculum content. He made the suggestion that "maybe mental maths needs to be mandatory, like I've said, or mandated a certain amount of hours per week" (Ethan, interview). His reaction suggested that unless this approach was officially endorsed in the school mathematics program, Ethan could not foresee a future for it in his classroom.

5.3.4.4. Arrow 4: Change in the Domain of Practice

At the beginning of the following term (Term 3) a second phase of the intervention was implemented by the Year 3 team. I observed the same lesson on mental computation in each classroom. The lesson provided some insights into aspects of the approach the teachers had started to internalise. In this subsection, what follows is a comparison and discussion of my interpretation of aspects of Ethan's and Fiona's lessons, illustrating the influence of cognitive dissonance.

It was evident that Ethan had conceptualised some of the fundamental ideas of the intervention, in the way he had assimilated aspects of the pedagogical approach into his teaching. He had adopted practices that promoted individual thinking time and enhanced opportunities for active participation from all students (Wiliam, 2011), for example, students using 'thumbs up' instead of 'hands up' to indicate they were ready to contribute to the discussion. He emulated an investigative approach to the lesson; the students were given the opportunity to explore, test and justify ideas. Subsequent to the discussion that ensued during the planning meeting regarding focus groups, Ethan incorporated focus groups into the lesson structure. The group sessions resembled the structure of the whole class discussion; a task was presented for students to complete individually before sharing and discussing ideas with the group. The focus group sessions reflected the outcome of the double dialectic that evolved during the planning session (refer to Section 5.4.2.2. for details on how the shared perspective on use of focus groups evolved). However, it was evident that Ethan also experienced some challenges with enacting his new knowledge. The lesson observation provided some insight into the concerns raised by Ethan in relation to managing the whole class discussion and time (refer to Arrow 3). During the lesson he made a conscious decision not to use visual representations to show student thinking and initiate discussion. Instead the discussion was facilitated based on oral explanations. Students' explanations were at times lengthy and difficult to comprehend without displaying visual representations. The students had access to mini-whiteboards during the lesson but they were not used to actively monitor student responses, an important component of orchestrating productive discussion (Stein et al., 2008). The students were not instructed to explain their thinking to a peer so there was not an opportunity for Ethan to monitor and select student ideas to contribute to the discussion. It seemed he was yet to establish clear routines and systems with his class. This resonates with the work of Brown and Coles (2013) who used the notion of relentless consistency to explain the need for establishing a classroom culture that fosters student agency and allows for ambitious teaching practices (Kazemi et al., 2009). Enacting such approaches effectively not only involves understanding the intricacies involved but requires time to practise and refine; Ethan was in the early stages of experimenting with this new approach.

Whilst Fiona also demonstrated enactment of various pedagogical changes such as allowing students individual thinking time and use of visual representations to show student thinking and initiate discussion, her lesson was orchestrated differently. She modified the approach to align with her perspective that students learn more effectively in ability groups. The lesson started with a short task to activate mathematical thinking and develop fluency. The students were seated on the floor in ability groups; Fiona directed certain questions to each group. It appeared that the brokering process had been less successful in guiding Fiona's pedagogy in line with some of the underpinning ideas of the approach.

The lesson highlighted the challenges with transitioning from a teacher-directed orientation towards co-constructed student-driven learning. Her students were invited to share their thinking but were not asked to comment or ask questions about the ideas their peers shared. The final authority on the efficiency of particular strategies seemed to rest with the teacher, Fiona. She seemed to have a clear trajectory in mind for the lesson; when a student offered an explanation that deviated from the strategies she intended to elicit, their contribution was quickly dismissed. It was not ascertained if such decisions were based on time constraints, concern that the discussion may confuse some students, or her uncertainty about the key learning points to elicit. Nonetheless, her reflection corroborated my interpretation that she found relinquishing a teacher-directed approach challenging:

> I have felt, like the stronger children, if I could have focused them. I don't know, maybe it's just the habit of the way we've been teaching but you know if you almost get them by the scruff of the neck early and point out, not point out 'try and elicit from them,' where they were just not quite on it. (Fiona, interview)

Fiona's reflection suggested she perceived it as her role to direct student learning; she felt a need for explicit teaching to move forward student learning.

Although she was keen to experiment with the ideas in her classroom, her comments indicated she was gravitating towards the idea of explicit teaching, rather than students learning from each other to construct their own ideas.

5.3.4.5. Summary

Both Ethan and Fiona experienced initial changes in their Personal Domain; the changes were predominantly in aspects of pedagogical knowledge. The opportunity to observe a sequence of modelled lessons provided a significant stimulus for both teachers; it fostered opportunities for brokering and bidirectional interactions between each teacher and myself. Both teachers described such opportunities as an important influence on their learning.

Although each teacher experienced changes in their practice, various internal and external factors seemed to constrain internalisation of the new approach. Ethan was experimenting with facilitating productive discussion for the first time. The challenges he described with the new approach were predominantly related to classroom management issues. Interestingly, he did not critically reflect on ways to modify his practice to overcome these challenges. It is possible his behaviour was indicative of his perception of himself as an experienced practitioner in the latter stages of his career. In addition, he appeared to experience some cognitive dissonance. Although he recognised the benefits of the approach in terms of student learning, he struggled to comprehend how he could fulfil his perceived curriculum responsibilities as a Year 3 teacher and allow students time to develop mental computation strategies within the given timeframe. His reflections implied that if the approach was endorsed in the school program, he would embrace it in his classroom in the future.

Observing Fiona teach in the final stages of the intervention, I found evidence that she had internalised aspects of the new approach to teaching. She was enacting learner-centred practices by allowing students individual thinking time and opportunity to explain and discuss strategies using visual representations. However, there was an element of her practice for which she was resistant to change. Differentiating by teaching in small ability groups was embedded in her classroom culture. She modified the approach of the intervention to incorporate ability grouping in such a way that her practice was misaligned with some of the underpinning goals of the intervention. There was a disjunction with our perspectives on differentiation; she recognised it as providing opportunities for differentiated teaching, whereas I perceived it as opportunities for differentiated learning. It should be recognised that Fiona had made some substantial changes to her pedagogical approach, but differentiating within a whole class approach was an aspect of the project with which she still appeared to be grappling.

It was apparent that both teachers experienced and expressed some level of cognitive dissonance that constrained further changes in their practice. They both valued the importance of students learning to compute mentally, explaining thinking and reasoning. Yet, they struggled with how this approach fitted within their current schema for teaching mathematics and their responsibilities as teachers.

5.3.5. Change sequences: A summary of teachers' learning

All four teachers perceived changes in aspects of their knowledge and practice; my observations and interactions with the teachers corroborated these changes. It emerged that there were two different change sequence patterns, and two pairs of teachers each experienced the same sequence. Deryn and Giselle followed a sequence in which changes within the domains were predominantly influenced by social dynamics (Arzarello et al., 2014). For Ethan and Fiona, the stimulation of cognitive dissonance influenced changes within domains and the linking of the domains.

Despite sharing the same change pattern, it was evident there were some differences in the changes that occurred within domains for each teacher. Changes for individual teachers were influenced by the interplay of internal and external factors within the change environment; a finding that resonates with those of Wilkie (2019). For example, Fiona was keen to develop her KCT as an early career teacher but changes in her practice appeared constrained by her disposition on how students learn best (ability grouping): a personal (internal) influence. Teaching by ability grouping was a practice previously endorsed by the school from involvement in a numeracy intervention program (external institutional factor). For Ethan and Fiona, the interplay of internal and external influences seemed to create some cognitive dissonance, which appeared to constrain their internalisation of new praxeologies. For example, Ethan recognised the benefits of the approach for student learning but external influences such as curriculum requirements resulted in his being in somewhat of a quandary with adopting the new approach. These findings reflect those of previous studies (Clarke & Hollingsworth, 2002; Goldsmith et al., 2014; Wilkie, 2019) that teacher professional learning is an idiosyncratic experience influenced by factors within the change environment.

There were various external stimuli that initiated and supported change for each teacher; all these changes were linked to components forming the Personal Domain. One possible explanation, which connects with findings of Justi and van Driel (2006), is that teachers were exposed to perspectives different to their usual practice; Justi and van Driel found this was an important consideration in designing the External Domain. The teachers in this study were experimenting with an approach that was distinctly different to their usual practice; they were learning to transition from predominantly teacher-directed approaches to learner-centred practices.

It was not simply the components of the External Domain that were important influences on learning opportunities, per se. The learning processes that occurred within each domain were also critical to instigating change (in knowledge, practice and disposition). In addition to mediating processes of reflection and enactment, the influence of social dynamics on learning was paramount (Arzarello et al., 2014). This was evidenced in the bidirectional interactions and brokering process that occurred between the teachers and myself, as well as among teachers. For two of the teachers, Deryn and Giselle, the brokering process and bidirectional interactions was particularly influential on their learning and change.

It appeared that changes in teachers' practice were both afforded and hindered by aspects of the institutional context. The influence of these institutional aspects on the professional learning of teachers will be explored in the following section.

5.4. The change environment: The institutional context

In this section, data concerning the second research sub-question are discussed, specifically, the institutional aspects of the change environment and how these seemed to influence opportunities to learn and change practice. The Meta-Didactical Transposition (MDT) model (Arzarello et al., 2014) was used to examine the influence of institutional affordances and constraints on changes to teachers' knowledge, practice and disposition (refer to Figure 3.2); these are discussed in the following subsection.

5.4.1. The influence of institutional affordances on change

The school environment had certain structures in place to afford opportunities for teachers to learn and develop their professional practice. These structures included: school leadership model and the role of the ML; systems to support teacher planning and teaching of mathematics; and the school perspective on professional learning. These influences correspond with two aspects that Clarke and Hollingsworth (2002) described as "fundamental" to teacher change: that teachers were seen as learners and the school was viewed as a learning community (p. 949). In this subsection, each of these aspects will be explored.

The school had established a system that allowed for Year groups of teachers to meet on a weekly basis to plan the teaching and learning of mathematics collaboratively. Whilst this seems to be common practice in most primary schools in Australia, the leadership structure at the school was designed to support this system as a professional learning experience for teachers. The ML attended weekly planning meetings with the purpose of providing subject expertise and guiding the planning process. The presence of the ML at meetings supported the structure of a learning community and ensured that meetings were used for the purpose intended:

> The idea of planning was that we plan together so we all have input and we all know what we are going to be teaching. Because if one person plans it, then you are all just teaching someone else's and it may not work for you or the children. (Hannah, interview)

The school expectation that planning was a collective learning experience was embedded as part of the school culture. The ML explained that although there were occasions when school events interfered or interrupted allocated planning time, in such situations "the idea is that we at least jot down a sequence and clean it up in the planners together" (Hannah, interview). The provision of structures to establish and support a learning community gave the impression that teachers were regarded as learners within the school.

From my perspective as a researcher, the ML's presence at planning meetings was advantageous in providing opportunities for the teaching team to see there was alignment between the goals of the research project and the perspectives of the ML on teaching and learning mathematics. The project supported the ML's goal to transition from ability group teaching towards an approach aligned with current research on effective pedagogy (Sullivan, 2011; Sullivan, Mousley & Zevenbergen, 2006). Consistency[25] with school policies and perspectives is considered an important factor that influences opportunities for teachers to learn (Desimone, 2009).

Active involvement of the ML in the research project was beneficial in terms of initiating and supporting change at an institutional level. This seemed necessary to allow some of the changes participating teachers were experimenting with, at a classroom level, to become embedded as future practice. For example, when the ML observed the modelled lessons in Giselle's classroom this raised awareness of challenges students were experiencing as a result of being introduced to written calculation methods prior to developing more sophisticated mental strategies. Subsequently, the ML implemented changes to the school mathematics program to support a future focus on teaching mental computation strategies at Year 3 level:

> Yes, so what we did after watching the modelled lesson, one of the ways that we've gone about the teaching of things is in Year 2 with those really, really high achieving students... we introduced the vertical algorithm as another strategy in their bank of strategies. However, they then attached to that because they see that as a sophisticated, adult, grown-up strategy

[25] Consistency is used here as a term related to the design of professional learning programs. It is considered one aspect of coherence (Desimone, 2009).

> because they see their parents do it and all of that, and then they become fixated on it. So looking at the Grade 2 planning of addition and subtraction we removed that for the high children. And have just kept it all mental, which did then challenge a lot of teacher's thinking because they felt like they weren't extending their students enough because they weren't showing them the algorithm. (Hannah, interview)

The ML described implementing changes in the curriculum at Year 2 level, which she felt were necessary to support learning of mental computation in Year 3 in the future. Her comment suggests that implementing this change created some challenges for the Year 2 teachers. The role of the ML was critical in terms of recognising the source of one of the challenges being faced by Year 3 students and teachers. Perhaps more importantly, she was able to instigate a change to address this issue. The ML's actions indicated that she was prepared to invest further time and energy to support the goals of the research project by working with teachers in another Year level:

> So I had to have lots of conversations with them [Year 2 teachers] about how that's not the most sophisticated strategy and it's just another approach and we should be building up in their head because that's just a procedure. So it changed our approach and our conversations around mental computation and I suppose teachers are quite fixated on it themselves, I think. Just like the kids are; I think they've grasped onto it through their learning and their past teaching days, I suppose you could say. So that has been a big change to our approach. (Hannah, interview)

In her interview, the ML articulated how it was necessary to extend professional conversations about the teaching of mental computation to the Year 2 teachers. Her comment resonates with the findings of Hiebert (2013) who suggests that the teaching of mathematics appears a cultural activity that has been passed down through generations. The school leadership structure, specifically the involvement of the ML, seemed to afford changes in practice aligned to the goals of the intervention.

An additional affordance to teacher learning and change was the school vision on teacher professional learning. It was a school requirement each year that teachers work collectively in Year groups on an action research project with a focus on improving an aspect of their teaching and learning. The teachers were required to engage in a certain amount of professional reading and present the findings of the project to the Principal. The decision of the Year 3 team to participate in this research project had a dual purpose of supporting teachers in meeting school professional learning requirements. The teachers requested additional reading on the subject content (mental computation) to support their school professional learning. This seemed a positive influence in terms of teachers recognising the value in a content focus of the intervention. It is possible that additional professional reading could have also supported their learning, based on findings of Wongsopawiro et al. (2017). They reported that science teachers who conducted a literature review and were involved in peer discussion experienced greater progress in learning and practice. The institutional context certainly provided these Year 3 teachers with a vested interest to engage with the research project. The teachers' collegial approach was evident in their action of creating a display board of student learning in mathematics, as a result of participation in the project (see Appendix E). This finding raises the question of whether these teachers might have responded differently to the project had they had not been subjected to such expectations and accountability from the school leadership.

Although there appeared to be clear affordances from implementing the intervention within this school context, such as: the leadership structure and role of the ML; an established collaborative planning system; and the school vision on professional learning, there also appeared to be some constraints to change. These constraints will be explored in the following section.

5.4.2. The influence of institutional constraints on change

Analysis of multiple data suggests that constraints to change could be categorised as three main themes: challenges with finding time in the curriculum, culture of learning, and challenges with the physical environment. Each of these will be explored in turn.

5.4.2.1. Challenges with finding time in the curriculum

The challenge of finding time to teach using this approach was highlighted as a concern by three of the participating teachers: Ethan, Deryn and Fiona. While their classroom experimentation was not adversely affected by this perceived challenge, it was a factor raised as a concern in terms of future implementation (ten references were made implicating time as a constraint to change). This challenge of finding time is highlighted in the following selected excerpts:

> I suppose that's my fine line as to 'have we spent too much time doing this?', if that makes sense. Whereas I know in the curriculum, there's probably a bit more we might have taught them. (Deryn, interview)

> Again, it's a time factor I guess… because we always feel, we've got to move onto the next unit. (Ethan, interview)

The excerpts above illustrate how teachers perceived time constraints imposed by curriculum demands (external and internal factor) and a busy school program (internal factor) as placing them under pressure to teach mathematics units within a certain timeframe. Limited time is commonly cited in the literature as a constraint to making changes to teaching and learning (Yurekili, Stein, Correnti, & Kisa, 2000). Interestingly, it was Deryn and Ethan who placed greatest emphasis on the challenge of time. As the Year 3 team leader, it is plausible that Deryn felt greater responsibility and pressure for ensuring her team met curriculum requirements. In Ethan's case, some of the issues he faced in terms of time pressures seemed related to challenges with classroom management. It is important to emphasise that teachers perceived time pressures as a constraint in terms of continued implementation with the approach in the future, rather than impeding their experimentation during the intervention.

5.4.2.2. Culture of learning

The teachers expressed some concerns with challenges related to culture of learning, namely differentiating to meet a range of student learning needs and initial resistance to the change in pedagogical approach from their students. Each of these concerns will be discussed in this subsection.

Teacher reflections conveyed that differentiating to meet the range of student learning needs acted as a constraint to changes in their pedagogical approach. The following excerpts have been selected to illustrate the concerns raised. It should be noted that two of examples were from a survey completed collectively by the Year 3 team[26] and may be representative of the views of a team member with a slightly dominant personality (Fiona), rather than the whole team. Fiona was the only teacher who explicitly discussed concerns with differentiation in her interview. The concerns related to…

> … the range of competencies in the room. (Fiona, interview)

> ... the range of learning needs in the room and being able to differentiate in whole a class discovery approach. (final survey – Year 3 Team)

> … not as much whole class teaching – there is still a need for group work with similar ideas involved. (final survey – Year 3 Team)

A key challenge the teachers faced was incorporating differentiation into an approach that predominantly revolved around whole class discussion. They might have regarded this as particularly challenging because of the existing culture of learning at the school; they were in the early stages of transitioning

[26] The intention was for the teachers to complete the final survey individually but due to limited time the team made an independent decision to complete this survey collectively during a meeting.

from an ability grouping approach. For some of the teachers in the team, namely Fiona and Ethan, ability-group teaching appeared an embedded culture. It is possible that the teachers' conceptualisation of differentiation magnified the challenge of meeting a range of student learning needs. The teachers appeared to see their role as providing differentiated *teaching* for students rather than differentiated *learning* experiences. The idea of whole class discussion, specifically teacher development of pedagogical skills to facilitate this effectively, seemed to constrain the implementation of the intervention as it was intended. However, this did not necessarily constrain teachers from experimenting with changes to their practice. A discussion on the issue of differentiation that ensued at the planning meeting in preparation for the second phase of the intervention, highlighted evolving meta-didactical praxeologies, and will be discussed next.

During the planning meeting, the teachers and I reflected on the challenges of differentiating. We engaged in a discussion on how a focus group could be incorporated into the lesson structure and the role of the teacher in this session. Pre-intervention, teachers' practice involved explicit teaching of students in ability groups, which they described as focus groups. My perception of a focus group and how to facilitate such a session had evolved from my professional learning experiences as a teacher in the UK, and differed slightly from the teachers' perspectives[27]. Whilst I recognised there could be some benefit to running a small group session, I was keen for this to be an opportunity for

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[27] In the UK these sessions were referred to as 'guided groups' and were promoted for Assessment for Learning purposes. Whilst the sessions could be used for explicit teaching, this was just one of many purposes of these sessions. They were also used for promoting reasoning and investigative approaches (Primary National Strategies CPD Materials, 2007).

students to articulate their thinking and develop reasoning skills. My vision was that the focus group would mirror the whole class session but with computation tasks varied to allow students to build on their existing knowledge. I had experimented with, and modelled this approach in Giselle's class. The discussion at the planning meeting highlighted the second-level dialectic, or double-level dialectic (Arzarello et al., 2014). Although there were some differences in opinion regarding how lower-achieving students in particular would learn best, the teachers were prepared to experiment with this approach in the second phase of the intervention. This was a shift in practice for the teachers, and an adaptation of the original lesson structure. It seemed that both of our praxeologies were beginning to evolve, with the potential for a shared praxeology to develop.

An additional constraint to changing teaching practice seemed to be the initial resistance the teachers encountered from students; this is widely reported in the literature as a hindrance to changes to approaches to teaching (Yurekili et al., 2000). Again, a challenge connected to learning culture, this time from a student perspective…

> … For the students to let go of a known strategy (e.g., vertical or split) and experiment with a new strategy. (final survey – Year 3 Team)
>
> Students are creatures of habit, they want to be right more than try something new, they might not have been exposed to other strategies, they see maths as having one answer and one process. Parental influence of

> using the vertical strategy, seen as the advanced or mature way to solve problems. (final survey – Year 3 Team)

Initially the change in approach to teaching and learning had met some resistance from students. It could be inferred that the students displayed characteristics of a fixed mindset to learning; they wanted to achieve success quickly and maintain self-preservation (Dweck, 2000; 2006). The teacher reflections imply that at the beginning, students did not perceive mathematics as a subject that involved exploring multiple strategies. It seems reasonable to assume that student attitudes were a reflection of the way they had been taught mathematics until this point in time. Although teachers reported a change in student attitude to learning during the implementation of the intervention, they still considered this a factor that curtailed initial progress with the change in pedagogical approach.

5.4.2.3. Challenges with the physical environment

At the time of the first phase of the intervention, the school was undergoing a significant building project. This entailed the construction of a two-storey facility with twelve new learning areas; the building was set to accommodate a further three hundred students. The scale of the project meant various types of heavy machinery, such as excavators, were in use throughout the school day. When asked to reflect on any constraints that may have affected the implementation of the program, Fiona and Giselle indicated issues with the physical school environment:

> My room! My room! (Fiona, interview)

> I think that construction is the biggest thing for us. The noise constraints and the focus of the kids, their attention on all the activities is disrupted. (Giselle, interview)

Fiona had temporarily moved to a room in the office area of the building, which was not designed for classroom purposes; limited space presented some challenges with seating the students on the floor for discussion and having a focal point, such as a whiteboard, to display student thinking. Giselle's comment was connected to the location of her temporary classroom on the school oval. The two-classroom block was located in the midst of the construction site, which at times presented a distraction to students. In addition, the building project meant that the Year 3 classroom was temporarily situated in different areas of the school. The physical distance between rooms limited opportunities for staff to have informal professional conversations between lessons; such conversations were only seen to occur in the staffroom during recess and lunch times.

5.5. Reflecting on the PL program: Refining the second cycle

The major issues encountered with implementation of the first intervention had been pre-empted in this cycle through the planning and early negotiation process. There were factors considered as constraints to teacher change, which although specific to School B's context, would be prudent not to disregard in the planning and implementation of future cycles. However, it seemed particularly valuable to reflect on and consider future suggestions from the teachers and ML to guide the design and implementation of the third cycle.

The provision of a teacher resource book and professional reading in the form of journal articles had been considered useful by the Year 3 team. These materials were provided at the start of the intervention; Fiona commented that availability of the materials in advance, i.e., when the project was introduced rather than commenced, would have been beneficial in allowing more time for her to familiarise herself with the ideas.

The ML (Hannah) emphasised it was the combination of all components of the program which led to positive outcomes for the teachers. In particular, she considered the integration of the cycle of planning, teaching and reflecting as crucial to the success of future professional learning programs for teachers. She suggested the integration of co-teaching, or a version of a lesson study, would further enhance opportunities for active learning between the teachers and researcher, or among colleagues.

5.6. Summary

In this chapter findings of the second intervention were presented and discussed in relation to the two research sub-questions. The data suggested there were two different change sequence patterns for the four participating teachers: one predominantly influenced by social dynamics (Deryn and Giselle) and the other by existence of cognitive dissonance (Ethan and Fiona). Particular external stimuli were considered critical for instigating learning processes for this group of teachers, namely modelled lessons and collective planning opportunities. These two external stimuli provided opportunity for different variables (variables constituting social dynamics) to exist: brokering, bidirectional interactions, and double dialectic (Arzarello et al., 2014). Such variables were critical to instigating change in knowledge, practice and disposition. For two teachers, Deryn and Giselle, the influence of brokering and bidirectional interactions (social dynamics) were critical for instigating learning. Whilst social dynamics were an important influence on learning for Ethan and Fiona, the stimulation of cognitive dissonance for both these teachers appeared to influence their internalisation of new praxeologies and change sequence pattern. For these two teachers, the interplay of internal and external factors was a critical influence on their learning experiences.

There were comparable and contrasting elements of change within each domain for the teachers sharing the same sequence. For example, after initial classroom experimentation, Deryn and Giselle both reflected on changes in student affect as a salient outcome but recognised additional different outcomes. The individualistic outcomes were possibly a reflection of distinct differences in years of teaching experience and their positions within the school (personal factor). This highlighted that although teachers may share the same change sequence within the same institutional context, their learning was an idiosyncratic process influenced by different personal (internal) factors.

It was apparent there were certain factors in the school environment, which afforded opportunities for teachers to learn and develop their professional practice. A key influential factor appeared to be the role of the Maths Leader (ML), Hannah, who had subject expertise to support and guide teachers with planning and teaching mathematics on a weekly basis. In addition, her vision for the school to transition from intervention ability grouping to an approach based on differentiated learning trajectories (Sullivan et al., 2006) was consistent with goals of this project. Her role initiating actions at an institutional level was critical to support and embed changes to teaching in future practice. In addition, the school leadership requirement for teachers to enact and demonstrate personal professional learning through action research, suggested that teachers were considered learners within this environment. Fundamentally, the school appeared a learning community in which the teachers were considered learners: two essential components for teacher change (Clarke & Hollingsworth, 2002, p. 949). There were factors within the school context, which also appeared to constrain aspects of teacher learning and change: differentiating to meet a range of student learning needs within a whole class approach; and disruption caused by a school building development program. Although the teachers articulated additional factors they considered constraints, the aforementioned were the ones interpreted as a hindrance to learning; other factors concerned potential constraints to future change. While the constraints created some hindrance to change, participation in the project had stimulated learning and change for all teachers, albeit to varying degrees.

6. Sharing a vision for learning and changing practice

Shared vision has to do with people in a school being able to hold a shared picture of the future they seek to create. (Thompson, Gregg, & Niska, 2004, p. 3)

This chapter reports on findings of the third intervention cycle conducted with a team of Year 3 teachers at a different school situated in the Greater Melbourne region (School C). Adoption of design-based research methodology meant data collected from previous iterations informed my decisions on the design and implementation of this third, subsequent cycle. Of particular importance was a focus on features of the intervention identified as critical in fostering and supporting teacher learning and change. These included integration of co-teaching, emphasis on 'in-the-moment' professional dialogue, and early provision of research-based professional reading. In terms of achieving this goal, this third cycle appeared most successful; since completing this intervention I have continued to support these teachers with professional learning. While the success may in part be explained by the cumulative effect of previous cycles of refinement, the distinct difference I perceived with this cycle, was that early into the intervention we all came to share a vision of effective teaching and learning of computation.

In the first section of this chapter, background information relating to the school context and participants is given. Following this, details concerning the refining of the intervention for this particular institutional setting are provided. In response to the first research sub-question (see Chapter 1, section 3), change sequences and discussion of the learning processes distinguishing this group of teachers from the preceding cycles is presented. The influence of the school setting (institutional context) on change processes is discussed in relation to the second research sub-question (see Chapter 1, section 3). Lastly, the evolving design of the professional learning program and future considerations for School C are reviewed.

6.1. School context and background

The third intervention cycle was conducted at an independent, co-educational school located in a Melbourne inner suburb. The school embraced the Jewish community and catered for students from early childhood through to secondary education. As an institution working in collaboration with Harvard Graduate School of Education for an extended period of time, it appeared an environment that welcomed innovative strategies for teaching and learning. The school was one of the pioneers in developing Cultures of Thinking[28] (CoT) and is renowned for achieving academic excellence in the region.

Students experienced a range of co-curricular opportunities and an extensive extra-curricular program. A range of specialist teachers and curriculum leaders were employed, including a numeracy coach (NC) to support the early years (EY) and primary classes. The classroom teachers in the EY and primary school were

[28] Cultures of Thinking (CoT) is the focus of a project started in collaboration with Harvard School of Education, Project Zero. The project builds on earlier work of Ron Ritchhart (2002) and focuses on honing collective and individual thinking in classrooms. Teachers are encouraged to use a variety of methods to promote and make thinking processes visible to enhance learning and collaboration. The project directs attention to eight cultural forces considered to influence a group learning situation: language, time, environment, opportunities, routines, modelling, interactions and expectations. http://www.pz.harvard.edu/projects/cultures-of-thinking

responsible for teaching mathematics; the NC was employed three days per week to support class teachers with the planning and teaching of mathematics.

6.2. Negotiating the implementation of the intervention

The methodological approach for the study meant taking into consideration the outcomes of the two previous iterations to refine the implementation of the third intervention cycle. Critical to the success of prior iterations was attention to ensuring collective participation at the school and aligning the timing of the intervention with the Year 3 mathematics program. Data also suggested the participants in the second intervention valued the external input throughout the planning and teaching cycle. In addition, informed by previous findings, the integration of co-teaching, an emphasis on professional dialogue in the teaching moment, and early provision of research-based readings were considered. The focus of this section is on describing the refining of the intervention for this particular school context.

6.2.1. Setting up the intervention

The decision to refine the introductory process was informed by time constraints experienced in the two preceding intervention cycles. Resources to support the introductory meeting were modified to incorporate some of the teaching content previously included in the professional learning session. Explanations of the instructional activities to promote student reasoning and a suggested lesson structure were integrated into the session. I anticipated the adaptations would be feasible based on the school's previous professional learning experiences. The school involvement in CoT, specifically promoting student thinking and making it visible, and the integration of challenging tasks[29] into the school mathematics program meant teachers had been exposed to innovative approaches to teaching and learning. It seemed reasonable to assume the teachers had some background knowledge that would complement this proposed approach to teaching mental computation. This minor modification afforded time for greater focus on how to use the instructional tools in a learning sequence and discussion of subject content during the professional learning session. In addition, the team was also provided with background reading (research papers on using the instructional tools) to give them time to process the key ideas underpinning the pedagogy and prepare questions to ask at the upcoming professional learning session.

Following the introductory meeting, I was invited to attend a planning session in which we negotiated how the sequence of lessons on mental computation could be integrated into the Year 3 unit plan on additive thinking. The usual practice at the school was to teach mathematics predominantly using tasks described as challenging. The NC, Laura (pseudonym), and Year 3 team were keen to look at how a new approach to teaching mental computation might complement their program and enrich student learning. The opportunity to have some input at this level of planning was useful for me to gain insights into their approach to planning, teacher thinking on the big ideas to include in a Year 3 unit on additive thinking, and the dynamics among team members. In the preceding

[29] The school initiated a professional learning day with Professor Peter Sullivan towards the end of 2017 to learn about teaching with sequences of challenging tasks. A task regarded as challenging, is one with which students do not initially know how to proceed and is complex. Such tasks require students make their own decisions of the method of solution, for which there can be multiple options that could be represented in multiple ways. The tasks should be within the Zone of Proximal Development (Vygotksy, 1978) for the student (Sullivan et al., 2015).

(second) intervention cycle at School B, the team leader and one of the teachers had raised concerns that they may not have covered sufficient curriculum content on addition and subtraction for Year 3, suggesting they had allocated too much time to mental strategies. (However, it should be noted that the concern was essentially that they had not focused on teaching formal written methods for computation, a view not shared by the Mathematics Leader at the school). The planning meeting for this third intervention cycle at School C was therefore an opportunity to unpack curriculum content descriptors with the teachers and discuss how to combine two approaches (mental computation sequences alongside challenging tasks) to teach the content and explore the proficiencies in the Australian curriculum. I considered this useful in preempting similar concerns with curriculum coverage at this school. In addition, the session provided opportunity to address potential issues with both the timing and adequate duration to implement the intervention. Devising the unit plan together allowed for bidirectional interactions between teachers and myself; it provided an opening for relationships to develop, a factor considered key to the success of design-based research (McKenney & Reeves, 2012).

Negotiating the assessment tool with the NC and teachers was an important part of setting up the intervention. The school was given the option to use a pre- and post-intervention assessment task designed specifically for the project. However, we discussed using one of the assessment tools the school had already adopted into their assessment schedule: the Mathematics Assessment Interview

(MAI)[30]. The purpose of the assessment tool was to inform planning and to measure student learning of the concepts considered important for the students to learn in the Australian curriculum. The school considered it important to use a tool already integrated into their assessment schedule; this was justified in terms of coherence with the school program for mathematics and reporting of student achievement to school leadership. From my perspective this was a viable option; the data needed to be useful for the school and to inform resource development for the intervention. Using a reliable tool that the teachers were already familiar with reduced complications with collecting assessment data. With student outcomes being considered influential on teacher attitudes and subsequent changes in classroom practice (Guskey, 2002), this also seemed important in relation to the goals of the project. An additional benefit was that Laura was prepared to conduct the assessment interviews herself with the Year 3 students. This was part of her usual role as NC, which meant participating in the project did not create additional workload for the class teachers. Although this tool had not been used in previous interventions of the study, I did not consider this as presenting a problem in the sense that the orientation for design-based research adopted for the study was conducting research *through* the intervention (McKenney & Reeves, 2012). With a focus on gaining insights

[30] The Mathematics Assessment Interview (MAI) was originally developed by researchers at Australian Catholic University (ACU) and Monash University as part of the Early Numeracy Research Project (ENRP). The ENRP was a three year (1999-2001) Prep – Year 2 research project involving 35 trial schools, predominantly government primary schools from across the state. The tool was designed to assess students' understanding of key ideas in mathematics in junior and middle primary school. Underpinning the assessment tool are growth points, which form a pathway/trajectory to key mathematical understandings (conceptual landscape). The assessment is conducted as a one-to-one interview (format).

into pathways for changing teaching, the importance was on collecting data useful for the school, to inform our planning and adjustment of resources.

As anticipated, based on preceding interventions, the time the school could allocate for the professional learning session was limited. The session was arranged in a lesson block when all three teachers in the team were released from teaching. During the session, the teachers experienced a number string in much the same way as it would be implemented with students in a classroom. However, there appeared some uncertainty with envisaging the structure of the lesson and how to elicit a mental strategy from student thinking. One of the teachers expressed an interest in observing the instructional tool being modelled. It was negotiated that a lesson would be modelled with one class for the teachers to observe. The main difference between this and prior interventions was the decision for the NC and I to co-teach the lesson. This modification evolved from suggestions of the Mathematics Leader (ML) in the previous school, who expressed the view that a co-teaching opportunity might strengthen the learning for teachers. One of the recognised the benefits of co-teaching in relation to teacher education, is the opportunity to scaffold learning experiences (Graziano & Navarrete, 2012). Co-teaching with the NC had the dual benefit of deepening her experience and understanding of the use of the instructional tool and opening opportunities for 'in-the-moment' professional conversations with teachers (Hunter, Hunter, Bills, & Thompson, 2016).

6.2.2. Establishing coherence with school goals

The project seemed to appeal to the NC, Laura, for two reasons. First, it was aligned with the schoolwide culture of promoting thinking and second, it supported her goal of students learning to compute mentally before being introduced to written algorithms. The latter was a goal Laura had already been implementing across the primary school. Despite having success with mental computation being filtered into teaching, she was not yet convinced that all teachers in the primary years shared her perspective on the importance of mental computation before formal written procedures (Laura, interview).

The collection of pre-intervention student assessment data indicated that participation in the project would be of benefit to the students. The results brought to attention the pitfalls of using the split strategy for subtraction and the difficulties students were experiencing with subtracting mentally (they were reliant on the split strategy)[31]. The NC commented that the data indicated the school was making good progress with the goal of focusing on mental computation and provided her with a good starting point for discussions about moving forward (Researcher's journal – email correspondence).

Flexibility with the early negotiation process and setting up of the intervention was important for establishing positive relationships with the participants (McKenney & Reeves, 2012) and pre-empting the occurrence of factors which had

[31] The MAI (Mathematics Assessment Interview) data indicated that most students fell within Growth Point 5 meaning that they could use derived mental strategies with 2-digit numbers. Most failed to achieve Growth Point 6 which involved computing with 3-digit and 4-digit numbers because they were reliant on using the split strategy for subtraction which was problematic (for Q26).

been found to constrain changes to practice in prior iterations. In addition, it was essential to establish coherence with the school goals for teaching and learning mathematics (Desimone, 2009).

In the following section, my interpretation of the change sequences that occurred and the associated learning processes are presented.

6.3. Change sequences

Data on teacher perspectives on the professional learning, outcomes they considered salient, perceived changes in their knowledge, disposition and practice, were analysed alongside my observations to gain insights into possible pathways for change (Clarke & Hollingsworth, 2002). The changes interpreted for these three teachers could be represented diagrammatically on two different change sequences. Similar to prior interventions, various stimuli in the External Domain initiated changes linked to components forming the Personal Domain. Significantly, it appeared the influence of aspects of social dynamics (Arzarello et al., 2014) within domains (Personal Domain and Domain of Practice) were critical in supporting learning processes for these teachers. To avoid repetition of discussion from preceding chapters, the focus of this section is to report on findings that distinguished learning and change for this group of teachers compared to those in the two previous interventions.

In the next subsection, a brief description of the contextual background of each teacher participant is presented. Following this, diagrammatical representations of the changes sequences and supporting explanations of the changes perceived by the teachers and by myself are given. In the final subsection, discussion is focused specifically on learning processes that occurred within this change environment (Arzarello et al., 2014; Clarke & Hollingsworth, 2002).

6.3.1. Contextual background of the teacher participants

First, background information relating to teaching experience, disposition and classroom practice is provided. This is followed by a discussion of my perceptions of the dynamics among the Year 3 team.

Details relating to years of teaching experience for each participant, including the number of years within middle primary, are presented in Table 6.1.

Table 6.1. *Teacher participants and years of teaching experience*

Teacher name (pseudonym)	Number of years teaching	Number of years in middle primary (Years 3 & 4)
Jenna	3	2
Kelsy	18	6
Ian	5	1

6.3.1.1. Jenna

Jenna was an early career teacher. She had been teaching for three years, two of which were at middle primary level. Jenna conveyed herself as being a confident member of the Year 3 team. In the planning sessions she was spontaneous in her interactions with colleagues, often being the one to instigate discussion and share her classroom experiences to clarify questions or queries with the pedagogical approach or subject content. For example, at a planning session when Kelsy expressed difficulty eliciting the focus strategy from students, Jenna explained how she selected an example of a student strategy elicited at the end of the first lesson and used this to steer the learning by getting the class to investigate the strategy (Researcher's journal – planning meeting 2). Although Jenna indicated that she was not entirely confident with planning and teaching curriculum content for the Number strand, she articulated that in such instances she consulted the NC (Laura) to address her questions (pre-intervention survey). She seemed to uphold positive relationships with both colleagues and the NC.

Jenna appeared to position herself as a learner and embraced the opportunity to experiment with new ideas:

> I think it was lovely to be part of a project externally to see what research is being done outside of the school and be part of that. (Jenna, interview)

Participating in this project encouraged her to investigate use of a different instructional tool; she commented that using number strings was new learning for her:

> The way you used the number strings, I haven't previously used number strings to elicit a certain response, so that would be something that would be new to me. (Jenna, interview)

Jenna described a typical lesson in her class as beginning with an equation or problem and using a thinking board to elicit strategies. She did not perceive the pedagogical approach underpinning the intervention as presenting a starkly different approach to her usual practice (Jenna, interview). My pre-intervention observation of Jenna corroborated this perspective. I observed her introducing a task to students with little prior instruction; through the use of questioning, she elicited the key ideas from the students.

6.3.1.2. Kelsy

Kelsy was an experienced practitioner with a teaching career spanning eighteen years; she had taught at middle primary level for six of those years. Kelsy was one of the leaders involved in initiating CoT across the school. When asked to reflect on her confidence with integrating the proficiencies from the *Australian Curriculum: Mathematics* (ACARA, 2014) into her planning and teaching, she indicated that this was an area of strength. She rated herself as 'confident' in applying the proficiencies of understanding, problem solving and reasoning to teaching mathematics (pre-intervention survey).

Although Kelsy perceived herself as pedagogically strong, she did express a lack of confidence in her mathematical knowledge:

> I don't enjoy teaching the more mathematically able children. I am not confident enough in my mathematical knowledge and I find learning alongside them less enjoyable than learning alongside support and mainstream learners. (Kelsy, pre-intervention survey)

Her lack of confidence in mathematical knowledge could possibly be attributed to her commencing her career as a Hebrew teacher, which did not involve teaching mathematics.

In addition to the potential challenge of teaching content with which she felt a 'little unconfident' (pre-intervention survey), the suggested pedagogy contrasted with her usual approach to teaching computation. When asked to describe how she would usually teach a computation strategy she responded with "model the strategy to the students" (pre-intervention survey).

Despite expressing concerns with her level of mathematical knowledge and indicating that the project would involve changes to her pedagogical approach for teaching computation, Kelsy experimented with the new approach in her classroom and welcomed the opportunity to learn. This was evident at the first planning session scheduled after the teachers had started teaching a sequence of three lessons. Kelsy had recorded student ideas elicited in the whole class discussion on sheets of poster paper. She brought these to the meeting to clarify why her students were adopting a particular strategy and the next steps for her as a teacher to progress student thinking (Researcher's journal – Planning meeting 2). Her actions exhibited the characteristics of a growth mindset towards learning (Dweck 2000; 2006).

6.3.1.3. Ian

Ian had commenced his teaching career five years prior and this was his first experience in the middle primary years. He had recently transitioned from Year 5 where there had been greater focus on students learning written algorithms. At our first planning meeting in which the focus was to map the key ideas for the unit plan on additive thinking, a brief discussion ensued about the teaching of written algorithms. A decision was made to remove this from the Year 3 planner.

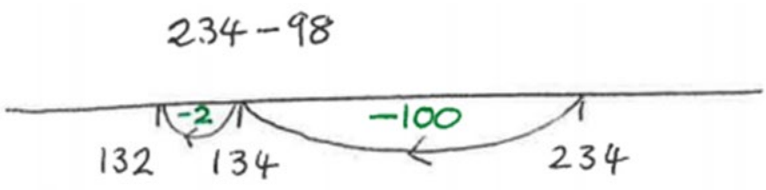
Although Ian did not explicitly express any concerns, immediately after the meeting the NC, Laura, commented that she was not convinced that Ian believed written algorithms should be removed from the Year 3 program. It appeared that he was yet to experience the benefits of students being exposed to mental computation strategies (Clarke, 2005; Kamii et al., 1991; McIntosh et al., 1997) and that participation in this project might provide a new learning opportunity.

Observing Ian pre-intervention, I noticed that there was a natural flow of discussion in the lesson, a characteristic that differentiated his classroom from the other Year 3 classes. He had clearly established a culture of learning that promoted student agency; there was a real sense of student ideas driving the direction of learning. However, it was evident there were not strategies or tools in place to manage the discussion, which led to some digression from the intended learning focus. It seemed the intervention would provide opportunity for Ian to refine his pedagogical practices, for example, encouraging him to use the Five Practices (Stein et al., 2008) to orchestrate purposeful discussion, rather than presenting him with a significantly *different* approach to teaching.

Reflecting on his confidence teaching mathematics, Ian indicated that he was a "little unconfident" in two key areas: mental computation strategies and integrating two of the Australian curriculum proficiencies (reasoning and fluency) into lessons (pre-intervention survey). This was corroborated at the initial planning meeting I attended to map out the content of the Year 3 unit on additive thinking. During the meeting a discussion evolved about devising an extending prompt for a task. One Senior School mathematics teacher (present in the room at the time) interjected to assist Ian with understanding how to extend the task to instigate learning of the key concepts. Through the discussion it emerged that this particular teacher had supported Ian in the previous year with planning and extending his Year 5 students (Researcher's journal – Planning meeting 1).

Ian displayed the characteristics of a growth mindset towards learning (Dweck, 2000; 2006). He was willing to take risks with his teaching to achieve learning goals and seemed uninhibited discussing mistakes with both colleagues and students. This was evidenced when I observed Ian teaching towards the end of the intervention. During the lesson he represented a student explanation with an incorrect visual representation, which he interpreted as being correct. This caused confusion and discussion amongst the students; the class appeared divided over the correct explanation[32]. A class discussion developed in which the misconception with the strategy was explored and thinking represented correctly on a new visual representation. Ian had established a culture of learning with his class in which he positioned himself as a fellow learner and was respected as such by his students.

[32] Ian posed the following question with the intention of eliciting use of the compensation strategy from students: 234 – 98. The student selected to share their thinking, explained they had rounded 98 to 100 to make the calculation easier. The student reasoned that because they had added 2 to 98 to round to 100, they needed to subtract 2 from the partial answer. Ian displayed this thinking visually on the board using an empty number line and indicated that the calculation was correct.



I interjected by posing a question to guide Ian and some students to reconsider their reasoning: *If you subtract 100 to make it easier, have you subtracted too many or too few?*

6.3.1.4. Year 3 team dynamics

From the beginning of the intervention it was apparent that the Year 3 team had formed positive, supportive working relationships with each other. The opportunity to observe a mathematics lesson in each class (pre-intervention) provided me with insights into the pedagogical approach, the classroom culture and the Year 3 team relationships. These insights are briefly discussed in this subsection.

During the pre-intervention observations I spent approximately twenty minutes in each classroom. It was evident that the team had planned collaboratively and had a shared pedagogical approach. As I moved from one classroom to another, I seemed to experience a continuation of the same lesson. It was apparent the teachers communicated well with each other; they were clear about the lesson goals, how they intended the students to learn, and their role in the lesson. My initial impression was that this was a team that worked closely; they had shared goals for student learning and were consistent in their approach to achieving these goals. One of the benefits of participating in this study's intervention seemed to be a chance to focus on how to extend student-centred pedagogy that enhanced opportunities for thinking and reasoning in connection with the teaching of computation.

The previous subsections presented contextual background to provide a backdrop for the findings presented in the next section: the change sequences I interpreted this group of teachers experienced.

6.3.2. Change sequences and learning processes: An overview

Two different change sequences emerged for the three teachers; these are represented diagrammatically in Figures 6.1 and 6.2. All teacher participants experienced initial changes in the Personal Domain; their learning was predominantly in aspects of PCK, namely KCT (Knowledge of Content and Teaching).

As stated earlier Jenna conveyed herself as a confident teacher, and did not perceive the approach suggested by the intervention as being starkly different to her usual practice. Although the subject content itself was new knowledge, this did not seem to perturb Jenna in experimenting with the approach. In contrast to the other two teachers, her self-reflection on her own professional learning was limited. Jenna predominantly reflected on changes in student learning and affect. Subsequently, the diagrammatical representation of her change sequence indicates that she recognised salient outcomes, which instigated an intention for future experimentation. The outcomes did not seem to lead her to reflect on changes in her own knowledge or disposition on teaching and learning. Her change sequence is depicted in Figure 6.1., an overview of Jenna's change sequence, including a summary of the learning processes, is presented in Table 6.2.

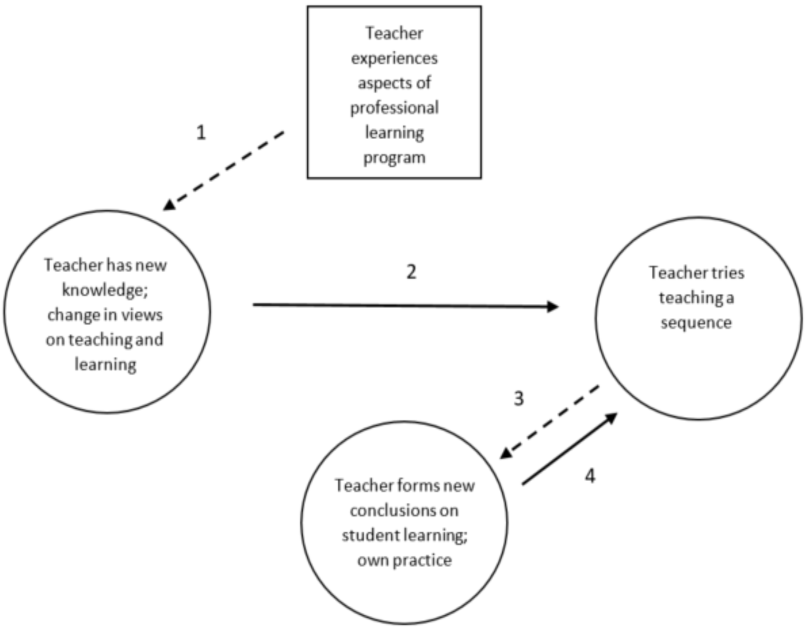


Figure 6.1. Change sequence for Jenna.

Table 6.2. *Summary of Jenna's change sequence*

Arrow	Domain Link[33]	Mediating process	Summary of learning processes
1	ED to PD	Reflection	Reflects on external stimuli: modelled lessons and teacher resource book. She has new PCK: how to use number strings as a tool to facilitate student centred learning of mental computation strategies.
2	PD to DP	Enactment	Enacts new Pedagogical Content Knowledge (PCK) to experiment with using number strings and number talks to facilitate learning of mental computation strategies.
3	DP to DC	Reflection	Reflects on changes in practice leading to positive outcomes for student learning: refining strategies for efficiency, articulating thinking, deepening thinking and reasoning for all students. Reflects on the positive impact on student motivation.
4	DC to DP	Enactment	Intends to focus on refining the pedagogical approach in future lessons to integrate greater focus on relational thinking and an investigative approach to lessons.

[33] Codes relating to the domains are: External Domain (ED), Personal Domain (PD), Domain of Practice (DP), Domain of Consequence (DC).

The same change sequence emerged for both Kelsy and Ian. These teachers expressed and displayed evidence of being slightly unconfident with their own knowledge about teaching computation. In addition, both indicated a level of uncertainty towards the intervention at the outset. Kelsy perceived the intervention as presenting her with a different pedagogical approach and Ian did not appear entirely convinced of the benefits for students of a focus on mental computation. However, they were self-reflective teachers who recognised salient outcomes from their classroom experimentation and subsequent changes in aspects of their Personal Domain.

