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Essays on Asset Pricing Theory

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ABSTRACT

In the finance research, studies of the relationship between different financial instruments and between financial instruments and real world financial activity provide important information for financial risk management and investment decision making. If we consider the diversification of portfolios, for instance, the correlation matrix of asset returns and the correlation between asset return and real variables are important in financial decision making. However, the information gleaned from these analytical tools is very sensitive to the methods and time horizons used. Some empirical tests also reveal puzzling relationships that are opposite to the predictions of economic theory. For example, stock returns and investment growth should have a positive contemporaneous relationship, according to Tobin's Q theory; however, much empirical literature reports a negative relationship. Another puzzling relationship can be found in the literature studying the stock returns and inflation. According to the Fisher hypothesis, stock returns should provide a hedge for inflation. In other words, stock returns have a positive correlation with inflation. However, most empirical studies report a negative relationship in the short horizon.

The central focus of my thesis is to examine various relationships between financial variables in the long-run. In my thesis, five relationships are identified and examined using various time series methods and using real business cycle model: (1) the relationship between stock return and real activities, proxied by industrial production; (2) the relationship between stock returns and inflation; (3) the multihorizon Sharpe ratio; (4) the relationship between risk and return; and (5) puzzling relationship between stock returns and investment growth.

TABLE OF CONTENTS

LIST OF TABLES

Chapter 7

LIST OF FIGURES

Chapter 2

Chapter 3

Chapter 4

Chapter 5

Chapter 6

Chapter 7

PART A: Suggested General Declaration

[This declaration to be modified as required and inserted in the front of the thesis.]

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Declaration for thesis based or partially based on conjointly published or unpublished work

General Declaration

In accordance with Monash University Doctorate Regulation 17 / Doctor of Philosophy and Master of Philosophy (MPhil) regulations the following declarations are made:

I hereby declare that this thesis contains no material which has been accepted for the award of any other degree or diploma at any university or equivalent institution and that, to the best of my knowledge and belief, this thesis contains no material previously published or written by another person, except where due reference is made in the text of the thesis.

This thesis includes 0 original papers published in peer reviewed journals and 1 unpublished publications. The core theme of the thesis is to examine the long-run relationship. The ideas, development and writing up of all the papers in the thesis were the principal responsibility of myself, the candidate, working within the Department of Accounting and Finance under the supervision of Dr Francis In.

The inclusion of co-authors reflects the fact that the work came from active collaboration between researchers and acknowledges input into team-based research.

In the case of chapter 3 my contribution to the work involved the following:

[If this is a laboratory-based discipline then there could follow a paragraph outlining the assistance given during the experiments, the nature of the experiments and an attribution to the contributors.]

Thesis chapter	Publication title	Publication status*	Nature and extent of candidate's contribution
3	The relationship between financial variables and real economic activity: Evidence from spectral and wavelet analyses	accepted	Setting up key idea, testing the econometric model, and writing the first draft.

[* For example, 'published' / 'in press' / 'accepted' / 'returned for revision']

Signed: ......[redacted]...

Date: ...12/12/2003.........

DISCLAIMER

The work contained in this research has not been previously submitted for a degree or diploma at any tertiary education. To the best of my knowledge and belief, the thesis contains no material previously published or written by any other person except where the due reference is made

Sangbae Kim

[redacted]

Signature

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The completion of this thesis would not have been possible without incorporating the knowledge, time and support that others have so generously shared with me. I am very pleased to acknowledge the assistance and supports as follows:

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Finally, I am forever truly indebted to my parents for their endless affection, unwavering motivation, and understanding throughout the challenging journey that has culminated in this present work. My work is dedicated to them with the deepest affection.

1 Introduction

In the finance research, studies of the relationship between different financial instruments and between financial instruments and real world financial activity provide important information for financial risk management and investment decision making. If we consider the diversification of portfolios, for instance, the correlation matrix of asset returns and the correlation between asset return and real variables are important in financial decision making. However, the information gleaned from these analytical tools is very sensitive to the methods and time horizons used. Some empirical tests also reveal puzzling relationships that are opposite to the predictions of economic theory. For example, stock returns and investment growth should have a positive contemporaneous relationship, according to Tobin's Q theory; however, much empirical literature reports a negative relationship. Another puzzling relationship can be found in the literature studying the stock returns and inflation. According to the Fisher hypothesis, stock returns should provide a hedge for inflation. In other words, stock returns have a positive correlation with inflation. However, most empirical studies report a negative relationship in the short horizon.

The central focus of this thesis is to examine the relationship between financial variables and the relationship between financial variables and real economic variables. This relationship has been studied in previous literature, mostly by paying attention to the short-run relationship. This thesis focuses on the long-run relationships rather than short-run. The natural question to arise is why we need to study not only the short-run relationship but also the long-run relationship. The research on stocks, for instance, provides one pointer to this need. Stocks are considered as a long-run investment, but previous studies have focused on the contemporaneous (short-run) relationships. This thesis investigates long-run relationships for two reasons. First, in the short-run, the relationship between variables could be affected by various factors, such as market frictions and investor's sentiments. That is, evidence at longer horizons may provide additional information. Second, from a practical point of view, many investors hold financial instruments over long holding periods. Therefore, it is important to know the manner in which stock returns move with the other variables over longer horizons. Another example can be found in investment in fixed income securities. The desired maturity for a fixed income security varies depending on the need of investors' investment horizon. Therefore, from investors' point of view, their needs are best met by information that is based on analysis of short-, intermediate- and long-run relationships. Consider investors who invest their funds for 10 years into a stock and a bond. If they try to minimize the total risk of their investment, they will choose a combination that has the lowest correlation between the two investments. However, if a short-run correlation is adopted for their investment, it may lead to suboptimal results. They should look at a particular correlation for their preferred investment horizon.

In my thesis, five relationships are examined using various time series methods and using real business cycle model: (1) the relationship between financial variables and real activities, proxied by industrial production; (2) puzzling relationship between stock returns and inflation; (3) the multihorizon Sharpe ratio; (4) the relationship between risk and return; and (5) the relation between stock returns and investment growth.

Overall, wavelet analysis provides a valuable platform to analyze the diverse relationships over various time scales. Examining the relationships over the various time scales shows that these relationships are not stable over the time scales, implying that the true dynamic structure of the relationship between variables varies over different time scales associated with those different horizons. However, most studies have been mostly restricted to two time scales (the short-run and the long-run). From this limitation, to comprehend only dynamic relationships of the financial markets, wavelet analysis is needed in the fields of economics and finance as its analytical tools are able to decompose data into more than two time scales.

In addition, our results indicate that due to presence of the investors who have different investment horizons, the portfolio managers (or investors) have difficulty making an investment decisions using contemporaneous data. However, decomposition of the movements of the financial markets into several time scales using wavelet analysis allows the portfolio managers (or investors) to make the right decision in the specific time scale. For example, consider an investment company with a large number of investors and money managers. Clearly, the investors and the money managers make decisions over different time scales. Suppose, for simplicity, that the investment horizon of an investor is one year and that the investment company reviews the performance of the money manager every quarter using the Sharpe ratio. The money manager will therefore focus on the three-month performance of a portfolio, while the investor will concentrate on the one-year performance. Thus, for this investor, the money manager may not provide the best service. To provide the best service for the diversified investors, the Sharpe ratio needs to be constructed over different investment horizons.

My thesis is composed of 8 chapters. Following this introduction, chapter 2 explains wavelet analysis and its use in the thesis. The main advantage of wavelet analysis is the ability to decompose the data into several time scales. Consider the large number of investors who participate in the stock market and make decisions over *different time scales*. Stock market participants are a diverse group and include intraday traders, hedging strategists, international portfolio managers, commercial banks, large multinational corporations, and national central banks. It is notable that these market participants operate on very different time scales. Another key distinctive features of wavelet analysis arises from the fact that wavelets gives us the ability not only to perform non-parametric estimations of highly complex structures without knowledge of the underlying functional form but also to accurately locate discontinuity and high frequency bursts in dynamic systems. In short, the major aspects of wavelet analysis are the ability to handle non-stationary data, localization in time, and the resolution of the signal in terms of the time scale of analysis (Ramsey, 1999).

Chapter 3 investigates the relationship between various financial variables and real activity, proxied by industrial production. Many empirical studies find financial variables possess a predictive power of the real activity. To examine this relationship, I adopt two time-series techniques: spectral analysis and wavelet analysis. Spectral analysis shows that US industrial production and financial variables have a common feature in the long-run and a varying lead-lag relationship depending on the business cycles. It implies that the relationship between US industrial production and financial variables is not fixed over time. This result is confirmed by the wavelet analysis. The lead-lag relationship, in the sense of Granger causality, varies depending on the time scale. From two time-series analyses (spectral analysis and wavelet analysis), it can be concluded that the lead-lag relationship between US financial variables and US industrial production varies depending on the time scale and frequency.

Chapter 4 tests the Fisher hypothesis, which states a positive relationship between nominal stock returns and inflation, and also provides a new perspective on the hypothesis. The new approach is based on a wavelet multiscaling method that decomposes a given time series on a scale-by-scale basis. Empirical results show that there is a positive relationship between stock returns and inflation at the shortest scale (1-month period) and at the longest scale (128-month period), while a negative relationship is shown at the intermediate scales. This indicates that the nominal return results are supportive of the Fisher hypothesis for risky assets in D1 and S7 of the wavelet domain, while stock returns do not play a role as an inflation hedge at the intermediate scales. The key empirical results show that time-scale decomposition provides a valuable means of testing the Fisher hypothesis, since a number of stock returns and inflation puzzles previously noted in the literature are resolved and explained by the wavelet analysis.

Chapter 5 studies the multihorizon Sharpe ratio. Previous studies focus on the contemporaneous Sharpe ratio, rather than the multihorizon Sharpe ratio, except for Hodges et al. (1997). They examine the multihorizon Sharpe ratio using randomized historical data from 1926 to 1993 and conclude that bonds outperform stocks over sufficiently long holding periods, which is inconsistent with general belief. Chapter 5 extends their study, following the three critiques of Siegel (1999), by adopting wavelet analysis to investigate the multihorizon Sharpe ratio. First, it examines the mean-reverting property of asset returns using the wavelet-based maximum-likelihood estimation of the long memory parameter. Second, it uses the real returns of stocks and bonds. Finally, it investigates the multihorizon Sharpe ratio using wavelet analysis. Adopting wavelet analysis does not require any assumption on the distribution of returns because wavelet analysis is non-parametric estimation and decomposes the unconditional variance into different time scales. The wavelet decomposition shows that the long memory parameter, calculated from the wavelet-based maximum-likelihood estimation, for all asset returns are less than 1 and close to 0, indicating that asset returns are mean-reverting. For the multihorizon Sharpe ratio, the Sharpe ratio of large-company stock portfolios is a higher value than the other three types of portfolios (small company stocks, long-term and intermediate-term government bonds) over all wavelet scales. In other words, large-company stock portfolios outperform the other portfolios over the different wavelet scales.

Chapter 6 examines the long-run relationship between stock returns and risk (volatility) using two newly developed methods: the King and Watson (1997) and

Den Haan (2000) approaches. Many previous studies do not show consistent results between stock returns and risk (volatility) and focus on the contemporaneous relationship. Using the King and Watson method, we find that long-run relationship highly depends on the contemporaneous relationships. For the VAR forecast correlation, proposed by Den Haan (2000), most industry portfolios show a negative relationship in the short-run as well as in the long-run. For the market portfolio, a negative relationship is dominant regardless of forecasting horizons. From these results, chapter 6 concludes that the long-run response of stock returns to a permanent volatility shock is sensitive to the assumed value of identifying parameters in each industry portfolio and the market portfolio and that, as in previous studies, the relationship between risk and return is mixed in the short-run. However, in the long-run, the negative relationship is dominant. A negative relationship in the long-run means that if investors feel that the risk of a portfolio is high in the future, the price of the portfolio rises to compensate the increased expected risk. Therefore, the future return of the portfolio decreases.

Chapter 7 investigates the puzzling relationship between investment growth and stock returns in a stochastic growth model. Many empirical studies find a negative relationship between current investment growth and current stock returns, while the theory predicts a positive relationship. When investment-specific technology and capital adjustment costs are adopted in the real business cycle (RBC) framework, investment-specific technology generates countercyclical movements of stock returns. A positive shock increases investment growth, while decreasing stock returns. Capital utilization plays an important role in transmitting the shock. For example, a positive shock decreases current stock returns because of an increased capital utilization, which causes more depreciation. In contrast, it increases investment. Therefore, the correlation coefficient has a negative value. In the long-run, the correlation coefficients are negative in the actual data. The benchmark model, which has a standard technology shock, generates positive correlation coefficients in the long-run as well as in the contemporaneous relationship, while the model, which includes the investment-specific technology shock and capital adjustment costs, generates negative correlation coefficients in the long-run. Chapter 8 concludes the thesis by summarizing the findings from the preceding chapters.

PART B: Suggested Declaration for Thesis Chapter

[This declaration to be completed for each conjointly authored publication and to be placed at the start of the thesis chapter in which the publication appears.]

Monash University

Declaration for Thesis Chapter 3

In the case of [insert chapter number], contributions to the work involved the following:

Name	% contribution	Nature of contribution
Sangbae Kim	60%	Setting up key idea, testing the econometric model, and writing the first draft.
Dr Francis In	40%	Initiation of research, development and extension of key idea, and writing up the final version.
[name 2] *		

Declaration by co-authors

The undersigned hereby certify that:

(1) they meet the criteria for authorship in that they have participated in the conception, execution, or interpretation, of at least that part of the publication in their field of expertise;
(2) they take public responsibility for their part of the publication, except for the responsible author who accepts overall responsibility for the publication;
(3) there are no other authors of the publication according to these criteria;
(4) potential conflicts of interest have been disclosed to (a) granting bodies, (b) the editor or publisher of journals or other publications, and (c) the head of the responsible academic unit; and
(5) the original data are stored at the following location(s) and will be held for at least five years from the date indicated below:

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Signature 1	[redacted]	Date 12/12/03
Signature 2		12/12/03
Signature 3		

[* Please include additional rows if there is insufficient space.]

Chapter 2 Methodology: Wavelet Analysis

2.1 Introduction

The multiscale relationship is important in economics and finance because each investor has a different investment horizon. Consider the large number of investors who participate in the stock market and make decisions over *different time scales*. Stock market participants are a diverse group, and include intraday traders, hedging strategists, international portfolio managers, commercial banks, large multinational corporations, and national central banks. It is notable that these market participants operate on very different time scales. In fact, due to the different decision-making time scales among traders, the true dynamic structure of the relationship between variables will *vary* over different time scales associated with those different horizons. However, most previous studies focus on a two-scale analysis - short-run and long-run. The reason being for this is mainly a lack of empirical tools. Recently, wavelet analysis has attracted attention in the fileds of economic and finance as a means of filling this gap.

Wavelet analysis is relatively new in economics and finance, although the literature on wavelets is growing rapidly. The studies, related to economics and finance, can be divided into four categories: general wavelet transformation, stationary process (long memory), denoising, and variance/covariance analysis. The first category includes Davidson et al. (1998), Pan and Wang (1998), Ramsey and Lampart (1998a, 1998b), and Chew (2001). Another stream of research is related to the long memory process of time series. Researchers in this field include

Ramsey et al. (1995) for self-similarity (long memory); Jensen (1999b), Tkacz (2001), Whitcher and Jensen (2000) and Jensen and Whitcher (2000) for wavelet OLS; Jensen (1999a, and 2000) for the wavelet maximum likelihood method; and Jamdee and Los (2003) and Kyaw et al. (2003) for the wavelet Hurst exponent.

The third category of recent wavelet analysis is the use of the wavelet denoising (detrending) method, for example, Fleming et al. (2000) and Capobianco (2003). The final category involves the application of wavelet analysis to multiscale variance/covariance analysis. This stream of wavelet analysis is mostly based on Percival and Walden (2000) and Gençay et al. (2002a). Applications of this method include Gençay et al. (2001), Gençay et al. (2003a and b) and In and Kim (2003).

The purpose of this chapter is to introduce wavelet analysis and to focus on what features of wavelet analysis can be applied to financial analysis. To examine wavelet analysis, this chapter begins with Fourier analysis, and then the chapter moves to the main features of wavelet analysis largely focusing on the application of time series analysis.

2.2 Fourier Analysis and Spectral Analysis

Fourier analysis and spectral analysis are used in modern signal processing and business cycle theory. This section introduces and investigates the properties of Fourier analysis and spectral analysis.

2.2.1 Fourier Analysis

In the history of mathematics, wavelet analysis has many different origins. One of them is Fourier analysis. The fundamental idea in Fourier analysis is that any deterministic function of frequency can be approximated by an infinite sum of trigonometric functions, called the Fourier representation.

Fourier's result states that any function $f \in L^2[-\pi, \pi]$[1] can be expressed as an infinite sum of dilated cosine and sine functions:

$$f(x) = \frac{1}{2}a_0 + \sum_{j=1}^{\infty}(a_j \cos(jx) + b_j \sin(jx)) \tag{2.1}$$

where an appropriately computed set of coefficients $\{a_0, a_1, b_1, \ldots\}$ is a complex sequence. As can be seen in equation (2.1), in Fourier transform, any signal can be expressed as a function of sines and cosines. The Fourier basis functions (sines and cosines) are very appealing when representing a time series that does not vary over time, i.e., a stationary time series (Gençay et al., 2002a, p97).

The equation (2.1) has to be interpreted with a caution. The equality is only meant in the L^2 sense, i.e.:

$$\int_{-\pi}^{\pi}\left[f(x) - \left(\frac{1}{2}a_0 + \sum_{j=1}^{\infty}(a_j \cos(jx) + b_j \sin(jx))\right)\right]^2 dx = 0 \tag{2.2}$$

[1] L^2 is a space of all functions with a well-defined integral of the sequence of the modulus of the function.

It is possible that f and its Fourier representation differ on a few points (this is the case at discontinuity points). The summation in equation (2.2) is up to infinity, but a function can be well-approximated (in the L^2 sense) by a finite sum with upper summation limit index J:

$$H_J(x) = \frac{1}{2}a_0 + \sum_{j=1}^{J}(a_j \cos(jx) + b_j \sin(jx)) \tag{2.3}$$

This Fourier series representation is highly useful in that any L^2 function can be expressed in terms of two basis functions: sines and cosines. This is because of the fact that the set of functions $\{\sin(j\cdot), \cos(j\cdot), j = 1, 2, \ldots\}$, together with the constant function, form a basis for the function space $L^2[-\pi, \pi]$ (Ogden, 1997).

The Fourier basis has three important properties. The first property is that it has an orthogonal basis. Orthogonality implies that the inner product of any two functions $f_1, f_2 \in L^2[a, b]$ is equal to zero, resulting from the sine and cosine functions.

The second property of Fourier transform is orthonormality, which means that the sequence of function f_j s are pairwise orthogonal and $\| f_j \| = 1$ for all j. Defining $u_j(x) = \pi^{-1/2}\sin(jx)$ for $j = 1,2,\ldots$ and $v_j(x) = \pi^{-1/2}\cos(jx)$ for $j = 1,2,\ldots$ with the constant function $v_0(x) = 1/\sqrt{2\pi}$ on $x \in [-\pi, \pi]$ makes the set of functions $\{v_0, u_1, v_1, \ldots\}$ orthonormal (Ogden, 1997).

Finally, the Fourier basis is a completely orthonormal system (Ogden, 1997). It is said that a sequence of function $\{f_j\}$ is a complete orthonormal system if the f_js are pairwise orthogonal, $\| f_j \| = 1$ for each j, and the only function orthogonal to each f_j is the zero function. Thus, the set $\{h_0, g_j, h_j: j = 1,2,\ldots\}$ is a complete orthonormal system for $L^2[-\pi, \pi]$.

From equation (2.1), the Fourier transform consists of sine and cosine functions, which are periodic functions. Therefore, if a function, $f(x)$, is a non-periodic signal, the expression of this function as the summation of the periodic functions (sine and cosine) does not accurately capture the movement of the signal. One could artificially extend the signal to make it periodic. However, it would require additional continuity at the endpoints. To avoid this problem, Gabor (1946) introduces the windowed Fourier transform (WFT, also called the short time Fourier transform) to measure the frequency variation of a signal. The WFT can be used to give information about signals simultaneously in the time and frequency domains. A real and symmetric window $u(t) = u(-t)$ is translated by k and modulated by the frequency ξ (Mallat, 1999):

$$u_{k,\xi}(t) = e^{i\xi t} u(t-k) \tag{2.4}$$

It is normalized $\| u \| = 1$ so that $\| u_{k,\xi} \| = 1$ for any $(k, \xi) \in \Re$. The resulting WFT of $f \in L^2(\Re)$ is:

$$Sf(u,\xi) = < f, u_{k,\xi} > = \int_{-\infty}^{+\infty} f(t)u(t-k)e^{-i\xi t}dt \qquad (2.5)$$

where $Sf(k,\xi)$ is the WFT. Therefore, with the WFT, the input signal $f(x)$ is divided into several sections, and each section is analyzed for its frequency content separately. The effect of the window is to localize the signal in time. The WFT represents a sort of compromise between the time- and frequency-based views of a signal. It provides some information about both when and at what frequencies a signal event occurs. However, we can only obtain this information with limited precision, determined by the size of the window. While the WFT compromise between time and frequency information can be useful, the drawback is that once you choose a particular size for the time window, that window is the same for all frequencies. Another drawback of the WFT is that it will not be able to resolve events if they happen to appear within the width of the window (Gençay, et al., 2002a, p99).

Another extension of the Fourier transform is the Fast Fourier Transform (FFT). To approximate a signal using the Fourier transform requires application of a matrix, the order of which is the number of sample points n. Since multiplying an $n \times n$ matrix by a vector costs in the order of n^2 arithmetic operations, the computational burden increases enormously with the number of sample points. However, if the samples are uniformly spaced, then the Fourier matrix can be factored into a product of just a few sparse matrices, and the resulting factors can be applied to a vector in a total of order $n \log n$ arithmetic operations. This is the so-called fast Fourier transform (Graps, 1995).

2.2.2 Spectral Analysis[2]

As discussed earlier, Fourier analysis transforms time domain data into frequency domain data. This feature naturally leads researchers to look for ways of determining which are dominant frequencies in a time series. In our research we have adopted one method that has proved successful in much researches, namely, power spectral density. The power spectral density of a time series x_t, denoted in $S_x(w)$, can be defined as the Fourier transform of the autocorrelation function.

$$S_x(w) = \int_{-\infty}^{\infty} R_x(\tau)e^{-j2\pi w\tau}d\tau \text{ where } R_x(\tau) = E\{x_t x_{t+\tau}\} \tag{2.6}$$

For example, suppose that we compute the power spectra of a covariance stationary stochastic process when we know the stochastic process, which has generated the time series. Note that any covariance stationary stochastic process can be given an infinite moving average representation or *Wold representation* as:

[2] In relation to the business cycle, the literature includes Howrey (1968), Sargent and Sims (1977), Baxter and King (1999), and recently Sarlan (2001), while in finance, the literature includes examination of the stock market (Fischer and Palasvirta, 1990; Knif et al., 1995; Lin et al., 1996; Asimakopoulos et al., 2000; Smith, 1999 and 2001), and investigation of the interest rates (Kirchgassner and Wolters, 1987; Hallett and Richter, 2001 and 2002). For a more extensive survey of the application of spectral analysis to economics and finance, see Ramsey and Thomson (1999).

$$x_t = \psi(L)\varepsilon_t = \sum_{j=0}^{\infty} \psi_j L^j \varepsilon_t \qquad (2.7)$$

where $\psi(L) = 1 + \psi_1 L + \psi_2 L^2 + \cdots$ is a polynomial in the lag operator, L, and $\psi_0 = 1$. The spectrum of the white noise process is $S_\varepsilon(w) = \sigma_\varepsilon^2 / 2\pi$, and equation (2.7) shows that x_t is generated by filtering the white noise process where $\psi(L)$ are the filter weights. The spectrum of x_t is thus the spectrum of the white noise process multiplied by the effect of the filter (Pedersen, 1999). This is easily computed taking the following steps:

1. Formulating the model in terms of its moving average representation (2.7) using the lag operator.

2. Substitute e^{-wt} for the lag operator L to get

   $$x_t = \sum_{j=0}^{\infty} \psi_j L^j \varepsilon_t = (1 + \psi_1 e^{-iw} + \psi_2 e^{-2iw} + \psi_3 e^{-3iw} + \cdots)\varepsilon_t$$

   This is the transfer function or the frequency response function of the filter (2.7)

3. Take the square of the absolute value of the frequency response function, called the power transfer function of the filter, denoted as:

   $$H(w) = \left|1 + \psi_1 e^{-iw} + \psi_2 e^{-2iw} + \psi_3 e^{-3iw} + \cdots\right|$$

4. The power spectral density function is the power transfer function of the filter multiplied by the spectrum of the white noise process

   $$S_x(w) = H(w) \cdot S_\varepsilon(w)$$

$$=\frac{1}{2\pi}\left|1+\psi_1 e^{-iw}+\psi_2 e^{-2iw}+\psi_3 e^{-3iw}+\cdots\right|\sigma_\varepsilon^2$$

For example, consider the following ARMA(2,2) process:

$$x_t=\theta_1 x_{t-1}+\theta_2 x_{t-2}+\varepsilon_t+\phi_1\varepsilon_t+\phi_2\varepsilon_{t-2}$$

To compute the power spectrum density, we follow four steps mentioned above.

Step 1. Reformulate above equation in terms of its moving average representation using the lag operator.

$$x_t=\frac{1+\phi_1 L+\phi_2 L^2}{1-\theta_1 L-\theta_2 L}\varepsilon_t$$

Step 2. Substitute for the exponential function instead of the lag operator.

$$x_t=\frac{1+\phi_1 e^{-iw}+\phi_2 e^{-2iw}}{1-\theta_1 e^{-iw}-\theta_2 e^{-2iw}}\varepsilon_t$$

Step 3. The corresponding frequency response function can be defined as follow:

$$h(e^{-iw})=\frac{1+\phi_1 e^{-iw}+\phi_2 e^{-2iw}}{1-\theta_1 e^{-iw}-\theta_2 e^{-2iw}}$$

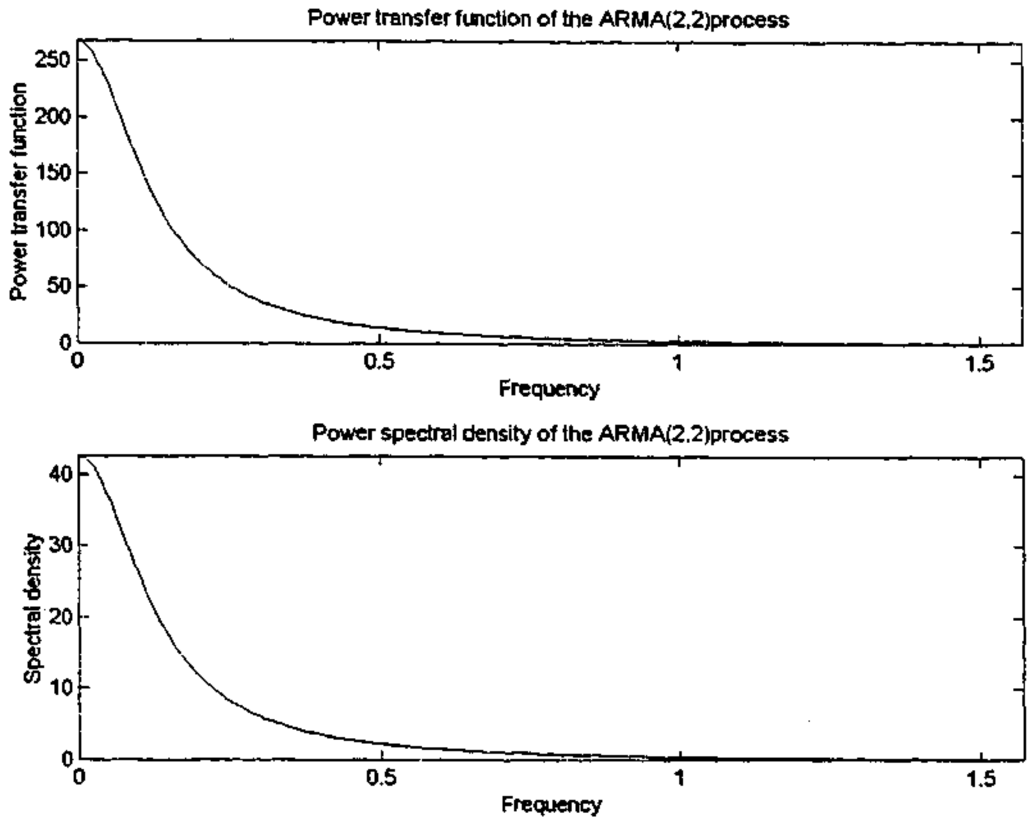
and the power transfer function becomes:

$$H(w)=\left|\frac{1+\phi_1 e^{-iw}+\phi_2 e^{-2iw}}{1-\theta_1 e^{-iw}-\theta_2 e^{-2iw}}\right|^2$$

Step 4. Finally, the power spectral density function can be defined as:

$$S_x(w)=\frac{1}{2\pi}\left|\frac{1+\phi_1 e^{-iw}+\phi_2 e^{-2iw}}{1-\theta_1 e^{-iw}-\theta_2 e^{-2iw}}\right|^2\sigma_\varepsilon^2$$

Figure 2.1 The Power Transfer Function and the Power Spectral Density Function of ARMA(2,2) Process



Note: This figure plots the power transfer function and the power spectral density function of ARMA(2,2) process. The upper figure presents the power transfer function, calculated by $H(w)$ in step 3. The bottom figure presents the power spectral density function calculated by step 4.

The power spectral density function of this ARMA(2,2) process is plotted in Figure 2.1 with the power transfer function.

This spectral analysis can be applied to the multivariate case. It is known as cross spectral analysis, and it allows us to examine the multivariate case in the frequency domain. This presents an alternative method to investigate the lead-lag relationship and comovements between time series. The cross spectrum is a complex quantity and can be reformed in terms of two real quantities, the cospectrum, $co_{ij}(w)$, and the quadrature spectrum, $qu_{ij}(w)$. Using Euler

Relations and DeMoire's Theorem, the coherence can be decomposed into two terms as follows.

$$S_{ij} = co_{ij}(w) + i\, qu_{ij}(w) \tag{2.8}$$

The cospectrum can be expressed by:

$$co_{ij}(w) = \frac{1}{2\pi} \sum_{s=-\infty}^{\infty} R_{ij}(s) \cos(ws) \tag{2.9}$$

where R_{ij} is defined as $E[(i_t - Ei_t)(j_t - Ej_t)]$. The cospectrum between two series i and j at frequency w can be interpreted as the covariance between two series i and j that is attributable to cycles with frequency w. The cospectrum can have either positive or negative values, since the autocovariances can be both positive and negative.

The quadrature spectrum is rewritten as follows:

$$qu_{ij}(w) = \frac{1}{2\pi} \sum_{s=-\infty}^{\infty} R_{ij}(s) \sin(ws) \tag{2.10}$$

The quadrature spectrum from time series i to time series j at frequency w is proportional to the portion of the covariance between two time series i and j due to cycles of frequency w. From this quadrature spectrum, we can observe which series has more out-of-phase cycles, because time series i may respond to an economic recession later than time series j.

Next we need to show how to derive the gain in spectral analysis. The gain has a feature which shows how a change of the regression coefficients in the time domain can affect the cross spectrum.

$$|G(w)| = \sqrt{(co_{ij}(w))^2 + (qu_{ij}(w))^2} \tag{2.11}$$

The function $|G(w)|$ is called the gain. The gain is equivalent to the regression coefficient for each frequency w. In other words, it measures the amplification of the frequency components of the j-process to obtain the corresponding components of the i-process.

The estimated coherence spectrum between two series for various frequencies is given by:

$$Coh(w) = \frac{|S_{ij}(w)|^2}{S_{ii}(w)S_{jj}(w)} \tag{2.12}$$

where S_{ii} and S_{jj} $(i \neq j)$ are the autospectrum estimated from:

$$S_{ii}(w) = \frac{1}{2\pi} \sum_{-(N-1)}^{N-1} \lambda_N(s) R_{ii}(s) e^{-iws} \tag{2.13}$$

where R_{ii} is defined as $E[(i_t - Ei_t)^2]$. The coherence is a real-valued function, which has a value between 0 and 1. The coherence between two time series measures the degree of which series are jointly influenced by cycles at frequency

w. In other words, as can be seen equation (2.12), coherence is the ratio of the squared cross spectrum to the product of two autospectrums, analogue to the squared coefficient of correlation. We can use the coherence between two or more time series to measure the extent to which multiple time series move together over the business cycle.

The lead-lag relationship between two time series can be captured by the phase. The phase, defined as $\varphi(w)$, can be expressed as a ratio between the cospectrum and quadrature spectrum.

$$\varphi(w) = \tan^{-1}\left(\frac{co_{ij}(w)}{qu_{ij}(w)}\right) \tag{2.14}$$

In addition, the phase gives the lead of one series over another series at frequency w. The phase graph gives information about the lag relationship between two time series. If the phase is a straight line over some frequency band, the slope is equal to the time lag and thus tells which series is leading and by how many periods (Pederson, 1999).

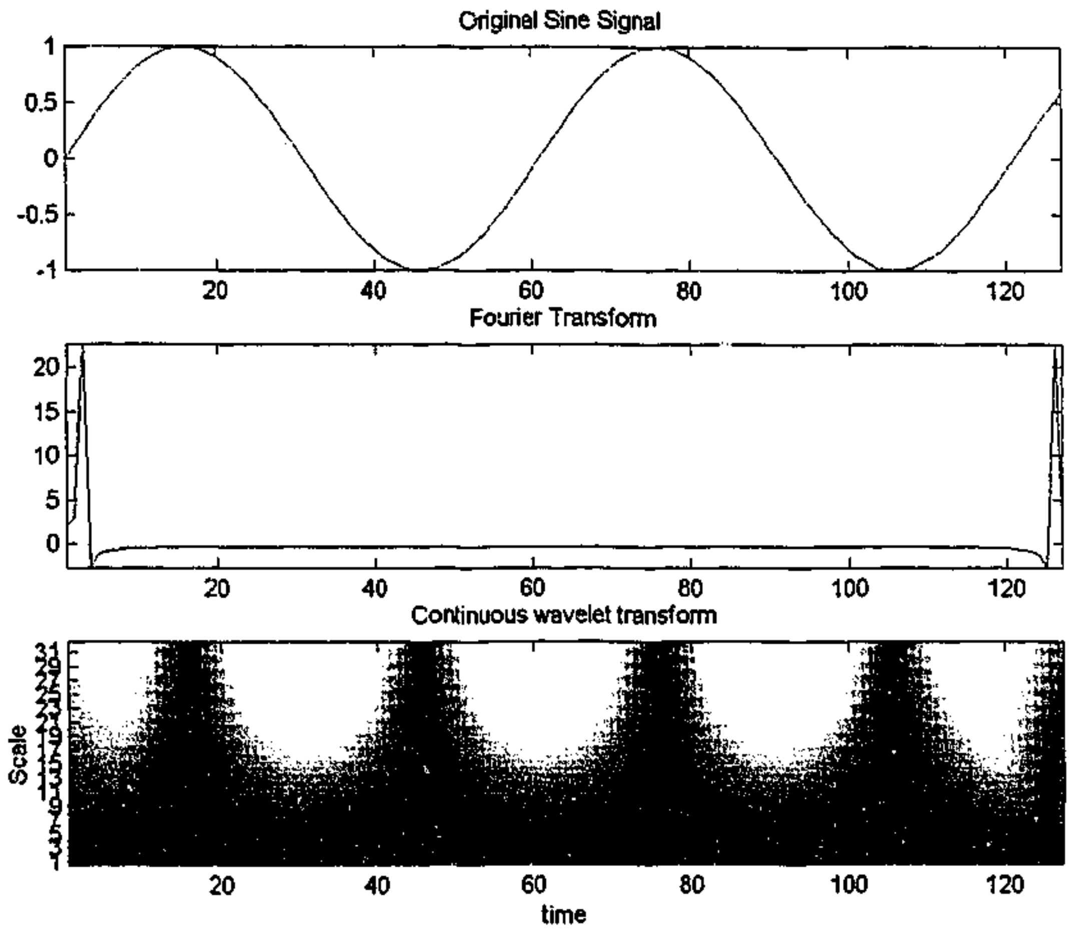
This reveals the lead and lag relationship between two variables at different frequencies. In other words, a positive phase slope indicates that the input variable leads the output variables, while a negative phase slope indicates that the input variable lags.

2.2.3 Comparison between Fourier Transform and Wavelet Transform

While Fourier analysis is one of the origins of wavelet analysis, the two methods have some points of similarity and some important points of difference. The first similarity is reversibility. Both transforms are reversible functions. That is, they allow going back and forward between the raw and transformed signals. Another similarity is that the basis functions are localized in frequency, making mathematical tools such as power spectra (how much power is contained in a frequency interval) and scalograms useful at picking out frequencies and calculating power distributions (Graps, 1995).

Even though they possess some similarities, the two transforms are different from each other. Through examining the difference between the two transforms, we can easily see why we need wavelet analysis instead of Fourier analysis. Wavelet analysis has three distinctive advantages over Fourier analysis. The first advantage is that wavelet analysis has the ability to decompose the data into several time scales instead of the frequency domain. This advantage allows us to examine the behavior of a signal over various time scales. The second advantage of wavelet transforms is that the windows vary. In order to isolate signal discontinuities, one would like to have some very short basis functions. At the same time, in order to obtain detailed frequency analysis, one would like to have some very long basis functions. In fact, wavelet transforms allow us to do both. The final advantage of wavelet transforms is their ability to handle the non-stationary data. Restricting to stationary time series would not be very promising and appealing since most interesting time series exhibit quite complicated patterns over time (trends, abrupt regime changes, bursts of variability, etc).

Figure 2.2 Comparison between Fourier Transformation and Wavelet Transformation



Note: This figure illustrates the original sine signal, the Fourier transformation, and the wavelet transformation. The second figure plots the Fourier transformation of the original signal. Clearly it indicates the original signal has a single frequency. The bottom figure indicates the continuous wavelet transformation. This figure has been constructed using Haar wavelet filter, which will be discussed later in this chapter.

To compare Fourier transformation and wavelet transformation, we decompose the sine signal ($s_t = \sin\left(\pi \cdot t/_{1.5}\right)$) using Fourier transformation and continuous wavelet transformation. To calculate the wavelet coefficients, the Haar wavelet filter has been adopted. More properties of the Haar wavelet filter will be discussed later. Figure 2.2 plots the original sine signal, Fourier coefficients, and wavelet coefficients.

As can be seen in Figure 2.2, a plot of the Fourier coefficients of this signal shows nothing particularly interesting: a flat spectrum with two peaks represents a single frequency. More specifically, the Fourier transformation picks up the low-frequency oscillation and lacks strong evidence of the discontinuity. In other words, the Fourier analysis only shows the global movements, not local movements. However, by giving up some frequency resolution, the wavelet transformation has the ability to capture events that are local in time. This makes the wavelet transformation an ideal tool for studying nonstationary or transient time series. In contrast to the Fourier transformation, as shown in Figure 2.2, the wavelet transformation clearly identifies the abrupt change in the function and the low-frequency sinusoid.

2.3 Wavelet Analysis

As a means of understanding the fundamentals of wavelet analysis, Daubechies (1992) provides an extensive look at the mathematical properties of wavelets. Chui (1992), and Strang and Nguyen (1996) are good introductions to wavelets. The text by Gençay et al. (2002a) gives a good discussion on how wavelets can be applied in economics and finance. Ramsey (1999 and 2002) and Schleicher (2002) also give some additional insights on how wavelet analysis can be adopted in economics and finance. In this section, we examine the properties of the continuous wavelet transform, and two discrete wavelet transforms (Discrete Wavelet Transform and Maximal Overlap Discrete Wavelet Transform).

2.3.1 Continuous Wavelet Transform

The continuous wavelet transform (CWT) is defined as the integral over all time of the signal multiplied by scaled, shifted versions of the wavelet function $\psi(scale, position, time)$:

$$C(scale, position) = \int_{-\infty}^{\infty} x_t \psi(scale, position, t) dt \tag{2.15}$$

The results of the CWT are many wavelet coefficients C, which are a function of scale and position. The scale and position can take on any values compatible with the region of the time series x_t. Multiplying each coefficient by the appropriately scaled (dilated) and shifted wavelet yields the constituent wavelets of the original signal. If the signal is a function of a continuous variable and a transform that is a function of two continuous variables is desired, the continuous wavelet transform (CWT) can be defined by (Burrus et al., 1998):

$$F(a,b) = \int x_t \, \psi\left(\frac{t-a}{b}\right) dt \tag{2.16}$$

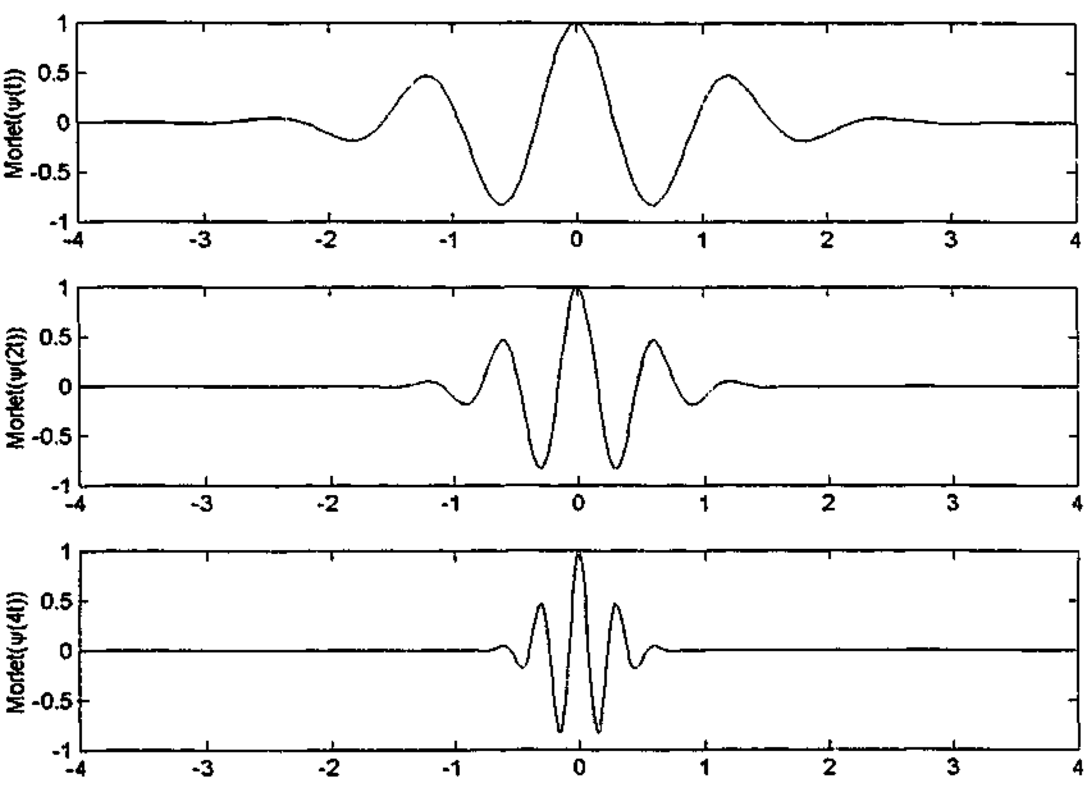
with an inverse transform of:

$$x_t = \int\int F(a,b)\,\psi\left(\frac{t-a}{b}\right) da\, db \tag{2.17}$$

where $\psi(t)$ is the basic wavelet and $a, b \in \mathbf{R}$ are real continuous variables.

To capture the high and low frequencies of the signal, the wavelet transformation utilizes a basic function (mother wavelet) that is stretched (scaled) and shifted.

