MONASH University

ESSAYS ON LOBBYING AND CRIME

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Abstract

We investigate whether and when the social network among legislators aid a lobby group in influencing voting decisions of legislators. The baseline model involves a group of legislators that are connected via an exogenously given network. Each legislator can vote for the status-quo policy or an alternative policy. A lobbyist can credibly promise payments to legislators if they vote for the alternative. The lobbyist chooses these payments to maximize the sum of legislators' probabilities of voting for the alternative policy subject to a budget constraint. Legislators value the payment they receive, and all legislators are assumed to have a common preference bias towards (or against) the status quo policy. The key feature of the model is that a legislator derives additional utility from voting in line with those legislators with whom she is directly connected in her network. We examine how the bias of the legislators and the network structure influence the payments by the lobbyist, and its resulting impact of the voting behaviour of the legislators.

We extend the baseline model by assuming that each legislator is affiliated to one of two different parties. Legislators in one party have a common level of bias towards the status quo policy, while the legislators in the other party has an equal and opposite bias towards the alternative policy. As in the baseline model, a legislator derives utility from voting in line with her neighbors within her own party. But, she also suffers a disutility from voting in line with legislators of the other party. This model leads to a variety of comparative statics results that help understand how changing the primitives of the model affect the payment received by a legislator in a party, the payment by the lobbyist to a party as a whole, and on the success of the lobbyist in influencing the legislators to vote for its preferred policy.

Finally, I investigate 'How to categorize an act as criminal or non-criminal?' While this question has been vigorously debated in law and philosophy, to the best of our knowledge it remains unresolved. We sketch one possible approach to resolve it using models of bilateral bargaining with complete and incomplete information where agents are either fully or partially informed about the type of the other agents. We use a simple bilateral interaction model with two agents where a stronger agent chooses whether or not to make a take-it-or-leave-it proposal to the weaker agent. The agents earn their outside option payoffs if the stronger agent does not make the proposal. The stronger agent has the power to reward the weaker agent if she accepts his proposal, and also the power to punish her if she rejects his proposal. The strong agent can potentially coerce the weak agent into accepting the proposal and make her worse off than her outside option. We try to determine the optimal level of legal policy that maximizes social welfare.

Declaration

This thesis is an original work of my research and contains no material which has been accepted for the award of any other degree or diploma at any university or equivalent institution and that, to the best of my knowledge and belief, this thesis contains no material previously published or written by another person, except where due reference is made in the text of the thesis.

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Chapter 1

Introduction

In United States, lobbying is authorized by the Lobbying Disclosure Act, 1995 which mandates registration of lobby firms. We have used the term ‘lobbying’ very loosely. We use it to refer to the interaction between an interest group and a legislator where the former actively persuades the latter to favour an alternative policy in return for campaign contributions. The role of an interest group is to promote the common interest of its members by influencing policy outcomes. In US, an interest group can fund electoral campaigns of legislators who support the group’s common interests. The legislators can use the campaign contributions for funding their campaign expenses.

Lobbying by interest groups is a common phenomenon and has been extensively studied in economics and political science (Snyder Jr. (1991)). A typical model assumes the lobbyist can ex-ante provide or can credibly promise to ex-post deliver money or some resource valued by a legislator if she supports a policy preferred by the lobbyist. The problem facing the lobbyist is how to optimally allocate its budget among different legislators. Legislators are assumed to maximize their utility which depends on these resources and an intrinsic bias towards or against the policy preferred by the lobbyist. The basic model has been extended in multiple directions (Austen-Smith and Wright (1992), Dekel et al. (2009)). The broad message of this literature, not surprisingly, is that money matters.

Battaglini and Patacchini (2018) enrich the basic model of lobbying in an interesting way. They ignore the party affiliation of legislators and instead emphasize the role of social connections between the legislators. Specifically, their model contains two equally resourceful lobby groups that prefer two different policies. Each lobbyist can make monetary transfers to the legislators in order to influence them to vote for its preferred policy. The key feature of their model is that each legislator not only cares about money, but also derives *additional* utility from voting in line with those legislators with whom she is directly connected, i.e., her 'neighbors' in the social network of the legislators. For analytical convenience, they assume each lobbyist chooses transfers to legislators in order to maximize the sum of the probabilities of the legislators voting for its preferred policy.

Social interactions among legislators have been studied since Rice (1927, 1928), and the potential impact of the social network among legislators (regardless of their political party affiliation) on their voting behavior has been noted at least since Truman (1951). Some recent studies have empirically demonstrated this possibility (Arnold et al. (2000) ; Fowler (2006) ; Cohen and Malloy (2014)). To the best of our knowledge, Battaglini and Patacchini (2018) is the first and only paper that examines the voting behaviour of legislators who care about how their 'neighbors' vote in the presence of lobbying.

Our model has some similarities with the issue of Net Neutrality in the United States which has been a topic of contention since the early 2000s. It means that all data packets on the internet should be treated equally with no internet service provider(ISP) possessing the power to discriminate or charge differently based on platform, source, method of communication, content, user or any other characteristic. In practice, ISPs may not intentionally block, slow down, or charge money for specific web contents which only some customers could afford. The supporters of net neutrality believe that the government has neglected individual freedom and security on the internet while the opponents are of the opinion that an intervention will impede free market innovation and investment. The Democrats launched their efforts to save net neutrality and the bill was approved on 2015 by Federal Communications Commission(FCC).

The biggest challengers (AT & T, Verizon, Comcast etc.) to net neutrality have lobbied three times harder than the proponents. In 2006 only, the opponents of net neutrality have spent approximately $71 million on lobbying and campaign contributions which is 18 times the expenditure of their counterparts. The efforts of the democrats to save net neutrality is analogous to the status quo bias of the legislators favouring a policy. Moreover, the campaign contributions from the supporters of net neutrality has been negligible compared to their oppositions and on 2017 majority favoured retaining the 2015 Open Internet Order[1]. This issue builds the premise to model a reduced form framework in Chapter 2 with a single lobby group convincing the legislators to oppose a policy in return for campaign contributions. We consider the companies opposing net neutrality as a single lobby group. For simplicity, we assume all the legislators have a status quo bias. Interestingly, the top three companies (AT & T, Verizon, Comcast) opposing net neutrality have spent approximately $20 million since 1989 on campaign contributions towards Democrats while the figure is $25 million for Republicans. This postulates the model discussed in Chapter 3, where a lobby group finances the campaigns of two opposing parties. Here we assume that members in each party have uniform and opposing bias towards a policy. This is a reasonable assumption, since majority of the Democrats favour net neutrality while the Republicans oppose it.

The first two chapters of my dissertation are best viewed as variants of Battaglini and Patacchini (2018) that modify two key features of their model. The basic idea behind the model in Chapter 2 is to study an environment with asymmetric lobby groups. We choose the simplest way to do so by assuming there is only one lobby group. Chapter 3 extends the model in Chapter 2 by distinguishing legislators on the basis of their party affiliation. Here, it is assumed that a legislator derives utility from voting in line with her 'neighbors' within her own party, but derives a *disutility* is she votes in line with the legislators of the other party.

The model in Chapter 2 assumes a finite set of legislators who can vote for either the status quo policy or an alternative policy. There exists an exogenously given social network among the legislators. A pair of legislators is either connected or disconnected in the network. Legislators have a common level

[1]Yet, the FCC voted in favour of repealing the Order, with effect from June 2018 despite efforts in Congress to stay the repeal

of bias towards (or, against) the status quo. The lobbyist prefers the alternative policy and chooses monetary transfers from a fixed budget to the legislators so as to maximize the sum of their probabilities of voting for the alternative policy. Each legislator cares about money, and also cares about voting in line with her neighbours in the social network.

We find the equilibrium payment by the lobbyist to a legislator depends on her pattern of connections in the network. Under our assumptions, the impact of the network structure on payments to legislators operates through a key network statistic, the *Bonacich Centrality*. Bonacich centrality was first proposed in the sociology literature (Bonacich (1987)). Ballester et al. (2006); Calv́o-Armengol et al. (2009) are the earliest papers to highlight its role in understanding the equilibrium outcomes of games with complementarity between agents' actions. The higher the Bonacich centrality of a legislator in the network, the larger the payment she receives from the lobby. The lobbyist benefits with an increase in its budget. It also benefits if the legislators' bias towards the status quo policy decreases.

The key comparative static result relates to the marginal impact of changes in the network structure on the lobby. Specifically, we consider a pair of networks where one network is 'denser' than another, i.e. the former network has at least one additional link relative to the latter network. If the legislators are biased towards the alternative policy, then the lobbyist is *always* relatively better off under a relatively denser network. However, if the legislators are biased towards the status-quo policy then whether and when a relatively denser network benefits the lobbyist depends on the size of the bias. If the bias of the legislators towards the status quo policy is sufficiently small (large), then denser networks benefit (hurt) the lobby. The marginal impact of a relatively denser network on the lobbyist is ambiguous at intermediate levels of the bias towards the status-quo.

The model described in Chapter 3 extends the baseline model described in Chapter 2. Firstly, we assume that legislators are affiliated to one of two different parties. Secondly, all legislators in one party have a common level of bias towards the status-quo, while all legislators in the other party have an equal and opposite bias against the status quo. Thirdly, each legislator in a party cares about voting in line with her neighbors within her own party, but also cares not to vote in line with the legislators in the other party. In this model we show that the equilibrium transfer by the lobbyist to a legislator is a function of her Bonacich centrality within her own party and the total resource allocated to her party. As the network of a party gets denser, the resource allocated by the lobbyist to the party as a whole increases. The model leads to a variety of comparative statics results that help understand how changes in the primitives of the model affect the payment received by a legislator, the payment received by a party as a whole, and the resulting impact on the lobby.

In chapter 3, we model a finite set of legislators voting for status quo or alternative policy. We consider two distinguishable political parties and each legislator is affiliated to either of those parties. Legislators in each party have uniform bias towards the status quo policy but the biases are distinguishable across parties. As mentioned earlier, the legislators is each party are connected via a network with members in their own party and benefit from conforming with their neighbours' voting decisions. But any legislator in a given party also derives a disutility from aligning their vote with a member from the opposing party. The amount of disutility to the legislator from such interactions is termed as the *degree of conflict*. We show that the equilibrium transfer to each legislator is a function of their Bonacich Centrality within their own party and the total resource allocated to that party. We also examine the marginal impact of any change in the network structure on the lobbyist. For any given party, ceteris paribus, the fund allocation to that party is increasing in the centrality of the network within the party. We examine the marginal impact of any change in the network structure on the lobbyist. The lobbyist can benefit from a denser network of the party with a bias towards the status quo policy, if the level of bias is reasonably small.

In both the chapters, legislators are paid according to their Bonacich Centrality and a denser network may be beneficial to the lobbyist. The main differences between the two models are, the uniform status quo bias in chapter 2 and the equal and opposing bias of the legislators in two different parties in Chapter 3. Moreover, for simplicity we consider strategic complementarity in legislator's action in the baseline model but later we introduce strategic substitutability to account for the conflict of aligning votes with opposing party members. Introducing conflict in the baseline model with opposing bias further enriches the result where we show that a lobbyist can make considerable campaign contribution to the party opposing her preferred policy. This result is in line with the scenario where top three oppositions of net neutrality have spent 44% of their campaign contributions on Democrats between 1989 - 2017.

One of the common applications of network models has been to study criminal networks (Liu et al., 2015). While following this literature, and the literature on the economic analysis of crime pioneered by Becker (1968), a more fundamental question arose: How to categorize an act as being criminal or non-criminal? The fourth chapter of my thesis proposes one possible approach to categorize an act as criminal or non-criminal. We use a simple bilateral interaction model with two agents where a stronger agent chooses whether or not to make a take-it-or-leave-it proposal to the weaker agent. The agents earn their outside option payoffs if the stronger agent does not make the proposal. The stronger agent has the power to reward the weaker agent if she accepts his proposal, and also the power to punish her if she rejects. The strong agent can potentially coerce the weak agent into accepting the proposal and make her worse off than her outside option. Our model does not a priori rule out the possibility that the interaction between the agents can potentially be Pareto improving relative to their outside options. We lay out the optimal social welfare maximizing penalty in a setting with both complete and incomplete information.

The remainder of this thesis is organised as follows. Chapter 2 presents our model of lobbying a network of agents with a common policy bias. In Chapter 3, we study an extension of the model discussed in Chapter 2. In Chapter 4 we discuss one possible approach about how to categorize whether an act is criminal or not. Final remarks and additional areas for research are presented in Chapter 5.

List of Symbols for Chapter 2

$N = \{1, 2, \cdots, n\}$	Set of Legislators
$\psi \in \{S, A\}$	Set of policies where S is status quo policy and A is new policy
$\mathbb{G} = [g_{ij}]$	Adjacency Matrix of a network where g_{ij} is the link between any two nodes i and j
$\Pi_i(\cdot)$	Total utility of any agent i
$u(m_i) \geq 0$	Utility of an agent i from money transfer m_i
$\mathbf{m} \in \mathbb{R}_+^n$	Monetary transfer vector to the legislators
$\mathbf{p} = (p_1 \cdots p_n)^T \in (0,1)^n$	Vector of probabilities of legislators voting for policy A where p_i is the individual probability.
$\mathbb{P}_A = \sum_j p_j \in (0, n)$	Sum of probabilities of legislators voting for policy A
$M = \sum_j m_i$	Total budget available to the lobbyist
$\beta \in \mathbb{R}$	Status Quo Bias
$\eta \in \mathbb{R}$	Network Spillover effect
$\epsilon_i \; U[-\frac{1}{2\theta}, \frac{1}{2\theta}]$	Uniformly distributed exogenous Shock parameter of agent i
θ	Density of the error term
$v_j(\psi)$	Indicator function, whether agent j votes for policy ψ
$b_i(\eta, \mathbb{G})$	Bonacich Centrality of a legislator i
$\mathbf{b}(\eta, \mathbb{G}) \in \mathbb{R}_+^n$	Bonacich Centrality vectors of legislators in graph $\mathbb{G}$
$\sigma = \sum_j b_j$	Sum of Bonacich Centrality of all legislators
$\zeta(\mathbb{G})$	Largest eigenvalue of matrix $\mathbb{G}$
$J_i[\cdot] = \frac{d}{dm_i}[\cdot]$	Jacobian matrix with respect to m_i
$\mathcal{Z}(\mathbb{G})$	Set of unconnected nodes in graph $\mathbb{G}$
$d_i(\mathbb{G})$	degree of node i in graph $\mathbb{G}$
$\mathcal{N}(\cdot)$	Cardinality of a set

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Chapter 2

Lobbying a network of agents with a common policy bias

2.1 Introduction

Our primary interest is to understand the relationship between connectedness among legislators and the likelihood that the lobbyist's preferred policy is adopted. The legislators value voting in line with their neighbours in a given network, we analyze the impact of any increase in connectedness among legislators(in some suitably defined sense) on lobbyist's objective.

We model a finite set of legislators who votes for either a status quo or an alternative policy. A pair of legislators is either connected or disconnected. The overall pattern of connections is the network among legislators. There exists a lobby group that prefers the alternative policy and seeks to influence the voting by legislators by promising monetary payments conditional on voting for the alternative policy. Monetary contributions has been used an incentive with which interest groups influence the legislators to choose their preferred policies Austen-Smith (1987). We assume the lobbyist chooses the payments to the legislators in order to maximize the sum of legislators' probabilities of voting for the alternative policy subject to a budget constraint.

The utility of a legislator i is additively separable in three components:

monetary payment from the lobby, an intrinsic bias towards or against the status quo policy, and the network payoff which is increasing in the number of legislators who vote as does i among the legislators with whom i is connected. We assume the level and direction of the bias is identical across all legislators. The network payoff is derived when a pair of connected legislators vote for the same policy, regardless of the policy.

We find the equilibrium payment to a legislator depends on the Bonacich centrality of all legislators. Bonacich centrality was first proposed in the sociology literature Bonacich (1987)). Ballester et al. (2006); Calv́o-Armengol et al. (2009) are the earliest papers to highlight its role in understanding the equilibrium outcomes of games with complementarity between agents' actions. The idea of using power and influence to measure an agent's position in the social network was discussed extensively in Katz (1953) and Bonacich (1987). The concept of using Centrality measures has been exploited by economists to quantify different aspects of the power, influence and position of an agent in any network. In other words, under our assumptions, the impact of the network structure on payments to legislators operates through this centrality measure and not any other structural feature of the network. The lobby benefits – i.e., the sum of equilibrium voting probabilities of legislators in favour of the alternative policy increases – if it has a larger budget[1]. We borrow this idea from Battaglini and Patacchini (2018), where two competing lobby groups only care about improving the sum of probabilities towards their preferred policy. In our model, we only consider one lobby group favouring the alternative policy A. We introduce competition through status quo bias and the winning policy is chosen by a super-majority rule. The same holds if the legislators' common level of bias in of the status quo decreases.

We examine the marginal impact of change in the network structure on the lobbyist. Specifically, we consider a pair of networks $\mathbb{G}$ and $\mathbb{G}^{\oplus}$ where $\mathbb{G}^{\oplus}$ is 'denser' than $\mathbb{G}$ in the specific sense that it contains at least one more connection in addition to all connections that are present under $\mathbb{G}$. When legislators are biased against the status quo, then the lobbyist is relatively better off un-

[1]The lobbyist derives no additional benefit from unused monetary resources. So efficient allocation of the total budget among the legislator is always rewarding to the lobby groups.

der denser network. When the legislators are biased in favour of the status quo, then denser networks benefit(hurt) the lobbyist if the magnitude of this bias is sufficiently small (large). The impact of a relatively denser network on the lobbyist is ambiguous at intermediate levels of the bias.

The remainder of the chapter is organized as follows. Section 2 presents our model on influencing votes through promises of monetary transfers. We find the equilibrium allocation of funds required to maximize the lobbyist objective and its relation to the voting preferences. In Section 3, we do a comparative study among two networks and find the effects on voting preferences for a larger network.

2.2 Model

The legislators are assumed to be self-interested, partisan, individual utility maximizers with diminishing returns. Each legislator cares about her own policy preferences and also take into account her neighbours decisions. We assume each legislator to be office motivated where she cares about her own vote and her neighbours. Throughout the extent of our analysis we assume all the n legislators to be office motivated[2].

There is an interest group whose objective is to influence the legislators to choose the new policy A in exchange of money. The interest group is endowed with monetary resources M. The legislators have an inherent valuation $\beta \in \mathbb{R}$ from voting for S.

We assume the network structure $\mathbb{G}$ is exogenously given and is common knowledge. Interest groups promise monetary payments to each legislator if they vote for lobbyist's preferred policy. The legislators calculate their expected utilities between voting for the new policy A or the status quo S. Legislators vote and the winning policy is chosen via the plurality rule and pay-off's are realised.

[2] Allowing for policy motivated legislators complicates the analysis and we may lose the closed-form expressions for equilibrium payments. Our results hold qualitatively if bias is replaced by outcome-contingent utility. (see Battaglini and Patacchini (2018)).

2.2.1 Definitions

There are a finite set of $N = \{1, 2, \cdots, n\}$ legislators. Each legislator simultaneously chooses between two alternative policies $\psi \in \{A, S\}$ where the status quo policy is S and the new policy is A. $\mathbb{G} = [g_{ij}]$ is a zero-diagonal, symmetric $n \times n$ matrix where $\mathbb{G}$ is also interpreted as the adjacency matrix of a graph. $\mathbb{G}$ is an unweighted, undirected[3], symmetric matrix where g_{ij} represents social connections between legislator i and j. For any two legislators i and j, $g_{ij} = g_{ji} = 1$ indicates that i and j are linked[4], otherwise $g_{ij} = g_{ji} = 0$. Legislators i and j are defined as neighbours iff $g_{ij} = 1$.

For simplicity, we assume $g_{ij} \in \{0, 1\}$[5] and we rule out strategic substitutability[6] or negative links. $g_{ij} > 0$ captures the strategic complementarity effect of j's action on i's vote.[7] The maximum number of direct links possible for any legislator i is $n-1$.[8] The assumption of non-negative g_{ij}'s can be interpreted as the legislators are politically aligned.[9] They are only directly linked with those whom they like or otherwise stay unconnected. No legislator is negatively influenced by the action of others i.e each legislator reaps the benefits of the positive links by conforming to other's action.

Bonacich Centrality: The Bonacich centrality of a given network $\mathbb{G}$ measures the importance of a given node based on its location and all possible walks between i and j. The adjacency matrix $\mathbb{G} = [g_{ij}]$ symbolizes all the direct links between players, such that $g_{ij} = 1$ if i is linked to j and $g_{ij} = 0$ otherwise.

[3]Modelling directed links will lead to different results and $\mathbb{G}$ will be an asymmetric square matrix.

[4]We rule out self loops or multiple links between a pair of nodes.

[5]Unlike Battaglini and Patacchini (2018) we do not normalize the sum of the social influence of each legislator i to 1. Assuming $\sum_j g_{ij} = 1$ makes their model more restrictive. Assuming $g_{ij} \in \{0, 1\}$ gives us more flexibility to do some network comparative statics and is less restrictive since $\sum_j g_{ij} \leq (n-1)$.

[6]Results of this model are not directly applicable for networks with some negative ties, $g_{ij} < 0$ and some positive ties $g_{ij} > 0$.

[7]While $g_{ij} < 0$ is the strategic substitutability effect of j's vote preference on i's action.

[8]$\mathbb{G}$ is an $n \times n$ matrix and $\mathbf{1}$ is a column vector of ones. For a graph with n nodes, the maximum number of direct links possible for any player in any network $\mathbb{G}$ is $\mathbb{G} \cdot \mathbf{1} = \sum_j g_{ij} \leq (n-1)$.

[9]That is, we study a lobbying towards a given side of the aisle.

While the k-th power of $\mathbb{G}$ keeps track of all the indirect connections of walk length k in $\mathbb{G}$ and is noted as $\mathbb{G}^k$. η is a non-negative scalar which is used as a discount(decay) factor of the indirect links with successive walks. For a given network $\mathbb{G}$, the generic form of Bonacich centrality vector of legislator i is:

$$\mathbf{b}(\eta, \mathbb{G}) = \eta^0 \mathbb{G}^0 \mathbf{1} + \eta^1 \mathbb{G}^1 \mathbf{1} + \eta^2 \mathbb{G}^2 \mathbf{1} + \cdots = \sum_{k=0}^{+\infty} \eta^k \mathbb{G}^k . \mathbf{1}$$

For any $\eta < 1$, the matrix $(\mathbb{I}_n - \eta \mathbb{G})^{-1}$ is invertible and the sum converges to a finite value. Thus, the largest eigenvalue of matrix $\mathbb{G}$ is $\zeta(\mathbb{G}) = \max\{\zeta_i(\mathbb{G})\}$. So the centrality vector $\mathbf{b}(\eta, \mathbb{G})$ exists. The Bonacich centrality of a legislator i is $b_i(\eta, \mathbb{G}) = \sum_{j=1}^{n} x_{ij}(\eta, \mathbb{G})$ where the term $x_{ij}(\eta, \mathbb{G}) \geq 1$ gives the sum of all possible walks from i to j. By definition, for any non-negative symmetric matrix $\mathbb{G}$ if all entries in $\mathbb{G}$ are real with $g_{ij} = g_{ji}$ and $\mathbb{G} \neq [0]$, then any increase in an entry in $\mathbb{G}$ will increase the Bonacich centrality of each individuals.

2.2.2 Payoffs

The utility of each legislator is contingent on the vote she casts, and the network externalities from voting in sync with her neighbours and exogenous factors. The utility of any legislator i for any policy $\psi \in \{A, S\}$ is represented by:

$$\Pi_i(\psi) = \begin{cases} u(m_i) + \eta \sum_j g_{ij} v_j(A) + \epsilon_{iA}, & \text{if legislator } i \text{ votes for } A \\ \beta + \eta \sum_j g_{ij} v_j(S) + \epsilon_{iS}, & \text{if legislator } i \text{ votes for } S \end{cases} \tag{2.1}$$

where $v_j(\psi)$ is a binary indicator of player j's vote,

$$v_j(\psi) = \begin{cases} 1, & \text{if legislator } j \text{ votes for } \psi \\ 0, & \text{otherwise} \end{cases}$$

The utility function is additively separable in the monetary benefit and network effects. $u(m_i)$ represents the direct utility of legislator i from the monetary contributions promised to i as a compensation for her vote for A. m_i is the amount assured to the legislator i for casting in favour of A. We assume $u(\cdot)$ to be a standard increasing, concave, continuous, twice differentiable utility function.

When legislator i is directly linked with j and her vote is similar to j's then she benefits from the peer-effects of social interactions through a exogenous spillover effect. The network spillover effect η is assumed to be homogeneous across all legislators i.

The second term describes the sum of the bilateral influence of all the direct links of agent i in $\mathbb{G}$. The strategic complementarity effect in votes is captured through $v_j(\psi)$ and is reflected through the network spillover term η when both i and j conform to each others' votes. If any legislator i's is directly aligned with other members of similar preferences then she is more influenced by her peers voting behaviour.

The last term is a stochastic parameter affecting player i's preference towards any policy ψ. The error term for player i for voting S is $\epsilon_{iS} = \epsilon_i$ where ϵ_i is independently uniformly distributed between $[-\frac{1}{2\theta}, \frac{1}{2\theta}]$ with mean 0 and a density of θ. The error term for player i for voting A is normalised to zero $\epsilon_{iA} = 0$, without loss of generality.

The conditions for uniqueness and existence of the pure strategy equilibrium based on the network structure. The pay-off dependence of the agents are based on her and her neighbours decisions. We consider a single interest group who lobbies to influence legislators away from the status quo policy. The objective of the interest group is to maximize the sum of probabilities of legislators' votes in favour of the new policy.

Assumption 2.1 (Invertability)**.** *The matrix* $(\mathbb{I} - \eta^* \mathbb{G}^T)^{-1}$ *is invertible.*

We assume θ is sufficiently small to ensure adequate uncertainty for uniqueness of solutions. Invertability is guaranteed if η^* is sufficiently small. We assume the matrix $(\mathbb{I} - \eta^* \mathbb{G}^T)^{-1}$ is invertible. We have a non-negative matrix of connections i.e. $g_{ij} \in \{0, 1\}$, so a sufficient condition for invertability of the matrix $(\mathbb{I} - \eta^* \mathbb{G})^{-1}$ is similar to the assumption made by Ballester et al. (2006). The following condition guarantees the inverse of the matrix exists for $\eta < \frac{1}{2\theta\zeta(\mathbb{G})}$

where $\zeta(\mathbb{G})$ is the largest eigenvalue of $\mathbb{G}$. The matrix $(\mathbb{I}-\eta^*\mathbb{G}^T)^{-1}$ has non-negative elements, so invertibility is sufficient to ensure positive Bonacich centrality. Therefore we can infer that $\left(\mathbb{I}-\eta^*\mathbb{G}^T\right)^{-1}\cdot\mathbf{1}>\mathbf{0}$.

2.2.3 Timeline

The game proceeds in the following stages:
(a) At time t_0, the connections between the legislators $\mathbb{G}$ are exogenously determined. $\mathbb{G}$ is common knowledge.
(b) Observing $\mathbb{G}$, the lobbyist announces the transfer vector $\mathbf{m}(\mathbb{G})$.
(c) At time t_1, each legislator observes a private preference shock ϵ_i.
(d) Legislators vote and payoffs are determined.

2.2.4 Lobbyist's Problem

The role of the interest group is to influence the legislator to vote in favour of new policy A in exchange of monetary contributions. The lobbyist cannot observe the individual shocks of the legislator. In the influence stage, the lobby group observes the network connections among n legislators and announces a vector of transfers. Based on the announced transfer and the shock, each legislator casts their vote. Hence the lobbyist will allocate funds optimally to maximize the aggregate probability of votes in favour of A. The lobbyist's problem is given as:

$$\max_{\mathbf{m}}\sum_j p_j(\mathbf{m}) \qquad s.t. \qquad \sum_j m_j \leq M$$

The feasible vector of payments is given by, $\mathbf{m}=(m_1,\cdots,m_n)$ such that $\sum_i m_i \leq M$ and $m_i \geq 0$, for all $i \in N$. M is the total available budget to the lobbyist. Let, $m_i \in \mathbf{m}_i$ and $\mathbf{m}_i$ be the vector of possible contributions available to legislator i. We know, $p_j = E(v_j(A))$ is the ex-ante probability of any legislator j for voting in favour of policy A.

The interest group doesn't know the preferences of the legislators with full certainty because of the exogenous shock parameter ϵ_i. Before making her voting decision each rational legislator i compares the expected pay-off from voting for A over S. She will vote for the new policy A iff:

$$E\Pi_i(S) - E\Pi_i(A) \leq 0 \tag{2.2}$$

From the utility equation 2.1, the utility of legislator i depends on the action of player j. Since, legislator i observes her own exogenous preference shock so, $E(\epsilon_i) = \epsilon_i$. Thus the above equation 2.2 becomes:

$$\begin{aligned} &\beta - u(m_i) + \eta \sum_j g_{ij}(1-p_j) - \eta \sum_j g_{ij}(p_j) + \epsilon_i \leq 0 \\ &\Leftrightarrow \epsilon_i \leq u(m_i) - \beta + \eta \sum_j g_{ij}(2p_j - 1) \end{aligned} \tag{2.3}$$

Thus the legislator votes for policy A if the above condition holds. Further calculations on the derivation of the individual probability for voting in favour of A is available in the Appendix 2.A.1.

Voting Stage

The legislators cast their vote based on their individual preferences, monetary transfers and the behaviour of the neighbours. The winning policy will be determined by a given voting rule based on the ballots. Legislators likelihood of voting for A are determined by the probabilities they place on neighbours' voting for A. Since the bias parameter β is exogenous, we obtain a linear system of simultaneous equations of legislators' individual probabilities of voting for A.