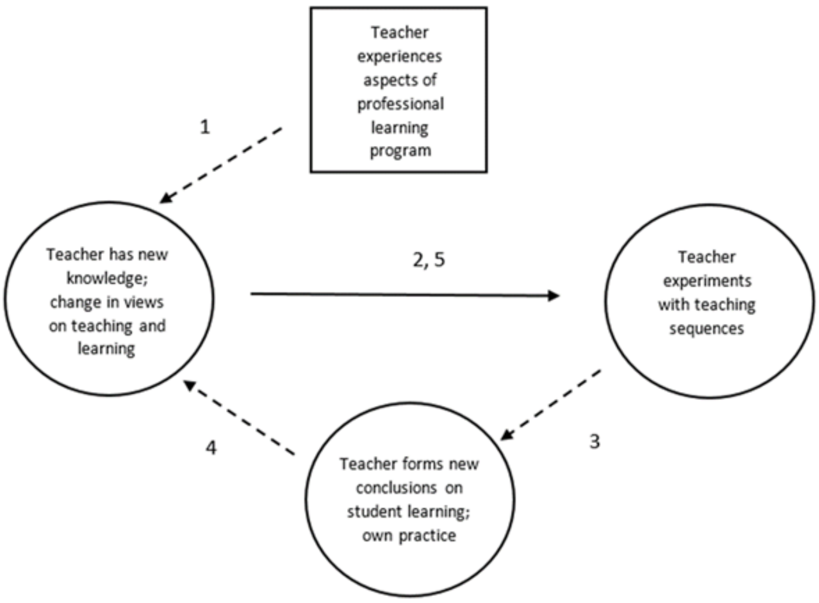


Figure 6.2. Change sequence for Kelsy and Ian.

An overview of change sequences, including a summary of the learning processes, is presented for Kelsy and Ian in Table 6.3.

Detailed analysis of the change sequence for each teacher is presented in Appendix E. In the following subsection the focus of the discussion is specifically on the learning processes that distinguished this group of teachers from preceding interventions.

6.3.3. Creating and supporting change: The interplay of external stimuli and social dynamics

The data indicated three external stimuli as significant in initiating change for this group of teachers: modelled (or demonstration[34]) lesson, teacher resource book and collective planning. In much the same way that the learning of mathematics is not a linear process consisting of micro-steps but a network of interconnected ideas, the teachers did not discuss the external stimuli as discrete, independent variables. The components were often perceived as supporting each other. For example, they valued seeing the ideas in action in the modelled lesson but emphasised their need to consult the teacher resource book to enable them to enact the ideas in their classrooms. In this subsection, the influence of each component (how it stimulated teacher learning) and the associated learning processes for the teachers is discussed.

[34] Laura used the term 'demonstration lesson' when referring to modelled lesson.

Table 6.3. *Summary of Kelsy's and Ian's change sequence*

Arrow	Domain Link	Mediating process	Summary of learning processes for Kelsy	Summary of learning processes for Ian
1	ED to PD	Reflection	Reflects on external stimuli: teacher resource book, planning meetings and associated professional dialogue. She develops aspects of her PCK, predominantly KCT connected to mental computation strategies.	Reflects on external stimuli: ***modelled lessons***, teacher resource book and professional dialogue during collective planning. He has new PCK: how to use new instructional tools to facilitate student learning of mental computation strategies.
2	PD to DP	Enactment	Enacts new PCK about teaching mental computation with conceptual understanding. She experiments with using number strings and number talks as new instructional tools.	Enacts new PCK to experiment with using number strings and number talks to facilitate student learning of mental strategies.
3	DP to DC	Reflection	Reflects on student learning outcomes: students using more efficient strategies, generalising and a metacognitive approach towards mental computation. Reflects on her exposure to external resources and opportunity to develop professionally.	Reflects on positive outcomes for student learning: learning of new mental strategies, refining strategies for efficiency and skills to articulate their thinking. ***Reflects on positive impact on student engagement and fostering of student agency.*** Reflects on positive impact on own professional learning about mental computation.
4	DC to PD	Reflection	Reflects on changes to her knowledge about teaching mental computation (KCT) as a result of students learning to articulate their thinking.	Reflects on development of his pedagogical knowledge (KCT) and changes in his pedagogical approach. ***The impact on student learning leads to a change in his attitude towards teaching of mental computation.***
5	PD to DP	Enactment	Identifies new learning goals and future focus for teaching of mental computation.	Intends to integrate the focus on teaching mental computation into future practice.

Note: Codes relating to the domains are: External Domain (ED), Personal Domain (PD), Domain of Practice (DP), Domain of Consequence (DC)

Text in ***bold italics*** highlights the differences interpreted in the learning processes for Ian.

6.3.3.1. Instigating learning: Modelled lessons

The modelled, or demonstration, lesson appeared an important influence on learning for this group of teachers. It should be noted that the use of a demonstration lesson had been modified for this particular context and involved the NC, Laura, and I, co-teaching the lesson. The modification was suggested as an option by Laura, which I welcomed based on feedback from the ML in the previous iteration, who had suggested incorporating co-teaching as a way to strengthen the PL program.

The first lesson in the sequence, in which number strings were used as an instructional tool, was modelled. Observing a modelled lesson was optional, the teachers at the school agreed it would be beneficial to see number strings being enacted. Two of the teachers (Ian and Jenna)[35] commented on the value of observing the use this instructional tool. Below are excerpts from Ian's interview:

> I think the modelling was really good, so watching you do a number string lesson. It's very different to reading it, and seeing it. I think that was very valuable. (Ian, interview)

> The modelling was really, is, really valuable. (Ian, interview)

Through the brokering process the teachers experienced how the instructional tool could be used to elicit student thinking and reasoning. This suggested a change in knowledge, specifically KCT, within the Personal Domain.

[35] Note that Kelsy was absent from school on the day the modelled lesson was scheduled.

The NC, Laura, substantiated the importance of the influence of brokering during the modelled lesson. In addition, Laura emphasised the opportunity for 'in-the-moment' professional dialogue within the modelled lesson. During the lesson, when the students independently explored number strings, I was able to converse with Ian and respond to his queries. He raised a concern with using questioning to elicit the strategy and what to do in a situation if the students did not contribute ideas aligned with the intended learning (Researcher's journal). From my perspective, co-teaching afforded the opportunity for such conversations. Laura monitored student progress and selected examples to help connect the ideas during the summary phase of the lesson, whilst I engaged in conversation with Ian.

> I think that the demonstration lesson was something that they could see and in that sense it was tangible, and what I loved about the demonstration lesson was, that there were in those moments, you had that lovely time to chat with Ian one-on-one, so as perhaps he observed the lesson he could come up with questions in that time and that could be addressed, during the session. (Laura, interview)

The excerpt from Laura's interview highlighted the value of teachers seeing something concrete to support development of their KCT, namely the brokering of the number strings. Her reflection draws attention to the value in teachers having accessibility to an external person, regarded as an expert, with whom they can interact and clarify questions or queries 'in-the-moment'. Elements of social dynamics, in particular brokering and interactions between Ian and myself (teacher and researcher) through professional dialogue, were an influential and integral part of the learning processes within the Personal Domain.

The modelled lessons were an important influence on changing teacher attitudes. In her interview, Laura commented how observing a lesson on mental computation appeared pivotal in instigating an initial change in disposition for Ian. As aforementioned, Laura expressed concerns at the beginning of the intervention that Ian was the one teacher in the Year 3 team who was yet to be convinced of the value of students learning to compute mentally:

> We had had lots of conversations about the benefits of delaying algorithms, that we've read papers about, there's a lovely Doug Clarke paper[36] about written algorithms in the primary years and how it can potentially undo all the good work. So we had unpacked that paper across primary but it wasn't really until he [Ian] saw firsthand the benefits of perhaps fully extracting a lesson, dedicating a lesson just to that one strategy that I think he was convinced. (Laura, interview)

It can be inferred from Laura's comment that it was the experience of the modelled lesson that initiated a change in Ian's perspective on the teaching of mental computation. In subsequent planning sessions, Ian certainly appeared genuinely interested and engaged in learning about this approach to teaching mental computation.

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[36] Clarke, D. M. (2005). Written algorithms in the primary years: Undoing the good work? In M. Coupland, J. Anderson, & T. Spence (Eds.), *Making mathematics vital.* Proceedings of the 20th biennial conference of the Australian Association of Mathematics Teachers *(pp. 93–98).* Adelaide: AAMT.

The modelled lesson was valuable for brokering the use of the instructional tool (number strings) and showing how the questions were designed to elicit a particular strategy through student reasoning. In the following excerpt, Laura expressed her perception of the most powerful influence on teacher learning throughout the intervention:

> We find the demonstration lessons particularly powerful… So maybe that has been most powerful. (Laura, interview)

Laura's reflection encapsulates the value attached to modelled (or demonstration) lessons for instigating learning opportunities within the primary year levels of the school. This concurs with findings of other studies that demonstration lessons are valuable for supporting teacher learning and change (e.g., Clarke et al., 2013; Grierson & Gallagher, 2009).

6.3.3.2. Instigating learning: Teacher resource book

The importance of the teacher resource book in supporting change was emphasised by the three teachers. The resource book had been modified for this intervention to include only those mental computation strategies appropriate for this school's cohort of students (based on the MAI data provided by the NC at the beginning of the intervention). Tasks had also been adapted, based on the student assessment data. A summary of the features of the book that appeared to support teacher experimentation with the new approach and supporting evidence is presented in Table 6.4.

Table 6.4. *Features of the teacher resource book that appeared to support change in classroom practice*

	Features of the Teacher Resource Book	**Supporting evidence**
Jenna	▪ Provision of a structured program. ▪ Suggestions for enabling and extending prompts to support differentiation.	*I think the way the lessons were sequenced so clearly was good for us to see as well. Often we've got a unit plan but it's not always as structured, as the program was, and the enabling and extend were very clearly identified as well.* (Jenna, interview)
Kelsy	▪ Includes examples of explanations to share with students and key mathematical vocabulary. ▪ Suggestions for enabling and extending prompts to support differentiation.	*The booklet was awesome as well because it helped with the language that I might not necessarily have been using. The teacher talk but then bringing it down to student understanding*. (Kelsy, interview) *Planning alongside colleagues and having a document with the necessary language and learning experiences definitely helps support planning (especially the enabling and extending prompts).* (Kelsy, pre-intervention survey)
Ian	▪ Easy to use and saves on preparation time. ▪ Includes teacher background information on each mental strategy (subject knowledge) and pedagogical considerations when discussing each strategy. ▪ Provides enough examples to build capacity to develop own resources in the future.	*I loved how the lessons were just there, the equations were there, everything was there, it made our lives a lot easier than having to come up with new equations.* (Ian, interview) *I liked the teacher background before as well, yeah that was really useful, you know all the understanding clearly exactly what the strategy was, the use for it and… and how to go forward with it. That was really valuable.* (Ian, interview) *I think they need to have the lessons very prescriptive, at first, so that the teachers have a really clear idea exactly how it should be run… that was really good because we would be able to create our own now, based off all those...So I think we need that.* (Ian, interview)

As illustrated in Table 6.4, all three teachers reflected on the importance of the teacher resource book in supporting their classroom experimentation. The differing parts of the book they each highlighted as being valued seemed to mirror their individual learning needs as teachers, reinforcing the idea that teacher learning is an idiosyncratic process (Wilkie, 2019). For example, in his pre-intervention survey, Ian indicated that he was a “little unconfident” with planning and teaching mental strategies. This connected with his reflection on the resource book: the beneficial inclusion of background information on the mental strategies themselves and pedagogical considerations.

The teachers all appreciated how the resource book and availability of additional resources provided a structured yet also flexible teaching program. This is captured in an interview comment by Kelsy:

> The organisation of having something as simple as that PowerPoint. I found it really valuable, it’s not informed my teaching but it’s just helped facilitate teaching that it’s really been there is a structure to it and there is room for flexibility. (Kelsy, interview)

The provision of sample resources such as PowerPoint slides to support the teaching of the lessons seemed valued in the sense that teachers could see how lessons might be structured and tasks presented to students. This resonates with findings of Desimone and Hill (2017), who described how providing teachers with intervention materials to support planning and teaching led to improvements in lesson structure and organisation, which ultimately increased student learning time.

Whilst all three teachers considered the provision of the teacher resource book as important to support planning and classroom experimentation, Kelsy contemplated *how* the team used the book. She reflected on weekly planning sessions in which discussions focused on how to teach; the resource book was used to stimulate these discussions. Kelsy highlighted the role of professional conversations as being critical in making meaningful use of the resource book:

> I think booklets are great but it's the *conversation* around the booklet; the booklet facilitates the conversation that grows the learning because reading in isolation doesn't necessarily… we teach what a thoughtful reader does and a thoughtful reader asks questions but it's being able to have that conversation about the questions, that I think our growth, that's where our growth and shifting thinking takes place. (Kelsy, interview)

For Kelsy, this experience was analogous to a student learning to read for meaning; the opportunity to ask and discuss questions was an essential part of the learning process. She implied that the teacher resource book could not be considered a discrete program component, as it was the interweaving of the professional dialogue that accentuated the value of the book. On the basis of her interview comments, it could be surmised that Kelsy held a situative perspective on learning (Sfard, 1998). It seemed that professional dialogue – the interactions among teachers and between teachers and myself, as we discussed the content of the resource book, and how to teach it – were critical processes supporting teacher learning and change within domains.

6.3.3.3. Instigating learning: Collective planning

The opportunity for collective planning was another component of the professional learning program that these teachers emphasised as being particularly influential on opportunities to learn and change practice. This was interesting in the sense that regular collective planning, with the support of the NC, Laura, was already an embedded practice within this institutional context. However, teacher reflections suggest that again, it was the particular learning processes that occurred during this meeting time that had a substantial influence on their learning. For example, it was during the second planning meeting that Ian shared his confusion about the purpose of the second lesson in the learning sequence. Specifically, he was unsure how the first and second lesson in the learning sequence connected and how presenting students with one equation could constitute a lesson (building a lesson on the concept of a number talk). The discussion that ensued illustrated the importance of dialogic interactions between Ian and myself. I explained that students would respond to the question with various strategies, and that he was to select from these examples to facilitate a discussion about appropriate, efficient and accurate strategies. The intention was to build on the ideas from the previous lesson, in which a number string was used to elicit a new strategy. Ian reflected on this incident and the value of this conversation during his interview:

> And then also you coming in for the meeting was really valuable as well, just to clarify after we tried it for a week, to be able to clarify. Like I was struggling with the number talk, thinking it was exactly the same as the

> number string, that from you coming in, I think it clarified that difference. (Ian, interview)

In the excerpt above, Ian commented on the value of having an external person present during meetings to clarify questions and queries, such as the use of different instructional tools on which the lessons were based. From my perspective, the conversation also highlighted the idiosyncratic nature of professional learning. Only the first lesson in the learning sequence (the lesson based on the instructional tool number strings) had been modelled for this group of teachers (in previous iterations the sequence of three lessons had been modelled). It was interesting how one teacher (in this case Ian) perhaps needed something more tangible, such as an actual modelled lesson to fully comprehend the purpose and structure of the second lesson, whilst others (Jenna and Kelsy) were able to interpret the intended purpose and structure from written documentation (in the resource book) and discussion in the professional learning session.

Kelsy reflected on the importance of opportunities for professional dialogue during meetings to support her learning and understanding of the mathematics:

> I think that it's important not to assume that everyone understands, that we're all on the same language page. There're complexities to mathematics and sometimes we just assume that we all know what we're talking about and it's those conversations that can identify whether, how well you

> understand the strategy that you're about to teach. Do you understand at all? (Kelsy, interview)

As a teacher who lacked confidence in her subject knowledge, Kelsy valued the opportunity to discuss the mathematical content of lessons. It could be interpreted that this gave her greater confidence to facilitate classroom discussion with her students. She further elaborated and emphasised the influence of these discussions during the planning meetings:

> I think the collaborative planning lit up the booklet. The professional conversation, I think, is important, that leading up to it but working through with the booklet and planning and discussing it, and nutting out together effective teaching methods properly, **because then you start to see everyone's approaches to how they might do it and learn from each other.** (Kelsy, interview)

> Have the conversations... I think it's so worthwhile having that, when you can nut out [sic] the planning stage and that you know what it is or what went wrong in one lesson... when the kids were just splitting away and it made sense, and then it was that conversation about keep the number intact, keep the one number intact, don't go splitting everything and that was for us a big learning curve. (Kelsy, interview)

The first excerpt above draws attention to the importance of the bidirectional interactions among the teachers in the meeting, the value of sharing how others would approach the task, and the intended learning to elicit from their students.

She suggested that these conversations and interactions illuminated the content of the teacher resource book. In the second excerpt, she reflects on the opportunity during planning meetings to share examples of student work samples or the mental jottings recorded during the lessons. As previously mentioned, in the second planning meeting Kelsy brought poster paper to share the student ideas she had recorded in the whole class discussion. She needed support with her students' fixation on partitioning both numbers in an equation based on place value and using the split strategy. This led to discussion about keeping one number whole to help reduce the number of steps students need to hold in their head when completing the calculation mentally. Kelsy commented how this was new learning for the teachers, as they were developing their knowledge of content and teaching (KCT). It seemed that the substantial time we allocated to planning, to work through mental computation tasks, to discuss the pedagogy and potential student misconceptions, was particularly beneficial for Kelsy.

Laura (NC) also commented on the value of the discussions at the planning meetings:

> Once we had populated that planning document, the follow-on discussions were a bit more free flowing in structure but actually really critical discussions, I thought, for the team to share their understandings and their observations. (Laura, interview)

The established planning meetings had usually been more structured: they had usually focused on completing planning documents. In this study, the provision of the teacher resource book – which outlined the key content to be learnt with examples of tasks and ideas on how to initiate learning – meant that meetings could be used instead for teachers to discuss their teaching experiences, difficulties students were experiencing, and how to progress with future lessons. Laura commented on the enhanced learning opportunities this created:

> The booklet really supported them in the content and the pedagogy as well and so that enabled us to focus upon more of a pedagogy tool within the class and the planning and I do feel that our planning doesn't often get to that level of discussion because often we are unpacking the big picture of curriculum, what does the content descriptor say, achievement standard, and we get bogged down in that, in what it means, not bogged down in a negative way, but often we don't get to the pedagogy. So that has been liberating in a sense that we got to have those discussions. (Laura, interview)

It seemed that providing teachers with the written outline of a structured program in the form of a teacher resource book, alongside opportunity for collective planning, was critical in enhancing their learning, in particular supporting teacher development of aspects of PCK, notably KCT. These components of the program afforded opportunities for bidirectional interactions among teachers and between the team and myself; it was through such interactions that the learning process emerged.

6.3.3.4. The influence of professional dialogue

The emergence of professional dialogue was a common thread running through the external components of the professional learning program and enhancing learning opportunities. Laura commented on the impact of the project in proliferating these conversations:

> Once I'm here they grab me with all their questions, and that certainly came true with this project; when I did see the team irrespective of whether it was just a lunch time break we would be chatting about the things that were happening in the classroom incidentally. So that's good, they became important conversations as well. (Laura, interview)

It seemed that the content of the project had ignited the interest of the teachers. Laura's comment suggests that classroom experimentation created valuable learning for both teachers and students; these were captured through professional dialogue (bidirectional interactions) between the teachers and the NC, when I was not present at school.

Laura also drew attention to the value of professional dialogue occurring within a lesson observation towards the end of the intervention:

> And then also those opportunities when Ian did teach and we could step in, so that was powerful as well, again because we could have those in-the-moment conversations. (Laura, interview)

The excerpt above refers to an occasion previously mentioned when Ian made a conceptual error with a visual representation of a mental strategy (refer to section 6.3.1.3.). By interjecting with a question and having a brief discussion during the lesson, the teacher and students were able to reason and correct the error.

Reflecting on the value of the bidirectional interactions in planning meetings (through which she gathered new ideas and developed pedagogical knowledge), Kelsy sought to articulate how the dialogue created opportunities to learn:

> So it's that slowing down and being able to bounce ideas off each other and then look towards that next learning opportunity a little bit deeper.
>
> (Kelsy, interview)

It seemed that participation in the project had opened opportunities for the teachers to have important discussions about the pedagogical approach and consider the key mathematical ideas to look for in student learning. In Kelsy's reference to "slowing down" there is a sense of removing the need to select or decide upon tasks to teach the curriculum content (since they were already provided in the resource book), that the team could dedicate their time to discussing the 'how' of teaching and learning in more depth.

6.3.3.5. Summary

Data indicated that there were three main components of the External Domain which were critical to instigating new learning: modelled lessons (co-taught with the NC), teacher resource book, and collective planning meetings. The aforementioned stimuli appeared to ignite an interest, or address a gap that teachers recognised, in their own learning, or student learning of computation. For Ian, the modelled lesson had been pivotal in stimulating a change in attitude and an interest in experimenting with a new approach to teaching computation. Initially, he was not entirely convinced of the merits of students learning to compute mentally.

Reflecting on their learning experiences, the participants articulated that whilst certain components of the program instigated initial learning opportunities, there were additional influences that supported and enhanced the learning process. Specifically, it was elements of social dynamics, namely brokering, dialogic interactions and bidirectional interactions, which the teachers and NC considered crucial in enhancing and developing their learning (Arzarello et al., 2014). These influences were afforded by the change environment and seemed a critical support mechanism to nurture the ideas initially planted. The ways in which the institutional context afforded these changes will be explored in the next section.

6.4. The change environment: The institutional context

The purpose of this section is to report on findings related to the second research sub-question; how aspects of the institutional (school) context influenced opportunities for teachers to learn and change practice. Analysis of the data were guided by the Meta-Didactical Transposition (MDT) model (Arzarello et al., 2014) (refer to Figure 3.2) and will be discussed in terms of influences that either afforded or constrained change within this school context.

It is acknowledged that data from the NC, a person in middle leadership responsible for leading change within the classrooms, predominantly informs this section. One of the benefits of interviewing the NC, Laura, was to draw on the experiences of someone who looked at the advantages of participating in the project from beyond the walls of their own classroom. Laura had a vision for mathematics across the primary years, and how the learning opportunities within the program might be extended beyond the Year 3 team. In addition, she had established a positive relationship with the teachers and had prior knowledge of their professional expertise, which was useful in relation to noticing changes in their classroom practice, attitudes, and interactions with each other.

6.4.1. The influence of institutional affordances on change

In this subsection aspects of the institutional context, which afforded opportunities for this group of teachers to learn and change practice, are discussed. Noticeably, there were some similarities between structures that afforded change in this school environment and those discussed in the preceding iteration. These structures, namely the influence of the NC, and established systems to support collective planning, are acknowledged but not discussed in depth to preclude repetition of content presented in Chapter 5. Instead, greater attention will be on the influence of aspects of the institutional context, which distinguish this school from the previous two contexts.

One of the features of this institutional context was the ethos that teachers appeared to regard themselves as learners situated within a learning community. This feature is recognised as fundamental to teacher professional learning (Clarke & Hollingsworth, 2002). The school collaboration with Harvard School of Education in connection with CoT indicated that it was an environment that embraced opportunities for innovative learning. The school ethos on this was further substantiated by evidence of internal professional learning within the school:

> So we have a similar program that we call Maths Lab here, where we do go and do demonstration lessons. It ends up being a bit of team teaching but we've developed this idea where just-in-time conversation actually is powerful professional learning... and often they end up being a team teach like we did, with other people in the room as well and we do take the liberty of having a conversation; even though the kids are still working we will take a moment to have a conversation then, as well as hopefully a post-lesson reflection. (Laura, interview)

The NC described some of the internal programs for mathematics within the school. I was not aware of this approach until I interviewed Laura at the end of the intervention, but the existence of such programs within the school was a possible explanation for the teachers welcoming the idea of experimenting with a new approach. Generally, the teachers were not perturbed by being observed and embraced 'in-the-moment' professional conversation during lessons (Hunter et al., 2016). It seemed that the teachers were already familiar with this style of collaborative learning; the intervention was another opportunity for external input within an established learning community.

An additional affordance of the school's involvement in CoT was that students were already familiar with exploring multiple strategies to solve a task, and explaining and justifying mathematical thinking (this was evidenced in the pre-intervention lesson observations). It has been suggested that students in CoT classrooms are more focused on thinking, learning and understanding as well as being more collaborative in nature than those not involved in CoT (Ritchhart, Church, & Morrison, 2011). In previous interventions at Schools A and B the students had taken time to adjust to a different approach to teaching and learning, whereas at this school the students were more adaptable to changes in their approach to learning. This flexibility with student approaches afforded more time for student learning.

The school had the affordance of having a NC, Laura, to support classroom teachers in the primary school with planning and teaching mathematics. Having an onsite expert was advantageous; it allowed opportunities for professional dialogue, which the teachers at this school valued. Laura was highly knowledgeable on the subject content of the intervention: mental computation strategies. In a previous role she had been involved in implementation of the Learning Framework in Number (LFIN) and an associated intervention program, which had a focus on students learning to compute mentally. Laura was able to support the teachers by clarifying questions or queries concerning the teaching of the strategies in my intervention, when I was not present at the school.

One of the advantages of implementing the intervention at this school, from my perspective as the researcher, was consistency with school goals. This is considered an important component of teacher professional learning (Desimone, 2009). The NC, Laura, had already discussed the benefits of delaying the introduction of formal written calculation methods (algorithms) with teachers in the primary years (Laura, interview). Participation in this project seemed advantageous in supporting Laura in her goals for teaching and learning of computation, particularly as she had expressed concerns at the beginning of the intervention that one of the Year 3 teachers (Ian) was unconvinced about the benefits of delaying the teaching of written algorithms.

The data suggested that this institutional context had some essential foundations already in place to support teacher professional learning, which is illustrated in the following comment by the NC:

> We've learnt so much and we've got so much more to learn too, which is good. It's a good place to be at. It's also nice just to document these learnings and feed back to the leadership team at school that there is an importance for the numeracy coach. (Laura, interview)

It appeared that Laura viewed herself and the teachers as learners situated within a learning community. The pre-existing positive relationships among Year 3 team members and Laura, evidenced throughout the intervention, provided a strong foundation on which to build new learning. Laura embraced the opportunity to learn and experiment with new pedagogies to enhance student learning but also the role this created for her in supporting teachers with ambitious approaches to teaching (Kazemi et al., 2009). In essence, the interplay between this team and aspects of the institutional context enhanced opportunities for learning and change in practice. The existence of internal contextual factors may have afforded opportunities for learning, but it was the interaction between these factors and the teachers (personal factors) which influenced their learning experiences. Participation in the project seemed to endorse the school's existing stance on innovative strategies for teaching and learning.

6.4.2. The influence of institutional constraints on change

The purpose of this subsection is to consider aspects of the institutional context which constrained opportunities for this group of teachers to learn and change practice. Throughout the intervention, the teachers were positive when discussing their classroom experimentation and learning experiences, a perspective shared by Laura. In the individual semi-structured interviews, the teachers were asked to reflect on constraints or challenges to their learning throughout the professional learning experience; the excerpts below indicate an absence of constraints:

> So there was nothing that constrained us, it was just a different style of teaching and a different style of learning for the students. (Jenna, interview)

> No, I haven't felt constrained by it. (Kelsy, interview)

Jenna commented on the different approach to teaching and learning participation in the project presented, in the sense that it was unlike teaching with challenging tasks. However, she made it clear that she did not consider this a constraint to learning or change. Similarly, Ian spoke of challenges he encountered with teaching new subject content matter (he was not familiar with teaching mental computation), but again was clear that this was not a constraint on his learning experience.

Kelsy suggested implementing the program at the beginning of the year would have been more beneficial in relation to student learning of mental computation:

> I think we had it towards the end of the term and I almost would want to start it at the beginning of the year rather than mid-year. It's just the timing of the project. But I think starting at the beginning of the year opens, it lends itself to games, it lends itself more for keeping that thread of the strategies alive, and then you build on those. (Kelsy, interview)

Kelsy's comment emphasised advantages of adjusting the timing of the project, such as allowing more time to embed strategies and further develop mental strategies. However, her comment was a suggestion for future implementation rather than a constraint on teacher professional learning.

From my researcher perspective, the timeframe for the professional learning session limited initial discussions about how to use the instructional tools in the context of the suggested lesson structure. This did not constrain teacher experimentation with the approach, but was a factor I reflected on during the second planning meeting – when Ian expressed confusion and uncertainty with developing the concept of a number talk into a lesson, previously discussed in this chapter. Essentially, this team of teachers did not recognise any constraints on opportunities to learn and change practice and I did not observe any; this was one of the fundamental factors distinguishing this institutional context from the two other participating schools.

6.4.3. Summary

Some of the affordances that enhanced learning opportunities and supported experimentation with a change in practice were characteristic of this particular institutional context. In this school it was apparent that teachers were seen as learners within a learning community. The teachers reflected on and contemplated changes they intended to continue to implement in their future teaching of mathematics. One of the distinct differences with this group of teachers, in comparison to the two other schools in the study, was that they appeared to have agency to make decisions about their pedagogical approach: specifically, how they were to use research-based pedagogy to inform their practice. The teachers did not appear to encounter any factors within the institutional environment that they considered constrained their learning experiences throughout the duration of the program.

6.5. Reflecting on the evolving design of the PL program

Components considered critical in creating learning opportunities in previous iterations were refined for this particular school context. The evolving design of the intervention was evidenced by the integration of co-teaching into the modelled lesson, emphasis on the use of ‘in-the-moment’ professional conversations, and early provision of background reading for teachers (relevant research papers). The focus of this section is to address what was learnt about implementing the teacher professional learning program at this school, and how this can be drawn upon to support future teacher learning and refinements of this type of professional learning program.

Two of the refinements to the intervention – co-teaching and a focus on professional dialogue – were important factors influencing teacher learning and change in this school context. These factors have been previously discussed in detail in this chapter; perhaps, what is important to consider here is why these refinements were feasible and supported the learning of this group of teachers. The school’s ongoing collaboration with Harvard Graduate School of Education in connection with CoT, was considered when refining the design of the professional learning. The teachers were already positioned within an innovative learning environment; the suggested pedagogical approach of this project did not present teachers with a significantly different approach. The teachers were also involved in ongoing onsite professional learning experiences that involved observations of each other’s teaching, co-teaching, and critically reflective debriefing sessions. Thus, introducing these components was not particularly disconcerting for this group of teachers: it was usual practice for them.

Within this school context, the role of an external expert seemed critical. This was indicated by analysis of various data to gain insights into how the three external stimuli (modelled lessons, teacher resource book, and collective planning) initiated change for this group of teachers. Laura, the NC, substantiated this finding, specifically in terms of supporting the adoption of new practices:

> So getting some input from the expert, so it's lovely to be able to when we have follow up conversations, say remember we had our researcher from Monash come and give us this advice. So that certainly has clout and I need a bit of that to get buy-in from people so there is a place for that outside expert coming in. (Laura, interview)

Laura discussed how affirmation from an external expert was helpful, and sometimes necessary, to endorse the direction in which she wanted to lead the teachers. Implementing the program at this school suggested that involvement of the researcher or external support throughout the duration of the program was instrumental in instigating and supporting change, a factor to be considered in the design of similar professional learning programs.

In terms of sustaining and supporting further change, the implementation of this program with Year 3 teachers brought to attention the importance of considering external support with curriculum mapping and developing school mathematics programs. Participation in the program had raised some questions for Laura, the NC who said, "this…

> ... raises for me lots of wonderings and questions about mapping out our year, and also what we do in other year levels." (Laura, interview)

For teachers to continue to apply their new knowledge and implement changes in practice, future consideration for external support with long term planning seemed important. This certainly appeared something for me to consider when designing professional learning programs of this nature in the future.

6.6. Summary

> ...Hugely positive...I think about what's the next step from what we've learnt, how to keep the momentum going. (Laura, interview)

The excerpt above encapsulates how the teachers and NC positioned themselves on a learning pathway. Participation in the professional learning program had instigated new learning for all teachers and a change in practice. Their learning was predominantly in aspects of their Pedagogical Content Knowledge (PCK), namely Knowledge of Content and Teaching (KCT). The intervention involved teachers refining and developing pedagogical practices for a particular content focus (mental computation) rather than implementing a significant change in pedagogical approach. For example, the teachers were already familiar with designing enabling and extending prompts to differentiate learning tasks in a whole class situation. All three teachers contemplated how they could continue to integrate this approach into their program for mental computation in the future.

The data suggested three components of the professional learning program (external stimuli) were critical for instigating initial change for these teachers: modelled lesson (co-taught with NC), teacher resource book, and collective planning. These stimuli were not regarded as discrete, independent factors. Instead, they were perceived as complementary in supporting learning. The stimuli appeared to address a gap that teachers recognised either in their own learning, or student learning. Of significance, were the additional influences that teachers recognised as supporting and enhancing their learning. Specifically, the elements of social dynamics such as brokering and bidirectional interactions were considered crucial for supporting and enhancing learning opportunities (Arzarello et al., 2014). It was the way Kelsy, and to a lesser extent Laura, were metacognitive in their thinking about the learning processes that distinguished the experiences at this school from prior interventions.

One of salient outcomes from teacher participation in the program was the change in attitude of one teacher, Ian, in relation to the value of teaching mental computation. At the beginning of the intervention, Ian was uncertain about delaying the teaching of formal written methods (algorithms). Through participation in the program and first-hand experience with students' reasoning about their mental strategies, he became convinced of the benefits. This outcome led the NC to reflect on how his transition in attitude was advantageous in supporting her with the changes she intended to embed across the primary school:

> And what a great advocate he'll be, to be able to share with other teachers, so he's a great resource now. (Laura, interview)

This institutional context had certain existing foundations, which appeared to afford such changes to practice. The teachers in this school appeared to be seen as learners and regarded themselves as learners within a learning community (Clarke & Hollingsworth, 2002). The pre-existing relationships between teachers and the NC were positive and the team was keen to experiment with innovative approaches to teaching and learning to improve student outcomes. The teachers and NC recognised the benefits from participating in the project, in terms of student learning and their own professional learning. At the end of the intervention they were in the process of contemplating how to continue this pathway of learning.

7. Discussion of the findings

It appears that fundamental to ''new'' perspectives on teacher change and teacher professional development that have learning as their core are views of ''teachers as learners'' and ''schools as learning communities''. (Clarke & Hollingsworth, 2002, p. 949)

The purpose of this chapter is to draw together key findings from the three interventions in different schools, in relation to the research questions and the recent literature on teacher professional learning. In comparing the change sequences for teachers from across the interventions, three change sequence types were identified. Commonalities with factors influencing the learning pathways of teachers experiencing the same change sequence types were noticed (Clarke & Hollingsworth, 2002). Fundamental to the changes that occurred seemed to be both how the teachers viewed themselves, and how they were viewed, as learners – in other words, the mindsets they appeared to exhibit (Dweck, 2000): if they acted as learners by self-reflecting on learning experiences; and to what extent they had opportunities to be active learners within a supportive learning community (Arzarello et al., 2014; Clarke & Hollingsworth, 2002).

This chapter is structured in two sections; the first (7.1) considers change sequence types and learning processes in relation to research sub-question 1 (see Chapter 1, section 4). The second section (7.2) focuses on the influence of institutional factors on opportunities for teachers to learn and change practice, to address research sub-question 2.

7.1. Comparing change sequences: Common pathways

In this section, the change sequences and learning processes for teachers from across the three interventions are examined in relation to the relevant literature. The Meta-Didactical Transposition model (MDT) (Arzarello et al., 2014) was used in conjunction with the Interconnected Model for Professional Growth (IMPG) (Clarke & Hollingsworth, 2002) to examine influences on learning processes within the domains of the IMPG. While it is recognised that the methodological approach adopted for this study is not intended for making generalisations as such (Cohen, Manion, & Morrison, 2011), and that professional learning experiences can be idiosyncratic in nature, there appeared common characteristics associated with teachers interpreted as sharing the same change sequence.

In drawing together findings from across the interventions, three change sequence types were identified. The names of Change Sequence Types 1 and 2, are indicative of commonalties with the teachers' learning experiences: *self-reflective learners* and *limited personal change*, respectively. One teacher was interpreted as experiencing the third change sequence type, *inhibited by professional circumstance.* The change sequence types are represented diagrammatically in Figure 7.1.

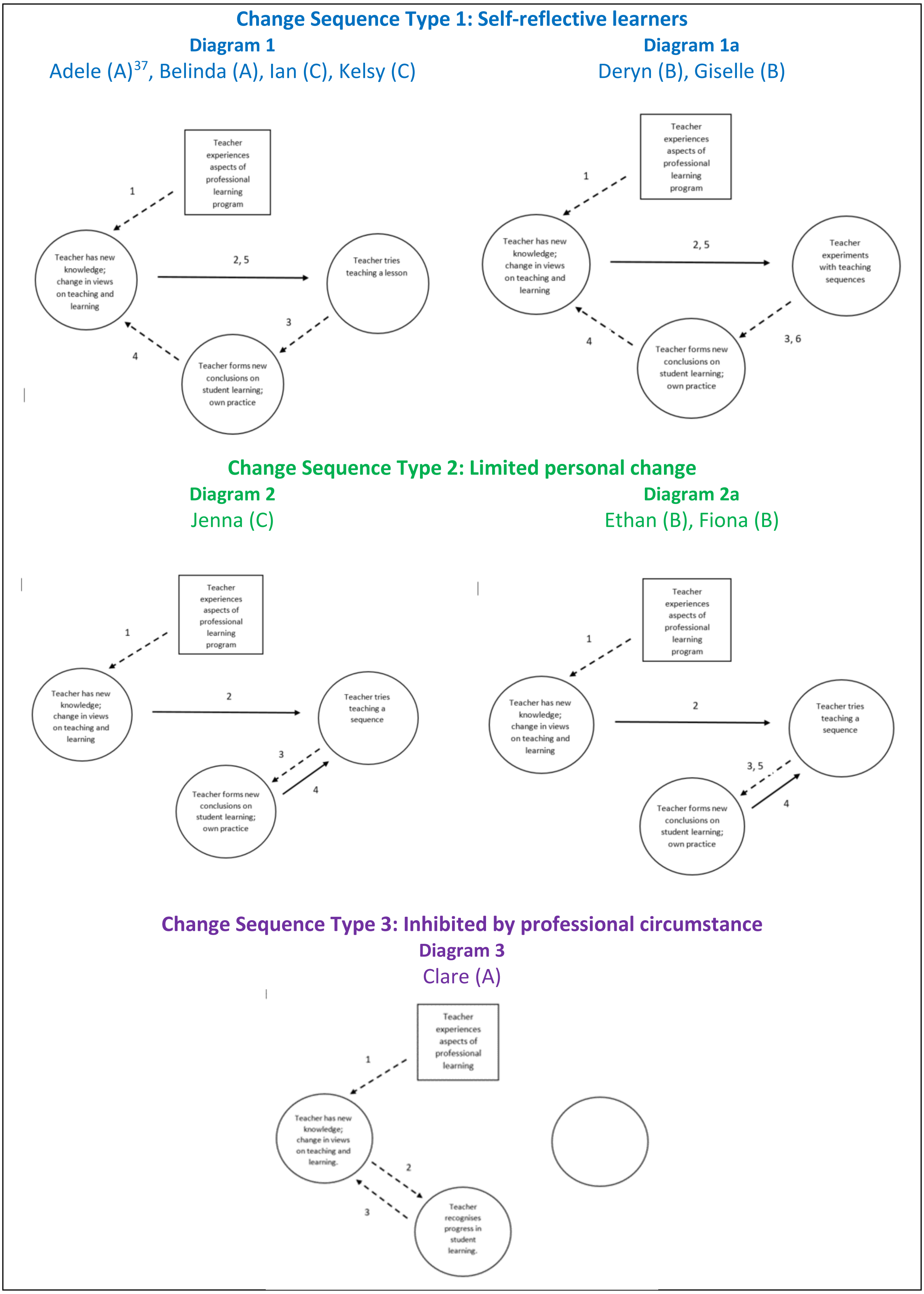

Figure 7.1. Change sequence types identified across the three interventions.

[37] The school context in which each teacher worked, is indicated by the letter following his or her name i.e. (A) represents school A.

As indicated by the diagrammatical representations of the change sequences (see Figure 7.1), groups of teachers (with the exception of one teacher, Clare) were interpreted as experiencing the same change pathway. Essentially, the same mediating processes linked the domains in a particular order. The teachers, categorised as following Change Sequence Types 1 and 2, were from across different institutional contexts. For Change Sequence Types 1 and 2, two change sequences diagrams are presented; the only difference between diagrams 1 and 1a is an additional mediating process of reflection from the Domain of Practice to the Domain of Consequence. The same explanation applies to diagrams 2 and 2a. A plausible explanation for the additional mediating process of reflection, is that teachers interpreted as experiencing this change sequence were from the second intervention (School B) where a second phase of the intervention was implemented at the beginning of the next school term. Teachers collectively engaging in an additional phase of classroom experimentation could explain this extended version of essentially the same sequence.

As displayed in Figure 7.1, all teachers in this study experienced initial changes linked to components forming the Personal Domain. Each teacher described a combination of stimuli in the External Domain as initiating changes in components of their Mathematical Knowledge for Teaching (MKT). An aspect of findings from a study by Justi and van Driel (2006), offers one possible explanation for all teachers in this study experiencing initial changes in the Personal Domain, following participation in professional learning experiences. They suggested one reason for the substantial impact on the Personal Domain of teachers in their study, was the design of the External Domain to provide opportunities for teachers to connect the professional learning experiences with aspects of their current practice, yet simultaneously present the teachers with distinctly different perspectives on teaching. Similarly, the teachers in this study all recognised the relevance of teaching mental computation, but the use of new instructional tools, namely number strings, presented them with a significantly different approach to teaching mental computation.

In analysing the data retrospectively, through the lens of a social constructivist, I initially drew upon the work of Ernest (1994) and his perspectives on learning to consider reflection as a process in which the teachers actively constructed and reconstructed their knowledge by reflecting on their own actions, and engaging with others to reflect on their actions. For this reason, adopting the conceptualisation of reflection defined by Hodgen and Johnson (2004) seemed most apt for the purpose of this study. They drew upon the work of Grimmet (1988) to conceptualise reflection as "the reconstruction of experience and knowledge" (p. 223). In their study on the professional change of six practising primary teachers, in which reflection was considered central, they focused on how teacher reflection on their actions led to significant changes in their practice.

The data suggested that teacher learning was influenced by the learner stance evidenced i.e., the characteristics they exhibited in relation to their mindset (Dweck, 2000; 2006), and if they acted as learners by self-reflecting on their professional learning experiences. In the following subsections, each of these influences will be discussed in connection with the three change sequence types identified for the teachers in this study and the influence on learning processes.

7.1.1. Change Sequence Type 1: Self-reflective learners

All teachers in the study evidenced experiencing initial changes in aspects of their knowledge. What distinguished the teachers, interpreted as following Change Sequence Type 1 (depicted in diagrams 1 and 1a, Fig. 7.1), were further changes in the Personal Domain. These changes are indicated by arrow 4 in both of the diagrams representing this Change Sequence Type 1. The additional changes these teachers experienced in the Personal Domain were in aspects of teacher knowledge and elements of their disposition. The teachers interpreted as experiencing Sequence Type 1 were from across the three different institutional contexts, with a wide range of years teaching experience (see Table 7.1).

Table 7.1. *Teachers interpreted as experiencing Change Sequence Type 1*

Teacher name (pseudonym)	School	Number of years teaching	Number of years in middle primary
Adele	A	10	0
Belinda	A	4	3
Deryn	B	16	4
Giselle	B	Graduate	0
Ian	C	4	1
Kelsy	C	18	6

Despite differences among the teachers in relation to their years of teaching experience and their school contexts, the data indicated they shared two common traits: their self-as-learner stance, in particular the characteristics they exhibited associated with a growth mindset (Dweck, 2000; 2006), and their propensity to self-reflect on professional learning experiences. Each of these factors will be discussed in turn, respectively.

The teachers, interpreted as experiencing Change Sequence Type 1, indicated that they perceived themselves as learners from the outset of the project. With the exception of Adele, they all expressed they were less confident in their own knowledge about planning and teaching mental computation strategies (pre-intervention surveys; Researcher's journal – initial professional learning session). Adele had a slightly different learning focus; she recognised a need to integrate the Australian mathematics curriculum proficiencies into her teaching and was keen to experiment with new instructional tools that could support this goal. By critically self-reflecting on their knowledge and practice at the beginning of the intervention, these teachers all identified a reason to engage in the professional learning experiences. This initial process of self-reflection suggested that the teachers perceived themselves as learners. Further to this, they all seemed to display characteristics of a growth mindset (Dweck, 2000; 2006) in relation to their own learning. (It is important to distinguish personal learning from student learning here because inconsistencies among the teachers in relation to fostering a growth mindset approach in students were noticed, with no evidence of this being promoted in Giselle's classroom). They were risk takers in the sense that they were keen to experiment with new ideas and instructional approaches. Recognition of themselves as learners seemed important, as it allowed them to identify aspects of their knowledge and/or practice to improve upon and seemed to provide an impetus to engage in the professional learning.

The other common characteristic exhibited by teachers interpreted as experiencing Change Sequence Type 1, was their actions as learners, in particular, the way they self-reflected on their learning experiences. These teachers were all interpreted as experiencing an additional arrow of reflection connecting back to the Personal Domain (depicted as arrow 4 in diagrams 1 and 1a). They all reflected on learning experiences related to a specific aspect of the new approach to teaching mental computation. They found themselves trying to make sense of new knowledge, a new practice or a classroom experience. Particular aspects of the professional learning led these teachers to reflect and reconstruct aspects of their knowledge and perspectives on teaching computation (Hodgen & Johnson, 2004). The learning experiences that stimulated deep reflection and subsequent changes in their Personal Domain are discussed next.

7.1.1.1. Professional learning experiences: Brokering process

Three of the teachers, categorised as experiencing this sequence type, namely Adele, Deryn and Giselle, reflected predominantly on changes in their classroom practice. For these teachers, engaging in the brokering process (Arzarello et al., 2014) was a significant influence on their learning of a particular aspect of instructional teaching; in other words it supported development of elements of their Pedagogical Content Knowledge (PCK). For Adele, it appeared that the brokering during modelled lessons supported changes in her pedagogical knowledge: specifically, how to use visual representations and the lesson structure to integrate more opportunities for student reasoning in her teaching. Similarly, Deryn and Giselle reflected on changes in aspects of their PCK from classroom situations in which the brokering process was experienced. Deryn reflected on learning to use number strings as an instructional tool, indicating a significant change in her Knowledge of Content and Teaching (KCT). She recognised that she had learned how to use the instructional tool to build on student knowledge and stimulate learning. Likewise, Giselle reflected on changes in her KCT: how to elicit student thinking and discuss the processes involved in mental computation strategies. While both of these teachers reflected on developing aspects of PCK predominantly connected to brokering opportunities, it should be acknowledged they also reflected on changes in their KCT associated with bidirectional interactions during both the modelled lessons and collective planning. Fundamentally, these three teachers all reflected on learning experiences that provided opportunities to reconstruct aspects of their PCK and led to substantial changes in their practice.

7.1.1.2. Professional learning experiences: Perspectives on practice

Adele, Deryn and Giselle predominantly reflected on experiences in which they had opportunities to develop aspects of their PCK. However, they also expressed changes in their perspectives on teaching mathematics – how their vision of effective teaching of mathematics, or in some cases specifically computation, had changed. When Adele reflected on her classroom experiences, she was fervent about the importance of instigating opportunities for students to explain their thinking and to reason. Her reflections suggested changes in her praxeology (Arzarello et al., 2014), that she had formed a new perspective on teaching and learning computation. At the beginning of the intervention she expressed an interest in learning about ways to integrate student reasoning into lessons; at the end she self-reflected on the value of students' reasoning. Her reflection seemed to consolidate her initial perspective on what effective teaching of mathematics involved; she was reflecting on the action of experimenting with this approach in her classroom. Giselle also indicated a change in her perception on how mathematics should be taught. She recognised the value in exposing students to multiple ways to solve a task, a significant shift from her direct teaching of one method. In contrast, Deryn reflected specifically on the value of students learning to compute mentally. She conveyed the view that attention needed to shift from a focus on written processes to mental strategies. It was apparent that these three teachers all reflected on experiences which led them to reconstruct aspects of their PCK, which at the same time resulted in changes or confirmation of perspectives on effective teaching of computation.

A summary of the learning processes, the external stimuli that instigated and supported learning, and the perceived changes in aspects of the Personal Domain for the three aforementioned teachers is presented in Table 7.2.

Table 7.2. *Summary of learning processes, external stimuli and changes in the Personal Domain for Adele, Deryn and Giselle*

	Teacher	School	Learning process	External stimuli	Changes in Personal Domain
Change Sequence Type 1	Adele	A	Brokering process	Modelled lessons PL session Professional reading Teacher resource book	PCK – predominantly KCT Perspective on teaching mathematics
	Deryn	B	Brokering process Bidirectional interactions	Modelled lessons Number strings Collective planning	PCK – KCT and SCK Perspective – value of students learning to compute mentally
	Giselle	B	Brokering process Bidirectional interactions	Modelled lessons Collective planning	PCK – KCT Perspective – value in exposing students to multiple strategies

7.1.1.3. Professional learning: Dialogic interactions to develop PCK

The other three teachers interpreted as experiencing Change Sequence Type 1 – Belinda, Ian and Kelsy – reflected on experiences during the program that supported changes in aspects of their knowledge, and perhaps more significantly aspects of their disposition. These teachers placed greater emphasis on the influence of professional dialogic interactions on their learning; what follows is a discussion of this influence on their learning.

There were similarities between the learning experiences of Belinda and Kelsy. Besides both perceiving themselves to have limited content knowledge about teaching mental computation at the start of the intervention, they also indicated pedagogical differences the new approach presented for them. The teacher resource book was designed to support the development of both aspects of this knowledge. Kelsy received additional support during collective planning in which there were rich, deep discussions about the pedagogical considerations for teaching the strategies and potential student misconceptions in learning. Belinda and I engaged in similar discussions following the modelled lessons in her classroom (although it was noted that she was limited to one-on-one conversations that focused on her students; she did not have the opportunity for such discussions with colleagues during collective planning).