Figure 2.3 Morlet Wavelets with Different Scales



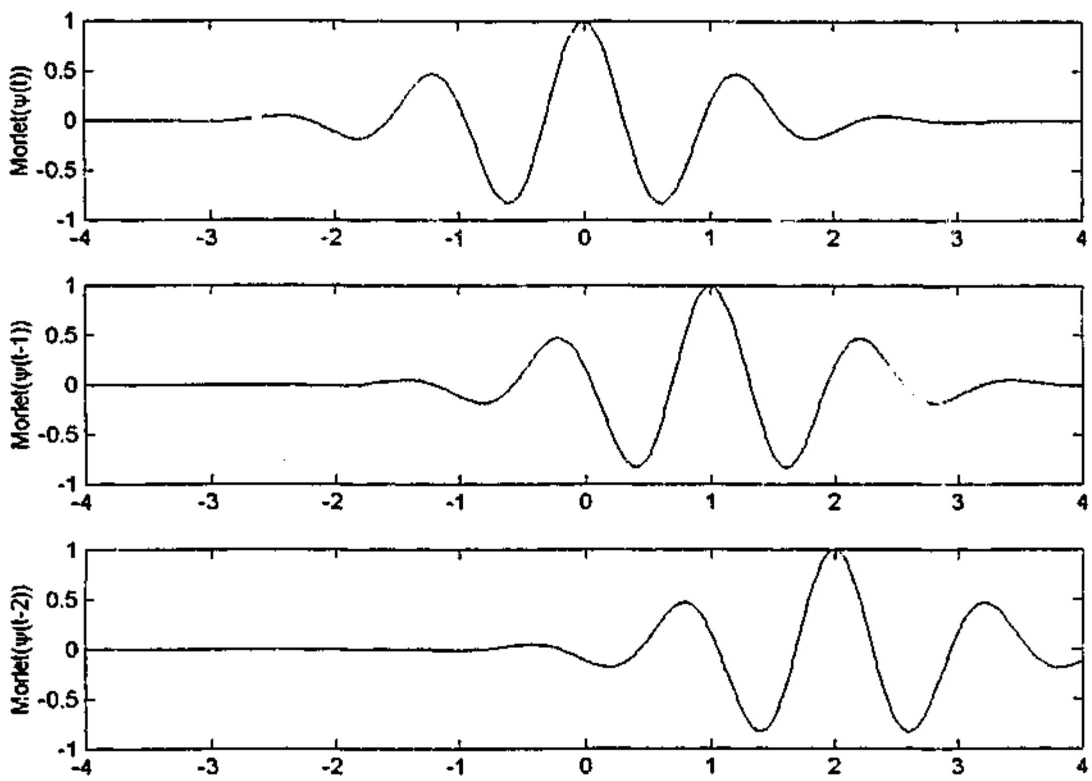
Note: This figure illustrates the effect of changing the scale b. It is observed that the smaller value of b generates the more compressed wavelet filter.

Scale of wavelets

Scaling a wavelet simply means stretching (or compressing) it. To go beyond colloquial descriptions such as "stretching", we introduce a scale factor, b, so that

$\psi_b(t) = \psi(t/b)$. The smaller the scale factor, the more "compressed" the wavelet (see Figure 2.3). It is natural to think about a correspondence between wavelet scales and frequency. A fine and small scale b generates a compressed wavelet. In turn, this compressed wavelet makes the details change rapidly. In consequence, a fine and small scale b can capture a high frequency oscillation. In contrast, a coarse and large scale b can capture low frequency movements.

Figure 2.4 Morlet Wavelets and Shifted Morlet Wavelets



Note: This figure represent the effect of changing a in equation (2.17).

Shifting of wavelets

Shifting a wavelet simply means advancing or delaying it. Mathematically, delaying a function $\psi(t)$ by a is represented by $\psi(t-a)$.

Using these two properties, the wavelet transformation intelligently adapts itself to capture features across a wide range of frequencies and thus has the ability to capture events that are local in time.

What conditions must wavelets satisfy?

A wavelet $\psi(t)$ is a simple function of time t that obeys some rules (admissibility, orthogonality, vanishing moments). First, the admissibility condition is:

$$C_\psi = \int_0^\infty \frac{|H(w)|}{w} dw < \infty$$

where $H(w)$ is the Fourier transform, a function of frequency w, of $\psi(t)$ in the CWT. This condition is only useful in theoretical analysis, and as in the Fourier transform, there is a necessary condition to satisfy the Dialect condition (i.e. continuous or only limited discontinuous points in the integration span). This condition ensures that $H(w)$ goes to zero as $w \to 0$ (Grossman and Morlet, 1984; Mallat, 1999).

In other words, to guarantee that $C_\psi < \infty$, we must impose the condition, $H(0) = 0$. This condition leads us to the first condition of a wavelet function.

$$\int_{-\infty}^{\infty} \psi(t)dt = 0 \tag{2.18}$$

If the energy of a function is defined as the squared function integrated over its domain, the second condition is that the wavelet function has unit energy.

$$\int_{-\infty}^{\infty} |\psi(t)|^2 dt = 1 \tag{2.19}$$

The second condition is orthogonality. As discussed in Fourier analysis, the wavelet also has an orthogonal property. Orthogonality means that the shifted functions in the same scale are orthogonal and also that the functions at different scales are orthogonal. In the implementation of a wavelet system by filter banks, orthogonality means that if the inverse of the analysis filter banks is exactly the transpose of itself, this wavelet is orthogonal. In this case, only one wavelet mother function is necessary for both analysis and synthesis. It does not have linear phase. If the wavelet is biorthogonal, the inverse of analysis filter bank is not necessarily the transpose of itself. In other words, there would be two mother functions for analysis and synthesis respectively. Some wavelet families can be both biorthogonal and orthogonal.

The final condition is related to vanishing moments. What is the relevance of vanishing moments? More vanishing moments means that the scale function is smoother. The number of vanishing moments comes from the defining wavelet equation.

2.3.2 Discrete Wavelet Transform

In time series analysis, the data has a finite length of duration. Therefore, only a finite range of scales and shifts are meaningful. In this section, we study the Discrete Wavelet Transform (DWT).

To describe the idea of multiresolution, it is better to start from the properties of scale function (father wavelet). A two-dimensional family of functions is generated from the basic scaling function by scaling and translation as follows:

$$\phi_{j,k}(t) = 2^{-\frac{j}{2}}\phi(2^{-j}t - k) = 2^{-j/2}\phi\left(\frac{t - 2^{j}k}{2^{j}}\right) \tag{2.20}$$

where 2^{j} is a sequence of scales. The term $2^{-j/2}$ maintains the norm of the basis functions $\phi(t)$ at 1. In this form, the wavelets are centered at $2^{j}k$ with scale 2^{j}. $2^{j}k$ is called the translation (shift) parameter. The change in j and k changes the support of the basis functions. 2^{j} is called the scale factor used for frequency partitioning. When j becomes larger, the scale factor 2^{j} becomes larger, and the function $\phi_{j,k}(t)$ becomes shorter and more spread out, and conversely when j gets smaller. Therefore, 2^{j} is a measure of the scale of the functions $\phi_{j,k}(t)$. The translation parameter $2^{j}k$ is matched to the scale parameter 2^{j} in the sense that as the function $\phi_{j,k}(t)$ gets wider, its translation step is correspondingly larger. This scaling function spans a space vector over k.

$$S_j = Span\{\phi_k(2^{j}t)\} \tag{2.21}$$

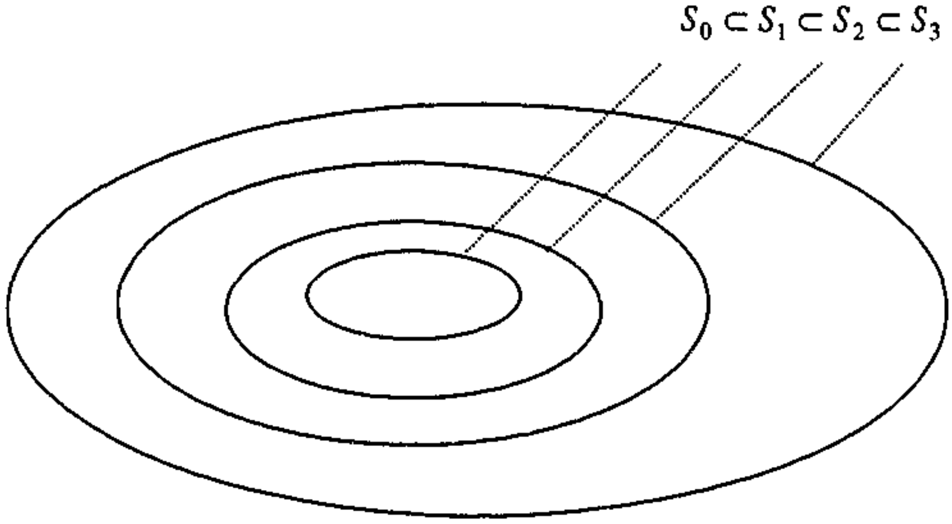
In order to describe multiresolution analysis (MRA) more specifically, the basic requirement of MRA can be formulated as follows:

$$\cdots \subset S_{-2} \subset S_{-1} \subset S_0 \subset S_1 \subset S_2 \subset \cdots \subset L^2 \tag{2.22}$$

with $S_{-\infty} = \{0\}$ and $S_{\infty} = L^2$

Each subspace S_j encodes the information of the signal at resolution level j, which can be represented by scale functions (Lee and Hong, 2001). This relationship, plotted in Figure 2.5, indicates that the space, which contains high resolution, also contains those of lower resolution.

Figure 2.5 Nested Vector Spaces Spanned by the Scaling Functions



Note: This figure illustrates the vector spaces spanned by the scaling functions. From this figure, it is observed that the higher scaling function nests the lower scaling functions.

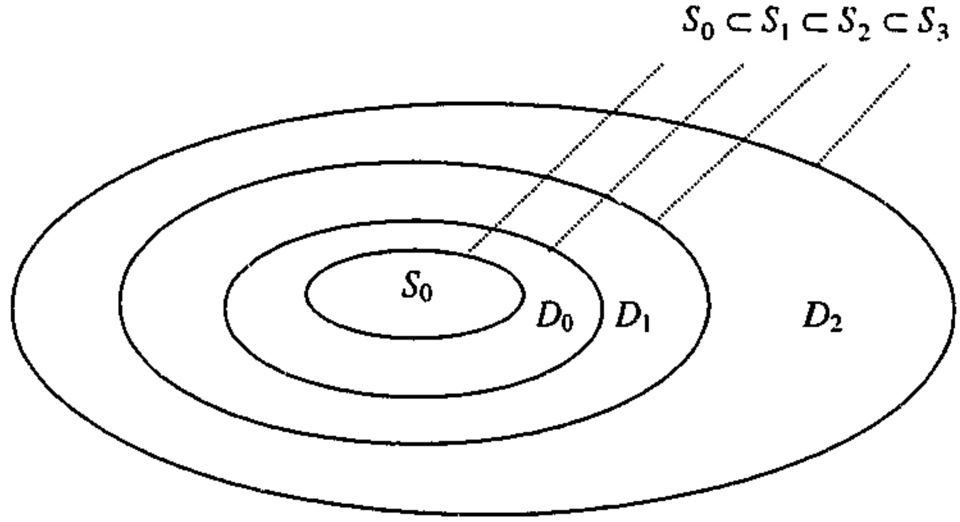
From Figure 2.5, we can find a relationship between two adjacent scaling functions such that if $\phi(t)$ is in V_0, it is also in V_1. This implies that $\phi(t)$ can be expressed as a weighted sum of shifted $\phi(2t)$. Therefore, MRA involves successively projecting all time series x_t to be studied into each of the approximation subspaces S_j.

$$\phi(t) = \sum_k g(k)\sqrt{2}\phi(2t - k), \quad k \in Z \tag{2.23}$$

where the coefficients $g(k)$ are a sequence of real (complex) numbers called the scaling function coefficients (low-pass filter) and $\sqrt{2}$ maintains the norm of the scaling function with the scale of two. This equation is called the refine equation, the MRA equation, or the dilation equation as it describes different interpretations or points of view (Burrus et al., 1998).

To this point, we have discussed some properties of scaling functions. These properties play an important role in describing the properties of the wavelet functions (mother wavelet, $\psi(t)$). The important features of time series can be captured better by defining a slightly different set of functions $\psi(t)$ that span the differences between two adjacent spaces, spanned by the various scales of the scaling functions. The relationship between vector spaces of the scaling functions and those of wavelet functions are plotted in Figure 2.6, based on the orthogonality condition of the scaling and wavelet functions (see section 2.3.1). This condition gives several advantages. Orthogonal basis functions allow simple calculation of expansion coefficients and have a Parseval's theorem that allows a partitioning of the signal energy in the wavelet transform domain (Burrus et al., 1998, p14).

Figure 2.6 Scaling Function and Wavelet Vector Spaces



Note: This figure illustrates the relationship between scaling functions and wavelet functions.

Combining this orthogonality with Figure 2.6, we can describe L^2 as follows:

$$L^2 = S_0 \oplus D_1 \oplus D_2 \oplus D_3 \oplus \cdots \quad (2.24)$$

where $\oplus$ denotes the orthogonal sum.

In equation (2.24), we can describe the relation of S_0 to the wavelet spaces as follows:

$$S_0 = D_{-\infty} \oplus \cdots \oplus D_{-1} \quad (2.25)$$

This relationship shows that the key idea of MRA consists in studying a signal by examining its increasingly coarser approximations as more and more details are cancelled from the data (Abry, et al., 1998).

Analogous to this relationship, the wavelets reside in the space spanned by the next narrower scaling function: i.e., $D_0 \subset S_1$. This leads us to express the wavelets as a weighted sum of shifted scaling function $\phi(2t)$, which is defined in equation (2.23), for some coefficients $h(k)$ (high-pass filter):

$$\psi(t) = \sum_k h(k)\sqrt{2}\phi(2t-k) \tag{2.26}$$

The function generated by equation (2.26) gives the mother wavelet $\psi(t)$, which has the following form[3]:

$$\psi_{j,k}(t) = 2^{-\frac{j}{2}}\psi(2^{-j}t-k) = 2^{-j/2}\psi\left(\frac{t-2^j k}{2^j}\right) \tag{2.27}$$

According to equation (2.24), any time series $x_t \in L^2$ could be written as a series expansion in terms of the scaling function and wavelets.

$$f(t) = \sum_{k=-\infty}^{\infty} s(k)\phi_k(t) + \sum_{j=0}^{\infty}\sum_{k=-\infty}^{\infty} d(j,k)\psi_{j,k}(t) \tag{2.28}$$

In this expression, the first expansion gives a function that is a low resolution or a coarse approximation of x_t. For each increasing index j in the second summation,

[3] Intuitively, a small j or a low resolution level can capture smooth components of the signal, while a large j or a high resolution level can capture variable components of the signal (Lee and Hong, 2001).

a higher or finer resolution function is added. The coefficients in this wavelet expansion are called the discrete wavelet transform (DWT[4]) of the signal x_t.

In another expression, the signal can be expressed as the sum of a finite set of high frequency parts and a residual low frequency part. The orthogonal wavelet series approximation up to scale J to a time series x_t is given by:

$$x_t \approx \sum_k s_{J,k}\phi_{J,k}(t) + \sum_k d_{J,k}\psi_{J,k}(t) + \sum_k d_{J-1,k}\psi_{J-1,k}(t) + \cdots + \sum_k d_{1,k}\psi_{1,k}(t) \tag{2.29}$$

with $s_{J,k} = \int \phi_{J,k}(t)x_t dt$ and $d_{j,k} = \int \psi_{j,k}(t)x_t dt$ where $j = 1, 2, \ldots, J$.

where J is the number of scales, and k ranges from 1 to the number of coefficients in the specified component. The coefficients $s_{J,k}$, $d_{J,k}$, ..., $d_{1,k}$ are the wavelet transform coefficients. J is the maximum integer such that 2^J is less than the number of data points. Their magnitude gives a measure of the contribution of the corresponding wavelet function to the approximation sum and wavelet series coefficients approximately specify the location of the corresponding wavelet function. More specifically, $s_{J,k}$ represents the smooth coefficients that capture the trend, while the detail coefficients $d_{j,k}$, ..., $d_{1,k}$, which can capture the higher

[4] Similar to the Continuous Wavelet Transform (CWT), the DWT is a two dimensional orthogonal decomposition of a time series that is well suited, and is in fact designed, to detect abrupt changes and fleeting phenomena. The important characteristic of the DWT is that its basis functions have compact support. Thus, they are able to pick up unique phenomena in the data (Goffe, 1994).

frequency oscillations, represent increasing finer scale deviations from the smooth trend.

Given these coefficients, the wavelet series approximation of the original time series x_t is given by the sum of the smooth signal $S_{J,k}$, and the detail signals $D_{J,k}$, $D_{J-1,k}$, ..., $D_{1,k}$:

$$x_t = S_{J,k} + D_{J,k} + D_{J-1,k} + \cdots + D_{1,k} \tag{2.30}$$

where $S_{J,k} = \sum_k s_{J,k}\phi_{J,k}(t)$,

$$D_{J,k} = \sum_k d_{J,k}\psi_{J,k}(t)$$

and $D_{j,k} = \sum_k d_{j,k}\psi_{j,k}(t)$, $j = 1, 2, ..., J\text{-}1$

The original signal components $S_{J,k}$, $D_{J,k}$, $D_{J-1,k}$, ..., $D_{1,k}$ are listed in the order of increasingly finer scale components. Signal variations on high scales are acquired using wavelets with large supports.

The DWT maps the vector $\mathbf{f} = (f_1, f_2, ..., f_n)'$ to a vector of n wavelet coefficients $\mathbf{w} = (w_1, w_2, ..., w_n)'$. The vector $\mathbf{w}$ contains the coefficients $s_{J,k}$, $d_{J,k}$, ..., $d_{1,k}$, $j = 1, 2, ..., J$ of the wavelet series approximation, equation (2.29).

The DWT is mathematically equivalent to multiplication by an orthogonal matrix $\mathbf{W}$:

$$\mathbf{w} = \mathbf{W}\mathbf{f} \tag{2.31}$$

where the coefficients are ordered from coarse scales to fine scales in the vector $\mathbf{w}$. In the case where n is divisible by 2^J:

$$\mathbf{w} = \begin{pmatrix} \mathbf{s}_J \\ \mathbf{d}_J \\ \mathbf{d}_{J-1} \\ \vdots \\ \mathbf{d}_1 \end{pmatrix} \tag{2.32}$$

where $\mathbf{s}_J = (s_{J,1}, s_{J,2}, \cdots, s_{J,n/2^J})'$

$$\mathbf{d}_J = (d_{J,1}, d_{J,2}, \cdots, d_{J,n/2^J})'$$

$$\mathbf{d}_{J-1} = (d_{J-1,1}, d_{J-1,2}, \cdots, d_{J-1,n/2^J})'$$

$$\vdots \qquad \qquad \vdots$$

$$\mathbf{d}_1 = (d_{1,1}, d_{1,2}, \cdots, d_{1,n/2^J})'$$

Each set of coefficients $\mathbf{s}_J$, $\mathbf{d}_J$, $\mathbf{d}_{J-1}$, ..., $\mathbf{d}_1$ is called a crystal. The term crystal is used because the wavelet coefficients in a crystal correspond to a set of translated wavelet functions arranged on a regular lattice.

An alternative way to think about the wavelet is to consider low- and high-pass filters, denoted in equations (2.23) and (2.26). The natural question is how to derive these filters so that they can be applied in wavelet analysis. The low- and high-pass filters can be obtained from the father and mother wavelets using the following relationships:

$$g(k) = \frac{1}{\sqrt{2}} \int \phi(t)\phi(2t-k)dt \tag{2.33}$$

$$h(k) = \frac{1}{\sqrt{2}} \int \psi(t)\psi(2t-k)dt \tag{2.34}$$

or

$$h(k) = (-1)^k g(k) \tag{2.35}$$

The relationship between filter banks and wavelets is extensively discussed in Strang and Nguyen (1996) and Percival and Walden (2000). The analysis indicates that one can approach the analysis of the properties of wavelets either through wavelets or through the properties of the filter banks (Ramsey, 2002). However, the introduction of filter banks reveals clearly the difficulty of handling boundary conditions, which will be discussed in the next section more extensively.

It is important to examine the properties of the wavelet filter, in a similar manner to the continuous case. The DWT has counterpart properties to the case presented in equations (2.18) and (2.19), which show integration to zero and unit energy. Let $h_l = (h_0, h_1, h_2, \cdots, h_{J-1})$ be a finite length discrete wavelet filter. This wavelet filter holds the same properties as the continuous wavelet function in as much as it sums to zero and has unit energy.

$$\sum_{l=0}^{J-1} h_l = 0 \text{ and } \sum_{l=0}^{J-1} h_l^2 = 1$$

In addition to these properties, since the wavelet filters are orthogonal to its shifts, the following property also has to hold for all wavelet filters.

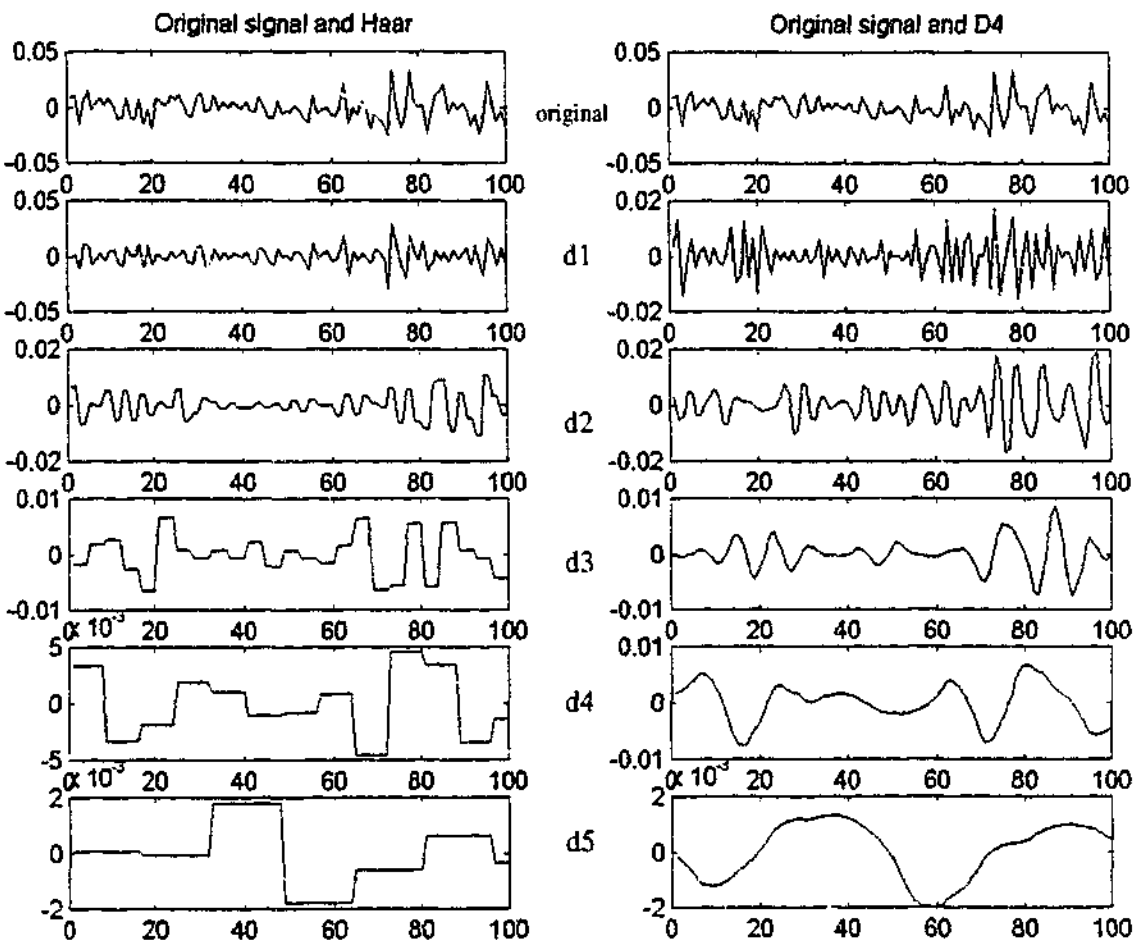
$$\sum_{l=0}^{J-1} h_l h_{l+2n} = 0 \text{, for all non-zero integers } n. \tag{2.36}$$

This implies that to construct the orthonormal matrix that defines the DWT, wavelet coefficients cannot interact with one another.

Up to now, we have studied the DWT. To get an idea about how the DWT is applied in the signal, in this example we use the stock index. We apply the DWT to the daily S&P 500 stock prices from June 29, 2000 to December 29, 2000. The return series are computed via the first difference of log-transformed prices – that is, $r_t = \log(P_t) - \log(P_{t-1})$. This series is plotted in the upper row of Figure 2.7. There is an obvious increase in variance in the returns toward the latter half of the series. The length of the returns series is N = 124, which is divisible by $2^5 = 32$, and therefore, we may decompose our returns series up J = 5.

The wavelet coefficient vectors d_1, ..., d_5 using the Haar wavelet are shown on the left hand side of Figure 2.7. The first scale of the wavelet coefficient d_1 is filtering out the high-frequency fluctuations by essentially looking at adjacent differences in the data. There is a large group of rapidly fluctuating returns between observations from 60 to 100. A small increase in the magnitude is also observed between 60 and 100, but smaller than the unit scale coefficients. This vector of wavelet coefficients is associated with changes of λ_1, equivalent to 2 – 4 days. Since the S&P 500 return series exhibits low-frequency oscillations, the higher (low-frequency) vectors of wavelet coefficients d_1 and d_5 indicate large variations from zero. Interestingly, as we noted above, the Haar wavelet is called a step function. As the wavelet time scale increases, the decomposed higher wavelet coefficients have a step-shape.

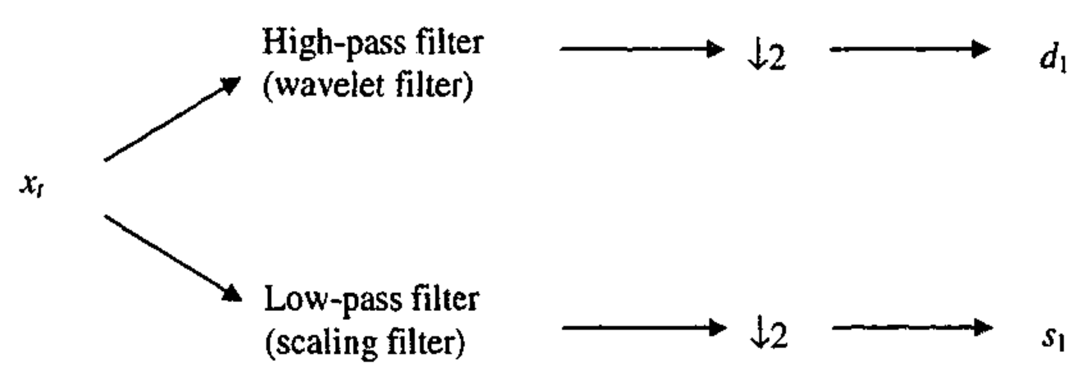
Figure 2.7 MRA using Haar and Daubechies 4 Wavelets



Note: This figure plots MRA using two different wavelet filters (Haar and D(4)) from June 29, 2000 to December 29, 2000 using daily frequency. The left hand side of this figure is constructed using the Haar wavelet filter, whereas the right-hand side of this figure is constructed by the Daubechies wavelet filter with length 4. We observe that the D(4) wavelet filter generates more smooth wavelet coefficients than the Haar wavelet filter.

The same decomposition was performed using the Daubechies extremal phase wavelet filter of length 4 (D(4)) and provided in Figure 2.7. The interpretations for each of the wavelet coefficient vectors are the same as in case of the Haar wavelet filter. The wavelet coefficients will be different given that the length of the filters is now four versus two, and should isolate features in specific frequency intervals better since the D(4) is a better approximation over the Haar wavelet. Compared with the Haar wavelet coefficients, the wavelet coefficient of D(4) are smoother than those of the Haar wavelet, as the wavelet time scale increases.

Figure 2.8 Flow Diagram Illustrating the Down-sampling



Note: This figure illustrates how a time series is down-sampled using the high- and low-pass filters and shows how to obtain the wavelet and scaling coefficients using a pyramid algorithm.

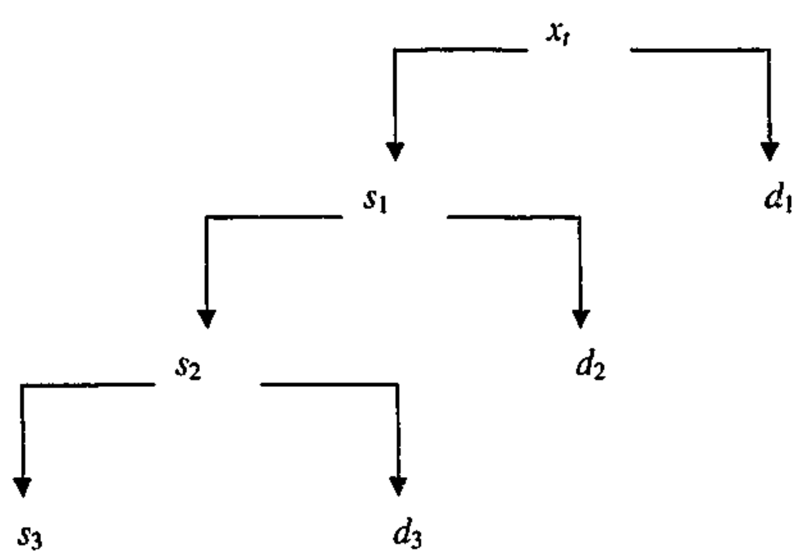
In practice, the DWT is implemented through a pyramid algorithm (Mallat, 1989), which starts with a time series x_t. The first step of the pyramid algorithm is to use the wavelet filter and scaling filter to decompose the time series against various wavelet scales. During this procedure, the time series are down-sampled by two. Suppose that the original signal x_t consists of N (=500) samples of data. Then the approximation and the detail signals will have 1000 samples of data, for a total of

$2N$ samples. To improve this sampling efficiency, we perform down-sampling. This simply means throwing away every second data point, down-sampled by 2, to produce the length $N/2^j$ wavelet coefficients vector d_j. In down-sampling, there is obviously the possibility of losing information, since half of the data is discarded. The effect in the frequency domain (Fourier transform) is called aliasing, which states that the result of this loss of information is mixing up of frequency components (Burrus et al., 1998).

Figure 2.8 gives a flow diagram for the first step of the pyramid algorithm, i.e., down-sampling. The symbol $\downarrow 2$ implies that every second value of the time series vector is removed. More precisely, Figure 2.8 illustrates the decomposition of x_t into the unit wavelet coefficients d_1 and the unit scale scaling coefficients s_1. The time series x_t is filtered using the wavelet filter and scaling filter and down-sampled by 2. Therefore, the N length vector of observations has been high- and low-pass filtered to obtain $N/2$ coefficients.

The second step of the pyramid algorithm is to treat the scaling coefficients series $\{s_1\}$ as our original time series, and repeat the filter and down-sampling procedure using wavelet and scaling filters. In other words, this decomposition process can be iterated, with successive approximations being decomposed in turn, so that the time series is broken down into many lower resolution components. For example, suppose that one wants to decompose the time series x_t into a third level. After decomposing and down-sampling the original time series in the first level, shown in Figure 2.8, the scaling coefficients are decomposed and down-sampled as we did in the original time series. Once we finish this procedure, we have the following length N decomposition $w=[d_1,d_2,s_2]^T$. After the third iteration of the pyramid algorithm (once again, we apply filtering procedure to s_2), the N length decomposition $w=[d_1,d_2,d_3,s_3]^T$ is obtained. This procedure may be repeated up to J times where $J=\log_2(N)$ and gives the vector of wavelet coefficients in equation (2.32). This is called the wavelet decomposition tree and is presented in Figure 2.9.

Figure 2.9 Analysis of a Time Series by a Wavelet Decomposition Tree



Note: This figure plots the wavelet decomposition tree. More specifically, the original time series can be decomposed into wavelet scaling coefficients and wavelet coefficients in the first step. In the next step, the scaling coefficients, obtained in the first step, is regarded as the original time series and decomposed as in the first step. This figure illustrates this procedure and continues to third step.

2.3.3 Maximal Overlap Discrete Wavelet Transform

We have examined the properties of the DWT as an alternative to the Fourier transform. In this sub-section, we examine the Maximal Overlap Discrete Wavelet Transform (MODWT). It is natural to ask why the MODWT is needed instead of the DWT. The motivation for formulating the MODWT is essentially to define a transform that acts as much as possible like the DWT, but does not suffer from the DWT's sensitivity[5] to the choice of a starting point for a time series (Percival and Walden, 2000).

Non-redundancy of the DWT is achieved by down-sampling the filtered output at each scale. (For detail, refer to Daubechies, 1992; Percival and Mofjeld, 1997). Importantly, the zero-phasing property of the MODWT permits meaningful interpretation of "timing" regarding the wavelet details. With this property, we can align perfectly the details from decomposition with the original time series. In comparison with the DWT, no phase shift will result in the MODWT.

The MODWT of level J for a time series x_t is a highly redundant non-orthogonal transform yielding the column vectors $\widetilde{D}_1, \widetilde{D}_2, \cdots, \widetilde{D}_J$ and $\widetilde{S}_J$, each of dimension N. The vector $\widetilde{D}_j$ contains the MODWT coefficients associated with changes in x_t between scale j-1 and j, while $\widetilde{S}_J$ contains the MODWT scaling coefficients associated with the smooth of x_t at scale J, or equivalently the variations of x_t at

[5] This sensitivity results from down-sampling the outputs from the wavelet and scaling filters at each stage of the pyramid algorithm (Percival and Walden, 2000).

scale J+1 and higher. The MODWT also follows the same pyramid algorithm as the DWT, while it utilizes the rescaled filters, instead of the wavelet and scaling filters in the DWT. These wavelet and scaling filters can be expressed as follows:

$$\tilde{h}_j = h_j / 2^j \quad \text{and} \quad \tilde{g}_j = g_j / 2^j \tag{2.37}$$

Utilizing its filtered output at each scale, a time series x_t can also be decomposed into its wavelet details and smooth as follows:

$$x_t = \sum_{j=1}^{J} \tilde{D}_j + \tilde{S}_J \tag{2.38}$$

However, this MRA of the MODWT provides some important features, which are not available to the original DWT. Percival and Walden (2000) present five important properties which distinguish the MODWT from the DWT:

(1) Although the DWT of level J restricts the sample size to an integer multiple of 2^J, the MODWT of level J is well defined for any sample size N.

(2) As in the DWT, the MODWT can be used to form an MRA. In contrast to the usual DWT, both the MODWT wavelet and scaling coefficients and the MRA are shift invariant in the sense that circularly shifting the time series by any amount will circularly shift by a corresponding amount the MODWT wavelet and scaling coefficients, details, and smooths. In other words, an MRA of the MODWT is associated with zero phase filters, implying that

events, which feature in the original time series x_t, may be poorly aligned with features in the MRA.

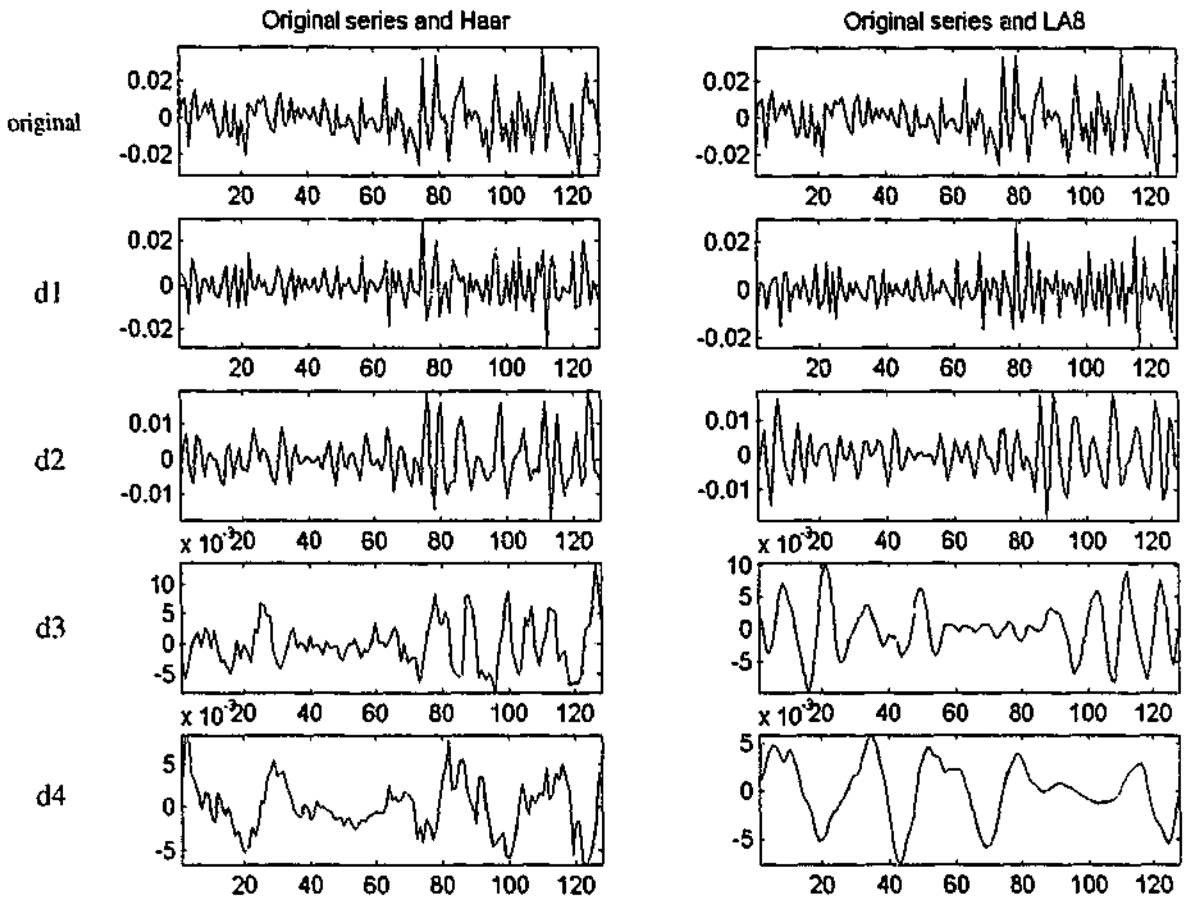
(3) In contrast to the DWT details and smooths, the MODWT details and smooth are associated with zero phase filters, thus making it possible to meaningfully line up features in an MRA with the original time series x_t.

(4) As is true for the DWT, the MODWT can be used to form an analysis of variance based on the wavelet and scaling coefficients. However, the MODWT wavelet variance estimator is asymptotically more efficient than the same estimator based on the DWT.

(5) Whereas a time series and a circular shift of the series can have different DWT-based empirical power spectra, the corresponding MODWT-based spectra are the same.

To provide an example of the MODWT, we construct the MRA of the S&P 500 returns. The data series are the same as those of Figure 2.7. The number of observations is 124. Note that with the MODWT, we are no longer limited to decomposing a sample size of dyadic length, i.e., a power of 2, while the only limiting factor is the overall depth of the transformation given by $J = \log_2(N) = 7$. However, in this example we choose a performance level $J = 4$ MODWT on the return series using Haar and the Daubechies least asymmetric wavelet filter of length 8 (LA(8)) for display purposes.

Figure 2.10 MRA analysis of the MODWT



Note: This figure plots the MODWT-MAR using two different wavelet filters (Haar and LA(8)) from June 29, 2000 to December 29, 2000 using daily frequency. The left hand side of this figure is constructed using the Haar wavelet filter, whereas the right-hand is constructed by LA(8). We observe that the LA(8) wavelet filter generates more smooth wavelet coefficients than the Haar wavelet filter.

The left-hand side of Figure 2.10 presents the MODWT coefficient vectors of r_t using the Haar wavelet filter. Note that there are N wavelet coefficients at each scale because the MODWT does not down-sample after filtering. Compared with Figure 2.7, the MODWT coefficients are smoother than those of the DWT. This is because the MODWT wavelet coefficients $\tilde{d}_j$ contain the DWT coefficients, scaled by $1/\sqrt{2}$, and also the DWT coefficients applied to x_t circularly shifted by one. Compared to the MODWT coefficients of the Haar wavelet, the coefficients filtered by LA(8) wavelet filter are even more smoother. The longer wavelet filter has induced significant amounts of correlation between the two adjacent coefficients, thus producing even smoother vectors of wavelet and scaling coefficients.

Practically, the wavelet coefficients are calculated using the wavelet filters. This introduces us another practical problem: how to choose a specific wavelet filter to implement wavelet analysis from the various wavelet filters. According to Gençay et al. (2002b), to choose an appropriate wavelet filter, there are three aspects to be considered: length of data, complexity of the spectral density function, and the underlying shape of features in the data. First, the length of the original data is an important factor because the distribution of wavelet coefficients computed by the boundary conditions will be very different from that of wavelet coefficients computed from complete sets of observations. The shorter the wavelet filter, the fewer wavelet coefficients produced.

Second, the complexity of the spectral density function has to be carefully considered to select a wavelet filter, since wavelet filters are finite in the time domain and thus infinite in the frequency domain. For example, if the spectral density function is quite dynamic, shorter wavelet filters may not be able to separate the activity between scales. In this case, longer wavelet filters would be more preferable to short wavelet filter. Clearly a balance between frequency localization and time localization is needed.

Finally, and most importantly, there is the issue of what the underlying features of the data look like. This is very important since wavelets are the basis functions of the data. If one chooses a wavelet filter that looks nothing like the underlying features, then the decomposition will be quite inefficient. Therefore, one should take care when selecting the wavelet filter and its corresponding basis function. Issues of smoothness and symmetry/asymmetry are the most common desirable characteristics for wavelet basis functions.

2.3.4 Boundary Condition

In addition to the problem of choosing a proper wavelet filter, another condition should be considered when undertaking an empirical analysis using wavelet: the boundary condition. As indicated in choosing the wavelet filter, the empirical data has a finite interval. This raises the issue of handling the boundaries[6]. In applying the DWT and the MODWT to finite length time series, there must be an established method for computing the remaining wavelet coefficients. Various techniques have been proposed to handle this problem, and three techniques in particular are briefly discussed in this section.

[6] Unser (1996) discusses some of the practical problems that arise when implementing these boundary conditions and argues that care must be given in coding the reconstruction algorithm to ensure that the original data can be recovered exactly.

Periodic boundary

The most natural method for dealing with the boundary is to assume that the length N series is periodic, and to grab observations from the other end to finish the computations. In other words, any time series $f(x)$ defined on [0, 1][7] could be expanded to live on the real line by regarding it as a periodic function with period one: $f(x) = f(x-[x])$ for $x \in R$. This is reasonable for some time series where strong seasonal effects are observed but cannot be applied universally in practice (Gençay et al., 2002a, p144). This technique is generally adapted in wavelet analysis, partly because it is very easy to implement, and partly because the resulting empirical wavelet coefficients are independent with identical variances.

Reflection boundary

The reflection boundary condition is used extensively in Fourier analysis to reflect the time series about the boundaries. This technique produces a time series of length $2N$. This reflected series is applied to the wavelet transformation under the assumption of periodic boundary conditions. More specifically, this technique consists of two methods: symmetric and antisymmetric reflection (see Ogden, 1997, p112 for more detail). In a symmetric reflection, it is required to extend the domain of the function beyond [0, 1] and define $f(x) = f(-x)$ for $x \in [-1, 0)$ and $f(x) = f(2-x)$ for $x \in (1, 2]$. This has an advantage over the periodic

[7] Since any interval can be translated, we consider here only the unit interval [0, 1] without loss of generality (Ogden, 1997, p110).

boundary condition in the sense that it preserves the continuity of the function, though discontinuities in the derivatives of f may be introduced.

In addition, antisymmetric reflection causes the function to be reflected antisymmetrically about the endpoints. In terms of mathematical notations, $f(x) = 2f(0) - f(-x)$ for $x \in [-1, 0]$ and $f(x) = 2f(1) - f(2-x)$ for $x \in (1, 2)$. This can preserve continuity in both the function and its first derivative. As with periodic boundary conditions, these methods impose their own alterations of the usual MRA. However, reflecting the time series does not alter the sample mean nor the sample variance, since all coefficients have been duplicated once (Gençay et al., 2002a).

Brick wall condition

Another way to handle the boundary is to impose the brick wall condition, which prohibits convolutions that extend beyond the ends of the series (Lindsay et al., 1996). In other words, this condition can be implemented to simply remove any wavelet coefficient computed involving the boundary. Imposing this condition requires care when we calculate the wavelet variance and covariance.

As indicated in Lindsay et al. (1996), the brick wall condition can be used in an analysis, where data compression and regeneration is not the goals in which any convolution that extends beyond the end of the data series is not permitted. This boundary condition is appropriate in an analysis when there is no compelling reason to assume that the data are periodic and symmetric in structure.

