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Explanations of Attitudes to Change: Colombian Mathematics Teachers' Conceptions of their Own Teaching Practices of Beginning Algebra

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To my beloved daughter, Paula, and to my parents, especially my mother who encouraged me to be a teacher

To the Colombian mathematics teachers who chose to participate in this study allowing me to learn so much from them

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Declaration

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university. To the best of my knowledge and belief, it contains no material previously published or written by any other person, except where due reference is made in the text of the thesis.

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Ana Cecilia Agudelo-Valderrama

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Abstract

This study investigated the relationship between Colombian mathematics teachers' conceptions of *beginning algebra* and their conceptions of their *own* teaching practices. The teachers' understandings of their teaching practices were explored with a view to unravelling their conceptions of change in their teaching. Focusing on the perspectives of teachers afforded opportunities that exposed the powerful role that the teachers' conceptions of social/institutional factors of teaching played in their conceptions of their practices. The degree to which they attributed these (external) factors as crucial determinants of their teaching provided the basis for a categorisation of (the) teachers' conceptions of their practices into four types of teachers, that goes from the (fully) 'external attributions' teacher to the (fully) 'internal attributions' teacher. From the findings, implications were identified for the creation of possibilities of change in the teaching of beginning algebra (and mathematics in general) in Colombia, as well as for developing the research on teachers' conceptions of mathematics and its teaching.

A two-phase case study research design was chosen for this study. In Phase 1, which aimed to identify a variety of conceptions from an initial group of teachers in order to select case studies, data were collected from a group of 13 mathematics teachers, who taught at six different (state and private) schools in Bogotá. The teachers varied in ages and teaching experience and were teaching in Grade 8. In Phase 2, a multi-case study with the participation of nine selected teachers was carried out.

In Phase 1, data were collected through the use of two questionnaires and an interview, taking place in the following sequence: Questionnaire 1, Questionnaire 1 follow-up interview, and Questionnaire 2. Descriptors used in the questionnaires were developed assuming that teachers' conceptions of the nature of mathematics might be very different, and might range from a traditional "instrumentalist" (Ernest, 1989) perspective of a collection of unrelated facts, rules and skills to one in which mathematics is a continually expanding field of human inquiry, where problem solving and understanding are central in mathematical activity (e.g., Cockcroft Report,

1982; Ernest, 1989, 1991). In Phase 2, data were collected through classroom observation, interviews, examination of curricular materials and a focus group session. The teachers dedicated part of the interviews to the construction of a concept map of the determinants of their practices.

Data analysis was conducted in the language (Spanish) of the data collection. Data collected through the different sources were reviewed and classified in order to identify i) the teachers' conceptions of beginning algebra, and ii) the teachers' conceptions of their own teaching practices. In identifying the teachers' conceptions of beginning algebra, the focus of the analysis was placed on data related to the fundamental components of teaching (i.e., answers to the *why*, *what* and *how* of the teaching of beginning algebra). In identifying the teachers' conceptions of their own teaching, the focus was placed, at one point, on *why the teachers taught Grade 8-algebra in the way they did* and, at another point, on *why they would (or would not) be willing to consider a different approach in their teaching of Grade 8-algebra.*

A significant contribution of this study is a typology of teachers' conceptions of their own practices which provides key insights to inform the provision of professional development and teacher education programs in Colombia. This contribution is particularly relevant to our understanding of the stability of mathematics teaching approaches in the Colombian context but has likely implications for a range of international education contexts. Another significant contribution of the study is represented by its theoretical implications for the development of the research into teachers' conceptions of mathematics and its teaching. The model of mathematics teachers' thought structures that emerged includes teachers' *social* 'knowledge, beliefs and attitudes' as an integral dimension of teachers' thinking.

Chapter 1: The origins and background to the study

In Colombia —and internationally— one of the most problematic areas of school mathematics learning is algebra (see, for example, Bednarz, Kieran & Lee, 1996; Booth, 1984; Cockcroft Report, 1982; Küchemann, 1981; MacGregor & Stacey, 1994; Zazkis, 2001). Data collected in the Colombian context on the learning (see Bonilla, 1994; Romero, Rojas & Rodriguez, 1996) and on the teaching of school algebra (see Agudelo-Valderrama, 2000, 2001) point to the great need to focus on the curriculum of this area where students' lack of motivation for work is a general problem, which produces high rates of failure and school drop out. Comparisons of performance in mathematics of students from 41 countries in the TIMSS[1], have ranked Colombian students in the 40th place (see, for example, Lokan, Ford & Greenwood, 1996). This means that Colombian school students are not developing the powerful mathematical thinking that is cultivated through the learning of algebra. Can this be related to the teaching approaches to which students are exposed?