Both of these teachers reflected on these opportunities in which they engaged in dialogic interactions, and on subsequent changes in aspects of their PCK, predominantly in KCT. The changes described by Belinda and Kelsy, are analogous to the findings of a study aforementioned, in which the IMPG was used to investigate the development of knowledge of beginning science teachers (Justi & van Driel, 2006). They concluded that the External Domain was particularly influential on the teachers' Personal Domain because of its relevance to teachers' practices, yet also presented them with new perspectives distinctly different to their usual way of teaching. Justi and van Driel (2006) also drew attention to multiple opportunities that teachers had, to interact with the researchers to discuss classroom experiences, uncertainties and changes in their practice. It is perhaps significant that Kelsy and Belinda both identified differences between their practice and the new approach, and were keen to engage in conversations to discuss the processes involved in enacting these changes. It was the dialogic interactions within the moment, which the teachers reflected on as supporting reconstruction of their knowledge.

Interestingly, Kelsy and Belinda both mentioned an increase in their confidence with teaching mathematics. This resonates with findings of Covay Minor et al. (2016), who looked at the development of two groups of middle school science teachers over a three-year period. One group was considered to have strong content knowledge and the other group weak content knowledge (note that the development of the two groups were compared against a control group). Their study suggested that teachers with initial weak content knowledge went on to develop greater confidence when provided with resources that were intended to support development of their pedagogical content knowledge. Parallels with the findings of their study, can be thus drawn with the findings of this study.

7.1.1.4. Idiosyncratic experiences: Changes in the Personal Domain

Although similarities can be drawn between the learning experiences of Kelsy and Belinda, it is apparent there were also distinct differences. Kelsy reflected more deeply on the processes of learning. She was metacognitive in thinking about how she had developed her KCT, in particular her development of clearer language to support students in articulating their thinking, and explaining strategies. This could perhaps be explained by her unique position as having a key role in the Cultures of Thinking (CoT) initiative adopted at the school context in which she worked. While Belinda valued extending her knowledge about mental computation strategies and developing her awareness of multiple ways a student may approach a task, it was changes in her perspective on student learning of mathematics which she was most ardent about – specifically, the importance of students articulating their thinking and learning sequentially (building on their knowledge); in other words her perspective on learning.

Belinda expressed dissatisfaction with the way she had learned mathematics as a student. She seemed to recognise that the procedural approach she had experienced was ineffective in achieving the goals of the current Australian curriculum, in which integration of the proficiencies emphasise the importance of students thinking and reasoning (Sullivan, 2011). It could be surmised that she connected her lack of confidence with teaching mathematics to the way she had been taught and therefore wanted to avoid repeating this experience for her own students.

7.1.1.5. Professional learning: Dialogic interactions and misconceptions

For Ian, a significant aspect of his learning concerned knowledge about how to represent mathematical ideas – an important aspect of teaching mathematics (Yackel & Cobb, 1996). Ian reflected on a particular classroom incident, in which he realised he held a conceptual misunderstanding in relation to teaching a mental strategy. The dialogic interactions between Ian and myself were pivotal in addressing the misconception (see Chapter 6, section 6.3.1.3) and developing his KCT (how to represent student thinking visually using an empty number line). Ian's reflection on this incident was particularly consequential in that it led to changes in the way he approached learning with his students. Following the incident, he placed emphasis on the value in learning from mistakes; this became a daily focal point of discussion in his class, "*Our favourite fail of the day"*, in which the discussion would centre on the 'fail' which created the greatest learning. This particular incident seemed to instigate a critical awareness about himself as a learner.

The dialogic interactions between Ian and myself during the modelled lesson, classroom teaching situations and the collective planning, were critical in supporting his learning. It was these incidences which stimulated his self-reflection and a reconstruction of aspects of his knowledge. He recognised changes in his questioning skills, to eliciting ideas from students and encouraging them to reason and justify strategies. Importantly, he reformed his perspectives on teaching computation – the importance of students learning to compute mentally before a focus on written algorithms.

A summary of the learning processes, the external stimuli that instigated and supported learning, and the perceived changes in aspects of the Personal Domain for the three aforementioned teachers is presented in Table 7.3.

Table 7.3. *Summary of learning processes, external stimuli and changes in the Personal Domain for Belinda, Ian and Kelsy*

	Teachers	School	Learning processes	External stimuli	Changes in Personal Domain
Change Sequence Type 1	Belinda	B	Professional dialogic interactions	Teacher resource book PL session Modelled lessons	PCK - KCT, SCK, KCS Disposition – confidence Perspective – importance of sequential learning for students
	Ian	C	Professional dialogic interactions	Modelled lesson Lesson observation – 'in-the-moment' conversations Collective planning Teacher resource book	PCK – KCT Perspective – value in students learning mental computation
	Kelsy	C	Professional dialogic interactions	Teacher resource book Facilitated planning	PCK – KCT Disposition - confidence

Each teacher who experienced Change Sequence Type 1, illustrated the role of self-reflection in their professional learning. These teachers all constructed their knowledge through a process of change when new ideas challenged their existing knowledge and led to restructuring, or in some cases their ideas were extended or their knowledge modified (Loucks-Horsley et al., 2010). While this group of teachers all shared the same change sequence pattern, and all experienced situations which seemed to trigger a moment of reflection and learning, each teacher's professional learning experience was also unique, thus highlighting the idiosyncratic nature of professional learning. In the following subsection, the common traits shared by teachers classified as experiencing Change Sequence Type 2, are discussed.

7.1.2. Change Sequence Type 2: Limited personal change

The teachers interpreted as experiencing Change Sequence Type 2, represented in Figure 7.1 as diagrams 2 and 2a, were from two different institutional contexts and had a range of years teaching experience, but interestingly they all had the same number of years' experience teaching middle primary grades (see Table 7.4).

Table 7.4. *Teachers interpreted as experiencing Change Sequence Type 2*

Teacher name (pseudonym)	School	Number of years teaching	Number of years in middle primary
Ethan	B	25	2
Fiona	B	2	2
Jenna	C	3	2

The common characteristic distinguishing this group of teachers from those experiencing Change Sequence Type 1, was they all seemed to view their participation in the program through the eyes of their students. They reflected on their classroom experiences and predominantly recognised changes related to student learning. Unlike teachers categorised as Change Sequence Type 1, this group of teachers did not seem to experience classroom situations which led them reflect and reconstruct aspects of their knowledge or reformulate their perspectives on effective teaching. Following participation in professional learning experiences at the beginning of the intervention, these teachers were not interpreted as experiencing any further changes in aspects of their Personal Domain. This difference is represented diagrammatically in Figure 7.1. Only one mediating process of reflection depicted by arrow 1 is connected to the Personal Domain.

When engaging in professional dialogue with each of these teachers individually, I found it difficult to encourage them to readjust their focus and self-reflect on their own learning experiences. When prompted, each teacher could identify a positive outcome in relation to their own professional learning (a salient outcome) but none of them reflected on a moment of learning that they connected to a change in perspective or in their knowledge. For example, Fiona expressed that participating the program had been valuable in the sense that it instigated more opportunities for professional conversations within the Year 3 team about *how t*o teach the content, but she did not elaborate further on the impact this had on her own knowledge or practice. Arguably, this is not unusual in that enabling teachers to reflect has been considered a far from simple task (Clarke, 1994; Hodgen & Johnson, 2004). In a longitudinal study that focused on professional change of six practising teachers in the UK, in which teachers were provided with formal opportunities for reflection, the researchers found that "deep and explicit reflection was an infrequent occurrence" (Hodgen & Johnson, 2004, p. 219).

In the remainder of this subsection, professional learning experiences of these teachers will be compared and contrasted in relation to the literature, to gain insights into their change pathways and learning processes. While Jenna and Ethan exhibited similarities and differences which can be compared, analysing the data retrospectively revealed that Fiona indicated perspectives that distinguished her from the other teachers experiencing Change Sequence Type 2.

7.1.1.6. Limited self-reflection on current practice

Although Jenna was an early career teacher, she exhibited spontaneity and confidence usually associated with slightly more experienced teachers (Huberman, 1992; 1995). In his work on teacher career cycles, Huberman described this as the 'stabilisation' phase (teachers typically have 4-6 years of experience at this stage). Jenna appeared actively engaged in the program. She compared experimentation with new ideas to her usual practice and recognised differences. In terms of content knowledge, in planning meetings Jenna indicated that the intervention presented her with new learning. The use of number strings also presented her with a new instructional tool. However, it was difficult to evoke self-reflection from Jenna on changes in her knowledge and practice. She gave one example when interviewed at the end of the intervention, in which she identified how integrating a focus on comparing efficiency of mental strategies added value to class discussions. She explained that prior to the intervention she would have only considered this an extension task. Loughran (2002) discussed one common element of reflection being linked to the notion of a problem (issue), or an interesting or perplexing incident. For Jenna, the professional learning experiences did not seem to challenge or disturb her current practice. It seemed she was refining elements of her practice rather than making significant changes.

In contrast, Ethan conveyed a calm, relaxed attitude towards teaching. The way he displayed a sense of confidence and self-acceptance about his teaching seemed analogous to the 'serenity' phase of Huberman's (1992; 1995) schematic model of the teaching career cycle. Despite emitting a sense of self-acceptance about his teaching, Ethan was keen to engage in conversations about student learning and discuss his views on mental computation. At the end of the intervention he recognised he had gained greater insights into student thinking processes, as well as gaining some new teaching ideas. He certainly indicated that he had developed his conceptualisation of mental computation beyond his initial ideas, which was focused on student fluency with basic facts. While Ethan identified positive outcomes from participating in the project, his thought processes did not extend further than recognising these as salient outcomes.

Contrary to Jenna, Ethan did encounter challenges with using visual representations to display student thinking and general classroom management skills. Although such situations gave him reason to experiment with changes in his practice and reconstruct his usual pedagogical approach, he instead modified the intended pedagogical approach (of the intervention) to teach in a way with which he was more comfortable. His behaviour suggested that he did not position himself as a learner. Perhaps he did not consider investing the energy in experimenting with such changes as worthwhile or achievable at a late ('serenity') phase in his career, a surmise, which connects to Huberman's (1992; 1995) conceptualisation of teaching career lifecycles.

7.1.1.7. Differing perspectives on effective and equitable teaching

Fiona indicated perspectives that distinguished her from the other teachers experiencing Change Sequence Type 2. It seemed that she perceived ability grouping as the optimal way to improve student achievement. When I observed her teaching towards the end of the intervention, she conveyed a sense of confidence and pride in the way she had modified the (intended) lesson structure to integrate ability grouping, suggesting she perceived this to be emulating good, equitable practice (in relation to student achievement). During whole class discussion, her students were seated in ability groups to allow her to direct questions she considered appropriate for certain groups of students more easily. Her actions indicated that she perceived ability grouping as a strategy to enhance equitable practice, in relation to student achievement, rather than one that perpetuates inequity (Taylor et al., 2019). Fiona's lesson modification certainly highlighted one of the methodological challenges associated with design-based research, since it essentially undermined some of the key principles underpinning the design of the intervention (Collins et al., 2004).

Findings from a recent large-scale, longitudinal study investigating how to support teachers' development of ambitious and equitable teaching practices (Cobb et al., 2018) offer potential insight into Fiona's actions. Their study highlighted that the vision teachers held of high-quality mathematics instruction influenced their instructional practice, including the changes they were prepared to make to their practice. This finding resonates with the actions displayed by Fiona, who perceived the best option for optimising student learning was to integrate ability grouping within the lesson structure. Of particular importance, was the finding from the study conducted by Cobb et al. (2018), that a focus on developing a *shared* vision of high-quality practice was important for changes in classroom practice to occur. "While MKT clearly matters, teachers' perspectives play a role in *how* that knowledge influences their instructional practice" (p. 56). Fiona's actions could in part be explained by the limited focus within the intervention on developing a shared vision of effective or high-quality teaching of mathematics.

A further characteristic of Fiona, which distinguished her from all teachers in this study, was the perspective she appeared to hold in relation to students' mathematical capabilities. Participation in the program raised new awareness of the learning needs of some of her students: she noticed a group of students who were inflexible with partitioning numbers to make the calculation easier (see Appendix E). After her initial efforts to adapt her teaching were unsuccessful, she indicated she was perplexed about how to address the issue. Subsequently, she attributed this inflexibility predominantly to student attitudes, i.e., an issue intrinsic to students that therefore could not be resolved by a change in her instructional practice. Fiona's response to a perplexing situation with student learning difficulties suggests she did not hold a productive view of students' mathematical capabilities (Cobb et al., 2018). The study conducted by Cobb et al. (2018) found that teachers who evidenced not holding a productive view of students' mathematical capabilities, were less likely to develop ambitious and equitable practices:

> That is, students were, on average, more likely to have opportunities to participate in discussions in which they provided reasoning for their solutions if their teacher explained student difficulty in terms of instruction, as opposed to in terms of student deficits. (Cobb et al., 2018, p. 58)

In the lesson observed, opportunities for students to explain their thinking and reason about mental strategies were tightly controlled by Fiona; it seemed she was not entirely convinced that engaging in such discussion would instigate learning of all students. This concurs with the findings of Cobb et al. (2018) that teachers' perspectives on high quality teaching, and whether they hold a productive view of student learning capabilities, are influential on the changes teachers make to their practice.

A summary of the learning processes, the external stimuli that instigated and supported learning, and the perceived changes in aspects of the Personal Domain for the three teachers categorised as experiencing Change Sequence Type 2 is presented in Table 7.5.

Table 7.5. *Summary of learning processes, external stimuli and initial changes in the Personal Domain for teachers experiencing Change Sequence Type 2*

	Teachers	School	Learning process	External stimuli	Initial changes in Personal Domain
Change Sequence Type 2	Ethan	B	Brokering Professional conversations (in modelled lessons)	Modelled lessons	PCK – predominantly KCS
	Fiona	B	Brokering Professional conversations	Number strings Modelled lessons Teacher resource book PL session	PCK – predominantly KCT (linked to new instructional tool – number strings)
	Jenna	C	Brokering	Number strings Modelled lessons Teacher resource book	PCK – KCT (linked to a new instructional tool – number strings)

7.1.3. Change sequence 3: Inhibited by professional circumstance

It has been suggested that "what teachers learn in PD varies significantly based on their prior knowledge and experience" (Covay Minor et al., 2016, p. 2). This resonates with the experiences of one of the graduate teachers in this study, Clare. The process of change involves teachers first having awareness of an idea, then experimenting with the idea and adapting it so it becomes sustainable (Loucks-Horsley et al., 2010). Although Clare expressed an interest in the intervention and was keen to participate in the study, it seemed she required additional support to progress to the experimentation stage. Her limited teaching experience and knowledge seemed to inhibit her willingness to experiment with the ideas in her classroom. She struggled to make sense of the instructional tools and the subject content from the beginning of the intervention. Hodgen (2003) suggested that teachers need to connect with new practices in some way, in order to make sense of the ideas and enact them. For Clare, there appeared to be a lack of connection with the new practices: they seemed beyond her current level of knowledge and her experiences both as a prospective teacher and a school student. The goals of the project, which essentially involved developing instructional practices considered ambitious (Kazemi et al., 2009), were perhaps outside her current zone of proximal development (Vygotsky, 1978).

Clare was in the early stages of developing aspects of her mathematical knowledge related to student development of additive thinking. In her semi-structured interview, Clare commented on her perception of computation with 2-digit numbers and use of empty number lines (ENL) as a tool to show thinking, as being appropriate for the younger years. She indicated that she had already taught the students traditional algorithms for addition, yet the majority of her class could not add or subtract 2-digit numbers mentally and had limited understanding of place value. As a graduate teacher, Clare indicated she was still forming a trajectory of how students learn to add and subtract: the key growth points (Clarke, 2008). Limited knowledge about the stages of learning would have made it difficult for Clare to anticipate student responses to the mental computation tasks and respond flexibly to their ideas, to steer the lesson towards the intended learning goal. Perhaps new awareness of student learning needs, highlighted in her observation of the modelled lessons, had raised awareness of some required expertise involved in this pedagogical approach and deterred her from experimentation.

The data indicated that Clare had a different level of entry knowledge and experience to the other participating teachers in her team (Hiebert & Grouws, 2007). In this sense, learning experiences of teachers appears analogous to those of students. The way the professional learning program was implemented at School A made it difficult to support Clare's learning needs. Perhaps if she had the opportunity to engage in dialogic interactions with colleagues, unpack the mathematics and discuss the instructional tools during collective planning, she would have been able to connect and transfer the new ideas into her own teaching. It is possible that she was operating in her zone of proximal development (Vygotsky, 1978) and simply required further support to experiment with the ideas. Unfortunately, a lack of flexibility with the implementation of the intervention at the school in which she worked meant that this was not possible.

A summary of the learning processes, the external stimuli that instigated learning, and the perceived changes in aspects of the Personal Domain for Clare is presented in Table 7.6.

Table 7.6. *Summary of learning processes, external stimuli and changes in the Personal Domain for the teacher experiencing Change Sequence Type 3*

	Teachers	School	Learning process	External stimuli	Changes in Personal Domain
Sequence Type 3	Clare	A	Brokering	PL session Modelled lessons	Perspective on value of teaching and learning mental computation. KCS and SCK – early changes but misconceptions evident.

Change Sequence Type 3 emphasised the significance of the institutional context on opportunities for teachers to learn and change practice. It can be surmised that this change sequence type may not have occurred had certain support mechanisms been available in the school context. In the next section, the influence of contextual factors on opportunities for teacher learning is discussed, including the extent to which the teachers had opportunities to be active learners within a supportive learning community.

7.2. Influence of the institutional context on learning

In this section findings related to the second research sub-question are discussed, specifically, how aspects of the teachers' institutional context constrain or afford opportunities for teachers to learn and change practice. The Meta-Didactical Transposition (MDT) model (Arzarello et al., 2014) was drawn upon to examine the influence of components of the institutional dimensions that are less explicit in the IMPG. Various factors, external and internal, either afforded or constrained opportunities for teachers to learn and change their practice within their school context. The following internal influences will be discussed: coherence with school goals, opportunities for collaborative learning, influence of an instructional coach, school ethos on professional learning, and accountability. Following this, the influence of an external factor – national assessment requirements (NAPLAN) – on professional learning opportunities will be addressed.

7.2.1. Coherence with school goals

There are two key aspects of coherence to consider: consistency with content of the professional learning and teachers' knowledge and beliefs; and consistency with school and state reforms (Desimone, 2009). As the first aspect has been addressed through analysis of the change sequence types, the focus of the discussion here is on the latter aspect: how consistency of the professional learning with the institutional visions for teaching and learning mathematics, influenced opportunities for teacher learning.

A lack of coherence between the school mathematics program and the goals of professional learning is often attributed as one of the main reasons for limited impact of interventions on teacher professional learning (Desimone & Pak, 2017). In this study, lack of coherence presented a constraint at the first intervention. At School A the challenge appeared to be a general lack of cohesion in relation to school goals for the teaching and learning of mathematics. The content focus of this study was new learning for both teachers and students; the Year 3 team leader was not aware of approaches to computation in earlier years or of specific goals for Year 3. It seemed that the content focus of the intervention did not form part of the student learning trajectory for additive thinking in the school program. Although School A had a mathematics leader (ML), she had classroom responsibilities and was located in a different area of the school; the program resources were shared with the ML but it was not feasible for her to be involved in the study.

The situation at School A was a contrast to Schools B and C, where the numeracy coach (NC) and ML, communicated clear goals for the teaching of mathematics across the school, which were coherent with the goals of the professional learning. At School B, the ML had a focus on a change in pedagogical approach (transition from ability group teaching to whole class approach). It was intended that participation of the Year 3 team in this study would contribute towards the achievement of that goal. In addition, the content focus of the teaching component of the program was aligned with the perspective held by the ML on students learning computation. At School C, the goals of the professional learning program supported the aims of the NC for students to be fluent and flexible with mental computation strategies before being introduced to formal algorithms. The pedagogical approach underpinning this study connected with key aspects of Cultures of Thinking (CoT), a pedagogy embedded across the school. At the two schools in which there was coherence between the goals of the intervention and school vision on teaching and learning of mathematics, there seemed more opportunities for dialogic interactions during collective planning, which focused on teaching and learning associated with these shared goals.

7.2.2. Opportunities for collaborative learning

Collective participation is arguably a critical component of professional learning. It can provide opportunities for professional dialogue and interaction; both of these elements of collective participation are recognised as powerful forms of teacher learning (Desimone, 2009; Desimone & Hill, 2017). In this study, various influences within each school context afforded or constrained elements of social dynamics, such as dialogic interactions and bidirectional interactions, which were crucial for enhancing and developing teacher learning (Arzarello et al., 2014). Subsequently, the collective learning experiences of teachers across the three interventions were variable. If the schools in this study were placed on a continuum in relation to opportunities for collaborative learning (as interpreted by the researcher), School A would be positioned at one end where opportunities were highest and School C at the opposite end where such opportunities were lowest. The institutional context of School B would be positioned in between, but closer to School C. The reasons for this hypothetical visual image will be discussed next.

Various influences within the institutional context of School A meant there was an absence of collective participation throughout much of the first intervention. The withdrawal of nearly half the teachers in the early stages of the project (the part-timers) had a detrimental impact on opportunities for collaborative learning. Subsequently, at School A, participation in the study appeared to provide teachers with learning opportunities at a mostly individual, rather than collective level. In comparison, aspects of the institutional contexts of Schools B and C afforded more opportunities for collective professional dialogue and interaction among teachers. Collaborative planning, for example, was embedded within the school culture.

Of the three schools, the institutional context of the third intervention, School C, enhanced comparably the most opportunities for collective professional dialogue and interaction between the participants. This is noteworthy, considering that in other studies, the advice teachers receive from colleagues has been found influential in supporting improvements in instruction. Cobb et al. (2018) referred to this as teachers' advice networks, which initially just involve networks within a school. In this study opportunities for interaction were not restricted to professional learning sessions, planning meetings or modelled lessons, they occurred at opportune moments inside and outside the classroom and seemed an integral part of the professional life of this group of teachers. The physical layout of the school, the visibility and availability of the NC who did not have classroom responsibilities, afforded these multiple interactions. The suggestion that the intensity of teacher engagement in the program can result in desirable outcomes for professional learning seems pertinent (Desimone, 2009). Perhaps also notable was the prevalent involvement of the NC, someone with strong subject knowledge and classroom experience to influence the quality of the interactions (Timperley et al., 2007).

In contrast to the two other schools, the pre-existing positive interpersonal relationships among teachers, and between teachers and the NC at School C, were seen to afford interactions and changes in teachers' practices. They had a close "community of colleagues" (Clarke & Hollingsworth, 2002, p. 955) with whom they could share and discuss their classroom experiences. It has been highlighted that "for collaborative time to support teachers in trying out and adapting particular practices, the team needs to develop a sense of trust and mutual support, as well as ways to reconcile different perspectives" (Cobb et al., 2018, p. 97).

The teachers at School C certainly conveyed a strong sense of trust and mutual support. The zeal and eagerness I observed of the teachers at School C seemed to differentiate their experiences from the teachers at School B. The teachers at School B highlighted that opportunities for interaction and professional dialogue per se, did not necessarily result in effective professional learning. It seemed that *who* constituted the learning community was a significant influence on the quality of the dialogue and the interactions among participants (Hodgen, 2003). At School B, a considerable part of collective planning time focused on revisiting the issue of differentiation. Fiona, who usually took a dominant social role, repeatedly diverted the conversation to this topic because it was of particular concern to her in her classroom. She conveyed a strong opinion on the need to teach students in ability groups; her dominant role seemed to influence the direction of pedagogical decisions made in collective planning meetings. This team, or maybe specifically the ML, seemed to need guidance on ways to moderate, yet respect the views of all team members if they were to develop a shared vision on effective teaching of mental computation. In contrast, teachers at School C conveyed the sense of a "shared vision, expectations, commitment, responsibility for student learning, and trust" which strengthened their engagement in opportunities for professional learning (Desimone & Pak, 2017, p.

7). They did not express any conflicting perspectives that distracted the teachers from the focus on ways to enhance student learning using ambitious practices. From my perspective the situation at School C highlights the positive influence of bi-directional interactions between a community of teachers and internal factors within the institutional context on professional learning and change.

7.2.3. Influence of an instructional coach

Active learning is recognised as occurring in many forms, such as "observing expert teachers or being observed, followed by interactive feedback and discussion; reviewing student work in the topic areas being covered; and leading discussions" (Desimone, 2009, p. 184). It is considered a critical component of effective of professional learning. Arguably, instructional coaching embodies active learning for teachers, commonly through one-on-one interactions that may occur inside and outside the classroom (Desimone & Pak, 2017). One of the recognised benefits of coaching is that it provides more opportunities for feedback and professional dialogue, which has been shown to be particularly influential in relation to the success of professional learning (Desimone & Pak, 2017). An internal factor for two of the schools involved in this study, which appeared to afford opportunities for active learning, was the role of the NC or ML, who in effect assumed the role of an instructional coach. Although these roles were effectively those of a coach, there was a perceived difference between the NC and ML in this study, which is worthy of mention. The ML at School B was associated with the school senior leadership team; her office was situated alongside other members of the leadership team and her role included leadership responsibilities. In contrast, the NC at School C, was located in a shared space with learning specialists, central to the primary classrooms. She was required to report to the senior leadership but was not considered part of the school leadership team. A discussion on how the roles of theses 'instructional coaches' influenced active learning across the institutional contexts follows next.

At School A, the ML was not involved in the intervention since she was teaching full time in another area of the school. To an extent it could be argued that I assumed the role of an instructional coach as an external expert, particularly during the modelling of the sequence of lessons. However, this also applied to the two other schools. In comparison with the two other institutions, opportunities for active learning were far fewer in School A. The impact of the intervention was constrained to individual teachers, in their classrooms. In contrast, Schools B and C each employed an ML or NC, without classroom responsibilities, to support teachers in the capacity of an instructional coach. The presence of an instructional coach at both of these schools seemed to afford opportunities for teachers to deepen their knowledge and improve their classroom practice (Cobb et al., 2018). The instructional coaches engaged in discussions which focused on: the teachers making sense of student explanations and reasoning of strategies; understanding misconceptions; and thinking about the 'where to next' with lessons. At School C, the subtle difference was that teachers more consciously and directly connected what the students were learning to the instruction they were given.

In terms of opportunities for active learning, there were many interactions between the ML, Hannah, and teachers within planning meetings at School B.

However, opportunities for interactions outside of these meetings were more limited than at School C, for various reasons. The Year 3 team at School B, was one class larger than at School C, which meant extra responsibilities, including mentoring of a graduate teacher. With regards to classroom support, during the intervention Hannah focused her attention on Giselle, a graduate teacher. Hannah's role also involved supporting an extra year level, in comparison with School C. There were four Year 6 classes at this School B, whereas at School C, Year 6 was included in the middle school and was not the responsibility of the primary years NC. As a resource, Hannah was stretched over a much larger group of teachers and students, and had to contend with a larger physical space. At School B the classrooms were spread over a large area due to a building project being underway, which presented another challenge. In addition, her office was located at the front of the school with the leadership team members – somewhat removed from the location of the classrooms. In contrast, at School C, the physical space in which the Year 3 classrooms were located, clustered around a Year 3 student communal area, and afforded interactions among team members. In addition, the Year 3 classrooms were within close proximity to Laura's office (the NC); she regularly passed through the Year 3 area during the school day, which again afforded opportunities for interactions among the teachers. At School B it seemed there were various influences, which reduced the intensity of interactions and opportunities for professional dialogue between the ML and teachers. In comparison, at School C there were multiple opportunities for one-on-one interactions between the NC and teachers; this factor was seen to distinguish the intensity of active learning (Desimone, 2009) at this school from the two other participating institutions.

In many respects the role the instructional coach had at School C seemed less challenging in comparison with School B. The ML at School B needed to draw upon skills beyond facilitating productive discussions and supporting teachers in learning activities: she needed to support some of the teachers with reconstructing their perspectives on what constituted effective teaching. This is a skill which Cobb et al. (2018) refer to as "facilitator press" (p. 108). In their study they found that instructional coaches who were particularly effective "pressed teachers to articulate rationales for instructional decisions" (Cobb et al., 2018, p. 108). This is perhaps one strategy that could have encouraged Fiona to develop a shared perspective on effective teaching of mathematics, by urging her to reflect critically on how her instructional actions connected to student learning and attitudes. The findings of this study suggest that the skill set required for an effective instructional coach encompasses more than sound Mathematical Knowledge for Teaching (MKT); it also involves being an effective facilitator and possessing strong interpersonal skills.

7.2.4. Professional learning, leadership and internal accountability

In this subsection the school ethos on professional learning, leadership structure and internal accountability, will be discussed in relation to their influence on opportunities for teachers to learn and change practice. Provision of professional learning opportunities and internal accountability are highlighted as key aspects of the school setting to take into consideration within an interpretative framework for professional learning study (Cobb et al., 2015).

All three schools involved in the study coordinated regular classroom release of their teachers for collective planning purposes. However, just two of the schools (Schools B and C) provided the support structure of an instructional coach – a person with expertise and knowledge in this subject area, to facilitate meetings. At these two schools the planning meetings were used for the intended purpose, whereas at School A collective planning time was used for administrative purposes. It has been suggested that school leaders influence "whether teacher collaboration meetings and coaches' work with teachers are productive." (Cobb et al., 2018, p. 5). This study appeared to confirm this finding.

The internal professional learning requirement for teachers at School B to participate in an action research project with their year level colleagues was a feature of the institutional context, which differentiated this school from the two others. It was seen to afford opportunities for teachers to learn, and certainly gave impetus for learning the subject content of the teaching component of the intervention. The teachers requested additional professional reading material to develop their content knowledge for teaching mental computation strategies. Discussion of their learning filtered into our planning sessions, as the teachers tried to make sense of student responses to learning to compute mentally. This internal school requirement also seemed to provide the teachers with further reason to engage purposefully and collaboratively. They collectively participated in additional activities associated with the professional learning program. For example, they created a display board of student learning on mental computation, to communicate their research focus to the school community (refer to Appendix E). The avid interest the group displayed could in part be explained by this internal accountability, a factor identified for consideration with professional learning studies (Cobb et al., 2015). The teachers were required to present the goals and outcomes – including student learning outcomes – to the school principal, and communicate the focus of their project to the school community. It is questionable as to what extent the teachers would have participated and developed their content knowledge, had they not been accountable to the school principal. Nonetheless, this internal school requirement enhanced opportunities provided in the professional learning program for teachers to develop aspects of their content knowledge and have a shared focus on student learning goals.

7.2.5. External factor: National level assessment requirements

The participatory experiences of some teachers, and my actions as a researcher, were constrained by an external influence: the implementation of the National Assessment Program – Literacy and Numeracy (NAPLAN)[38]. The extent to which the assessments influenced opportunities for teachers to participate in the program, and for me to implement the program as intended, varied across the three school contexts. Findings of this study resonate with those of Thompson and Harbaugh (2013) in which survey results from in-service teachers were

[38] The National Assessment Program – Literacy and Numeracy (NAPLAN) is implemented in annually in schools in Australia in Years 3, 5, 7 and 9. Students in these year levels are expected to participate in tests in reading, writing, language conventions (spelling, grammar and punctuation) and numeracy.

analysed from three school systems, across two states (Western and South Australia). The surveys indicated that teachers from government schools perceived NAPLAN to have a greater negative impact on curriculum and pedagogy in comparison with teachers from Catholic and independent schools (there was no significant difference between results from the latter two school systems).

Whilst the findings from this study correlate with those of Thompson and Harbaugh (2013), it is important to acknowledge that at the time the intervention was implemented at School C, NAPLAN had already been completed for the year. Nevertheless, the school had been involved in another research project during Semester one, when NAPLAN was conducted. This suggests that NAPLAN did not cause any particular disruption to school programs at School C. Findings of a more recent study, on the impact of NAPLAN on independent schools in Australia, further substantiate those of this study. “NAPLAN was largely considered just another test within a battery of school assessments that does not hold any special consequences for the students, teachers or schools as a whole” (Rogers, Barblett, & Robinson, 2016, p. 340). Their study concluded “that in general, greater importance is placed upon NAPLAN achievement by principals in government schools” (Rogers et al., 2016, p. 340). Given that the results of this study concurred with findings in the aforementioned literature, the last part of this subsection is focused on gaining further insights into the impact of the external requirements on opportunities for teachers at School A to engage in the professional learning program.

At School A almost half the Year 3 teachers withdrew from the intervention in the early stages, the main reason being pressure to prepare their students to perform well in NAPLAN. With only a few Year 3 teachers participating, this effectively removed the option for collective planning of mathematics during Year 3 team meetings. (As aforementioned in Chapter 4, in an effort to improve the well-being of teachers, school leadership had restricted additional after-school meetings, thus making alternative arrangements for collective planning difficult). Some of the perceived pressure by teachers at School A could be attributed to working at a government school located in a high socioeconomic status area with a reputation for high student achievement in relation to NAPLAN scores. For this group of teachers, it seemed imperative that high achievement levels and the reputation of the school were maintained. This interpretation concurs with a study by Polesel et al. (2014) in which online survey data from over 8000 educators across Australia indicated that 95% felt that poor results would create a negative image of the school in the media; 96% believed that poor results would be detrimental to the reputation of the school in the community. After spending a week interacting with the teachers at School A on multiple occasions, I surmised that it was not simply about maintaining the school reputation; it was also about individual teachers feeling the need to protect their reputation within the school and parent community. Many of their students received private tuition outside school hours to enhance their achievement, suggesting that there was substantial pressure on teachers from the parent community.

In relation to external influences on the social context of the institutional setting, findings of this study concurred with those of other studies that the impact of NAPLAN on school programs may reflect a systemic issue (Rogers et al., 2016; Thompson & Harbaugh, 2013). For two of the schools in this study, the Catholic school and independent school, it was found that implementation of NAPLAN did not affect opportunities for teachers to learn and change practice. For the government school, NAPLAN appeared to severely constrain opportunities for teacher learning.

7.3. Summary

The focus of the first section of this chapter was to discuss the change sequences and learning processes experienced by all the teacher participants, in relation to the first research sub-question. The teachers in this study were categorised as following three change sequence types: self-reflective learners, limited personal change, and inhibited by professional circumstance. The names of the sequence types were indicative of commonalities with the learning experiences of the teachers categorised from the data analysis as following the same pathway. Fundamental to the change sequence teachers followed seemed the learner stance they adopted, the characteristics exhibited in relation to their mindset as a learner (Dweck, 2000; 2006), and if they acted as learners by self-reflecting on their professional learning experiences. For teachers whose thinking or practice was challenged during the intervention, and who displayed attributes of a growth mindset, such experiences created opportunities for them to self-reflect and enhance their learning. Generally, when teachers developed a shared perspective or vision for effective teaching of mental computation that aligned with the goals of the intervention, it tended to influence the changes in practice they were prepared to adopt to support achieving the shared vision. This aspect of the findings concurred with those of Cobb et al. (2018).

Although each teacher suggested a combination of stimuli instigated and supported their learning, two particular components of the professional learning program appeared critical: modelled lessons and facilitated collective planning. Both enhanced opportunities for elements of social dynamics, in particular, the brokering process and bidirectional interactions. Social dynamics were a significant influence on learning and change within the domains of the change environment. Using the MDT to examine influences on learning processes within domains of the IMPG, highlighted the idiosyncratic nature of professional learning. Teachers interpreted as following the same change sequence nonetheless experienced different learning processes and changes within domains.

The second section of this chapter focused on the influence of institutional factors on opportunities for teachers to learn and change practice. Internal support structures such as an instructional coach, facilitated collective planning with an instructional coach, and a school ethos on professional learning in which accountability was integral, had potential to afford opportunities for teachers to learn and develop practice. However, it seemed that for such structures to be effective in supporting learning and change, teachers needed opportunities to be active learners within a supportive, innovative learning community. Strong interpersonal relationships, where there was mutual respect among team members, afforded positive, rich professional interactions. In addition, the extent to which the teachers and the instructional coach, developed a shared vision of effective and equitable teaching of mental computation that was aligned with the goals the intervention, influenced the changes teachers made to their practice. Consistency between the intervention and the school vision seemed critical. While professional learning can be influenced by aspects of the institutional context, this study highlights that teachers also play an important role in shaping learning experiences. It emphasises the significance of bidirectional interactions between the teaching community and institutional context in shaping professional learning experiences.

In the next chapter, conclusions are drawn and the implications of this study discussed. The theoretical implications for researchers studying change processes are considered and practical suggestions for those involved with implementing professional learning in schools are outlined. The limitations of this study and possible avenues for future research are also addressed.

8. Implications and conclusions

We've learnt so much and we've got so much more to learn too, which is good. It's a good place to be at. (Laura, interview)

The aim of this study was to investigate the processes involved in teachers learning sophisticated instructional practices through onsite professional learning experiences: specifically, how teachers learn and develop practice within the institutional context in which they work. The teaching and learning of mental computation with conceptual understanding provided the context in which to study the change processes of teachers, and the variability issue in relation to different outcomes for individual teachers participating in the same professional learning experiences (Goldsmith et al., 2014). The intention was not to validate a particular approach or find a solution to the challenges of providing professional learning for practising teachers. Instead, it was to gain insights into learning processes: ways to foster and support teacher learning and change. It was found that all teachers participating in this study learned and developed aspects of their practice, albeit to varying degrees. However, each intervention raised as many new questions as it answered and highlighted the complexity of changing teaching practice within different institutional settings.

In this chapter I consider how findings from the intervention, implemented across three different schools, might inform future design and implementation of onsite professional learning experiences, and guide further research analysing learning processes involved in developing teaching practice. In the first section the implications of this study are discussed and practical suggestions given for those involved in planning and implementing professional learning in schools. The theoretical implications and considerations for researchers studying change processes are also addressed. This is followed by a comment on the contribution this study makes to the field, discussion of the limitations, and possible avenues for future research.

8.1. Implications for schools and future research

Adoption of a design-based research methodology meant the implications of this study were both pragmatic and theoretical (Cobb et al., 2015). Pragmatically, the study involved refining a design to support teachers with developing aspects of their teaching practice. Essentially teachers were learning to use sophisticated instructional tools to stimulate student arithmetical reasoning and relational thinking (Wright et al., 2012) to orchestrate purposeful class discussion (Stein et al., 2008). Theoretically, the study involved developing, implementing and refining an intervention to support teacher learning processes. The purpose was to gain insights into *how* teachers learn to use sophisticated instructional tools, in other words develop ambitious teaching practices (Kazemi et al., 2009), and the support structures teachers required. The focus of this section is to consider what has been learnt about implementing onsite teacher professional learning, and how this can be drawn upon to effectively shape future professional learning programs and research.

In the following subsections, the implications of this study are articulated as practical suggestions for those in schools responsible for planning and implementing professional learning experiences. Theoretical considerations for researchers conducting studies on professional learning processes are also outlined. The intention is to present ideas that may be useful to consider when planning and implementing either future professional learning programs or research studies on teacher professional learning in mathematics. The suggestions have been shaped by my knowledge of the research field, personal experiences, and convictions: essentially elements that influence my professional judgement (Loucks-Horsley et al., 2010).

8.1.1. Suggestions for school leadership: Planning and implementing professional learning

This study highlighted the complexity of designing and implementing onsite professional learning for practising teachers. The findings indicated a multitude of factors which can afford or constrain opportunities for teachers to learn and change their practice. These included: whether a school was viewed as a learning community by both teachers and leadership; whether the teachers evidenced a learner stance and acted as learners; internal and external influences within the school context; and the range of external stimuli and support mechanisms available for individual teacher learning needs. Drawing upon findings from this study, three key considerations emerged for schools when designing and implementing onsite professional learning programs. These were: forming a shared vision for effective and equitable teaching of mathematics; considering aspects of the institutional context; and promoting a professional learning culture. Each of these will be discussed respectively.

8.1.1.1 Forming a vision for effective and equitable teaching of mathematics

The findings from across the three interventions drew attention to the importance of teachers holding a shared vision for effective and equitable teaching of mathematics (Cobb et al., 2018). When a team appeared to share a vision on effective teaching of mental computation and hold a productive view of student learning (Cobb et al., 2018), the dialogic interactions during collective planning were focused on how teachers could develop their instructional practice to improve learning for students. In contrast, when conflicting perspectives existed between team members, or there seemed an absence of a vision for effective teaching of computation, this constrained opportunities for teacher learning and change. Thus, forming a shared vision might help diminish the negative influence of divergent views and challenges presented in relation to teachers adopting new practices, or internalising praxeologies (Arzarello et al., 2014). Based on the findings from this study, it seems imperative for schools to consider what effective and equitable teaching of mathematics might look like in classrooms, as well as strategies that might support them in moving towards achieving such a vision.

Formulating a vision of effective and equitable teaching in mathematics could be achieved by drawing upon research-based knowledge (Loucks-Horsley et al., 2010). It is suggested that schools consider:

- the nature of mathematics teaching and learning e.g., promoting conceptual understanding (Kazemi & Stipek, 2001; Parrish, 2011);

- ambitious instructional tools e.g., number strings (Kazemi, et al., 2009; Lambert et al., 2017);

- research on learning of mathematics e.g., student-centred practices or strategies (Hiebert et al., 1997);

- research on effective pedagogical practices in mathematics e.g., Five practices for stimulating productive discussion in mathematics (Stein et al., 2008); use of Talk Moves (Kazemi & Hintz, 2014);

- equitable practices e.g., effective differentiation to allow all students everyone access to tasks/learning opportunities (Sullivan et al., 2006).

To assist with forming a vision for effective and equitable teaching of mathematics, a suggested list of questions for schools to consider when reviewing the research-based knowledge aforementioned are provided in Table 8.1.

Table 8.1. *Questions for school leadership to consider when forming a vision for effective teaching of mathematics*

Formulating a vision for effective teaching of mathematics: Questions to consider
▪ What are the different elements that will form our vision for effective and equitable teaching of mathematics?
- *What will the nature of mathematics look like in our school? e.g., an approach with a focus on conceptual thinking, cognitively challenging tasks*
- *What instructional tools will we adopt to support putting our vision into action? e.g., number strings, number talks*
- *What student-centred practices for learning will support our vision for effective teaching? e.g., cooperative learning strategies*
- *What are strategies will help us differentiate student learning in heterogenous classrooms? e.g., enabling and extending prompts*
▪ What knowledge bases (research) will inform our vision?
▪ What will our vision of effective mathematics teaching look like in practice?
▪ How will we develop this vision *with* teachers?
▪ How will we communicate this vision to all stakeholders involved in working towards achieving the vision?
▪ How will we continue to foster teacher engagement and support in working towards our vision?

The process of formulating a vision seems equally as important as forming the vision itself. “The literature suggests that a leader cannot dictate a vision, no matter now lofty or appropriate the vision may be. The vision must be truly shared” (Thompson et al., 2004, p. 3).

Ensuring teachers have ownership and there is a “shared sense of purpose” (Timperley et al., 2007, p. 91) seem imperative for a vision to become embedded within the school culture. Forming the vision collaboratively with teachers through a professional learning community, would be one approach to foster and share a visual perception of effective teaching. Scheduling opportunities for continuous professional learning experiences such as establishing professional learning communities; planning regular professional learning experiences; sharing good practice; and seeking external support with working towards the vision, are actions that might support schools with embedding a shared vision of effective practice in mathematics.

Forming a vision for effective and equitable teaching of mathematics seems an important foundational step in preparation for professional learning in mathematics. The findings from this study indicated that when a team developed a shared perspective or vision for teaching and learning mathematics, as was the situation with the third intervention cycle, professional learning time was focused on achieving this goal collaboratively and opportunities for learning enhanced. In addition, it is suggested that schools consider the institutional context in which the vision will be implemented, the aspects which will afford and constrain opportunities for teachers to learn and change practice.

Suggestions for considering the institutional context will be discussed in the next subsection.

8.1.1.2 Considering aspects of the institutional context

As with a review of the literature on professional learning conducted by Goldsmith et al. (2014) and a recent study on professional learning by Wilkie (2019), this study's findings confirmed aspects of the institutional context can significantly influence opportunities for teachers to learn and change practice. Certain aspects were essentially mechanisms for change and enhanced opportunities for teachers to learn, for example: internal structures such as an instructional coach; facilitated collective planning; a culture of professional learning in which accountability was integral; and provision of ongoing innovative professional learning activities. It should be noted that the aforementioned internal structures were not all within in any one institution in this study, but were present across the three schools and were seen to afford learning opportunities and change.

One of the implications of this finding is to consider how to moderate the variability issue within a school, in other words, how to address the issue of significantly different outcomes for individual teachers participating in the same professional learning experiences. As suggested, a crucial first step for schools intending to implement professional learning to achieve their vision for effective and equitable teaching of mathematics, is to carefully consider how institutional aspects may influence (afford or constrain) opportunities for their teachers to learn and improve practice. Recognising such aspects in their own institutional context might allow school leaders to manage these more effectively, or perhaps consider possible changes that could improve their institutional context for teacher learning.

Drawing upon the findings of this study and the literature on teacher professional learning (Arzarello et al., 2014; Clarke & Hollingsworth, 2002; Cobb et al., 2015; Desimone, 2009), I have identified key aspects of the institutional context considered important when planning and implementing professional learning activities. These aspects are presented in Tables 8.2 and 8.3 with suggested questions for schools to consider. Paying attention to a school's internal influences, such as professional requirements and support structures for example, might support the school with being viewed as a learning community by teachers: a fundamental consideration for supporting teacher learning and change (Clarke & Hollingsworth, 2002).

Table 8.2. *Internal influences to consider when planning and implementing professional learning within an institutional context*

Internal aspects		
Key aspect	***Components***	***Questions to consider***
School level administration	Policies – mathematics, learning and teaching, assessment Curriculum programs	*Is our vision for effective and equitable teaching of mathematics clearly communicated in our policies?* *Does our mathematics policy outline our approach to teaching mathematics?* *Is our vision communicated to the wider school community to elicit parental support?*
Professional requirements (internal accountability)	Planning documentation Assessment requirements Professional learning requirements Mathematics instruction requirements	*To what extent do our planning documents support our pedagogical approach to teaching and learning?* *How do our assessment requirements support student learning and align with our pedagogical approach?* *Do we have professional learning requirements, in which accountability is integral, that support teachers in working towards our vision?* *Are our expectations for mathematics instruction consistent with our mathematics policy and vision for effective and equitable teaching?*
Professional support structures	School ethos on professional learning Instructional coaches Colleagues Collective (facilitated) planning Professional learning communities	*Do we have a school ethos on professional learning?* *What internal support structures are in place to support teachers and enhance their opportunities for professional learning?* *What resources do we have available to support professional learning?*
Learning environment	Physical classroom space, classroom resources, IT resources, maths resources, school physical environment	*In what ways do aspects of our current learning environment afford or constrain achieving our vision for effective and equitable teaching and learning of mathematics?*
Student demographics	Student academic assessment data, socioeconomic influences	*Based on our students' learning needs, what are our school's current priorities for teacher and leader professional learning?* *What are our goals for student learning and are they aligned with our vision for effective and equitable teaching of mathematics?* *What do we know about student learning needs that should shape our goals for working towards our vision of effective and equitable teaching of mathematics?*

Two of the internal key aspects listed in Table 8.2., namely professional support structures and school level administration, and some of the associated components will be discussed further in the following subsections.

Table 8.3. *External influences to consider when planning and implementing professional learning within an institutional context*

External aspects		
Key aspect	***Components***	***Questions to consider***
National level	Policies, curriculum documents, assessment requirements, initiatives to raise student standards	*To what extent does our vison for effective and equitable teaching align with current government initiatives?*
State level	Policies, curriculum documents, professional learning requirements Professional support from Department of Education consultants / Catholic Education learning consultants	*To what extent does our vision for effective and equitable teaching of mathematics support current state level initiatives?* *What initiatives do we have in place, and what might we need, to meet state level professional learning requirements for mathematics?* *What external professional support is available to support us in achieving our vision?* *To what extent might external professional support our vision?*

8.1.1.3 Promoting a professional learning culture

In much the same way that teachers are encouraged to initiate a culture of learning within their classrooms (Brown & Coles, 2013; S. Clarke, 2008; 2014), the findings from this study suggest that initiating a professional learning culture within a school is an important foundation to enhance opportunities for collaborative professional learning. At the school in which the third intervention cycle was implemented, there was a sense of teachers being situated within a professional learning culture. There was an ongoing program organised by the numeracy coach to develop teacher knowledge across the primary years. The team had established professional, supportive relationships and embraced opportunities to learn; they emulated a growth mindset in both professional meetings and in their classrooms.

Perhaps an important initial step in this process would be to consider ways in which teachers are given opportunities to develop as learners, to convey the message that the school values the ongoing growth of teachers as professionals. Findings from this study suggest that for changes in knowledge, and/or disposition and practice, it is essential that teachers consider themselves as learners, act as learners i.e., display elements of a growth mindset (Dweck, 2006), and are supported with opportunities to operate within a learning community. Thus, it seems an important implication for schools to consider integrating opportunities for teachers to focus on the value of a growth mindset in relation to their own professional learning and practice. This suggestion could be incorporated into a strategic plan for promoting a professional learning culture. There are readily available resources for teachers to learn about the importance of a growth mindset in relation to both the teaching and learning mathematics. The work of Dweck (2000; 2006; 2010) provides support for teachers in developing their own mindsets, as well as those of their students. The questionnaire and implicit theories scales developed by Dweck (2000) provide a useful tool to encourage teachers to self-reflect on themselves as learners. Teacher online courses available through the YouCubed website (Youcubed, 2017), co-founded by Jo Boaler and Cathy Williams, provide a resource to stimulate thinking and support learning related to some of the key challenges experienced in this project i.e., a focus on multiple strategies to nurture a growth mindset in students.