2.4 Wavelet Variance, Covariance and Correlation

Variance, covariance and correlation are used to provide useful statistical information to researchers, and hence they are applied to many financial theories. In this section, we explain how the wavelet variance, covariance and correlation are derived in the wavelet domain.

2.4.1 Wavelet Variance

In addition to the features of wavelet transforms (the DWT, the MODWT) stated in sections 2.3.2 and 2.3.3, an important characteristic of wavelet transform is its ability to decompose or analyze the variance of a stochastic process. When we derive the wavelet coefficients using the DWT (or the MODWT), these wavelet coefficients indicate the changes at a particular scale. Using these coefficients, the wavelet variance on a particular scale can be obtained. In other words, the basic idea of the wavelet variance is to substitute the notion of variability over certain scales for the global measure of variability estimated by the sample variance (Percival and Walden, 2000). We first examine how the wavelet variances would be related to a sample variance. This can be seen by examining the sample variance of the time series y. The sample variance can be expressed as follows:

$$\begin{aligned}\hat{\sigma}_y^2 &= \frac{1}{N}\sum_{i=1}^{N}[y_i - \bar{y}]^2 \\ &= \frac{1}{N}\sum_{i=1}^{N}[y_i]^2 - \bar{y}^2 \qquad (2.39)\end{aligned}$$

where $\bar{y}$ is the sample mean. Using orthogonality of the wavelet basis vectors, the sum of the series can be expressed as the sum of the squares of the wavelet coefficients.

$$\sum_{i=1}^{N}[y_i]^2 = \sum_{j=1}^{L}\sum_{k=1}^{n_j} d_{j,k}^2 + N\bar{y}^2 \tag{2.40}$$

Therefore, substituting equation (2.40) into equation (2.39) makes the variance a function of the wavelet coefficients.

$$\hat{\sigma}_y^2 = \frac{1}{N}\sum_{j=1}^{L}\left(\sum_{k=1}^{n_j} d_{j,k}^2\right) \tag{2.41}$$

Let d and $\tilde{d}$ be the DWT and MODWT coefficient vectors, respectively. Equation (2.39) can be expressed as a vector notation under the assumption that a time series y has a zero mean.

$$\| y \|^2 = \| d \|^2 = \| \tilde{d} \|^2 \tag{2.42}$$

The relationship in equation (2.42) provides a decomposition of variance between the original series and either the DWT or MODWT wavelet coefficients (Gençay et al., 2002a).

The wavelet variance for a time series X with a dyadic length $N = 2^J$ is estimated using the DWT coefficients for scale $\lambda_j \equiv 2^{j-1}$ through:

$$\hat{\sigma}_X^2(\lambda_j) \equiv \frac{1}{\hat{N}_j} \sum_{t=L_j'}^{N} [d_{j,t}^X]^2 \tag{2.43}$$

where $\hat{N}_j = N/2^j - L_j'$ is the number of wavelet coefficients at scale λ_j unaffected by the boundary[8] and $L_j' = [(L-2)(1-2^{-j})]$ is the number of the DWT coefficients computed using the boundary. While the spectral density function decomposes the process variance on a frequency-by-frequency basis, the wavelet variance decomposes the variance of X_t on a scale-by-scale basis.

We denote the MODWT coefficients of $X_1, \cdots, X_N$ as $\tilde{d}_{j,t}$ for $j = 1, \ldots, J$ and $t = 1, \ldots, N/2^j$. Similar to the variance of the DWT coefficients, the wavelet variance estimated by the MODWT coefficients for scale λ_j is as follows:

$$\tilde{v}_X^2(\lambda_j) \equiv \frac{1}{\tilde{N}_j} \sum_{t=L_j}^{N} [\tilde{d}_{j,t}^X]^2 \tag{2.44}$$

where $\tilde{N}_j = N - L_j + 1$ is the number of coefficients unaffected by the boundary, and $L_j = (2^j - 1)(L-1) + 1$ is the length of the scale λ_j wavelet filter. The

[8] How the boundary condition (especially the brick wall condition) affects the number of wavelet coefficients is explained in section 2.3.4.

decomposition of a time series as a sum of wavelet variances indicates which scales are important contributors to the time series variance (Percival and Walden, 2000; Serroukh and Walden, 2000). Percival (1995) and Percival and Walden (2000, p309) provide the asymptotic relative efficiencies for the wavelet variance estimator based on the orthogonal DWT compared to the estimator based on the MODWT using a variety of power law processes. In their studies, they find that the DWT-based estimator can be rather inefficient – in the worst case, its large sample variance is twice that of the MODWT-based estimator.

To this point we have examined how to derive the wavelet variances. For statistical inference, the confidence interval for the wavelet variance is required. Percival (1995) develops a theory for determining the uncertainty in the wavelet variance estimate for wavelet filters of various lengths under a Gaussian assumption. Under the assumption that the estimates of the wavelet variance of the DWT and the MODWT are unbiased and asymptotically normally distributed[9] (Lindsay et al., 1996), the approximate 100(1–2p)% confidence interval for the DWT estimate, $\hat{\sigma}_X^2(\lambda_j)$ and the MODWT estimate, $\tilde{v}_X^2(\lambda_j)$ can be derived:

$$\left[\hat{\sigma}_X^2(\lambda_j) - \Phi^{-1}(1-p)\sqrt{\mathrm{var}(\hat{\sigma}_X^2(\lambda_j))},\ \hat{\sigma}_X^2(\lambda_j) + \Phi^{-1}(1-p)\sqrt{\mathrm{var}(\hat{\sigma}_X^2(\lambda_j))}\right] \quad (2.45)$$

$$\left[\tilde{v}_X^2(\lambda_j) - \Phi^{-1}(1-p)\sqrt{\mathrm{var}(\tilde{v}_X^2(\lambda_j))},\ \tilde{v}_X^2(\lambda_j) + \Phi^{-1}(1-p)\sqrt{\mathrm{var}(\tilde{v}_X^2(\lambda_j))}\right] \quad (2.46)$$

[9] Serroukh, Walden and Percival (2000) prove the wavelet variance is asymptotically normally distributed in three cases of the original time series: Non-linear Processes, Non-Gaussian Linear Process, and Non-stationary Processes and Differencing.

where $\Phi^{-1}(1-p)$ is the $(1-p)\times 100\,\%$ point for the standard normal distribution.

2.4.2 Wavelet Covariance and Correlation

In many economic and financial analyses, the temporal structure of the covariance between two series is of interest. This covariance structure can be applied to the wavelet analysis. The wavelet covariance is firstly compared to the Fourier cross spectra by Hudgins et al. (1993) using atmospheric surface-layer measurements of the horizontal and vertical velocities and the vertical velocity and temperature. In finance literature, the calculation of the wavelet covariance is a relatively new technique. Only a few researchers adopt this technique (see Gençay et al., 2001, 2003a and b; In and Kim, 2003).

The wavelet analysis of univariate time series can be generalized to multiple time series by defining the concept of the wavelet covariance between X_t and Y_t. As in standard statistics, the wavelet covariance can be defined as the covariance between the wavelet coefficients of X_t and Y_t at scale λ_j.

The sample covariance between X_t and Y_t is:

$$\begin{aligned}\sigma_{XY} &= \frac{1}{N}\sum_{i=1}^{N}\left(X_{i,t}-\overline{X}\right)\left(Y_{i,t}-\overline{Y}\right)\\ &= \frac{1}{N}\sum_{i=1}^{N}X_{i,t}Y_{i,t}-\overline{X}\,\overline{Y}=\frac{<XY>}{N}-\overline{X}\,\overline{Y}\end{aligned} \tag{2.47}$$

The inner product of two vectors in the last equality can be expressed in terms of their wavelet decompositions as follows:

$$< XY >= \left\langle \sum_{j=1}^{L}\sum_{k=1}^{n_j} d_{j,k}^{X}\psi_{j,k} + \overline{X}1, \sum_{j=1}^{L}\sum_{k=1}^{n_j} d_{j,k}^{Y}\psi_{j,k} + \overline{Y}1 \right\rangle \tag{2.48}$$

Therefore, using the orthogonality properties of the vectors $\psi_{j,k}$, the sample covariance of the series may be written in terms of the wavelet coefficients.

$$\hat{\sigma}_{XY} = \sum_{i=1}^{L}\left\{\frac{1}{N}\sum_{k=1}^{n_j} d_{j,k}^{X} d_{j,k}^{Y}\right\} \tag{2.49}$$

As with the wavelet variance for univariate time series, the wavelet covariance also decomposes the covariance between two stochastic processes on a scale-by-scale basis. The term in the bracket in equation (2.49) indicates the contribution to the covariance associated with each scale λ_j. More specifically, we can express the wavelet covariance at scale λ_j as follows:

$$\hat{\sigma}_{XY,j} = \frac{1}{N}\sum_{k=1}^{n_j} d_{j,k}^{X} d_{j,k}^{Y} \tag{2.50}$$

If the brick wall boundary conditions are imposed, i.e., if the wavelet coefficients affected by the boundary are removed, the DWT estimate for the wavelet covariance can be derived as follows, analogous to the wavelet variance:

$$\hat{\sigma}_{XY,j} = \frac{1}{2^j \hat{N}_j} \sum_{k=L_j}^{\hat{N}_j} d_{j,k}^X d_{j,k}^Y \qquad (2.51)$$

The MODWT wavelet covariance can also be expressed in terms of the MODWT wavelet coefficients:

$$\tilde{\sigma}_{XY,j} = \frac{1}{2^j \tilde{N}_j} \sum_{k=L_j}^{\tilde{N}_j} \tilde{d}_{j,k}^X \tilde{d}_{j,k}^Y \qquad (2.52)$$

The MODWT method allows a more accurate determination of the covariance associated with each scale (Lindsay et al., 1996). Note that the estimator does not include any coefficients that make explicit use of the periodic boundary conditions. We can construct a biased estimator of the wavelet covariance by simply including the MODWT wavelet coefficients affected by the boundary and renormalizing.

As shown in equations (2.51) and (2.52), the wavelet decomposition of the covariance is determined by the product of the coefficients from the two decompositions performed separately. Lindsay et al. (1996) show that the MODWT estimator $\tilde{\sigma}_{XY,j}$ is asymptotically normally distributed with mean $\sigma_{XY,j} \equiv 2^{-j} E\{\tilde{d}_j^X \tilde{d}_j^Y\}$ and variance

$$\operatorname{var}(\tilde{\sigma}_{XY,j}) = \frac{1}{2^{2j+1} \tilde{N}_j} \left[\int_{-1/2}^{1/2} | S_{d_j^X d_j^Y}(w) |^2 \, dw + \int_{-1/2}^{1/2} (S_{d_j^X}(w) S_{d_j^Y}(w)) dw \right] \qquad (2.53)$$

where $S_x(w)$ indicates the power spectral density at the frequency w of variable x. Based on these findings, an approximate $100\times(1-2p)\%$ the confidence interval for the wavelet covariance can be constructed as follows:

$$\left[\sigma_{XY,j} - \Phi^{-1}(1-p)\sqrt{\mathrm{var}(\sigma_{XY,j})}, \sigma_{XY,j} + \Phi^{-1}(1-p)\sqrt{\mathrm{var}(\sigma_{XY,j})}\right] \tag{2.54}$$

where $\Phi^{-1}(1-p)$ is the $(1-p)\times 100\,\%$ point for the standard normal distribution. Because it is well known that the covariance does not take into account the variation of the univariate time series, it is natural to introduce the concept of the wavelet correlation. Although the wavelet covariance decomposes the covariance between two stochastic processes on a scale-by-scale basis and indicates a comovement between two series to some extent, in some situations it would be more informative to normalize the wavelet covariance by the variability calculated from the observed wavelet coefficients. Statistically, it is necessary to calculate the wavelet correlation. The wavelet correlation is simply made up of the wavelet covariance for $\{X_t, Y_t\}$, and wavelet variances for $\{X_t\}$ and $\{Y_t\}$. The wavelet correlation can be expressed as follows:

$$\tilde{\rho}_{XY}(\lambda_j) \equiv \frac{Cov_{XY}(\lambda_j)}{\tilde{v}_X(\lambda_j)\tilde{v}_Y(\lambda_j)} \tag{2.55}$$

As with the usual correlation coefficient between two random variables, $|\tilde{\rho}_{XY}(\lambda_j)| < 1$. The wavelet correlation is analogous to its Fourier equivalent, the complex coherency (Gençay et al., 2002a, p 258).

We now turn our attention to the confidence interval of the wavelet correlation. Given the inherent non-normality of the correlation coefficient for small sample sizes, a non-linear transformation is sometimes required in order to construct a confidence interval. Let $h(\rho) \equiv \tanh^{-1}(\rho)$ define Fisher's z-transformation. For the estimated correlation coefficient $\hat{\rho}$, based on N independent samples, $\sqrt{N-3}\left[h(\hat{\rho}) - h(\rho)\right]$ is approximately distributed as a Gaussian with mean zero and unit variance. Based on these findings, an approximate 100×(1 – 2p)% the confidence interval for the wavelet correlation can be constructed as follows:

$$\left[\tanh\left\{h[\hat{\rho}_{XY}(\lambda_j)] - \frac{\Phi^{-1}(1-p)}{\sqrt{\hat{N}_j - 3}}\right\}, \tanh\left\{h[\hat{\rho}_{XY}(\lambda_j)] + \frac{\Phi^{-1}(1-p)}{\sqrt{\hat{N}_j - 3}}\right\}\right] \qquad (2.56)$$

where $\hat{N}_j$ is the number of wavelet coefficients associated with scale λ_j computed via the DWT – not the MODWT. This assumption of uncorrelated observations in order to use Fisher's z-transformation is only valid if we believe no systematic trends or non-stationary features exist in the wavelet coefficients at each scale.

2.4.3 Cross Wavelet Covariance and Correlation

The cross correlation is a more powerful tool for examining the relationship between two time series. The cross correlation function considers the two series not only simultaneously (at lag 0), but also with a time shift. The cross correlation reveals causal relationships and information flow structures in the sense of Granger causality. If two time series were generated on the basis of a synchronous information flow, they would have a symmetric lagged correlation function, $\rho_\tau = \rho_{-\tau}$; the symmetry would be violated only by insignificantly small, purely stochastic deviations. As soon as the deviations between ρ_τ and $\rho_{-\tau}$ become significant, there is asymmetry in the information flow and a causal relationship that requires an explanation.

The cross correlation can be constructed utilizing the wavelet cross covariance. It is straightforward to derive the cross covariance, once the wavelet covariance is derived. For $N \geq L_j$, a biased estimator of the wavelet cross covariance based on the MODWT is given by:

$$R_{XY,\tau} = \begin{cases} \frac{1}{\tilde{N}} \sum_{t=L_j-1}^{N-\tau-1} \tilde{d}_{j,t}^{X} \tilde{d}_{j,t+\tau}^{Y} & \text{for } \tau = 0, \cdots, \tilde{N}_j - 1 \\ \frac{1}{\tilde{N}} \sum_{t=L_j-1}^{N-\tau-1} \tilde{d}_{j,t}^{X} \tilde{d}_{j,t+\tau}^{Y} & \text{for } \tau = -1, \cdots, -(\tilde{N}_j - 1) \\ 0 & \text{otherwise} \end{cases} \tag{2.57}$$

Allowing the two processes to differ by an integer lag τ, the wavelet cross correlation can be defined as:

$$\rho_{\tau,XY}(\lambda_j) \equiv \frac{R_{XY,\tau}(\lambda_j)}{\tilde{v}_X(\lambda_j)\tilde{v}_Y(\lambda_j)} \tag{2.58}$$

2.5 Long Memory Estimation Using Wavelet Analysis

Recently, several works have found evidence of stochastic long memory behavior in the financial time series. The presence of long memory dynamics, which is a special form of non-linear relationships, indicates non-linear dependence in the first moment of the distribution, and hence provides a potentially predictable component in the series dynamics. In this section, we describe and summarize the estimation procedure of the long memory parameter using wavelet analysis, based on the studies of Jensen (1999a, b and 2000). More specifically, in section 2.5.1, the definition of long memory and the meaning of long memory parameter are described. We present the Wavelet Ordinary Square (WOLS) in section 2.5.2, while in section 2.5.3, we explain how to derive the maximum-likelihood estimator for the long memory parameter.

2.5.1 Definitions of Long Memory

There are several possible definitions of the property of long memory. According to McLeod and Hipel (1978), a discrete time series x_t, with autocorrelation function, ρ_j at lag j, possesses long memory if the quantity

$$\lim_{n\to\infty}\sum_{j=-n}^{n}|\rho_j| \tag{2.59}$$

is non-finite. Equivalently, the spectral density $S(w)$ will be unbounded at low frequencies. A stationary and invertible ARMA process has autocorrelations, which are geometrically bounded, and hence is a short memory process (Baillie, 1996). Fractionally integrated processes are long memory processes given the definition in equation (2.59).

Let x_t denote a fractionally integrated process, $I(d)$, defined by:

$$(1-L)^d x_t = e_t$$
$$x_t = (1-L)^{-d} e_t \tag{2.60}$$

where L is the lag operator, $e_t \sim i.i.d.\, N(0,\sigma_e^2)$, and d is the fractional differencing parameter, which is allowed to assume any real value in (0, 1). The process x_t is covariance-stationary for $0<d<1/2$, but not otherwise[10]. While the process x_t is covariance-stationary when $0<d<1/2$, its autocovariance function declines hyperbolically to zero, making x_t a long-memory process. If $1/2<d<1$, x_t has an infinite variance, but still has a mean-reverting property in the very long run.

2.5.2 Wavelet Ordinary Least Square

As shown in Jensen (1999b), the wavelet coefficient from an $I(d)$ process has a variance that is a function of the scaling parameter, j, but is independent of the translation parameter, k. McCoy and Walden (1996) and Jensen (1999b) demonstrate that wavelet coefficients have a normal distribution with zero mean and variance, $\sigma^2 2^{-2jd}$. Taking logarithms on the wavelet coefficient's variance yields the following relationship.

$$\ln R(j) = \ln \sigma^2 - d \ln 2^{2j} \tag{2.61}$$

where $R(j)$ denotes the wavelet coefficient's variance and is linearly related to $\ln 2^{-2j}$ by the fractional differencing parameter, d. However, note that due to the restriction of the DWT (Discrete Wavelet Transform), the number of observations for the underlying process, x_t, must be a power of 2. Using Monte Carlo experiments, Jensen (1999b) demonstrates that the small and large properties of

[10] When $d<1$, the process is called 'mean reverting', although this terminology needs to be used with care, since the existence of the mean is not easily shown when the variance is undefined.

the wavelet OLS estimator are superior to the GPH estimator. More specifically, the WOLS estimator has a lower MSE (Mean Squared Errors) than the GPH estimator.

2.5.3 Approximate Maximum-likelihood Estimation of the Long Memory Parameter

Wavelet-based maximum likelihood estimation procedures, related to economic and finance research, have been studied by Jensen (1999a and 2000). Although least squares estimation is popular because of its simplicity to program and compute, it produces much larger mean square errors when compared to maximum likelihood methods. Another advantage of the wavelet-based MLE is that the long memory estimator, d, is unaffected by the unknown μ, since the wavelet coefficients autocovariance function is invariant to μ (Jensen, 2000). The approximate maximum likelihood methodology[11], proposed in Jensen (1999a and 2000), overcomes the difficulty of computing the exact likelihood by replacing the covariance matrix of the process with an approximation using the DWT.[12] This is possible through the ability of the DWT to decorrelate the long memory process.

[11] Cheung and Diebold (1994) find that the approximate MLE can be an efficient and attractive alternative to the exact MLE when μ is unknown.

[12] The wavelet MLE enjoys the advantage of having both the strengths of an MLE and a semiparametric estimator, but does not suffer their known drawbacks (Jensen, 2000).

If x_t is a length $N = 2^J$ FDP with mean zero and covariance matrix given by Ω_x, then the likelihood can be expressed as follow (see Gençay et al., 2002a, p172):

$$L(d, \sigma_\varepsilon^2 \mid x) = (2\pi)^{-N/2} \mid \Omega_x \mid^{-1/2} \exp\left(-\frac{1}{2} x^T \Omega_x^{-1} x\right) \tag{2.62}$$

The quantity $\mid \Omega_x \mid$ is the determinant of Ω_x. The maximum-likelihood estimators (MLEs) of the parameters (d and σ_ε^2) are those quantities that maximize equation (2.62). As in Gençay et al. (2002a), to avoid the difficulties in computing the exact MLEs, we use the approximation of the DWT as applied to FDPs. In other words, the covariance matrix Ω_x is expressed by:

$$\Omega_x \approx \tilde{\Omega}_x = \mathrm{W}^T \Sigma_x \mathrm{W} \tag{2.63}$$

where W is the orthogonal matrix defining the DWT and Σ_x is a diagonal matrix containing the variances of DWT coefficients. Using equation (2.62), we try to find the values of d and σ_ε^2 that minimize the following log-likelihood function.

$$\tilde{L}(d, \sigma_\varepsilon^2 \mid x) = -2\log\left(\tilde{L}(d, \sigma_\varepsilon^2 \mid x)\right) - N\log(2\pi) = \log\left(\mid \tilde{\Omega}_x \mid\right) + x^T \Omega_x^{-1} x \tag{2.64}$$

In section 5.4, we apply the approximate maximum likelihood estimator for long memory parameter to examine the mean reverting property of stock and bond returns.

Chapter 3 The Relationship between Financial Variables and Real Economic Activity: Evidence from Spectral and Wavelet analyses[1]

3.1 Introduction

In the past literature, many empirical studies find that financial variables possess a predictive power over real economic activity. It is generally accepted that one of the financial variables that predict real activity is stock prices, For example, typically, the discounted-cash-flow valuation model states that stock prices reflect investors' expectations about future real economic variables such as corporate earnings, or its aggregate proxy, industrial production. If expectations are correct on average, lagged stock returns should be correlated with the contemporaneous growth rate of the future evolution of industrial production (Choi et al., 1999). Fama (1990) and Schwert (1990) examine the relationship between stock returns and industrial production of the US and find that stock returns have a predictive power for the US industrial production using R^2 values of simple OLS (Ordinary Least Square). However, the limitation of their papers is in using an in-sample procedure such as OLS. To overcome this limitation, Choi et al. (1999) adopt several time-series methodologies such as ECM (Error Correction Model) and a cointegration test for in-sample procedures using G-7 data. They find that most countries enhance predictions of future industrial production.

[1] This chapter has been accepted in Studies in Nonlinear Dynamics and Econometrics.

Another example can be found in interest rate spread, such as the difference between commercial paper and Treasury bill rates or the difference between risky bond yields and risk-free Treasury bill rates. Previous literature (Stock and Watson, 1989; Friedman and Kuttner, 1992; Kwark, 2002) finds that the interest spread has a highly predictive power for future business conditions. As for stock prices, the explanation for the interest rate spread being a leading indicator over the business cycle is based on investors' perceptions of the future economy. If investors expect that the future economic growth is favorable and bankruptcy risk will be reduced for economic growth, they might want relatively small compensation, i.e., risk premium, compared to economic recession. Accordingly, the interest rate spread would be decreased. That is, a decrease of interest rate spread is associated with an increase of output. Kwark (2002) shows that the interest rate spread between risky bond loan rates and risk-free rates has a predictive power for subsequent fluctuations in real output using general equilibrium model, which incorporates heterogeneity among firms by introducing idiosyncratic shocks as well as aggregate shock.

The reason being why the financial variables have a predictive power, or are a leading indicator, of real activities is that if investors expect economic recession and an increased risk in their investment, they want more risk premium for their investment as compensation.

The chapter contributes the existing literature by examining the predictive power of the financial variables over real activity, not only over various frequencies but also over various time scales. To do so, first, we examine interest rate spread and stock prices to provide more profound understanding of the relationship between the financial variables and real activities. Second, we adopt the frequency domain analysis to analyze the relationship between real activity and financial variables. Finally, we introduce a new approach to the investigation of the relationship between financial variables and real economic activity over various time scales.

The purpose of examining the financial variables is not to determine which variable is most effective and a leading indicator, but to examine how the relationship differs from variable to variable. To examine this relationship more concretely, two established methods are used. First, spectral analysis is adopted to examine their comovements, and the lead-lag relationship. Frequency domain analysis has been used in macroeconomics, especially in the business cycle and finance literature since Granger (1966). In relation to the business cycle, the literature includes Howrey (1968), Sargent and Sims (1977), Baxter and King (1999), and recently Sarlan (2001), while in finance, the literature includes examination of the stock market (Fischer and Palasvirta, 1990; Knif et al., 1995; Lin et al., 1996; Asimakopoulos et al., 2000; Smith, 1999 and 2001), and investigation of the interest rates (Kirchgassner and Wolters, 1987; Hallett and Richter, 2001 and 2002)[2].

In wavelet analysis, the result of frequency domain analysis shows that US industrial production and financial variables share long-term features. However, in terms of the lead-lag relationship between US industrial production and financial

[2] For more extensive survey of the application of spectral analysis to economic and financial data, see Ramsey and Thomson (1999, p. 57).

variables, the results show that financial variables lead US industrial production in longer cycles, while US industrial production leads financial variables in shorter cycles. It implies that the relationship between US industrial production and financial variables is not fixed over time. This result is confirmed by wavelet analysis. The lead-lag relationship, in the sense of Granger causality, varies depending on the time scale. More specifically, we find that at the finest time scale (scale 1), most financial variables show a feedback relationship with US industrial production. Second, at the intermediate time scale, the results depend on the variables. Finally, at the long-term trend, feedback relationships are observed in most variables.

The remainder of the chapter is organized as follows. Section 2 describes the data used in the study and the basic statistics. In section 3, we describe spectral analysis and discuss the empirical findings. Section 4 presents the wavelet analysis and discusses the associated results. A summary and concluding remarks are presented in Section 5.

3.2 Data and Basic Statistics

The main purpose of this study is to investigate whether financial variables have a predictive power over industrial production over various frequency domains and time scales. Previous studies use various financial variables to examine whether they contain a great deal of information about future economic conditions. Bernake (1983) has used the difference between Baa corporate bond yields and long-term US government bonds to examine whether this variable has a predictive power. Friedman and Kuttner (1992) adopt the Granger causality test to investigate whether the difference between the commercial paper rate and the Treasury bill rate contains significant information about future output. They find that the spread is a better predictor of economic activities than money, interest rates, or any other financial variables. Fama and French (1989) and Choi et al. (1999) use stock prices to check whether the stock return can predict future economic activity. To enable comparison with these previous studies, we also use these variables to examine whether financial variables have a predictive power.

Our data set is composed of monthly observations of the aggregate stock price index and the industrial production index taken from the International Financial Statistics of the IMF (International Monetary Fund). For other financial variables, we use the 3-month commercial paper rate, the 3-month Treasury bill rate of the secondary market, the prime rate for short-term business loans, Moody's Aaa and Baa rates, and the CPI index, which are taken from the US Federal Reserve. To calculate the real value of each variable, the CPI index has been used. The data ranges from 1959:1 to 2001:5, except for the commercial paper rate, which ranges from 1971:4 to 1997:8.[3] We construct the four financial variables using the above data series. We define CORSP as the difference between the Moody's Aaa-rated and Baa-rated corporate bond interest rates, CPTB3 as the difference between the difference between the 3-month commercial paper rate and the 3-month Treasury bill rate, PTB3 as the difference between the Prime rate and the 3-month Treasury

[3] Since August 1997, the commercial paper rate has been divided into two categories: financial and non-financial. To avoid inconsistency of data, we use the period indicated.

bill rate, respectively. The final variable, Shares, is defined as the first backward difference of the stock prices.

Table 3.1 KPSS Unit Root Test

		L=0	L=10	L=20
IP	level	5.586	0.539	0.305
	difference	0.164	0.071	0.066
CORSP	level	7.421	0.739	0.421
	difference	0.040	0.036	0.039
CPTB3	level	0.612	0.085	0.059
	difference	0.013	0.023	0.032
PTB3	Level	1.324	0.176	0.117
	Difference	0.010	0.017	0.024
Shares	Level	10.792	1.023	0.561
	difference	0.061	0.044	0.051

Note: Test statistic is $T^{-2}\Sigma S_t^2 / S^2(L)$ where $S_t = \Sigma_t$, $t = 1, 2, \ldots, T$. $S^2(L) = T^{-1}\Sigma_t^2 + 2T^{-1}\Sigma(1 - S/L+1)\Sigma_{t-s}$. The null hypothesis of stationarity is rejected if the test statistics are greater than the critical values. The critical values are 0.176 at the 5% significance level, 0.216 at the 1% significance level. This test includes a constant and trend. IP is industrial production, CORSP is the difference between the Moody's Aaa-rated and Baa-rated bond interest rates. CPTB3 is the difference between the 3-month commercial paper rate and the 3-month Treasury bill rate. PTB3 indicates the difference between Prime rate and the 3-month Treasury bill rate. Finally, Shares are defined as the first backward difference of the stock prices.

The data must be stationary to perform spectral analysis. To test unit root, we adopt the method (hereafter KPSS) proposed by Kwiatkowski et al. (1992). Using the KPSS test, stationarity is rejected for the levels, while stationarity cannot be rejected for the difference as presented in Table 3.1.

Table 3.2 presents the sample moments and the cyclical behavior of various financial variables and industrial production, focusing on the lead-lag relationship and cross correlation of variables with real industrial production. Three kinds of interest rate spreads are provided as measures of default risk. First, we use

CORSP. This is appropriate because this interest differential reflects a risk premium due to a difference in default risk. Second, we consider CPTB3. This interest rate difference has been adopted in our studies because Stock and Watson (1989) suggest this interest rate difference as a leading indicator. Finally, PTB3 is considered. As discussed in Kwark (2002), the bank loan rate would be a correct risky loan rate. However, due to the available measure of bank loan rates, he used the prime loan rate as a risky loan rate. We follow his view and adopt the prime rate for our analysis. One problem of using the prime rate is that this interest rate does not change frequently.

The sample moments of each variable are presented in Panel A of Table 3.2. Means of all variables are close to zero. The standard deviation ranges from 0.009 (IP) to 0.323 (CPTB3), indicating that the interest rate difference between commercial paper and Treasury bills is most volatile during the sample period. The measures for skewness and kurtosis statistics are also reported to check whether monthly data are normally distributed. These statistics indicates that all data are not normally distributed.

Table 3.2 Sample Moments and Cyclical Behavior of Financial Variables
Panel A Basic Statistics

	Mean	Std.Dev.	Rel. Std.	Skewness	Kurtosis
IP	0.003	0.009	1.000	-0.089	6.101
CORSP	0.000	0.082	9.400	0.442	2.339
CPTB3	-0.002	0.323	39.579	-0.371	0.969
PTB3	0.002	0.188	21.565	0.560	6.360
Shares	0.006	0.035	4.025	-0.647	2.183

Note: Skewness and kurtosis are defined as $E[(R_t - \mu)]^3$ and $E[(R_t - \mu)]^4$, where μ is the sample mean. Std. Dev. and Rel. Std. indicate the standard deviation and relative standard deviation, respectively, calculated using the standard deviation of IP.

Panel B Cross Correlation with Industrial Production

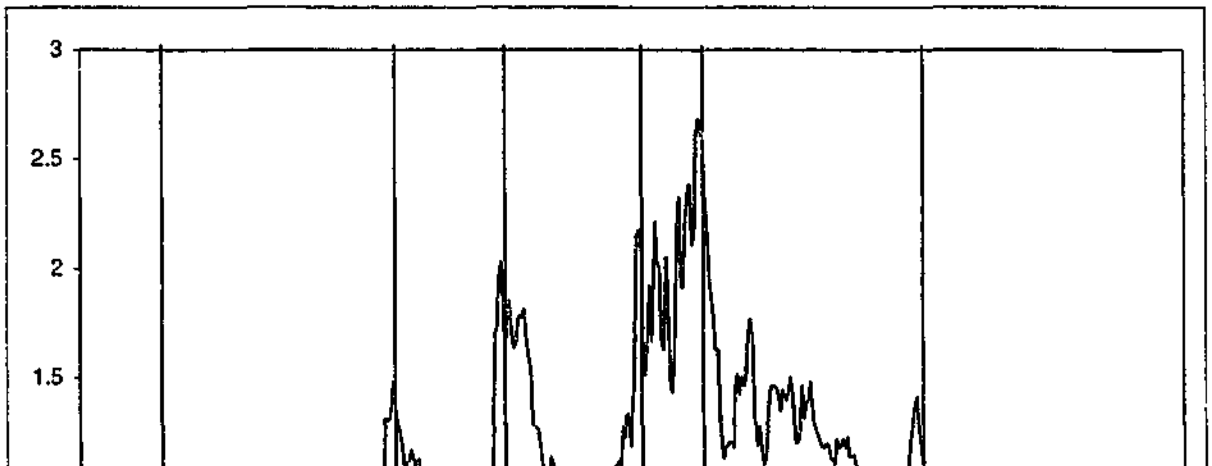
	Corr(ΔIP_t, Δx_t)						
x=	-3	-2	-1	0	1	2	3
IP	0.138	0.204	0.380	1.000	0.380	0.204	0.138
CORSP	-0.147	-0.207	-0.253	-0.119	-0.014	-0.009	-0.029
CPTB3	-0.119	-0.045	0.056	0.178	0.190	0.096	0.129
PTB3	-0.139	-0.151	-0.089	-0.039	0.060	0.126	0.144
Shares	0.138	0.148	0.056	0.010	-0.039	-0.086	-0.084

Sources: International Financial Statistics (IFS) CD-ROM (June 2002) from IMF, and Federal Reserve System statistical release (H.15). All variables are calculated as a real term using CPI index.

Note: The data period of CPTB3 is from 1971:4 to 1997:8 and other variables have a period from 1959:1 to 2001:5. Corr(ΔIP_t, Δx_t) is the cross correlation coefficients between IP and financial variables, where x_t = IP, CORSP, CPTB3, PTB3, and Shares.

The cross correlation of variables with industrial production is reported in Panel B of Table 3.2. The lagged cross correlations between industrial production and all previous financial variables are negative except for shares and CPTB3 with first lagged IP, implying the leading behavior of the interest rate spread is common for all three measures of interest rate spread. The cross correlations between industrial production and shares show different pattern from the other correlations. This is because the share index has a different movement from the interest rate spreads. When the economy is expected to grow, the interest rate spread is expected to decrease, because the interest rate for risky loans decreases. However, the share index increases in an economic boom. This cross correlation structure simply indicates that the financial variables play a role as a leading indicator of real activity, implying that if the interest rate spread (share index) decreases (increases) today, then output is expected to increase in future. In other words, the financial variables lead the real activity. This tendency is by Figure 3.1.

Figure 3.1 Business Cycle and Interest Rate Spread



Note: The x-axis indicates the interest rate spread between the Moody's Aaa-rated and Baa-rated bond yields, which are obtained from Federal Reserve System statistical release (H.15). The vertical lines indicate the troughs of the business cycle obtained from NBER (http://www.nber.org/cycles.html/). The corresponding months are February 1961, November 1970, March 1975, July 1980,November 1982, and March 1991, respectively.

Figure 3.1 shows the movements of interest rate spread between Baa and Aaa over the period 1959:1 to 2001:5. The vertical lines indicate the troughs of business cycles, obtained from NBER[4]. As found in the cross correlation structures, this figure gives further evidence that financial variables are a leading indicator of real activity, showing that the peak of interest rate spread widens before output decreases. From this simple analysis, we conclude that financial variables can

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[4] The date of business cycle expansions and contractions can be found on http://www.nber.org/cycles.html/.

predict real activity. In the next sections this evidence will be examined more thoroughly using various time series techniques.

3.3 Empirical Results of Spectral Analysis

Utilizing the cross spectral analysis, presented in section 2.2.2, Table 3.3 presents the summary statistics for the gain, the coherence and the phase. As discussed in the previous sub-section, the gain measures the degree by which a change of the regression coefficients in the time domain can affect the cross spectrum. During the sample period, the gains are varying depending on the frequency and variables. For example, the simple mean of the gain between industrial production and CORSP has higher values (0.036) than those of the others, showing that the change of CORSP has more amplitude than the other variables on industrial production. The construction of the coherence as a measure of association for different frequencies is evident from equation (3.1). In fact, the coherence can be considered as a square of a correlation coefficient. During the sample period, the coherence varies at different frequencies. For example, the mean value of coherence between industrial production and CORSP is 0.349, while the maximum is 0.783 and the minimum is 0.023. The results of the phase also show the varying property at different frequencies.

Variance, skewness, and kurtosis are also reported in Table 3.3. These statistics indicate that the coherences and the phases are characterized by varying degrees of skewness and excess kurtosis.

Table 3.3 Descriptive Statistics for Gain, Coherence, and Phase

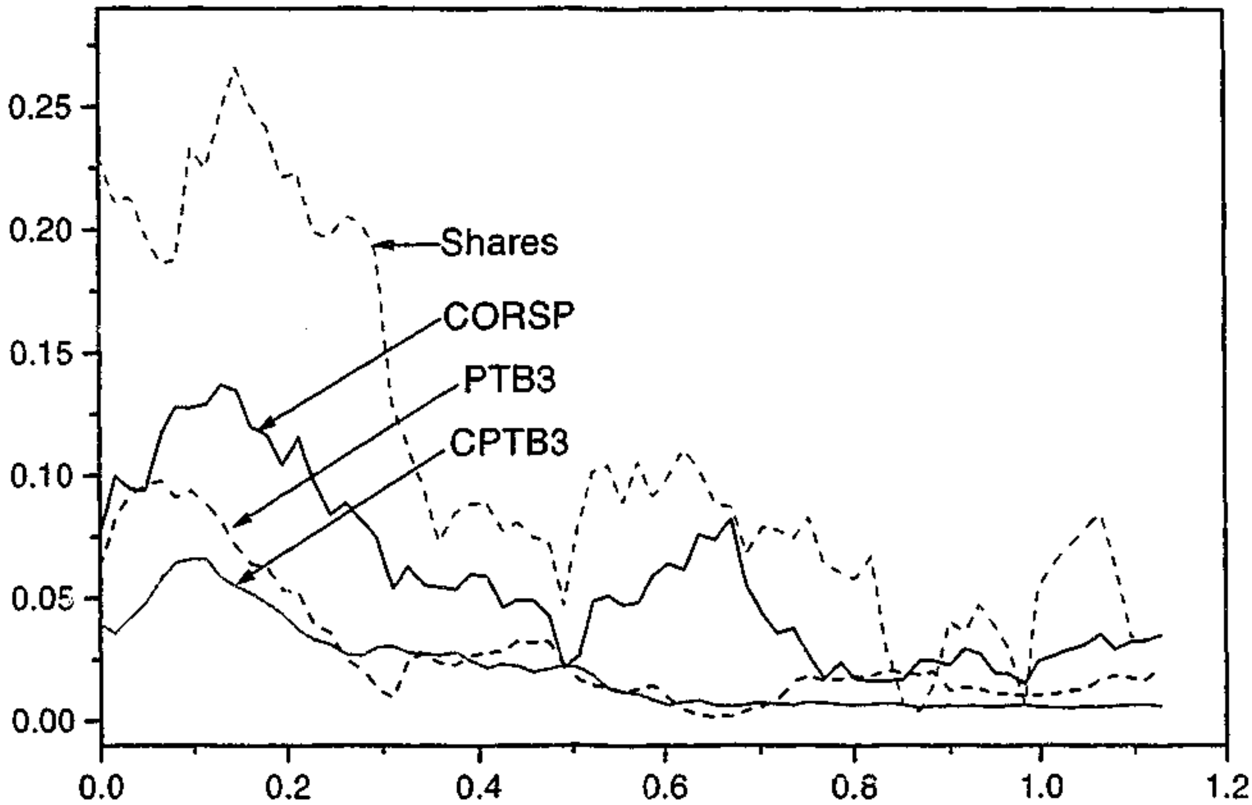
	Mean	Variance	Skewness	Kurtosis	Maximum	Minimum
			gain			
CORSP	0.036	0.001	1.875	3.260	0.137	0.002
CPTB3[a]	0.012	0.000	2.501	6.111	0.066	0.000
PTB3	0.017	0.000	2.684	7.403	0.098	0.001
Shares	0.079	0.003	1.602	2.577	0.266	0.004
			coherence			
CORSP	0.349	0.026	0.296	-0.214	0.783	0.023
CPTB3[a]	0.377	0.028	0.488	-0.277	0.814	0.024
PTB3	0.326	0.034	0.230	-1.063	0.739	0.036
Shares	0.339	0.026	0.693	1.004	0.833	0.015
			phase			
CORSP	-0.954	2.204	0.953	0.250	3.069	-3.142
CPTB3[a]	-0.356	1.642	0.810	0.930	3.100	-3.141
PTB3	-0.458	3.067	0.513	-0.908	3.127	-3.142
Shares	-0.016	3.602	-0.229	-1.280	3.072	-3.134

Note: The gain is calculated as $|G(w)|=\sqrt{(co_{ij}(w))^2+(qu_{ij}(w))^2}$, the coherence is calculated as $Coh(w)=|S_{ij}(w)|^2/S_{ii}(w)S_{jj}(w)$, and the phase is calculated as $\varphi(w)=\tan^{-1}\left(co_{ij}(w)/qu_{ij}(w)\right)$. Skewness and kurtosis are defined as $E[(R_t-\mu)]^3$ and $E[(R_t-\mu)]^4$, where μ is the sample mean.

[a] The data period of CPTB3 is from 1971:4 to 1997:8 and other variables have a period from 1959:1 to 2001:5.

Figure 3.2 shows the calculated gains of CORSP, CPTB3, PTB3, and shares and the impact of the US industrial production. In fact, the changes in financial variables set up the long and short cycles in industrial production, with industrial production responding to the movements of financial variables with movements less than one-third of their original sizes. As can be observed in Table 3.3, the highest value of the gain is 0.137 between IP and CORSP. However, the short cycles (less than 6 months in length) are clearly weaker than long cycles (more than 1 year). Comparing responses to various financial variables is of interest. First, the most striking feature of this graph is that the effect of shares is stronger than the other financial variables. In this figure, the movements of gain between US industrial production and shares are placed at the top of the four lines, and can be seen to be very unstable.

Figure 3.2 Estimated Gains of CORSP, PTB3, CPTB3, and Shares



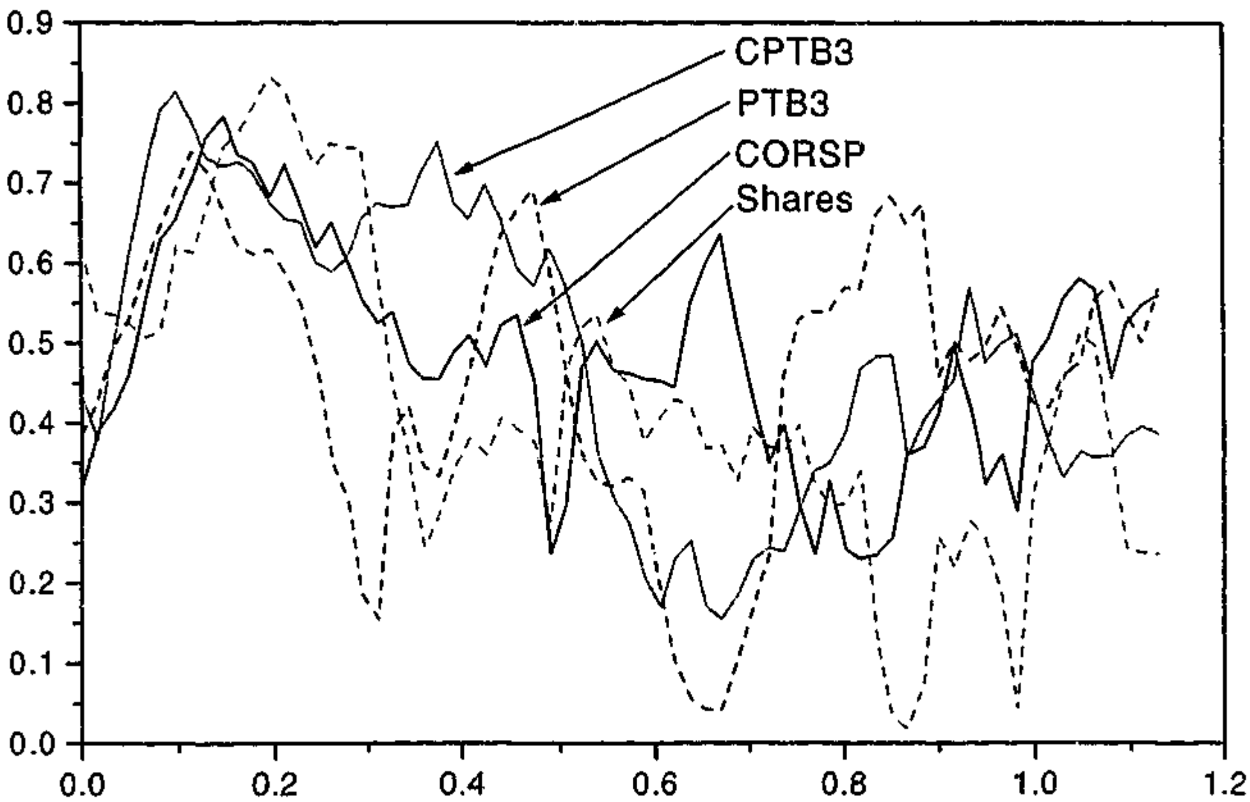
Note: The x-axis indicates the frequencies, while the y-axis indicates the gain, which is calculated as $|G(w)| = \sqrt{(co_{ij}(w))^2 + (qu_{ij}(w))^2}$. CORSP is defined as the difference between the Moody's Aaa-rated and Baa-rated corporate bond interest rates. CPTB3 indicates the difference between the 3-month commercial paper rate and the 3-month Treasury bill rate. PTB3 is defined as the difference between the Prime rate and the 3-month Treasury bill rate. Finally, Shares are defined as the first backward difference of the stock prices.