Algebra involves ways of thinking and seeing real-life situations that go beyond facts and computation with particular numbers. It focuses attention on the relational aspects of operations and the underlying mathematical structures present in contextual problem situations. Some of the capabilities observed in algebra-competent students include an awareness of the mathematical structures that govern relationships between the quantities that operate in specific problems or contextual situations being explored, and the abilities to generalise and to represent (in different forms) the identified relationships (cf. Roberts, 2002). This powerful mathematical thinking, which can be developed by young students (see, for example, Sutherland 1991; Kaput & Blanton, 2001), and has been characterised as algebraic thinking (Mason, 1999; Mason, Pimm, Graham & Gowar, 1985), is useful for individuals in the analysis of real-life situations and decision making and is, therefore, required for their active participation in democratic societies. Further, with the advance of technology, individuals are being

[1] The Third International Study in Mathematics and Science was carried out in 41 countries during the period of 1991-1995. This study involved Colombian students belonging to Population 2 level (i.e., Grades 7 and 8 students) and to Population 3 level students (i.e., Grade 11 students).

increasingly required to be algebra-competent citizens. However, as noted above, students experience great difficulties in the learning of algebra. Although some students manage to pass exams and move on to higher education courses, many are unsuccessful. When students are unsuccessful in learning algebra, they dislike the subject and they are more likely to become marginalised (Agudelo-Valderrama, 1996) and disadvantaged in society (Usiskin, 1999). Unfortunately, through my experience and work with mathematics teachers in Colombia, I have learnt that many teachers are not interested in changing their teaching approaches in school algebra; that is, in school mathematics generally.

My previous research in Colombia, in a project (*PROMECA*[2]), which studied the teaching-learning patterns taking place in four schools —in what was called the transition from arithmetic to algebra work— showed that *algebra* lacked meaning not only for pupils but also for teachers (González & Pedroza, 1999). The PROMECA study showed that the teaching of algebra focused on an "absolutist" (Lerman, 1990), "instrumental" (Ernest, 1989), 'transmissive' (e.g., Brown, 2000; Heaton, 2000) delivery approach of a list of topics set for Grades 8 and 9. The teachers emphasised the great difficulties and the low motivation that their pupils showed in the learning of algebra. However, most teachers showed very low interest in participation in professional development, despite their awareness of the pupils' difficulties and of the high rates of mathematics failure, and despite the support offered by their head teachers for their participating in the development program. The emphasis placed in central policy (see Subsection 1.1.2.2) on teachers becoming "*continuous* constructors of the curriculum" (National Ministry of Education 'MEN', 1994, 1996 & Mathematics National Curricular Guidelines, 1998, p. 1), and to encourage the active participation of pupils in their learning did not seem to represent a call for change in their teaching approaches for most of them.

The literature shows that the phenomenon of the persistence of 'instrumental' and 'transmissive' mathematics teaching approaches is not unique to the Colombian context,

[2] PROMECA, in Spanish, stands for "Promoting teaching for understanding in elementary algebra". The piloting of the 'Exploration phase' of the data collection (i.e., the data collection for identification of the teaching-learning patterns followed in Grades 7 and 8) of this study was carried out from March to June 1998. The main study, which included the 'Exploration phase' and the Professional Development project that was called PROMECA, started in August 1998 and ended in July 2000.

for these classroom approaches in school mathematics have also been reported in other educational systems. For example, in the United States of America, Smith III (1996) pointed to the widespread practice of teaching mathematics "by telling", in contrast to the principle put forward by the curricular reform of teaching by enabling students' mathematical activity rather than by telling and demonstrating procedures to them. Gregg (1995) also highlighted "the durability and stability" of traditional practices in the teaching of mathematics, and Price and Ball (1997) argued that classroom practices in many places in the United States continue to be "as conventional as ever", despite the fact that "contemporary reforms in the US urge deep changes in mathematics teaching and learning" (p. 637).