Another suggestion for promoting a professional learning culture, is to establish professional learning communities as a way to embed continuous professional learning. This seems pertinent when it is considered that professional learning communities are recognised as valuable for enhancing a strong collaborative and supportive culture (Loucks-Horsley et al., 2010). In schools at which interpersonal issues exist or there are dominating team members, this would provide a long-term strategy to help address such situations. Establishing such communities gives reason to establish social norms for the group and could therefore provide a step for moderating behaviours that can otherwise constrain productive professional dialogue (Thompson et al., 2004). This seems particularly important when it is considered that a large number of teachers in this study placed emphasis on the importance of dialogic interactions in supporting their learning.

Establishing professional learning communities could provide an opportunity to enhance professional relationships among teachers, promote a culture in which professional learning is seen as continuous, and make effective use of resources in a way that maximises opportunities for teacher learning. In Australian primary schools, leaders providing time release for teacher collective planning is standard practice. In two of the schools in this study it was facilitated by staff with specialist subject knowledge, and this was found to afford opportunities for teacher learning and change. However, releasing a year level team for facilitated collective planning requires careful coordination and reasonable level of resource input i.e., coordinating specialist subject release time for classroom teachers or the employment of casual relief teachers. Considering the resource implication of supporting facilitated collective planning, it seems prudent to maximise potential opportunities for teacher learning.

Reconceptualising ‘planning meetings’ as a professional learning community gathering (rather than an administrative meeting), might increase the prominence of the school’s vision for professional learning. This may be beneficial for helping to ensure that time and resources are invested in actual professional learning, and not simply for the purpose of administrative task completion or teacher compliance with planning requirements (Cobb et al., 2018). It also opens opportunities for learning beyond the planning process of a sequence of learning activities, for example, to follow-up collaborative analysis of student work samples, found to be a valuable strategy for promoting teacher learning (Kazemi & Franke, 2004; Wilkie, 2016). Such activities may not ordinarily be considered if collaborative professional time is specifically labelled ‘planning’.

A suggested set of questions to guide school leadership in considering the current professional learning culture of their school is presented in Table 8.4. It is intended the questions will stimulate an evaluation of internal structures currently provided to support a professional learning culture, and ways to enhance resources to embed a collaborative professional culture of learning across the school.

Table 8.4. *Questions to guide schools planning to establish a professional learning culture*

Promoting a professional learning culture: Questions to consider
▪ Is our school perceived as a learning community by our teachers?
▪ What support structures are currently in place to promote mutual respect for colleagues as professionals? e.g., professional learning communities, social norms, professional protocol
▪ What structures are in place to support collaborative teacher learning? e.g., professional learning communities, facilitated collective planning
▪ What are our goals and expectations for teachers in relation to professional learning?
▪ What are the essential components of a learning environment to support teachers in achieving their professional learning goals?
▪ Do we have a school vision for teacher professional learning?
▪ What resources do we need to support a school vision for professional learning?
▪ What internal accountability structures might we consider?
▪ How will we communicate our vision for a professional learning culture?

In the previous subsections, 'big picture' suggestions for school leadership in relation to planning and implementing professional learning were outlined. In the following subsections, the implications of this study in relation to the role of the instructional coach in schools are addressed.

8.1.2. Suggestions for school instructional coaches

First, it is perhaps important to clarify the interpretation of the term 'instructional coach' used here. Instructional coaches can be appointed internally within schools, or be an external resource supporting a school with a particular aspect of a professional learning program. In this section, the focus is on providing suggestions for school-based instructional coaches facilitating professional learning.

This study confirmed findings of other studies (Fishman et al., 2003; Loucks-Horsley et al., 2010; Covay Minor et al., 2016), that professional learning experiences need to be flexible and adaptable to meet the individual learning needs of teachers. Although there were teachers in this study who shared the same change sequence, the findings indicated that learning was an idiosyncratic process influenced by different personal factors. It was apparent that each teacher's prior knowledge, experiences, and mindset towards teaching and learning mathematics (Boaler, 2016; Dweck, 2010) influenced their professional learning and which components of the professional learning they considered valuable. Although the study indicated that a range of external stimuli were necessary to meet the learning needs of teachers, there were two stimuli found to exert a critical influence on teacher learning and change: modelled lessons and collective planning. Both were found to enhance various elements of the teachers' social dynamics, such as, brokering and bidirectional interactions (Arzarello et al., 2014). These social dynamics were considered crucial for changes in teacher knowledge, practice, and disposition. These findings have important implications for those involved in designing and implementing professional learning in schools, in particular for instructional coaches who assume a facilitatory role in relation to providing active learning experiences e.g., professional discussions, and collective planning, to support teachers with developing effective instructional practice (Desimone, 2009; Desimone & Pak, 2017). Essentially, instructional coaches need to be mindful of the notion presented in chapter one, that teacher change is influenced by an interplay of learning experiences, personal, and contextual factors (Covay Minor et al., 2016).

A suggested set of questions, drawn from my experiences facilitating teacher interactions and working with school-based coaches throughout this study, is presented in Table 8.5. The questions address the need for flexibility in supporting the individual learning needs of teachers. It is intended the questions will stimulate thinking about how to support individual learning needs of teachers and capitalise on available resources.

Table 8.5. *Questions to guide school instructional coaches with planning and implementing flexible professional learning*

Implementing flexible professional learning: Questions for instructional coaches to consider
▪ What are the individual learning needs of each teacher?
▪ What knowledge and skills do teachers need to enact our vision for effective and equitable teaching of mathematics?
▪ What are the changes in practice needed to achieve our vision?
▪ What strategies might be appropriate to support individual teacher learning needs? i.e., Who would be better supported by observing modelled lessons? Who would benefit more from co-teaching opportunities?
▪ Are there multiple opportunities for teachers to interact and engage in professional dialogue to support their learning?
▪ What resources are available to support learning? i.e., professional reading, guidance documents
▪ Is it possible to support learning through engaging in off-site, external professional development?
▪ Are there external consultants available to support the program and enhance learning?
▪ What internal structures are already place? How could these be improved to support teacher learning?
▪ What resources do we need to support continuous professional learning experiences?

In relation to the mathematics content focus of the interventions, findings from this study suggest the need for the school mathematics policy to outline explicitly the approach and expectations for teaching computation across the primary years. The implication of developing such a document, is perhaps an additional role for the instructional coach, and is the focus of the following section.

8.1.2.1 Developing a Mathematics policy and guidance documents

For two of the teachers participating in the second intervention cycle, it is surmised that a mathematics policy, clearly outlining an approach to teaching mental computation, might have supported them in a shift from the experimentation phase to internalising new praxeologies. For the team leader at this school, who was keen to continue teaching in the ways advocated in the intervention, such documentation would perhaps have assisted her in convincing team members to develop a shared vision for effective teaching of computation. Such documentation may also have offered support for teachers at the first intervention, who indicated that they were unsure about the school stance on computation and how it was being taught in other year levels. Drawing upon the findings of this study, it is suggested that developing a mathematics policy that clearly outlines an approach to teaching computation, might be useful in endorsing a school vision for effective teaching of mathematics.

In addition to a mathematics policy, the findings suggest that providing support documents with information to guide the teaching of computation and development of trajectories for learning computation strategies, would be valuable. The teachers in this study all indicated that they needed to develop aspects of their Mathematical Knowledge for Teaching (MKT) in relation to mental computation strategies, and nearly all commented on the value of having a teacher resource book that provided both key content knowledge and pedagogical considerations for teaching. The teacher resource book designed for this study (by the researcher) provided teachers with a framework to support their designing of hypothetical learning trajectories. Thus, it seems valuable for schools to build on this idea to produce documentation that would outline a progression of computation strategies and guidance on the effective teaching of such strategies. This suggestion may not seem new; over recent years various countries have produced such documents to support implementation of the curriculum, for example, Principles and Standards for School Mathematics (PSSM) from the United States (NCTM, 2000), the Numeracy Development Projects (NDP) resources from New Zealand (Ministry of Education [MOE], 2008), the National Numeracy Strategy (NNS) from the UK (QCA, 1999). In Australia, such resources were developed through a three-year research project (2001-2003) in which the University of Tasmania worked in partnership with the Department of Education, Tasmania, Catholic Education and the Department of Education in the Australian Capital Territory (ACT). However, these resources were developed prior to the current Australian National Curriculum for Mathematics (ACARA, 2014) and the ideas have not been explicitly integrated into the new curriculum documentation.

In the following subsection, the focus shifts from professional learning suggestions for schools to theoretical implications of this study.

8.1.3. Theoretical implications for studies on professional learning processes

One of the key challenges for researchers exploring professional learning to change teaching practice, is the need for theories that explain processes and help support teachers in developing their practice (Cobb et al., 2015; Desimone, 2009). In this study two theoretical models, considered complementary, were combined to form a theoretical framework to examine learning processes of teachers: the

Interconnected Model of Professional Growth (IMPG) (Clarke & Hollingsworth, 2002) (see Figure 3.1) and the Meta-Didactical Transposition (MDT) model (Arzarello et al., 2014) (see Figure 3.2). This combination of models was initially investigated by Wilkie (2019). The IMPG is comprised of four domains, which constitute the personal and professional world of the teacher. The domains are situated in the Change Environment: the context in which teachers work. The MDT model was used as a lens to examine the learning processes within two particular domains of the IMPG, the External Domain and Domain of Practice.

While some changes the teachers experienced could be explained through the mediating processes of reflection and enactment in the IMPG, the data indicated there were additional influences on the change processes within the domains of the IMPG. A component of the professional learning program (External Domain) that seemed a critical influence on learning for most teachers in the study were the modelled lessons. Although this external stimulus led to changes in aspects of knowledge in the Personal Domain for most teachers in the study, it seemed there were other influences on learning within the External Domain, which will be explained next.

The process of brokering the instructional tools during the modelled lessons provided opportunity for teachers to internalise the purpose of the tools and the new pedagogical approach. In addition, many teachers emphasised the importance of 'in-the-moment' professional conversations during the modelled lessons. These conversations were typically bidirectional interactions between the teacher and myself, as we selected examples of student thinking (recorded on mini-whiteboards) to orchestrate whole class productive discussion (Stein et al., 2008). These bidirectional interactions appeared to support teachers with internalising the process of facilitating productive discussion. The examples of brokering and bidirectional interactions aforementioned, illustrate with the MDT model how aspects of social dynamics (Arzarello et al., 2014) were influential on the learning processes within the External Domain. It appeared that it was not simply the opportunity to observe and reflect on the modelled lesson that instigated new learning. Evolving social dynamics that occurred between the teachers and myself, described as brokering and bidirectional interactions, seemed a critical influence on learning.

Another component of the professional learning program that the data indicated as a crucial influence on teacher learning, was the opportunity for facilitated collective planning. During the second and third interventions, I facilitated collective planning each week. It was during these sessions that bidirectional interactions occurred between the teachers and myself, and also among teachers. In some sessions we drew upon each other's knowledge to adapt and co-construct lesson resources; in others we analysed and discussed student learning and the next steps to move forward learning. Discussions about the pedagogical approach and differentiated learning, which involved brokering and a potential shared praxeology, ensued in the second intervention cycle (Arzarello et al., 2014). The interactions that occurred during these meetings, among the teachers and between teachers and myself, seemed compelling influences on their learning processes. Essentially these aspects of social dynamics influenced the learning processes that occurred later in the intervention, within the Domain of Practice.

Although affordances and constraints are conceptualised within the change environment in the IMPG, the model does not explicitly address the influence of aspects of the institution on the change sequences. For example, in the second intervention cycle, the school ethos on professional learning seemed to afford opportunities for the teachers to request and engage in additional professional reading to extend their content knowledge on approaches to teaching mental computation. At each intervention in this study, there were aspects of the institution that influenced changes occurring within sequences for individual teachers (as discussed in detail in Chapters 4, 5 and 6). The use of the MDT model was necessary to analyse the influence of these factors within change sequences.

In Figure 8.1 blue arrows indicate how the MDT model (the entire model) provided a lens to analyse changes specifically within the External Domain and the Domain of Practice from the IMPG. The combination of the two theoretical models depicted in Figure 8.1. provided a framework to explain the learning processes and support mechanisms teachers require to change their practice within their institutional context.

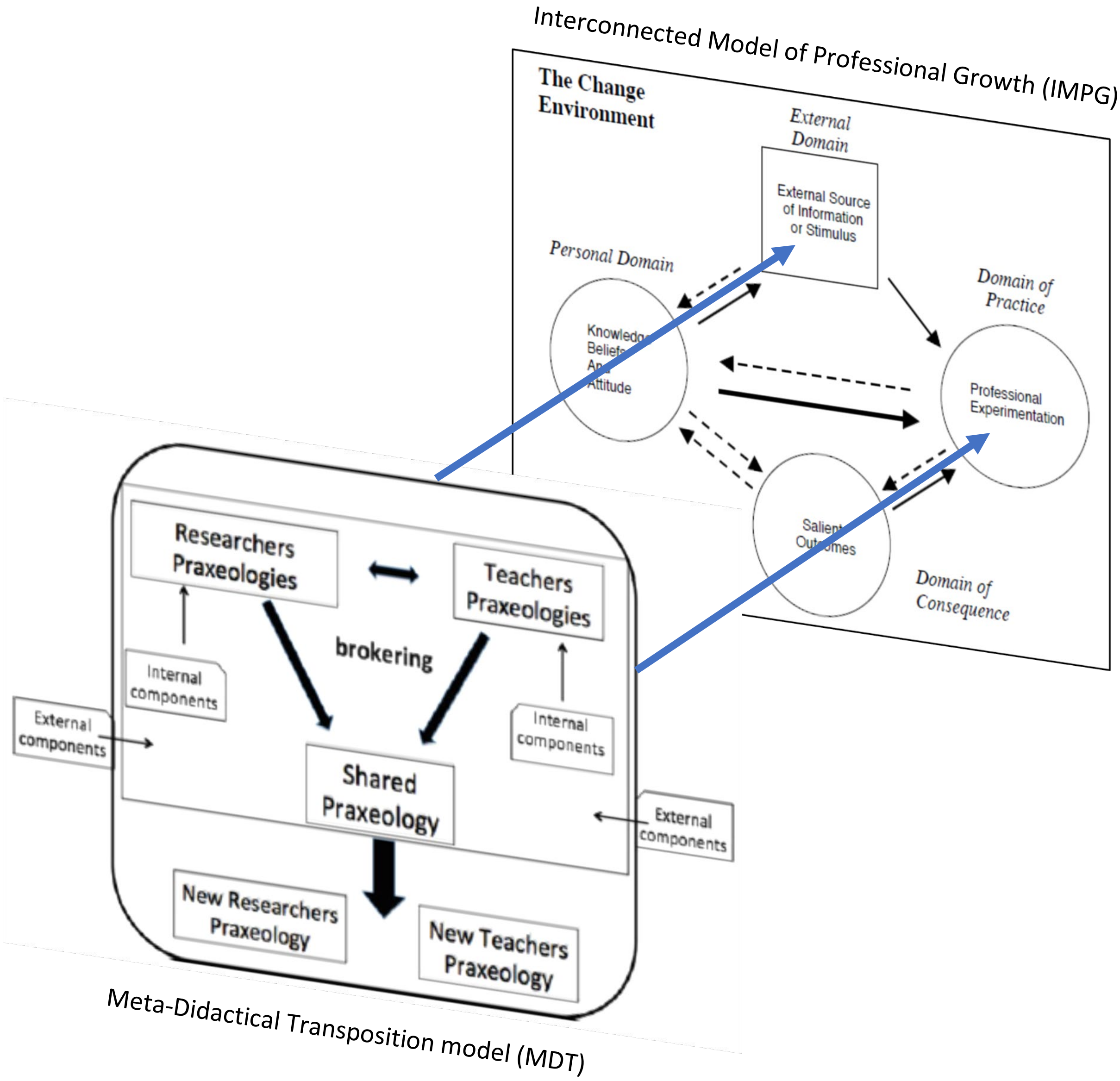

Figure 8.1. Using the MDT as a lens to analyse influences on learning processes within domains of the IMPG.

8.2. Contribution of this study

The findings of this study contribute to the developing empirical and theoretical knowledge base on professional learning of primary teachers in mathematics. Design-based research methodology was adopted to investigate learning processes and ways to foster and support teacher learning and change through an intervention. This orientation of the methodology allowed detailed description and analysis of change pathways and learning processes experienced by each teacher. The study highlighted reasons for the variation issue – why there is variation in individual teachers in relation to what is learned and how new knowledge is transformed into practice. Moreover, the study included explanation of how the institutional context influenced opportunities for teacher learning and a change in practice. The research was conducted on practice, which is associated with local level theory, and makes an original contribution to the development of practice-specific professional learning theory. Acknowledging the small-scale size of the study, it would perhaps be more accurate to describe it as contributing to the 'building blocks' of local theory (McKenney & Reeves, 2012). Nonetheless, the study was in-depth and thus important for fine-grained theoretical analysis. A summary of the key contributions of this study to the research knowledge base on teacher professional learning is provided in Table 8.6.

Table 8.6. *Summary of the contribution of this study to the empirical and theoretical knowledge base on teacher professional learning*

Theory	**Practice**
This study further investigated a theoretical framework (Wilkie, 2019) to examine learning processes and the influence of aspects of the institutional context on learning, by combining two complementary theoretical models: ▪ Interconnected Model of Professional Growth (IMPG) developed by Clarke & Hollingsworth (2002) ▪ Meta-Didactical Transposition model (MDT) developed by Arzarello et al. (2014)	Guidance for school leadership when planning and implementing professional learning; three key considerations were identified and questions formulated to guide schools with implementing each of the following suggestions: ▪ Actively involve teachers in collaboratively forming a vision for effective and equitable teaching of mathematics; ▪ Consider how institutional aspects may afford or constrain opportunities for teachers to learn and improve structures; ▪ Establish a professional learning culture to promote the school as a learning community, enhance opportunities for learning and social dynamics through professional learning communities and include opportunities for teachers to develop as learners. Guidance for school-based instructional coaches on planning and facilitating professional learning experiences, that include consideration for moderating the variability issue: ▪ Plan multiple different strategies, in particular those that enhance elements of social dynamics; ▪ Incorporate flexibility with the program to support individual learning goals of teachers; ▪ Develop school-based mathematics policy and guidance documents to support development of aspects of MKT in relation to content of the professional learning.

8.3. Limitations and directions for future research

This study left some questions unanswered, as well as raised new questions on aspects of teacher professional learning that I had not anticipated. In this section, I critically evaluate the design of the study, consider its limitations, and consider how future research might build on the learning documented in this thesis.

8.3.1. Quality of design-based studies on professional learning

Design-based research was specifically chosen for this study because it provides a systematic way to investigate and support teachers' learning of sophisticated, new instructional practices within the school context, thus making a direct contribution to both theory and practice. However, as a comparatively new methodology in learning sciences, there are some limitations associated with it being recently established (Cobb et al., 2015). One of the main limitations with adopting design-based methodology is the current lack of "explicitly codified standards" to determine the quality of design studies on professional learning, referred to as argumentative grammar (Cobb et al., 2015, p. 497). In more established methodologies, argumentative grammars link the research questions to data, data to analysis, and the analysis to the final claims (Cobb et al., 2015, p. 489). Although this shortcoming can make it potentially difficult to design a study that builds on the findings of previous studies, identifying this issue allows future research to consider this challenge at the design stage.

Many design-based studies in the literature have been considered problematic in focusing on teacher *participation* in professional learning rather than *changes* in their practice (Cobb et al., 2015). When designing this study, a conscious decision was made to incorporate lesson observations as a data collection method to allow for analysis of changes in teacher practice. In addition, attention was also given to documenting the details of the institutional context, in response to criticism of design-based studies providing limited specific details specifically related to the institutional context. In this study details of the institutional context were considered important to analyse potential connections among aspects of the context, teacher learning and changes in practice i.e., institutional aspects that were necessary to both support learning and mediate learning. Having critically evaluated aspects of the design quality, next I discuss some possible directions for future research in connection to the aforementioned issues.

This study investigated change pathways and learning processes of teachers, using a theoretical framework comprised of two models in the literature conceptualising teacher professional learning in mathematics. It highlighted the critical influence of creating opportunities for social dynamics i.e., brokering, bidirectional interactions, and double dialectic, as mechanisms to support and enhance teacher learning and change in practice. It appeared that different aspects of social dynamics and the extent to which they influenced teacher learning for different teachers was affected by personal factors i.e., teacher experience, their learner stance (mindset), aspects of their disposition, as well as aspects of the institutional context, in particular the social context in which they were working. However, it is recognised that this was a small-scale in-depth study; it involved ten teachers and two instructional coaches, across three different school contexts. The focus on change processes, is also a relatively new area for studies on teacher professional learning. With this in mind, it is suggested that the use of the MDT model as a lens to examine the influences on individual learning processes and how change within domains of the IMPG is shaped by aspects within different institutional contexts, would be worthy of further exploration.

Through analysing the change pathways of ten teachers across three institutional contexts and with successive interventions, three noticeable change sequence types transpired. The names of the change sequence types emerged from interpretation of the teachers' stance as learners, and if they acted as learners within the institutional context in which they worked. An additional potential future research direction would be to analyse the learning processes of teachers over a wider range of contexts and across more teachers, to investigate the potential for additional change sequence types.

8.3.2. Equity

It seems important to acknowledge that given the complex nature of studying teacher learning within a school context, it is extremely difficult to document everything that materialises throughout the duration of such a study (Cobb, Confrey, Disessa, Lehrer, & Schauble, 2003). A recognised limitation of this study, and one typically associated with design-based methodology, is the issue of equity in relation to the research participants (Cobb et al., 2015). In this study, monitoring the duration teachers were engaged in activities directly associated with professional learning, presented some challenges. During the second intervention there was also some variation with the direct interaction time I had with individual teachers within the school; this was largely the influence of institutional factors. For example, one teacher's classroom was located close to the staffroom where I based myself when I was not supporting in classrooms. It was common for her to seek me out to engage in professional dialogue and clarify questions or queries prior to teaching or to share and reflect on her classroom experience post-teaching.

Another recognised potential limitation connected to documenting specific details was largely due to variation in the intensity of teacher engagement in the program. It has been suggested this is a significant influence on the outcomes for professional learning (Desimone, 2009) and is therefore an important factor to consider. In this study, there was variation in the time teachers engaged in activities associated with the professional learning program, in which I was not directly involved. It was difficult to discern how much additional time the teachers at both of these schools engaged in activities that could be considered as active learning in my absence. For example, the teachers at School C shared how they spent additional time discussing and teaching the content beyond the sequence of lessons we planned collaboratively. One strategy which may help researchers keep track of useful professional conversations or additional activities, would be for participating teachers to keep a reflective journal or log. Teachers could be given prompts to reflect on and record interactions with colleagues and the focus topic of the professional dialogue, on a daily basis.

8.3.3. Sustainability and pragmatic viability for future studies

This study highlighted the importance of considering the capacity of school staff to support their own teachers' learning i.e., the involvement of a school-based instructional coach, and the influence this might have on the viability and sustainability of an intervention. The importance of considering the capacity and influence of others within the institutional context is a recognised issue associated with design-based methodology, raised by Cobb et al. (2015). This potential limitation is accentuated when the focus of the intervention is on researching instructional practices or pedagogical approaches that are distinctly different from teachers' usual practice and potentially beyond the experiences of the school-based instructional coach, from whom teachers may require additional support during the intervention.

In this study, the two schools who were supported by the involvement of an onsite instructional coach with strong knowledge in teaching mental computation, indicated that the intervention was more viable and potentially sustainable. However, at the first school, the absence of an onsite instructional coach appeared a constraint on opportunities for teachers to learn during the intervention. This indicated that the pragmatic viability and sustainability of the suggested approach to teaching mental computation at this school was questionable because teachers did not have the support of someone onsite with suitable expertise when they encountered challenges or situations they needed to clarify. Thus, an important consideration for those planning professional learning programs, is recognising the need to have the support of an instructional coach with strong subject knowledge related to the particular practice being addressed in an intervention (such as mental computation in this study), alongside some experience of teaching practices referred to as ambitious (Kazemi et al., 2009) for the approach to be viable and sustainable (Cobb et al., 2018).

Recognising the importance of an instructional coach may offer a solution to the sustainability issue, but raises a question in relation to the support instructional coaches require to fulfill their role successfully. This study also brought to attention the idiosyncratic nature of teacher professional learning and the need to have flexibility within a program to meet individual learning needs of teachers. Further investigation into how programs can be adapted to meet individual teaching learning needs, how coaches successfully adapt programs, and the skills needed by coaches, is warranted. With limited research evident in the literature on how to support the learning of instructional coaches and others within a similar capacity, the intention would be to seek insights into the role of instructional coaches in implementing professional learning programs (Cobb et al., 2015).

8.3.4. Duration of this study

For changes in knowledge and pedagogy to occur, it is suggested that teachers need to be engaged in activities associated with professional learning for an adequate time period i.e., number of days, hours or school terms (Desimone, 2009). Yet there remains some dispute regarding specification of the duration of programs for them to be considered effective (Desimone & Pak, 2017). One factor influencing the design of this intervention, specifically the duration of cycles, was the nature of teacher planning in the timetabled program for teaching computation in the participating schools. The planning of mathematics in units to focus on a specific aspect of the curriculum for a duration of 1-3 weeks, meant that the teaching component of the intervention had to be implemented within this timeframe. At the schools in which the second and third cycles of the intervention were conducted, units on additive thinking were taught twice a year. Although it was feasible to revisit mental computation strategies for another phase in the second intervention cycle, this was not possible in the third intervention cycle (being in the second semester). Investigating implementation of a second phase at each school, so that the duration aligns more closely with recommendations in larger studies (Desimone, 2009), would be worthwhile considering in future research.

8.3.5. A professional learning culture and accountability

A situation in this study, which differentiated the second intervention cycle from the others and seems worthy of further investigation, concerns the accountability of teachers to school leadership for conducting their own action research – a factor integral to the professional learning culture at the school. The effect was positive in the sense that it gave teachers an impetus to engage and a reason to deepen their content knowledge (the teachers at School B requested additional research papers on the teaching and learning of mental computation for their action research project). It could be surmised that this accountability encouraged the teachers to persevere with the ideas, perhaps for longer than if they had not been required to report on the findings to the school principal. This raises a question about whether it is valuable for some element of accountability to be an integral part of a professional learning culture at a school; what the potential benefits might be; and the extent to which it is a positive influence. In a broader sense, further examination of elements of a professional learning culture that are essential, or most influential, in relation to teacher learning and change may be fruitful avenues for future research.

8.3.6. Mechanisms of change and creating cognitive dissonance

One of the findings from this study was that for a change in practice, individual teachers seemed to need an experience to instigate a mechanism of change, one that created some level of cognitive dissonance or reconstruction of knowledge. This raised questions related to how to engineer situations to instigate change, to create some level of cognitive dissonance or moment of self-reflection.

For one teacher in the study, her reflections and actions indicated that she held a non-productive view of student learning (Cobb et al., 2018). She did not connect student learning with her choice of instructional practice i.e., that changes to her instructional practice may, over time, have an impact on student learning. This led me to contemplate how to effectively change a teacher's view of student learning, and how a professional learning program could instigate and support such a change. I considered if involving teachers with a non-productive view of student learning, in constructing a shared school vision of effective equitable teaching of mathematics, might be helpful in supporting this change. If such a vision could be developed over time and involve professional learning experiences in which teachers observed the pedagogy being enacted successfully with their students, perhaps, this would instigate a sufficient level of cognitive dissonance for change in practice. Further investigation of working with teachers to create a shared vision for effective and equitable teaching of mathematics, and the learning processes this may or may not instigate, could be worthwhile.

8.3.7. Supporting learning of heterogeneous groups of students

A specific issue with this study was experimentation with instructional practices that participating teachers were confident would support learning of a heterogeneous group of students (Cobb et al., 2015). Although the issue of differentiated learning was considered at the design stage, on reflection I had not anticipated the extent to which establishing a shared understanding of differentiated learning, might influence changes to practice some teachers were prepared to adopt during the second intervention cycle. Certainly, future studies would be prudent to pay attention to issues related to differing opinions of differentiated learning and consider strategies to manage similar situations. In this study, retrospective analysis indicated that strategies to create some dissonance for the teacher, with which this was an issue might have been beneficial, to provide an opportunity for her to reframe her views on equitable instructional practices. Facilitated discussion on this particular issue, or others that emerge, and provision of adequate research-based evidence may perhaps be needed in future research, alongside more explicit modelling of how such instructional practices can be successfully implemented in classrooms. Such suggestions require further investigation.

8.4. Conclusion

> The ability to learn prodigiously from birth to death sets human beings apart from other forms of life…
>
> Whether we are teachers, principals, professors… our primary responsibility is to promote learning in others and ourselves. (Barth, 2002, p. 9)

The aim of this study was to investigate the processes involved in teachers learning sophisticated instructional practices through onsite professional learning experiences: specifically, how teachers learn and develop practice within the institutional context in which they work. The intent was to gain insights into learning processes, and ways to foster and support teacher learning and change. With recent increased attention on improving student achievement in mathematics, and effective teaching practice recognised as a significant factor influencing student learning (Askew et al., 1997; Hiebert & Grouws, 2007), a focus on teacher professional learning seemed critical.

Although each teacher suggested a combination of stimuli instigated and supported their learning, the modelled lessons and facilitated collective planning appeared crucial. These two components of the professional learning enhanced opportunities for brokering, bidirectional interactions and double dialectic, aspects of social dynamics teachers indicated were a critical influence on learning and change (Arzarello et al., 2014).

Fundamental to change seemed whether the teachers evidenced a learner stance and if they acted as learners by self-reflecting on their learning experiences. For teachers whose knowledge or practice was challenged during the intervention, they had opportunities to self-reflect and restructure their knowledge. In some incidences, new ideas extended knowledge and led to modification of knowledge and practice instead.

This study also indicated that certain internal structures within the school context were essential to support individual teacher learning, namely the availability and relevant expertise of an instructional coach. Another institutional factor – a school ethos on professional learning in which accountability measures were integral – was instrumental for teacher engagement. The provision of such internal structures to support professional learning and change, suggested teachers were valued as professionals and the school considered a learning community by the leadership. However, for such structures to be effective in supporting learning and change, teachers needed opportunities to be active learners within a supportive, professional learning community. In addition, it appeared that the extent to which teachers had developed a shared visual perception of effective teaching of mental computation influenced the changes they made to their practice. The findings indicated that when team members held perspectives on teaching and learning that were not aligned with the rest of the team, or the goals of the professional learning program, opportunities for learning and change were constrained.

Implications for those in schools responsible for planning and implementing professional learning for teachers in mathematics include: working towards developing a shared vision for effective and equitable teaching of mathematics; considering the internal and external influences within the school environment at the planning stage; and establishing a professional learning culture in which professional learning communities are integrated. In addition to practical implications were theoretical implications for researchers investigating change processes. The combination of two theoretical models, the IMPG (Clarke & Hollingsworth, 2002) and the MDT model (Arzarello et al., 2014), as a framework to study change processes is worthy of future investigation.

This study highlighted the idiosyncratic nature of teacher professional learning and the complexity with implementing onsite collaborative programs. Yet, the study also found that opportunities for teacher learning and change are enhanced when teachers view themselves as learners; are situated within an innovative, supportive learning community; and are given opportunities to engage in collaborative professional learning experiences. It is particularly important that opportunities for professional learning are attuned to an evolving shared vision for effective and equitable teaching of mathematics within a school.

8.5. Afterword

Loucks-Horsley et al. (2010) describe skilled designers of professional learning as having one foot planted in theory i.e., research knowledge-base, and the other in action i.e., working within the institutional context to plan and implement professional learning. When I commenced this study, I had the experience of having one foot firmly in action, with the toes of the other foot dipped in theory. Through my training and experience as an Aid worker and providing teacher professional learning over several years, I had learned about the importance of building relationships of trust and respect, about adapting to and working within the context, for change in teaching practice to become sustainable. Yet the skills I developed as a facilitator of teacher professional learning, across different capacities as an educator, predominantly evolved through a process of risk-taking and mistake-making: an experimentation with new ideas. While the learning processes of teachers seem analogous to those of students in many ways, the inconsistency I had experienced with learning and change across different classrooms, led me to question what else I needed to learn about adult professional learning. I did not set out with the aim of becoming a skilled designer of professional learning. Rather, I sought to develop my knowledge base of teacher professional learning, and in particular, ways to foster learning and support change processes within school contexts, and to contribute to the evolving field as a researcher. Through developing my knowledge and skills, I have learned much about the deeply complex process of teacher professional learning and the importance of theoretical knowledge in guiding that learning. It is the complexity and challenge, alongside the overarching goal of improving student learning, that upholds the zeal and interest to learn more. For it seems that the world of design research

> … is more art than science. It is fuelled by vision and passion; requires great skill, knowledge and creativity; and continues to evolve… (Loucks-Horsley et al., 2010, p. 21).

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APPENDIX A

Categorisation of mental computation strategies

Table A.1 Overview of the categorisation of mental computation strategies

Table A.1 Overview of the categorisation of mental computation strategies

The strategies presented in this table have been classified into three basic categories: decomposition strategies; sequential strategies; varying strategies, based on a review of the literature for mental computation (addition and subtraction) with 2-digit and 3-digit numbers (Torbeyns, Verschaffel, & Ghesquière, 2006; Torbeyns & Verschaffel, 2013).

Mental computation strategy labels used when teaching and/or analysing student responses with teachers in this study, are nested within each of the three basic categories (displayed in the first table column).

Category	**Links to the literature**		
	Categorisation	*Reference*	Examples given in literature
Sequential strategies			
Strategy label used in this study: **Jump**	Jump (or sequential)	Wright, Ellemor-Collins and Tabor (2012); Torbeyns, Verschaffel and Ghesquière (2006); Torbeyns, De Smedt, Gheesquière and Verschaffel (2009)	
	Sequencing	Thompson (2000)	
	N10	Beishuizen (1993); Beishuizen, Van Putten and Van Mulken (1997); Blöte, Klein and Beishuizen (2000)	
	Break up one number using place value	Harnett (2007)	
	B1, B2	Reys, Reys, Nohda and Emori (1995)	Addition examples for the sum: 79 + 26 Hold one addend constant B1 - First addend (79 + 20 = 99; 99 + 6 = 105) B2 - Second addend (26 + 70 = 96; 96 + 9 = 105)
	Aggregation	Cooper, Heirdsfield and Irons (1996); Heirdsfield and Cooper (2004)	Right to left: 28 + 35:28 + 5 = 33, 33 + 30 = 63 52 - 24: 52 - 4 = 48, 48 – 20 = 28 Left to right: 28 + 30 = 58, 58 + 5 = 63 52 - 24: 52 – 20 = 32, 32 – 4 = 28

Category	Links to the literature		
	Categorisation	*Reference*	Examples given in literature
			Heirdsfield and Cooper (2004) use the terms u-N10 and N10 respectively when referring to right to left; left to right calculations.
	Stepwise	Selter (2001); Heinze, Marschick and Lipowsky (2009); Csíkos (2016)	Calculation examples for 527 + 399; 701 - 698 527 + 300; + 90; + 9 701-600; - 90; -8
Strategy label used in this study: **Bridging through multiples of ten**	Jump to the Decuple	Wright, Ellemor-Collins and Tabor (2012)	Begin from one number, jump to the nearest decuple, jump tens then jump remaining ones.
	A10	Beishuizen, Van Putten and Van Mulken (1997); Blöte, Klein and Beishuizen (2000)	The A refers to adding on to the next round ten.
	Bridging through 10	Thompson (1999)	
Decomposition – *Collections based strategies*			
Strategy label used in this study: **Split strategy**[39]	Split (or decomposition)	Wright, Ellemor-Collins and Tabor (2012); Torbeyns, Verschaffel and Ghesquière (2006); Torbeyns, De Smedt, Gheesquière and Verschaffel (2009); Heinze, Marschick and Lipowsky (2009); Csíkos (2016)	
	Partitioning	Thompson (2000)	
	1010	Beishuizen (1993); Beishuizen, Van Putten and Van Mulken (1997); Blöte, Klein and Beishuizen (2000)	
	Break up two numbers using place value	Hartnett (2007)	
	A1, A2	Reys, Reys, Nohda and Emori (1995)	Addition examples for the sum: 79 + 26 A1 – left to right (tens first) (70 + 20 = 90; 9 + 6 = 15; 90 + 15 = 105)

[39] This strategy was not a teaching focus during the intervention but was discussed when analysing student work samples.

Category	Links to the literature		
	Categorisation	*Reference*	Examples given in literature
			A2 – right to left (ones first) (9 + 6 = 15; 70 + 20 = 90; 15 + 90 = 105)
	Separation	Cooper, Heirdsfield and Irons (1996); Heirdsfield and Cooper (2004)	Examples below from Cooper, Heirdsfield and Irons (1996): *Right to left:* 28 + 35: 8 + 5 = 13, 20 + 30 = 50, 13 + 50 = 63 52 – 24: 12-4=8, 40-20 = 20. 2O + 8 = 28 *Left to right:* 28 + 35: 20 + 30 = 50, 8 + 5 = 13, 50 + 13 = 63 52 – 24: 50 – 20 = 30, 2 – 4= 2 down, 30 - 2=28 Heirdsfield and Cooper (2004) use the terms u-1010 and 1010 respectively when referring to right to left; left to right calculations.
	Hundreds, tens, units (htu)	Selter (2001)	Calculation examples for 527 + 399; 701 - 698 500 + 300; 20 + 90; 7 + 9; 800 + 110; +16 700-600; 0-90; 1-8; 100-90; -7
Strategy label used in this study: **Split-jump**[40]	Split-jump	Wright, Ellemor-Collins and Tabor (2012);	Split tens and ones, add/subtract tens, add first ones, jump second ones.
	A3	Reys, Reys, Nohda and Emori (1995)	Addition example for the sum: 79 + 26 A3 – cumulating sums (70 + 20 = 90; 90 + 9 = 99; 99 + 6 = 105)
	10s (1010-sequential or 1010 stepwise)	Beishuizen, Van Putten and Van Mulken (1997); Blöte, Klein and Beishuizen (2000)	Adaptation of 1010: after splitting off and dealing with the tens, the units are handled in a sequential way.
	Hybrid / mixed method	Thompson (2000)	

[40] This strategy was not a teaching focus during the intervention but was discussed when analysing student work samples.

Category	Links to the literature		
	Categorisation	*Reference*	Examples given in literature
	Cumulative sum or difference	Heirdsfield and Cooper (2004)	
	Htu and stepwise	Selter (2001)	Calculation examples for 527 + 399; 701 – 698 500 + 300; +27; +90; +9 700-600; +1; -90; -8
Varying strategies			
Strategy label used in this study: **Compensation[41]:** *adjust one number and compensate*	N10C	Blöte, Klein and Beishuizen (2000)	
	C1, C2	Reys, Reys, Nohda and Emori (1995)	Addition examples for the sum: 79 + 26 C1 – First addend (80 + 26 = 106; 106 – 1 = 105) C2 – Second addend (79 + 30 = 109; 109 – 4 = 105)
	Wholistic - compensation	Cooper, Heirdsfield and Irons (1996); Heirdsfield and Cooper 2004)	Examples below from Cooper, Heirdsfield & Irons (1996): 28 + 35: (28 + 2) + 35 = 30 + 35 = 65, 65 – 2 = 63 52 - 24: 52 - (24 + 6) = 52 – 30 = 22, 22 + 6 = 28 Heirdsfield and Cooper (2004) use the reference N10C to identify compensation.
	Over-jump	Wright, Ellemor-Collins, Tabor (2012);	Begin from one number, over shoot the jump then compensate.
	Compensation	Thompson (1999; 2000); Torbeyns, Verschaffel & Ghesquière (2006); Torbeyns, De Smedt, Gheesquière, Verschaffel (2009); Wright, Ellemor-Collins, Tabor (2012); Heinze, Marschick & Lipowsky (2009)	
	Adjust and compensate	Hartnett (2007)	
	Auxiliary task	Selter (2001)	Calculation examples for 527 + 399; 701 - 698 *527 + 400 - 1*

41 This strategy was simply referred to as compensation strategy.

Category	Links to the literature		
	Categorisation	*Reference*	Examples given in literature
			701-700; +2
Strategy label used in this study: **Compensation:** *adjust two numbers and compensate*	C3, D1	Reys, Reys, Nohda & Emori (1995);	Addition examples for the sum: 79 + 26 C3 – Both addends (80 + 30 = 110; 110 - 1 – 4 = 105) D. Round both addends to multiple of five, then adjust D1 (75 + 25 = 100; 100 + 4 + 1 = 105)
	Adjust two numbers and compensate	Hartnett (2007)	
	Compensation	Wright, Ellemor-Collins, Tabor (2012)	
Strategy label used in this study: **Transformation**	Wholistic - levelling	Cooper, Heirdsfield & Irons (1996); Heirdsfield & Cooper (2004)	Examples below from Cooper, Heirdsfield & Irons (1996): 28 + 35: 30 + 33 = 63 52 - 24: 58 – 30 = 28 Heirdsfield and Cooper (2004) describe levelling as adjusting both numbers e.g. calculating 28 + 35 as 30 + 33
	Transformation	Wright, Ellemor-Collins, Tabor (2012);	Change both numbers while preserving the result, then add or subtract.
	Adjust & compensate – adjust two numbers.	Hartnett (2007)	
	Balancing	Thompson (1999)	Transforming to retain equivalence e.g. 7 + 9 is the same as 6 + 10
	Simplifying	Selter (2001); Heinze, Marschick & Lipowsky (2009); Csíkos (2016)	526 + 400 700 - 697
Strategy label used in this study:	Complementary addition	Thompson (2000); Wright, Ellemor-Collins, Tabor (2012)	Finding the difference when the numbers are close together.

Category	Links to the literature		
	Categorisation	*Reference*	Examples given in literature
Indirect addition	Short jump	Blöte, Klein, Beishuizen (2000); Torbeyns, Verschaffel & Ghesquière (2006)	Blöte, Klein, & Beishuizen (2000) refer to bridging the difference in subtraction problems like 71-69 in one or two steps. Torbeyns, Verschaffel, & Ghesquière (2006) use the terms short jump or complementary addition to describe finding the difference when the numbers are close together.
	Indirect addition	Torbeyns, De Smedt, Gheesquière, & Verschaffel (2009); Heinze, Marschick, & Lipowsky (2009); Csíkos (2016)	Used for subtractions when there is a small difference between the integers
	Adding up	Selter (2001)	Calculation example for: 701 - 698 698 + □ = 701
	Count on to subtract	Hartnett (2007)	

Note:

Hartnett (2007) and Thompson (1999) both refer *near doubles* as a strategy that could be categorised under the main heading of *varying strategies*; this has not been included in the table above because of limited reference to the strategy in the literature reviewed.

Appendix A - References

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APPENDIX B

Resources designed to support teacher professional learning

Teacher resource book	*Sample of a learning sequence (3 lessons) from the book*
Lesson resources	*Sample of PowerPoint slides for a modelled lesson*
Lesson plan	*Sample of a plan used for a modelled lesson*
Student strategies	*Sample of student strategies elicited in whole class discussion*
Professional learning session PowerPoint	*Example of slideshow used in the session at School C*

MENTAL STRATEGY FOCUS: Jump strategy - addition

RATIONALE FOR THE SEQUENCE OF LESSONS

The focus of this sequence of lessons is on finding the sum of two numbers mentally using a sequential strategy, commonly referred to as the jump strategy (Wright, Ellemor-Collins, & Tabor, 2012; Torbeysn, Verschaffel, & Ghesquière, 2006; Torbeyns, De Smedt, Ghesquière, & Verschaffel, 2009). Although the jump strategy is initially more challenging for students to learn, it is less prone to errors with calculations that require regrouping and is therefore particularly useful when the calculation involves regrouping (Wright et al., 2012). The jump strategy is generally more efficient than the split, or partitioning strategy, because it only requires breaking apart one number in the calculation.

REFERENCE TO THE AUSTRALIAN CURRICULUM

This sequence of lessons addresses the following content descriptors from the Australian curriculum:

> Apply place value to partition, rearrange and regroup numbers to at least 10 000 to assist calculations and solve problems (ACMNA053)
>
> Recall addition facts for single-digit numbers and related subtraction facts to develop increasingly efficient mental strategies for computation (ACMNA055)

The lessons address the following proficiency strands from the Australian curriculum:

> *Understanding:* It is intended that students will develop understanding of number relationships
>
> *Reasoning:* It is intended that students will develop reasoning by explaining strategies
>
> *Fluency:* It is intended that students will become more fluent with mental calculations

PRIOR LEARNING

This sequence of lessons builds on content descriptors from Year 2. When finding the sum of numbers mentally using the jump strategy, students need to be secure in and use the following foundational knowledge:

- Partition numbers using place value
- Add multiples of ten to any number (conceptual place value) e.g. 54 + 30
- Combinations (addition facts) with numbers up to 10 e.g. 8 = 5 + 3, 8 = 4 + 4, 8 = 6 + 2

Short focused activities to develop fluency in some of the key foundational knowledge listed above, could be incorporated into the ACTIVATE part of the lesson.

Year Level: 3

LESSON 1: Finding the sum
MENTAL STRATEGY FOCUS: Jump strategy
INSTRUCTIONAL ACTIVITY: Number Strings

LEARNING FOCUS:

The jump strategy involves keeping one addend whole and partitioning the second addend into parts to make the calculation easier to do mentally.

addend + addend = sum

Adding from left to right (adding the hundreds first) is useful in maintaining the quantities of the numbers and the place value of the digits. When students visualise the traditional algorithm the place value can get lost and there is greater tendency for errors (Wright et al., 2012). The jump strategy is less prone to error than using the partitioning strategy (splitting both numbers in accordance with place value) and more efficient because it only requires splitting one addend.

In this lesson, students should also develop an understanding of the empty number line (ENL) as a mathematical model.

KEY MATHEMATICAL VOCABULARY:

partition, split, jump, count on, add, sum, place value, efficient

LAUNCH *the learning*[42]:

45 + 20
45 + 24
53 + 30
53 + 36
64 + 27

How the number string was designed:
The intention is for students to keep the first addend whole and partition the second addend into chunks that make the addition easier to do mentally. The first question in the string is a warm-up question. The second question is paired to the first and is just four more, so could be represented visually on the same empty number line (ENL). The third and fourth questions are paired, the third question is the support question for the fourth question. The visual representations for the third and fourth question could be shown on the same ENL. The fifth question is presented as a challenge; it involves regrouping ones and does not have any prior questions supporting it so students need to think how they can make the calculation easier to do mentally.

[42] Note the design of this number string has been adapted from the work of Fosnot and Uittenbogaard (2007).

EXPLORE *mental strategies*[43]:

35 + 30
35 + 33
54 + 40
54 + 45
67 + 24

How the number string was designed:
The number string for the EXPLORE phase is a slight variation of the string presented in the LAUNCH phase of the lesson. The first question in the string is a warm-up question. The second question is three more than the first question so could be represented visually on the same empty number line. The third and fourth questions are paired; the fourth question is five more than the third question. The fifth question is presented as a challenge; it involves regrouping of ones and does not have any prior questions supporting it so students need to think how they can make the calculation easier to do mentally.

PEDAGOGICAL CONSIDERATIONS:

- The final question in both strings involve regrouping ones. If students are computing mentally by partitioning both addends, or visualising the algorithm, a computation error is common at this stage in the calculation. For students choosing to partition both addends to calculate the answer, it should be highlighted that this strategy is less efficient as it involves more steps.
- Initially young children tend to approach addition working from left to right (adding the tens first), it is only after the written algorithm has been introduced that this approach is generally cast side by students (Kamii, Lewis, & Jones, 199). For students who are visualising the written algorithm it is worth emphasising that mental strategies and written algorithms are two different approaches for computation; it is not the case that one approach is more superior to the other. The numbers in the question should determine which approach, mental or written, is more appropriate. Visualising a written algorithm for a question in which the numbers are appropriate for working with mentally is generally less efficient and more prone to error.
- Using an Empty Number Line (ENL) supports the idea of adding sequentially. Eventually students should be able to record their thinking without the ENL model. Refer to the section below on possible student strategies for examples of displaying student thinking using an ENL.
- The design of the number string is based on variation theory, which means that one thing has to stay the same (invariant) and one thing changes (variant) to allow students to observe a pattern in the questions. In the examples given, the invariant is the partitioning of the second addend based on place value; the variant is the first addend.
- When posing further questions to encourage students to use the jump strategy, it is better to devise questions in which the addends are not close to a multiple of 10.

EXPLAINING THE LESSON TO STUDENTS:

[43] Note the design of this number string has been adapted from the work of Fosnot and Uittenbogaard (2007).

There are many different ways to find the sum of two numbers. You are going to work out efficient strategies for finding the sum of two numbers mentally (in your head). You will practise explaining your thinking to others.

When sharing the learning intention with students this could be recorded as:

L.I. Use mental strategies to calculate the sum of two numbers

The following are suggestions for generating success criteria with the students:

- ✓ I have partitioned the numbers efficiently (split the numbers the least number of times)
- ✓ I have used a visual representation and/or mental jottings to show my thinking
- ✓ I have checked that my calculation makes sense (I have explained it visually to a partner)
- ✓ I have checked that the answer is correct (I have compared and discussed with a partner)

ENABLING PROMPT(S):

- Practise questions that build on adding multiples of ten to a number, progressing to adding in chunks (tens and ones) with 2-digit numbers initially. In the table below, are examples of strings that get progressively more challenging (reading the table from left to right):

No regrouping of ones	Regrouping ones
42 + 30	55 + 30
42 + 33	55 + 38
42 + 40	47 + 40
42 + 45	47 + 44

EXTENDING PROMPT(S):

- Extend by presenting questions that involve regrouping tens and ones. For example:

Regrouping tens and ones
66 + 50
66 + 57
78 + 50
78 + 55

- Can you think of another way to show your thinking visually?