Second, it is of interest to compare the movements of the gains between longer cycles (between frequencies 0 and 0.5, which is equivalent to greater than 1-year cycle) and shorter cycles (between frequencies 0.5 and 1.0, which is equivalent to the cycles between 6 months and 1 year). Generally, the change of the financial variables has greater effects on US industrial production in longer cycles than in shorter cycles. The gains of CORSP are greater than the other interest rate spreads during shorter and longer cycles. This is because the default rate of lower credit-graded companies is increasing when the economy is in recession.

Focusing on the CPTB3, which has been used in the study of Friedman and Kuttner (1992) as a better predictor of real activities, the movements of the gain of CPTB3 are quite stable, compared to the interest rate spreads and shares. The change in the CPTB3 influences US industrial production, and its amplitude is less than one tenth of the size of the original movements in CPTB3. Compared to the movements of CORSP and PTB3, its amplitude is less in both longer and shorter cycles, indicating that CPTB3 has less impact on US industrial production than the other interest rate spreads. This result would imply that US industrial production influences CPTB3 less than the other financial variables in terms of its amplitude.

The coherence plays a role as a correlation coefficient defined at frequency w. Note that the coherence can be regarded as a square of a correlation coefficient. Figure 3.3 presents the coherence of four financial variables with US industrial production. In particular, the coherence for CORSP shows that CORSP explains 80% of US industrial production at very low frequency (frequency around 0.14), which is about 0.89 measured as a correlation coefficient. It can be interpreted that both variables (US industrial production and CORSP) have similar long-term features. However, the coherence of CORSP starts to decrease to frequency 0.28. After this, it shows similar cycle as in low frequencies. The other interest rate spreads show a similar pattern with CORSP, while the coherence for shares reaches its peak at shorter cycles than those of the other variables. At frequency 0.20, the coherence is 0.83, which is equivalent to 0.91 of correlation coefficient. Overall, the coherences for financial variables have a highest value in low frequencies, indicating that the financial variables and US industrial production have a similar long-term movement.

Figure 3.3 Estimated Coherences of CORSP, PTB3, CPTB3, and Shares



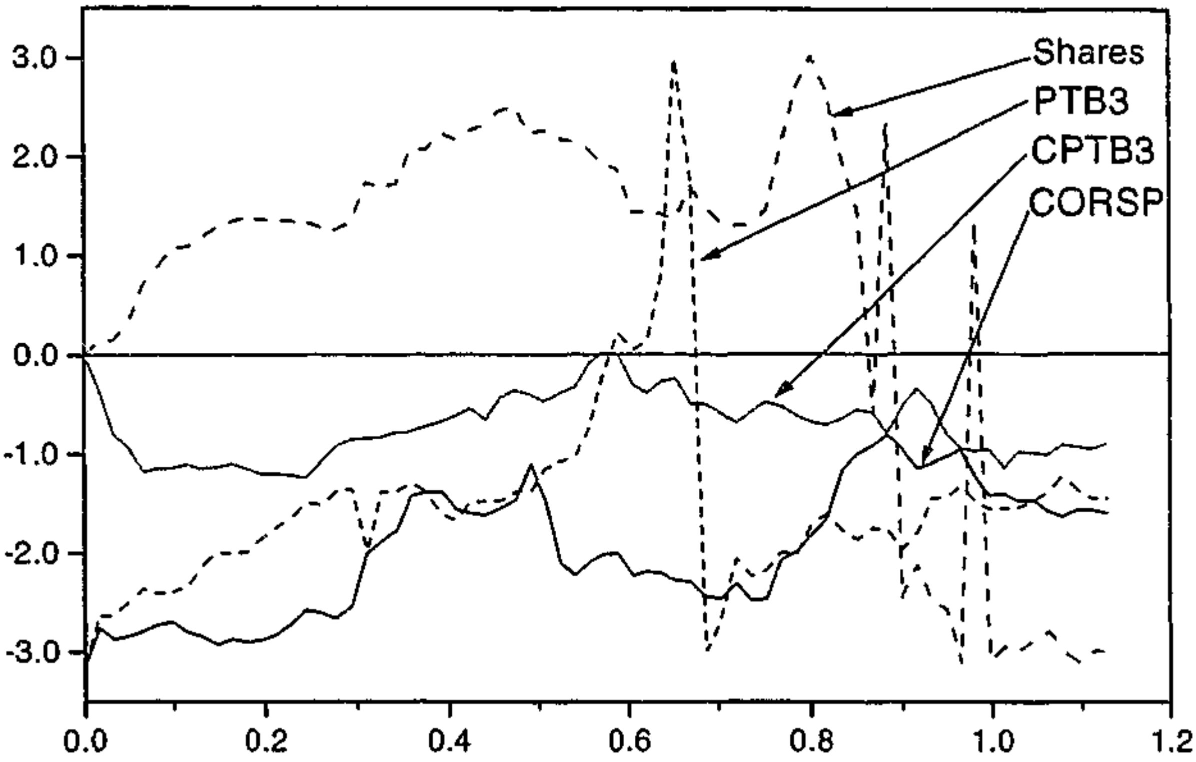
Note: The x-axis indicates the frequencies, while the y-axis indicates the coherence, which is calculated as $Coh(w) = |S_{ij}(w)|^2 / S_{ii}(w)S_{jj}(w)$. CORSP is defined as the difference between the Moody's Aaa-rated and Baa-rated corporate bond interest rates. CPTB3 indicates the difference between the 3-month commercial paper rate and the 3-month Treasury bill rate. PTB3 is defined as the difference between the Prime rate and the 3-month Treasury bill rate. Finally, Shares are defined as the first backward difference of the stock prices.

The phase of the cross-spectrum is graphically presented in Figure 3.4. Note that the phase of shares has positive values between frequencies 0 to 0.59, while those of the interest spreads are negative. This is because of the different movement of shares when economic recession is expected. When economic recession is expected, the interest rate spread between risky bonds and risk-free bonds increases, while the shares index decreases. We note this opposite response of shares in interpreting our results. All phases show the different cycles. However, overall, all variables have a positive slope in longer cycles, but a negative slope in shorter cycles. For instance, the phase of CORSP increases from frequency 0 to frequency 0.376 (greater than a 16-month cycles), showing a positive slope. After this point, it starts to decrease and has a negative slope until frequency 0.75 (around an 8-month cycle), implying that in longer cycles (greater than a 16-months cycles) US industrial production leads CORSP, while in shorter cycles (between 8 months and 16 months), US industrial production lags CORSP. In the less than 8-month cycle (from frequency 0.75 to 0.1), it shows a positive slope again.

PTB3 shows a longer cycle compared to CORSP. PTB3 shows a positive slope between frequency 0 and frequency 0.6 (greater than 10-month cycle), while it has a negative slope for a short period. After frequency 0.70, the slope is positive, indicating that US industrial production leads PTB3. From this, it can be observed that the phase cycle is around 6 months.

Turning to CPTB3, we see a negative slope in longer cycles and a positive slope in shorter cycles. The phase cycle is much longer than those of the other interest rate spreads. For instance, the length of negative slope is between frequency 0.59 and 1.18 (between a 5-month cycle and a 10-month cycle), while CORSP is between frequency 0.40 and 0.75.

Figure 3.4 Estimated Phases of CORSP, PTB3, CPTB3, and Shares



Note: The x-axis indicates the frequencies, while the y-axis indicates the phase, which is calculated as $\varphi(w) = \tan^{-1}\left(co_{ij}(w)/qu_{ij}(w)\right)$. CORSP is defined as the difference between the Moody's Aaa-rated and Baa-rated bond corporate interest rates. CPTB3 indicates the difference between the 3-month commercial paper rate and the 3-month Treasury bill rate. PTB3 is defined as the difference between the Prime rate and the 3-month Treasury bill rate. Finally, Shares are defined as the first backward difference of the stock prices.

As indicated before, the phase of shares has a positive value from the starting point. The slope of shares also shows a positive value until frequency 0.47 (around a 1-year cycle). After this frequency, the slope is negative, implying that the US industrial production lags the shares.

The results from the analysis of phase indicate that all variables do not lead US industrial production at all frequencies. At some frequencies, US industrial production leads the financial variables, depending on the variables. Another finding is that the slope of phase is varying, also depending on which financial variables have been used. This result implies that the leading or lagging period of financial variables is also varying.

Overall, the spectral analysis shows that the relationship is not constant over frequencies, but varying. More specifically, the results from the gain show that the effects of financial variables on US industrial production are stronger in longer cycles (at lower frequencies) than in shorter cycles (in higher frequencies). From the phase analysis, it is found that in longer cycles US industrial production leads the financial variables, while in shorter cycles, US industrial production tends to lag. However, the time-length of leads and lags depends on the financial variables. The coherence indicates that US industrial production shares long term features with the financial variables.

3.4 Empirical Results Using Wavelet Analysis

There is a large volume of literature on the existence of the lead-lag relationship between financial variables and real variables, such as output or industrial production. This relationship can often be captured by the causality test – Granger or Sim's causality. In the study of Ramsey and Lampart (1998a), they decompose money and income into various time scales using wavelet analysis and find that at the finest scale (D_1) income, Granger causes money but at the intermediate scales (D_2 to D_4) money Granger causes income. Feedback relationships exist at the higher time scales. Gençay et al. (2002a) test this relationship in the UK and find similar results. As in the previous studies, we also adopt the Granger causality test to examine the relationship between financial variables and US industrial production.

Since our focus is on the lead-lag relationship between financial variables and US industrial production using the wavelet analysis, the relationship is examined when the variation in each variable has been restricted to a specific scale.

Table 3.4 reports the results of the Granger causality test for the raw data. As expected, all financial variables lead industrial production, while only PTB3 has a feedback relationship with industrial production.

Table 3.4 Granger Causality Test for Raw Data

$x =$	CORSP	CPTB3	PTB3	Shares
$x \rightarrow$ IP	12.795*	9.248*	5.461*	4.401*
	(0.000)	(0.000)	(0.000)	(0.002)
IP $\rightarrow x$	0.696	1.779	3.625*	1.046
	(0.595)	(0.133)	(0.006)	(0.383)

Note: significance levels are in parentheses. * indicates significance at 5% level.

Utilizing the MODWT wavelet coefficients in section 2.3.3, the results from wavelet analysis are presented in Table 3.5. Overall, there are two findings worth noting. First, there is no consistent direction of causality. The causal relationship varies depending on the time scale. Second, in the long-term trend (indicated as S_6), all financial variables show the feedback relationship, except for CPTB3, which is consistent with the results of spectral analysis. Comparing the results with the coherence, all financial variables in the low frequencies show higher values than in the high frequencies. This implies that the movements between US industrial production and financial variables are getting closer.

Table 3.5 Granger Causality Test for Wavelet Analysis

$x=$		D_1	D_2	D_3	D_4	D_5	D_6	S_6
CORSP	$x \to$ IP	7.161*	0.798	1.029	1.493	2.869*	1.246	4.208*
		(0.000)	(0.527)	(0.392)	(0.203)	(0.023)	(0.291)	(0.002)
	IP $\to x$	0.558	0.804	0.544	0.394	0.362	3.442*	4.655*
		(0.693)	(0.523)	(0.704)	(0.813)	(0.835)	(0.009)	(0.001)
CPTB3	$x \to$ IP	3.133*	0.424	0.244	0.401	1.661	1.306	5.388*
		(0.015)	(0.792)	(0.913)	(0.808)	(0.159)	(0.268)	(0.000)
	IP $\to x$	4.296*	4.000*	0.568	1.540	1.062	3.630*	1.587
		(0.002)	(0.004)	(0.686)	(0.190)	(0.375)	(0.007)	(0.178)
PTB3	$x \to$ IP	2.582*	0.088	0.511	0.825	0.619	0.282	3.009*
		(0.037)	(0.986)	(0.728)	(0.509)	(0.649)	(0.889)	(0.018)
	IP $\to x$	3.754*	7.184*	0.841	1.942	0.628	4.371*	5.473*
		(0.005)	(0.000)	(0.499)	(0.102)	(0.643)	(0.002)	(0.000)
Shares	$x \to$ IP	6.807*	2.616*	0.766	2.572*	7.497*	3.719*	4.274*
		(0.000)	(0.035)	(0.548)	(0.037)	(0.000)	(0.005)	(0.002)
	IP $\to x$	4.221*	4.497*	0.451	0.642	4.667*	1.933	2.334*
		(0.000)	(0.001)	(0.771)	(0.633)	(0.001)	(0.104)	(0.055)

Note: The original data has been transformed by the wavelet filter (LA(8)) up to time scale 6. The significance levels are in parentheses. * indicates significance at 5% level. The first detail (wavelet coefficient) D_1 captures oscillations with a period length 2 to 4 months. Equivalently, D_2, D_3, D_4, and D_5 capture oscillations with a period of 4-8, 8-16, 16-32 and 32-64 months, respectively. The last detail D_6 captures oscillations with a period of 64 to 128 days. The wavelet smooth S_6 captures the oscillations with a period of longer than 218 months.

More specifically, at the finest time scale, CORSP Granger causes US industrial production, while at the longer time scale, scale 6 (equivalent to 16 – 32 months), CORSP Granger causes US industrial production. Finally, at the long term trend, two variables (CORSP and the US industrial production) show a feedback relationship. In the case of PTB3, at the finest time scale, they show a feedback relationship, while at scales 2 and 4, US industrial production Granger causes PTB3. Again, at the long term trend, they show a feedback relationship.

For CPTB3, they show a similar pattern to PTB3. At time scales 2 and 6, US industrial production Granger causes CPTB3, while at the long term trend, a feedback relationship is observed. The relationship between share prices and US industrial production shows a significantly varying relationship among the financial variables chosen in our study. As in the other variables, at the finest time scale, both variables show a feedback relationship. At time scale 2 (equivalent to 2 – 4 months), US industrial production Granger causes share prices, while at longer time scales (scales 4 and 5) share prices Granger cause US industrial production. Finally, at time scale 6 (equivalent to 32 – 64 months) and the long term trend, both variables again show a feedback relationship.

In sum, our results indicate that financial variables Grangers cause US industrial production in the raw data. In contrast, the financial variables and US industrial production show a feedback relationship at the lower scales and at the higher scales. This may result from the capacity utilization of the firms and the investment lag[5] and the efficient market hypothesis of the financial variables. In other words, in the short run, the financial variables absorb all related information to determine the equilibrium prices, while industrial production may need a sufficient time to adjust its capacity to absorb the relevant information. Therefore,

[5] The evidence of the existence of investment lags has been reported in much research on the relationship between investment and stock return. For example see McConnell and Muscarella (1985), Chan et al. (1990), and Lamont (2000). For a more detailed explanation of capacity utilization, see Burnside and Eichenbaum (1996)

it is natural for financial variables to lead US industrial production at time t. However, at the lower scales, the firms adjust their capacity for responding to new information. In other words, the firms increase (decrease) their capacity following the good (bad) news. Therefore, at the lower scales, financial variables and US industrial production show a feedback relationship. As the time scale increases, the firms increase their investment to keep their optimum capacity, and need time for the effect of investment to be realized in the production. During these time scales, this procedure may cause there to be no relationship between financial variables and US industrial production. However, in the long-run, firms can adjust their production and capacity utilization without any time delays. Therefore, at the higher time scales, financial variables and US industrial production show a feedback relationship.

Our wavelet analysis shows that no financial variable has a constant relationship with US industrial production, which is a similar result to that of Ramsey and Lampart (1998a) and Gençay et al. (2002a). The relationship is varying depending on the time scale.

3.5 Summary and Concluding Remarks

In this chapter, we investigate the relationship between real activities and financial variables using two time series analyses: frequency domain analysis and wavelet analysis. In these analyses, it is found that financial variables lead US industrial production at the raw data, and overall that the relationship between financial variables and US industrial production is not constant depending on the time scale and frequency.

The results of frequency domain analysis are more complicated than the previous time series analysis. However, three main findings are worth noting. First, from the gain of cross spectral analysis, the effects of financial variables on US industrial production are stronger in longer cycles (at lower frequencies) than in shorter cycles (in higher frequencies). Among four financial variables, the effect of shares is strongest. Second, from the phase analysis, in longer cycles, US industrial production leads financial variables, while in shorter cycles, US industrial production lags. Overall, the time-length of leads and lags depends on financial variables. This result implies that the relationship between US industrial production and financial variables is varying along the business cycle. Finally, the coherence analysis indicates that US industrial production moves along with financial variables.

In contrast to the frequency domain analysis, the wavelet analysis has an ability to decompose the data into various time scales, which allows us to investigate the relationship in different time scales and locations. In the wavelet analysis, first, at the finest time scale (scale 1), most financial variables show a feedback relationship with US industrial production. Second, at the intermediate time scale, the results are mixed: (1) there is no relationship between financial variables and US industrial production, PTB3, and CPTB3; (2) financial variables Granger cause US industrial production; (3) US industrial production Granger causes US financial variables. Finally, at the long-term trend (indicated as S_6), feedback relationships are observed in most variables, except for CPTB3.

Overall, from Tables 3.4 and 3.5, our results indicate that financial variables Granger cause US industrial production in the raw data, while financial variables and US industrial production show a feedback relationship at the lower and higher time scales. This finding may result from the capacity utilization and the investment lag. More specifically, in the short run, financial variables absorb all related information to determine the equilibrium prices, while industrial production may need a sufficient time to adjust its capacity to absorb the relevant information. Therefore, it is natural for financial variables containing more information to lead US industrial production containing less information at time t. However, at the lower time scales, the capacity utilization of the firms plays a role in their ability to adjust their production in response to new information, while at the higher time scales, firms adjust their production without any time delays.

From two time series analyses (spectral and wavelet analyses), it can be concluded that the lead-lag relationship between US financial variables and US industrial production is varying depending on the time scale and frequency. However, both analyses show that US industrial production and financial variables move along together with a common long-term trend. Our results are consistent with previous studies, such as those of Ramsey and Lampart (1998a, 1998b) and Gençay et al. (2002a).

Chapter 4 The Relationship between Stock returns and Inflation: New Evidence from Wavelet Analysis

4.1 Introduction

According to the Fisher hypothesis, in its most common version, the expected nominal asset returns should move one for one with expected inflation. Essentially, this implies that real stock returns are determined by real factors independently of the rate of inflation. However, most past empirical literature shows that stock returns are negatively correlated with inflation[1] (see Fama and Schwert, 1977; Gultekin, 1983; recently Barnes et al., 1999). A negative relationship implies that investors, whose real wealth is diminished by inflation, can expect this effect to be compounded by a lower than average return on the stock market (Choudhry, 2001)[2].

[1] Jovanovic and Ueda (1998) examine the relationship between stock returns and inflation rate based on a principal-agent economy and find that unexpected inflation shifts real income from firms (the principals) to workers (the agents), and thereby lowers stock returns.

[2] Fama (1981) explains the negative short-run correlation between stock returns and inflation by the negative short-run correlation between inflation and real activity (the 'proxy' hypothesis). The proxy hypothesis is that the main determinant of stock prices is the company's future earning potential. If inflation and future expected output in the economy are negatively correlated, then inflation may proxy for future real output. Recently, Gallagher and Taylor (2002) examine the proxy hypothesis and find that their model supports strongly Fama's 'proxy hypothesis' in the US over the last 40 years.

Most empirical studies, examining the relationship between stock returns and inflation, focus on relatively short horizons, typically less than a year. However, examining the long-run relationship is important in at least two aspects. First, from a practical point of view, many investors hold stocks over long holding periods. Therefore, it is important to know the manner in which stock prices move with inflation over longer horizons (Boudoukh and Richardson, 1993). Second, the relationship between stock returns and inflation at the long horizon is of particular interest given that at the short horizon, the true long-run relationship could be obscured by short-term noise, which might derive from agents trading for portfolio rebalance or unexpected immediate consumption need reasons (Harrison and Zhang, 1999). Along with these aspects, Boudoukh and Richardson (1993), Solnik and Solnik (1997), Engsted and Tanggaard (2002), and Schotman and Schweitzer (2000) examine the relationship between stock returns and inflation over long-horizons, and their results support the Fisher hypothesis as the horizon increases.

In short, most previous studies examine the relationship between stock returns and inflation, either at the short horizon or at the long horizon. Hence, previous researches have often presented a limited understanding of the true dynamic relationship between stock returns and inflation, due to the limited time scale. However, important and interesting questions arise in considering to what extent stock returns and inflation move together negatively or positively over the different time horizons and whether expected nominal stock returns, compared to the real returns, correspond differently to the inflation over the different time horizons. In other words, the central issue in studying the true dynamic relationship between stock returns and inflation is "timing".

The main purpose of this chapter is to propose a new approach, wavelet analysis, for investigating the relationship between nominal stock returns and inflation over different time scales. The new approach is based on a wavelet multiscaling method that decomposes a given time series on a scale-by-scale basis. The main advantage[3] of the wavelet analysis is the ability to decompose the data into several time scales. Consider the large number of investors who trade in the security market and make decisions over different time scales. One can visualize traders operating minute-by-minute, hour-by-hour, day-by-day, month-by-month, year-by-year. In fact, due to the different decision-making time scales among traders, the true dynamic structure of the relationship between the stock returns and inflation itself will *vary* over different time scales associated with those different horizons. Economists and financial analysts have long recognized the idea of several time periods in decision making, while economic and financial analyses have been restricted to at most two time scales (the short-run and the long-run), due to the lack of analytical tools to decompose data into more than two time scales.

Our study extends the current literature in two important ways. First, to the best of our knowledge, this chapter is the *first* to investigate the Fisher hypothesis and its

[3] The major aspects of wavelet analysis are the ability to handle non-stationary data, localization in time, and the resolution of the signal in terms of the time scale of analysis. Among these aspects, the most important property of wavelet analysis is decomposition by time scale (Ramsey, 1999).

examination of the relationship between stock returns and inflation, using wavelet analysis. This study helps to deepen our understanding of the true relationship between nominal and real stock returns and inflation over the different time scales. The results therefore should be of interest to both international and local investors, as well as monetary and regulatory authorities. Furthermore, to deepen understanding of the true relationship between stock returns and inflation, we investigate and analyze variance, covariance of nominal and real returns, and inflation. Correlations and cross-correlations between nominal and real returns and inflation are calculated for the different time scales.

Second, we also examine the long-run relationship between stock returns and inflation not only in nominal but also in real terms. While the previous studies examine the relationship using nominal stock returns, we are interested in studying real returns because a security is an inflation hedge if and only if its real return is independent of the inflation rate (Bodie, 1976). A test of the relationship between real returns and the inflation rate may provide further evidence on the ability of stock returns to act as a hedge against inflation.

The main results from the empirical analysis can be summarized as follows. First, in regression analysis based on wavelet domains, the results are different from those of long-horizon regression in both the nominal and real returns. The wavelet correlation also shows a similar result to regression analysis based on wavelet analysis, which shows a positive relationship at the shortest horizon and the longest horizon, and a negative relationship at the intermediate horizon.

The remainder of the chapter is organized as follows: Section 4.2 presents the data and basic statistics. In section 4.3, we discuss the empirical results. In section 4.4, a summary and concluding remarks are presented.

4.2 Data and Basic Statistics

We use monthly nominal and real stock returns and inflation rates for the US in the period January 1926 to December 2000. Data were collected from Stocks, Bond, Bills, and Inflation (SBBI) sourced from Ibbotson Associates (2001). Table 4.1 presents several summary statistics for the monthly data of nominal and real stock returns, and inflation. As shown in Panel A of Table 4.1, all sample me~~s are positive and close to 0.

Table 4.1 Basic Statistics
Panel A Descriptive Statistics

$x =$	real stock return	nominal stock return	inflation
Mean	0.007	0.010	0.003
Variance	0.003	0.003	0.000
Skewness	-0.071	0.356	1.232
Kurtosis	12.870	9.941	14.608
JB	6212.489	3724.703	8229.679
	(0.000)	(0.000)	(0.000)
ρ	0.077	0.099	0.556
LB(15) for x	37.848	53.567	1708.520
	(0.000)	(0.000)	(0.000)
ρ^2	0.194	0.266	0.209
LB(15) for x^2	240.517	484.619	146.191
	(0.000)	(0.000)	(0.000)

Note: Data used are monthly U.S. nominal and real stock returns, and inflation for the period January 1962 to December 2000. Data were collected from Stocks, Bonds, Bills and Inflation (SBBI) sourced from Ibbotson Associates (2001). Significance levels are in parentheses. LB(n) is the Ljung-Box statistic for up to n lags, distributed as χ^2 with n degrees of freedom. ρ and ρ^2 indicates the first-order autocorrelations of returns and squared returns, respectively. Skewness and kurtosis are defined as $E[(R_t - \mu)]^3$ and $E[(R_t - \mu)]^4$, where μ is the sample mean. JB indicates the Jarque-Bera statistics.

Panel B Correlation Matrix

	real stock return	nominal stock return	inflation
real stock return	1.000	0.955	-0.084
nominal stock return		1.000	0.008
inflation			1.000

Among the stock returns and inflation, first-order autocorrelation of monthly data ranges from 0.077 (real stock return) to 0.556 (inflation), implying that inflation is more persistent than stock returns, and the Ljung-Box statistics indicate the persistence of linear dependency of each set of data. For the squared data, the first-order serial correlations vary between 0.194 (real stock return) and 0.266 (nominal stock return), and the Ljung-Box statistics show strong evidence of non-linear dependency in all data. The measures for skewness, kurtosis and Jarque-Berra statistics are also reported to check whether monthly data are normally distributed. These statistics indicate that all data are not normally distributed.

We report the unconditional contemporaneous correlation coefficients among three variables – real stock returns, nominal stock returns and inflation – in Panel B of Table 4.1. The striking feature is the difference between the correlations of real and nominal stock returns. The correlation between nominal stock returns and inflation shows a positive value, while the correlation between real stock returns and inflation is negative. This result is in contrast to that of Boudoukh and Richardson (1993) in nominal terms, while consistent with Choudhry (2001) in real terms. The result suggests that when the monthly data have been adopted, the results can be altered because of the different time scale.

4.3 Empirical Results of Wavelet Analysis

A major innovation of this chapter is the introduction of a new approach to study the Fisher hypothesis, as it provides a unique approach to addressing the stock returns inflation puzzle. The stock market consists of thousands of traders and investors, all with different time scales when it comes to making an investment. Wavelet analysis is a natural tool used to investigate the relationship between stock returns and inflations, as it enables us to decompose the data on a scale-by-scale basis.

Table 4.2 Regressions in Wavelet Domain

Panel A Nominal Stock Returns

	d1	d2	d3	d4	d5	d6	d7	s7
b_1	0.899	-0.530	0.219	-1.501*	-0.958*	-0.558*	-0.372*	1.968*
	(0.491)	(0.452)	(0.383)	(0.314)	(0.246)	(0.130)	(0.105)	(0.080)
R^2	0.004	0.002	0.000	0.026	0.019	0.025	0.046	0.539

Note: Data used are monthly U.S. nominal stock returns and inflation for the period January 1962 to December 2000. Data were collected from Stocks, Bonds, Bills and Inflation (SBBI) sourced from Ibbotson Associates (2001). Significance levels are in parentheses. * indicates the significance at 5% level. The wavelet coefficients are calculated using Daubechies 4 (D(4)) wavelet filter.

Panel B Real Stock Returns

	d1	d2	d3	d4	d5	d6	d7	s7
b_1	0.083	-1.514*	-0.724	-2.559*	-1.969*	-1.448*	-1.301*	0.910*
	(0.521)	(0.469)	(0.391)	(0.323)	(0.241)	(0.131)	(0.120)	(0.085)
R^2	0.000	0.012	0.004	0.069	0.077	0.148	0.194	0.180

Note: Data used are monthly U.S. real stock returns and inflation for the period January 1962 to December 2000. Data were collected from Stocks, Bonds, Bills and Inflation (SBBI) sourced from Ibbotson Associates (2001). Significance levels are in parentheses. * indicates the significance at 5% level. The wavelet coefficients are calculated using Daubechies 4 (D(4)) wavelet filter.

In this section, we analyze and report the empirical results of the relationship between stock returns (nominal and real) and inflation using wavelet analysis.

Considering the sample size and the length of the wavelet filter, we settle on the MODWT based on the Daubechies extremal phase wavelet filter of length 4 (D(4)), while our decompositions go to level 7 (equivalent up to the 64-month period). First, we analyze the relationship based on the Fisher model, using decomposed data through the MODWT.

As can be seen in Table 4.2, through decomposition of the MODWT, the 7 levels' data can be generated. We report the estimated coefficient of b_1, corresponding standard errors and R^2. First, the values of R^2 are increasing as the time scale increases. In d1 (equivalent to a 1-month period), the relationship between nominal stock returns and inflation is significantly positive, implying that stocks are a good hedge against inflation. The real stock return also has a positive value of b_1. As the time scale increases, the coefficient, b_1, shows a positive value at d3 (equivalent to a 4-month period) and s7 (equivalent to greater than a 128-month period), while showing a negative value at the d2, d4, d5, d6, and d7 (equivalent of 2, 8, 16, 32, and 64-month period) in nominal return. In the real return, the values of b_1 show a similar pattern with those of the nominal stock return. However, the results of the nominal stock return are different, showing a negative relationship in most time horizons, based on the wavelet regression. Overall, the absolute values for coefficient b_1 have the highest value at the intermediate time scales (d4 both in nominal and real returns), indicating that the degrees of correlation between two variables are increasing up to d4 and decreasing after d4. Overall, our results are consistent with the results of Boudoukh and Richardson (1993), Schotman and Schweitzer (2000), and Engsted and Tanggaard (2002), since our results support the Fisher hypothesis as the horizon increases using the nominal term. More specifically, the result from the regression analysis is statistically supportive in d1 and s7 (equivalent to a 1-month period and greater than a 128-month period).

Next, we move to the variances of nominal and real stock returns, and inflation. Figure 4.1 illustrates the MODWT-based wavelet variance of nominal and real stock returns, and inflation. The black line indicates the wavelet variance and the black dotted line indicates the 95% confidence interval against the various time scales.[4] There is an approximate linear relationship between the wavelet variance and the wavelet scale, indicating the potential for long memory in the volatility series. The variance decreases as the wavelet scale increases. Notice that the wavelet variance has a highest value at the first scale. Similarly, the gray line in Figure 4.1 displays the wavelet variances and their corresponding confidence interval of the real return against the wavelet scale. The movement of wavelet variance of the real return is quite similar to that of the nominal return. The results of inflation are quite striking, indicated as the thick black line in Figure 4.1. The wavelet variances of inflation are very stable and close to zero over the wavelet scale. This result is generally accepted in the literature. As indicated in Schotman and Schweitzer (2000), the volatility of stock returns is usually much higher than the volatility of inflation.

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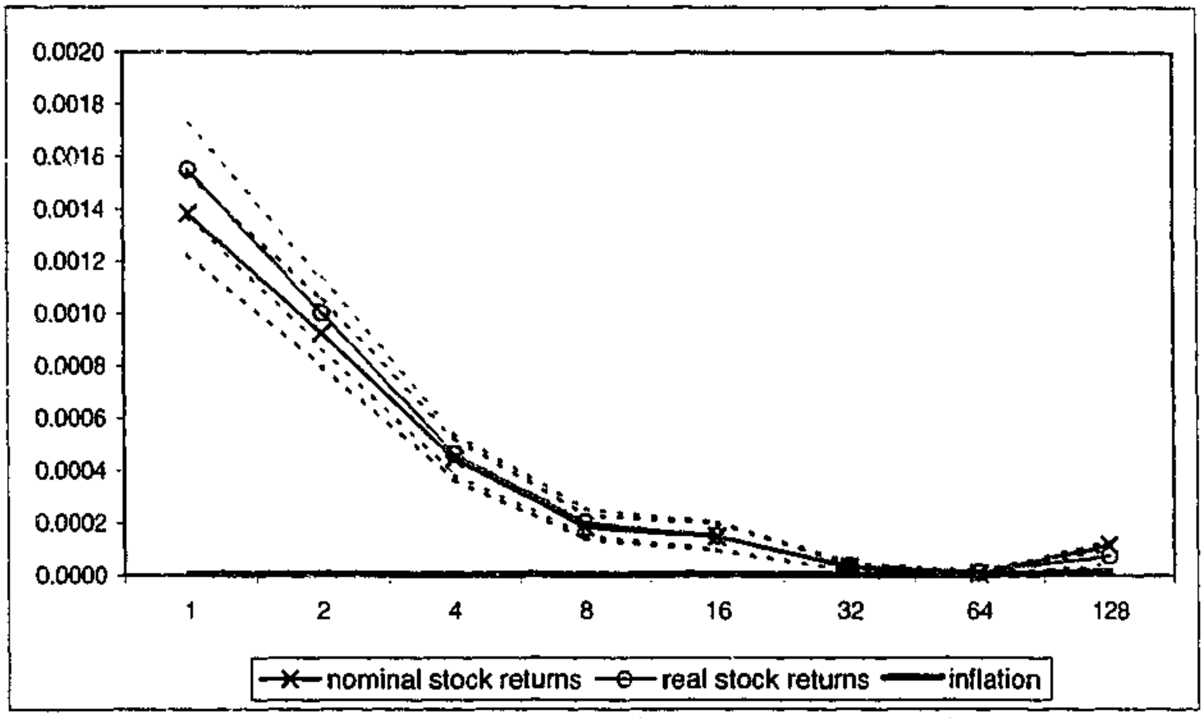
[4] For a detailed explanation of how to construct the confidence interval of wavelet variance, see Gençay et al., (2002a, p. 242).

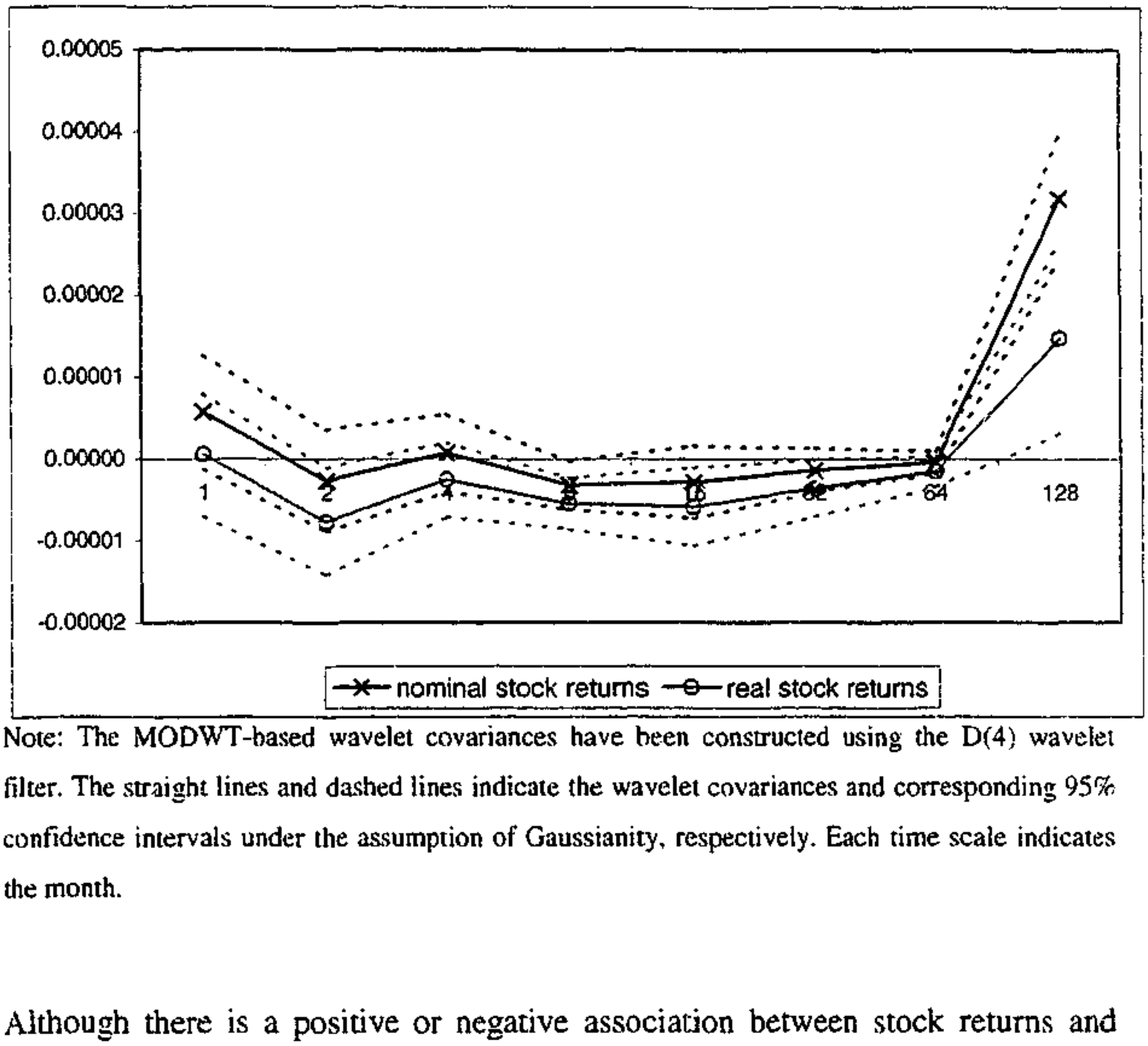
Figure 4.1 Estimated Wavelet Variance of Nominal Stock Returns, Real Stock Returns and Inflation from January 1926 to December 2000 Using Monthly Frequency



Note: The MODWT-based wavelet variances of nominal and real returns, and inflation have been constructed using the D(4) wavelet filter. The straight lines and dashed lines indicate the wavelet variances and corresponding 95% confidence intervals under the assumption of Gaussianity, respectively. Each time scale indicates the month. For example, wavelet scale 8 indicates 8 month.

In addition to examining the variances of three time series, we construct the wavelet covariance to examine how two series are associated with one another (between nominal stock returns and inflation, and between real stock returns and inflation). Figure 4.2 shows the MODWT-based wavelet covariances of nominal and real returns using the D(4) wavelet filter. Approximate confidence intervals are also presented. Roughly speaking, the movements of covariance between the nominal stock returns and inflation have a W-shape. In d1, the covariance shows a positive value; however, it decreases up to d4 (equivalent to an 8 month period). After d4, its value starts to increase. Up to d7 (equivalent to a 64 month period), it shows a negative value. In s7 (equivalent to greater than a 128-month period), the wavelet covariance is positive.

Figure 4.2 Estimated Wavelet Covariance of Nominal Stock Returns, and Real Stock Returns with Inflation from January 1926 to December 2000 Using Monthly Frequency

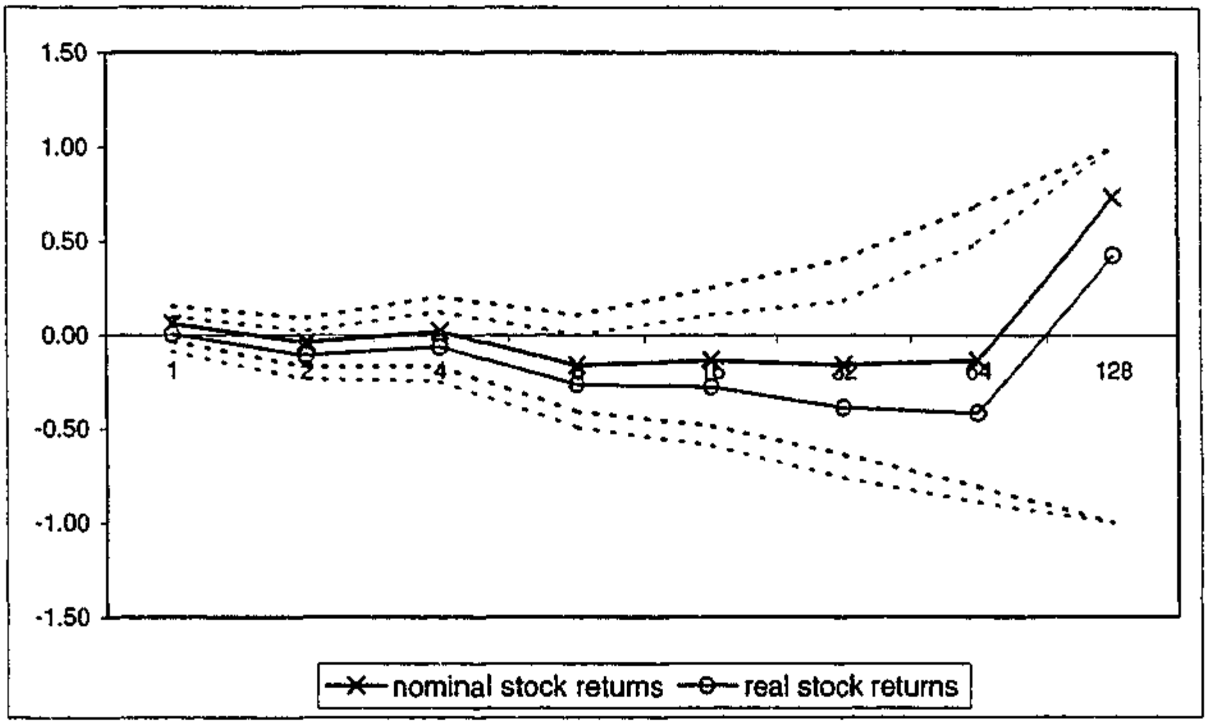


Note: The MODWT-based wavelet covariances have been constructed using the D(4) wavelet filter. The straight lines and dashed lines indicate the wavelet covariances and corresponding 95% confidence intervals under the assumption of Gaussianity, respectively. Each time scale indicates the month.

Although there is a positive or negative association between stock returns and inflation, it is difficult to compare the wavelet scales due to the different variabilities exhibited by them. In this case, the wavelet correlation should be constructed to examine the magnitude of the association of each series.

Figure 4.3 illustrates the correlation of stock returns with inflation against the wavelet scales. As can be seen in Figure 4.3, the significant positive relationship can be observed in d1 (1-month period) and s7 (greater than a 128-month period), while in the rest of the wavelet scale, the negative relationship can be found in both nominal stock and real stock returns, except d3 (4-month period) of nominal return. Another thing to note is that the confidence intervals are significantly increased given the amount of variability in the estimated wavelet variances. Overall, the result of wavelet correlation is consistent with the wavelet regression analysis.