Equilibrium Voting

The probability for any player i to vote for A is denoted by $p_i \in [0,1]$ where $\mathbf{p} = (p_1, p_2, \cdots, p_n)^T$ is a vector of the probabilities[10] of all the players such that $\mathbf{p} : \mathbf{m}_1 \times \cdots \times \mathbf{m}_n \to [0,1]^n$. For any $\mathbf{m}$ the above probability vector is a linear mapping from the probability vector $\mathbf{p}(\mathbf{m})$ to itself where $F(\mathbf{m}, \mathbf{p}(\mathbf{m}))$ is a linear

[10] $\mathbf{p}$ is a mapping from a set of possible transfers to a set of probabilities.

transformation of $\mathbf{p}(\mathbf{m})$. For a small θ, the set F is closed, convex and continuous in $\mathbf{p}$ as it is a contraction mapping from $[0,1]^n$ to itself. So, a unique equilibrium exists. For a larger value of θ the equation may not be well behaved or unique.

The probability of legislators choosing the new policy A is represented as:

$$\mathbf{p}(\mathbf{m}) = \begin{pmatrix} p_1(\mathbf{m}) \\ p_2(\mathbf{m}) \\ \vdots \\ p_n(\mathbf{m}) \end{pmatrix} = \begin{pmatrix} \frac{1}{2} + \theta[u(m_1) - \beta + \eta \sum_j g_{1j}(2p_j(\mathbf{m}) - 1)] \\ \vdots \\ \frac{1}{2} + \theta[u(m_n) - \beta + \eta \sum_j g_{nj}(2p_j(\mathbf{m}) - 1)] \end{pmatrix} \tag{2.4}$$

or, (Alternative form)

$$\mathbf{p}(\mathbf{m}) = (\frac{1}{2} - \theta\beta) \cdot \mathbf{1} + \theta \cdot u(\mathbf{m}) + 2\theta\eta \cdot \mathbb{G} \cdot \mathbf{p}(\mathbf{m}) - \theta\eta \cdot \mathbb{G} \cdot \mathbf{1}$$

where $\mathbf{1}$ is an $n \times 1$ column vector of 1's, $u(\mathbf{m})$ is an $n \times 1$ vector of the direct utility from the transfers promised to the legislators, $\mathbf{u} = u(\mathbf{m}) = (u(m_1), u(m_2), \cdots, u(m_n))^T$.

Solving for the equilibrium probabilities from equation 2.4 we get,

$$(\mathbb{I}_n - \eta^* \mathbb{G}) \cdot \mathbf{p} = \frac{1}{2}(\mathbb{I}_n - \eta^* \mathbb{G}) \cdot \mathbf{1} + \theta\mathbf{u} - \theta\beta.\mathbf{1} \qquad [\because 2\theta\eta = \eta^*]$$

Pre-multiplying both sides by $(\mathbb{I}_n - \eta^* \mathbb{G})^{-1}$,

$$\mathbf{p} = \frac{1}{2} \cdot \mathbf{1} + \theta(\mathbb{I}_n - \eta^* \mathbb{G})^{-1} \cdot \mathbf{u} - \theta\beta \cdot (\mathbb{I}_n - \eta^* \mathbb{G})^{-1}.\mathbf{1} \tag{2.5}$$

The Bonacich centrality vector in our analysis is written as $\mathbf{b}(\eta^*, \mathbb{G}) = (\mathbb{I}_n - \eta^* \mathbb{G}^T)^{-1} \cdot \mathbf{1}$. Using equation 2.5 we get an unique equilibrium probability vector $\mathbf{p}(\mathbf{m})$ for a fixed $\theta < \theta^*$ such that the sum of the probabilities $\sum_j p_j(\mathbf{m})$ is as follows:

$$\mathbf{p}^T \cdot \mathbf{1} = \frac{1}{2} \cdot \mathbf{1}^T \cdot \mathbf{1} + \theta \cdot \mathbf{u}^T \cdot \mathbf{b}(\eta^*, \mathbb{G}^T) - \theta \cdot \beta \cdot \mathbf{b}(\eta^*, \mathbb{G}^T) \cdot \mathbf{1}$$

Alternative form,

$$\sum_j^n p_j(\mathbf{m}) = \frac{n}{2} + \theta\big[\sum_j u(m_j) b_j - \beta \sum_j b_j\big] \tag{2.6}$$

We now address the equilibrium probabilities from solving the linear system of equations. The sum of the probabilities $\sum_j^n p_j$ in the above equation are continuous, increasing and differentiable with respect to the monetary transfer m_i for all i and converges to n.

Initial Stage

The interest group is allocated with a budget M. The main objective of the interest group is to optimize the allocation of resources among the legislators such that the sum of probabilities is maximized. The interest group will maximise their objective function given a budget constraint:

$$\max_{\mathbf{m}} \sum_j p_j(\mathbf{m}) \qquad s.t. \qquad \sum_j m_j \leq M \tag{2.7}$$

In the previous section we are ensured a pure strategy solution when θ is sufficiently small. Then there exists a θ^*, such that $\theta < \theta^*$ then the solution is unique. So for any sufficiently small value of θ, we do the constrained optimization problem in 2.7 and get the following first order conditions:

$$\sum_j \frac{\partial p_j}{\partial m_i} = J_i[\mathbf{p}]^T \cdot \mathbf{1} = \lambda, \qquad \text{and} \qquad \sum_j m_j = M \tag{2.7a}$$

for all $j \in N$. Using the above first order conditions in equation 2.7a, we solve for the equilibrium level of monetary transfer. We define, $J_i[\mathbf{p}] = [\frac{\partial p_1}{\partial m_i}, \cdots, \frac{\partial p_n}{\partial m_i}]^T$ as the Jacobian matrix or the first order derivative of the vector of probabilities with respect to the transfer made to the legislator i. Differentiating the optimal probability distribution vector in equation 2.5 with respect to the monetary transfer made to individual legislator m_i, we get $J_i[\mathbf{p}] = \theta(\mathbb{I}_n - \eta^*\mathbb{G})^{-1} J_i[\mathbf{u}]$. We know that any individual j benefits from her own transfers $u(m_j)$. Thus the effect of marginal change in m_i on the direct utility of monetary is $u'(m_j)$ if $j = i$ and 0 otherwise. $J_i[\mathbf{u}]$ is a vector of zero's except for the i-th term which is $\frac{\partial u_i}{\partial m_i}$.[11]

By assumption $(\mathbb{I}_n - \eta^*\mathbb{G})^{-1}$ exists and we show that the sum of probabilities $\mathbb{P}$ is differentiable and increasing in m_i. The first order condition from

[11]Based on initial assumptions on utility, $J_i[\mathbf{u}] = (0\ 0\ \cdots \frac{\partial u_i}{\partial m_i}\ \cdots\ 0)^T$.

equation 2.6 yields, $\sum_j \frac{\partial p_j}{\partial m_i} = \theta \sum_j x_{ij} u'(m_j)$, is positive[12] as $u'(.)$ is increasing. The second order sufficiency condition gives, $\sum_j \frac{\partial^2 p_j}{\partial m_i^2} = \theta \sum_j x_{ij} u''(m_j)$ the sum of probabilities to be concave because of the diminishing marginal returns from money.

Above results are conditional on the magnitude of θ and relatively small values of θ will ensure adequate uncertainty for existence and uniqueness of solutions. From equation 2.5, we solve for a linear system of equation for an unique probability vector $\mathbf{p}^*$. Since the equilibrium sum of probabilities of A is increasing and concave in m_i and $p_i \in [0,1]$ for all i and as m_i grows large, the sum of probabilities $\sum_j^n p_j$ converges to n.

From definition of Bonacich centrality and differentiating equation 2.6 (details in appendix 2.5) we get the following,

$$J_i[\mathbf{p}]^T \cdot \mathbf{1} = \theta \cdot J_i[\mathbf{u}]^T \cdot \mathbf{b}(\eta^*, \mathbb{G}^T)$$

Using the above condition in equation 2.7a, we show the Lagrangian multiplier is proportional to the equilibrium transfer m_i^* (appendix 2.A.2b)to the legislator. The equilibrium $m_i^*(\mathbf{b}, M)$ is conditional on the available budget M. The marginal cost of resources in equilibrium is dependent on the Bonacich centrality and marginal utility of direct transfer $\lambda^* = \frac{\lambda}{\theta} = u'(m_i) \cdot b_i(\eta^*, \mathbb{G}^T)$. For any equilibrium vector of transfers $\mathbf{m}^*$, we calculate the indirect utility function:

$$\mathbb{P}_A(\beta, \eta, \theta, M, \mathbb{G}) = \sum_j p_j(\mathbf{m}^*) \tag{2.8}$$

$$\Leftrightarrow \max_{\mathbf{m}} \sum_j p_j(\mathbf{m}) = \max_{\mathbf{m}} \Big[\frac{n}{2} - \theta\beta \sum_j b_j + \theta \sum_j u(m_j) \cdot b_j(\eta^*, \mathbb{G}^T) \Big]$$

Replacing the optimal transfer $m_i^*(\mathbf{b}, M)$ in equation 2.6, we get the total probability of the legislators to vote in favour of the new policy A.

Additionally, we assume the budget M to the lobbyist below some critical level M^*. It is set so that $M < M^*$ the network effects are not overwhelmed by the lobbyist's transfer. Moreover, for any M, the objective of the lobby group is to maximize the sum of probabilities of votes in favour of policy A. If the budget

[12] Only when $x_{ij} > 0$ which holds iff $g_{ij} > 0$ for all i, j's. But, when some g_{ij} are negative then some x_{ij} may be negative.

M is high as M^*, then policy A wins with certainty. For the rest of the analysis we assume $M < M^*$.

Proposition 2.1. *For any graph $\mathbb{G}$, **(a)** equilibrium transfer to each legislator $m_i^*(\mathbf{b}, M)$ depends on the Bonacich centrality vector, and **(b)** equilibrium $\mathbb{P}_A$ is increasing in M and decreasing in β.*

Part (a) in proposition 2.1 entails that in equilibrium each legislator i receives monetary contribution according to the Bonacich centrality vector. The proof is available in the appendix 2.5. If we assume a *logarithmic* utility function then each legislator receives transfer proportional to their individual weighted Bonacich centrality i.e. $m_i^* = \frac{b_i}{\sum_j b_j} M$. For more details see example 2.2.4.

Part (b) explains that more money never disadvantages the legislator. If the budget allocation of the lobbyist improves then they will always use it in their favour to improve upon the sum of probabilities of votes for A. Secondly, any increase in the legislator's bias towards the status quo policy always impedes the lobbyist's objective.

By definition, the sum of probabilities $\mathbb{P}_A$ for any given $\mathbb{G}$ would lie between 0 and n. Moreover, for any $M < M^*$, the objective of the interest group is to maximize the sum of probabilities of votes in favour of policy A. Thus any increase in the budget allocation for lobbying activities will lead to an increase in vote share. Differentiating the sum of probabilities $\mathbb{P}_A$ with respect to M we get, $\frac{d\mathbb{P}_A}{dM} > 0$.

Each individual has a bias towards some favoured policy. By assumption, the bias β among the individual legislators are assumed to be homogeneous. For any given β, an increase in the legislator's bias towards status quo in equation 2.6 gives $\frac{d\mathbb{P}_A}{d\beta} < 0$. The result is quite straightforward, an increase in the status quo bias will hinder the interest group's objective of influencing legislators towards alternative policy. Any increase in bias β directly increases the utility of the legislator voting in favour of policy S. She also benefits from the effects of positive spillover from the complementarity in her neighbours voting towards S. Hence, any increase in bias will disadvantage the lobbyist.

The main objective of the lobbyist is to earn votes in favour of the policy A. Let's assume the lobbyist's budget M is separable in m_i^*. For any given $\mathbb{G}$

and fixed M the equilibrium transfer $m_i^*(\mathbf{b}, M)$ is conditional on their Bonacich centrality. Ceteris paribus, for any increase in M, the centrality vector $\mathbf{b}$ is unaffected (by construction) and equilibrium transfer increases $\frac{\partial m_i^*}{\partial M} > 0$ for all i. High value of M implies larger m_i^*, $J_i[\mathbf{p}] > 0$ and λ is positive (from equation 2.7a). We have already established in the previous subsection that $\mathbb{P}_A$ is increasing and concave in m_i and $p_j(\mathbf{m}) \in (0,1)$ is bounded. For sufficiently large m_i, the individual probability p_j converges to 1.[13]

Example: Using a logarithmic utility function $u(m_i) = \log m_i$ in equation 2.8 and $m_i > 0$, we solve the first order condition yields the marginal cost of resources. We get,

$$\lambda^* = \frac{b_i}{m_i} \qquad \text{and} \qquad \sum_i m_i = M$$

for all $i \in N$. By algebraic manipulation, the equilibrium transfer is $m_i^* = \frac{b_i}{\sum_j b_j} M$ where $\sum_j b_j = \sigma$. Notice that the ratio of equilibrium transfers between two legislators is proportional to their Bonacich centrality i.e. $\frac{m_i^*}{m_k^*} = \frac{b_i}{b_k}$. Ceteris paribus, any improvement in the centrality of any two legislators i and j will increase their centrality and their equilibrium transfer. From the indirect utility function, the sum of probabilities in equilibrium is:

[13] By definition of probabilities $p_i \in (0,1)$ and from equation 2.5, we get $-\frac{1}{2\theta} \le \sum_j x_{ij} u_j - \beta b_i \le \frac{1}{2\theta}$ where x_{ij}'s are elements of the matrix $(\mathbb{I} - \eta^* \mathbb{G})^{-1}$. If $M = 0$, then $u(0) = 0$ and for a totally disconnected graph the following boundary conditions apply, $-\frac{1}{2\theta} \le \beta \le \frac{1}{2\theta}$. Again, for a fully connected graph the following boundary conditions apply, $-\left(\frac{1}{2\theta} - (n-1)\eta\right) \le \beta \le \left(\frac{1}{2\theta} - (n-1)\eta\right)$.
(i) Thus, the sufficient condition for existence of an equilibrium in any network and no money is $-\left(\frac{1}{2\theta} - (n-1)\eta\right) \le \beta \le \left(\frac{1}{2\theta} - (n-1)\eta\right)$.
For any $M > 0$, define $u_i(M) = u^{\max}$. The vector of utility function in equation 2.5 is $\mathbf{u} \in [0, u^{\max}]^n$. By assumption 2.1, in a network with no strategic substitutability the entries in the inverse matrix are non-negative. If individual i gets all the money, then $u_i = u^{\max}$ and $u_j = 0$ for all $i \ne j$. The individual benefits more from money than network externalities, if the following holds:-
(ii) $\frac{\eta^*}{1-(n-2)\eta^*-(n-1)\eta^{*2}} \sum_{j \ne i} u_j + \frac{(1-(n-2)\eta^*)}{1-(n-2)\eta^*-(n-1)\eta^{*2}} u_i \le \frac{(1-(n-2)\eta^*)}{1-(n-2)\eta^*-(n-1)\eta^{*2}} u^{\max} \le \frac{1}{2\theta} + \frac{\beta}{1-(n-1)\eta^*}$.
This automatically implies the sum of probabilities $\mathbb{P}_A \in (0, n)$.

$$\mathbb{P}_A = \frac{n}{2} + \theta \sum_j b_j \cdot \log(\frac{M}{\sigma}.b_i) - \theta\beta\sigma \tag{2.8a}$$

We have already shown that the optimal transfer to the legislator is proportional to their Bonacich centrality. Differentiating the sum of probabilities of the lobbyist for A with respect to the parameters, $\frac{d\mathbb{P}_A}{dM} > 0$ and $\frac{d\mathbb{P}_A}{d\beta} < 0$ conforms our proposition 2.1.

2.3 Network Comparative Statics

In this section we consider the network comparative statics where $\mathbb{G}^{\oplus} \supset \mathbb{G}$ for any $\mathbb{G}, \mathbb{G}^{\oplus} \in \mathcal{G}$. Thus graph $\mathbb{G}^{\oplus}$ has more links or is denser than $\mathbb{G}$. Now while comparing between two graphs if $\mathbb{G} \subset \mathbb{G}^{\oplus}$, then by definition $\zeta(\mathbb{G}^{\oplus}) > \zeta(\mathbb{G})$. A denser network complementary effect implies higher maximum eigenvalues $\max\{\zeta_i(\mathbb{G}^{\oplus})\} > \max\{\zeta_i(\mathbb{G})\}$ and thus a strict increase in Bonacich centrality vector $\mathbf{b}^{\oplus}(\eta^*, \mathbb{G}^{\oplus T}) > \mathbf{b}(\eta^*, \mathbb{G}^T)$ such that $b_i^{\oplus} > b_i$ for all i.

If we compare two given graphs $\mathbb{G}$ and $\mathbb{G}^{\oplus}$, the equilibrium transfer vectors are $\mathbf{m}^*$ and $\mathbf{m}^*_{\oplus}$ respectively where $m_{i\oplus} \geq m_i$ for some i and $m_{i\oplus} < m_i$ for others. Using the sum of probabilities for A for both graphs $\mathbb{P}_A(\mathbb{G})$ and $\mathbb{P}_A(\mathbb{G}^{\oplus})$ in equilibrium, we compare the different networks with the equilibrium level of transfers. Here we establish a relation between the density of two networks, the transfer vector and their effect on the sum of probabilities $\mathbb{P}_A$ of votes for A.

To begin, let's assume that legislators have an inherent bias $\beta \leq 0$ away from the status quo or towards the new policy A. The interest groups maximize their sum of probabilities by promising equilibrium transfers and the voting decision of the legislators. Let's consider a pair of networks $\mathbb{G}$ and $\mathbb{G}^{\oplus}$ where $\mathbb{G}^{\oplus}$ is ' denser' than $\mathbb{G}$, i.e. it contains at least one more connection in addition to all connections that are present under $\mathbb{G}$. Thus $\{\mathbb{G}, \mathbb{G}^{\oplus}\} \in \mathcal{G}$ where one is a subset of the other $\mathbb{G} \subset \mathbb{G}^{\oplus}$, the equilibrium the optimal transfer vectors are $\mathbf{m}^*$ and $\mathbf{m}^*_{\oplus}$. The budget is fixed at M such that $\sum_j m_j^* = \sum_j m_{j\oplus}^* = M$.

Setting $\beta \leq 0$ and plugging the equilibrium transfer vector in equation 2.8 we get the equilibrium sum of probabilities for both the graphs $\mathbb{G}$ and $\mathbb{G}^{\oplus}$:

$$\mathbb{P}_A(\mathbf{m}^*, \mathbb{G}) = \frac{n}{2} + \theta \sum_j u(m_j^*) b_j - \theta\beta \sum_j b_j \tag{2.8b}$$
$$\mathbb{P}_A(\mathbf{m}_\oplus^*, \mathbb{G}^\oplus) = \frac{n}{2} + \theta \sum_j u(m_{j\oplus}^*) b_j^\oplus - \theta\beta \sum_j b_j^\oplus$$

Proposition 2.2. *For any two graphs $\mathbb{G}$ and $\mathbb{G}^\oplus$, if $\mathbb{G} \subset \mathbb{G}^\oplus$ and $\beta \leq 0$, then $\mathbb{P}_A(\mathbb{G}^\oplus \big| \beta) > \mathbb{P}_A(\mathbb{G} \big| \beta)$.*

If the legislators have a positive bias towards the alternative policy A or a negative bias away from the status quo S then a denser network always benefits the lobbyist. A negative value of the bias acts in favour of the lobbyist as shown in equation 2.8. So a comparatively well connected graph improves the sum of probabilities in favour of A when the legislators are biased towards the alternative policy.

Proof. By definition, when $\mathbb{G} \subset \mathbb{G}^\oplus$ then, $b_j < b_j^\oplus$ for all j. Here we compare the sum of probabilities of votes in favour of A for different graphs. Take any two graph $\mathbb{G}$ and $\mathbb{G}^\oplus$ where one is a subset of the other $\mathbb{G} \subset \mathbb{G}^\oplus \in \mathcal{G}$. Let, the equilibrium the optimal transfer vectors are $\mathbf{m}^*$ and $\mathbf{m}_\oplus^*$.

For a graph $\mathbb{G}$ with $\mathbf{m}^*$ as the optimal transfer, the sum of probabilities is given by $\mathbb{P}_A(\mathbf{m}^*, \mathbb{G}) = \max_{\mathbf{m}} \{\mathbb{P}_A(\mathbf{m}, \mathbb{G})\} \geq \mathbb{P}_A(\mathbf{m}_\oplus^*, \mathbb{G})$ (by definition). Similarly, for graph $\mathbb{G}^\oplus$ with $\mathbf{m}_\oplus^*$ optimal transfer, the sum of probabilities of voting for A is given by $\mathbb{P}_A(\mathbf{m}_\oplus^*, \mathbb{G}^\oplus) = \max_{\mathbf{m}} \{\mathbb{P}_A(\mathbf{m}, \mathbb{G}^\oplus)\} \geq \mathbb{P}_A(\mathbf{m}^*, \mathbb{G}^\oplus)$ (by definition). We already mentioned, $\mathbb{P}_A(\mathbf{m}^*, \mathbb{G}) \geq \mathbb{P}_A(\mathbf{m}_\oplus^*, \mathbb{G})$ and using $\mathbf{m}^*$ as transfer vector in graph $\mathbb{G}^\oplus$, we get the following $\mathbb{P}_A(\mathbf{m}^*, \mathbb{G}) < \mathbb{P}_A(\mathbf{m}^*, \mathbb{G}^\oplus)$, since $b_j < b_j^\oplus$ for all j (see Appendix 2.A.3 and 2.A.4). But by optimization, $\mathbf{m}_\oplus^*$ is the optimal transfer vector for graph $\mathbb{G}^\oplus$. Hence, $\mathbb{P}_A(\mathbf{m}^*, \mathbb{G}^\oplus) \leq \mathbb{P}_A(\mathbf{m}_\oplus^*, \mathbb{G}^\oplus)$. Therefore the sum of probabilities for the alternative policy always increase with a denser network. *(Q.E.D)* □

The explanation holds true for the following cases, (a) the legislators have no inherent bias $\beta = 0$ towards any policy and (b) when the legislators have a negative bias or bias away from the status quo policy S i.e. $\beta < 0$. Using the above proof we see that a denser network is beneficial for the lobbyist i.e.

$\mathbb{P}_A(\mathbf{m}^*, \mathbb{G}^\oplus) \leq \mathbb{P}_A(\mathbf{m}^*_\oplus, \mathbb{G}^\oplus)$. What happens to the payments to legislator i for a denser network $\mathbb{G}^\oplus$? The result is ambiguous. The equilibrium transfer vector for the two graphs $\mathbb{G}$ and $\mathbb{G}^\oplus$ can be either same for both the graphs i.e. $\mathbf{m}^* = \mathbf{m}^*_\oplus$ or separate i.e. $\mathbf{m}^* \neq \mathbf{m}^*_\oplus$. When $\mathbf{m}^* = \mathbf{m}^*_\oplus$, every legislator i gets the same payment for both network structure. Since the lobbyist faces the same budget constraint M, so there cannot be any strict improvement in the payment vector. When the equilibrium payment vector for the graphs $\mathbb{G}$ and $\mathbb{G}^\oplus$ is $\mathbf{m}^* \neq \mathbf{m}^*_\oplus$, then the payment of legislator i is $m^*_{i\oplus} \geq m^*_i$ for some i and $m^*_{i\oplus} < m^*_i$ for the others. If the legislators are unbiased or have a positive bias towards the alternative policy, then denser network always benefits the lobbyist's objective.

But what happens when we compare the sum of probabilities under $\mathbb{G}$ and $\mathbb{G}^\oplus$ for any given $\beta > 0$? The result is not so straight forward and only holds under certain restrictions. Given any graph $\mathbb{G} \subset \mathbb{G}^\oplus$ where the legislator has a bias $\beta > 0$ towards policy S, the sum of probabilities doesn't necessarily increase for the denser network. For a given budget M, the equilibrium monetary transfer depends on the Bonacich centrality.

Lemma 2.3. *For any two graphs $\mathbb{G}$ and $\mathbb{G}^\oplus$, if $\mathbb{G} \subset \mathbb{G}^\oplus$ then $\mathbb{P}_A(\mathbb{G}^\oplus, \mathbf{m}^*_\oplus \big| \beta) > \mathbb{P}_A(\mathbb{G}, \mathbf{m}^* \big| \beta)$ if $0 < \beta < \beta_c(\mathbb{G}, \mathbb{G}^\oplus)$.*

A denser network benefits the lobbyist if the status quo bias of the legislator is relatively small. The lobbyist prefers the alternative policy A and maximises the sum of probabilities for A. In this lemma we measure the impact of a denser network on $\mathbb{P}_A$ when the status quo bias of the legislator is positive. In proposition 2.2 we have established that a relatively well-connected network aids lobbyist's objective if the legislators have a negative status quo bias. network if the aggregate utility from equilibrium monetary transfer to the legislators dominates the status quo bias. This is due to the strategic complementarity in the legislators voting decisions. A denser network implies an increase in the complementarity effect which implies increased benefits to the lobbyist. But the intuition is slightly different in lemma 2.3. Here we show that the lobbyist benefits from a denser network if the status quo bias(positive) is relatively small in a suitably defined sense.

For any given budget M, a denser network is valuable to the lobbyist i.e.

$$\max\left\{\mathbb{P}_A(\mathbf{m}^*_\oplus,\mathbb{G}^\oplus),\mathbb{P}_A(\mathbf{m}^*,\mathbb{G}^\oplus)\right\} > \mathbb{P}_A(\mathbf{m}^*,\mathbb{G})$$

if $\beta < \beta_c(\mathbb{G},\mathbb{G}^\oplus)$ where $\beta_c(\mathbb{G},\mathbb{G}^\oplus) \in \max\{\beta_c^{\mathbf{mm}},\beta_c^{\mathbf{mm}_\oplus}\}$ (see Appendix 2.A.5a and 2.A.5b). We know that, if $\mathbb{P}_A(\mathbf{m}^*,\mathbb{G}^\oplus) \leq \mathbb{P}_A(\mathbf{m}^*_\oplus,\mathbb{G}^\oplus)$ then $\beta_c^{\mathbf{mm}} \leq \beta_c^{\mathbf{mm}_\oplus}$. In this case the lobbyist will choose $\mathbf{m}^*_\oplus$ as the optimal transfer vector. The lobbyist chooses the transfer that yields her the higher sum of probabilities in voting for A. Here we compare the marginal impact of the denser network on equilibrium $\mathbb{P}_A$. In this case, the lobbyist benefits if $\mathbb{P}_A(\mathbf{m}^*,\mathbb{G}) < \mathbb{P}_A(\mathbf{m}^*_\oplus,\mathbb{G}^\oplus)$ which automatically implies $\beta_c(\mathbb{G},\mathbb{G}^\oplus) = \beta_c^{\mathbf{mm}_\oplus}$.

Consider a special case of the previous lemma 2.3 where the two graphs $\mathbb{G}$ and $\mathbb{G}^\oplus$ are symmetric. If $\mathbb{G} \subset \mathbb{G}^\oplus$ then the Bonacich centrality is $b_i^\oplus = c \cdot b_i$ for all $i \in N$ where $c > 1$ is a scalar. The equilibrium transfer vector that maximises the sum of probabilities for both the denser and sparser graphs is $\mathbf{m}^*$. One example of comparing two symmetric graphs is, a complete graph $com(\mathbf{G})$ where each legislator is connected with all other members and, a totally disconnected graph $disc(\mathbf{G})$ where none of the legislators are connected to each other. By definition, the Bonacich centrality of each individual in $com(\mathbf{G})$ is identical, i.e. $b_i\big(com(\mathbf{G})\big) = b_j\big(com(\mathbf{G})\big)$ for all $i \neq j$ and the Bonacich centrality of each individual in $disc(\mathbf{G})$ is $b_i\big(disc(\mathbf{G})\big) = b_j\big(disc(\mathbf{G})\big)$ for all $i \neq j$. Also the centrality vector of the complete graph strictly dominates the centrality of the disconnected graph, i.e. $\mathbf{b}\big(com(\mathbf{G})\big) > \mathbf{b}\big(disc(\mathbf{G})\big)$. Since $com(\mathbf{G})$ and $disc(\mathbf{G})$ are symmetric, the equilibrium transfer vector that maximises the sum of probabilities for A is $\mathbf{m}^*$. Each legislator gets the same amount of transfer in both cases but the lobbyist will benefit in the denser network if the status quo bias is reasonably small. In general, we compare the sum of probabilities of votes for A in both the networks $\mathbb{P}_A(\mathbb{G}^\oplus,\mathbf{m}^*\,|\,\beta)$ and $\mathbb{P}_A(\mathbb{G},\mathbf{m}^*\,|\,\beta)$. For any two symmetric graphs the optimal transfer vector is $\mathbf{m}^*_\oplus = \mathbf{m}^*$. A denser graph is advantageous to the lobbyist if the actual value of the legislators bias does not exceed the critical value i.e. $\beta < \beta_c^{\mathbf{mm}} = \beta_c$.(see Appendix 2.A.5a).

Now, if we consider two asymmetric graphs $\mathbb{G}$ and $\mathbb{G}^\oplus$ where $\mathbb{G} \subset \mathbb{G}^\oplus$ such that the Bonacich centrality of legislator i for $\mathbb{G}$ and $\mathbb{G}^\oplus$ is not proportional for all i's. In other words, $b_i^\oplus \neq c \cdot b_i$ for all $i \in N$ where $c > 1$. The equilibrium transfers that maximises the sum of probabilities for the respective graphs $\mathbb{G}$ and $\mathbb{G}^{\oplus}$ are $\mathbf{m}^*$ and $\mathbf{m}^*_{\oplus}$. The denser network is valuable to the lobbyist i.e. $\mathbb{P}_A(\mathbf{m}^*_{\oplus}, \mathbb{G}^{\oplus} \big| \beta) > \mathbb{P}_A(\mathbf{m}^*, \mathbb{G} \big| \beta)$, if $\beta < \beta_c(\mathbb{G}, \mathbb{G}^{\oplus})$. (for results see Appendix 2.A.5b). If the legislators have a small bias towards policy S and is paid according to the equilibrium transfer vectors $\mathbf{m}^*_{\oplus}$ then the lobbyist benefits more from denser graph if the above condition holds. A lobbyist is better-off with a less-connected network if the legislators status quo bias is at least as good as the critical value $\beta_c^{\mathbf{m}\mathbf{m}_{\oplus}}$.

