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Just-In-Time Replenishment and Component Substitution Decisions for Assemble-To-Order Manufacturing when Capital is Investor-Supplied

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Declaration

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university or other institution. To the best of my knowledge and belief, this thesis contains no material previously published or written by another person, except where due reference is made in the text.

[redacted]

John M. Betts

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Abstract

This thesis presents a new approach to reconciling the inventory reduction aspect of Just-In-Time (JIT) replenishment and component substitution with inventory cost-minimisation approaches to batch sizing. Previous attempts to analyse JIT replenishment from a management science perspective have typically required extreme setup cost reduction in order to justify the very small batch sizes recommended by Just-In-Time practitioners, making it unlikely that these approaches capture the essential economic reasons why JIT is successful.

It is argued that a limitation of these analyses of JIT replenishment is that they consider inventory policy decisions independently of the financial position or capitalisation of the company by implicitly assuming that the company is financed with unlimited borrowed capital. In practice, companies are financed principally by owners or investors. Thus the amount of capital available for investment in inventory is finite and determined by the availability of capital or an investor's preference, and consequently, is an additional variable in inventory replenishment decisions. The existence of a finite limit on the level of capital invested in a company also introduces the risk that a company may fail by exhausting all available working capital as an additional factor in replenishment policy decisions.

The decision to adopt JIT replenishment for certain components used in Assemble-To-Order manufacture is analysed as a function of investment level under a series of inventory models that reflect varying assumptions about the demand for finished products, and the nature of the constraint imposed on the inventory investment, in order to progressively expose various cost factors affecting the replenishment policy decision. The decision to reduce inventory by the substitution of one component with an over-specification alternative held in inventory is also considered. The analysis under each model is illustrated by determining the investment levels at which a case study company would adopt JIT replenishment and the substitution of various components.

The analysis shows that when capital is constrained, the elimination of a component from inventory by JIT replenishment releases the capital formerly invested in that component which can be reinvested in the remaining batch-replenished components to increase the efficiency with which they are replenished. The resulting savings in inventory costs can then be used to offset the cost of JIT replenishment making this method of inventory reduction, when investment is sufficiently constrained, cost-effective even when the cost of replenishment *per item* is increased under JIT. Similar benefits also arise from component substitution when invested capital is sufficiently constrained. When demand is stochastic, additional benefits of JIT emerge because replenishment of the JIT component within the manufacturing lead time eliminates safety stock and the cost of lost sales attributable to the JIT component. It is also shown that JIT replenishment reduces the degree to which capital invested in inventory fluctuates. This in turn lowers the risk that a company may fail by exhausting all working capital, and presents a further benefit of JIT replenishment.

The analysis of the optimal replenishment policy for the case study company shows that the adoption of JIT replenishment and component substitution allow a company to remain profitable with reduced investment in inventory. As a consequence, these inventory reduction strategies make operation possible with less capital and with higher returns on the investment of this capital.

Table of Contents

List of Figures

List of Tables

Notation

A_i	Annual demand for finished product i
α_j	Weighting factor for component j
b_j	Cost of back ordering one unit of component j
B_j	Cost of lost sales attributable to component j
C_j	Unit cost for component j
C_j^+	Cost per unit overstocked for component j
$C_{COGI}(T)$	Cost of goods consumed from inventory over time T
D_j	Annual demand for component j
δ_j	Indicator signifying that component j is batch replenished
F	Annual fixed costs
$f_j(q_j)$	The probability density function of inventory level for each component.
$f_K(\cdot)$	The probability density function (PDF) of total investment in inventory
$F_K(\cdot)$	The cumulative distribution function (CDF) of total investment in inventory
$g_j(x)$	PDF of the demand through lead time for component j
$G_j(x)$	CDF of the demand through lead time for component j
$h_j(\xi)$	PDF of the demand through review period for component j
$H_j(\xi)$	PDF of the demand through review period for component j
I	Interest rate
J	Inventory holding costs other than interest
J_j	Inventory holding costs other than interest for component j
k_j	The capital invested in each component
κ_j	The safety factor (in standard deviations) for each component j
K	The amount of capital invested in inventory

$\overline{K}$	The average investment in inventory
K^{*}	The maximum permitted investment in inventory
$\widetilde{K}$	The average investment in inventory for which original or alternative replenishment policy are equally profitable
$\widetilde{K}^{*}$	The total investment in inventory for which original or alternative replenishment policy yield equal Risk-Discounted Profit
K_i	The capital invested in inventory immediately before the i^{th} replenishment
ΔK_i	The amount by which inventory is increased at the i^{th} replenishment
$\dfrac{\langle \Delta K_i \rangle}{\langle \Delta T_i \rangle}$	Rate at which capital is consumed for the purchase of inventory
λ	Lagrange multiplier
ℓ_j	Increase in inventory of component j resulting from lost sales of finished products
$L(r_j)$	Expected number of lost sales attributable to component j during its reorder cycle
m	The number of component types used
n	The number of finished product types made
v_{ij}	The number of units of component j used to assemble finished product i
ρ_i	The probability that inventory level, after the i^{th} replenishment, does not exceed K^{*}
p_i	Probability of a lost sale of finished product i
P	Annual profit
$\hat{P}$	Annual profit of simulated companies
P_K	Augmented profit equation incorporating constraint
$\wp$	Probability of company survival
$\hat{\wp}$	One-year survival probability of simulated companies
$\phi(\cdot)$	Normal PDF
$\Phi(\cdot)$	Normal CDF
q_j	The inventory level of each component at a randomly chosen time.
Q_j	Batch size for component j

$Q_j^{'}$	Batch size of component j after JIT replenishment or component substitution
r_j	Reorder point for component j
$r_j^{'}$	Reorder point for component j after JIT replenishment or substitution
R_j	Replenishment cost *(per batch)* for component j
$R_j^{'}$	Replenishment cost *(per unit)* for component j replenished Just-In-Time
$\Re$	Uniformly distributed real number on the interval $(0,1)$
s_j	Target level for component j
S_i	Sale price of finished product i
σ_j	Standard deviation of the demand through lead time for component j
$\sigma_{\overline{K}}^2$	Variance of investment in inventory when average investment is $\overline{K}$
t	Arbitrary point in time
T	Arbitrary period of time
T_{K^*}	The duration of company lifetime when maximum investment is K^*.
$T_{K^*}(y)$	Company lifetime when inventory level is reviewed every y periods
ΔT_i	The duration between the $(i-1)^{th}$ and the i^{th} replenishment
TRC	Total relevant (inventory) costs
TRC_λ	Total relevant (inventory) costs subject to a constraint
θ_j	Average demand through lead time $(\overline{x}_j)$, for component j
w	The number of intervals between replenishments in a company lifetime.
W	The number of intervals between periodic reviews in a company lifetime.
x	Demand through lead time
$\overline{x}_j$	Expected demand through lead time for component j
ξ	Demand through review period
y	Duration between periodic inventory level reviews
z	Standard Normal variate

Acronyms and Abbreviations

ATO	Assemble-To-Order
BOM	Bill-Of-Materials
EOI	Economic Order Interval
EOQ	Economic Order Quantity
JIT	Just-In-Time
MRP	Material Requirements Planning
PMF	Product Master File
RDP	Risk-Discounted Profit
RDROI	Risk-Discounted Return On Investment
ROI	Return On Investment

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Chapter 1.

Introduction

This thesis studies inventory reduction decisions for components used in Assemble-To-Order (ATO) manufacture from the perspective of a company manager or investor in the company. Two methods of inventory reduction are investigated. They are replenishing a component Just-In-Time (JIT), and the substitution of one component with an alternative already held in inventory. It is assumed that the investment in the company is finite, set at a level determined by the investor, and is thus an additional variable in the inventory reduction decision. Taking an investor's view also has the consequence that the effect of inventory reduction decision on business performance measures such as Return On Investment (ROI) and company survival are as important as their effect on company profit. Under this new analysis, benefits arise from JIT replenishment and component substitution in addition to those previously reported in the management science literature. The case study examples illustrate how inventory reduction may be employed to increase profitability when investment in inventory is reduced, resulting in increased ROI at lower investment levels than could be considered under traditional analysis.

The following section describes the context of the research. The research objective is then stated in Section 1.2. The research method is outlined in Section 1.3 and departures from previous research are presented in Section 1.4. Section 1.5 presents key definitions and defines the scope of the research. Section 1.6 presents an outline of the thesis, which concludes the chapter.

1.1. Research context

The success of post-war Japanese manufacturing, attributable partly to the adoption of the Just-In-Time philosophy, has strongly influenced current Western manufacturing practice and rhetoric (Womack *et al.* 1990). One of the most visible elements of JIT is inventory reduction, with 'zero inventory' (Hall 1983) and replenishment batch sizes of one unit (Schonberger 1982) as theoretical ideals. In contrast to this, there is a history of analysing inventory policy in the management science literature by determining the optimal trade-off between the competing costs of procuring and maintaining inventory in order to minimise total inventory costs (Hopp and Spearman 1996). The Economic Order Quantity (EOQ) (Harris 1913; Wilson 1934) is the most familiar of these models, which minimises the sum of replenishment and holding costs of inventory under the assumption of deterministic demand. Treating demand for inventory items as stochastic, the continuous review (Q,r) model (Hadley and Whitin 1963; Tersine 1988; Hopp and Spearman 1996) seeks to minimise total replenishment, holding and lost order costs. Because the per-batch replenishment cost of stock is typically much greater than the per-unit holding cost for most typical inventory systems, both models recommend large batch sizes, as well as holding safety stock when demand is stochastic, which are at odds with the JIT approach.

Attempts at reconciling the reduced batch sizes of JIT manufacturing/replenishment with the cost trade-off approach of the EOQ have typically advocated setup cost reduction. This follows from an elementary analysis of the EOQ, and is usually cited as a necessary condition for the implementation of JIT (Hall 1983; Shingo 1985; Groenevelt 1993; Leschke 1997). An alternative means of justifying the reduced batch sizes under JIT operations is through a fuller accounting of total costs and its effect on the conditions for inventory reduction (Jones 1991). For example, when the cost of each factor that JIT attempts to control is included in the total cost of holding inventory, batch sizes recommended by the EOQ reduce to those advocated by

JIT practitioners (Schonberger and Schniederjans 1984). These costs include those arising from shop-floor material handling and storage, failing to make quality improvements between batches, uncertain demand and the interest cost of financing the inventory itself. Other authors have included the cost of damages (Chyr *et al.* 1990), quantity discounts (Fazel *et al.* 1998), increased worker competence through repeated setups (Replogle 1988), the synchronisation of production with demand (d'Ouville *et al.* 1992), and the reduction in warehouse space resulting from the implementation of JIT replenishment (Schniederjans and Cao 2000b; Schniederjans and Cao 2000a).

Another approach investigated by some authors is the investment of additional capital into setup cost reduction in order to reduce total inventory costs over numerous replenishment or production cycles. This may take the form of a one-off purchase of plant or machinery (Porteus 1985), or take the form of a periodic investment such as a rental or lease agreement (Billington 1987). In either case, the opportunity cost of investing in reduced setups, when added to the holding cost of the original EOQ cost function leads to smaller batch sizes. In a similar vein, a large order contract may be placed with a supplier in order to receive a stream of small replenishment orders JIT (Ramasesh 1990; Hong and Hayya 1992; Baker *et al.* 1994). Additionally, certain authors have also considered investing in reducing the variability of demand through lead time, when demand is stochastic, as a means of reducing inventory (Gerchak and Parlar 1991; Silver 1992; Ouyang *et al.* 1999).

Although these models yield certain insights into the conditions required for inventory reduction, because of their particular assumptions, large decreases in setup costs are usually required to justify the reduced reorder quantities advocated by JIT practitioners, suggesting that it is unlikely that these analyses capture the essential economic reasons why JIT is successful. As explanations of the efficacy of JIT, these models depend on complicating the original EOQ model whose appeal has always been its simplicity as an idealisation.

Similar assumptions also occur in the discussion of component substitution, where the benefits resulting from increased commonality such as a reduction in total inventory related costs have been identified for the deterministic demand case (Drezner *et al.* 1995). When demand is stochastic, the benefits of risk-pooling accruing from the increased commonality that result from component substitution may lead to a reduction in safety stock (Collier 1982), a reduction in total inventory (Baker *et al.* 1986), and increase service levels (Baker 1985; Gerchak *et al.* 1988). However, these benefits are typically not so compelling as to recommend the complete elimination of a component from inventory (Eynan and Rosenblatt 1996; Gurnani and Drezner 2000; Hillier 2000; Hillier 2002).

In this thesis it is argued that a limitation of these analyses is that they consider inventory costs independently of the financial position or capitalisation of the company as a whole. Many models assume that a company has unlimited access to borrowed capital at the prevailing interest rate, which is apparent from the implicit assumption that a firm has sufficient funds to purchase inventory in economical batch sizes. In practice companies are financed principally with equity capital provided by the owners or investors in the company, and to a lesser degree, by investor-secured borrowed funds, (Brigham and Gapenski 1991; Mason and Harrison 1993; Whyley 1998). Although borrowed capital can be used as an alternative, and sometimes temporary source of finance, component inventory has a collateral value that is only a small proportion of its cost to the company (Gitman *et al.* 1995). Thus, the inventory itself can only secure a small proportion of the capital that can be borrowed to finance inventory purchases. Consequently, for all practical purposes, the amount of capital invested in the company is finite, even when capital is borrowed. This is especially true for small business, (McMahon *et al.* 1993), where undercapitalisation and limited access to borrowed capital are frequently cited causes of failure (Welsh and White 1981; Hall and Young 1993; Gallagher 1999).

Taking an investor's view to company inventory policy suggests that objectives other than traditional profit maximisation can be used to evaluate inventory policy. Some authors have proposed maximising Return On Investment (ROI) (Schroeder and Krishnan 1976; Trietsch 1995) or residual income (defined as profit less the opportunity cost of capital invested in inventory) (Morse and Scheiner 1979) as criteria for inventory decisions. Maximising ROI or residual income will necessarily lead to recommended batch sizes smaller than those of the EOQ model by requiring that the return on capital invested is greater than the risk-free interest rate (Trietsch 1995; Otake *et al.* 1999). The use of ROI as a performance measure may also encourage a more efficient use of assets in the case of companies subject to a capital ceiling, or investment limit, because managers will only invest if a suitable return can be obtained (Arcelus and Srinivasan 1987). Furthermore, investors might choose a certain level of investment because of limitations in available capital, to increase ROI, to balance their portfolio or to increase operational flexibility. Thus, the cost minimisation approach, without regard to capital requirements or the company's ROI, is more representative of the view that a production manager might hold, as opposed to that of a general manager or investor in the company who would treat the investment in the company as an additional variable to be controlled.

1.2. Research objective

This thesis analyses the implications for inventory replenishment policy decisions when it is assumed that inventory is financed with investor-supplied capital. The objective of the research is to determine whether to adopt JIT replenishment for certain components or to increase component commonality by substituting some components with other over-specification alternatives.

Thus, in what follows, it will be assumed that the amount of capital invested in inventory is constrained, and set at a level determined by the manager/investor. The level of investment may be sufficiently large to allow the company to exploit economical replenishment quantities, or may be deliberately kept small in order to increase ROI, or reduced because of the limited availability of capital. By viewing inventory decisions from the perspective on an investor, the amount of capital invested in inventory is introduced as an independent variable in the decision analysis. The consequences of this assumption for the nature of the decision process, the appropriate business performance measures and the optimality of JIT replenishment will then be explored. As a result of this analysis the conditions under which particular components should be substituted or replenished JIT, and the effect of the decision on the whole company performance are seen in a new way.

1.3. Research approach

1.3.1. Sequence of investigation

Three investigations are reported in the thesis, which analyse JIT replenishment and component substitution decisions using three analytical models that reflect varying assumptions about the demand for finished products and the nature of constraint imposed on the inventory investment. The first model presented considers the case when the demand for finished products is deterministic and a constraint is imposed on the average inventory investment. In modelling terms this constraint is weak, or non-binding, as it permits the investment in inventory to exceed the constraint at times. The second model extends the deterministic case to now consider the case where the demand for finished products is stochastic. However, this model preserves the assumption that a constraint is imposed on the average inventory only, which again allows the inventory level to fluctuate above the average at times. The third model analyses the JIT and component substitution decisions under stochastic demand, but now with a binding, or hard, constraint imposed on inventory. When the constraint is binding, the company may exhaust capital required to replenish inventory leading to the cessation of manufacturing operations.

This introduces risk of company failure, which reduces the effective returns of a business when seen from an investor's point of view, and introduces company survival as an additional concern into the inventory replenishment decision. By presenting the models in this way, new factors in the replenishment policy decision are progressively introduced in order to show the consequences of the various assumptions for the decision process.

1.3.2. The role of discrete-event simulation within the investigation

The research described in this thesis was initiated as a case-based investigation into the means by which a small manufacturing company could reduce its investment in inventory. The case study company was predominantly self-funded with a very small overdraft facility. The general manager wanted to reduce the level of investment in component inventory for the range of products manufactured at the time in order to invest the excess capital into research and development with a view to producing a wider range of products. The company requested the author evaluate a small number of specific scenarios relating to reduced replenishment batch sizes and JIT replenishment of certain key components. This was undertaken by modelling the financial and operational performance of the company under several different scenarios using discrete-event simulation. Within the simulation framework, it was not possible to seek globally optimal scenarios due to the extensive computation required. However, this earlier study has motivated the research undertaken in this thesis through the unique insight into the company's problem gained by working with it at the general manager level.

In order to address the limitations of the earlier case study, discrete-event simulation is replaced by a series of increasingly realistic analytical models which build up to the complexity of the original simulation model but allow a more detailed analytical solution to be obtained. This in turn has yielded greater insights into the nature of the problem. The analytical models developed in the thesis make the effect of each cost component in the decision equations explicit, as well as making the calculation of optimal replenishment policies at varying investment levels tractable. Simulation is used in this thesis only to illustrate and justify assumptions made at various points in the remaining chapters. Specifically, the results of the earlier case study are not reported (see (Betts and Johnston 1997; Betts *et al.* 1998; Betts and Johnston 1998)), since the insights gained from this study are developed in a more general and systematic way through the analytical analysis. However, the case company data is used to illustrate inventory replenishment policy decisions throughout the thesis. In this way, the original questions of the case study company have been completely answered and some results of greater generality obtained.

1.4. Points of departure from previous research on inventory reduction

The consideration that capital is investor-supplied introduces several important changes to the determination of an optimal inventory replenishment policy which differ from the traditional analysis where it is assumed that a manager has access to unlimited borrowed capital, at a cost determined by the interest rate, typified by the unconstrained EOQ. Firstly, the capital available for investment is finite, set at a certain level decided by the investor, and is thus an additional independent variable of the policy decision. Secondly, taking an investor's view of company inventory policy suggests that objectives other than the traditional profit maximisation can be used to evaluate inventory policy. It is argued in this thesis that all investment levels, ranging from the minimum required for profitable operation, which is also close to that required to maximise ROI, to a maximum set by the condition that ROI be equal to the interest rate, could be considered by an investor in practice.

Although inventory models under investment constraint have been considered previously, for example, (Hadley and Whitin 1963), an economic analysis of JIT replenishment decisions when investment is constrained has not been reported by previous authors. Although some authors analyse the marginal utility of investing *additional* capital in order to obtain JIT replenishment, for example (Porteus 1985), this is done without regard to a base-level investment in the company. The analysis of inventory replenishment policy decisions, from the perspective of a company manager or investor, introduces new factors into the decision, which increase the cost effectiveness of inventory reduction strategies, particularly as total investment decreases. The benefits that are shown to arise from inventory reduction then permit JIT replenishment or substitution of components at costs that would be deemed to be uneconomical under traditional analyses because the value of money is higher under constraint.

The finiteness of capital available for investment also introduces the possibility that the company may become insolvent by exhausting all available capital required for the purchase of materials and payment of operating expenses (Gitman *et al.* 1995). The inability to meet debts as they fall due is typically a trigger for bankruptcy proceedings against a company to commence, (Pound *et al.* 1983; Gallagher 1999), and is a major cause of company failure (McMahon *et al.* 1993), often with little or no return to investors (Hall and Young 1993; Whyley 1998). At any realistic level of capitalisation, all companies face some risk of failure through insolvency, which potentially reduces the value of expected profits. Because the purchase of inventory is the principal means by which capital is used in the models considered in this thesis, the choice of inventory replenishment policy is a factor that contributes to this risk. Thus, one research issue investigated in this thesis is to incorporate the risk of company failure into an inventory model in order to determine optimal replenishment policies taking into account the risk of company failure through capital exhaustion. Although previous authors have incorporated risk into inventory models by valuing profit as though it were a financial instrument (Anvari and Kusy 1990; Singhal *et al.* 1994), none have included the possibility of company failure, with no return to an investor. Thus, the effect on company survival of inventory reduction strategies such as JIT replenishment and component substitution remains unexamined.

1.5. Key definitions and scope of the research

This section outlines the scope of the research by defining the type of manufacturing system, product structure, replenishment modes, and financial assumptions that underlie the models developed in subsequent chapters. These assumptions permit the adoption of models that simplify the determination of inventory costs in order to make the analysis in subsequent chapters tractable. The assumptions made in modelling are consistent with the practice of the case study company, whose data is used to illustrate the analysis. However, each model is derived independently of the case study company and thus similarity to the case company is not required for validation of the model. To varying degrees, the generality of each of the conclusions obtained are limited by the assumptions underlying each model. These limitations, and others subsequently introduced by the modelling process, are discussed as part of the conclusion of each chapter in which a model is derived (Chapters 4, 5 and 6).

1.5.1. Manufacturing Method

It is assumed that the manufacturing system is one where a company assembles finished products one-at-a-time, to customer order, from components either held in inventory or replenished JIT. It is also assumed that, when all required components are available, finished products can be manufactured in a single stage. This assumption allows the composition of each finished product to be treated as a single level bill-of-materials. The purchasing, assembly and distribution operations assumed are shown schematically in Figure 1.1.

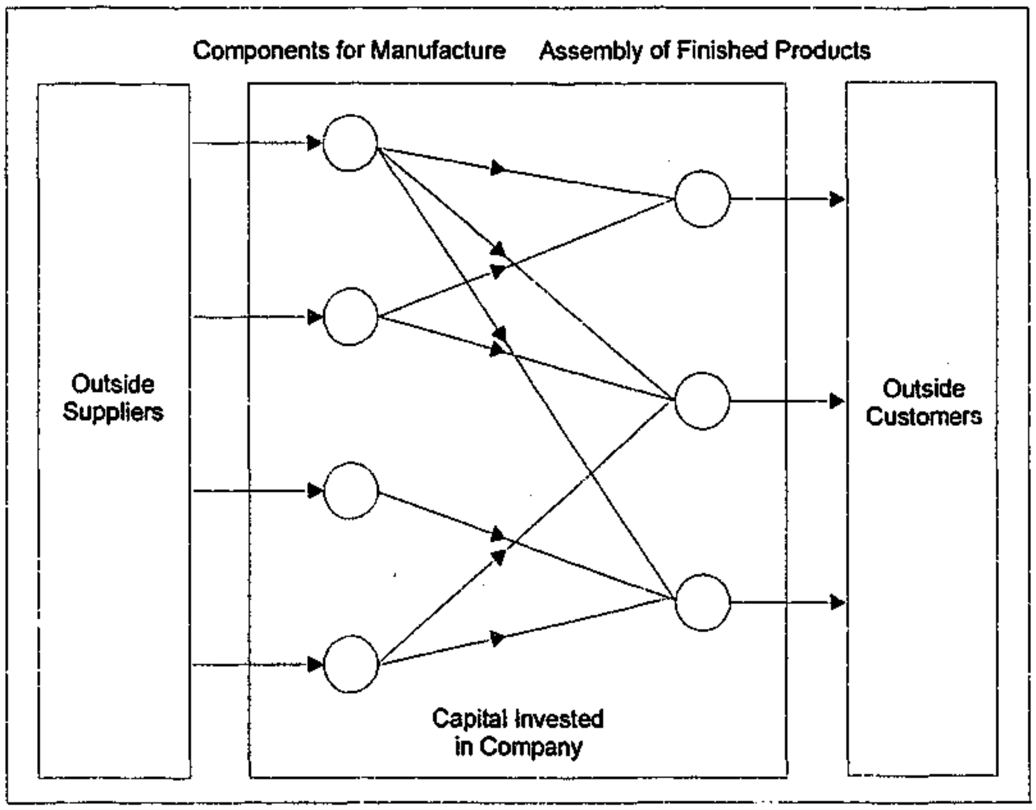

Figure 1.1: The manufacturing operation assumed in the models developed in the thesis

The effect of manufacturing to customer order means that no inventory of finished products is held. The single level bill-of-materials means that demand for components is lot-for-lot with the demand for finished products. The manufacture of finished products is assumed to occur as a single operation, from a single level bill-of-materials. The models developed also assume that no inventory of work-in-progress or partially completed products is held. One consequence of this last assumption is that the total investment in the company is available for the purchase of component inventory.

1.5.2. Approach to modelling inventory

The analysis undertaken in this thesis is based on inventory models which treat the inventory of components for ATO manufacture as if it is a multi-item inventory of components having independent demand (such as would be the case of an inventory for components for sale). This approach thus assumes a 'Base Stock Control System' for inventory management is employed whereby replenishments for items at each level of a multi-level echelon inventory system are made independently of all other levels, based on actual end-item demand (Silver *et al.* 1998). Although 'order-up-to-S' policies are typically employed for determining the replenishment quantities when inventory is periodically reviewed, as would be the case for batch manufacture, (see, for example, (van Houtum *et al.* 1996)), the models under stochastic demand in this thesis assume that inventory is continuously reviewed, which permits the use of the more general (Q,r) replenishment policy (Axsäter and Rosling 1994). Taking this approach, the replenishment of components are individually managed, with the consequence that the inventory levels of different components are uncorrelated over the long term. The assumption of independently replenished components permits the use of models well known in the literature for the determination of inventory costs, (Hadley and Whitin 1963; Tersine 1988), and new insights into the cost-tradeoff of the JIT decision under constraint are gained.

The approach of modelling components as independent items was adopted because of the relative simplicity and tractability of determining the optimal inventory replenishment policies for independent components over production planning and scheduling approaches for multi-item assembly. The complexity of these approaches make the determination of an optimal replenishment policy computationally intensive, even in the deterministic demand case (Lawler *et al.* 1993). Uncertain demand introduces further complexity, and even though stochastic programming methods can be applied to the optimisation of inventories under these conditions, (Birge and Mulvey 1996), the intractability of exact approaches for problems of a realistic scale requires that simplifying approximations be made to permit solution within a feasible time frame (Federgruen 1993). A further consideration is that mathematical models of multi-stage production inventories are typically complex, and although exact models have been derived for the simplest serial and assembly inventory systems (see for example, (De Bodt and Graves 1985; Schmidt and Nahmias 1985; Gurani *et al.* 2000)) the intractability of the cost function prohibits the extension of the exact model to the case of more components. Thus very few cases corresponding to an actual manufacturing situation are exactly solvable under stochastic demand (Axsäter 1993; Federgruen 1993).

The inventory models employed for the analysis of replenishment policy decisions in the case of stochastic demand thus simplify the interaction between components due to the assembly operations and necessarily introduce certain approximations due to the intractability of using exact cost functions. However, these models are complex enough to represent each of the inventory cost factors relevant to the decision analysis. To a degree of error that is small for operations typified by the case company, the models derived in these chapters permit some analysis of the conditions under which it is profit-maximising for an investor to adopt JIT replenishment or component substitution under stochastic demand when capital is constrained. These models also provide a tractable means of computing the optimal replenishment policy from a range of alternatives at varying investment levels for the case data in order to illustrate the analysis.

1.5.3. Just-In-Time Replenishment

In its broadest interpretation, JIT defines a management philosophy of producing or obtaining items only when needed, in the smallest possible quantities, with minimal waste of human and other resources, (Monden 1983; Lubbin 1988; Groenevelt 1993; Hallihan *et al.* 1997). The analysis of JIT implementations, particularly in the practitioner-oriented literature, has considered reform of all facets of the manufacturing process with the general aim of achieving improved material flow and handling as well as greater quality control (Vokurka and Davis 1996). Successful JIT manufacturing operations have been attributed to factors such as increased worker involvement and autonomy, continuous improvements in process design, close control of suppliers and setup cost/time reduction (Sugimori *et al.* 1977; Schonberger 1982; Hall 1983; Suzaki 1985). One expression of the ideals of a perfectly functioning JIT manufacturing system is found in the 'seven zeros' of JIT: zero defects; zero setup (time and/or cost); zero lot size; zero handling (of goods in and out of inventory); zero surging (stable demand and synchronisation between production and demand); zero breakdowns; culminating in zero lead time (Edwards 1983; Ohno 1988). As a consequence, the term JIT has become synonymous with terms such as 'zero inventory' and 'stockless production' (Hall 1983; Groenevelt 1993; Hopp and Spearman 1996).

In contrast to the broad view, the interpretation of JIT in the 'narrow sense' (Hall 1983; Groenevelt 1993) has focussed on those elements of a JIT system that affect inventory and material flow. For example, in describing JIT under the name Zero Inventories (ZI), Zangwill (Zangwill 1987) writes: 'Under ZI there is little or no buffer stock. Instead stock arrives just when it is required for the next stage of production, that is, "Just-In-Time".' Lubbin (1988), espouses a similar view in describing JIT as 'having *just what is needed, just when it is needed.*' Similarly, JIT purchasing requires that deliveries are synchronised with a buyer's production schedule (possibly by a kanban system), small purchase lots, with deliveries in the exact quantities required (Ansari and Modaress 1990; Waters-Fuller 1995; González-Benito 2002). The operational benefits resulting from JIT purchasing, are similar to those observed in JIT manufacturing systems more generally, that is, reduction of raw material and work-in-progress inventories, lead time reduction, and reduced material handling (Groenevelt 1993; Germain and Dröge 1998).

JIT replenishment within the scope of the thesis refers to JIT in the 'narrow sense', that is, JIT replenished components arrive when required for manufacture in order that inventory holding and material handling are minimised. Thus, the replenishment of components JIT in the models developed means that a purchase order for a component is made after the receipt of an order for a finished product requiring that component. This is, in essence, equivalent to the way a kanban is used in a JIT manufacturing operation. The supply of the JIT component then occurs in a sufficiently short time to permit manufacture of the finished product within the customer lead time (the amount of time required to supply a customer order from start to finish (Hopp and Spearman 1996)). The adoption of JIT replenishment for any component, thus, means that the component will be replenished within the customer lead time. Because finished products are assembled from a single level bill-of-materials with no work-in-progress held, these JIT components never form part of inventory. As the demand for finished products is assumed to be one-at-a-time for the case data, JIT components are replenished one-at-a-time. However, this assumption does not limit the generality of the models developed, which only require that JIT replenished components are consumed immediately on receipt. For components having a long lead time when batch-replenished, the change to a JIT replenishment policy, and subsequent reduction in replenishment lead time may incur a significant increase in annual replenishment costs.

It should also be noted that in the following decision analysis, the company may only choose between batch replenishment or JIT replenishment for certain components. That is, if the company adopts JIT replenishment for a component, batch replenishments for that component cease. Specifically, JIT replenishment is not used as an additional replenishment mode to augment a batch replenishment policy in order to circumvent potential stock outs as would be the case when emergency orders are permitted (Chiang and Gutierrez 1998; Mohebbi and Posner 1999; Teunter and Vlachos 2001).

1.5.4. Component Substitution

The possibility of substituting certain components for alternatives within the suite of components used for manufacture is also investigated. It is implicit that when substitution is adopted an over-specification alternative is used. Consequently, substitution always results an increase in cost for the substitute component. Some researchers have investigated partial substitution, that is, the possibility of using substitute components as an emergency ordering system in order to circumvent stock outs, for example, (Hillier 2002). However, this approach is not taken in this thesis, where the adoption of a substitute component means that the original component is eliminated from inventory, and the demand for the original component transferred to the substitute.

1.5.5. Accounting and financial considerations

All models developed assume that the total investment in the company is used only to finance the purchase of component inventory. Consequently, the value of any stock on hand at any time, plus any cash reserves held represent the current assets of the company (Yorston *et al.* 1986). This assumption thus excludes any additional investment in property, plant, or other assets by the company. The assumption has no effect on the replenishment policy decision analysis in the following chapters because all policy alternatives are compared at equal levels of investment in inventory. Although the implementation of a JIT manufacturing system may require some additional investment in plant or machinery, it is assumed that the adoption of JIT replenishment requires no infrastructure changes. Consequently, the adoption of JIT replenishment does not require that the company change the proportional investment in inventory, plant and machinery.

Because invested capital is used only to finance the stock of components, the models developed treat all other expenses, including the cost of replenishing and holding stock as operating costs, which are financed by the sale of finished products. This is in keeping with the approach of separating revenue (profit and loss) accounts from the general equity (balance sheet) accounts (Yorston *et al.* 1986; Tersine 1988). Thus, any increase in operating costs, for example, when a component is replenished JIT, results in reduced profit unless balanced by a reduction in the replenishment cost of other components.

The models developed also assume that the company pays fixed operating costs, which include a component for the rental of factory space and plant plus labour costs. Operating costs are assumed to be independent of the level of investment in inventory because the level of production is assumed to be constant (excluding lost sales) at all investment levels. Because it is also assumed that the manufacturing method is unaffected by replenishment policy, the assumption of fixed operating costs has no effect on the decision models described in Chapters 4 and 5, as any fixed cost terms cancel in the replenishment policy decision analysis. The value of fixed operating costs is introduced as a factor in the policy decisions discussed in Chapter 6 through its effect on expected profit. However, because fixed operating costs typically represent a small proportion of the cost of purchasing and replenishing components the effect on replenishment policy decisions is small except for very highly constrained investments near the threshold of profitability.

A final simplifying assumption implicit in the discrete-event simulation of the company, and in the analytical model of Chapter 6, is that the investment in inventory is incremented and decremented instantaneously with components being replenished or consumed for manufacture. This approach treats all financial transactions as though they are cash-based, and eliminates the effect of 'trade credit' (Brigham and Gapenski 1991), or the cost to the company of the delay between the outlay of capital for manufacture and the receipt of income from sales. Thus, in subsequent models, invested capital is not used to finance the float between debtors (customers) and creditors (the company). It has been the experience of the case study company that the average investment in inventory outstanding per day of delay is constant, and valued at approximately 0.2% of annual sales per day. However, the level of trade credit, while independent of replenishment policy, is dependent on sales which are assumed to be constant, and is thus omitted from the analysis. The second effect of assuming cash based transactions is that the model in Chapter 6 assumes that inventory investment level is reviewed continuously, thus preventing total investment to be exceeded at any time. The effect of relaxing this condition by permitting inventory to exceed the maximum, except at certain review points corresponding to a monthly or quarterly reconciliation of accounts, is investigated in Appendix A.

1.6. Outline of the thesis

Chapter 2 introduces the approach taken in this thesis to modelling a multi-item inventory of components for assemble-to-order manufacture subject to a capital constraint. The chapter presents a review of the literature on multi-item inventories, which motivates the models used and derived in this thesis.

Chapter 3 introduces the case study company, the specifications of the products made and the constituent components.

Chapter 4 considers the conditions under which a company would adopt JIT replenishment or component substitution when it is assumed that the demand for finished products is deterministic. The case study example is then used to illustrate the theoretical analysis by determining the profit maximising inventory investment policy as a function of capital invested.

The material presented in this chapter has been published as:

> Betts, J.M. and Johnston, R.B. (2001a). "Just-In-Time Replenishment Decisions for Assembly Manufacturing with Investor-Supplied Finance." *Journal of the Operational Research Society* 52(7): 750-761.

A decision support model derived from this chapter is published as:

> Betts, J.M. and Johnston, R.B. (1999). "Adopting Just-In-Time Replenishment Policies under Capital Constraint: Analysis and Decision Support Tools." *Proceedings of 5th International Conference of The International Society for Decision Support Systems*, Melbourne. 6A2 pp 1 - 18.

Chapter 5 extends the analysis presented in Chapter 4 by considering inventory replenishment policy decisions when demand for finished products is stochastic. The decision model of this chapter requires that an approximation of the multi-product (Q,r) model be derived for

Assemble-To-Order manufacturing where the product structure is defined by a single level bill-of-materials. The model derived then permits the efficient computation of the optimal replenishment batch sizes, enabling the exhaustive comparison of different replenishment policies at any investment level. The profit maximising replenishment policy at all investment levels is again determined for the case study data to illustrate the preceding analysis.

The material in this chapter forms the basis of the journal article:

> Betts, J.M. and Johnston, R.B. "Just-In-Time Component Replenishment Decisions for Assemble-To-Order Manufacturing under Capital Constraint and Stochastic Demand." *International Journal of Production Economics: (submitted 2001, in second stage review).*

Chapter 6 further extends the previous models by changing the nature of the capital constraint to now represent a limit on the amount of capital invested in inventory which cannot be exceeded in the course of operations. This is in contrast to the models of Chapters 4 and 5 in which the constraint defines the average investment in inventory about which the actual level of inventory fluctuates over time. This new model requires that the average investment in inventory be smaller than the constraint in order to permit the fluctuation of inventory investment level. The inventory model of this chapter defines the risk of company failure as the probability that the inventory level will exceed the capital constraint over a finite period in order to represent the case that a real company may fail by exhausting all working capital. The objective of the model is to maximise the expected profit of an ensemble of similar investments. Consequently, the optimal inventory replenishment policy requires the joint maximisation of profit and the probability of company survival, and thus introduces the probability of company survival as an additional factor in determining replenishment policy.

A preliminary account of part of the material in this chapter has been published as:

> Betts, J.M. and Johnston, R.B. (2001b). "Risk-Discounted Return-On-Investment as a Measure of Operational Performance in Inventory Decisions." *Proceedings of Eurosim 2001*, Delft.

Chapter 7 summarises the results of the previous chapters and discusses the effect of the various assumptions about customer demand and the type of constraint on the effectiveness of both component substitution and JIT replenishment as means of achieving inventory reduction. The main conclusions and effect of the study on the case study company are outlined, as are limitations of the current research and directions for future research.

Chapter 2.

Multi-item Inventory Modelling

The fundamental problems addressed by all inventory models are when to replenish inventory and how much to order for the replenishment (Hadley and Whitin 1963). Most inventory models attempt to determine the optimal timing and quantity of replenishments with the objective of minimising the associated costs of inventory, which typically include the costs of procuring stock and holding stock. In the case of stochastic demand, lost sales or the placement of back orders when demand is not met may be included as an additional cost. The problem under investigation in this thesis requires the replenishment of multiple components for assemble-to-order (ATO) manufacture be modelled, subject to a resource constraint. Thus, one requirement of the multi-item inventory models used in this thesis is that the timing and quantity of replenishments across all components must not result in a total inventory investment that violates the constraint. In the case when demand is stochastic, the probability of failing to manufacture a finished product is a function of the inventory level of groups of components, which requires that a suitable determination of joint service levels be made.

This chapter introduces the approach taken in this thesis to determining the quantity and timing of replenishment orders in a multi-item inventory of components for ATO manufacture subject to a constraint on invested capital. A review of multi-item inventory models, which motivate the modelling approaches used in the thesis, is presented. Because inventory modelling is such a large area of research, the following review is necessarily restricted to research that is directly relevant to models in this thesis. More general surveys of inventory models are presented in (Chikán 1990; Porteus 1990; Lee and Nahmias 1993; Silver *et al.* 1998). The following section considers the case when demand is deterministic. Constrained multi-item inventories under stochastic demand are then investigated in Section 2.2. This section considers the means by which previous authors have determined cost-minimising replenishment policies subject to constraint as well as reviewing approaches to determining service levels when multiple items jointly determine the reliability an inventory system, such as for ATO manufacture. Section 2.3 describes the modelling approach undertaken in the thesis, which concludes the chapter.

2.1. Deterministic demand

In the case of deterministic demand, the management of a multi-item inventory subject to a constraint requires that inventory related costs be minimised subject to satisfying the constraint. Two approaches to interpreting the constraint exist in the literature. The first approach considers the constraint to be binding, that is, the constraint represents a limit that cannot be exceeded. The best example of such a constraint is one where a physical limit, such as on warehouse space exists. A second approach is a non-binding constraint, where the constraint represents an optimal level (for example, the average inventory investment) that may be exceeded at times. The following subsection presents various approaches to determining replenishment policies under both interpretations of the constraint.

2.1.1. Batch sizing when inventory is subject to a constraint

Some of the earliest analysis of deterministic multi-item inventories subject to a binding constraint appear in Buchan and Koenigsberg, (1963), Hadley and Whitin (1963), Lewis (1970), and Johnson and Montgomery (1974). Two types of constraint are generally presented in this literature; one is a constraint on total available storage space, and the other is a constraint on total capital invested in inventory. Generally speaking, the constrained model is similar in each case. However, as will be shown in the thesis, the formulation of the constraint terms greatly affects the ease with which the model can be optimised. All authors take a similar approach to determining the replenishment quantities that minimise total inventory costs, which are obtained by modifying the single item EOQ model to include multiple items and to introduce a constraint. The adaptation for the case of multiple products requires that the total relevant inventory costs are calculated as the sum of the inventory costs for each item in the single product EOQ case. The introduction of a capital constraint is achieved by augmenting the total inventory cost through the introduction of a penalty for the total inventory level exceeding the constraint using a Lagrange multiplier. Because the basic method employed was first studied by Joseph-Louis Lagrange (1736 - 1813) (Kuhn 1991), this method of optimisation is often referred to as the 'Lagrangian approach'. Taking this approach, and using the notation adopted in this thesis, the annual total relevant costs for an m-item inventory subject to a constraint on the use of resource α, TRC_λ, are derived in Hadley and Whitin (1963, (Equation 2-52)), for example, as

$$TRC_\lambda = \sum_{j=1}^{m} \frac{D_j}{Q_j} R_j + \frac{J_j}{2} \sum_{j=1}^{m} Q_j C_j + \lambda \left(\sum_{j=1}^{m} Q_j \alpha_j - \alpha^* \right) \tag{2.1}$$

where

D_j = *Annual demand for component j*

Q_j = *Batch size for component j*

R_j = *Replenishment cost (per batch) for component j*

J_j = *Inventory holding costs other than interest for component j*

C_j = *Unit cost for component j*

λ = *Lagrange multiplier*

α_j = *The use of resource* α *per unit of component j*

α^* = *The maximum permitted level of resource* α

Constrained optimisation is then achieved by solving

$$\frac{dTRC_\lambda}{dQ_j} = -\frac{D_j}{Q_j^2}R_j + \frac{J_jC_j}{2} + \lambda\alpha_j = 0 \quad for\ j = 1,2,\ldots,m, \tag{2.2}$$

and

$$\frac{dTRC_\lambda}{d\lambda} = \sum_{j=1}^{m} Q_j\alpha_j - \alpha^* = 0, \tag{2.3}$$

which yield the total cost minimising replenishment quantities of each item subject to the constraint, Q_j^*, as

$$Q_j^* = \sqrt{\frac{2D_jR_j}{J_jC_j + 2\lambda^*\alpha_j}} \quad for\ j = 1,2,\ldots,m, \tag{2.4}$$

where λ^* denotes the cost-minimising value of the Lagrange multiplier. The typical approach of finding the optimal Q_j^* for a given K^* as reported in the literature, is by varying λ by trial and error until a suitably close approximation of K^* is obtained (Hadley and Whitin 1963; Tersine 1988). Trial and error approaches are recommended because the form of Equation 2.4 does not permit an expression for λ to be formed that allows an analytical solution to be found. As a consequence of these difficulties, some authors (for example, (Maloney and Klein 1993; Rosenblatt and Rothblum 1994)) have investigated more efficient algorithms for the determination of the optimal Lagrange multiplier for general cases under deterministic demand. Other authors have investigated alternative solution techniques such as Non-Linear Goal Programming (Padmanabhan and Vrat 1990).

Although the total inventory costs shown in Equation 2.1 present a difficult optimisation problem, in the case where a capital constraint is imposed, and holding costs are imposed as a constant proportion of inventory level for all items, the optimal replenishment quantities for a given constraint can be determined analytically (Rosenblatt 1981). In this case, for a constraint on total investment of K^*, Equation 2.1 becomes

$$TRC_\lambda = \sum_{j=1}^{m} \frac{D_j}{Q_j} R_j + \frac{J}{2} \sum_{j=1}^{m} Q_j C_j + \lambda \left(\sum_{j=1}^{m} Q_j C_j - K^* \right) \tag{2.5}$$

where

J = *Inventory holding costs other than interest*

having optimality condition

$$Q_j^* = \sqrt{\frac{2D_j R_j}{C_j (J + 2\lambda^*)}} \quad for\ j = 1,2,\dots,m. \tag{2.6}$$

It will be shown subsequently in Chapter 4, that the form of Equation 2.6 now leads to a simple expression for $\sum_{j=1}^{m} Q_j C_j$ which enables a solution for Q_j^* to be found analytically.

The form of Equations 2.1 and 2.5 shows that an assumption of the models presented thus far is to set the constraint at the maximum possible inventory level that would be attained if replenishments of all items coincided, for example, in the case of a capital constraint this is $\sum_{j=1}^{m} Q_j C_j$. For an inventory consisting of many components, this approach to setting the constraint requires that replenishment batch sizes be small enough to accommodate the rare event that all replenishments coincide. Thus, one disadvantage of this approach is that batch sizes may be set at a level that under-utilises the available capital resource at the expense of causing increased replenishment costs. For example, when replenishment quantities are determined with a constraint set at the maximum inventory level, the amount of capital invested in inventory on average is $\frac{1}{2} \sum_{j=1}^{m} Q_j C_j$. In other words, the available capital resource is only

50% utilised on average. As a consequence, other authors have proposed alternative strategies for determining the optimal replenishment quantity/policy for multi-item inventories subject to a capital constraint that increases the utilisation of the constrained resource in pursuit of reduced total inventory costs.

One approach to increasing replenishment quantities under a Lagrangian approach advanced by some authors is to assume that the constraint is imposed on the average inventory (Tersine 1988; Trietsch 1995). This approach follows from the assumption that when an inventory consists of many items being independently replenished, it is likely that the aggregate inventory will be close to the average inventory for a large proportion of the time. Thus, the inventory constraint consists of the sum of one half of a batch of each component, appropriately weighted for its use of the constraint. In the case of a constraint on total capital invested, the constraint term is $\frac{1}{2}\sum_{j=1}^{m} Q_j C_j$. However, such a constraint must be viewed as non-binding since it must be violated 50% of the time. Viewed in this way the constraint represents an ideal, or 'optimal inventory investment' (Tersine 1988), and not an absolute limit on inventory. However, one advantage of setting the constraint in this way is that it permits a fuller use of the available resource, with replenishment quantities that are double that determined by the Lagrangian method as originally presented, which results in a reduction in total inventory costs.

Implicit in the Lagrangian approach is the assumption that inventory items are independently replenished, which introduces the possibility of peaks in inventory level occurring when replenishments coincide. Alternative approaches include co-ordinating inventory replenishments, or varying the size of replenishments in order to prevent a constraint being exceeded. Such approaches do permit the use of larger, and hence more economical, replenishment batch sizes than those determined by the Lagrangian approach. One such approach to managing inventory is staggering the receipt of replenishment orders to prevent peaks in inventory level occurring due to different items being replenished simultaneously. The basic technique for implementing this method requires that a suitable (equal or integer multiple) length replenishment cycle be determined for all items in inventory. Replenishments of each item are then time-phased, in order that the maximum inventory of each occurs at different times. Because of the use of constant reorder intervals, this approach is commonly referred to as the 'fixed cycle' approach to managing inventory replenishments. Several variations of the basic method have been reported in the literature.

One means by which a cyclic replenishment policy is obtained is to first determine the Economic Order Interval (EOI), for all components and then calculate the degree of staggering, or time-phasing, of orders that minimises the maximum inventory volume (Homer 1966; Page and Paul 1976). Page and Paul show that in some cases this method may lead to increased total inventory costs compared with the Lagrangian approach of determining the optimal average inventory as the replenishment quantity for many components recommended by the EOI may vary significantly from the replenishment quantity determined by the EOQ. To partially overcome this limitation, Page and Paul propose an improvement to their original method whereby groups of products having a similar optimal reorder cycle length as determined by the EOQ are identified. The Economic Order Interval is then determined for all items within each group. Each group of items is then replenished in a staggered cycle to minimise the maximum inventory attributable to the group. The optimal assignment of components to groups, as well as the optimal allocation of the proportion of total investment (or warehouse space) to each group is computationally intensive. The authors propose a heuristic method for the determination of the optimal product groups which permits the more efficient calculation of the optimal replenishment quantities.

Goyal (1978) presents an extension of Page and Paul's work to show graphically that it is possible to minimise the maximum use of an inventory resource (for example, volume or investment) by staggering the purchase of components having a different number of replenishment cycles over a given time provided that the replenishment cycle length of each component is an integer multiple of some base period (taken to be the EOI of the most frequently ordered component in Goyal's paper). The significance of this result is that the heuristic of Page and Paul can be improved by time-phasing the replenishment cycles between component groups in order to further minimise the maximum use of the resource under consideration.

Assuming a cyclic replenishment mode of equal length for all components, Zoller (1977) independently of both Page and Paul, and Goyal, determines analytical expressions for the minimum storage space, and the associated optimal delivery schedule. The conclusions of Goyal and Zoller are confirmed by Hartley and Thomas (1982), who show that, for the two product case, a periodic replenishment cycle can be determined for each component, that minimises total inventory cost, subject to the constraint. Hartley and Thomas' method assumes that the replenishment cycle length for each component are in rational proportions in order that a time-phased fixed cycle can exist. The model developed by Hartley and Thomas has both the Lagrangian, and the fixed cycle approach of Page and Paul, as special cases, and thus, this model can be viewed as an integration of both methods. The authors consider the two product case, and note that the total inventory cost function is highly non-linear, and not easy to solve. They do, however, present an algorithm for determining the optimal replenishment quantities and staggering interval for the two-product case in a later paper (Thomas and Hartley 1983). Taking a similar approach as Hartley and Thomas, Matsuyama (1992) derives an algorithm for the optimal replenishment quantity and staggering interval for the case of three or more products.

Gallego *et al.* (1992) show that the problem of determining the optimal sequence of replenishments, as typified by the approach of Hartley and Thomas, and Zoller, is NP-complete, meaning that a polynomial time algorithm for solution does not exist (Garey and Johnson 1979).

However, Gallego *et al.* (1996) derive a heuristic method for determining the optimal replenishment batch sizes and staggering cycle in which the reorder interval for every component is a power-of-two fraction, that is, 2^{-1} 4^{-1} ..., of a certain base cycle length. The heuristic method solves the problems analysed by Hartley and Thomas to a small degree of error, and permits the investigation of cases with a greater number of components. Teo *et al.* (1998) also develop a heuristic solution to the staggering problem for the case where the replenishment intervals of components are integer multiples.

Rosenblatt (1981) compares the Lagrangian and fixed cycle approaches and derives an algorithm to determine which approach reduces total inventory costs for a given case. The author also shows analytically for two examples, that as inventory constraint increases, the fixed cycle approach yields lower total inventory costs than the Lagrangian approach. These analytical results accord with observations made by Page and Paul across a range of simulated problems.

A further approach to minimising inventory costs subject to a constraint is by the adoption of a non-stationary ordering policy, that is, a policy where the replenishment quantity for any component can be varied in order to prevent the constraint being violated. Güder *et al.* (1995) propose an algorithm for determining the optimal replenishment schedule and corresponding replenishment quantities over a finite period which requires that the when an item is to be replenished, it should be replenished in the largest quantity that does not exceed the amount determined by the unconstrained EOQ or violate the constraint. The minimum quantity that will be replenished at any time is determined by the constrained EOQ, that is, by the Lagrangian approach. The authors note that over any finite time period, the total inventory costs resulting from the non-stationary approach will always be smaller than those obtained by the Lagrangian approach. This is because the non-stationary algorithm has an opportunistic approach to ordering by increasing replenishment quantities when there are sufficient resources available, with a consequent reduction in overall replenishment costs.

Güder *et al.* compare the total inventory costs resulting from the non-stationary approach against those obtained using both the Lagrangian and fixed cycle approaches for a series of randomly generated problems for inventories varying size and under varying degrees of constraint. These results are summarised in Table 2.1. Although the relative performance of all three methods is related to the degree of inventory constraint and the number of components in inventory, the actual difference in total inventory costs under all three models is generally small. The non-stationary ordering policy always results in inventory costs lower than that of the Lagrangian approach for the reason given previously. However, the cost increase resulting from the use of the Lagrangian approach over the non-stationary approach is less than 7% for small problems having between 2 and 10 components, and less than 2% for cases having 20 components or more.

Table 2.1: Comparison of total inventory costs for three methods of determining resource constrained replenishment quantities (adapted from (Güder *et al.* 1995))

Number Of Components	Constraint Tightness (Percentage of resource required in equivalent unconstrained case)								
	90%			70%			50%		
	N	L	F	N	L	F	N	L	F
2 - 10	1.000	1.003	1.136	1.000	1.027	1.068	1.000	1.065	1.035
20 - 70	1.000	1.001	1.127	1.000	1.006	1.086	1.000	1.013	0.967

N = Non-Stationary, L = Lagrangian, F = Fixed-Cycle

The results of Güder *et al.* also show that the non-stationary approach resulted in the lowest inventory costs for all cases except when inventory consisted of a large number of components and was highly constrained, in which case, the fixed cycle approach performed best. The optimality of the fixed cycle approach for highly constrained operations also confirms the findings of Hartley and Thomas, and Rosenblatt. However, when the fixed cycle approach was optimal, the reduction in inventory costs under this approach was small, of the order of 3%. By contrast, under the minimum constraint, the fixed cycle approach performed poorly, with total cost increases of the order of 13% over either the Lagrangian or non-stationary approaches. Thus, these results show that the performance of the fixed cycle approach is highly sensitive to the degree of constraint, with clearly sub-optimal performance under minimum constraint. The authors show in a subsequent paper, that these general conclusions also hold for the case of multiple resource constraints, for example a constraint on total capital invested and total warehouse space (Güder and Zydiak 1999).

2.1.2. Discussion

This section has presented three approaches for determining the optimal replenishment batch sizes for a multi-item inventory, subject to a binding constraint, when demand is deterministic. These are the Lagrangian, fixed cycle, and non-stationary approaches. A comparison of total costs resulting from each of the three basic approaches, as summarised in Table 2.1., shows that the differences between each method are small across a wide range of problems. A major difference between the three approaches however, is their computational tractability.

Page and Paul (1976) show that the fixed cycle approach is simple to compute in its basic form using the EOI. However, the authors note that this approach is likely to yield sub-optimal replenishment quantities when there is a wide variety in product cost characteristics. Although the authors present an alternative method of determining optimal component groupings in order to form sub-cycles within an overall replenishment scheme, this approach is computationally intensive, notwithstanding the heuristic method given. Exact approaches for determining the optimal replenishment quantities and associated replenishment cycles are is intractable across a large number of products. Hartley and Thomas note the difficulty in calculating the optimal policy even for the two product case. Although Gallego *et al.* (1996) present an heuristic method for determining a feasible replenishment schedule subject to constraint, solution is still computationally intensive. The complex form of the inventory cost equation, and optimal replenishment quantities, under the cyclic approach also prohibits an analytical comparison of alternative replenishment policies.