Figure 4.3 Estimated Wavelet Correlation of Nominal Stock and Real Stock Returns with Inflation from January 1926 to December 2000 Using Monthly Frequency



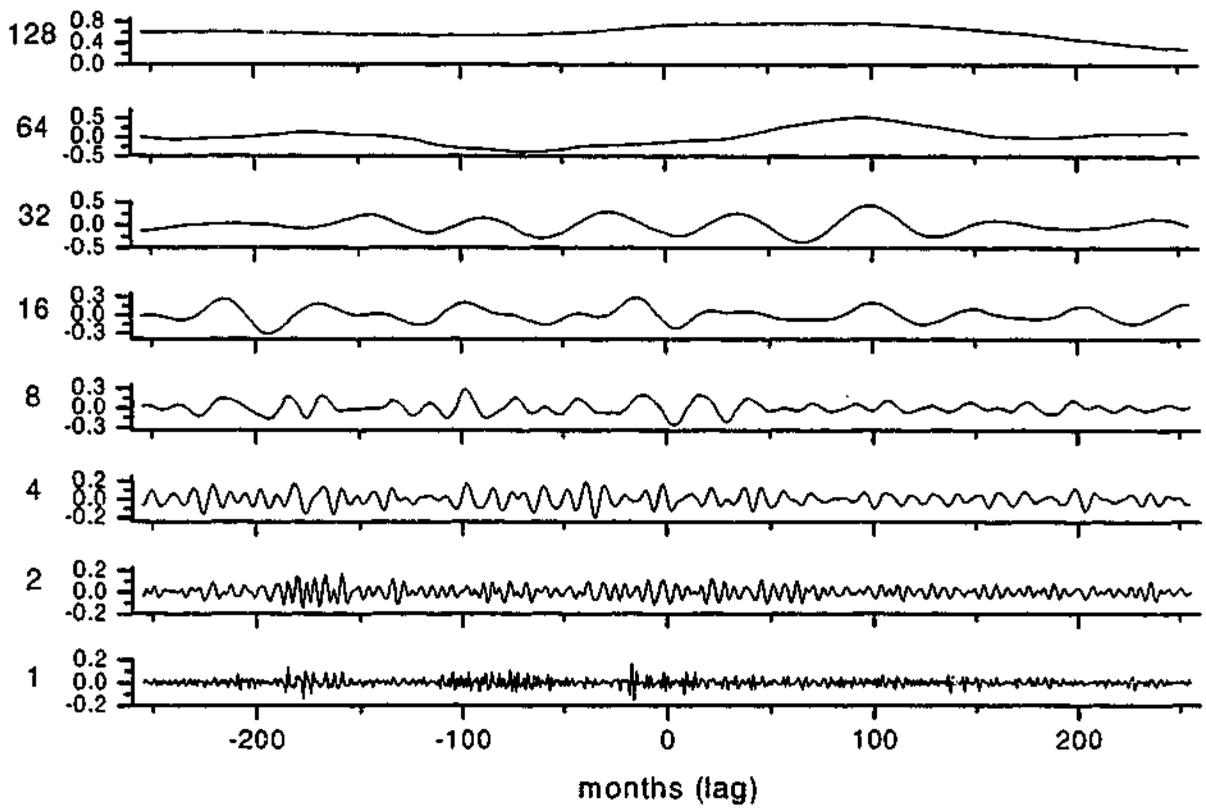
Note: The MODWT-based wavelet correlations have been constructed using the D(4) wavelet filter. The straight lines and dashed lines indicate the wavelet correlations and corresponding 95% confidence intervals, respectively. The approximate confidence interval for the estimated wavelet correlation does not utilize any information regarding the distribution of the wavelet correlation. Therefore, these confidence intervals are robust to non-Gaussianity. Each time scale indicates the month.

It is of interest to compare our results with those of Boudoukh and Richardson (1993), who find a positive relationship in the long run (5 years). A 5-year relationship can be captured by analyzing the results of d7 in Figures 4.3 and 4.4.

In contrast to Boudoukh and Richardson (1993), our results show a negative relationship in the 5-year cycle. In terms of cross correlation, we also observe a negative correlation. However, in the higher cycle, the stock return and inflation have a positive relationship, consistent with the previous studies (see Boudoukh and Richardson, 1993; Engsted and Tanggaard, 2002). Overall, the Fisher model holds at the shortest scale (d1) and the longest smooth scale (s7) in the wavelet analysis (regression analysis based on wavelet domain and wavelet correlation)[5]. The cross correlations of nominal and real stock returns with inflation are plotted in Figures 4.4 and 4.5. Both figures show a similar pattern regardless of the wavelet scales. At the point of lag zero, the cross correlation shows a negative value. All wavelet scales appear to be asymmetric. More specifically, the first three scales, associated with periods of 1, 2, and 4 months, indicate the small number of cross-correlations and then a group of lags (both negative and positive) that show a slight positive wavelet correlation. The wavelet cross-correlation function on d4, associated with 8-month period, is also asymmetric where the oscillations are much higher in the negative lags than in the positive lags.

[5] In the terminology of Schotman and Schweitzer (2000), as can be seen in Table 1, inflation is persistent. This persistence leads the correlation to be positive at the long horizon.

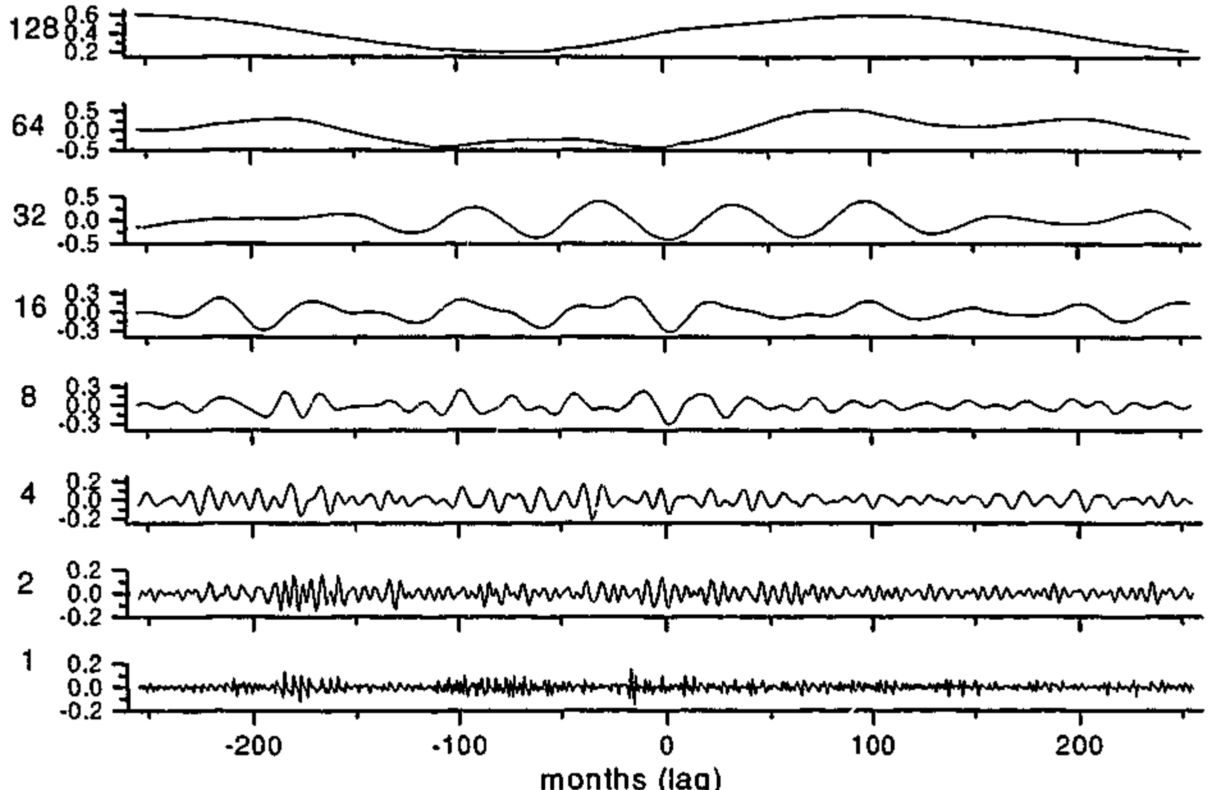
Figure 4.4 Estimated Wavelet Cross-correlation between Nominal Stock Returns and Inflation from January 1926 to December 2000 Using Monthly Frequency



Note: The straight lines indicate the wavelet cross correlations. Each time scale indicates the month.

Overall, the relationship between the nominal stock returns and inflation is not very different to that between the real stock return and inflation. The regression analysis based on wavelet domain and wavelet correlation show a positive relationship at the first level (1-month period) and s7 (greater than a 128-month period), but a negative relationship at the rest of the levels regardless of nominal or real stock returns. This result indicates that based on our data, the relationship between stock return and inflation is positive at the longest horizon, which is consistent with the findings of Boudoukh and Richardson (1993) and Engsted and Tanggaard (2001).

Figure 4.5 Estimated Wavelet Cross-correlation between Real Stock Returns and Inflation from January 1926 to December 2000 Using Monthly Frequency



Note: The straight lines indicate the wavelet cross correlations. Each time scale indicates the month.

4.4 Concluding Remarks

The main innovation of this chapter is the introduction of a new approach to study the Fisher hypothesis. This new approach is based on a wavelet multiscaling method that decomposes a given time series on a scale-by-scale basis. To test the relationship, we have adopted regression analysis in the wavelet domain, and wavelet covariance/correlation analysis.

However, the results of the regression analysis in the wavelet domain and the wavelet correlation show that the relationship is positive at the short horizon, i.e., d1, equivalent to a 1-month period, while in the rest of the wavelet scale, a negative relationship is observed. From our wavelet correlation analysis, it is found that at the long horizon (around 5 years) the positive relationship between stock returns and inflation still exists, consistent with the result of Boudoukh and Richardson (1993), Schotman and Schweitzer (2000), and Engsted and Tanggaard (2002). Consistent with the results of Rapach (2002), we find that there is a positive relationship in the long horizon between real stock returns and inflation. The nominal stock returns results provide empirical evidence supportive of the Fisher hypothesis for risky assets in d1 and s7 of the wavelet domain, equivalent to a 1-month period and greater than a 128-month period.

When we explore whether there is a difference between the relationship between real and nominal stock returns with inflation, our results indicate that in all regression analyses, real returns have a significant negative relationship with inflation except for d1 and s7 in wavelet analysis. More specifically, at the shortest time scale (d1) and the longest smooth scale (s7), a positive relationship is observed in real returns, while a negative relationship is observed in the rest of the time scales. With regard to the findings of Bodie (1976), our results show that stock returns do not play a role as an inflation hedge during our sample period, except in the shortest scale (d1) and the longest smooth scale (s7) in wavelet analysis.

The calculated wavelet variance shows that the wavelet variance of stock returns (nominal and real returns) decreases as the wavelet scale increases, while the wavelet variance of inflation is quite stable. Due to the stability of the variance of inflation, the movements of the wavelet variance of nominal returns are quite similar to those of real returns.

In conclusion, our results provide some support for the importance of time scale decomposition in explaining the inconsistent results in the relationship between stock returns and inflation in the literature on the Fisher hypothesis. In other words, our key empirical results shows that time-scale decomposition provides considerable insight into testing of the Fisher hypothesis, since a number of stock returns and inflation puzzles previously noted in the literature are resolved and explained by the wavelet analysis.

Chapter 5 Multihorizon Sharpe Ratio

5.1 Introduction

Strategic asset allocation is the process used to identify the optimal portfolio for a given investor over his or her investment horizon. To evaluate the performance of a portfolio, the Sharpe ratio is widely used in a mean-variance framework. As indicated in Levy (1972), the Sharpe ratio is closely related to the investment horizon. The holding period that is relevant for portfolio allocation is the length of time investors hold any stocks or bonds, no matter how many changes are made among the individual issues in their portfolio (Siegel, 1998, p29). In other words, the horizon sensitivity of the Sharpe ratio is very important to evaluate the performance of one or more portfolios. An investor might not be interested in short-term performance of portfolios at all. Institutional investors like pension funds have a very long investment horizon. Therefore, it is interesting to examine the long-term performance of the investments when the investment horizon increases.

Numerous empirical studies (Nielsen and Vassalou, 2003; Pedersen and Satchell, 2000; Miller and Gehr, 1978 among others) have been conducted on the Sharpe ratio, but there is only one published study of the multihorizon Sharpe ratio (Hodges et al., 1997). They examine the multiperiod Sharpe ratio using randomized historical data from 1926 to 1993. However, contrary to general belief, they conclude that bonds outperform stocks in sufficiently long holding periods.

Siegel (1999) summarizes three problems in Hodges et al. (1997). First, their analysis omits a key feature of long-term stock data—the mean reversion of equity returns. Second, because economic agents try to maximize their real consumption, the examination of the multihorizon Sharpe ratio should be based on the *real* returns, not *nominal* returns. Using nominal data to calculate risk and return improperly favors bonds, whose returns are fixed in many terms. Finally, an assumption about the properties of the returns of stocks and bonds can causes a more serious problem. In their study, they assume that returns are independently and identically distributed across time. This means that the probability distribution of R_t is identical to that of R_s for any dates t and s and that R_t and R_s are statistically independent for all $t \neq s$. While this assumption enables us to calculate the multihorizon Sharpe ratio easily, previous literature (Poterba and Summers, 1988; Siegel, 1998) clearly indicates that real equity returns have strong mean reversion, while real returns on fixed-income securities do not. The presence of a mean-reversion process makes the standard deviation of multihorizon returns much lower for equities than those found under the independence assumption (Siegel, 1999).

Another performance measure in the multihorizon setting can be found in Dacorogna et al. (2001). They propose two new performance measures, named as X_{eff} and R_{eff} respectively, and report that these new measures are numerically more stable than the Sharpe ratio and exhibits fewer deficiencies[1]. However, the two new risk-adjusted measures show three drawbacks. First, it requires knowledge of the explicit value of the risk aversion coefficients, that is the proper value of the risk aversion coefficient is controversial. In the literature on risk aversion, the various degrees of risk aversion are estimated and reported (Kroner and Sultan 1993, p545). Second, their approach is based on the trading model of the exchange markets using the nominal returns, implying that this model also suffers similar criticism to that of Hodges et al. (1997). Finally, to measure the multihorizon performance, they use the overlapping return series to calculate long horizon returns. Calculating the long horizon return in this way can reduce the number of data, resulting in a biased estimator (In and Kim, 2003).

Compared with previous studies, this section employs a new approach, using wavelet analysis to investigate the multihorizon Sharpe ratio. This section aims to contribute to the literature on the study of the multihorizon Sharpe ratio. Three *innovations* are introduced compared to previous studies. First, we examine the

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[1] Dacorogna et al. (2001) summarize three drawbacks of the Sharpe ratio to evaluate the market model in foreign exchange markets. First, the definition of the Sharpe Ratio puts the variance of the return into the denominator which makes the ratio numerically unstable at extremely large values when the variance of the return is close to zero. This creates a lack of identification between the return and its volatility. Second, the Sharpe Ratio is unable to consider the clustering of profits and losses. An even mixture of profit and loss trades is usually preferred to clusters of losses and clusters of profits, provided the total set of profit and loss trades is the same in both cases. Third, the Sharpe Ratio treats the variability of profitable returns (which are unimportant to investors) the same way as the variability of losses (which are an investor's major concern).

mean-reverting property of asset returns using the wavelet-based maximum-likelihood estimation of the long-memory parameter. Second, since consumers maximize real consumption, we calculate the risk and returns of stocks and bonds in real, not nominal, terms. Finally, to the best of our knowledge, no previous study has investigated the multihorizon Sharpe ratio using wavelet analysis. Adopting wavelet analysis does not require any assumption on the distribution of returns, because wavelet analysis is a nonparametric estimation and decomposes the unconditional variance into different time scales.

Our wavelet decomposition shows that the long-memory parameter, calculated from the wavelet-based maximum-likelihood estimation, for all asset returns is less than 1 and close to 0, indicating that asset returns are mean-reverting. For the multihorizon Sharpe ratio, the Sharpe ratio of a large-company stock portfolio is higher value than the other three portfolios (small company stocks, long-term and intermediate-term government bonds) over all wavelet scales. In other words, a large-company stock portfolio outperforms the other portfolios over the wavelet scales.

The rest of the chapter is organized in the following manner. Section 5.2 discusses the Sharpe ratio. The data and the basic statistics are discussed in section 5.3. In setion 5.4, we present and discuss the empirical results. Finally, section 5.5 presents the summary and concluding remarks.

5.2 Sharpe ratio

It is well known that the Sharpe ratio is an approach to evaluating portfolio performance in a mean-variance framework.[2] In a mean-variance world, this Sharpe ratio captures the expected excess return per unit of risk associated with the excess return. Since it gives risk estimates before decisions are actually taken, the *ex ante* Sharpe ratio can be very useful for decision-making (e.g., choosing investments), while the *ex post* Sharpe ratio can be used for evaluation of investments. In other words, the Sharpe ratio captures both risk and return (actual or expected, depending on the circumstances) in a single measure. A rising (falling) excess return or a falling (rising) standard deviation leads to an increase (decrease) in the Sharpe ratio. Therefore, if investors face an exclusive choice among a number of portfolios, then they can unambiguously rank them on the basis of their Sharpe ratios. A fund with higher Sharpe ratio will be enable all investors to achieve a higher expected utility (Nielsen and Vassalou, 2003). If we choose investments before the event among the various alternatives, one would choose that investment with the highest *ex ante* Sharpe ratio

[2] The traditional approach to decide and/or to evaluate investments is to use a Sharpe ratio. Suppose we have a portfolio, *p*, with a return, R_p and a risk-free rate of interest, and a benchmark portfolio (i.e., US Treasury bills), denoted by *f*, with a return R_f. The Sharpe ratio can be defined by: $SR_p = \left(\overline{R}_p - \overline{R}_f\right)/\sigma_p$, where SR_p is the Sharpe ratio for a portfolio, $\overline{R}_p$ is the mean return on the portfolio, $\overline{R}_f$ is the mean return on risk-free rate of interest and σ_p is the standard deviation of the portfolio return.

It is important to appreciate that the Sharpe ratio always refers to the *differential* between two portfolios. We can think of this differential as reflecting a self-financing investment portfolio with the two components: one component represents the acquired asset and the other reflects the short position taken to finance that acquisition. As Sharpe (1994) explains, the usefulness of the Sharpe ratio is the fact that a differential return represents the result of a zero-investment strategy. This can be defined as any strategy that involves a zero outlay of money in the present and returns either a positive, negative, or zero amount in the future, depending on circumstances (Dowd, 2000).

While the Sharpe ratio has gained considerable popularity, the relationship between the Sharpe ratio and the investment horizon has received little attention from researchers. One of the exceptions is Hodges et al. (1997). As mentioned in Hodges et al. (1997), a Sharpe ratio computed using short (monthly, quarterly, annual) return intervals to evaluate portfolios or make asset allocation decisions will be biased for long-term investors and may lead to suboptimal results. Traditionally, to construct the multihorizon excess return and variance of a portfolio, the n-period expected return and variance are calculated using the following equations under the assumption that the returns are independently and identically distributed across time:

$$R_n = (1+R_1)^n - 1 \tag{5.1}$$

$$\sigma_n^2 = \left\{\left[\sigma_1^2 + (1+R_1)^2\right]^n - (1+R_1)^{2n}\right\} \tag{5.2}$$

where R_1 and σ_1^2 are the portfolio one-period expected return and variance, and R_n and σ_n^2 are the n-period return and variance of the portfolio. From equations (5.1) and (5.2), the Sharpe ratio for an n-period investment horizon can be calculated as follows:

$$SR_n = \frac{R_n - R_{f,n}}{\sqrt{\sigma_n^2}} = \frac{(1+R_1)^n - (1+R_f)^n}{\sqrt{\left[\sigma_1^2 + (1+R_1)^2\right]^n - (1+R_1)^{2n}}} \tag{5.3}$$

This equation shows that the multihorizon Sharpe ratio is a complex, nonlinear function of the one-period expected return, the one-period standard deviation, and n, the length of the holding period (Hodges et al., 1997).

The problems with using these equations are two-fold: the first and more serious problem concerns the assumption of the distribution. The previous studies show that the real return on the stock has a property of mean reversion, which indicates that the stock return is not independently and identically distributed. The second problem concerns the construction of an n-period return and variance. As can be seen in equations (5.1) and (5.2), the construction of a long-period return leaves us a handful observation. Therefore, this may result in a biased estimator.

Wavelet analysis enables us to overcome these shortcomings of the previous method to generate the mean and variance without imposing any assumption over the multihorizon and losing observations. In the next section, we briefly discuss wavelet analysis.

Given the wavelet variance, equation (2.61) and the mean returns at scale λ_j, the

Sharpe ratio at scale λ_j can be calculated as:

$$SR_j^w = \frac{\tilde{R}_j(\lambda_j) - \tilde{R}_f(\lambda_j)}{\sqrt{\tilde{v}_R^2(\lambda_j)}} \tag{5.4}$$

In this specification, SR_j^w indicates the wavelet multihorizon Sharpe ratio, which can be varying depending on the wavelet scales (i.e., investment horizons).

5.3 Data and basic statistics

The data employed are large and small companies' monthly stock returns, and long-term and intermediate-term government bond returns, which are measured by real term from Stocks, Bonds, Bills, and Inflation (SBBI), Ibbotson Associates (2001). As indicated in the introduction and in Siegel (1999), to examine the multihorizon Sharpe ratio, the real return is preferable to the nominal return because the investors (consumers) are concerned with real returns rather than nominal returns. The data periods range from April 1958 to December 2000.

Table 5.1 presents several summary statistics for the monthly data of real stock returns (large and small companies) and real bond returns (long-term and intermediate-term government bonds). As shown in Panel A of Table 5.1, means are all positive and close to 0. These figures clearly show that the stock returns are higher than the bond returns, indicating the equity premium puzzle. Another noticeable statistic is that the mean and variance of the small company stock returns are higher than those of the large company stock returns, implying that the higher the return, the higher the risk because the investors require more return to compensate their risk exposure.

Table 5.1 Basic Statistics

Panel A Descriptive Statistics

$x=$	Treasury bill	Large stocks	Small stocks	Long-term government bonds	Intermediate-term government bonds
Mean	0.0012	0.0069	0.0087	0.0023	0.0022
Variance	0.0000	0.0018	0.0045	0.0008	0.0002
Skewness	-0.1395	-0.3889	-2.0230	0.5523	0.4355
Kurtosis	1.6655	1.9861	19.5523	2.8185	5.6193
JB	60.8374	97.0555	8504.8188	195.5051	689.8134
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
ρ	0.4720	0.0194	0.1542	0.0971	0.2077
LB(15) for x	964.7112	11.8166	25.3824	39.4452	50.1044
	(0.0000)	(0.4605)	(0.0131)	(0.0001)	(0.0000)
ρ^2	0.4380	0.0915	-0.0112	0.2406	0.1330
LB(15) for x^2	330.4971	29.4918	1.0548	150.5148	128.5617
	(0.0000)	(0.0033)	(0.9999)	(0.0000)	(0.0000)

Note: Panel A in Table 1 presents the basic statistics for the monthly returns of the Treasury bill, large and small stocks, long-term and intermediate-term government bonds over the period from April 1958 to December 2000. Significance levels are in parentheses. LB(n) is the Ljung-Box statistic for up to n lags, distributed as χ^2 with n degrees of freedom. ρ and ρ^2 indicates the first-order autocorrelations of returns and squared returns, respectively. Skewness and kurtosis are defined as $E[(R_t-\mu)]^3$ and $E[(R_t-\mu)]^4$, respectively, where μ is the sample mean.

Panel B. Correlation Matrix

	Treasury bill	Large company stocks	Small company stocks	Long-term government bonds	Intermediate-term government bonds
Treasury bill	1.000	0.190	0.113	0.281	0.353
Large company stocks		1.000	0.690	0.322	0.259
Small company stocks			1.000	0.133	0.102
Long-term government bonds				1.000	0.853
Intermediate-term government bonds					1.000

Among the real returns, first-order autocorrelation (ρ) of monthly data ranges from 0.0832 (large company stocks) to 0.4784 (Treasury bill), and the Ljung-Box statistics indicate the persistence of linear dependency of all data. For the squared level data, the first-order serial correlations (ρ^2) vary between 0.1535 (intermediate bonds) and 0.2798 (large company stocks), and the Ljung-Box statistics show strong evidence of nonlinear dependency in all data.

The measures for skewness, kurtosis and Jarque-Berra statistics are also reported to check whether monthly data are normally distributed. The sign of skewness varies; however, the magnitudes depend on the particular security. In general, if the values of kurtosis are larger than 3.0, then monthly returns are more peaked and have fatter tails than normal distributions. The Jarque-Berra statistics also indicate that all data are not normally distributed. The LB(15) for squared return series is highly significant for all assets, suggesting the possibility of the presence of autoregressive conditional heteroskedasticity.

We report the unconditional contemporaneous correlation coefficients for the Treasury bill, large and small companies' stock returns, and long-term and intermediate-term government bond returns in Panel B of Table 5.1. The correlation structure of the five variables is probably the most important feature from the point of view of investors and portfolio managers. Hedging and diversification strategies invariably involve some measure of correlation. As is shown in Panel B of Table 5.1, the correlations between stock returns and between bonds are very high; 0.690 between large and small companies' stocks and 0.853 between long- and intermediate-term bonds. The correlation between different securities is very low, indicating stock and bond returns do not move together.

5.4 Main empirical results

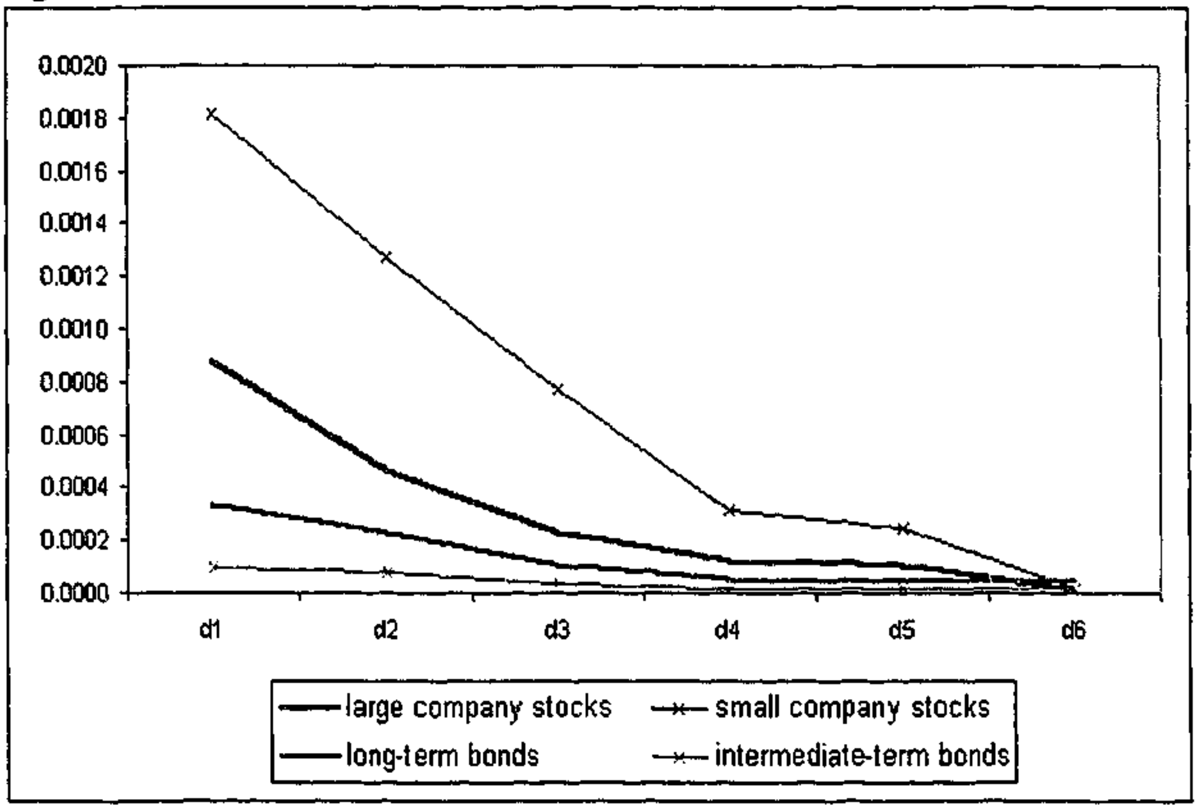
The main purpose of this section is to examine the relationship between the Sharpe ratio and the investment horizon for portfolios of small company stocks, large company stocks, intermediate-term government bonds, and long-term government bonds. To investigate the multihorizon Sharpe ratio using wavelet analysis we need to consider the balance between the sample size and the length of the wavelet filter. We settle with the MODWT based on the Daubechies least asymmetric wavelet filter of length 8 (LA(8)), while we decompose our data up to level 6. We also conducted the test using different wavelet filter such as Daubechies D(4). The results are not sensitive to the choice of the wavelet family. In our analysis, the data set has a finite length, namely, 512 observations.[3] This restriction naturally brings the issue of boundary conditions into the computation procedure. We simply choose the periodic boundary condition.[4] Since we use monthly data, scale 1 represents 2-4 month period dynamics. Equivalently, scales

[3] As indicated in section 4, MODWT does not require that the length of data be equal to a factor of 2. However, we use 512 observations ($=2^9$) due to the estimation of the mean reversion property of asset returns.

[4] We also test the reflection boundary condition, while the results are not qualitatively different.

2, 3, 4, 5, 6, and 7 correspond to 4-8, 8-16, 16-32, 32-64, 64-128, and 128-256 month period dynamics, respectively.

Figure 5.1 Estimated Wavelet Variance



Note: Figure 5.1 represents the wavelet variance of the four portfolio returns. The y-axis indicates the wavelet variance and the x-axis indicates the wavelet time scale. To calculate the wavelet variance of each portfolio, we decompose each time series up to level 6, using the Daubechies least asymmetric wavelet filter of length 8 (LA(8)).

First, we examine the variances of the four returns against various time scales. An important characteristic of the wavelet transform is its ability to decompose (analyze) the variance of the stochastic process. According to Siegel (1998), the risk of holding stocks and bonds depends crucially on the holding period. He finds that over a 20-year holding period, the risk of holding stocks is less than that of bonds. Figure 5.1 illustrates the MODWT-based wavelet variance of four series against the wavelet scales.[5] There is an approximate linear relationship between the wavelet variance and the wavelet scale, indicating the potential for long memory in the volatility series. The variances of both the stock and bond markets decrease as the wavelet scale increases. Note that the variance-versus-wavelet scale curves show a broad peak at the lowest scale (d1) in both markets. More specifically, a wavelet variance in a particular time scale indicates the contribution to sample variance (Lindsay et al., 1996, p. 778). The sample variances are 0.0018 for large company stocks, 0.0045 for small company stocks, 0.0008 for long-term bonds and 0.0002 for intermediate-term bonds, respectively, and 48.8%,[6] 40.6%, 43.3%, and 37.4% of the total variances of the four portfolio returns, respectively, are accounted for by the lowest scale (d1). Overall, the returns on stocks are more volatile than those of bonds over all wavelet scales, which is different from Siegel (1998), except for the scale 6. More specifically, the small company stock return possesses the highest volatility among four series. The lowest volatility belongs to the intermediate government bond return. These findings are expected from the preliminary tests and the property of the wavelet variance.

It is of interest to examine whether asset returns follow a random walk over various wavelet scales. Mathematically, the random walk assumption of an asset is determined by how fast the risk of average returns should decline as the holding period lengthens. A random walk is a process where future returns have no

[5] For the statistical inference, we also calculate the confidence interval for four series using equation (5.10). However, we do not plot the confidence intervals for a clear presentation.

[6] This figure can be calculated by the normalization of wavelet variance using the sample variance. For more detail, see Lindsay et al. (1996).

relation to (or are completely independent of) past returns (Siegel, 1998). Table 5.2 illustrates the wavelet standard deviation and the risk predicted under the random walk assumption of the four series.

Table 5.2 Holding Period Risk at the Wavelet Scales

	raw data	d1	d2	d3	d4	d5	d6
Large company stocks							
Risk	0.0423	0.0296	0.0216	0.0150	0.0111	0.0102	0.0021
Expected value	0.0423	0.0299	0.0212	0.0150	0.0106	0.0075	0.0053
Small company stocks							
Risk	0.0670	0.0427	0.0357	0.0278	0.0177	0.0158	0.0047
Expected value	0.0670	0.0473	0.0335	0.0237	0.0167	0.0118	0.0084
Long-term government bonds							
Risk	0.0278	0.0183	0.0150	0.0103	0.0070	0.0068	0.0066
Expected value	0.0278	0.0197	0.0139	0.0098	0.0070	0.0049	0.0035
Intermediate-term government bonds							
Risk	0.0158	0.0096	0.0088	0.0062	0.0042	0.0038	0.0040
Expected value	0.0158	0.0111	0.0079	0.0056	0.0039	0.0028	0.0020

Note: Table 5.2 provides the holding period risk at the wavelet scales to compare the variances under the random walk hypothesis. In this table, risk implies the standard deviation of raw data and the wavelet variance in various wavelet scales. The expected value has been calculated under the assumption that the standard deviation falls as the square root of the length of the holding period (Siegel, 1998, p. 33).

Our wavelet decomposition shows that the risks of all returns decline faster than predicted at the first wavelet scale, indicated as d1. In the intermediate scales (d2 and d5), the risks of all returns are greater than/similar to the expected risks. Finally, the risk, which is calculated from the wavelet variance, is lower than expected at the final wavelet scale, d6 (equivalent to 64 to 128 month period). This result is different from the previous studies. Siegel (1998, 1999) reports that the stock return shows mean reversion, while the bond return shows a mean-aversion property. Overall, our result supports the mean-reversion property of asset returns.

To further examine the mean-reversion property of asset returns, we adopt the long memory of asset returns using the maximum-likelihood estimator based on wavelets, proposed by Jensen (2000). Although the wavelet OLS estimator for the long-memory parameter (see Jensen, 1999b) is simple to program and compute, it produces much larger mean-square errors when compared to maximum-likelihood methods. In this section, we use the maximum-likelihood estimator for the long-memory parameter.[7] (see, chapter 2, section 2.6.3)

Table 5.3 Estimated Long-memory parameter

	Haar	D(4)	D(8)	LA(8)	LA(16)
Large company stocks	0.034	0.040	0.027	0.024	0.030
Small company stocks	0.084	0.082	0.076	0.058	0.074
Long-term government bonds	0.001	0.017	0.059	0.027	0.032
Medium-term government bonds	0.031	0.036	0.086	0.047	0.055

Note: Table 5.3 reports the long-memory parameter, calculated by the wavelet-based maximum-likelihood method. The asymptotic Cramer-Rao lower bound for the MSE of unbiased estimators of d is $6/(N\pi^2)$. In our study, $N = 512$. Hence, the asymptotic root of MSE for d is 0.001188.

Mean reversion occurs so long as $d<1$. It follows that a test for fractional integration can be used to determine the existence of mean reversion (Cheung and Lai, 2001). The results are presented in Table 5.3. If our data is mean-reverting, the long-memory parameter has to be less than 1.

To examine the sensitivity of the wavelet filters and the robustness of our results, we also conduct the ML test for long-memory parameter using different wavelet filters (i.e., Haar, D(4), D(8), LA(8), and LA(16)). However, the results are qualitatively similar regardless the wavelet filters. To construct an approximate

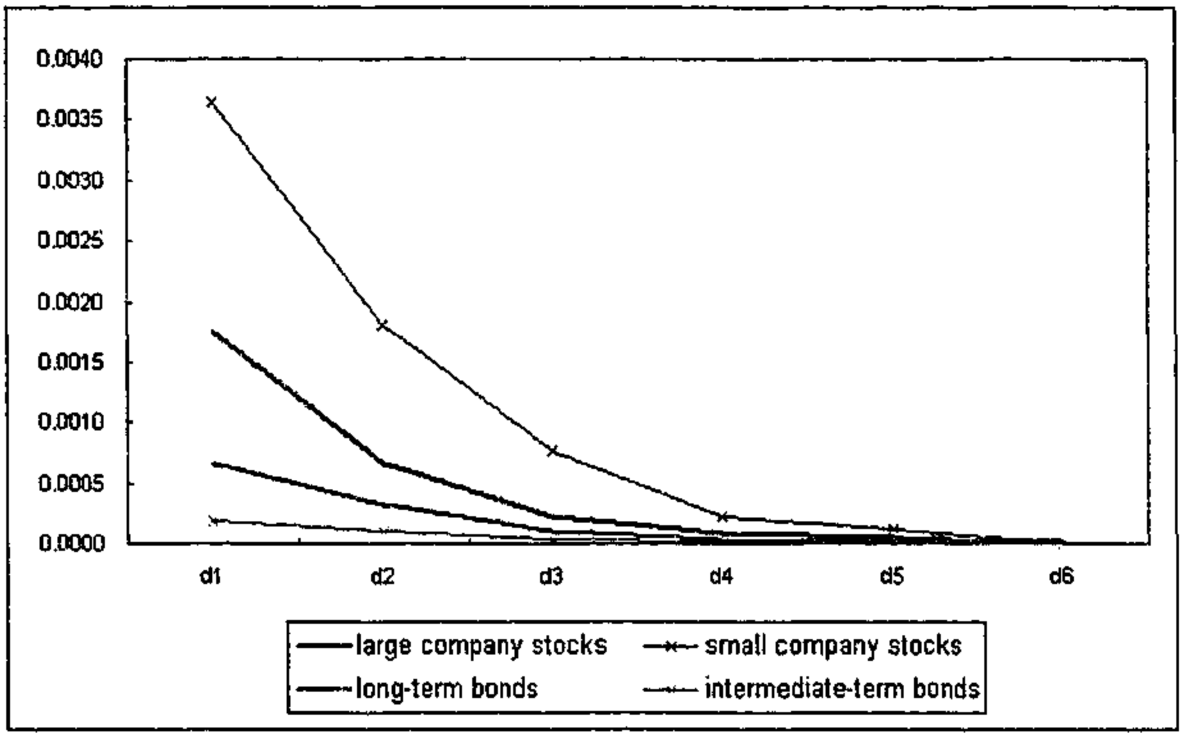
[7] For concrete discussion and estimation procedures about the wavelet-based maximum likelihood estimation, refer to Gençay et al. (2002a, p. 172).

95% confidence interval for statistical inference, we use the Cramer-Rao lower bound for the MSEs of unbiased estimators of the long-memory parameter, *d*. According to Kashyap and Eom (1988), the Cramer-Rao lower bound can be calculated as $6/(N\pi^2)$. Hence, in our case, the Cramer-Rao lower bound is 0.001188. Using this value, the 95% confidence interval can be constructed by adding and subtracting 0.068 from our estimated long-memory parameters. Clearly, the long-memory parameters for all asset returns are less than 1 and close to 0, implying that the asset returns are mean-reverting.

This test can also be interpreted in terms of a long memory in asset returns. Our results are consistent with recent empirical findings; for example, with Lo (1991) and Cheung and Lai (1995), who conclude that the stock returns do not possess the long-memory property.

It is of interest to compare the annualized wavelet variances (risks) over the different time scales. In financial valuation models, it is commonly required to calculate an annualized risk coefficient. Under the random walk model, the risk of an asset at any time scale is estimated by the linear rescaling of the risk from other time scales. Conventionally, the risk is scaled at the square root of time. Our results are presented in Figure 5.2. As observed in the figure, the annualized variances decrease as the time scale increases, with a smoother decreasing pattern. This result implies that an investor with a short investment horizon has to respond to every fluctuation in the realized returns, while for an investor with a much longer horizon, the long-run risk is significantly less.

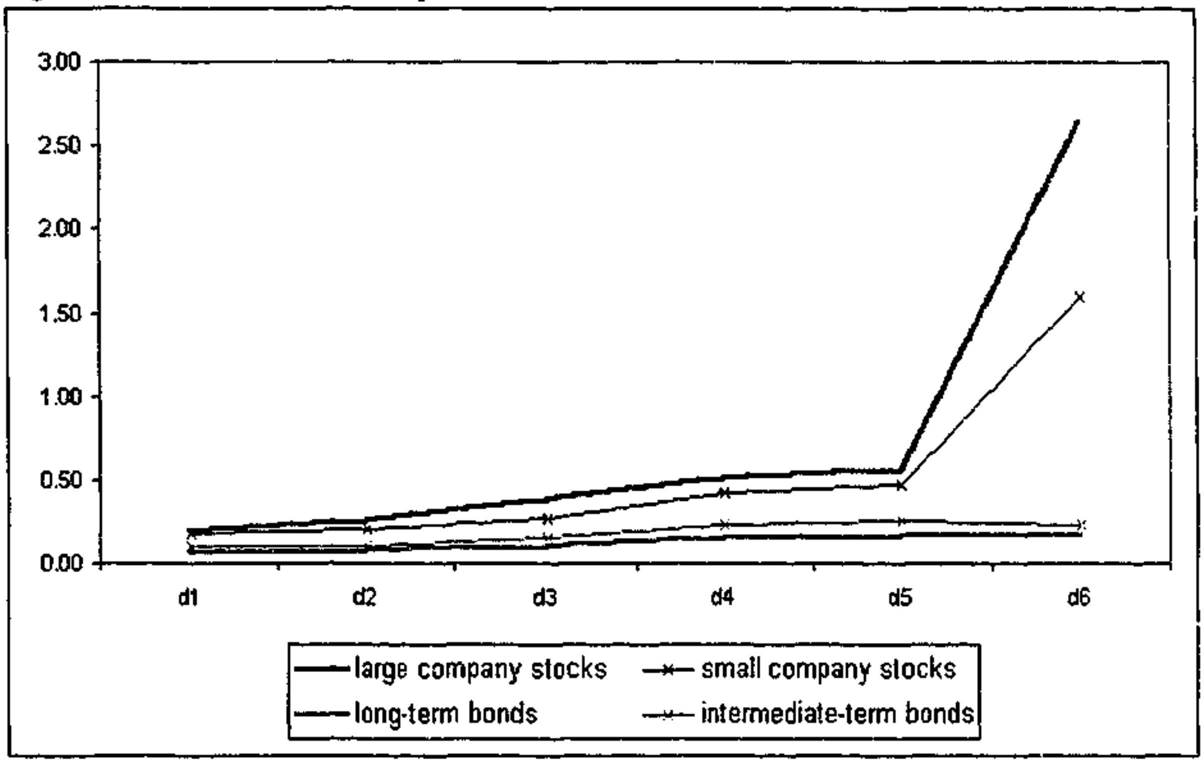
Figure 5.2 Estimated Annualized Wavelet Variance



Note: Figure 5.2 represents the annualised wavelet variance of the four portfolio returns. The y-axis indicates the wavelet variance and the x-axis indicates the wavelet time scale. To calculate the wavelet variance of each portfolio, we decompose each time series up to level 6, using the Daubechies least asymmetric wavelet filter of length 8 (LA(8)). To calculate the annualized variance, the variance is scaled at the square root of time.

Turning to our main purpose, the multihorizon Sharpe ratios, calculated from wavelet analysis, are plotted in Figure 5.3. Figure 5.3 shows that the Sharpe ratio for each portfolio increases as the holding period is extended. The Sharpe ratio for the large company stock portfolio is 0.19 for the first wavelet scale, equivalent to a 2-4 month period, and increases to 2.64 at the longest wavelet scale, equivalent to a 64-128 month period. A similar pattern is observed in the other asset returns. For example, the Sharpe ratio of long-term government bonds starts at 0.06 at the first wavelet scale, and rises to 0.17 at the longest wavelet scale. Figure 5.3 shows the relative rankings of the stock and bond portfolios over the wavelet scales. The Sharpe ratios for the stock portfolios are greater than those of bond portfolios at all time scales. Overall, our results are different from those of Hodges et al. (1997), who find that the Sharpe ratio eventually declines in all cases and the counterintuitive result that bonds become more attractive than stocks for long holding periods. However, our results support the conventional wisdom of money managers that for investors with long horizons, a greater share of portfolio assets should be allocated to stocks (Siegel, 1998, 1999).

Figure 5.3 Multihorizon Sharpe Ratio



Note: Figure 5.3 indicates the multihorizon Sharpe ratio calculated from the wavelet variance. The y-axis indicates the multihorizon Sharpe ratio and the x-axis indicates the wavelet time scale. To calculate the multihorizon Sharpe ratio of each portfolio at scale λ_j, we decompose each time series up to level 6, using the Daubechies least asymmetric wavelet filter of length 8 (LA(8)).

This result is closely related to the mean-reverting property of stock returns. As indicated in Samuelson (1991), mean reversion increases stock holdings if investors have a risk-aversion coefficient greater than unity. Furthermore, Barberis (2000) finds that mean reversion in returns slows the growth of conditional variances of long-horizon returns, as shown in Figure 5.1. This makes the equities appear less risky at long horizons, and hence more attractive to the investor.

In sum, our results indicate that the Sharpe ratio of a large-company stock portfolio is a higher value than the other three portfolios (small-company stocks, long-term and intermediate-term government bonds) over all wavelet scales. In other words, a large-company stock portfolio outperforms the other portfolios over the wavelet scales.

5.5 Concluding remarks

Although the Sharpe ratio has become an important part of modern financial analysis, its applications have not been accompanied by examination of the investment horizon, which is an important factor for investors. In this section, we examine the multihorizon Sharpe ratio using the newly developed method of wavelet analysis. Wavelet analysis has the advantage of being able to decompose the time series over the various time scales. This advantage allows us to investigate the behavior of our data over multiple horizons, not just short- and long-run behavior.