CONSOLIDATING TASK:

The consolidating task is a set of similar questions which can be completed on the worksheet provided or in a work book. The worksheet format has been designed to allow students to show their thinking.

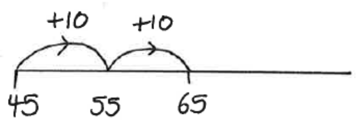
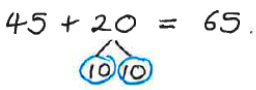
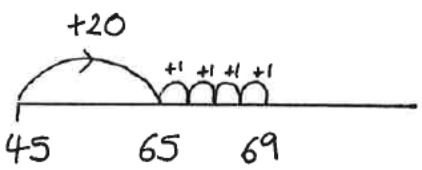
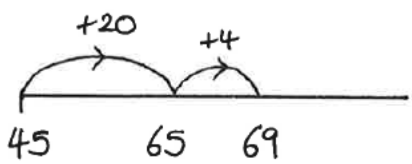
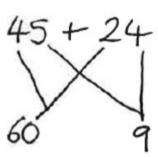
Below are two suggestions for number strings that could be used to consolidate student learning, depending on student responses to tasks in the previous parts of the lesson.

56 + 20	62 + 30
56 + 23	62 + 34
42 + 40	46 + 20

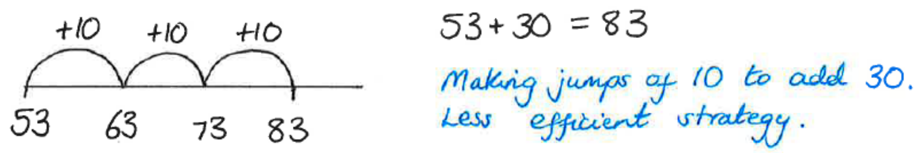
42 + 47	46 + 25
45 + 36	54 + 37

POSSIBLE STUDENT RESPONSES:

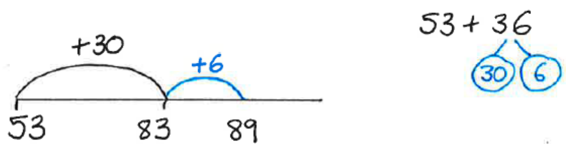
- Some students may choose to partition both numbers, particularly for calculations that do not involve regrouping ones. In this situation, it is important to discuss the efficiency of the calculation.
- In questions that involve regrouping ones, some students may partition the ones in the second addend to bridge through a multiple of ten to make the calculation easier. As students develop fluency, this step is eliminated thus reducing the number of steps and making the calculation more efficient.

LAUNCH	Possible Visual Recordings
45 + 20 45 + 24	  Showing thinking visually on the same ENL for the paired questions will support students making connections between the questions.   More efficient adding of ones 45 + 24 = 69 40 + 20 + 4 = 69  60 + 9 = 69 Partitioning strategy. Less efficient than jump. Some students may choose to partition both numbers, although this is not the intention for the lesson.

53 + 30

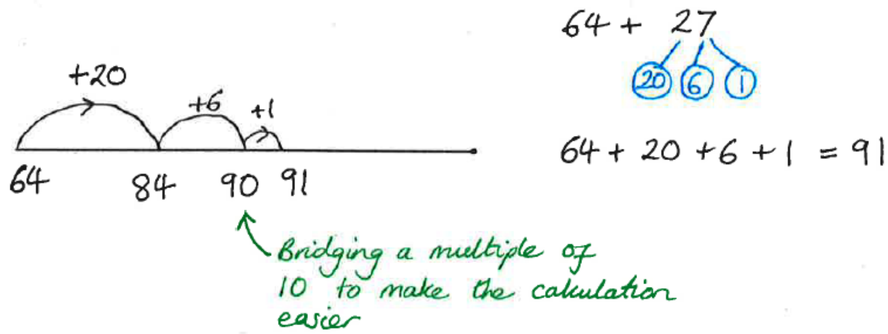


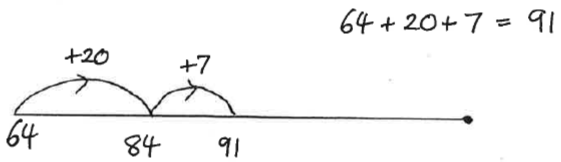
53 + 36



If one efficient jump of 30 is made, the addition of the ones could be shown on the same ENL.

64 + 27





FINDING WAYS TO ADD IN YOUR HEAD

Each line in the table is for a question. First, work out the answer in your head and then show how you did it.

	Question	Answer	Show how you worked out the answer in your head
1			
2			
3			
4			
5			
6			

Can you find another way to work out the answer mentally? Can you explain it visually?

1) Look at the final question. Can you find another way to work out the answer mentally? Can you explain it visually? Show your thinking below.

2) Draw a ☺ next to the more efficient method (choose between the first way you worked it out and the way shown below). Can you explain why this method is more efficient?

3) Which method do find easiest to use to find the sum of the numbers? Explain why it is easier for you.

LESSON 2: Finding the sum
MENTAL STRATEGY FOCUS: Jump strategy
INSTRUCTIONAL ACTIVITY: Number Talk

LEARNING FOCUS:

The jump strategy involves keeping one addend whole and partitioning the second addend into parts to make the calculation easier to do mentally.

addend + addend = sum

Adding from left to right (adding the hundreds first) is useful in maintaining the quantities of the numbers and the place value of the digits. When students visualise the traditional algorithm, or compute using single digits, the place value can get lost and there is greater tendency for errors (Wright, Ellemor-Collins, & Tabor, 2012). The jump strategy is less prone to error than using the partitioning strategy (splitting both numbers in accordance with place value) and more efficient because it only requires splitting one addend.

In this lesson, students should also develop an understanding of the empty number line (ENL) as a mathematical model.

KEY MATHEMATICAL VOCABULARY:

partition, split, jump, count on, add, sum, place value, efficient

LAUNCH *the learning*:

FINDING WAYS TO ADD IN YOUR HEAD

Work out how to find the sum of these numbers in your head:

45 + 37

Can you explain your thinking visually?

PEDAGOGICAL CONSIDERATIONS:

- When posing further questions to encourage students to use the jump strategy, it is better to devise questions in which the addends are not close to a multiple of 10.
- Using an Empty Number Line (ENL) supports the idea of adding sequentially. Eventually students should be able to record their thinking without the ENL model. Refer to the section below on Possible Student Responses, for examples of ways to display student thinking using an ENL.

EXPLAINING THE LESSON TO STUDENTS:

There are many different ways to find the sum of two numbers. You are going to investigate the strategies we have discussed as a class to work out efficient strategies to find the sum of two numbers mentally (in your head). You will practise explaining your thinking to others.

When sharing the learning intention with students this could be recorded as:

L.I. Use mental strategies to calculate the sum of two numbers

ENABLING PROMPT(S):

- Practise questions that do not involve regrouping of ones initially. For example:
  42 + 33
  45 + 22

EXTENDING PROMPT(S):

- If you had to teach another Year 3 student how to use the jump strategy to find the sum of two numbers, what would you tell them? Use an example of a question to help explain your thinking and record your ideas as a list of Top Tips. Use drawings or mental jottings to explain your ideas visually.
- Extend by presenting questions that involve regrouping tens and ones when adding 2-digit numbers. For example: Work out 75 + 46 mentally

CONSOLIDATING TASK:

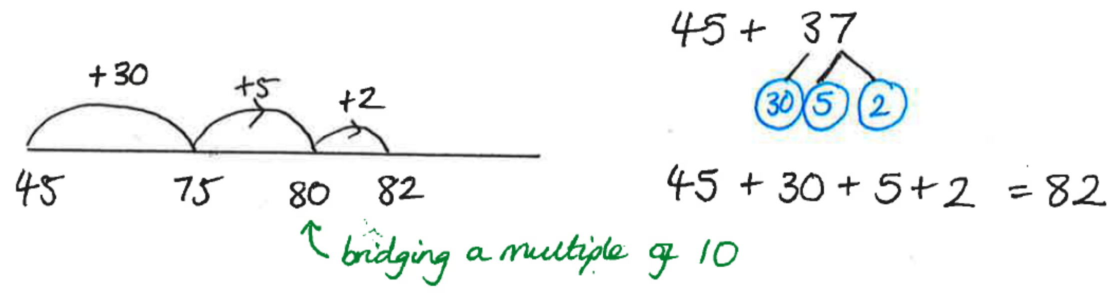
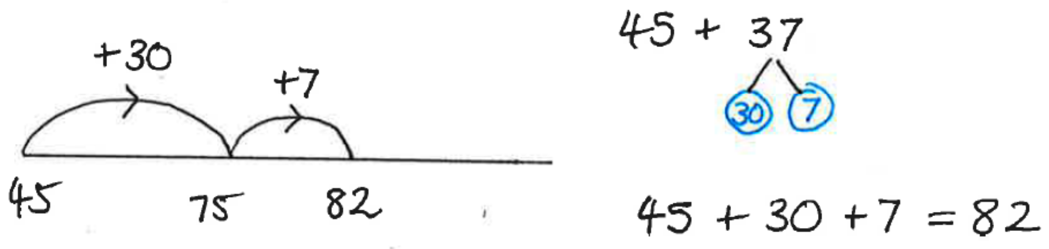
The consolidating task is a set of similar questions which can be completed on the worksheet provided or in a work book. The worksheet format has been designed to allow students to show their thinking.

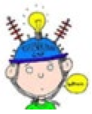
The table below contains examples of questions that are increasingly challenging, reading the table from left to right (more stars indicate more challenge). The class could be presented with a set of questions from the table below to investigate, or be given the option to select questions that are challenging for them to compute mentally.

★	★★	★★★
47 + 22	37 + 24	77 + 25
36 + 42	54 + 37	86 + 25
66 + 33	46 + 25	66 + 45
45 + 34	64 + 36	57 + 46
72 + 26	45 + 36	74 + 37
54 + 45	63 + 28	83 + 28

POSSIBLE STUDENT RESPONSES:

- Some students may partition the ones in the second addend to bridge through a multiple of ten to make the calculation easier. As students develop fluency, this step is usually eliminated thus making the calculation more efficient.
- Some students may choose to partition both numbers, particularly for calculations that do not involve regrouping ones. In this situation, it is important to discuss the efficiency of the calculation.

LAUNCH 45 + 37	Possible Visual Recordings
	
	



FINDING WAYS TO ADD IN YOUR HEAD

Each line in the table is for a question. First, work out the answer in your head and then show how you did it.

	Question	Answer	Show how you worked out the answer in your head
1			
2			
3			
4			
5			
6			

If you had to teach another Year 3 student how to use the jump strategy to find the sum of two numbers, what would you tell them?

Use an example of a question to help explain your thinking. You can make up the question or use one of the questions on this recording sheet. Record your ideas as a list of Top Tips. Include mental jottings to explain your ideas visually.

LESSON 3: Finding the sum
MENTAL STRATEGY FOCUS: Jump strategy
INSTRUCTIONAL ACTIVITY: Number Talk in context

LEARNING FOCUS:

The jump strategy involves keeping one addend whole and partitioning the second addend into parts to make the calculation easier to do mentally.

addend + addend = sum

Adding from left to right (adding the hundreds first) is useful in maintaining the quantities of the numbers and the place value of the digits. When students visualise the traditional algorithm, or compute using single digits, the place value can get lost and there is greater tendency for errors (Wright, Ellemor-Collins, & Tabor, 2012). The jump strategy is less prone to error than using the partitioning strategy (splitting both numbers in accordance with place value) and more efficient because it only requires splitting one addend.

In this lesson, students should also develop an understanding of the empty number line (ENL) as a mathematical model.

KEY MATHEMATICAL VOCABULARY:

partition, split, jump, count on, add, sum, place value, efficient

LAUNCH *the learning*:

> How many tickets?
>
> Sea View Primary hosted a Family Fun Day to raise money for new outdoor play equipment. If 47 adult tickets and 55 child tickets were sold for the Family Fun Day, how many people attended the event?
>
> Work out the answer in your head.
> Can you explain your thinking visually?

PEDAGOGICAL CONSIDERATIONS:

- The whole class discussion should include the commutative property of addition and how it is easier to change the order of the addends to that presented in the problem context i.e. make the number of child tickets the first addend.
- Using an Empty Number Line (ENL) supports the idea of adding sequentially. Eventually students should be able to record their thinking without the ENL model. Refer to the section below on possible student strategies for examples of displaying student thinking using an ENL.

- When posing further questions to encourage students to use the jump strategy, it is better to devise questions in which the addends are not close to a multiple of 10.

EXPLAINING THE LESSON TO STUDENTS:

There are many different ways to find the sum of two numbers. You are going to work out efficient strategies to find the sum of two numbers mentally (in your head). You will practise explaining your thinking to others.

When sharing the learning intention with students this could be recorded as:

L.I. Use mental strategies to calculate the sum of two numbers

ENABLING PROMPT(S):

- Practise questions that do not involve regrouping of ones initially. For example:
  42 + 33
  45 + 22

EXTENDING PROMPT(S):

- Look at the final question you worked out the answer to in your head. Can you think of other ways to work out the answer mentally? Record your thinking visually. Explain which method you think is more efficient.
- Extend by presenting questions that involve regrouping tens and ones when adding 2-digit numbers. For example: Work out 75 + 46 mentally

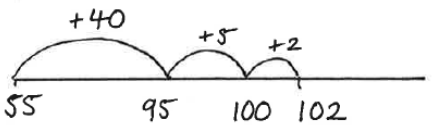
CONSOLIDATING TASK:

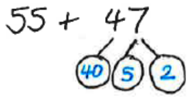
The consolidating task is a set of similar questions, which can be completed on the worksheet provided or in a work book. The worksheet format has been designed to allow students to show their thinking.

POSSIBLE STUDENT RESPONSES:

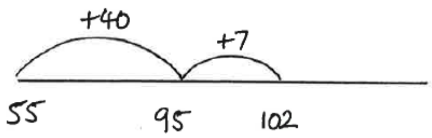
- Some students may partition the ones in the second addend to bridge through a multiple of ten to make the calculation easier. As students develop fluency, this step is usually eliminated thus making the calculation more efficient.
- Some students may choose to partition both numbers, particularly for calculations that do not involve regrouping ones. In this situation, it is important to discuss the efficiency of the calculation.

LAUNCH 55 + 47	Possible Visual Recordings



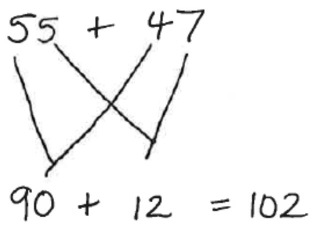


55 + 40 + 5 + 2 = 102



A more efficient way to partition the second addend

55 + 40 + 7 = 102



50 + 40 = 90
5 + 7 = 12
90 + 12 = 102

Some students may choose to partition both numbers. This method involves more steps and is less efficient that the methods shown above.

HOW MANY TICKETS?

Each line in the table is for a question. First, work out the answer in your head and then show how you did it.

	Number of adult tickets	Number of child tickets	How many tickets sold?	Show how you worked out the answer in your head
1	26	35		
2	37	66		
3	25	47		
4	26	55		
5	57	45		
6	74	38		

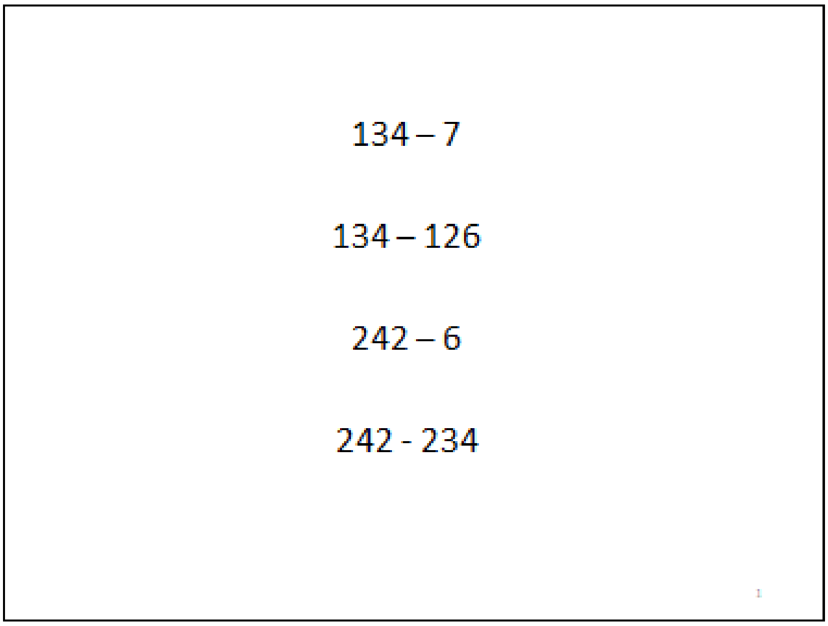
Look at the final question you worked out the answer to in your head. Can you think of other ways to work out the answer mentally?

Record your thinking visually. Explain which method you think is more efficient.

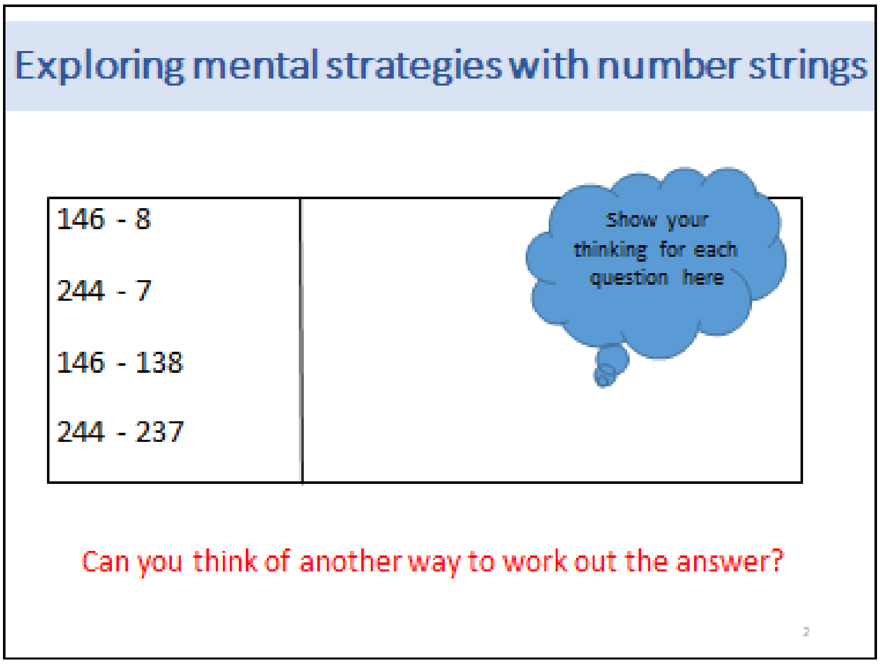
SAMPLE OF LESSON SLIDESHOW: MODELLED LESSON AT SCHOOL C

Strategy focus: Indirect addition

Slide 1



Slide 2



Slide 3

Using mental strategies to find the difference

Mild	Spicy	Hot
64 – 8	221 – 7	1001 – 997
64 – 58	221 – 197	1001 – 7
93 – 6	255 – 248	2003 – 998
93 – 86	255 – 15	2003 – 9
101 - 6	334 – 327	2007 - 1996
101 - 96	334 - 16	2007 - 15

Design a number string to help another Year 3 student find the difference between two numbers mentally

Note:
The design of the number strings on these slides have been adapted from the work of Fosnot and Uittenbogaard (2007).

SAMPLE OF LESSON PLAN: FOR THE MODELLED LESSON AT SCHOOL C

		Key Questions & Maths Language	Anticipated responses & Teaching Points
ACTIVATE **5 -10 mins**	Activity: Numbers rule Display a list of numbers. Decide on a rule e.g. pairs to 100; add/subtract a multiple of 10). The class must say the modified numbers on the list. The list can be read starting top or bottom.	• How many more to make…100? number bonds friendly numbers	Looking for patterns with pairs of numbers that make 10, 100, 1000 (friendly numbers) are important for making efficient jumps with this strategy. Students should be able to use knowledge of number bonds to 10 or 20 to derive pairs that make 100. **Explaining ACTIVATE**: We are going to warm up our brains by practising recall of number bonds. This knowledge may be useful for developing some mental strategies later in the lesson.
LAUNCH **Approx. 10 - 15 mins**	**Explaining the lesson:** *There are many different ways to solve subtraction problems. We are going to find efficient strategies for solving problems mentally (in your head).* Present each question visually, one at a time. 134 – 7 134 - 126 242 - 6 242 - 234 Individual thinking time. Use thumbs up to indicate when ready with an answer. Students to share thinking with the class as work through the number string. Display student strategies on the board using ENL and notation. Record class observations about the strategies on board. Explain that as mathematicians we need to further explore to prove if our generalisations are correct and to see if we can refine (improve) our initial ideas.	• How did you decide your first move? • Did anyone use the same strategy but use different jumps/steps? • Which strategy do you think is most efficient? • What makes this strategy more efficient? • How can you prove your answer for this question is correct? Can you explain it visually? • What do you notice is the same/different with the strategies we have used to work out the difference? difference / partition / bridge / friendly numbers / count up / add up / count back / efficient / ENL /	Some students may count back in ones rather than bridging through a multiple of 10. Model both strategies on same number line and discuss efficiency of the two approaches. Generalisations: when numbers are close together it is usually easier to count up/add up; when the numbers are far apart it is easier to count back. Enabling prompts may be needed so lower achieving students can access the questions. Extending prompt: Can you think of another way to work out the answer mentally?

		Key Questions & Maths Language	Anticipated responses & Teaching Points
		generalisations	
EXPLORE **Approx. 10-15 mins**	Present a string of calculations for students to work out the difference mentally. Students to record answer first, then show working out and visual representations to prove the answer is correct. 146 – 8 244 – 7 146 – 138 244 - 237 Check answers and compare working out with partner. Pairs to discuss if the generalisations from the LAUNCH phase are true and if the initial ideas can be refined. Students to think about the visual representations they will use to explain and justify their thinking in the SUMMARY.	• Do you notice any similarities/differences between your mental strategies? • How can you prove your answer for this question is correct? Can you explain it visually? • Has your partner got a more efficient strategy which you could test on future calculations? difference / partition / bridge / friendly numbers / count up / add up / count back / efficient / ENL / generalisations	See suggested enabling and extending prompts at the end of the document. Circulate to carefully select students who will be asked to explain strategies in the SUMMARY phase of the lesson.
SUMMARY **Approx. 15 mins**	Students share examples of strategies for questions from the EXPLORE phase. Record on board visually – use ENL and notation. Compare strategies for efficiency – focus on use of bridging and bigger jumps for efficient strategies. Share and explain sorting of calculations into two groups: count back or count up / add up? Record and sort calculations in a T-chart on board. Revisit generalisations: When we are finding the difference between two numbers mentally, why is it helpful to look carefully at the numbers in the question?	• Why does that strategy work? • What makes this strategy more efficient? • Did you make any interesting mistakes? • How do you know when to count up or count back? difference / partition / bridge / friendly numbers / efficient	It may be useful to use the part/whole model to explain the relationship between addition and subtraction – how the parts are related to the whole. whole part | part 146 8 | ? Discuss how the strategy depends on the numbers in the question; if the numbers are close together then it's easier to count up/add up; when the numbers are far apart and it's usually easier to count back. Efficient calculations have less jumps; bridging through a multiple of ten or hundred makes the calculation easier and more efficient.

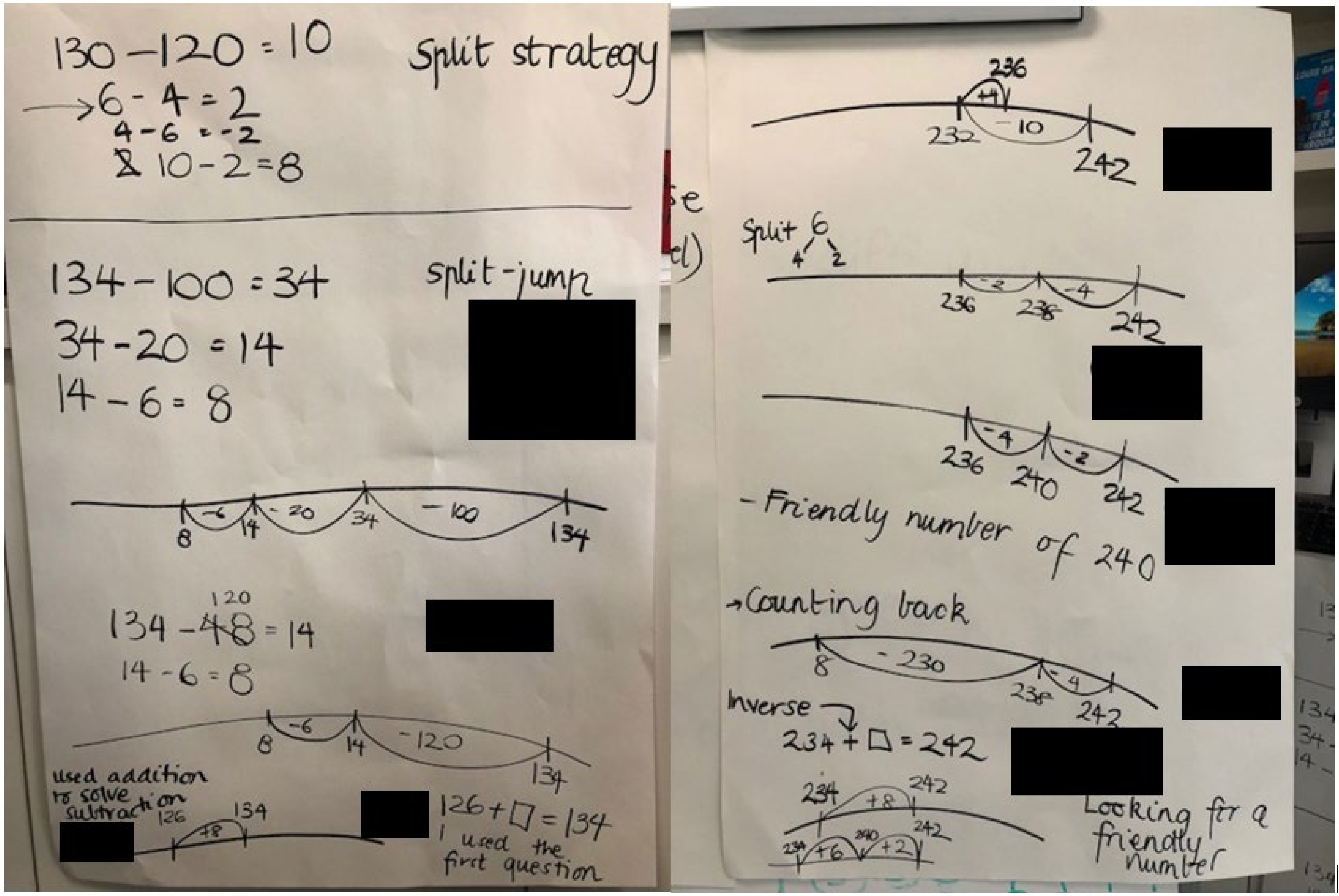
		Key Questions & Maths Language	Anticipated responses & Teaching Points
CONSOLIDATE **Approx. 15-20 mins**	Suggestions for possible number strings to consolidate student learning: ***Option 1*** / ***Option 2*** / ***Option 3*** 64 - 8 / 221 - 7 / 1001 – 997 64 - 58 / 221 – 197 / 1001 – 7 93 - 6 / 255 – 248 / 2003 – 998 93 - 86 / 255 – 15 / 2003 – 9 101 - 6 / 334 - 327 / 2007 – 1996 101 - 96 / 334 - 16 / 2007 - 15		Recording the learning intention: LI: Use mental strategies to calculate the difference between two numbers

Suggestions for enabling prompts:
Practise some questions that start with the whole as a multiple of 10 or 100 and the subtrahend close to a multiple of 10

50 - 8	100 - 7
50 - 4	100 - 4
50 - 48	100 - 97
50 - 38	100 – 77
50 - 34	100 – 84

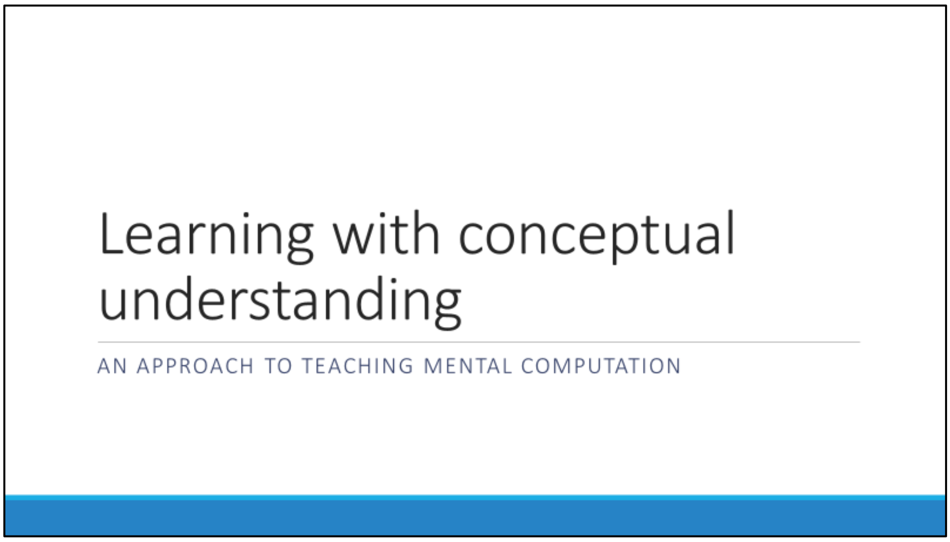
Note: The design of the number strings in this lesson plan have been adapted from the work of Fosnot and Uittenbogaard (2007).

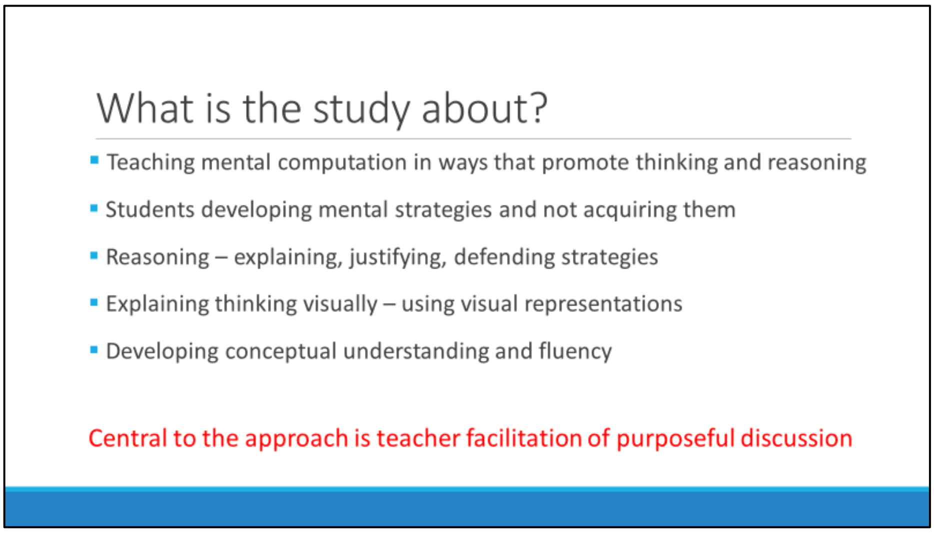
EXAMPLES OF STUDENT STRATEGIES SHARED DURING WHOLE CLASS DISCUSSION – MODELLED LESSON AT SCHOOL C



Note: Student names were recorded next to the strategy they explained. Names have been covered in the photo above to protect the identity of the students and class.

PROFESSIONAL LEARNING SESSION: SAMPLE OF POWERPOINT USED AT SCHOOL C





Sequence of instructional activities

- **Lesson 1** – Number strings: Eliciting a particular mental strategy
- **Lesson 2** – Number talk: Investigating the strategy further
- **Lesson 3** – Number talk (in context): Using a context to support learning

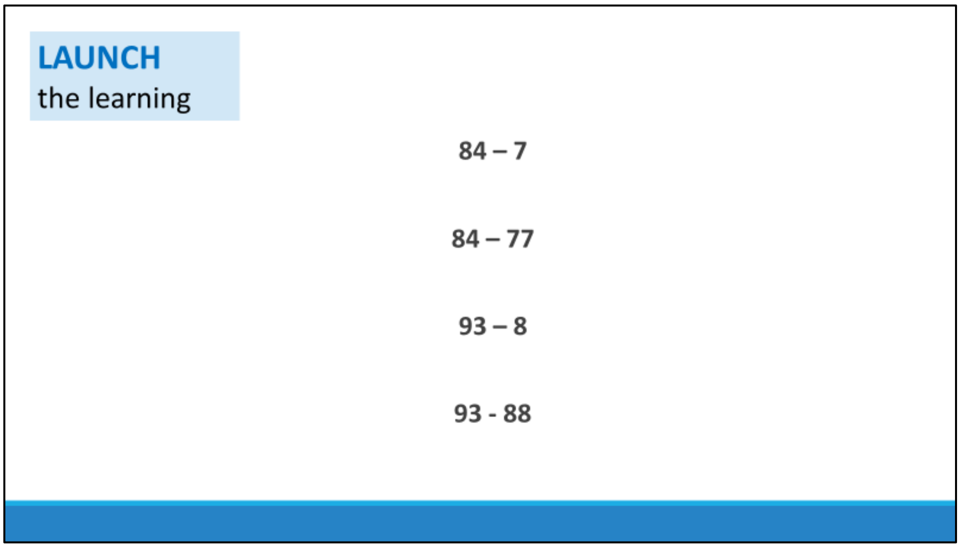
One mental strategy is the focus for 3 lessons

Exploring the instructional activities

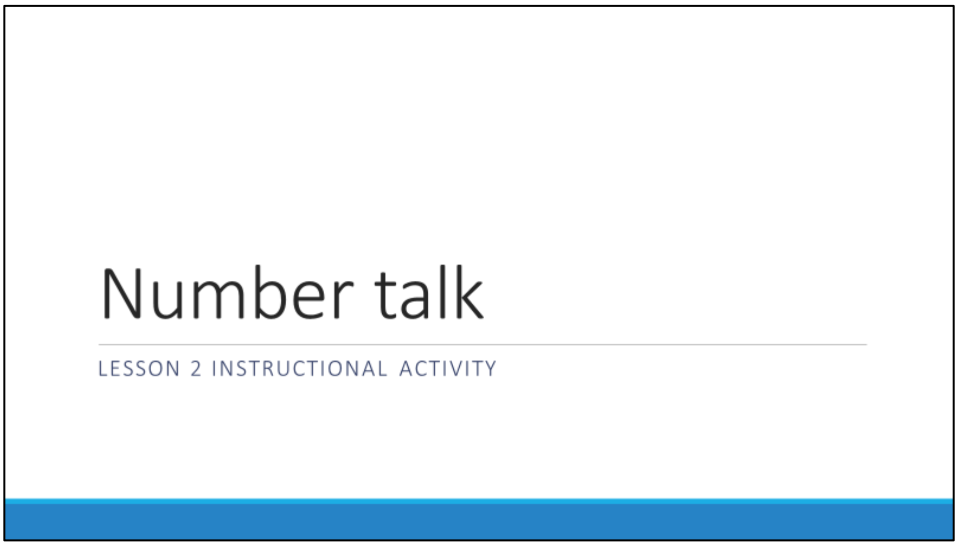
NUMBER STRINGS & NUMBER TALKS

Number strings

LESSON 1 INSTRUCTIONAL ACTIVITY



Note: The design of the number string above was adapted from the work of Fosnot and Uittenbogaard (2007).



FINDING THE DIFFERENCE

Work out how to find the difference between these numbers in your head:

134 - 127

How can your prove your answer is correct?

Can you explain it visually?

Number talk in a context

LESSON 3 INSTRUCTIONAL ACTIVITY

HOW MANY MORE POINTS?

In the computer game ***Galaxy Hunter,*** players need 155 points to reach Level 8.
So far you have scored 148 points.
How many more points do you need to reach Level 8?

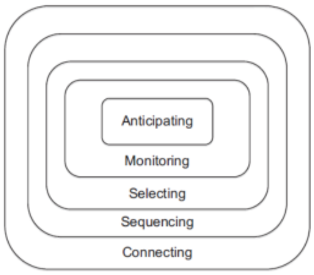
Explain how you could work this out in your head.

Purposeful discussion

MANAGING CLASSROOM DISCUSSION

Five practices

1) Anticipating
2) Monitoring
3) Selecting
4) Sequencing
5) Connecting

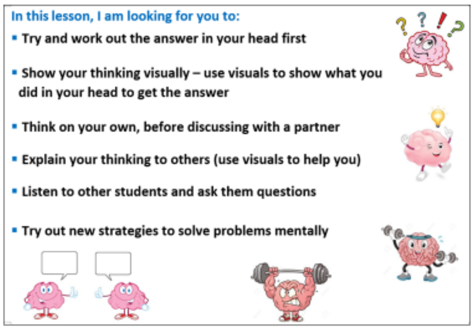


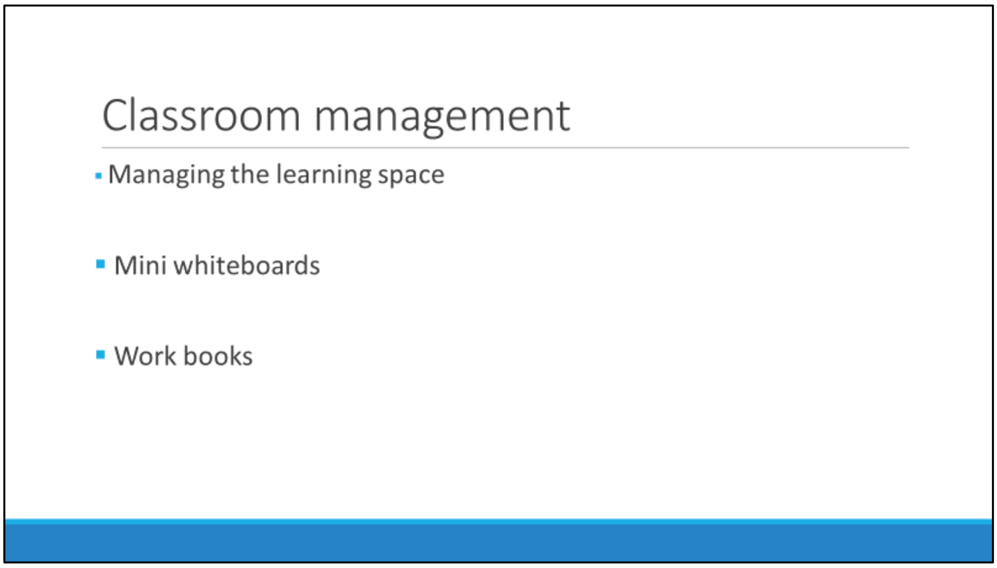
(Stein, Engle, Smith and Hughes, 2008, p. 322)

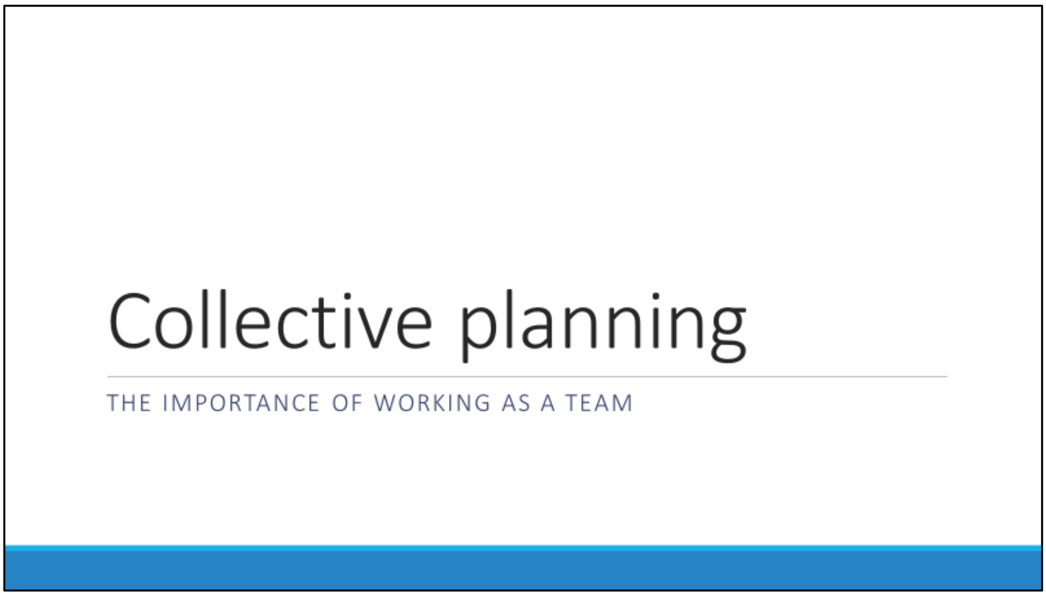
Learning environment

A THINKING AND REASONING CLASSROOM

Making pedagogy explicit




Classroom management
Managing the learning space
Mini whiteboards
Work books


Collective planning
THE IMPORTANCE OF WORKING AS A TEAM

Scoring with a Four-Point Rubric			
GOT IT		NOT YET	
4 Excellent: Full accomplishment	3 Proficient: Substantial Accomplishment	2 Marginal: Partial accomplishment	1 Unsatisfactory: Little accomplishment
Strategy and execution meet the content, process and qualitative demands of the task. Communication is judged by effectiveness, not length. May have minor errors.	Could work to full accomplishment with minimal feedback. Errors are minor, so teacher is confident that understanding is adequate to accomplish the objective.	Part of the task is accomplished, but there is a lack of evidence of understanding or evidence of not understanding. Direct input or further teaching is required.	The task is attempted and some mathematical effort is made. There may be fragments of accomplishment but there is little or no success.

(Van de Walle, 2007, p. 84)

APPENDIX C

Ethics documentation

Explanatory statement for teachers

Teacher consent form

Explanatory statement for students

Student consent form

Explanatory statement for parents

Parent consent form

Numeracy coach consent form

Ethics approval certificate from Monash University

MONASH University

EXPLANATORY STATEMENT

Year 3 Teachers

Project: Teacher professional learning and teacher change: the critical role of Mathematics Knowledge for Teaching

Researcher: Sally Hughes
PhD Candidate
Faculty of Education
Phone: 0401015769
email: sally.hughes@monash.edu

Chief Investigator: Dr. Karina Wilkie
Senior Lecturer, Mathematics Education
Faculty of Education
Phone: 9904 4227
email: karina.wilkie@monash.edu

You are invited to take part in this study. Please read this Explanatory Statement in full before deciding whether or not to participate in this research. If you would like further information regarding any aspect of this project, you are encouraged to contact the researchers via the phone numbers or email addresses listed above.

What does the research involve?

This research involves the investigation of professional learning of middle primary teachers for teaching mental computation with conceptual understanding. It involves a collaborative teaching experiment with other Year 3 teachers in your school in which the focus is on teaching mental computation is ways that promote student thinking and reasoning. Insights gained from the study will contribute to the development of accessible resources that support teaching of mental computation.

As a research participant, you will be asked to contribute approximately 8 hours of time outside of class. Your participation involves:

- a pre- and post-questionnaire about your experiences teaching mental computation;
- attending a professional learning session on mental computation (approx. 2 hours) and completing a questionnaire at the end of the session;
- attending weekly planning meeting with team members and the researcher throughout the research period (the meeting will be approximately 30 mins and will be audio recorded);
- a lesson observation at the beginning and end of the research project;
- a 30 minute individual interview with the researcher (this will be audio recorded);
- conducting a class assessment task devised with the researcher, with your class pre- and post- the study (you will be asked to share anonymised task results with the researcher);
- conducting an individual assessment (interview) on mental computation with a selection of students from your class (this will be audio recorded).

You will also be given the opportunity to do some team teaching with the researcher should you wish to do so; this is optional and is not a requirement of participating in the project.

Why were you chosen for this research?

You have been chosen for this research because you are teaching a Year 3 class.

Consenting to participate in the project and withdrawing from the research

After reading this information if you are willing to participate, please sign the accompanying consent form and return it to Sally Hughes.
Your participation in the research project is completely voluntary. Should you wish to withdraw from the project at any stage before the final given date for withdrawal, you will be able to do so by contacting the researcher.

Possible benefits and risks to participants

Your expertise and experiences are valuable for ongoing research efforts on the teaching and learning of mental computation. Your involvement in the project is also intended to contribute to your professional learning and provide you with resources and strategies for teaching mental computation.

Weekly planning meetings will be as usual, you will just be asked to contribute examples of student learning and share your experiences of teaching mental computation. There will be an opportunity to do some team teaching but this is entirely optional. The individual interview is an informal semi-structured chat about your experiences and recommendations for teaching mental computation. With your permission, the meetings and interviews will be audio-recorded to ensure that the data is used accurately. All of the data will be used in a way that minimises the risk of any teachers or students being identified by others in reports about the research.

Confidentiality

The data collected for this research project will be confidential. Pseudonyms and codes will be used to protect your confidentiality. All the data collected from the project (questionnaires, meeting and interview recordings, student work samples) will be used as part of a research degree; any presentations and publications will not identify the school or any of the participants involved in the project. The audio recordings are for the researcher to ensure accuracy in reporting findings and will not be provided to the school or to other researchers.

Storage of data

All data collected (recordings, questionnaires, files) will be securely stored in accordance with Monash University regulations and will be kept for five years before being destroyed.

Results

The findings will be presented as part of a research degree, at academic conferences and in journals. A summary of the findings will be emailed to participants upon completion of the research project report.

Complaints

Should you have any concerns or complaints about the conduct of the project, you are welcome to contact the Executive Officer, Monash University Human Research Ethics (MUHREC):

Executive Officer
Monash University Human Research Ethics Committee (MUHREC)
Room 111, Chancellery Building E,
24 Sports Walk, Clayton Campus
Research Office
Monash University VIC 3800

Tel: +61 3 9905 2052 Email: muhrec@monash.edu Fax: +61 3 9905 3831

Thank you,

Sally Hughes
Researcher / PhD Candidate

MONASH University

CONSENT FORM

Year 3 Teachers

Project: Teacher professional learning and teacher change: the critical role of Mathematics Knowledge for Teaching

Researcher: Sally Hughes
Chief Investigator: Dr Karina Wilkie

I have been asked to take part in the Monash University research project specified above. I have read and understood the Explanatory Statement and I hereby consent to participate in this project.

I consent to the following:	Yes	No
Participation in a professional learning session on teaching mental computation with conceptual understanding (to be held after school hours) and completion of a questionnaire at the end of the session		
Completion of a pre- and post-questionnaire about teaching mental computation		
Participation in a 30 minute individual interview with Sally at the end of the research project to share my experiences		
Participation in weekly planning meetings with Year 3 team members and Sally throughout the project period		
A lesson observation at the beginning and end of the research project		
Team teaching a mental computation lesson with Sally (this is optional rather than a requirement of participation in the study)		
Audio recording of the planning meetings and my interview with Sally		
My Year 3 students completing a pre- and post-assessment task developed in consultation with Sally and the teacher participants;		
Sharing anonymised results from the student pre- and post-assessment task with Sally		
Conducting an individual assessment in the form of an interview with a selection of my Year 3 students which will be audio recorded (interview questions developed in consultation with Sally and the teacher participants)		
Sally using the data that I provide, in ways that protect my privacy, to report on findings from the research		

Name of Participant ____________________

Participant Signature ____________________ Date ____________

MONASH University

EXPLANATORY STATEMENT

Students

Project: **Teacher professional learning and teacher change: the critical role of Mathematics Knowledge for Teaching**

Chief Investigator: Dr. Karina Wilkie
Senior Lecturer, Mathematics Education
Faculty of Education
Phone: 9904 4227
email: karina.wilkie@monash.edu

Researcher: Sally Hughes
PhD Candidate
Faculty of Education
Phone: 0401015769
email: sally.hughes@monash.edu

You are invited to take part in a research project with Monash University. Your teacher will read you this Explanatory Statement about the project before you decide whether or not you wish to participate in the project. You will be able to ask your teacher or the researcher any questions about the project once you have listened to the Explanatory Statement.

What is the research project about?
The Year 3 teachers and students at (NAME OF SCHOOL) are being invited to participate in a research project with Monash University. The aim of the project is to investigate how best to teach Year 3 students to solve addition and subtraction problems mentally (in your head) so that everyone can learn to be good at mental maths.

Your class teacher will teach most of the lessons on mental maths. Sometimes the researcher, Sally Hughes, might also be involved in teaching you some new mental maths strategies. The researcher will come and watch some of the lessons on mental maths to find out how everyone is learning.

The researcher is an experienced primary school teacher with expertise in mathematical learning in the primary years. She is studying the teaching and learning of mathematics for her PhD at Monash University.

Why have you been chosen to participate in this research project?
You have been chosen to participate in this research because you are a Year 3 student.

What will you have to do in this research project?
You will participate in maths lessons in the same way you do usually. The maths lessons in this project will be about ways to add and subtract mentally (in your head). You will be asked to take part in discussions and explain some of your mathematical thinking to the class.

You will complete two assessment tasks very similar to other assessments you do in maths. The assessments will help your teacher find out what you know about mental maths so they can plan lessons to help you get even better at maths. Your class teacher will be asked to share the assessment results for the whole class with the researcher, your names will be removed from the assessment papers so the researcher will not know whose assessment she is reading.

You may get the opportunity to do an assessment interview with your class teacher, which will be audio recorded. The researcher will ask to listen to the interview to find out how you worked out the answers to some of the questions. The researcher will not be told the name of the student when she listens to the voice recording.

The researcher will ask to look at some of your work on mental strategies, the work that is shared with the researcher will not have your names on so she will not know whose work she is looking at.