As will be discussed in Chapter 2, a substantive amount of quantitative research on the teaching of school mathematics has focused on the identification of consistency (or inconsistency) between teachers' 'beliefs' or 'conceptions' of mathematics and their conceptions of mathematics teaching. However, studying teachers' beliefs about mathematics and its teaching out of the actual context of teachers' classroom practices does not tell us much about the difficulties of teacher change. The few qualitative studies in which mathematics teachers' actual practices have been researched have also focused attention on identifying the consistency or otherwise between teachers' professed beliefs and their actual practices. From these qualitative studies we learn that teachers identify contextual factors to explain the inconsistencies between their professed beliefs and their actual practices, but we do not learn about *why* the teachers think those factors impact on their practices and *how* those factors influence their understanding of their *own* teaching practices. How do teachers see the relationship between their conceptions of mathematics and their conceptions of their own teaching practices —as opposed to how researchers seem to see it?

If we want to gain some understanding of the phenomenon of the persistence or 'stability' of mathematics teaching and about the possibilities of teacher change in Colombia —or in any other context— what counts is the teachers' conceptions of their *own* teaching practices, which must take into account their understandings of what they see as barriers to change in their specific contextual situations. We need to study not just the teachers' conceptions of mathematics and mathematics teaching but also their conceptions of their 'own' teaching practices. We have very little knowledge about teachers' understandings of their own teaching practices (e.g., their conceptions of their roles as teachers, the contextual situations of their teaching and their change). Therefore, the overall aim of this study is to investigate the relationship between mathematics teachers' conceptions of *beginning algebra* (mathematics) and their conceptions of their own teaching practices with a view to unravelling their conceptions of change in their practices.

The term *conceptions* in this study was defined to encompass teachers' *knowledge, beliefs* and *attitudes* as it has been shown that teachers' beliefs and attitudes have a "powerful impact on teachers' make-up and approach" (Ernest, 1989, p. 25). The Colombian teachers in the PROMECA project provided strong evidence that their attitudes to their teaching represented key components of their conceptions. 'Attitudes' as components of the affective dimension of teachers' conceptions needed to be recognised and understood if I was to gain insight into their resistance to change.

'Beginning algebra', as will be explained further in Chapter 2 (Section 2.2), refers to the processes through which the teachers introduced algebra work to pupils. Therefore, of special attention were the teachers' conceptions of the concept of variable, as this study assumes that this concept is at the centre of algebraic thinking and therefore of mathematical thinking.

The teachers' understandings of what they did in their teaching and their explanations for 'why they taught in the way they taught' represent the teachers' conceptions of their own teaching practices. Focusing on the teachers' perspectives provided opportunities to identify the influences that their conceptions of the social/institutional context of teaching played in their conceptions of their teaching practices and, therefore, of their change. The teachers' understandings of their own teaching practices were explored with a view to unravelling their attitudes to change in their teaching practices, which illuminated possibilities for teacher change.

1.1 Background to the study

Having provided an overview of the nature and purpose of this present study, this section presents more detailed information about the context from which the study originated and about the Colombian educational system.

1.1.1 Findings from the PROMECA project

The Colombian project PROMECA, from which this present study arose, followed a naturalistic approach (Guba & Lincoln, 1981, 1994), which comprised a 20-month longitudinal study exploring the teaching-learning patterns that took place in the classrooms of seven teachers who were teaching in both Grades 7 and 8. I chose to work with teachers who were teaching in both Grades 7 and 8 because, according to them, "algebra starts in Grade 8". Therefore, it was important to focus both on how 'algebra work began in Grade 8' and on what work preceded algebra (in Grade 7).

Data[3] from the 5-month 'Phase of Exploration of the situation' or 'Exploration phase' showed that teachers characteristically presented algebra as a set of formal definitions and rules to operate with given symbolic expressions. An incomplete understanding of the concept of variable was observed on the part of the teachers, which limited their capacity to organise classroom work in a different form from the one proposed in the textbooks being followed. The data also showed that pupils disliked the subject. They declared a lack of understanding and relevance of algebra to their lives. The pupils' lack of motivation for the study of algebra was also a main problem identified by the teachers.