Based on this advantage of wavelet analysis, we examine the multihorizon Sharpe ratio. First, we examine the mean-reverting property of four asset returns using the wavelet-based maximum-likelihood estimation of the long-memory parameter.

Second, we adopt real returns, not nominal returns, because that is what investors really care about. Finally, as stated in Siegel (1999), while the assumption that the return is distributed identically and independently over time makes it easy to calculate long-horizon return and standard deviation, it does not reflect mean reversion on stock returns. Our wavelet analysis does not impose any assumption on distribution of returns and therefore is robust to misspecification, which may originate from assuming a particular distribution of returns.

Our wavelet decomposition shows that the risks of all returns is lower than or similar to the expected value by a random walk assumption of asset returns, while in the intermediate scales, d2 to d5 (equivalent to a 4-8 months period and 32-64 months period, respectively), the risks of all returns are greater than expected. Overall, our result supports the mean-reversion property of asset returns. In addition, the long-memory parameters for all asset returns, using the wavelet-based maximum-likelihood estimation, are less than 1 and close to 0, implying that the asset returns are mean-reverting.

The wavelet variance and the annualized wavelet variance show that the annualized variances decrease as the time scale increases, implying that that an investor with a short investment horizon has to respond to every fluctuation in the realized returns, while for an investor with a much longer horizon, the long-run risk is significantly less.

Turning to our main purpose, our results are different from those of Hodges et al. (1997), who find that the Sharpe ratio eventually declines in all cases and the counterintuitive result that bonds become more attractive than stocks for long holding periods. However, our results support the conventional wisdom of money managers that for investors with long horizons, a greater share of portfolio assets should be allocated to stocks (Siegel, 1998 and 1999).

In short, our results indicate that the Sharpe ratio of a large-company stock portfolio is a higher value than the other three portfolios (small-company stocks, long-term and intermediate-term government bonds) regardless of wavelet time scales. In other words, a large-company stock portfolio outperforms the other portfolios over the wavelet scales. This result is closely related to the mean-reverting property of stock returns. As indicated in Samuelson (1991) and Barberis (2000), a mean-reversion property in stock returns slows the growth of conditional variances of long-horizon returns. This makes equities appear less risky at long horizons, and hence more attractive to the investor.

Chapter 6 The Long-run Relationship between Risk and Return Using Industry Data

6.1 Introduction

The relationship between risk and return is fundamental to equilibrium asset pricing. The return on market portfolio plays a central role in the capital asset pricing model (CAPM), whose implication is that investors are only compensated for bearing the systematic covariance risk (Chiao, et al., 2003). However, the intertemporal properties of stock returns are not yet fully understood. For example, there is an ongoing debate in the literature about the relationship between stock market risk and return and the extent to which stock market volatility moves stock prices. To find the relationship between risk and return, much published research has used CAPM or Merton's (1973) ICAPM. In that research estimates of the simple risk-return relationship vary from significant positive (Harvey, 1989) to significant negative (Campbell, 1987).[1]

The failure to reach an agreement on the risk-return relationship can be attributed to three factors. First, neither the conditional return nor the conditional variance is directly observable. To overcome this problem, instrumental variable (IV) models and the autoregressive conditional heteroskedasticity (ARCH) models are popularly adopted popularly as identification methods. However, empirical results

[1] Backus and Gregory (1993) have studied the relationship between risk and return in a dynamic asset pricing model and found that the relationship can be either positive or negative. Recently,

are sensitive to these restrictions. For example, Campbell (1987) finds that the results depend on the choice of instrumental variables. In particular, the nominal risk-free rate is negatively related to the return and positively related to the variance, and "these two results together give a perverse negative relationship between the conditional mean and variance for common stock" (Campbell, 1987). As for the ARCH model, if the conditional distribution of the return shock is changed from normal to student-*t*, the significant positive relationship disappears (see Baillie and DeGennaro, 1990). Second, as indicated in Whitelaw (1994), no functional form is imposed on the relationship between the mean and volatility of return. Finally, while the stock is considered to be a long-term investment, the previous studies have focused on the short-run relationship.

Our study extends the previous studies in three aspects. First, to avoid the sensitivity inherent in choosing the conditional econometric model (GMM or ARCH type model) to get the conditional return and volatility, we adopt the method of Campbell et al. (2001) and construct the monthly market return and volatility directly from the data. Second, we focus on the long-run relationship between risk and return, adopting the methods of King and Watson (1997) and Den Haan (2000). Finally, the industry-level data has been constructed and used to examine the long-run relationship in contrast to the previous studies, which use

Elsas et al. (2003) examine the relationship between beta and return for the German stock market and find a significant relationship between beta and return.

the market portfolio. In our study, we construct monthly 13-industry portfolios[2] and a market portfolio from the daily individual industry data calculated ain the s same way of Campbell et al. (2001).

We are interested in the long-run relationship for two reasons. First, in the short-run, the relationship between risk and return could be affected by various factors, such as market frictions and investors' sentiments. That is, evidence at these longer horizons may provide additional information regarding explanations for a negative or positive correlation between excess stock return and variance. Second, from a practical point of view, many investors hold stocks over long holding periods. Therefore, it is important to know the manner in which stock excess returns move with variance over longer horizons.[3]

In our study, it is found that the long-run relationship highly depends on the contemporaneous relationship. More precisely, the long-run response of stock return to a permanent volatility shock relies on the value of the mean-in-volatility effect in the sense of Brandt and Kang (2001). The other results, which include

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[2] The importance of industry factors is emphasized in early literatures (Lessard, 1974 and 1976). Recently, the importance of industry factors has emerged in relation to momentum in stock returns (Moskowitz and Grinblatt, 1999; Griffin and Stulz, 2001).

[3] Few studies have examined the long-run relationship between risk and return. Harrison and Zhang (1999) find the significantly positive relation at long holding horizons, particularly one and two years. Whitelaw (2000) examines the relationship in a general equilibrium framework and finds that the long-run relationship between expected returns and volatility is less obvious because of potential time variation in the conditional correlation, the conditional volatility of the marginal rate of substitution, and the riskless rate.

two identifying parameters ($\lambda_{s\sigma}$ and $\gamma_{\sigma s}$) and measure the mean-in-volatility effect and the long run response of volatility to a permanent stock return shock, also indicates that the long-run response of stock returns to a permanent volatility shock depends on the two identifying parameters. For the VAR forecast correlation, proposed by Den Haan (2000), most industry portfolios show a negative relationship in the short-run as well as in the long-run. However, the construction industry shows a positive relationship in the short-run and negative relationship in the long-run. For the market portfolio, a negative relationship is dominant regardless of forecasting horizons.

This chapter is organized as follows. In section 6.2, we describe the derivation of the monthly return and volatility from the industry data. The data set and basic statistics are presented in section 6.3. In section 6.4, the King and Watson's (1997) method is presented and the empirical results are discussed. The long run correlation coefficients from the VAR forecast error are discussed in section 6.5. Finally, we present our concluding remarks in section 6.6.

6.2 Data and Basic Statistics

In our study, the monthly volatility is estimated by the square of the sum of the difference between daily stock return and monthly average. This method is adopted in Duffee (1995) and Campbell et al. (2001).[4]

$$\sigma_{i,t}^2 = \sum_{j=1}^{st} (R_{i,j} - \mu_i)^2 \tag{6.1}$$

where $R_{i,j}$ is the industry return on the jth day of month t, st denotes the number of trading days in the specific month, and μ_i indicates the average return in that month.

The daily industry stock prices are obtained from Datastream. Three-month Treasury bill yields for the risk-free rate are from the US Federal Reserve system (H.15). Taking the natural logarithm of one plus the return and subtracting the natural logarithm of one plus the monthly Treasury bill yields divided by trading days in a month yield the daily excess return. This monthly data has the advantage of reducing noise and the disadvantage of being slightly stable. From this

[4] Another method to calculate the volatility, proposed by French et al. (1987), is the volatility estimate that adjusts for first-order autocorrelation in returns: $\sigma_{i,t}^2 = \sum_{j=1}^{st} (R_{i,j} - \mu_i)^2 + 2\sum_{j=1}^{st} (R_{i,j} - \mu_i)(R_{i,j+1} - \mu_i)$. As noted in Duffee (1995), this method can generate a negative variance estimate if the first-order autocorrelation of daily returns in a given month is less than -0.5.

calculation the total data ranges from January 1973 to June 2000 with 330 observations.[5]

Following Li (1998), we reorganize this data from 102 industrial industries into 13 portfolios, and the stock prices of 13 industries are calculated as a weighted average at time t. Each industry's market value has been adopted as the weight of the industry.[6]

In Figure 6.1, the movements of market return and volatility are plotted during the sample period. The figure shows the high volatility during the 1970s due to the oil and food shock and following the stock market crash in October 1987. During those periods, high volatility is accompanied by low returns. However, there is no distinguishable trend in both return and volatility, consistent with the results of Campbell et al. (2001). This allows us to use no trend in our VAR method. In Figure 6.2, we plot the movements of returns of two industries: petroleum and

[5] To check the stationarity of our data, two statistics for unit root tests (ADF and KPSS proposed by Kwiatkowski et al., 1992) are adopted. From both tests, stationary cannot be rejected for our data.

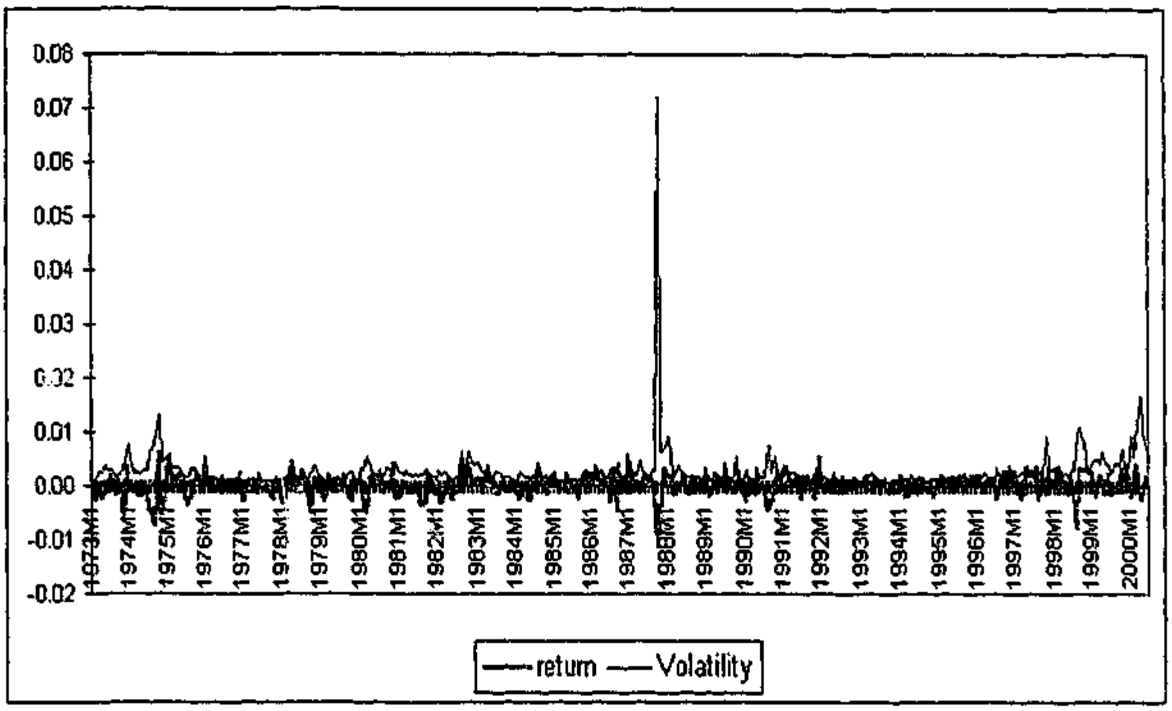
[6] Li (1998) in fact actually uses 12 categories, but we add one more industry named "diversified industries". According to Datastream, the diversified industry category is defined as industrial companies engaged in three, or more, classes of business that differ substantially from each other, no one of which contributes 50%, or more, of pre-tax profit, nor less than 10%. This industry is hard to divide following the SIC code. Furthermore, the market value of this industry is bigger than those of transportation and construction at the end of June 2000. For these reasons, we consider this industry as an individual industry.

Table 6.1 Industry Groups

Industry	Sub-industry
Basic Industry	Gold Mining, Packaging, Paper, Chemicals, Advanced Materials, Chemicals (Commodity), Chemicals (Speciality), Household Products, Personal Products, Engineering Fabricators, Non-Ferrous Metals, Steel, Forestry, Other Mineral Extractors & Mines, Other Business.
Capital Goods	Computer Hardware, Engineering General, Semiconductors, Tyres & Rubbers, Medical Equipment & Supplies, Photography Broacasting Contractors, Electrical Equipment, Electricity, Electronic Equipment,
Consumer Durable	Household Appliances & Housewares, Telecom Equipment, Aerospace, Automobiles, Autoparts, Commercial Vehicles & Trucks, Defence, Builders Merchants, Distributors of Industrial Components & Equipment, Vehicle Distribution
Construction	House Building, Other Construction, Engineering-Contractors, Building & Construction Materials
Finance/Real Estate	Banks, Consumer Finance, Mining Finance, Mortgage Finance, Other Financial, Asset manager, Investment Banks, Investment Company, Insurance Brokers, Insurance-Non-life, Life Assurance, Re-insurance, Other Insurance, Property Agencies, Real Estate Holding & Development, Real Estate Investment Trusts
Food/Tobacco	Beverages-Brewers, Beverages-Distillers & Vintners, Food & Drug Retailers, Soft Drinks, Tobacco, Farming & Fishing
Diversified Industrials	-
Leisure	Publishing & Printing, Restaurants & Pubs, Hostel, Gaming, Home Entertainment, Leisure Equipment, Leisure Facilities,Photography
Petroleum	Oil & Gas-Exploration & Production, Oil-Services, Oil-Integrated
Services	Funerals & Cemeteries, Laundries & Cleaners, Business Support Services, Computer Services, Furnishing & Floor Coverings, Internet, Security & Alarm Services, Software, Health Maintenance Organizations, Hospital Management & Long Term Care, Other Health Care, Education, Business Training & Employment Agencies, Retailers e-commerce, Retailers-Hardlines, Retailers-Multi Department, Retailers-Soft Goods, Medical Equipment & Supplies, Pharmaceuticals, Environmental Control
Textiles/Trade	Clothing & Footwear, Other Textiles & Leather Goods, Distributors-Other, Discount & Super Stores and Warehouses,
Transportation	Shipping & Ports, Airlines & Airports, Rail, Road & Freight
Utilities	Cable & Satellite, Fixed-Line Telecommunication Services, Media Agencies, Wireless Telecommunication Services, Gas Distribution

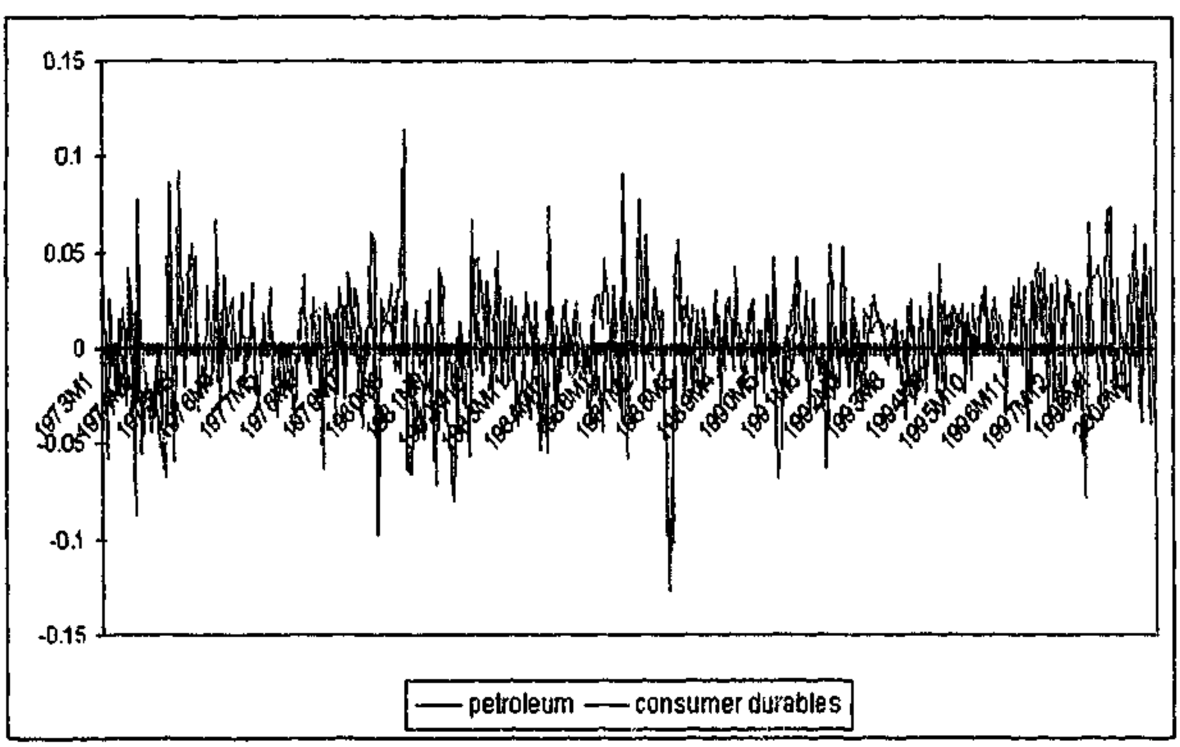
Note: This table shows the 13-industry groups using 102 industry sub-groups using the SIC code. This table has been constructed using Li (1998) except for Diversified industrial. According to Datastream, the diversified industry is defined as industrial companies engaged in three, or more, classes of business.

Figure 6.1 Movements of Market Returns and Volatility



Note: This figure shows the movements of monthly market risk and returns during sample period from January 1973 to June 2000. Clearly the peak has been observed in October 1987.

Figure 6.2 Movements of the returns of Petroleum and Consumer Durables



Note: This figure shows two industry returns during sample periods. From this figure, we can observe that even though the same shock affects the stock market, each industry has a different reaction on the shock.

consumer durables. This figure shows the different peaks and troughs during the sample period. It is evident that the movement of each industry is affected not only by whole market shocks but also by individual industry shocks. As an example of a whole market shock, the figure shows that when stock market crashes in October 1987, both returns decrease severely. An example of a individual industry shock occurs the Persian Gulf War in January 1991. Note that the petroleum companies are severely affected by the oil supply from the Persian region, and that the consumer durable industry includes defence as a sub-industry. In January 1991, the annualized (multiplied by 12) return of the petroleum industry is –0.120, while that of the consumer durable industry is 0.117. When the same shock hits the stock market, it affects the industry differently. Figure 2 provides evidence as to why the analysis of individual industry is important.

Table 6.2 shows us the basic statistics for industry and market returns. The most profitable industry during the sample period is the services industry, while the construction industry is the worst. In terms of volatility of excess return, transportation is observed as a most volatile industry. The measures for skewness and kurtosis are also reported to check whether monthly returns are normally distributed. The sign of skewness varies; however, the magnitudes depend on the particular industry.

Among the 13 industry portfolios and the market portfolio, the first-order autocorrelation of monthly returns ranges from –0.039 (utility) to 0.126 (leisure), and the Ljung-Box statistics indicate the persistence of linear dependency of each set of data. For the squared level data, the first-order serial correlations vary

Table 6.2 Basic Statistics

Panel A Descriptive Statistics

	Mean	Variance	Skewness	Kurtosis	JB	Significance level	ρ	LB(15)	Significance level	ρ^2	LB(15)	Significance level
Market	0.008	0.046	-0.751	2.805	139.165	0.000	0.017	11.219	0.510	0.197	27.154	0.007
Basic	0.000	0.064	-0.613	3.393	178.923	0.000	0.009	13.577	0.329	0.047	15.397	0.221
Capital	0.010	0.073	-0.343	1.380	32.641	0.000	0.002	10.323	0.588	0.215	30.867	0.002
Consumer	0.008	0.049	-0.452	1.906	61.179	0.000	0.051	11.705	0.470	0.063	25.361	0.013
Construction	-0.003	0.115	-0.257	1.475	33.538	0.000	0.104	21.954	0.038	0.100	33.580	0.001
Finance	0.013	0.073	-0.436	1.708	50.547	0.000	0.100	21.367	0.045	0.096	14.101	0.294
Food	0.016	0.065	-0.632	2.531	110.109	0.000	0.082	28.187	0.005	0.268	62.611	0.000
Industrial	0.014	0.073	-0.607	3.560	194.443	0.000	-0.020	8.658	0.732	0.143	38.810	0.000
Leisure	0.005	0.076	-0.553	2.159	80.915	0.000	0.126	34.970	0.000	0.194	47.174	0.000
Petroleum	0.001	0.068	-0.072	1.167	19.029	0.000	-0.045	11.675	0.472	0.314	89.250	0.000
Services	0.023	0.072	-0.437	1.408	37.756	0.000	-0.015	11.400	0.495	0.245	39.935	0.000
Textile	0.007	0.109	-0.439	1.922	61.358	0.000	0.112	26.830	0.008	0.105	14.981	0.243
Transportation	0.005	0.094	-0.520	2.822	124.418	0.000	0.042	10.472	0.575	0.114	14.102	0.294
Utility	0.006	0.043	-0.172	0.513	5.241	0.073	-0.039	15.994	0.192	0.124	32.063	0.001

Note: Significance levels are in parentheses. LB(n) is the Ljung-Box statistic for up to n lags, distributed as χ^2 with n degrees of freedom. ρ and ρ^2 indicate the first-order autocorrelations of returns and squared returns, respectively. Skewness and kurtosis are defined as $E[(R_t-\mu)]^3$ and $E[(R_t-\mu)]^4$, where μ is the sample mean. JB indicates the Jarque-Bera statistics.

Panel B Correlation Matrix

	Basic	Capital	Consumer	Construction	Finance	Food	Industrial	Leisure	Petroleum	Service	Textile	Transportation	Utility	Market
Basic	1.000	0.709	0.827	0.771	0.785	0.737	0.810	0.555	0.737	0.775	0.708	0.554	0.774	0.885
Capital		1.000	0.764	0.627	0.616	0.621	0.754	0.451	0.792	0.609	0.592	0.471	0.676	0.839
Consumer			1.000	0.777	0.806	0.693	0.795	0.517	0.787	0.729	0.728	0.692	0.734	0.939
Construction				1.000	0.753	0.636	0.725	0.576	0.648	0.777	0.650	0.548	0.695	0.819
Finance					1.000	0.745	0.780	0.503	0.748	0.716	0.707	0.634	0.704	0.863
Food						1.000	0.774	0.397	0.745	0.570	0.697	0.556	0.704	0.787
Industrial							1.000	0.427	0.809	0.681	0.748	0.560	0.758	0.870
Leisure								1.000	0.416	0.595	0.285	0.426	0.488	0.638
Petroleum									1.000	0.638	0.734	0.580	0.738	0.882
Service										1.000	0.573	0.501	0.659	0.799
Textile											1.000	0.513	0.621	0.760
Transportation												1.000	0.533	0.713
Utility													1.000	0.806
Market														1.000

between 0.047 (basic industry) and 0.314 (petroleum), and the Ljung-Box statistics show strong evidence of non-linear dependency in all data

The correlation structure among the industries (Panel B of Table 6.2) is probably the most important feature from the point of view of investors and portfolio managers. Hedging and diversification strategies invariably involve some measure of correlation. As is shown in Panel B of Table 6.2, the correlations of excess return range from 0.285 between petroleum and textiles to 0.827 between basic industry and consumer durables.

6.3 King and Watson Approach

In this section, we present the King and Watson (1997) methodology for examining the long-run relationship between stock returns and volatility. The King and Watson methodology relies on a bivariate VAR model in stationary variables.

6.3.1 VAR Model

Let $s_{i,t}$ and $\sigma^2_{i,t}$ be the ith industry (market) excess return and volatility at time t, respectively. Consider the following bivariate VAR model:

$$s_{i,t} = \lambda_{s\sigma}\sigma_t + \sum_{j=1}^{p}\alpha_{j,ss}s_{i,t-j} + \sum_{j=1}^{p}\alpha_{j,s\sigma}\sigma^2_{i,t-j} + \varepsilon^s_t \qquad (6.2)$$

$$\sigma_{i,t}^2 = \lambda_{\sigma s} s_t + \sum_{j=1}^{p} \alpha_{j,\sigma s} s_{i,t-j} + \sum_{j=1}^{p} \alpha_{j,\sigma\sigma} \sigma_{i,t-j}^2 + \varepsilon_t^{\sigma} \qquad (6.3)$$

where ε_t^s and ε_t^{σ} are white-noise stock excess returns and volatility of excess returns structural shocks, respectively. This form of equation (6.2) for the stock excess return can be analogous to a generalized version of the usual volatility-in-mean model, where this effect can be captured by the coefficient, $\lambda_{s\sigma}$. In addition, the model can also capture the lag-volatility-in-mean effect, depending on the coefficients $\alpha_{j,s\sigma}$. If this term is positive (negative), the lagged volatility has a positive (negative) effect on the stock excess returns. It is of interest because it reflects the volatility feedback effect studied by Campbell and Hentschel (1992), and Bekaert and Wu (2000) among others. The volatility feedback effect states that a large realization of news (positive or negative) increases both current and future volatility due to the persistence of volatility (Brandt and Kang, 2001). Intuitively, the lagged volatility effect is important because it reflects the compensation of investors for the increased risk, implying the positive intertemporal relationship between stock returns and volatility. Analogous to equation (6.2), equation (6.3) contains two effects: the mean-in-volatility effect and the lagged-mean-in-volatility effect. Contrary to the previous studies, except for Brandt and Kang (2001), our model can capture both the contemporaneous effect through mean-in-volatility and the lagged effect through the lagged-mean-in-volatility effect. The system can be written more compactly by:

$$\alpha_{ss}(L)s_t = \alpha_{s\sigma}(L)\sigma_t^2 + \varepsilon_t^s$$

$$\alpha_{\sigma\sigma}(L)\sigma_t^2 = \alpha_{\sigma s}(L)s_t + \varepsilon_t^\sigma$$

where $\alpha_{ss}(L) = 1 - \sum_{j=1}^{p} \alpha_{j,ss} L^j$, $\alpha_{s\sigma}(L) = \lambda_{s\sigma} + \sum_{j=1}^{p} \alpha_{j,s\sigma} L^j$,

$\alpha_{\sigma\sigma}(L) = 1 - \sum_{j=1}^{p} \alpha_{j,\sigma\sigma} L^j$, and $\alpha_{\sigma s}(L) = \lambda_{\sigma s} + \sum_{j=1}^{p} \alpha_{j,\sigma s} L^j$. Letting $\varepsilon_t = (\varepsilon_t^s, \varepsilon_t^\sigma)'$, we define $E(\varepsilon_t, \varepsilon_t)' = \Sigma_\varepsilon$, the variance-covariance matrix for the structural shocks. We are interested in the long-run (infinite-horizon) stock prices response to a volatility shock. This can be expressed in terms of the long-run multipliers, $\gamma_{s\sigma} = \alpha_{s\sigma}(1)/\alpha_{ss}(1)$, which give the percentage increase in stock returns for each percentage point increase in volatility resulting from a permanent volatility shock. To obtain a consistent estimate of $\gamma_{s\sigma}$, it is required to identify the system of equations (6.2) and (6.3). As the system now stands, it is underidentified. King and Watson (1997) and Rapach (2002) use the three identifications for each estimate of $\gamma_{s\sigma}$. In other words, for the estimation of $\gamma_{s\sigma}$, three coefficients should be known; (1) $\lambda_{s\sigma}$ (2) $\lambda_{\sigma s}$ (3) $\gamma_{\sigma s}$. For our purposes, another assumption is that Σ_ε is diagonal. This is a standard assumption in structural VAR modelling and is tantamount to assuming that the structural shocks are contemporaneously uncorrelated. Note that as long as $\lambda_{s\sigma}$ or $\lambda_{\sigma s}$ does not equal zero, the restriction that Σ_ε is diagonal does not preclude both stock excess returns and volatility of excess returns from responding contemporaneously to either structural shock. We next discuss the economic interpretation of each restriction.

In addition to the restriction that Σ_ε is diagonal, the first identification scheme assumes that $\lambda_{s\sigma}$ is known, which restricts the contemporaneous response of stock excess returns to a permanent volatility shock. In this case, the natural question is how the volatility affects stock returns. The coefficient $\lambda_{s\sigma}$ measures the contemporaneous relationship between stock excess return and volatility. It nests the volatility-in-mean model to study the contemporaneous effect of volatility on excess returns. Previous studies state that $\lambda_{s\sigma}$ can be negative or positive.

Assuming that $\lambda_{\sigma s}$ is known, as in the second identification scheme, the contemporaneous volatility response to a permanent stock excess returns shock is restricted. This coefficient captures the effect of mean-in-volatility. While Brandt and Kang (2001) find that the sign of this effect is negative, they argue that the mean-in-volatility effect mirrors the volatility-in-mean effect. Based on this, we assume that the sign of $\lambda_{\sigma s}$ can be negative or positive.

Finally, assuming that $\gamma_{\sigma s} = \alpha_{\sigma s}(1) / \alpha_{\sigma\sigma}(1)$ is known, as in the third identification scheme, the volatility's long-run response to a permanent stock market shock is restricted. As noted in definition of $\gamma_{\sigma s}$, it contains the contemporaneous effect and intertemporal effect of stock excess returns on the volatility. While the intertemporal effect (the lagged-mean-in-volatility effect) is negative, the contemporaneous effect (the mean-in-volatility effect) is ambiguous. Therefore, the sign of $\gamma_{\sigma s}$ can be negative or positive.

Once the system of equations is identified using one of the three schemes, the parameters of (2) and (3) can be consistently estimated using an instrumental variables procedure. With consistent estimates of the system's parameters in hand, a consistent estimate $\gamma_{s\sigma}$ can be generated. Following King and Watson (1997) and Rapach (2002), we genera·: estimates of $\gamma_{s\sigma}$ using each of the three identification schemes. Instead of generating $\gamma_{s\sigma}$ estimates for a single assumed value of $\lambda_{s\sigma}$, $\lambda_{\sigma s}$ and $\gamma_{\sigma s}$, as in King and Watson (1997) and Rapach (2002), $\gamma_{s\sigma}$ estimates are generated for a range of $\lambda_{s\sigma}$, $\lambda_{\sigma s}$ and $\gamma_{\sigma s}$ values about zero. This checks the robustness of long-run inferences and allows the readers to decide which estimates are most plausible.

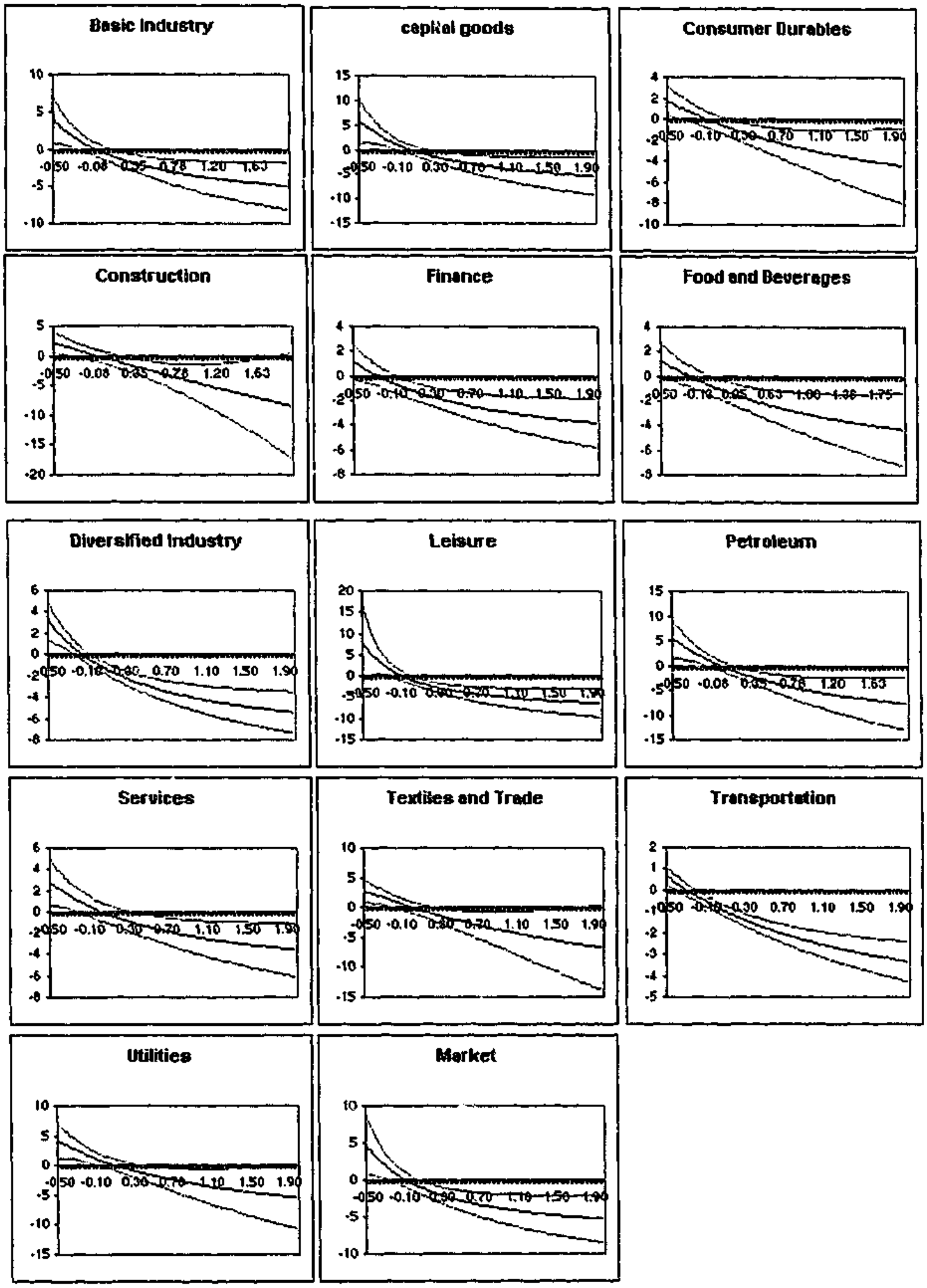
6.3.2 Empirical Results

In this subsection, we discuss the empirical results derived from the King and Watson (1997) method. Because the VAR model requires the stationarity of each variable, we do the unit root tests of our data set.

Figure 6.3 presents point estimates of $\gamma_{s\sigma}$ based on the first identification scheme (Σ_{ε} is diagonal and $\lambda_{\sigma s}$ is known) for each industry and the market portfolio. The grey lines in each panel of Figure 6.3 delineate 95 % confidence bands for the $\gamma_{s\sigma}$ estimates.

We see from Figure 6.3 that the $\gamma_{s\sigma}$ point estimate is decreasing in the assumed value of $\lambda_{\sigma s}$ for each industry. For a number of industries, a different value of $\lambda_{\sigma s}$

Figure 6.3 $\gamma_{s\sigma}$ Estimates for Different Assumed $\lambda_{\sigma s}$ Identifying Values

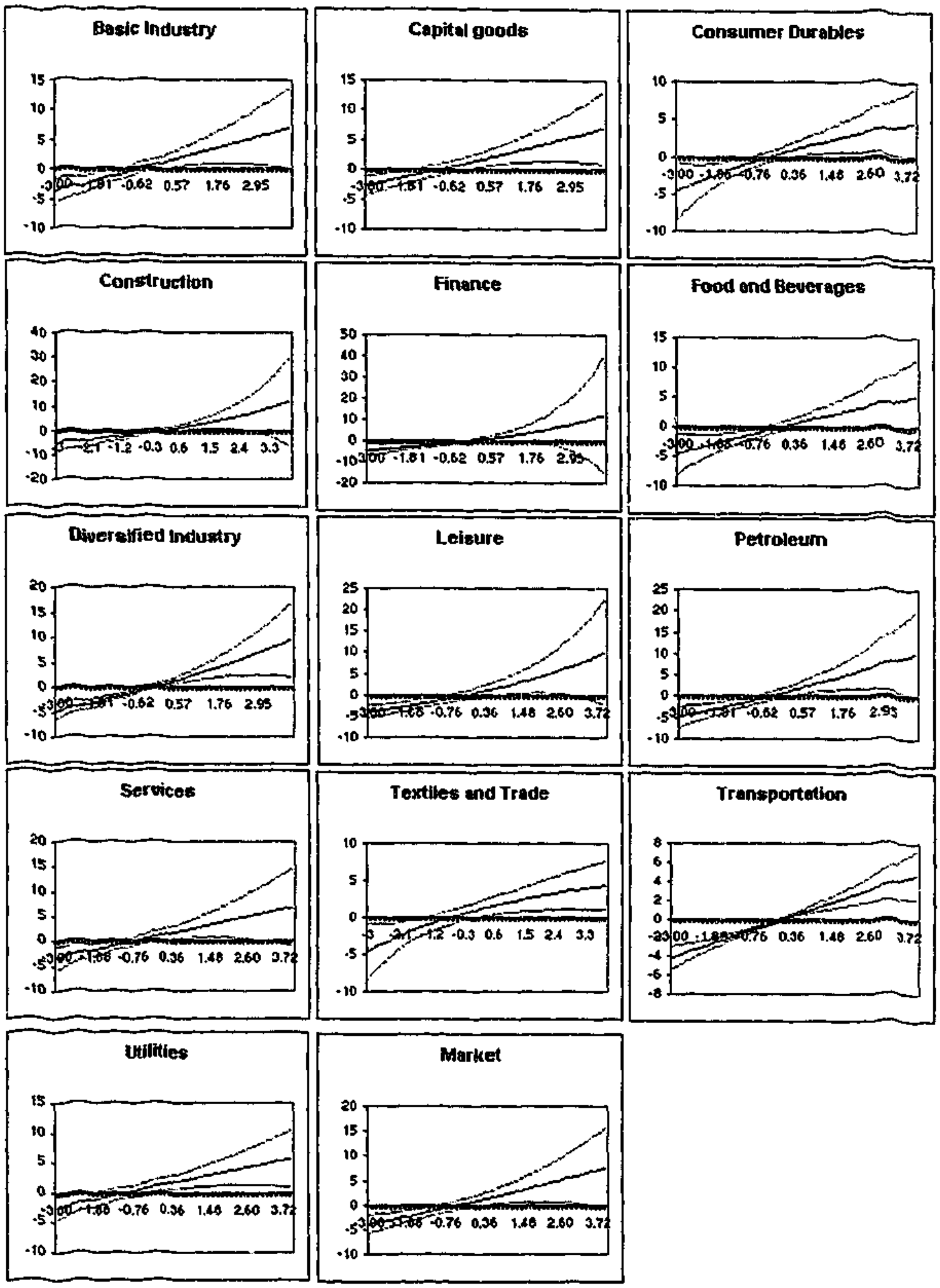


Note: This figure presents point estimates of $\gamma_{s\sigma}$ based on the first identification scheme (Σ_ε is diagonal and $\lambda_{\sigma s}$ is known) for each industry and market portfolio. The grey lines delineate 95% confidence bands for the $\gamma_{s\sigma}$ estimates.

produces a different value of $\gamma_{s\sigma}$ estimates, which are varying from negative to positive. For example, consider the basic industry. If we assume that $\lambda_{\sigma s} = -0.06$, then the $\gamma_{s\sigma}$ point estimate equals 0.11, which is significant according to the 95% confidence bands. A $\lambda_{\sigma s}$ value of –0.06 means that the volatility decreases contemporaneously by 0.6 percentage points for each 10 percent increase in the real stock returns in equation (6.3), while a $\gamma_{s\sigma}$ value of 0.11 means that long-run stock returns increase by 0.11% for each percentage point increase in volatility, resulting from a permanent volatility shock. As discussed above, theory suggests that the sign of $\lambda_{\sigma s}$ is unambiguous (negative or positive) so that $\lambda_{s\sigma}$ values between –0.05 and 0.20 do not appear quantitatively implausible.

For each industry, there is a range of $\lambda_{\sigma s}$ values for which the $\gamma_{s\sigma}$ estimate is not significantly different from zero, in line with long-run relationship with respect to portfolio returns. These ranges typically include $\lambda_{\sigma s}$ values between approximately –0.25 and 0.25. Also observe that $\gamma_{s\sigma}$ estimates are significantly negative for all industries except services and utility for a range of positive $\lambda_{\sigma s}$ values. However, given that the previous studies suggest that $\lambda_{\sigma s}$ is ambiguous, the sign of $\gamma_{s\sigma}$ highly depends on the values of $\lambda_{\sigma s}$. In other words, in the sense of Brandt and Kang (2001), the long-run effect of volatility on stock returns depends on how stock returns affect the volatility contemporaneously (mean-in-volatility effect).

Figure 6.4 $\gamma_{s\sigma}$ Estimates for Different Assumed $\lambda_{s\sigma}$ Identifying Values

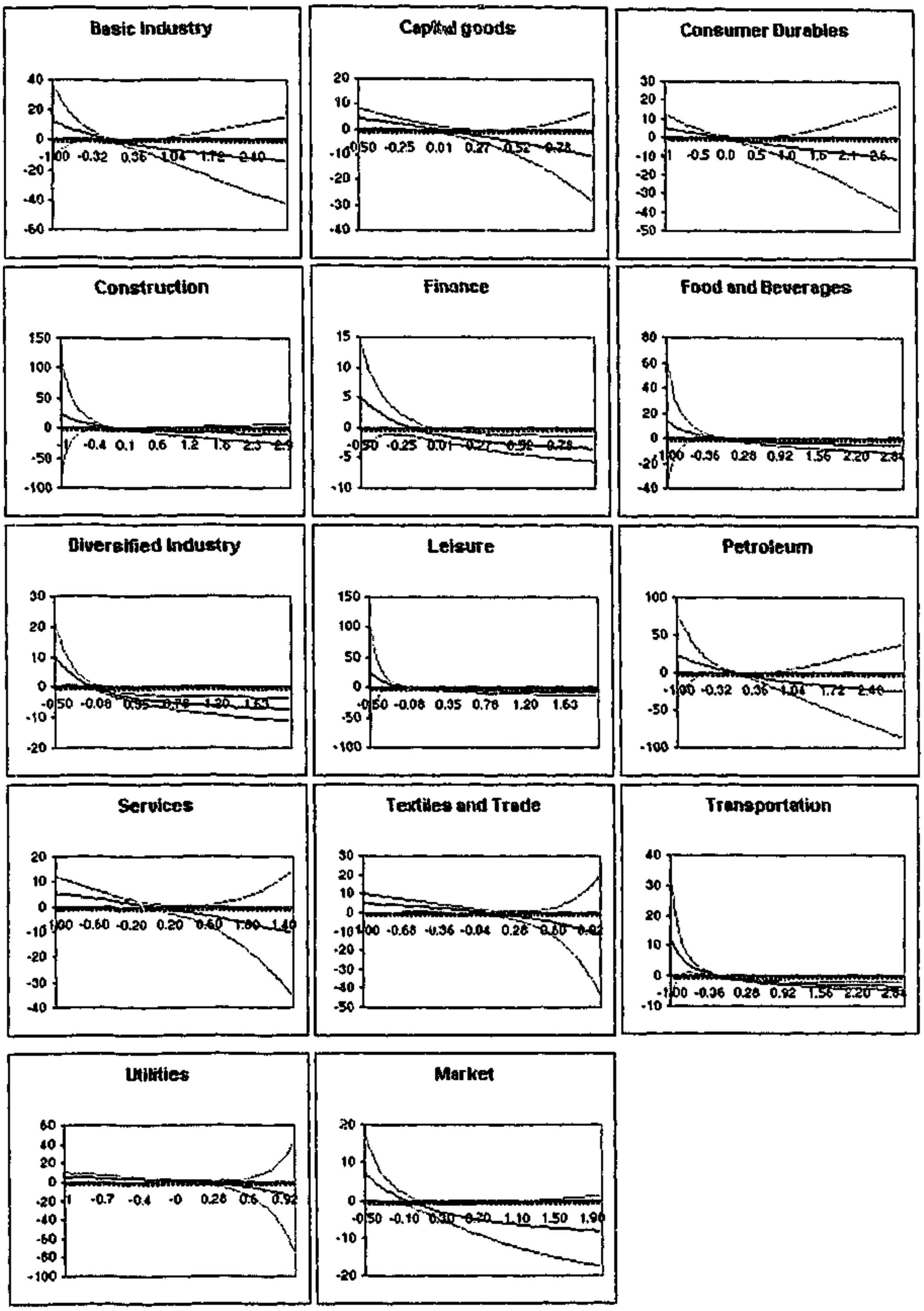


Note: This figure reports $\gamma_{s\sigma}$ point estimates based on the second identification scheme (Σ_{ε} is diagonal and $\lambda_{s\sigma}$ is known). The grey lines delineate 95% confidence bands for the $\gamma_{s\sigma}$ estimates.