Confidentiality
All of the data collected will be useful to help teachers find out more about how mathematics is effectively taught to, and learnt by, primary school students. Any work that is shared with the researcher, Sally Hughes, will not have your name recorded on it.

Consenting to participate and withdrawing from the research
You will be asked to sign a consent form so that the researcher knows if you would like to share your work on mental maths with her as part of the research project. If you choose not to participate in the project you will still take part in maths lessons as usual, it just means that your work and your assessment results will not be shared with the researcher.

You are allowed to change your mind about participating in the research project, you just need to let your class teacher know.

Thank you,

Dr. Karina Wilkie
Chief Investigator

MONASH University

ASSENT FORM

Students

Project: **Teacher professional learning and teacher change: the critical role of Mathematics Knowledge for Teaching**

Chief Investigator: Dr. Karina Wilkie
Senior Lecturer, Mathematics Education
Faculty of Education
Phone: 9904 4227
email: karina.wilkie@monash.edu

Researcher: Sally Hughes
PhD Candidate
Faculty of Education
Phone: 0401015769
email: sally.hughes@monash.edu

I have been asked to join in this Monash University research project. The letter that explained everything about this project has been read to me and I have had a chance to ask questions about it. I understand what this research project is about and would like to join in.

I understand that being in this research project is my choice and that I can change my mind and choose not to be part of this project at any time and that no one will be annoyed with me if I change my mind. I know that if I have any questions I can ask my teacher or the researcher at any time.

I agree to:	Yes	No
Share my work about learning to add and subtract mentally with the researcher	☐	☐
Let my teacher talk to the researcher about my learning of mental maths	☐	☐
Be observed by the researcher	☐	☐
Let my teacher share an audio recording of me solving problems using mental strategies (my name will not be given to the researcher)	☐	☐

Name ______________________________ Date ____________

MONASH University

CONSENT FORM

Numeracy Co-ordinator

Project: Teacher professional learning and teacher change: the critical role of Mathematics Knowledge for Teaching

Researcher: Sally Hughes
Chief Investigator: Dr Karina Wilkie

I have been asked to take part in the Monash University research project specified above. I have read and understood the Explanatory Statement and I hereby consent to participate in this project.

I consent to the following:	Yes	No
Participation in a 20 minute individual interview with Sally at the end of the research project to share my perceptions of the professional learning program		
Audio recording of my interview with Sally		
Sally using the data that I provide, in ways that protect my privacy, to report on findings from the research		

Name of Participant ______________________________

Participant Signature ______________________________ Date ______________

Monash Ethics Certificate

MONASH University

Monash University Human Research Ethics Committee

Approval Certificate

This is to certify that the project below was considered by the Monash University Human Research Ethics Committee. The Committee was satisfied that the proposal meets the requirements of the *National Statement on Ethical Conduct in Human Research* and has granted approval.

Project Number: 9695
Project Title: Teacher professional learning and teacher change: the critical role of mathematical knowledge for teaching
Chief Investigator: Dr Karina Wilkie
Expiry Date: 14/08/2022

Terms of approval - failure to comply with the terms below is in breach of your approval and the *Australian Code for the Responsible Conduct of Research*.

1. The Chief Investigator is responsible for ensuring that permission letters are obtained, if relevant, before any data collection can occur at the specified organisation.
2. Approval is only valid whilst you hold a position at Monash University.
3. It is responsibility of the Chief Investigator to ensure that all investigators are aware of the terms of approval and to ensure the project is conducted as approved by MUHREC.
4. You should notify MUHREC immediately of any serious or unexpected adverse effects on participants or unforeseen events affecting the ethical acceptability of the project.
5. The Explanatory Statement must be on Monash letterhead and the Monash University complaints clause must include your project number.
6. Amendments to approved projects including changes to personnel must not commence without written approval from MHUREC.
7. Annual Report - continued approval of this project is dependent on the submission of an Annual Report.
8. Final Report - should be provided at the conclusion of the project. MUHREC should be notified if the project is discontinued before the expected completion date.
9. Monitoring - project may be subject to an audit or any other form of monitoring by MUHREC at any time.
10. Retention and storage of data - The Chief Investigator is responsible for the storage and retention of the original data pertaining to the project for a minimum period of five years.

Thank you for your assistance.

Professor Nip Thomson

Chair, MUHREC

CC: Miss Sally Hughes

List of approved documents:

Document Type	File Name	Date	Version
Explanatory Statement	explanatory-statement_Sally Hughes	16/06/2017	1
Explanatory Statement	Email to school seeking permission	16/06/2017	1

Department of Education Ethics Certification

Department of
Education & Training

2 Treasury Place
East Melbourne Victoria 3002
Telephone: 03 9637 2000
DX210083

2017_003581

Miss Sally Hughes
Faculty of Education
Monash University
Wellington Road
CLAYTON 3800

Dear Miss Hughes

Thank you for your application of 5 December 2017 in which you request permission to conduct research in Victorian government schools titled *Teacher professional learning and teacher change: the critical role of Mathematics Knowledge for Teaching.*

I am pleased to advise that on the basis of the information you have provided your research proposal is approved in principle subject to the conditions detailed below.

1. Department approved research projects currently undergoing a Human Research Ethics Committee (HREC) review are required to provide the Department with evidence of the HREC approval once complete.

2. The research is conducted in accordance with the final documentation you provided to the Department of Education and Training.

3. Separate approval for the research needs to be sought from school principals. This is to be supported by the Department of Education and Training approved documentation and, if applicable, the letter of approval from a relevant and formally constituted Human Research Ethics Committee.

4. The project is commenced within 12 months of this approval letter and any extensions or variations to your study, including those requested by an ethics committee must be submitted to the Department of Education and Training for its consideration before you proceed.

5. As a matter of courtesy, you advise the relevant Regional Director of the schools or governing body of the early childhood settings that you intend to approach. An outline of your research and a copy of this letter should be provided to the Regional Director or governing body.

6. You acknowledge the support of the Department of Education Training in any publications arising from the research.

VICTORIA
State
Government

Ethics approval – Catholic Education Melbourne

MONASH University

Sally Hughes <sally.hughes@monash.edu>

Research Application 0726: Outcome (Approved)

research@cem.edu.au <research@cem.edu.au> 19 February 2018 at 08:35
To: sally.hughes@monash.edu
Cc: research@cem.edu.au

Dear Miss Hughes

Congratulations, your research application, 0726 - 'Teacher professional learning and teacher change: the critical role of Mathematical Knowledge for Teaching', has been approved by Catholic Education Melbourne.

I am pleased to advise that your research application is approved in principle subject to the eight standard conditions outlined below.

1. The decision as to whether or not research can proceed in a school rests with the school's principal, so you will need to obtain their approval directly before commencing any research activity. You should provide the principal with an outline of your research proposal and indicate what will be asked of the school. A copy of this email of approval, and a copy of notification of approval from your organisation's/university's Ethics Committee, should also be provided.

2. A copy of the approval notification from your institution's Ethics Committee must be forwarded to this Office (if not already provided), together with any modifications to your research protocol requested by the Committee. You may not start any research in Catholic schools until this step has been completed.

3. A Working with Children (WWC) check – or registration with the Victorian Institute of Teaching (VIT) – is necessary for all researchers visiting schools. Appropriate documentation must be shown to the principal before starting the research in the school.

4. No student is to participate in the research study unless s/he is willing to do so and consent is given by a parent/guardian.

5. Any substantial modifications to the research proposal, or additional research involving use of the data collected, will require a further research application to be submitted to Catholic Education Melbourne.

6. Data relating to individuals or the school are to remain confidential and protected in line with the Privacy Act 1988 (Commonwealth).

7. Since participating schools have an interest in research findings, you should consider ways in which the results of the study could be made available for the benefit of the school community.

8. At the conclusion of the study, a copy or summary of the research findings should be forwarded to Catholic Education Melbourne. It would be appreciated if you could submit this via email to research@cem.edu.au.

I wish you well with your research study. The information provided in your proposal is now closed for further changes. If you have any queries concerning this matter or need to make amendments in the future, please contact research@cem.edu.au.

Yours sincerely

Mr Jim Miles
DIRECTOR ENTERPRISE SERVICES
Catholic Education Melbourne

APPENDIX D

Research instruments

Teacher surveys – Pre-intervention, Post-professional learning session, Post-intervention

Teacher semi-structured interview schedule

Numeracy coach interview schedule

Focus group questions

Student assessment tasks

PRE-INTERVENTION SURVEY

Section A The questions in this section are about your expertise and experience teaching mathematics.

1) For how many years have you been teaching?

2) For how many years have you been teaching middle primary students (Year 3 and/or Year 4)?

3) Is mathematics an area of specialism in terms of your own qualifications or in your teaching career?

YES / NO

If you answered YES, please provide details.

4) a. What aspects of teaching mathematics do you enjoy? Please give reasons.

b. What aspects of teaching mathematics are not enjoyable for you? Please give reasons.

5) Please indicate how you would rate your confidence in planning and teaching the following aspects of the mathematics curriculum to Year 3 students:

	Unconfident	Little unconfident	Confident	Very confident

Understanding of the prescribed curriculum content for the sub-strand Number and place value				
Mental strategies for addition and subtraction				
Written methods for addition and subtraction				
Mental strategies for multiplication and division				
Written methods for multiplication and division				
Assessing student learning for the sub-strand Number and place value				

	Unconfident	Little unconfident	Confident	Very confident
Integration of the following proficiency strands in lesson planning for sub-strand Number and place value:				
Understanding				
Reasoning				
Fluency				
Problem solving				

Section B The questions in this section are about student learning goals.

6) What are the main learning goals for students in your class, in regards to mental computation?

7) What strategies are in place to help students achieve these goals?

Section C The questions in this section are about your approaches to teaching mental computation.

8) How would you usually introduce a new mental strategy to students in your class?

9) What **teaching** strategies do you think have been most effective for student learning of mental computation in your class?

10) Can you give an example of a mental computation for addition or subtraction, appropriate for the Year 3, which has proven difficult for **most** students in your class?
How did you support students in overcoming this difficulty?

11) What resources do you find most useful in supporting the planning and teaching of mental computation? i.e. curriculum documents, published resources, colleagues.

POST-PROFESSIONAL LEARNING SESSION SURVEY

Reflection on Professional Learning Sessions *(planning meeting and modelled lessons)*

1) How useful was the professional learning in preparing you to trial a sequence of instructional activities as an approach to teaching mental computation?

1	2	3	4
Not useful	Moderately useful	Useful	Very useful

Please give a reason for your rating:

2) What was the most useful thing you learnt from participating in the sessions?

3) How effective do you think the sequence of activities will be in improving student learning of mental computation in your class?

1	2	3	4
Not effective	Moderately effective	Effective	Very effective

Please give a reason for your response:

4) What do you anticipate will be the biggest challenge with trialling this approach to teaching mental computation with your class?

5) Will trialling this approach to mental computation require you to make any significant changes to your usual teaching practice for mathematics?
YES / NO

If you responded, YES, please indicate what changes will be required.
If you responded, NO, please indicate reasons.

Mental Computation Professional Learning Project: Final Teacher Survey

Q1. What do you think was the most valuable thing your students learned from participating in the project? Explain why.

Q2. What was most challenging for your students in learning mental computation in this way?

Why do you think this was challenging?

Q3. What was the biggest challenge you faced with implementing this approach to teaching mental computation?

Why was this?

Q4. What changes, if any, have you made to the way you teach mathematics since participating in this project?

Explain why.

Q5. Reflecting on your participation in the project, is there anything you would consider changing in your future teaching of mental computation?

Explain the reasons for these changes.

Q6. What advice would you give to a teacher in another school, who was considering participating in this project?

Q7. In what ways has participation in the project contributed to your professional learning in teaching mental computation?
Which components of the program were most helpful?

Q8. If you were given the opportunity to participate in a professional learning program on mental computation next year, what would you like to see included in the program?
Please tick or shade in which of the components listed below you would like to see included:

modelled lessons	☐	professional reading materials	☐
co-teaching opportunity	☐	lesson resources i.e. PowerPoints or IWB resources	☐
provision of teacher resource book	☐	collaborative planning	☐
lesson observations and feedback	☐	collaborative analysis of student work samples	☐

Do you have any other suggestions not included in the list above?

What do you consider the most valuable component of a professional learning program?

Why do you think this?

INTERVIEW SCHEDULES & FOCUS GROUP QUESTIONS

SEMI-STRUCTURED INTERVIEW

1. What do you think is the most valuable thing your students learned from participating in this project?

   Did anything surprise you in relation to student learning of mental computation?

2. Can you describe a learning experience that has been most effective for student development of mental computation strategies?

3. Based on your experience with this project, what are your main learning goals for students in your class with regards to mental computation?

4. How do you think this approach to teaching mental computation has impacted on student learning? I.e. has it impacted on attitude? Confidence? Skills in explaining mathematical thinking or strategies?

5. What are some of the common difficulties or misconceptions students in your class have with mental computation? How have you helped them overcome these?

   What has proven to be most effective for student learning in your class?

6. Since participating in this project, have you used any new mathematical representations or ways to explain strategies to support student learning of mental computation?

7. What was the most valuable thing you learned from participating in this project?

8. What changes, if any, have you made to your teaching since participating in this project?

9. Reflecting on your experience with the project…
   a) is there anything you would consider changing in your future teaching of mental computation?
   b) is there anything you would consider changing in your general teaching of mathematics?

      Why would you make these changes?

10. What were the biggest challenges you faced with implementing this approach to teaching mental computation?

11. What advice would you give to another teacher or school if they were considering getting involved in this project?
    Are there any factors which constrained the changes or implementation of the approach?

12. To what extent has participation in the project contributed to your professional learning?
    Which components of the program were most helpful?
    Why?

    professional reading materials

PL learning session at start
modelled lesson / co-teaching
teacher resource book
collaborative planning
If Independent Schools Victoria, asked your advice on key components of future mathematics PL programs what you recommend they include? Why?

Any other suggestions, ideas or information you would like to share?

NUMERACY COACH: SEMI-STRUCTURED INTERVIEW

Perceptions of professional learning

1. Reflecting on the experiences of the Year 3 team, are there any changes you would consider making for future teaching of mental computation in Year 3? Why would you make these changes?

   1.1 In other year levels?

2. The professional learning program incorporated various components such as, modelled lesson, provision of a teacher resource book, professional reading materials, professional learning session, collaborative planning. Which components do you think were most helpful in terms of teachers' professional learning?

   2.1 Are there any components, which you think should have had greater emphasis? Why?
   2.2 Is there anything else which would have been helpful to include in the program?

3. If Independent Schools Victoria, approached you for advice on the development of future professional learning program for number, what would you recommend as essential components of the program and why?

4. Are there any other suggestions, ideas or information you would like to share?

FINAL FOCUS GROUP QUESTIONS

Focus on teacher perceptions of their professional learning experiences

What was the most valuable thing you learned about teaching and learning of mental computation?

Did anything surprise you in relation to student learning of mental computation?

Reflecting on your experience with the project as a team, what changes would you consider making for future teaching of mental computation? Why would you make these changes?

The professional learning program incorporated various components such as, modelled lesson, co-teaching, provision of a teacher resource book, professional reading materials, lesson resources i.e. PowerPoints, collaborative planning. Which components were most helpful in terms of your professional learning? Explain why.

If CEM approached you for advice on the development of future professional learning units on mental computation or number, what would you recommend as essential components of the program and why?

Are there any other suggestions, ideas or information you would like to share?

STUDENT ASSESSMENT TASKS

Written Pre-Assessment- Analysis of questions **SCHOOL B**

Questions		Possible student strategies	
1	38 + 6	Jumping across a decuple	
2	75 - 8		
3	83 - 50	Subtracting by tens off decuple	
4	57+ □ = 60	Jump forward to the decuple	
5	80 - 47	Jump back from a decuple	
6	25 + 13	No regrouping – 2 digits	
7	32 + 43		
8	45 - 13		
9	68 - 24		
10	127 + 62	No regrouping - 3-digit and 2-digit	
11	265 - 34		
12	25 + 36	Regrouping	
13	82 - 77	Regrouping	Counting up / adding up would be easier / efficient
14	75 - 39	Regrouping	Compensation strategy would make this easier
15	61 - 36	Regrouping	Compensation or constant difference (transformation)
16	159 + 26	Regrouping	Compensation
17	148 + 34	Regrouping	Bridging or jump strategy
18	123 - 46	Regrouping	Jump strategy would make this easier

Student Assessment Interview (Post-Intervention)

You will need:

- A device to audio record
- Questions on pieces of card / paper
- Some paper and a pencil (in case a student needs to clarify their verbal explanation)

The interview will be audio recorded so that the class teacher does not have to take notes during the interview.

The class teacher will show the student the question on a card. Once the student has given a response the teacher will ask the student to explain how they got the answer.

The class teacher will read the follow script:

> *I am going to ask you some questions. I will show you each question on a card. I would like you to try and work out the answer in your head.*

After the student responds with an answer:

> *Can you tell me how you worked out that answer?*

The class teacher will ask the student to explain how they worked out the answer before proceeding to the next question. If the explanation is unclear, ask the student to show their thinking as a mental jotting on a piece of paper.

Question 1	54 + 37
Question 2	93 - 46
Question 3	148 + 26
Question 4	142 - 27
Question 5	84 - 78
Question 6	155 + 47
Question 7	124 - 36
Question 8	136 + 29

Question 7	224 - 36
Question 8	266 + 29

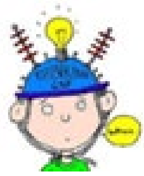
54 + 37
93 - 46
148 + 26
142 - 27
84 - 78

155 + 47
124 - 36
136 + 29

SCHOOL B WRITTEN POST-ASSESSMENT – CO-CONSTRUCTED WITH TEACHERS

Finding Ways to Add and Subtract in Your Head

First work out the answer in your head, then show how you did it.



Name:
Class:

48 + 25 =	**68 – 24 =**

159 + 36 =	**265 - 34 =**

94 – 87 =	**75 – 39 =**

61 – 36 =	**158 + 34 =**

123 – 46 =	**188 + 72 =**

134 + 87 =	**235 – 87 =**

242 – 54 =	**327 + 255 =**

296 + 125 =

APPENDIX E

Data analysis documentation

Table E.1	*Overview of data collected from each school*
Example interview transcript	*Transcript before coding*
Example teacher surveys	*Pre-intervention, Post-post professional learning, Post-intervention*
Example lesson observation	*Observation field notes*
NVivo codebook	*Final coding hierarchy used in NVivo*
Example coding using NVivo	*Interview transcript coding*
Change sequence analysis	*School B participants - analysis of change sequences* *School C participants - analysis of change sequences*
Display board photographs	*Photographs of student learning on a display board created by teachers at school B*

TABLE E.1 OVERVIEW OF THE METHODS OF DATA COLLECTION AT EACH SCHOOL

Research Stage	Research methods	Schools		
		A	B	C
PRE- intervention	Teacher survey		✓	✓
	Lesson observation – snapshot	✓	✓	✓
	Student assessment task (written)	✓	✓	✓
DURING intervention	Professional learning session	✓	✓	✓
	Post-professional learning session survey[44]	✓	✓	✓
	Modelled lessons and lesson debriefs	✓	✓	✓
	Focus groups: Weekly collaborative planning meetings		✓	✓
POST- intervention	Student assessment task (written)		✓	
	Student assessment interviews		✓	✓
	Individual teacher semi-structured interviews	✓	✓	✓
	Individual teacher surveys[45]	✓		✓
	Focus group: reflection on the intervention		✓	
	Individual semi-structured interviews with ML or NC (within schools who were actively involved in the program)	n/a	✓	✓

[44] At school A one teacher (Adele) did not complete the survey; at school C two teachers did not complete the survey (Ian, Jenna).
[45] At school B the teachers completed the survey collectively rather than individually (note they were requested to complete it individually); School C one teacher (Jenna) did not complete the survey.

EXAMPLE INTERVIEW TRANSCRIPT

EXAMPLE INTERVIEW TRANSCRIPT

Semi-structured interview **SCHOOL C**

Date: 18/09/18

Participant: Kelsy

Project: Mental Computation Professional Learning Project

What do you think is the most valuable thing your students learned from participating in the project?

I hope that it's the explicit conversation and the banter that you can have about having a toolbox of strategies but really being able to be metacognitive about which strategy you are actually using. So that's my hope that they'll take away that. A more metacognitive approach towards mental computation.

Did anything surprise you, in terms of student learning?

I think it surprised that sometimes the kids who are quicker at mental computation aren't as efficient as they think they are. Or that they will over complicate, or make something more complex when it's not necessary. That I found quite surprising.

Is there any part of the project that you think has been particularly effective in getting the students to learn mental strategies?

Definitely the number strings. The number strings component just identified for them what it is we were doing.

Having had the experience with this project, what are the main learning goals for students in your class with regards to mental computation?

My learning goals would be that we have mental computation strategies but really naming and noticing, which is the most effective strategy for the set of numbers we're working with.

Yes, it's the idea of thinking about the numbers rather than gazing around the room and looking for what tool you could use.

Yes. How well do I know the numbers that are in front of me, really how well do I know. And it goes across all operations because it's, I mean there are different strategies that you can use for multiplication and division, but it's really recognising the role that number plays within the operation that is being used.

Yes, and the effect the operation has on the numbers.

Yeah.

How do you think this approach to teaching mental computation has impacted on student learning? Do you think it has impacted on their attitude, their skills or confidence in explaining mathematical thinking?

I think that, bar one child in my class who's a very at risk student, I feel like the entry point for every child is achievable so in terms of the learning every child had an entry point, in fact I felt that it almost took away any of the anxiety that they might sometimes have towards a board full of numbers. They all had an entry point, just one child who is a unique case. But the rest of them, even my other at risk students felt willing to give it a go. I don't think that they have synthesised the whole process but they have stopped counting back by ones. So for them that's big growth, I think.

I noticed when I was in your classroom, that the students who were started jumping in tens were trying to push themselves and count by 20 for example.

Yes, it gave them a greater systematic approach to how they could do it. It provided them with a structure.

Did you notice any common difficulties or misconceptions with mental computation?

I think that the common one I noticed were the kids, and they were more on the enabling end, they don't generalise. For them, they are still not sure when it comes to compensation why they would do that because they just don't feel like they have that number fluency. Whereas those children that are working at class level or above, with a logical mind working with numbers that way helped them, whereas some of them, looking at them if you gave it to them now, as today we are going to create some equations which strategy would be best those children still don't recognise that compensation is going to be the most fluent? But I think they would split in tens rather than going back in ones and they might even split the 9 into a 4 and a 5, or a 3 and a 6, whereas before they weren't doing that. So they've got greater confidence but they're not taking it…that's the difference between working towards student on a report card and an at level student on a report card.

Yes, that cognitive load is still with the basic number fluency.

Yes.

Do you think there has been anything that has been particularly effective in helping those students overcome those difficulties?

I think over time more consistent use of it and really still keeping those strategies alive, where they'll begin to recognise if they hear it more often…I am compensating because 29 is like 30. If they hear it more, they'll begin to make those connections. And possibly some real life situations where they hear it being used to estimate an answer, that's where they might start to feel some purpose behind it.

Did you experience any new representations or strategies to explain ideas?

Not new but more effective ways of explaining. I don't think I would have explained, I don't think I've ever named compensation, compensation. We never named it that way and never really taught it as explicitly so it's not new but it's more effective I think, efficient ways. And also I think that mental computation is solving in your head and then explaining your strategy, it's not about explaining your strategy to show how you got your answer so that's really shifted my thinking.

And how would you have taught it before?

I don't think I recognised it, yeah they've got great mental computation skills but never actually slowed down to identify it as explicitly. So yeah, it was always there, present, but not noticed and named.

Do you think it's not really emphasised in the curriculum?

I don't think **we've** emphasised it, we've often talked about there are strategies, what strategy have you used? What strategy have you used? That's a great strategy! But not really identified it in terms of effective and efficient, which one could be used by others. So and so uses these great strategies but not necessarily pushed other children into why don't you try it that way.

What do you think is the most valuable thing you have learned from participating in the project?

The organisation of having something as simple as that PowerPoint. I found it really valuable, it's not informed my teaching but it's just helped facilitate teaching that it's really been there is a structure to it and there is room for flexibility. But that organisation. The booklet was awesome as well because it helped with the language that I might not necessarily have been using. The teacher talk but then bringing it down to student understanding.

Do you think you have made any changes to your teaching as a result of participating in this project? Or perhaps there are changes you might consider in the future?

I think setting that expectation, I always thought my expectations were clear, I think it's clarified my expectations for how learning could look, that when we're exploring something whether it's a mental computation strategy or just showing your thinking being explicit about the possibilities and how that helps you with your learning. Absolutely guide where I go with my construction and I think, yeah, just stopping and recognising the mental computation that exists and what's that strategy we're using and really keeping it at the forefront of their minds. Just that schema just keeping it there, keeping that connections so when you do identify complexities, for some it might be more complex than for others, but maybe the student teaching thing, the student can teach the other student because they've got the language.

What do you think are the biggest challenges that this approach presents?

When I was working with my at risk student, but then that's a unique situation where he's one-to-one correspondence of number was so poor that the equations were too difficult for him sometimes for those children but then like I said that's a unique set of circumstances and he needed to do one-on-one with possibly a different teacher, I couldn't attend to his learning needs as well as keeping an eye on everyone else. Sometimes thinking about those children who could do with a push. Did they really need bigger numbers to work with? What's the set of numbers that would draw the best learning for them?

Do you mean in terms of how high to go?

Yes, and is it really necessary because once they've got that strategy, do they need bigger numbers? Will they use that strategy because if they're compensating with 2-digt and they compensate with 3-digit and they compensate with 4-digit do they need to go higher? They're effectively mastered that skill, or is that the actual strategy that's going to work best for them?

Yes, that's a valid point because I think once you've got the concept the size of the number doesn't matter.

Although, they are "I'm using 5-digit numbers.." (chuckles). There's an ego there!

Sometimes for higher achieving kids, although this does not appear to be an issue with these students, they can believe that only written algorithms can be used for bigger numbers so showing them how you can use a mental strategy to work easily with 5-digit numbers in your head is useful in getting them to see the purpose of mental computation.

One of the children in yesterday's class, her bridging in the thousands threw her, she could create this number that she was going to subtract with but when it came to splitting it or just working her way backwards her bridging got completely thrown and she just completely bamboozled herself so that was interesting to see.

The project was also a professional learning program for teachers, there was one aspect that you missed out on but the program included a session where I explain the project and we discussed the big ideas, there was the modelled lesson, professional reading materials, teacher resource book and collaborative planning (which is part of your usual practice) but which I attended. Are there any particular components which you feel were most useful?

I think the collaborative planning lit up the booklet. The professional conversation, I think, is important, that leading up to it but working through with the booklet and planning and discussing it and nutting out together effective teaching methods probably, because then you start to see everyone's approaches to how they might do it and learn from each other.

Yes, and I thought the discussion we had in one of those meetings was really rich. We really got into the depths of how you could teach it and potential challenges presented by students.

I think that it's important not to assume that everyone understands that we're all on the same language page, there's complexities to mathematics and sometimes we just assume that we all know what we're talking about and it's those conversations that can identify whether, how well you understand the strategy that you're about to teach. Do you understand at all, and those sort of things.

And also I think experience with it as well because a lot of my learning about it has come from experimentation and trying it and unless you've had those experiences you can't always anticipate what could happen.

Absolutely, but I think booklets are great but it's the conversation around the booklet, the booklet facilitates the conversation, that grows the learning because reading in isolation doesn't necessarily…we teach what a thoughtful reader does and a thoughtful reader asks questions but it's being able to have that conversation about the questions, that I think our growth, that's where our growth and shifting thinking takes place.

If you had to give advice to another school considering participating in this project what would you say?

Have the conversations. It's so, I think the learning is so worthwhile for the students but as a…I wouldn't want to be a teacher on my own doing it, I think it's so worthwhile having, that when you can nut out the planning stage and that you know what it is or what went wrong in one lesson and how you can then, even just that idea of that very first start when the kids were just splitting away and it made sense, and then it was that conversation about keep the number intact, keep the one number intact, don't go splitting everything and that was for us a big learning curve, and then we put in that expectation for the children so it's those kind of conversations you're bound to end up, not creating misconceptions but what we were doing wasn't wrong but it was not as effective as keeping one number intact so it's that slowing down and being able to bounce ideas off each other and then look towards that next learning opportunity a little bit deeper.

Yeah, and I also think that by having those conversations and pulling out that one idea, without telling students explicitly what to do or modelling it you're just putting in that extra piece of information. So by keeping that number intact how are you going to do it?

Yeah, I think it's slowed us down as well and I think that it's richer than three lessons. That's what I would probably say, the pressure to, if you want to keep it in three lessons you won't be able to keep it in three lessons. If it's all going well it goes beyond three lessons. I don't think you get to the heart of it in three lessons.

That's interesting. Do you think there is anything in the school environment that has been a constraint to you being able to teach it exactly how you want to?

No, I haven't felt constrained by it.

Okay, I just wondered when you said it could sometimes go beyond the three lessons…

Oh….I think we had it towards the end of the term and I almost would want to start it at the beginning of the year rather than mid-year. It's just the timing of the project. But I think starting at the beginning of the year opens, it lends itself to games it lends itself more for keeping that thread of the strategies alive, and then you build on those. What other strategies do we know? What other strategies can we use? When do they work? When do they not work? That conversation can build itself, you're embedding the language more effectively.

Yes, I think it's something you can spread out (I have done in the past) over a whole year. So you could have a week or 10 days on it in Term 1 and spread it across the terms. Actually you stepped in with the more sophisticated strategies, partly because of the timing whereas a school I worked with in Term 1 focused on more basic jump strategies.

And it ties in beautifully with place value, it's all of those sort of connections that we tend to do at the beginning of the year. But I think it's also that building on stamina, maths stamina, giving things a go. Sitting in that time of confusion and that's okay, building that into it rather than….

Yes, the whole approach of what it's like to think and…

Yeah, and it's not just activities it's learning. You haven't given us a set of activities to do, it's richer than that. That's why I think it's something that could be applied effectively across a primary school and the professional learning I've done I could use in entry points for other year levels, it's got value there.

EXAMPLE TEACHER SURVEYS

Section A The questions in this section are about your expertise and experience teaching mathematics.

12) For how many years have you been teaching?

5

13) For how many years have you been teaching middle primary students (Year 3 and/or Year 4)?

1

14) Is mathematics an area of specialism in terms of your own qualifications or in your teaching career?

YES / NO

If you answered YES, please provide details.

15) a. What aspects of teaching mathematics do you enjoy? Please give reasons.

Watching students enthusiastically grapple with challenge. Particularly challenges connected to real-life context.

b. What aspects of teaching mathematics are not enjoyable for you? Please give reasons.

16) Please indicate how you would rate your confidence in planning and teaching the following aspects of the mathematics curriculum to Year 3 students:

	Unconfident	Little unconfident	Confident	Very confident
Understanding of the prescribed curriculum content for the sub-strand Number and place value				✓
Mental strategies for addition and subtraction		✓		
Written methods for addition and subtraction			✓	
Mental strategies for multiplication and division		✓		

Written methods for multiplication and division			✓	
Assessing student learning for the sub-strand Number and place value			✓	

	Unconfident	Little unconfident	Confident	Very confident
Integration of the following proficiency strands in lesson planning for sub-strand Number and place value:				
Understanding			✓	
Reasoning		✓		
Fluency		✓		
Problem solving			✓	

Section B The questions in this section are about student learning goals.

17) What are the main learning goals for students in your class, in regards to mental computation?

- Always show thinking.
- You can choose whichever strategy best works for you.
- Aim for efficiency.

18) What strategies are in place to help students achieve these goals?

- Planning and implementing open ended tasks.
- High expectations on book work – if no thinking is recorded I send it back.
- Regularly invite students to share their thinking with the rest of the class.

Section C The questions in this section are about your approaches to teaching mental computation.

19) How would you usually introduce a new mental strategy to students in your class?

- Provide a question which could target that strategy.

20) What **teaching** strategies do you think have been most effective for student learning of mental computation in your class?

- Introducing the Number Line.
- Invite students to share their strategies.
- If possible, focus on the student who is demonstrating the chosen strategy.
- If no student has demonstrated the strategy, explicitly model it.

21) Can you give an example of a mental computation for addition or subtraction, appropriate for the Year 3, which has proven difficult for **most** students in your class?
How did you support students in overcoming this difficulty?

We have only taught jump and split strategies on a number line and the students have a firm understanding of this.

22) What resources do you find most useful in supporting the planning and teaching of mental computation? i.e. curriculum documents, published resources, colleagues.

Our Number Coach, Laura, is our main resource for identifying mental computation strategies.

POST-PROFESSIONAL LEARNING SURVEY **Fiona – SCHOOL B**

6) **How useful was the professional learning in preparing you to trial a sequence of instructional activities as an approach to teaching mental computation?**

1	2	3	4
Not useful	Moderately useful	Useful	Very useful

Please give a reason for your rating:

Further time to explore resource before unit would have been good. Perhaps a visual flow diagram of unit (e.g. LP x 3 x Strat 1 Add'n; LP x 3 x Strat 1 Sub; then…

7) **What was the most useful thing you learnt from participating in the sessions?**

Number strings was new…can now see how to plan a string. Value of initial 'game' to engage mental fluency; time management!!

8) **How effective do you think the sequence of activities will be in improving student learning of mental computation in your class?**

1	2*	3	4
Not effective	Moderately effective	Effective	Very effective

*****variable prior knowledge and skills of students

Please give a reason for your response:

I predict students will vary quite widely as they have entered with highly variable prior knowledge and maths confidence.

9) **What do you anticipate will be the biggest challenge with trialling this approach to teaching mental computation with your class?**

Range of student learning needs and attitudes.

Time and environment impact structure and students are quite unfamiliar with visual representation.

10) **Will trialling this approach to mental computation require you to make any significant changes to your usual teaching practice for mathematics?**
YES / NO

If you responded, YES, please indicate what changes will be required.

If you responded, NO, please indicate reasons.

Extended sequence that is substantially whole class teaching. Differentiated questions but 'whole' teacher led conversation.

Lower level of student lead discussion and/or shared 'anchor chart' summary of key points

POST-INTERVENTION SURVEY **KELSY– SCHOOL C**

This survey was partially completed by the teacher; this was typical of all of post-intervention surveys. The participants only tended to respond to questions they thought they had not commented on sufficiently in their interviews.

Mental Computation Professional Learning Project: Final Teacher Survey

Q1. What do you think was the most valuable thing your students learned from participating in the project? Explain why.

Q2. What was most challenging for your students in learning mental computation in this way?

Why do you think this was challenging?

Q3. What was the biggest challenge you faced with implementing this approach to teaching mental computation?

I think the biggest challenge was ensuring that I understood the strategy we were teaching and ensured that I was using the correct language.

Why was this?

This was important because the students needed to use a common language when discussing the specific strategies that were being taught- as long as I was clear in my understanding of the strategies, misconceptions made by the students could be shifted and change more efficiently.

Q4. What changes, if any, have you made to the way you teach mathematics since participating in this project?

Explain why.

Q5. Reflecting on your participation in the project, is there anything you would consider changing in your future teaching of mental computation?

I think I would still see what strategies the children know before I begin teaching- then explore the more well-known strategies to ensure there is a common language and understanding. I would look at each strategy separately and ensure that there is an understanding as to why we use the different strategies. The 3-step process of number strings, number talk and numbers in real life context is something that I would like to implement more.

Explain the reasons for these changes.

Q6. What advice would you give to a teacher in another school, who was considering participating in this project?

Q7. In what ways has participation in the project contributed to your professional learning in teaching mental computation?

Participation in this project has highlighted the importance of mental computation and how we can teach it more effectively.

Which components of the program were most helpful?

The conversation with my fellow teachers, working with you and the booklet was great support and resource.

Q8. If you were given the opportunity to participate in a professional learning program on mental computation next year, what would you like to see included in the program?
Please tick or shade in which of the components listed below you would like to see included:

modelled lessons		professional reading materials	
co-teaching opportunity		lesson resources i.e. PowerPoints or IWB resources	
provision of teacher resource book		collaborative planning	
lesson observations and feedback		collaborative analysis of student work samples	

EXAMPLE LESSON OBSERVATION

END OF INTERVENTION – LESSON OBSERVATION **DERYN– SCHOOL B**

Lesson Observation Notes **23/07/18**

Lesson 2: Finding the difference

Teacher: Deryn (team leader)

Lesson time: 50 minutes

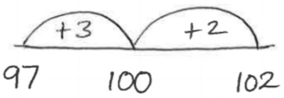
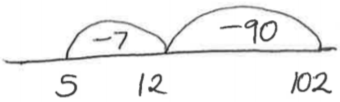
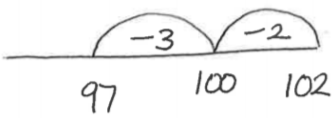
Note: Lesson 1 of the sequence taught on Friday 20 July

<table>
<tr><th></th><th>Observations</th><th>Comments</th></tr>
<tr><td>Activate</td><td>Students seated on floor with mini-whiteboards and pens.

Recap on learning from lesson 1 in the sequence.
T: On Friday we looked at two mental strategies for finding the difference.
T: What was the name of one of the mental strategies we looked at?
Students put up hands to answer question.
S: Count up
T records response on board.
T: When do we use this?
Students put up hands to answer. T repeats response given. Records on board.
T: Who can give me an example of when we might use it?
S: 29 – 20
T records on board.
T: Probably need bigger numbers than that.
T repeats question to elicit two more examples from students to record on the board in a t-chart.

T: What was the name of the other one?
S: Count down
T: When do we use this strategy?
S: when numbers are far apart
T repeats response so everyone can hear it.
T: Who can give me an example of when we might use it?
Students put hands up to share answers.
T records on board in T-chart.

<table><tr><th>count up</th><th>count down</th></tr><tr><td>numbers close
29 – 20
98 – 88
102 - 95</td><td>numbers far apart
107 – 9
88 – 2
99 – 7
106 - 15</td></tr></table></td><td>This was a general recap and lesson introduction rather than ‘activate’ session as described in the resource book. This was possibly because the first in the sequence lesson was taught on Friday.</td></tr>
<tr><td>Whole class 3 phase session 1</td><td>PowerPoint displayed – key content for the lesson.
T reads the learning intention displayed on the first slide:
Use mental strategies to calculate the difference between two numbers
First task – bare number question – displayed on board.
T: I want you to work out the answer, then show your working out.
Students were given considerable thinking time.
T suggested that they write which strategy they were using.
T prompted students to revise their thinking: Have you used the most efficient?
T moved across front of the group, looking across at some of the mini-whiteboards. This was mainly restricted to students at the front of the room; didn’t really circulate the group.</td><td></td></tr>
</table>

	T: What answers did you get? Students put up hands to share answers. T selects a student to share. T: Do we agree that's the answer? Student was asked to explain their strategy. T interpreted and drew a visual representation on the board, paraphrasing the procedure so it was clear for all students to hear and see.  102 − 97 = 5 Who did it a different way? T selected another student to share. Student explained orally; teacher displayed thinking visually.  Does anyone have a different way to share? Third student selected; T represented thinking that was described orally on the board.  T looked at third visual and commented that it was the same as the first one but in reverse. T seemed a little surprised by this and confirmed with me that it was the same but counting back. She said that some students had worked out other questions like that. T: Which one is most efficient? Students put up hands to answer. T: Who agrees? Student with hand up selected to answer. The visual representations for task 1 were left on the board.	This appears to be 'counting down to' the subtrahend. A strategy used for tasks when the answer is given but the subtrahend is missing.
Whole class 3 phase session 2	Second task displayed on the PowerPoint. 105 – 8 Students were asked to work out the answer and then show their working out. T assisted a student at the front of the group. They had subtracted 5 from the minuend but were unsure what to do next. T prompted: What's 100 – 3? You can use friends of 10. T: What's the answer? Who got a different answer? All students seemed to agree on the same answer (nothing written on mini w/bs to confirm thinking of others. Student was asked to shared his whiteboard. He stood up, showed the board and explained orally how he worked out the answer.	

<table>
<tr><td></td><td>S: I took away 10, then added 2.
T: Why did you take away 10 and then add back 2?
S: I took away 10 because that was easier.
There was a long silence.
In the end T verbalised for the student because he could not explain why he added back the 2.
T described the strategy the count down strategy because the student had counted back 10.

Another student was asked to share her strategy.
She described how she counted back 5 and then 3.
T summarised and said this was the same as S1's strategy, it was the count down strategy.

T: Why wouldn't it make sense to count up from 8 to 105 to find the difference?
With some teacher prompting, it was discussed how it would take a very long time because the numbers were far apart.

T: What maths word do we use to describe this?
The word efficient was elicited.</td><td></td></tr>
<tr><td>Consolidating task</td><td>Three levels of questions (mild, spicy, hot) were displayed on board for students to select their level.
(T commented that the questions had been adapted slightly from the ones in the Teacher Resource Book).
Students moved to their seats and recording seats were distributed.

<table>
<tr><th>Mild</th><th>Spicy</th><th>Hot</th></tr>
<tr><td>55 – 47
77 – 68
84 – 78
74 – 8
94 – 87
65 - 7</td><td>104 – 96
113 – 7
106 – 89
113 – 97
111 – 8
121 - 98</td><td>120 – 97
124 – 98
135 – 95
142 – 15
150 – 98
127 - 9</td></tr>
</table>

T: Is there anyone who would like to come to the floor and do some of the Mild questions with me?
Focus group was run by the teacher with 5 students. Focus group worked on mini-w/bs on floor. Teacher posed questions to guide the calculations:
Are the numbers far apart or close together?
Which strategy shall we use?
Can you count up in your head?
Count in your head first, before you draw it.

The focus group gradually dispersed as they achieved success with the questions.

Students had access to an extension task. This was a sheet containing football scores. Students were asked to find the difference between the scores.</td><td></td></tr>
<tr><td>Summary</td><td>T: Who did mild?
Students put up hands to indicate.
74 – 8
T: Did you count up or down? Why?
Students shared explanations orally (visual representations were not used to</td><td></td></tr>
</table>

	Who did spicy? Who did 113 – 97? What strategy did you use? This was repeated for hot. Students described the strategies they used orally.	

QSR NVIVO CODEBOOK

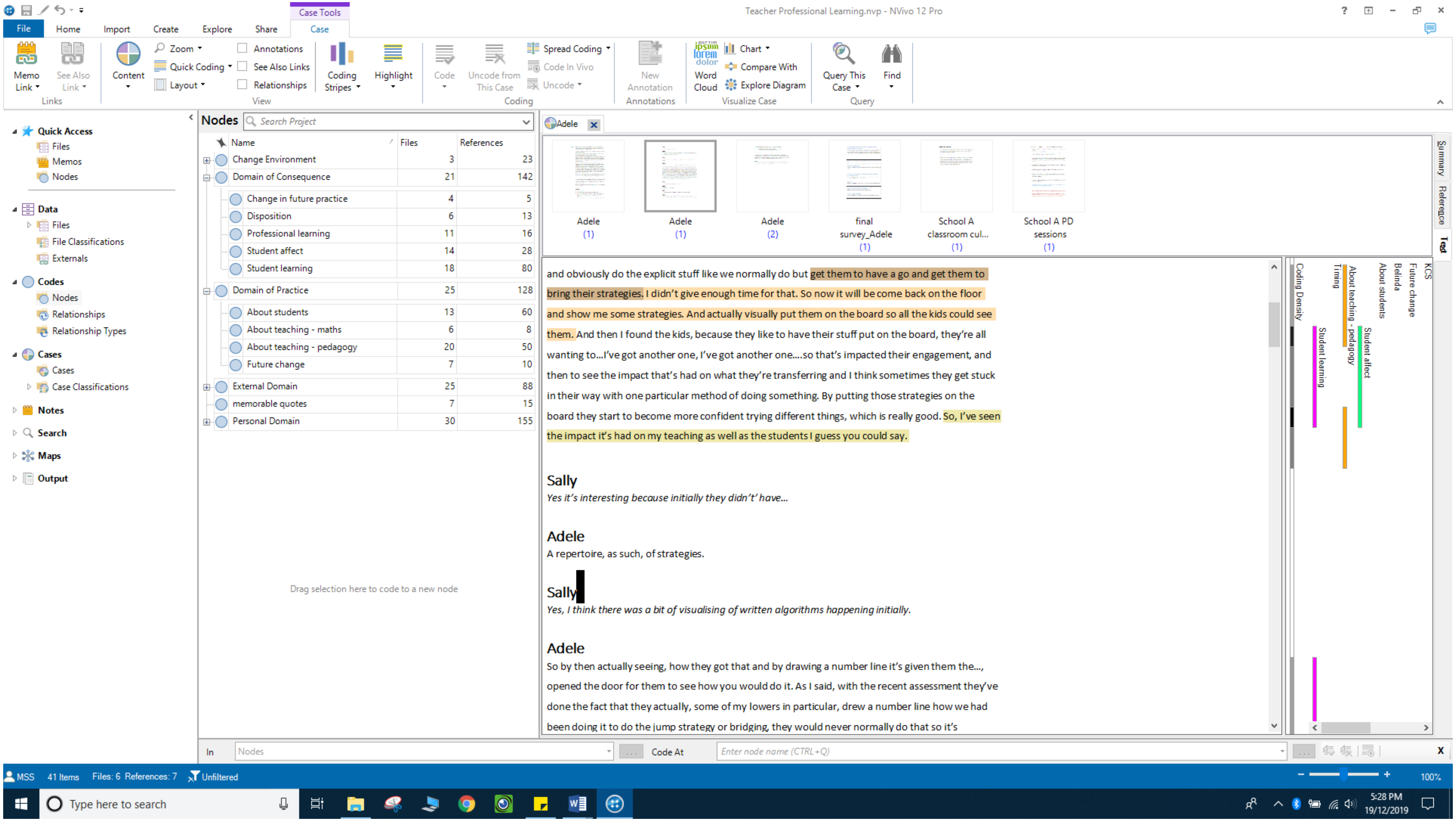
Teacher Professional Learning

NODE & *sub-nodes*	DESCRIPTION
Change Environment	Affordances and constraints to change within the institution; changes at an institutional level both during the intervention and planning for the future.
Affordances	Affordances existing within the change environment.
Constraint	Teacher description of constraints to changing approaches to teaching; external factors that influence teaching decisions.
Communication	Communication issues (interpersonal issues) constraining opportunities to learn and/or make changes to practice.
Consistency	Issues with consistency within the Year 3 team and across the school.
Other	Other factors constraining opportunities to learn and change practice.
Range of learning needs	Challenge of differentiating learning constraining changes to practice.
Student disposition	Student disposition acting as a constraint to learning mental computation strategies.
Time	Limited time to implement the intervention; limited time to teach using this new pedagogical approach.
Timing	The timing of the intervention constraining opportunities to learn and change practice.
Institutional change	Changes to planning and teaching beyond classrooms of the participants.
Domain of Consequence	Teacher reflections on outcomes considered salient as a result of classroom experimentation.
Change in future practice	Reflection on an outcome leading to consideration for a future change in practice.
Disposition	Reflection on aspects of disposition (attitudes, anxiety, confidence, ideas relating to the conceptualisation of maths) resulting from participation in the project. This is not necessarily a change, in many cases a reflection and affirmation of a pre-existing aspect of their disposition.
Professional learning	Teacher reflecting on their own learning as a result of the intervention.
Student affect	Change in student affect (engagement, attitudes) as a result of classroom experimentation with this new approach to teaching.
Student learning	Change in student learning (positive outcome) as a result of classroom experimentation; new awareness of student learning needs.
Domain of Practice	Teacher enactment of new learning and experimentation with the approach in their

	classroom.
About students	Teacher reflection on student learning (through classroom experimentation).
About teaching - maths	Teacher reflection on the mathematics content being taught.
About teaching - pedagogy	Teacher reflection and enactment of the pedagogical approach to teaching mental computation.
Future change	Description of changes teachers consider implementing in the future based on salient outcomes resulting from classroom experimentation.
External Domain	How aspects of the professional learning program create opportunities to learn and develop teacher knowledge and practice.
Future changes	Suggestions for future iterations of the professional learning program.
Intervention design - PL program	Reflection on the combination of multiple components considered important in the design of the PL program.
Intervention design - teaching component	Reflections on the design of the teaching intervention (teaching of mental computation).
Modelled lessons	Reflection on the opportunity to observe the researcher; observe students.
Professional conversations	Reflection on the importance of professional conversations during lessons, planning meetings and between lessons.
Professional learning session	Reflection on learning in this session.
Professional readings	Reflections on the importance of professional reading materials in developing knowledge and classroom practice.
Teacher resource book	Reflection on the usefulness of the resource book in supporting planning and teaching.
memorable quotes	
Personal Domain	Reflection on changes in aspects of knowledge and disposition resulting from participation in the professional learning project.
Disposition - anxiety	Teacher anxiety towards teaching mathematics.
Disposition - attitudes	Teacher attitudes including enjoyment, interest and enthusiasm for teaching mathematics, as well as attitude regarding maths in the classroom (how students should learn) and in everyday life.
Disposition - conceptualisation	How mathematics is conceptualised by teachers in terms of understanding (problem solving, interconnecting truths, unrelated facts and skills) but also including the actions of maths that frame it as a subject e.g. activities completed.
Disposition - confidence	Confidence to teach or experiment with teaching mathematics using this approach.