To summarize, we have shown that when the legislators' bias $\beta \leq 0$, the denser network always benefits the lobbyist. Intuitively, legislators bias towards policy A works along with the lobbyist's budget to influence legislator's voting decision. A legislator directly benefits from the monetary transfer in voting for A as opposed to a disutility of β (if negative) if they vote for S. This induces individual voting decision towards policy A. Additionally, the strategic complementarity from neighbours voting decisions towards A further assists the alternative policy to be chosen. In that scenario, a denser network will diffuse the spillover effect better for policy A aiding the lobbyist.

On the other hand, if $\beta > 0$, a sparser or a less connected graph can favour the lobbyist $\mathbb{P}_A(\mathbb{G}) > \mathbb{P}_A(\mathbb{G}^{\oplus})$ if the actual value for the bias β is beyond the critical limit β_c. The result only holds for reasonably small status quo bias because of two opposing forces. Here a positive status quo bias works against the lobbyist's budget. A legislator can directly benefit from either the monetary transfer if they vote for A or from β (positive) if votes for S. Further, she draws additional utility from the spillover effects of neighbours votes. If the budget available to the lobbyist is too inadequate to overcome the effect of the status quo bias, an equilibrium with policy S can be sustained. Under such circumstances, a sparser network will favour by the lobbyist. A sparser network will dampen the diffusion of the spillover effect of the legislators votes towards A. A numerical example is provided in the later section. Moreover, a lobbyist with adequate resources can overcome the effect of the status quo bias and benefit from a denser network only if the bias is below a critical value.

2.4 Effect to Lobbyist Under Different Networks

In this subsection we characterize the conditions where adding a new link between any two legislator helps or hurts the lobbyist. By definition, if we add any non-negative link to the given graph implies complementarity effect in the network increases. By keeping the number of legislators unchanged at n we ensure that the topology of the network remains unaffected. Assume an incomplete network $\mathbb{G} \in \mathscr{G}$ and add a link between two agents i and k who are not connected in $\mathbb{G}$ but are connected in $\mathbb{G}^{\oplus}$. In other words, $\mathbb{G}^{\oplus} = \mathbb{G} + \{ik\}$ where $\{ik\} \notin \mathbb{G}$ but $\{ik\} \in \mathbb{G}^{\oplus}$. Let $d_i(\mathbb{G}) = \mathscr{N}\left\{ j \quad | \quad g_{ij} = 1 \right\}$ be the degree or the cardinality of direct links of legislator i in network $\mathbb{G}$. We assume undirected links i.e. $ij = ji$. Any additional link between legislator i and k will change the network from $\mathbb{G}$ to $\mathbb{G}^{\oplus}_{ik}$. Thus the degree of agent i and k in $\mathbb{G}^{\oplus}_{ik}$ increase by one unit i.e. $d_i(\mathbb{G}^{\oplus}_{ik}) = d_i(\mathbb{G}) + 1$ and $d_k(\mathbb{G}^{\oplus}_{ik}) = d_k(\mathbb{G}) + 1$ but $d_j(\mathbb{G}^{\oplus}_{ik}) = d_j(\mathbb{G})$ for all $j \neq i, k$.

We assume an incomplete graph $\mathbb{G}$, thus there exist at least one unconnected link between any two agents i and j such that $g_{ij} = 0$. Let $\mathscr{Z}(\mathbb{G})$ be the set of all unconnected links in graph $\mathbb{G}$ such that $\mathbb{G} \cup \mathscr{Z}(\mathbb{G}) = com(\mathbf{G})$ where $com(\mathbf{G})$ is a complete graph i.e. every legislator is connected to everyone else. The degree of every agent i in $com(\mathbf{G})$ is $(n-1)$ and the set of unconnected nodes in a complete graph is empty i.e. $\mathscr{Z}(com(\mathbf{G})) = \emptyset$.

Let the cardinality of set of unconnected links in graph $\mathbb{G}$ is $z(\mathbb{G})$, i.e. $\mathscr{N}(\mathscr{Z}) = z(\mathbb{G})$. Adding a new link $\{ij\} \in \mathscr{Z}$ to graph $\mathbb{G}$ gives us $\mathbb{G}^{\oplus}_{ij} = \mathbb{G} + \{ij\}$. For given parameter values, we compare two networks $\mathbb{G}$ and $\mathbb{G}^{\oplus}_{ij}$ where an additional link $\{ij\}$ is beneficial to the lobbyist i.e. $\mathbb{P}_A(\mathbb{G}) < \mathbb{P}_A(\mathbb{G}^{\oplus}_{ij})$ iff $\beta < \beta_c^{ij}$. This is a direct application of the result from the previous subsection which leads us to our next proposition. The minimum critical value of status quo bias below which adding a new link to $\mathbb{G}$ always helps the lobbyist and the maximum critical value of bias above which adding a new link always hurts the lobbyist.

Proposition 2.4. *For any incomplete $\mathbb{G}$, there exist $\underline{\beta}_c(\mathbb{G})$ and $\overline{\beta}_c(\mathbb{G})$ such that adding any new link to $\mathbb{G}$ cannot decrease the equilibrium payoff of the lobbyist if $\beta \leq \underline{\beta}_c(\mathbb{G})$; and cannot increase the equilibrium payoff of the lobbyist if $\beta \geq \overline{\beta}_c(\mathbb{G})$.*

Proposition 2.4 establishes that adding a new link to any incomplete net-

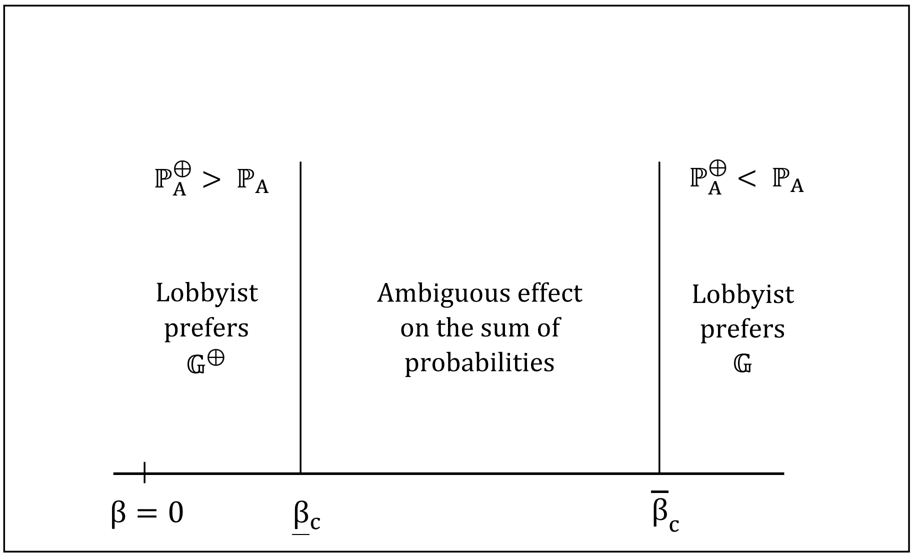
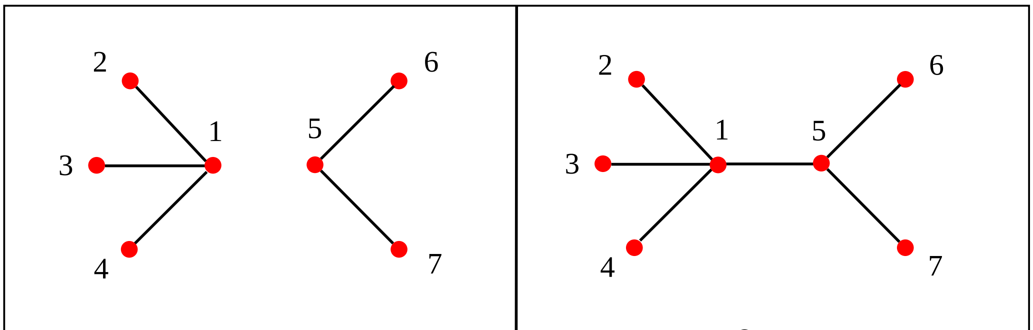


Figure 2.1: Critical value of bias

work is only beneficial to the lobbyist if the legislators are slightly biased towards the status quo policy while adding a new link hurts them if the legislators are highly biased towards status quo S.

Example: Figure 2.2(a) illustrates a simple network $\mathbb{G}$ with seven legislators i.e. $n = 7$. The right panel (b) represents adding a link {15} to the network $\mathbb{G}$. The new graph is represented as $\mathbb{G}_{15}^{\oplus} = \mathbb{G} + \{15\}$. The legislators have the same logarithmic utility function of $u(m_i) = \log m_i$. The total budget available to the lobbyist for distributing among the legislators is given by $M = 100$. For given parameter values of $\theta = 0.1$, $\eta = 0.2$ we can calculate the Bonacich centrality of each legislator in graph $\mathbb{G}$ and their equilibrium transfer vector $\mathbf{m}^*$. The lobbyist maximizes the sum of probabilities $\mathbb{P}_A(\mathbb{G}, \mathbf{m}^*)$. If the value of the status quo bias β of the legislator is relatively small i.e. if $\beta = 2$, the sum of probability in favour of policy A is 3.9902. We have shown that adding a link {15}, {67} or {23} to graph $\mathbb{G}$ increases $\mathbb{P}_A$ as shown in table 2.1. In this case adding any link to the network $\mathbb{G}$ benefits lobbyist. If β is small, the lobbyist uses the transfer to offset the legislators bias towards the status quo policy. Monetary transfer has a direct effect on the utility of each legislator, which favours the lobbyist's objective. Ceteris paribus, additional money never hurts legislator, which improves the probability of an individual voting for A. The indirect ef-



(a) Graph: $\mathbb{G}$

(b) Graph: $\mathbb{G}_{15}^{\oplus} = \mathbb{G} + \{15\}$

Figure 2.2: Adding Links to network $\mathbb{G}$

fect of money advances through the network spillover effect. In other words, any increase in the individual probability for voting in favour of A may influence the vote of her neighbours. On the other hand, for higher value of status quo bias, a denser network may undermine the lobbyist's objective. As shown in our example, if $\beta = 3$ additional links decreases the sum of probabilities from $\mathbb{P}_A(\mathbb{G}|\beta = 3)$ to $\mathbb{P}_A(\mathbb{G}_{15}^{\oplus})$, $\mathbb{P}_A(\mathbb{G}_{67}^{\oplus})$ and $\mathbb{P}_A(\mathbb{G}_{23}^{\oplus})$ respectively. But if $\beta = 2.67$, adding links has an ambiguous effect on the sum of probabilities. In this case, adding the link $\{15\}$ to graph $\mathbb{G}$ increases the sum of probabilities from $\mathbb{P}_A(\mathbb{G}|\beta = 2.67)$ to $\mathbb{P}_A(\mathbb{G}_{15}^{\oplus})$ while adding the link $\{67\}$ to graph $\mathbb{G}$ decreases the sum of probabilities to $\mathbb{P}_A(\mathbb{G}_{67}^{\oplus})$.

Table 2.1: Comparative statics of network $\mathbb{G}, \mathbb{G}^{\oplus}$ and status quo bias β

Graph	$\mathbb{P}_A$	$\beta = 2$	$\beta = 2.67$	$\beta = 3$
$\mathbb{G}$	$\mathbb{P}_A(\mathbb{G})$	3.9902	3.4923	3.2471
$\mathbb{G}_{15}^{\oplus} = \mathbb{G} + \{15\}$	$\mathbb{P}_A(\mathbb{G}_{15}^{\oplus})$	3.9973	3.4927	3.2441
$\mathbb{G}_{67}^{\oplus} = \mathbb{G} + \{67\}$	$\mathbb{P}_A(\mathbb{G}_{67}^{\oplus})$	3.9961	3.4922	3.2440

Note: The above values are for given parameter values of $\eta = 0.2, \theta = 0.1$ and $M = 100$

We compare two networks $\mathbb{G}$ and $\mathbb{G}_{rs}^{\oplus}$ where an additional link $\{rs\}$ is beneficial to the lobbyist i.e. $\mathbb{P}_A(\mathbb{G}) < \mathbb{P}_A(\mathbb{G}_{rs}^{\oplus})$ iff $\beta < \beta_c^{rs}$. The critical value of the bias is given by β_c^{rs} (using lemma 2.3). With slight abuse of notation, if instead we join two other nodes $\{st\} \in \left\{\mathcal{Z} \middle| \{rs\}\right\}$ in $\mathbb{G}$ where $r \neq t$. In this case, we compare the networks $\mathbb{G}$ and $\mathbb{G}^{\oplus}_{st}$. Adding this new link is beneficial to the lobbyist i.e. $\mathbb{P}_A(\mathbb{G}) < \mathbb{P}_A(\mathbb{G}^{\oplus}_{st})$ iff $\beta < \beta^{st}_c$. Using this argument we can get critical value of status quo bias for every unconnected node in $\mathcal{Z}$. Without any loss of generality, we rank all the critical values of status quo bias in $\mathcal{Z}$. The minimum critical value of the bias is $\underline{\beta_c} = \beta^{ij}_c = \min\limits_{rs \in \mathcal{Z}} \left\{\beta_c rs\right\}$ where $\{ij\} \in \mathcal{Z}$. Thus adding the link $\{ij\}$ in $\mathbb{G}$ will be beneficial to the lobbyist i.e. $\mathbb{P}_A(\mathbb{G}^{\oplus}_{ij}) > \mathbb{P}_A(\mathbb{G})$ when $\beta < \underline{\beta_c}$. This implies adding any link $\{rs\} \in \mathcal{Z}$ benefits the lobbyist, if the actual value of the status quo bias is relatively small i.e. $\beta \leq \underline{\beta_c}$. A visual representation is available in Figure 2.1. In Proposition 2.2 and Lemma 2.3, we show that the critical value of the bias β_c is contingent on the structure of the graphs in comparison $\mathbb{G}$ and $\mathbb{G}^{\oplus}$. While adding a single link to $\mathbb{G}$, we look for a graph $\mathbb{G}^{\oplus}_{ij}$ that yields least critical value of the bias $\underline{\beta_c}$. Using this result we infer, if the actual value of the bias β is less than the critical value $\underline{\beta_c}$, then the lobbyist will always be better off with a denser network.

Similarly using the same argument, there must exist a link $kl \in \mathcal{Z}$ which yields the maximum critical value of bias $\overline{\beta_c} = \beta^{kl}_c = \max\limits_{rs \in \mathcal{Z}} \left\{\beta^{rs}_c\right\}$. Thus, adding the new link $\{kl\}$ will hurt the interest group i.e. $\mathbb{P}_A(\mathbb{G}^{\oplus}_{kl}) < \mathbb{P}_A(\mathbb{G})$ when $\beta > \overline{\beta_c}$. Using the similar argument as above, we get a graph $\mathbb{G}^{\oplus}_{kl}$ that has the maximum critical value of the bias $\overline{\beta_c}$. Thus, adding a new link to the graph $\mathbb{G}$ will always be harmful for the lobbyist iff $\beta > \overline{\beta_c}$ and the interest group benefits from a sparser graph. In other words, for a relatively large status quo bias beyond $\overline{\beta_c}$ adding any new link hurts lobbyist. We can further extend this argument for adding two links anywhere in the graph $\mathbb{G}$ and more.

From the above analysis we see that for any given $\mathbb{G}$ and M, if inherent bias of the legislator is $\beta < \underline{\beta_c}$, then adding any link will always be favourable to the interest group and for any $\beta > \overline{\beta_c}$ it will be detrimental. But for any value of the bias $\beta \in [\underline{\beta_c}, \overline{\beta_c}]$, the effect of adding links will be ambiguous and will be contingent of the specific links added.

2.5 Concluding Remarks

In this chapter, we present a theory on the role of promises of transfer on voting decisions of connected legislators. We exploit the idea of the lobbyist using money as a tool to influence connected legislators by paying them according to the Bonacich centrality vector. We measure the sum of voting probabilities for the new policy and its responsiveness to the changes in the budget constraint and the legislator's bias. This approach provides us with a premise to do a comparative study between voting outcomes under two networks. We see that for a relatively well connected graph, the legislators' bias towards new policy never hurts the lobbyist and also provide a positive sharp cut-off for the critical value of bias beyond which the lobbyist cannot gain from a denser network. We show that adding links is beneficial to the lobbyist if the bias is smaller than a minimum critical value and hurts her when the actual value of the bias is beyond the maximum critical value.

In the previous part of our analysis we have focused on equilibrium transfers and graph comparison under strategic complementarities among the legislators. In the next chapter we extend our analysis and results by including strategic substitutability within the legislators. This would improve our understanding of the voting behaviour and decision in a more general setting. Moreover we can do a comparative static analysis between two networks. Including negative ties in our framework where $g_{ij} \in \{-1, 0, 1\}$ provides an insight on the impact of conflict within legislators and it's impact on the agent's centrality and their voting decisions.

Appendix 2.A

[2.A.1] The probability $p_j(\mathbf{m})$ of each legislator j for voting in favour of the new policy is derived from the cumulative distribution function of the uniform error distribution $\epsilon_i \sim U[-\frac{1}{2\theta}, \frac{1}{2\theta}]$:

$$
\begin{aligned}
& -\frac{1}{2\theta} \leq \epsilon_i \leq u(m_i) + \eta \sum_j g_{ij}(2p_j - 1) \\
C.D.F. \Leftrightarrow\ & p_j(\mathbf{m}) = \theta\big[u(m_j) - \beta + \eta \sum_j g_{ij}(2p_j(\mathbf{m}) - 1) - (-\frac{1}{2\theta})\big] \qquad (2.A.1) \\
\Leftrightarrow\ & p_j(\mathbf{m}) = \frac{1}{2} + \theta\big[u(m_j) - \beta + \eta \sum_j g_{ij}(2p_j(\mathbf{m}) - 1)\big]
\end{aligned}
$$

[2.A.2] **Proof of Proposition 2.1(a):** Assuming that the sum of probabilities is differentiable and the inverse $(\mathbb{I}_n - \eta^* \mathbb{G}^T)^{-1}$ exists, we get:

$$
\begin{aligned}
& J_i[\mathbf{p}] = \theta.(\mathbb{I}_n - \eta^* \mathbb{G})^{-1}.J_i[\mathbf{u}] \\
\Leftrightarrow\ & J_i[\mathbf{p}]^T = \theta.J_i[\mathbf{u}]^T.(\mathbb{I}_n - \eta^*.\mathbb{G}^T)^{-1} \\
\Leftrightarrow\ & J_i[\mathbf{p}]^T.\mathbf{1} = \theta.J_i[\mathbf{u}]^T.(\mathbb{I}_n - \eta^*.\mathbb{G}^T)^{-1}.\mathbf{1} \\
\Leftrightarrow\ & J_i[\mathbf{p}]^T.\mathbf{1} = \theta.J_i[\mathbf{u}]^T.\mathbf{b}(\eta^*, \mathbb{G}^T)
\end{aligned} \qquad (2.A.2a)
$$

Solving the optimal value of transfer m_i^* from equation 2.7a and appendix 2.A.2a we get,

$$
\begin{aligned}
& \lambda = \theta.J_i[\mathbf{u}]^T.\mathbf{b}(\eta^*, \mathbb{G}^T) = J_i[\mathbf{p}]^T.\mathbf{1} \\
\Leftrightarrow\ & \lambda^* = J_i[\mathbf{u}]^T.\mathbf{b}(\eta^*, \mathbb{G}^T) = u'(m_j).b_j; \qquad \forall j \in N \\
\Leftrightarrow\ & \lambda^* = u'(m_1).b_1 = \cdots = u'(m_n).b_n
\end{aligned}
$$

Without any loss of generality we can include the constant θ in the Lagrangian parameter λ^*. Incorporating the above result $m_j = u'^{-1}\big(u'(m_i).\frac{b_i}{b_j}\big)$ in the budget equation of the lobbyist. We get,

$$
\begin{aligned}
& M = \sum_j m_j = \sum_j u'^{-1}\big(u'(m_i).\frac{b_i}{b_j}\big) \\
\Leftrightarrow\ & m_j^* = m_j^*(M, \mathbf{b})
\end{aligned} \qquad (2.A.2b)
$$

[2.A.3] For any given M, when $\beta = 0$ if we use the same optimal transfer $\mathbf{m}^*(\mathbf{b}, \mathbb{G})$ for both graphs $\{\mathbb{G}, \mathbb{G}^{\oplus}\} \in \mathcal{G}$ where $\mathbb{G} \subset \mathbb{G}^{\oplus}$. We verify whether a denser graph is beneficial for the interest group in the absence of bias. The relation between the sum of probabilities $\mathbb{P}_A(\mathbb{G}, m^*)$ and $\mathbb{P}_A(\mathbb{G}^{\oplus}, m^*)$ is as follows:

$$\begin{aligned}
&\mathbb{P}_A(\mathbb{G},\mathbf{m}^*) < \mathbb{P}_A(\mathbb{G}^{\oplus},\mathbf{m}^*) \\
\Leftrightarrow\ & \frac{n}{2} + \theta\sum_j b_j \cdot u(m_j^*) < \frac{n}{2} + \theta\sum_j b_j^{\oplus} \cdot u(m_j^*) \\
\Leftrightarrow\ & \sum_j b_j \cdot u(m_j^*) < \sum_j b_j^{\oplus} \cdot u(m_j^*) \qquad [\because b_j < b_j^{\oplus} \qquad \forall j \in N] \\
\Leftrightarrow\ & 0 < \sum_j (b_j^{\oplus} - b_j).u(m_j^*) \qquad [\because u(\cdot) > 0]
\end{aligned} \tag{2.A.3}$$

The aggregate Bonacich centrality in a denser network is greater, i.e. $\sigma^{\oplus} > \sigma$ where $\sigma^{\oplus}$ and σ are the sum of the Bonacich centralities of $\mathbb{G}^{\oplus}$ and $\mathbb{G}$ respectively.

[2.A.4] For a given M, when $\beta < 0$ if we use the same optimal transfer $\mathbf{m}^*(\mathbf{b},\mathbb{G})$ for both graphs $\{\mathbb{G},\mathbb{G}^{\oplus}\} \in \mathscr{G}$ where $\mathbb{G} \subset \mathbb{G}^{\oplus}$. We verify whether a denser graph is beneficial for the interest group in presence of negative bias. The relation between the sum of probabilities $\mathbb{P}_A(\mathbb{G},\mathbf{m}^*)$ and $\mathbb{P}_A(\mathbb{G}^{\oplus},\mathbf{m}^*)$ is as follows:

$$\begin{aligned}
&\mathbb{P}_A(\mathbb{G},\mathbf{m}^*) < \mathbb{P}_A(\mathbb{G}^{\oplus},\mathbf{m}^*) \\
\Leftrightarrow\ & \frac{n}{2} + \theta\sum_j b_j.u(m_j^*) - \theta\beta\sum_j b_j < \frac{n}{2} + \theta\sum_j b_j^{\oplus}.u(m_j^*) - \theta\beta\sum_j b_j^{\oplus} \\
\Leftrightarrow\ & \sum_j b_j.u(m_j^*) - \beta\sum_j b_j < \sum_j b_j^{\oplus}.u(m_j^*) - \beta\sum_j b_j^{\oplus} \qquad [\because \sum_j b_j < \sum_j b_j^{\oplus}] \\
\Leftrightarrow\ & \beta\sum_j (b_j^{\oplus} - b_j) < \sum_j (b_j^{\oplus} - b_j).u(m_j^*) \qquad [\because u(\cdot) > 0]
\end{aligned} \tag{2.A.4}$$

Since $\beta < 0$, the L.H.S is negative and R.H.S is positive so the above relation $\mathbb{P}_A(\mathbb{G},\mathbf{m}^*) < \mathbb{P}_A(\mathbb{G}^{\oplus},\mathbf{m}^*)$ holds.

[2.A.5] **Proof of Proposition 2.3 Case(a)**: When the legislators have a positive bias $\beta > 0$ for the status quo policy S, given the same optimal transfer $\mathbf{m}^*$ we compare the sum of probabilities of votes in favour of A for the graphs $\{\mathbb{G},\mathbb{G}^{\oplus}\} \in \mathscr{G}$ where $\mathbb{G} \subset \mathbb{G}^{\oplus}$. The relation between the sum of probabilities $\mathbb{P}_A(\mathbb{G},\mathbf{m}^*)$ and $\mathbb{P}_A(\mathbb{G}^{\oplus},\mathbf{m}^*)$ is as follows:

$$
\begin{aligned}
&\mathbb{P}_A(\mathbb{G},\mathbf{m}^*) < \mathbb{P}_A(\mathbb{G}^\oplus,\mathbf{m}^*)\\
\Leftrightarrow\ &\sum_j b_j.u(m_j^*) - \beta\sum_j b_j < \sum_j b_j^\oplus.u(m_j^*) - \beta\sum_j b_j^\oplus \qquad [\because \sum_j b_j < \sum_j b_j^\oplus]\\
\Leftrightarrow\ &\beta\sum_j (b_j^\oplus - b_j) < \sum_j (b_j^\oplus - b_j).u(m_j^*) \qquad [\because u(.) > 0]\\
\Leftrightarrow\ &\beta < \frac{\sum_j (b_j^\oplus - b_j).u(m_j^*)}{\sum_j (b_j^\oplus - b_j)} \qquad \left[\because w_j(\mathbf{b},\mathbf{b}^\oplus, m_j^*) = \frac{(b_j^\oplus - b_j).u(m_j^*)}{\sum_j (b_j^\oplus - b_j)}\right]
\end{aligned}
\tag{2.A.5a}
$$

Since $\beta > 0$, both the R.H.S and L.H.S in the above equation 2.A.5a is positive. Thus the relation between the sum of probabilities, $\mathbb{P}_A(\mathbb{G},\mathbf{m}^*) < \mathbb{P}_A(\mathbb{G}^\oplus,\mathbf{m}^*)$ holds true iff $\beta < \beta_c^{\mathbf{mm}} = \sum_j w_j(\mathbf{b},\mathbf{b}^\oplus, m_j^*)$ for all $j \in N$. When the legislators are slightly biased towards the status quo policy S and are paid according to the same transfer vector $\mathbf{m}^*$ for both the graphs $\mathbb{G}$ and $\mathbb{G}^\oplus$, then the lobbyist benefits more from a well-connected graph if the above restriction holds. In other words, a lobbyist is better-off with a sparser network if the legislators bias towards S is above the weighted sum of utilities $\sum_j w_j(\mathbf{b},\mathbf{b}^\oplus, m_j^*)$ where $w_j(\mathbf{b},\mathbf{b}^\oplus, m_j^*)$ is the ratio of difference of individual Bonacich centrality of legislator j for $\mathbb{G}$ and $\mathbb{G}^\oplus$ to the sum of the differences. When the legislators bias towards the status quo policy is too high, monetary contribution from the interest group is not enough to compensate for the bias in a relatively well connected network.

Case(b) When the legislators have a positive bias $\beta > 0$ for the status quo policy S, if separate optimal transfer $\mathbf{m}^*$ and $\mathbf{m}_\oplus^*$ maximizes the sum of probabilities of votes in favour of A for the graphs $\{\mathbb{G},\mathbb{G}^\oplus\} \in \mathscr{G}$ where $\mathbb{G} \subset \mathbb{G}^\oplus$. The relation between the sum of probabilities $\mathbb{P}_A(\mathbf{m}^*,\mathbf{G})$ and $\mathbb{P}_A(\mathbb{G}^\oplus,\mathbf{m}_\oplus^*)$ is as follows:

$$
\begin{aligned}
&\mathbb{P}_A(\mathbb{G},\mathbf{m}^*) < \mathbb{P}_A(\mathbb{G}^\oplus,\mathbf{m}^*_\oplus)\\
\Leftrightarrow &\sum_j b_j \cdot u(m^*_j) - \beta\sum_j b_j < \sum_j b^\oplus_j . u(m^*_{j\oplus}) - \beta\sum_j b^\oplus_j \qquad [\because \sum_j b_j < \sum_j b^\oplus_j]\\
\Leftrightarrow &\beta\sum_j (b^\oplus_j - b_j) < \sum_j \left(b^\oplus_j u(m^*_{j\oplus}) - b_j . u(m^*_j)\right) \qquad [\because u(.)>0]\\
\Leftrightarrow &\beta < \frac{\sum_j \left(b^\oplus_j u(m^*_{j\oplus}) - b_j . u(m^*_j)\right)}{\sum\limits_j (b^\oplus_j - b_j)} \qquad \left[\because y_j(\mathbf{b},\mathbf{b}^\oplus, m^*_{j\oplus}) = \frac{\left(b^\oplus_j u(m^*_{j\oplus}) - b_j . u(m^*_j)\right)}{\sum\limits_j (b^\oplus_j - b_j)}\right]
\end{aligned}
\tag{2.A.5b}
$$

Since $\beta > 0$, both the R.H.S and L.H.S in the above equation 2.A.5b is positive. Thus the relation between the sum of probabilities, $\mathbb{P}_A(\mathbb{G},\mathbf{m}^*) < \mathbb{P}_A(\mathbb{G}^\oplus,\mathbf{m}^*_\oplus)$ holds true iff $\beta < \beta_c^{\mathbf{mm}_\oplus} = \sum\limits_j y_j(\mathbf{b},\mathbf{b}^\oplus, m^*_{j\oplus}) = \beta_c(\mathbb{G},\mathbb{G}^\oplus)$. If the legislators have a small bias towards policy S and is paid according to the equilibrium transfer vectors $\mathbf{m}^*$ and $\mathbf{m}^*_\oplus$, then the lobbyist benefits more from denser graph if the above condition holds. A lobbyist is better-off with a less-connected network if the legislators bias towards S is above the sum of the weighted difference of utilities $\sum_j y_j(\mathbf{b},\mathbf{b}^\oplus, m^*_{j\oplus})$ where $y_j(\mathbf{b},\mathbf{b}^\oplus, m^*_{j\oplus})$ is the ratio of difference of individual Bonacich centrality of legislator j for $\mathbb{G}$ and $\mathbb{G}^\oplus$ to the sum of the differences of Bonacich centrality. By definition, the bonacich centrality vector in a denser network is always larger $\mathbf{b}^\oplus > \mathbf{b}$. Manipulating equations 2.8b we get, $(\mathbf{u}^{\oplus T} - \beta\cdot\mathbf{1}^T)\mathbf{b}^\oplus \geq (\mathbf{u}^T - \beta\cdot\mathbf{1}^T)\mathbf{b}$ where $\mathbf{u}$ and $\mathbf{u}^\oplus$ are the utility of money from the equilibrium transfers $\mathbf{m}^*$ and $\mathbf{m}^*_\oplus$ respectively. If the utility from monetary contribution to the legislator dominates their status quo bias β, a well-connected network aids lobbyist. In other words, the lobbyist can offset a small positive bias of the legislators with monetary contribution for a relatively denser network.