The main disadvantage of the non-stationary approach is also that there is no analytical form for determining total costs, as replenishment quantities may vary. As a consequence, replenishment policies have to be run over a fixed duration and total costs calculated. Thus, the determination of costs in this case is quite lengthy. Because replenishment quantities vary over time, this approach does not permit any comparisons between replenishment policies or investments at varying levels to be made analytically.

The Lagrangian approach as originally presented in its most general form has the disadvantage of requiring a trial-and-error solution approach which increases the computational complexity of this approach. However, for the case of a capital constraint, and fixed holding costs across all components, the model of Rosenblatt yields to a straightforward analytical solution. The value of the Lagrange multiplier also has the well-known interpretation as the 'shadow price', or change in total inventory costs per unit of resource used, which permits an analysis of the utility of the resource employed (Hadley 1964).

For the purpose of investigating the research objectives, the Lagrangian approach has advantages over the other approaches by permitting an analytical determination of replenishment quantities, which confers several benefits. Firstly, the evaluation of replenishment quantities subject to constraint is efficient. Secondly, the simpler form of the total inventory cost model and expressions for the optimal replenishment quantities permit an analytical investigation of the JIT or component substitution decision under deterministic demand. This in turn permits insights into implications of these decisions for management to be gained, that would be less easily observed under more complex models. It has been shown that the difference in inventory costs between these approaches is small, thus the choice of the method of determining replenishment quantities would not significantly change the conclusions of the investigation. Furthermore, the choice of an inventory management strategy where items are replenished independently, introduces no additional assumptions into the inventory model to those that underlie the EOQ, and which would otherwise reduce the generality of the conclusions drawn from the model.

2.2. Stochastic demand

The assumption of stochastic demand introduces the additional inventory costs of either lost sales or the back ordering of stock resulting from unsatisfied demand, to the costs present in the deterministic case. Thus, the problem of determining the optimal quantity (and timing) of replenishments now requires the consideration of fluctuating demand, which, in general terms, requires that a safety factor (or stock in excess of average demand) be held to address uncertainty. In the simplest models, a safety factor is introduced by increasing the replenishment quantity. More complex models permit the replenishment quantity and safety factor to be jointly determined. In the case of assembly manufacture, the output of finished products is reliant on the supply of the components. Thus, a further consideration in the case of stochastic demand is the service levels of finished products manufactured from an inventory of components be determined. The following sub-sections consider first, approaches to determining replenishment policies that minimise holding and lost/back order costs subject to a constraint, and secondly, approaches to determining service levels when sales are due to the supply of multiple components.

2.2.1. Resource constraint

The simplest multi-product inventory model for independent items in the case of stochastic demand is the multi-product Single Period model, in which each item is replenished (in quantity Q_j) at the beginning of a review period in order to meet demand over the review period. This model assumes that unsold items at the end of a review period are discarded, and the process starts afresh at the beginning of the next review period (which justifies the name of the model). The objective of the model is to minimise the total cost of a stockout subject to satisfying a capital or warehouse space constraint. Rewriting the example in Hadley and Whitin (1963 Equation (6-9)), using the notation adopted in this thesis and setting a capital constraint instead of a volume constraint, total variable costs subject to the constraint are

$$TRC_\lambda = \sum_{j=1}^{m} B_j \int_{Q_j}^{\infty} (\xi - Q_j) h_j(\xi) d\xi + \lambda \left(\sum_{j=1}^{m} Q_j C_j - K^* \right) \tag{2.7}$$

where

$$\begin{aligned} B_j &= \text{Cost of lost sales attributable to component } j \\ \xi &= \text{Demand through review period} \\ h_j(\xi) &= \text{Probability Density Function (PDF) of demand through review period for component } j \end{aligned}$$

This model assumes that the cost of replenishments, which occur for each item at the beginning of each period are fixed, and are not included in the formulation of total variable costs. Thus, this model only attempts to minimise the cost of lost sales, subject to constraint. Because all components are replenished at the beginning of each review period, the maximum inventory level, set at $\sum_{j=1}^{m} Q_j C_j$ in Equation 2.7, always occurs. The cost-minimising batch size for each component is determined by the Lagrangian approach. Solving $\frac{dTRC_\lambda}{dQ_j} = 0$ gives

$$\frac{B_j}{C_j} H_j(Q_j) = \lambda \tag{2.8}$$

where $H_j(\xi)$ is the Cumulative Distribution Function (CDF) of the demand through the review period for component j. The form of the cost-minimising expression for Q_j in Equation 2.8 shows that the condition for optimality in this case requires that the expected cost of lost sales as a proportion of the unit cost be constant for each component (and equal to λ). Because an analytical solution for λ cannot be obtained under most demand distributions, and because each Q_j is a function of λ, the usual approach to determining the optimal Q_j for a given K^* reported in the literature is trial and error, varying λ until a close enough approximation to K^* is obtained (Hadley and Whitin 1963; Silver *et al.* 1998; Erlebacher 2000).

Another, more general formulation of the previous model is the Newsboy (Newsperson or Newsstand) Model. Under this model, inventory costs result from lost sales and from the scrapping of unsold goods. Thus, this model includes a penalty for being either understocked or overstocked. Using C_j^+ to represent the cost of each unit overstocked of item j, in any period, the model, given in Lau and Lau (1996), adapted for the case of a capital constraint is

$$TRC_\lambda = \sum_{j=1}^{m} C_j^+ \int_0^{Q_j} (Q_j - \xi) h_j(\xi) d\xi + B_j \int_{Q_j}^{\infty} (\xi - Q_j) h_j(\xi) d\xi + \lambda \left(\sum_{j=1}^{m} Q_j C_j - K^* \right), \qquad (2.9)$$

where the cost of being understocked, $\int_{Q_j}^{\infty} (\xi - Q_j) h_j(\xi) d\xi$ is augmented by the cost of being overstocked $C_j^+ \int_0^{Q_j} (Q_j - \xi) h_j(\xi) d\xi$. The optimal replenishment quantities are again determined using the Lagrangian approach. The cost-minimising values of Q_j subject to constraint are given by

$$Q_j^* = H_j^{-1} \left(\frac{B_j - \lambda C_j}{B_j + C_j^+} \right). \qquad (2.10)$$

As with the previous model, the form of Equation 2.10 prohibits a closed form expression for λ in most cases, necessitating trial-and-error approaches for determining the cost minimising replenishment quantities. As a consequence, more efficient solutions using exact equations for some demand distributions and heuristic methods for a range of other distributions have been investigated by a range of authors, including, (Nahmias and Schmidt 1984; Ben-Daya and Raouf 1993; Lau and Lau 1996; Lau and Lau 1997; Erlebacher 2000; Moon and Silver 2000; Vairaktarakis 2000).

A variation of the previous stochastic models is one where the excess stock in each period is not discarded, but carried over into the next review period. One replenishment policy, with largely the same mathematical properties as the previous model is the 'order-up-to-S', or target level policy, whereby an amount is ordered each period to bring the inventory level of component j up to a target level, s_j. A formulation, which includes the cost of holding stock is given by Oral (Oral 1981). Again, the total inventory cost function, and constraint are the sum of equivalent single item cases and the cost of replenishment is assumed to be fixed. Total variable costs subject to the constraint are

$$TRC_\lambda = \sum_{j=1}^{m}\left(J_j\int_0^{s_j}(s_j-\xi)h_j(\xi)d\xi + B_j\int_{s_j}^{\infty}(\xi-s_j)h_j(\xi)d\xi\right)+\lambda\left(\sum_{j=1}^{m}s_jC_j-K^*\right) \qquad (2.11)$$

The total cost minimising values of s_j are given by

$$s_j^* = H_j^{-1}\left(\frac{B_j-\lambda C_j}{B_j+J_j}\right) \quad for \quad J_j \le \lambda C_j \le B_j, \qquad (2.12)$$

the similarity with Equation 2.10 is evident. Determination of the optimal target level for each component subject to satisfying the constraint is by trial and error.

The form of Equations 2.8, 2.10 and 2.12 shows that the effect of increased constraint, through increased λ is a reduction in the service level. Thus, in addition to increased replenishment costs due to smaller replenishment quantities, which also occur in the deterministic case under increased constraint, in the case of stochastic demand, constraint introduces the additional cost of increased unmet demand. In subsequent chapters, it will be shown that the increased risk of lost sales under increased constraint increases the effectiveness of JIT inventory reduction methods through their effect on constraint reduction.

The preceding stochastic inventory models all assume that Q_j or s_j are determined as a function of the available investment, which in turn establishes the service level of each item. However, an alternative approach of determining the optimal investment in a multi-item inventory, is to selectively manipulate the service level of items or groups of items in order to satisfy a constraint on the total investment in inventory. Using this approach, the safety levels of all or certain components may be varied according to some criterion, or subjectively. As an example of this approach, Ramani and Krishnan-Kutty (1985) group a large multi-item spare parts inventory by, firstly, an ABC (Pareto) analysis, and secondly, by a VED (Vital/Essential/Desirable) classification to form nine subgroups. The authors vary the service level of each group of components selectively in order to reduce total relevant inventory costs. However, through its effect on determining the reorder point for each component, their approach implicitly determines the total level of investment in inventory.

Gerson and Brown (1970) relax the requirements of previous models that replenishments for each component occur at equal intervals and derive a safety stock, reorder quantity (s,Q) model, where both the reorder quantity Q_j and a safety factor are jointly optimised. Let component j have demand in any period ξ with PDF $h_j(\xi)$, CDF $H_j(\xi)$ and standard deviation σ_j. A safety factor is defined as κ_j and the safety stock of each component is $\kappa_j\sigma_j$. The authors define $L(\kappa_j)=\int_{\kappa_j}^{\infty}(\xi-\kappa_j)h_j(\xi)d\xi$ as the partial expectation of κ_j, where the quantity $\sigma_j L(\kappa_j)$ is the expected number of orders short per replenishment cycle. Using the notation previously given, total inventory costs subject to the constraint are

$$TRC_{\lambda} = \sum_{j=1}^{m} \frac{D_j}{Q_j}\left(R_j + b_j\sigma_j L(\kappa_j)\right) + \lambda\left(\sum_{j=1}^{m} C_j\left(\kappa_j\sigma_j + \frac{Q_j}{2}\right) - K^*\right) \tag{2.13}$$

Where b_j is a back order cost. Taking partial derivatives $\frac{dTRC}{dQ_j}$ and $\frac{dTRC}{d\kappa_j}$ gives

$$Q_j = \sqrt{\frac{2\left(b_j s_j L(\kappa_j) + R_j D_j\right)}{\lambda C_j}} \tag{2.14}$$

$$\frac{b_j D_j\left(1 - H(\kappa_j)\right)}{C_j Q_j} = \lambda \tag{2.15}$$

The optimal values of Q_j and κ_j are determined for a particular value of λ using Newton's iteration method. Equation 2.13 shows that the constraint is set at $\sum_{j=1}^{m} C_j\left(\kappa_j\sigma_j + \frac{Q_j}{2}\right)$ which follows a similar approach to that taken by Tersine and Trietsch in the deterministic case of assuming that the total inventory level is, on average, one half of the replenishment quantity of each item (plus the safety stock in this case). The assumption is valid in this case since the model assumes that items are independently replenished.

2.2.2. Joint service level

The models of the previous sections have treated inventory items independently except for their competition for the use of a shared resource. This section now considers approaches taken by previous authors in determining the joint service level resulting from an inventory of multiple items, where assembly or supply requires that a given suite of components be available simultaneously, as, for example, is required in the case of ATO manufacture. The models presented in this section do not consider a capital constraint explicitly. However, the imposition of a service level constraint may implicitly determine a particular level of investment.

The most straight-forward approach to determining the joint service level of a number of items is to assume that the inventory level and demand for each item are independent, in which case the probability of no shortage in all items is the product of the probability of no shortage in each item. One of the earliest models following this approach is a single-period model attributed to Ryzhikov (1969) and reported in Chikán (1990 (Model 276)). The model assumes that the probability that no shortage occurs is evaluated over a duration of T, where $\alpha_j T$ denotes the duration of the review period of each component, and $0 < \alpha_j T \leq T$. Thus, this model assumes that inventory items may have independent review periods, however the model requires that the probability of no shortage is determined over at least one complete replenishment cycle for each item. The demand for item j has PDF over the period T of $h_{jT}(\xi)$. The service constraint requires that the probability of no shortage in any item be greater than β, and is determined as

$$\prod_{j=1}^{m} \int_{0}^{s_j} h_{jT}(\xi) d\xi \geq \beta \tag{2.16}$$

The total inventory cost equation, consisting only of holding costs over the duration T is

$$TRC = \alpha_j T \sum_{j=1}^{m} J_j \int_{0}^{s_j} (s_j - \xi) h_{jT}(\xi) d\xi . \tag{2.17}$$

Total costs are minimised subject to the constraint by Lagrange multiplier methods described previously. Because the constraint is now determined as a product, the partial derivatives with respect to s_j and λ are not separable in j. Consequently, solution of the system of equations generated by the partial derivatives may be quite difficult.

A similar approach is used by Hopp and Spearman (1993) in determining the service level for the manufacture of end items assembled from j components. In the case of each component having a demand through lead time of x, stock level at the beginning manufacturing period of

$\bar{x}_j + \kappa_j\sigma_j$, of which $\kappa_j\sigma_j$ represents safety stock. Assuming Normally distributed demand with PDF denoted by $\phi(\cdot)$ and CDF denoted by $\Phi(\cdot)$ having mean $\bar{x}_j$ and standard deviation σ_j, the service level constraint, which requires that the probability of being able to assemble finished products without delay is greater than β is

$$\prod_{j=1}^{m}\Phi(\kappa_j) \geq \beta . \tag{2.18}$$

The total inventory cost (which in this case is just holding cost) equation subject to constraint is

$$TRC_\lambda = \sum_{j=1}^{m} J_j\sigma_j\left(\kappa_j\Phi(\kappa_j) + \phi(\kappa_j)\right) - \lambda\left(\prod_{j=1}^{m}\Phi(\kappa_j) - \beta\right). \tag{2.19}$$

The determination of the cost minimising values of κ_j subject to the constraint is by the Lagrangian method, with the value of λ found by iteration.

These authors also present an adaptation of the previous model for the case when an assembly consists of a large number of components. The intention is to reduce the large number of items into a smaller number of item groups in order to make computation more tractable. In this case, components are grouped into h categories, with the number of items in each group being n_k, and each component in a category having the same safety factor κ_k. The service constraint is now

$$\prod_{k=1}^{h}\Phi^{n_k}(\kappa_k) > \beta , \tag{2.20}$$

where $\Phi^{n_k}(\cdot)$ indicates that each item in the assembly contributes to the determination of the service level. The cost minimising values of κ_k are determined as for the previous model.

When independence between inventory items cannot be assumed, the determination of the service level given by a group of items requires that the joint probability distribution for demand over a review period be known. The determination of the multivariate PDF thus requires that the demand distribution for each item, and the interrelationships between demand for different items be known. Consequently the resulting multivariate steady-state distribution typically has a complex form (Cheung and Hausman 1995; Song *et al.* 1999; Song 2002). For certain more commonly studied multivariate distributions, such as the multivariate normal distribution, efficient calculation methods have been developed (Tong 1990). However the intractability of problems of a practical size typically require that the dimensionality of the problem be reduced or approximations to the multivariate distribution be derived (Prékopa 1965; Kelle 1988; Srinivasan *et al.* 1992). For example, Hausmann *et al.* (1998) consider the joint demand fulfilment probability over a finite time interval for a multi-item inventory using an order-up-to-S replenishment policy, with all items having equal length review periods and demand defined by the multivariate normal distribution. The authors determine the optimal exact joint demand fulfilment probabilities for sample problems of up to 10 items. However, the complexity of the multiple integration required for the calculation of the joint CDF requires that the dimensionality of the problem be reduced. The authors propose an equal fractile heuristic, that is, an order-up-to-S policy where all items have an equal probability of a stockout, which yields a close approximation to the exact solution at joint service levels greater than 85%.

2.2.3. Discussion

The review of multi-item inventory models under stochastic demand has shown that the Lagrangian approach is the predominant method employed for the determination of optimal replenishment quantities and safety stock levels. The basic assumption of single period models, that items are supplied at the beginning of a period for use during that period, tends to eliminate alternative approaches such as cyclic replenishment or non-stationary ordering policies. For cases where items are replenished independently, the complexity of the inventory cost function makes cyclic approaches intractable, and the unpredictability of future demands prevents a decisive evaluation of an optimal replenishment policy over a certain time horizon for non-stationary approaches. In the case of single period and newsboy models, the available resource is fully used at the beginning of each review period, when all replenishments coincide. Thus, the adoption of a non-binding constraint would be invalid for these models. By contrast, the (s,Q) model of Gerson and Brown, assumes independent replenishment cycles for all items which permits the imposition of a non-binding constraint.

The joint service level across multiple items is most easily determined when it can be assumed that the inventory levels of items are independent. In these cases, the Multiplication Law for independent events allows the joint probability of no stockout across multiple items to be calculated as the product of no stockout in each item. Hopp and Spearman (1993) presented an adaptation of this law for the case of groups of items. The complexity of multivariate distributions precluded the exact determination of service levels in cases of more than a few products, prohibiting the application these models to problems of a realistic scale.

This section has shown that the determination of the optimal replenishment policy under stochastic multi-item inventory models is more complex than for the deterministic case. To some extent this is due to costs (such as back order or lost sales) being imposed probabilistically, where the non-existence of a closed-form expression for the inverse CDF of many probability distributions prevents an exact analytical expression for the optimal replenishment quantity. The absence of an analytical solution in turn further complicates the determination of replenishment quantities when it is also required that a constraint be satisfied. In cases where safety stock is held, the joint determination of replenishment quantity and safety factor requires that an iterative approach be taken as each variable is a function of the other, further complicating any analysis. Thus, the increased complexity of inventory models under stochastic demand introduces the potential for the solution of the model to obscure the investigation of the research objectives.

2.3. The approach to modelling in this thesis

In order to investigate the research objective, the requirements of the inventory models adopted in subsequent chapters are:

- Models must include all relevant inventory costs, but present the simplest formulation of total costs in order to permit an analytical investigation of the research objective where possible and to make the determination of replenishment policy for specific cases tractable.
- Be indicative of inventory management approaches that would be encountered in general practice.
- Be representative of the approach taken by the case study company.
- Where possible, models should have sufficient generality to extend the conclusions of the research beyond the assumptions of the case study

The Lagrangian method is used to determine optimal replenishment quantities and reorder levels in the multi-item inventory models of the following chapters. The Lagrangian method is adopted for several reasons. Firstly, in the case of deterministic demand, when inventory is subject to a capital constraint, and holding cost is a fixed proportion of the replenishment quantity, the Lagrangian method yields the most easily computed replenishment quantities of all three approaches considered. Under these conditions, a simple expression for the optimal Q_j permits an analytical investigation into the research objectives. Furthermore, the Lagrangian approach introduces no assumptions about the co-ordination of replenishments. By contrast, cyclic approaches, complicate the determination of optimal replenishment by requiring that both the replenishment quantity, and the staggering cycle be jointly optimised.

In the case of stochastic demand, the literature review has shown that Lagrangian approach is the most common method of optimisation as the complexity of the inventory cost equation, and the inherent variability of demand, makes the implementation of alternative approaches (such as staggered replenishments) more difficult. It is thus assumed that components are independently replenished, and hence, apart from the competition between components for their share of the capital resource (which determines the timing and quantity of replenishments), there is no other interaction. The adoption of the Lagrangian approach with staggered replenishments in both the deterministic and stochastic demand cases also maintains a consistency between inventory modelling approaches under different assumptions of customer demand. The analysis undertaken in the thesis also requires that the multi-item inventory models be used for the determination of the optimal replenishment policies across a wide range of investment levels. Consequently, analytical simplicity and computational tractability of the Lagrangian approach over other methods is an important criterion for adopting this approach over alternatives.

The form of the inventory cost equation used in the deterministic case is that of Equation 2.5. In the case of stochastic demand, a multi-item (Q,r) model is employed, which presents a simpler cost interpretation of the optimality conditions (by determining safety stock instead of a safety factor) than that of the model of Gerson and Brown (1970) (Equation 2.13). The multi-item (Q,r) model is also adopted for the stochastic demand case because it extends the model in the deterministic case without introducing any additional assumptions underlying the deterministic model other than the variability of demand (Hopp and Spearman 1996). By contrast, the single period and Newsboy models introduce the additional assumption that replenishment occurs periodically, to service demand within a finite period. The multi-item (Q,r) model also reflects the method of inventory management practised by the case company.

The multi-item inventory models of subsequent chapters consider the case of both a non-binding and a binding constraint on investment in inventory. In the case of a non-binding constraint, the constraint is defined as the sum of the average capital invested in each component, which in the deterministic case is $\frac{1}{2}\sum_{j=1}^{m} Q_j C_j$. Setting the constraint based on the average inventory is consistent with the assumption that components are replenished in independent cycles in both the deterministic and stochastic cases, and thus the inventory level of different components are uncorrelated in the long term. A binding constraint on inventory investment is introduced in Chapter 6, whereby the replenishment quantity and reorder point of components are determined by the Lagrangian approach, with the constraint on inventory investment set at the average level. The probability of exceeding a predetermined maximum investment (at a level greater than the average investment) introduces a penalty for capital exceeding the constraint. The maximisation of profit or ROI subject to this penalty introduces a new approach to determining replenishment policies subject to constraint.

The assumption that inventory levels of components are uncorrelated over the long-term has the consequence, in the stochastic demand case, that instances of a stockout in any component are independent. This in turn permits the determination of the probability of a lost sale of finished products to be determined from the probability of a stockout in components using a multiplicative rule. In Chapter 5, an approach similar to that taken by Hopp and Spearman (1993) (Equations 2.18 and 2.20) for determining the joint service level due to a number of components required for ATO manufacture is introduced.

2.4. Conclusion

This chapter has introduced the approach to modelling undertaken in the thesis through a review of multi-item inventory models. The chapter has presented existing approaches to determining the cost minimising replenishment quantities subject to constraint, and the determination of joint service levels, which form the basis for developing the models presented in subsequent chapters. The choice of models on which this research is based have also been justified in terms of their relationship to the purpose of the investigation.

Chapter 3.

The Case Study Company

This chapter introduces the case study company, whose data forms the basis of the decision analysis examples in subsequent chapters. This chapter describes the products made, their constituent components and manufacturing method, and outlines the assumptions about the company's operating costs and manufacturing operations present in the subsequent decision analysis.

3.1. The company

The case study company, Cash Engineering Research Pty. Ltd., is a privately owned company that designs and manufactures air compressors and related machinery. The company has been in existence for approximately 60 years, and over that time has evolved from mainstream manufacture to a predominately research and development enterprise in the mid 1980s. In the early 1990s, the company restarted manufacturing air compressors in order to expose practical research problems as a means of initiating patent-worthy inventions. Under the current management the company engages in both manufacturing and research. These activities complement each other as the company's innovations are implemented in their compressors and the operating factory initiates research and provides the infrastructure for testing designs.

Although the company derives a portion of its income from the sale of licences, the manufacturing arm of the business operates as an independent profit-making financial unit.

The company manufactures a small range of air-compressors to customer order and had sales of approximately 100 compressors per year valued at about $500,000 at the commencement of this study. The data used to illustrate subsequent decision analysis was gathered at this time (1997-1998). Consumer behaviour is known from a long history of participation in this industry. Similarly, the cost of staff and other overhead costs are well known from experience. The activities undertaken in the normal course of business were easily observed due to the small scale of the business and because of the manager's first hand involvement in the design and manufacturing process. The processes of ordering and receiving components, building and delivering compressors as well as other administrative tasks were discrete and easily articulated. These factors, in addition to the general manager's commitment to reducing investment in inventory, made this company particularly appropriate for the modelling undertaken in this thesis.

3.2. The product and components

The compressors made by the case study company are for light to medium industries and are novel in that they use a pair of helical screws as the compressing element instead of the more commonly used piston. The compressors are made in a range of 7 sizes, from approximately 90 components. Although the inventories of major components are independently replenished, some of the smaller components are purchased as pre-assembled sub-assemblies (for example, the piping and oil separator assembly and the wiring and electronic controls). As a consequence, each compressor is assumed to be composed of 9 generic sub-assembly groups that preserve the unit cost, lead time and replenishment cost variety present in individual components. Figure 3.1 shows the structure of the compressors from each of the 9 component groups.

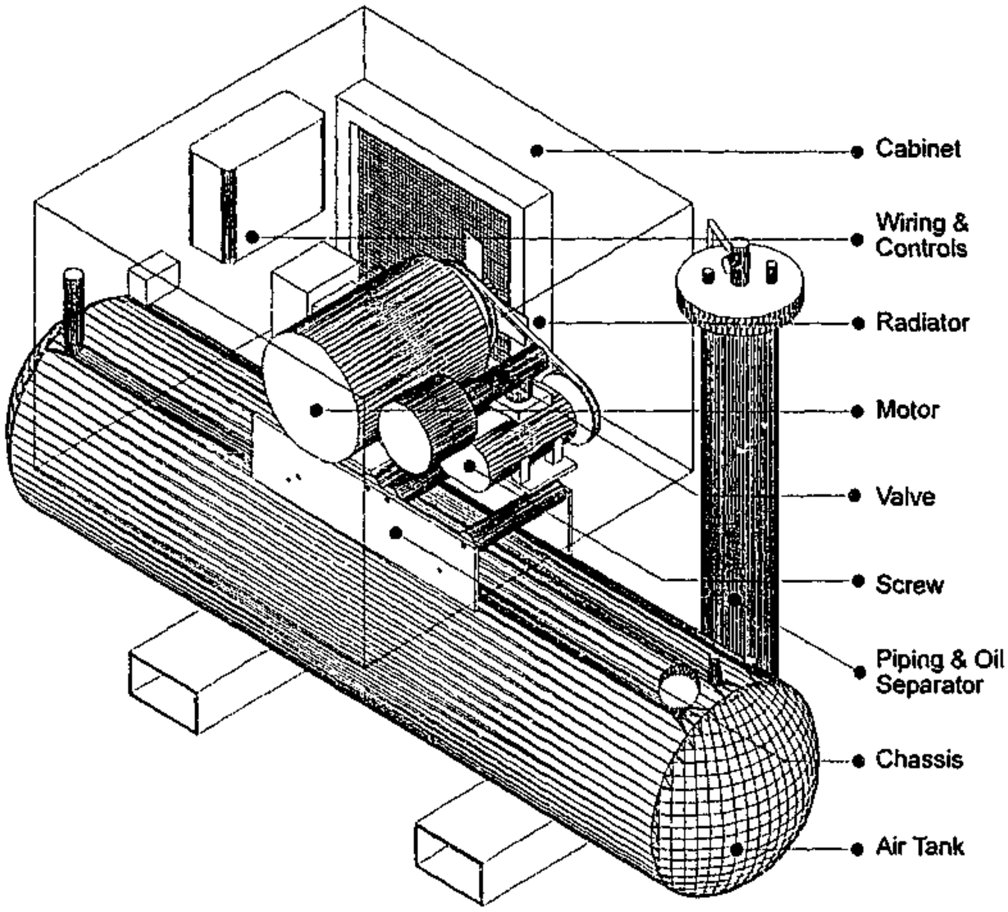

Figure 3.1: The composition of a typical air compressor showing the nine component groups used for manufacture. Source: Nick Murray, Cash Engineering Research Pty. Ltd.

The average annual demand and sale price of each model air compressor are shown in Table 3.1. The components used in the manufacture of each model compressor are given in Table 3.2. The different degrees of commonality for each component type are evident in Table 3.2, with the motors for example, which determine the power output of the compressors being unique to each model, whereas the chassis on the other hand is common to all models.

Table 3.1: The average annual demand and sale price of each model of air compressor

	Model Size (Horsepower)						
	3	5	7.5	10	15	20	25
Mean Annual Demand	12	8	8	25	25	7	15
Sale Price *($)*	2,800	3,000	3,300	3,900	5,200	6,000	6,500

Table 3.2: Components used in the manufacture of each model air compressor

		Model Size (Horsepower)						
		3	5	7.5	10	15	20	25
Component	Air Tank 1	•	•	•	•			
	Air Tank 2					•	•	•
	Cabinet 1	•	•					
	Cabinet 2			•	•			
	Cabinet 3					•	•	•
	Chassis	•	•	•	•	•	•	•
	Motor 1	•						
	Motor 2		•					
	Motor 3			•				
	Motor 4				•			
	Motor 5					•		
	Motor 6						•	
	Motor 7							•
	Piping 1	•	•	•	•			
	Piping 2					•	•	•
	Radiator 1	•	•					
	Radiator 2			•	•			
	Radiator 3					•	•	
	Radiator 4							•
	Screw 1	•	•	•	•	•		
	Screw 2						•	•
	Valve 1	•	•	•				
	Valve 2				•	•	•	•
	Wiring 1	•	•	•	•			
	Wiring 2					•	•	•

In the models of Chapters 5 and 6, which analyse the stochastic demand case, it is assumed that the demand for finished products is a stationary Poisson process, with the rate for each model given by the average annual demand in Table 3.1. Consequently, because the demand for each component results from the superposition of a finite number of Poisson processes, the demand for each component is also a Poisson distributed random variable, with mean determined as the sum of the average demand for each model air compressor that the component forms part of (Nelson 1995). Following the same assumptions for the case of a single component, in the case of component substitution, the demand for the substitute component is also a Poisson distributed random variable with mean demand now being the sum of the demand for the original and substitute components. A simplifying assumption in the stochastic case is that there is no variability in the lead time of components. As a consequence, the probability of a stockout in any component is only due to the variability of customer demand for finished products.

The unit cost, lead time, mean annual demand, batch replenishment cost and the JIT replenishment cost of each component are given in Table 3.3. Where a component may be substituted by another, this is also given. The quantity in which the company typically replenished components at the commencement of the study are also shown to permit a comparison of the costs, per unit, for batched and JIT replenishment. In subsequent models, however, the quantity in which each component is batch replenished is varied in order to satisfy a constraint on the investment in inventory. Note that because finished products are manufactured in batches of one unit, the JIT replenishment cost is for a single component, thus, for certain components JIT replenishment attracts a significant cost penalty. For example, using the replenishment quantities in Table 3.3, replenishing Screw 1 in batches of 25 units costs $750 per batch, or $30 per item, whereas replenishing Screw 1 JIT costs $100 per item. The average demand for each component is calculated from Tables 3.1 and 3.2 as the sum of the average demand for each model air compressor that the component forms part of.

As can be seen in Table 3.3, there is a wide variation in the unit cost, lead time and replenishment cost characteristics between each of the component groups. For example, the screw element is purchased from a European supplier and freighted by sea with a lead time of approximately 100 working days. This is the most expensive single component in the compressor and thus makes a significant contribution to inventory. As well, the high cost of freight has meant that the company has traditionally purchased these components in large batch sizes, further contributing to the large inventory of these products. A major concern of the inventory reduction decision analysis in subsequent chapters is to investigate the effectiveness of JIT replenishment for the screw elements using air transport at an increased unit cost.

Table 3.3: Annual demand, lead time, unit cost, batch-replenishment and JIT replenishment costs and substitutes for each component

Component	Substitute Component	Mean Annual Demand	Standard Lead Time *Days*	Unit Cost *($)*	Standard Replenishment Cost *($) per Batch*	Standard Replenishment Batch Size	Just-In-Time Replenishment Cost *($) per Item*
Air Tank 1	Air Tank 2	53	15	80	260	10	260
Air Tank 2	-	47	15	430	260	10	260
Cabinet 1	-	20	1	100	50	5	50
Cabinet 2	-	33	1	100	50	5	50
Cabinet 3	-	47	1	150	50	5	50
Chassis	-	100	2	75	30	10	30
Motor 1	-	12	1	200	0	1	0
Motor 2	-	8	1	250	0	1	0
Motor 3	-	8	1	300	0	1	0
Motor 4	-	25	1	400	0	1	0
Motor 5	-	25	1	500	0	1	0
Motor 6	-	7	1	600	0	1	0
Motor 7	-	15	1	700	0	1	0
Piping 1	Piping 2	53	2	75	30	10	30
Piping 2	-	47	2	125	30	10	30
Radiator 1	Radiator 2	20	30	100	1,050	20	1,050
Radiator 2	-	33	30	150	1,050	20	1,050
Radiator 3	Radiator 4	32	30	200	1,050	20	1,050
Radiator 4	-	15	30	350	1,050	20	1,050
Screw 1	-	78	100	1,000	750	25	100
Screw 2	-	22	100	2,000	750	10	400
Valve 1	Valve 2	28	20	110	2,050	50	1,025
Valve 2	-	72	20	195	3,050	50	1,525
Wiring 1	-	53	1	150	0	1	0
Wiring 2	-	47	1	200	0	1	0

The valves and radiators also represent a large contribution to inventory due to their high setup cost under batch manufacturing, although these components have a low unit production cost. For these components, the substitution with over-specification alternatives (for example substituting Valve 2 for Valve 1) is investigated. By contrast, both the motors and wiring assemblies are replenished within one day from local suppliers with no cost for delivery, and consequently, can always be replenished in single units at no additional cost.

Although the arrival of customer orders is unpredictable, the company is able to make the compressors one-at-a-time to order provided components are either in stock or available JIT, because the customers accept delivery within several days of ordering. As a consequence of this, the company maintains no inventory of finished products. Compressors are generally built on a first-come-first-served basis, by one or two people, and if all components are present, a compressor can be manufactured from scratch in one day. As well, because each model is manufactured according to a standard sequence of operations, with a high degree of similarity between components of varying sizes, the time taken to manufacture any compressor is independent of the model. Consequently, labour costs are constant across all models.

In addition to the cost of replenishing components, the models of subsequent chapters assume that the company pays a fixed operating cost of $85,000 *per annum*. This fixed cost includes a component for the rental of factory space and labour costs, and is assumed to be independent of the level of investment in inventory because the level of production is assumed to be constant (excluding lost sales) at all investment levels.

3.3. Conclusion

This chapter has introduced the case study company, the products made and constituent components. The relatively small size of the company and the willing participation of employees and management enabled all aspects of the company's operations to be easily observed. This, in turn, permitted several models of company profit to be developed, which are presented in subsequent chapters. The company, product and component data form the basis of all decision analysis illustrations. Although the case study company is small and its manufacturing process is simple, it will be shown in subsequent chapters that, to varying degrees, the models based on this company allow statements about the efficacy of JIT that can be shown to apply to manufacturing more generally.

Chapter 4.

Inventory Replenishment Policy Decisions under Deterministic Demand

In this chapter, a batch sizing model using only the traditional cost terms is formulated for the case when the capital which funds inventory holdings is provided by the investors in the company. This model considers the case when demand for finished products is deterministic and introduces two important modifications to the traditional EOQ model. Firstly, because capital is investor supplied, the interest cost of capital is not a component of the company's operating cost. Secondly, the amount of capital available for investment must be viewed as having a fixed value.

The next section describes the conditions under which a manufacturer would adopt JIT replenishment or component substitution policies for the standard multi-product, multi-component batch sizing model of a manufacturing company operating with unlimited borrowed capital, and is included for later comparison with the investor-financed case. Batch sizing under constraint using investor-supplied capital is then considered. The theoretical analysis is then illustrated by determining an inventory reduction strategy for the case study company.

4.1. JIT replenishment decisions with borrowed capital

The case of a company operating with an unlimited amount of borrowed capital is first considered. Profit is revenue minus expenses. Revenue is the proceeds from the sale of finished products. Expenses consist of: the cost of components or raw materials, the variable cost of replenishment, the cost of holding stock, and fixed overhead costs which include the rental of property or plant, labour costs for the manufacture of finished products, and the fixed costs of processing of replenishment orders. In the case where stock is purchased with borrowed capital, the cost of holding stock must also include the cost of borrowing the necessary funds. Thus, for a company manufacturing a suite of n products from a range of m components, the profit, when demand for finished products is deterministic is

$$\textit{Profit} = \textit{Sales} - \textit{Component Costs} - \textit{Replenishment Costs} - \textit{Holding Costs} - \textit{Fixed Costs}$$

that is,

$$P = \sum_{i=1}^{n} A_i S_i - \sum_{j=1}^{m} D_j C_j - \sum_{j=1}^{m} \frac{D_j}{Q_j} R_j - \frac{(J+I)}{2} \sum_{j=1}^{m} Q_j C_j - F \tag{4.1}$$

where

P = Annual profit
A_i = Annual demand for product i
S_i = Selling price for product i
D_j = Aggregate demand for component j
C_j = Unit cost for component j
Q_j = Batch size for component j
R_j = Replenishment cost (per batch) for component j
J = Holding costs other than interest
I = Interest charge
F = Fixed annual overhead costs.

Maximising P with respect to each Q_j yields the Economic Order Quantity, (Harris 1913; Wilson 1934)

$$Q_j = \sqrt{\frac{2D_jR_j}{C_j(J+I)}}. \tag{4.2}$$

The term $C_j(J+I)$ is the holding cost for each component and includes the interest cost, I.

The case of a company operating with borrowed capital deciding whether to change the replenishment policy of certain components to JIT, or opting to decrease the range of components employed by the substitution of components is now analysed. A JIT policy for component replenishment requires that the components arrive just as they are needed, resulting in an average inventory holding close to zero, but possibly incurring greater delivery costs to reduce lead time and/or handle smaller quantities (Fazel *et al.* 1998; Schniederjans and Cao 2000). This compares with the typical average stock holding of $Q_j/2$ for reorder point methods using large batch sizes, or MRP methods where the replenishment lead time is greater than the planning horizon (Tersine 1988). Inventory savings can also be achieved by substituting for a component in one machine, a similar component of superior specification already used in another machine. In both cases, the effect of a policy change may result in an increase in the cost of the finished products, although this may be offset by savings in holding or purchasing costs. For a company maintaining an inventory with borrowed funds, these decisions simply involve a cost tradeoff between component replenishment and holding costs.

Although JIT rhetoric advocates batch sizes of 'one unit' as the ideal replenishment quantity (Hall 1983), JIT replenishment actually only requires that stock arrive when required for production in order that no buffer stock or inventory be held (Zangwill 1987). In the case of ATO manufacture, JIT replenishment requires that components are replenished lot-for-lot with demand, with the replenishment quantity being determined by the manufacturing batch size.

Thus, any batch sizes could fit within the general framework of JIT replenishment as long as they were supplied when and in the quantity required and consumed immediately on receipt. For the case study company, finished products are manufactured one-at-a-time and thus the JIT replenishment batch size in the following model is one unit, however the model could be adapted to incorporate JIT replenishment quantities of any size.

In the case of a manufacturer deciding whether to change to a JIT replenishment policy for Component 1, to be replenished in batches of one unit, at a replenishment cost *per item* of R_1', the new profit is

$$P_{JIT} = \sum_{i=1}^{n} A_i S_i - \sum_{j=1}^{m} D_j C_j - \sum_{j=2}^{m} \frac{D_j}{Q_j} R_j - \frac{(J+I)}{2} \sum_{j=2}^{m} Q_j C_j - D_1 R_1' - F \tag{4.3}$$

A change to the JIT policy would be desirable when $P_{JIT} > P$, giving

$$D_1 R_1' - \frac{D_1}{Q_1} R_1 < \frac{(J+I)}{2} Q_1 C_1 . \tag{4.4}$$

Thus, when inventory is financed with borrowed money, a manufacturer would choose to change the replenishment policy of a component to JIT when the increase in the replenishment cost is justified by the saving in inventory holding cost that results.

Similarly, when Component 2 is substituted for Component 1 the new profit is

$$\begin{aligned} P_{SUB} = \sum_{i=1}^{n} A_i S_i - \sum_{j=3}^{m} D_j C_j - (D_1 + D_2) C_2 - \\ \sum_{j=3}^{m} \frac{D_j}{Q_j} R_j - \frac{(D_1 + D_2)}{Q_2'} R_2 - \frac{(J+I)}{2} \left(\sum_{j=3}^{m} Q_j C_j + Q_2' C_2 \right) - F \end{aligned} \tag{4.5}$$

where

$$Q_1 = 0 \tag{4.6}$$

and

$$Q_2^{'} = \sqrt{\frac{2(D_1 + D_2)R_2}{C_2(J + I)}}. \tag{4.7}$$

In this case, the cost of replacing Component 1 with the nominally more expensive Component 2 needs to be offset by the reduction in holding and replenishment costs achieved by the elimination of Component 1. This decision requires $P_{SUB} > P$, giving

$$D_1C_2 - D_1C_1 + \frac{(D_1 + D_2)}{Q_2^{'}}R_2 - \frac{D_1}{Q_1}R_1 - \frac{D_2}{Q_2}R_2 < \frac{(J + I)}{2}\left(Q_2^{'}C_2 - Q_1C_1 - Q_2C_2\right). \tag{4.8}$$

The preceding discussion has presented an analysis of inventory reduction approaches typical of those presented in the technical literature, where capital is assumed (at least implicitly) to be borrowed and unlimited. In such cases, the issue at stake in either case is economy or profit maximisation, rather than the benefits that may result from a redistribution of invested capital. Changes to the replenishment policy of individual or pairs of components have no effect on the costs of the remaining components. The effect of both changes is to reduce inventory investment by the elimination of one component from inventory. One consequence of this change is to reduce the amount of capital borrowed to finance the purchase of inventory. However, in the case of unlimited borrowing potential, the consequent benefit of such a change is only the saving in the interest cost of money, and since both alternatives in practice require either the use of more expensive components, or greater replenishment costs, they are unlikely to be appealing under this scenario. Furthermore, the economy offered by large batch sizes under the EOQ policy makes the transition to JIT unlikely while $R^{'}$ is large. This is why much previous research has focussed on setup cost reduction as a necessary step in the implementation of JIT.

4.2. Inventory management with investor-supplied capital

4.2.1. The model

In practice, companies operate with capital contributed by investors. Companies may borrow from banks, although the amount that can be borrowed is based on the security offered. Borrowing money to purchase component inventory typically presents a problem because the components have little intrinsic redeemable value and can only contribute a small proportion of their own security. Thus, the amount of capital available to a company generally, and for inventory in particular, is limited, (McMahon *et al.* 1993). Accordingly, the change in batch sizing and other inventory policy decisions that result from the imposition of a fixed limit on the amount of capital available to invest in inventory are now considered.

In the case of investor-supplied capital, the holding cost in the profit equation is modified since the cost of purchasing inventory does not include an interest component as the money is not borrowed.

$$P=\sum_{i=1}^{n} A_i S_i-\sum_{j=1}^{m} D_j C_j-\sum_{j=1}^{m} \frac{D_j}{Q_j} R_j-\frac{J}{2} \sum_{j=1}^{m} Q_j C_j-F . \tag{4.9}$$

Note that Equation 4.9 is now representative of the case study company's costs under the simplifications described in the previous chapter where it is assumed that inventory is the only capital item: plant and premises are rented and included in F. No inventory of finished products or work in progress is held because products are assembled directly from components to customer order. Following this approach $J=0$ and all warehousing costs are included in F. However, J is retained for completeness and in order to follow the approach taken by other authors (for example, (Trietsch 1995)).

For a company with an amount K to invest in inventory, the profit maximising batch sizes under this model can be determined using Equation 4.9 under the constraint imposed by the inventory investment. It is assumed that replenishment orders for different components occur independently so that the average inventory held for each component is one half of the batch quantity. This is in keeping with the approach taken by previous authors, (Tersine 1988; Trietsch 1995), and is justified in detail in Section 5.1.2. It is further assumed that all of K is invested in inventory: that is, there is no portion of K invested elsewhere (for example, in a bank). This assumption is justified in Section 4.2.3. As the model considers a deterministic case, safety stock has not been included. The constraint is therefore

$$K = \frac{1}{2}\sum_{j=1}^{m} Q_j C_j . \tag{4.10}$$

Using constrained optimisation of P with a Lagrange multiplier λ, yields

$$Q_j = \sqrt{\frac{2D_j R_j}{C_j(J+\lambda)}} . \tag{4.11}$$

Multiplying both sides of 4.6 by $\frac{C_j}{2}$ shows the capital invested in each component to be

$$\frac{Q_j C_j}{2} = \sqrt{\frac{D_j R_j C_j}{2(J+\lambda)}} . \tag{4.12}$$

Summing both sides of 4.7 over j gives

$$K = \sum_{j=1}^{m} \sqrt{\frac{D_j R_j C_j}{2(J+\lambda)}} \tag{4.13}$$

hence

$$\lambda = \frac{\left[\sum_{j=1}^{m}\sqrt{D_j R_j C_j}\right]^2}{2K^2} - J . \tag{4.14}$$

Equation 4.11 shows that the optimal batch sizes maintain the same proportion to each other for any K. This leads to a simple computational method for their evaluation, as shown by Parsons (1966), and Rosenblatt (1981), which is obtained by rewriting Equation 4.14 as

$$\sqrt{\lambda + J} = \frac{\sum_{j=1}^{m}\sqrt{D_j R_j C_j}}{\sqrt{2}K} \tag{4.15}$$

and substituting Equation 4.15 into 4.11 to give

$$Q_j = \frac{2K\sqrt{D_j R_j}}{\sqrt{C_j}\sum_{j=1}^{m}\sqrt{D_j R_j C_j}} . \tag{4.16}$$

4.2.2 Interpretation of the Lagrange multiplier

Equation 4.12 shows that the investment in each component is proportional to $\sqrt{D_j R_j C_j}$. Thus, the form of Equation 4.15 shows that λ can be viewed as a measure of the degree of constraint, as $\sqrt{\lambda + J}$ is the ratio of the invested capital (K) to the profit maximising unconstrained investment. λ can also be interpreted as the marginal return on investment, or the opportunity cost of capital invested in inventory, since it represents the change in income that results from an increase or decrease in investment in inventory. (Although the interpretation of λ as a shadow cost (Hadley and Whitin 1963, Appendix 1) is familiar in constrained optimisation, it is worth emphasising it here because it helps in the interpretation of extra terms in the JIT decision analysis when capital is constrained).

$$
\begin{aligned}
\frac{dP}{dK} &= \frac{d}{dK}\left[\sum_{i=1}^{n} A_i S_i - \sum_{j=1}^{m} D_j C_j - \sum_{j=1}^{m} \frac{D_j}{Q_j} R_j - \frac{J}{2}\sum_{j=1}^{m} Q_j C_j - F\right] \\
&= \frac{d}{dK}\left[-\sum_{j=1}^{m} \frac{D_j}{Q_j} R_j - \frac{J}{2}\sum_{j=1}^{m} Q_j C_j\right] \\
&= \frac{d}{dK}\left[-\left(\sum_{j=1}^{m}\sqrt{D_j R_j C_j}\right)^2 / 2K - JK\right] \qquad (4.17) \\
&= \frac{\left(\sum_{j=1}^{m}\sqrt{D_j R_j C_j}\right)^2}{2K^2} - J = \lambda
\end{aligned}
$$

In fact, the reduced batch sizes under constrained capital, given by Equation 4.11, can be understood through this interpretation of λ. Note that in the batch size equation, $C_j(J+\lambda)$ now replaces the inventory holding cost term $C_j(J+I)$ of the standard EOQ formulation in Equation 4.1. Thus, the optimal batch size in the capital constrained case can be obtained from the formula for the unconstrained case by viewing λ as the *effective* cost of money, and $C_j(J+\lambda)$ as the *effective* inventory holding cost. In other words, the batch sizes given by Equation 4.11 are consistent with the view that the stocks are financed with money borrowed from the firm *itself* at a rate equal to the firm's marginal rate of return: $\frac{dP}{dK} = \lambda$. It is shown in subsequent sections that the view that money invested in component stocks must be valued at the opportunity cost of the firm in constrained inventory policy making can also be applied to the JIT and component commonality decisions, and results in new insights.

4.2.3. Inventory decisions based on investment level

Inventory decision models have typically attempted to optimise the operations of inventory warehouses without consideration of the financial position of the company as a whole. Consequently, approaches to batch sizing such as the EOQ model have determined batch sizes on the implicit assumption that a company can obtain, at the current interest rate, sufficient resources to finance the policy. Thus total inventory investment, $K = \frac{1}{2}\sum_{j=1}^{m} Q_j C_j$, is determined by the batch sizing decision taken by the company. By contrast, when inventory investment is owner-financed, it cannot be assumed that there will always be sufficient resources to finance the inventory required for profit maximisation. Consequently, the capital available for investment must be regarded as an extra decision variable, with batch sizes depending on available capital. For any given investment level choice, the best inventory policy is the one that maximises profit. However, an owner-investor might choose a *given* investment level on criteria other than absolute profit maximisation: the total capital available may be limited and less than that required for maximum profit, or it may be deliberately kept to a low level to improve the rate of return on investment, $ROI = \frac{P}{K}$, despite lower absolute profit levels. ROI is a widely used measure of the performance of an investment. Such lean, high-performing investments could also be part of a wider investment portfolio, or could be chosen to increase operating flexibility in the face of change.

Figure 4.1 shows the variation of profit and ROI as a function of investment level for the case study data. Clearly, the lowest sensible level of investment is determined by the condition that $P > 0$. The following argument establishes the largest sensible investment as that for which $\lambda = \frac{dP}{dK} = I$.

Consider a company with an amount K to invest which is deciding whether it would be sensible to retain any of K as cash and invest it at the interest rate, I. Let K' be the part which is invested in inventory, $\delta K'$ be a contemplated increment in K', and $P(K')$ be the profit earned for investment of K' in inventory. One would invest $\delta K'$ in the company to purchase inventory, rather than as a cash investment, if $P(K' + \delta K') > P(K') + I\delta K'$. But

$$P(K' + \delta K') = P(K') + \delta K' \frac{dP}{dK} \text{ as } \delta K' \to 0 \qquad (4.18)$$

so an investor would put all capital into inventory (for this deterministic case) provided $\frac{dP}{dK} > I$. Thus, the case where $\lambda = I$ defines the largest sensible investment in component stock, because further investment in stock would yield returns lower than the interest rate, and the company could earn more by investing the extra funds in a bank. Interestingly, the batch sizes at this investment level, according to Equation 4.11, are exactly the EOQ batch sizes determined by Equation 4.1, despite the fact that this K does not maximise P in Equation 4.9 (see Figure 4.1). Thus, financing the EOQ inventory policy represents the most conservative investment of capital.

This analysis puts the adoption of inventory policies under the EOQ in an interesting light, not widely articulated in the inventory management literature. Setting the replenishment batch size at the EOQ is indicative of the view that a production manager might hold as it minimises total inventory costs based on an assumed cost of capital at the interest rate but does not consider the consequences of this decision for the company. Furthermore, such a decision is independent of the amount of capital available to finance the company. By contrast, the objective of an investor or company manager is to maximise returns subject to a given level of available investment capital. With the exception of ROI maximising batch sizing models (Schroeder and Krishnan 1976; Morse and Scheiner 1979; Trietsch 1995), the analysis of inventory decisions from an investor's point of view has seldom been undertaken, and the appropriateness of such an approach is an insight gained from first-hand involvement with the case study company. Taking an investor's view, it is argued that all investment levels, ranging from the minimum required for profitable operation (which is also close to the ROI maximising policy) to a maximum set by profit maximisation, could be considered in practice. An investor/manager may consider a reduced investment because of limited access to capital, to minimise an investment in inventory in order to maintain flexibility in the face of change, or in the pursuit of increased ROI.

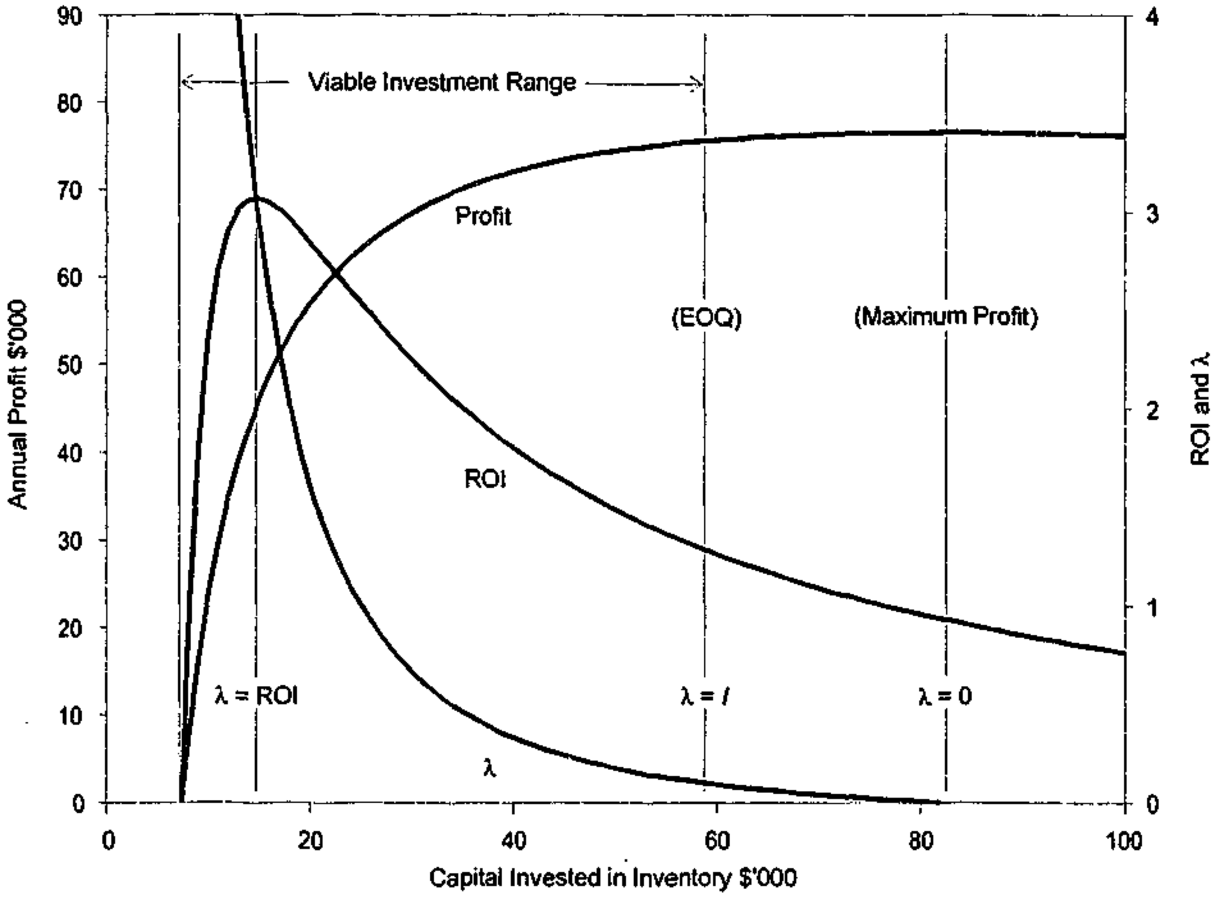

Figure 4.1: Profit, ROI and λ for case study data as a function of capital invested in inventory

In the next section, the decision to adopt JIT replenishment and component substitution is revisited, this time when investment in inventory is constrained. It is shown that adoption of these inventory reduction strategies becomes optimal when capital is limited, due to the high effective value of money invested in inventory, and also improves the relative merit of small investments strategies, judged in terms of ROI.