KCS	Knowledge of content and students, including how well they know individual students and the curriculum content students generally find challenging.
KCT	Development of knowledge of content and teaching such as the use of visual representations to explain student thinking and use of effective questioning to facilitate learning.
Knowledge at math horizon	Commonly described as knowing where content will be used in future learning but also including knowledge of typical learning trajectories.
SCK	Specialised content knowledge

EXAMPLE CODING USING NVIVO



CHANGE SEQUENCE ANALYSIS

Change sequence analysis: Using criteria to establish relations between domains in the IMPG **DERYN - SCHOOL B**

Arrow	Domain Link	Mediating process	Criteria	Summary	Examples of learning processes
1	ED to PD	Reflection	When something that was done or discussed during one of the learning experiences modified teachers' initial knowledge or disposition on teaching mental computation.	Reflects on external stimuli: professional learning session, modelled lessons and related professional conversations, teacher resource book. She has new PCK; how to use number strings as a tool to facilitate student centred learning of mental computation strategies. She experiences a change in her disposition; she is enthused about using new instructional tools.	**PROFESSIONAL LEARNING SESSION**: The change in pedagogical approach in relation to differentiating student learning seemed to instigate an interest and a desire to experiment with the new pedagogical approach in her classroom. This enthusiasm appeared to be stimulated by an anticipated positive response from her students in terms of engagement (Researcher's Journal). **MODELLED LESSONS**: Throughout the first modelled lesson, Deryn was extremely attentive. She observed the instructional part of the lesson closely, making notes on a printed copy of the lesson plan provided. In our debrief at the end of the lesson, she commented that observing had clarified her understanding of the number string, she elaborated by explaining that she had formed the wrong impression during the professional learning session and by observing realized she had confused the idea of the number string and the sequence. Although I led the instructional teaching, Deryn was an active participant in the lesson. She was involved in orchestrating the class discussions, by selecting examples of student thinking (recorded on mini-whiteboards) to address misconceptions but also highlight more efficient approaches. Through the brokering process it seemed that observing the researcher helped Deryn internalise new knowledge about using Number Strings as an instructional tool. Although we had worked through an example of a Number String in the professional learning session, it seemed that observing the process in action with her students was necessary for her to internalise the purpose and design of a number string (internalise new pedagogical knowledge, specifically Knowledge of Content and Teaching KCT). The significance of observing the modelled lessons was corroborated when she reflected in her semi-structured interview: *…I really like other people coming in and modelling for me. I like listening to others, I like the way as you said, listening to someone else's questioning, listening to someone else's way of doing things. I really enjoy that and I get a lot out of that. I suppose in my classroom with my kids, things like that I have found very beneficial.* Following observation of two modelled lessons, the researcher and team met for a weekly collective planning session. Deryn was enthused at the planning meeting, she shared that she was keen to experiment with a Number Strings lesson with her class since gaining insight into how the instructional tool could be used. It seemed that observing the modelled lessons had not only stimulated learning of new pedagogical knowledge but also initiated a change in her disposition; in the professional learning session she had not displayed this level of enthusiasm and interest for using number strings. Deryn also shared how she had noticed a change in student attitudes towards learning in the modelled sessions; she seemed to recognise the benefits of this approach to teaching in terms of student learning. **TEACHER RESOURCE BOOK:** seemed an important external source of information for Deryn, in terms of supporting planning and teaching of lessons. In her semi-structured interview she described how the book had been used to create lesson resources. In addition, she commented on the usefulness of the

Arrow	Domain Link	Mediating process	Criteria	Summary	Examples of learning processes
					information provided in the book: *…I think it was good to go through it. We made our own PowerPoints from it, like for today, because I said to them I like having a PowerPoint I like having it presented visually. So it was good that it was all in line and we could pick it up very easily and teach it. It would be a great resource for me now if I go back and teach the bridging strategy for example, let's go back and refresh my memory on the problems that relate to it and how to actually teach it, what are the steps before teaching it. (Interview)* Using a slideshow to present the questions in the Number String one at a time, or the main problem in the Number Talk, was an approach used in the modelled lessons. It was a new idea for Deryn and one she seemed to internalise. Use of the resource book to look at problems that relate to, or would elicit a particular strategy from students, suggests that a focus on teaching mental strategies involved development of new specialised content knowledge (SCK) for Deryn. **PROFESSIONAL CONVERSATIONS**: Deryn seemed to value the professional conversations in planning meetings and the opportunity to plan lessons with an external source of support, she placed emphasis on this component during her interview and also later in the focus group at the end of the intervention: *…Yeah, and it was good even watching how you modified it. You listened to us say, how kids aren't ready for that or those numbers are too high and things like that. It was great seeing that you could actually modify it. So there was a basic skeleton of it that you could go higher or lower with it…* *Yeah…so having those discussions in between…* *Yeah, having those discussions in between was good. (Interview)* *…it was a great resource book but it wasn't set in stone either. You know, you listened as we talked in planning meetings about the needs of our kids and where our kids were at, to be able to modify it. (Focus Group)* The value Deryn attached to the discussions (professional conversations) in the planning meetings concurs with findings from synthesis of the literature on professional learning of practicing teachers conducted by Goldsmith, Doerr and Lewis (2014). They found that "professional conversations can provide teachers with the encouragement and support that is need to begin to experiment with new approaches to teaching (Britt et al., 2001 as cited in Goldsmith et al., p. 15). It is also noteworthy that her mention of the researcher listening and responding flexibly by working with the teachers to modify resources was considered important. From my perspective as the researcher, there was a sense of working of team work, we drew on each other's knowledge to co-construct ideas and resources to move forward student learning. Although it appeared that opportunity to observe the researcher model a sequence of lessons was

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					significant in stimulating new learning, specifically development of KCT alongside an initial change in disposition, it seemed that evolving social dynamics with the researcher also played a critical role. The opportunity for professional conversations with colleagues and the researcher appeared an important stimulus for Deryn to enact her new knowledge in the classroom.
2	PD to DP	Enactment	When a specific aspect of teachers' knowledge or disposition influenced something that occurred in their teaching practices.	Enacts new knowledge about teaching mental computation with conceptual understanding. She begins teaching a new sequence: using the jump strategy to subtract mentally. She focuses on allowing time for students to think mentally before recording their thinking visually. Reflects on the importance of the professional conversations following lessons and at planning.	**NEW PEDAGOGICAL APPROACH:** Deryn described a significant change in her teaching practice, allowing time for students to think mentally and explain their thinking rather than model a process for students to follow: *…I like the fact that we're actually trying to make them figure out the answer first before the working out, whereas mathematically processing usually we say, this is how you work it out. Whereas we're trying to get them to transfer that into their head, I like that. But you've also got the working out to check but you're trying to get them to do it in their head first, I really like that. (Interview)* Deryn's comments indicate that she was experimenting with a new pedagogical approach, one that gave opportunity for students to develop agency. Her classroom actions suggest early changes in her praxeology. **ENACTING NEW KNOWLEDGE (SCK & KCT)**: Immediately after the lesson, Deryn was keen to share her teaching experience. She was excited and positive about student learning in the lesson (Researcher's Journal). She shared an example of student thinking she was particularly impressed with, a student who had applied their knowledge of number facts to subtract rather than relying on place value to partition and subtract parts of the number mentally e.g. 84 – 46 as 84 – 44 – 2. The question discussed was the final question in the string and had purposively been designed to ignite student thinking about relational thinking driving the strategies used to compute. Deryn seemed unaware that the student had applied the bridging strategy to jump through a friendly number, suggesting that some of the content knowledge was new learning (SCK). Deryn had purposively found me after the lesson, not only to share the positive learning experience, but also to discuss some uncertainty with pedagogical content knowledge, specifically KCT. She was unsure if she had used the empty number line (ENL) correctly and wanted to clarify the direction of the jump and the position of the minuend when subtracting. This information was provided in the teacher resource book, which suggested that perhaps she had only had time to access the lesson resources discussed and prepared at the planning meeting. Her questions suggested that although she had gained confidence to experiment with the approach in her classroom, both the content and pedagogy were new learning, she was at the beginning of a learning process.
3	DP to DC	Reflection	When the teacher noticed and/or reflected on	Reflects on student learning outcomes: student affect; student development of mathematical	**STUDENT LEARNING – LANGUAGE TO ARTICULATE THINKING**: She highlighted student development of mathematical vocabulary to articulate thinking and to reason as positive outcomes: *…I think it's great that the kids can now define words like efficient and mental, things like that. I think it's*

Arrow	Domain Link	Mediating process	Criteria	Summary	Examples of learning processes
			something that they or their students did in their teaching practice that caused specific outcomes e.g. student learning, teacher use of visual representations, student motivation.	language articulate thinking; improvement in accuracy in mental computation. Reflects on outcomes in terms of her own professional learning: how to use number strings as a tool to build on student knowledge.	*great that they're using some of that technical vocab. For some of the kids that need a lot of repetitive learning, it's been really good for them. (Interview)* *…And figuring out an answer first…and then reasoning how you got to that answer in your head. Not just, 'I just did it'. So getting kids to actually explain how they got the answer. (Interview)* The concept of efficiency and critical discussion of strategies was a new learning experience for the students. Initially students had struggled to articulate their thinking. Deryn enjoyed retelling experiences at the recent Parent-Student-Teacher interviews, it seemed that the parents were equally impressed with the way the children were able to articulate their thinking and explain their learning about mental strategies for addition and subtraction. She commented on how many students discussed their learning at the parent evening with enthusiasm, the unit had been a positive learning experience (Researcher's Journal). **STUDENT AFFECT (engagement, enjoyment, enthusiasm)**: At a planning meeting Deryn commented on student engagement and enjoyment in the lessons, she noted that it was the opportunity for whole class discussion, in particular the comparing of different strategies which they seemed to enjoy most. She had also noticed that students were beginning to look carefully and the numbers in questions and think about how they might solve a problem. **STUDENT LEARNING (ACCURACY & RELATIONAL THNKING):** The results from the student assessment task completed at the end of the term, several weeks after the end of the intervention, confirmed Deryn's classroom observations of student learning and appeared another salient outcome for Deryn: *…Yeah, and just seeing your kids succeed. Like when you said, this many percent answered accurately whereas before…that's great reinforcement that you are on the right track and doing the right thing and that the kids are learning. (Interview)* The students demonstrated significant improvement in accuracy with mental computation tasks. In addition, students also demonstrated relational thinking (Wright et al., 2012) showing appropriate use of different strategies for particular questions.
4	DC to PD	Reflection	When teachers reflected on a specific outcome, thus changing a specific aspect of their knowledge or disposition on teaching. When a teacher's	Deryn reflected on changes in her Pedagogical Content Knowledge (PCK), specifically changes in KCT and SCK, as a result of classroom experimentation. Her reflections also indicated change in her disposition.	**CHANGE IN PCK:** Reflecting on the most valuable thing she had learned, Deryn considered development in her PCK (KCT), which she largely attributed to learning to use Number Strings as an instructional tool: *…I suppose for me it's been the building on knowledge. I liked that we've actually gone quite in depth and we've built on. I like the strings that you start with an easier version and then make it harder, if that makes sense…but I suppose the number strings is the bit I've taken out of it, as how to build on their knowledge if that makes sense. (Interview)* She recognised the value in building on existing student knowledge to foster connections with new strategies (Murphy, 2004). This reflection indicates that she is internalising a new approach to teaching mental computation and that her praxeology on how students learn to compute mentally was beginning

Arrow	Domain Link	Mediating process	Criteria	Summary	Examples of learning processes
			evaluative reflection on salient outcomes led to a change in knowledge or disposition.		to align with that of the researcher. She further elaborated giving examples of questions she might ask to elicit student thinking, a significant change from explicit modelling approach her students were more familiar with: *…Or building on it and going, "Well, what's this step? How can I use that to work out this?" (Interview)* **CHANGE IN HER DISPOSTION:** Deryn's reflection on salient outcomes in relation to improvement in student learning of mental computation and student affect, seemed to connect to a change in her disposition. Her comment suggests a change in her attitude regarding maths in the classroom and everyday life: *…I just think actually teaching it. We get bogged down by the processes too much sometimes that we forget about the mental part, when you go into a shop and you give a kid a dollar and twenty cents and they can't add that on….does that make sense? I think that's when you realise the real world is why you teach the mental. But I think sometimes it gets lost or it's just a Tools session. So I think the importance of actually teaching it. (Interview)* Deryn appeared to reconsider the purpose and value in teaching mental computation which seemed to result in the conclusion that there was too much focus on written processes.
5	PD to DP	Enactment	When a specific aspect of teachers' knowledge or disposition influenced something that occurred in their teaching practices.	Enacts new learning about mental computation strategies (KCT) to teach a new sequence of lessons on indirect addition.	**USING NEW PEDAGOGY**: Deryn described how sharing examples of student thinking with the class could be used as a teaching strategy to move forward learning. The students were required to think and make their own decisions about the efficiency of mental strategies: *…you are exposing them to the different strategies makes them go 'aha' that's a more efficient way of doing it than my strategy. That's where your Number Talks and your Number Strings came into it, practising this one strategy and then giving them…I think the process was really good if that makes sense.* *The Number Talks were good for thinking out but then the strings make them practise one certain strategy and then you could see whether they put that into practice when you gave them a real life problem. (Interview)* This pedagogical approach seemed new for Deryn, it was a contrast to explicit modelling of strategies for students to practice, it required students to think and reason about efficiency of strategies. She elaborated further, commenting on the design of the sequence of lessons, in particular the purpose of the different instructional tools. She seemed to recognise value in using strings first to elicit and practise a strategy whereas Number Talks could be used for thinking and reasoning. Deryn commented on changes she perceived in her practice in relation to the process of teaching computation: *…So I suppose it was a way of changing the way we delivered our lessons rather than making it a process, it was more answer first and then reasoning whereas I suppose in the past we've done process then answer. (Focus group)* *…I think it's probably made us more aware of actually making it part of processes, actually doing it in your*

Arrow	Domain Link	Mediating process	Criteria	Summary	Examples of learning processes
					head and valuing the importance of doing it in your head. Not just teaching the process…(Focus group) This new approach involved asking students to thinking mentally first, followed by reasoning to justify their strategy. This appeared to contrast to usual practice of modelling the process or strategy to students first. Deryn's facilitation of the main part of the lesson indicated that she had internalised the approach of allowing students thinking time without explicit instruction. She made use of mini-whiteboards to assist her in selecting examples to feed into the whole class discussion and displayed student thinking visually for the class to discuss. However, when a student shared a strategy she had not anticipated she seemed hesitant with her response. It seemed that aspects of her knowledge (KCT) limited discussion, or curtailed opportunity for a teaching point. In response to the question, 102 – 97, the student explained that they had counted back from 102 (minuend) to the subtrahend (97). Deryn represented the student thinking visually. There was a brief comment that this strategy was counting back but she was not convinced that this was correct. The subsequent whole class discussion was refocused onto the first two examples shared which were what she had anticipated or planned to elicit during the lesson.
6	DP to DC	Reflection	When a teacher noticed and/or reflected on something that they or their students did in their teaching practice that caused specific outcomes e.g. student learning, teacher use of visual representations, student motivation	Reflects on salient outcomes in terms of her own professional learning: deepening of her own knowledge about mental strategies (SCK); student learning and changes in her own disposition.	**PROFESSIONAL LEARNING:DEVELOPMENT OF OWN SCK:** Deryn reflected on her classroom experimentation and commented on development of her knowledge of mental strategies: *…I think it was really explicit for me. I've taught the strategies before but have probably got a lot more used to when to use those strategies, that's what I got out of it the most. I've taught jump and all that but being explicit about the type of problems you can use this strategy for. Whereas trying to make them up in your head sometimes can be like…then you present to the kids and go, 'hang on, you're not supposed to use that strategy for that! (Interview)* *…And I suppose I like the fact that, I've learned a bit more about some of the strategies too as to when you would use them and things like that, that's been really good. Without skipping over, I think quite often in a busy curriculum we're just here's a strategy use it and off you go. And here's the next strategy, use it and off you go. Whereas I suppose this has been a little bit more in depth. (Interview)* In the first excerpt, she reflects on how experimentation with the strategies in her classroom has deepened her knowledge about when to use the strategies, in particular the design of questions to elicit a particular strategy. In the second excerpt, she again reflects on development of her knowledge of when to use particular strategies suggested that she considers this a salient outcome from her teaching. **STUDENT LEARNING:** Deryn described positive outcomes in terms of student learning:

Arrow	Domain Link	Mediating process	Criteria	Summary	Examples of learning processes
					And make kids more aware, or more verbal in actually being able to explain it. (Focus Group) It seemed that students recognised situations when it was appropriate to use mental strategies, as well as being able to explain their thinking. Deryn also reflected on some of the challenges students faced during the lessons. Classroom experimentation with this new approach seemed to highlight the importance of teaching mental computation: *…And they've lost the ability to mentally solve it, I believe. I think that's something we all identified as being a need. That they have actually lost that ability to be able to work it out. (Focus Group)* *…And valued more…(Focus Group)* Her comments suggest that further classroom experimentation reinforced her disposition that mental computation should be considered an important part of the curriculum. This interpretation that her disposition on the importance of mental computation had been internalised was fortified by a further comment during the focus group session: *…And even now the kids are still going back to vertical and I'm like, 'Why?' You can jump 20 and 3 more and you've got the answer. You don't need to be doing vertical. (Focus Group)* It would seem that her experiences had resulted in her forming the view that when faced with a problem, students should first make a decision on whether size of the numbers or complexity are appropriate for a mental or written strategy.

Change sequence analysis: Using criteria to establish relations between domains in the IMPG **ETHAN - SCHOOL B**

Arrow	Domain Link	Mediating process	Criteria	Summary	Examples of learning processes
1	ED to PD	Reflection	When something that was done or discussed during one of the learning experiences modified teachers' initial knowledge or disposition on teaching mental computation.	Reflected on external stimuli: observing the researcher model a sequence of lessons and the opportunity to engage in professional conversations.	**MODELLED LESSONS:** Ethan reflected on the modelled lessons, placing emphasis the importance of observing his students. He explained that with over twenty years of experience as a classroom teacher, it was not observing another practitioner that was of value but the opportunity to observe students (Researcher's Journal). During the modelled lessons he listened carefully to student responses. He focused on connecting the verbal explanations with the mental jottings students recorded on mini-whiteboards to explain and justify their thinking. When reflecting on the most useful thing he learned from participating in the professional learning sessions he commented: *"Seeing the misconceptions of some students*" (Post-professional learning survey). It seemed that this experience had provided him with time to notice, contemplate and discuss student misconceptions. Observing his students appeared to support development of his knowledge about how students learn to compute mentally, conceptualized by Hill, Ball and Schilling (2008) as Knowledge of Content and Students (KCS). It seemed that opportunity to see how to use student thinking and ideas to drive purposeful discussion and move forward learning was also of value. In the post-professional learning survey, Ethan reflected on the sessions and commented that they, *"Made me think more about utilizing student's knowledge/strategies."* He seemed to recognise that sharing and discussing student thinking was a powerful tool to instigate learning of others, he discussed this observation fervently in his semi-structured interview: *...definitely seeing their peers because often as teachers you don't do that. You might work in small groups and sometimes we might show them, whereas you drew it out of them a lot more. (Interview)* The excerpt above suggests that his usual pedagogical approach was more teacher-centred, with him assuming responsibility for providing explanations or modelling of ideas. Ethan also seemed to recognise that sharing and discussing student strategies with the class was an approach that supported students in refining their thinking and developing more efficient strategies for computing mentally: *…Students were able to see other students' methods and this enabled them to understand that there are efficient strategies. (Post-professional learning survey).* Although Ethan had been keen to emphasise that it was the opportunity to observe his students that was most important, it seemed that observing someone teach had also developed aspects of his pedagogical knowledge. When asked about which components of the program had been most useful in terms of his own professional learning, Ethan reflected on the opportunity to learn new teaching ideas: *…I liked you modelling for us, when I saw you do a few lessons early on there were one or two things I picked up on, particularly that game, that auction game. That was only simple but I would never have thought of that before. I guess seeing how you did it, knowing your experience working in a lot of schools and it's an area you are really strong with. So probably that for me. (Focus group)* In the excerpt above, Ethan referred to game designed for students to develop fluency in counting forwards and back in multiples of ten, this was an aspect of learning which challenged the majority of the class in the first lesson.

Arrow	Domain Link	Mediating process	Criteria	Summary	Examples of learning processes
					Through the brokering process, Ethan seemed to gain new pedagogical content knowledge (PCK) by observing and interacting with the students and researcher. The opportunity to observe the modelled lessons seemed to provide Ethan with new pedagogical knowledge. **PROFESSIONAL CONVERSATIONS**: Whilst engaging in professional conversations appeared to stimulate learning for Ethan, it also seemed to activate some uncertainty in Ethan's disposition. There appeared some internal conflict in relation to his disposition about students learning mathematics; he seemed uncertain how this new approach fitted within his current schema for teaching. Ethan asked for some clarification on the end goals for the project, in terms of student learning, and seemed slightly disconcerted with students being exposed to a range of strategies. He expressed this concern through an analogy of a sports person learning to play golf. He explained that when someone learns a new golfing technique this does not necessarily lead to a positive outcome, the pressure of the game often results in the player reverting back to familiar, less effective techniques. It seemed that Ethan was yet to notice the subtle difference between the classroom situation and his analogy; in the classroom students were not being shown how to perform a new technique or strategy, they were constructing the ideas themselves by thinking and reasoning about the ideas of their peers. They were essentially learning based on social constructivist principles (Fosnot, 1996).
2	PD to DP	Enactment	When a specific aspect of teachers' knowledge or disposition influenced something that occurred in their teaching practices.	Ethan enacted new pedagogical content knowledge (PCK) in the classroom. He experimented with teaching lessons focused on the jump strategy to subtract and bridging to add mentally. He focused on facilitating whole class purposeful discussion, specifically students articulating thinking verbally and using visual representations to stimulate learning of peers. He expressed some negativity towards change, his classroom management skills seemed to interfere with implementation of the new approach.	**NEW PCK – Students articulating thinking & use of visual representations to stimulate learning**: Ethan focused on applying new pedagogical knowledge to encourage students to explain strategies to stimulate further learning. He described how the change in practice required students to explain their thinking and reason: *…A lot of the kids had trouble actually….ah…I just know it. And they weren't actually sure what to write down so I think more of that… I sometimes use the word Martian, someone who lands on earth, and they've got no idea, you have to go from scratch, so by them doing that they have to really think what they're doing. So I think that's really valuable. (Interview)* Although some students initially found this challenging, Ethan seemed to consider the approach a valuable learning process, it required students to think deeply and justify their processes. This suggested that through classroom experimentation, Ethan was beginning to internalise the new approach. Ethan described how changes in his pedagogical approach, specifically sharing student thinking visually, allowed opportunities for students to learn from each other: *…I guess, maybe, seeing different strategies that other kids use and a couple of kids swapped because of that. I think they saw their peers doing things a certain way...And then they had seen some of their peers, their friends do things shorter so they got a lot from seeing what others had done. (Interview)* He described changes in how students were learning; they were beginning to experiment with new ideas shared by their peers and refine their thinking to become more efficient. An approach that promoted students learning from their peers through whole class discussion seemed to contrast with Ethan's usual practice. His regular routine was to provide direct instruction to students in small groups, which were engineered based on assessment results. **NEW PCK – Transition to whole class teaching:** Ethan experimented with enacting new pedagogical knowledge to teach using whole class discussion. His experimentation appeared to stimulate some negativity towards change. The following excerpts suggest that Ethan faced some challenges with his transition from small group

Arrow	Domain Link	Mediating process	Criteria	Summary	Examples of learning processes
					teaching to a whole class approach. The second excerpt is connected to the first, and shows his justification for teaching small groups: *…I guess, maybe I would use it more in small groups at the start…it is time consuming. In the ideal world it would be great to do that for longer but I'd probably just do it as group work. (Interview)* *…I liked the sequence. I guess, again it's another managerial thing, it would be a lot easier if it was done with half the class even…..or not in groups. I've got them in four groups, which varies a bit each time we test, I group my kids. (Interview)* His comments suggest that the culture of teaching in small groups, an approach that was adopted by the school as a result of their involvement in a Math Recovery research project in unison wit Catholic Education Melbourne (CEM), seemed embedded in his classroom. **NEW PCK – negative attitude to transition to whole class teaching:** A whole class approach to teaching, seemed a new pedagogical approach for Ethan, one that presented him with some challenges in relation to classroom management: *…and I know this is just a management thing in my class but using the boards can be a bit distracting at times. I use the boards with a small group on the floor where it is easy to control. It's good, I like the idea of it but sometimes some kids can fiddle around so it can be a bit distracting. (Interview)* *…because if you've got 26 kids holding up their boards and waiting you are going to lose half of them. It's just a managerial side of it and that's not a criticism, it's great what's in here but I think it would be best to target a smaller group. (Interview)* Although Ethan reflected positively when he observed someone else implement the approach with his class, his experimentation seemed to evoke some negative attitudes towards change. The approach modelled involved students using the boards to explain their thinking visually to a partner, after they had individually engaged with the problem mentally. The mini-whiteboards were useful in allowing the teacher to circulate and select appropriate examples, including misconceptions, to orchestrate whole class discussion. His reflection suggests classroom management skills interfered with using mini-whiteboards as a tool to support the pedagogical approach. He was yet to establish use of techniques such as learning partners to help manage whole class discussion more effectively, or establish systems that did not involve the whole class expecting to share their examples of thinking individually. Implementing such systems might also support more effective time management of whole class discussion (Clarke, 2014; Wiliam, 2011).
3	DP to DC	Reflection	When the teacher noticed and/or reflected on something that they or their students did in their teaching practice that caused specific outcomes e.g. student learning,	Reflected on salient outcomes in relation to student learning, namely their development of a range of computation strategies and improved efficiency. Reflected on outcomes in	**STUDENT LEARNING OUTCOMES:** Reflecting on student learning outcomes, Ethan seemed impressed by the multitude of strategies that students explained: *…I was slightly surprised, no pleasantly surprised at how many strategies the kids were using…. I think that's a good thing. It was a bit of an insight into the way kids think a little, I thought that was good. (Interview)* His reflection indicated a little uncertainty about the range of strategies being a positive outcome. Instead it seemed that the opportunity to develop his own knowledge about how students think and learn to compute mentally, KCS, appeared a more positive outcome. He elaborated on this further, in his comments about

Arrow	Domain Link	Mediating process	Criteria	Summary	Examples of learning processes
			teacher use of visual representations, student motivation.	relation to his own professional learning, specifically extending aspects of his pedagogical knowledge (KCS). Positive student learning outcomes appeared to create internal conflict with how he could best meet student learning needs and perceived curriculum responsibilities, within a given time frame.	developing awareness of student misconceptions. **DEVELOPING KCS:** Ethan's perception of the impact of a change in his pedagogical approach seemed to raise awareness of misconceptions in student learning he had not anticipated: *…To see how kids think. Yeah, there were a few surprises there and a few misconceptions which I would never have thought before to investigate. (Interview)* In his interview he appeared to view this as a positive outcome, he was intrigued about different ways students think and he seemed keen to extend his knowledge on how students learn. Ethan reflected further on outcomes in relation to student learning. He seemed to be in the early stages of developing his knowledge about how students learn to compute mentally (KCS). During the interview he highlighted how many students had made a common error with subtraction because they tend to swap over digits in the ones place value position in both the minuend and subtrahend. He explained that in swapping the digits they are able to subtract the smaller digit form the larger one. However, the errors the students were making were predominantly the result of an over reliance on partitioning both numbers in the equation based on place value and not understanding how to regroup the number. In the example discussed in the interview, it was not a case of a student swapping the digit in the ones place so they could subtract but a misconception that all numbers must be subtracted when the operation in the equation is subtraction. The example discussed in the interview is shown below in Figure 5.3. **DISPOSITION – Positive attitude towards mental computation:** Experimenting with teaching mental computation led Ethan to reflect on the importance of student's learning to compute mentally: *…but maybe mental maths needs more of a priority. And I am a believer in it because of that statistic I've been told, that 80% of our maths we use in real life is mental maths. So it doesn't get 80% of our maths time, we need to do a lot more of it. So maybe, when it is in a program, almost in a program set out in a lesson plan that's something we could incorporate into maths. We teach number, we teach chance and data, we teach measurement, why not mental maths? Does it need to be recognised as a stand alone? Does that sound too grandiose? But it certainly makes sense to me. (Interview)* *…Maybe mental maths needs to be mandatory, like I've said, or mandated a certain about of hours per week, I think. (Interview)* His reflection indicated that he recognised value in teaching students to compute mentally, suggesting that mental computation needed a greater focus in the mathematics program to ensure that students were given this opportunity. It seemed that classroom experimentation had led to a change in his disposition, indicating that he had developed his understanding that learning 'mental maths' involves more than simply recalling knowledge of number facts. Whilst Ethan recognised the benefits for students from learning to compute mentally, the enactment of the new approach appeared to create internal conflict in relation to his disposition: *…Obviously we'll have to do more on written forms, which we do have to teach. (Interview)*

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					He seemed to perceive the teaching of written calculation methods as his responsibility as a teacher. In his interview response he placed emphasis on the words, "*we do have to teach".* His comment seemed to reflect his conceptualisation of mathematics, rather than the perspective shared by the Mathematics Leader who was an advocate of the Learning Framework in Number developed by Wright and colleagues. It is possible that his disposition concerning the conceptualisation of mathematics was influenced by institutional aspects. This concurs with the argument presented by Hiebert (2013) that mathematics teaching of mathematics can be viewed as a cultural activity; in this case the teaching of written methods in middle primary years is a practice that has been passed down through generations in Australia. It seemed that this conflict was intensified by a limited time frame in which to teach addition and subtraction. When asked to reflect explicitly on constraints with the approach, he responded with: *…The number one reason would be time constraint. I don't know what it's like in England but they always use the word the 'crowded curriculum' but maybe mental maths needs more of a priority. (Interview)* Whilst he recognised the benefits for students of learning to compute mentally, it seemed that time constraints meant it was difficult to comprehend how mental computation could be integrated into the mathematics program when there was a perceived requirement to teach written methods. Ethan reflected on the outcomes of his classroom experimentation recognising benefits in terms of student learning and his own professional learning. He recognised that students were developing a range of computation strategies. Enacting his new pedagogical knowledge appeared to instigate opportunities for him to develop his KCS, he reflected on extending his knowledge about how students learn and their misconceptions. His experimentation with this new approach seemed to create internal conflict in relation to his disposition, it led him to reflect on curriculum content and his responsibilities as a teacher. He perceived the teaching of written methods as his responsibility and seemed perplexed about how both could be integrated into the teaching program within the given time frame. Recognising the benefits to students from learning to compute mentally seemed to have left him in a quandary in terms of meeting these needs and fulfilling his perceived teacher responsibilities.
4	DC to DP	Enactment	When a specific outcome made the teacher change their practice at that moment (reflection-in-action). When a specific outcome made teachers state how they would modify the associated teaching practice in the future.	Enacted new knowledge of pedagogical approaches to teach a new sequence of lessons on indirect addition.	**CHANGE IN PEDAGOGICAL APPROACH – lesson structure:** Ethan appeared to have internalised the lesson structure, an indication that his praxeology was changing. This was evident by his decision to start the lesson with a short task to activate mathematical thinking and develop fluency. Ethan had reflected on the fluency tasks designed for the modelled lessons, specifically how they were effective engaging the students and providing an opportunity to develop fluency with basic knowledge and skills (refer to interview excerpt in section 5.2.4.1.). It seemed that through the brokering process Ethan had adopted this pedagogical approach. Although the task he designed was only loosely connected with the learning focus for the lesson, it did foster student engagement and capture interest for the rest of the lesson. **CHANGE IN PEDAGOGICAL APPROACH – utilising student thinking:** Allowing students individual thinking time to engage with a task before asking them to contribute to whole class discussion was another pedagogical approach Ethan was seen to assimilate in his teaching. He had adopted the practice of asking students to indicate they were ready to contribute a response by showing a 'thumbs up' close to their chest, showing awareness of the importance of strategies that enhanced opportunities for active participation from all students (Wiliam, 2011).

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					CHANGE IN PEDAGOGICAL APPROACH – facilitating whole class discussion: A key focus of the project had been to use visual representations to show student thinking, initiate discussion and move forward learning. This was a component Ethan reflected on as being important when observing the modelled lessons, and one he had discussed experimenting with during the first phase of the intervention. However, in the lesson observed Ethan chose not to implement this component, instead the discussion was based on oral explanations. Students explanations were at times lengthy and difficult to comprehend without the use of mental jottings or visual representations to clarify ideas, it was therefore not surprising that some students found it difficult to remain focused. Whilst Ethan may have been clear about the intended learning goal, orchestrating the discussion towards achieving this goal presented some challenges. He did not appear to implement the five practices described by Stein et al. (2008) for facilitating productive discussion (anticipate, monitor, select, sequence, connect) which seemed to make it difficult to move the discussion beyond an ad hoc selection of student ideas. Although the students had access to mini-whiteboards there was not a system in place to use these to actively monitor student responses. Students were also not instructed to explain their thinking to a partner (talk partner or learning partner) following individual thinking time so there was not an opportunity for Ethan to circulate and monitor student thinking using this strategy. It was possible that challenges he described earlier with managing whole class discussion and resources had deterred him from adopting this approach, he was yet to establish clear routines and systems with his class. This resonates with the work of Brown and Coles (2013) who used the notion of relentless consistency to establish classroom practices or a classroom culture that foster student agency and allow ambitious teaching practices (Kazemi et al., 2009). It should be emphasised that Ethan was in the early stages of experimenting with this new approach, components of which have been referred to as "ambitious instruction" (Kazemi et al., 2009). It therefore seems reasonable to expect that enacting such approaches effectively would not only involve understanding the intricacies involved but require time to practice and refine.
5	DP to DC	Reflection	When a teacher noticed and/or reflected on something that they or their students did in their teaching practice that caused specific outcomes e.g. student learning, teacher use of visual representations, student motivation	Reflects on salient outcomes, he recognised the benefits of a change in practice in terms of both student learning and his own learning.	Ethan reflected on the outcome of the changes he perceived in his practice: *…So it made us make them to tell us what they actually did. So they never thought about it. (Focus group)* *…It made me think more about how many different strategies kids used, I probably wouldn't have gone as deep before. A couple of kids surprised me how they thought. (Focus group)* The first excerpt suggests that the approach required students to think differently, it seemed that explaining thinking and reasoning about strategies was not something that Ethan had asked students to do pre-intervention. The second excerpt indicates that asking students to explain strategies and reason about efficiency provided an opportunity to gain greater insights into student thinking processes. Ethan seemed to recognise that changing his pedagogical approach was beneficial for students and something he should consider to use more in the future: *…but now it would probably make me think a bit more into extracting from the kids how did they get their answer a little bit more. And share it more with the others. (Focus group)* *…I think the kids have learnt off others more now, that's my observation. (Focus group)* He reflected on how this change in practice involved eliciting student thinking processes and using student ideas

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					to drive whole class discussion. His comment indicates an intention to do this more regularly, suggesting that he recognised value in the change in approach. The idea of students constructing their ideas by looking for patterns, critically reflecting and discussing efficiency seemed to appeal to Ethan as an effective way for students to learn: *…I guess, I know it's only addition and subtraction, but I'd like them to be able to think and apply this to multiplication and division. I know they are different but I still think we can use the word 'efficient,' we can look for patterns. So we could take some lessons from this and use it in other parts of maths. (Interview)* His reflection suggests that there are aspects of the pedagogical approach that Ethan would like to see applied to other areas of mathematics, an indication that outcomes from his classroom experimentation were salient to influence changes in future practice.

Change sequence analysis: Using criteria to establish relations between domains in the IMPG **FIONA - SCHOOL B**

Arrow	Domain Link	Mediating process	Criteria	Summary	Examples of learning processes
1	ED to PD	Reflection	When something that was done or discussed during one of the learning experiences modified teachers' initial knowledge or disposition on teaching mental computation.	Reflected on external stimuli: observing the researcher model a sequence of lessons and the opportunity to engage in professional conversations.	**MODELLED LESSONS:** Ethan reflected on the modelled lessons, placing emphasis the importance of observing his students. He explained that with over twenty years of experience as a classroom teacher, it was not observing another practitioner that was of value but the opportunity to observe students (Researcher's Journal). During the modelled lessons he listened carefully to student responses. He focused on connecting the verbal explanations with the mental jottings students recorded on mini-whiteboards to explain and justify their thinking. When reflecting on the most useful thing he learned from participating in the professional learning sessions he commented: *"Seeing the misconceptions of some students*" (Post-professional learning survey). It seemed that this experience had provided him with time to notice, contemplate and discuss student misconceptions. Observing his students appeared to support development of his knowledge about how students learn to compute mentally, conceptualized by Hill, Ball and Schilling (2008) as Knowledge of Content and Students (KCS). It seemed that opportunity to see how to use student thinking and ideas to drive purposeful discussion and move forward learning was also of value. In the post-professional learning survey, Ethan reflected on the sessions and commented that they, *"Made me think more about utilising student's knowledge/strategies."* He seemed to recognise that sharing and discussing student thinking was a powerful tool to instigate learning of others, he discussed this observation fervently in his semi-structured interview: *...definitely seeing their peers because often as teachers you don't do that. You might work in small groups and sometimes we might show them, whereas you drew it out of them a lot more. (Interview)* The excerpt above suggests that his usual pedagogical approach was more teacher-centred, with him assuming responsibility for providing explanations or modelling of ideas. Ethan also seemed to recognise that sharing and discussing student strategies with the class was an approach that supported students in refining their thinking and developing more efficient strategies for computing mentally: *…Students were able to see other students' methods and this enabled them to understand that there are efficient strategies. (Post-professional learning survey).* Although Ethan had been keen to emphasise that it was the opportunity to observe his students that was most important, it seemed that observing someone teach had also developed aspects of his pedagogical knowledge. When asked about which components of the program had been most useful in terms of his own professional learning, Ethan reflected on the opportunity to learn new teaching ideas: *…I liked you modelling for us, when I saw you do a few lessons early on there were one or two things I picked up on, particularly that game, that auction game. That was only simple but I would never have thought of that before. I guess seeing how you did it, knowing your experience working in a lot of schools and it's an area you are really strong with. So probably that for me. (Focus group)* In the excerpt above, Ethan referred to game designed for students to develop fluency in counting forwards and back in multiples of ten, this was an aspect of learning which challenged the majority of the class in the first lesson.

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					Through the brokering process, Ethan seemed to gain new pedagogical content knowledge (PCK) by observing and interacting with the students and researcher. The opportunity to observe the modelled lessons seemed to provide Ethan with new pedagogical knowledge. **PROFESSIONAL CONVERSATIONS**: Whilst engaging in professional conversations appeared to stimulate learning for Ethan, it also seemed to activate some uncertainty in Ethan's disposition. There appeared some internal conflict in relation to his disposition about students learning mathematics; he seemed uncertain how this new approach fitted within his current schema for teaching. Ethan asked for some clarification on the end goals for the project, in terms of student learning, and seemed slightly disconcerted with students being exposed to a range of strategies. He expressed this concern through an analogy of a sports person learning to play golf. He explained that when someone learns a new golfing technique this does not necessarily lead to a positive outcome, the pressure of the game often results in the player reverting back to familiar, less effective techniques. It seemed that Ethan was yet to notice the subtle difference between the classroom situation and his analogy; in the classroom students were not being shown how to perform a new technique or strategy, they were constructing the ideas themselves by thinking and reasoning about the ideas of their peers. They were essentially learning based on social constructivist principles (Fosnot, 1996).
2	PD to DP	Enactment	When a specific aspect of teachers' knowledge or disposition influenced something that occurred in their teaching practices.	Ethan enacted new pedagogical content knowledge (PCK) in the classroom. He experimented with teaching lessons focused on the jump strategy to subtract and bridging to add mentally. He focused on facilitating whole class purposeful discussion, specifically students articulating thinking verbally and using visual representations to stimulate learning of peers. He expressed some negativity towards change, his classroom management skills seemed to interfere with implementation of the new approach.	**NEW PCK – Students articulating thinking & use of visual representations to stimulate learning**: Ethan focused on applying new pedagogical knowledge to encourage students to explain strategies to stimulate further learning. He described how the change in practice required students to explain their thinking and reason: *…A lot of the kids had trouble actually….ah…I just know it. And they weren't actually sure what to write down so I think more of that… I sometimes use the word Martian, someone who lands on earth, and they've got no idea, you have to go from scratch, so by them doing that they have to really think what they're doing. So I think that's really valuable. (Interview)* Although some students initially found this challenging, Ethan seemed to consider the approach a valuable learning process, it required students to think deeply and justify their processes. This suggested that through classroom experimentation, Ethan was beginning to internalise the new approach. Ethan described how changes in his pedagogical approach, specifically sharing student thinking visually, allowed opportunities for students to learn from each other: *…I guess, maybe, seeing different strategies that other kids use and a couple of kids swapped because of that. I think they saw their peers doing things a certain way...And then they had seen some of their peers, their friends do things shorter so they got a lot from seeing what others had done. (Interview)* He described changes in how students were learning; they were beginning to experiment with new ideas shared by their peers and refine their thinking to become more efficient. An approach that promoted students learning from their peers through whole class discussion seemed to contrast with Ethan's usual practice. His regular routine was to provide direct instruction to students in small groups, which were engineered based on assessment results. **NEW PCK – Transition to whole class teaching:** Ethan experimented with enacting new pedagogical knowledge to teach using whole class discussion. His experimentation appeared to stimulate some negativity towards change. The following excerpts suggest that Ethan faced some challenges with his transition from small group

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					teaching to a whole class approach. The second excerpt is connected to the first, and shows his justification for teaching small groups: *…I guess, maybe I would use it more in small groups at the start…it is time consuming. In the ideal world it would be great to do that for longer but I'd probably just do it as group work. (Interview)* *…I liked the sequence. I guess, again it's another managerial thing, it would be a lot easier if it was done with half the class even…..or not in groups. I've got them in four groups, which varies a bit each time we test, I group my kids. (Interview)* His comments suggest that the culture of teaching in small groups, an approach that was adopted by the school as a result of their involvement in a Math Recovery research project in unison wit Catholic Education Melbourne (CEM), seemed embedded in his classroom. **NEW PCK – negative attitude to transition to whole class teaching:** A whole class approach to teaching, seemed a new pedagogical approach for Ethan, one that presented him with some challenges in relation to classroom management: *…and I know this is just a management thing in my class but using the boards can be a bit distracting at times. I use the boards with a small group on the floor where it is easy to control. It's good, I like the idea of it but sometimes some kids can fiddle around so it can be a bit distracting. (Interview)* *…because if you've got 26 kids holding up their boards and waiting you are going to lose half of them. It's just a managerial side of it and that's not a criticism, it's great what's in here but I think it would be best to target a smaller group. (Interview)* Although Ethan reflected positively when he observed someone else implement the approach with his class, his experimentation seemed to evoke some negative attitudes towards change. The approach modelled involved students using the boards to explain their thinking visually to a partner, after they had individually engaged with the problem mentally. The mini-whiteboards were useful in allowing the teacher to circulate and select appropriate examples, including misconceptions, to orchestrate whole class discussion. His reflection suggests classroom management skills interfered with using mini-whiteboards as a tool to support the pedagogical approach. He was yet to establish use of techniques such as learning partners to help manage whole class discussion more effectively, or establish systems that did not involve the whole class expecting to share their examples of thinking individually. Implementing such systems might also support more effective time management of whole class discussion (Clarke, 2014; Wiliam, 2011).
3	DP to DC	Reflection	When the teacher noticed and/or reflected on something that they or their students did in their teaching practice that caused specific outcomes e.g. student learning,	Reflected on salient outcomes in relation to student learning, namely their development of a range of computation strategies and improved efficiency. Reflected on outcomes in	**STUDENT LEARNING OUTCOMES:** Reflecting on student learning outcomes, Ethan seemed impressed by the multitude of strategies that students explained: *…I was slightly surprised, no pleasantly surprised at how many strategies the kids were using…. I think that's a good thing. It was a bit of an insight into the way kids think a little, I thought that was good. (Interview)* His reflection indicated a little uncertainty about the range of strategies being a positive outcome. Instead it seemed that the opportunity to develop his own knowledge about how students think and learn to compute mentally, KCS, appeared a more positive outcome. He elaborated on this further, in his comments about

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			teacher use of visual representations, student motivation.	relation to his own professional learning, specifically extending aspects of his pedagogical knowledge (KCS). Positive student learning outcomes appeared to create internal conflict with how he could best meet student learning needs and perceived curriculum responsibilities, within a given time frame.	developing awareness of student misconceptions. **DEVELOPING KCS:** Ethan's perception of the impact of a change in his pedagogical approach seemed to raise awareness of misconceptions in student learning he had not anticipated: *…To see how kids think. Yeah, there were a few surprises there and a few misconceptions which I would never have thought before to investigate. (Interview)* In his interview he appeared to view this as a positive outcome, he was intrigued about different ways students think and he seemed keen to extend his knowledge on how students learn. Ethan reflected further on outcomes in relation to student learning. He seemed to be in the early stages of developing his knowledge about how students learn to compute mentally (KCS). During the interview he highlighted how many students had made a common error with subtraction because they tend to swap over digits in the ones place value position in both the minuend and subtrahend. He explained that in swapping the digits they are able to subtract the smaller digit form the larger one. However, the errors the students were making were predominantly the result of an over reliance on partitioning both numbers in the equation based on place value and not understanding how to regroup the number. In the example discussed in the interview, it was not a case of a student swapping the digit in the ones place so they could subtract but a misconception that all numbers must be subtracted when the operation in the equation is subtraction. The example discussed in the interview is shown below in Figure 5.3. **DISPOSITION – Positive attitude towards mental computation:** Experimenting with teaching mental computation led Ethan to reflect on the importance of student's learning to compute mentally: *…but maybe mental maths needs more of a priority. And I am a believer in it because of that statistic I've been told, that 80% of our maths we use in real life is mental maths. So it doesn't get 80% of our maths time, we need to do a lot more of it. So maybe, when it is in a program, almost in a program set out in a lesson plan that's something we could incorporate into maths. We teach number, we teach chance and data, we teach measurement, why not mental maths? Does it need to be recognised as a stand alone? Does that sound too grandiose? But it certainly makes sense to me. (Interview)* *…Maybe mental maths needs to be mandatory, like I've said, or mandated a certain about of hours per week, I think. (Interview)* His reflection indicated that he recognised value in teaching students to compute mentally, suggesting that mental computation needed a greater focus in the mathematics program to ensure that students were given this opportunity. It seemed that classroom experimentation had led to a change in his disposition, indicating that he had developed his understanding that learning 'mental maths' involves more than simply recalling knowledge of number facts. Whilst Ethan recognised the benefits for students from learning to compute mentally, the enactment of the new approach appeared to create internal conflict in relation to his disposition: *…Obviously we'll have to do more on written forms, which we do have to teach. (Interview)*

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					He seemed to perceive the teaching of written calculation methods as his responsibility as a teacher. In his interview response he placed emphasis on the words, "*we do have to teach".* His comment seemed to reflect his conceptualisation of mathematics, rather than the perspective shared by the Mathematics Leader who was an advocate of the Learning Framework in Number developed by Wright and colleagues. It is possible that his disposition concerning the conceptualisation of mathematics was influenced by institutional aspects. This concurs with the argument presented by Hiebert (2013) that mathematics teaching of mathematics can be viewed as a cultural activity; in this case the teaching of written methods in middle primary years is a practice that has been passed down through generations in Australia. It seemed that this conflict was intensified by a limited time frame in which to teach addition and subtraction. When asked to reflect explicitly on constraints with the approach, he responded with: *…The number one reason would be time constraint. I don't know what it's like in England but they always use the word the 'crowded curriculum' but maybe mental maths needs more of a priority. (Interview)* Whilst he recognised the benefits for students of learning to compute mentally, it seemed that time constraints meant it was difficult to comprehend how mental computation could be integrated into the mathematics program when there was a perceived requirement to teach written methods. Ethan reflected on the outcomes of his classroom experimentation recognising benefits in terms of student learning and his own professional learning. He recognised that students were developing a range of computation strategies. Enacting his new pedagogical knowledge appeared to instigate opportunities for him to develop his KCS, he reflected on extending his knowledge about how students learn and their misconceptions. His experimentation with this new approach seemed to create internal conflict in relation to his disposition, it led him to reflect on curriculum content and his responsibilities as a teacher. He perceived the teaching of written methods as his responsibility and seemed perplexed about how both could be integrated into the teaching program within the given time frame. Recognising the benefits to students from learning to compute mentally seemed to have left him in a quandary in terms of meeting these needs and fulfilling his perceived teacher responsibilities.
4	DC to DP	Enactment	When a specific outcome made the teacher change their practice at that moment (reflection-in-action). When a specific outcome made teachers state how they would modify the associated teaching practice in the future.	Enacted new knowledge of pedagogical approaches to teach a new sequence of lessons on indirect addition.	**CHANGE IN PEDAGOGICAL APPROACH – lesson structure:** Ethan appeared to have internalised the lesson structure, an indication that his praxeology was changing. This was evident by his decision to start the lesson with a short task to activate mathematical thinking and develop fluency. Ethan had reflected on the fluency tasks designed for the modelled lessons, specifically how they were effective engaging the students and providing an opportunity to develop fluency with basic knowledge and skills (refer to interview excerpt in section 5.2.4.1.). It seemed that through the brokering process Ethan had adopted this pedagogical approach. Although the task he designed was only loosely connected with the learning focus for the lesson, it did foster student engagement and capture interest for the rest of the lesson. **CHANGE IN PEDAGOGICAL APPROACH – utilising student thinking:** Allowing students individual thinking time to engage with a task before asking them to contribute to whole class discussion was another pedagogical approach Ethan was seen to assimilate in his teaching. He had adopted the practice of asking students to indicate they were ready to contribute a response by showing a 'thumbs up' close to their chest, showing awareness of the importance of strategies that enhanced opportunities for active participation from all students (Wiliam, 2011).