[2.A.6] Proof of Proposition 2.4 : Let's compare two networks $\mathbb{G}$ and $\mathbb{G}^\oplus_{rs}$ where an additional link $\{rs\}$ is beneficial to the lobbyist i.e. $\mathbb{P}_A(\mathbb{G}) < \mathbb{P}_A(\mathbb{G}^\oplus_{rs})$ if $\beta < \beta_c^{rs}$. The critical value of the bias is given by β_c^{rs} (using lemma 2.3). With slight abuse of notation, if instead we join two other nodes $\{st\} \in \left\{\mathcal{I}\middle|\{rs\}\right\}$ in $\mathbb{G}$ where $r \neq t$. In this case, we compare the networks $\mathbb{G}$ and $\mathbb{G}^\oplus_{st}$. Adding this new link is beneficial to the lobbyist i.e. $\mathbb{P}_A(\mathbb{G}) < \mathbb{P}_A(\mathbb{G}^\oplus_{st})$

if $\beta < \beta_c^{st}$. Using this argument we can get critical value of status quo bias for every unconnected node in $\mathcal{Z}$.

Without any loss of generality, we rank all the critical values of status quo bias in $\mathcal{Z}$. The minimum critical value of the bias is $\underline{\beta_c} = \beta_c^{ij} = \min_{rs \in \mathcal{Z}} \left\{ \beta_c^{rs} \right\}$ where $\{ij\} \in \mathcal{Z}$. Thus adding the link $\{ij\}$ in $\mathbb{G}$ will be beneficial to the lobbyist i.e. $\mathbb{P}_A(\mathbb{G}_{ij}^{\oplus}) > \mathbb{P}_A(\mathbb{G})$ when $\beta < \underline{\beta_c}$. This implies adding any link $\{rs\} \in \mathcal{Z}$ benefits the lobbyist, if the actual value of the status quo bias is relatively small i.e. $\beta \le \underline{\beta_c}$.

Similarly, there must exist a link $\{kl\} \in \mathcal{Z}$ which yields the maximum critical value of bias $\overline{\beta_c} = \beta_c^{kl} = \max_{rs \in \mathcal{Z}} \left\{ \beta_c^{rs} \right\}$. Thus, adding the new link $\{kl\}$ will hurt the lobby group i.e. $\mathbb{P}_A(\mathbb{G}_{kl}^{\oplus}) < \mathbb{P}_A(\mathbb{G})$ when $\beta > \overline{\beta_c}$. Using the similar argument as above, we get a graph $\mathbb{G}_{kl}^{\oplus}$ that has the maximum critical value of the bias $\overline{\beta_c}$.

List of Symbols for Chapter 3

$\tau \in \{L, R\}$	Set of parties where L is left and R is right
$\psi \in \{S, A\}$	Set of policies where S is status quo policy and A is new policy
n_τ	Number of members in party τ
$g^\tau_{ij} \in \{0, 1\}$	Connections between two legislators where i and j belongs to party τ
$g^{\tau\tau'}_{ij} \in \{-1\}$	Connections between two legislators where $i \in \tau$ and $j \in \tau'$
$\mathbb{G}_\tau = [g^\tau_{ij}]$	Adjacency Matrix of connections among legislators in party τ
$\mathbb{G} = [g_{ij}]$	Adjacency Matrix of the whole network
$\Pi_i(\cdot)$	Total utility of any agent i
$u(m_i) \geq 0$	Utility of an agent i from money transfer m_i
$\mathbf{m} \in \mathbb{R}^n_+$	Monetary transfer vector to the legislators
$p_i \in (0, 1)$	Probability of an agent i to vote for policy A
$\mathbf{p} \in (0, 1)^n$	Vector of probabilities of legislators voting for policy A
$\mathbb{P}_A = \sum_j p_j \in (0, n)$	Sum of probabilities of legislators voting for policy A
$M = \sum_j m_i$	Total budget available to the lobbyist
$\beta_\tau \in \mathbb{R}$	Status Quo Bias of legislators in party τ
$\eta \in \mathbb{R}$	Network Spillover effect
κ_τ	Degree of conflict of legislators of party τ
$\epsilon_i \; U[-\frac{1}{2\theta}, \frac{1}{2\theta}]$	Uniformly distributed exogenous Shock parameter of agent i
θ	Density of the error term
$v_j(\psi)$	Indicator function, whether agent j votes for policy ψ
$b_i(\mathbb{G}_\tau)$	Unweighted Bonacich Centrality of a legislator $i \in \tau$
$\mathbf{b}(\eta, \mathbb{G}_\tau)$	Bonacich Centrality vectors of legislators in party τ with weights ω_τ
$\boldsymbol{b}(\mathbb{G}) = \omega_\tau \mathbf{b}_\tau$	Weighted Bonacich Centrality vector of legislators in party τ
$\sigma_\tau = \sum_j b^\tau_j$	Sum of unweighted Bonacich Centrality of legislators in party τ
$J_i[\cdot] = \frac{d}{dm_i}[\cdot]$	Jacobian matrix with respect to m_i

Chapter 3

Lobbying networks of agents with opposing policy bias

3.1 Introduction

In network literature, a social connection is a relation between two agents. In Chapter 2, we have studied the effect of positive ties on the voting decisions of the legislators. We have shown that the interpersonal ties among legislators plays a role in allocation of resources among legislators. In our model, the lobbyist exploits the positive ties among legislators towards his preferred policy. Now we introduce negative ties among legislators to introduce the notion of conflict, mistrust or incompatibility between the agents. Studying the effect of friendship of legislators is not uncommon in among political scientists and sociologists Fowler (2006). In this chapter we model conflict among legislators through negative ties and any legislator voting in line with a negatively linked neighbour gives her disutility.

We model n legislators vote for the status quo or the alternative policy. Legislators are affiliated to either the 'left' L or the 'right' R party. The legislators in party L has a bias towards the status quo policy and the legislators in party R is biased towards the alternative policy. The legislators care about the promised fund they can obtain from the lobbyist if they vote for the lobbyist's preferred policy and her neighbours' vote. One critical difference from the previous chapter which makes this model a more interesting application of our baseline study is the negative links among legislators across parties. We assume that each legislator is either connected or unconnected with a member of his own party. Also, each legislator in party L is negatively linked with every member in R and vice versa. Including this feature in our model helps capture both the strategic substitutability and complementarity in legislators actions. On one hand each legislator receives a positive benefit if she votes in line with 'friends' from her own party. At the same time, she gets a disutility if her vote matches with legislators from the opposition. In other words, each legislator benefits η by aligning with their friends decisions and suffers κ for voting in line with opposition legislators. η is the positive effect from conforming with 'friends' vote and κ can be interpreted as the degree of conflict or cost from aligning with the 'oppositions' vote. Hence every legislator wants to distinguish or distance themselves from their opposition's ballot to avoid the disutility. Our aim is to understand how the lobbyist with a given budget, uses the network structure to his advantage to efficiently allocate funds.

Under our assumptions, the equilibrium transfer to each legislator is a function of their relative position within the party and the total resource allocated to that party. The lobbyist benefits if it has a larger budget. For any legislator in a given party, any increase in the degree of conflict improves their centrality in the network and their equilibrium transfers and deteriorates the centrality of the members of the opposition along with their respective transfers. Again, if the magnitude of the status quo bias of the legislators is sufficiently small (large), then the lobbyist benefits(disadvantages) from a denser network. We show that any increase in the degree of conflict of a given party has a positive impact on the individual and the overall equilibrium payments of the legislators within the party. We establish that any increase in the degree of conflict of a given party will have a positive impact on the individual and the overall equilibrium payments of the legislators within the party. This increase in conflict will worsen the weighted Bonacich Centrality of the opposition legislators because of the substitutability effect across parties. We also examine the marginal impact of any change in the network structure on the lobbyist. Regarding that, we find that any additional links in a given party improves the overall fund allocation to that party because of the increased network activity. We see that for a relatively well connected graph, the legislators' bias towards new policy hurts the lobbyist if the status quo bias is above a critical threshold.

In the previous chapter we have studied the effect of promised monetary transfer of a single lobbyist on the votes of connected legislators. We examine the marginal impact of a change in the network structure on maximising the lobbyist's objective. In this chapter, we've further enriched the baseline model from Chapter 2 by introducing two opposing parties connected by negative links. For an extended portion of our analysis we have assumed a logarithmic utility function for sharper results. We aim to study the impact of lobbying on the equilibrium payment of the legislators in different parties and the total resource allocation to political parties under different network setting. Assuming a concave utility function of money gives us closed form solutions. Our analysis can be extended to a broader class of utility functions with minor restrictions.

The remainder of the chapter is organized as follows. Section 2 presents our model on influencing legislators' votes through promises of monetary transfer from the lobbyist. We find the equilibrium allocation of funds required to maximize the lobbyist objective and its relation to the voting preferences. In Section 3, we do a comparative study among two networks and find the effects on voting preferences for a larger network.

3.2 Model

The legislators are assumed to be self-interested, partisan, individual utility maximizers with diminishing returns. Each legislator cares about her own policy preferences but also take into account her neighbours decisions. We assume each legislator to be office motivated where she cares about her own vote and her neighbours' vote.

There is an interest group whose objective is to influence the legislators to choose the new policy A in exchange of money. The interest group is endowed with monetary resources M. Interest groups promise monetary payments to each legislator if they vote for the lobbyist's preferred policy. The legislator's calculate their expected utilities between voting for the new policy A or the status quo S. Legislators vote for their favoured policy $\psi \in \{S, A\}$ and the winning policy is chosen via the plurality rule and payoff's are realised.

The legislators are divided into two political parties, left L and right R. Each party $\tau \in \{L, R\}$ has $n_\tau > 0$ members where $\sum_\tau n_\tau = n$. The members of the parties are mutually exclusive. If we merge the two political parties, we get back our initial model in Chapter 2. The legislators of party L has a uniform positive bias β_L towards the status quo policy and the members of party R has a uniform negative bias away from status quo policy. In other words, members in party L has a bias β_L towards S and members in party R has a bias β_R towards A. The legislators are connected by an exogenously given network structure.

3.2.1 Definitions

Consider a legislature with n members where n_L members belong to party L and n_R in party R. Each legislator simultaneously choose between two alternative policies $\psi \in \{A, S\}$ where the status quo policy is S and the new policy is A.

$\mathbb{G}_\tau = [g^\tau_{ij}]$ is a zero-diagonal, symmetric $n_\tau \times n_\tau$ matrix where $\mathbb{G}_\tau$ is also interpreted as the intra-party adjacency matrix between the members of each party τ. $\mathbb{G}_\tau$ is an unweighted, undirected[1], symmetric matrix where g^τ_{ij} represents social connections between legislator i and j from party τ. For any two legislators of the same party τ, $g^\tau_{ij} = g^\tau_{ji} = 1$ indicates that i and j are linked[2], otherwise $g^\tau_{ij} = g^\tau_{ji} = 0$. Legislators i and j are defined as *compatible* neighbours if $g^\tau_{ij} = 1$ and $g^\tau_{ij} > 0$ captures the strategic complementarity effect of j's action on i's vote. For example, if legislator i of party L is connected to another member j of the same party, i.e. $g^L_{ij} = 1$, then i and j are compatible neighbours. $\mathbb{G}_L$ and $\mathbb{G}_R$ are the adjacency matrices of all the compatible neighbours of the legislators of party L and party R respectively.

We also assume each member of party τ is negatively linked to every mem-

[1]Modelling directed links will lead to different results and $\mathbb{G}_\tau$ will be an asymmetric square matrix.

[2]We rule out self loops or multiple links between a pair of nodes.

ber of party τ' i.e. $g_{ij}^{\tau\tau'} = \{-1\}$ where i is a member of party τ and j is from party τ'. In such cases legislators i and j are defined as *conflicting* neighbours. $g_{ij}^{\tau\tau'} < 0$ is the strategic substitutability effect of the action of j's action on i's vote. Again, if a legislator i of party L is connected to another legislator j of party R, i.e. $g_{ij}^{LR} = -1$, then i and j are conflicting neighbours. Each legislator in party L is negatively linked to every member in R, so the adjacency matrix of the conflicting neighbours of the legislators of party L, $\mathbb{G}_{LR}$ is a $n_L \times (n - n_L)$ matrix of ones. Similarly, $\mathbb{G}_{RL}$ is a $(n - n_L) \times n_L$ the adjacency matrix of the conflicting neighbours of R.

For simplicity, we assume $g_{ij} \in \{-1, 0, 1\}$ where the link between legislators are unweighted. Here we study both strategic substitutability and complementarity effect on the legislators behaviour. Each legislator is both positively and negatively influenced by the action of others i.e each legislator reaps the benefits of the positive links by conforming to compatible neighbours action but also suffers from discomfort from conflicting neighbours action. The maximum number of direct links possible for any legislator i is $n - 1$.

Here we consider a complex network with $\mathbb{G}$ to represent the connections among legislators. For a given network $\mathbb{G}$ with both positive and negative ties among legislators a simple way to interpret $(\mathbb{I} - \mathbb{G}^T)^{-1} \cdot \mathbf{1} = \boldsymbol{b}$ is the position of a legislator in the entire network. While $(\mathbb{I} - \mathbb{G}_R)^{-1} \cdot \mathbf{1} = \mathbf{b}_R$ and $(\mathbb{I} - \mathbb{G}_L)^{-1} \cdot \mathbf{1} = \mathbf{b}_L$ represents the vectors of Bonacich Centralities of legislators of party R and L respectively.

3.2.2 Payoffs

The utility of each legislator is contingent on the vote she casts and the spillover from voting in sync with her neighbours and exogenous shock. The utility function Π_i of any legislator i in party $\tau \in \{L, R\}$ if she votes for the alternative policy A policy is given by:

$$\Pi_i(A) = u(m_i) + \eta \sum_{j=1}^{n_\tau} g_{ij}^{\tau} v_j(A) + \sum_{j'=(n_\tau+1)}^{n} \kappa_\tau g_{ij'}^{\tau\tau'} v_{j'}(A) \tag{3.1a}$$

where j, j' votes for A. The utility of any legislator i if she votes for S is:

$$\Pi_i(S) = \beta_\tau + \eta \sum_{j=1}^{n_\tau} g_{ij}^{\tau} v_j(S) + \sum_{j'=(n_\tau+1)}^{n} \kappa_\tau g_{ij'}^{\tau\tau'} v_{j'}(S) + \epsilon_i \tag{3.1b}$$

where j, j' votes for S. $v_j(\psi)$ is a binary indicator function of each player j's vote for ψ,

$$v_j(\psi) = \begin{cases} 1, & \text{if legislator } j \text{ votes for } \psi \\ 0, & \text{otherwise} \end{cases}$$

The utility function is additively separable in the monetary benefit and network effects. We assume $u(\cdot)$ to be a standard increasing, concave, continuous, twice differentiable utility function. The first term of the first equation $u(m_i)$ represents the monetary contributions promised to i in lieu of her vote for A. m_i is the amount assured to the legislator i for casting in favour of A. If legislator i is directly linked with another member j from her own party τ and her vote is similar to j's then she benefits from the peer-effects of social interactions through a exogenous spillover effect. The network spillover effect η is assumed to be homogeneous across all legislators i.

Assumption 3.1 (Normalization)**.** $\beta_L + \beta_R = 0$

β_τ is the bias of any individual $i \in \tau$ towards the status quo policy. The status quo bias is uniform across all legislators for a given party. We assume $\beta_L + \beta_R = 0$ where any legislator from party L has a positive bias β_L towards S and any member in party R has a negative bias away from status quo policy. The level and direction of the bias is identical across all legislators in any given party. In other words, the value of biases for two parties are assumed to be perfect substitutes and are normalised to zero[3].

The second element describes the sum of the bilateral influence of all the direct links of agent i within her party τ in $\mathbb{G}_\tau$. The strategic complementarity effect in votes is captured through $v_j(\psi)$ in the second term and is reflected through the network spillover term η when both i and j conform to each others'

[3]The normalization is a simplification and can be manipulated by setting $\beta_L + \beta_R = B$, where B is non-zero.

votes. If any legislator i's is directly aligned with other members of similar preferences then she is more influenced by her compatible peers voting behaviour.

The utility of legislator i also captures strategic substitutability effect the votes of conflicting neighbours' through the indicator function $v_{j'}(\psi)$. The negative spillover effect $\kappa_\tau \in \{\kappa_L, \kappa_R\}$ can be interpreted as the degree of conflict. If a member j' of the opposing party votes in line with i, she receives a disutility κ_τ for conforming with an opposing party. κ_τ is interpreted as the degree of conflict or aversion agent i gets for voting in line with a member of the opposing party. κ_τ is common knowledge. Every member in party L loathes voting in line with legislators of the opposing party R by a magnitude of κ_L. Similarly, legislators in R receives disutility of κ_R for conforming votes with any member in L. The net effect of spillovers on a legislator depends on her party affiliation and the action of her neighbours.

The last term in the second equation is the stochastic parameter ϵ_i affecting player i's preference towards policy S where ϵ_i is independently uniformly distributed between $[-\frac{1}{2\theta}, \frac{1}{2\theta}]$ with mean 0 and density θ. Without any loss of generality, we normalise the exogenous shock parameter affecting preference towards A to be 0.

Assumption 3.2 (Invertability). *The matrix $(\mathbb{I} - \mathbb{G}^T)^{-1}$ is invertible and the sub-matrices $\left(\mathbb{I}_{(n_L)} - \eta^* \mathbb{G}_L^T\right)^{-1}$ and $\left(\mathbb{I}_{(n-n_L)} - \eta^* \mathbb{G}_R^T\right)^{-1}$ are also invertible.*

We assume θ to be considerably small but a positive real number such that considerable uncertainty is present in the environment and for uniqueness of solutions. Invertability is guaranteed if η^* is sufficiently small. We assume the matrix $(\mathbb{I} - \mathbb{G}^T)^{-1}$ is invertible. We assume non-negative entries in the adjacency matrix of each party $\tau \in \{L, R\}$. The legislators within each party are politically aligned and is captured through $\mathbb{G}_L$ or $\mathbb{G}_R$. Invertability of the matrices $\left(\mathbb{I}_{(n_L)} - \eta^* \mathbb{G}_L^T\right)^{-1}$ and $\left(\mathbb{I}_{(n-n_L)} - \eta^* \mathbb{G}_R^T\right)^{-1}$ ensures that the inverse exists[4] and is

[4]We have assumed a non-negative matrix of connections within a party i.e. $g_{ij}^\tau \in \{0,1\}$, so a sufficient condition for invertability of the matrix $(\mathbb{I} - \eta^* \mathbb{G}_\tau)^{-1}$ is similar to the assumption made by Ballester et al. (2006). The following condition guarantees the inverse of the sub-matrices exists for $\eta < \frac{1}{2\theta\zeta(\mathbb{G}_\tau)}$ where $\zeta(\mathbb{G}_\tau)$ is the largest eigenvalue of $\mathbb{G}_\tau$.
But the matrix $(\mathbb{I} - \mathbb{G}^T)^{-1}$ has negative elements since the links between any two members of

positive. Furthermore we can infer that $\left(\mathbb{I}_{(n_L)} - \eta^* \mathbb{G}_L^T\right)^{-1} \cdot \mathbf{1} > \mathbf{0}$ and $\left(\mathbb{I}_{(n-n_L)} - \eta^* \mathbb{G}_R^T\right)^{-1} \cdot \mathbf{1} > \mathbf{0}$.

Assumption 3.3 (Non-Negativity)**.** *The weighted Bonacich centrality vector is positive: If* $\kappa_L < \frac{1}{2\theta\sigma_L}$ *and* $\kappa_R < \frac{1}{2\theta\sigma_R}$*, then* $(\mathbb{I} - \mathbb{G}^T)^{-1} \cdot \mathbf{1} > \mathbf{0}$.

Assumption 3.3 follows from our previous assumption 3.2. Additionally, to ensure the weighted Bonacich centrality vector to be positive we assume $\kappa_L < \frac{1}{2\theta\sigma_L}$ and $\kappa_R < \frac{1}{2\theta\sigma_R}$. The degree of conflict should not be too large and is dependent on the inverse of the sum of party-specific Bonacich centralities. The weighted Bonacich centrality vector $(\mathbb{I} - \mathbb{G}^T)^{-1} \cdot \mathbf{1} > \mathbf{0}$ is positive since it is logical and it is easier to interpret the impact of a marginal increase in centrality on the sum of probability of A. For further calculations see appendix 3.A.2 and 3.B.1. We allow both positive and negative externalities in adjacency matrix $\mathbb{G}$ and to ensure positive values of the centrality of each legislator we need $(1 - \kappa_R^* \sigma_R) > 0$ and $(1 - \kappa_L^* \sigma_L) > 0$. Since κ_L, κ_R and θ is positive it also implies $(1 - \kappa_R^* \kappa_L^* \sigma_R \sigma_L) > 0$.

The conditions for uniqueness and existence of the pure strategy equilibrium based on the network structure. The pay-off dependence of the agents are based on her and her neighbours decisions. We study the allocation of funds to the members of different parties and the comparative statics associated with the change in network $\mathbb{G}_\tau$, the degree of conflict κ_τ and the exogenous bias β_τ.

3.2.3 Timeline

The game proceeds in the following stages:
(a) Each legislator has a bias β_L or β_R which determine their party affiliations.
(b) At time t_0, the connections between the legislators $\mathbb{G}_\tau$ are exogenously determined. $\mathbb{G}_\tau$ is common knowledge.
(c) Observing $\mathbb{G}_\tau$, the lobbyist announces the transfer vector $\mathbf{m}(\mathbb{G}_\tau)$.
(d) At time t_1, each legislator observes a private preference shock ϵ_i.
(e) Legislators vote and payoffs are determined.

opposing parties are $g_{ij}^{\tau\tau'} = \{-1\}$, so invertibility is not sufficient to ensure positive Bonacich centrality.

3.2.4 Problem

The role of the interest group is to influence the legislator to vote in favour of new policy A in exchange of monetary contributions. The lobbyist cannot observe the individual shocks of the legislator. In the influence stage, the lobby group observes the network connections among n legislators and announces a vector of transfers. Based on the announced transfer and the shock, each legislator casts their vote. Hence the lobbyist will allocate funds optimally to maximize the aggregate probability of votes in favour of A. The lobbyist's problem is given as:

$$\max_{\mathbf{m}} \sum_j p_j^\tau(\mathbf{m}) \qquad s.t. \qquad \sum_j m_j \leq M$$

The feasible vector of payments is given by, $\mathbf{m} = (m_1, \cdots, m_n)$ such that $\sum_i m_i \leq M$ and $m_i \geq 0$, for all $i \in N$. M is the total available budget to the lobbyist. Let, $m_i \in \mathbf{m}_i$ and $\mathbf{m}_i$ be the vector of possible contributions available to legislator i. We know, $p_j = E(v_j(A))$ is the ex-ante probability of any legislator j for voting in favour of policy A.

The interest group doesn't know the preferences of the legislators with full certainty because of the exogenous shock parameter ϵ_i. Before making her voting decision each rational legislator i compares the expected pay-off from voting for A over S. She will vote for the new policy A iff:

$$E\Pi_i^\tau(S) - E\Pi_i^\tau(A) \leq 0$$

From the utility equation 3.1a and 3.1b, the utility of legislator $i \in \tau$ depend on the actions of conforming neighbour j and conflicting neighbour j'. Since, legislator i only observes her own exogenous preference shock so, $E(\epsilon_i) = \epsilon_i$. The above equation can be rewritten as:

$$-\frac{1}{2\theta} \leq \epsilon_i \leq (u_i - \beta_\tau) + \eta \sum_{j=1}^{n_\tau} g_{ij}^\tau (2p_j - 1) + \sum_{j'=(n_\tau+1)}^{n} \kappa_\tau g_{ij'}^{\tau\tau'} (2p_{j'} - 1) \tag{3.2}$$

Thus the legislator votes for policy A if the above condition holds. Further calculations on the derivation of the individual probability for voting in favour of A is available in the Appendix 3.A.1.

Voting Stage

The legislators cast their vote based on their individual preferences, monetary transfers and the behaviour of the neighbours. The winning policy will be determined by a given voting rule based on the ballots. Legislators likelihood of voting for A are determined by the probabilities they place on neighbours' voting for A. Since the bias parameter β_τ is party specific and exogenous, we obtain a linear system of simultaneous equations of legislators' individual probabilities of voting for A.

Equilibrium Voting

The probability for any player i from party τ to vote for A is denoted by $p_i^\tau \in [0,1]$ where $\mathbf{p} = (p_1^L, \cdots, p_n^R)^T$ is a vector of the probabilities[5] of all the players such that $\mathbf{p} : \mathbf{m}_1 \times \cdots \times \mathbf{m}_n \rightarrow [0,1]^n$. For any $\mathbf{m}$ the above probability vector is a linear mapping from the probability vector $\mathbf{p}(\mathbf{m})$ to itself where $F(\mathbf{m}, \mathbf{p}(\mathbf{m}))$ is a linear transformation of $\mathbf{p}(\mathbf{m})$. For a small θ, the set F is closed, convex and continuous in $\mathbf{p}$ as it is a contraction mapping from $[0,1]^n$ to itself. So, a unique equilibrium exists. For a larger value of θ the equation may not be well behaved or unique.

The probability of legislators choosing the new policy A is represented as:

$$\mathbf{p}(\mathbf{m}) = \begin{pmatrix} p_1^L(\mathbf{m}) \\ \vdots \\ p_n^R(\mathbf{m}) \end{pmatrix} = \begin{pmatrix} \frac{1}{2} + \theta\big[u(m_1) - \beta_L + \eta \sum_j g_{1j}^L (2p_j^L(\mathbf{m}) - 1) + \kappa_L \sum_{j'} g_{1j'}^R (2p_{j'}^R(\mathbf{m}) - 1)\big] \\ \vdots \\ \frac{1}{2} + \theta\big[u(m_n) - \beta_R + \kappa_R \sum_j g_{nj}^L (2p_j^L(\mathbf{m}) - 1) + \eta \sum_{j'} g_{nj'}^R (2p_{j'}^R(\mathbf{m}) - 1)\big] \end{pmatrix} \tag{3.3}$$

or,

$$\mathbf{p} = \frac{1}{2} \cdot \mathbf{1} + \theta\big(u(\mathbf{m}) - \boldsymbol{\beta}\big) + 2\theta \cdot \mathbb{G}'\mathbf{p} - \theta \cdot \mathbb{G}'\mathbf{1}$$

where $\mathbf{1}$ is an $n \times 1$ column vector of 1's, $u(\mathbf{m})$ is an $n \times 1$ vector of the direct utility from the transfers promised to the legislators, $\mathbf{u} = u(\mathbf{m}) = \big(u(m_1), \cdots, u(m_n)\big)^T$.

[5] $\mathbf{p}$ is a mapping from a set of possible transfers to a set of probabilities.

Also $\boldsymbol{\beta} = (\beta_L, \cdots, \beta_R)^T$ is the vector of bias towards S where β_L is the left party bias for the first n_L legislators and β_R is the right bias for the rest of the $(n - n_L)$ members. $\mathbb{G}'$ is the modified adjacency matrix(see Appendix 3.A.2). We solve for the equilibrium probabilities from equation 3.3 to get the vector of individual probabilities of voting in favour of A(see Appendix 3.A.3). The sum of probabilities of voting in favour of policy A is given by:

$$\mathbb{P}_A = \frac{1}{2} \cdot \mathbf{1}^T \mathbf{1} + \theta(\mathbf{u}^T - \boldsymbol{\beta}^T) \cdot \boldsymbol{b} \tag{3.4}$$

We address the equilibrium probabilities of voting in favour of A from solving the linear system of equations. The sum of the probabilities $\sum_j^n p_j$ in the above equation are continuous, increasing and differentiable with respect to the monetary transfer m_i for all i and converges to n.