4.3. JIT replenishment decisions with investor-supplied capital

4.3.1. Just-In-Time replenishment of components

The decision of whether to adopt a JIT policy for Component 1 is again analysed, this time where capital for inventory is owner-supplied and the level of inventory capital is a variable of the decision. The adoption of a JIT policy means that the investment in Component 1 inventory is close to zero but the cost for replenishments of a single unit may be larger due to the cost of reducing the component lead time. The decision to adopt this policy requires $P_{JIT} > P$. P_{JIT} is given by Equation 4.3, without the interest component in the holding cost, and under the inventory investment constraint

$$K = \frac{1}{2}\sum_{j=2}^{m} Q_j C_j \,. \tag{4.19}$$

The constrained optimisation proceeds as before, giving

$$Q_1^{'} = 1 \tag{4.20}$$

and

$$Q_j^{'} = \sqrt{\frac{2D_j R_j}{C_j\left(J + \lambda_{JIT}\right)}} \quad for\ j = 2,3,\ldots,m, \tag{4.21}$$

where

$$\lambda_{JIT} = \frac{\left[\sum_{j=2}^{m}\sqrt{D_j R_j C_j}\right]^2}{2K^2} - J \tag{4.22}$$

The reduced value of λ_{JIT} reflects that the inventory investment is spread over fewer components. Consequently, the batch sizes of the $m-1$ non-JIT components are larger than those when all components are non-JIT for the same inventory investment K, and accordingly, the annual replenishment costs for these components are smaller. This effect introduces a new factor into the decision, namely that, when inventory investment is fixed, elimination of a JIT component from the inventory investment pool results in the remaining components being replenished more efficiently using the liberated capital. This new factor is decisive when the opportunity cost of inventory investment is high, that is, when capital is highly constrained.

To see this more clearly, consider the case where the JIT decision for Component 1 is just viable for an investment of $\tilde{K}$ dollars, that is $P_{JIT}(\tilde{K}) = P(\tilde{K})$. In what follows, $\tilde{K}$ is referred to as the *indifference* investment level for the policy decision. Component 1 currently requires $Q_1C_1/2$ dollars to finance. If the JIT decision is adopted, this money can be reinvested in the firm at a marginal return rate of $\lambda_{JIT}(K)$ to bring the total capitalisation back to $\tilde{K}$ dollars. These returns accrue due to the relief of the constraint on the remaining components and have magnitude

$$\begin{aligned}
\int_{\tilde{K}-\frac{Q_1C_1}{2}}^{\tilde{K}} \lambda_{JIT}\, dK &= \int_{\tilde{K}-\frac{Q_1C_1}{2}}^{\tilde{K}} \frac{dP_{JIT}(K)}{dK} dK \\
&= P_{JIT}(\tilde{K}) - P_{JIT}\left(\tilde{K} - \frac{Q_1C_1}{2}\right) \\
&= P(\tilde{K}) - P_{JIT}\left(\tilde{K} - \frac{Q_1C_1}{2}\right) \\
&= R_1'D_1 - \frac{R_1D_1}{Q_1} - \frac{J}{2}Q_1C_1
\end{aligned} \tag{4.23}$$

where the last step follows from the fact that the batch size for the non-JIT components at the lower terminal of integration is the same as the batch size of the same components prior to eliminating Component 1 from the inventory investment pool. That is,

$Q_j^{'}\left(\widetilde{K}-\frac{Q_1C_1}{2}\right)=Q_j\left(\widetilde{K}\right)$ *for* $j=2,3,...,m$. In order that $P_{JIT}\left(\widetilde{K}\right)>P\left(\widetilde{K}\right)$, the decision to adopt a JIT policy for Component 1 now requires that

$$R_1^{'}D_1-\frac{R_1D_1}{Q_1}<\frac{J}{2}Q_1C_1+\int_{\widetilde{K}-\frac{Q_1C_1}{2}}^{\widetilde{K}}\lambda_{JIT}dK \tag{4.24}$$

Comparing Equation 4.24 with Equation 4.4, it can be seen that a company will adopt a JIT policy for a component when the increase in replenishment costs is justified by the saving in holding costs for that component *plus the returns from reinvesting into the firm the capital formerly invested in inventory of that component, to improve the efficiency of replenishment of the remaining components.* As the capital available to the firm is reduced, the increase in λ_{JIT} causes the last factor to become decisive and leads to the viability of JIT for certain components *even without any attempt to reduce the JIT replenishment cost* $R_1^{'}$. An additional contributing factor is that the difference in the JIT and non-JIT replenishment costs is smaller than that for the EOQ because the constrained batch sizes are now smaller. Note that the integral term in Equation 4.24 is approximately $\frac{Q_1C_1}{2}\lambda_{JIT}$ and thus Equation 4.24 has approximately the same form as Equation 4.4 for the borrowed capital case, but with I replaced by λ_{JIT}, further justifying the interpretation of λ as the cost of capital.

While Equation 4.24 shows clearly how the JIT decision is affected by the capital constraint compared to the case where capital is borrowed, it is not so convenient for deciding which components should be replenished JIT and at what levels of inventory investment. This can be accomplished more easily by determining the indifference level of investment $\widetilde{K}$, where $P\left(\widetilde{K}\right)=P_{JIT}\left(\widetilde{K}\right)$. Equating Equation 4.9 and Equation 4.3 without the interest component for borrowed capital gives, after the elimination of common terms,

$$\sum_{j=1}^{m} \frac{D_j}{Q_j} R_j + \frac{J}{2} \sum_{j=1}^{m} Q_j C_j = \sum_{j=2}^{m} \frac{D_j}{Q_j^{'}} R_j + \frac{J}{2} \sum_{j=2}^{m} Q_j^{'} C_j + D_1 R_1^{'} . \tag{4.25}$$

However, $\sum_{j=1}^{m} Q_j C_j = \sum_{j=2}^{m} Q_j^{'} C_j$, thus

$$\sum_{j=1}^{m} \frac{D_j}{Q_j} R_j = \sum_{j=2}^{m} \frac{D_j}{Q_j^{'}} R_j + D_1 R_1^{'} . \tag{4.26}$$

Substituting Q_j with Equation 4.16, and $Q_j^{'}$ by its equivalent expression gives

$$\frac{\left(\sum_{j=1}^{m} \sqrt{D_j R_j C_j} \right)^2}{2K} = \frac{\left(\sum_{j=2}^{m} \sqrt{D_j R_j C_j} \right)^2}{2K} + D_1 R_1^{'} , \tag{4.27}$$

hence, the point of indifference is given by

$$\widetilde{K} = \frac{1}{2 D_1 R_1^{'}} \left(\left(\sum_{j=1}^{m} \sqrt{D_j R_j C_j} \right)^2 - \left(\sum_{j=2}^{m} \sqrt{D_j R_j C_j} \right)^2 \right) . \tag{4.28}$$

Rewriting $\sum_{j=1}^{m} \sqrt{D_j R_j C_j}$ as $\sqrt{D_1 R_1 C_1} + \sum_{j=2}^{m} \sqrt{D_j R_j C_j}$, allows Equation 4.28 to be rewritten as

$$\widetilde{K} = \frac{1}{2 D_1 R_1^{'}} \left(D_1 R_1 C_1 + 2 \sqrt{D_1 R_1 C_1} \sum_{j=2}^{m} \sqrt{D_j R_j C_j} \right) , \tag{4.29}$$

Thus

$$\widetilde{K} = \frac{1}{R_1^{'}} \left(\frac{R_1 C_1}{2} + \frac{\sqrt{R_1 C_1}}{\sqrt{D_1}} \sum_{j=2}^{m} \sqrt{D_j R_j C_j} \right) . \tag{4.30}$$

When the inventory consists of a large number of components having a similar investment in each, that is $\sqrt{D_j R_j C_j}$ is similar for each component, Equation 4.30 can be approximated by

$$\widetilde{K} \approx \frac{\sqrt{R_1 C_1}}{R_1' \sqrt{D_1}} \sum_{j=2}^{m} \sqrt{D_j R_j C_j} \tag{4.31}$$

as the ratio $\frac{R_1 C_1}{2R_1'}$ only accounts for a small proportion of the value of $\widetilde{K}$.

Although Equation 4.29 shows how $\widetilde{K}$ may be determined exactly, Equation 4.31 shows that as a general principle, JIT replenishment is favourable at large investment levels for components with low demand or low JIT replenishment cost and also for components with a large contribution to total investment through having a large unit cost and/or replenishment cost. For a company considering the order in which to adopt JIT replenishment for components, a ranking from highest to lowest $\widetilde{K}$, evaluated using Equation 4.29, would provide a basis for decisions (Betts and Johnston 1999), as will be illustrated with the case study. The ratio $\frac{\sqrt{R_1 C_1}}{R_1' \sqrt{D_1}}$ in Equation 4.31 also has the interpretation as the proportional investment in each component divided by the cost of JIT replenishment. This ratio also provides a good approximate means of ranking candidate components for JIT replenishment as will also be shown in the following case study.

4.3.2. Component substitution

An alternative approach is to substitute components. Again, the opportunity value of the capital released from inventory needs to be considered in the decision. In the case where Component 2 is substituted for Component 1, substitution requires $P_{SUB} > P$. P_{SUB} is given by Equation 4.5, again without the interest component in holding cost and under the constraint

$$K = \frac{1}{2}\left(Q_2'C_2 + \sum_{j=3}^{m} Q_j'C_j\right). \tag{4.32}$$

Constrained optimisation yields

$$Q_1' = 0, \tag{4.33}$$

$$Q_2' = \sqrt{\frac{2(D_1 + D_2)R_2}{C_2(J + \lambda_{SUB})}} \tag{4.34}$$

and

$$Q_j' = \sqrt{\frac{2D_jR_j}{C_j(J + \lambda_{SUB})}} \quad for\ j = 3,4,\ldots,m, \tag{4.35}$$

where

$$\lambda_{SUB} = \frac{\left[\sum_{j=3}^{m}\sqrt{D_jR_jC_j} + \sqrt{(D_1 + D_2)R_2C_2}\right]^2}{2K^2} - J. \tag{4.36}$$

A similar argument to that given for the JIT case gives the decision criterion as

$$\begin{aligned} D_1C_2 - D_1C_1 + \frac{(D_1 + D_2)}{Q_2'}R_2 - \frac{D_1}{Q_1}R_1 - \frac{D_2}{Q_2}R_2 \\ < \frac{J}{2}\left(Q_2'C_2 - Q_1C_1 - Q_2C_2\right) + \int_{K' - \frac{Q_2'C_2 - Q_1C_1 - Q_2C_2}{2}}^{K'} \lambda_{SUB}\,dK. \end{aligned} \tag{4.37}$$

Again, the final term is the result of reinvesting in the firm the freed inventory capital from Component 1. Note that when component substitution is adopted, the increased demand for the substitute results in a greater optimal batch size for this component (see Equation 4.31). This both increases the investment in the substitute component and reduces the amount of capital that would be reinvested in the remaining batch-replenished components were JIT replenishment to be adopted (see Equations 4.24 and 4.34). Thus, the substitution of a component may reduce the benefit resulting from the reinvested capital compared to JIT replenishment for the same component. Consequently, component substitution is most effective when the cost increase of substitution is relatively small, or under decreasing investment, when the opportunity cost of capital is greater. It will be shown in subsequent chapters that substitution is generally an inferior approach to inventory reduction.

As with the JIT decision, the indifference level of investment $\widetilde{K}$, for a component substitution, is determined by evaluating $P_{SUB}(\widetilde{K}) = P(\widetilde{K})$, which gives

$$\widetilde{K} = \frac{1}{2D_1(C_2 - C_1)}\left[\left(\sum_{j=1}^{m}\sqrt{D_jR_jC_j}\right)^2 - \left(\sum_{j=3}^{m}\sqrt{D_jR_jC_j} + \sqrt{(D_1 + D_2)R_2C_2}\right)^2\right]. \quad (4.38)$$

Rewriting Equation 4.38 as

$$\widetilde{K} = \frac{1}{2D_1(C_2 - C_1)}\left[\left(\sum_{j=3}^{m}\sqrt{D_jR_jC_j} + \sum_{j=1}^{2}\sqrt{D_jR_jC_j}\right)^2 - \left(\sum_{j=3}^{m}\sqrt{D_jR_jC_j} + \sqrt{(D_1 + D_2)R_2C_2}\right)^2\right] \quad (4.39)$$

and multiplying out the squared terms gives

$$\begin{aligned}\widetilde{K} = \frac{1}{2D_1(C_2 - C_1)}\Bigg[&2\sum_{j=3}^{m}\sqrt{D_jR_jC_j}\left(\sum_{j=1}^{2}\sqrt{D_jR_jC_j} - \sqrt{(D_1 + D_2)R_2C_2}\right) \\ &+ D_1(R_1C_1 - R_2C_2) + 2\sqrt{D_1R_1C_1}\sqrt{D_2R_2C_2}\Bigg].\end{aligned} \quad (4.40)$$

When inventory consists of a large number of components having a similar investment in each, the terms multiplied by $\sum_{j=3}^{m}\sqrt{D_j R_j C_j}$ are significant in Equation 4.40, which permits the approximation

$$\tilde{K} \approx \frac{\sqrt{D_1 R_1 C_1} + \sqrt{D_2 R_2 C_2} - \sqrt{(D_1 + D_2) R_2 C_2}}{D_1 (C_2 - C_1)} \sum_{j=3}^{m} \sqrt{D_j R_j C_j} . \tag{4.41}$$

Equation 4.41 shows that component substitution is profit maximising at greater investments when the increase in component costs is low. This occurs when the demand for the original component (D_1) is low or the unit cost increase due to the substitute, $(C_2 - C_1)$, is small. The terms in the numerator of Equation 4.41 show the change in the proportional investment in components resulting from the substitution, that is, the original investment in Components 1 and 2, less the investment in Component 2 after substitution is made. These terms show that substitution is effective at increased investments when the substitute results in the greatest reduction in the proportional investment in components. Thus, suitable candidate components for substitution are those requiring a large investment in inventory due to high replenishment and/or unit cost, but having low demand and/or a reasonably inexpensive substitute available. Radiator 1 is such a component, and it will be shown in the case study example following that the substitution of this component with Radiator 2 is the substitute that is cost effective at the greatest investment level. Following the same approach as for the JIT replenishment decision, the ratio $\frac{\sqrt{D_1 R_1 C_1} + \sqrt{D_2 R_2 C_2} - \sqrt{(D_1 + D_2) R_2 C_2}}{D_1 (C_2 - C_1)}$ could form the basis for ranking substitution alternatives.

Comparing equations 4.31 and 4.41, it is evident, that each of the ratios, $\frac{\sqrt{R_1C_1}}{R_1'\sqrt{D_1}}$ and

$\frac{\sqrt{D_1R_1C_1}+\sqrt{D_2R_2C_2}-\sqrt{(D_1+D_2)R_2C_2}}{D_1(C_2-C_1)}$ have a similar derivation. Under the assumption that there are many components in inventory in order that the difference between $\sum_{j=2}^{m}\sqrt{D_jR_jC_j}$ and $\sum_{j=3}^{m}\sqrt{D_jR_jC_j}$ is small, these ratios can be treated as having a similar scale. In the case study example, in Table 4.1, these ratios are presented together for the case data, and the high level of consistency between them is evident. Both ratios have the same interpretation, as each represents the change in the proportional investment in inventory due to the replenishment policy change divided by the cost of the change. This observation further justifies the use of these ratios for a joint ranking of replenishment policy change alternatives.

The use of JIT replenishment policies or component substitution has the potential to reduce the amount of investment in inventory, which may be desirable when capital is limited or when the minimum investment in inventory is sought for other reasons such as operating flexibility or portfolio management. This section has shown that for either method of inventory reduction, there is a point of indifference where both the original and modified replenishment policies offer the same profit. Below this point, profit is increased for either inventory reduction policy. For an investor evaluating the return of the company operating with a reduced investment, the increase in profit carries two beneficial consequences. The company remains profitable at lower investment levels and, since the company has increased profit, the ROI is greater under these leaner policies. This is evident in Figure 4.2, which shows a comparison of the Profit and ROI for the case study company using the optimal JIT replenishment and component substitution policies across the range of investment levels, as described in Table 4.1, with the Profit and ROI obtained under the original (batched) replenishment policy.

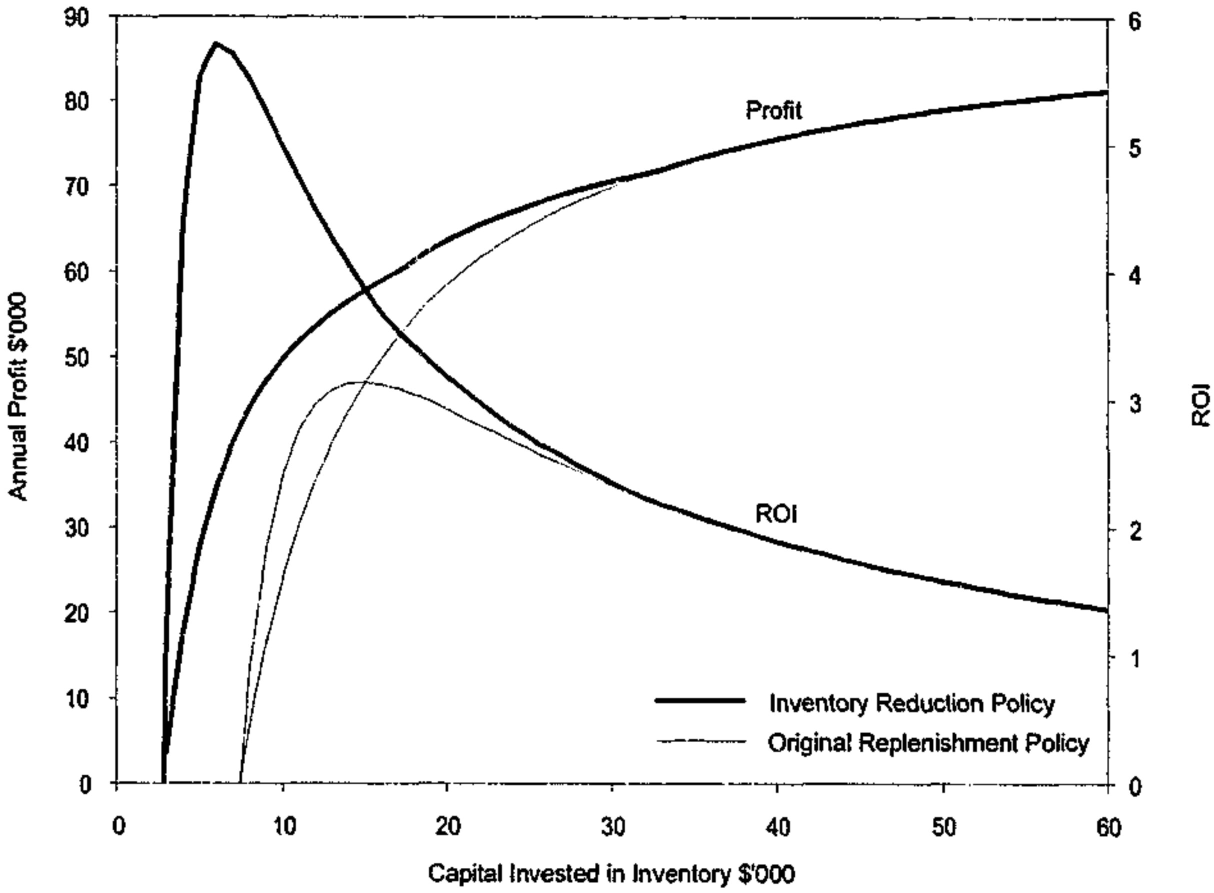

Figure 4.2: Profit and ROI for the original policy and with the adoption of the optimal inventory reduction policies at each investment level as given in Table 4.1

4.4. Case study example

4.4.1. Calculation methods

In order to observe the effect of changing the amount of capital available for inventory investment, a model of the company's profits based on Equations 4.3, 4.5 and 4.9 under constraint was constructed. The model included the JIT and component substitution options listed in Table 3.2. Two methods were used to calculate the profit of policy combinations in the analysis that follows. Firstly, the company's profit for invested capital, ranging from $1,000 to $100,000, was evaluated for all policy combinations at intervals of $1,000. The batch sizes, profit, ROI and replenishment policy for the most profitable policy combination at each investment level was calculated by full enumeration. As this method was computationally intensive, particularly for determining the exact investment levels at which policy transitions occurred, an alternative method, based on the indifference values of policy changes was also used. This was an iterative scheme, as described in Figure 4.3.

Repeat steps 2 to 4 until all policy changes have been accepted	1	Start with no policy changes.
	2	Evaluate the indifference investment level, $\widetilde{K}$, for each of the proposed policy changes using Equations 4.28 or 4.38.
	3	Accept the policy having the greatest $\widetilde{K}$.
	4	Update terms in Equations 4.28 or 4.38 to reflect cost changes due to accepted policy

Figure 4.3: Iterative scheme for calculating replenishment policy changes

The sequence of changes determined by the iterative scheme accords with the policy changes resulting from the evaluation of all possible policy combinations for the case study data. It should be noted however, that this method is not completely equivalent to the full enumeration method if one wishes to find the most profitable policy combination at a particular inventory investment level, because it does not recognise that certain other policy decisions might be more profitable when adopted if certain policy decisions made at a higher investment levels were to be reversed. An exhaustive analysis demonstrated that this situation did not occur with the present data so the iterative method was used to obtain more precise values for the following analysis.

4.4.2. Results and discussion

Table 4.1 shows the indifference investment level for all policy changes calculated by the iterative method and sorted from highest to lowest. The type of policy change and the profit at each level are also shown. The cost of each policy change is also given, for example, in the case of the decision to replenish Screw 1 JIT, the total annual cost of individually replenishing each screw is $7,800. In order to illustrate the following discussion, $\sqrt{D_j R_j C_j}$, which is proportional to the investment in each component has been calculated. The ratios describing the change in the proportional investment in components due to each policy change as a proportion of the cost of the policy change are also given. These ratios are $\frac{\sqrt{R_1 C_1}}{R_1' \sqrt{D_1}}$ and $\frac{\sqrt{D_1 R_1 C_1} + \sqrt{D_2 R_2 C_2} - \sqrt{(D_1 + D_2) R_2 C_2}}{D_1 (C_2 - C_1)}$ for the JIT and component substitution cases respectively. With this table it is possible to determine the policy changes that would be worth considering for adoption by the company for any given capital available. The effect of the policy changes is cumulative, and so, for example, the substitution of Valve 1 with Valve 2 becomes cost effective at $12,808 and assumes that the policy changes for Screw 1, Radiator 1 and Screw 2 have taken place.

Table 4.1: Indifference values of investment for policy changes using the iterative solution method showing the type and cost of change, proportional investment in component, and profit evaluated at the indifference value

Policy Groups	Policy Transition	Component	Substitute	$\sqrt{D_j R_j C_j}$	Cost of Policy Change ($)	$\frac{\sqrt{R_j C_j}}{R_j^{'} \sqrt{D_j}}$	$\frac{\sum_{j=1}^{2}\sqrt{D_j R_j C_j} - \sqrt{(D_1+D_2)R_2 C_2}}{D_1(C_1-C_2)}$	$\tilde{K}$ ($)	Type of Change	Profit ($)
Always Just-In-Time Reordering	1	Motor 1	-	0	-	-	-	82,272	JIT	84,862
	2	Motor 2	-	0	-	-	-	82,272	JIT	84,862
	3	Motor 3	-	0	-	-	-	82,272	JIT	84,862
	4	Motor 4	-	0	-	-	-	82,272	JIT	84,862
	5	Motor 5	-	0	-	-	-	82,272	JIT	84,862
	6	Motor 6	-	0	-	-	-	82,272	JIT	84,862
	7	Motor 7	-	0	-	-	-	82,272	JIT	84,862
	8	Wiring 1	-	0	-	-	-	82,272	JIT	84,862
	9	Wiring 2	-	0	-	-	-	82,272	JIT	84,862
Policy Based on Investment Level	10	Screw 1	-	7,649	7,800	0.98	-	32,548	JIT	71,731
	11	Radiator 1	Radiator 2	1,449	1,000	-	0.84	24,309	Substitute	67,240
	12	Screw 2	-	5,745	8,800	0.65	-	16,281	JIT	59,238
	13	Valve 1	Valve 2	2,513	2,380	-	0.56	12,808	Substitute	54,915
	14	Cabinet 1	-	316	1,000	0.32	-	6,168	JIT	35,644
	15	Piping 2	-	420	1,410	0.30	-	5,697	JIT	32,772
	16	Cabinet 3	-	594	2,350	0.25	-	4,706	JIT	25,074
	17	Cabinet 2	-	406	1,650	0.25	-	4,463	JIT	22,735
	18	Piping 1	Piping 2	345	2,650	-	0.06	3,855	JIT	15,786
	19	Air Tank 2	-	2,292	12,220	0.19	-	3,083	JIT	4,722
Policy Never Adopted	20	Radiator 3	Radiator 4	2,592	4,800	-	0.16	2,434	Substitute	-7,570
	21	Chassis	-	474	3,000	0.16	-	1,970	JIT	-18,990
	22	Radiator 4	-	2,348	15,750	0.15	-	1,647	JIT	-29,516
	23	Air Tank 1	Air Tank 2	1,050	13,780	-	0.00	712	JIT	-98,423
	24	Radiator 2	-	2,280	34,650	0.07	-	506	JIT	-133,798
	25	Valve 2	-	6,544	109,800	0.06	-	195	JIT	-228,753

Based on the indifference value and associated profit, three groups of component type can be identified. Firstly, there are components which would always be replenished JIT, regardless of the amount of capital available for investment. Secondly, there are components whose replenishment policy is determined by the amount of capital available for investment. Finally, there are components which would never be considered for JIT replenishment or substitution, without setup cost reduction. The following discussion looks at each of these component groups in turn.

The components that are always ordered JIT are the motors and wiring which are replenished at a negligible cost. Hence, a JIT policy is feasible at any investment level. This is why the company can maintain a large range of motors, and why component rationalisation does not arise. For these components, the investment level at the EOQ, and the corresponding profit, are listed in Table 4.1.

Figure 4.4 shows the batch sizes for a number of key components whose policy changes are dependent on the amount of invested capital. The graph shows that the proportion of available capital invested in each component remains constant as capital varies. This is because the investment in all non-JIT components is proportional to $\sqrt{D_j R_j C_j}$, while that for JIT components is 0. Each policy change is maintained for all levels of investment below its indifference level $\widetilde{K}$. For example, in the case of Screw 1 or Screw 2, there is no reversion to batch-replenishment at investments below the indifference point for the adoption of JIT replenishment.

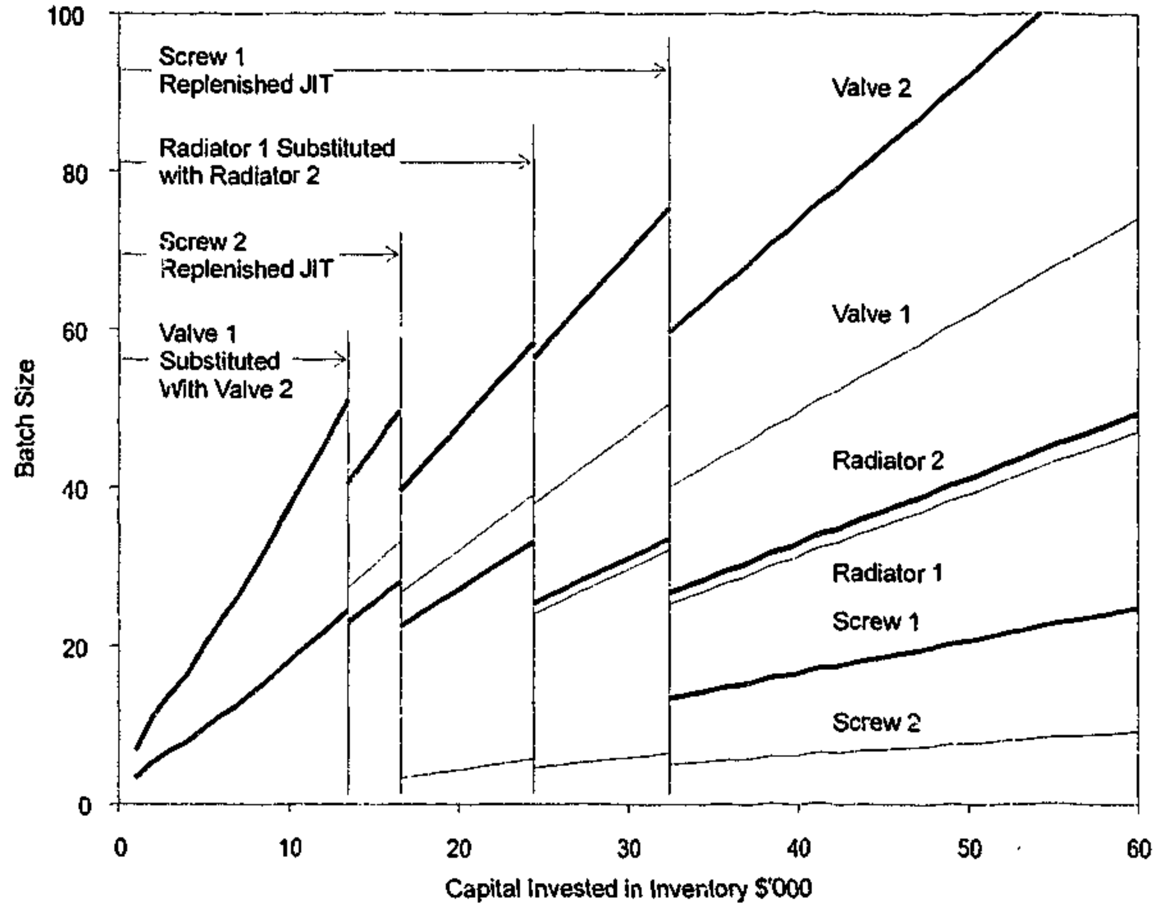

Figure 4.4: Change in replenishment batch sizes accompanying policy changes for several components evaluated by full enumeration as a function of investment level

The policy change that becomes viable at the greatest inventory investment is the JIT replenishment of Screw 1. Its indifference level is calculated with Equation 4.28 to be $32,548. The effect of relaxing the constraint on the remaining non-JIT components is evident in Figure 4.4. When this policy is adopted for Screw 1 the batch sizes for the remaining non-JIT components are now the same as they were under the non-JIT policy at an investment of about $42,000. The consequent reduction in the replenishment cost for the non-JIT components justifies the increased replenishment cost for Screw 1.

The benefit of liberating inventory capital is seen clearly when the cost components in both the constrained and unconstrained cases, based on Equations 4.24 and 4.4 respectively, are articulated at the indifference point $\widetilde{K}$, as shown in Table 4.2. It should be noted that, for the case study, $J = 0$ was used although the results generalise for a non-zero J. Note also, that Table 4.2 presents non-integer values of Q_1 obtained at an investment of $\widetilde{K}$ to illustrate the theoretical discussion. For determining replenishment quantities in a practical case, Q_1 must be integer-valued, obtained by rounding $\widetilde{K}$ to the nearest value of K that permits integer replenishment quantities of all components.

Table 4.2: Comparison of costs for the decision to replenish Screw 1 JIT for the constrained and unconstrained case. The unconstrained case uses the EOQ.

Expression	Unconstrained	Constrained
$\widetilde{K}$	-	\$32,548
Q_1	34.2	13.5
$R_1^{'}D_1$	\$7,800	\$7,800
$\frac{R_1D_1}{Q_1}$	\$1,710	\$4,350
$\frac{J}{2}Q_1C_1$	\$0	\$0
$\frac{I}{2}Q_1C_1$	\$1,710	-
$\frac{1}{2}Q_1C_1$	\$17,100	\$6,725
$\int_{\widetilde{K}-\frac{Q_1C_1}{2}}^{\widetilde{K}}\lambda_{JIT}dK$	-	\$3,450
$P_{JIT}-P$	-\$4,379	\$0

In the unconstrained case, inventory costs are independent of invested capital and Q_1 is the EOQ. In this case a decision to change policy would be unprofitable since $R_1^{'}D_1-\frac{R_1D_1}{Q_1}>\frac{(J+I)}{2}Q_1C_1$. Indeed, a policy change under these conditions would yield a loss of \$4,379 *per annum*. For the constrained case, it can be seen that Q_1 is smaller than the EOQ, with a consequently increased replenishment cost. The batch replenishment cost of Component

1 in the constrained case is $2,640 *per annum* greater than in the unconstrained case. However, the elimination of Screw 1 from inventory permits the redistribution of $6,725 across the remaining components in order to increase the batch sizes of replenishments, resulting in an annual saving of $3,450 in the replenishment cost of these components. It is thus the value of the redistributed capital that is the decisive factor when considering the policy changeover. Furthermore, as K decreases, the benefit arising from the redistribution of inventory increases, hence for $K < \widetilde{K}$, the inequality in Equation 4.24 continues to hold and the JIT replenishment policy prevails.

The next policy change to occur is the substitution of Radiator 2 for Radiator 1. Evaluating Equation 4.38 with the JIT replenishment of Screw 1 included gives an indifference investment level of $24,309. The increase in batch size for Radiator 2 reflects the increased demand for this component. The increase in batch size for the remaining components, resulting from the redistribution of the investment in Radiator 1, is evident. The next two policy changes, JIT replenishment of Screw 2 and the substitution of Valve 1 with Valve 2, can also be seen in Figure 4.4.

In Table 4.1, the policy changes that are profit-maximising at the greatest investment levels are those changes for which the ratios $\frac{\sqrt{R_1 C_1}}{R_1' \sqrt{D_1}}$ and $\frac{\sqrt{D_1 R_1 C_1} + \sqrt{D_2 R_2 C_2} - \sqrt{(D_1 + D_2) R_2 C_2}}{D_1 (C_2 - C_1)}$ are greatest. Thus components that are suitable for JIT replenishment are those with a high contribution to inventory investment value, indicated by $\sqrt{D_j R_j C_j}$, such as Screw 1 and Screw 2. The substitution of Radiator 1 with Radiator 2 and Valve 1 with Valve 2 are also profit-maximising at large investments because the cost increase due to the substitutions are small enough to be offset by the release of the investment in these components. The next group of components to be considered for JIT replenishment are the cabinets and piping. Although these represent small contributions to the total inventory, the cost of changing policy is sufficiently small to make these policy changes attractive when investment is sufficiently reduced.

The last group of policy changes to consider are those which would never be implemented. These are numbered as policy changes 20 to 25 in Table 4.1. These would be unacceptable as the indifference point for each occurs at investment levels that are below the threshold of profitability. Components in this category include Radiators 2, 3, and 4, and Valve 2. For these components, the cost of JIT replenishment is too great to offset the benefit from constraint reduction through policy change. Replenishing these components always requires economies of scale, and they could only be brought into a JIT regime after operating or manufacturing changes that reduced their JIT replenishment costs. The substitution of Radiator 3 by Radiator 4 and Air Tank 1 by Air Tank 2 are both unacceptable as the cost of substitution outweighs the benefits from eliminating these components from inventory at investment levels at which the company is profitable. In the case of the radiators, the cost of substituting Radiator 3 by Radiator 4 is large because of the relatively large annual demand for Radiator 3 (32 per annum) and because of the large unit cost increase ($150 per item). By contrast, the substitution of Radiator 2 by Radiator 1 was profit maximising at a relatively large investment because of the smaller cost increase due to this substitution as a result of the lower demand (20 per annum) and smaller unit cost increase ($50) resulting from the substitution.

It is evident in Table 4.1 that the ranking of policy changes by the ratios $\frac{\sqrt{R_1C_1}}{R_1'\sqrt{D_1}}$ and

$\frac{\sqrt{D_1R_1C_1}+\sqrt{D_2R_2C_2}-\sqrt{(D_1+D_2)R_2C_2}}{D_1(C_2-C_1)}$ ratios accords with the actual sequence of policy change adoption in all cases except for the substitution of Piping 1 with Piping 2. For these components, the very small proportional investment in these components $\left(\sqrt{D_jR_jC_j}\right)$ makes the indifference investment highly dependent on the actual pool of remaining components after the higher-ranked policy changes have occurred. Thus, these components are substituted at a higher indifference investment (rank) than the ratio $\frac{\sqrt{D_1R_1C_1}+\sqrt{D_2R_2C_2}-\sqrt{(D_1+D_2)R_2C_2}}{D_1(C_2-C_1)}$ indicates. Although it appears that these ratios compare favourably against the exhaustive method of ranking the sequence of policy changes for the case study data, their accuracy for more general cases has not been confirmed, and remains a matter for subsequent investigation.

The effect of implementing the policy changes is now illustrated by comparing the company operating under its original policy with it after the adoption of the policy changes described in Table 4.1. Figure 4.5 shows profit and λ for the company operating under both scenarios as the amount of investment in inventory is varied. A comparison of the profit curves shows that the successive application of JIT and substitution strategies yields greater profits than the original policy for inventory investments less than \$32,548 when JIT replenishment first becomes viable. In addition to increasing profitability at lower investment levels, adopting these inventory reduction strategies lowers the minimum investment for which the company's operations become profitable from approximately \$8,000 to \$3,000. As a consequence of both these factors, the ROI for the company operating under JIT policies is greatly increased, (see Figure 4.2), with a maximum nearly double that of the unmodified policy. Even with an increased unit or replenishment cost, adoption of these policies would be very appealing to an investor seeking to maximise ROI.

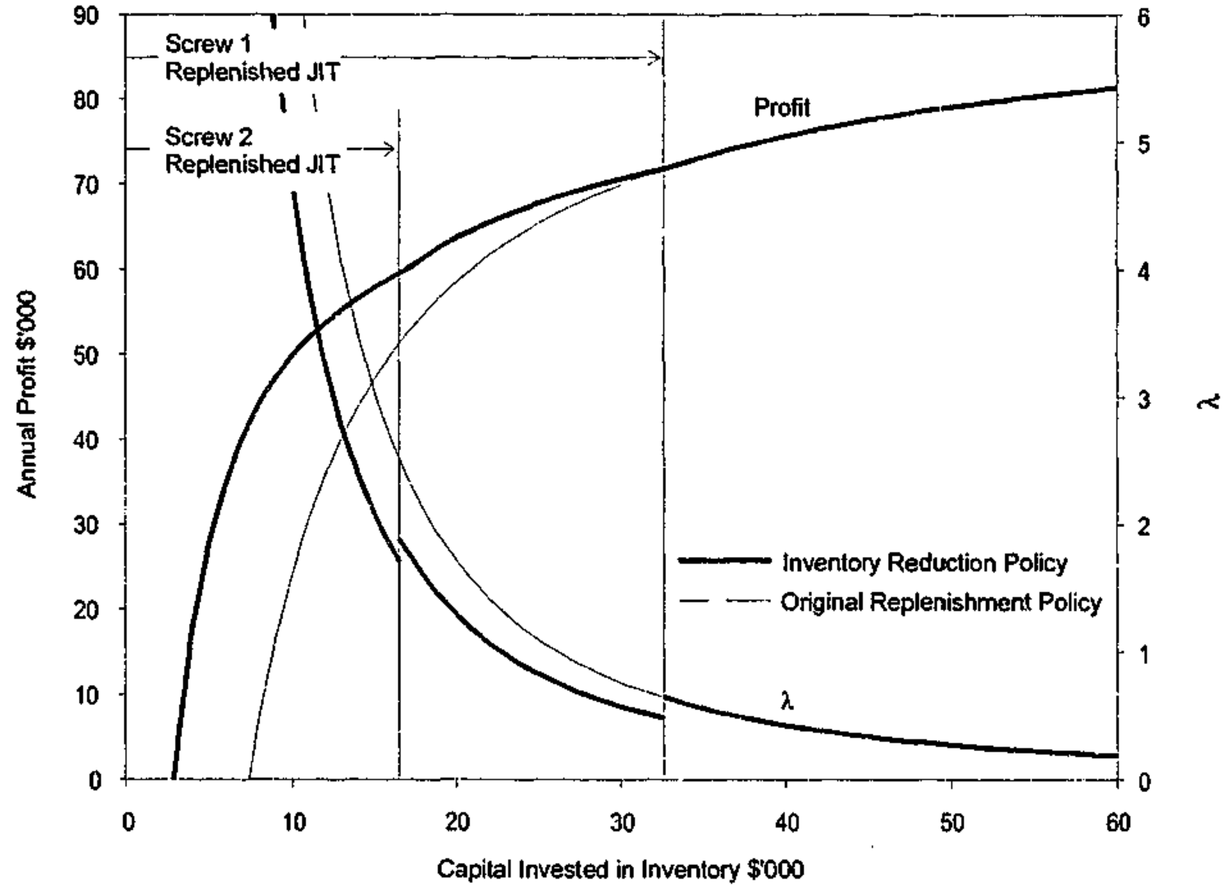

Figure 4.5: Profit and λ for the original policy and after adopting inventory reducing policies

A comparison of λ under both the fixed and modified policies illustrates the mechanism by which JIT policies work to increase profitability at reduced investment. The policy changes reduce λ, which is evident at $32,000 and $16,000 where the effect of the JIT replenishment of the screws has a large effect. The effect of substitutions is too small to be seen on the graph. Since $\lambda = \frac{dP}{dK}$, a lower λ indicates a smaller increase in profit for a given increase in investment. For a company seeking to increase its investment for profit maximisation, the larger original λ value is beneficial, but conversely, for a company seeking to reduce its investment, the smaller λ values lead to a reduced loss in profit for a given decrease in investment, and higher values of ROI. Thus, the adoption of these inventory reduction strategies decreases the sensitivity of the company's profit to decreases in operating capital.

4.5. Conclusion

This chapter has shown that JIT replenishment can be justified by a traditional 'cost tradeoff' approach, even in the absence of large setup cost reduction, when the finiteness of available capital for inventory is considered. The key idea is that money invested in inventory has a greater value when investment is constrained because it can potentially be reinvested in the company at its marginal rate of return. These returns take the form of reduced replenishment costs for the remaining non-JIT components due to the ability to finance the inventory resulting from larger batches of these. The substitution of a component with an over-specification alternative also permits the investment formerly in the eliminated component to be reinvested into the remaining components to reduce the replenishment costs of the remaining batch-replenished components. When the effect of reinvested capital is included in the decision analysis, inventory reduction initiatives become increasingly attractive as investment is more tightly constrained.

This chapter has shown how JIT replenishment and component substitution can be selectively employed to determine the profit-maximising replenishment policy at any feasible investment level. Figure 4.5 shows that the application of inventory reduction strategies to the case study company results in increased profit at reduced investments compared with that obtained under their original replenishment policy. Furthermore, by also reducing the minimum feasible investment for the case study company, the adoption of JIT replenishment and component substitution produced a greater ROI than could previously be contemplated by the company.

The model employed to determine the optimal replenishment quantities for a given investment, and to determine the investment levels below which JIT replenishment and component substitution are profit maximising has assumed that components are replenished independently. Thus, the current analysis does not consider the case where the replenishment of components is managed in a way that reduces the occurrences of peaks in inventory such as the cyclic and non-stationary approaches described in Chapter 2. However, both the adoption of staggered replenishments or non-stationary ordering approaches reduce the degree of inventory constraint for a given investment. Consequently, under these approaches the value of capital redistributed at each replenishment policy change is diminished. This has the likely effect of reducing the indifference investment level $\left(\widetilde{K}\right)$ for which each of the policy changes proposed in this chapter are profit-maximising. However, a comparison of the inventory costs under each of these approaches in Chapter 2 showed that the differences between them were small, indicating that the change in the degree of constraint was small. Thus, it is unlikely that the conclusions of this chapter would be changed significantly by these alternative approaches to inventory management.

Concerning the generality of these conclusions, the model has employed a simple manufacturing scenario, which assumes that finished products are manufactured from a single level bill-of-materials with no work in progress or finished goods inventory. The analysis concerns the setting of replenishment batch sizes for purchased components with a view to controlling the investment in their inventory. However, the insight gained from this analysis, namely, that money tied up in inventory may be reinvested in the company at its marginal rate of return, can be applied to work-in-progress or finished goods inventories in order to increase profit or ROI when investment is constrained. That some items reside in partly or completely assembled products does not affect the total investment in inventory. The decision of whether to assemble some of these items into stocked components affects the company's direct labour cost, which is an operating cost of the business, and assumed to be independent of replenishment policy under the current model. Thus, to some extent, inventory investment decisions are independent of assembly decisions. However, assembling components or finished products to stock does introduce additional interactions between batch sizing decisions for bought items and work-in-progress inventory that has not been investigated. In the present analysis all interactions between bought parts are taken into account in the value of λ. Therefore, the present analysis is only rigorous and tractable under the assumptions made here. Despite these limitations, the model in dealing with the inventory investment aspect of manufacturing yields indicative insights useful for more complex manufacturing scenarios.

Chapter 5.

Inventory Replenishment Policy Decisions under Stochastic Demand

The analysis of the previous chapter is now extended to the case where the customer orders have stochastic arrival times. The model introduced in this chapter is a good representation of the case study company whose data is used to illustrate the analysis. The model also captures many features relevant to JIT policy setting in general, particularly in regard to control of inventory investment levels, while still being tractable. It is shown that this model can be transformed into a constrained multi-item (Q,r) inventory model by attributing to each component a stock-out cost penalty which, when summed, reproduces the expected cost of lost sales of finished products to a good approximation. This allows optimal values of replenishment quantity and reorder point to be determined, using an iterative scheme, for each component for any total investment level assuming batch replenishment. For each possible combination of batch replenishment, JIT replenishment or component substitution, the cost-minimising replenishment quantity and reorder point can be calculated for the non-JIT components and the optimum policy combination determined for a given investment level.

Despite much work on the effect of JIT on the performance of production systems, (Groenevelt 1993), particularly on Kanban systems, little analysis of the economic justification of JIT replenishment for inventory systems under stochastic demand has been undertaken (Schniederjans and Olson 1999). Additionally, studies describing successful JIT implementations tend to be empirical and factory specific (Gunasekaran *et al.* 1993). This is particularly true of multi-product and assembly systems where the complexity of the cost function limits the tractability of these models, for example, (De Bodt and Graves 1985; Schmidt and Nahmias 1985; Gurani *et al.* 2000). As a consequence, the potential of JIT replenishment to mitigate the effect of supply or demand uncertainty upon total inventory costs has been largely overlooked. The development in this chapter of a multi-product stochastic inventory model which can be solved efficiently now makes possible the analysis of JIT replenishment and component substitution decisions when inventory is subject to stochastic demand.

Section 5.1 presents the multi-product constrained inventory model for batch-replenished components that are sold as assemblies to customer order. It is then shown how the model, under certain assumptions, can be cast into a familiar form by attributing a portion of the cost of lost sales of finished products to each component. The cost-minimising replenishment quantities and reorder points are then determined. The modifications to the original model introduced by the JIT replenishment or substitution of certain components are then shown. Section 5.2 describes the conditions for the viability of JIT replenishment or substitution of a given component. Analysis of the general model for the case company is presented in Section 5.3 in order to illustrate the potential impact of JIT replenishment and component substitution on company performance in a realistic setting. Section 5.4 concludes the chapter with a discussion of the new factors that the consideration of stochastic demand introduces into the inventory reduction decision.

5.1. Multi-item reorder-point, replenishment-quantity model

5.1.1. The model with investor supplied capital

In the case of a company operating with investor-supplied capital, manufacturing n types of finished products from a range of m purchased components using assemble-to-order production and facing stochastic order arrival times, annual profit can be written as

$$P = \sum_{i=1}^{n} A_i S_i (1 - p_i) - \sum_{j=1}^{m} D_j C_j - \sum_{j=1}^{m} \frac{D_j}{Q_j} R_j - J \sum_{j=1}^{m} C_j \left(\frac{Q_j}{2} + r_j - \bar{x}_j + \ell_j \right) - F \qquad (5.1)$$

under the constraint

$$K = \sum_{j=1}^{m} C_j \left(\frac{Q_j}{2} + r_j - \bar{x}_j + \ell_j \right) \qquad (5.2)$$

where

p_i = *Probability of lost sale of product i*
r_j = *Reorder point for component j*
$\bar{x}_j$ = *Expected demand through lead time for component j*
ℓ_j = *Increase in inventory of component j resulting from the lost sales of finished products*

The first term in Equation 5.1 represents the income that results from the sale of finished products, and is the annual demand less the expected number of lost sales. The next terms represent respectively: the cost of purchased components, the variable cost of batch replenishing components, inventory holding costs, and fixed costs. These have been derived from the single-item (Q,r) inventory model (Tersine 1988) summed over m components. Because demand for finished products is stochastic, D_j now represents the expected annual demand. Also, because it is assumed that all capital is investor supplied, there is no interest charge on the capital used to purchase inventory items. If all capital were borrowed at interest rate I, then the holding cost would be $(J+I)$. The expected increase in the inventory of each component resulting from the lost sales of finished products has been denoted as ℓ_j. Because production is assemble-to-order, there are no additional holding costs for work-in-progress. Because the company is financed with investor-supplied capital, the amount available for investment in inventory, K, is finite, and set as the sum of the average holding of each component multiplied by its unit cost.

Because many components may cause a lost sale of each type of finished product in Equation 5.1, it is not possible to evaluate independently the profit maximising values of Q and r for each component when the equation is in this form. The following section shows how, under certain assumptions, the model can be recast in a more familiar form (Hadley and Whitin 1963) where the cost of a lost sale is evaluated as a function of the probability of a component being out of stock. This approach eliminates the need for explicitly evaluating p_i, and ℓ_j, and allows the profit maximising values of Q_j and r_j to be found using partial derivatives.

5.1.2. The cost of lost sales

The cost of lost sales of finished products resulting from stockouts of components is now expressed in terms of cost penalties that can be attributed to each component. The model assumes that components are assembled to order into finished products, based on a single level bill-of-materials with no work-in-progress held. Products are manufactured on a first-come-first-served basis, and are not constructed until all required components are available. Finished products are ordered one-at-a-time, which means that any instance of a lost sale is only for a single product. The model is a good representation of the manufacturing practice of the case study company, and now includes extra realism by recognising the variability of component demand through lead time. Also, the simple demand structure for finished products and manufacturing system permits a tractable analysis.

It is also assumed that the inventory levels of each component, forming part of a final product are uncorrelated over the long-term with the consequence that a stockout in any component is independent of the inventory level of all other components. This follows from the observation that components with different lead times, replenishment quantities and annual demand will be replenished independently of each other. To illustrate this point, Figure 5.1 shows the pair-wise scatterplots of the inventory levels for the seven batch-replenished components that comprise the 3 horsepower air compressor, observed in a discrete-event simulation of the case study company over a duration of 1000 days. Each plot shows the inventory levels for pairs of components over each of 1000 days of operation for a single instance of a simulated company. Details of the discrete-event simulation trial will be presented in Section 6.1 and details of discrete-event simulation model are given in Appendix C. Although the scales have been omitted from each plot, the inventory levels in each vary from a minimum of approximately $r_j - \bar{x}_j$ to a maximum of $r_j - \bar{x}_j + Q_j$, with Q_j and r_j determined when the average investment in inventory is $K =$ \$60,000 (these are given in Table C.2). The remaining components used to manufacture the 3 horsepower air compressor (Motor 1 and Wiring 1) are ordered in units of one, and consumed immediately for manufacture, and thus never form part of the inventory.

It is evident in Figure 5.1 that some pairs of components, for example Radiator 1 and Valve 1, have correlated inventory levels over short periods as a result of these components being required for a similar group of finished products. However, because components have replenishment cycles of different length, and are replenished in different quantities, each sequence of correlated inventory levels ceases when a replenishment order for either component is received. The net effect is that all components have uncorrelated inventory levels over the long term. This is evident in each scatterplot, which shows that that all combinations of inventory levels between any pair of components are theoretically possible.