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5	DP to DC	Reflection	When a teacher noticed and/or reflected on something that they or their students did in their teaching practice that caused specific outcomes e.g. student learning, teacher use of visual representations, student motivation	Reflects on salient outcomes, he recognised the benefits of a change in practice in terms of both student learning and his own learning.	Ethan reflected on the outcome of the changes he perceived in his practice: *…So it made us make them to tell us what they actually did. So they never thought about it. (Focus group)* *…It made me think more about how many different strategies kids used, I probably wouldn’t have gone as deep before. A couple of kids surprised me how they thought. (Focus group)* The first excerpt suggests that the approach required students to think differently, it seemed that explaining thinking and reasoning about strategies was not something that Ethan had asked students to do pre-intervention. The second excerpt indicates that asking students to explain strategies and reason about efficiency provided an opportunity to gain greater insights into student thinking processes. Ethan seemed to recognise that changing his pedagogical approach was beneficial for students and something he should consider to use more in the future: *…but now it would probably make me think a bit more into extracting from the kids how did they get their answer a little bit more. And share it more with the others. (Focus group)* *…I think the kids have learnt off others more now, that’s my observation. (Focus group)* He reflected on how this change in practice involved eliciting student thinking processes and using student ideas

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					to drive whole class discussion. His comment indicates an intention to do this more regularly, suggesting that he recognised value in the change in approach. The idea of students constructing their ideas by looking for patterns, critically reflecting and discussing efficiency seemed to appeal to Ethan as an effective way for students to learn: *…I guess, I know it's only addition and subtraction, but I'd like them to be able to think and apply this to multiplication and division. I know they are different but I still think we can use the word 'efficient,' we can look for patterns. So we could take some lessons from this and use it in other parts of maths. (Interview)* His reflection suggests that there are aspects of the pedagogical approach that Ethan would like to see applied to other areas of mathematics, an indication that outcomes from his classroom experimentation were salient to influence changes in future practice.

Change sequence analysis: Using criteria to establish relations between domains in the IMPG **GISELLE - SCHOOL B**

Arrow	Domain Link	Mediating process	Criteria	Summary	Examples of learning processes
1	ED to PD	Reflection	When something that was done or discussed during one of the learning experiences modified teachers' initial knowledge or disposition on teaching mental computation.	Reflects on external stimuli: modelled lessons, professional conversations, teaching resources and collaborative planning. She has new pedagogical content knowledge to facilitate student centred learning of mental computation strategies.	**MODELLED LESSONS & PROFESSIONAL CONVERSATIONS:** The opportunity to observe a sequence of modelled lessons and engage in professional conversations appeared to provide an external stimulus for Giselle. *…I think that's why having you in the classroom was nice because we could sort of say, well you know we need a pull out group or we need the lower kids on the floor so to have you in there, I felt that was useful as a grad, because you know sometimes you need to adapt your lessons or whatever so to be able to have you in there and talk us through what you're doing and then we can talk about what we think, just the discussion was useful. (Focus Group)* Reflecting on this opportunity, it is interesting that Giselle predominantly focused on the professional discussions that occurred during the lessons. Whilst the instructional teaching was modelled, I was conscious of articulating and inviting Giselle into the decision-making process throughout the lesson. Being aware that she was a graduate teacher, the intention was to provide her with some insight into the pedagogical approach, namely use of student thinking and ideas to drive the direction of the lesson and instigate further learning. The brokering process seemed to support her internalisation of new pedagogical knowledge and develop confidence to experiment with the approach in her classroom (seems that experimentation is necessary to really develop a shared praxeology). This interpretation concurs with findings of Goldsmith, Doerr & Lewis (2014) who suggested that "professional conversations can provide teachers with the encouragement and support that is needed to begin to experiment with new approaches to teaching" (p. 15). In her Post-Professional Learning Survey, Giselle reflected on her observation of students in the modelled sessions and described what she considered most useful: *…Getting students to verbalise their thinking before documenting their answer.* *How important the discussion of their mental computation is.* She seemed to recognise value in students thinking first and articulating their thinking verbally before using visual representations or mental jottings to record their ideas. She also reflected on the importance of students having opportunities to share and discuss their ideas as part of the learning process. **TEACHER RESOURCES & COLLABORATIVE PLANNING:** Collaborative planning and creation of teaching resources seemed an additional external stimulus for Giselle. In her Pre-intervention Survey, which it is noted was part completed during the intervention, she commented that "*Sharing lesson plans with other Year 3s and designing lesson PowerPoints"* was an important in supporting her professional learning. It was interesting that Giselle commented on this as an important part of the program because regular team planning meetings with the Mathematics Leader was an established practice within the school. However, this reflection seemed corroborated by the Year 3 team leader, Deryn, who also valued the usefulness of time to produce the lesson resources (Semi-structured Interview). The Mathematics Leader had explained that due to time constraints planning meetings can sometimes just focus on outlining a sequence of tasks on the planner, with teachers independently organising resources. Providing teachers with an outline of a sequence and lesson structure seemed to allow time for teachers to discuss how they would teach the content, adapt and create

					resources for their classes. It seemed that it was the opportunity to discuss the mathematics with the team and develop resources that Giselle valued, and which seemed to provide opportunity to develop her pedagogical knowledge and confidence to enact the new approach. This interpretation seems to reflect that conveyed by Timperley et al. (2007) that opportunities to discuss mathematics and how to teach it seemed to influence teachers' learning.
2	PD to DP	Enactment	When a specific aspect of teachers' knowledge or disposition influenced something that occurred in their teaching practices.	Enacts new knowledge, predominantly PCK. She teaches two differences sequences. She focuses on developing her skills to facilitate whole class discussion and opportunities for students to think and reason.	Giselle explained that as a graduate teacher she was enacting and experimenting with both new pedagogical content knowledge and new subject matter knowledge: *…I think that because I am a new teacher everything is new for me so all these strategies are new…(Interview)* Her reflection suggested that not only was the pedagogical approach new but the subject content was something she was learning for the first time. She described how enacting the new pedagogical approach and her new knowledge about mental strategies created some initial challenges for student learning: *…I think for a lot of them they really struggled with their ability to be flexible in thinking about different strategies. They have one strategy in their head for how to solve it and when you show them a new strategy they can't adjust to it because they say 'no that's not how I work it out'. I work it this way…so I think they are a bit rigid in their thinking (Interview)* *They were asking, 'why are we learning a different strategy?' And I said, well they can be applied to different questions, so you know what I mean, you are never just going to use that one strategy because sometimes if you have to go over the decuple and it throws you, so you use a different strategy like the bridging. (Interview)* It seemed that the change in pedagogical approach created some challenges for students, they were used to receiving direct instruction from the teacher. It appeared that considering multiple options or using multiple strategies to solve a problem, rather than mastering one strategy was a new experience for them. She described student inflexibility with thinking as one of the main challenges with them learning to compute mentally. Giselle described teaching experiences which indicated she was enacting and developing her new Specialised Content Knowledge (SCK): *…say they had to do 68 + 6 they split the 6 into 3 and 3. And I said that's pointless if you've at 68 because you only need jump 2 to bridge to the 70 you don't want 3, that throws you off. And even splitting when you are bridging, to split that number and understand that if you take 2 from that 6 and put it over there you only have 4 left, you don't have 6. Some of them were still putting the 6 on. (Interview)* Her description of the teaching moment illustrates that she had developed her knowledge and understanding of the mental strategies, teaching mental computation was a new experience for her as a teacher. Her recount demonstrates/displays her efforts to interact and interpret student thinking. Whilst this suggests that through experimentation she is beginning to internalise new knowledge and approach to teaching, the example also implies/indicates her tendency to revert/default to 'telling' rather than eliciting thinking. Experimentation with facilitating whole class discussion led her to reflect and identify challenges related to

general pedagogical skills. In the planning meeting she shared difficulty of ensuring active participation by all students in the fluency-based tasks at the beginning of the lesson. She was also concerned at how she could assess the difficulties of individual students or ascertain which students were making good progress when she had the whole class to monitor. We discussed different options for adapting the activities so she could gain a snapshot of individual thinking in a whole class situation, such as asking students to record the final number in the sequence or the next three numbers in the sequence on their mini-whiteboards. In addition, we discussed strategies to encourage all students to actively participate, such as asking them to listen to their peers saying the counting sequences and to record this on mini-whiteboards so they could look for patterns and contribute to the discussion at the end of the counting task. Giselle was receptive to such suggestions, she appeared keen to experiment with them in her classroom (Researcher's Journal). These challenges were related to her developing general pedagogical skills, rather than issues with mathematical content.

Experimentation with a new pedagogical approach also led her to reflect on the difficulties the students were experiencing with learning to compute mentally. In the planning meeting she described how it took students some time to compute mentally and followed an arduous process of adding and subtracting by counting forwards and back in steps of ten instead of multiples of ten (Researcher's Journal). Although she noticed this seemed a long and inefficient approach she was unsure how to support students to think differently and move forward their learning.

Giselle seemed to recognise the benefits to her students from learning to compute mentally using this approach and contemplated ways she could improve her teaching of the content. She reflected on her teaching of bridging strategy and how she could improve her teaching in the future by thinking about the tasks presented to students.

…But if you are looking at a number say 62 plus 23 to bridge that doesn't make sense. So we only looked at bridging of numbers like 68, 49 whatever so you think if you threw that in, would they start to try and bridge? (Interview)

She reflected on the examples and appeared to consider if she had provided enough opportunities for the students to develop their relational thinking and reasoning skills (Wright et al., 2012). Her reflection on student learning suggests that she is beginning to internalise the pedagogical approach.

Despite experiencing some initial challenges with implementing the new approach, in terms of student learning and development of her own pedagogical skills, Giselle persevered with classroom experimentation.

…I guess it's just a different approach for the kids in mathematics and that would probably be the biggest thing for them. They were unsure about how the lessons worked, especially at the start, and what they are expected to do and the different strategies they are expected to show. (Interview)

However, it seemed that her persistence resulted in her recognising some salient outcomes from the change in her practice. Her identification of positive results were noticed following her experimentation with the teaching of the second sequence of lessons which involved eliciting bridging as a mental strategy. This concurs with the notion of relentless consistency (Brown & Coles, 2013) to establish a learning culture that fosters

					student agency and allows for learning through teaching practices that have been described as ambitious (Kazemi et al., 2009).
3	DP to DC	Reflection	When the teacher noticed and/or reflected on something that they or their students did in their teaching practice that caused specific outcomes e.g. student learning, teacher use of visual representations, student motivation.	Reflects on salient outcomes in relation to student learning and affect: improvement in engagement, confidence, attitudes to learning and flexibility in thinking. Reflects on her own professional learning, namely developing knowledge about how students learn and facilitating productive mathematical discussion to move forward student learning.	**STUDENT ENGAGEMENT & PARTICIPATION:** A salient outcome for Giselle seemed to be student engagement and participation during the Activate session at the beginning of the lesson and in whole class discussion, two aspects of the lesson structure which initially raised concerns for her during early classroom experimentation. *…I think they thrive when they're on the floor and using their whiteboards or you're doing the auctions at the start, that sort of lead in to what they're going to be doing, I feel that for them that's the strongest part of the lesson, when they are actively engaged in it…So I feel that when they are on the floor doing things like the Number Talks that's when it's most valuable for them. I think they get the most out of it. (Interview)* She perceived opportunities for whole class discussion as the strength of the lesson in terms of student thinking and engagement. **STUDENT CONFIDENCE (& participation):** Giselle reflected further on the outcomes of changes in her practice on students. She described a positive influence on levels of student confidence and participation in mathematics: *…I think the confidence goes up, especially when they are on the floor, when you ask the kids to respond not verbally just visually, give me a thumbs up once you've got it. I feel we are getting responses from students who wouldn't normally say anything because they don't want to verbalise the answer. It's in their head but they won't verbalise it. So the whole put your thumb up if you've got it or write it down on your board it's still sort of anonymous for them as in they don't have to speak in front of the class. So I think that for those learners who are a little bit less confident, something like that just gets them in with everyone else and helps them join the group. (Interview)* It seemed that changes in her pedagogical approach, implementing a strategy such as 'thumbs up' instead of 'hands up' was having a positive impact on the classroom culture and encouraging active participation and learning from students who had previously lacked confidence in whole class situations. Use of such strategies has been advocated as tools for effective teaching practice (Wiliam, 2011). **STUDENT ATTITUDES:** She reflected on the impact of the changes in her pedagogical approach on student attitudes: *…The kids loved it, they loved it. I think it was engaging for them too and you know challenged them in different ways. I think they're more open minded about things now. (Interview)* In addition to increased levels of engagement, which Giselle reiterated in her reflection, it seemed she perceived a change in student attitudes to learning. At the beginning of the intervention she described the students as being rigid in their thinking, they wanted to use one learned method to solve equations rather than think about the numbers and be flexible in their thinking. **PROFESSIONAL LEARNING:** Giselle reflected on her own professional learning and considered development of aspects of her knowledge as most valuable: *...I guess being flexible in understanding how the kids learn and the different ways that they think, and taking on their suggestions and their approaches because most of the time they say something and I put it up on the*

					board then other kids say, yes that's how I worked it out too. So yeah, making a bit more of a conversation about it, rather than just saying that's the answer and that's how we get there. (Interview) She reflected on developing understanding about how students learn to compute mentally. She also seemed to recognise development of her pedagogical skills, namely how to use student thinking to facilitate productive discussion and move forward student learning.
4	DC to PD	Reflection	When teachers reflected on a specific outcome, thus changing a specific aspect of their knowledge or disposition on teaching. When a teacher's evaluative reflection on salient outcomes led to a change in knowledge or disposition.	Reflects on changes in aspects of her knowledge and her disposition towards teaching.	**CHANGE PCK (KCT) & possibly SCK:** Giselle reflected on her learning and seemed to recognise a significant development in aspects of her knowledge: *...But I think even just the ways to represent addition and subtraction for me I think about how I did it in school, which was probably just partitioning and vertical addition. But to look at these different strategies that I do in my head but I'd never thought about it explaining ideas in that way to the kids. So as an adult you think it's (clicks her fingers) because it's so fluid you are not really thinking about how you get to the answer. (Interview)* She reflected on her perception of the knowledge he had at the start of the intervention, her reflection implies/it could be inferred that her ideas about this would have been based on the way she learned during her school years rather than any learning she had experienced as a pre-service teacher. It is possible that she had been exposed to new learning in her course but that she had not considered applying it to her own classroom, a view shared by Sfard (2005) who argued that prospective teachers do not seem to transfer new learning about teaching to their own classrooms. It appeared that Giselle had developed aspects of her PCK, specifically KCT. It is possible that she also developed aspects of her Subject Matter Knowledge, namely SCK, as her understanding of when to use various mental strategies developed. **CHANGE IN DISPOSITON:** Giselle reflected on salient outcomes in relation to student learning, it seemed that changes in student engagement, participation levels and flexibility in their thinking connected to a change in her disposition towards teaching mathematics: *…I think it's made me think about maths a little bit differently in the ways that I approach it…I think that the way I think about showing them possible solutions to get to the answer is different for me. So I've become a bit more flexible because for me if I am thinking about mental computation I think of it as how I get there, rather than how they get there. So I feel that because there are so many different ways for these kids to get there, it's sort of opens it up as a different way for us to explore one question. So yeah….I guess I'll approach tasks slightly differently, maybe with a bit more of an open minded approach, rather than being so narrow and specific and this is just how we get there and that's, that. (Interview)* Giselle described changes in her pedagogy, anticipating different ways students may approach a task rather than the one method she considered as most appropriate. It seemed she was changing her perception on how mathematics could be taught; she was starting to recognise that there are often multiple ways to solve a task. It seemed that she had internalised the new approach to teaching, her praxeology was changing, and she now considered direct teaching of one method as too "narrow." *...So I guess it was about me being flexible as a teacher as well, and thinking about the content I would have to present, but the different ways that it could be presented to different learners. Because it's easy to get caught doing the same thing for the same learners and it doesn't work. (Interview)*

					She appeared to experience changes in her disposition in relation to student learning: *...Yes and you need to be flexible, you can't be rigid. Because if you are rigid in your thinking you get it wrong...(Interview)* At the start of the intervention she seemed unsure about students being exposed to different strategies, she thought this may confuse them (Researcher's Journal). However, recognition/reflection salient outcomes in relation to student learning seemed to result in a change in her disposition on how students best learn to compute mentally.
5	PD to DP	Enactment	When a specific aspect of teachers' knowledge or disposition influenced something that occurred in their teaching practices.	Enacts new knowledge about mental computation strategies (KCT) to teach a new sequence of lessons on indirect addition.	Giselle was observed teaching the second lesson in a sequence on using mental strategies to find the difference. The lesson provided some insight into aspects of the approach she appeared to internalise, what follows is a discussion of the researcher's interpretation of the lesson. Giselle appeared to have internalised aspects of the lesson structure. The lesson commenced with a game to engage students and develop fluency with basic number facts. She seemed to have adopted the three phase cycle which involved launching a task, students thinking and exploring the ideas then summarising through whole class purposeful discussion. It seemed that she recognised the benefits of students learning using this pedagogical approach. Noteworthy was the way the lesson structure as adapted to include the use of focus groups. Following discussions at planning meetings concerning differentiation, the integration of focus groups based on the three phase cycle was negotiated and a shared praxeology evolved. The focus groups evolved based on the concept of it being a small scale version of the whole class discussion rather than as an opportunity for small group direct teaching moment. The small groups allowed for the teacher to get a good snapshot of individual student learning and elicit discussion on student thinking processes. It seemed the interactions between researcher and teachers in planning meetings had allowed for meta-didactical praxeology concerning differentiated learning to evolve. Giselle continued to experiment with her new PCK in the classroom. She was observed enacting a whole class discussion. She allowed students individual thinking time before inviting them to contribute their ideas. She made good use of visual representations and mental jottings to share student thinking, listening carefully to the explanations to show their thinking accurately. The visual representations were used to initiate discussion, mainly about the efficiency of the different strategies students had used. She facilitated the discussion by inviting students who she anticipated would have the least efficient approaches to contribute their ideas (students did not use whiteboards to articulate their thinking to a partner before contributing to whole class discussion) and worked towards ending the discussion with the more efficient approaches. This reflected the approach that had been modelled in her classroom and one of the suggestions articulated in the work of Stein et al. (2008) on orchestrating productive discussions. It is possible that her decision not to utilize the mini-whiteboards until much later in the lesson was just a small lapse in her thinking, the students had the boards in front of them and she posed a rhetorical question during the second three phase lesson cycle, "We didn't use the boards did we?" Her decision not to use the boards did not appear to have an adverse effect on the flow of the discussion, illustrating that she knew her students well enough to be able to anticipate their thinking. It should be considered that Giselle was a graduate teacher enacting new knowledge to teach using an approach

that has been described as ambitious teaching (Kazemi et al. 2009). The change she enacted seemed a significant contrast to her usual practice:

...And I think their instinct is just to pick up a pencil and solve it. To do it in their head and put their thumbs up, I had to ask a lot of kids not to talk, don't touch your pen because their instinct is to solve it first rather than immediately go to the head…so sort of like a reverse operation of how they were doing it, that's what I felt. (Focus Group)

Giselle's reflection suggests that the practice of asking students to think mentally is both a new approach and new subject content for the students. They are not familiar with making decisions about how to perform a calculation, the instinct was to compute using learned written methods. This interpretation is reinforced by the following excerpt from the Focus Group session:

...I think that's why I stressed that, you know, I'm teaching a new strategy today, it's just another strategy to add to your collection of strategies…(Focus Group)

Giselle's comment indicates/suggests that it was not usual practice to expose students to multiple strategies, students were instead familiar with a written method as the one main method to solve problems that involved computation. Observing the whole class discussion indicated that Giselle's new pedagogical knowledge, such as allowing students individual thinking time before inviting them to share their ideas, using visuals to share student thinking and initiate discussion was moving/had moved from an external to an internal component of her praxeologies.

It appeared that Giselle's praxeology was in transition, she was internalising components of a new approach and adopting learner-centred techniques. There were aspects of her practice which were in early stages of development, for example the whole class discussion was fast paced and teacher directed in the sense that the teacher was asking all the questions and selecting the students to contribute. There were not opportunities for students to ask their peers questions about their mental methods, or moments for students to make decisions to contribute to the discussion (opt into the discussion). At times some of the questions Giselle posed were quite leading, she was keen to steer the direction of the lesson towards the intended goal. For example, immediately after a student articulated their thinking in response to an equation presented Giselle posed the question about the strategy used, "Do the numbers have to be close together or far apart?" rather than a more open question that invited the student to justify their strategy. However, it is possible that this approach was purposeful due to the lesson being scheduled in a fifty minute block. Giselle was keen to reinforce key learning points and had a tendency to summarise these points herself rather than invite students to do this, for example, "So, if the numbers are far away from each other we count back and if they are close together we count up." It was also interesting that she expressed her opinion on the strategy she would use which could be interpreted as an indirect way of telling the students how to compute the answer mentally: "I would do this because it's easier for my brain." At the end of the summary phase, just before the students worked on some tasks to consolidate some of the ideas from the lesson, Giselle posed a question and explained which strategy she would use to work out the answer mentally and why. This final part of the lesson did not involve any interaction with the students. It would be interpreted as her final opportunity to explicitly convey the key learning points before the students worked independently. It seemed that she thought it was her role to teach at some point in the lesson, almost as if allowing the students to do the explaining meant the lesson was

					deficit. In previous excerpt from the Focus Group session, she used the words, "…I'm teaching a new strategy today…" Whilst it may have been an unconscious choice of words from Giselle, it seemed to reflect the role she assumed as teacher in the lesson. (she seemed to reflect the role of a teacher she assumed in the lesson).
6	DP to DC	Reflection	When a teacher noticed and/or reflected on something that they or their students did in their teaching practice that caused specific outcomes e.g. student learning, teacher use of visual representations, student motivation	Reflects on outcomes of changes in pedagogical approach in relation to student learning, predominantly the positive impact on lower achieving students.	**STUDENT LEARNING – differentiation:** When she reflected on student learning throughout the intervention, she reiterated her newly formed view that about the importance of students thinking mentally: *...And when they are doing it in their heads, when they are at the floor at the start it just forces those lower students to actually think. (Focus Group)* Her comment suggests that the pedagogical approach to teaching mental computation provides all students with an opportunity to engage in the lesson, it allows for active learning across different achievement levels. Giselle reflected on the adaptation of the lesson structure to include differentiated teaching using focus groups. This was an approach that was negotiated between the researcher and the teachers. *...I think at the start we didn't have that focus group and there were a lot of students who were going back to their desks and because their maths maybe wasn't to the standard of the other kids, they were falling behind or they are unsure of the strategies. So I think if I was to repeat it again, I would absolutely have a focus group... (Interview)* It seemed that integrating the focus group into the lesson structure allowed an opportunity for some guided learning and discussion of strategies.

Change sequence analysis: Using criteria to establish relations between domains in the IMPG **IAN - SCHOOL C**

Arrow	Domain Link	Mediating process	Criteria	Summary	Evidence of learning processes *NVivo coding indicated in bold font (uppercase)*
1	ED to PD	Reflection	When something that was done or discussed during one of the learning experiences modified teachers' initial knowledge or disposition on teaching mental computation.	Reflects on external stimuli: modelled lessons, teacher resource book and professional dialogue during collective planning. He has new PCK: how to use new instructional tools to facilitate student learning of mental computation strategies.	*...I think the modelling was really good, so watching you do a number string lesson. It's very different to reading it, and seeing it. I think that was very valuable.* **ED-MODELLED** *...The modelling was really, is, really valuable. And even with the prescriptive lesson I liked the teacher background before as well, yeah that was really useful, you know all the understanding clearly exactly what the strategy was, the use for it and.....and how to go forward with it. That was really valuable*. **ED-MODELLING** *...I loved how the lessons were just there, the equations were there, everything was there, it made our lives a lot easier than having to come up with new equations.* **ED-TEACHER RESOURCE BK** *...I think they need to have the lessons very prescriptive, at first, so that the teachers have a really clear idea exactly how it should be run so that the data is consistent, that was really good. Because we would be able to create our own now, based off all those three. So I think we need that.* **ED-TEACHER BK** **Book:** Planning meeting 2, conversation highlighted that Ian had been reading the teacher resource book. He commented on students progressing from using an ENL to record their thinking. *...And then also you coming in for the meeting was really valuable as well, just to clarify after we tried it for a week, to be able to clarify. Like I was struggling with the number talk, thinking it was exactly the same as the number string, that from you coming in, I think it clarified that difference.* **ED-PROF CONVERSATIONS** **Planning meeting 2:** Importance of bi-directional illustrated. Ian was confused about the lesson based on idea/concept of a number talk – how it would evolve into a lesson (he had only seen the number string lesson modelled). Jenna supported by backing up with an examples of using a student strategy to drive forward the lesson and next question (Researcher's journal - field notes). Ian reflected on this in his interview and the value of this conversation.
2	PD to DP	Enactment	When a specific aspect of teachers' knowledge or disposition influenced something that occurred in their teaching practices.	Enacts new PCK to experiment with using number strings and number talks to facilitate student learning of mental strategies.	*...we gave each of them, each of them their own whiteboards, and for them all to have a go and for them to share with me, and then listening to each other, I think that was a really good structure for them, to be able to just stop and listen. Quite often when it comes to the reflection part of the lesson they switch off because they feel like it's done and they don't actually listen to their peers, whereas being at the beginning and for the majority of the lesson it's about sharing your strategies, and really there not being a right or a wrong, and as a teacher we're just kind of guiding them towards the strategy that we want them to practise.* **DP-ABOUT TEACHING-PED** *...I love how with the questions or the structure of the table there was the question column, answer column, then how I got the answer. I loved how it's structured like that. Normally I would flip it so I'll go question, solving the answer, how you solve it and then the answer. But it flips it so it makes it more reliant on mental computation as a pose to written because I think that simple shift there shows the kids you need to do it in your head first. And a lot of these kids are experiencing a lot of success because it is differentiated.* **DP-ABOUT TEACHING - PED**

...because it was a new thing I needed to put really clear expectations on it. Like that thumbs up on the chest when they have figured it out in their head, and then I waited for the majority of the class to be ready and then I say okay you can write it down. And that's when they all would start writing and then they knew to hold it up and then I would look around to see what everyone has got. And if I noticed misconceptions I could sit down with a kid who just had not idea and just whisper with them, what do you think that…while everyone's doing it at the same time which was good. And then when they put it down they knew to put it down with the whiteboard marker on the thing so they could actually stop and listen which was…yeah, I had to get to that stage so now it's a tool we can use all the time. **DP-ABOUT TEACHING – PED**

Experimenting with the approach led to understanding of the value in the approach and depth of strategies (not about fluency): Ian seemed to recognise that this wasn't all about fluency with numbers, he realised there was conceptual understanding involved for students to be able to explain and justify strategies. In the second planning meeting he raised that he need more than the allocated three lessons for students to discuss and explore ideas.

...And a lot of these kids are experiencing a lot of success because it is differentiated. **DP-ABOUT STUDENTS**

...Yeah, they did struggle a lot with this, even with the simpler prompts, just basic subtracting and adding by multiples of 10 they struggled with, so what it did was highlighted for me as a teacher, wow, this is what they need to work on. So now I've been working on, just with that group, on just simple partitioning of single digit numbers we're focusing on that. **DP-ABOUT STDS**

Planning meeting 2: seemed that student agency – choosing the questions and then testing out something that had tried engaged students (he thought). Also similarity in task meant there was a sense of achievement (Researcher's journal – field notes)

...I think I am going to utilise those little whiteboards and make them more part of my every day classroom tools that the kids know to use. I think just having the kids flick up their whiteboard and then me looking around and choosing two kids in my head that will share, it allows their thoughts still to be seen but also for me to be quite clever to be able to choose where the conversation goes. Because previously he can be difficult when you are roaming around the classroom to choose two kids to share, I think in the moment with the whiteboards that was really efficient so I'm definitely going to use that. **DP-FUTURE CHANGE**

The strategy he reflects on above encourages active participation by all students, through giving everyone a voice and opportunity it was diffusing the dominant characters who were quick to respond vocally.

Lesson observation: Experimented with lesson structure – thought carefully about the purpose and

					learning needs of students. Engaging and fun. The power of the 'in the moment' professional conversation. During the lesson observation students and teacher had a misconception with the strategy. A moment for the conversation in the lesson resulted in revisiting and discussion of the concept with the whole class. The power of an observer asking a prompt question: Have you subtracted too many or too few?
3	DP to DC	Reflection	When the teacher noticed and/or reflected on something that they or their students did in their teaching practice that caused specific outcomes e.g. student learning, teacher use of visual representations, student motivation.	Reflects on positive outcomes for student learning: learning of new mental strategies, refining strategies for efficiency and skills to articulate their thinking. Reflects on positive impact on student engagement and fostering of student agency. Reflects on positive impact on own professional learning about mental computation.	*...I think it enlightened a lot of them, just having more tools for mental computation strategies for doing these sums in their head which they never even contemplated before. Quite often their strategy was always the jump strategy and we tended to not have it as a focus, doing it in your head. It was always draw it on a number line, it was always written strategies so having a focus on being able to do it mentally and equipping them with these different strategies I think that's valuable for all of them.* **DC-STD LEARNING** *...Also extending, I've got some really bright kids in my class, and extending them can be a challenge but this was a challenge. Being able to do this mentally in your head and use different strategies actually extended those kids quite well. Like they were engaged the whole time.* **DC-STD LEARNING** *...And you know what the majority of them didn't use those strategies before, maybe one or two did, and then it opened up the minds of those other extension kids and then they got it, got it, got it (clicks fingers) and they flew.* **DC – STD LEARNING** *...I think they started to really be able to verbally communicate what is a more efficient strategy.* **DC-STD LEARNING** *...Explaining their thinking, definitely. I think through them just constantly sharing with one another, their thinking, I think their articulation has definitely improved with how to describe what was going on in their head. It's no longer, how did you get the answer, well I just got it. They realise that there are steps that are happening in their head that are getting them to the answer, which is really good.* **DC-STD LEARNING** *...they seemed to be really into it and they loved the choice involved.* **DC-STD AFFECT** *...if I find it's easy then I'll move up and make it more and more challenging so they were really mature about it.* **DC-STUD AFFECT** *...But actually that led to an awesome conversation with the kids about failure and it not being a bad thing. I sat with the kids afterwards and we spoke about failure and how you can actually learn from it, and me as a teacher I was put in that situation where I did fail. But now I am equipped with new knowledge I never thought about that before, or the way that you made one comment that shifted my thinking and I spoke to the kids about that, and they could see that if the teacher was okay to fail and learn from it then it's such a bad thing. And so we talked about good fails and bad fails and we implemented our favourite fail of the day so the kids go…what's the fail of the day that you learnt the most from?* **DC-PROF LEARNING & STUDENT AFFECT** *...They just didn't think about it before and even myself, I didn't use a lot of these strategies before and it opened up my eyes to mental arithmetic as well, as a teacher.* **DC – PROF. LEARNING** *...Yeah, and I got it in the end and I think for me that was a big learning curve as well. I just went in and I assumed it was exactly the same as the other one because this is new for me as well and now…yeah, it*

					just shifted and it made me learn from that. **DC – PROF LEARNING** *...if you don't learn anything from the fail then you are just going to keep making mistakes and it's an issue. Whereas if you actually shift your thinking and think okay how do I not fail next time, or how do I improve on this then next time, then it's useful, it's a good thing. And that I think goes with the growth mindset philosophy as well.* **DC-PROF LEARNING & DISPOSITION** *...think that's a challenge, I don't know how to make this integrated, in general, in the year. As an addition and subtraction unit we could 100% have time for these mental strategies. I don't know, we'll have to figure that out as a team, see how we can integrate it because it's valuable. It's clearly valuable, especially at this age when they're not so reliant on the vertical algorithms*. **DC-FUTURE PRACTICE**
4	DC to PD	Reflection	When teachers reflected on a specific outcome, thus changing a specific aspect of their knowledge or disposition on teaching. When a teacher's evaluative reflection on salient outcomes led to a change in knowledge or disposition.	Reflects on development of his pedagogical knowledge (KCT) and changes in his pedagogical approach. Impact on student learning leads to a change in his attitude towards teaching of mental computation.	*...see how we can integrate it because it's valuable. It's clearly valuable, especially at this age when they're not so reliant on the vertical algorithms.* **PD-DISPOSITION-ATTITUDES & CONCEPTUALISATION** *...Yeah, with the number line, when we were talking about how to show, using a number line, how to show subtraction on it being under the line. We did it all so it was under the line and addition was over which made sense to a lot of the kids but it wasn't until you showed me an example with compensation with subtraction where it makes more sense both of them being at the top. And I showed the kids and some of them were like, 'ah yeah that does make sense. It does make it clearer.' I think it's just using a tool, getting comfortable with it and then being able to use it flexibly.* **PD-KCT** *...I never, we rarely used the thinking line, the number line. And that's been a really good one for me just to help the kids show their thinking more. I think that's been a really good tool.* **PD-KCT** *...The most valuable, I think equipping the students, I think it's putting them, showing value in mental computation. I think that has been really important. I think that's going to rub off on all areas of the mathematics curriculum and the importance of mental computation I think for me, I think we need to put more emphasis on it. And then the other thing, also the kids sharing their thinking and me not saying this is the better way, it's more well posing a different way, or questioning is this the most efficient way or how else can we do it? I think for me that's been really valuable. It's so tempting just to tell them but research tells us that they get they actually take away more when they discover.* **PD-DISPOSITION-CONCEPTUALISATION** *...Yeah, and I got it in the end and I think for me that was a big learning curve as well. I just went in and I assumed it was exactly the same as the other one because this is new for me as well and now…yeah, it just shifted and it made me learn from that.* **PD-DISPOSITION-ATTITUDE**
5	PD to DP	Enactment	When a specific aspect of teachers' knowledge or disposition influenced something that	Intends to integrate the focus on teaching mental computation into future practice.	*...the next thing is how do we use this approach which has been successful...* **DP – FUTURE CHANGE**

	occurred in their teaching practices.	

IAN - CONTEXTUAL BACKGROUND INFORMATION

- In the conversations during the modelled session, Ian contemplated the pedagogical approach. He commented that he could find it difficult to allow wait time for the students to think and connect the ideas. He said that he would have persevered with more pointed questions to elicit the ideas earlier in the lesson. He wanted them to reach the generalisations much quicker. (Modelled lesson – field notes). Interestingly on his pre-intervention survey he described "watching students grapple with challenge" as an aspect of mathematics he enjoyed.

- Ian had been teaching for five years, this was his first year in middle primary.

- Observing Ian pre-intervention, there was a natural flow of discussion in his class. There was a culture of learning that promoted student agency; student ideas pushed forward the lesson. At one stage during the lesson, two students presented conflicting ideas. Ian did not resolve the issue, he recorded the two ideas on the board and said the class would return to these ideas later. I was unsure is this was a pedagogical approach or whether Nathan was unsure how move forward the discussion appropriately. (Pre-intervention lesson observation field notes).

- Reflecting on his own confidence with teaching mathematics, he indicated that he was a 'little unconfident' with planning and teaching mental computation strategies. He described having experience of teaching jump and split strategies (students had a firm understanding of these strategies). He identified the numeracy coach as his main source of support for teaching mental computation (Pre-intervention survey).

- He indicated that he was a little unconfident in regards to integrating some proficiencies into lessons, namely reasoning and fluency into lessons. He expressed that he was confident with integrating understanding and problem solving (Pre-intervention survey).

- This lack of confidence in teaching mathematics was further reinforced during the first planning meeting I attended. When completing tasks in the term planner, there was some discussion regarding extending prompts. A senior school maths teacher was present in the planning room and intervened to support Ian in his uncertainty with creating two step problems as an extending prompt (adding more variables). This teacher had supported Ian last year when he was challenged to extend students in his Year 5 class. During the planning meeting he also commented on how he preferred the more recent Peter Sullivan book because it had examples of enabling and extending prompts (field notes – planning meeting 1).

- Ian seemed a little perturbed by some boys in his class who were particularly strong at maths. He explained that they can usually work out an answer quickly and are resistant to explaining their thinking or finding ways to be systematic. He described these boys being resistant to engage in task that were open-ended with multiple answers. These students want to reach the task quickly and complete the task (field notes – planning meeting 1).

- He described his usual teaching as involving students 'regularly' sharing their thinking with the class. He also described high expectations for recording thinking in books (pre-intervention survey).

Change sequence analysis: Using criteria to establish relations between domains in the IMPG **JENNA - SCHOOL C**

Arrow	Domain Link	Mediating process	Criteria	Summary	Evidence of learning processes *NVivo coding indicated in bold font (uppercase)*
1	ED to PD	Reflection	When something that was done or discussed during one of the learning experiences modified teachers' initial knowledge or disposition on teaching mental computation.	Reflects on external stimuli: modelled lessons and teacher resource book. She has new PCK: how to use number strings as a tool to facilitate student centred learning of mental strategies.	*...I think it was lovely to be part of a project externally to see what research is being done outside of the school and be part of that.* **EXTERNAL – PL PROG DESIGN** *...I think the way the lessons were sequenced so clearly was good for us to see as well. Often we've got a unit plan but it's not always as structured, as the program was, and the enabling and extend were very clearly identified as well.* **EXTERNAL – RESOURCE BOOK** *...the way you used the number strings, I haven't previously used number strings to elicit a certain response so that would be something that would be new to me.* **DP ABOUT TEACHING - PED**
2	PD to DP	Enactment	When a specific aspect of teachers' knowledge or disposition influenced something that occurred in their teaching practices.	Enacts new PCK to experiment with using number strings and number talks to facilitate student learning of mental strategies.	Commented on how the approach was different to usual practice and how it seemed valuable for the students. Usually they work through a recording process to get the answer and the focus is on this. (Planning meeting – field notes) *...I think, the interactive element when they're sitting on the floor, having a good sharing their thinking, everyone's having a go at the same time. Choosing people to share, documenting their thinking myself. I think they love that interactive element. I think efficiency as I've talked about.* **DP ABOUT TEACHING – PED** **Experimenting with a new approach; how the teaching is different**...*Sometimes that would go downhill if they confused the rest of the class. So number talk now, so you share your thinking how you got it and I will document it and then if I feel like we are starting to go downhill I can stop it before anyone else gets confused.* **DP ABOUT TEACHING - PED** **Planning meeting conversation**: When Kelsy shared that she had difficulty in eliciting the focus strategy, she shared how she used Noam's strategy as the anchor to steer the class in the second lesson. (Planning meeting 2 – field notes) **DP ABOUT TEACHING - PED** **Planning meeting conversation**: Shared example of student with complex working – she wasn't sure if this student was using the compensation strategy or not. She drew a diagram to explain. The student compensates at the beginning of the calculation. From her diagram Laura and I thought that she was adjusting both numbers and using the transformation strategy. E.g. if she had 30 + 29 she would subtract 1 first then add 30 or 55 + 29 (55 -1, 54 + 30 = 84). Jenna said she didn't correct it or try and get her to change the method – she was unsure what she was actually doing. (Planning meeting 2 – field notes) **DP ABOUT TEACHING - PED** **Lesson observation:** Expresses some concerns over HA students. When observed the student ideas

					provide the content but her questioning is directed at eliciting information quickly. This means her whole class discussion time finishes earlier hence students complete questions quickly. She asks them to reflect and make connections with previous lessons. She drives the generalisations....”we need to make a claim here.” And collects evidence. She asks them to contribute ideas for equations that you would this strategy for. She bypassed suggestions that she thought would take the discussion elsewhere e.g. You have purposively chosen a number that is about halfway haven’t you? She provided success criteria for what she was looking for....as mathematicians what should we do to find the most efficient strategy? Didn’t have an end summary but she explained how she would end the lesson.
3	DP to DC	Reflection	When the teacher noticed and/or reflected on something that they or their students did in their teaching practice that caused specific outcomes e.g. student learning, teacher use of visual representations, student motivation.	Reflects on changes in practice leading to positive outcomes for student learning: refining strategies for efficiency, articulating thinking, deepening thinking and reasoning for all students. Reflects on positive impact on student motivation.	*...I think the awareness that often there is one strategy that is preferred, or more efficient, or proven to be more efficient. We always talk about having many strategies, when would you choose a strategy and why but I don’t know that we necessarily zoom in and articulate that. Often there really is just one strategy that by far surpasses the rest. So the realisation that maybe you might have many ways to solve it but to stop and consider really what is the most efficient.* **DC – STD LEARNING** *...showing what’s going on in your mind. Showing how you get to the answer. I know at the beginning of the project I said to you that I have a lot of kids that just do the mental comp in their head, which is fine, but then when they get the wrong answer they are unable to articulate where they made the mistake. So that whole thing of if you can show your thinking, show how you get to the answer...* **DC – STD LEARNING** *...I think the use of the empty number line. Does that answer that question? I think that as a strategy to scaffold and show their thinking has been really good.* **DC – STD LEARNING** *...I think explaining their thinking, yes.* **DC – STD LEARNING** *...and some kids will be invited to come up and show what they’ve done motivated a lot of them to actually put their thinking down on paper.* **DC – STD AFFECT** **When asked to reflect on what she had learned most from the project Jenna stated this.** *...The emphasis on efficiency because often we might have an equation on the board, like multiplication very often, ‘Here’s a multiplication equation, what are the different ways you can solve it?’ It could be skip counting, it could be an array, it could be repeated addition. But then often as the next stage, for extension or for when you’re finished, then go back and think about what was the most efficient strategy. But maybe that needs to be part of the learning more than an add on. So the focus on efficiency.* **DC – PROF LEARNING** ...But at the same time in order for me to understand how they’re solving things sometimes they need to write down what’s going on in their head so it feels like there’s a bit of a contradiction there. So that’s something that I’m still navigating myself as a teacher. **DC – DISPOSITION**
4	DC to DP	Enactment	When a specific outcome made teachers state how they would modify the associated	Intends to focus on refining the pedagogical approach in future lessons	*...And I feel like if I was to go and do it again or to consolidate it further, it would be, you know we made claims…I’m going to use compensation because I’m taking away a number that’s just a little bit less than a friendly number. But maybe there would be a bit more of a focus on the numbers before making decisions about which strategy to use.* **DC – FUTURE CHANGE**

			teaching practice in the future.		

Change sequence analysis: Using criteria to establish relations between domains in the IMPG KELSY - SCHOOL C

Arrow	Domain Link	Mediating process	Criteria	Summary	Evidence of learning processes NVivo coding indicated in bold font (uppercase)
1	ED to PD	Reflection	When something that was done or discussed during one of the learning experiences modified teachers' initial knowledge or disposition on teaching mental computation.	Reflects on external stimuli: teacher resource book; planning meetings and associated professional conversations. She develops aspects of her PCK, predominantly KCT on mental computation strategies.	...The organisation of having something as simple as that PowerPoint. I found it really valuable, it's not informed my teaching but it's just helped facilitate teaching that it's really been there is a structure to it and there is room for flexibility. But that organisation. The booklet was awesome as well because it helped with the language that I might not necessarily have been using. The teacher talk but then bringing it down to student understanding. **ED – TEACHER RESOURCE BOOK** ...I think the collaborative planning lit up the booklet. The professional conversation, I think, is important, that leading up to it but working through with the booklet and planning and discussing it, and nutting out together effective teaching methods probably, because then you start to see everyone's approaches to how they might do it and learn from each other. **ED – PROF CONVERSATIONS** ...I think that it's important not to assume that everyone understands that we're all on the same language page, there's complexities to mathematics and sometimes we just assume that we all know what we're talking about and it's those conversations that can identify whether, how well you understand the strategy that you're about to teach. Do you understand at all, and those sort of things. **ED-PROF CONVERSATIONS** ...Absolutely, but I think booklets are great but it's the conversation around the booklet, the booklet facilitates the conversation that grows the learning because reading in isolation doesn't necessarily…we teach what a thoughtful reader does and a thoughtful reader asks questions but it's being able to have that conversation about the questions, that I think our growth, that's where our growth and shifting thinking takes place. **ED-PROF CONVERSATIONS** ...Have the conversations. It's so, I think the learning is so worthwhile for the students but as a…I wouldn't want to be a teacher on my own doing it, I think it's so worthwhile having that, when you can nut out the planning stage and that you know what it is or what went wrong in one lesson and how you can then, even just that idea of that very first start when the kids were just splitting away and it made sense, and then it was that conversation about keep the number intact, keep the one number intact, don't go splitting everything and that was for us a big learning curve, and then we put in that expectation for the children so it's those kind of conversations you're bound to end up, not creating misconceptions but what we were doing wasn't wrong but it was not as effective as keeping one number intact so it's that slowing down and being able to bounce ideas off each other and then look towards that next learning opportunity a little bit deeper. **ED-PROF CONVERSATIONS** Pre-intervention survey: "Planning alongside colleagues and having a document with the necessary language and learning experiences definitely helps support planning (especially the enabling and extending prompts)".
2	PD to DP	Enactment	When a specific aspect of teachers' knowledge or disposition influenced	Enacts new PCK about teaching mental computation with conceptual understanding. She experiments with using number	New pedagogical approach based on Kelsy's responses on the Pre-intervention survey. She described her usual approach for introducing a new strategy: "model the strategy to the students." ...I think it surprised that sometimes the kids who are quicker at mental computation aren't as efficient as

			something that occurred in their teaching practices.	strings and number talks as new instructional tools.	they think they are. Or that they will over complicate, or make something more complex when it's not necessary. That I found quite surprising. **DP-ABOUT STDS** ...Yes. How well do I know the numbers that are in front of me, really how well do I know. And it goes across all operations because it's, I mean there are different strategies that you can use for multiplication and division, but it's really recognising the role that number plays within the operation that is being used. **DP-ABOUT TEACHING MATHS** Lesson observations: Lesson was not about investigating a strategy and testing it out; it was about eliciting and then practising a strategy. Note that Kelsy did not observe a modelled lesson (she was absent on PD the day it was done). The discussion time was limited because Kelsy was nervous about being observed; she experiences maths anxiety as a teacher. She selected the first example shared, which was the most efficient way to compute mentally. She did not explore other examples. Showing of student work part way through the lesson was to emphasise how a student had taken a risk and pushing themselves to work mentally with harder numbers. Emphasis was also on clearly showing thinking using an ENL. Excellent summary of the lesson: good incorporation of the concept of rounding. The word was elicited in discussion about shifting a number to a friendly ten. Pulled together the idea that compensation was about rounding one of the digits in the equation. This was a vocal 'aha' moment for many students.
3	DP to DC	Reflection	When the teacher noticed and/or reflected on something that they or their students did in their teaching practice that caused specific outcomes e.g. student learning, teacher use of visual representations, student motivation.	Reflects on student learning outcomes: students using more efficient strategies, generalising and a metacognitive approach towards mental computation. Reflects on exposure to external resources and opportunity to develop professionally.	...I hope that it's the explicit conversation and the banter that you can have about having a toolbox of strategies but really being able to be metacognitive about which strategy you are actually using. So that's my hope that they'll take away that. A more metacognitive approach towards mental computation. **DC-STD LEARNING** ...Definitely the number strings. The number strings component just identified for them what it is we were doing. **DC-STD LEARNING** ...But the rest of them, even my other at risk students felt willing to give it a go. I don't think that they have synthesised the whole process but they have stopped counting back by ones. So for them that's big growth, I think. **DC-STD LEARNING** ...Yes, it gave them a greater systematic approach to how they could do it. It provided them with a structure. **DC-STD LEARNING** ...I think that the common one I noticed were the kids, and they were more on the enabling end, they don't generalise. For them, they are still not sure when it comes to compensation why they would do that because they just don't feel like they have that number fluency. Whereas those children that are working at class level or above, with a logical mind working with numbers that way helped them. **DC-STD LEARNING** ...I feel like the entry point for every child is achievable so in terms of the learning every child had an entry point, in fact I felt that it almost took away any of the anxiety that they might sometimes have... **DC-STD AFFECT** ...Yeah, and it's not just activities it's learning. You haven't given us a set of activities to do, it's richer than that. That's why I think it's something that could be applied effectively across a primary school and

					the professional learning I've done I could use in entry points for other year levels, it's got value there. **DC-PROF LEARNING**
4	DC to PD	Reflection	When teachers reflected on a specific outcome, thus changing a specific aspect of their knowledge or disposition on teaching. When a teacher's evaluative reflection on salient outcomes led to a change in knowledge or disposition.	Reflects on changes to her knowledge about teaching mental computation (KCT) as a result of students learning to articulate their thinking.	...Not new but more effective ways of explaining. I don't think I would have explained, I don't think I've ever named compensation, compensation. We never named it that way and never really taught it as explicitly so it's not new but it's more effective I think, efficient ways. And also I think that mental computation is solving in your head and then explaining your strategy, it's not about explaining your strategy to show how you got your answer so that's really shifted my thinking. **PD-KCT** ...I don't think I recognised it, yeah they've got great mental computation skills but never actually slowed down to identify it as explicitly. So yeah, it was always there, present, but not noticed and named**. PD-SCK** ...I think setting that expectation, I always thought my expectations were clear, I think it's clarified my expectations for how learning could look, that when we're exploring something whether it's a mental computation strategy or just showing your thinking being explicit about the possibilities and how that helps you with your learning. **PD-KCT** ...just stopping and recognising the mental computation that exists and what's that strategy we're using and really keeping it at the forefront of their minds. Just that schema just keeping it there, keeping those connections... **PD-DISP-CONCEPTUALISATION**
5	PD to DP	Enactment	When a specific aspect of teachers' knowledge or disposition influenced something that occurred in their teaching practices.	Identifies new learning goals and future focus for teaching of mental computation.	...My learning goals would be that we have mental computation strategies but really naming and noticing, which is the most effective strategy for the set of numbers we're working with. **DP – FUTURE CHANGE** ...I think over time more consistent use of it and really still keeping those strategies alive, where they'll begin to recognise if they hear it more often…I am compensating because 29 is like 30. If they hear it more, they'll begin to make those connections. And possibly some real life situations where they hear it being used to estimate an answer, that's where they might start to feel some purpose behind it. **DP-FUTURE CHANGE**

PHOTOS OF A DISPLAY BOARD CREATED BY TEACHERS AT SCHOOL B