Initial Stage

The objective of the interest group is to optimize the allocation of resources among the legislators such that the sum of probabilities is maximized. The interest group will maximise their objective function given a budget constraint:

$$\max_{\mathbf{m}} \sum_j p_j(\mathbf{m}) \qquad s.t. \qquad \sum_j m_j \leq M \tag{3.5}$$

In the previous section we are ensured a pure strategy solution when θ is sufficiently small. For any sufficiently small value of θ, the constrained optimization problem in equation 3.5 and we get the following first order conditions:

$$\sum_j \frac{\partial p_j}{\partial m_i} = J_i[\mathbf{p}]^T \cdot \mathbf{1} = \lambda, \qquad \text{and} \qquad \sum_j m_j = M \tag{3.5a}$$

for all $j \in \{L, R\}$. Using the above first order conditions in equation 3.5a, we solve for the equilibrium level of monetary transfer. We define, $J_i[\mathbf{p}] = [\frac{\partial p_1}{\partial m_i}, \cdots, \frac{\partial p_n}{\partial m_i}]^T$ as the Jacobian matrix or the first order derivative of the vector of probabilities with respect to the transfer made to the legislator $i \in \tau$. Differentiating the optimal probability distribution vector in equation 3.5 with respect to the monetary transfer made to individual legislator m_i, we get $J_i[\mathbf{p}] = \theta(\mathbb{I} - \mathbb{G})^{-1} J_i[\mathbf{u}]$(see Appendix 3.A.4). We know that any individual j benefits from her own transfers

$u(m_j)$. The effect of marginal change in m_i on the direct utility of monetary is $u'(m_j)$ if $j = i$ and 0 otherwise. $J_i[\mathbf{u}]$ is a vector of zero's except for the i-th term which is $\frac{\partial u_i}{\partial m_i}$.[6]

By assumption $(\mathbb{I} - \mathbb{G})^{-1}$ exists and we show that the sum of probabilities $\mathbb{P}_A$ is differentiable and increasing in $m_{i\in L}$. The first order condition from equation 3.5a yields, $J_i[\mathbf{p}]^T\mathbf{1}$ is positive as $u'(.)$ is increasing. The second order sufficiency condition gives, $J_i^2[\mathbf{p}]^T\mathbf{1}$ the sum of probabilities to be concave because of the diminishing marginal returns from money. From equation 3.4, we solve for a linear system of equation for an unique probability vector $\mathbf{p}^*$. From definition of Bonacich centrality and differentiating equation 3.4 (details in appendix 3.A.5) we get the following,

$$J_i[\mathbf{p}]^T \cdot \mathbf{1} = \theta \cdot J_i[\mathbf{u}]^T \boldsymbol{b}$$

Using the above condition in equation 3.5a, we show the Lagrangian multiplier is proportional to the equilibrium transfer m_i^* (appendix 3.A.3)to the legislator. The equilibrium $m_i^*(\boldsymbol{b}, M)$ is conditional on the available budget M. The marginal cost of resources in equilibrium is dependent on the Bonacich centrality and marginal utility of direct transfer $\lambda^* = \frac{\lambda}{\theta} = u'(m_i) \cdot b_i$. Assuming that inverse of the $u'(.)$ exists, the equilibrium transfer to any individual i affiliated to party τ is $m_i^* = m_i^*(M, \boldsymbol{b})$ and the vector of transfers is given by $\mathbf{m}^*$. The indirect utility function(see Appendix 3.A.4) and replacing the optimal transfer $m_i^*(\boldsymbol{b}, M)$ in equation 3.4, we get the total probability of the legislators to vote in favour of the new policy A:

$$\begin{aligned}
\mathbb{P}_A(\mathbf{m}^*) =& \frac{n}{2} + \frac{\theta(1-\kappa_R^*\sigma_R)}{(1-\kappa_R^*\sigma_R\kappa_L^*\sigma_L)} \sum_{j\in L} u(m_j^*) b_j + \frac{\theta(1-\kappa_L^*\sigma_L)}{(1-\kappa_R^*\sigma_R\kappa_L^*\sigma_L)} \sum_{j'\in R} u(m_{j'}^*) b_{j'} \\
&+ \frac{\theta\beta_L}{(1-\kappa_R^*\sigma_R\kappa_L^*\sigma_L)}(\sigma_R - \sigma_L) + \frac{\theta\beta_L\sigma_R\sigma_L}{(1-\kappa_R^*\sigma_R\kappa_L^*\sigma_L)}(\kappa_R^* - \kappa_L^*) \\
=& \frac{n}{2} + \theta\omega_L \sum_{j\in L} u(m_j^*) b_j + \theta\omega_R \sum_{j'\in R} u(m_{j'}^*) b_{j'} - \theta\Big(\beta_L\omega_L\sigma_L + \beta_R\omega_R\sigma_R\Big)
\end{aligned} \tag{3.6}$$

[6]Based on initial assumptions on utility, $J_i[\mathbf{u}] = (0\ 0 \cdots \frac{\partial u_i}{\partial m_i} \cdots 0)^T$.

The weighted Bonacich Centrality vector in our analysis is given by $\boldsymbol{b}$ which can be further decomposed into conflict weighted centrality of party members of L and R. The vector of centralities of n_L members of party L is given by $\omega_L \mathbf{b}_L = \big(\frac{1-\kappa_R^*\sigma_R}{1-\kappa_R^*\sigma_R\kappa_L^*\sigma_L}\big)\mathbf{b}_L$ where $\mathbf{b}_L$ is the Bonacich centrality vector of L. Similarly, $\omega_R \mathbf{b}_R = \big(\frac{1-\kappa_L^*\sigma_L}{1-\kappa_R^*\sigma_R\kappa_L^*\sigma_L}\big)\mathbf{b}_R$ is the weighted centrality vector of $(n-n_L)$ legislators of party R. $\omega_\tau\big(\kappa_\tau,\kappa_{\tau'}\big|\mathbb{G},\eta,\theta\big)$ is the magnitude by which the centrality of the each legislator $i \in l$ is affected and σ_τ is the sum of the Bonacich centrality of each legislator i affiliated to party τ.

Additionally, we assume the budget M to the lobbyist below some critical level M^*. It is set so that $M < M^*$ the network effects are not overwhelmed by the lobbyist's transfer. For most of our study we assume the utility from money to be logarithmic[7] $u(m_i) = \log m_i$ and $m_i > 0$, then for any two individuals i and j, we solve the first order condition which yields the marginal cost of resources(see Appendix 3.A.5). We get,

$$\omega_L \cdot \frac{b_i}{m_i} = \omega_R \cdot \frac{b_j}{m_j}$$

for all $i \in L$ and $j \in R$. By algebraic manipulation, we acquire the equilibrium transfers:

$$m^*_{i\in L} = \frac{(1-\kappa_R^*\sigma_R)b_i \cdot M}{(1-\kappa_R^*\sigma_R)\sigma_L + (1-\kappa_L^*\sigma_L)\sigma_R} = \left(\frac{\omega_L}{\omega_L\sigma_L + \omega_R\sigma_R}\right)b_i M = \left(\frac{b_i}{\sigma_L}\right)\cdot M\rho_L \tag{3.7a}$$

$$m^*_{i'\in R} = \frac{(1-\kappa_L^*\sigma_L)b_{i'} \cdot M}{(1-\kappa_R^*\sigma_R)\sigma_L + (1-\kappa_L^*\sigma_L)\sigma_R} = \left(\frac{\omega_R}{\omega_L\sigma_L + \omega_R\sigma_R}\right)b_j M = \left(\frac{b_{i'}}{\sigma_R}\right)\cdot M\rho_R \tag{3.7b}$$

where $\rho_L = \left[\frac{1}{1+\left(\frac{(1/\sigma_L)-\kappa_L^*}{(1/\sigma_R)-\kappa_R^*}\right)}\right]$ and $\rho_R = \left[\frac{1}{1+\left(\frac{(1/\sigma_R)-\kappa_R^*}{(1/\sigma_L)-\kappa_L^*}\right)}\right]$ are the proportions that

[7] This class of result will also hold for iso-elastic or CRRA utility function:

$$u(m_i) = \begin{cases} \dfrac{(m_i)^{1-\gamma}-1}{1-\gamma} & if \quad \gamma \neq 1 \\ \log(m_i) & if \quad \gamma = 1 \end{cases}$$

where $u'(m_i) = (m_i)^{-\gamma}$

determines the total monetary allocation to each party. The equilibrium transfer to each legislator i in party $\tau \in \{L, R\}$ is a function of their relative position within the party and the portion of the total budget allocated to the party. An individual's relative position within their party is given by $\frac{b_i}{\sigma_\tau}$. For any individual $i \in L$ her relative position is $\frac{b_i}{\sigma_L}$ where σ_L is the sum of individual Bonacich Centrality $b_i(\eta^*, \mathbb{G}_L)$ of all members in party L. The proportion of budget allocated to party L is given by $\rho_L M$. The total monetary transfer made to any party τ is represented by $M_\tau = \sum_{i\in\tau} m_i$. M_τ is the sum of individual transfers made to legislators of party τ by the lobbyist. The share of the total budget to each party is given by:

$$(M_L/M) = \frac{\omega_L\sigma_L}{\omega_L\sigma_L + \omega_R\sigma_R} = \rho_L \tag{3.8a}$$

$$(M_R/M) = \frac{\omega_R\sigma_R}{\omega_L\sigma_L + \omega_R\sigma_R} = \rho_R \tag{3.8b}$$

Budget allocated to each party τ can be rewritten as $M_\tau = \rho_\tau M$ and equilibrium transfers are independent of the status quo bias β_τ.

Proposition 3.1. *For any given graph $\mathbb{G}$, **(a)** equilibrium transfer $m_i^*(\boldsymbol{b}, M)$ to each legislator i from party τ depends on their Bonacich centrality vector within the party, **(b)** equilibrium $\mathbb{P}_A$ is increasing in M, **(c)** equilibrium $\mathbb{P}_A$ is increasing in the status quo bias β_L if κ_R is sufficiently high.*

Part (a) of the above proposition entails that in equilibrium each legislator i affiliated to party τ receives monetary contribution according to their Bonacich centrality vector in their party. For a logarithmic utility function each legislator receives transfer proportional to their weighted individual Bonacich centrality $\left(\frac{b_i}{\sigma_L}\right) \cdot M\rho_L$. From equations 3.7a we see that the equilibrium transfer of each legislator is a function of their relative centrality in the party and total share of funds available to the party. In equilibrium, members in party L gets $\left(\frac{b_i}{\sigma_L}\right) \cdot M_L$ and legislators in R receives $\left(\frac{b_i}{\sigma_R}\right) \cdot M_R$.

Part (b) explains that more money never hurts the legislator. If the budget allocation of the lobbyist improves then they will always use it in their favour to improve upon the sum of probabilities of votes for A. Thirdly, any increase in the legislator's status quo bias will have an ambiguous effect on the sum of probabilities $\mathbb{P}_A$. Any increase in the budget allocation for lobbying activities will lead to an increase in vote share. Differentiating the sum of probabilities $\mathbb{P}_A$ with respect to M we get, $\frac{d\mathbb{P}_A}{dM} > 0$. By assumption 3.3, the Bonacich centrality is always positive and utility is increasing in money, so $\mathbb{P}_A$ is increasing with M. For any given $\mathbb{G}$ and fixed M the equilibrium transfer $m_i^*(\boldsymbol{b}, M)$ is conditional on the Bonacich centrality within their party. Ceteris paribus, for any increase in M, the centrality vector $\boldsymbol{b}$ is unaffected (by construction) and equilibrium transfer increases $\frac{\partial m_i^*}{\partial M} > 0$ for all $i \in \tau$ given $(1 - \kappa_\tau^* \sigma_\tau) > 0$. High value of M implies larger m_i^*, $J_i[\mathbf{p}] > 0$ and λ is positive (from equation 3.A.5).

The legislators in party L have a status quo bias $\beta_L > 0$ while the members in party R members have equal and opposite bias towards policy A. The bias β_τ among the individual legislators in any given party are assumed to be homogeneous across all legislators in the party. Any increase in β_L implies a decrease in bias towards A for the opposition party legislators. $\mathbb{P}_A$ increases with increase in status quo bias i.e. $\frac{d\mathbb{P}_A}{d\beta_L} > 0$ if $\left(\frac{\sigma_R - \sigma_L}{\sigma_R \sigma_L}\right) \geq 2\theta(\kappa_L - \kappa_R)$. This happens if the degree of conflict κ_R is sufficiently higher than κ_L i.e. $\kappa_R \geq \kappa_L + \frac{\sigma_L - \sigma_R}{2\theta\sigma_L\sigma_R}$. If the legislators in party L is better connected(higher Bonacich Centrality) than party R, the conflict of R should be sufficiently large for policy A to be implemented.

Intuitively, an increase in status quo bias β_L increases the direct benefits of the legislators of L choosing policy S and increases the positive spillover effect within the party because of the complementarity in actions. Simultaneously, the direct benefits of the legislators of R voting for policy A also increases. If the conflict κ_R is sufficiently high then the legislators in party R are likely to choose policy A to benefit from the complementarity effect and reduce the substitutability effect from opposition members voting for S. In equilibrium, the lobbyist uses a considerable portion of the available funds efficiently among the legislators in L to influence their decisions towards A.

We provide a few possible scenarios where an increase in the status quo bias can benefit the lobbyist. If the legislators of party L has no conflict against the legislators in R, i.e. $\kappa_L = 0$ then $\omega_R = 1$ i.e. the Bonacich centrality of the legislators of party R is unaffected by their connections with their opposing legislators. In other words, the utility of any legislator in L is unaffected for voting in line with conflicting neighbours of R. For any positive κ_R the centrality of the legislators of party L is affected by a magnitude of $(1 - \kappa_R^* \sigma_R)$. Thus the centrality of the legislators of party L is dampened by a factor of the sum of Bonacich Centrality σ_R of the party R legislators. The reverse is true if $\kappa_R = 0$ and $\kappa_L > 0$ i.e. the Bonacich centrality of the legislators of party L is unaffected by their connections with members of party R and there is a decrease in the centrality of legislators of party R.

We notice that the degree of conflict parameter κ_τ of the conflicting neighbours affect the weighted Bonacich Centrality of any agent $i \in \tau, \tau'$. Any increase in the degree of conflict κ_τ of party τ improves the centrality of its own members but will have an ambiguous effect on the members of other party. If κ_τ rises, the centrality of the legislators of party τ goes up because of the negative linkage between conflicting neighbours.

If both parties have the same degree of conflict[8] $\kappa_L = \kappa_R > 0$ then any increase in the status quo bias will aid to the interest group's objective if $\sigma_R \geq \sigma_L$. Here, any increase in the status quo bias, increases the sum of probabilities depending on the structure of the network of the two political parties i.e. only if the sum of Bonacich centrality of party R is greater than σ_L then $\omega_R \leq \omega_L$. As the status quo bias increases, the members in party L will have a higher incentive to vote for alternative policy only if the joint effect of money and the net effect of network externalities supersedes the effect of bias which is possible if the network of party R is 'denser' than that of party L in some suitably defined sense (here, it is the sum of Bonacich centrality) i.e. $\sigma_R \geq \sigma_L$. Again, if both parties have similar sum of Bonacich Centralities i.e. $\sigma_R = \sigma_L$ then any increase in the status quo bias will help promoting lobbyist's objective policy if the conflict parameter κ_R of party R is larger than κ_L.

Proposition 3.2. *Assuming a logarithmic utility function, in equilibrium;*
*[**A**] transfers made to any individual $i \in L$ is monotonically decreasing in κ_R and monotonically increasing in κ_L,*
*[**B**] the total contribution to party L is monotonically decreasing in κ_R and increasing in κ_L.*

For logarithmic utility function any increase in the degree of conflict pa-

[8]If both κ_L and κ_R is 0, we get back the unweighted Bonacich centrality results with no negative externalities in the previous section.

rameter κ_L will always improve the equilibrium transfer to $i \in L$ and any increase in the conflict parameter of R will be detrimental to the equilibrium payoffs of i since $\frac{dm_i^*}{d\kappa_L} > 0$ and $\frac{dm_i^*}{d\kappa_R} < 0$. The first half of the proposition describes that any increase in the the degree of conflict κ_L improves the centrality of the legislators of L and hence their individual budget allocation increases. Legislators in L are connected via negative links with their conflicting neighbours and gets disutility for conforming with oppositions' votes. If the degree of conflict of the opposition party κ_R increases, legislators in L are negatively affected which is reflected in their equilibrium payoff.

Using equation 3.8a we can comment on the budget allocation of the lobbyist for different parties. The total amount of fund allocated to the legislator's in party L exceeds R's if $2\theta(\kappa_L - \kappa_R) \geq \left(\frac{\sigma_R - \sigma_L}{\sigma_R \sigma_L}\right)$. If both groups have the same degree of conflict i.e. $\kappa_L = \kappa_R$, then party L gets a larger share if the sum of Bonacich of L is greater. In other words, for similar level of conflict among two parties, the party with relatively more connections gets a larger share of the lobbyist's funds. Again, if both party L and R have the same sum of Bonacich centrality i.e. $\sigma_L = \sigma_R$ then members in party L will receive a greater share of the lobbyist's budget if $\kappa_L \geq \kappa_R$. In this case a higher level of conflict of L will dampen the centrality of members in R because of the substitutability effect and thus affects the equilibrium payoff of legislators in R, even in a 'fairly similar' network.

For a given budget M, any increase in κ_L will escalate the total budget allocated to the members of party L. As κ_L increases, ω_R is affected which weakens the centrality of every legislator from party R by a fixed proportion and aids the budget allocation to party L. Notice that in ρ_L any increase in κ_L the total share of funds contributed to party L increases and hence individual share for all members in L increases. Similarly if κ_R increases the proportion of funds going to party L decreases. As the degree of conflict of given party increases, the Centrality of the opposition legislators are worsened because of negative linkage between parties. Any increase in κ_L impacts the utility of any legislator $i \in L$ as they want to distinguish their votes from the oppositions'. This increase in κ_L has a negative impact on the centrality of opposing party legislators and positively affects their own centrality because of the strategic substitutability effect.

Any increase in conflict of party L increases the disutility of the legislators in L because of the negative spillover effects from similar votes by conflicting neighbours. Following the increase in conflict of L, the lobbyist's transfers to legislators in L increases to maintain their interest in the alternative policy A. This result is quite intuitive. Analogously, any increase in κ_R will improve the total fund allocation to party R.

3.3 Network Comparative Statics

In this section we study the effects of different parameters and network structure on the lobbyist's preferred policy. Consider the model in the previous chapter where the legislators have uniform status quo bias β. If no budget available to the lobbyist $M = 0$, the alternative policy will win by plurality only if $\mathbb{P}_A \in (\frac{n}{2}, n]$. Since $u(0) = 0$, policy A is chosen if the sum of probabilities is $\left(\frac{n}{2} - \theta\beta\sigma\right) \geq \frac{n}{2}$ or the status quo bias is $-\left(\frac{n}{2\theta\sigma}\right) \leq \beta \leq 0$. For any positive status quo bias β and $M = 0$, the sum of probabilities $\mathbb{P}_A < \frac{n}{2}$. In the absence of any promises of transfer from lobbyist and a positive status quo bias, policy A will not be chosen by supermajority.

In this chapter we verify if policy A can be chosen by plurality rule in absence of any promises of monetary transfers. For models with party-specific status quo bias β_τ and no money, the lobbyist's preferred policy A will be implemented if $\mathbb{P}_A \in (\frac{n}{2}, n]$. Using Assumption 3.1 and equation 3.6 the above condition can be rewritten as:

$$\frac{n}{2} \leq \frac{n}{2} + \theta\beta_L\big(\omega_R\sigma_R - \omega_L\sigma_L\big) \leq n$$
$$\implies 0 \leq \beta_L \leq \frac{n}{2\theta}\Big(\frac{1}{\sigma_R\omega_R - \sigma_L\omega_L}\Big) \tag{3.6.a}$$

We know that legislators in party R has a bias β_R towards the alternative policy A. For $M = 0$, policy A can be implemented if $\kappa_R \geq \kappa_L + \frac{1}{2\theta}\big(\frac{1}{\sigma_R} - \frac{1}{\sigma_L}\big)$, i.e. the degree of conflict of party R reasonably higher than the conflict of party L when L is well-connected.

Given any graph $\mathbb{G}$, the legislators from the left-party contributes a larger share to the sum of probabilities of voting for policy A if the following holds:

$$\begin{aligned}
&\left(\frac{\mathbb{P}_A^L(\mathbb{G})}{\mathbb{P}_A}\right) \geq \left(\frac{\mathbb{P}_A^R(\mathbb{G})}{\mathbb{P}_A}\right) \\
\Leftrightarrow \beta_L \leq \frac{(n_L - n_R)}{2\theta(\omega_L\sigma_L + \omega_R\sigma_R)} &+ \left(\frac{\rho_L}{\sigma_L}\right) \cdot \sum_{j\in L} u_j b_j - \left(\frac{\rho_R}{\sigma_R}\right) \cdot \sum_{j'\in R} u_{j'} b_{j'} \qquad (3.9)
\end{aligned}$$

where $\mathbb{P}_A^\tau$ is the sum of probability of legislators from party τ to vote for A. We know that legislators in L has a status quo bias β_L and legislators in party R are biased towards policy A. Here we give the conditions for which party L contributes to a larger portion of total votes for A.

In the absence of budget M for transfer, $u(0) = 0$ the above equation 3.9 becomes:

$$\beta_L \leq \frac{(n_L - n_R)}{2\theta(\omega_L\sigma_L + \omega_R\sigma_R)}$$

If party L has same or fewer number of legislators than party R i.e. $n_L \leq n_R$, legislators in L can contribute a larger share of votes for alternative policy if $\beta_L \leq 0$. This case is trivial. If $\beta_L \leq 0$ then $\beta_R > 0$ and as $n_L \leq n_R$, fewer legislators are biased towards the status quo than A. Using equations 3.1a and 3.1b, we compare the expected utility of any agent $i \in L$ from voting for A over S. The effect of strategic substitutability far out weighs the effect complementarity since each legislator in L is negatively linked with members in R. So the negative effect of conflicting neighbours voting in line with a legislator in L supersedes the positive spillover effect from compatible neighbours. The opposite happens for any legislator $i' \in R$, as their expected utility from voting for S dominates $E\Pi_{i'}(A)$. They receive a positive utility from the bias β_R in voting for S and as $n_R \geq n_L$, the positive spillover from conforming with compatible neighbour outweighs the negative effect of conflict. Thus, legislators in party L will contribute more to total votes for A than members in R.

The above results also holds true for considerably small but positive value of status quo bias β_L if $n_L > n_R$. The left party can contribute to a larger share of votes for A even with a positive status quo bias for given parameter values. From equations 3.1a and 3.1b, we see that any legislator $i \in L$ will choose policy

A over S if the expected utility from $E\Pi_{i'}(A)$ subdues $E\Pi_{i'}(S)$. In other words, the net effect from status quo bias β_L, spillover η, conflict κ_L and the exogenous shock in A dominates that of S. For very large value of β_L, the net benefit from voting for status quo will always dominate that of A. To summarize, in the absence of any transfer to legislators the party with a positive status quo bias can contribute to a greater share of total votes for A, only if the bias is reasonably small.

3.4 Benefit to Lobbyist under different party networks

Here we consider the network comparative statics where $\mathbb{G}_\tau^\oplus \supset \mathbb{G}_\tau$ for any $\mathbb{G}_\tau, \mathbb{G}_\tau^\oplus \in \mathscr{G}$. Thus for any party τ, graph $\mathbb{G}_\tau^\oplus$ has more links or is denser than $\mathbb{G}_\tau$. While comparing between two graphs with non-negative links if $\mathbb{G}_\tau^\oplus \supset \mathbb{G}_\tau$, then by definition $\zeta(\mathbb{G}_\tau^\oplus) > \zeta(\mathbb{G}_\tau)$. A denser network complementary effect implies higher maximum eigenvalues $\max\{\zeta_i(\mathbb{G}_\tau^\oplus)\} > \max\{\zeta_i(\mathbb{G}_\tau)\}$ and thus a strict increase in the unweighted Bonacich centrality vector $\mathbf{b}_\tau(\eta^*, \mathbb{G}_\tau^\oplus) > \mathbf{b}_\tau(\eta^*, \mathbb{G}_\tau)$ such that $b_j^\oplus > b_j$ for all $j \in \tau$.

By construction $\mathbb{G}_L, \mathbb{G}_R$ and $\mathbb{1}$ are sub-matrices of a larger adjacency matrix $\mathbb{G}$ where $\mathbb{G}_L$ is a symmetric adjacency matrix of order n_L, $\mathbb{G}_R$ is of order $(n - n_L) \times (n - n_L)$ and $\mathbb{1}$ is a matrix of ones(for details see Appendix 3.A.2). For any incomplete graph $\mathbb{G}_L$, if $\mathbb{G}_L^\oplus \supset \mathbb{G}_L$ then $\mathbb{G}^\oplus \supset \mathbb{G}$. With slight abuse of definition we consider $\mathbb{G}_L^\oplus$ to be a *denser network* than $\mathbb{G}_L$. We have already established that legislators within any party τ can either be connected via positive links or stay unconnected, i.e. $g_{ij}^\tau = \{1, 0\}$ while each legislator in party τ is connected with everyone in τ' by negative links, i.e. $g_{ij'}^{\tau\tau'} = \{-1\}$.

Lemma 3.3. *Take any two graphs $\mathbb{G}, \mathbb{G}^\oplus \in \mathscr{G}$ such that $\mathbb{G} \subset \mathbb{G}^\oplus$ and $\mathbb{G}_R = \mathbb{G}_R^\oplus$ then*
[A] *$\omega_L^\oplus > \omega_L$ and $\omega_R^\oplus < \omega_R$, and*
[B] *the weighted centrality of every member of L increases and that of all legislators of R decreases.*

Ceteris paribus, if the density of a graph of party L increases, then the weighted Bonacich centrality of the members of party L increases and weighted

Bonacich centrality of the members of party R decreases. Let's consider the case $\mathbb{G}_L \subset \mathbb{G}_L^{\oplus}$ where $\mathbb{G}_L^{\oplus}$ has more positive links than $\mathbb{G}_L$ then by definition $\mathbf{b}_L < \mathbf{b}_L^{\oplus}$ where $\mathbf{b}_L, \mathbf{b}_L^{\oplus}$ are $n_L \times 1$ unweighted Bonacich centrality vector without the effect of the conflict parameter. Naturally the sum of unweighted Bonacich centrality in the denser network is larger than that of $\mathbb{G}_L$ i.e. $\mathbf{b}_L^T \cdot \mathbf{1} < \mathbf{b}_L^{\oplus T} \cdot \mathbf{1}$ implies $\sigma_L < \sigma_L^{\oplus}$. Assuming that $\mathbb{G}_R$ remains unchanged, the unweighted centrality of the members of party R, $\mathbf{b}_R$ is unaffected for any increase in network density of L. Since there are no positive links between any members of L and R, our result stands. As a consequence of the above assumption we can conclude $\mathbb{G} \subset \mathbb{G}^{\oplus}$ which will affect the overall weights of the centrality. We can conclude that more links in party L increases the weights of the centrality vector from $\omega_L(\mathbb{G})$ to $\omega_L^{\oplus}(\mathbb{G}^{\oplus})$. For detailed proof see Appendix 3.A.6. For any $\mathbb{G}_L \subset \mathbb{G}_L^{\oplus}$, we have $\boldsymbol{\omega}_L^T \cdot \mathbf{b}_L \leq \boldsymbol{\omega}_L^{\oplus T} \cdot \mathbf{b}_L^{\oplus}$ where $\boldsymbol{\omega}_L$ and $\boldsymbol{\omega}_L^{\oplus}$ are $n_L \times 1$ vectors of weights of the left party i.e. the centrality of each member of party L increases.

As the connections in party L becomes denser, the centrality of the agents in party L improves to $\mathbf{b}_L^{\oplus}$ along with their sum of party Bonacich centrality $\sigma_L^{\oplus}$. Each member of party R is connected by a negative link with every member of the opposition L and suffers a punishment of κ_R^* for voting in line with opposition. By construction, centrality is calculated by summing the results of infinite walks between two agents i and j depreciated with a decay factor in each period. Intuitively, denser network in party L will aid to the weight of centrality of party L and hurt the weights of party R (see Appendix 3.A.7). Hence, the weights of each legislator in party R falls, i.e. $\omega_R^{\oplus} < \omega_R$. Since, $\mathbb{G}_R$ is unchanged, the unweighted centrality $\mathbf{b}_R$ is fixed and $\boldsymbol{\omega}_R^T \cdot \mathbf{b}_R \geq \boldsymbol{\omega}_R^{\oplus T} \cdot \mathbf{b}_R$ where $\boldsymbol{\omega}_R$ and $\boldsymbol{\omega}_R^{\oplus}$ are $(n - n_L) \times 1$ vectors of weights of the right party i.e. the centrality of each member of party R reduces.

Conversely, for any given $\mathbb{G}_L$ and $\mathbb{G}_R \subset \mathbb{G}_R^{\oplus}$ the weights of the centrality vector of party R improves from $\omega_R(\mathbb{G})$ to $\omega_R^{\oplus}(\mathbb{G}^{\oplus})$ and the weights of the centrality of party L decreases from $\omega_L(\mathbb{G})$ to $\omega_L^{\oplus}(\mathbb{G}^{\oplus})$. Thus $\omega_R < \omega_R^{\oplus}$ and $\omega_L^{\oplus} > \omega_L$. Following from the previous results, $\boldsymbol{\omega}_R^T \cdot \mathbf{b}_R \leq \boldsymbol{\omega}_R^{\oplus T} \cdot \mathbf{b}_R^{\oplus}$ and $\boldsymbol{\omega}_L^T \cdot \mathbf{b}_L \geq \boldsymbol{\omega}_L^{\oplus T} \cdot \mathbf{b}_L$ i.e. the centrality of each member of party R increases by a fixed ratio and that of party L falls by a fixed proportion.

Figure 3.1 illustrates a simple network $\mathbb{G}$ with two parties L and R where

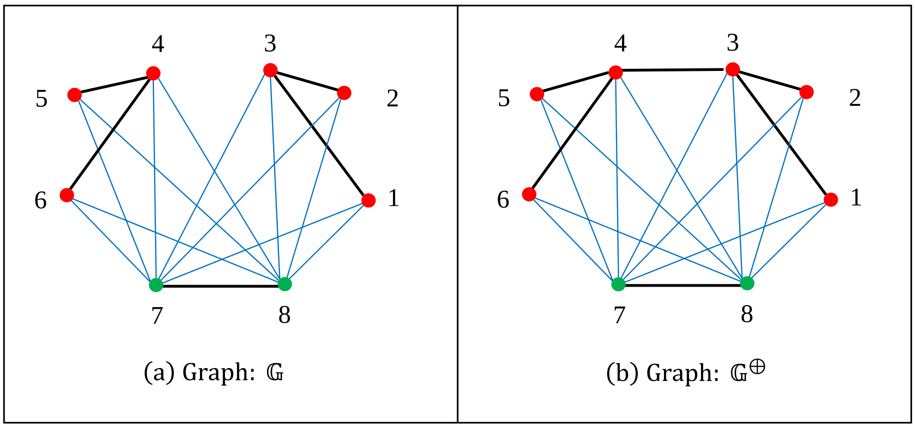


Figure 3.1: Graph Comparison

$n_L = 6$ and $n_R = 2$. The nodes in red represent the legislators of left party and the nodes in green are members of party R. The edges in black characterizes the connections between compatible neighbours within party while the edges is blue are the negative links between conflicting neighbours between parties. We assume that $\mathbb{G}^{\oplus}$ has an additional link between the legislators 3 and 4 i.e. $\mathbb{G}^{\oplus} = \mathbb{G} + \{34\}$.