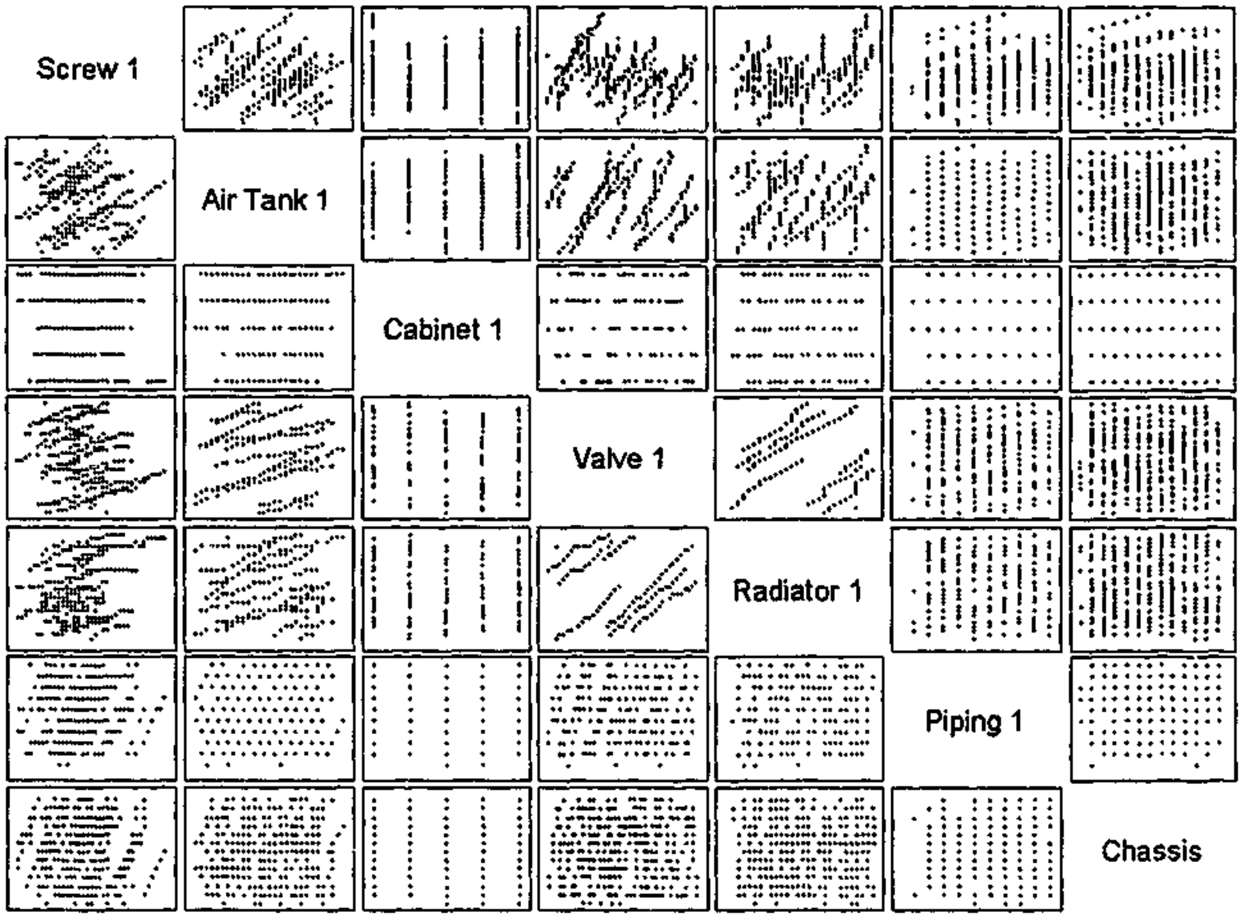

Figure 5.1: Pair-wise scatterplots of the inventory level for each batch-replenished component of compressor 1 over 1000 days of simulated operation

Let the probability that any component suffers a stockout during a replenishment order cycle be that proportion of total component demand that is not available during each order cycle, namely $\frac{L(r_j)}{Q_j}$, where $L(r_j)$ represents the expected number of components that are not available to assemble finished products in each replenishment order cycle for a component having a reorder point r_j. $L(r_j) = \int_{r_j}^{\infty} (x - r_j) g_j(x) dx$ where $g_j(x)$ is the distribution of demand through lead time of component j (Hadley and Whitin 1963). Because demand is stochastic, Q_j now represents the expected demand for component j during each order cycle. Treating the inventory level of each component as an independent random variable, and following a similar approach to that taken by Hopp and Spearman (1993), the probability of a lost sale of finished product i is

$$p_i = 1 - \prod_{j=1}^{m} \left(1 - \frac{L(r_j)}{Q_j}\right)^{v_{ij}} \tag{5.3}$$

where

v_{ij} = the number of units of component j used to manufacture product i

Equation 5.3 can be expanded as

$$\begin{aligned} 1 - \prod_{j=1}^{m} \left(1 - \frac{L(r_j)}{Q_j}\right)^{v_{ij}} &= \sum_{j=1}^{m} \frac{v_{ij} L(r_j)}{Q_j} \\ &- \sum_{j=1}^{m} \sum_{k=1}^{m} \frac{v_{ij} L(r_j)}{Q_j} \frac{v_{ik} L(r_k)}{Q_k}, \ for\ k < j \\ &+ \sum_{j=1}^{m} \binom{v_{ij}}{2} \frac{L(r_j)^2}{Q_j^2}, \ for\ v_{ij} > 1 \\ &\ldots \end{aligned} \tag{5.4}$$

When the expected number of lost sales in any replenishment cycle is small, that is, $L(r_j)/Q_j \to 0$, Equation 5.3 can be approximated by the sum

$$p_i = \sum_{j=1}^{m} \frac{v_{ij} L(r_j)}{Q_j} \pm O\left(\frac{v_{ij} L(r_j)}{Q_j}\right)^2, \tag{5.5}$$

which permits the lost sales cost of finished products to be expressed approximately as a sum of costs indexed on j. This in turn yields an approximation for B_j, the cost of a stockout of component j.

$$\sum_{i=1}^{n} A_i S_i p_i \approx \sum_{j=1}^{m} \frac{L(r_j)}{Q_j}\left(\sum_{i=1}^{n} v_{ij} A_i S_i\right)$$
$$= \sum_{j=1}^{m} \frac{D_j}{Q_j} B_j L(r_j) \text{ where } B_j = \frac{\sum_{i=1}^{n} v_{ij} A_i S_i}{D_j}. \tag{5.6}$$

The form of B_j in Equation 5.6 shows that the cost of a stockout of any component (attributable to lost sales of finished products) is valued at the weighted average sale price of the finished product of which its part. This determination of B_j, also indicates that every component should be regarded as critical for manufacture, as a stockout in any component results in lost sales.

The expansion of Equation 5.3 shows that the approximation in Equation 5.4 overestimates the probability of a lost sale as a consequence of implicitly assuming that stockouts of multiple components do not occur simultaneously. For example, the second term in the expansion of Equation 5.3 shows the over-estimation due to double counting the joint probability of a stockout in two different components simultaneously. The approximation is reasonable however when the probability of a stockout in any single component is very small, as then the probability of a lost sale due to a stockout in multiple components simultaneously is negligible. The error introduced by the approximation is also reduced for the case study example, which assumes that only one unit of any component is used in the assembly of finished products. Thus, by assuming that v_{ij} = 0 or 1, the errors due to a stockout of multiple units of a single component, as shown by the third term in Equation 5.4, are eliminated. For a make-to-order from stock policy, as used in the case study, the stock out probability will be small because sacrificing sales in order to reduce inventory costs is an ineffective strategy. Thus, the probability of a stockout in any component is small for the case study company at all investment levels except where inventory is very highly constrained, when it becomes cost-effective to sacrifice sales in order to reduce safety stock. Such an operating regime, however, corresponds to the company operating below the threshold of profitability, as shown in Figure 5.4.

Table 5.1 compares the service level determined theoretically from Equation 5.7 with those obtained in a series of simulations of the case study company at varying levels of investment in inventory. (see Section 6.1 and appendix C) For both the simulated companies and the analytical model, the service level was determined by expressing the expected value of lost sales as a proportion of total expected sales.

Table 5.1 Service levels determined using Equation 5.1 including the approximation of Equation 5.5, and from simulated companies as a function of investment in inventory

Average Investment in Inventory	λ	Service Level Determined Analytically	Service Level Simulated Companies
35,000	4.64	0.93	0.96
40,000	3.14	0.95	0.97
45,000	2.08	0.97	0.98
50,000	1.49	0.98	0.99
55,000	1.07	0.98	0.99
60,000	0.80	0.99	0.99
65,000	0.64	0.99	0.99
70,000	0.50	0.99	0.99
75,000	0.40	0.99	0.99
80,000	0.33	1.00	0.99
85,000	0.28	1.00	-
90,000	0.24	1.00	-
95,000	0.20	1.00	-
100,000	0.17	1.00	-

When investment in inventory is unconstrained, the service levels obtained in both cases are in close agreement, and close to 1. As the investment in inventory decreases, and the level of inventory constraint increases, the service level for both cases decreases, with the service level predicted by the analytical model decreasing at a greater rate as a result of the approximation in Equation 5.5. It is evident however, that even at greatly reduced investment, the difference between the analytical and simulated cases is small. Thus, for all practical purposes, the effect of this approximation on the conclusions drawn from the model is also small.

The determination of a stockout cost for components now permits the cost of lost sales of finished products in Equation 5.1 to be calculated from the probability of a stockout of components. Thus annual profit is now

$$P = \sum_{i=1}^{n} A_i S_i - \sum_{j=1}^{m} D_j C_j - \sum_{j=1}^{m} \frac{D_j}{Q_j}\left(R_j + B_j L(r_j)\right) - J\sum_{j=1}^{m} C_j\left(\frac{Q_j}{2} + r_j - \bar{x}_j + L(r_j)\right) - F \quad (5.7)$$

under the constraint

$$K = \sum_{j=1}^{m} C_j\left(\frac{Q_j}{2} + r_j - \bar{x}_j + L(r_j)\right). \quad (5.8)$$

Consequently, all costs that vary as a function of Q and r are now expressed in terms of component costs, simplifying the solution for optimal values of each Q_j and r_j by permitting the decomposition of the profit equation into m single component cases.

5.1.3. Determination of reorder-point and replenishment-quantity

The profit maximising values of Q_j and r_j are now evaluated for Equation 5.7. Using the familiar method of Lagrange multipliers (Hadley and Whitin 1963; Tersine 1988), the augmented profit equation incorporating the constraint, Equation 5.8, is

$$\begin{aligned} P_K = {} & \sum_{i=1}^{n} A_i S_i - \sum_{j=1}^{m} D_j C_j - \sum_{j=1}^{m} \frac{D_j}{Q_j}\left(R_j + B_j L(r_j)\right) \\ & - J\sum_{j=1}^{m} C_j\left(\frac{Q_j}{2} + r_j - \bar{x}_j + L(r_j)\right) - F - \lambda\left(\sum_{j=1}^{m} C_j\left(\frac{Q_j}{2} + r_j - \bar{x}_j + L(r_j)\right) - K\right), \end{aligned} \quad (5.9)$$

The profit maximising values of Q_j and r_j are determined by solving $\frac{dP_K}{dQ_j} = 0$ and $\frac{dP_K}{dr_j} = 0$

respectively for each j, as well as by solving $\frac{dP_K}{d\lambda}=0$. These are

$$Q_j=\sqrt{\frac{2D_j\left(R_j+B_jL(r_j)\right)}{C_j(J+\lambda)}} \tag{5.10}$$

and

$$G_j(r_j)=\frac{\frac{D_jB_j}{Q_j}}{\frac{D_jB_j}{Q_j}+C_j(J+\lambda)}, \tag{5.11}$$

which follows from $L'(r_j)=G_j(r_j)-1$, where $G_j(r_j)=\sum_{x=0}^{r_j}g_j(x)$. Solving $\frac{dP_K}{d\lambda}=0$ gives

$$\lambda=\frac{\left(\sum_{j=1}^{m}\sqrt{C_jD_j\left(R_j+B_jL(r_j)\right)}\right)^2}{2\left(K-\sum_{j=1}^{m}C_j\left(r_j-\bar{x}_j+L(r_j)\right)\right)^2}-J. \tag{5.12}$$

As for the deterministic case in the previous chapter, λ can be interpreted as the marginal return on investment $\frac{dP}{dK}$, or the opportunity cost of capital invested in inventory, since it is the change in income that results from a unit increase or decrease of inventory investment. λ also indicates the degree of resource constraint and consequently λ increases as K decreases.

The profit maximising values of Q_j and r_j are solved for a particular value of λ by iterating Equations 5.10 and 5.11 until convergence is obtained. The solution of Q_j and r_j for a particular value of K, however, requires that the corresponding value of λ be known. A closed form expression for λ as a function of K does not exist. Consequently, the solution method used for the case study is to determine Q_j and r_j based on an initial estimate of λ, to calculate K, and then to iterate the value of λ using Newton's method until the values of Q_j and r_j which yield the desired value of K are found.

5.1.4. The model for JIT replenishment

The case where the company replenishes Component 1 in a Just-In-Time manner is now considered. This requires that Component 1 is ordered lot-for-lot with the demand for finished products and that delivery occurs within the customer lead time, that is, the time a customer is prepared to wait from the placement of an order to the receipt of finished goods (Hopp and Spearman 1996). Thus for components replenished JIT, $L(r_j)=0$ and the cost of lost sales is now

$$\sum_{i=1}^{n} A_i S_i p_i' \approx \sum_{j=2}^{m} \frac{D_j}{Q_j} B_j L(r_j) \text{ where } B_j = \frac{\sum_{i=1}^{n} \delta_{ij} A_i S_i}{D_j}. \tag{5.13}$$

Note that p_i' represents the new probability of a lost sale of finished product i when Component 1 is replenished JIT.

Using R_1' to represent the new replenishment cost for the JIT replenishment of Component 1, the profit under JIT replenishment, P_{JIT}, is

$$\begin{aligned} P_{JIT} = \sum_{i=1}^{n} A_i S_i - \sum_{j=1}^{m} D_j C_j - \sum_{j=2}^{m} \frac{D_j}{Q_j}\left(R_j + B_j L(r_j)\right) \\ - J \sum_{j=2}^{m} C_j \left(\frac{Q_j}{2} + r_j - \bar{x}_j + L(r_j) \right) - R_1' D_1 - F \end{aligned} \tag{5.14}$$

under the constraint

$$K=\sum_{j=2}^{m} C_j\left(\frac{Q_j}{2}+r_j-\bar{x}_j+L(r_j)\right). \tag{5.15}$$

The profit maximising values of $Q_j^{'}$ and $r_j^{'}$, the new replenishment quantity and reorder point for components when Component 1 is replenished JIT, are determined in a similar way to the previous case as

$$Q_j^{'}=\sqrt{\frac{2D_j\left(R_j+B_jL(r_j^{'})\right)}{C_j\left(J+\lambda_{JIT}\right)}} \tag{5.16}$$

and

$$G_j(r_j^{'})=\frac{\dfrac{D_jB_j}{Q_j^{'}}}{\dfrac{D_jB_j}{Q_j^{'}}+C_j\left(J+\lambda_{JIT}\right)} \tag{5.17}$$

for $j=2,3,\ldots,m$, where

$$\lambda_{JIT}=\frac{\left(\sum_{j=2}^{m}\sqrt{C_jD_j\left(R_j+B_jL(r_j^{'})\right)}\right)^2}{2\left(K-\sum_{j=2}^{m}C_j\left(r_j^{'}-\bar{x}_j+L(r_j^{'})\right)\right)^2}-J. \tag{5.18}$$

Note that for Component 1, $Q_1^{'}=1$ and $G_1(r_1^{'})=1$. By comparing Equations 5.12 and 5.18, it can be seen that for a fixed value of K, $\lambda>\lambda_{JIT}$. The reduced value of λ_{JIT} reflects that the inventory investment is spread over fewer components.

5.1.5. The model with component substitution

An alternative method for reducing the investment in inventory is by the substitution of one component with another over-specification component already required for the manufacture of another finished product in order to reduce the component range. For example, when Component 2 is substituted for Component 1 the new profit equation in the case of investor supplied capital is

$$\begin{aligned} P_{SUB} = \sum_{i=1}^{n} A_i S_i - (D_1 + D_2)C_2 - \sum_{j=3}^{m} D_j C_j - \frac{(D_1 + D_2)}{Q_2^{'}}\left(R_2 + B_2 L(r_2^{'})\right) \\ - \sum_{j=3}^{m} \frac{D_j}{Q_j^{'}}\left(R_j + B_j L(r_j^{'})\right) - J\sum_{j=2}^{m} C_j\left(\frac{Q_j^{'}}{2} + r_j^{'} - \bar{x}_j + L(r_j^{'})\right) - F \end{aligned} \tag{5.19}$$

The profit maximising values of $Q_j^{'}$ and $r_j^{'}$ under the inventory investment constraint

$$K = \sum_{j=2}^{m} C_j\left(\frac{Q_j^{'}}{2} + r_j^{'} - \bar{x}_j + L(r_j^{'})\right) \tag{5.20}$$

are determined using methods similar to those described previously as

$$Q_2^{'} = \sqrt{\frac{2(D_1 + D_2)(R_2 + B_2 L(r_2^{'}))}{C_2(J + \lambda_{SUB})}} \tag{5.21}$$

and

$$G_2(r_2^{'}) = \frac{\frac{(D_1 + D_2)B_2}{Q_2^{'}}}{\frac{(D_1 + D_2)B_2}{Q_2^{'}} + C_2(J + \lambda_{SUB})} \tag{5.22}$$

for Component 2, where the increased demand for this component is now $(D_1 + D_2)$.

$Q_j^{'}$ and $r_j^{'}$ for the remaining components $(j = 3,4,\ldots,m)$ are

$$Q_j^{'} = \sqrt{\frac{2D_j(R_j + B_j L(r_j^{'}))}{C_j(J + \lambda_{SUB})}} \tag{5.23}$$

and

$$G_j(r_j^{'}) = \frac{\dfrac{D_j B_j}{Q_j^{'}}}{\dfrac{D_j B_j}{Q_j^{'}} + C_j(J + \lambda_{SUB})}, \tag{5.24}$$

where

$$\lambda_{SUB} = \frac{\left(\sqrt{C_2(D_1 + D_2)(R_2 + B_2 L(r_2^{'}))} + \sum_{j=3}^{m} \sqrt{C_j D_j (R_j + B_j L(r_j^{'}))}\right)^2}{2\left(K - \sum_{j=2}^{m} C_j (r_j^{'} - \bar{x}_j + L(r_j^{'}))\right)^2} - J. \tag{5.25}$$

The value of $Q^{'}$ and $r^{'}$ are determined for each component j at a certain value of K in the same way as for the original and JIT cases. $Q^{'}$ and $r^{'}$ are evaluated using iteration for a particular value of λ_{SUB} with the value of λ_{SUB} varied until convergence on the required value of K is obtained.

5.2. Conditions for increased profit with JIT replenishment

It can be seen from Equations 5.12 and 5.18 that, when inventory is financed with investor supplied capital, the values of Q_j, r_j, $Q_j^{'}$ and $r_j^{'}$ are dependent on the amount of capital available for investment. Hence, the case where the JIT decision for Component 1 is just viable is analysed at the indifference investment level of $\widetilde{K}$ dollars, that is, $P_{JIT}(\widetilde{K}) = P(\widetilde{K})$. Component 1 currently requires $C_1\left(\dfrac{Q_1}{2} + r_1 - \bar{x}_1 + L(r_1)\right)$ dollars to finance. If the JIT decision is adopted, this money can be reinvested in the firm at a marginal return rate of $\lambda_{JIT}(K)$ to bring the total capitalisation back to $\widetilde{K}$ dollars. These returns accrue due to the relief of the constraint on the remaining components and have magnitude

$$
\begin{aligned}
\int\limits_{\left(\tilde{K}-C_1\left(\frac{Q_1}{2}+r_1-\bar{x}_1+L(r_1)\right)\right)}^{\tilde{K}} \lambda_{JIT}dK &= \int\limits_{\left(\tilde{K}-C_1\left(\frac{Q_1}{2}+r_1-\bar{x}_1+L(r_1)\right)\right)}^{\tilde{K}} \frac{dP_{JIT}(K)}{dK}dK \\
&= P_{JIT}\left(\tilde{K}\right)-P_{JIT}\left(\tilde{K}-C_1\left(\frac{Q_1}{2}+r_1-\bar{x}_1+L(r_1)\right)\right) \\
&= P\left(\tilde{K}\right)-P_{JIT}\left(\tilde{K}-C_1\left(\frac{Q_1}{2}+r_1-\bar{x}_1+L(r_1)\right)\right) \\
&= D_1R_1^{'}-\frac{D_1}{Q_1}\left(R_1+B_1L(r_1)\right)-JC_1\left(\frac{Q_1}{2}+r_1-\bar{x}_1+L(r_1)\right)
\end{aligned}
\tag{5.26}
$$

where the last step follows from the fact that Q_j and r_j for each of the non-JIT components at the lower terminal of integration, is the same as the Q_j and r_j values of the same components prior to eliminating Component 1 from the inventory investment pool, that is:

$$
Q_j^{'}\left(\tilde{K}-C_1\left(\frac{Q_1}{2}+r_1-\bar{x}_1+L(r_1)\right)\right)=Q_j\left(\tilde{K}\right) \tag{5.27}
$$

and

$$
r_j^{'}\left(\tilde{K}-C_1\left(\frac{Q_1}{2}+r_1-\bar{x}_1+L(r_1)\right)\right)=r_j\left(\tilde{K}\right) \tag{5.28}
$$

for $j=2,3,\ldots,m$. In order that $P_{JIT}\left(\tilde{K}\right)>P\left(\tilde{K}\right)$, the decision to adopt a JIT policy for Component 1 requires that

$$
D_1R_1^{'}-\frac{D_1}{Q_1}R_1<JC_1\frac{Q_1}{2}+JC_1\left(r_1-\bar{x}_1+L(r_1)\right)+\frac{D_1}{Q_1}B_1L(r_1)+\int\limits_{\left(\tilde{K}-C_1\left(\frac{Q_1}{2}+r_1-\bar{x}_1+L(r_1)\right)\right)}^{\tilde{K}}\lambda_{JIT}dK\ . \tag{5.29}
$$

Because JIT replenishment assumes that components are supplied within the customer lead time, the risk of a stock-out of the JIT component is eliminated. Consequently, Equation 5.29 shows that a company will adopt a JIT policy for a component when the increase in replenishment costs is justified by the total saving in the holding cost of cycle stock, $JC_1\frac{Q_1}{2}$, plus the holding cost of safety stock, $JC_1(r_1 - \bar{x}_1 + L(r_1))$, plus the cost of lost sales attributable to stock-outs of Component 1, $\frac{D_1}{Q_1}B_1L(r_1)$ in addition to the cost saving achieved when replenishing the fewer remaining stocked components with the same available capital, $\int_{\left(\tilde{K}-C_1\left(\frac{Q_1}{2}+r_1-\bar{x}_1+L(r_1)\right)\right)}^{\tilde{K}} \lambda_{JIT} dK$.

Note that the JIT condition for investor-supplied capital under deterministic demand of the previous chapter may be obtained from Equation 5.29 by setting $L(r_1) = 0$ and $r_1 = \bar{x}_1$, giving

$$D_1R_1' - \frac{D_1}{Q_1}R_1 < JC_1\frac{Q_1}{2} + \int_{\left(\tilde{K}-C_1\frac{Q_1}{2}\right)}^{\tilde{K}} \lambda_{JIT} dK \,. \tag{4.7}$$

In contrast to the JIT decision under constraint, if the company were financed by borrowed capital, the decision to adopt JIT replenishment requires that

$$D_1R_1' - \frac{D_1}{Q_1}R_1 < (J+I)C_1\frac{Q_1}{2} + (J+I)C_1(r_1 - \bar{x}_1 + L(r_1)) + \frac{D_1}{Q_1}B_1L(r_1), \tag{5.30}$$

where the cost of holding inventory now includes the interest cost of borrowed funds denoted by I. Comparing Equations 5.29 and 5.30, it is evident that two factors make JIT replenishment more favourable when capital is constrained. Firstly, the replenishment cost of the remaining batch-replenished components is reduced as a consequence of replenishing fewer stocked components with the same capital. Secondly, when capital is constrained. Q_j, and r_j are smaller than in the unconstrained case, which increases both the batch replenishment cost,

$\frac{D_1}{Q_1}R_1$, and the lost order cost, $\frac{D_1}{Q_1}B_1L(r_1)$, for component 1, which increases the savings that result from eliminating this component from inventory.

Comparing Equation 5.30 for the stochastic, unconstrained, borrowed-capital case with Equation 4.4 for the deterministic, unconstrained, borrowed-capital case,

$$D_1R_1' - \frac{D_1}{Q_1}R_1 < \frac{(J+I)}{2}Q_1C_1, \tag{4.4}$$

the conditions for JIT replenishment are more favourable in the stochastic case because of the saving in the cost of holding safety stock $(J+I)C_1(r_1 - \bar{x}_1 + L(r_1))$ and the elimination of the cost $\frac{D_1}{Q_1}B_1L(r_1)$ of the portion of the risk of lost sales attributable to stockouts of Component 1. Thus, even in the unconstrained case, JIT replenishment is partly justified by its ability to control the effects of supply uncertainty.

The decision to adopt JIT replenishment under stochastic demand (Equation 5.29) is again dependent on the level of capital available, as was the case under deterministic demand, (Equation 4.20) although this time in two ways. As before, the ability to reinvest the capital formerly invested in the inventory of Component 1 to improve the efficiency of replenishing the remaining stocked components (by increasing each remaining Q_j and r_j) makes the JIT decision more favourable when capital is scarce. But also, the effect of JIT in mitigating the effect of demand uncertainty is greater when capital is scarce. Increasing constraint increases the probability of lost sales due to component stockouts, resulting in a greater proportion of investment in safety stock to cycle stock for Component 1 when batch-replenished. Consequently, the reduction of uncertainty becomes an increasingly important component of the JIT decision as the available capital for investment decreases. Comparing the integral terms in Equations 5.29 and 4.24 shows that, when capital is constrained, JIT replenishment under stochastic demand now permits the redistribution of the investment in cycle stock *and* safety stock of the JIT component. This makes the JIT decision particularly cost-effective for components with a large investment in safety stock, such as components with a long lead time, large lost order cost or large replenishment cost. Each of these contributions to justifying the JIT decision will be illustrated numerically in the discussion of the case study company.

For a company operating with finite capital, the cost effectiveness of JIT replenishment is further enhanced under stochastic demand because in this case the provision of safety stock increases capital constraint, and hence increases the utility of liberated capital, compared with the deterministic case. The increased capital constraint (λ) in the stochastic case is shown in Figure, 5.2, which also shows profit (excluding lost order costs for the stochastic case) for the case study company under stochastic and deterministic demand. For both cases, $J = 0$, and therefore the decrease in profit under stochastic demand is due only to the provision of safety stock resulting in smaller replenishment batches.

The increased cost of servicing stochastic demand is apparent in the reduced profitability, with the consequence that the minimum investment required for the threshold of profitability under stochastic demand is approximately $10,000 greater than when demand is deterministic. As well, the differences in both the level of constraint and the profitability between the two demand cases increase as the capital invested decreases, indicating that the relative cost of meeting stochastic demand increases as total investment reduces, and further increasing the effectiveness of JIT replenishment at reduced investment levels.

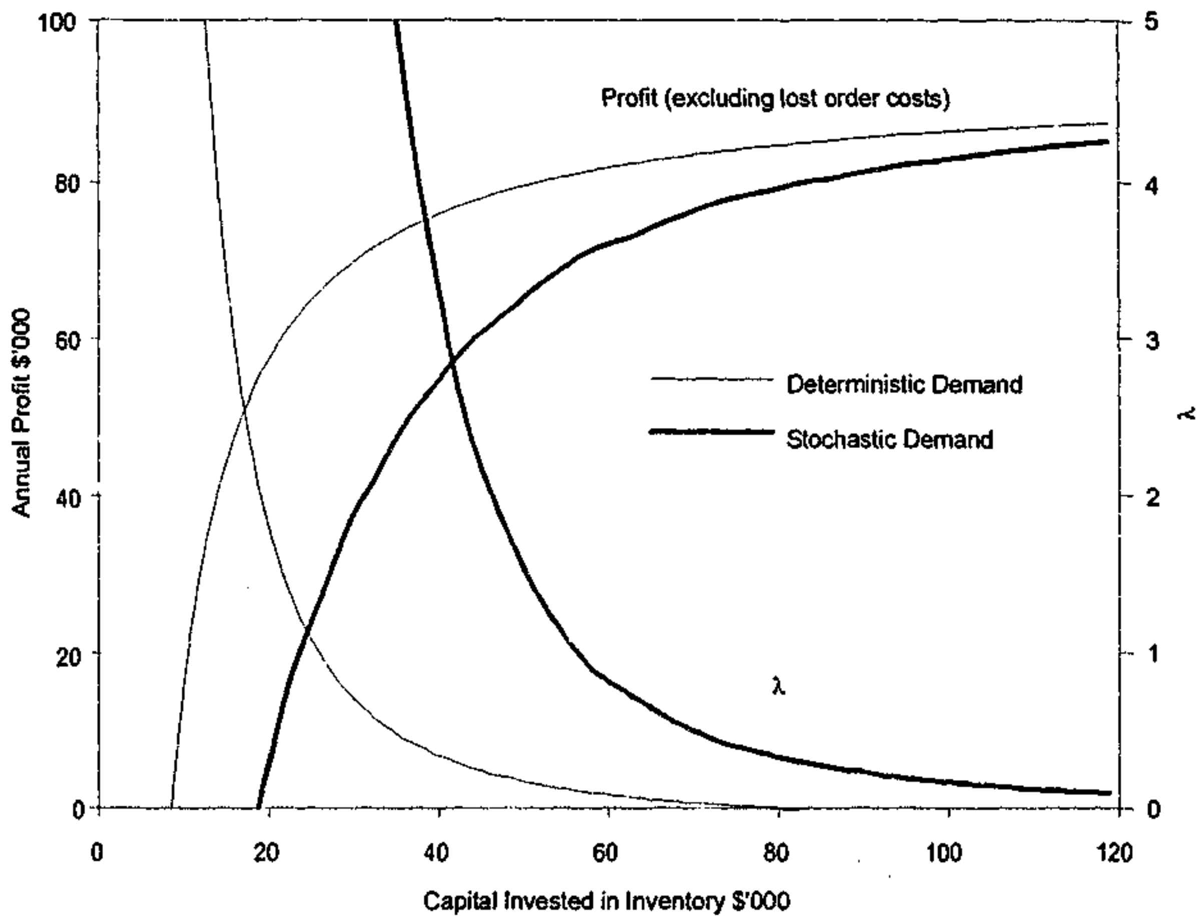

Figure 5.2: Profit (excluding lost order costs) and λ as a function of capital invested in inventory for the cases when demand is deterministic and stochastic

5.3. Case study example

5.3.1. Calculation methods

Equations 5.7, 5.14 and 5.19, under the assumption of investor-supplied capital, were used to determine the profit of the company as a function of the level of investment in inventory and of the inventory policy adopted in the following analysis. The adoption of a multi-item (Q,r) model required that two assumptions were made. Firstly, as reported in Chapter 3, the demand for each type of air compressor was an independently distributed Poisson random variable. Consequently, the demand through-lead time distribution for each component was also Poisson, with expected demand equal to the sum of demand for each of the air compressors of which the component is a part. Hence $g_j(x)=\frac{e^{-\theta_j}\theta_j^x}{x!}$, where θ_j is the mean demand through lead time for component j. Secondly, the (Q,r) model assumes that both the manufacture of finished products and the JIT replenishment of components occurs simultaneously with demand whereas, in practice, customers would wait several days for finished compressors. Consequently, the model slightly overestimates the probability of a lost order due to a stockout in a batch-replenished component, over that observed in practice, but correctly models JIT replenishment as occurring within the customer lead time. Because the probability of a stockout for all components is very small except under extremely high constraint, this assumption has negligible effect on the accuracy of the model. The unit cost, substitute components, batch replenishment, JIT replenishment costs and supply lead time are given in Table 2.2. The demand through lead time and lost order cost for each component based on the sales value of finished products, determined by Equation 5.6, are shown in Table 5.2.

Table 5.2: Expected demand through lead time and lost order cost attributable to each component

Component	Expected Demand Through lead time θ_j	Lost Order Cost Attributable to Component B_j (\$)
Air Tank 1	3.15	3,425
Air Tank 2	2.80	5,734
Cabinet 1	0.08	2,880
Cabinet 2	0.13	3,755
Cabinet 3	0.19	5,734
Chassis	0.79	4,510
Motor 1	0.05	2,800
Motor 2	0.03	3,000
Motor 3	0.03	3,300
Motor 4	0.10	3,900
Motor 5	0.10	5,200
Motor 6	0.03	6,000
Motor 7	0.06	6,500
Piping 1	0.42	3,425
Piping 2	0.37	5,734
Radiator 1	2.38	2,880
Radiator 2	3.93	3,755
Radiator 3	3.81	5,375
Radiator 4	1.79	6,500
Screw 1	30.95	3,994
Screw 2	8.73	6,341
Valve 1	2.22	3,000
Valve 2	5.71	5,097
Wiring 1	0.21	3,425
Wiring 2	0.19	5,734

Two methods were used to calculate the profit of policy combinations in the analysis that follows. Firstly, the company's profit for invested capital ranging from \$1,000 to \$120,000 was evaluated for all combinations of replenishment policy and component substitution at intervals of \$1,000. The replenishment quantities, profit, ROI and replenishment policy for the most profitable policy combination at each investment level was recorded. Using this method, the investment level at which each of the inventory reduction strategies became cost-effective was determined approximately. It was observed that if an inventory reduction strategy was profit-maximising at a particular investment level, it was for all investment levels below this. A second series of calculations was then performed to determine more precisely the indifference value at which each policy change occurred. The indifference value, $\tilde{K}$, for each of the policy changes was determined using Newton's method, by varying K until the profit under both the original policy and the policy with the inventory reduction incorporated were equal. Because the sequence of policy changes had been determined in the first series of calculations, the determination of indifference values was achieved by sequentially testing each policy alternative, from greatest indifference level to smallest, incorporating each policy change for investment below its indifference level.

5.3.2. Results and discussion

Table 5.3 shows the indifference investment levels for all policy changes, sorted from highest to lowest. The type of policy change, the cost of making the change by substitution or by JIT replenishment, the cost of lost sales, and the profit at each level are also shown. With this table it is possible to determine the policy changes that would be worth considering for adoption by the company for any given available capital. The effect of the policy changes is cumulative: for example, the substitution of Radiator 1 with Radiator 2 becomes cost effective at $27,068 and assumes that JIT replenishment for Screw 1 and Screw 2 has also been adopted at this investment level.

As was the case under deterministic demand, three groups of component type can be identified based on the indifference level, and associated profit, at each policy change. Firstly, there are components that should be always replenished JIT, regardless of the amount of capital available for investment. Secondly, there are components whose replenishment policy is determined by the amount of capital available for investment. Finally, there are components that would never be considered for JIT replenishment or substitution, without setup cost reduction. This last group always requires economies of scale when ordering. The following discussion looks at each of these component groups in turn.

Table 5.3: Sequence of replenishment policy changes showing the type and cost of each change, $\widetilde{K}$, cost of lost sales and profit at each change

Policy Groups	Policy Transition	Component	Substitute	Cost of Policy Change ($)	$\widetilde{K}$ ($)	Type of Change	Cost of Lost Sales ($)	Profit ($)
Always Just-In-Time Reordering	1	Motor 1	-	-	120,866	JIT	726	84,132
	2	Motor 2	-	-	120,866	JIT	726	84,132
	3	Motor 3	-	-	120,866	JIT	726	84,132
	4	Motor 4	-	-	120,866	JIT	726	84,132
	5	Motor 5	-	-	120,866	JIT	726	84,132
	6	Motor 6	-	-	120,866	JIT	726	84,132
	7	Motor 7	-	-	120,866	JIT	726	84,132
	8	Wiring 1	-	-	120,866	JIT	726	84,132
	9	Wiring 2	-	-	120,866	JIT	726	84,132
Policy Based on Investment Level	10	Screw 1	-	7,800	82,462	JIT	542	77,399
	11	Screw 2	-	8,800	49,913	JIT	226	69,218
	12	Radiator 1	Radiator 2	1,000	27,068	Substitute	1,187	60,958
	13	Valve 1	Valve 2	2,380	21,453	Substitute	1,808	55,448
	14	Piping 2	-	1,410	19,241	JIT	2,030	52,241
	15	Air Tank 2	-	12,220	13,389	JIT	3,006	37,127
	16	Piping 1	-	2,650	11,590	JIT	3,783	31,810
	17	Cabinet 1	-	1,000	10,040	JIT	5,209	25,526
	18	Chassis	-	3,000	9,461	JIT	5,488	22,301
	19	Cabinet 3	-	2,350	8,019	JIT	11,578	11,577
	20	Radiator 4	-	15,750	7,938	JIT	4,256	10,923
	21	Cabinet 2	-	1,650	7,133	JIT	4,895	6,137
Policy Never Adopted	22	Radiator 3	-	4,800	4,356	JIT	9,212	-34,927
	23	Air Tank 1	-	13,780	3,601	JIT	9,317	-48,806
	24	Radiator 2	-	34,650	2,294	JIT	13,793	-93,807

The components that are always ordered JIT are the motors and wiring which are replenished at a negligible cost. Hence, a JIT policy is feasible at any investment level. This is why the company can employ a large range of motors, and component rationalisation does not occur. For these components, the indifference point is set at the maximum investment that a company would reasonably consider, which occurs when $\lambda = I$. For the case study, I was set at 10% *per annum*.

The next group of policy changes are those which are cost-effective when the capital available for investment is less than the maximum reasonable investment. Figure 5.3 shows the average inventory level, $\left(\frac{Q_j}{2} + r_j - \bar{x}_j + L(r_j)\right)$, for a number of key components whose policy changes are dependent on the amount of invested capital. The policy change that becomes viable at the greatest inventory investment is the JIT replenishment of Screw 1. Its indifference level is calculated as \$82,642. The effect of relaxing the constraint on the remaining non-JIT components is evident in Figure 5.3, which shows that, when JIT replenishment is adopted for Screw 1, the replenishment quantities for the remaining non-JIT components are the same as they were under the non-JIT policy at an investment of about \$111,000. The consequent reduction in the replenishment cost for the non-JIT components justifies the increased replenishment cost for Screw 1. It can also be seen that the replenishment policy change is maintained for all levels of investment below the indifference level $\tilde{K}$, indicating that it is not cost-effective to adopt batch based replenishment for this component at investments below the indifference level.

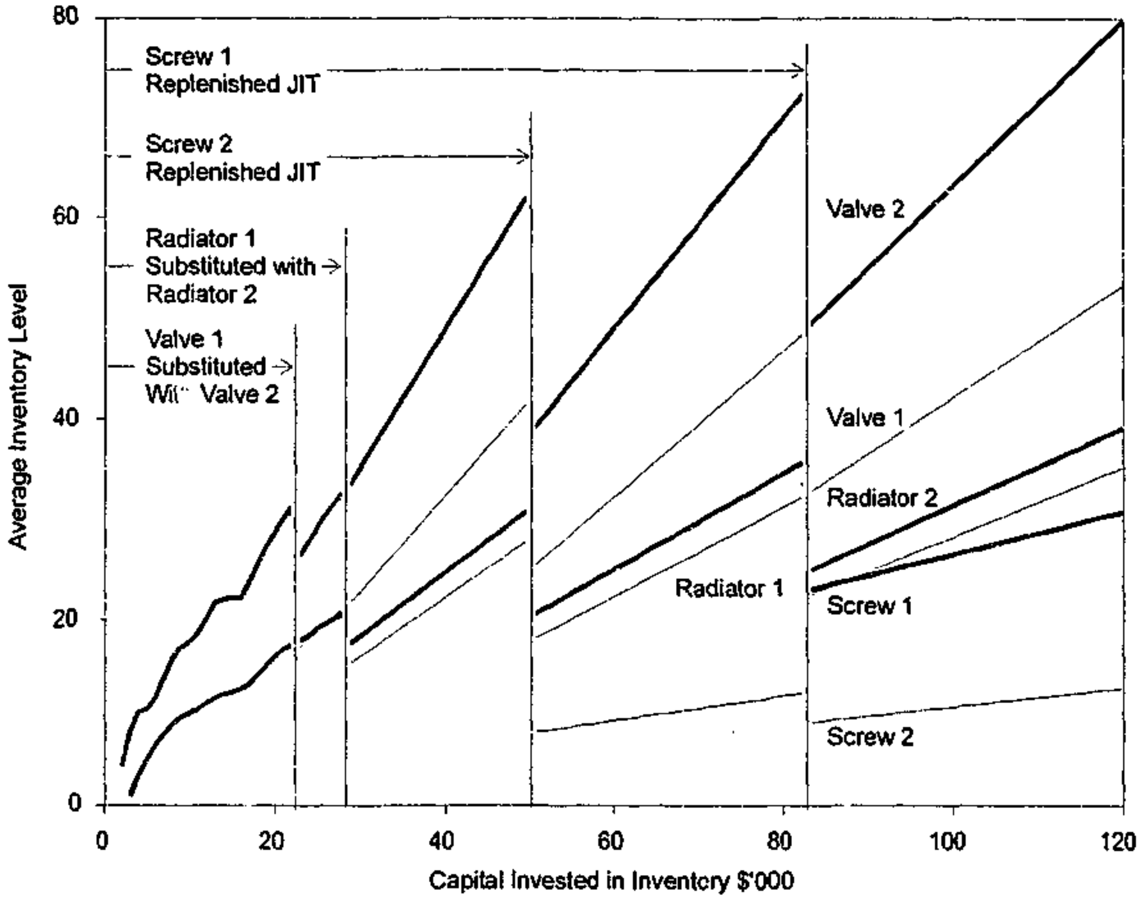

Figure 5.3: Change in the average inventory level of several components as a function of investment level

Table 5.4 shows the cost components which form the basis of the decision to replenish Screw 1 JIT for the constrained and unconstrained cases, based on Equations 5.29 and 5.30 respectively. For the unconstrained case, the costs are independent of invested capital, and Q_1, r_1 and all replenishment costs are based on the maximum reasonable investment, which occurs when $\lambda = I$. For the unconstrained case, Q_1 and r_1 are evaluated at the indifference investment level $\widetilde{K}$. It should also be noted that, for the case study $J = 0$ was used, although the results generalise for a non-zero J.

Table 5.4: Q, r and replenishment costs for batch and JIT replenishment of Screw 1, evaluated at the indifference value, $\widetilde{K}$, for constrained and unconstrained cases.

Expression	Unconstrained	Constrained
$\widetilde{K}$	-	\$82,462
Q_1	36.7	21.2
r_1	44	43
$C_1\left(\frac{Q_1}{2}+r_1-\bar{x}_1+L(r_1)\right)$	-	\$22,709
$R'_1 D_1$	\$7,800	\$7,800
$\frac{D_1}{Q_1}R_1$	\$1,595	\$2,755
$\frac{D_1}{Q_1}B_1L(r_1)$	\$239	\$643
$JC_1\left(\frac{Q_1}{2}+r_1-\bar{x}_1+L(r_1)\right)$	\$0	\$0
$IC_1\left(\frac{Q_1}{2}+r_1-\bar{x}_1+L(r_1)\right)$	\$3,141	-
$\int_{\left(\widetilde{K}-C_1\left(\frac{Q_1}{2}+r_1-\bar{x}_1+L(r_1)\right)\right)}^{\widetilde{K}}\lambda_{JIT}dK$	-	\$4,402
$P_{JIT}-P$	-\$2,825	\$0

The decision to adopt JIT replenishment of Screw 1 for the constrained case, at an investment of \$82,462, occurs because the increase in the replenishment cost for JIT replenishment, $\left(R'_1 D_1-\frac{D_1}{Q_1}R_1\right)$, is equal to the cost savings that result from the order policy change. These cost savings come from two sources. Firstly, the elimination of Screw 1 from inventory permits the redistribution of $22,709 *per annum* to purchase the remaining components, resulting in a saving of replenishment and lost order costs totalling $4,402 annually. Secondly, by obtaining replenishment within the customer lead time, the JIT replenishment of Screw 1 saves an additional $643 annually in lost sales attributable to this component. This additional cost saving, resulting from increased reliability, makes inventory reduction cost-effective at a greater replenishment cost than it would be under deterministic demand. Furthermore, the large investment in an expensive component such as Screw 1 shows that when finished products are made from stock, avoiding lost sales is very costly. By comparison, JIT replenishment is relatively inexpensive.

Because the batch replenishment cost, lost order cost, and the value of redistributed capital all increase as the investment level decreases, the JIT decision remains cost-effective for any investment below the indifference level. The increasing risk of lost sales as capital is reduced makes the elimination of a lost order cost a significant contributor to the JIT decision at low investment levels. By contrast, in the unconstrained case, the JIT replenishment of Screw 1 results in a net loss of $2,825 *per annum*. In this case, the capital liberated by the elimination of Screw 1 from inventory cannot yield improved economy across the remaining components because replenishment quantities and reorder points for all components are independently set at their profit maximising level, which is determined by the prevailing interest rate. In these circumstances, JIT replenishment could only be justified by a significant increase in the cost of capital, or by a significant reduction in replenishment cost.

The redistribution of capital invested in Screw 1 across the remaining components reduces the degree of constraint and the proportion of capital invested in safety stock. This is a consequence of the level of safety stock being less sensitive than cycle stock to decreasing total investment, thus, the proportional investment in safety stock increases as constraint increases. Figure 5.4 shows safety stock expressed as a proportion of the average stock for Screw 1 plotted as a function of the total investment in inventory. At the maximum feasible investment (when $\lambda = I$), safety stock represents approximately 40% of the investment in Screw 1. As the level of investment decreases, the proportion of safety stock increases until investment is sufficiently reduced so that deliberately losing sales orders in order to reduce component replenishment costs becomes a profit maximising strategy. However, in many cases, for example, the case study company, this coincides with a company operating with insufficient capital to achieve profitability. As capital is further decreased, the proportion of safety stock decreases as orders are deliberately sacrificed.

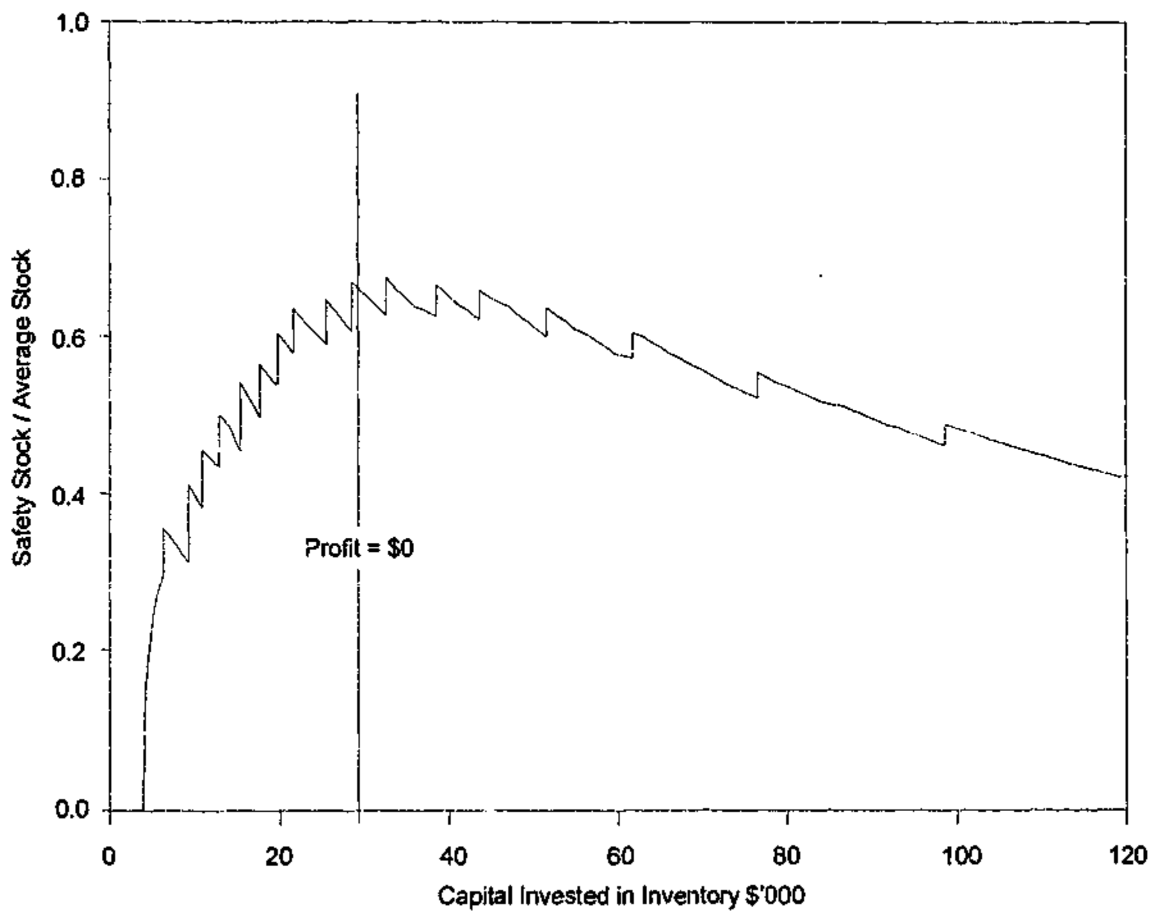

Figure 5.4: Safety stock expressed as a proportion of average stock for Screw 1 as a function of investment level

Through the elimination of a component from inventory, both JIT replenishment and component substitution generally reduce the proportion of capital that is invested in safety stock at a given level of investment. Safety stock is always reduced when components are replenished using JIT. This occurs because the capital invested in both the cycle stock and safety stock is redistributed across the remaining components to reduce constraint and, consequently, the overall proportion of capital invested in safety stock. In the case of component substitution, the proportional investment in safety stock is typically reduced, except when high demand and a high unit cost for the substitute component lead to an increase in the proportion of safety stock overall.

The policy change that takes place at the next lowest investment level is the JIT replenishment of Screw 2. Again, the effect of eliminating the component from inventory is seen as an increase in the investment in each of the components. Table 5.3 shows that when Screw 2 is replenished JIT, at an investment of $49,913, the expected cost of lost sales decreases to $226. The highly reduced expected cost of lost sales is due to the replenishment within the customer lead time of the two components to which the greatest cost of lost sales is attributable. The substitution of Radiator 2 with Radiator 1 and Valve 1 with Valve 2 are the next changes to take place. Although the substitution of a component results in one component being eliminated from inventory, it also results in an increased safety stock and replenishment quantity of the substitute component. Consequently, the amount of capital available for redistribution across the remaining components is small. As a result, both of these component substitutions do not become profit-maximising until the capital for inventory is highly constrained, even though the cost of making either of these component substitutions is relatively small ($1,000 *per annum* for Radiator 1 and $2,380 for Valve 2).

The last group of policy changes to consider are those which would never be implemented. These are numbered as policy changes 22 to 24 in Table 5.3. These changes would be unacceptable as the indifference point for each occurs at investment levels that are below the threshold of profitability. Looking at the components in this category, which include Radiators 2, and 3, and Air Tank 2, it is evident that the cost of JIT replenishment is too great to offset the benefit from constraint reduction through policy change. For each of these components, the high cost of individual replenishment orders prohibits the change to a JIT replenishment policy at any feasible investment level. As they now stand, these components always benefit from economies of scale when being replenished and this would only be addressed by radical changes to the replenishment cost structure. A number of options for component substitution, for example, Radiator 4 for Radiator 3, Piping 2 for Piping 1 and Tank 2 for Tank 1 would never be adopted as the indifference value for these changes is less than the value for JIT policy change.

The present analysis of the case study, which treats demand as stochastic, reaffirms the basic results of the previous chapter, which used the same case data but treated demand as deterministic, and illustrates the effect that the differing assumptions of deterministic and stochastic demand have on the JIT decision. The inventory reduction strategy that was cost-effective at the greatest investment level was Screw 1 in both the deterministic and stochastic cases. The sequence of policy changes that occurred as capital reduced was similar in both cases with a few exceptions. The policy change occurring at the second greatest investment level in the stochastic case was the JIT replenishment of Screw 2. This compares with the substitution of Radiator 2 for Radiator 1 in the deterministic case. This change of sequence results from a number of causes. Firstly, the high unit cost and long lead time of Screw 2 results in a large investment in safety stock that is available for redistribution when Screw 2 is replenished JIT, making this policy change desirable at a greater investment level. Secondly, component substitution is less cost-effective under low constraint in the stochastic case where the additional demand for the substitute component increases both Q *and* r for this component, which in turn reduces the amount of capital to be released for reinvestment across the remaining components. JIT replenishment of the cabinets, which were supplied with a short lead time and required no safety stock, occurred at a lower rank order in the stochastic case.

The effect of implementing the policy changes is now illustrated by comparing the company profit operating under its original batch replenishment policy with its profit after the adoption of the inventory reduction policy changes described in Table 5.3. Figure 5.5 shows profit, ROI, and λ for the company operating under both scenarios as the amount of investment in inventory is varied. A comparison of the profit curves shows that the successive application of JIT and substitution strategies yields greater profits than the original policy for inventory investments of less than \$82,462 when JIT replenishment first becomes viable. As well as increasing profitability at lower investment levels, adopting these inventory reduction strategies lowers the minimum investment at which the company's operations become profitable, from approximately \$32,000 to \$8,000. As a consequence of both these factors, the ROI for the company operating under JIT policies is greatly increased with a maximum nearly double that of the unmodified policy. Even with an increased unit or replenishment cost, adoption of these policies would be very appealing to an investor seeking to maximise ROI.

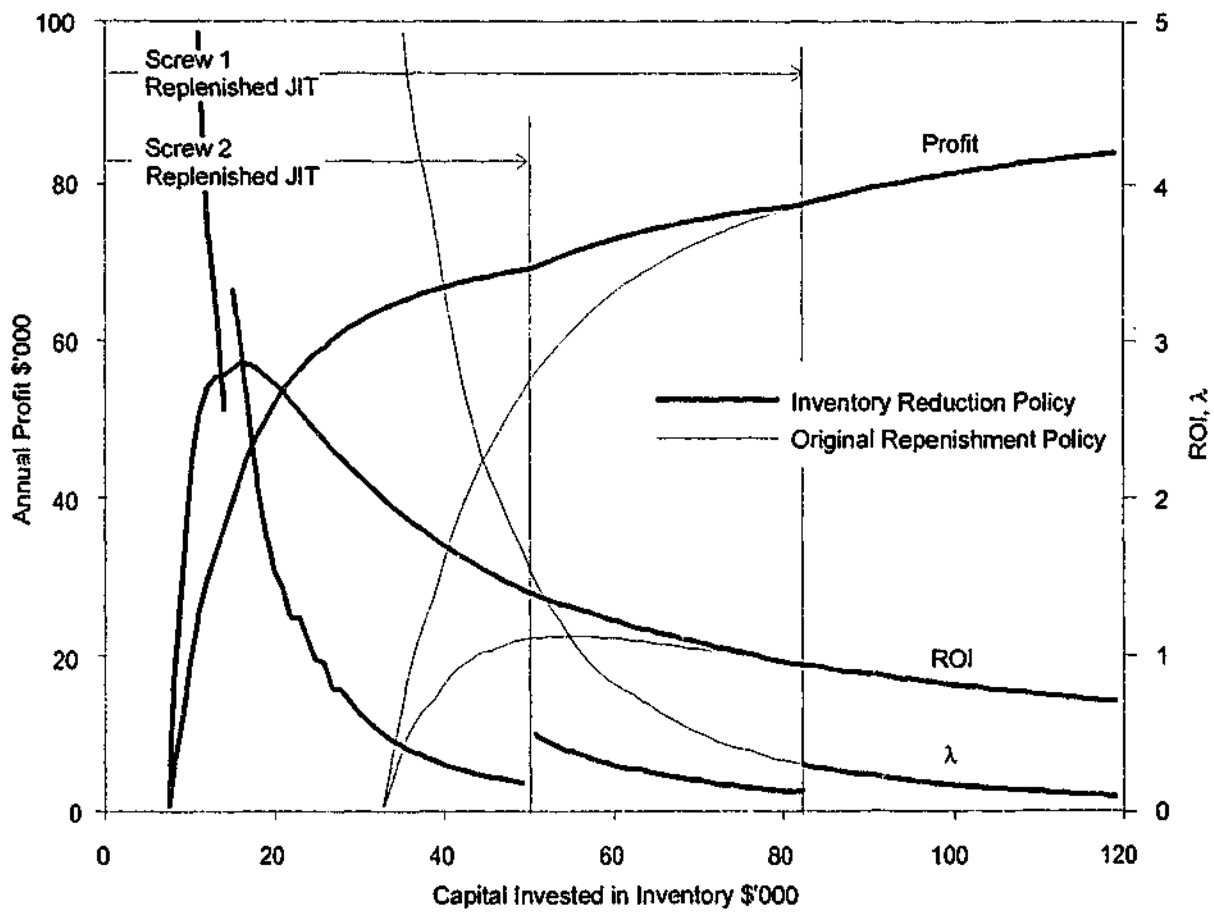

Figure 5.5: Profit, ROI and λ for case study company operating under original investment policy, and having adopted inventory reduction strategies

A comparison of λ under both the original and JIT policies illustrates the mechanism by which the adoption of JIT replenishment works to increase profitability at reduced investment. The policy changes reduce λ, which is evident at \$82,000 and \$50,000 where the effect of the JIT replenishment of Screw 1 and Screw 2 has a large effect. The effect of substitutions is too small to be seen on the graph. Since $\lambda = \frac{dP}{dK}$, a lower λ indicates a smaller increase in profit for a given increase in investment. For a company seeking to increase its investment to maximise the absolute profit level, the larger λ of the batch-oriented policies is beneficial, but conversely, for a company seeking to reduce its investment for ROI maximisation, the smaller λ_{JIT} values lead to a reduced loss in profit for a given decrease in investment, and higher values of ROI. Thus, the adoption of these inventory reduction strategies decreases the sensitivity of the company's profit to decreases in operating capital.