For comparison with previous studies, we discuss the relationship between market returns and volatility more precisely. The last figure in Figure 6.3 illustrates $\gamma_{s\sigma}$ estimates for different assumed $\lambda_{\sigma s}$ identifying values. The general shape of plot is the same as the other industry portfolio returns and volatilities. The $\gamma_{s\sigma}$ estimate is not significantly different from zero in the range of $\lambda_{\sigma s}$ between -0.25 and 0.05, while the most positive values of $\lambda_{\sigma s}$ generate the significantly negative values of $\gamma_{s\sigma}$.

Figure 6.4 reports $\gamma_{s\sigma}$ point estimates based on the second identification scheme (Σ_{ε} is diagonal and $\lambda_{s\sigma}$ is known). The $\gamma_{s\sigma}$ estimates are increasing in $\lambda_{s\sigma}$ for each industry. Recall that positive $\lambda_{s\sigma}$ values correspond to a volatility-in-mean effect. For most industries, a $\lambda_{s\sigma}$ value less than approximately -1 yields a significantly negative $\gamma_{s\sigma}$ estimate except for the food and beverages, petroleum, and transportation industries. A $\lambda_{s\sigma}$ value of -1 means that portfolio returns decrease contemporaneously by 1 percent for each percentage point increase in volatility in equation (6.2). For $\lambda_{s\sigma}$ values near zero, the $\gamma_{s\sigma}$ estimates are typically not significantly different from zero for all industries except for consumer durables, capital, and food and beverages industries. A $\lambda_{s\sigma}$ value greater than approximately 0.05 produces a significantly positive $\gamma_{s\sigma}$ estimate. Using the basic industry again as an example, if we assume a $\lambda_{s\sigma}$ value of 0.5, then the $\gamma_{s\sigma}$ point estimate equals 1.518.

Fig 6.5 $\gamma_{s\sigma}$ Estimates for Different Assumed $\gamma_{\sigma s}$ Identifying Values



Note: This figure presents $\gamma_{s\sigma}$ estimates based on the third identification scheme (Σ_{ε} is diagonal and $\gamma_{\sigma s}$ is known). The grey lines delineate 95% confidence bands for the $\gamma_{s\sigma}$ estimates.

As in Figure 6.3, we again discuss market returns and volatility. The estimates of $\gamma_{s\sigma}$ for different assumed $\lambda_{s\sigma}$ identifying values are plotted in the last row in Figure 6.4. The shape of movements is the same as the for other industry portfolios. The $\gamma_{s\sigma}$ estimate is not significantly different from zero in range of $\lambda_{s\sigma}$ between –0.83 and 0.5, while the most positive values of $\lambda_{s\sigma}$ generate the significantly negative values of $\gamma_{s\sigma}$. The contemporaneous relationship between stock return and volatility can be captured by the coefficients $\lambda_{s\sigma}$, which is the volatility-in-mean effect, in the sense of Brandt and Kang (2001) and Campbell and Hentchel (1992). From this figure, it can be concluded that the long-run relation between stock return and volatility depends highly on the contemporaneous relationship. Roughly speaking, the positive value of contemporaneous relationship brings the positive value of long-run relationship regardless of the lagged effect of mean and volatility.

Figure 6.5 presents $\gamma_{s\sigma}$ estimates based on the third identification scheme (Σ_ε is diagonal and $\gamma_{\sigma s}$ is known). For each industry, the $\gamma_{s\sigma}$ estimates are decreasing in $\gamma_{\sigma s}$. For most industries, $\gamma_{\sigma s}$ values near zero produce $\gamma_{s\sigma}$ estimates that are not significantly different from zero except for finance, food and beverages and transportation industries. As noted above, $\gamma_{\sigma s} = 0$ corresponds to no effect of the change of stock returns on the volatility. If one is willing to make this assumption, then $\gamma_{s\sigma}$ is plausibly zero for most industries. For some industries, some negative

$\gamma_{\sigma s}$ values produce $\gamma_{s\sigma}$ estimates that are significantly positive, while for other industries some positive $\gamma_{\sigma s}$ values generate significantly negative $\gamma_{s\sigma}$ estimates.

6.4 Den Haan's VAR Forecast Correlations

In this section, we examine the relationship between risk and return at the short-run and the long-run using the VARs. To examine the long-run relationship between risk and return, the VAR method, proposed by Den Haan (2000), is adopted in our study. In section 6.4.1, we show how to derive correlation coefficients at different forecast horizon using VARs. In section 6.4.2, we discuss the forecasting relationship between risk and return estimated using this procedure.

6.4.1 Calculation of Correlation Coefficients Using VARs.

We use the implied covariance, derived in VAR model (6.2) and (6.3) to derive the correlation coefficients. In our chapter, the second method is used. In the second way, the correlation coefficients are calculated as follows. Simply, the system can be written as the following first-order VAR system with T observations and N variables.

$$Z_T = Z_{T-1}B' + u_T \qquad (6.4)$$

where Z_T is $(T \times LN)$ matrix equal to $[X_T, X_{T-1,} \cdots, X_{T-L+1}]$, u_T is a $(T \times LN)$ matrix equal to $[e_T, 0_{T,N,} \cdots, 0_{T,N}]$, and

$$B' = \begin{bmatrix} A_1^{'} & I_N & O_N & \cdots & O_N \\ A_2^{'} & O_N & I_N & \cdots & O_N \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_L^{'} & O_N & O_N & \cdots & I_N \end{bmatrix}$$

where I_N is an $(N \times N)$ identity matrix and O_N is an $(N \times N)$ zero matrix. Let COV(K) now denote the $(LN \times LN)$ variance covariance matrix of the K-period ahead forecast errors. Then, it is easy to derive the following equation:

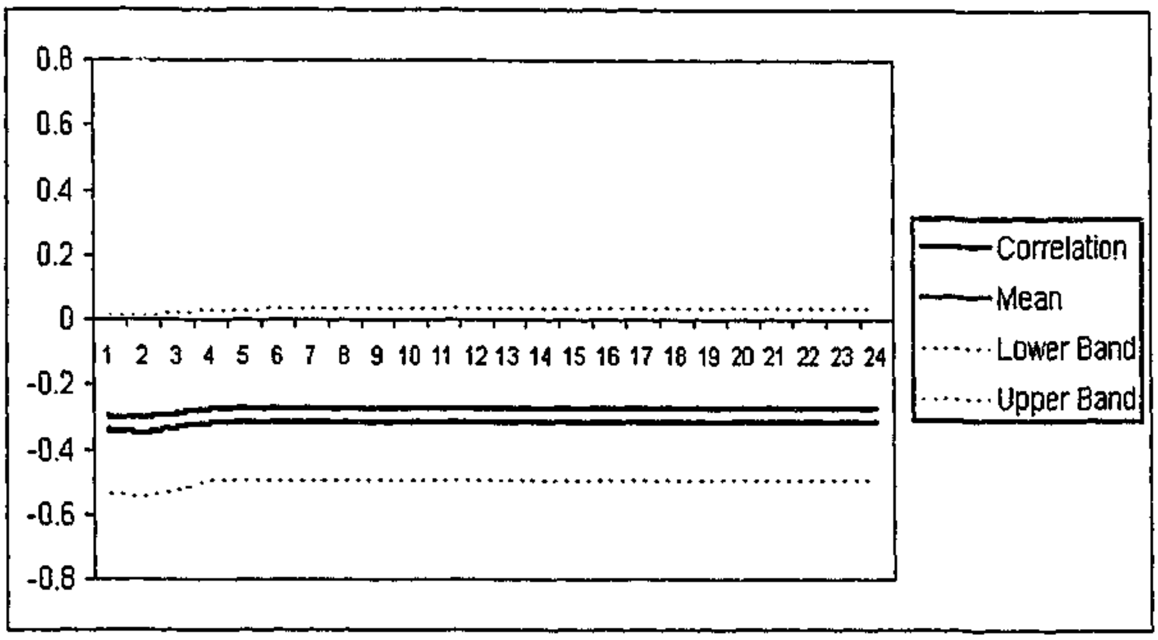
$$COV(K) = {E[Z_{T+K} - Z_T B'^K]'[Z_{T+K} - Z_T B'^K]} \big/ T = \sum_{i=0}^{K-1} B^i \Omega B'^i$$

where $F^0 = I_{NL}$ and $\Omega = {E(u_T^{'} u_T)} \big/ T$.

6.4.2 Empirical Results of VAR Forecast Correlation

In this subsection, we discuss the correlation coefficients derived from the VAR model. The comovement of return and volatility using the VAR forecast errors is presented in Figures 6.6 and 6.7. First we will discuss the results when the 13 monthly industry portfolios are used, and then we will discuss the results of the market portfolio.

Figure 6.6 Correlation Coefficients for Market Returns and Volatility

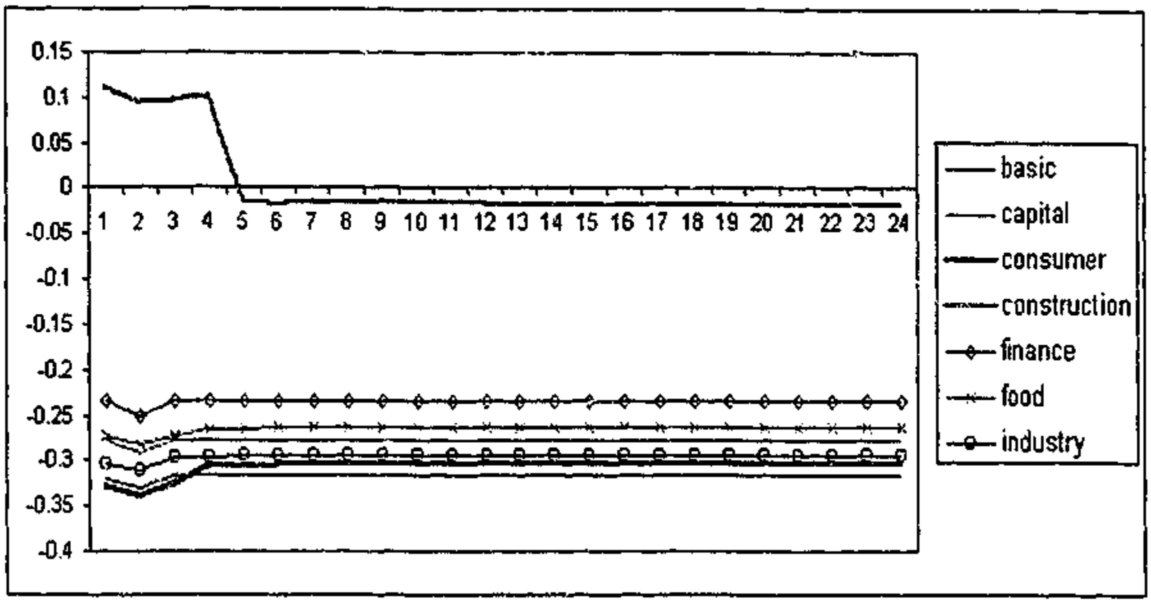


Note: In this figure, the grey dotted lines indicate the 95% confidence interval calculated using bootstrap methods. The average correlation coefficients, indicated as a mean is the average correlation coefficients calculated from bootstrap method.

The results of the market portfolio are shown in Figure 6.6, in which we report the confidence intervals as well as the average correlation coefficients across replications. As mentioned in Den Haan (2000), the calculated correlation coefficients are subject to sampling variation because they are based on the estimated VARs. For this reason, we calculate the confidence level using bootstrap methods. More specifically, the estimated VAR and its bootstrapped errors are used to generate 3000 economies. The correlation coefficients with up to 24 months forecasting horizons are calculated for each economy. The average correlation coefficients are plotted in Figure 6.6 as a mean. As can be seen in the figure, the average across replications is very similar to the original estimates, which means that no correction for small-sample bias is needed. The correlation coefficients between risk and returns are negative in all forecasting horizons.

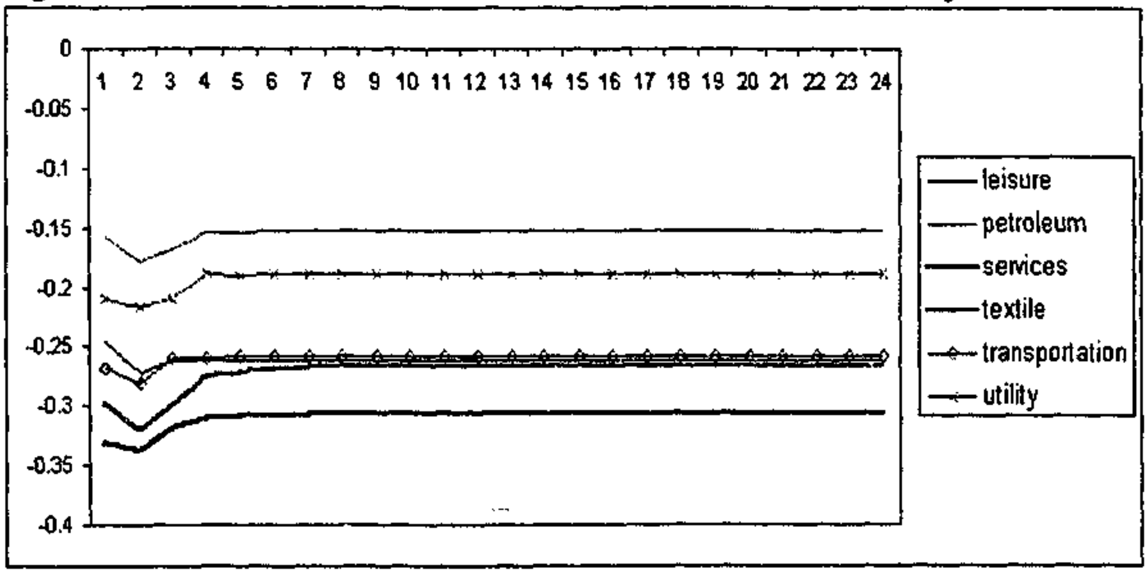
From the results of industry portfolios and the market portfolio, we can conclude that the relationship between risk and return is mixed from significantly negative to insignificantly positive in the short-run, while negative in the long-run.

Figure 6.7a Comovements of Returns and Volatilities of 7 Industry Portfolios



Note: Each line indicates the VAR forecast correlation of each industry. The symbols on the line indicate significance at 5% level.

Figure 6.7b Comovements of Returns and Volatilities of 6 Industry Portfolios



Note: Each line indicates the VAR forecast correlation of each industry. The symbols on the line indicate significance at 5% level.

Figures 6.7a and 6.7b plot the results for the comovement between risk and returns of individual industry for forecasting horizons up to 24 months. For all 13 industry portfolios, the correlation coefficients at long-run forecast horizons are negative and lies within 5% confidence interval.[7] The correlation coefficients of the construction industry show the most interesting movements. In the short-run, the correlation coefficients are positive. However, as the forecasting horizon increases, the correlation coefficients are negative. From this result, it is concluded that in the long-run, the negative correlation coefficients dominates.

6.5 Concluding Remarks

In our study, we adopt two newly developed methods of King and Watson (1997) and Den Haan (2000), to examine the long-run relationship between stock returns and volatility. Our study is different from the previous studies in three aspects. First, to avoid sensitivity problems depending on the choice of the model, we construct monthly market excess returns and volatility, adopting the method of Campbell et al. (2001). Second, we focus on the long-run relationship between risk and returns. Finally, the industry level data has been constructed and used to examine the long run relationship. In examining the relationship between risk and returns, previous studies use the market portfolio. However, in our study, we

[7] In Figures 6.7a and 6.7b, the individual confidence levels are not reported due to the clear presentation. However, they are available on request.

construct the monthly 13-industry portfolios and a market portfolio from the daily individual industry data calculated in the same manner as Campbell et al. (2001).

In the methodology of King and Watson (1997), which measures long-run stock return responses to a permanent volatility shock in portfolios for 13 industries and the market, the estimation results turn out to be sensitive to the assumed value of identifying parameters in each of the 13 industry portfolios and the market portfolio.

Focusing on the relationship between market returns and volatilities, given that previous studies and theory suggest that the relationship can be negative or positive, we find that the long-run relationship highly depends on the contemporaneous relationship. More precisely, the long-run response of stock return to a permanent volatility shock relies on the value of the mean-in-volatility effect in the sense of Brandt and Kang (2001). The other results, which include two identifying parameters ($\lambda_{s\sigma}$ and $\gamma_{\sigma s}$) and measure the mean-in-volatility effect and the long-run response of volatility to a permanent stock return shock, also indicate that the long run response of stock returns to a permanent volatility shock depends on the two identifying parameters.

For the long-run relationship, the VAR forecast error has been used to construct the correlation coefficients between risk and return. From the industry portfolio data, most industry portfolios show a negative relationship in the short-run as well as in the long run. However, the construction industry shows the positive relationship in the short-run, and a negative relationship in the long-run. For the market portfolio, a negative relationship is dominant regardless of forecasting horizons.

From these results, we conclude that the long run response of stock returns to a permanent volatility shock is sensitive to the assumed value of identifying parameters in each industry portfolios and the market portfolio, and that, as in previous studies, the relationship between risk and returns is mixed in the short-run. However, in the long run, a negative relationship is dominant. A negative relationship in the long run means that if investors feel that the risk of a portfolio is high in the future, the price of the portfolio rises to compensate for the increased expected risk. Therefore, the future return of the portfolio decreases.

Chapter 7 On the Relationship between Investment and Stock Returns: the Case of Investment-specific Technology and Adjustment Costs

7.1 Introduction

Literature that was initiated by Tobin (1969) relates investment to Q, expressed as the ratio of the market valuation to the cost of acquiring new capital. In this expression, the important source of variation in the numerator, market valuation of capital, is directly related to the change of stock prices. Therefore, Q theory asserts a positive contemporaneous relationship between stock returns and investment. However, empirical tests (Barro, 1990; Blanchard et al., 1993 among others) show that investment and stock returns have a significantly negative contemporaneous covariation, and investment and future stock returns have a covariation that is not significantly different from zero.

From the finance perspective, the relationship between stock returns and investment is important partly because the investment decisions of a firm influence the investors' perception of a particular firm. Therefore, the stock prices of the firm are also affected by investment decision (McConnell and Muscarella, 1985; Chan et al., 1990; Lamont, 2000). From the macroeconomic perspective, the relationship is also important in that the stock prices, like other asset prices, show a leading behavior for the economic fluctuations (Fama, 1990; Choi et al., 1999).

Most asset pricing studies in the real business cycle (RBC) can be divided into two categories. The studies of Rouwenhorst (1995), Boldrin et al. (1995), Jermann (1998), and Lettau (2003) examine equity premium, while Beaudry and Guay (1996) and Chapman (1997) investigate the relationship between asset prices and real variables.

The main objective of this paper is to investigate the relationship between stock returns and investment growth in the RBC framework. This investigation is important because this observed inverse relationship could be driven by some economic fundamentals. There are many papers, including the paper mentioned above, that investigate this relationship, whereas there is no research based on a stochastic growth model. The contribution of this paper is to provide an understanding of the relationship between stock returns and investment in the general equilibrium framework, deriving the correlation between investment growth and stock returns in the stochastic growth model. We derive the closed-form solutions for stock returns and calculate the correlation between investment and stock returns using the method proposed by Campbell (1994).

To investigate our proposition, we extend the general RBC model in two ways. Firstly, investment-specific technology is incorporated.[1] Our decision is motivated

[1] One of the reasons for the negative relationship between investment growth and stock returns is existence of investment lag (delivery lag, planning lag, construction lag, etc.). According to Greenwood et al. (2000), under the investment-specific technology, a positive shock raises the return on the investment in the current period. This stimulates the investment and hence a higher equipment stock in the next period. That is, the shocks to investment are modelled as current technological changes that affect the productivity of new capital goods only, leaving the

by the recent study of In and Yoon (2001). In their studies, adopting investment-specific technology and capital adjustment costs in the general equilibrium model could help to explain the equity premium puzzle. Another reason can be found in Boldrin et al, (1995). They argue that when investment-specific technology has been adopted in their model, stock prices are countercyclical.[2] The second extension to the RBC model is to adopt capital adjustment costs. Many empirical studies of investment support the presence of adjustment costs in capital (see Shapiro, 1986). Recently, Jermann (1998) has found that incorporating adjustment costs in the RBC model has improved the ability to explain the equity premium puzzle.

In this paper, we use a simple analytical approach to the stochastic growth model, namely, the method of undetermined coefficients proposed by Campbell (1994). Using this method, we derive the elasticity of the real variables including stock returns. Given these elasticities, the correlation between investment growth and stock return can be expressed as a function of state variables, in our case, two state variables – capital and investment shock. This technique has been used in a few papers; Campbell (1994), Campbell and Ludvigson (1998), Lettau (2003), Lettau et al. (2001) and In and Yoon (2001).[3]

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productivity of existing capital unchanged. Because of this time-to-build delay, only the productivity of future capital is affected.

[2] This cyclical behavior of stock prices is also reported in Campbell and Cochrane (1999).

[3] In regard to the accuracy of the log-linearization technique, see Kydland and Prescott (1982), King et al. (1988), Christiano (1990), Campbell (1994) and Den Haan and Marcet (1994) among others.

To explore and develop our extended model, first we construct the basic, benchmark model, which has a labor-augmenting productivity shock. Then we compare the results with investment-specific technology and capital adjustment cost. For further examination of the relationship between investment growth and stock return, we calculate the long-run correlation coefficients of the data, benchmark model and our extended model, and then the results are compared.

The main result of the paper is that the introduction of investment-specific technology and adjustment cost to capital considerably improves the model's performance. Generally, the results of the benchmark model are not consistent with the actual data, but are consistent with previous theoretical studies. With the incorporation of investment-specific technology, the results are consistent with actual data. However, the relative standard deviation between output and investment is too high compared to the actual data, implying that solely changing the production function does not provide an answer to the puzzling relationship between stock returns and investment growth. However, the results with capital adjustment costs show the same sign as actual data. In the calculated long-run correlation coefficients, the actual data shows a negative long-run relationship, while the benchmark model generates a positive long-run relationship. In contrast to the benchmark model, our model shows negative long-run correlation coefficients with similar movements.

This paper proceeds as follows: Section 7.2 shows some stylized facts about the relationship between investment and stock returns. The model is presented in section 7.3, and the empirical results are discussed in the section 7.4. Finally, in section 7.5, we present our conclusion.

7.2 Stylized Facts

The data we use to examine these properties are annual US observations over the period 1947 – 2000. The variables used correspond to the logarithms of per-capita GDP, per-capita real consumption (non-durable and services) and real per capita private investment taken from the NIPA and stock returns taken form Ibbotson Associates (2001). Table 7.1 summarizes some key facts on the relationship between investment growth and stock returns of the US economy. A successful model should be consistent with these basic statistics. Panel A of Table 7.1 illustrates a well-known puzzling relationship between investment growth and stock returns, which is consistent with a large volume of empirical results. The correlation between investment growth and lagged stock returns is 0.470, while the correlation between investment growth and current stock returns is –0.279 in the sample period. The correlation with future stock returns is –0.106.

The correlation coefficient between future stock returns and investment growth indicates the results consistent with the theory. Lettau and Ludvigson (2002) show the negative relationship between future stock return and optimal rate of investment in the framework of simple log-linearized marginal Q. This relationship indicates that an increase in expected future stock returns is associated with a smaller drop in today's stock price than the increase in stock returns in the near future. In the case of correlation with lagged stock returns, the positive coefficient on lagged returns does not imply that investment responds to changes in discount rates. This is because returns reflect changes in future cash flows as well as changes in future discount rates (Campbell, 1991 explain the reason for this). Thus, the relationship between lagged returns and investment growth could be driven only by changes in expected future cash flows, such as changes in the productivity of capital.

Table 7.1 Correlation with Stock Returns, 1947 - 2000

Panel A Correlation Coefficient

	Lagged return (t-2)	Lagged return (t-1)	Current return	Led return (t+1)	Led return (t+2)
Investment growth	0.173	0.470	-0.279	-0.106	0.038

Panel B Relative Standard Deviation

x =	Output ($\Delta \log Y_t$)	Consumption ($\Delta \log C_t$)	investment($\Delta \log I_t$)
σ_x / σ_Y	1.000	0.603	2.144

Source: Investment is nonresidential fixed investment from NIPA and stock returns are taken from the Stocks, Bonds, Bills and Inflation (Ibbotson Associates, 2001)

Note: Investment and stock returns are calculated as logarithm of real investment and stock returns. Investment growth is the backward difference of log-investment. To calculate the real term, GDP deflator has been used.

In sum, Table 7.1 shows that the results are consistent with the theory except for the contemporaneous relationship. In order to facilitate comparisons with simulated data, Panel B of Table 7.1 reports the relative standard deviation, which will be used to check the performance of our model.

7.3 Model

Because our purpose is to investigate how investment-specific technology affects the relationship between investment growth and stock returns through the investment shock and how the capital adjustment costs influence the relationship, we use the three different models: (1) a benchmark model with an aggregate, labor-augmenting productivity shock, (2) a model with investment-specific technology shock, (3) a model with investment technology shock and adjustment costs in investments. To solve our models, the method of undetermined coefficients proposed by Campbell (1994), has been adopted.[4]

7.3.1 Economic Model and Solving the System

To illustrate the usefulness of our approach, this section starts with the model specification of the utility function and production function. In this paper, we adopt the CRRA utility, which has been widely used in the finance context. This utility function has a property of being scale-invariant. There is a representative

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[4] Since the procedure of solving the model is similar to that of Campbell (1994), the procedure will not be discussed in this paper. However, the derivation of asset pricing and correlation between investment growth and stock returns, which is our focus in this paper, will be presented and discussed.

agent in the economy who maximizes the expected value of lifetime utility as given by:[5]

$$E_t \sum_{i=0}^{\infty} \beta^i U(C_{t+i}, 1 - N_{t+i}) = E_t \sum_{i=0}^{\infty} \beta^i \left(\frac{C_t^{1-\gamma}}{1-\gamma} + \theta \log(1 - N_t) \right), 0 < \theta < 1 \qquad (7.1)$$

where β indicates the fixed discount rate, γ is the risk aversion coefficient. C_t and N_t represent consumption and labor, respectively. This specification has also been used in Baxter and King (1991) to examine the demand shocks and externalities.

Second, the production function is taken from Greenwood et al. (2000). Using the notation Y_t for output, A_t for technology, K_t for capital, and U_t for capacity utilization, the production function is

$$Y_t = (U_t K_t)^{1-\alpha} N_t^{\alpha} - \frac{\phi((K_{t+1}/A_t) - G(K_t/A_t))^2}{K_t/A_t} \qquad (7.2)$$

where G represents the steady state growth rate. Note that this production function is different from the standard neoclassical specification in two points. First, it allows the capital adjustment costs in the specification itself. Second, it includes a

[5] For the sake of brevity, we do not present all models but the third model, which includes investment-specific technology and adjustment costs. However, the simulation results of three models will be discussed.

variable rate of capacity utilization. The parameter 1-α is referred to as the capital share. In the context of investment-specific technology proposed by Greenwood et al. (1988), the variable U_t determines the flow of capital services and represents how intensively the capital is used. The capital utilization times the stock of capital can be defined as effective capital services. In equilibrium, firms will choose to hoard capital; that is, they will set capital utilization rates to less than capacity (Burnside and Eichenbaum, 1996). This allows firms to adjust their ability to adapt to an economic shock.

The second term of the right-hand side of equation (7.2) represents the capital adjustment costs. The function is equal to zero only along the steady state. Adjustment costs will influence only the investment decision along small fluctuations around the stationary growth path (Schmitt-Grohé, 1998). Naturally, this utilization is related to the capital accumulation process, as will be shown below:

$$K_{t+1} = (1-\delta_t)K_t + I_t A_t \tag{7.3}$$

where $\delta_t = \frac{1}{w} U_t^w$ (7.4)

where I_t denotes gross investment, and δ_t is a non-negative function of capital utilization. This specification means that the more use the capital, i.e., the higher capital utilization rate, the more depreciated. Another notable thing is that in this specification, the capital accumulation and production capacity in period t+1 depend on both investment and investment-specific technology shock (A_t) affecting the productivity of new capital goods.

The fourth equation is market equilibrium. All produced goods are either consumed or invested.

$$Y_t = C_t + I_t \qquad (7.5)$$

The real business cycle model is a system of nonlinear equations in the logs of investment-specific technology, output, and consumption outside of steady state. Because of this, we first derive an approximate analytical solution by transforming the model into a system of approximate loglinear difference equations. Since our focus is on the deviation from steady state, all constant terms will be dropped in the approximate model.

Like the other method to solve the RBC model, all real variables, including stock returns, can be expressed as a function of state variables; in our case, the current capital stock and investment-specific technology shock. To derive the solution, we start by assuming that the consumption and technology shock have the following form. We use the notation η_{yx} for the partial elasticity of y with respect to x.

$$c_t = \eta_{ck} k_t + \eta_{ca} a_t \qquad (7.6)$$

$$a_t = \rho a_{t-1} + \varepsilon_t \qquad -1 \leq \rho \leq 1 \qquad (7.7)$$

where AR(1) coefficient, ρ measures the persistence of investment-specific technology shocks, with the extreme case of $\rho = 1$ being a random walk for technology.

Under this assumption, our focus is to find the values of the elasticities of consumption, η_{ck} and η_{ca}. After finding these elasticities of consumption, it is straightforward to express the remaining real variables as a function of state variables. The results can be expressed as follow:[6]

$$n_t = \eta_{nk} k_t + \eta_{na} a_t \tag{7.8}$$

$$y_t = \eta_{yk} k_t + \eta_{ya} a_t \tag{7.9}$$

$$i_t = \eta_{ik} k_t + \eta_{ia} a_t \tag{7.10}$$

$$k_{t+1} = \eta_{kk} k_t + \eta_{ka} a_t \tag{7.11}$$

7.3.2 Stock Returns

The goal in this section is to derive explicit solutions of stock returns. The equations are written in terms of the fundamental parameters of the RBC model, such as risk aversion and the persistence of investment-specific technology shock, and the elasticities of the endogenous variables from the loglinear solution of the RBC model.

[6] We do not present the explicit solution for the elasticities of real variables. However, it is available on request.

It is a common assumption that with competitive markets, stock returns will be the net marginal product of capital (Corsetti, 1997). Thus, stock returns can be expressed as follows:

Benchmark

$$R_t^B = \alpha \frac{Y_t}{K_t} + (1 - \delta) \tag{7.12}$$

Investment-specific technology

$$R_t^I = (1-\alpha)\frac{Y_t}{K_t} + \left(1 - \frac{1}{w} U_t^w\right)\frac{1}{A_t} \tag{7.13}$$

Investment-specific technology with capital adjustment cost

$$R_t^C = (1-\alpha)\frac{(U_t K_t)^{1-\alpha} N_t^\alpha}{K_t} + \left(1 - \frac{1}{w} U_t^w\right)\frac{1}{A_t} + \frac{\phi}{A_t}\left(\frac{K_{t+1}}{K_t}\right)^2 - \frac{\phi}{A_t} G^2 \tag{7.14}$$

From these three equations, it is interesting that in the general standard growth model stock return is not related to current investment but related to last period investment through the capital accumulation. However, two other stock returns show that it is related to current investment-specific shock through A_t. It is obvious that when the firm decides more investments at time t, stock returns at time t will be affected by this decision. Another interesting point is that the capital utilization rates also affect stock returns. Under the assumption of $A_t > 0$, the higher utilization of capital results in a lower return. It is related to replacement cost. If a capital has been used intensively for producing output, this capital experiences a large depreciation, and then it must be soon replaced. This causes investment expenditure as new capital replaces old capital.. This could generate the countercyclical behavior of stock returns. Intuitively, it is expected that capital should be used more intensively during economic booms when its marginal product is high and less intensively during recessions when its marginal product is low.

In the case of stock returns with investment-specific technology and capital adjustment costs, it contains next period capital and the next period capital affects stock returns positively. This is because if the firms invest for the next period, the shareholder regards this news as a positive impact on their share prices. However, at the steady state growth path, the next period capital does not affect stock returns because of last term in right-hand side of equation (7.14), which exactly offsets the capital growth.

Using the loglinearization technique, stock returns can be expressed as follows:

Benchmark

$$r_t^B = \eta_{rk} k_t + \eta_{ra} a_t \qquad \text{where, } \eta_{rk} = (\eta_{yk} - 1),\ \eta_{ra} = \eta_{ya} \qquad (7.15)$$

From this equation, it is seen that stock returns are closely related to output. This is because we assume that stock returns are equal to the net marginal product of capital. However, this derivation tells that stock returns decrease if capital increases mainly because the elasticity of stock returns with respect to capital is less than unity. However, the technology shock affects stock returns positively. The net effect of technology shock is ambiguous depending on the fundamental parameters of the model.

Investment-specific technology

$$r_t^I = \eta_{rk} k_t + \eta_{ra} a_t \tag{7.16}$$

where $\eta_{rk} = \Omega_1(\eta_{yk} - 1)$, $\eta_{ra} = \Omega_1 \eta_{ya} + \Omega_1 - 1$ and $\Omega_1 = \dfrac{(1+g)(1+r)-1}{(1+g)(1+r)}$

Compared with stock returns of the benchmark model, the elasticities of capital are different even though we assume that both η_{yk} s are same, but the elasticity of investment-specific technology (η_{rk}) has a lesser value than that of the benchmark model because the value of Ω_1 is less than 1. This is because of the transmission mechanism of investment-specific technology. Greenwood et al. (2000) state that a technology shock in investment-specific technology does not directly affect the production function in the current period. Current output is affected only to the extent that the shock can elicit increased employment of capital and labor in response to changed investment opportunities (see Greenwood et al., 2000). Stock returns are also affected by changing of employment of capital and labor in production function and also the capital utilization speed. The existence of capital utilization rates in stock returns could be interpreted in two different ways: positive and negative. The positive aspect is through the marginal product of capital services, which is increased by the higher utilization of capital.

On the other hand, the negative effect is from the capital depreciation rate. The higher utilization rate causes the higher depreciation rate. This increased depreciation rate affects stock returns negatively. However, if the capital increases, stock returns decrease because the elasticity of return with respect to capital is less than unity. Like the benchmark case, the elasticity of return with respect to technology shock is ambiguous depending on the fundamental parameters.

Investment-specific technology with capital adjustment cost

$$r_t^C = \eta_{rk} k_t + \eta_{ra} a_t \tag{7.17}$$

where $\eta_{rk} = (1-\alpha)\Omega_1 (B_1 \eta_{ck} + B_2) - \Omega Z_1 \eta_{nk} + (\Omega_2 - 1)\eta_{kk}$ and

$$\eta_{ra} = (1-\alpha)\Omega_1 (B_1 \eta_{ca} + B_3) + \alpha\Omega_1 \eta_{na} + \Omega_2 \eta_{ka} + \Omega_1 - 1$$

and where $\Omega_1 = \dfrac{(1+g)(1+r)-1}{(1+g)(1+r)}$ and $\Omega_2 = \dfrac{\phi(1+g)}{(1+r)}$

Note that stock returns equation of investment-specific technology with adjustment costs contains the future value of capital, reflected by the coefficient, η_{kk} and the capital adjustment parameter in equation (7.17), which can be interpreted as a cost to capital adjustment. The higher the value of capital adjustment costs, the higher the elasticity of return with respect to capital. If the cost of capital adjustment is higher, the contribution of capital increases in production. The more the contribution of capital, the higher stock returns. This is because a time when adjustment costs are high is a good time to lower investment, because the firm can sell a larger quantity of the consumption good for every unit by which it lowers the capital stock (Cochrane, 1991). However, the effect of technology change on stock returns is ambiguous depending on the fundamental parameters of the model.

7.3.3 Correlation between Stock Returns and Investment Growth

The goal of this subsection is to derive the correlation coefficient between stock returns and investment growth. To derive the correlation coefficients, the covariance between investment growth and stock returns, and the variances of investment growth and stock returns have to be calculated first. To derive these terms (covariance and two variances), we begin by expressing investment growth and stock returns as an ARMA form. From equation (7.10), investment growth can be expressed as follows:

$$\Delta i_t = \eta_{ik}\Delta k_t + \eta_{ia}\Delta a_t \tag{7.18}$$

where Δ indicates the backward difference. Using equations (7.7) and (7.11), investment growth becomes

$$\Delta i_t = \frac{(1-L)L\eta_{ik}\eta_{ka}}{(1-\rho L)(1-\eta_{kk}L)}\varepsilon_t + \frac{(1-L)\eta_{ia}}{1-\rho L}\varepsilon_t$$

$$\Delta i_t = \frac{1-L}{1-\rho L}\left(\eta_{ia} + \frac{\eta_{ik}\eta_{ka}L}{(1-\eta_{kk}L)}\right)\varepsilon_t \tag{7.19}$$

where L denotes the lag operator. Technology shocks (labor-augmenting or investment specific technology shocks) affect investment growth through two channels. The term involving η_{ia} measures the direct effect of the shock on investment growth, holding capital constant. However, the technology shock also increases the capital stocks in the next period, which in turn causes investment to grow. Therefore, the indirect effect through the capital stock causes investment growth to increase. Depending on the parameters of the model, one of these effects could dominate. The shock persistence parameter plays an important role. As mentioned in Lettau (2003), examining two extreme cases might give some insights; completely transitory ($\rho = 0$) and permanent ($\rho = 1$) shocks.

First, consider a completely transitory case ($\rho = 0$). Hence, equation (7.19) becomes

$$\Delta i_t = (1-L)\left(\eta_{ia} + \frac{\eta_{ik}\eta_{ka}L}{(1-\eta_{kk}L)}\right)\varepsilon_t \tag{7.20}$$

$$\Delta i_t = (1-L)\eta_{ia}\varepsilon_t \tag{7.21}$$

We can further simplify this expression by assuming $\eta_{ka} \approx 0$. If shocks die out instantaneously, the capital stock will not be affected very much. Hence the elasticity of the capital stock with respect to the shock is very small (see

Campbell, 1994). In this case investment growth is just a backward difference of the white noise process of shock. In other words, investment growth is determined by how investment responds to the shock. If investment responds to the shock negatively, investment growth also reacts negatively.

The second special case is that the shock is permanent ($\rho = 1$). When the shock is permanent, equation (7.19) becomes

$$\Delta i_t = \left(\eta_{ia} + \frac{\eta_{ik}\eta_{ka}L}{(1-\eta_{kk}L)} \right)\varepsilon_t \tag{7.22}$$

After simplifying, investment growth follows the ARMA(1,1) process because an AR root cancels with the MA root. The reaction of investment growth after a positive technology shock is divided into two parts, direct and indirect effects. Like the general case, the effect of a positive technology shock is ambiguous depending on the parameters of the models.

For the computation of variance and covariance, it will be useful to rewrite the ARMA(2,2) process of investment growth (7.19) in its MA(∞) representation.

$$\Delta i_t = \frac{\eta_{ia}\varepsilon_t(1-L)}{(1-\rho L)(1-\eta_{kk}L)}[1+\theta_1 L]$$

$$= \frac{1}{\rho - \eta_{kk}} \sum_{s=0}^{\infty} \left\{ \left[(\rho + \theta_1 - 1)\rho^s - (\eta_{kk} + \theta_1 - 1)\eta_{kk}^s \right] \mu_1 \varepsilon_{t-s} - \theta_1(\rho^2 - \eta_{kk}^s)\mu_1 \varepsilon_{t-1-s} \right\}$$

(7.23)

where $\theta_1 = (\eta_{ik}\eta_{ka} - \eta_{kk}\eta_{ia})/\eta_{ia}$. This MA representation is also useful for computing the unconditional variance of investment growth. After rearranging equation (7.23), the variance becomes

$$Var(\Delta i_t) = \left(\frac{\eta_{ia}}{\rho - \eta_{kk}}\right)^2 \sigma_\varepsilon^2 \{\left[(\rho+\theta_1-1)^2 + (\eta_{kk}+\theta_1-1)^2 - 2(\rho+\theta_1-1)(\eta_{kk}+\theta_1-1)\right]$$
$$+\frac{(\rho(\rho+\theta_1-1)-\theta_1)^2}{1-\rho^2} + \frac{(\eta_{kk}(\eta_{kk}+\theta_1-1)-\theta_1)^2}{1-\eta_{kk}^2}$$
$$-2\frac{(\rho(\rho+\theta_1-1)-\theta_1)(\eta_{kk}(\eta_{kk}+\theta_1-1)-\theta_1)}{1-\rho\eta_{kk}}\}$$

(7.24)

The derivation of variance of stock returns follows the same procedure as that of investment growth. In contrast to investment growth, stock returns follow an ARMA(2,1) process.

$$r_t^{eq} = \frac{1}{(1-\rho L)}\left(\eta_{ra} + \frac{\eta_{rk}\eta_{ka}L}{(1-\eta_{kk}L)}\right)\varepsilon_t \tag{7.25}$$

As can be seen in equation (7.25), the effect of a positive technology also has two channels; direct and indirect effects. Therefore, the effect is ambiguous, depending on the parameters of the model. The resulting the MA(∞) representation of stock returns is as follows:

$$r_t^{eq} = \frac{1}{\rho - \eta_{kk}} \sum_{s=0}^{\infty} \left[(\rho + \theta_2)\rho^s - (\eta_{kk} + \theta_2)\eta_{kk}^s \right] \eta_{ra} \varepsilon_{t-s} \tag{7.26}$$

where $\theta_2 = (\eta_{rk}\eta_{ka} - \eta_{ra}\eta_{kk}) / \eta_{ra}$. The unconditional variance of stock returns is

$$Var(r_t^{eq}) = \left(\frac{\eta_{ra}}{\rho - \eta_{kk}} \right)^2 \sigma_\varepsilon^2 \left[\frac{(\rho + \theta_2)^2}{1 - \rho^2} + \frac{(\eta_{kk} + \theta_2)^2}{1 - \eta_{kk}^2} - 2 \frac{(\rho + \theta_2)(\eta_{kk} + \theta_2)}{1 - \rho\eta_{kk}} \right] \tag{7.27}$$

After substituting the equation (7.24) and (7.26) into the definition of covariance, the covariance between investment growth and stock returns becomes

$$\begin{aligned} Cov(\Delta i_t, r_t^{eq}) = \frac{\eta_{ia}\eta_{ra}}{(\rho - \eta_{kk})^2} \sigma_\varepsilon^2 \{ & \frac{(\rho + \theta_1 - 1)(\rho + \theta_2)}{(1 - \rho^2)} + \frac{(\eta_{kk} + \theta_1 - 1)(\eta_{kk} + \theta_2)}{(1 - \eta_{kk}^2)} \\ & - \frac{1}{1 - \rho\eta_{kk}} ((\eta_{kk} + \theta_1 - 1)(\rho + \theta_2) + (\rho + \theta_1 - 1)(\eta_{kk} + \theta_2)) \\ & - \theta_1 (\frac{\rho(\rho + \theta_2)}{(1 - \rho^2)} + \frac{\eta_{kk}(\eta_{kk} + \theta_2)}{(1 - \eta_{kk}^2)} \\ & - \frac{\rho(\rho + \theta_2)}{(1 - \rho\eta_{kk})} - \frac{\eta_{kk}(\eta_{kk} + \theta_2)}{(1 - \rho\eta_{kk})}) \} \end{aligned} \tag{7.28}$$

Apparently, the sign of covariance is determined by the elasticities of investment and stock returns with respect to technology shock under the assumption that the remaining terms are all positive. If both elasticities respond in the same direction to the technology shock, the covariance has a positive sign. The covariance of led stock returns can be calculated as follows:

$$Cov(\Delta i_t, r_{t+1}^{eq}) = \frac{\eta_{ia}\eta_{ra}}{(\rho-\eta_{kk})^2}\sigma_\varepsilon^2\{\frac{\rho(\rho+\theta_2)(\rho+\theta_1-1)}{(1-\rho^2)} + \frac{\eta_{kk}(\eta_{kk}+\theta_2)(\eta_{kk}+\theta_1-1)}{(1-\eta_{kk}^2)}$$

$$-\frac{1}{1-\rho\eta_{kk}}\left[\rho(\eta_{kk}+\theta_1-1)(\rho+\theta_2)+\eta_{kk}(\rho+\theta_1-1)(\eta_{kk}+\theta_2)\right]$$

$$-\theta_1\left(\frac{\rho^2(\rho+\theta_2)}{(1-\rho^2)}+\frac{\eta_{kk}^2(\eta_{kk}+\theta_2)}{1-\eta_{kk}^2}-\frac{\left(\rho^2(\rho+\theta_2)+\eta_{kk}^2(\eta_{kk}+\theta_2)\right)}{(1-\rho\eta_{kk})}\right)\}$$

In the same manner, the covariance with lagged stock returns becomes

$$Cov(\Delta i_t, r_{t-1}^{eq}) = \frac{\mu_1\mu_2}{(\rho-\eta_{kk})^2}\sigma_\varepsilon^2\{\frac{\rho(\rho+\theta_2)(\rho+\theta_1-1)}{(1-\rho^2)} + \frac{\eta_{kk}(\eta_{kk}+\theta_2)(\eta_{kk}+\theta_1-1)}{(1-\eta_{kk}^2)}$$

$$-\frac{1}{1-\rho\eta_{kk}}\left[\eta_{kk}(\eta_{kk}+\theta_1-1)(\rho+\theta_2)+\rho(\rho+\theta_1-1)(\eta_{kk}+\theta_2)\right]$$

$$-\theta_1\left(\frac{(\rho+\theta_2)}{(1-\rho^2)}+\frac{(\eta_{kk}+\theta_2)}{1-\eta_{kk}^2}-\frac{\left((\rho+\theta_2)+(\eta_{kk}+\theta_2)\right)}{(1-\rho\eta_{kk})}\right)\}$$

Using these results, we derive the contemporaneous correlation, and the correlation coefficients with led and lagged stock returns.