The legislators have the same logarithmic utility function of $u(m_i) = \log m_i$. $M = 100$ is the total budget available to the lobbyist for distributing among the legislators. For given parameter values of $\eta = 0.7, \theta = 0.1, \kappa_L = 0.2, \kappa_R = 0.3$, the following table constitutes the weighted Bonacich Centrality $\boldsymbol{b}_i$ of the legislators and their equilibrium transfers:

Using the above Lemma, an additional link between 3 and 4 increases the Bonacich centrality of the legislators in party L and reduces the centrality of the legislators in R. This new link will improve the equilibrium transfers of 3 and 4 but reduces the transfers to other members in L and R. The additional link among the central players in L improve the total fund allocation to party L from $M_L = 79.58$ to $M_L^{\oplus} = 81.52$. This leads us to our next proposition where any additional link between two members in party L increases the transfers to all members in the left party increases if $(\kappa_L^* + \kappa_R^*)\sigma_R > 1$. This condition does not hold in the above example and thus the transfer to the central players only increase.

Table 3.1: Comparison of Individual Bonacich Centrality and Equilibrium transfers of graph $\mathbb{G}$ and $\mathbb{G}^{\oplus}$

Legislator(Party)	Centrality ($\boldsymbol{b}_i$)	Centrality ($\boldsymbol{b}_i^{\oplus}$)	Transfer (m_i)	Transfer ($m_i^{\oplus}$)
1 (Left)	1.065	1.097	12.74	12.43
2 (Left)	1.065	1.097	12.74	12.43
3 (Left)	1.196	1.405	14.31	15.90
4 (Left)	1.196	1.405	14.31	15.90
5 (Left)	1.065	1.097	12.74	12.43
6 (Left)	1.065	1.097	12.74	12.43
7(Right)	0.853	0.799	10.21	9.38
8(Right)	0.853	0.799	10.21	9.38

Note: The above values are for given parameter values of $\eta = 0.7, \theta = 0.1, \kappa_L = 0.2, \kappa_R = 0.3$ and $M = 100$

Proposition 3.4. *Assume a logarithmic utility function for all legislators and a given budget M to the lobbyist. For any two graphs $\mathbb{G}, \mathbb{G}^{\oplus} \in \mathcal{G}$ such that $\mathbb{G}_L \subset \mathbb{G}_L^{\oplus}$ and $\mathbb{G}_R = \mathbb{G}_R^{\oplus}$,*
***[A]** the equilibrium transfer **(i)** to any legislator i in party L is increasing in σ_L if $(\kappa_L^* + \kappa_R^*)\sigma_R > 1$ and **(ii)** to any legislator $i \in R$ is monotonically decreasing in σ_L,*
***[B]** in equilibrium, the fund allocated to party L is monotonically increasing in σ_L and fund allocated to party R is monotonically decreasing in σ_L, and*
***[C]** the sum of probabilities $\mathbb{P}_A(\mathbb{G}^{\oplus}, \mathbf{m}_{\oplus}^* \big| \beta_L) \geq \mathbb{P}_A(\mathbb{G}, \mathbf{m}^* \big| \beta_L)$ if $\beta_L \leq \beta_L^c(\mathbb{G}, \mathbb{G}^{\oplus})$.*

Assuming a logarithmic utility function for all legislators and holding the budget of the lobbyist fixed at M we do some analysis of the effects of increasing in graph density of the legislators. Take any two incomplete graphs $\mathbb{G}_L, \mathbb{G}_L^{\oplus} \in \mathcal{G}$ such that $\mathbb{G}_L \subset \mathbb{G}_L^{\oplus}$ and $\mathbb{G} \subset \mathbb{G}^{\oplus}$. We define additional positive links between two legislators in a given party τ as increase in graph density. One simple example with one additional link is $\mathbb{G}_L^{\oplus} = \mathbb{G}_L + \{ij\}$ where $i, j \in L$ and legislator i and j were previously not linked. There can be multiple additional links. If $\mathbb{G}_R$ is unchanged, then $\mathbb{G} \subset \mathbb{G}^{\oplus}$.

The first half of the proposition demonstrates that the equilibrium transfer made to any individual i from party L is increasing with the increase in density of the graph σ_L if $(\kappa_L^* + \kappa_R^*)\sigma_R > 1$. The sum of Bonacich Centrality among legislators of party L increases with any additional links between them. By definition, increase in graph density of L, the unweighted sum of Bonacich centrality σ_L increases. Any increase in the sum of unweighted Bonacich centrality of party L increases has an ambiguous effect on equilibrium transfers made to individual i, depending on the conflict parameter and how connected her neighbours(both conflicting and conforming) are. If we compare the equilibrium payoffs of an individual $i \in L$ for $\mathbb{G}$ and $\mathbb{G}^{\oplus}$, we get $m_i^*(M,\mathbb{G})$ and $m_{i\oplus}^*(M,\mathbb{G}^{\oplus})$. The equilibrium transfer to an individual unambiguously increases iff their degree of conflict is sufficiently high, i.e. $\kappa_L \geq \frac{(1-\kappa_R^*\sigma_R)}{2\theta\sigma_R}$.

In the second half of the proposition [A], we measure the cross-effect of the effect of sum of Bonacich of legislators of party R on the equilibrium monetary transfer to any individual $i \in L$. We notice that $\frac{dm_{i\in R}^*}{d\sigma_L} < 0$ i.e. any increase in the density of graph among legislators of L will be worsen the equilibrium transfers to all the legislators of R. In equilibrium, the transfers to every individual $i \in R$ unambiguously decreases. Thus, the total fund allocated to party R falls. Hence, the results in part [B] are straightforward, any increase in the sum of centrality of party L will improve the total fund allocation to party L i.e. $\frac{dM_L}{d\sigma_L} > 0$ and any increase in connections among legislators in L will reduce the equilibrium fund allocation to R, $\frac{dM_R}{d\sigma_L} < 0$.

Part [C] of the proposition shows that the sum of probabilities of voting for A increases with increase in graph density of L if the status quo bias is below a critical level β_L^c. Take any two graphs $\mathbb{G}_L$ and $\mathbb{G}_L^{\oplus}$ where $\mathbb{G}_L \subset \mathbb{G}_L^{\oplus}$ and hence $\mathbb{G} \subset \mathbb{G}^{\oplus}$. Since the lobbyist is office motivated, they only care about the sum of probabilities. We know that the equilibrium transfers $\mathbf{m}^*(\sigma_L, \sigma_R)$ and $\mathbf{m}_{\oplus}^*(\sigma_L^{\oplus}, \sigma_R)$ are functions of the sum of Bonacich Centrality of all the legislators. For any graph $\mathbb{G}$, the optimal transfer vector $\mathbf{m}^*$ maximizes the sum of probabilities of votes in favour of policy A i.e. $\mathbb{P}_A(\mathbb{G}, \mathbf{m}^*) = \max\{\mathbb{P}_A(\mathbb{G}, \mathbf{m})\}$. The equilibrium transfer vector that maximises the sum of probabilities for graph $\mathbb{G}^{\oplus}$ is $\mathbf{m}_{\oplus}^*$ but the lobbyist might derive higher $\mathbb{P}_A(\mathbb{G}^{\oplus})$ from transfer $\mathbf{m}^*$ depending on the nature of utility function and the centrality of the legislators. For any $\mathbb{G}^{\oplus}$ lobbyist will choose the transfer vector $\mathbf{m}_{\oplus}^*$ or $\mathbf{m}^*$ based on the equilibrium value of the sum of probabilities $\max\left\{\mathbb{P}_A(\mathbf{m}^*_\oplus), \mathbb{P}_A(\mathbf{m}^*)\right\}$.

For a given status quo bias β_L, a denser network is valuable to the lobbyist i.e. $\max\left\{\mathbb{P}_A(\mathbf{m}^*_\oplus, \mathbb{G}^\oplus), \mathbb{P}_A(\mathbf{m}^*, \mathbb{G}^\oplus)\right\} > \mathbb{P}_A(\mathbf{m}^*, \mathbb{G})$ if $\beta_L < \beta^c_L(\mathbb{G}, \mathbb{G}^\oplus)$ where $\beta^c_L(\mathbb{G}, \mathbb{G}^\oplus) \in \max\{\beta^{c\mathbf{mm}}_L, \beta^{c\mathbf{mm}_\oplus}_L\}$ (for a detailed analysis see Appendix 3.A.8 and 3.A.9). This result is analogous to our lemma 2.3 in Chapter 2. It follows from above that $\beta^{c\mathbf{mm}}_L \leq \beta^{c\mathbf{mm}_\oplus}_L$ if $\mathbb{P}_A(\mathbf{m}^*, \mathbb{G}^\oplus) \leq \mathbb{P}_A(\mathbf{m}^*_\oplus, \mathbb{G}^\oplus)$. In this case the lobbyist will choose $\mathbf{m}^*_\oplus$ as the optimal transfer vector.

For any graph $\mathbb{G}^\oplus$ if $\mathbb{P}_A(\mathbf{m}^*_\oplus) \geq \mathbb{P}_A(\mathbf{m}^*)$, the optimal transfer vector to the legislators is $\mathbf{m}^*_\oplus$. The lobbyist is better off with a denser network i.e. $\mathbb{P}_A(\mathbf{m}^*_\oplus, \mathbb{G}^\oplus) > \mathbb{P}_A(\mathbf{m}^*, \mathbb{G})$ if $\beta_L < \beta^{c\mathbf{mm}_\oplus}_L$ (see Appendix 3.A.8). If the legislators of party L have a small bias towards policy S and is paid according to the equilibrium transfer vectors $\mathbf{m}^*_\oplus$ then the lobbyist benefits more from denser graph if the above condition holds. A lobbyist is better-off with a less-connected network if the legislators status quo bias is at least as good as the critical value $\beta^{c\mathbf{mm}_\oplus}_L$.

Similarly, for any graph $\mathbb{G}^\oplus$ if $\mathbb{P}_A(\mathbf{m}^*) \geq \mathbb{P}_A(\mathbf{m}^*_\oplus)$, the optimal transfer vector to the legislators is $\mathbf{m}^*$ which is similar to the optimal transfer in graph $\mathbb{G}$. A denser graph is advantageous to the lobbyist if the actual value of the legislators bias doesn't exceed the critical value i.e $\beta_L < \beta^{c\mathbf{mm}}_L$ (see Appendix 3.A.9).

If the legislators have a small bias towards policy S and is paid according to the equilibrium transfer vectors $\mathbf{m}^*$ or $\mathbf{m}^*_\oplus$, then the lobbyist benefits more from denser graph if $\beta_L \leq \beta^c_L(\mathbb{G}, \mathbb{G}^\oplus)$. The lobbyist can offset a small status quo bias of the left party legislators with monetary contribution for a relatively denser network. In other words, a lobbyist is better-off with a sparser network if the actual status quo bias exceeds the maximum critical value $\beta_L > \max\{\beta^{c\mathbf{mm}}_L, \beta^{c\mathbf{mm}_\oplus}_L\}$.

For any given budget M, the lobbyist chooses the transfer that yields her the higher sum of probabilities in voting for the alternative policy i.e. $\mathbb{P}_A(\mathbf{m}^*, \mathbb{G}) \leq \max\left\{\mathbb{P}_A(\mathbf{m}^*_\oplus), \mathbb{P}_A(\mathbf{m}^*)\right\}$ which automatically implies higher cut-off values i.e. $\beta^c_L = \max\{\beta^{c\mathbf{mm}}_L, \beta^{c\mathbf{mm}_\oplus}_L\}$. Thus the lobbyist chooses the transfer vector based on the critical values of β_L. He chooses $\mathbf{m}^*_\oplus$ if $\beta^{c\mathbf{mm}}_L \leq \beta^{c\mathbf{mm}_\oplus}_L$ and $\mathbf{m}^*$ otherwise. To summarize, we have shown that when the legislators' bias β_L doesn't exceed the maximum critical value β^c_L, a denser network can favour the lobbyist. But if $\beta_L > \beta^c_L$ i.e. the actual value of the status quo bias is beyond the maximum critical value, a sparser graph or a fewer number of compatible neighbours among legislators can favour the lobbyist.

3.5 Concluding Remarks

In this chapter, we have further enriched our understanding of the role of monetary transfer on voting decisions of legislators connected by both positive and negative ties. We infer that the lobbyist can influence connected legislators by paying them according to the Bonacich centrality vector. We have considered strategic substitutability and complementarity among legislators' ties. Any increase in the degree of conflict of a given party improves the individual and the overall equilibrium payments of the party. The increase in conflict will worsen the position (centrality) of the opposition legislators because of the substitutability effect. We find that any additional links in a given party improves the overall fund allocation to that party. We see that for a relatively well connected graph, the legislators' bias towards new policy disadvantages the lobbyist if the status quo bias is above a critical threshold.

In both the chapters, legislators are paid according to their position in the network and a denser network may be beneficial to the lobbyist. The main differences between the models are, the uniform status quo bias among the legislators in Chapter 2 and the equal and opposing bias of the legislators in two different parties in Chapter 3. We have considered strategic complementarity in legislators actions in the baseline framework but later we introduce strategic substitutability to account for the conflict of aligning votes with opposing party members. Using conflict in the further enriches the result where we show that a lobbyist can make considerable campaign contributions to the party opposing her preferred policy. We show that the party with a moderate status quo bias opposing the lobbyist's objective can benefit from a denser network because of the positive spillover from the complementarity in neighbours' votes.

Appendix 3.A

[3.A.1] The probability $p_j(\mathbf{m})$ of any legislator $j \in \tau$ for voting in favour of the new policy is derived from the cumulative distribution function of the uniform error distribution $\epsilon_i \sim U[-\frac{1}{2\theta}, \frac{1}{2\theta}]$:

$$\begin{aligned}
&-\frac{1}{2\theta} \leq \epsilon_i \leq u(m_i) - \beta_\tau + \eta \sum_j g_{ij}^{\tau}(2p_j - 1) + \kappa_{\tau'} \sum_{j'} g_{ij'}^{\tau\tau'}(2p_{j'} - 1) \\
&\Leftrightarrow p_j(\mathbf{m}) = \theta\big[u(m_j) - \beta_\tau + \eta \sum_j g_{ij}(2p_j(\mathbf{m}) - 1) + \kappa_\tau \sum_{j'} g_{ij'}^{\tau}(2p_{j'}(\mathbf{m}) - 1) - (-\frac{1}{2\theta})\big] \\
&\Leftrightarrow p_j(\mathbf{m}) = \frac{1}{2} + \theta\big[u(m_j) - \beta_\tau + \eta \sum_j g_{ij}(2p_j(\mathbf{m}) - 1) + \kappa_\tau \sum_{j'} g_{ij'}^{\tau}(2p_{j'}(\mathbf{m}) - 1)\big]
\end{aligned} \tag{3.A.1}$$

[3.A.2] The adjacency matrix between agents is given by $\mathbb{G}$ where $\mathbb{G} = 2\theta\mathbb{G}'$. For the rest of the analysis we partition the matrix $\mathbb{G}$ into the following:

$$\hat{\mathbb{G}} = \left[\begin{array}{c|c} \mathbb{G}_L & \mathbf{0} \\ \hline \mathbf{0} & \mathbb{G}_R \end{array}\right]; \quad \mathbb{K} = \left[\begin{array}{c|c} \mathbf{0} & -\kappa_L \mathbb{1} \\ \hline -\kappa_R \mathbb{1} & \mathbf{0} \end{array}\right]; \quad \mathbb{G} = \left[\begin{array}{c|c} \eta^* \mathbb{G}_L & -\kappa_L^* \mathbb{1} \\ \hline -\kappa_R^* \mathbb{1} & \eta^* \mathbb{G}_R \end{array}\right] \tag{3.A.2}$$

where $\eta^* = 2\eta\theta$ and $\kappa_\tau^* = 2\kappa_\tau\theta$ and $2\theta = \theta^*$. The matrix $\mathbb{G}$ is symmetric if $\mathbb{G}_L = \mathbb{G}_R$, $\kappa_L = \kappa_R$ and $\mathbb{1}^T = \mathbb{1}$ where $\mathbb{1}$ is a symmetric matrix of ones.

[3.A.3]By manipulating the equation 3.A.1 we represent the above equation in matrix form:

$$\begin{aligned}
\mathbf{p} &= \frac{1}{2} \cdot \mathbf{1} + \theta\big(u(\mathbf{m}) - \boldsymbol{\beta}\big) + 2\theta \cdot \mathbb{G}'\mathbf{p} - \theta \cdot \mathbb{G}'\mathbf{1} \\
\mathbf{p} &= \frac{1}{2} \cdot \mathbb{I}\mathbf{1} + \theta\big(u(\mathbf{m}) - \boldsymbol{\beta}\big) + 2\theta \cdot \mathbb{G}'\mathbf{p} - \frac{1}{2} 2\theta \cdot \mathbb{G}'\mathbf{1} \\
(\mathbb{I} - \mathbb{G})\mathbf{p} &= \frac{1}{2}(\mathbb{I} - \mathbb{G})\mathbf{1} + \theta\big(u(\mathbf{m}) - \boldsymbol{\beta}\big) \\
\mathbf{p} &= \frac{1}{2} \cdot \mathbf{1} + \theta(\mathbb{I} - \mathbb{G})^{-1}\big(u(\mathbf{m}) - \boldsymbol{\beta}\big)
\end{aligned} \tag{3.A.3}$$

where $\mathbf{p}$ is the probability vector of voting in favour of policy A.

[3.A.4] Using the result from Appendix 3.B.2 we can calculate the sum of probabilities of voting in favour of policy A

$$
\begin{aligned}
\mathbb{P}_A &= \mathbf{p}^T \cdot \mathbf{1} \\
&= \frac{1}{2} \cdot \mathbf{1}^T \mathbf{1} + \theta(\mathbf{u}^T - \boldsymbol{\beta}^T)(\mathbb{I} - \mathbb{G}^T)^{-1} \cdot \mathbf{1} \\
&= \frac{1}{2} \cdot \mathbf{1}^T \mathbf{1} + \theta(\mathbf{u}^T - \boldsymbol{\beta}^T) \cdot \boldsymbol{b} \qquad [\because \beta_L + \beta_R = 0] \\
&\text{In Equilibrium,} \\
&= \frac{n}{2} + \frac{\theta(1-\kappa_R^*\sigma_R)}{(1-\kappa_R^*\sigma_R\kappa_L^*\sigma_L)}\Big(\sum_{j\in L} u(m_j^*)b_j - \beta_L \sum_{j\in L} b_j\Big) + \\
&\frac{\theta(1-\kappa_L^*\sigma_L)}{(1-\kappa_R^*\sigma_R\kappa_L^*\sigma_L)}\Big(\sum_{j'\in R} u(m_{j'}^*)b_{j'} - \beta_R \sum_{j'\in R} b_{j'}\Big) \\
&= \frac{n}{2} + \frac{\theta(1-\kappa_R^*\sigma_R)}{(1-\kappa_R^*\sigma_R\kappa_L^*\sigma_L)} \sum_{j\in L} u(m_j^*)b_j + \frac{\theta(1-\kappa_L^*\sigma_L)}{(1-\kappa_R^*\sigma_R\kappa_L^*\sigma_L)} \sum_{j'\in R} u(m_{j'}^*)b_{j'} \\
&+ \frac{\theta\beta_L}{(1-\kappa_R^*\sigma_R\kappa_L^*\sigma_L)}(\sigma_R - \sigma_L) + \frac{\theta\beta_L\sigma_R\sigma_L}{(1-\kappa_R^*\sigma_R\kappa_L^*\sigma_L)}(\kappa_R^* - \kappa_L^*)
\end{aligned} \tag{3.A.4}
$$

[3.A.5] Proof of Proposition 3.1: Assuming that the sum of probabilities are differentiable and the inverse $(\mathbb{I} - \mathbb{G}^T)^{-1}$ exists, we get:

$$
\begin{aligned}
&J_i[\mathbf{p}] = \theta \cdot (\mathbb{I} - \mathbb{G})^{-1} \cdot J_i[\mathbf{u}] \qquad \Big[\because J_i[\cdot] = \frac{d}{dm_i}\Big] \\
\Leftrightarrow\ & J_i[\mathbf{p}]^T = \theta \cdot J_i[\mathbf{u}]^T \cdot (\mathbb{I} - \mathbb{G}^T)^{-1} \\
\Leftrightarrow\ & J_i[\mathbf{p}]^T \cdot \mathbf{1} = \theta \cdot J_i[\mathbf{u}]^T \cdot (\mathbb{I} - \mathbb{G}^T)^{-1} \cdot \mathbf{1} \\
\Leftrightarrow\ & J_i[\mathbf{p}]^T \cdot \mathbf{1} = \theta \cdot J_i[\mathbf{u}]^T . \boldsymbol{b} \qquad \text{[using Equation 3.B.2]}
\end{aligned} \tag{3.A.5}
$$

Solving the optimal value of transfer from equation 3.6.a we get,

$$
\begin{aligned}
&\lambda = \theta \cdot J_i[\mathbf{u}]^T . \boldsymbol{b}(\eta^*, \kappa_L^*, \kappa_R^*, \mathbb{G}^T) = J_i[\mathbf{p}]^T \cdot \mathbf{1} \\
\Leftrightarrow\ & \lambda^* = J_i[\mathbf{u}]^T \boldsymbol{b} = u'(m_i).b_i; \qquad \forall i \in \{L, R\} \\
\Leftrightarrow\ & \lambda^* = \frac{(1-\kappa_R^*\sigma_R)}{(1-\kappa_R^*\sigma_R\kappa_L^*\sigma_L)} u'(m_1) \cdot b_1 = \cdots = \frac{(1-\kappa_L^*\sigma_L)}{(1-\kappa_R^*\sigma_R\kappa_L^*\sigma_L)} u'(m_n) \cdot b_n
\end{aligned}
$$

We assume the utility from money to be logarithmic $u(m_i) = \log m_i$ and $m_i > 0$, then for any two individuals i and j, we solve the first order condition which yields the marginal cost of resources(see Appendix 3.A.5).

We get, $\omega_L \cdot \frac{b_i}{m_i} = \omega_R \cdot \frac{b_j}{m_j}$ for all $i \in L$ and $j \in R$. By algebraic manipulation, we acquire the equilibrium transfers:

$$m^*_{i \in L} = \frac{(1-\kappa^*_R\sigma_R)b_i \cdot M}{(1-\kappa^*_R\sigma_R)\sigma_L + (1-\kappa^*_L\sigma_L)\sigma_R} = \left(\frac{\omega_L}{\omega_L\sigma_L + \omega_R\sigma_R}\right) b_i M = \left(\frac{b_i}{\sigma_L}\right) \cdot M\rho_L$$
$$m^*_{i' \in R} = \frac{(1-\kappa^*_L\sigma_L)b_{i'} \cdot M}{(1-\kappa^*_R\sigma_R)\sigma_L + (1-\kappa^*_L\sigma_L)\sigma_R} = \left(\frac{\omega_R}{\omega_L\sigma_L + \omega_R\sigma_R}\right) b_j M = \left(\frac{b_{i'}}{\sigma_R}\right) \cdot M\rho_R$$

where $\rho_L = \left[\frac{1}{1+\left(\frac{(1/\sigma_L)-\kappa^*_L}{(1/\sigma_R)-\kappa^*_R}\right)}\right]$ and $\rho_R = \left[\frac{1}{1+\left(\frac{(1/\sigma_R)-\kappa^*_R}{(1/\sigma_L)-\kappa^*_L}\right)}\right]$ are the proportions that determines the total monetary allocation to each party.

[3.A.6] **Proof of Lemma 3.3**: For party L, given any two graphs $\mathbb{G}_L, \mathbb{G}^{\oplus}_L \in \mathcal{G}$, where $\mathbb{G}_L \subset \mathbb{G}^{\oplus}_L$, the unweighted Bonacich centrality vector of the graphs are $\mathbf{b}_L$ and $\mathbf{b}^{\oplus}_L$ where $\mathbf{b}^{\oplus}_L > \mathbf{b}^T_L$ and $\sigma^{\oplus}_L > \sigma_L$. If $\mathbb{G}_R$ is unchanged, then the unweighted centrality of the graph $\mathbb{G}_R$ is given by $\mathbf{b}_R$. We know that $\mathbb{G} \subset \mathbb{G}^{\oplus}$, thus using Appendix 3.B.2 we can infer that the weights of the graph will be affected for any change in $\mathbb{G}$ to $\mathbb{G}^{\oplus}$. We compare the change in weights of the centrality of the members of party L when the sum of the centrality of L increase from σ_L to $\sigma^{\oplus}_L$. Ceteris paribus, if σ_L increase ω_L increases. In other words as connections within the members of party L becomes denser, the weights of their centrality increases from $\omega_L(\mathbf{b}_L, \mathbf{b}_R|\kappa_L, \kappa_R)$ to $\omega^{\oplus}_L(\mathbf{b}^{\oplus}_L, \mathbf{b}_R|\kappa_L, \kappa_R)$. Since σ_L and σ_R are large, our result will holds for a sufficiently small θ, κ_R and κ_L where $(1-\kappa^*_R\sigma_R) \geq 0$ and $(1-\kappa^*_L\sigma_L) \geq 0$.

$$\sigma_L < \sigma^{\oplus}_L \implies \left(\frac{1-\kappa^*_R\sigma_R}{1-\kappa^*_R\sigma_R\kappa^*_L\sigma_L}\right) < \left(\frac{1-\kappa^*_R\sigma_R}{1-\kappa^*_R\sigma_R\kappa^*_L\sigma^{\oplus}_L}\right) \implies \omega_L < \omega^{\oplus}_L$$
$$\text{So,} \quad \boldsymbol{\omega}^T_L \cdot \mathbf{b}_L \leq \boldsymbol{\omega}^{\oplus T}_L \cdot \mathbf{b}_L \implies \boldsymbol{\omega}^T_L \cdot \mathbf{b}_L \leq \boldsymbol{\omega}^{\oplus T}_L \cdot \mathbf{b}^{\oplus}_L \quad [\because \mathbf{b}^{\oplus}_L > \mathbf{b}_L] \tag{3.A.6}$$

where $\boldsymbol{\omega}_L$ and $\boldsymbol{\omega}^{\oplus}_L$ are $n_L \times 1$ vectors of weights of the left party. Similarly if $\mathbb{G}_L$ is fixed and $\mathbb{G}_R \subset \mathbb{G}^{\oplus}_R$, then $\omega_R < \omega^{\oplus}_R$.

Given the above situation, as connections become denser for party L, we compare the effects on the weights of the centrality of the legislators of party R. Since the connections in $\mathbb{G}_R$ is fixed, $\mathbf{b}_L$ and σ_R are unchanged. Let's consider the weights $\omega_R(\mathbf{b}_L, \mathbf{b}_R|\kappa_L, \kappa_R)$ of the centrality of the other party R improves, then

$$\begin{aligned} \omega_R < \omega_R^{\oplus} \implies \quad & \left(\frac{1-\kappa_L^*\sigma_L}{1-\kappa_R^*\sigma_R\kappa_L^*\sigma_L}\right) < \left(\frac{1-\kappa_L^*\sigma_L^{\oplus}}{1-\kappa_R^*\sigma_R\kappa_L^*\sigma_L^{\oplus}}\right) \\ \implies \quad & (1-\kappa_R^*\sigma_R) \not< 0 \qquad [\because (1-\kappa_R^*\sigma_R) \geq 0] \\ \implies \quad & \boldsymbol{\omega}_R^T \cdot \mathbf{b}_R \geq \boldsymbol{\omega}_R^{\oplus T} \cdot \mathbf{b}_R \end{aligned} \tag{3.A.7}$$

where $\boldsymbol{\omega}_R$ and $\boldsymbol{\omega}_R^{\oplus}$ are $(n-n_L)\times 1$ vectors of weights of the right party. Hence, $\omega_R \geq \omega_R^{\oplus}$. Analogously if $\mathbb{G}_L$ is fixed and $\mathbb{G}_R \subset \mathbb{G}_R^{\oplus}$, then $\omega_L \geq \omega_L^{\oplus}$.