5.4. Conclusion

This chapter extends the results of the previous chapter that considered only the deterministic case and has shown that the cost-effectiveness of JIT replenishment is further enhanced under stochastic demand as a result of the effectiveness of JIT replenishment in addressing uncertainty. In the case of stochastic demand, JIT replenishment of a component permits the redistribution of the investment in both the cycle stock *and* safety stock in order to make the replenishment of the remaining components more efficient. This redistribution of capital becomes an increasingly important benefit of JIT replenishment as capital becomes more limited, and is consistent with the observation that the effective cost of money under constraint is the return on investment of the firm. JIT replenishment also eliminates the risk of lost sales due to a stockout in that component. Finally, by reducing the constraint on inventory, JIT replenishment reduces the overall proportion of capital invested in safety stock. In general terms, inventory reduction through component rationalisation/substitution can be justified in the same way. However the present stochastic analysis shows that component substitution is less effective than JIT replenishment because the increase in demand for the substitute component may result in increased investment in safety stock which partly offsets the savings that result from stocking fewer components .

This chapter has also extended the application of stochastic inventory modelling to the case where components are assembled from stock to order and sold as finished products using a single level bill-of-materials. This new multi-product, batch production model allows for the analysis of the general conditions required for JIT replenishment to be adopted, and forms the basis of a tractable method of calculating the specific investment levels at which inventory reduction policies would be cost-effective under the manufacturing scenario typified by the case study company. The case study example illustrates how a profit-maximising replenishment policy may be determined at any feasible investment level by iteration. The results of this chapter reinforce the idea introduced in Chapter 4: that inventory reduction policies assume greater cost-effectiveness as capital becomes increasingly constrained. The sequence in which replenishment policy changes become profit-maximising (Table 5.3) also reaffirms the previous results, allowing for the differences that the consideration of stochastic demand introduces. (The differences in replenishment policy changes under each model are compared in the concluding chapter.)

A limitation of analysis under the current model, as it applies to the case study company, is that due to the assumptions of the (Q,r) model, demand is treated as being instantaneous whereas, it was noted previously that customers are prepared to wait several days for supply. Thus the current model does not consider the possibility that, for components with a supply lead time less than the customer lead time, batch replenishment orders could be made after the receipt of orders for finished products. Assuming a customer lead time of several days for the case study company, components in this category include the cabinets, the chassis, and piping. The consequence of replenishing in this mode is threefold. Firstly, there is no need to hold safety stock of these components. Secondly, the immediate consumption of one unit of the component on receipt would mean essentially that the inventory level that component was only incremented by $(Q_j - 1)$ at any replenishment with a consequent reduction in the average investment in inventory for that component. Finally, replenishment of components within the customer lead time also has the consequence that there are no lost sales attributable to that component, as observed with JIT replenishment. Whilst all of these factors would free up additional capital for investment, and reduce lost sales, because of the short lead time of these components, the amount of safety stock, and the cost of lost sales attributable to these components under the current model is small. Thus, although these additional considerations may affect the investment levels at which policy changes become profit-maximising, and eliminate the need to consider certain components as candidates for JIT replenishment, their effect on the conclusions of the chapter are not significant.

By assuming a single level bill-of-materials, the present model avoids the largely intractable problem of jointly optimising batch sizing and scheduling decisions, as would be the case under multi-stage manufacture. Whilst the results obtained in this chapter are rigorous for the type of manufacturing performed at the case study company, namely, make-to-order-from-stock, they are less applicable to manufacturing in general for two reasons. Firstly, as for the deterministic demand case, it is assumed that finished products are made to order through a single level bill-of-materials and that the cost of manufacture is fixed whereas staged manufacture may introduce variations in the manufacturing cost. Secondly, multi-stage manufacturing under stochastic demand introduces the interaction between the investment in component inventory, work-in-progress, and finished goods inventory on the consequent risk of lost sales. Thus, under stochastic demand, the cost of lost sales is more sensitive to the structure of the bill-of-materials, and the relative investment in each manufacturing stage. The generality of the model is further limited by the assumption that the probability of a stockout in any component is small in order to justify the approximation in Equation 5.5. Whilst this assumption is valid for the case study company because it manufactures high valued products from a stock of mainly relatively low valued components, this is not indicative of all manufacturing. It is most likely the case that, under other finished product/component cost scenarios, there are other optimal hedges against uncertainty. Nevertheless, the analysis given here of this simpler case offers useful insights into important aspects of these more complex scenarios that relate to the effective use of limited available capital in pursuit of increased profit and ROI.

Chapter 6.

Inventory Replenishment Policy Decisions under the Risk of Failure

The inventory models in previous chapters have assumed that the constraint on total capital invested applies to the average investment in inventory, and that the actual level of inventory investment at any time fluctuates about the average. Under this assumption, the inventory constraint represents the ideal inventory investment (Tersine 1988), but does not actually place a limit on the maximum level that investment in inventory may obtain. By contrast, this chapter introduces an inventory model in which the constraint represents the maximum permitted investment, as would be the case for a self-financed company with a predetermined limit on capital available for investment in inventory. Thus, following the approach taken elsewhere in this thesis in which the investment in a company is only in inventory, the effect of the constraint is to prohibit investment in inventory exceeding a predetermined level. The model derived in this chapter assumes that, if the inventory level exceeds the constraint following the normal process of replenishments, then the company is deemed to have failed. The risk that the company may fail is then introduced into the batch sizing model by adjusting the profit that the company would earn if there were no risk of failure, for the probability that the company may fail over a certain duration. As a consequence, this new constraint requires that, in addition to maximising profit, inventory replenishment policy decisions must also be made with a view to reducing to an acceptable level, the probability that the replenishment policy will result in the capital constraint being exceeded.

The model in this chapter attempts to present the simplest representation of a company failing due to the exhaustion of all available working capital or by exceeding a fixed credit limit. The effect of a binding constraint on the company's operation is that the replenishment of components would be prohibited if this resulted in the total investment in inventory exceeding the constraint. This is exactly the same as the company having insufficient funds for the purchase of component inventory since at any instant the total investment in the company is a mixture of inventory investment and cash available for component replenishment. Where components are critical for manufacture, the inability to replenish may prevent the manufacture of any finished products, leading to a cessation of all production. For the purpose of model building, this event indicates that the company has failed by having insufficient liquid capital. In this sense, the model captures the effect of the company being unable to meet a critical payment, (Welsh and White 1981), or costs as they fall due, (Whyley 1998; Gallagher 1999), both indicating the financial distress of the company, which may lead to a company being wound up as insolvent. Although the model ignores the many other factors that contribute to company failure (which are beyond the scope of this investigation) it introduces a formal relationship between replenishment policy and company liquidity which has not been investigated by previous authors. Furthermore, although it is assumed that a company be profitable in order to survive, it is observed that this does not guarantee survival, as even profitable companies may fail through having insufficient working capital, (Welsh and White 1981; McMahon *et al.* 1993). This suggests that profit and survival be determined as separate (but not completely independent) attributes of any inventory investment decision, as they are in the model which follows.

Previous attempts to incorporate risk into inventory models have been based on the assumption that risk refers only to the volatility of future returns (usually profit), which treats company performance as though it were a stock or other financial instrument. In their review of inventory models incorporating risk, (Anvari and Kusy 1990), describe three approaches, where the intention in setting inventory policy is to maximise any of the following: a linear function of the expected profit and the variance of profit (in which case profit is discounted by a proportion of the standard deviation of profit); the probability of reaching a certain profit target; or a market determined risk adjusted measure of profit. As an example of the last type, (Singhal *et al.* 1994), use the Capital Asset Pricing Model to value the fluctuating cash flow resulting from inventory decisions in order to determine the replenishment quantity and reorder point that minimise the net present value of total costs. Although such approaches do address the variability of company profit over time, a serious limitation is that they do not explicitly consider the risk that a company may fail, often with little or no return to investors, (Hall and Young 1993; Whyley 1998).

The evaluation of company performance, when there is a possibility that it may fail, with little or no return to investors, requires that a new measure be adopted in order to value both profit and risk. It is assumed that the profit that the company earns in any period is constant and that the lifetime of a company has a definite probability distribution. The return that investors would expect to earn from their investment over a certain period of time is therefore earnings-per-period, multiplied by the probability of survival for a cohort of identical investments over the same duration. This is consistent with the standard practice of valuing the return on risky investments at their expected value, (Brigham and Gapenski 1991). The new measure of return is the profit that the company would earn if survival were certain, discounted for the risk of company failure over a fixed period, that is, the *expected* profit. In this chapter the term Risk-Discounted Profit (RDP), is used to denote profit adjusted for the risk of company failure. The term 'expected profit' has not been used in order to make the connection between risk and RDP explicit, and to avoid confusion with other instances where mathematical expectation is used.

In order to accommodate fluctuating levels of inventory investment when the maximum level is subject to a binding constraint, it follows that the company should adopt an inventory replenishment policy determined by an average investment in inventory that is sufficiently lower than the constraint in order to allow the actual inventory fluctuations to remain below the level of the constraint at all times. The total investment in inventory, for the purpose of evaluating ROI for example, is the level of the constraint, (that is, the total available cash) since this capital must always be available for investment in inventory. One straightforward approach to setting replenishment batch sizes would be to assume that components are replenished independently and set the constraint at the maximum level that the inventory could possibly attain. That is, after the simultaneous replenishment of all components, $K = \sum_{j=1}^{m} C_j \left(Q_j + r_j\right)$. However, this approach results in sub-optimal profitability in order to accommodate the very rare occurrence of all components being replenished simultaneously, as is shown subsequently for the case study example in Figure 6.10. Taking this approach, the retention of a large proportion of the investment in inventory, as a buffer against capital exhaustion, is an inefficient use of resources. On the other hand, setting the average investment in inventory too high may lead to an unacceptable risk of company failure. Therefore, for every chosen total investment, there will be a particular average investment that optimises risk adjusted for profit. Using this approach, the model introduced in this chapter permits the calculation of the optimal inventory replenishment policy when the risk of company failure is included. The model then permits the analysis of inventory replenishment policy decisions under risk, where it is shown that the ability to control the amount of fluctuation in inventory level is a previously unexamined benefit of JIT replenishment.

To motivate the later analytical models, the chapter begins with a simulation of the case study company having a fixed maximum investment in inventory, but with replenishment quantities set to maintain varying average investments in inventory, in order to observe profitability and risk of company failure across a range of feasible policies. The profit of simulated companies is discounted for the risk of failure of the cohort at each investment level to determine the RDP-maximising investment strategy. An analytical model for calculating RDP is then introduced as a more efficient means of calculating the expected return on risky investments. The derivation of a model for RDP shows that that every replenishment policy decision needs to be analysed in terms of its effect on profit and risk. The factors affecting a company's decision to adopt JIT replenishment or component substitution for certain components are then analysed. It is shown that the introduction of the risk of failure introduces the change in the company's expected profit as an extra factor into the replenishment policy decision equation. The chapter concludes with an analysis of the optimal replenishment policy for the case study company, at varying investment levels, which illustrates the preceding analysis.

6.1. Constrained total investment in inventory

6.1.1. Design of the simulation trial and parameter settings

This section presents a discrete-event simulation of the manufacturing operations of the company, where finished products are assembled to customer order from an inventory of purchased components. Details of the discrete-event simulation program are given in Appendix C. The specifications of finished products and components were the same as those used in all models presented in the previous chapters. The sale value of finished products and the cost of components are given in Tables 3.1 and 3.3 respectively, the composition of finished products is given in Table 3.2. The demand for each type of finished product was modelled as a Poisson distributed random variable, with annual demand given in Table 3.1.

All simulation trials were run with the simulated company having a maximum permitted investment in inventory of \$100,000. This was a binding constraint on inventory, and replenishments were prohibited if this would result in the constraint being exceeded. The factor that was varied across different trials was the replenishment policy used by simulated companies, which were set to maintain an average investment in inventory at different levels. For the trials reported, the replenishment policies were set to maintain average investments in inventory varying from \$35,000 to \$100,000 in \$5,000 increments. Multiple trials were run at each of these levels. The average investment in inventory determined the values of Q_j and r_j used by the simulated company in each trial. In the case of an average investment of \$60,000, Q_j and r_j were determined from Equations 5.7 to 5.12, with K = \$60,000, in order that the average inventory during the normal course of operations would fluctuate about \$60,000. The Product Master File for this case, showing the values of Q_j and r_j is presented in Appendix C as Table C.2. Each trial began with the inventory level of each batch-replenished component set at its average level (rounded down to the nearest integer), $\left\lfloor \frac{Q_j}{2} + r_j - \bar{x}_j + L(r_j) \right\rfloor$, also shown in Table C.2. Thus, at the commencement of each trial the investment in inventory was $\sum_{j=1}^{m} C_j \left\lfloor \frac{Q_j}{2} + r_j - \bar{x}_j + L(r_j) \right\rfloor$, which approximates the average inventory investment. The difference between the investment in inventory at commencement, and the maximum permitted investment in inventory of \$100,000 thus determined the degree to which the investment in inventory could increase over the normal course of replenishments without exceeding the capital constraint.

In order to eliminate the effect of a company failing during its start-up phase, and to randomise the inventory position of each component between trials, an initialisation period, set at two years of operation (excluding non-production days) was run, with instances of the simulated companies that were still viable (manufacturing) at this stage included in the analysis of the simulation trial. The counting of lifetime and recording of manufacturing output and profit began after this period. Trials were terminated after a further 8 years. In each trial, the proportion of orders that were supplied, profit earned, and the day in which the last machine was manufactured, were recorded. The last day on which a finished machine was manufactured established the duration over which the company had survived. The number of trials at each average investment level was varied in order to obtain a sufficiently large cohort for determining the one-year survival probability at each level, with a greater number of trials where the rate of company failure was greater. In the case of trials where the replenishment policy was set to maintain an average investment in inventory of $100,000, the inability to replenish inventory without exceeding the capital limit of $100,000 meant that no instances of simulated companies survived start-up. (This result was expected, but tested and included in the results for completeness, and for the purpose of comparing with the analytical model in a later section.) The number of trials run under each replenishment policy (based on average investment level) are shown in Table 6.1.

6.1.2. Results and analysis

Figure 6.1 shows a boxplot of annual profit earned by each of the simulated companies still in operation at the termination of the trial. Results are grouped by the average investment that each replenishment policy was set to maintain. Thus the boxplot at $60,000 shows the results for all surviving trials operating with Q_j and r_j set to maintain an average investment of $60,000, as shown in Table C.2. For the purpose of comparison, the profit determined by the constrained multi-item (Q,r) model has also been plotted as a function of the average investment in inventory from Equation 5.7 subject to the constraint in Equation 5.8. At an average investment of between $35,000 and $80,000, the median profit obtained by simulation is close to that determined theoretically by the (Q,r) model (see Equation 5.7). It is evident, however, that at very low investment levels, the theoretical model overestimates the cost of lost sales. As the investment in inventory increases, the company's profit increases, with additional investment in inventory increasing the economy with which components are batch-replenished. The profit of simulated companies is close to that determined theoretically at $85,000, although the profit at this level is based on a very small cohort of three surviving companies. At average investment levels of $90,000 and $95,000, no simulated companies survived until the cessation of the trial.

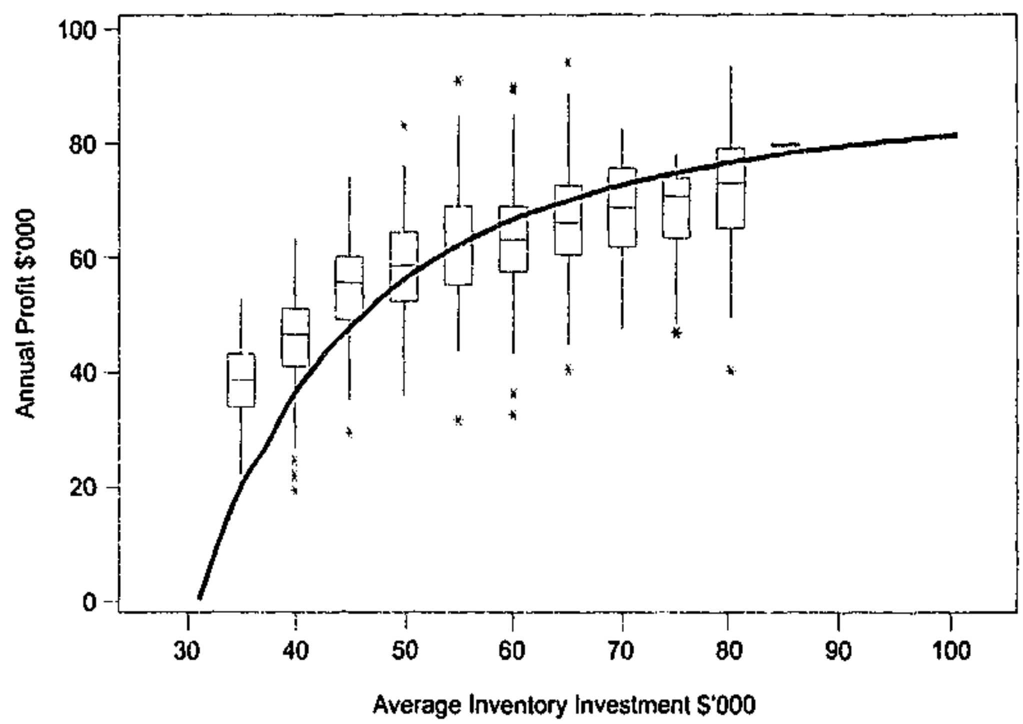

Figure 6.1: Annual profit for simulated companies at varying average investment levels and determined by the constrained multi-item (Q, r) model

The previous analysis of company performance was based on data gathered from simulated companies that survived until the end of the trial. Thus, the measures of company performance used so far only report on successfully operating simulated companies. At all investment levels however, a proportion of simulated companies ceased production before their trial was terminated because they had insufficient capital for replenishing components that were critical for manufacture. In the following analysis, these simulated companies are deemed to have failed through having exhausted available capital. Because a certain proportion of companies failed during each trial, the measurement of profit presented previously overestimates the return that an investor could expect as it does not account for the likelihood that a certain proportion of companies will fail and yield no return to an investor.

Table 6.1: The number of simulated companies surviving each year and the one-year survival probabilities for simulated companies

Average Investment in Inventory	Number of Simulated Companies Surviving Start-up	Number of simulated companies surviving each year. One-year survival probability indicated by ( ).								Average One-Year survival probability
		1	2	3	4	5	6	7	8	
$35,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1.00
		(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	
$40,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1.00
		(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	
$45,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1.00
		(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	
$50,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1.00
		(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	
$55,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1.00
		(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	
$60,000	1,000	1,000	998	998	998	997	992	991	990	1.00
		(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(0.99)	(1.00)	(1.00)	
$65,000	5,000	4,957	4,940	4,904	4,854	4,817	4,757	4,707	4,684	0.99
		(0.99)	(1.00)	(0.99)	(0.99)	(0.99)	(0.99)	(0.99)	(1.00)	
$70,000	5,000	4,669	4,386	4,081	3,835	3,601	3,349	3,135	2,945	0.94
		(0.93)	(0.94)	(0.93)	(0.94)	(0.94)	(0.93)	(0.94)	(0.94)	
$75,000	5,000	4,084	3,148	2,519	1,933	1,526	1,180	926	692	0.79
		(0.82)	(0.77)	(0.80)	(0.77)	(0.79)	(0.77)	(0.78)	(0.75)	
$80,000	5,000	3,014	1,596	934	514	285	163	105	67	0.57
		(0.60)	(0.53)	(0.59)	(0.55)	(0.55)	(0.57)	(0.64)	(0.64)	
$85,000	5,000	1,912	666	226	91	33	17	7	3	0.39
		(0.38)	(0.35)	(0.34)	(0.40)	(0.36)	(0.52)	(0.41)	(0.43)	
$90,000	10,000	2,990	1,000	302	104	27	7	1	-	0.30
		(0.30)	(0.33)	(0.30)	(0.34)	(0.26)	(0.26)	(0.14)	-	
$95,000	20,000	6,369	2,136	417	112	19	4	1	-	0.25
		(0.32)	(0.34)	(0.20)	(0.27)	(0.17)	(0.21)	(0.25)	-	
$100,000	-	-	-	-	-	-	-	-	-	0.00
		-	-	-	-	-	-	-	-	

The one-year survival probability, or probability that a simulated company, in operation at the commencement of one year is still in operation at the end of the year, is derived from the data for each investment level and given in Table 6.1. For each average investment level, the one-year survival probability is constant over successive years, these are shown in Figure 6.2 as a function of the average investment in inventory. When the average investment in inventory is low, there is sufficient buffer capital to make it very unlikely that capital is exhausted. As the average investment increases and the buffer is reduced, the risk of company failure increases, with no probability of company survival at an average investment of $100,000.

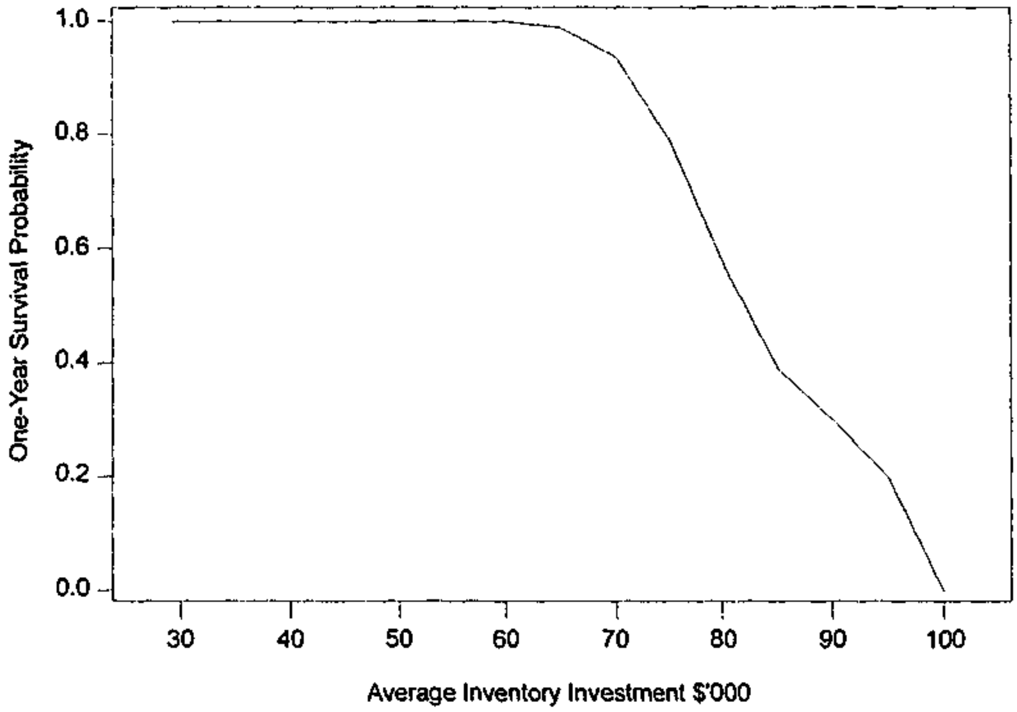

Figure 6.2: One-year survival probability for simulated companies as a function of average investment in inventory

The expected, or risk-discounted, profit of simulated companies is determined by multiplying the profit of each simulated company by the average one-year survival probability of the cohort operating under replenishment policies set to maintain the same average inventory. The choice of evaluating the probability of survival over one year is arbitrary, but consistent with the evaluation of other measures of performance on an annual basis. Figure 6.3 shows risk-discounted profit *per annum* as a function of the average investment in inventory. The data for Figure 6.3 was derived by multiplying the profit of surviving companies under each replenishment policy (used to produce Figure 1) by the one-year survival probability of the cohort. For example, the profit of companies operating with a replenishment policy set to maintain an average investment in inventory of $80,000 are multiplied by 0.57. The reduction in profit is apparent for investments at greater average investments. The average investment in inventory of $65,000 has the greatest median RDP as the increased probability of company failure at investments greater than this outweighs the economy resulting from an increased average investment in inventory. At investment levels of $80,000 and $90,000, the high probability of company failure renders operations based on such large average inventories highly infeasible.

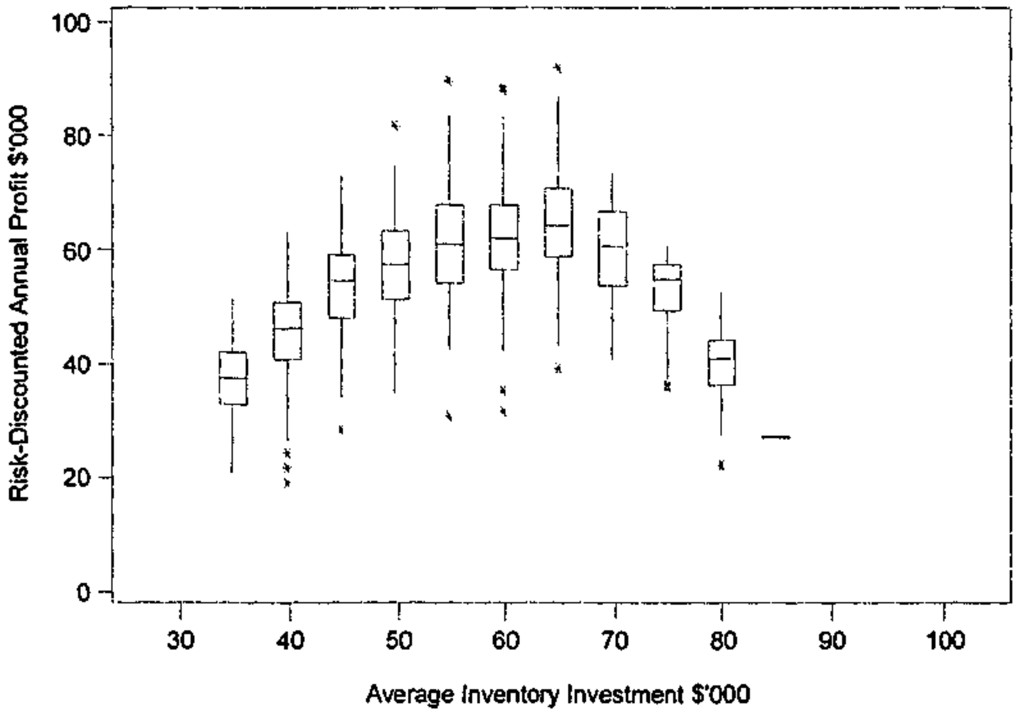

Figure 6.3: Annual risk-discounted profit for simulated companies as a function of the average investment in inventory

6.1.3. Summary and discussion of results

The simulation study illustrates that, when the constraint on total inventory level is binding, additional capital is required to permit the fluctuation of inventory above the average level in order to realise the service level and profit determined theoretically by the constrained (Q,r) model. The study has also shown the sensitivity of RDP to the choice of average investment in inventory. When the investment in average inventory is low, profit is low due to the increased cost of small batch replenishments, despite a high probability of survival. At the other extreme, the benefit of reduced replenishment costs at increased average investment are obtained at the cost of a greater risk of company failure due to capital exhaustion. In this study, the RDP-maximising total investment of $100,000 required that the average investment in inventory be $65,000. In this section, the optimal investment policy was determined from an analysis of results obtained by the computationally-intensive process of discrete-event simulation. In the following sections a more efficient analytical method for determining RDP is proposed to calculate the optimal average investment in inventory at any total investment in order to analyse RDP-maximising inventory replenishment policies at varying investment levels.

6.2. A batch sizing model

The risk-discounted profit, RDP, for a total investment of K^*, of which $\overline{K}$ is the average investment in inventory, is determined by maximising

$$P = \sum_{i=1}^{n} A_i S_i - \sum_{j=1}^{m} D_j C_j - \sum_{j=1}^{m} \frac{D_j}{Q_j}\left(R_j + B_j L(r_j)\right) - J \sum_{j=1}^{m} C_j \left(\frac{Q_j}{2} + r_j - \overline{x}_j + L(r_j)\right) - F \quad (6.1)$$

subject to the constraint

$$\overline{K} = \sum_{j=1}^{m} C_j \left(\frac{Q_j}{2} + r_j - \overline{x}_j + L(r_j)\right), \quad (6.2)$$

(compare with Equations 5.7 and 5.8). The constraint is now denoted by $\overline{K}$ to indicate that the inventory policy settings are determined by the average investment. Using $\wp$ to represent the probability of company survival over a certain duration t, risk-discounted profit is defined as

$$RDP = \wp\left(K^*, \overline{K}, t\right) P. \quad (6.3)$$

Although it will be shown subsequently that Q_j and r_j are also arguments of $\wp$, these have been omitted for notational clarity. In the following calculation of RDP, the iterative calculation of the optimal values of Q_j and r_j for a given value of $\overline{K}$, requires that they be calculated independently of $\wp$. Thus, to avoid the intractable problem of jointly optimising Q_j, r_j and $\wp$, the following model ignores the interaction between batch sizing and the subsequent risk of failure. The calculation of RDP requires that the function defining $\wp$ be derived.

6.3. The survival probability

In this section, $\wp$ is derived for the case of an assemble-to-order manufacturer having a fixed, maximum investment in component inventory. The model determines the risk of company failure by requiring a replenishment order that would lead to the maximum permitted investment in inventory being exceeded. It is assumed that the demand for components is sufficiently independent, and that the inventory consists of enough components to allow the fluctuation of the total investment in inventory over time to be treated as a random process. It then follows that the company lifetime, or duration until the inventory level is exceeded, is a random variable. The probability of the company surviving over any duration can then be determined from the resulting lifetime distribution.

The model development begins by describing the underlying assumptions. The probability of company survival is derived when demand for components is deterministic. The model is then generalised to include the case where demand for components is stochastic, and the case where total inventory includes components that are replenished JIT. The method by which the RDP-maximising investment is evaluated at any investment level is then described. The RDP obtained using the model presented in Section 2 is then compared with the results obtained in the simulation trial of the previous section and shows a close agreement.

6.3.1. Assumptions and approximations

The lifetime of a company is defined as the duration over which it operates with the total investment in inventory remaining below a pre-determined maximum denoted by K^*. The company's operations terminate when a replenishment order is required that would result in the capital constraint being exceeded. The only assumption made about the initial level of inventory investment in any component is that the total investment does not exceed the constraint. Figure 6.4 shows the total inventory level obtained from one simulation of the case study company, indicating the maximum investment level permitted, and illustrating the simulated company operating over a lifetime of T_{K^*} days. Operations cease when a replenishment order that would cause inventory investment level to exceed K^* is required.

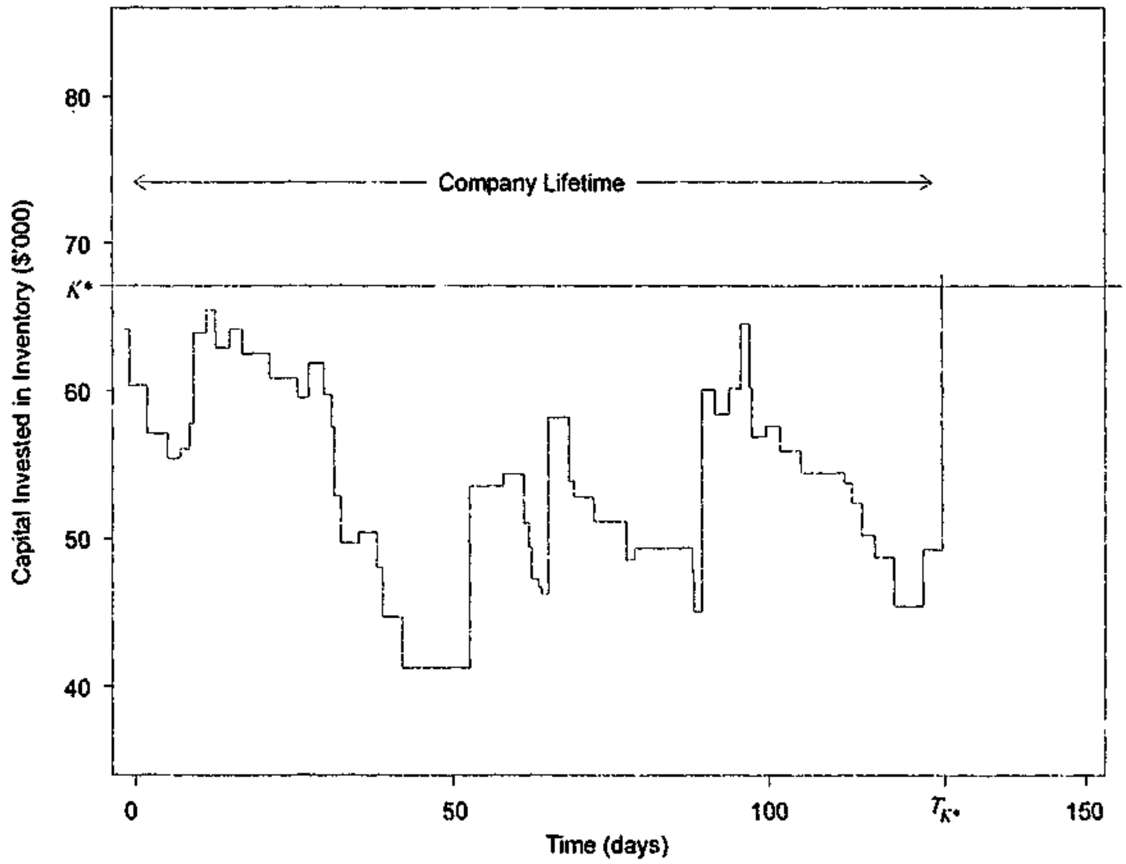

Figure 6.4: The total investment in inventory for a simulated company showing the company's operations terminating when a replenishment order exceeds the capital constraint

In modelling the level of investment in inventory over time it is assumed that the rate at which total inventory is decremented as components are consumed for manufacture is equal to the rate at which total inventory is incremented when components are replenished over the long term. This results in the average inventory level remaining constant over time, having value $\overline{K}$, and is consistent with a company adhering to an inventory control program consisting of planned replenishments. As a consequence of these assumptions, the term 'buffer capital', which has had an informal interpretation until this stage, can now be defined more formally as $K^* - \overline{K}$, that is the difference between the average and the maximum permitted investment in inventory. In addition, because inventory is incremented and decremented in whole units, according to a planned replenishment policy, it is assumed that the investment in inventory, when observed at a random instant, is a discrete random variable with finite upper and lower bounds.

Because the inventory consists of many components having different replenishment quantities and/or aggregate demand, the inventory levels of all components are uncorrelated over the long term. Although, over a short duration, the inventory level of any pairs of components may be correlated as a consequence of each experiencing a similar sequence of decrements, the independence of each component's replenishment cycle prevents correlation prevailing over a long duration (see, for example, Components 1 and 2 in Figure 6.5).

The model assumes that a company fails when sufficient coinciding or closely occurring replenishment orders result in inventory investment exceeding K^*. It follows from this definition that failures only occur at instants when a component is replenished. Exploiting this property, the following analysis assumes that company lifetime is composed of a sequence of time intervals between the successive replenishment of *any* components, terminating when a replenishment would result in K^* being exceeded. Although the inventory replenishment cycles of all components may be deterministic, when the inventory consists of sufficient components with uncorrelated inventory levels, the instant at which any component is replenished, coincides with a random stage of the replenishment cycle of all other components. This assumption requires leads to two approximations. Firstly, at the instant that any component is replenished the inventory levels of all other components, and hence the total investment in inventory, can be approximated by a random variable. Secondly, the duration until the next replenishment of any component occurs can also be approximated by a random variable. (see Figure 6.5). Together, these approximations permit the approximation by a random process of what is essentially an aggregation of multiple uncorrelated deterministic processes.

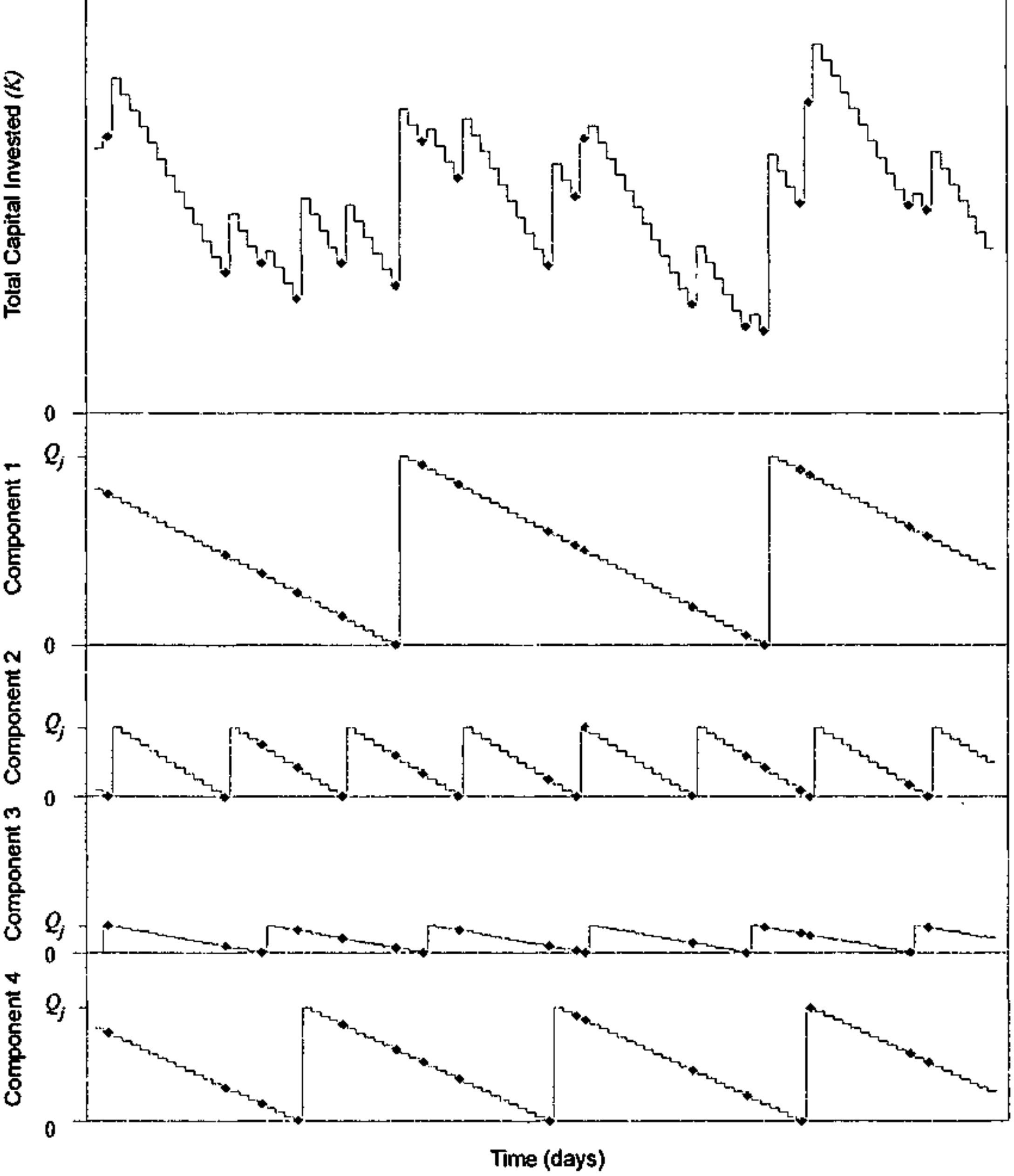

Figure 6.5: The variation in the total investment in an inventory consisting of four components under deterministic demand. The inventory level of each component and total investment has been highlighted at instances where any component is replenished.

Company lifetime, T_{K^*}, is treated as being composed of w intervals between successive replenishments, each of duration ΔT_i, and incrementing the investment in inventory by ΔK_i. The inventory level immediately before the i^{th} replenishment is received is denoted by K_i (see Figure 6.6). Because the inventory process is approximated by a random process, as described earlier, K_i, ΔK_i, ΔT_i and hence T_{K^*} are random variables. Company operations terminate when a replenishment order of magnitude ΔK_w results in the limit on total capital invested being exceeded. No assumptions are made about the distribution of ΔK_i except that, when the inventory consists of a large number of batch-replenished components, $\Delta K_i << K^*$. No assumption about the distribution of ΔT_i is made. However it is assumed that ΔT_i is uncorrelated with either K_i or ΔK_i.

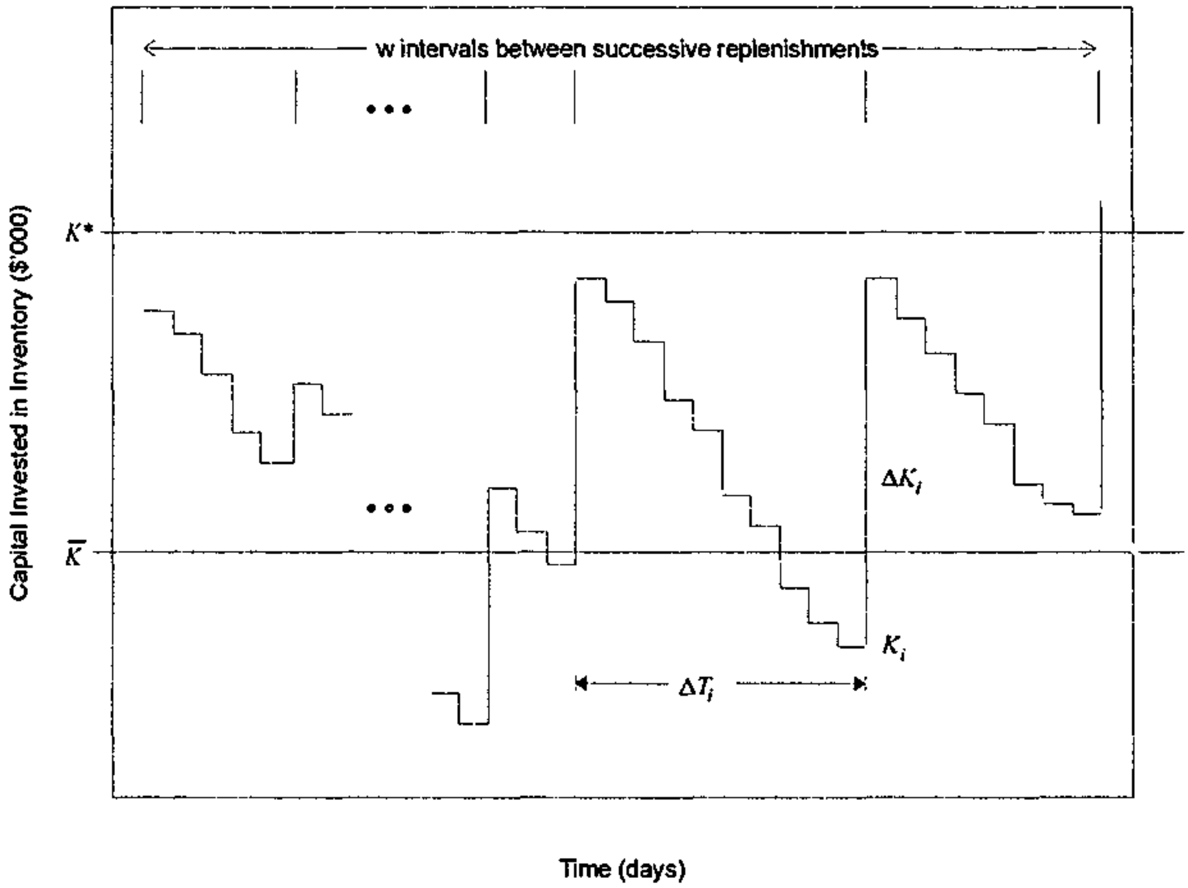

Figure 6.6: Capital invested in inventory over the lifetime of a company, showing that lifetime is composed of w replenishment cycles

The model assumes that the investment in inventory is reviewed continuously, which prohibits investment in inventory exceeding K^* at any time. This approach assumes that financial transactions occur at the same time as each physical transaction, that is, as components are consumed or replenished, which would be the case if the company only replenished components and sold finished products on a cash-on-delivery basis. In Appendix A, the model is further developed for the case when the inventory level is reviewed periodically, which permits investment in inventory to exceed K^* at all times except at review points, as would be the case if the company made a periodic reconciliation of accounts. There it is shown that under periodic review, the probability of company failure is reduced, although the behaviour of the RDP model, and the conclusions obtained from the present analysis, are unchanged.

6.3.2. Survival probability under deterministic demand

When demand is deterministic, the inventory level of any single component over time is cyclical, with inventory levels decreasing at a constant rate until all stock of the component is exhausted, at which point, the inventory is replenished by the reorder quantity, Q_j. (see Fig 6.5) Because the inventory level of each component, q_j, assumes each of the values from 0 to Q_j for an equal proportion of time, then q_j, when observed at random intervals (such as when the replenishment of other components occur), is a discrete uniformly distributed random variable having the probability distribution function, such that

$$f_j(q_j) = \frac{1}{Q_j + 1} \quad for \quad q_j = 0,1,\ldots,Q_j, \tag{6.4}$$

with mean

$$\langle q_j \rangle = \frac{Q_j}{2}, \tag{6.5}$$

and variance

$$Var(q_j) = \frac{Q_j^2 + 2Q_j}{12}. \tag{6.6}$$

Consequently, the mean and variance of the investment in inventory, k_j, for each batch-replenished component are given by

$$\langle k_j \rangle = \frac{Q_j C_j}{2} \tag{6.7}$$

and

$$Var(k_j) = \left(\frac{Q_j^2 + 2Q_j}{12}\right)C_j^2. \tag{6.8}$$

Because the inventory consists of a large number of components having uncorrelated inventory levels, the total amount of capital invested in inventory can be approximated by a normally distributed random variable, denoted by $f_K(K)$, having mean

$$\langle K \rangle = \sum_{j=1}^{m} \frac{Q_j C_j}{2} \tag{6.9}$$

and variance

$$Var(K) = \sum_{j=1}^{m} \left(\frac{Q_j^2 + 2Q_j}{12}\right)C_j^2, \tag{6.10}$$

which introduces the additional approximation of the discrete bounded distribution for K by a continuous, unbounded distribution. This approximation now permits $f_K(K)$ to be determined for any K. Additionally, the probability is very small that K, under this approximation, exceeds the true maximum level that inventory could possibly obtain, $\sum_{j=1}^{m} C_j Q_j$, if all components were replenished simultaneously. Consequently, this approximation has negligible effect on the conclusions drawn from the model.

As illustrated in Figure 6.6, the lifetime of a company consists of a sequence of w intervals between successive replenishments, terminating when $K_w + \Delta K_w > K^*$. The value of w is thus a random variable with distribution determined by the probability that inventory does not exceed K^* after each replenishment. Denoting the probability that total investment in inventory does not exceed K^* at the i^{th} replenishment by ρ_i, where

$$1-\rho_i = P\left(K_i + \Delta K_i > K^* \mid K_i < K^*\right), \tag{6.11}$$

the probability distribution function for w is given by

$$f_w(w) = \prod_{i=1}^{w-1} \rho_i \left(1-\rho_w\right). \tag{6.12}$$

An approximation for $f_w(w)$ is obtained by replacing each ρ_i by its expected value, giving

$$f_w(w) \approx \langle \rho_i \rangle^{w-1} \langle 1-\rho_n \rangle, \tag{6.13}$$

which is PDF of a geometric distribution having mean

$$\langle w \rangle = \frac{1}{\langle 1-\rho_i \rangle}. \tag{6.14}$$

The replacement of each ρ_i with $\langle \rho_i \rangle$ represents the major approximation of the model. The error introduced into the value of w by the approximation vanishes for $\rho_i \to 1$ or $\rho_i \to 0$. It will be shown in a later section that it is only values of $\rho_i \to 1$ that yield survival probabilities large enough to be of practical significance in determining an RDP-maximising investment policy. Consequently, the error introduced by the approximation is minimised for combinations of $\overline{K}$ and K^* that approach optimality. Thus, the conclusions drawn from the model are largely unaffected by the approximation.

The evaluation of $\langle w \rangle$ requires that $\langle 1-\rho_i \rangle$ be determined. From Equation 6.11,

$$\begin{aligned} 1-\rho_i &= \frac{P\left(K^* - \Delta K_i < K_i < K^*\right)}{P\left(K_i < K^*\right)} \\ &= \frac{F_K\left(K^*\right) - F_K\left(K^* - \Delta K_i\right)}{F_K\left(K^*\right)}. \end{aligned} \tag{6.15}$$

When the total inventory consists of a large number of components, any single increment in inventory level will be small relative to the maximum permitted investment, that is, $\frac{\Delta K_i}{K^*} << 1$. The approximation of the total investment in inventory by a continuous distribution permits Equation 6.15 to be approximated as

$$1-\rho_i = \frac{f_K\left(K^*\right)\Delta K_i}{F_K\left(K^*\right)} + O\Delta K_i^2. \tag{6.16}$$

Because the approximated distribution of capital invested in inventory is continuous and theoretically unbounded, ρ_i can be evaluated for all K_i.

The expected value of $(1-\rho_i)$ is

$$\langle 1-\rho_i \rangle = \frac{f_K\left(K^*\right)}{F_K\left(K^*\right)} \langle \Delta K_i \rangle. \tag{6.17}$$

Substituting Equation 6.17 into Equation 6.14 gives

$$\langle w \rangle = \frac{F_K\left(K^*\right)}{f_K\left(K^*\right)\langle \Delta K_i \rangle}. \tag{6.18}$$

The expected value of T_{K^*} is determined by recalling that

$$T_{K^*} = \sum_{i=1}^{w} \Delta T_i , \qquad (6.19)$$

thus

$$T_{K^*} = w \langle \Delta T_i \rangle . \qquad (6.20)$$

Taking expected values and diving by $\langle \Delta T_i \rangle$ gives

$$\langle w \rangle = \frac{\langle T_{K^*} \rangle}{\langle \Delta T_i \rangle} . \qquad (6.21)$$

Equating 6.12 and 6.14 gives

$$\langle T_{K^*} \rangle = \frac{F_K(K^*)\langle \Delta T_i \rangle}{f_K(K^*)\langle \Delta K_i \rangle} . \qquad (6.22)$$

It was shown in Equation 6.13 that w could be approximated as a geometrically distributed random variable. Because ΔT_i is uncorrelated with w and of an indeterminate distribution, it follows that T_{K^*} is also geometrically distributed, with the mean given by Equation 6.22. Adopting the exponential distribution as the continuous analogue of the geometric distribution for the calculation of survival probabilities, (Evans *et al.* 1993), the probability that a company survives over t periods when T_{K^*} is known is

$$\wp(t; T_{K^*}) = p(T_{K^*} > t) = e^{-t/\langle T_{K^*} \rangle} , \qquad (6.23)$$

which permits $\wp$ to be expressed as a function of the original parameters

$$\wp\left(K^*,\overline{K},t\right)=e^{-t\frac{f_K\left(K^*\right)\left\langle\Delta K_i\right\rangle}{F_K\left(K^*\right)\left\langle\Delta T_i\right\rangle}}\,. \tag{6.24}$$

It is noteworthy that the exponential distribution of company lifetime under this model is consistent with the constant rate of failure for simulated companies shown in Table 6.1, and accords with the observation that real companies fail at a constant rate (Pattinson and Tozer 1997).

The evaluation of $\wp$ now requires that $\dfrac{\left\langle\Delta K_i\right\rangle}{\left\langle\Delta T_i\right\rangle}$ be determined. Figure 6.6 shows that ΔK_i measures the downward drift in capital invested in inventory as components are consumed from inventory for the manufacture of finished products over the interval ΔT_i. Consequently, $\dfrac{\left\langle\Delta K_i\right\rangle}{\left\langle\Delta T_i\right\rangle}$ represents the average rate at which the capital invested in inventory decreases as batch-replenished components are consumed for manufacture. Because the capital invested in inventory maintains a constant average level over the long term, the rate at which capital is invested in inventory (as batch-replenished component orders are received) is equal to the rate at which capital is consumed for manufacture.

Define T as any arbitrary time period consisting of a sequence of n replenishment intervals such that

$$\sum_{i=1}^{n}\Delta T_i=T \tag{6.25}$$

then

$$n\left\langle\Delta T_i\right\rangle=T\,. \tag{6.26}$$

Over the corresponding time interval, the amount expended on components, that is the cost of goods consumed from inventory, $C_{COGI}(T)$, is

$$\sum_{i=1}^{n} \Delta K_i = C_{COGI}(T) \tag{6.27}$$

and

$$n\langle \Delta K_i \rangle = C_{COGI}(T). \tag{6.28}$$

Equating 6.17 and 6.18 gives

$$\frac{\langle \Delta T_i \rangle}{\langle \Delta K_i \rangle} = \frac{T}{C_{COGI}(T)}. \tag{6.29}$$

Adopting one year $(T = 1)$ as a convenient time frame, and noting that $C_{COGI}(1)$ corresponds to the sum of the annual demand for each component multiplied by its cost gives

$$\frac{\langle \Delta T_i \rangle}{\langle \Delta K_i \rangle} = \frac{1}{\sum_{j=1}^{m} C_j D_j}, \tag{6.30}$$

which permits the calculation of $\wp$ using Equations 6.9, 6.10 and 6.24.

6.3.3. Survival probability under stochastic demand

The evaluation of $\wp$ is now extended to the stochastic case when the inventory is replenished following the continuous review (Q, r) model. The model developed in this section assumes that customer orders not supplied when due are lost, as described in Chapter 5. The assumption of stochastic demand introduces several differences to the deterministic case. Firstly the distribution of capital invested in inventory must account for safety stock being held, the effect of lost sales and the variability of demand through lead time on inventory level. The possibility of lost sales also reduces the rate of replenishment. Because the demand for components from inventory is now stochastic, T_{K}. *is* a random variable with a value determined by the sequence of demand for components in addition to the random duration of intervals between successive replenishments.

Under a continuous review policy, for each batch-replenished component j, the reorder quantity Q_j, is ordered when inventory level falls to r_j. Demand through lead time for each type of component is stochastic, with discrete valued probability distribution $g(x_j)$, having mean $\bar{x}_j$ and variance denoted by $Var(x_j)$. The expected number of components not supplied in each order cycle is given by the partial expectation of r_j, $L(r_j)$.