Of particular interest was the lack of motivation observed on the part of some teachers, for the teaching of "Grade 8-algebra", and the lack of meaning given to letters in algebra. The teachers showed no awareness of the multifaceted character of variable and, therefore, lacked the flexibility to handle the different uses of variable, which affected their understanding of algebraic expressions in general. Gains in understanding of the mathematical concept of variable by the teachers who participated in the

[3] During the 'Exploration of the situation', information was collected through teachers' interviews, classroom observation, examination of curricular materials and pre-test and interviews applied to pupils.

development program (PROMECA) and who completed it, resulted in an increased capacity to design and organise classroom activities that provided pupils with opportunities to engage in the establishment of connections between the concepts explored and their own mathematical ideas (see Agudelo-Valderrama, 2000).

Furthermore, the teachers who followed the Development Program through, identified *their knowledge* of subject matter as playing a *crucial* role in their capacity to improve their own practice (see González & Pedroza, 1999). Although mathematics content knowledge has been identified as crucial for teachers to be able to teach (Leinhardt, Putnam, Stein & Baxter, 1991) and to teach for understanding (Ball, 1991; Fennema & Franke, 1992; Schifter & Fosnost, 1993; Shulman, 1986, 1987), the findings of the study made it clear to me that knowledge of mathematics content is not the only component of teachers' conceptions that accounts for teachers' change of practices. These teachers were successful in achieving change because they were interested in looking for alternative teaching approaches, as they were concerned with the learning of their pupils. They had positive attitudes towards changing and improving their practices, but this was not the case with other teachers who withdrew part way from the Development Program. Improvement of practice in their case, involved two aspects: (i) the development of their capacity to design and monitor their own classroom activities, rather than to follow what was posed in the textbook, and (ii) the observed change of attitudes of their pupils who, according to the project-reports of the teachers, showed a motivation for the learning of algebra that they had never seen before.

My observation of the impact of teachers' attitudes on their teaching practices pointed to the crucial need to include teachers' *attitudes* as a component of their conceptions in a study that is concerned with the identification of possibilities for teacher change.

1.1.2 The Colombian education system

Colombia has had a history of strong centralisation in educational organisation and decision making. Education delivery has taken place within a highly prescriptive system that focuses on an academic, formalistic curriculum designed with the needs (as established by the system) of the student population who moves on to higher education.

Although an increase in educational coverage has taken place over the years, Colombia has yet to achieve universal primary school enrolment and a satisfactory level of retention. In 1987, 30% of the population aged 7-11 did not have access to basic primary education, and 50% of the children who enrolled in the public primary school did not finish (MEN, 1987). According to the 1998 UNESCO Report on Latin America and the Caribbean Region, 11% of the school-age children (6-14 year-olds) were not enrolled in the school system, and dropout rates were high. The report highlights that Colombia is one of a number of countries with some of the most extreme inequalities in the distribution of income in the world. The inequalities in income are mirrored by inequalities in access to schooling and attendance:

> The rural poor and indigenous populations are at the extreme disadvantage relative to other groups... On average, two out of every five children in rural areas (as compared with one in six in urban areas) fail to finish primary school or are at least two years behind when they do so. (1998 UNESCO Report on Latin America and the Caribbean Region, p. 13)

In 1994, as a consequence of the reformed Constitution of 1991, The General Law of Education was issued, introducing a new framework for the organisation and provision of basic education. The central priorities emphasised by this law are:

- The decentralisation of the administration of education, and
- the flexibility of the curriculum

According to this law, each school needs to design its own 'Institutional Education Plan', which should include the principles and aims of the institution, the curriculum, and the school's organisational and management plan. Teachers are urged to participate actively in the construction of the school curriculum, as education is to be improved by attending to the needs of the specific communities which schools serve (Article 78). The new curriculum guidelines emphasise a shift in teaching methods from the traditional "chalk and talk" to a "hands-on" approach with more active participation on the part of the students (Díaz, Solarte & Arce, 1997). As in previous curricular reforms (see MEN, 1975, 1990), the General Law of Education establishes as one of the *fundamental aims* of basic education, "the development of the individual's critical, reflective and analytical capacities" which are needed for his/her active participation in a *democratic* society (Articles 5 & 21).