[3.A.8] Proof of Proposition 3.4 (c): Using equation 3.A.4, for any given graph $\mathbb{G}$, the sum of probabilities of A with equilibrium transfer $\mathbf{m}^*$ is given by:

$$\mathbb{P}_A(\mathbb{G}, \mathbf{m}^*) = \frac{n}{2} + \theta\omega_L \sum_{j\in L} u(m_j^*)b_j + \theta\omega_R \sum_{j'\in R} u(m_{j'}^*)b_{j'} - \theta\beta_L[\omega_L\sigma_L - \omega_R\sigma_R] \tag{3.A.8a}$$

where m_j^* is the equilibrium transfer to an individual $j \in L$ and $m_{j'}^*$ is the equilibrium transfer to an individual $j' \in R$. Similarly, for any given graph $\mathbb{G} \subset \mathbb{G}^{\oplus}$ where $\mathbb{G}_L \subset \mathbb{G}_L^{\oplus}$ and $\mathbb{G}_R$ is fixed the weighted Bonacich centrality vector changes from $\boldsymbol{b}$ to $\boldsymbol{b}^{\oplus}$. From Appendix 3.A.6 and 3.A.7 we know the following results, $\mathbf{b}_R$ is unchanged, $\mathbf{b}_L < \mathbf{b}_L^{\oplus}$, $\omega_L < \omega_L^{\oplus}$ and $\omega_R > \omega_R^{\oplus}$. The sum of probabilities of A with equilibrium transfer $\mathbf{m}_{\oplus}^*$ is given by:

$$\mathbb{P}_A(\mathbb{G}^{\oplus}, \mathbf{m}_{\oplus}^*) = \frac{n}{2} + \theta\omega_L^{\oplus} \sum_{j\in L} u(m_{j\oplus}^*)b_j^{\oplus} + \theta\omega_R^{\oplus} \sum_{j'\in R} u(m_{j'\oplus}^*)b_{j'} - \theta\beta_L[\omega_L^{\oplus}\sigma_L^{\oplus} - \omega_R^{\oplus}\sigma_R] \tag{3.A.8b}$$

where $m_{j\oplus}^*$ is the equilibrium transfer to an individual $j \in L$ and $m_{j'\oplus}^*$ is the equilibrium transfer to an individual $j' \in R$. Comparing equations 3.A.8a and 3.A.8b, we show that the sum of probabilities increases with an increase in density if the following condition holds:

$$
\begin{aligned}
& \mathbb{P}_A(\mathbb{G}^{\oplus}, \mathbf{m}^*_{\oplus}) \geq \mathbb{P}_A(\mathbb{G}, \mathbf{m}^*) \\
\Longrightarrow \quad & \omega_L^{\oplus} \sum_{j\in L} u(m^*_{j\oplus}) b_j^{\oplus} + \omega_R^{\oplus} \sum_{j'\in R} u(m^*_{j'\oplus}) b_{j'} - \beta_L[\omega_L^{\oplus}\sigma_L^{\oplus} - \omega_R^{\oplus}\sigma_R] \\
\geq \quad & \omega_L \sum_{j\in L} u(m^*_j) b_j + \omega_R \sum_{j'\in R} u(m^*_{j'}) b_{j'} - \beta_L[\omega_L\sigma_L - \omega_R\sigma_R] \\
\beta_L \leq & \frac{\left(\omega_L^{\oplus} \sum_{j\in L} u(m^*_{j\oplus}) b_j^{\oplus} - \omega_L \sum_{j\in L} u(m^*_j) b_j\right) + \left(\omega_R^{\oplus} \sum_{j'\in R} u(m^*_{j'\oplus}) b_{j'} - \omega_R \sum_{j'\in R} u(m^*_{j'}) b_{j'}\right)}{(\omega_L^{\oplus}\sigma_L^{\oplus} - \omega_L\sigma_L) + (\omega_R\sigma_R - \omega_R^{\oplus}\sigma_R)} \\
& = \beta_L^c(\mathbf{m}^*, \mathbf{m}^*_{\oplus}, \mathbb{G}, \mathbb{G}^{\oplus}) = \beta_L^{\mathbf{cmm}_{\oplus}}
\end{aligned}
\tag{3.A.8}
$$

We know that $\mathbb{P}_A(\mathbb{G}^{\oplus}, \mathbf{m}^*_{\oplus}) = \max\{\mathbb{P}_A(\mathbb{G}^{\oplus}, \mathbf{m}_{\oplus})\}$. But the new optimal transfer vector $\mathbf{m}^*_{\oplus}$ might lead to a $\mathbb{P}_A(\mathbb{G}^{\oplus}, \mathbf{m}^*_{\oplus})$ lower than $\mathbb{P}_A(\mathbb{G})$ based on the network. The equilibrium transfers $\mathbf{m}^*(\sigma_L, \sigma_R)$ and $\mathbf{m}^*_{\oplus}(\sigma_L^{\oplus}, \sigma_R)$ are functions of the sum of Bonacich Centrality of all the legislators. If the lobbyist sticks to their previous transfer vector $\mathbf{m}^*$, the sum of probabilities for A is given by:

$$
\mathbb{P}_A(\mathbb{G}^{\oplus}, \mathbf{m}^*) = \frac{n}{2} + \theta\omega_L^{\oplus} \sum_{j\in L} u(m^*_j) b_j^{\oplus} + \theta\omega_R^{\oplus} \sum_{j'\in R} u(m^*_{j'}) b_{j'} - \theta\beta_L[\omega_L^{\oplus}\sigma_L^{\oplus} - \omega_R^{\oplus}\sigma_R]
\tag{3.A.8c}
$$

Lobbyist will prefer the transfer vector $\mathbf{m}^*$ only if $\mathbb{P}_A(\mathbb{G}^{\oplus}, \mathbf{m}^*) \geq \mathbb{P}_A(\mathbb{G}^{\oplus}, \mathbf{m}^*_{\oplus})$. Comparing equations 3.A.8a and 3.A.8c, we show that the sum of probabilities increases with an increase in density if the following condition holds:

$$
\begin{aligned}
& \mathbb{P}_A(\mathbb{G}^{\oplus}, \mathbf{m}^*) \geq \mathbb{P}_A(\mathbb{G}, \mathbf{m}^*) \\
\Longrightarrow \quad & \omega_L^{\oplus} \sum_{j\in L} u(m^*_j) b_j^{\oplus} + \omega_R^{\oplus} \sum_{j'\in R} u(m^*_{j'}) b_{j'} - \beta_L[\omega_L^{\oplus}\sigma_L^{\oplus} - \omega_R^{\oplus}\sigma_R] \\
\geq \quad & \omega_L \sum_{j\in L} u(m^*_j) b_j + \omega_R \sum_{j'\in R} u(m^*_{j'}) b_{j'} - \beta_L[\omega_L\sigma_L - \omega_R\sigma_R] \\
\beta_L \leq & \frac{\left(\omega_L^{\oplus} \sum_{j\in L} u(m^*_j) b_j^{\oplus} - \omega_L \sum_{j\in L} u(m^*_j) b_j\right) + \left(\omega_R^{\oplus} \sum_{j'\in R} u(m^*_{j'}) b_{j'} - \omega_R \sum_{j'\in R} u(m^*_{j'}) b_{j'}\right)}{(\omega_L^{\oplus}\sigma_L^{\oplus} - \omega_L\sigma_L) + (\omega_R\sigma_R - \omega_R^{\oplus}\sigma_R)} \\
& = \beta_L^c(\mathbf{m}^*, \mathbf{m}^*, \mathbb{G}, \mathbb{G}^{\oplus}) = \beta_L^{\mathbf{cmm}}
\end{aligned}
\tag{3.A.9}
$$

Appendix 3.B

[3.B.1] To calculate the groupwise Bonacich Centrality from the inverse $(\mathbb{I} - \mathbb{G}^T)^{-1} \cdot \mathbf{1}$, we use the simplified Binomial Inverse Theorem:

$$(\mathbb{A} + \mathbb{B})^{-1} = \mathbb{A}^{-1} - \mathbb{A}^{-1}(\mathbb{I} + \mathbb{B}\mathbb{A}^{-1})^{-1}\mathbb{B}\mathbb{A}^{-1}$$

where $\mathbb{A}$ and $\mathbb{B}$ are non-singular matrices. The left party Bonacich centrality is represented by $(\mathbb{I} - \eta^* \mathbb{G}_L)^{-1} \cdot \mathbf{1} = \mathbf{b}_L$ where $\mathbf{b}_L$ is an $n_L \times 1$ matrix and similarly $(\mathbb{I} - \eta^* \mathbb{G}_R)^{-1} \cdot \mathbf{1} = \mathbf{b}_R$ is the column vector of the right party Bonacich Centrality with $(n - n_L)$ elements.

[3.B.2] Assuming that both $(\mathbb{I} - \eta^* \mathbb{G}_L)^{-1}$ and $(\mathbb{I} - \eta^* \mathbb{G}_R)^{-1}$ are invertible we can say that:

$$(\mathbb{I} - \eta^* \mathbb{G}_L)^{-1} \cdot \mathbb{1} = \begin{bmatrix} b_1 & \dots & b_1 \\ \vdots & \ddots & \vdots \\ b_{n_L} & \dots & b_{n_L} \end{bmatrix} \quad \text{and} \quad (\mathbb{I} - \eta^* \mathbb{G}_R)^{-1} \cdot \mathbb{1} = \begin{bmatrix} b_{(n_L+1)} & \dots & b_{(n_L+1)} \\ \vdots & \ddots & \vdots \\ b_n & \dots & b_n \end{bmatrix}$$

where $\mathbb{1}$ is a symmetric matrix of ones of order n_L and $(n - n_L)$ respectively. The sum of the Bonacich centrality of any party $\tau \in \{L, R\}$ is given by $\sigma_\tau = \sum_{j=1}^{n_\tau} b_j^\tau$.

[3.B.3] Using the formulae for Inverse of Partitioned matrix we compute the party specific weighted Bonacich conditional on the parameter values(degree of conflict, spillover effect and the graph):

$$(\mathbb{I} - \mathbb{G}^T)^{-1} \cdot \mathbf{1} = \left[\begin{array}{c|c} (\mathbb{I} - \eta^* \mathbb{G}_L^T) & \kappa_R^* \mathbb{1} \\ \hline \kappa_L^* \mathbb{1} & (\mathbb{I} - \eta^* \mathbb{G}_R^T) \end{array} \right]^{-1} \cdot \mathbf{1} \qquad (3.B.1)$$

Using the formulae of Inverse of Block matrices, equation 3.B.1 is inverted and represented in terms of party-specific centrality as follows:

$$\left[\begin{array}{c|c} \left((\mathbb{I}-\eta^*\mathbb{G}_L^T)-\kappa_R^*\kappa_L^*\mathbb{1}\cdot(\mathbb{I}-\eta^*\mathbb{G}_R^T)^{-1}\cdot\mathbb{1}\right)^{-1} & -\kappa_R^*(\mathbb{I}-\eta^*\mathbb{G}_L^T)^{-1}\cdot\mathbb{1}\left((\mathbb{I}-\eta^*\mathbb{G}_R^T)-\kappa_R^*\kappa_L^*\cdot\mathbb{1}(\mathbb{I}-\eta^*\mathbb{G}_L^T)^{-1}\cdot\mathbb{1}\right)^{-1} \\ \hline -\kappa_L^*(\mathbb{I}-\eta^*\mathbb{G}_L^T)^{-1}\mathbb{1}\cdot\left((\mathbb{I}-\eta^*\mathbb{G}_L^T)-\kappa_R^*\kappa_L^*\mathbb{1}\cdot(\mathbb{I}-\eta^*\mathbb{G}_R^T)^{-1}\cdot\mathbb{1}\right)^{-1} & \left((\mathbb{I}-\eta^*\mathbb{G}_R^T)-\kappa_R^*\kappa_L^*\mathbb{1}\cdot(\mathbb{I}-\eta^*\mathbb{G}_L^T)^{-1}\cdot\mathbb{1}\right)^{-1} \end{array}\right]\cdot\mathbf{1}$$

$$= \left[\begin{array}{ccc} \frac{1}{1-\kappa_R^*\sigma_R\kappa_L^*\sigma_L}(\mathbb{I}-\eta^*\mathbb{G}_L^T)^{-1}\cdot\mathbf{1}_{n_L} & + & \frac{-\kappa_R^*\sigma_R}{1-\kappa_R^*\sigma_R\kappa_L^*\sigma_L}(\mathbb{I}-\eta^*\mathbb{G}_L^T)^{-1}\cdot\mathbf{1}_{n_L} \\ \frac{-\kappa_L^*\sigma_L}{1-\kappa_R^*\sigma_R\kappa_L^*\sigma_L}(\mathbb{I}-\eta^*\mathbb{G}_R^T)^{-1}\cdot\mathbf{1}_{(n-n_L)} & + & \frac{1}{1-\kappa_R^*\sigma_R\kappa_L^*\sigma_L}(\mathbb{I}-\eta^*\mathbb{G}_R^T)^{-1}\cdot\mathbf{1}_{(n-n_L)} \end{array}\right]$$

$$\Longrightarrow \left(\frac{1}{1-\kappa_R^*\sigma_R\kappa_L^*\sigma_L}\right)\begin{bmatrix} (1-\kappa_R^*\sigma_R)b_1 \\ \vdots \\ (1-\kappa_R^*\sigma_R)b_{n_L} \\ (1-\kappa_L^*\sigma_L)b_{n_L+1} \\ \vdots \\ (1-\kappa_L^*\sigma_L)b_n \end{bmatrix} = \boldsymbol{\omega}^T\cdot\mathbf{b} = \boldsymbol{b} \tag{3.B.2}$$

The $n\times 1$ vector of weights are given by $\boldsymbol{\omega} = \{\omega_L \cdots \omega_L\ \omega_R \cdots \omega_R\}^T$ where $\omega_L = \left(\frac{1-\kappa_R^*\sigma_R}{1-\kappa_R^*\sigma_R\kappa_L^*\sigma_L}\right)$, $\omega_R = \left(\frac{1-\kappa_L^*\sigma_L}{1-\kappa_R^*\sigma_R\kappa_L^*\sigma_L}\right)$ and $\mathbf{b}$ is the unweighted party specific centrality of the legislators.

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Chapter 4

How to define a criminal act ?

4.1 Introduction

Becker (1968) pioneered the modern economic analysis of crime and proposed that the optimal mix of law enforcement policies calls for maximizing a social welfare function. Following Beccaria (1864) and Bentham (1879), Becker's framework assumes that the gains to the "criminal" from the "crime" should be counted positively in the social welfare function. Stigler (1970, pp. 526-527) was perhaps the first to question this assumption.

The determination of this social value is not explained, and one is entitled to doubt its usefulness as an explanatory concept: what evidence is there that society sets a positive value upon the utility derived from a murder, rape, or arson? In fact, the society has branded the utility derived from such activities as illicit.

The ensuing debate has not led to a consensus regarding whether, when and how to count the gains to the "criminal" (Klevorick, 1985; Lewin and Trumbull, 1990; Dau-Schmidt, 1990). In fact, it has only led to perhaps a more fundamental debate about how to determine whether an act is criminal or not. For instance, Husak (2007) points out that

We cannot say that the fruits of criminal activities do not count in given utilitarian calculations without knowing what conduct is criminal – the very question we want our deliberations to answer.

In short, while it is debatable whether and how to count the gains to the criminal from the act that has been deemed criminal while determining the optimal legal penalty, the more fundamental question is how to categorize an act as criminal or non-criminal. To the best of our knowledge, there is no compelling answer. This chapter develops *one* way to approach this question.

We conduct our analysis using a simple bilateral interaction with the following salient features. The *stronger* agent chooses whether or not to make a take-it-or-leave-it proposal to the *weaker* agent. The agents earn their *outside option* payoffs if the stronger agent does not make the proposal. The strength of the stronger agent lies in that he has the power to reward the weaker agent if she accepts his proposal, and also the power to punish her if she rejects his proposal.

Most importantly, the weak agent may accept the proposal even if her payoff upon acceptance is less than her outside option in order to avoid the punishment she will suffer upon rejection. Thus, the strong agent can potentially coerce the weak agent into accepting the proposal and make her worse off than her outside option. In short, the best response by the weaker agent upon receiving the proposal may or may not generate a Pareto improvement relative to the agents' outside options.

While this broad outline encompasses salient features of many interactions, the exact payoff specification we choose is best interpreted as an attempt to capture the possibility of *harassment* of the weaker agent by the stronger agent. As we describe in the following section, our model does not *a priori* rule out the possibility that the interaction between the agents can potentially be Pareto improving relative to their outside options. In contrast, much of the economic analysis of crime *begins* with the implicit assumption that the act under consideration is not Pareto improving.

4.2 Model

Consider the game illustrated in Figure 4.1. For ease of exposition we shall refer to the first mover as the man (M) and the second mover as the woman (W). First, the man chooses whether or not to make a proposal to the woman. If the man does not make the proposal, then both agents earn their outside option utility payoffs which are normalized to zero. Or,

$$U_m(NP, \cdot) = U_w(NP, \cdot) = 0.$$

If the man makes the proposal, then the woman can either accept or reject the proposal. If the woman rejects, then the utility payoff of the man is assumed to be zero.[1] The payoff of the woman upon rejecting the man's proposal is

$$U_w(P, R) = -\theta_m,$$

where $\theta_m \in (0, M]$ is the type of the man. We interpret θ_m as the likelihood that the man punishes the woman for rejecting his proposal. The disutility caused by the man to the woman in such a case is normalized to unity, such that θ_m can be interpreted as the expected disutility to the woman from rejecting the man's proposal.

Now consider the scenario where the man makes the proposal and the woman accepts. The payoff of the woman is assumed to be

$$U_w(P, A) = k\theta_m - \theta_w.$$

$k \geq 0$ is the reward that the man confers upon the woman for accepting his proposal. As mentioned above, $\theta_m \in [0, M]$ is the *type* of the man. We assume it also represents the likelihood of the man conferring this reward on the woman when she accepts his proposal. Thus, the term $k\theta_m$ represents the expected benefit to woman from accepting the man's proposal. The negative term in the woman's payoff following the acceptance of the man's proposal captures

[1] The normalization to zero is for analytical convenience. The idea we want to capture is that in the absence of any legal penalty, the man prefers the woman to accept rather than reject his proposal.

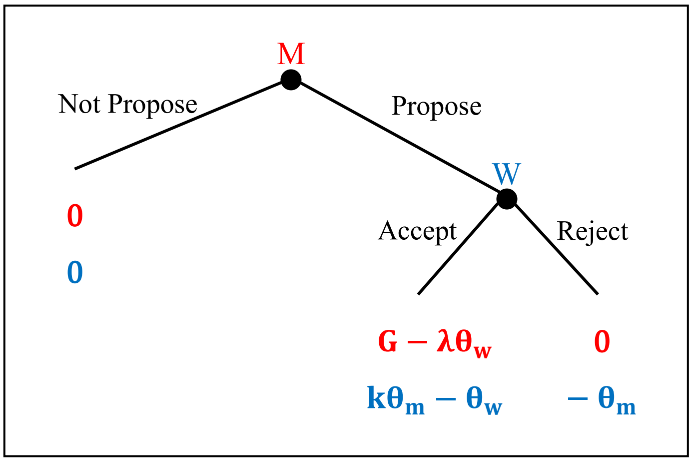

Figure 4.1: Game Tree

the idea that there is something inherently indecent about the proposal from the woman's perspective. Thus, while there are benefits from accepting the proposal, she also suffers a disutility from choosing to accept the proposal. $\theta_w \in [0,1]$ represents the type of the woman such that her disutility from accepting the proposal is increasing in her type.

The payoff obtained by the man when the woman accepts his proposal is

$$U_m(P,A) = G - \lambda\theta_w.$$

Here, $G > 0$ is the gratification utility the man obtains if the woman accepts his proposal. As mentioned above, $\theta_w \in [0,1]$ is the *type* of the woman. $\lambda \geq 0$ is the legal penalty the man suffers for making the proposal. As mentioned above, the higher the type of the woman, the larger the disutility she suffers from accepting the proposal. Hence, we assume that θ_w also indicates the likelihood that the woman seeks legal redress after accepting the proposal. Thus, the disutility term in the man's payoff, $\lambda\theta_w$, may be interpreted as the expected legal penalty the man suffers if the woman of type θ_w accepts his proposal.[2] The overall payoff specification leads to the following four salient features of the model.

[2]Of course, if λ is zero, then there is no penalty; and, determining the optimal value of λ is the second central question.

[F1] The outcome resulting from the woman rejecting the man's proposal is Pareto dominated by the outcome that results when the man does not make the proposal.

[F2] When λ is zero, then (i) making the proposal is the weakly dominant strategy for every type of man, regardless of the type of the woman, and (ii) the man is better off if the woman accepts rather than rejects his proposal.

[F3] The punishment power of the man implies that every type of the woman (either weakly or strictly) prefers not to receive the proposal rather than reject the proposal by any type of man.

[F4] Even when the woman's best response is to accept rather than reject the proposal made by the man, she may be worse off relative to her outside option payoff of zero. Specifically, the woman of type θ_w is strictly worse off upon accepting the proposal by the man of type θ_m rather than not receiving the proposal if $k\theta_m - \theta_w < 0$, i.e., if

$$\theta_w > k\theta_m.$$

The marginal gain to the woman of type θ_w from accepting rather than rejecting the proposal from the man of type θ_m is

$$k\theta_m - \theta_w + \theta_m \;=\; (1+k)\theta_m - \theta_w$$

Thus, the marginal gain to the woman from accepting rather than rejecting the proposal is increasing in the type of the man who makes the proposal and decreasing in her own type. We maintain the following assumption throughout the analysis.

[A1] Assumption 1. $G > 0$, $k \geq 0$, $M > 0$, and $(1+k)M < 1$.

$G > 0$ implies the man derives strictly positive gratification if the woman accepts his proposal. $k \geq 0$ implies the man has the power to reward the woman for accepting his proposal. $M > 0$ implies the likelihood that some types of the man have a strictly positive likelihood of rewarding (punishing) the woman for accepting (rejecting) his proposal.

The most crucial assumption is $(1+k)M < 1$. It implies that some types of the woman, $\theta_w \in ((1+k)M, 1]$, will reject the proposal from every type of the man. One may view this as saying that some types of the woman find it repugnant to accept the proposal regardless of the type of the man who makes the proposal. Since $k \geq 0$, this necessarily restricts M, the highest possible type of the man, to be strictly less than one. As the type of the man indicates his likelihood of rewarding or punishing the woman for accepting or rejecting his proposal, $M < 1$ implies that no type of the man punishes (rewards) the woman for rejecting (accepting) his proposal with certainty.

In the following we consider two variants of the model outlined above. In both variants we assume there exists a unit mass of men with types uniformly distributed over $[0, M]$, and a unit mass of women with types uniformly distributed over $[0, 1]$. The first variant is the *complete information* model where agents' types are assumed to be common knowledge prior to the interaction.

The second variant is the *incomplete information* model where each agent knows his or her type prior to the interaction but not the type of the agent he or she is interacting with. Whenever required, we shall also assume that interactions between all types of men and women are equally likely. Whenever required, we shall also assume that interactions between all types of men and women are equally likely.[3]

4.3 Equilibrium

We first describe the equilibrium in the setting with complete information and then analyze the setting with incomplete information.

[3] We assume uniform distributions of types and equally likely interactions between all types for analytical convenience. As we shall elaborate, the conceptual point we raise is independent of these assumptions.

4.3.1 Complete information

Consider a unit mass of men with types uniformly distributed over $[0, M]$, and a unit mass of women with their types uniformly distributed over $[0, 1]$. Suppose agents' types are common knowledge prior to the interaction. Given any pair of types $\theta_m \in [0, M]$ and $\theta_w \in [0, 1]$, the woman of type θ_w will accept rather than reject the proposal by the man of type θ_m if and only if

$$k\theta_m - \theta_w \ \geq\ -\theta_m \quad \Rightarrow\ \theta_m \ \geq\ \frac{\theta_w}{1+k}.$$

We are assuming a woman accepts the proposal when she is indifferent between accepting and rejecting. We also assume a man will offer the proposal to any woman who will accept the proposal, if and only if the man's payoff upon acceptance by the woman is strictly positive. Formally, a man of type θ_m will offer the proposal to the woman of type θ_w if and only if

$$k\theta_m - \theta_w \ \geq\ \theta_m \ \text{ and } \ G - \lambda\theta_w \ >\ 0$$

$$\Rightarrow\ \theta_w \ \leq\ \min\{(1+k)\theta_m,\ \frac{G}{\lambda}\}.$$

Let λ_{PC} solve $\frac{G}{\lambda} = (1+k)M$, such that $\lambda_{PC} = \frac{G}{(1+k)M} > G$. If $\lambda \leq \lambda_{PC}$, then the woman will accept the proposal from the man of type θ_m if and only if she is of type $\theta_w \leq (1+k)\theta_m$. If $\lambda > \lambda_{PC}$, then the woman will accept the proposal from the man of type θ_m if and only if she is of type $\theta_w \ \leq\ \min\{(1+k)\theta_m,\ \frac{G}{\lambda}\}$. These observations lead to the following result.

Proposition 1. Fix any tuple $\beta = (G, k, M, \lambda)$ and suppose assumption A1 holds. The subgame perfect equilibrium of the interaction between the man of type $\theta_m \in [0, M]$ and the woman of type $\theta_w \in [0, 1]$ is as follows.

(a) If $\lambda \leq \lambda_{PC}$, then the man of type θ_m offers the proposal to the woman if and only if she is of type $\theta_w \ \leq\ (1+k)\theta_m$; and, the woman accepts the proposal.

(b) If $\lambda > \lambda_{PC}$, the man of type θ_m offers the proposal to the woman if and only if she is of type $\theta_w \le \min\{(1+k)\theta_m, \frac{G}{\lambda}\}$; and, the woman accepts the proposal.

The first mover advantage of the man implies that the equilibrium payoff of any type of man cannot be strictly less than his outside option payoff of zero, regardless of the type of the woman he interacts with. However, the equilibrium payoff of women may be zero, strictly positive, or strictly negative depending upon their own type and the type of the man.

The woman of type θ_w will prefer to accept rather than reject the proposal by the man of type θ_m if $\theta_w \le (1+k)\theta_m$. However, only women with type $\theta_w \le k\theta_m$ are weakly or strictly better off upon accepting the proposal relative to their outside option payoff of zero. The woman of type $\theta_w \in (k\theta_m, (1+k)\theta_m]$ accepts the proposal by the man of type θ_m but ends up *strictly worse off* relative to her outside option payoff of zero.

Note that the equilibrium of the interaction may or may not involve a proposal by the man. Based on the above discussion, we can categorize the nature of the *equilibria that involve a proposal* on the basis of agents' equilibrium payoffs relative to their outside options. Given any tuple $\beta = (G, k, M, \lambda)$, let $U_j^*(\theta_m, \theta_w|\beta)$ denote the equilibrium payoff of agent $j \in \{m, w\}$ when the type of the man is θ_m and the type of the woman is θ_w.

Definition 1. Fix any tuple $\beta = (G, k, M, \lambda)$. If the equilibrium of the (θ_m, θ_w)-*interaction* involves a proposal, then the equilibrium proposal is

(a) Pareto *dominated* if $U_m^*(\theta_m, \theta_w|\beta) \le 0$ and $U_w^*(\theta_m, \theta_w|\beta) < 0$.
(b) Pareto *conflicting* if $U_m^*(\theta_m, \theta_w|\beta) > 0$ and $U_w^*(\theta_m, \theta_w|\beta) < 0$.
(c) Pareto *improving* if $U_m^*(\theta_m, \theta_w|\beta) \ge 0$ and $U_w^*(\theta_m, \theta_w|\beta) \ge 0$, with at least one inequality being strict.

Definition 1 in conjunction with the equilibrium characterized in Proposition 1 leads to the following categorization of the interactions depending upon whether the equilibrium of the interaction involves a proposal, and if so then the type of the equilibrium proposal.

Proposition 2. Fix any tuple $\beta = (G, k, M, \lambda)$. The equilibrium of the (θ_m,θ_w)-*interaction*

(a) never involves a Pareto *dominated* proposal.
(b) involves a Pareto *conflicting* proposal if $\theta_w \in (k\theta_m,\ \min\{(1+k)\theta_m,\ \frac{G}{\lambda}\})$.
(c) involves a Pareto *improving* proposal if $\theta_w \in [0,\ \min\{k\theta_m,\ \frac{G}{\lambda}\}]$.
(d) involves no proposal if $\theta_w \in [\min\{(1+k)\theta_m,\ \frac{G}{\lambda}\}, 1]$.

In the equilibrium at any given level of the legal penalty $\lambda \geq 0$, the woman accepts if she receives the proposal from the man, and the man makes the proposal to every woman who will accept rather than reject conditional on receiving the proposal. An increase in the legal penalty up to $\lambda = \lambda_{PC}$ has no impact on the structure of the equilibrium relative to $\lambda = 0$. However, as the legal penalty exceeds λ_{PC}, it deters some types men from making Pareto conflicting proposals to some types of women. Further increase in the legal penalty will also deter some types of men from making Pareto improving proposals to some types of women. To see this, let us define λ_{PI} as the solution to

$$\frac{G}{\lambda} = kM$$

$$\Rightarrow \quad \lambda_{PI} = \frac{G}{kM} > \lambda_{PC}.$$

If $\lambda \in (\lambda_{PC}, \lambda_{PI}]$, then a man of type $\theta_m \in (\frac{G}{(1+k)\lambda},\ M]$ will not offer the proposal to a woman of type $\theta_w \in (\frac{G}{\lambda},\ (1+k)\theta_m]$ even though the woman will accept the proposal if the man were to offer the proposal. However, the woman would be worse off upon accepting the proposal relative to her outside option payoff of zero. Hence, an increase in the legal penalty from zero to some value in the interval $(\lambda_{PC}, \lambda_{PI}]$ eliminates some Pareto *conflicting* proposals (see Figure 4.2).

Once λ exceeds λ_{PI}, it also eliminates some Pareto *improving* proposals. Formally, if $\lambda > \lambda_{PI}$, then the man of type $\theta_m \in (\frac{G}{(1+k)\lambda},\ M]$ will not offer the proposal to the woman of type $\theta_w \in (\frac{G}{\lambda},\ (1+k)\theta_m]$ even though she will accept the proposal if the man were to offer the proposal. Some of these types of women – $\theta_w \in (k\theta_m, (1+k)\theta_m]$ – would be strictly worse off upon accepting the proposal

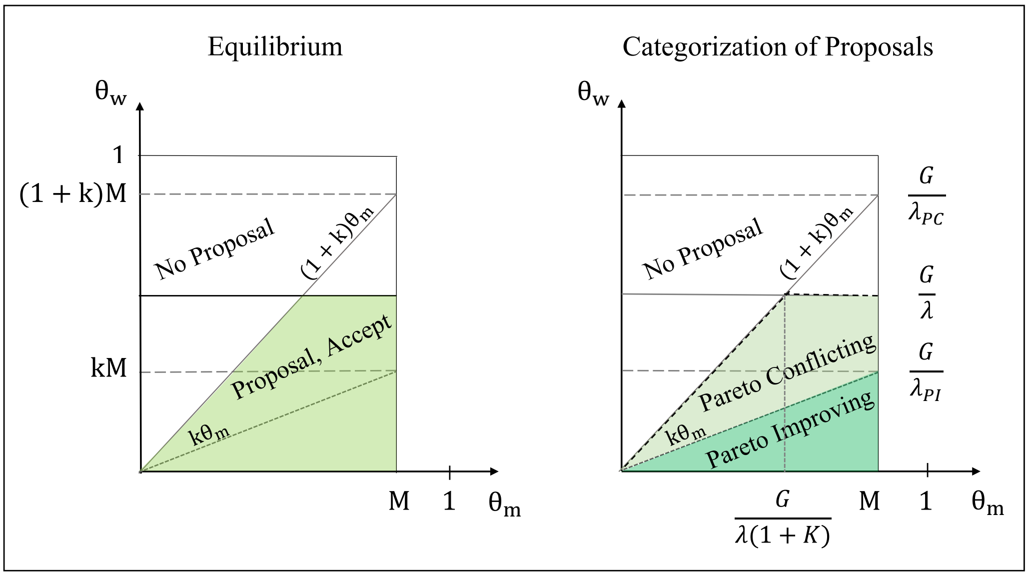

Figure 4.2: Equilibrium and categorization of the interactions under complete information

if it was offered, but they would accept nonetheless to avoid the punishment upon rejection if the proposal was offered. The remainder – $\theta_w \in (\frac{G}{\lambda},\ k\theta_m]$ – would be weakly or strictly better off upon accepting the proposal relative to the outside option of zero if they received the proposal. Hence, an increase in the legal penalty from zero to some value strictly larger than λ_{PI} eliminates not only some some Pareto conflicting proposals, but some Pareto improving proposals as well.