Figure 6.7 shows the inventory level of a component under stochastic demand. The inventory consists of a series of separate cycles, with each beginning when the inventory level reaches the reorder point. The minimum level of stock in each replenishment cycle occurs immediately before a replenishment order is received, and is a random variable defined as $\max\left[r_j - g(x_j),0\right]$. The maximum inventory level occurs immediately after replenishment and has magnitude $\max\left[r_j - g(x_j),0\right] + Q_j$. Thus in each replenishment cycle the inventory level assumes each of the discrete values from 0 to Q_j units above a base level of $\max\left[r_j - g(x_j),0\right]$. Based on this observation, the amount of stock held in inventory can be treated as the sum of two random variables, one determining the minimum stock level in each cycle and a second describing the distribution of inventory above the minimum in each cycle.

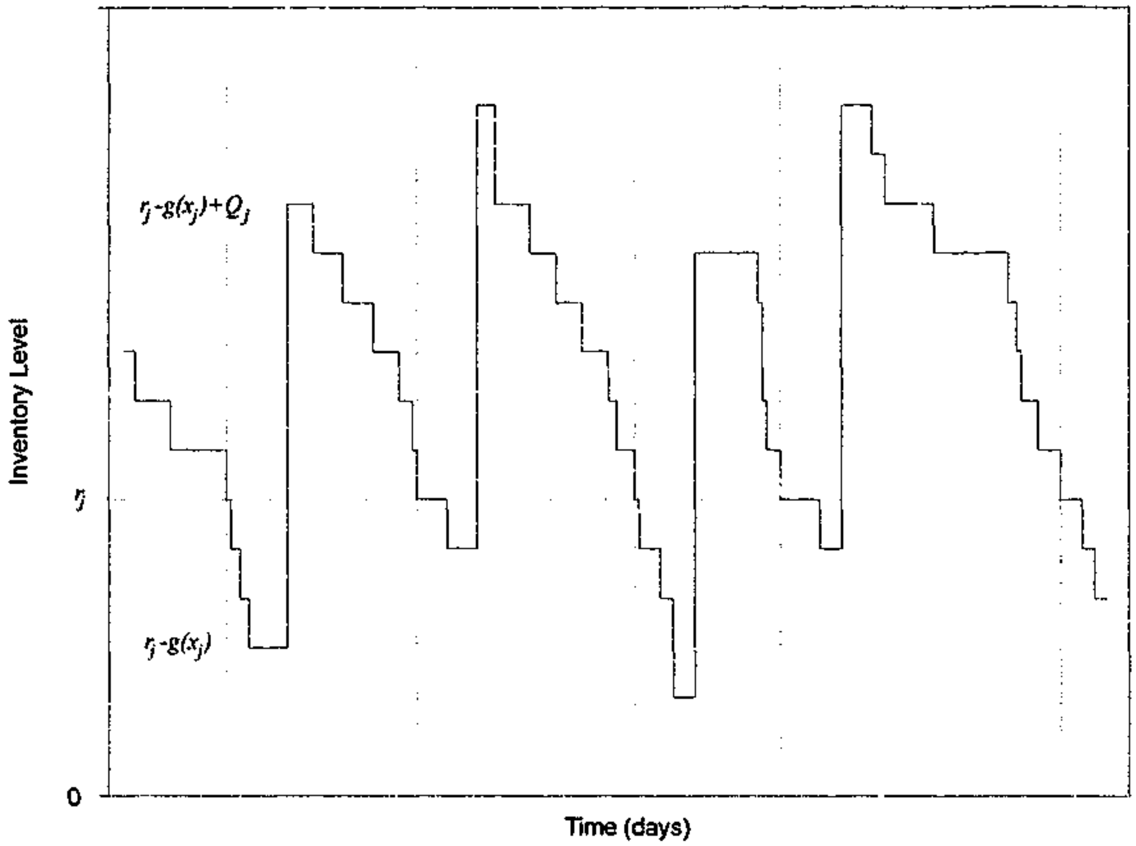

Figure 6.7: Inventory level of a generic component when demand is stochastic with replenishments determined by a (Q, r) policy

The expected value of the minimum stock level is given by the safety stock, $r_j - \bar{x}_j + L(r_j)$, where the term $L(r_j)$ represents the increase in average inventory level that results from lost sales. The variance of the minimum stock level is determined by the variance of the demand through lead time denoted by $Var(x_j)$. The amount of stock above the minimum level during any replenishment cycle has a discrete distribution assuming the values from 0 to Q_j. Because the intervals between each change in component level are uncorrelated with inventory level, the distribution of inventory level above the base level in any cycle, when observed at random times, has the same discrete uniform distribution described in the deterministic case with mean and variance given by Equations 6.5 and 6.6 respectively.

The mean and variance of the inventory level for each batch-replenished component are the sum of, respectively, the means and variances of the random variables describing the minimum stock level in any cycle, and the variation within each replenishment cycle. Consequently, the mean and variance of the investment in inventory for each component, K_j, are given by

$$\langle k_j \rangle = \left(r_j - \bar{x}_j + L(r_j) + \frac{Q_j}{2} \right) C_j \tag{6.31}$$

$$Var(k_j) = \left(Var(x_j) + \frac{Q_j^2 + 2Q_j}{12} \right) C_j^2 . \tag{6.32}$$

For a company stocking many components with independent inventory levels, the total capital invested in inventory can be approximated by a normal distribution having the following mean and variance.

$$\langle K \rangle = \bar{K} = \sum_{j=1}^{m} \left(r_j - \bar{x}_j + L(r_j) + \frac{Q_j}{2} \right) C_j \tag{6.33}$$

$$Var(K) = \sum_{j=1}^{m} \left(Var(x_j) + \frac{Q_j^2 + 2Q_j}{12} \right) C_j^2 . \tag{6.34}$$

The probability of a lost sale attributable to any component in any reorder cycle is $\left(1 - \frac{L(r_j)}{Q_j}\right)$ (see Section 5.1.2.). Using the same approach to derive Equation 6.30, $\frac{\langle \Delta K_i \rangle}{\langle \Delta T_i \rangle}$ is found to be

$$\frac{\langle \Delta K_i \rangle}{\langle \Delta T_i \rangle} = \sum_{j=1}^{m} C_j D_j \left(1 - \frac{L(r_j)}{Q_j} \right), \tag{6.35}$$

showing that the value of $\frac{\langle \Delta K_i \rangle}{\langle \Delta T_i \rangle}$ under stochastic demand is less than would be the case if demand were deterministic as a result of lost sales of finished products. The calculation of $\wp$ is now the same as for the deterministic case, with the condition that $f_K(K^*)$ and $F_K(K^*)$ have mean and variance given by Equations 6.33 and 6.34 respectively, and $\frac{\langle \Delta K_i \rangle}{\langle \Delta T_i \rangle}$ in Equation 6.35.

6.3.4. JIT replenishment

Because JIT components are supplied within the customer lead time and are used immediately, they are never held in inventory and thus should not be included in the calculation of either the distribution of capital invested in inventory or the rate at which capital is consumed from inventory for the manufacture of finished products. Consequently, Equations 6.33, 6.34 and 6.35 can be expressed more generally to include the case of a company utilising both batch-replenished and JIT-replenished components as

$$\langle K \rangle = \overline{K} = \sum_{j=1}^{m} \left(r_j - \overline{x}_j + L(r_j) + \frac{Q_j}{2} \right) C_j \delta_j \tag{6.36}$$

$$Var(K) = \sigma_K^2 = \sum_{j=1}^{m} \left(Var(x_j) + \frac{Q_j^2 + 2Q_j}{12} \right) C_j^2 \delta_j \tag{6.37}$$

and

$$\frac{\langle \Delta K_i \rangle}{\langle \Delta T_i \rangle} = \sum_{j=1}^{m} C_j D_j \left(1 - \frac{L(r_j)}{Q_j} \right) \delta_j \tag{6.38}$$

$$where\ \delta_j = \begin{cases} 0 & if\ component\ replenished\ JIT \\ 1 & otherwise. \end{cases}$$

6.3.5. The complete RDP calculation

For a company with a total investment of K^*, of which $\overline{K}$ set as the average investment in inventory, and a review period over which the probability of failure is determined of t periods, $RDP\left(\overline{K}, K^*, t\right)$ is calculated in the following stages, as outlined in Figure 6.8.

1 Annual profit, P, and the profit maximising values of Q_j and r_j are determined from Equations 6.1 and 6.2 by iteration.

2 The variance of the normal approximation for the amount of capital in inventory is evaluated as $\sigma_{\overline{K}}^2 = \sum_{j=1}^{m}\left(Var\left(x_j\right) + \frac{Q_j^2 + 2Q_j}{12}\right)C_j^2\delta_j$.

3 The rate at which capital is consumed from inventory is calculated as $\frac{\langle \Delta K_i \rangle}{\langle \Delta T_i \rangle} = \sum_{j=1}^{m} C_j D_j \left(1 - \frac{L(r_j)}{Q_j}\right)\delta_j$.

4 The probability of the company surviving over duration t is $\wp\left(K^*, \overline{K}, t\right) = e^{-t\frac{f_K\left(K^*\right)\langle \Delta K_i \rangle}{F_K\left(K^*\right)\langle \Delta T_i \rangle}}$.

5 $RDP = \wp\left(K^*, \overline{K}, t\right)P$.

Figure 6.8: The calculation of RDP

The calculation of the RDP-maximising inventory policy for a certain investment, K^*, requires that P and $\wp$ be jointly optimised. It is evident, however, from the calculation scheme in Figure 6.8, that both P and $\wp$ cannot be simultaneously determined. However, both P and $\wp$ are dependent on $\overline{K}$, which permits a near optimal solution for RDP to be obtained exhaustively in the following case study by evaluating RDP for values of $\overline{K} < K^*$ at discrete intervals, following the process described in Figure 6.8.

Figure 6.9 shows the RDP calculated according to the method described in Figure 6.8 for values of $\overline{K}$ at intervals of \$1,000, superimposed on the boxplot of RDP obtained in the simulation study of Section 6.1. The close agreement between the theoretical and simulated values in the region where RDP is maximised, that is, for an average investment in the region of \$60,000 - \$70,000, is evident. RDP is actually maximised at an average investment in inventory of \$63,000. Table D.1 in Appendix D shows that the goodness of fit between the simulated results and the theoretical model is statistically significant when the average investment in inventory is \$60,000 and \$65,000. The discrepancy between the theoretical and simulated values for investments below \$60,000 is due to the (Q,r) model in Equation 5.7 overestimating the cost of lost sales, as previously discussed. The discrepancy at investments above \$70,000 is due to the approximation of ρ_i by $\langle \rho_i \rangle$ in Equation 6.13.

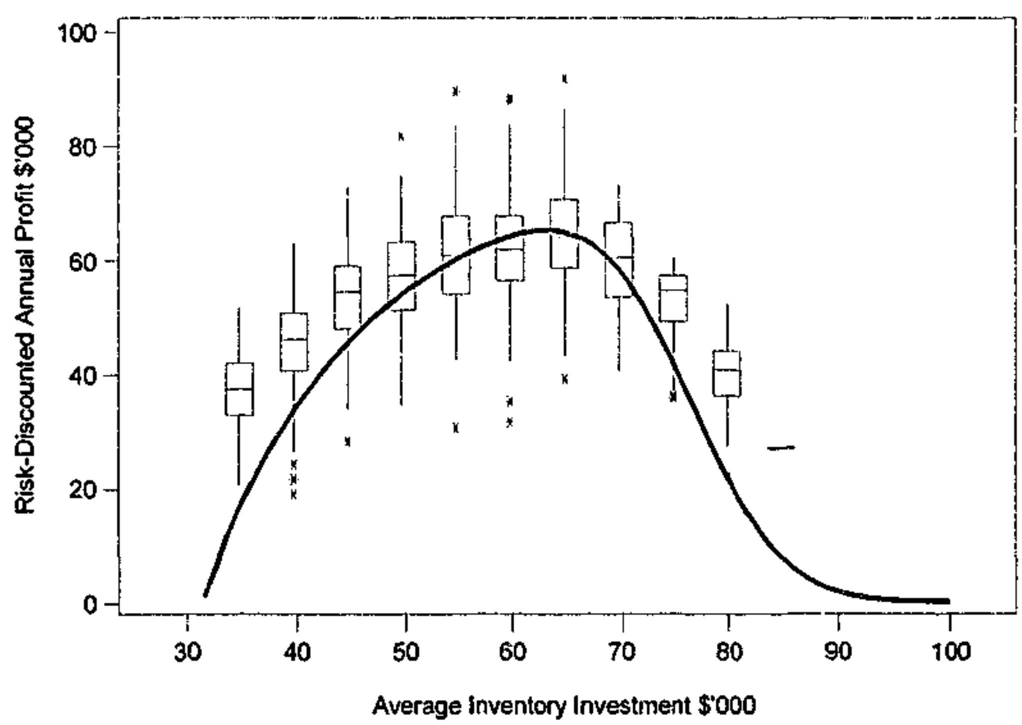

Figure 6.9: Comparison of RDP obtained by simulation with values obtained analytically

6.3.6. RDP approach *vs* Lagrangian approach

The determination of an optimal replenishment policy subject to a binding constraint could be simplified by using the method of Chapter 5, but setting the constraint at the maximum inventory that could occur when all replenishments coincide. However, such an approach to batch setting represents the most conservative choice of setting a constraint as it tolerates no risk of exceeding a capital constraint, even though this risk may be small.

Figure 6.10 shows RDP as a function of total investment calculated according the method described in Figure 6.8 for the case study company. The profit obtained using the traditional Lagrangian approach is also given. Under the Lagrangian approach, the constraint on inventory investment is set at the maximum inventory level that could possibly occur when all replenishments coincide. Taking this approach, Q_j and r_j are determined by maximising profit as defined by Equation 5.7, following the process described in Chapter 5, but with the constraint set as $K = \sum_{j=1}^{m} C_j\left(Q_j + r_j\right)$. The level of profit determined in this way is not discounted as there is no risk of exceeding the capital constraint.

Comparing the profit obtained under the traditional Lagrangian approach and the RDP approach, it is evident that ensuring that the constraint is never exceeded with certainty is expensive. The reduced profit under the traditional approach is due to a reduction in Q_j and r_j at any given investment compared with the RDP approach, which increases replenishment costs and the risk of lost sales. As a consequence of these increased costs, the company's operations require a greater investment in inventory to reach the threshold of profitability under the traditional approach compared with the RDP approach.

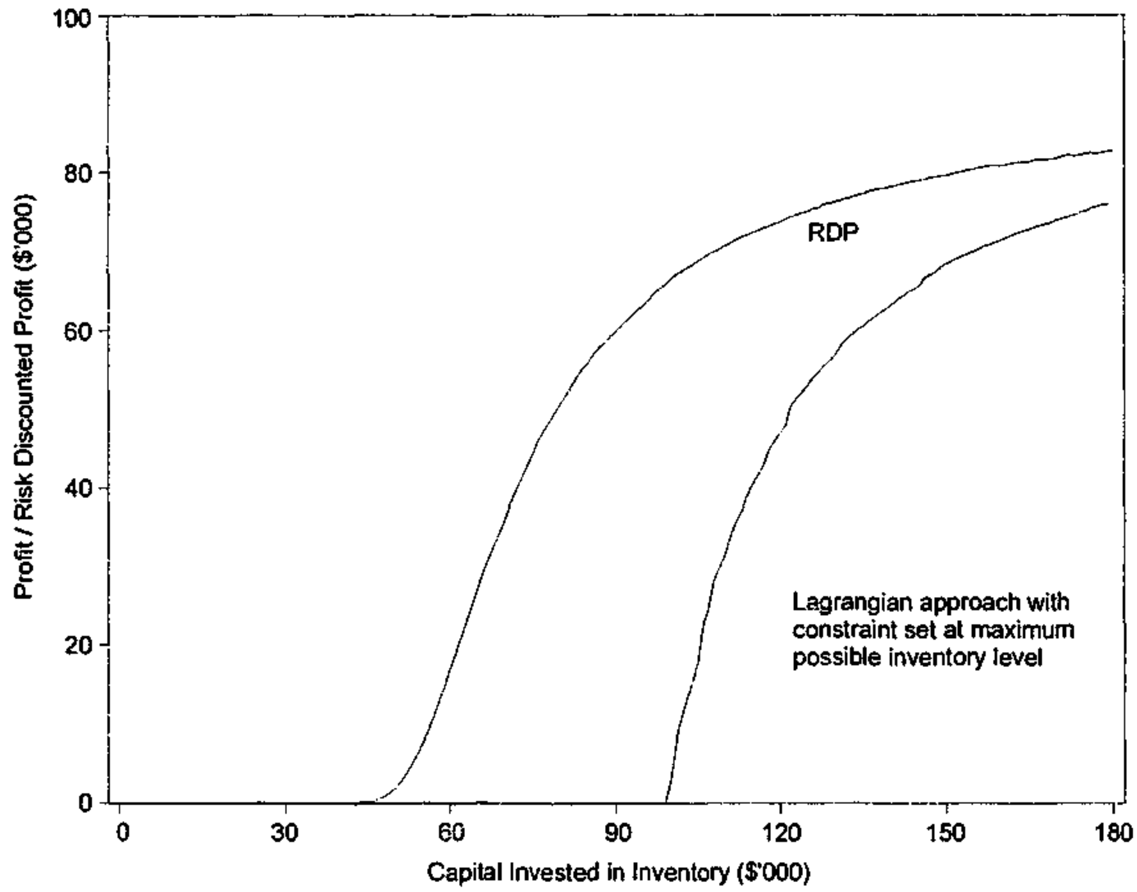

Figure 6.10: Comparison of profit obtained using RDP approach and using Lagrangian approach with constraint set at maximum possible inventory level

The RDP approach as presented establishes replenishment policies under a binding constraint by determining the maximum level of risk for failure through capital exhaustion that it is cost-effective to finance. Thus, this section has shown that by accepting the negligible risk of failure due to the very rare instances of multiple replenishments of components coinciding, the new model permits increased replenishment quantities and safety stock of components for any given level of capital, which increases the profitability of the company's operations over the traditional Lagrangian approach.

6.4. The optimal average investment in inventory

The risk-discounted profit is now evaluated for the case study company as K^* varies from \$0 to \$180,000 (see Figure 6.11). The maximum investment that an investor would reasonably consider is now determined by setting $\frac{dRDP}{dK^*} = I$ as the marginal discounted return on investments greater than this are less than the risk free interest rate. For the case study data, $I = 0.1$, and the maximum feasible investment occurs when K^* = \$156,700. Risk-discounted return on investment, RDROI, evaluated as

$$RDROI = \wp \frac{P}{K^*}, \tag{6.39}$$

has also been calculated, and is maximised at an investment of approximately \$92,000.

Figure 6.11 shows that directing increased resources to increase the probability of company survival is the RDP-maximising strategy at low investment levels. This is evident from the rapid increase in $\wp$ as K^* increases over the lower end of the feasible investment range. As K^* increases further, $\wp$ approaches a maximum of 1. For all values of K^* within the range that an investor would reasonably consider, that is, investments within the range that maximises either RDROI or RDP, $\wp > 0.99$. The requirement that $\wp$ be maximised, requires in turn that $f_K(K^*)$ be maximised, and consequently that $\rho_i \to 1$. This last expression has the simple interpretation, that for the risk of company failure to be acceptably low, the probability of exceeding the constraint at any replenishment is close to zero. The requirement that $\rho_i \to 1$ justifies the approximation made in Equation 6.13, that requires either that $\rho_i \to 1$ or $\rho_i \to 0$ in order that the error introduced by the approximation be minimised.

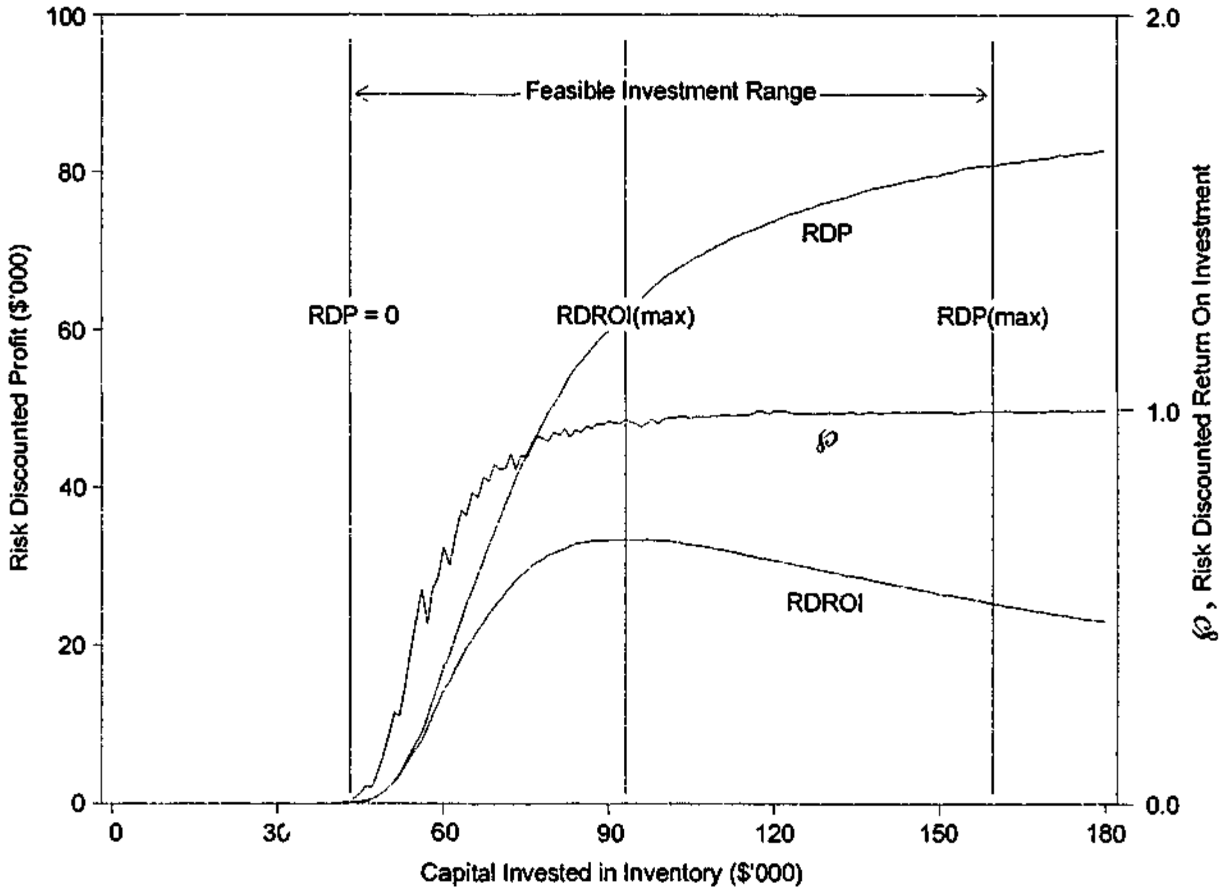

Figure 6.11: RDP, RDROI and ℘ as a function of capital invested in inventory

The capital invested in cycle stock, safety stock and buffer capital $\left(K^* - \overline{K}\right)$ for the RDP-maximising investment are shown as a proportion of the total investment in inventory in Figure 6.12. As K^* increases from the threshold of profitability, an increasing proportion of the total investment is allocated as buffer capital, resulting in the rapid increase in $\wp$. As the total investment increases above approximately \$60,000, the proportion of capital invested as a buffer against exceeding the constraint remains constant at approximately 37% of the total investment. This is because, under a fixed investment policy (for example, all components batch-replenished), the optimal level of buffer capital is roughly proportional to the standard deviation of the capital invested in inventory $\sigma_{\overline{K}}$, which in turn is roughly proportional to $\overline{K}$ at all values of K^*, except when investment is highly constrained.

Figure 6.12 also shows that it is only under very high constraint that it becomes RDP-maximising to reduce the investment in buffer capital in order to invest in cycle and safety stock. These very low investment levels, however, correspond to the company operating in a sub-optimal investment range, where both RDP and RDROI are very low, making it unlikely that a company would choose to invest in this range. It is shown in Section 6.7 that inventory reduction strategies make it possible to maintain a sufficient proportion of buffer capital at very low investment levels which extends the effective investment range that a company could reasonably consider.

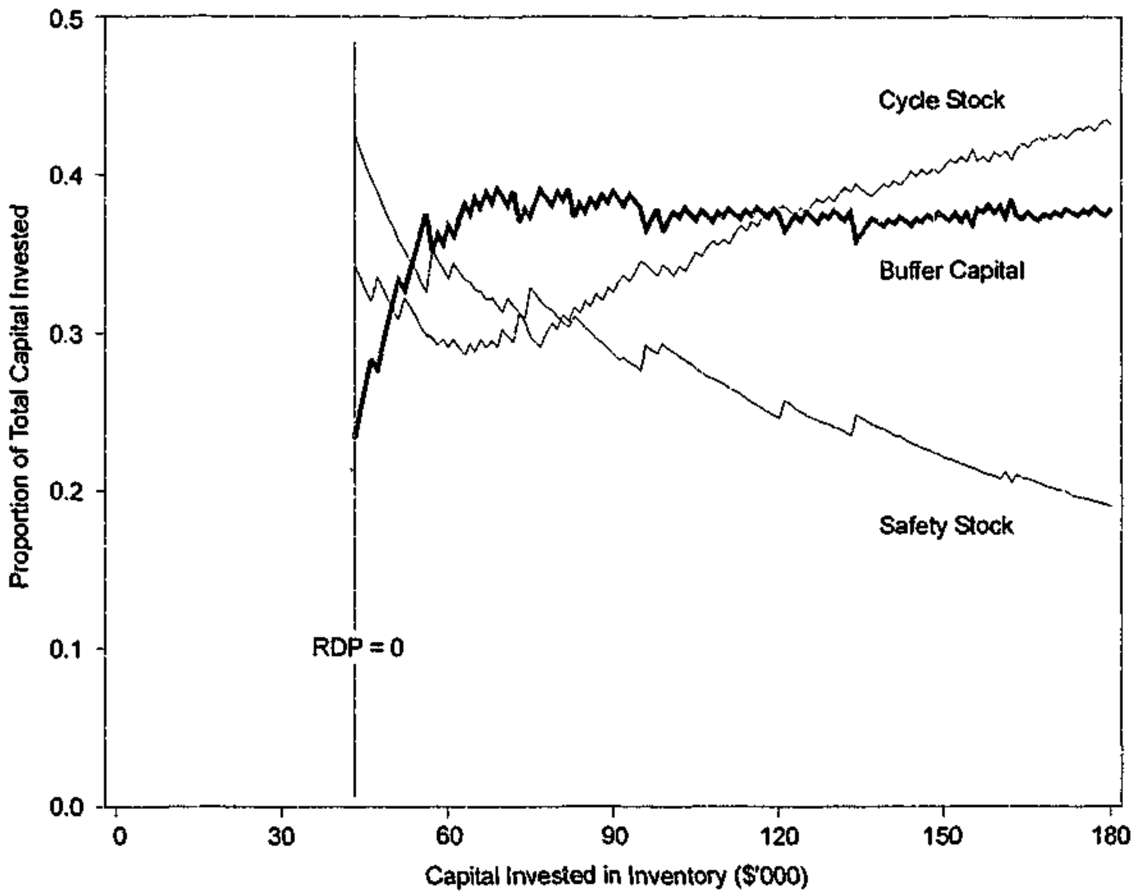

Figure 6.12: Proportion of capital invested in cycle stock, safety stock and buffer capital as a function of capital invested in inventory

6.5. Sensitivity of the survival probability

Equations 6.36, 6.37 and 6.38 summarise everything that can be determined about the fluctuation in inventory level over time from the assumptions of the inventory process described in Section 6.3. The variance of the investment in inventory, when observed at random instants in time, $\sigma_{\overline{K}}^2$, which is a function of $\overline{K}$, determines the probability of the inventory level exceeding K^* when a replenishment order is received. The rate at which the investment in inventory is decremented as batch-replenished components are consumed for manufacture, $\frac{\langle \Delta K_i \rangle}{\langle \Delta T_i \rangle}$, establishes the average frequency of replenishment orders. Both $\sigma_{\overline{K}}^2$ and $\frac{\langle \Delta K_i \rangle}{\langle \Delta T_i \rangle}$ are measures of the *volatility* of the inventory process, that is, the degree to which the investment in inventory fluctuates over time, and in addition to $\overline{K}$ and t determine the value of $\wp$ for a certain K^*. The sensitivity of $\wp$ to these factors is now discussed. Recall Equation 6.24

$$\wp\left(K^*, \overline{K}, t\right) = e^{-t\frac{f_K\left(K^*\right)\langle \Delta K_i \rangle}{F_K\left(K^*\right)\langle \Delta T_i \rangle}}. \qquad (6.24)$$

The rate at which capital is consumed from inventory appears explicitly, where it is readily observed that increasing $\frac{\langle \Delta K_i \rangle}{\langle \Delta T_i \rangle}$ leads to a decrease in $\wp$. Thus, increasing the rate at which capital is consumed for the batch replenishment of inventory results in more frequent replenishments, increasing the risk of company failure.

$\sigma_{\overline{K}}^2$ and $\overline{K}$ are introduced into the calculation of $\wp$ through their effect on the value of $f_K\left(K^*\right)$ and $F_K\left(K^*\right)$. Note that $f_K\left(K^*\right) = f\left(z^*\right)$ and $F_K\left(K^*\right) = F\left(z^*\right)$, where z^* is a standard normal variate, and $z^* = \frac{K^* - \overline{K}}{\sigma_{\overline{K}}}$. When $K^* > \overline{K}$, increasing $\sigma_{\overline{K}}^2$ reduces z^* for a fixed $\overline{K}$.

This reduction in z^* leads to an increase in the ratio $\frac{f_K(K^*)}{F_K(K^*)}$, and consequently reduces $\wp$. It follows that increasing the variance of the investment in inventory increases the probability that K^* will be exceeded at any replenishment, resulting in an increased risk of company failure. Reducing $\overline{K}$ has the similar effect of reducing z^* with a consequent increase in $\wp$. It is shown in the following case study that the change to JIT replenishment or component substitution is accompanied by a reduction in $\overline{K}$ which effectively increases $\wp$ at each decision point.

This section has illustrated the effect of increased inventory volatility on reducing the probability of company survival. It will be shown subsequently that the risk of company failure requires that $\wp$ be introduced as an additional factor in the decision to adopt inventory reduction strategies. As a consequence of this, $\frac{\langle \Delta K_i \rangle}{\langle \Delta T_i \rangle}$ and $\sigma_{\overline{K}}^2$ are introduced into the replenishment policy decision through their effect on $\wp$. Equations 6.37 and 6.38 showed that JIT replenishment leads to a reduction in the two factors that determine inventory volatility. The resultant decrease in the risk of company failure is shown in subsequent sections to introduce an additional benefit of JIT replenishment not previously included in the JIT decision.

6.6. The JIT replenishment and component substitution decision

The decision to either replenish Component 1 JIT, or to substitute Component 1 with an over-specification Component 2, are now analysed for a given level of total investment K^*, where maximising risk-discounted profit is the decision criterion. The probability that a company may fail requires that revenue and expenses be evaluated at their expected value: that is, their actual value discounted for the risk of company failure. The evaluation of income and inventory related costs at their expected value extends the decision model of the previous chapter to account for the risk of company failure. It will also be shown that changes in replenishment policy actually affect the risk of company survival introducing new factors into the policy decision.

The section begins with an analysis of the expected cost factors present in the JIT replenishment and component substitution decision equations. Although the form of the decision equations does not allow for explicit solutions, the previous section has shown that RDP-maximising investments require that $\wp$ be close to 1. The effect of each policy change on $\wp$ is then analysed by examining the effect of each replenishment policy decision on the two factors determining inventory volatility, $\sigma_{\bar{K}}^2$ and $\frac{\langle \Delta K_i \rangle}{\langle \Delta T_i \rangle}$, and thus $\wp$. Although it is evident that any replenishment policy change will affect both the inventory cost factors and the risk of company failure simultaneously, these factors are examined separately in order to make the analysis tractable. The case study in the following section illustrates the interaction of these factors by a numerical analysis of the replenishment policy decisions.

6.6.1. JIT replenishment

RDP has been defined previously for the case in which Component 1 is batch-replenished. When Component 1 is replenished JIT, profit is determined by maximising

$$\begin{aligned} \wp_{JIT} = \sum_{i=1}^{n} A_i S_i - \sum_{j=1}^{m} D_j C_j - \sum_{j=2}^{m} \frac{D_j}{Q_j}\left(R_j + B_j L(r_j)\right) \\ - J\sum_{j=2}^{m} C_j\left(\frac{Q_j}{2} + r_j - \bar{x}_j + L(r_j)\right) - F - R_1' D_1 \end{aligned} \tag{6.40}$$

subject to

$$\bar{K}_{JIT} = \sum_{j=2}^{m} C_j\left(\frac{Q_j}{2} + r_j - \bar{x}_j + L(r_j)\right), \tag{6.41}$$

(compare with Equations 6.1 and 6.2). Maximisation of risk-discounted profit is again determined by iteration, which yields the profit-maximising values of P_{JIT}, $\wp_{JIT}$, $\overline{K}_{JIT}$, $Q_j^{'}$ and $r_j^{'}$. Because the RDP policy is independently optimised for the batch-replenished and JIT-replenished cases, it cannot be assumed that $\overline{K}_{JIT} = \overline{K}$. Consequently, it is not assumed that Q_j and $Q_j^{'}$, r_j and $r_j^{'}$ are equal for components 2 to m in the following analysis.

The decision to replenish JIT, at a certain fixed investment level, K^*, requires that

$$P_{JIT}\wp_{JIT} > P\wp . \tag{6.42}$$

Note that K^* is an argument of the function $\wp$ and thus remains a factor in the JIT decision.

Substituting the terms of the profit equations under the batched and JIT replenishment of Component 1 into Equation 6.42, the JIT replenishment of Component 1 is RDP-maximising when

$$\begin{aligned}
\wp_{JIT} D_1 R_1^{'} - \wp \frac{D_1}{Q_1} R_1 < {} & \wp J C_1 \left(\frac{Q_1}{2} + r_1 - \overline{x}_1 + L(r_1) \right) + \wp \frac{D_1}{Q_1} B_1 L(r_1) \\
& + (\wp_{JIT} - \wp)\left(\sum_{i=1}^{n} A_i S_i - \sum_{j=1}^{m} D_j C_j - F \right) \\
& + \wp \left(\sum_{j=2}^{m} \frac{D_j}{Q_j}\left(R_j + B_j L(r_j)\right) + J \sum_{j=2}^{m} C_j \left(\frac{Q_j}{2} + r_j - \overline{x}_j + L(r_j) \right) \right) \\
& - \wp_{JIT} \left(\sum_{j=2}^{m} \frac{D_j}{Q_j^{'}}\left(R_j + B_j L(r_j^{'})\right) + J \sum_{j=2}^{m} C_j \left(\frac{Q_j^{'}}{2} + r_j - \overline{x}_j + L(r_j^{'}) \right) \right).
\end{aligned} \tag{6.43}$$

The decision to replenish Component 1 JIT when RDP is to be maximised, now requires that the risk-discounted value of each of the cost factors present in the stochastic case be evaluated. Consequently the effect of JIT replenishment on survival probability introduces a new factor into the JIT decision through its effect on expected profit.

The JIT replenishment of Component 1 is RDP indifferent when the replenishment cost increase due to reordering Component 1 JIT,

$$\wp_{JIT} D_1 R_1' - \wp \frac{D_1}{Q_1} R_1, \tag{6.44}$$

is less than the sum of:

- The expected saving in holding cost, and the cost of lost sales due to Component 1 being eliminated from inventory

$$\wp\left(JC_1\left(\frac{Q_1}{2} + r_1 - \bar{x}_1 + L(r_1) \right) + \frac{D_1}{Q_1} B_1 L(r_1) \right). \tag{6.45}$$

- The change in replenishment, lost order, and holding costs of Components 2 to m, including the efficiency gains obtained by redistributing the investment formerly in Component 1 across the remaining components,

$$\begin{aligned} &\wp\left(\sum_{j=2}^{m} \frac{D_j}{Q_j}\left(R_j + B_j L(r_j)\right) + J\sum_{j=2}^{m} C_j\left(\frac{Q_j}{2} + r_j - \bar{x}_j + L(r_j) \right) \right) \\ &- \wp_{JIT}\left(\sum_{j=2}^{m} \frac{D_j}{Q_j'}\left(R_j + B_j L(r_j')\right) + J\sum_{j=2}^{m} C_j\left(\frac{Q_j'}{2} + r_j' - \bar{x}_j + L(r_j') \right) \right). \end{aligned} \tag{6.46}$$

- The expected change in income less fixed costs,

$$\left(\wp_{JIT} - \wp\right)\left(\sum_{i=1}^{n} A_i S_i - \sum_{j=1}^{m} D_j C_j - F \right). \tag{6.47}$$

The cost factors shown in Equations 6.44, 6.45 and 6.46 are simply the risk discounted value of the costs present in the JIT decision under stochastic demand, (see Section 5.2). Equation 6.47, however, introduces the expected value of a change in survival probability as a proportion of income less the cost of components and any fixed costs. This is a new factor in the decision to adopt JIT replenishment, which attributes an additional cost (or saving) of the decision through its effect on the risk of company failure. A major issue in the following analysis is which of the two factors determining RDP, the change of risk of company failure, or profit, is the most influential in determining inventory replenishment policies. Because the form of Equation 6.43 does not permit this question to be answered decisively by analytical means, a numerical analysis of the case study data will be undertaken in a later section to determine the relative importance these two factors.

The form of Equation 6.47 suggests, firstly, that a possible pre-condition for JIT replenishment is that $\wp_{JIT} > \wp$, in order that the increase in expected profit less fixed costs be an additional factor to offset any increase in replenishment costs due to JIT replenishment. Secondly, when $\wp_{JIT}$ is greater than $\wp$, the increase in expected profit less fixed costs permits the adoption of JIT replenishment at a greater total investment level than would be cost-effective if profit alone were considered. Although these hypotheses cannot be verified analytically, the following example, and subsequent analysis in Section 6.7.3 confirms that they hold for the case study data.

The effect of $\wp_{JIT} > \wp$ on the replenishment policy decision is illustrated in Figure 6.13, which shows both profit and risk-discounted profit for the case study company across a range of investment levels. In one instance, the company has adopted JIT replenishment for Screw 1 but has maintained batched replenishment for Screw 2. In the other, the company has adopted JIT replenishment for both Screw 1 and Screw 2. The effect of each replenishment method on company survival is evident by the amount of discounting of profit that occurs under each alternative. For values of K^* ranging from \$80,000 to \$85,000, the probability of survival with Screw 2 batch-replenished was 0.985. Over the same interval, the survival probability with Screw 2 JIT-replenished was 0.995. The difference in survival probability of approximately 0.01 resulted in profit being discounted by an additional \$700 when Screw 2 was batch-replenished. The reduced risk-discounting that occurs when Screw 2 is JIT-replenished counts as an additional cost saving when JIT replenishment is adopted, making this policy change cost-effective at an investment level approximately \$3,000 greater than would be the case if the JIT decision were based on undiscounted profit.

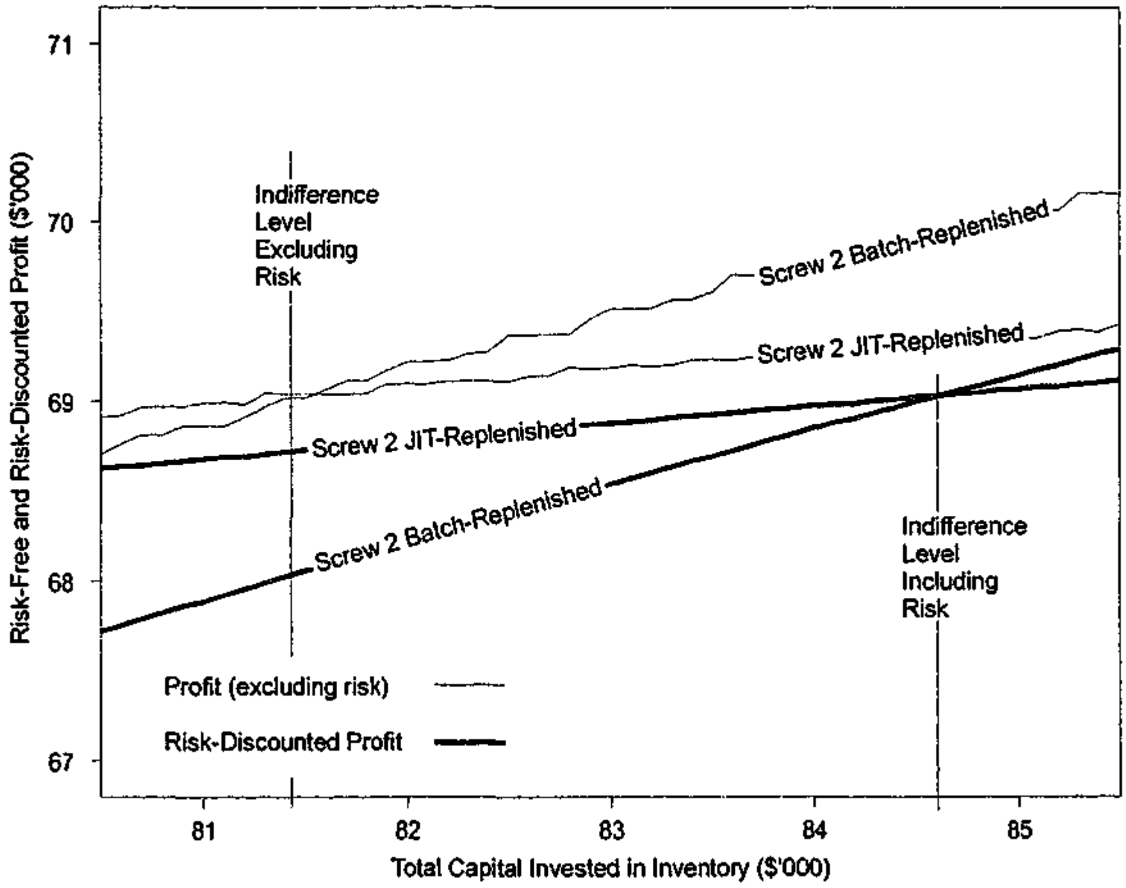

Figure 6.13: The decision to adopt JIT replenishment of Screw 2 when maximising either profit or risk discounted profit is the decision criteria

6.6.2. Component substitution

The profit when Component 1 is substituted with Component 2 is given by

$$P_{SUB} = \sum_{i=1}^{n} A_i S_i - (D_1 + D_2)C_2 - \sum_{j=3}^{m} D_j C_j - \frac{(D_1 + D_2)}{Q_2^{'}}\left(R_2 + B_2 L(r_2^{'})\right)$$
$$- \sum_{j=3}^{m} \frac{D_j}{Q_j^{'}}\left(R_j + B_j L(r_j^{'})\right) - J\sum_{j=2}^{m} C_j\left(\frac{Q_j^{'}}{2} + r_j - \bar{x}_j + L(r_j^{'})\right) - F \qquad (6.48)$$

subject to

$$\overline{K}_{SUB} = \sum_{j=2}^{m} C_j \left(\frac{Q_j^{'}}{2} + r_j - \overline{x}_j + L(r_j^{'}) \right) \tag{6.49}$$

having survival probability $\wp_{SUB}$. Maximisation of risk-discounted profit again yields the profit-maximising values of $\wp_{SUB}$, $\overline{K}_{SUB}$, $Q_j^{'}$ and $r_j^{'}$. By treating the investment in inventory as consisting of only the components from 2 to m, the resulting profit under component substitution again implicitly includes the reduction in replenishment, holding and lost order costs that accrue from the redistribution of the capital formerly invested in Component 1.

The decision to substitute Component 1 with Component 2, at a certain fixed investment level, K^{*}, is RDP-maximising when

$$\begin{aligned}
\wp_{SUB}\frac{D_1}{Q_2^{'}}R_2 - \wp\frac{D_1}{Q_1}R_1 &< \wp\left(\frac{D_j}{Q_j}B_1L(r_1) + JC_1\left(\frac{Q_1}{2} + r_1 - \overline{x}_1 + L(r_1)\right)\right) \\
&+ \wp_{SUB}\left(\sum_{i=1}^{n} A_iS_i - D_1C_2 - \sum_{j=2}^{m} D_jC_j - F\right) - \wp\left(\sum_{i=1}^{n} A_iS_i - \sum_{j=1}^{m} D_jC_j - F\right) \\
&+ \wp\left(\sum_{j=2}^{m}\frac{D_j}{Q_j}\left(R_j + B_jL(r_j)\right) + J\sum_{j=2}^{m} C_j\left(\frac{Q_j}{2} + r_j - \overline{x}_j + L(r_j)\right)\right) \\
&- \wp_{SUB}\left(\frac{D_1}{Q_2^{'}}B_2L(r_2^{'}) + \sum_{j=2}^{m}\frac{D_j}{Q_j^{'}}\left(R_j + B_jL(r_j^{'})\right) + J\sum_{j=2}^{m} C_j\left(\frac{Q_j^{'}}{2} + r_j - \overline{x}_j + L(r_j^{'})\right)\right).
\end{aligned} \tag{6.50}$$

The component substitution decision under risk of company failure is similar to the decision under stochastic demand (see Equation 5.19). However, the risk of company failure again introduces the expected value of income less fixed costs into the replenishment policy decision. The change in income less fixed costs is now

$$\left(\wp_{SUB} - \wp\right)\left(\sum_{i=1}^{n} A_iS_i - \sum_{j=2}^{m} D_jC_j - F\right) + \wp D_1C_1 - \wp_{SUB}D_1C_2, \tag{6.51}$$

where it also suggests that a condition for component substitution to be RDP-maximising is that $\wp_{SUB} > \wp$.

6.6.3. The effect on the survival probability

It has been shown in the previous subsections that the prospect of company failure introduces the risk of failure as an extra decision variable in an analysis of replenishment policy decisions. It was shown in Section 6.4 that the risk of failure is sensitive to inventory volatility. In this section, the effect of each type of policy change on the two measures of inventory volatility, σ_K^2 and $\frac{\langle \Delta K_i \rangle}{\langle \Delta T_i \rangle}$, are now examined in order to determine the effect of each type of policy change on $\wp$.

The change in $\frac{\langle \Delta K_i \rangle}{\langle \Delta T_i \rangle}$ after Component 1 has been eliminated from inventory, by adopting JIT replenishment, is

$$\frac{\langle \Delta K_i \rangle}{\langle \Delta T_i \rangle}_{JIT} - \frac{\langle \Delta K_i \rangle}{\langle \Delta T_i \rangle} = \sum_{j=2}^{m} C_j D_j \left(1 - \frac{L(r_j')}{Q_j'}\right) - \sum_{j=1}^{m} C_j D_j \left(1 - \frac{L(r_j)}{Q_j}\right) \tag{6.52}$$

The change in $\frac{\langle \Delta K_i \rangle}{\langle \Delta T_i \rangle}$ after Component 1 is substituted with Component 2 is

$$\begin{aligned} \frac{\langle \Delta K_i \rangle}{\langle \Delta T_i \rangle}_{SUB} - \frac{\langle \Delta K_i \rangle}{\langle \Delta T_i \rangle} = {} & C_2 (D_1 + D_2)\left(1 - \frac{L(r_2')}{Q_2'}\right) - \sum_{j=3}^{m} C_j D_j \left(1 - \frac{L(r_j')}{Q_j'}\right) \\ & - \sum_{j=1}^{m} C_j D_j \left(1 - \frac{L(r_j)}{Q_j}\right) \end{aligned} \tag{6.53}$$

Q_j' and r_j' represent the new replenishment quantity and reorder point for each batch-replenished component after JIT replenishment or component substitution has been implemented. When $\overline{K}$ is large enough to make the probability of lost sales a small proportion of total costs, in the case of JIT replenishment, Equation 6.53 can be simplified to

$$\frac{\langle \Delta K_i \rangle}{\langle \Delta T_i \rangle}_{JIT} - \frac{\langle \Delta K_i \rangle}{\langle \Delta T_i \rangle} \approx -C_1 D_1 . \tag{6.54}$$

For the case where Component 2 is substituted for Component 1, the change in the rate at which capital is consumed for the purchase of inventory is

$$\frac{\langle \Delta K_i \rangle}{\langle \Delta T_i \rangle}_{SUB} - \frac{\langle \Delta K_i \rangle}{\langle \Delta T_i \rangle} \approx D_1 (C_2 - C_1). \tag{6.55}$$

From Equation 6.54 it can be seen that the effect of JIT replenishment is to always reduce $\frac{\langle \Delta K_i \rangle}{\langle \Delta T_i \rangle}$ by eliminating a component from inventory. By contrast, the condition that $C_2 > C_1$ in Equation 6.55 means that component substitution always increases $\frac{\langle \Delta K_i \rangle}{\langle \Delta T_i \rangle}$.

The effect of either inventory reduction strategy on $\sigma^2_{\overline{K}}$ is sensitive to the quantity in which components are batch-replenished, which is determined by the demand, replenishment, holding and lost order costs for each component, and is a function of the average investment in capital $(\overline{K})$. This makes it impossible to categorically specify the effect of either policy change on $\sigma^2_{\overline{K}}$. Some analysis, however, can indicate the general behaviour of each inventory strategy as well as the general properties of components whose elimination from inventory is most likely to lead to a reduction in $\sigma^2_{\overline{K}}$.

The change in $\sigma_{\bar{K}}^2$ after Component 1 has been replenished JIT is

$$\sigma_{\bar{K}JIT}^2 - \sigma_{\bar{K}}^2 = \sum_{j=2}^{m}\left(\frac{Q_j'^2 + 2Q_j'}{12}\right)C_j^2\delta_j - \sum_{j=1}^{m}\left(\frac{Q_j^2 + 2Q_j}{12}\right)C_j^2\delta_j - C_1^2 Var(x_1). \qquad (6.56)$$

Thus, for the JIT replenishment of Component 1 to reduce the variance of the inventory process, it is necessary that the increase in variance due to the increased reorder quantities of the remaining batch-replenished components is less than the variance originally attributable to Component 1.

According to Equation 6.32, the variance due to each batch-replenished component, under stochastic demand, is the sum of two factors: the variance of demand through lead time, $C_j^2 Var(x_j)$, which is independent of $\bar{K}$, and the variance based on the size of the replenishment quantity, $\frac{C_j^2\left(Q_j^2 + 2Q_j\right)}{12}$ which is a function of $\bar{K}$. It is the relative magnitude of these components that determines whether total inventory variance will be reduced by JIT replenishment. When the investment in inventory is large, one consequence is that Q_j and Q_j' also tend to be large, outweighing the effect of eliminating the variability of demand through lead time of Component 1. The JIT replenishment of Component 1 in such cases may actually result in an increase in the variance of the investment in inventory, as occurs in the case study when Screw 1 is replenished JIT. However, as the total investment in inventory decreases, replenishment quantities for all components are reduced and the significance of $C_1^2 Var(x_1)$ in Equation 6.32 increases. In these circumstances, the elimination of Component 1 from inventory typically leads to a reduction in the variance in inventory investment. (For the case study data, the JIT replenishment of Screw 1 leads to a reduction in the variance of inventory investment when the total investment is less than \$119,000.) Components that effectively reduce the variance of inventory investment are those which have a large variance in demand through lead time. Thus candidate components for JIT replenishment are those with a large unit cost and/or a long lead time, for example, Screws 1 and 2.

When Component 2 is substituted for Component 1, the change in the variance of capital invested in inventory is

$$\begin{aligned}\sigma^2_{K\,SUB} - \sigma^2_{K} = \sum_{j=2}^{m}\left(\frac{Q_j'^2 + 2Q_j'}{12}\right)C_j^2\delta_j + C_2^2 Var(x_1) \\ - \sum_{j=1}^{m}\left(\frac{Q_j^2 + 2Q_j}{12}\right)C_j^2\delta_j - C_1^2 Var(x_1).\end{aligned} \tag{6.57}$$

The demand for Component 2 increases to $D_1 + D_2$, which increases the proportional investment in Q_2' relative to the remaining batch-replenished components. The redistribution of capital formerly invested in Component 1 also results in increased batch sizes for the remaining components. Component substitution always results in an increase in the component of variance due to demand through lead time attributable to Component 1, of magnitude $\left(C_2^2 - C_1^2\right)Var(x_1)$. Consequently, it is necessary that the reduction in the component of variance due to component replenishment, $\sum_{j=2}^{m}\left(\frac{Q_j'^2 + 2Q_j'}{12}\right)C_j^2\delta_j - \sum_{j=1}^{m}\left(\frac{Q_j^2 + 2Q_j}{12}\right)C_j^2\delta_j$, be greater than this for a reduction in total variance to occur. Favourable conditions for reducing variance under substitution occur when Component 2 has a shorter lead time and/or reorder cost than Component 1 in order that the increased investment in Component 2 does not result in unduly large Q_2. However, as total investment decreases, the reduced replenishment quantity for all components means that any reduction in the variance due to component replenishment is less effective in compensating for the increase in variance attributable to Component 1. This means that component substitution may actually lead to an increase in variance at reduced investment.

It was shown in Section 6.4 that decreasing inventory volatility, by decreasing either the variance of the capital actually invested in components, $\sigma_{\overline{K}}^2$, or the rate at which capital is consumed for the purchase of components, $\frac{\langle \Delta K_i \rangle}{\langle \Delta T_i \rangle}$, led to an increased probability of company survival. These factors indicate that JIT replenishment is an effective means of increasing the probability of company survival by reducing $\frac{\langle \Delta K_i \rangle}{\langle \Delta T_i \rangle}$, and $\sigma_{\overline{K}}$ at reduced investment levels. Because component substitution always results in an increase in $\frac{\langle \Delta K_i \rangle}{\langle \Delta T_i \rangle}$, and may increase $\sigma_{\overline{K}}$ at reduced investment, it is less effective in increasing survival probability.

6.7. Case study

In this section, RDP maximisation is used as the criterion for adopting the inventory reduction strategies of JIT replenishment and component substitution for the case study company as the total capital invested in inventory varies. The results of the case study are then used to illustrate the preceding discussion and analysis. Data obtained at each policy decision change illustrate a discussion of the way in which the risk of failure affects the decision to adopt either strategy.

6.7.1. Method of calculation

In order to provide a reference against which company performance after adopting inventory reduction strategies could be compared, it was necessary to determine the optimal average investment in inventory for the company operating with batch replenishment of all components except motors and wiring. The optimal RDP policy was determined for the case study company at total investment levels varying from \$1,000 to \$150,000 in \$1,000 increments. This range of investment levels encompassed the minimum capital required for the company to achieve profitability, and extended beyond the total investment level for which the change in replenishment policy became RDP-maximising. The RDP-maximising policy was determined at each K^* exhaustively by evaluating RDP following the process described in Figure 6.8, for all values of the average investment in inventory, $\overline{K}$, less than K^* in \$1,000 increments, and recording the optimal RDP and corresponding value of $\overline{K}$.