7.4 Empirical Results

To simulate our models, we calibrate the values of fundamental parameters, such as g, the log technology growth rate; r, the log real return on investment; α, the exponent on labor, equivalent to labor's share of output; w, the elasticity of depreciation with respect to the capacity utilization rate. We use the various parameters for the persistence of shock, ρ and the coefficient of risk aversion γ.

For the purpose of calibrating the model, benchmark values for these parameters are set as g = 1.24%, r = 0.06, α = 0.70, δ = 0.1, w = 1.50 or 2.32, which are mainly taken from Greenwood et al. (2000). In this section, we analyze the results of all three models mainly focusing on the correlation between stock returns and investment growth.

7.4.1 Benchmark Model

The results of the benchmark model can be seen in Table 7.2. In order to check the performance of benchmark model, we report the relative standard deviations. Compared to the same statistics from the actual data, when the risk aversion coefficient is set to 1 and the persistence of shock is set to 0.95, the model mimics the actual data well.

To consider the model in more depth, first, consider a case with risk aversion of unity and fairly persistent technology shock ($\rho = 0.95$). The sign of correlation coefficients with lagged stock returns is negative and the others are positive while the actual data shows a positive relationship with lagged stock returns and a negative relationships with led and current stock returns. The current relationship is consistent with the Q theory. This result suggests that current investment takes time to be realized in stock returns, and that it needs some modification to fit the actual data.

Next, consider the varying persistence parameter while keeping risk aversion at 1. In all cases, the sign patterns are the same (negative for correlation with lagged stock returns and positive for the correlations with current and led stock returns), implying that the positive labor-augmenting technology shock increases investment and stock return.

Table 7.2 Moments Inferred from the Benchmark Model
Panel A Correlation Coefficients

		Stock Return				
γ	ρ	-2	-1	0	1	2
0.01	0.00	0.252	-0.835	0.530	-0.013	-0.003
	0.50	-0.039	-0.753	0.553	0.095	0.008
	0.95	-0.131	-0.582	0.672	0.146	0.031
1.00	0.00	0.012	-0.685	0.815	-0.051	-0.036
	0.50	-0.213	-0.474	0.702	0.259	0.065
	0.95	-0.173	-0.241	0.562	0.383	0.257
10.0	0.00	0.000	-0.578	0.816	-0.064	-0.047
	0.50	-0.143	-0.288	0.687	0.235	0.038
	0.95	-0.005	-0.017	0.409	0.273	0.174
100.0	0.00	-0.001	-0.558	0.811	-0.066	-0.049
	0.50	-0.128	-0.253	0.674	0.227	0.033
	0.95	0.043	0.045	0.316	0.205	0.124

Note: This table reports the simulated cross correlation between investment growth and stock returns using cor(ΔI_t, Δr_{t-i}) where i = -2, -1, 0, 1, 2. To construct this simulated cross correlation, the varying values of risk aversion coefficient, γ and shock persistence, ρ, have been considered.

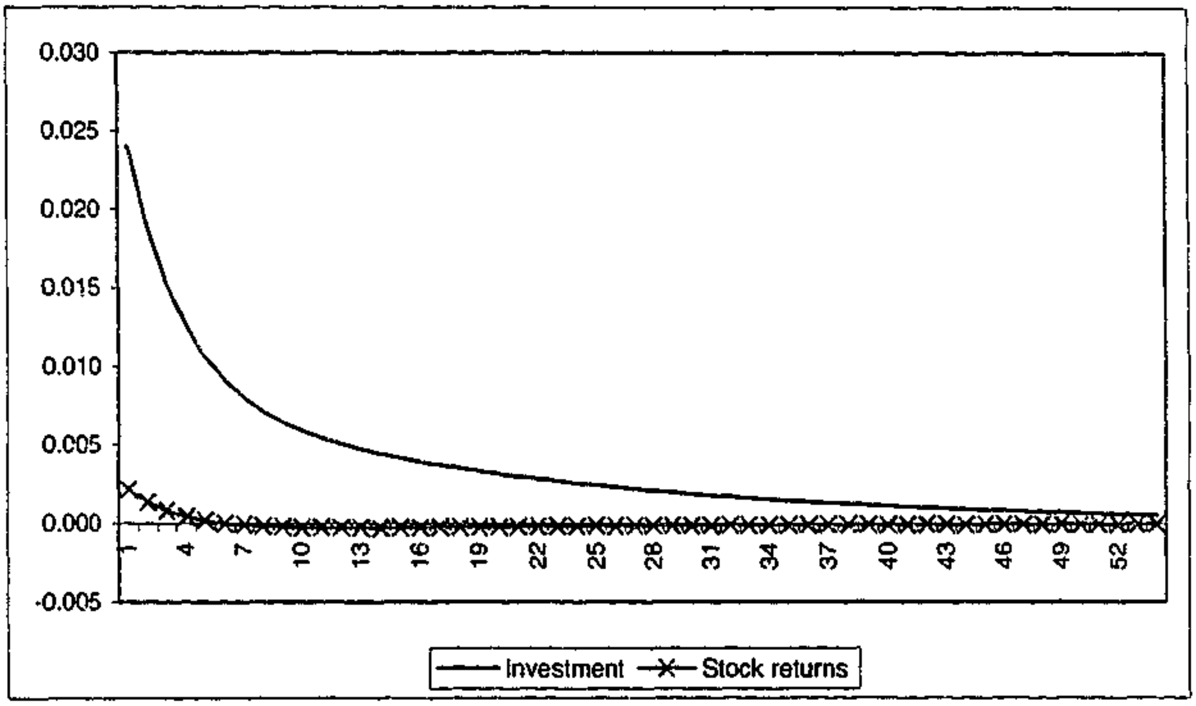
Panel B Relative Standard Deviations

γ	ρ	Output ($\Delta \log Y_t$)	Consumption ($\Delta \log C_t$)	Investment ($\Delta \log I_t$)
1.00	0.95	1.000	0.534	2.604

Note: This table reports the simulated relative standard deviation of output, consumption, and investment. To construct this simulated cross correlation, the varying values of risk aversion coefficient, γ and shock persistence, ρ, have been considered. However, we report only the case where $\gamma = 1$ and $\rho = 0.95$. Overall pattern is similar regardless of risk aversion and shock persistence: investment is more volatile than consumption. The results are available on request.

This relationship can be seen clearly in Figure 7.1, which plots the impulse responses of investment and stock returns to a positive technology shock. In this figure, a positive shock affects both of them positively. It is natural for stock returns to respond minimally to a positive shock. The real business cycle model assumes that the supply of capital is infinitely elastic, so that its equilibrium price is a constant. As is well known, the payoff on capital (its rental rate) in the standard business cycle model fluctuates very little (Boldrin et al., 1995). Stock returns of the standard RBC model are determined by the payoff on capital. As a result (see equation (7.12)), the response of stock returns to a positive productivity shock is very little.

Figure. 7.1 Investment and Stock Returns: Impulse Responses to a Shock in Productivity for the Benchmark Model



Notes: To construct the impulse response analysis, $\gamma = 1.0$ and $\rho = 0.95$ has been used. This figure shows that both investment and stock returns respond to shock positively, implying that the correlation between investment growth and stock returns is positive.

Turning to changes in the risk aversion, higher risk aversion implies a lower elasticity of substitution, in our case $\sigma = 1/\gamma$, and so consumers want to smooth their consumption paths. In this context, this relationship has a strong implication for stock returns and investment. In the case of stock return, the greater desire to smooth consumption leads stock returns to actually decline over all horizons in response to a positive productivity shock. For investment, investors who want to smooth their consumption path over time will attempt to smooth out fluctuations in their investment arising from time variations in expected returns. Consider the following example. When returns are expected to be higher in the future, forward-looking investors will allow consumption out of total wealth to rise above its long-term trend with wealth, and so when the returns are expected to rise, investment may decrease due to an increase in consumption. In contrast, when such investors expect returns to fall in the future, they will reduce their consumption and increase their investment. This intuitive expectation can be captured by considering the following loglinearized Q, taken from Lettau and Ludvigson (2002). After substituting that $\mu + \gamma E_t \Delta c_{t+1} = E_t r_{t+1}$ where γ is the risk aversion.[7]

$$q_t \approx E_t\left[\sum_{j=0}^{\infty} \rho_q^j [(1-\rho_q) m_{t+1+j} - \gamma \Delta c_{t+1+j}]\right] \tag{7.29}$$

where E_t is the conditional expectation operator based on information at time t and m_t is defined as $\log(M_t) = \log((1-\delta)\partial \pi_t / \partial K_t)$ where π_t is net cash flow at

[7] To derive this relationship, we simply assume that investors have a power utility for consumption and the consumption growth and stock returns are conditionally homoskedastic.

time t.[8] From this expression, if the risk aversion of investors is high, they want higher return to compensate their consumption. Therefore, the higher future stock returns leads investment to decrease. So future stock returns and investment growth have a negative correlation.

However, as can be seen in Panel A of Table 7.2, the correlations with extremely high risk aversion all show positive correlation, which is different to our intuitive expectation. The failure of the benchmark model to explain the correlation between investment growth and stock returns could have two reasons. Firstly, the benchmark model does not consider the capital adjustment costs in the model. The importance of capital adjustment cost can be found in Jermann (1998) and Cochrane (1991). Jermann (1998) finds that incorporating capital adjustment costs in the RBC model slightly helps to resolve the equity premium puzzle. Cochrane (1991) gives us a more intuitively explanation. In his paper, he argues that a time when adjustment costs are high is a good time to lower investment, because the firm can sell a larger quantity of the consumption good for every unit by which it lowers the capital stock. This means that the high capital adjustment costs are related to high stock returns due to increased output.

Another reason can be found in the intertemporal substitution effect. If the intertemporal substitution is important in an economic setting, the risk aversion coefficients also play an important role in determining the asset prices. Indirect

[8] This equation is based on the derivation of Abel and Blanchard (1986) under the assumption that the firm chooses I_t in order to maximize the value of the firm at time t, and the marginal cost of investment must be equal to the expected present value of marginal profits to capital.

evidences for a reduction in the intertemporal substitution effect can be found in the notion of habit formation. In the habit model, whatever the habit, the higher previous consumption, the bigger the habit, and the higher must be the current level of consumption to deliver the same effect (Deaton, 1992). The higher level of current consumption due to the consumption habit reduces the intertemporal substitution effect. Therefore, the role of the risk aversion coefficient also reduces. For a more intuitive explanation, compare the power utility and habit utility. With the power utility and positive consumption growth, the future marginal utility of consumption is low compared with the marginal utility of present consumption. Increasing γ intensifies this, so that a higher interest rate is required to discourage households from attempting to reallocate consumption from the future to the present. However, existing habit persistence has the effect of increasing the future habit stock, and raises the marginal utility of future consumption, reducing the incentive to reallocate consumption toward the present (Boldrin et al., 1995). Investment-specific technology proposed by Greenwood et al. (1988) also reconciles the intertemporal substitution effect.

Based on this finding, we first adopt the technology proposed by Greenwood et al. (1988) and then we extend this model by incorporating the capital adjustment costs.

7.4.2 Investment-specific Technology

The transmission mechanism of investment-specific technology is different from that of a production technology shock. In investment-specific technology, the utilization of capital plays an important role in transmitting the technology shock. Although the technology shock affects only new capital, the current utilization of capital and the productivity of labor are also affected by the current shock through changing the utilization cost of installed capital.

Table 7.3 Moments Inferred from Investment-specific Technology
Panel A Correlation Coefficients

		Stock Return				
γ	ρ	-2	-1	0	1	2
0.01	0.00	-0.233	0.555	-0.016	-0.152	-0.044
	0.50	-0.010	0.505	0.026	-0.103	-0.085
	0.95	0.066	0.217	0.011	-0.012	-0.017
1.00	0.00	-0.032	0.744	-0.551	-0.010	-0.008
	0.50	0.235	0.572	-0.279	-0.153	-0.087
	0.95	0.186	0.242	-0.018	-0.022	-0.025
10.0	0.00	-0.028	0.744	-0.562	-0.008	-0.006
	0.50	0.242	0.573	-0.289	-0.155	-0.086
	0.95	0.196	0.250	-0.012	-0.016	-0.018
100.0	0.00	-0.027	0.744	-0.563	-0.007	-0.006
	0.50	0.242	0.573	-0.289	-0.155	-0.086
	0.95	0.197	0.252	-0.011	-0.014	-0.017

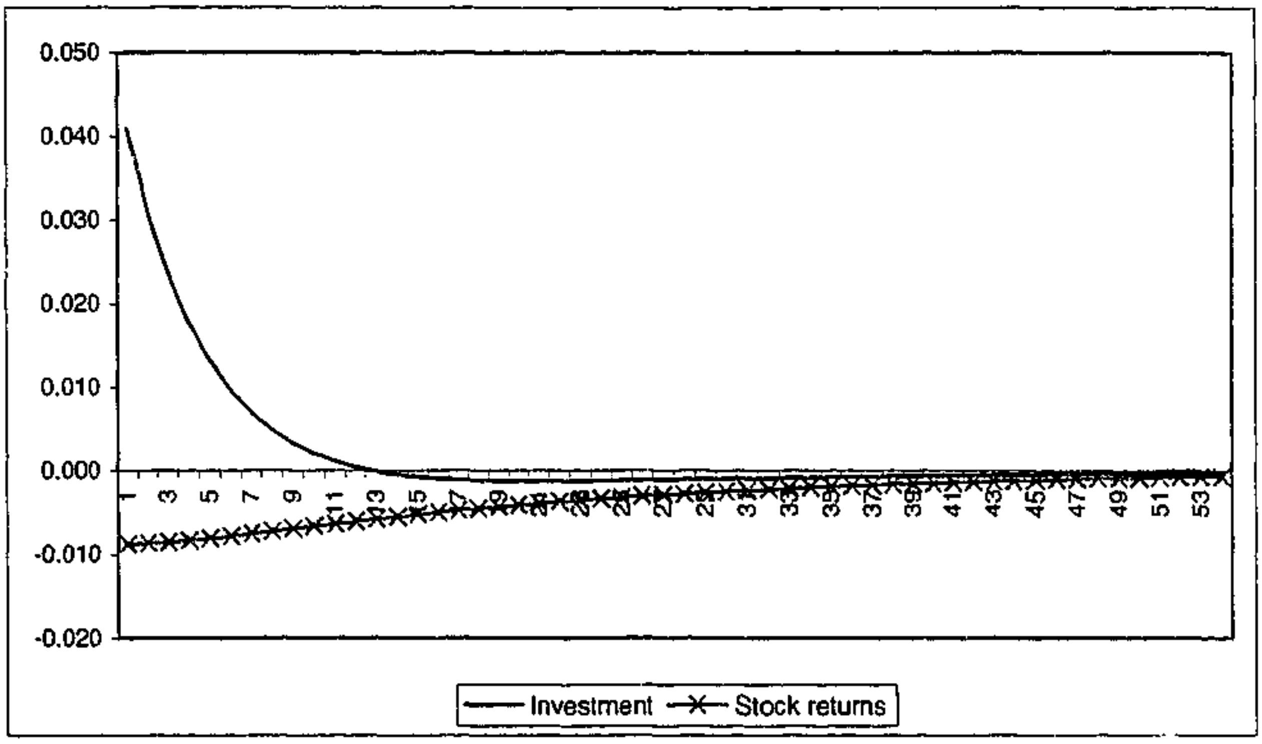
Note: This table reports the simulated cross correlation between investment growth and stock returns using cor(ΔI_t, Δr_{t-i}) where i = -2, -1, 0, 1, 2. To construct this simulated cross correlation, the varying values of risk aversion coefficient, γ, and shock persistence, ρ, have been considered.

Panel B Relative Standard Deviations

γ	ρ	Output ($\Delta\log Y_t$)	Consumption ($\Delta\log C_t$)	investment($\Delta\log I_t$)
1.00	0.95	1.000	0.251	5.339

Note: This table reports the simulated relative standard deviation of output, consumption, and investment. To construct this simulated cross correlation, the varying values of risk aversion coefficient, γ, and shock persistence, ρ, have been considered. However, we report only the case where $\gamma = 1$ and $\rho = 0.95$. Overall pattern is similar regardless of risk aversion and shock persistence: investment is more volatile than consumption. The results are available on request.

Figure. 7.2 Impulse Responses of Investment and Stock Returns for Investment-specific Technology



Notes: To construct the impulse response analysis, $\gamma = 1.0$ and $\rho = 0.95$ has been used. This figure shows that investment responds to shock positively, while stock returns responds negatively. This result implies that the correlation between investment growth and stock returns is negative.

The results of adopting investment-specific technology are presented in Table 7.3. First consider a case that the risk aversion is set to unity and the shock is fairly persistent ($\rho = 0.95$). The result is very different from that of the benchmark model: a positive relationship with lagged stock returns, and a negative relationship with current and led stock return.

In contrast to the traditional technology shock, a positive shock affects stock returns negatively, as can be seen in Figure 7.2. This is because a positive shock increases the current flow of capital services endogenously, since it lowers the replacement costs of existing capital. Therefore, this shock causes a higher utilization rate from existing capital. The increased service of capital contributes more output, while it decreases stock returns due to the increased depreciation rate. Christiano and Fisher (1998) also find that the stock prices respond countercyclically to investment-specific shocks, confirmed by Figure 7.2. However, this transmission mechanism affects investment growth positively. Through this transmission mechanism, the current period correlation between investment growth and stock returns has a negative value.

Next, consider varying the shock persistence while keeping the risk aversion at unity. The correlation with current stock returns is decreasing in ρ because the increasing persistence tends to increase the variances of investment growth and stock returns. Another reason can be seen in the indirect channel through capital accumulation (see equations (7.20) and (7.25)). As the persistence of shock increases, the indirect channel plays a more important role. The key is in the capital utilization. Because of capital hoarding in equilibrium (Burnside and Eichenbaum, 1996), a firm can immediately adjust its output to a shock that affects the marginal product of capital. A positive technology shock leads the firm to increase its utilization rate, which in turns decreases stock return due to increased depreciation rate. The correlation with lagged stock returns is also increasing in ρ because the increasing persistence tends to increase the variances of investment growth and stock returns.

It is of interest to consider the correlation with future stock return and investment growth. In this case, the correlation between investment growth and stock return shows a negative value even though the value is very small. This is related to the question of which channel dominates: direct or indirect channels. When the persistence of shock is transitory ($\rho = 0$), the capital accumulation channel is dying out quickly. Therefore, the direct channel plays an important role in this relationship. When the shock is completely transitory, current investment growth responds positively to a positive technology shock. As the persistence of shock increases, the indirect channel through capital accumulation and the capital utilization rate plays a more important role. A current positive shock leads the next period capital to increase. The representative firm uses the increased capital fully in the next period. The high capital utilization leads to high marginal productivity of capital services while the depreciation rate increases and the replacement costs increases. Through this mechanism, next period stock returns decrease.

Finally, consider varying the risk aversion. Interestingly, in contrast to the benchmark model, the risk aversion coefficients do not play a role at all. In all cases of risk aversion, the results show the same pattern. This is because of the difference between two models. The standard macroeconomic model, like our benchmark model, presumes that the intertemporal substitution effect, which is generated by a technological shift, will dominate the income effect. To see this, consider the Euler equation of our model. The final step to solve this dynamic model is to use the equation (7.7). If the intertemporal substitution effect dominates, risk aversion plays an important role at this dynamic setting. However, in the case of investment-specific technology, the intratemporal substitution plays a greater role than the intertemporal substitution. This effect leads the representative consumer away from leisure and towards consumption (see

Greenwood et al., 1988). In other words, if the intratemporal substitution dominates the intertemporal substitution, the risk aversion coefficient does not play a significant role in this model. Therefore, the changing risk aversion does not affect the relationship between investment growth and stock returns.

Turning to Panel B of Table 7.3, the relative standard deviation shows that the adoption of an investment-specific technology shock in our model generates high volatility of investment. The relative standard deviation of consumption in investment-specific technology is lower than those of Tables 7.1 and 7.2. However, the relative standard deviation of investment is markedly higher than those of Tables 7.1 and 7.2. This is caused by the mechanism of the investment-specific technology shock. As Greenwood et al. (1988) argues, if the intratemporal substitution dominates the intertemporal substitution in investment-specific technology, the consumer wants to smooth the consumption path. Therefore, the variance of consumption decreases, while the variance of investment increases.

7.4.3 Incorporating Capital Adjustment Costs

When the parameters of our model are calibrated, the size of the adjustment cost, ϕ, is determined. However, the proper value of the adjustment cost is controversial as is the size of the risk aversion coefficient. The adjustment cost is obviously an important parameter for evaluating the cyclical properties of the model (Greenwood at al. 2000). However, estimates of ϕ based on regressions, Euler equations, or other techniques often result in higher values, implying that implausibly large fractions of output are lost to adjustment costs (Cochrane 1997). In our model, we adopt the values of capital adjustment costs from Greenwood et al. (2000): 1.50 and 2.32.

Table 7.4 Moments Inferred from Investment-specific Technology with Capital Adjustment Costs (ϕ = 1.50)

Panel A Correlation Coefficients

		Stock Return				
γ	ρ	-2	-1	0	1	2
0.01	0.00	0.002	-0.166	0.349	-0.008	-0.008
	0.50	0.166	0.327	-0.176	-0.095	-0.055
	0.95	0.125	0.129	-0.103	-0.100	-0.098
1.00	0.00	-0.002	-0.054	0.257	-0.007	-0.007
	0.50	0.186	0.375	-0.192	-0.102	-0.057
	0.95	0.164	0.175	-0.079	-0.077	-0.075
10.0	0.00	-0.003	-0.007	0.217	-0.006	-0.006
	0.50	0.193	0.394	-0.197	-0.104	-0.057
	0.95	0.188	0.204	-0.058	-0.056	-0.055
100.0	0.00	-0.004	0.000	0.211	-0.006	-0.006
	0.50	0.194	0.397	-0.198	-0.104	-0.057
	0.95	0.193	0.209	-0.053	-0.052	-0.050

Note: This table reports the simulated cross correlation between investment growth and stock returns using cor(ΔI_t, Δr_{t-i}) where i = -2, -1, 0, 1, 2. To construct this simulated cross correlation, the varying values of risk aversion coefficient, γ, and shock persistence, ρ, have been considered.

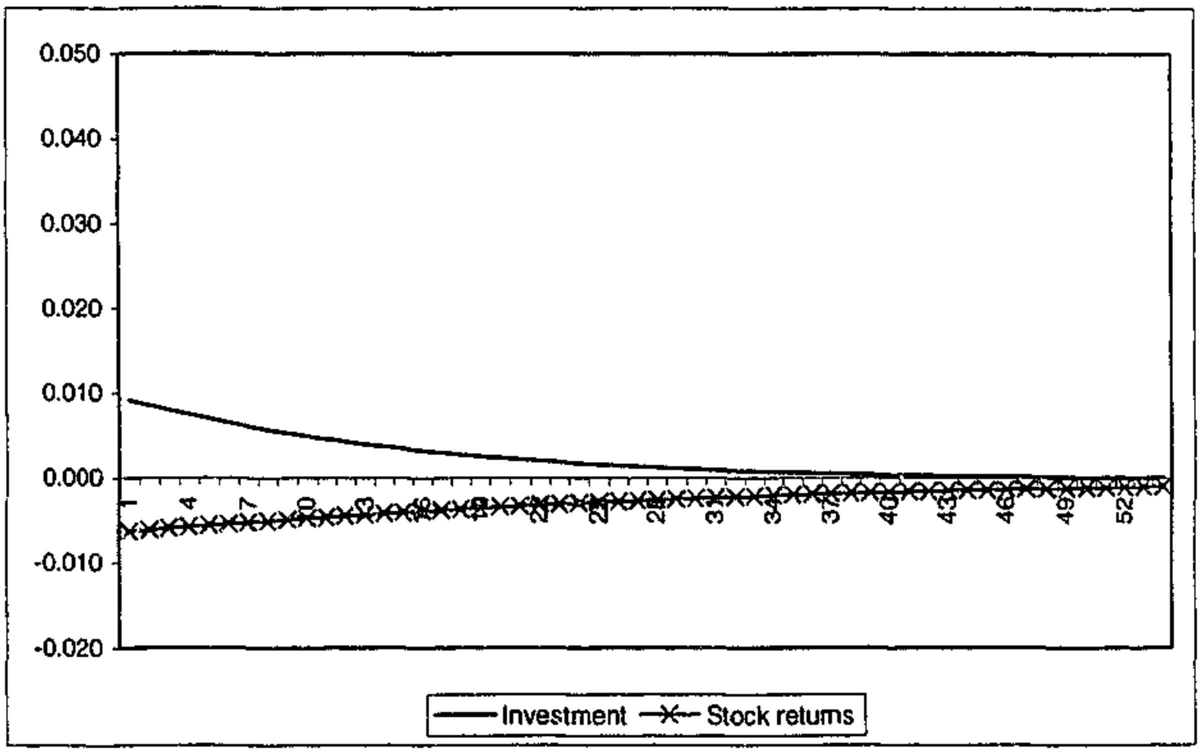
Panel B Relative Standard Deviations

γ	ρ	Output ($\Delta\log Y_t$)	Consumption ($\Delta\log C_t$)	investment($\Delta\log I_t$)
1.00	0.95	1.000	0.541	2.787

Note: This table reports the simulated relative standard deviation of output, consumption, and investment. To construct this simulated cross correlation, the varying values of risk aversion coefficient, γ, and shock persistence, ρ, have been considered. However, we report only the case where γ = 1 and ρ = 0.95. Overall pattern is similar regardless of risk aversion and shock persistence: investment is more volatile than consumption. The results are available on request.

The results of the model with the capital adjustment costs are presented in Table 7.4 (ϕ= 1.50) and 5 (ϕ= 2.32). Overall, the results of our model mimic the actual data quite well. Even though the values of correlation are slightly different from those of the actual data, the signs of the correlation coefficients are the same as those of the actual data regardless of the value of adjustment costs, except for the transitory shock ($\rho = 0$).

Figure. 7.3 Impulse Responses of Investment and Stock Returns for Investment-specific Technology with Capital Adjustment Costs (ϕ= 1.50)



Notes: to construct the impulse response analysis, $\gamma = 1.0$ and $\rho = 0.95$ has been used. This figure shows that investment responds to shock positively, while stock returns responds negatively. This result implies that the correlation between investment growth and stock returns is negative. Compared to Figure 7.2, the existence of capital adjustment cost reduces the volatilities of investment and stock returns.

First, consider the case in which the risk aversion is set to unity and the shock is fairly persistent ($\rho = 0.95$). The signs of the correlation coefficients are same as the actual data. Compared to the investment-specific shock case, the sign patterns are the same. However, the magnitude of correlation with lagged stock returns has been decreased. The presence of capital adjustment costs in the model changes investment behavior in that it ceases to be volatile in response to the shock. As can be seen in Figure 7.3, investment responds positively to a positive shock with small fluctuations.

Next, consider changes to the risk aversion coefficients. The results are similar to those of the model with investment-specific technology. The risk aversion coefficients do not play a role in this model.

Finally, consider changes to the persistence of shock and keeping the risk aversion at unity. Changing the persistence shows different movements in correlation with led, current and lagged stock returns. As mentioned in section 7.4.2, this is mainly caused by which effect (direct or indirect effects) dominates in this model.

When both results (Panel A of Tables 7.4 and 7.5) are compared, the current and future correlations of Panel A of Table 7.4 ($\phi = 1.50$) are slightly lower in magnitude than those of Panel A of Table 7.5 ($\phi = 2.32$). This is because when the ϕ is relatively small, a given technology shock will generate more investment and output than when ϕ is relatively high (Greenwood et al. 2000). This causes the variances of investment growth and stock returns in Panel A of Table 7.4 to be bigger than those of Panel A of Table 7.5.

In terms of the performance of the model, the different size of capital adjustment costs shows different volatility. The volatilities of consumption and investment are close to those of the actual data and the benchmark model. The existence of capital adjustment costs in the model makes new investment more costly. Therefore, the response of investment to a positive technology shock is smoother in investment-specific technology with capital adjustment costs than in investment-specific technology. Therefore, the variance of investment is smaller than that of Panel B of Table 7.3.

Table 7.5 Moments Inferred from Investment-specific Technology with Capital Adjustment Costs ($\phi = 2.32$)
Panel A Correlation Coefficients

		Stock Return				
γ	ρ	-2	-1	0	1	2
0.01	0.00	0.002	-0.323	0.443	-0.003	-0.003
	0.50	0.213	0.423	-0.392	-0.198	-0.101
	0.95	0.108	0.113	-0.167	-0.160	-0.153
1.00	0.00	0.000	-0.465	0.573	-0.002	-0.002
	0.50	0.214	0.427	-0.347	-0.175	-0.090
	0.95	0.139	0.146	-0.142	-0.135	-0.129
10.0	0.00	0.000	-0.500	0.600	-0.002	-0.002
	0.50	0.214	0.430	-0.326	-0.165	-0.084
	0.95	0.167	0.176	-0.114	-0.108	-0.103
100.0	0.00	0.000	-0.505	0.603	-0.002	-0.002
	0.50	0.214	0.430	-0.323	-0.163	-0.083
	0.95	0.173	0.184	-0.106	-0.101	-0.097

Note: This table reports the simulated cross correlation between investment growth and stock returns using cor(ΔI_t, Δr_{t-i}) where i = -2, -1, 0, 1, 2. To construct this simulated cross correlation, the varying values of risk aversion coefficient, γ and shock persistence, ρ, have been considered.

Panel B Relative Standard Deviations

γ	ρ	Output ($\Delta \log Y_t$)	Consumption ($\Delta \log C_t$)	investment($\Delta \log I_t$)
1.00	0.95	1.000	0.782	1.835

Note: This table reports the simulated relative standard deviation of output, consumption, and investment. To construct this simulated cross correlation, the varying values of risk aversion coefficient, γ and shock persistence, ρ, have been considered. However, we report only the case where $\gamma = 1$ and $\rho = 0.95$. Overall pattern is similar regardless of risk aversion and shock persistence: investment is more volatile than consumption. The results are available on request.

7.4.4 Comovements between Investment Growth and Stock Returns

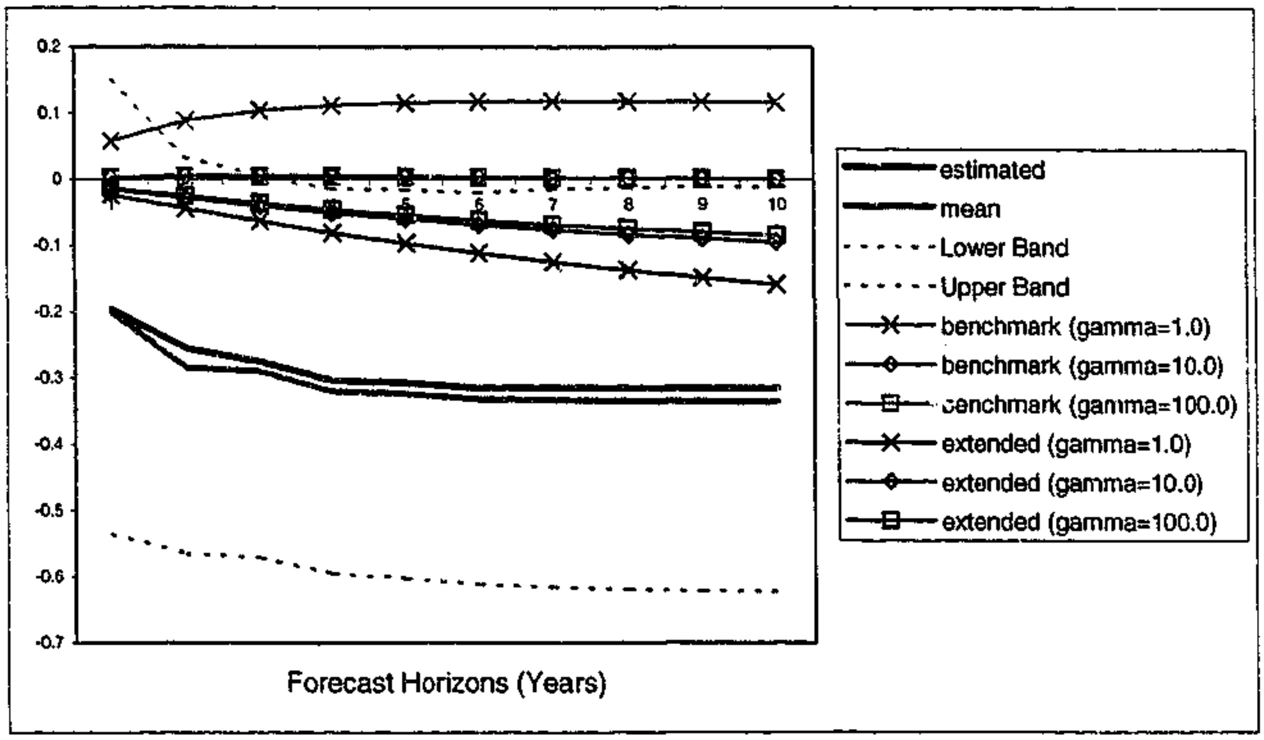
Traditionally, in the RBC literatures by focusing on only the unconditional correlation, one is losing valuable information about the dynamic aspects of the comovement of variables. Therefore, examining the long-run relationship between variables is essential to determine whether the economic models can explain the actual economy or not. In this section, we analyze the comovements of investment growth and stock returns using the VAR forecast correlations developed by Den Haan (2000).[9]

The forecast correlation coefficients between investment growth and stock return are plotted up to 10 years ahead in Figure 7.4. In this figure, we plot four correlation coefficients: estimated, mean, benchmark model and extended model with various values of γ (investment-specific technology and capital adjustment) with a 95% confidence interval. As mentioned in Den Haan (2000) and Bekaert et al. (1997), the calculated correlation coefficients are subject to sampling variation because they are based on the estimated VARs. For this reason, we calculate the confidence interval using bootstrap methods. More specifically, the estimated VAR and its bootstrapped errors are used to generate 3000 economies. The correlation coefficients with up to 10 years forecast horizons are calculated for each economy. The average correlation coefficients are plotted in Figure 7.4 as a mean. As can be seen in Figure 7.4, the average across replications is very similar

[9] For an explanation of how to derive the forecast correlations, see Den Haan (2000).

to the original estimates, which means that no correction for small-sample bias is needed.

Figure 7.4 Forecast Correlation Coefficients between Investment Growth and Stock Returns



Notes: When the correlation coefficients of the benchmark and the extended model are calculated, the parameters used: $\rho = 0.95$, $\phi = 2.32$ and the values of γ are indicated in the figure. This figure shows that the standard growth model can not generate negative forecast correlations, while the actual data shows negative forecast correlations. However, our extended model generates negative forecast correlations over all forecast horizons.

As noted in Figure 7.4, the long-run correlation between investment and stock returns in the actual economy is negative. To check which model economy better mimics the actual comovements of investment growth and stock returns, we also plot two simulated long-run correlation coefficients of the benchmark model and extended model. The long-run correlation coefficients in the benchmark model are positive, while our extended model shows negative correlation coefficients regardless of time horizons. This test shows that our extended model performs better than the benchmark model in the long-run.

7.5 Conclusion

The purpose of this paper is to gain more insight into the determination of correlation between investment growth and stock returns in the RBC framework and to examine how two modifications of a benchmark model (investment-specific technology and capital adjustment costs) work to mimic the actual data. We present the closed-form solutions for the correlation with led, current and lagged stock returns based on the approximate log-linear solution of Campbell (1994). This procedure allows us to express the correlation coefficients as a function of the fundamental parameters of the model. To achieve our purpose, we extend the model two ways: first, adopting investment-specific technology and second incorporating the capital adjustment costs in investment-specific technology.

Since the current correlation between investment growth and stock returns in the benchmark model shows a similar result of that predicted by Q theory, the interpretation from the benchmark model is that the standard real business cycle model fails to explain the relationship between investment growth and stock returns. In fact, the standard RBC model (our benchmark model), when driven by persistent technology shocks predicts a highly procyclical movement in stock returns, while the actual data does not support this claim. This is partly because the intertemporal substitution effect dominates the income effect. In the case of investment-specific technology, the risk aversion of consumer does not play a role in mimicking the data even though it also failed to mimic our actual data in terms of variability of variables. This is because in the transmission mechanism of investment-specific technology, the intratemporal substitution of consumption, and the intertemporal substitution work together.

While the model with investment-specific technology generates very volatile investment and smooth consumption, the model with investment-specific technology and capital adjustment costs generates a similar relative standard deviation to output. This is because the existence of capital adjustment costs in the model makes investment costly. Therefore, investment can be smoother than seen in the model with investment-specific technology.

The modified model with investment-specific technology and capital adjustment costs shows the same correlation sign as the actual data set. Investment growth and stock returns show different movements to a shock. The positive shock increases investment growth and decreases stock returns. The capital utilization has played an important role in transmitting the shock. For example, the positive shock decreases current stock returns because of increased capital utilization, which causes more depreciation. In contrast, it increases investment. Therefore, the correlation coefficient has a negative value.

Finally, in the long-run, the correlation coefficients are negative in the actual data. The benchmark model, which has a standard technology shock, generates positive correlation coefficients in the long-run and in the contemporaneous relationship, while the our model that includes the investment-specific technology shock and capital adjustment costs generates negative correlation coefficients in the long-run.

One of the important empirical findings of this chapter is that the relationship between investment growth and stock returns appears puzzling partly because the standard model does not consider capital adjustment costs, and partly because the intertemporal substitution effect dominates the income effect.

Chapter 8 Concluding Remarks and Summary

This research examines the relationship between several variables – it mostly focuses on stock returns using various time-series methods and the RBC model. Studying the relationship is of interest because it is important for investment decision making and risk management in finance. In contrast with previous studies, which show that the relationships are sensitive to the methodologies and models used for financial analysis. This thesis examines not only the short-run relationship but also the long-run relationship. The long-run relationship is also important for investors, who have various investment horizons. To investigate both short- and long-run relationships, thesis adopts some newly developed methods: wavelet analysis, King and Watson approach, VAR forecast correlation, and spectral analysis, and a standard growth model, which is extended to incorporate investment-specific technology and capital adjustment costs. The findings can be summarized as follows:

Chapter 3 studies the relationship between various financial variables and real activity, proxied by industrial production. Many empirical studies find that financial variables possess a predictive power of real activity. To examine this relationship, chapter 3 adopts two time-series techniques: spectral analysis and wavelet analysis. The result of spectral analysis shows that US industrial production and the financial variables have a common feature in the long-run and a varying lead-lag relationship depending on the cycles. It implies that the relationship between US industrial production and financial variables is not fixed over time. This result is confirmed by the wavelet analysis. The lead-lag relationship, in the sense of Granger causality, varies depending on the time scale. From the two time-series analyses (spectral analysis and wavelet analysis), it can be concluded that the lead-lag relationship between US financial variables and US industrial production varies depending on the time scale and frequency.

Chapter 4 tests the Fisher hypothesis, which states a positive relationship between nominal stock returns and inflation, and also provides a new perspective on the hypothesis. The new approach is based on a wavelet multiscaling method that decomposes a given time series on a scale-by-scale basis. Empirical results show that there is a positive relationship between stock returns and inflation at the shortest scale (1-month period) and at the longest scale (128-month period), while a negative relationship is shown at the intermediate scales. This indicates that the nominal return results are supportive of the Fisher hypothesis for risky assets in d1 and s7 of the wavelet domain, while stock returns do not play a role as an inflation hedge at the intermediate scales. The key empirical results show that time-scale decomposition provides a valuable means of testing the Fisher hypothesis, since a number of stock returns and inflation puzzles previously noted in the literature are resolved and explained by the wavelet analysis.

Chapter 5 examines the multihorizon Sharpe ratio. This study extends Hodges et al. (1997), following the argument of Siegel (1999). The wavelet decomposition shows that the risks of all returns is lower than or similar to the expected value by random walk assumption of asset returns, while at the intermediate scales, d2 to d5 (equivalent to 4-8 months period and 32-64 months period, respectively), the risk of all returns are greater than the expected. Overall, the result supports the mean reversion property of asset returns. In addition, the long memory parameters, using the wavelet-based maximum-likelihood estimation, for all asset returns are less than 1 and close to 0, implying that the asset returns are mean-reverting. Turning to the main purpose of this chapter, the results are different from those of Hodges et al. (1997), who find that the Sharpe ratio eventually declines in all cases and the counter-intuitive result that bonds become more attractive than stocks for long holding periods. However, the key results support the conventional wisdom of money managers that for investors with long horizons, a greater share of portfolio assets should be allocated to stocks (Siegel, 1998, 1999).

In short, these results indicate that the Sharpe ratio of large-company stock portfolios is a higher value than the other three types of portfolios (small-company stocks, long-term and intermediate-term government bonds) over all wavelet scales. In other words, large-company stock portfolios outperform the other portfolios over the wavelet scales. This result is closely related to the mean-reverting property of stock returns. As indicated in Samuelson (1991) and Barberis (2000), the mean-reversion property in stock returns slows the growth of conditional variances of long horizon returns. This makes the equities appear less risky at long horizons, and hence more attractive to the investor.

Chapter 6 investigates the long-run relationship between stock returns and risk (volatility) using two newly developed methods: King and Watson (1997) and Den Haan (2000). Many previous studies do not show the consistent results and focus on the contemporaneous relationship. Chapter 6 supports that the long-run relationship highly depends on the contemporaneous relationship. For the VAR forecast correlation, proposed by Den Haan (2000), most industry portfolios show a negative relationship in the short-run as well as in the long-run. For the market portfolio, a negative relationship is dominant regardless of forecasting horizons. From these results, it is concluded that the long-run response of stock returns to a permanent volatility shock is sensitive to the assumed value of identifying parameters in each industry portfolio and the market portfolio and that, as in the previous studies, the relationship between risk and return is mixed in the short-run. However, in the long-run, a negative relationship is dominant. A negative relationship in the long-run means that if investors feel that the risk of a portfolio is high in the future, the price of the portfolio rises to compensate the increased expected risk. Therefore, the future return of the portfolio decreases.

In chapter 7, the puzzling relationship between investment growth and stock returns is examined in an extended stochastic growth model. Many empirical studies find a negative relationship between current investment growth and current stock returns, while the theory predicts a positive relationship. A key to an explanation of this negative contemporaneous relationship may lie is in the countercyclical movement of stock returns. The standard growth model cannot generate the countercyclical movement of stock returns, whereas the extended model, which incorporates investment-specific technology and capital adjustment costs does generate the countercyclical movements of stock returns. The positive shock increases investment growth while decreasing stock returns. Capital utilization has played an important role in transmitting the shock. For example, a positive shock decreases current stock returns because of increased capital utilization, which causes more depreciation. In contrast, it increases the investment. Therefore, the correlation coefficient has a negative value. In the long-run, correlation coefficients are negative in the actual data. The benchmark model, which has a standard technology shock, generates positive correlation coefficients in the long-run and in the contemporaneous relationship, while the model, which includes an investment-specific technology shock and capital adjustment costs, generates negative correlation coefficients in the long-run.

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