In anticipation of the forthcoming analysis it is useful to note the probability of the four types of interactions mentioned in Proposition 2 for the specific case of no legal penalty, i.e., when $\lambda = 0$.

Proposition 3. Fix any tuple $\beta = (G, k, M, \lambda = 0)$ and suppose A1 holds. The *ex-ante probability* that the equilibrium of the interaction involves

(a) a Pareto *dominated* proposal is 0.
(b) a Pareto *conflicting* proposal is $\frac{M}{2}$.
(c) a Pareto *improving* proposal is $\frac{kM}{2}$.
(d) no proposal is $(1 - \frac{(1+k)M}{2})$.

4.3.2 Incomplete information

We now consider the setting where the types of men and women are uniformly distributed over $[0, M]$ and $[0,1]$, respectively. These distributions are common knowledge. Agents privately know their types but do not know the type of the agent they are interacting with.[4]. A strategy for men is a mapping from type to an action, i.e., propose or not propose. Similarly, the strategy for women is a mapping form type to an action, i.e., accept or reject, conditional on receiving the proposal.

The expected payoff of a woman from accepting the proposal increases as her type decreases, whereas her expected payoff from rejecting the proposal is independent of her type. Hence, for any strategy of men, if type θ_w woman accepts the proposal, then the woman of type $\hat{\theta}_w < \theta_w$ will also accept the proposal. Consequently, the equilibrium strategy for women will be a *threshold* strategy: women up to a threshold type will accept the proposal, and those above the threshold will reject the proposal.

Given any threshold strategy of women, the expected payoff of every type of man from proposing will be identical since the payoff of a man is independent of his type. The expected payoff of every type of man from not proposing is also identical, and equal to zero. Hence, in equilibrium, either all types of men will propose or no type of man will propose. Consequently, depending upon the parameters, the equilibrium will take one of two possible forms: (1) all types of men propose and only the women up to a threshold type accept, or (2) no type of man proposes.

Suppose the parameters are such that the equilibrium where all types of men propose exists. Let $\theta_w^a \in [0,1]$ denote the threshold type of the woman such that a woman accepts the proposal if and only if $\theta \in [0, \theta_w^a]$. If so, type θ_w^a woman will be indifferent between accepting and rejecting the proposal.

[4]As before, whenever required, we shall assume that all types of interactions between different types of men and women are equally likely

Formally, θ_w^a solves

$$\begin{aligned}
\mathbb{E}(U_w(A|P)) &= \mathbb{E}(U_w(R|P)) \\
\Rightarrow \int_{\theta_m=0}^{\theta_m=M} \left[k\theta_m - \theta_w\right] \cdot \frac{d\theta_m}{M} &= \int_{\theta_m=0}^{\theta_m=M} \left[-\theta_m\right] \cdot \frac{d\theta_m}{M} \\
\Rightarrow \theta_w^a &= \frac{1}{2}(1+k)M
\end{aligned}$$

The expected payoff to a man of any type from making the proposal will thus be

$$\begin{aligned}
\mathbb{E}(U_m(P)) &= \int_{\theta_w=0}^{\theta_w=\theta_w^a} \left[G - \lambda\theta_w\right] \cdot d\theta_w = G\theta_w^a - \lambda\frac{(\theta_w^a)^2}{2} \\
\Rightarrow \mathbb{E}(U_m(P)) &= \frac{1}{2}G(1+k)M - \lambda\frac{(1+k)^2M^2}{8}
\end{aligned}$$

As the expected payoff to every type of man from not proposing is zero, the equilibrium where all types of men propose exists if and only if $\mathbb{E}(U_m(P) > 0$, i.e., if and only if[5]

$$\lambda \leq \overline{\lambda}_{INC} = \frac{4G}{(1+k)M}.$$

Proposition 4. Consider any tuple $\beta = (G, k, M, \lambda)$ and suppose A1 holds.

- If $\lambda \in [0, \overline{\lambda}_{INC})$, then the equilibrium is such that
  - every type of man proposes; and,
  - the woman accepts if and only if her type is $\theta_w \in [0, \theta_w^a]$.
- If $\lambda \geq \overline{\lambda}_{INC}$, then no type of man proposes in equilibrium.

[5]As mentioned before, we assume that the man does not propose when he is indifferent between proposing and not proposing, and the woman accepts when she is indifferent between accepting and rejecting the proposal.

All agents earn their outside option payoffs of zero when $\lambda \geq \overline{\lambda}_{INC}$ since no proposals are made in equilibrium. At any $\lambda < \overline{\lambda}_{INC}$, the expected payoff of a man from proposing is non-negative and independent of his type. The equilibrium expected payoff of any type of woman who rejects the proposal is strictly negative. In equilibrium, a woman accepts the proposal if and only if she is of type $\theta_w \leq \theta_w^a$. However, only a subset of the types that accept *expect* to be better off upon accepting the proposal relative to their outside option of zero. The highest type, θ_w^b, whose equilibrium expected payoff is equal to the outside option payoff of zero is obtained by solving

$$\begin{aligned} \mathbb{E}(U_w(A)) &= \int_{\theta_m=0}^{\theta_m=M} \left[k\theta_m - \theta_w^b\right] \cdot \frac{d\theta_m}{M} = 0 \\ \Rightarrow \quad \theta_w^b &= \frac{kM}{2} \end{aligned}$$

As in the complete information case, we can categorize the interaction as Pareto dominated, Pareto improving, or Pareto conflicting. Here, we focus on the ex-ante perspective of agents, i.e., when they know their own type but not the type of the agent they are interacting with. Given any tuple $\beta = (G, k, M, \lambda)$, let $\mathbb{E}(U_m^*(\theta_m|\beta))$ denote the expected equilibrium payoff of the man of type $\theta_m \in [0, M]$. Proposition 1 implies that for any feasible β, regardless of the type of the man,

$$\mathbb{E}(U_m^*(\theta_m|\beta)) \geq 0.$$

Hence, the expected payoff of men, $\mathbb{E}(U^* m(\beta))$, can never be strictly negative. However, the expected equilibrium payoff of women can be strictly positive, zero, or strictly negative depending on their type.

Definition 2. Fix any tuple $\beta = (G, k, M, \lambda)$. From the ex-ante perspective of the woman of type $\theta_w \in [0, 1]$, the *equilibrium proposal* is

(a) Pareto *dominated* if $\mathbb{E}(U_m^*(\beta)) \leq 0$ and $\mathbb{E}(U_w^*(\theta_w|\beta)) < 0$.

(b) Pareto *conflicting* if $\mathbb{E}(U_m^*(\beta)) > 0$ and $\mathbb{E}(U_w^*(\theta_w|\beta)) < 0$.

(c) Pareto *improving* if $\mathbb{E}(U_m^*(\beta)) \geq 0$ and $\mathbb{E}(U_w^*(\theta_w|\beta)) \geq 0$, with at least one

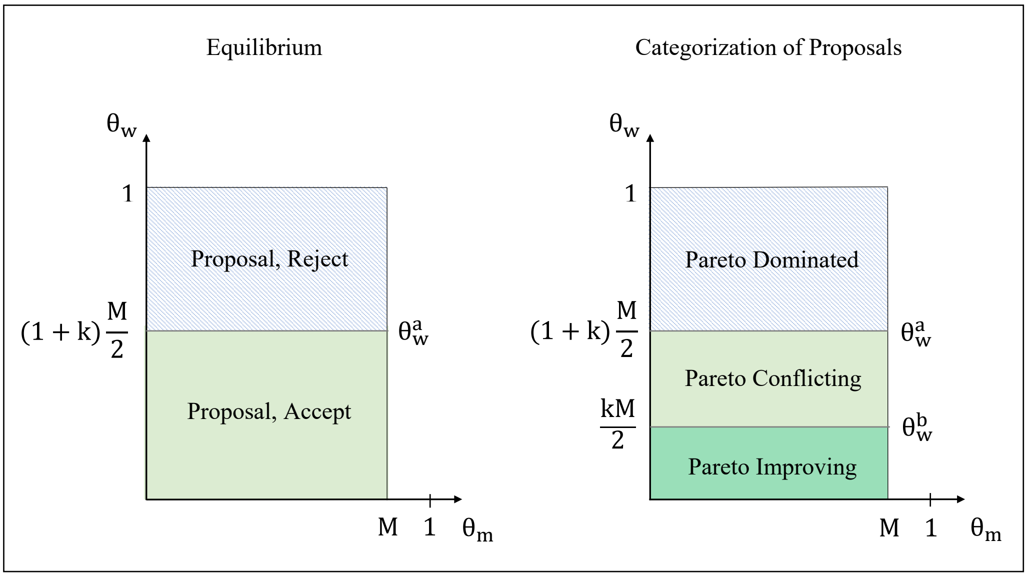

Figure 4.3: Equilibrium and categorization of equilibrium proposals under incomplete information

inequality being strict.

Proposition 5. Fix any tuple (G, k, M) and suppose A1 holds. Consider the equilibrium at any $\lambda < \lambda_{INC}$ where all types of men propose. From the ex-ante perspective, the *equilibrium proposal* is

(a) Pareto *dominated* for women of type $\theta_w \in [\theta_w^a, 1] = [\frac{(1+k)M}{2}, 1]$.
(b) Pareto *conflicting* for women of type $\theta_w \in (\theta_w^b, \theta_w^a) = (\frac{kM}{2}, \frac{(1+k)M}{2})$.
(c) Pareto *improving* for women of type $\theta_w \in [0, \theta_w^b] = [0, \frac{kM}{2}]$.

Further, within the context of the model itself, the man of every type knows Proposition 5. The left panel in Figure 4.3 summarizes the equilibrium of the interaction and the right panel summarizes the ex-ante categorization of equilibrium proposals (as per Propositions 4 and 5, respectively). The following proposition notes the *ex-ante probabilities* of whether and which type of proposal arises in equilibrium.

Proposition 6. Fix any tuple $\beta = (G, k, M, \lambda = 0)$ and suppose A1 holds. The *ex-ante probability* that the equilibrium of the interaction is believed to involve

(a) a Pareto *dominated* proposal is $(1 - \frac{(1+k)M}{2})$.
(b) a Pareto *conflicting* proposal is $\frac{M}{2}$.
(c) a Pareto *improving* proposal is $\frac{kM}{2}$.
(d) no proposal is 0.

4.4 Optimal penalty via social welfare maximization

In this section we consider the conventional approach in the economic analysis of law. The conventional approach does not address the first question regarding whether an act is a crime or not. It directly jumps to the second question of determining the optimal legal policies by maximizing social welfare. In the following, we lay out the optimal social welfare maximizing penalty in the setting with complete information, and in the setting with incomplete information. The simple message is that *zero* penalty maximizes social welfare if G, the gratification a man derives when the woman accepts his proposal, is beyond a threshold.

4.4.1 Complete information

Suppose the type of the woman is drawn from the uniform distribution over $[0, 1]$, the type of the man is drawn from the uniform distribution over $[0, M]$, and the drawn types become common knowledge prior to the interaction. The welfare calculations need to account for the fact that the structure of the equilibrium depends on whether legal penalty is lower or higher than λ_{PC}.

The welfare of the women for any $\lambda \in [0, \lambda_{PC})$ is

$$\Pi_w(\lambda|\lambda \in [0,\lambda_{PC})) = \int_{\theta_w=0}^{\theta_w=(1+k)M} \int_{\theta_m=\frac{\theta_w}{1+k}}^{\theta_m=M} \Big[k\theta_m - \theta_w\Big] \cdot \frac{d\theta_m}{M} \cdot d\theta_w$$

$$\Rightarrow \Pi_w(\lambda|\lambda \in [0,\lambda_{PC})) = \frac{M^2}{6}\Big[k^2 - 1\Big]$$

In contrast, the welfare of women for any $\lambda \geq \lambda_{PC}$ is

$$\Pi_w(\lambda|\lambda \geq \lambda_{PC}) = \int_{\theta_w=0}^{\theta_w=\frac{G}{\lambda}} \int_{\theta_m=\frac{\theta_w}{1+k}}^{\theta_m=M} \Big[k\theta_m - \theta_w\Big] \cdot \frac{d\theta_m}{M} \cdot d\theta_w$$

$$\Rightarrow \Pi_w(\lambda|\lambda \geq \lambda_{PC}) = \frac{M^2}{6}\Big[\frac{3kG}{M} \cdot \frac{1}{\lambda} - \frac{3G^2}{M^2} \cdot \frac{1}{\lambda^2} + \frac{(2+k)G^3}{(1+k)^2 M^3} \cdot \frac{1}{\lambda^3}\Big]$$

Figure 4.4 illustrates the welfare of women as a function of the legal penalty λ. The legal penalty that maximizes the welfare of women is

$$\lambda_w^{opt} = (\frac{2+k}{1+k}) \cdot \frac{G}{kM} > \frac{G}{kM} = \lambda_{PI}.$$

Further, the welfare of women at λ_w^{opt} is

$$\Pi_w(\lambda_w^{opt}) = \frac{M^2}{6}\Big[\frac{k^2(3+4k+2k^2)}{(2+k)^2}\Big].$$

The following proposition gathers the key points of the above calculations.

Proposition 7. If we focus exclusively on the welfare of women, then the optimal legal penalty is strictly positive since $\lambda_w^{opt} > \lambda_{PI} > 0$. The welfare of women at λ_w^{opt} is independent of G, the gratification benefit that a man derives when a woman accepts his proposal.

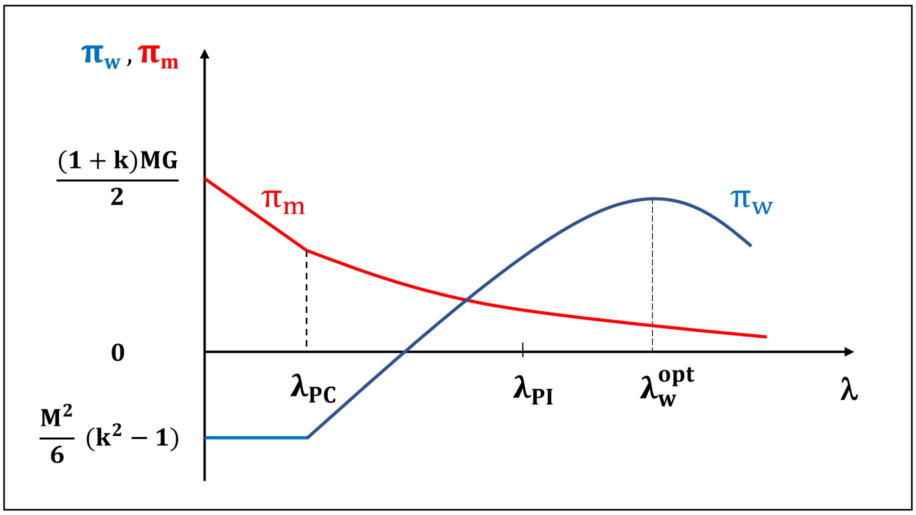

Figure 4.4: Welfare under complete information

The figure has been drawn for $k < 1$, which implies the welfare of women is strictly negative at least up to λ_{PC}.

Turning to the welfare of men, first note

$$\Pi_m(\lambda|\lambda \in [0, \lambda_{PC})) = \int_{\theta_m=0}^{\theta_m=M} \int_{\theta_w=0}^{\theta_w=(1+k)\theta_m} \left[G - \lambda\theta_w\right] \cdot d\theta_w \cdot \frac{d\theta_m}{M}$$

$$\Rightarrow \Pi_m(\lambda|\lambda \in [0, \lambda_{PC})) = \frac{M^2}{6}\left[\frac{3(1+k)G}{M} - \lambda(1+k)^2\right]$$

For values of $\lambda \geq \lambda_{PC}$, we find

$$\Pi_m(\lambda|\lambda \geq \lambda_{PC}) = \int_{\theta_m=0}^{\theta_m=\theta_m^{np}} \int_{\theta_w=0}^{\theta_w=(1+k)\theta_m} \left[G - \lambda\theta_w\right] \cdot d\theta_w \cdot \frac{d\theta_m}{M} + \int_{\theta_m=\theta_m^{np}}^{\theta_m=M} \int_{\theta_w=0}^{\theta_w=\frac{G}{\lambda}} \left[G - \lambda\theta_w\right] \cdot d\theta_w \cdot \frac{d\theta_m}{M}$$

where $\theta_m^{np} = \frac{G}{\lambda(1+k)}$ is the lowest type of man who does not propose to the woman if her type is greater than $\frac{G}{\lambda}$.

$$\Rightarrow \quad \Pi_m(\lambda|\lambda \geq \lambda_{PC}) \quad = \quad \frac{M^2}{6}\left[\frac{3G^2}{M^2}\cdot\frac{1}{\lambda} - \frac{G^3}{(1+k)M^3}\cdot\frac{1}{\lambda^2}\right]$$

Straightforward calculations suggest that for every $\lambda \geq 0$,

$$\frac{\partial \Pi_m}{\partial \lambda} \; < \; 0 \quad \text{and} \quad \frac{\partial^2 \Pi_m}{\partial \lambda^2} \; \geq \; 0.$$

Proposition 8. Suppose A1 holds and the planner chooses the legal penalty to maximize social welfare, i.e., the sum of the wlefare of men and the welfare of women.[6] Then, there exists a threshold gratification benefit for men, G^{COM}, such that social welfare is maximized at

(a) some $\lambda \in (\lambda_{PC}, \lambda_w^{opt})$ if $G \leq G^{COM}$.
(b) $\lambda = 0$ if $G \geq G^{COM}$.

The proof of the first part follows from two simple observations. First, the optimal value of the penalty cannot be greater than λ_w^{opt} because the welfare of both men and women decreases with an increase in the penalty beyond λ_w^{opt}. Second, the optimal penalty cannot lie in the interval $(0, \lambda_{PC}]$ as social welfare at $\lambda = 0$ is strictly higher. The second part follows from the fact that (i) as G increases, the welfare of the men at $\lambda = 0$, but the maximum welfare of women is independent of G. Hence, if G is sufficiently high, then the social welfare will be maximized at $\lambda = 0$.

4.4.2 Incomplete information

Proposition 4 implies that the equilibrium under the incomplete information setting involves no proposal by any type of man when the legal penalty reaches a threshold given by $\overline{\lambda}_{INC}$. Hence,

[6] We ignore any costs of enforcing the legal penalty as it has no substantive effect on our analysis. Including these costs will simply imply that the optimal penalty will be lower than the one we find.

$$\Pi_w(\lambda|\lambda \geq \overline{\lambda}_{INC}) \;=\; \Pi_w(\lambda|\lambda \geq \overline{\lambda}_{INC}) \;=\; 0.$$

The equilibrium at $\lambda < \overline{\lambda}_{INC}$ involves a proposal by every type of man and acceptance by women of types up to θ_w^a. Hence, the welfare of women is

$$\begin{aligned}
\mathbb{E}(\Pi_w(\lambda|\lambda < \overline{\lambda}_{INC})) \;&=\; \int\limits_{\theta_w=0}^{\theta_w=\theta_w^a} \int\limits_{\theta_m=0}^{\theta_m=M} \Big[k\theta_m - \theta_w\Big] \cdot \frac{d\theta_m}{M} \cdot d\theta_w \\
&\quad + \int\limits_{\theta_w=\theta_w^a}^{\theta_w=1} \int\limits_{\theta_m=0}^{\theta_m=M} \Big[-\theta_m\Big] \cdot \frac{d\theta_m}{M}
\end{aligned}$$

$$\Rightarrow \;\mathbb{E}(\Pi_w(\lambda|\lambda < \overline{\lambda}_{INC})) \;=\; \frac{M}{8}\Big[(1+k)^2 M - 4\Big]$$

The welfare of men is

$$\mathbb{E}(\Pi_m(\lambda|\lambda < \overline{\lambda}_{INC})) \;=\; \int\limits_{\theta_m=0}^{\theta_m=M} \int\limits_{\theta_w=0}^{\theta_w=\theta_w^a} \Big[G - \lambda\theta_w\Big] \cdot d\theta_w \cdot \frac{d\theta_m}{M}$$

$$\Rightarrow \;\mathbb{E}(\Pi_m(\lambda|\lambda < \overline{\lambda}_{INC})) \;=\; \frac{1}{2}G(1+k)M \;-\; \lambda\frac{(1+k)^2 M^2}{8}$$

The welfare of women is independent of λ. The welfare of men linearly decreases with an increase in λ till $\overline{\lambda}_{INC}$, and is zero thereafter (see Figure 4.5). Hence, under incomplete information, the social welfare maximizing level of legal penalty will be either zero or $\overline{\lambda}_{INC}$ as summarized in the following result.

Proposition 9. For any k and M there exists a threshold gratification benefit for men, $G^{INC}(k, M)$, such that

- social welfare is maximized at $\lambda = \overline{\lambda}_{INC}$ if $G \leq G^{INC}(k, M)$.
- social welfare is maximized at $\lambda = 0$ if $G > G^{INC}$.

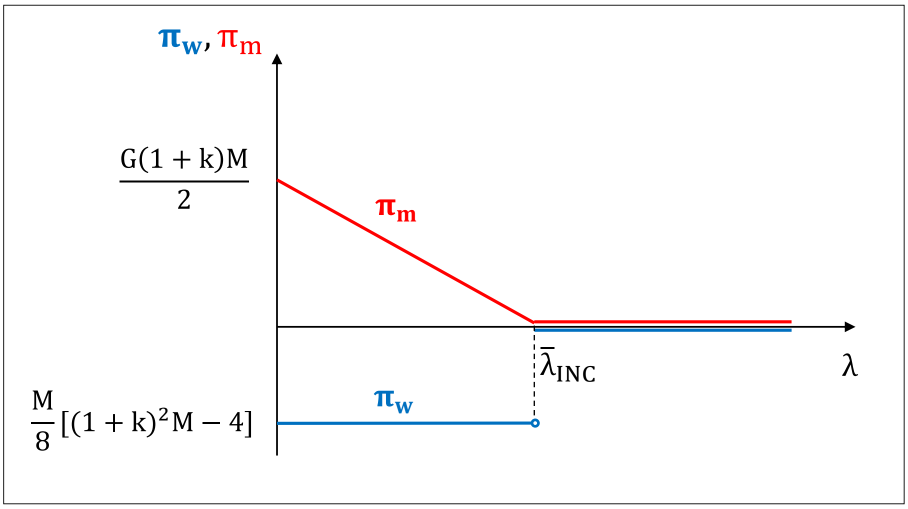

Figure 4.5: Welfare under incomplete information

Propositions 8 and 9 formally express the simple point that if the gratification benefit of men is sufficiently high, then a legal policy maker who is guided by the standard economic approach of maximizing social welfare will find the socially optimal level of legal punishment for men is *zero*. The higher the value of *G*, the more likely it is that no penalty is the optimal penalty.

4.5 An alternative approach

In contrast to the conventional approach, we first define when the act of making a proposal will be considered a "crime", or simply a legally punishable act.

Criminal Act. *The act of making the proposal is a criminal act if the ex-ante probability of Pareto improving proposals is strictly less than the ex-ante probability of Pareto conflicting proposals.*

The hallmark of a typical market interaction is that agents will not initiate an interaction unless it leads to a Pareto improvement from the ex-ante perspective. The ex-post outcomes resulting from the interaction may make one or both the agents worse off relative to no interaction. The above definition accommodates this idea, and takes it one step further.

As per Propositions 3 and 6, the act of making the proposal would be a crime under both the complete and incomplete information variants of the model if $k < 1$. The difference in probabilities of Pareto improving and Pareto conflicting proposals under both the complete and incomplete information settings is $(k-1)\frac{M}{2}$, which is strictly negative if $k < 1$.

We are unable to make a strong case for a particular way to determine the optimal penalty once the act has been deemed criminal. One possibility is to broadly follow Stigler (1970) and set the optimal penalty to maximize *exclusively* the welfare of women. If $k < 1$ such that making the proposal is a criminal act, then the optimal penalties as per the complete and incomplete information models will be λ_w^{opt} and $\overline{\lambda}_{INC}$, respectively. Note that putting all the weight on maximizing the welfare of agents on one side of the interaction is not alien to economic analysis of law. The prime example is antitrust law, and “law and economics" scholars have long argued that the appropriate objective of antitrust laws should be to exclusively maximize consumer surplus, not the sum of consumer and producer surplus (Farell and Katz, 2006).

It is worth stressing that conventional social welfare maximization would be a questionable way to determine the optimal penalty for a criminal act. It seems internally inconsistent to first distinguish between Pareto improving and Pareto conflicting proposals to determine criminality, and then completely ignore the normative difference between such proposals while determining the optimal penalty. It may be worth exploring the merits of choosing weights on agents' utilities by accounting for whether the interaction is Pareto improving, Pareto conflicting, or Pareto dominated.

Note that the *signs* of the utilities of both agents are already accounted appropriately in the standard social welfare function for Pareto improving and Pareto dominated interactions. Both agents gain in the former and their realized positive utilities increase social welfare, whereas neither agent gains in the latter and their realized negative utilities decrease social welfare. The tricky case involves Pareto conflicting interactions where one agent strictly gains and the other strictly loses. The loss of the agent who loses is already accounted appropriately. The question is whether the gain of the agent who gains in a Pareto conflicting interaction can truly be considered a gain: should it be counted at all, and if so, should it be counted positively or negatively? While there is no obvious answer, theoretical analyses can at the very least explore the implications of all the three options.

Some key features of our approach in relation to the conventional approach are as follows.

[1] Our approach does not demand any more information than the conventional approach, and takes an ex-ante perspective as is the norm in the economic analysis of law.

[2] In the determination of whether the act is a crime or not, the primitive unit of account is the *interaction*, and not the *agent*. We ask whether the *interaction* is more likely to be Pareto improving or Pareto conflicting; and, this calculation pays to attention to the size of utility gains and losses experience by the agents. Thus, the determination of a criminal act does not require us to engage in interpersonal utility comparisons.

[3] Lewin and Trumbull (1990) suggest that an act should be regarded criminal if social welfare is negative in the absence of any legal penalty for the act. Recall, Propositions 8 and 9 in Section 4 highlight a potential weakness of this suggestion. It cannot escape the implication that if the man finds acceptance of the proposal more gratifying, then it becomes less likely that he act of making the proposal will be deemed criminal.

[4] The criterion we propose for determining an act as criminal seems consistent with our intuitive understanding of the exemplars of criminal acts (rape, murder and arson): arguably, the likelihood that such acts are Pareto improving is lower than the likelihood that they are Pareto conflicting.

[5] When the act of making the proposal is not a "criminal" act, it does not necessarily mean that it should go unpunished. Most legal systems prescribe punishments for some "non-criminal" acts as well. In the context of our model we may define the act of making a proposal to be *non-criminal but punishable* if

- the act of making a proposal is not criminal; but,
- the ex-ante probability of Pareto improving proposals is no more than the *sum* of the ex-ante probabilities of Pareto conflicting proposals and Pareto dominated proposals.

In our incomplete information setting, this implies that the act of making a proposal is non-criminal but punishable if $k \geq 1$.[7] Perhaps, the conventional approach of finding the optimal legal penalty by maximizing the standard social welfare function is appropriate for non-criminal but punishable acts. We hope to develop these ideas further in our future work.

[7] The relevant difference in probabilities as per Proposition 6 is $(kM - 1)$, which is always strictly negative under Assumption A1. Also note that the notion of non-criminal but punishable acts is vacuous in the setting with complete information since Pareto dominated proposals never arise.

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Chapter 5

Conclusion

We conclude with a brief summary of the main findings and outline some possible extensions. The main purpose of our research was to investigate whether and when the social network among legislators aids a lobby group in influencing the voting decisions of legislators.

In Chapter 2, we model a set of legislators who vote for either a status quo or an alternative policy. We assume that legislators have a common bias towards the status quo policy. The lobbyist chooses transfer payments to the legislators in order to maximise the sum of probabilities of the legislators voting for the alternative policy. Each legislator cares about money and derives utility from voting in line with her neighbors in her social network. We find that the equilibrium payment to the legislator depends on her Bonacich centrality in the network. The lobbyist benefits from a decrease in the status-quo bias of the legislators. The lobbyist can offset reasonably small status-quo biases with monetary transfers. We also provide some comparative statics result showing the marginal impacts of changes in the network structure. If the legislators are biased towards the alternative policy then a lobbyist is always better-off with a bigger network. The lobbyist also benefits from a bigger network if the legislators have a sufficiently small status quo bias.

In Chapter 3, we extend the baseline model by assuming that the legislators are affiliated to one of two different parties. Legislators in each party are connected via a within-party network. All the legislators in one party have a common bias towards the status-quo policy while the legislators in the other party have an equal and opposite bias towards the alternative policy. A legislator derives utility from voting in line with her neighbors in her own party, but also receives a disutility if she ends up voting in line with legislators from the other party. The disutility of a legislator from aligning her vote with a member from the opposition party is interpreted as the degree of conflict. All legislators in a given party have a common degree of conflict towards the other party, but the degree of conflict may differ across parties. Any increase in the degree of conflict of the legislators in a party towards the other party increases the equilibrium payment to the former by the lobbyist. If both parties have the same network structure, then total payment by the lobbyist to the party with a higher degree of conflict is relative larger. If both parties have the same degree of conflict, then a larger share of the lobbyist's payments goes to the party with the greater sum of Bonacich centralities of its legislators. We also clarify which type of networks make it easier for the lobbyist to influence the legislators. There is considerable scope for further extending the models we have studied. One may examine a model with strategic rather than probabilistic voting, incomplete or asymmetric information, and endogenize the degree of conflict between the parties. The ideas about determination of criminal acts in the previous chapter can be further refined and contrasted with existing views in legal philosophy. We hope to purse some of these questions in our future work.

In the final chapter, we try to answer a question widely debated in law and economics on whether, when and how to count the gains to a "criminal" and the optimize the level of penalty. To categorize an act as criminal or non-criminal we use a model with bilateral interactions between a stronger agent who chooses whether or not to make a take-it-or-leave-it proposal to a weaker agent. Our model does not rule out the possibility that the interaction between the agents can potentially be Pareto improving relative to their outside options. We lay out the optimal social welfare maximizing penalty in the setting with complete information, and in the setting with incomplete information. Additionally we determine the optimal legal penalty and provide an alternative approach to define a criminal act.

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