The RDP-maximising replenishment policy at each investment level were similarly determined by evaluating all combinations of replenishment options, for each value of $\overline{K} < K^*$, and recording both the maximum RDP, and the RDP-maximising inventory replenishment policy at each K^*. All inventory reduction options that were cost effective at investment levels over which the company was operating profitably in the previous case study (Table 5.2), under stochastic demand, were considered in this trial. Total investment level and average inventory were varied in increments of \$1,000 as a first approximation for the investment levels at which inventory reduction policies were adopted. Subsequently, a local search was undertaken by enumerating possible combinations at \$10 intervals within the neighbourhood of K^* and $\overline{K}$ found in the first series of calculations. The values of K^* and $\overline{K}$, for which it became cost-effective to replenish Screw 1, were determined by using local search at \$1 intervals in order to complete Table 6.3.

6.7.2. Results

Table 6.2 lists each of the replenishment policies tested in the order of adoption, from greatest total investment to least. $\widetilde{K}^*$ denotes the greatest investment level for which each policy change became cost effective. Each was RDP-maximising for all investment levels below $\widetilde{K}^*$, with the exception of the substitution of Radiator 1 with Radiator 2, which ceased to be RDP-maximising for K^* < \$13,900. As was the case in the preceding chapters, three groups of components can be identified. These are components that are always replenished JIT, those whose replenishment policy depends on the total investment level, and finally, those for which it is never cost-effective to replenish JIT or substitute.

Components that were always replenished JIT were the Motors and Wiring. They were obtained locally, on a next-day-delivery basis, with no delivery cost. For these components, the total investment was set at the level for which the marginal return on additional funds invested, $\frac{dRDP}{dK^*} = 0.1$. This is the greatest investment that an investor would reasonably consider if the risk-free rate of return earned on other types of investments was 10%.

The next group of components were those for which JIT replenishment became cost effective at total investments levels below the maximum feasible investment. These have been ranked in order of the maximum investment for which each was RDP-maximising. The succession of policy changes is cumulative: whereby each policy changes assumes that all other changes at a higher investment level have also been adopted.

Table 6.2: Sequence of replenishment policies that become RDP-maximising as the capital available for investment decreases. *Substitution of Radiator 1 ceases to be RDP-maximising for investments below $13,900.

Policy Groups	Policy Transition	Component	Policy Change	Cost of Change	Total Investment $\widetilde{K}^*$	Inventory Investment $\overline{K}$	Profit Before Discount	One-Year Survival Probability $\wp$	RDP	RDROI
			($)	($)	($)	($)	($)		($)	
Always Just-In-Time Reordering	1	Motor 1	-	-	156,700	98,200	81,351	0.992	80,700	0.515
	2	Motor 2	-	-	156,700	98,200	81,351	0.992	80,700	0.515
	3	Motor 3	-	-	156,700	98,200	81,351	0.992	80,700	0.515
	4	Motor 4	-	-	156,700	98,200	81,351	0.992	80,700	0.515
	5	Motor 5	-	-	156,700	98,200	81,351	0.992	80,700	0.515
	6	Motor 6	-	-	156,700	98,200	81,351	0.992	80,700	0.515
	7	Motor 7	-	-	156,700	98,200	81,351	0.992	80,700	0.515
	8	Wiring 1	-	-	156,700	98,200	81,351	0.992	80,700	0.515
	9	Wiring 2	-	-	156,700	98,200	81,351	0.992	80,700	0.515
Policy Based on Investment Level	10	Screw 1	JIT	7,800	133,337	85,800	78,289	0.981	76,802	0.576
	11	Screw 2	JIT	8,800	84,613	51,399	69,954	0.987	69,045	0.816
	12	Piping 2	JIT	1,410	26,910	17,520	47,375	0.972	46,049	1.711
	13	Radiator 1*	SUB	1,000	25,260	16,600	45,043	0.958	43,151	1.708
	14	Valve 1	SUB	2,380	24,050	15,280	42,490	0.964	40,960	1.703
	15	Air Tank 2	JIT	12,220	20,800	12,800	35,391	0.941	33,303	1.601
	16	Piping 1	JIT	2,650	16,800	10,230	25,832	0.925	23,895	1.422
	17	Chassis	JIT	3,000	14,800	9,310	20,871	0.823	17,177	1.161
	18	Cabinet 1	JIT	1,000	14,400	8,920	19,057	0.843	16,065	1.116
	19	Radiator 4	JIT	15,750	12,300	7,880	9,847	0.703	6,922	0.563
Policy Never Adopted	20	Air Tank 1	JIT	13,780	-	-	-	-	-	-
	21	Cabinet 2	JIT	1,650	-	-	-	-	-	-
	22	Cabinet 3	JIT	2,350	-	-	-	-	-	-
	23	Radiator 2	JIT	34,650	-	-	-	-	-	-
	24	Radiator 3	JIT	33,600	-	-	-	-	-	-
	25	Valve 2	-	-	-	-	-	-	-	-

The inventory reduction policy adopted at the greatest investment level was the JIT replenishment of Screw 1, which was equally profitable under either replenishment policy when the total investment in inventory was $\widetilde{K}^{*}$ = \$133,337. The factors required to calculate profit under both replenishment alternatives for Screw 1 have been evaluated at $\widetilde{K}^{*}$, and are shown, at both their actual value (that is, undiscounted), and discounted for the risk of company failure, in Table 6.3.

Table 6.3: Cost factors for JIT replenishment of Screw 1 evaluated at indifference level showing actual values and discounted for the risk of company failure

	Screw 1 Batch-replenished		Screw 1 JIT-replenished	
Cost	Actual Value	Discounted Value $\wp = 0.981$	Actual Value	Discounted Value $\wp_{JIT} = 0.995$
	\$	\$	\$	\$
Income less component and fixed costs	93,140	91,369	93,140	92,674
Reorder, Holding and Lost order costs of Component 1	(3,143)	(3,083)	(7,800)	(7,761)
Reorder, Holding and Lost order costs of components 2 to m	(11,708)	(11,486)	(8,153)	(8,112)
Profit	78,289	76,801	77,187	76,801

If there is no risk of company failure, the decision to adopt the JIT replenishment of Screw 1 is determined by the cost terms present in the JIT decision under stochastic demand (see Section 5.2). The elimination of Screw 1 from inventory results in a saving of \$3,143 in replenishment, holding and lost order costs, which only partly offsets the increased cost of JIT replenishment (\$7,800). The redistribution of the investment formerly in Screw 1 across the remaining batch-replenished components results in a reduction in the replenishment, holding and lost order costs of these components from \$11,708 to \$8,153, which also partly offsets the increased replenishment cost of Component 1. The net effect of JIT replenishment however is a decrease in profit of \$1,102 due to the increased replenishment costs. The JIT replenishment decision when RDP is to be maximised, however, requires that each of the terms in the decision equation be discounted for the risk of company failure. JIT replenishment results in an increase in the probability of company survival from 0.981 to 0.995. The expected increase in reorder, holding and lost order costs, when Screw 1 is replenished JIT, is now \$1,304. However, the increased survival probability when Screw 1 is replenished JIT however, results in an increase in expected profit of \$1,304, which balances the increase in inventory costs due to JIT replenishment, with the result that both policies have the same RDP.

The policy change that takes place at the next greatest investment level is the JIT reordering of Screw 2, which is profit-maximising for investments below \$84,613. As is the case for Screw 1, the high unit cost and long lead time of this component make JIT replenishment cost-effective at a large total investment. The remaining policy changes all occur when the total investment is greatly reduced, and investment in inventory is highly constrained. The third-ranked policy change is the JIT replenishment of Piping 2, which is risk-discounted profit-maximising for total investments less than \$26,910. The substitution of Radiator 1 with Radiator 2 and Valve 1 with Valve 2 then follow as the next policy changes as total investment decreases.

The final group of components are those that are never cost-effective to replenish JIT. These included Cabinets 2 and 3, Radiators 2 and 3, Air Tank 1 and Valve 2. For the cabinets, radiators and tank, the JIT replenishment cost *per-item* is the same as the *per-batch* replenishment cost, making JIT replenishment of these components prohibitively costly. The high unit cost of JIT replenishment of Valve 2 also makes this component unsuitable for inventory reduction.

Figure 6.14 shows RDP and RDROI as a function of total investment level for the case study company operating under both a batch replenishment policy, and adopting JIT replenishment and substitution of components, following the sequence described in Table 6.2. The investment levels for which it was cost-effective to replenish Screw 1, Screw 2, and Piping 2 JIT are indicated, and show that the difference in the company's RDP over a large investment range is due to the JIT replenishment of Screw 1 and Screw 2 only, highlighting the importance of these components for inclusion in an inventory reduction program.

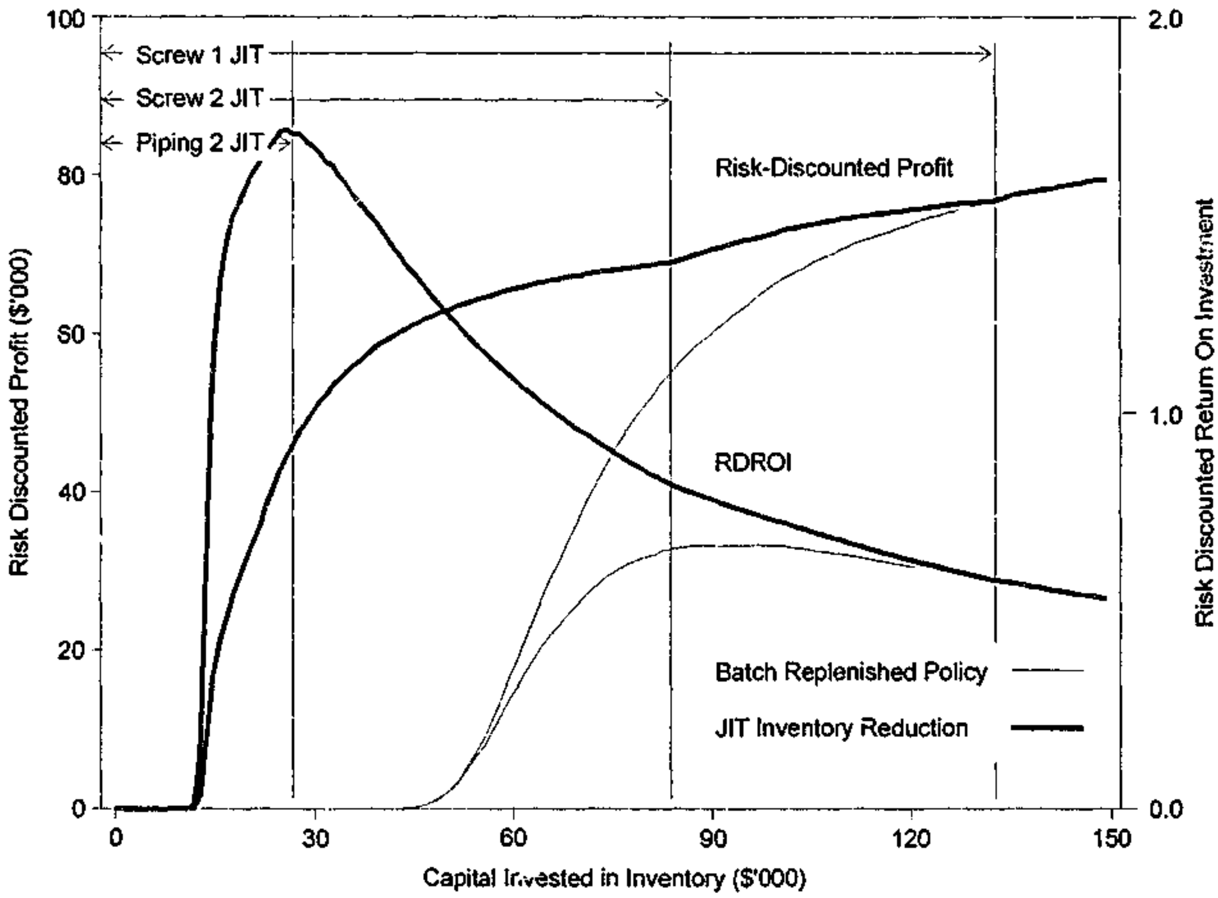

Figure 6.14: RDP and RDROI as a function of capital invested in inventory for batch-replenished policy and with JIT inventory reduction

The increased risk-discounted profit that results from the adoption of inventory reduction policies is evident in Figure 6.14, where it can also be seen that the difference in risk-discounted profit between the batch-replenished and JIT policies increases as the level of total investment decreases. This indicates that JIT becomes an increasingly effective strategy for maximising company performance at reduced investment levels. The adoption of JIT also results in a reduction of the minimum amount of capital that could possibly be invested profitably. In the case of a batch-replenished policy, the minimum investment level for which the company is profitable is $43,000. By contrast, the adoption of JIT replenishment allows for investment as low as $13,000.

By increasing the risk-discounted profit at reduced investment levels, and by lowering the minimum possible investment level, JIT replenishment offers increased RDROI. Under a batch-replenished policy, the maximum annual RDROI is approximately 0.67 at an investment of $91,000. By adopting JIT replenishment, a maximum RDROI of 1.7 is obtained on an investment of $26,000. For a self-funded investor under capital constraint, the high yield on capital invested makes the adoption of JIT an appealing option. As well as allowing for lower levels of investment, the JIT-replenished policy is less sensitive to reduction in investment levels and remains highly profitable at reduced investment. For instance, with a total investment of between $40,000 and $80,000, the company, by adopting JIT replenishment for Screw 1 and Screw 2 only, earns a risk-discounted profit of at least 75% of the maximum possible. Over the same interval, the batch-replenished policy ranges from the threshold of profitability to a risk-discounted profit of approximately 60% of the maximum possible.

6.7.3 Discussion

The effect of the risk of company failure on the replenishment policy decision is now analysed by comparing the inventory decisions under risk with the same decisions in the absence of risk obtained in the previous chapter. The analysis shows that the important new factor in the policy decision is the effect of each change on inventory volatility. The ability of JIT replenishment to reduce inventory volatility, reducing the rate of capital flow through inventory and the variance of capital invested in inventory, increases the effectiveness of this method of achieving inventory reduction. By contrast, it is shown that component substitution is of reduced effectiveness for reducing inventory because of its inability to reduce inventory volatility.

It is first shown that each RDP-maximising policy maintains a probability of company survival that is as large as practically possible. Additionally, each change in replenishment policy is accompanied by an increase in $\wp$. It is then observed that, in order to maintain a suitably large $\wp$, less capital is redistributed across the remaining components when a component is eliminated from inventory. The reduction in capital for redistribution requires a greater degree of constraint for the replenishment policy change to be cost effective, reducing the average investment at which each would be contemplated. The effect of risk on each method of inventory reduction is then shown by a comparison of the reduction in average inventory in the risk-discounted case with that required in the absence of risk. The analysis also shows that it is more effective to increase $\wp$ than to increase profit in order to obtain the RDP-maximising investment at each policy change.

Figure 6.15 shows $\wp$ as a function of total capital invested for the case study company operating under batch replenishment, and after adopting the JIT replenishment policies described in Table 6.2. The amount of buffer capital expressed as a proportion of the total investment, $\left(\frac{\widetilde{K}^{*} - \overline{K}}{\widetilde{K}^{*}}\right)$, is also shown. Recall that the purpose of buffer capital is to permit the investment in inventory to fluctuate without exceeding the capital constraint. The requirement that survival probability be high in order to maximise RDP is shown in several ways. Firstly, when capital is relatively unconstrained, the RDP-maximising policy is one that sets the probability of survival as high as is practically possible. For the case study company, this was approximately 0.99 and required that approximately 38% of the total investment in inventory be directed to buffer capital. This corresponded to a value of $\widetilde{K}^{*}$ being approximately 3.7 standard deviations above the mean inventory level. Secondly, even with the adoption of inventory reduction strategies, a probability of company survival close to 1 was still required for the RDP-maximising investment and was maintained at significantly reduced investments. The rapid increase in buffer capital, as total investment increases from the threshold of probability, shows that under very high constraint, it is RDP-maximising under either policy to direct resources to buffer capital in order to increase $\wp$. This last point suggests that, even when profit is highly reduced, the most cost-effective way to increase RDP is by increasing the probability of company survival. It is now shown that it is the requirement that $\wp$ always be maximised, which makes inventory reduction strategies that reduce inventory volatility the most effective when RDP maximisation is the decision criterion.

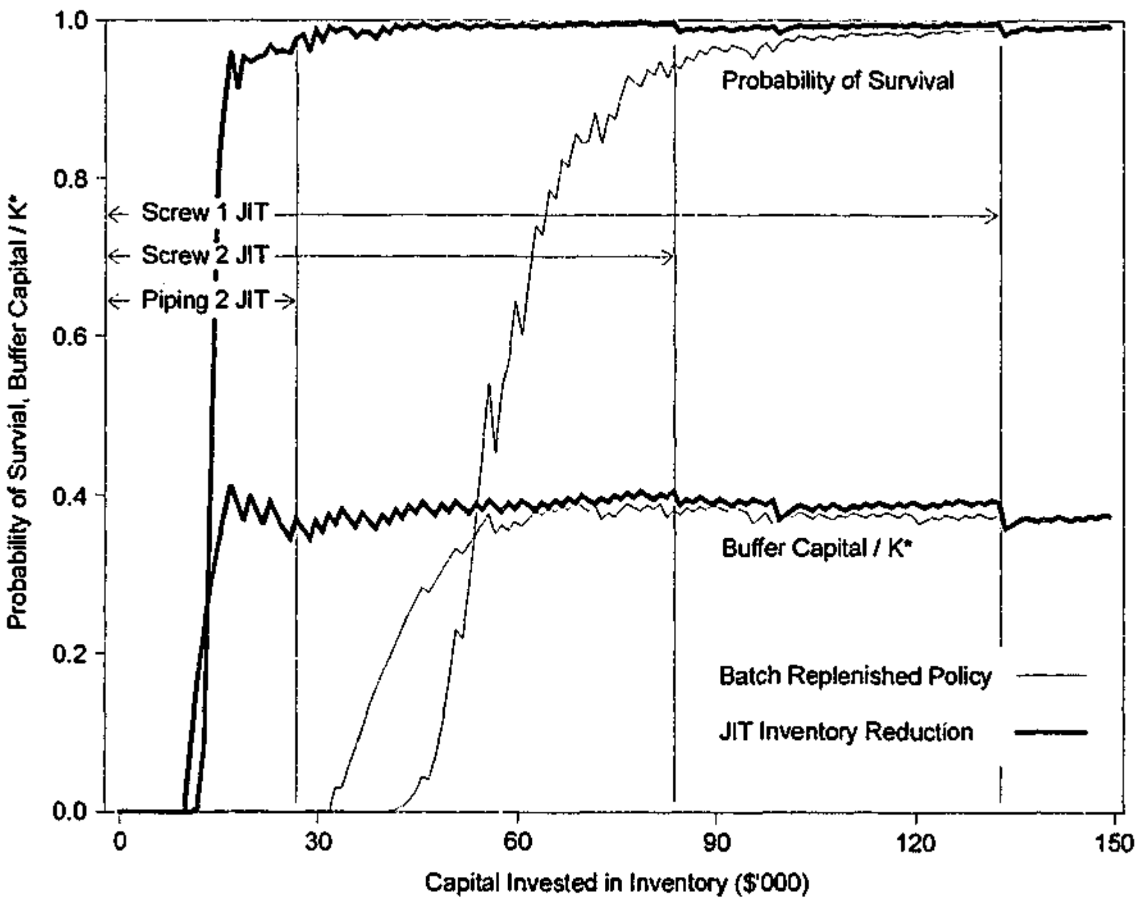

Figure 6.15: The probability of survival and proportional investment in buffer capital as a function of capital invested in inventory

The effect of adopting risk-discounted profit as the decision criterion over simply evaluating profit can be observed by comparing the inventory investment at which each change became cost-effective under stochastic demand, $\widetilde{K}$, (see Table 5.3) with $\overline{K}$ from the current case study. If it were the case that discounting for the risk of company failure (by its effect on $\wp$) had no effect on the replenishment policy change, then it would be reasonable to expect that $\overline{K} = \widetilde{K}$ as the policy decision would be determined only by the cost factors analysed in the stochastic case of the previous chapter. Table 6.4 shows each replenishment policy change ranked in descending order of $\widetilde{K}^*$, showing $\overline{K}$, $\wp$ and $\frac{\langle \Delta K_i \rangle}{\langle \Delta T_i \rangle}$ for both the adoption and non-adoption of each policy, and $\widetilde{K}$, the indifference level for each policy change under stochastic demand but excluding risk.

Table 6.4: Replenishment policy changes showing, $\overline{K}$, $\widetilde{K}$ (for the stochastic case), $\wp$ and $\frac{\langle \Delta K_i \rangle}{\langle \Delta T_i \rangle}$ for adoption/non-adoption of each policy change

Component	Replenishm't Policy	$\wp$	$\frac{\langle \Delta K_i \rangle}{\langle \Delta T_i \rangle}$	$\sigma_{\overline{K}}$	$\overline{K}$	$\widetilde{K}$
Screw 1	-	0.981	789	13,986	85,800	82,462
	JIT	0.995	481	14,242	81,100	
Screw 2	-	0.987	479	9,547	51,399	49,913
	JIT	0.996	307	9,178	50,617	
Piping 2	-	0.972	305	2,713	17,520	19,241
	JIT	0.975	282	2,767	17,300	
Radiator 1	-	0.958	282	2,670	16,600	27,068
	SUB	0.973	313	2,692	16,100	
Valve 1	-	0.964	312	2,578	15,280	21,453
	SUB	0.966	322	2,730	14,750	
Air Tank 2	-	0.941	320	2,346	12,830	13,389
	JIT	0.966	244	2,519	12,210	
Piping 1	-	0.925	243	2,072	10,230	11,590
	JIT	0.927	227	2,117	10,110	
Chassis	-	0.823	227	1,918	9,310	9,461
	JIT	0.883	197	1,941	9,030	
Cabinet 1	-	0.843	197	1,913	8,920	10,040
	JIT	0.852	197	1,911	8,830	
Radiator 4	-	0.703	195	1,826	7,880	7,938
	JIT	0.776	177	1,867	7,590	

It is evident that the adoption of each policy change is accompanied by an increase in $\wp$, as indicated by Equations 6.47 and 6.20. In some cases the increase in $\wp$ is very small, although the absence of any case in which replenishment policy change leads to a reduction in $\wp$ suggests (at least for the case study company) that it is always more cost-effective to direct resources to maintain a large $\wp$. The effect of JIT replenishment in reducing $\frac{\langle \Delta K_i \rangle}{\langle \Delta T_i \rangle}$, and of component substitution in increasing $\frac{\langle \Delta K_i \rangle}{\langle \Delta T_i \rangle}$, are also apparent. Each policy change, with the exception of JIT replenishment of Screw 2, leads to an increase in $\sigma_{\overline{K}}$ at $\widetilde{K}^{*}$,as the redistribution of the capital invested in the component eliminated from inventory is invested across the remaining components, increasing Q_j, and hence $\sigma_{\overline{K}}$. In order that an increase in $\sigma_{\overline{K}}$ and/or $\frac{\langle \Delta K_i \rangle}{\langle \Delta T_i \rangle}$ does not lead to a reduction in $\wp$, each policy change is also accompanied by a reduction in $\overline{K}$, which effectively increases the level of buffer capital. Because $\overline{K}$ is reduced under the policy change, there is less capital available for redistribution across the remaining components with which to obtain more efficient replenishments. As a consequence of this, a greater degree of inventory constraint is required in order that the benefits of inventory reduction (outlined in the stochastic case, Section 5.2) are large enough to justify the policy change. This is why $\widetilde{K} > \overline{K}$ in almost all cases.

It should be noted, however, that the current analysis is based on the parameter settings of the case study company, and although the range of component costs, lead time and demand distributions are quite typical of those that may be encountered in practice, the analysis may not be true for all product/cost combinations. The relative effectiveness of increasing profit over decreasing risk in any replenishment policy change decision, will always hinge on the marginal utility of additional investment required for either. For the case study company, inventory volatility is sufficiently controllable that minimising the risk due to this factor is cost-effective in all but the most highly constrained investment settings. However, it is possible to imagine a situation where the demand through lead time of components was sufficiently volatile to make additional investment to increase $\wp$ ineffective. A more thorough analysis of the optimal investment strategy for all possible component types is however beyond the scope of the thesis.

The JIT replenishment of Screw 1 and Screw 2 become RDP-maximising at a greater average investment in inventory under risk because the elimination of each component from inventory reduces inventory volatility. The large investment in each Screw makes the reduction in $\frac{\langle \Delta K_i \rangle}{\langle \Delta T_i \rangle}$ significant at each change. The JIT replenishment of Screw 1 results in a small increase in $\sigma_{\overline{K}}$ at $\widetilde{K}^*$ and the elimination of Screw 2 results in a reduction in $\sigma_{\overline{K}}$. However, the reduction in inventory investment volatility permits a large increase in $\wp$ at each policy change. The replenishment of these components within the customer lead time, eliminates the holding and lost order costs attributable to them. The redistribution of the large investment in cycle and safety stock of these components makes possible the more economical replenishment of the remaining batch-replenished components. For these components you get every benefit arising from inventory reduction outlined in this thesis! For most of the remaining JIT-replenished components, the indifference level of decision occurs at a slightly smaller investment than in the stochastic case. This is because, although $\frac{\langle \Delta K_i \rangle}{\langle \Delta T_i \rangle}$ is always reduced, an increase in $\sigma_{\overline{K}}$ results in a reduced $\overline{K}$. The consequent reduction in the amount of capital for redistribution requires increased constraint to make the JIT replenishment of these components cost effective. For most components this occurs when $\overline{K}$ was approximately 90% of $\widetilde{K}$.

Component substitution is of greatly reduced effectiveness under a RDP-maximising model. The substitution of one component with a more expensive alternative results in an increase in $\frac{\langle \Delta K_i \rangle}{\langle \Delta T_i \rangle}$, in addition to increasing $\sigma_{\overline{K}}$. Both factors increase inventory volatility, requiring that $\overline{K}$ be decreased in order to maintain a sufficiently large $\wp$. Also, because the increased demand for the substitute component results in a smaller amount of capital to be redistributed across remaining components, the effectiveness of component substitution in freeing up capital for redistribution across the remaining components is always reduced. Together, both factors require that an extreme increase in constraint be required to justify change. This is why the average investment in inventory at which substitution becomes RDP-maximising is approximately 65% of that required for the policy change in the stochastic case.

6.8. Summary and conclusion

This chapter has extended the analysis of inventory reduction decisions presented in the previous chapters by modelling the case where a company's investment in inventory is subject to a binding constraint, with a consequent risk that it may fail by exhausting the capital required for inventory replenishments. The model derived in this chapter has presented a new measure of company performance, which discounts the profit that a company would earn if survival were certain for the risk that the company may fail, with no return to an investor. This new model then permits inventory replenishment policy decisions to be evaluated in terms of their effect on both profitability and risk of company failure. This new model required that profit be calculated using the (Q,r) model of the previous chapter. The risk of company failure was then determined as a function of the volatility of the inventory process, the review period, and the maximum investment permitted. The volatility of the inventory process was determined by two factors: the variance of the investment in inventory and the rate at which capital was consumed for the purchase of inventory. It has been shown that any change in replenishment policy affected both profit and inventory volatility, and hence the risk of failure. As a consequence, in any determination of the optimal inventory replenishment policy, reducing risk and increasing profit were competing factors. By accepting the negligible risk of failure when it ceases to be cost-effective to invest in prevention, the new model yields replenishment policies that increase the profitability of the company's operations over those determined by the traditional Lagrangian approach

The analysis of inventory replenishment policy decisions in this chapter has reaffirmed the conclusions of Chapters 4 and 5. However, the introduction of the risk of company failure, as an extra factor to be controlled has meant that the effect of each type of replenishment policy change on company survival be an additional factor in all policy decisions. Under these new conditions the relative cost-effectiveness of JIT replenishment has increased because the elimination of a component from inventory reduces the rate of capital consumption for the purchase of inventory and the volatility due to the demand through lead time fluctuation for the eliminated component. Each of these factors reduces total inventory volatility and consequently reduces the risk of company failure. On the other hand, the substitution of one component with a more expensive alternative increases the rate of capital consumption and the variance of demand through lead time for the substitute component and consequently increases the risk of company failure, thus reducing the attractiveness of this inventory reduction initiative.

In the cases of Screw 1 and Screw 2, both the demand through lead time, and the rate of capital consumption for these components were significant. Consequently, the elimination of these components from inventory was cost effective at a greater average investment in inventory than for the risk-free case. The JIT replenishment decision analysis for these components shows that the reduction in inventory volatility is an additional benefit of JIT replenishment, which can be valued as the increase in expected profit under the policy change. That the JIT replenishment of these components increases the probability of company survival at reduced investment enhances the other benefits described in previous chapters. Figures 6.11 and 6.12 show that much of the benefit arising from inventory reduction over the company's feasible investment range is due to the JIT replenishment of these components.

For the case study company it was RDP-maximising to maintain a high probability of company survival at all but the most highly constrained investment levels. The replenishment policy decisions reinforce this by requiring that all changes be accompanied by an increase in probability of company survival. The requirement that survival probability increase at each policy change reduced the amount of capital available for redistribution as each component was eliminated from inventory. Consequently the indifference investment level for most inventory policy changes occurred at a smaller average inventory investment than was the case when risk was not a factor in the decision. This effect was greatest for component substitution where the resulting increase in inventory volatility increased the risk of company failure attributable to the replenishment policy change.

Concerning the generality of the results, this chapter has introduced a model for the distribution of company lifetime, based on the probability of the company exceeding a predetermined capital limit. The model is representative of a company, having investment only in inventory, becoming illiquid as a result of all capital being tied up in component inventory. The model makes few assumptions about the underlying inventory process, except that the demand for each type of components is uncorrelated in the long-term. It was shown in the previous chapter that this is a reasonable assumption. The exponential form of the resulting lifetime distribution accords with the observation that real companies fail at a constant rate. The model presented in this chapter assumes a single level bill-of-materials, and thus the effect of holding work-in-progress or finished product inventories on inventory volatility have not been investigated. The current model determines the probability of exceeding a capital constraint as a function of the replenishment policy and customer demand. Thus, the model determines the risk of company failure due to the variability of the external demand for finished products. The effect of WIP or finished product inventory on the probability of exceeding a capital constraint depends on the company's management of these inventories. Thus, these inventories are subject to internal control. Holding any additional inventory over that of components will increase the base level (or average) inventory, and increase the variance of the total investment in inventory over time. However, it is also foreseeable that WIP and finished goods inventory, being less sensitive to volatility of demand for finished products could be managed strategically to minimise any increase in risk. However, this is a function of the proportional investment in each type of inventory and the company's policy for the management of these inventories, which remain to be more fully investigated.

Because the decision equations derived in this chapter do not yield analytical solutions, the analysis of whether profit-maximisation or risk-reduction is the more significant factor in the replenishment policy decision could not be fully completed. For the case study company, maintaining a high probability of company survival was a RDP-maximising strategy. The generality of the conclusions reached in this chapter are thus limited by this assumption. In a different situation, where the demand for finished products results in greater volatility of the inventory process, it may not be cost-effective to maintain a probability of company survival at the levels of the present study. However, whether policy changeovers are always associated with a decrease in risk is a function of component parameters and is an open question.

Chapter 7.

Synthesis and Conclusion

This thesis has explored the consequences for inventory replenishment policy decisions of viewing inventory as being financed with investor-supplied capital. Adopting this point of view has meant that inventory investment must be assumed to be constrained, with replenishment policy decisions being a function of the level of inventory capitalisation. The finite nature of the capital invested in the company has also introduced the risk of company failure through capital exhaustion as an additional factor in replenishment policy decisions. The thesis has presented an analysis of JIT replenishment and component substitution decisions under three different modelling assumptions. These models have presented an increasingly complete model of the case study operations. However, the direct application of the models in this thesis to other specific manufacturing operations is limited by the assumptions made about inventory management and manufacturing method described in Chapter 1. Notwithstanding these limitations, the models derived in this thesis are representative of inventory management decisions in assemble-to-order manufacturing environments more generally. The use of increasingly complex models has also progressively introduced the benefits resulting from these inventory reduction strategies in order to make the effect of each explicit. This chapter summarises these benefits, compares the effect of different modelling assumptions on the inventory reduction decision, and shows the increasing effectiveness of JIT replenishment under increasing uncertainty. The chapter concludes by describing themes for future investigation.

7.1. Summary of results

Table 7.1 presents a brief summary of the assumptions about the demand for finished products and the type of constraint imposed in each model of the previous chapters, in addition to the main conclusions obtained under each model.

Table 7.1: Assumptions and main results obtained under each model

Chapter	Demand for finished products	Type of constraint on inventory	Effect of JIT replenishment	Effect of component substitution
4	Deterministic	Average investment	Increased cost of JIT replenishment	Greater demand for substitute increases component costs
			Redistribution of capital formerly invested in component	Redistribution of capital formerly invested in component
			Increased profitability at reduced investment yields increased ROI	Increased profitability at reduced investment yields increased ROI
5	Stochastic	Average investment	Redistribution of capital formerly invested in cycle and safety stock	Redistribution of capital formerly invested in cycle and safety stock
			Eliminates the risk of lost sales attributable to component	Cost of lost sales attributable to original component transferred to substitute
			Reduced overall proportion of investment in safety stock	Increased safety stock for substitute component
6	Stochastic	Maximum investment	Reduced rate of capital consumption for component inventory	Increased rate of capital consumption for component inventory
			Variance of demand through lead time attributable to component eliminated	Variance of demand through lead time attributable to substitute increased
			Reduced risk of company failure	Increased risk of company failure

The deterministic model of Chapter 4 shows that when capital is investor-supplied and hence constrained, the elimination of a component from inventory permits the capital formerly invested in the component to be reinvested in the company at its marginal rate of return. These returns accrue from increased batch sizes of the remaining batch-replenished components resulting in reduced replenishment costs. The resulting cost saving may thus offset any increase in the cost of JIT replenishment or component substitution. Thus, the redistribution of the capital formerly invested in the JIT component presents an additional benefit of JIT replenishment to those identified by other authors. As the level of investment decreases, the effective value of reinvested capital increases, and under highly constrained conditions, it may be possible to justify JIT replenishment or component substitution at greatly increased replenishment or component costs. The adoption of these inventory reduction strategies also allows the business to remain profitable at low total investment levels making operation possible with less capital and with higher returns on investment of this capital.

In Chapter 5, the introduction of stochastic demand requires that part of the investment in inventory be used for safety stock as a hedge against lost sales resulting from component stockouts. It was shown that under stochastic demand, the replenishment of a component JIT allows for the reinvestment of the capital formerly invested in both the cycle stock and safety stock of the component into the remaining batch-replenished components. JIT replenishment also eliminates the risk of a lost sale due to a stockout of the JIT component. Finally, by reducing the level of inventory constraint, JIT replenishment reduces the overall proportion of capital invested in safety stock. Thus, JIT replenishment presents additional benefits when demand is stochastic to those shown in the deterministic case, which result from the impact of JIT on controlling the cost of uncertainty. Component substitution also results in the elimination of a component from inventory, which permits the reinvestment of the capital formerly invested in this component as it did when demand was deterministic. However, substitution results in the demand for the eliminated component being transferred to another (usually more expensive) substitute. This in turn results in increased safety stock for the substitute component. The cost of lost sales attributable to the eliminated component are also transferred to the substitute. Consequently, the effectiveness of component substitution diminishes under stochastic demand.

The imposition of a binding constraint on inventory investment in the Risk-Discounted model of Chapter 6 introduces the possibility that a company may fail by exhausting all available capital required for inventory replenishments. The presence of the risk of company failure means that the change in the company's expected profit under the new replenishment policy is an extra factor in the policy decision. It was shown that the risk of company failure is determined by the volatility of capital invested in inventory. This introduces an additional benefit of JIT replenishment through its potential to lower the risk of company failure by reducing inventory volatility. The elimination of a component from inventory reduces the rate at which capital is consumed for the purchase of component inventory, and eliminates the variance of demand through lead time attributable to the component. Each of these factors reduces inventory volatility and consequently reduces the risk of company failure. Conversely, component substitution is of diminished effectiveness under a risk-discounted profit model because the increased demand for the substitute component increases the rate at which capital is consumed from inventory and also the variance of demand through lead time attributable to the original component.

The introduction of additional assumptions in successive models has progressively exposed the benefits accruing from inventory reduction strategies. However, successive models have necessarily increased in complexity, which has in turn required the introduction of more assumptions and thus reduced the generality of the conclusions. The deterministic model of Chapter 4 assumes that finished products are assembled-to-order from stock, one-at-a-time, from a single-level bill-of-materials and that no work-in-progress or finished products are held, in order that all invested capital be available for investment in inventory. The stochastic model of Chapter 5 further assumes that a relatively expensive finished product is constructed from cheaper components in order to justify the approximation made in deriving an expression for the cost of lost sales attributable to components. In Chapter 6 it is further assumed that the RDP-maximising strategy for a company, at all but the most highly constrained investment, is one that has a very low risk of company failure. Thus the final model assumes that reserving a significant portion of the total investment in inventory as a buffer against capital exhaustion is a cost-effective investment strategy. It was shown in Section 6.3.6 that this assumption was true for the case study company. Although successive models have introduced assumptions that limit their rigorous application to manufacturing operations typified by the case study company, each model has presented insights about different aspects of the efficacy of JIT that can usefully guide inventory management practice more generally.

7.2. The increased effectiveness of JIT replenishment under uncertainty

The most important discovery of this work is that under the assumption of constrained capital, the capital formerly invested in the component eliminated from inventory can be reinvested into the remaining batch replenished components to increase the economy with which each is replenished. The consequent reduction in inventory costs can then be used to offset the increased cost of JIT replenishment. This insight was a consequence of simultaneously observing both the financial and operational management of the case company, and had not been recognised by previous authors. Additional benefits, also not discussed in the literature were found when uncertainty was increased, which made JIT replenishment even more cost-effective through its ability to reduce the cost to the company of addressing uncertainty.

The models of Chapters 4 to 6 assume different degrees of uncertainty in the company's operating environment. The deterministic model of Chapter 4 represents complete certainty about the demand for finished products. Uncertainty is introduced as stochastic demand for finished products in Chapter 5, which introduced the costs of lost sales and holding safety stock. The risk that a company may fail by exceeding a predetermined limit on investment in Chapter 6, makes the company's operating environment more uncertain and requires that a portion of the total investment be held in reserve as a buffer against the risk of company failure. The cost of increased uncertainty, as shown by a reduction in profit and increased capital constraint, under successive models is illustrated in Figure 7.1, which shows profit and λ obtained under the models of each chapter. Thus in the deterministic case, profit is calculated from Equation 4.9 subject to the constraint in Equation 4.10. Similarly in the stochastic case, profit is determined by Equation 5.7 subject to the constraint in Equation 5.8. Risk-discounted profit is calculated by following the steps outlined in Figure 6.8. Thus in Figure 7.1, the capital invested in inventory represents the average inventory for the deterministic and stochastic models, and represents the total investment when the risk of failure is introduced in the model of Chapter 6.

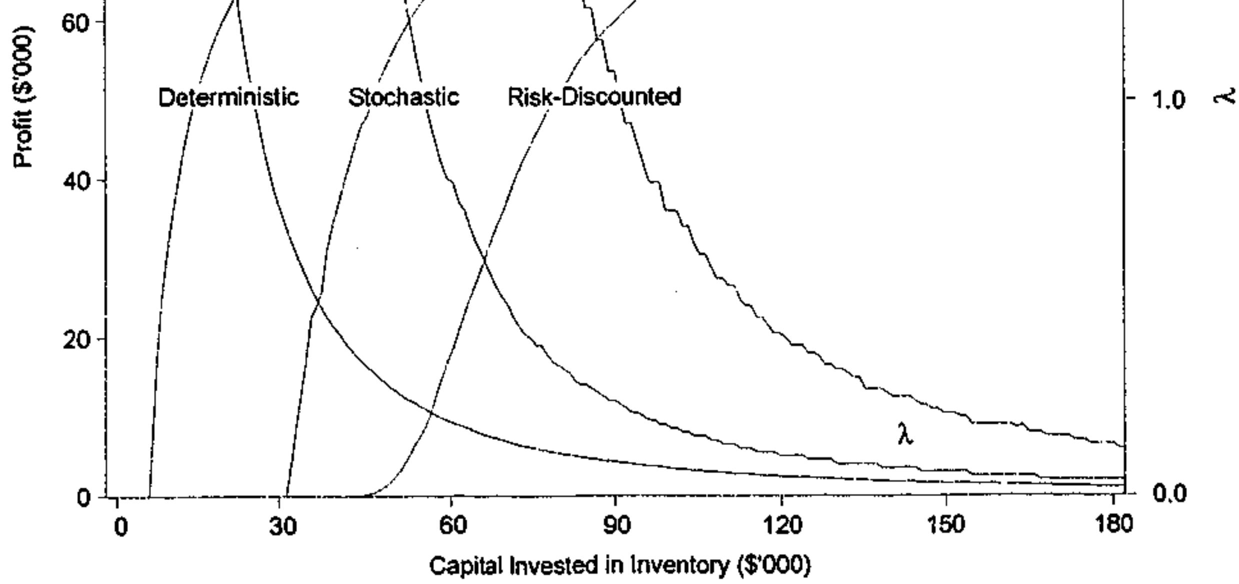

Figure 7.1: Profit and inventory constraint as a function of investment level for each model

The elimination of the costs of dealing with uncertainty presents further benefits of inventory reduction policies through their potential to reduce uncertainty. It has been shown that the effectiveness of JIT replenishment as a means of inventory reduction is enhanced under increasing uncertainty because JIT replenishment leads to a reduction in the costs of addressing stochastic demand and reduces inventory volatility. This is in contrast to component substitution, where the increased demand for a more expensive substitute component leads to greater costs of addressing uncertainty, which reduce the benefits arising from eliminating a component from inventory.

In order to see more clearly the effect of increased uncertainty on each method of inventory reduction, the replenishment policy decisions under each model are now compared. Figure 7.2 shows the indifference investment level for each policy change under each model expressed as a proportion of the maximum reasonable investment that an investor would consider under the assumption of each model when the risk-free rate of return is 10%. The data from which Figure 7.2 is derived was originally presented in Tables 4.1, 5.3 and 6.2, where the same limit on the maximum investment was set. A summary of this data appears in Appendix B as Table B.1. Although the maximum investment in inventory is different under the assumptions of each of the three models, the purpose of Figure 7.2, Table B.1, and subsequent discussion is to compare the effectiveness of different policy changes as a proportion of the maximum reasonable investment under each model. Consequently, the data is presented together on the same scale. Although the choice of the maximum reasonable investment is sensitive to the choice of return sought by an investor, the proportional investment level at which the policy changes are cost-effect would change in a consistent way. Thus the conclusions drawn from Figure 7.2 and Table B.1 are not affected by the choice of a maximum investment.

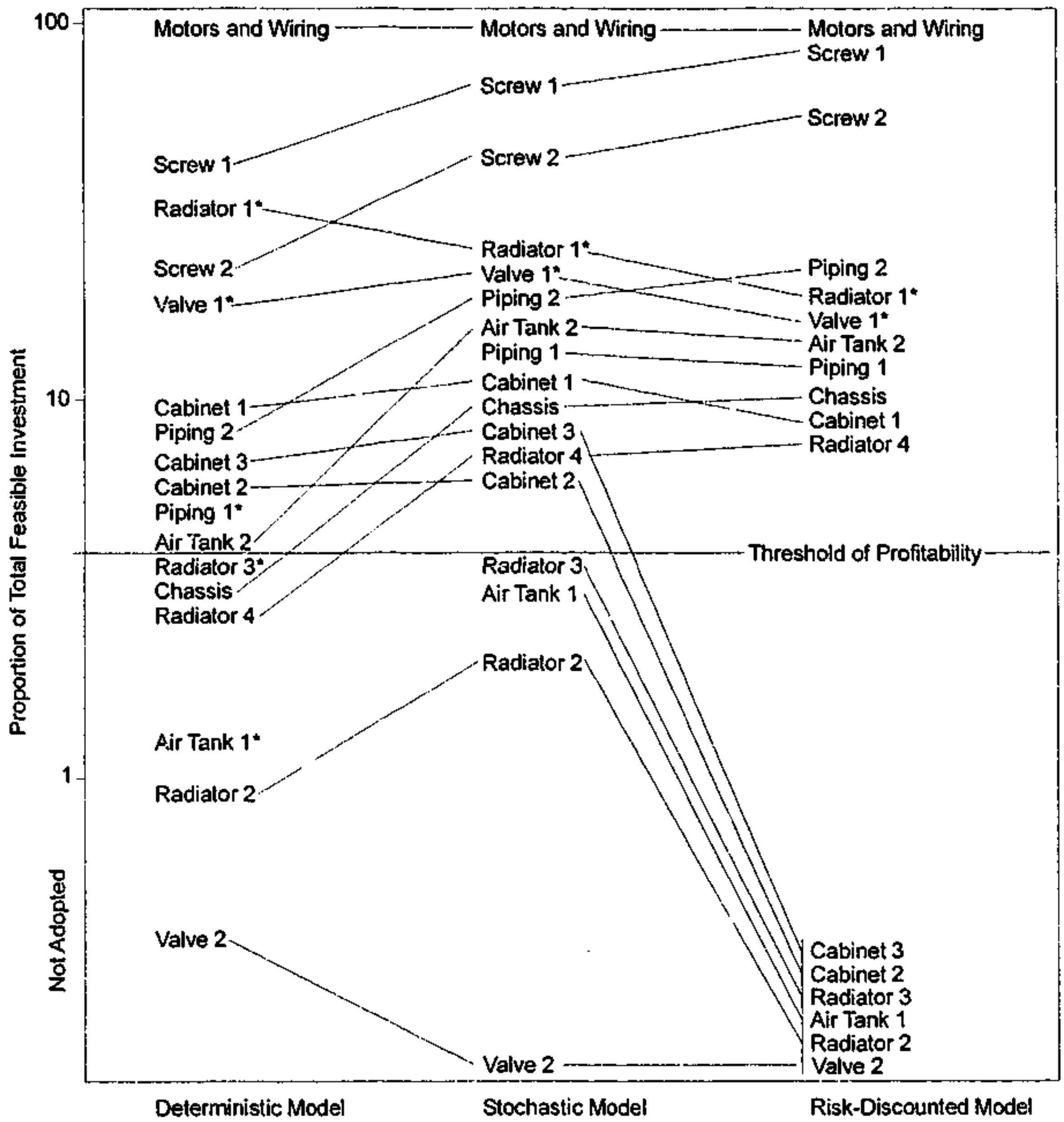

Figure 7.2: Indifference investment levels for each policy change expressed as a proportion of the maximum reasonable investment for each model. * Indicates component substitution.

The JIT replenishment of Screws 1 and 2 are profit maximising at an increasing proportion of total investment as uncertainty increases. It was observed in Chapter 5 that the JIT replenishment of these components confers every benefit arising from JIT replenishment described in this thesis. These components remain effective candidates for JIT replenishment at increased investment levels under the stochastic and risk-discounted models because the level of inventory constraint is increased, which in turn increases the value of the capital reinvested into the remaining batch replenished components. Because of their high unit cost, high replenishment cost and long lead time when batch-replenished, these components account for a significant proportion of inventory and consequently it is cost-effective to replenish them JIT at relatively large investments. The JIT replenishment of several other components is also profit maximising at increased proportional investments under increasing uncertainty. These are Piping 2, the Chassis, and Radiator 4. By contrast, the substitution of Valve 1 and Radiator 1 become profit maximising at decreasing proportional investments as uncertainty increases. This is because the increased demand for the more expensive substitute results in increased safety stock under stochastic demand and increased inventory volatility when the risk of exceeding a capital constraint is a consideration.

The additional benefits of inventory replenishment decisions introduced in successive models are also evident in the greater benefits that result from adopting inventory reduction under each model. Figure 7.3 shows company profit under each model as a function of investment level both with and without the adoption of the inventory reduction strategies described in Tables 4.1, 5.3 and 6.2. Under each successive model, the increased profitability and lower threshold of profitability that result from the adoption of the inventory reduction strategies are evident. It can be seen that the magnitude of both the profit increase, and the reduction in the threshold of profitability, are greater under increasing uncertainty. As a consequence of this, the increase in maximum ROI that can be obtained under each model is greatly increased under increasing uncertainty. These reduced levels of investment may be desirable for an investor with limited access to capital, wishing to minimise total investment in order to maintain flexibility, or to pursue greater ROI.

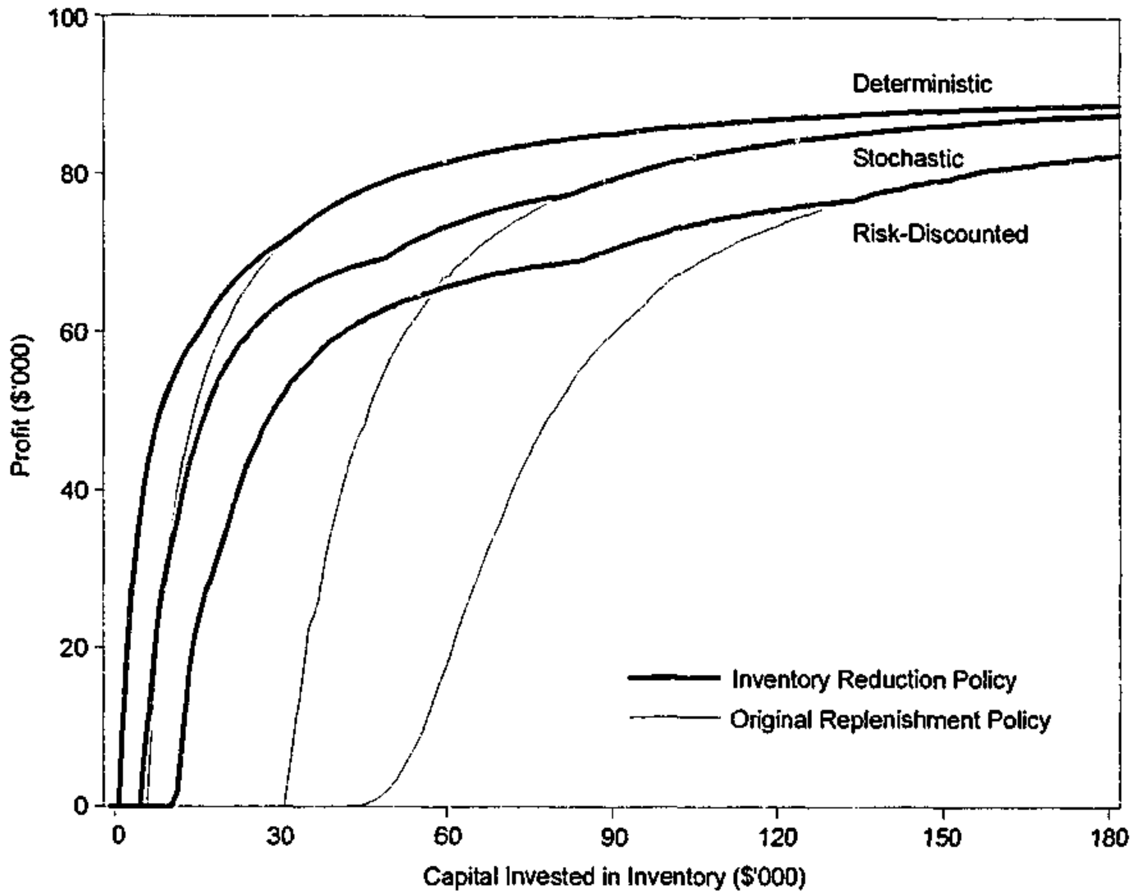

Figure 7.3: Profit with and without the adoption of inventory reduction strategies as a function of investment level under different models

The effect of adopting JIT replenishment and component substitution on the maximum ROI, and the minimum investment that could be considered under each model are shown in Table 7.2. Generally speaking, the adoption of inventory reduction strategies allows for a reduction in the minimum possible investment to approximately half that of the original (batched) replenishment policy. Table 7.2 also shows that the maximum ROI that can be earned is approximately doubled when inventory reduction strategies are adopted.

Table 7.2: Minimum possible investment level and maximum ROI, with and without the adoption of inventory reduction strategies under different models

	Minimum Possible Investment			Maximum Return on Investment		
Model	Original Policy *($)*	Inventory Reduction *($)*	Percent Change %	Original Policy	Inventory Reduction	Percent Change %
Deterministic	8,000	3,000	-63	3.14	5.78	184
Stochastic	32,000	7,000	-78	1.02	2.86	280
Risk-Discounted	43,000	12,000	-72	0.67	1.71	257

7.3. Major contributions of the research

7.3.1. Investigation of research objective

The research results show that when the capital invested in inventory is constrained, a number of benefits resulting from JIT replenishment and component substitution emerge, which have not previously been identified:

i. *Redistribution of the investment formerly in the JIT component.* When capital is investor-supplied, the elimination of a component from inventory by adopting JIT replenishment of that component releases the capital formerly invested in that component which can be reinvested in the remaining batch-replenished components to increase the efficiency with which each of these components is replenished. The resulting savings in total inventory costs of the remaining batch-replenished components can be used to offset a proportion of the cost of JIT replenishment, making JIT replenishment cost-effective, even under increased replenishment costs. The magnitude of the benefit increases as constraint increases, assuming greater effectiveness as the capital invested in inventory diminishes.

ii. *Increased ROI from inventory reduction using JIT replenishment.* Because the adoption of JIT replenishment for certain components increases profitability at reduced investment and reduces the minimum investment required for a company's operations to be profitable, it greatly increases the ROI earned by the company at reduced investments levels, over a non-JIT policy.

iii. *Elimination of costs due to uncertain demand under JIT.* When demand for finished products is stochastic, the elimination of a component from inventory, by adopting JIT replenishment, also eliminates the capital invested in the safety stock of the JIT component which may also be invested in the remaining batch-replenished components. By replenishing within the time a customer will wait for the manufacture of an end-item, the cost of lost sales attributable to the JIT component is also eliminated. Furthermore, by reducing the level of constraint, the elimination of a component from inventory reduces the overall proportion of capital invested in the safety stock of the batch-replenished components, thus, further increasing the efficiency with which batch-replenished components are purchased.

iv. *Reduction of inventory volatility through JIT replenishment.* When inventory is subject to a binding constraint, the elimination of the JIT component from inventory reduces the variance of the inventory process attributable to that component. The rate at which capital is consumed for the manufacture of products is also reduced by eliminating the JIT component from inventory. These factors reduce the volatility of the inventory process, and consequently reduce the risk of company failure, which introduces an additional benefit by increasing the company's expected profitability at a given investment level.

v. *Redistribution of the investment formerly in the substituted component.* Under deterministic demand, component substitution introduces similar benefits to JIT replenishment by eliminating a component from inventory, although the increased demand for the substitute component increases the optimal replenishment batch size for this component. Therefore, the amount of capital reinvested into the remaining batch-replenished components is lower than if JIT replenishment were adopted.

The initial analysis under deterministic demand shows that component substitution is of similar effectiveness as JIT replenishment as a strategy for inventory reduction. However, the effectiveness of substitution diminishes when stochastic demand and the risk of company failure are introduced:

vi. *Effectiveness of substitution reduced under stochastic demand.* The effectiveness of component substitution diminishes under stochastic demand where the demand for the component eliminated from inventory is transferred to the substitute component, and increases the safety stock and variance of demand through lead time for this component. As a consequence, less capital is redistributed to increase the economy with which remaining batch-replenished components are purchased.

vii. *Increased inventory volatility through component substitution.* Component substitution fails to reduce the variance of capital invested in inventory, and the substitution of one component with a more expensive alternative increases the rate at which capital is consumed for the purchase of inventory items. These factors increase the volatility of the inventory process. Consequently, component substitution fails to reduce the risk of company failure, which further diminishes the effectiveness of this method of inventory reduction.

7.3.2. Developments in inventory modelling

This research also introduces several new developments in inventory modelling:

viii. *Adaptation of (Q,r) model for Assemble-To-Order manufacture.* The multi-item (Q,r) inventory model is modified for the case where components are assembled-to-order from stock under a single level bill-of-materials, and sold as finished products. The model requires that the lost order cost for each component be approximated from the cost of lost sales of finished products attributable to that component. The model permits a fast, iterative solution to multi-item batch sizing, which has not previously been reported.

ix. *Inventory model incorporating the risk of company failure.* A new multi-item inventory model is developed for the case in which the investment in inventory is subject to a binding constraint. This model determines the risk-discounted profit, that is, profit if survival is certain, discounted for the risk that the company may fail by exceeding the capital constraint over a certain duration. The model thus integrates the determination of business risk and operational performance for a given investment policy. This approach to determining inventory policy has not been reported by previous authors.

7.4. Significance of the research

The development of the model in Chapter 5 has provided an exact solution to the problem initially posed by the manager of the case study company. The model also allows for the determination of optimal investment policies under different product/component/investment scenarios. The response of the case company to the insights gained by this research has been to progressively apply the inventory reduction methods described, where feasible, to all components. Since the original case study was undertaken, the company has experienced a threefold increase in demand for air compressors and has increased its range of compressors to include several larger models. However, the company has been able to accommodate increased demand and product variety without any increase in either the physical space in the factory for manufacture and storage, or any increase in capital invested. The company has maintained the batched replenishment of Screw 1 and Screw 2 but has adopted JIT reordering of the screw elements for the new larger models. JIT replenishment is also used for the supply of cabinets, air tanks and piping. A significant outcome of the project for the company was the acceptance of the idea that replenishing the high cost screw component one-off by air transport could be cost effective under certain conditions: at the beginning of the project this was viewed as an absurdity. The company is moving towards franchising their manufacturing system. The model described here has proved useful in demonstrating to investors that a broader range of investment levels and high returns are possible.

The analysis has a number of implications for practitioners. It shows that JIT initiatives should be directed at *specific* components. The remaining components are best managed by traditional economy-of-scale methods. This is a new insight since current JIT rhetoric generally advocates an all-or-nothing approach. Also, the analysis has shown how JIT replenishment may be adopted without requiring any commitment to the broader reforms of JIT, for example, production smoothing and quality improvements. The analysis also shows how to identify candidate components for JIT policy change with a model that makes realistic assumptions about customer demand and the finite nature of invested capital. It also shows that a manager can make correct decisions about inventory policy changes by mentally valuing the cost of money tied up in inventory at the marginal rate of return of the company as well as other cost savings that occur when components are eliminated from inventory. Although these factors may not be known precisely, this change in thinking can lead to a more informed understanding about what is best. Furthermore, this type of analysis can also be applied to work in progress and finished goods inventories. Finally, it shows how to select inventory policies that maximise either return on investment, absolute returns, or a combination of both, given the investor's preference and available capital.

This thesis has introduced developments in inventory modelling that now make possible the analytical determination of the optimal replenishment policy for ATO manufacture under stochastic demand at a predetermined level of inventory capitalisation set by a general manager or investor. Previous attempts to determine an optimal investment strategy for the case company required the trial-and-error testing of particular scenarios using discrete-event simulation. Thus the analytical method presents both computational efficiency and the guarantee of optimality (within the limitations of the model). Specific contributions are the introduction in Chapter 5 of an adaptation of the (Q,r) model for a case in which batch-replenished components are sold as assemblies, and in Chapter 6, a model for determining the optimal investment in inventory when the risk of company failure by exceeding a capital constraint exists. The new model resulted in increased profitability for the case study example over traditional approaches by determining when it was cost-ineffective to invest further in risk reduction. In Chapter 4, a method for determining the profit-maximising investment at a given investment was presented, which could form the basis of a decision support module.

In addition to showing the benefits that emerge from JIT replenishment under capital constrained investments, this research has shown how JIT replenishment may be used to reduce total investment, in the pursuit of greater ROI. It has also shown how, by both increasing profitability and reducing the minimum feasible investment, JIT replenishment permits far greater ROI that could be obtained using batch replenishment. Finally, by showing how replenishment batch sizes reduce under capital constraint, the thesis has presented a reconciliation of the EOQ and JIT approaches to determining replenishment policy.

7.5. Directions for future research

Future research motivated by this thesis includes:

i. *Investigation of the benefits of JIT replenishment during company start-up:* The extension of the model in Chapter 6 to consider the case of a company during its start-up phase. This is in contrast to the model presented in this thesis, which assumed that the inventory demand/replenishment process had continued for a sufficiently long duration in order that the inventory levels of components can be assumed to be independent. In the case of a company start-up, the independence of components cannot be assumed, and the analysis of the inventory levels for simulated companies operating with no initial inventory has shown that a clearly identifiable peak in inventory occurs at start-up. The longest lead time of the batch-replenished components establishes a minimum duration until production can commence. These factors introduce several potential benefits resulting from JIT replenishment over those described in the thesis.

Firstly, the elimination of a JIT component from inventory may reduce the degree to which the inventory peaks at start-up, and the consequent risk of company failure. Secondly, in the case of components with a long lead time, JIT replenishment may permit the company to begin manufacturing operations earlier, which has consequences both for profitability, and for the commencement of the inventory demand in order that the peak occurring at start-up be reduced. Consequently, one line of investigation is to consider the benefits of JIT replenishment at company start-up, both in terms of risk reduction and through the earlier commencement of operations. Due to the difficulty of modelling this situation analytically, discrete-event simulation, using the model described in this thesis, and the case study data, is proposed as an initial approach.

ii. *The one-off purchase of machinery to permit the manufacture of components JIT:* The case study company uses certain components that, under their current method of production, incur replenishment (setup) costs that prohibit JIT replenishment. The radiators and valves are good examples, as both require considerable tooling up before manufacture of these components can commence. As a consequence, both components have always been replenished in economical batches. However, the manager of the case study company has expressed interest in the purchase of numerical-controlled machinery to permit the manufacture of the radiators in-house on a just-in-time basis. As the investment required for this is significant, the company wishes to undertake a cost-benefit analysis of this purchase. It is proposed to develop a model similar to that in Chapter 5, but to include a one-off purchase cost for the machinery in order to determine the investment level at which the company could justify the purchase.

iii. *Further investigation of the ranking of components for JIT replenishment or substitution:* Chapter 4 introduced two ratios for the ranking of the indifference investment levels at which JIT and component substitutions were profit maximising.

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iii. *Further investigation of the ranking of components for JIT replenishment or substitution:* Chapter 4 introduced two ratios for the ranking of the indifference investment levels at which JIT and component substitutions were profit maximising.

These ratios were $\frac{\sqrt{R_1 C_1}}{R_1^{'}\sqrt{D_1}}$ and $\frac{\sqrt{D_1 R_1 C_1}+\sqrt{D_2 R_2 C_2}-\sqrt{(D_1+D_2)R_2 C_2}}{D_1(C_2-C_1)}$ for the JIT and component substitution cases respectively. Although these ratios were derived from approximations to the expression for the actual indifference investment level, the consistency of the predicted rank of policy changes to the actual rank of changes determined exhaustively was high. The high predictive power of these ratios suggests that the current exhaustive method of determining the optimal sequence of JIT replenishment and component substitution may be hastened by evaluating proposed policy changes in the sequence determined by these ratios. Such an approach would potentially increase the efficiency with which the optimal sequence of replenishments could be determined for large inventories. However, a fuller investigation into the accuracy of these ranking methods in relation to component parameters and inventory size remains a subject for further investigation.

Appendix A.

The RDP-Maximising Replenishment Policy when Inventory is Reviewed Periodically

The derivation of the lifetime distribution in Chapter 6 is based on the assumption that the total inventory investment level is continuously reviewed, with the company being deemed to fail at the first instance of K exceeding K^*. This approach implicitly treats all financial transactions as though they are cash based. In this appendix, the distribution of company lifetime is determined for the case when the total inventory investment level is reviewed at discrete time intervals and inventory investment is permitted to exceed K^* (indicating over-investment) except at the review times. This more general model corresponds to the case where inventory investment level is reconciled periodically, for example, monthly. It is shown that increasing the duration between successive reviews reduces the risk of company failure. Consequently, the effect of the review period on the optimal investment policy can be explained in more general terms through its effect on the underlying risk of company failure. An analysis of the effect of review period on the RDP-maximising policy shows that under decreasing risk of failure the optimal level of buffer capital decreases and the optimal survival probability increases. The analysis of the decision to replenish Screw 1 JIT however, shows that the cost factors underlying this decision are largely unaffected by changing risk.

A.1. The survival probability

When the investment in inventory is reviewed periodically, it is required that for a company to fail that an instance of $K > K^*$ must coincide with an instant at which total inventory investment level is being reviewed. This is illustrated in figure A.1, which is an adaptation of figure 5.4, plotted from the same data to now show the lifetime for a simulated company when inventory review occurs periodically at the times indicated. The lifetime of a simulated company is indicated, showing the commencement of operations at t^0 and failing at t^3 when $K > K^*$ is observed. In the following analysis, company lifetime under periodic review is denoted as $t_{K^*}(y)$, where y is the duration between successive observations.

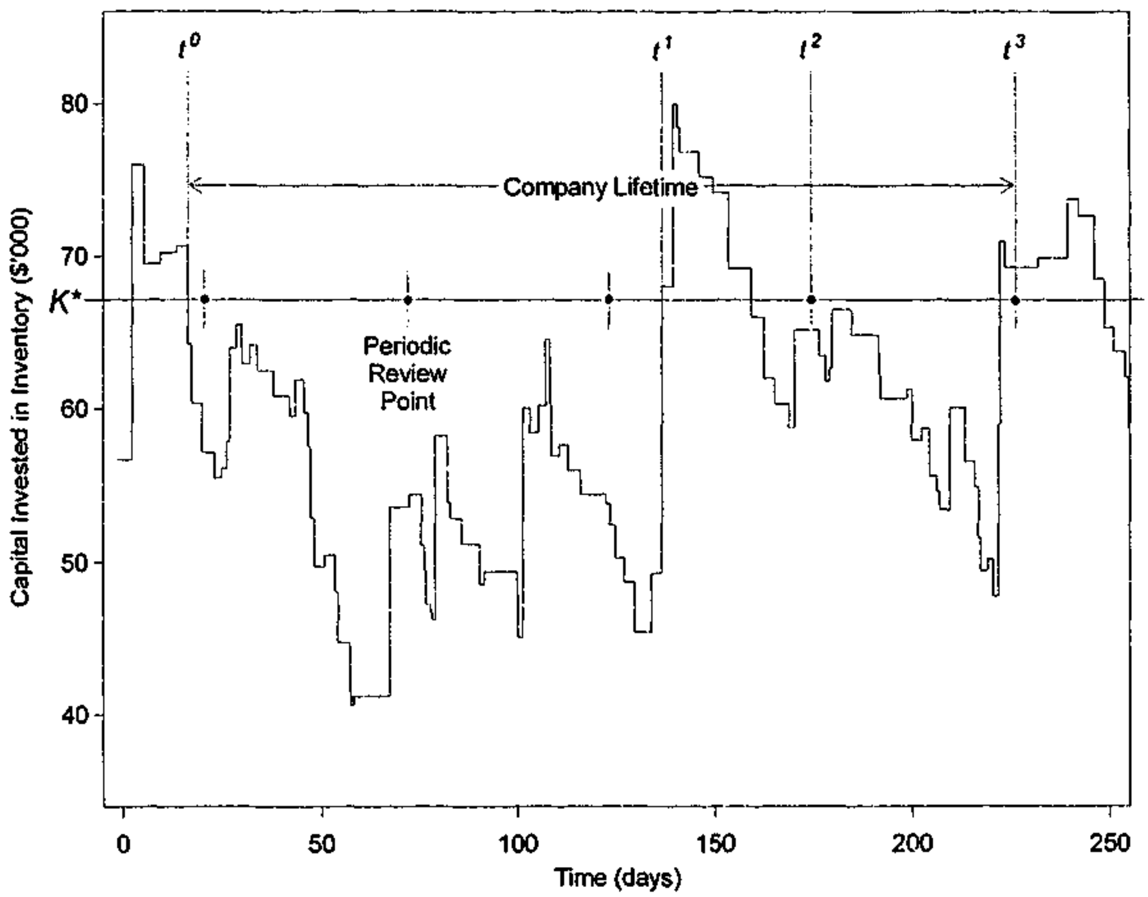

Figure A.1: Capital invested in inventory over time showing the lifetime for a simulated company based on an investment of K^* and periodic review of inventory level

A.1.1. Assumptions

$t_{K^*}(y)$ is assumed to be composed of three component time intervals, with each being a random variable. The first interval, t^0 to t^1, is the duration until the first instant where $K > K^*$. The second interval, t^1 to t^2 marks the period of time between the company's investment in inventory first exceeding K^* and the next periodic review. The third interval, t^2 to t^3, is the duration that company operates under periodic monitoring of K until $K > K^*$ is detected.

It is assumed that the duration between consecutive inventory level reviews are large relative to the interval between successive inventory fluctuations. This would be the case if the inventory were reviewed at one or two month intervals, say, for the case study company. When the number of inventory fluctuations between successive observations is large, the probability that $K < K^*$ in any subsequent observation of the inventory investment assumed to be independent of the value of K at the previous periodic observation. As a consequence, the probability that $K < K^*$ at any observation is given by the long run probability $F_K(K^*)$.

It is also assumed that $F_K(K^*) > \frac{1}{2}$, which corresponds to the case where $K^* > \overline{K}$, that is, buffer capital be greater than zero, which is identified in Chapter 6 as a basic condition for a non-zero probability of survival at all total investment levels (see figure 6.12).

A.1.2. Distribution of company lifetime

The duration of the interval t^0 to t^1 was determined in section 5.3, and is exponentially distributed with mean $\frac{F_K(K^*)}{f_K(K^*)}\frac{dt}{dK}$. Because the time at which a company fails under continuous review is independent of the timing of the periodic review points, the duration $t^2 - t^1$ is a uniformly distributed random variable on the interval $[0, y]$ having mean $\frac{y}{2}$.

In order to determine the distribution of $t^3 - t^2$, let the duration from t^2 to t^3 consist of W sub-intervals between consecutive observations. The probability that $K > K^*$ is observed at any periodic review point is $F_K(K^*)$. Thus, probability that $K > K^*$ is first observed at the W^{th} periodic observation after t_2 is

$$f_W(W) = F_K(K^*)^W(1 - F_K(K^*)) \quad for \quad W = 0,1,2... \tag{A.1}$$

which defines a geometric distribution having mean $\frac{F_K(K^*)}{1-F_K(K^*)}$. Note that the distribution of $f_W(W)$ includes the possibility that the company may be observed to have failed at t_2. Because the interval t^2 to t^3 is composed of W periodic observations, each occurring at intervals of y periods, the duration t^3 - t^2 is also geometrically distributed having mean $\frac{yF_K(K^*)}{1-F_K(K^*)}$. Table A.1 summarises the means and distributional form of the three elements of $t_{K^*}(y)$.

Table A.1: The elements of $t_{K^*}(y)$

Element	Interval	Distribution	Mean
1	t^0 to t^1	Exponential	$\frac{F_K(K^*)}{f_K(K^*)}\frac{dt}{dK}$
2	t^1 to t^2	Uniform	$\frac{y}{2}$
3	t^2 to t^3	Geometric	$\frac{yF_K(K^*)}{1-F_K(K^*)}$

$t_{K^*}(y)$ is a mixture distribution consisting of the three component distributions in table A.1. Although the form of $t_{K^*}(y)$ is indeterminate, when y is small, say $y \approx 0$, the means of elements 2 and 3 are small relative to the mean of element 1, hence the resultant distribution can be approximated by the exponential distribution. Conversely, when y is large, say $y > 10$ for the case study company, the mean of element 3 is large relative to the means of elements 1 and 2, hence, the resultant distribution is approximately geometric, which has the exponential distribution as a continuous analogue. Thus for either y large or small, the distribution of $t_{K^*}(y)$ can be approximated by the exponential distribution, with mean given by

$$\langle t_{K^*}(y)\rangle = \frac{F_K(K^*)}{f_K(K^*)}\frac{dt}{dK} + \frac{y}{2} + \frac{yF_K(K^*)}{1-F_K(K^*)}, \tag{A.2}$$

thus the probability of a company surviving t periods is

$$\wp = p(t_{K^*}(y) > t) = e^{-t/\langle t_{K^*}(y)\rangle} \tag{A.3}$$

Note that Equation 6.22 is a special case of Equation A.2 with $y = 0$. Equation A.2 shows that for a fixed investment policy, increasing y will increase $\langle t_{K^*}(y)\rangle$. Equation A.3 shows that increasing $\langle t_{K^*}(y)\rangle$ results in an increased $\wp$ when investment policy and t are fixed. Thus, keeping other factors constant, increasing the interval between inventory level reviews leads to a reduction in the underlying risk of company failure due to capital exhaustion.

A.2. Sensitivity analysis

In order to illustrate the effect of changing the inventory level review period on the RDP-maximising investment policy, the optimal policy across a range of investment levels and the indifference investment level for the adoption of JIT replenishment of Screw 1 are determined under varying review periods. It was shown in the previous section that increasing the interval at which inventory was reviewed reduced the risk of failure for a given policy. Thus the following analysis of the effect of varying y has a more general interpretation as the effect of varying the underlying risk of company failure on the RDP-maximising replenishment policy.

In the following analysis, $y =$ 0, corresponding to continuous review, and $y =$ 21, 42, 63, corresponding to one, two and three month intervals between reviews respectively. The RDP-maximising inventory replenishment policy is evaluated for each parameter combination at total investment levels between \$30,000 and \$150,000 at intervals of \$100.

A.2.1. Variation in buffer capital and survival probability

The proportion of capital invested in buffer stock for RDP-maximising investment is shown for each value of y as a function of the total investment in figure A.2. It is evident that as the review period increases, and the consequent risk of company failure decreases, the optimal proportion of the investment in buffer capital decreases.

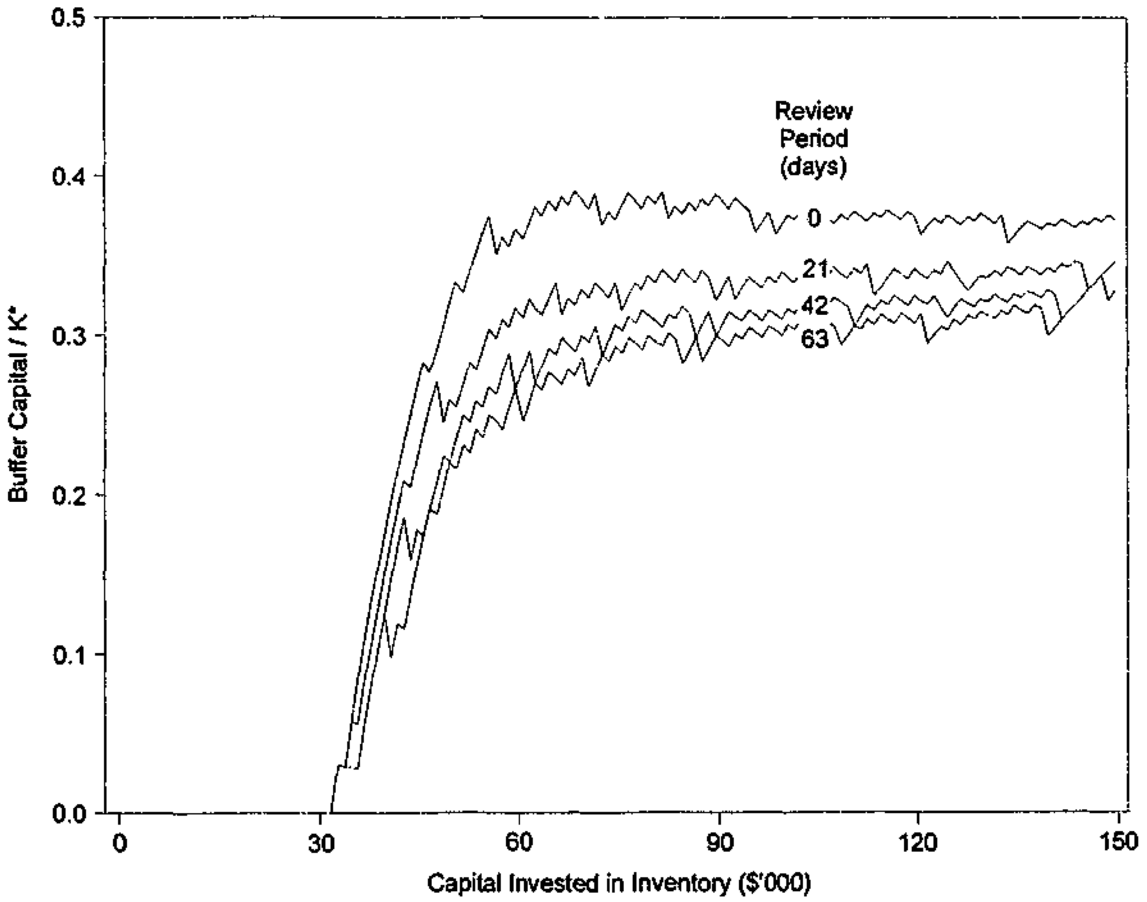

Figure A.2: Buffer capital as a function of the total investment in inventory under different review periods.

Figure A.3 shows the optimal value of $\wp$ for the RDP-maximising investment as a function of total investment level. The RDP-maximising value of $\wp$ increases as the as the interval between periodic reviews increases, with the function defining $\wp$ having the same form for each value of y, but being displaced horizontally (corresponding to the changing investment in buffer capital). Decreasing the underlying risk of failure leads to an increase in the RDP-maximising

$\wp$. The increase in $\wp$ is greatest at reduced investment levels where the competition between the investment in inventory and buffer capital is greatest.

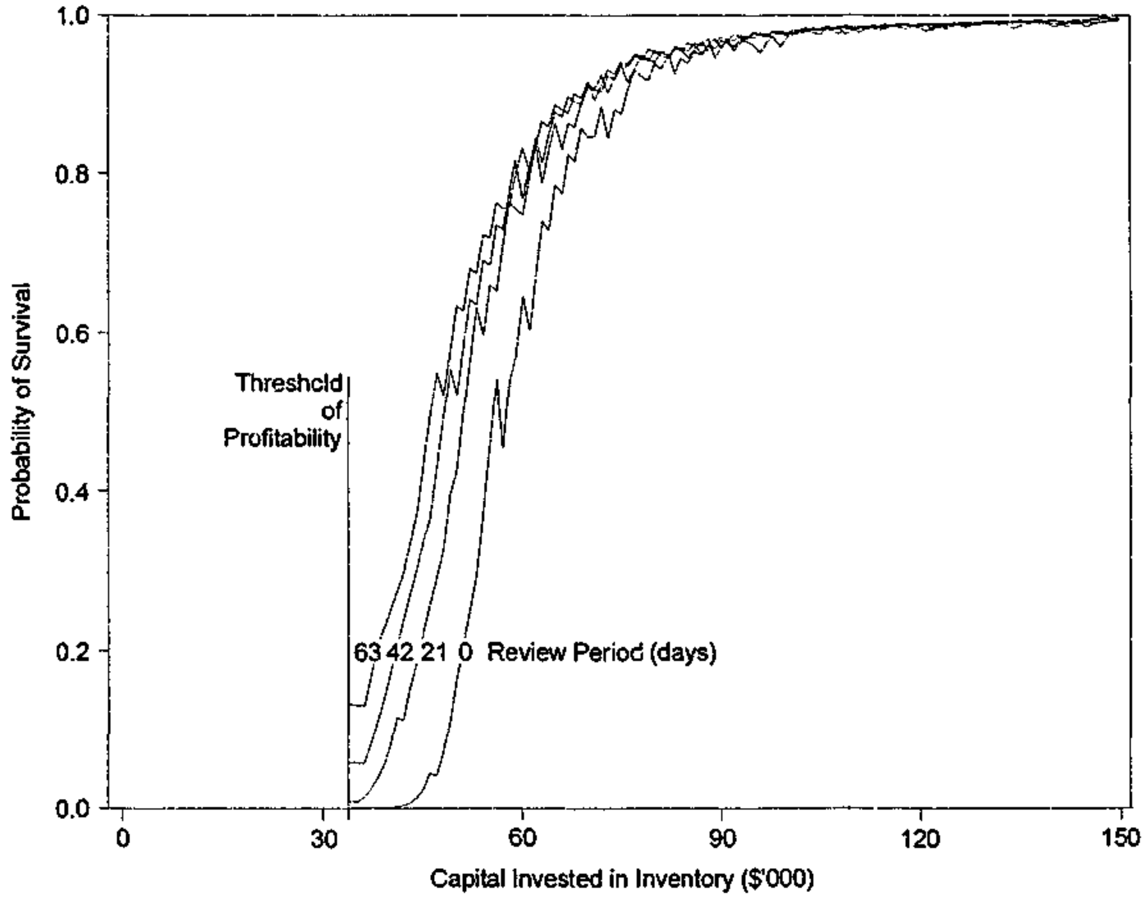

Figure A.3: The optimal $\wp$ as a function of the total investment in inventory under different review periods.

The net effect of both increasing $\wp$ and decreasing the proportional investment in buffer capital is to increase RDP. This is shown in figure A.4., where the effect of increasing the interval between inventory reviews results in the greatest increase in RDP at reduced investments.

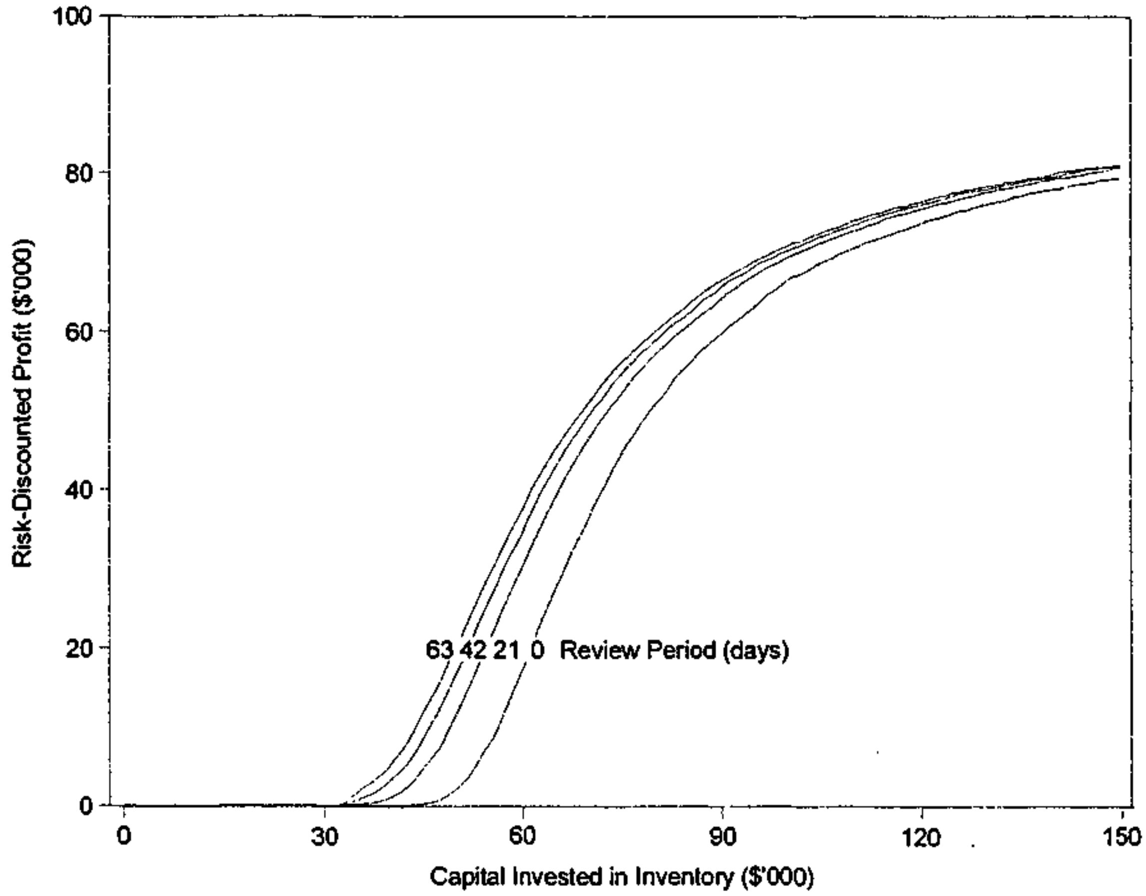

Figure A.4: RDP as a function of the total investment in inventory under different review periods

A.2.2. Decision to adopt JIT replenishment for Screw 1

The decision to adopt JIT replenishment for Screw 1 is now analysed as a function of y. The indifference investment level, RDP, the average investment in inventory and probability of company survival for the JIT replenishment decision have been evaluated at each value of y and are shown in table A.2.

Table A.2:. y, $\widetilde{K}^*$, RDP, $\overline{K}$, $\overline{K}_{JIT}$, $\wp$ and $\wp_{JIT}$ for the decision to adopt JIT replenishment of Screw 1 under different levels of risk

y	$\widetilde{K}^*$	RDP	$\overline{K}$	$\overline{K}_{JIT}$	$\wp$	$\wp_{JIT}$
63	119,900	76,496	82,500	78,700	0.987	0.995
42	121,600	76,486	82,400	78,900	0.988	0.995
21	124,500	76,568	82,400	79,100	0.989	0.995
0	132,800	76,758	83,000	80,900	0.989	0.995

Table A.2 shows that the values of $\overline{K}$, $\overline{K}_{JIT}$, $\wp$ and $\wp_{JIT}$ show very little variation as y changes. This indicates that the factors determining the replenishment policy change decision in Equation 5.28 are largely unaffected by different levels of underlying risk. This is because under changing levels of risk, the optimal values of $\wp$ and $\wp_{JIT}$ require similar increases/decreases in the proportional investment in buffer capital. This is illustrated in figure A.3 as each function defining $\wp$ is a displacement by $\$K$ of an essentially generic form. Furthermore, the increase in buffer capital is required to optimise $\wp$ and $\wp_{JIT}$, but has little effect on $\overline{K}$ (a slight increase in $\overline{K}_{JIT}$ as $\widetilde{K}^{*}$ increases), which means that the cost terms in Equation 5.28 remain unchanged. The increased investment in buffer capital under increasing risk explains the increased value of $\widetilde{K}^{*}$ as y decreases.

A.3. Summary

The preceding analysis has shown that the interval at which inventory is periodically reviewed is a factor in determining the risk of company failure. As a result of this observation, the effect of the review period on the optimal proportion of buffer capital and probability of survival for a given investment can be seen more generally through their relationship to the underlying risk of company failure. It was shown that increasing the interval at which inventory was reviewed decreases the risk of company failure. Decreasing the risk of failure resulted in a reduced investment in buffer capital, and an increased probability of survival for the optimal investment policy. Additionally, the effect of the inventory review period on the decision to adopt JIT replenishment of Screw 1 could also be explained in terms of underlying risk. This decision is essentially unaffected by the underlying risk of failure, except through its effect on buffer capital which explains the increase in $\widetilde{K}^{*}$ as the interval between successive reviews decreases.

Appendix B.

Comparison of Replenishment Policy Changes under each Model

Table B.1 shows the indifference investment level for each policy change under each of the three inventory models, originally presented in Tables 4.1, 5.3 and 6.2.

Table B.1: Policy changes for each component under each model in Chapters 4, 5 and 6, showing the indifference investment level for each change, which is also expressed as a percentage of the maximum feasible investment when $\lambda = I = 0.1$. Component substitutions are indicated by *

Deterministic Demand			Stochastic Demand			Risk-Discounted		
Component	$\widetilde{K}$	$\frac{\widetilde{K}}{\widetilde{K}_{max}}\%$	Component	$\widetilde{K}$	$\frac{\widetilde{K}}{\widetilde{K}_{max}}\%$	Component	$\widetilde{K}^*$	$\frac{\widetilde{K}^*}{\widetilde{K}^*_{max}}\%$
Motor 1	82,272	100.0	Motor 1	120,866	100.0	Motor 1	156,700	100.0
Motor 2	82,272	100.0	Motor 2	120,866	100.0	Motor 2	156,700	100.0
Motor 3	82,272	100.0	Motor 3	120,866	100.0	Motor 3	156,700	100.0
Motor 4	82,272	100.0	Motor 4	120,866	100.0	Motor 4	156,700	100.0
Motor 5	82,272	100.0	Motor 5	120,866	100.0	Motor 5	156,700	100.0
Motor 6	82,272	100.0	Motor 6	120,866	100.0	Motor 6	156,700	100.0
Motor 7	82,272	100.0	Motor 7	120,866	100.0	Motor 7	156,700	100.0
Wiring 1	82,272	100.0	Wiring 1	120,866	100.0	Wiring 1	156,700	100.0
Wiring 2	82,272	100.0	Wiring 2	120,866	100.0	Wiring 2	156,700	100.0
Screw 1	32,548	39.6	Screw 1	82,462	68.2	Screw 1	133,337	85.1
Radiator 1*	24,309	29.5	Screw 2	49,913	41.3	Screw 2	84,613	54.0
Screw 2	16,281	19.8	Radiator 1*	27,068	22.4	Piping 2	27,000	17.2
Valve 1*	12,808	15.6	Valve 1*	21,453	17.7	Radiator 1*	25,300	16.1
Cabinet 1	6,168	7.5	Piping 2	19,241	15.9	Valve 1*	23,800	15.2
Piping 2	5,697	6.9	Air Tank 2	13,389	11.1	Air Tank 2	20,800	13.3
Cabinet 3	4,706	5.7	Piping 1	11,590	9.6	Piping 1	16,800	10.7
Cabinet 2	4,463	5.4	Cabinet 1	10,040	8.3	Chassis	14,800	9.4
Piping 1*	3,855	4.7	Chassis	9,461	7.8	Cabinet 1	14,400	9.2
Air Tank 2	3,083	3.7	Cabinet 3	8,019	6.6	Radiator 4	12,300	7.8
Radiator 3*	2,434	3.0	Radiator 4	7,938	6.6	Air Tank 1	-	-
Chassis	1,970	2.4	Cabinet 2	7,133	5.9	Cabinet 2	-	-
Radiator 4	1,647	2.0	Radiator 3	4,356	3.6	Cabinet 3	-	-
Air Tank 1*	712	0.9	Air Tank 1	3,601	3.0	Radiator 2	-	-
Radiator 2	506	0.6	Radiator 2	2,294	1.9	Radiator 3	-	-
Valve 2	195	0.2	Valve 2	-	-	Valve 2	-	-

In order to compare the indifference investment levels under each model on a similar scale in section 6.2, each has also been expressed as a proportion of the maximum investment an investor would consider under each model when the risk-free rate of return is 10%, as was assumed in the case study examples of previous chapters. Thus, for the deterministic and stochastic cases, $\widetilde{K}_{max}$ was evaluated by setting $\frac{dP}{dK} = \lambda = 0.1$. $\widetilde{K}^{*}_{max}$ was determined by setting $\frac{dRDP}{dK} = 0.1$ for the risk-discounted case.

Appendix C.

The Discrete-Event Simulation Model

A discrete-event simulation model of the case study company was developed in the pilot study, and is used in this thesis to motivate and illustrate the analytical modelling in Chapters 5 and 6. This appendix first gives a brief introduction to the program. Details of input parameters and variables are then described. The appendix concludes with a more detailed description of the operation of the discrete-event simulation by describing each stage of the model.

C.1. Overview

The simulation model reproduces the daily operations of the case study company engaged in manufacturing activities subject to a constraint on capital invested in inventory in order to observe the profitability of successfully operating companies and the duration, or lifetime, over which the company successfully operates.

The program is written in Borland Delphi, and employs database tables managed by the Borland Database Engine to store information such as the Bill-Of-Materials and the specifications of components and finished products. Database tables are also used to record dynamic information such as the inventory level of components, future customer and replenishment orders as well as monthly operating profit.

The discrete-event simulation uses the day as the basic unit of time. Months are then used to establish the accounting framework, with summary of profit or loss calculated at the end of each month. Holding costs and overhead costs are expressed as yearly amounts and applied *pro rata* in each daily cycle. As there is no provision in the simulation for non-production days such as weekends and holidays a year of 252 days with 12 months of 21 days is used for all of the trials reported. Additionally, a cash based accounting method is used, with every financial transaction settled at the time it occur, that is, sales and purchases are paid for immediately, operating costs are applied at each period. The company's financial records are updated at every transaction and at the end of each period. The company pays no taxation, interest or dividends on profits, thus the expected annual profit earned from operations is equivalent to that given by Equation 5.7. In addition to financially oriented data, a record of all orders received and supplied is kept, which permits the determination of the service level of simulated companies.

C.2. Parameters and variables

The simulation program requires that the specifications of finished products and their constituent components are defined at initialisation from data stored as tab-delimited text files. Additionally, a number of parameters that describe the duration of phases of the simulation and factors such as the level of capital constraint on inventory are also required at initialisation. This section describes these input parameters, and variables used throughout the simulation.

C.2.1. Input parameters

Capital_constraint: The limit imposed on the amount of capital that may be invested in inventory. This variable was set at $100,000 for the main trials reported in this thesis.

Fixed_costs: Annual costs that are independent of demand or production, (this cost appears as F in the analytical models, and was set at $85,000 in all trials).

Initialisation_phase: The run-in period, or duration over which the simulated company operates without any performance statistics being recorded. This variable was set at 504 days for the trials reported in this thesis.

Termination_period: The period at which trials in which the simulated company is still operating successfully are censored. This variable was set at 2016 days after the Initialisation_phase for the trials reported in this thesis.

C.2.2. Input files

Three input files are used to describe the finished products and their constituent components at initialisation, these files are, the Product Master File (PMF), Finished Products File and Bill-Of-Materials. The Finished Products File contains information about the finished products. The file used for all simulation trials in this thesis is reproduced as Table C.1. The (average) annual demand and sale price of each product is given. Additionally, the customer lead time, or number of days that a customer would wait for their order to be filled is also set from this file. In the trials presented in this thesis, the customer wait time was set to 1 day, that is, the finished products were to be supplied the next-day or the order would be lost, as this was the closest approximation of the (Q,r) model possible with the discrete-event simulation.

Table C.1: The Finished Products File which describes the parameters of the manufactured products

Model Size *(horsepower)*	Annual Demand	Sale Price *($)*	Customer Lead Time *(days)*
3	12	2,800	1
5	8	3,000	1
7	8	3,300	1
10	25	3,900	1
15	25	5,200	1
20	7	6,000	1
25	15	6,500	1

The Product Master File describes the component specifications, including unit cost, replenishment cost, replenishment quantity, reorder point and lead time. Because the trials reported in the thesis were initialised with stock levels being set at their average level (see Section 6.1), this data is also included also. The PMF for the case of an average investment in inventory of $60,000 is shown in Table C.2.

Table C.2: Sample Product Master File for the case when the average investment in inventory is $60,000

Component Number	Unit Cost	Replenish't Cost	Replenish't Quantity	Lead Time	Stock at Initialisation	Reorder Point
	($)	*($)*		*(days)*		
1	1,000	750	15	100	17	10
2	200	0	1	1	0	0
3	80	260	22	15	15	5
4	100	50	5	1	2	0
5	110	2,050	37	20	21	3
6	100	1,050	23	30	15	4
7	75	30	8	2	6	3
8	75	30	11	2	8	3
9	150	0	1	1	0	0
10	250	0	1	1	0	0
11	300	0	1	1	0	0
12	100	50	6	1	3	0
13	150	1,050	25	30	16	4
14	400	0	1	1	0	0
15	190	3,050	55	20	32	5
16	500	0	1	1	0	0
17	430	260	9	15	8	4
18	150	50	6	1	3	0
19	200	1,050	22	30	15	4
20	125	30	6	2	5	3
21	2,000	750	6	100	8	5
22	600	0	1	1	0	0
23	700	0	1	1	0	0
24	350	1,050	11	30	8	3
25	200	0	1	1	0	0

The Bill-Of-Materials described the constituent components of each finished product using ordered pairs consisting of (Model Size, Component Number) as shown in Table C.3.

Table C.3: The Bill-Of-Materials for the assembly of finished products

Model Size	Component Number
3	1
3	2
3	3
3	4
3	5
3	6
3	7
3	8
3	9
5	1
5	3
...	...

C.2.3. Description of key variables

A number of variables are used in the simulation to determine, and control the reporting of, operational performance, including:

Company_account: The amount of capital (cash) held by the company. This variable is incremented at all sales, and decremented with all costs. Thus, this variable maintains a running record of the company's financial fortunes, and, assuming that the company's operation is successful, increases throughout the simulation. This variable is not used in the analysis of company profit presented in Chapter 6, and is only included for completeness.

Last_machine: This variable recorded the period in which the last finished product was made. This variable was updated as each product was made, and represented the productive lifetime of the simulated company.

Monthly_expenses: This includes the cost of purchasing components, their procurement costs, and operating costs. Note that, consistent with the analysis in the main text, the proportional cost of holding stock (J) was set to zero. Thus, no holding costs were included in the calculation of Monthly_expenses for the trials reported in this thesis.

Monthly_income: The income from sales over each month.

Monthly_profit: Monthly_income less Monthly_expenses. Monthly_profit also records the change in Company_account over each month and is used to determine the operating performance of simulated companies in subsequent analysis.

Orders_made: The number of finished products produced over the simulated company's lifetime after the Initialisation_phase.

Orders_received: The number of customer orders received over the simulated company's lifetime after the Initialisation_phase.

Sales_over_month: This variable records whether any sales of finished products are made over the current month.

C.3. The program

This section describes the operation of the discrete-event simulation program. Figure C.1 presents a flow diagram of the program. Details on each of the stages then follow. In addition to initialisation and termination activities, the simulation model progresses through a fixed sequence of manufacturing operations in each daily cycle, which are indicated in Figure C.1. Although the discrete-event simulation model is designed to run multiple trials with varying inputs, the procedures responsible for controlling multiple trials have been omitted for clarity. Thus the model shown in Figure C.1 shows only the stages required for running a single instance of a simulated company.

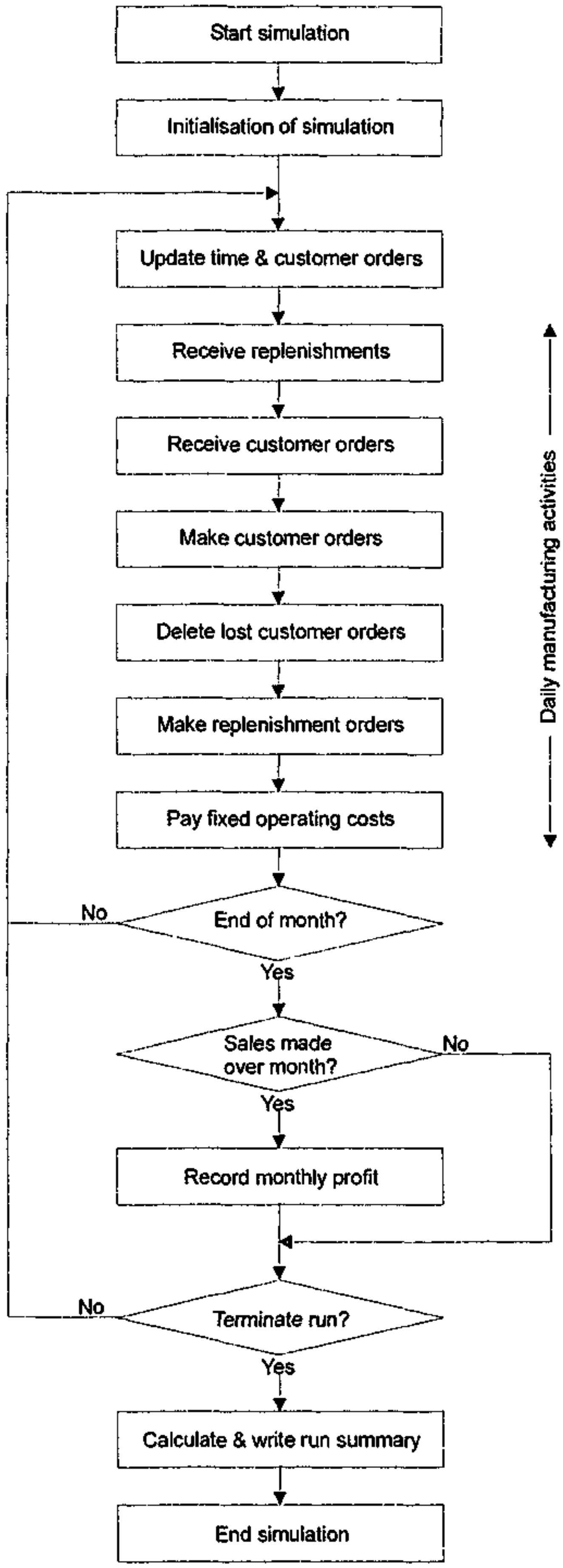

Figure C.1: Flow diagram of the discrete-event simulation model

C. 4. Description of the main stages of the simulation

Initialisation of simulation: At this stage, a file listing parameter values, and the names of files describing the specifications of finished products and components is read. The variables defined are: Capital_constraint, Company_account, Fixed_costs, Initialisation_phase, Termination_period. Other variables are set to zero. The files read are Bill-Of-Materials, Product Master File, Finished Products File. The current time, t, is set to day zero.

The time at which the first customer orders are received are also set at this stage. Let the time, in days, of the n^{th} customer order for each type of finished product be denoted as t_{ni}. Note that although t, is discrete-valued, t_{ni} is a continuous variable. Because the demand for finished products is a Poisson distributed random variable, the inter-order times are for finished products are exponentially distributed. Thus, the dates at which the first customer orders for each type of finished product are received are

$$t_{1i} = -\frac{252}{A_i}\ln(\Re), \qquad \text{(C.1)}$$

where $\Re$ is a uniformly distributed random real number between 1 and 0, and the average demand for each model of finished product is A_i.

Update time & customer orders: The current period is incremented by one unit. The timing of the next customer orders are observed each period to ensure that at least one customer order for each type of finished product extends at least one day beyond the current period. This is consistent with the standard approach in queuing simulations of maintaining a Future Event List (Pidd 1993; Banks *et al.* 1996). The time of successive customer orders for each model of finished product are determined using the pseudocode described in Figure C.2 and stored in a database table of future customer orders.

If $t_{ni} < t+1$ then

Repeat

$$t_{(n+1)i} = t_{ni} - \frac{252}{A_i}\ln(\Re)$$

Add order to future events list

Until $t_{(n+1)i} > t + 1$

Figure C.2: Pseudocode for generating successive customer orders for each model of finished product

Receive replenishments: Replenishment orders that are due to arrive in the current period are added to component inventory if this does not result in the value of the total inventory now exceeding the Capital_constraint. The value of components and procurement costs are added to the Monthly_costs. These costs are also deducted from the Company_account,.

Receive customer orders: Any customer orders for which $t \le t_{ni} < t+1$ are deemed to be received, that is, the simulated company is now aware of them for the purpose of manufacturing, and for determining the inventory position of constituent components. If the initialisation phase has been completed, then Orders_received is incremented by one unit.

Make customer orders: If all constituent component are present, then the finished product is 'made'. That is, one unit of stock is decremented from the inventory of all constituent components, and the sale value of the finished product is added to the Company_account and Monthly_income. If the initialisation phase has been completed, then Orders_supplied and Sales_over_month are incremented by one unit. The value of the variable Last_machine is set as the current period.

Delete overdue customer orders: Any customer orders that cannot be made and have exceeded the customer lead time are recorded as lost sales.

Make replenishment orders: Replenishment orders for components to be received in later periods are made using a standard reorder point method (Hopp and Spearman 1996). A replenishment quantity Q is ordered when the level of inventory reaches the reorder point r. The details of these replenishments are stored in a list of future replenishments (which is reviewed each period during the 'Receive replenishments' stage).

Pay fixed operating costs: these are deducted from the company's account and also added to the running total of monthly expenses at the daily rate.

End of month?: at the end of each month (set at 21 days in the trials reported) monthly profit is calculated as total monthly income less total monthly expenses.

Sales made over month?: This stage tests that the number of products sold over the month is greater than zero. The purpose of this stage is to ensure that monthly profit is only recorded while the simulated company is engaged in manufacturing. This test prevents the monthly profit (loss) being recorded in the case where a simulated company is unable to replenish inventory. If no sales have been made over the month, the variables monthly_income, monthly_expenses and sales_over_month are reset to zero and the simulation continues.

Record monthly profit: If monthly sales are greater than zero, and the initialisation phase completed then monthly profit is recorded. The variables monthly_income, monthly_expenses and sales_over_month are reset to zero.

Terminate run?: Two tests for terminating runs are used. Firstly, if no production has occurred for at least twice the longest lead time (200 days) then the run is terminated. Alternatively, for simulated companies still in operation at the censoring time (2016 days after the initialisation phase for the trials reported) then the run is terminated.

Calculate & write run summary: At the end of the run average monthly profit is calculated from stored values. If the company has survived the initialisation phase, that is,

$$\text{Last\_machine} > \text{Initialisation\_phase}, \qquad \text{(C.2)}$$

then the input parameter values, names of files used, summary statistics such as the average monthly profit, Last_machine, Orders_received and Orders_made are written to an output file.

Appendix D.

Comparison of Analytical and Simulated Results

This appendix compares the values of Risk-Discounted Profit obtained in the simulation trial, presented in Section 6.1, with those obtained from the theoretical model as defined in Figure 6.8. The results show that the values of RDP obtained theoretically are within two standard deviations of those obtained from the discrete-event simulation when the average investment in inventory is \$60,000 and \$65,000, indicating that the analytical model is a good approximation of the simulation model at these values.

The value of RDP obtained from the simulation study is the product of two performance measures: annual profit, and the one-year survival probability for simulated companies. Let $\hat{P}$ denote the annual profit for simulated companies, and $\hat{\wp}$ denote the one-year survival probability for simulated companies. The expected profit, $E(\hat{P})$ and the standard error of the expected profit, $\sigma_{E(\hat{P})}$, are determined from the simulated companies still in operation at the termination of the simulation trial. The expected one-year survival probability, $E(\hat{\wp})$, and its standard error, $\sigma_{E(\hat{\wp})}$, are determined from the one-year survival probabilities presented in Table 6.1. These statistics are summarised in Table D.1.

Risk-Discounted Profit is determined for simulated companies as expected profit multiplied by the one-year survival probability. Thus, for simulated companies, $RDP = \hat{\wp}\hat{P}$. It is assumed, to make the following analysis tractable, that the one-year survival probability and the profit of surviving simulated companies are independent. Thus the mean and variance of RDP are given by the following expressions (Mood *et al.* 1974).

$$E\left(\hat{\wp}\hat{P}\right)=E\left(\hat{\wp}\right)E\left(\hat{P}\right) \tag{D.1}$$

$$\sigma^2_{\left(\hat{\wp}\hat{P}\right)}=Var\left(\hat{\wp}\hat{P}\right)=\left(E\left(\hat{\wp}\right)\right)^2 Var\left(\hat{P}\right)+\left(E\left(\hat{P}\right)\right)^2 Var\left(\hat{\wp}\right)+Var\left(\hat{P}\right)Var\left(\hat{\wp}\right) \tag{D.2}$$

$E\left(\hat{\wp}\hat{P}\right)$ and $\sigma^2_{\left(\hat{\wp}\hat{P}\right)}$ are presented in Table D.1.

The RDP obtained theoretically at a total investment in inventory of $100,000 and an average investment at the levels reported in the simulation trial, obtained using the method in Figure 6.8, are also shown in Table D.1. The standardised RDP, denoted by RDP_s, and calculated from the data as $RDP_s=\dfrac{RDP-E\left(\hat{\wp}\hat{P}\right)}{\sigma_{E\left(\hat{\wp}\hat{P}\right)}}$, are also given.

Table D.1: Expected values of RDP obtained theoretically and by discrete-event simulation

Average Investm't in Inventory	Simulated Values						Theoretical Values	
	$E\left(\hat{P}\right)$	$\sigma_{E\left(\hat{P}\right)}$	$E\left(\hat{\wp}\right)$	$\sigma_{E\left(\hat{\wp}\right)}$	$E\left(\hat{\wp}\hat{P}\right)$	$\sigma_{E\left(\hat{\wp}\hat{P}\right)}$	RDP	RDP_s
35,000	40,470	221	1.000	0.000	40,470	221	16,400	-109.00
40,000	47,364	240	1.000	0.000	47,364	240	33,300	-58.60
45,000	53,787	250	1.000	0.000	53,787	250	46,000	-31.10
50,000	58,849	272	1.000	0.000	58,849	272	54,000	-17.80
55,000	61,934	285	1.000	0.000	61,934	285	60,900	-3.60
60,000	64,429	296	0.999	0.000	64,348	315	64,500	0.50*
65,000	66,365	137	0.992	0.000	65,826	240	66,200	1.60*
70,000	67,705	151	0.936	0.000	63,371	311	61,600	-5.70
75,000	67,972	267	0.786	0.000	53,447	1,473	39,100	-9.70
80,000	73,072	1,281	0.566	0.002	41,338	3,114	21,500	-6.40
85,000	79,551	161	0.392	0.003	31,164	4,487	4,200	-6.00
90,000	-	-	0.300	0.014	-	-	-	-
95,000	-	-	0.250	0.005	-	-	-	-
100,000	-	-	-	-	-	-	-	-
* indicates that the goodness of fit between simulated and theoretical model is significant at the 95% level								

It is shown in Table D.1 that the difference between the simulated results and those obtained analytically fall within a 95% confidence interval $(\pm 1.96\sigma)$ when the average investment in inventory is $60,000 and $65,000. Thus, within this range, the both models are in close agreement. For both models, these investment levels correspond to a probability of survival close to 1, and also to a low probability of lost sales of finished products, which explains the high degree of fit.

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