1.1.2.1 Structure of the school system

The school system is composed of four stages: pre-school (1 year), primary school (5 years), basic secondary school (4 years), and secondary vocational school (2 years) as shown in Table 1.1. According to the law, the first four years of Secondary (i.e., Grades 6, 7, 8 & 9) which belong to the basic 9-year cycle of education is *compulsory*. The last two years of Secondary (i.e., Grades 10 & 11) belong to vocational education. Most secondary schools offer the *academic* vocational mode (i.e., the one that is required for entrance to higher education). The full secondary school cycle is called *Baccalaureate*. According to the law, one year of preschool education is to be compulsory for five-year olds (Article 17). However, while in rural schools there is no provision yet of pre-school education, private schools in capital cities normally offer two years of pre-school education (Kinder and Transition). Table 1.1 shows the structure of the school system established by the 1994 policy.

Table 1.1 The structure of the school system in Colombia according to current policy

Level	*No. of years*	*Grades*		
Pre-school	1 year	Pre-school		
		1		
		2	*Basic cycle*	
Primary	5 years	3		
		4		
		5		
		6		
		7		
		8		
Secondary	6 years	9		*Baccalaureate*
		10		
		11		

State schools offer two school sessions a day for two different groups of students: the morning session from 7:00 am to 12:30 pm, and the afternoon session from 12:45 pm to 5:45 pm. However, some state schools in the capital cities offer an extra session (the evening session) designed for students who work during the day. All secondary school leavers must sit the National External Examination carried out by ICFES[4], where standardised tasks are used in order to control admission to higher education. The examination is also used as an indicator of the academic quality of the schools.

1.1.2.2 Mathematics education in the Colombian education system

Before the issuing of the General Law of Education in 1994, the teaching of mathematics followed the National Curriculum which was based on the formalistic aspects of a "Pure mathematics model" (Robitaille & Dirks, 1982). According to the curriculum document mathematics teaching in schools had to follow a list of hierarchically organised topics that had been compartmentalised into different areas of mathematics. Consequently, algebra was a packaged course to be taught in Grades 8 and 9. The programs for each school grade came organised into 'units' of study, and these, in turn, into topics and subtopics with their corresponding specific objectives, teaching methods, number of hours to be spent in each unit, and indicators of pupils' assessment. Curriculum statements were translated into pupils' textbooks that became the sole focus of teaching and learning all around the country (Mockus, 1985). This type of textbook, designed for the pupils, continues to be the only curricular material available for teachers (Diaz, Solarte & Arce (1997). This fact was corroborated by the findings of PROMECA. Table 1.2 outlines the curriculum content for the different grades of the school system.

Table 1.2 The focus of school mathematics in the Colombian educational system

Level	*Area of mathematics.*	*Grades*
Primary		1
	Arithmetic	2
	and	3
	Notions of geometry	4
		5
Secondary	Arithmetic &	6
	geometry	7
	Algebra	8
	Algebra	9
	Trigonometry	10
	Calculus	11

[4] ICFES: The Colombian Institute for the Promotion of Higher Education

Although current textbooks name the corresponding school grade they are designed for (i.e., 'Grade 7 mathematics' or 'Grade 8 mathematics', and not 'Arithmetic' or 'Algebra' as in old textbooks) the content and focus of the work follows the traditional list of topics of the National Curriculum that was in force until 1994.

Curricular reform

Curriculum design and development have traditionally been the responsibility of the central government. The 'Modern Maths' Curriculum in Colombia, and generally in Latin America, was adopted in the early seventies through imported ideas especially from the School Mathematics Study Group, Papy and others (Silva D'Ambrosio, 1991). The importation of ideas was a consequence of Colombian scholars studying mathematics overseas, who became the experts and specialists in the field of curriculum design. In Colombia curriculum development has been associated with the addition of new abstract topics to the original list of topics assigned to each school grade. For example, the topic of Logical connectives, that in the programs of 1975 belonged to Grade 8, in the reform of 1988 were included in the primary grades. According to the framework of the 1988 reform, the inclusion of topics of Logic from the first year of the primary school is justified because:

> Computers embody in their circuits the symbolic Logic of Boole... This Logic occupies today a privileged place in the design and use of calculators and computers, and in general, in all areas of informatics. (Mathematics curriculum document, MEN, p. 9).

As outlined before, according to the General Law of Education issued in 1994, every school must design its own Institutional Education Plan that includes the school curriculum, as schools are autonomous in the design of their curricula (Article 77). The role of the Ministry of Education is "to provide general guidelines that constitute the core and common curriculum for all schools..." (Article 3, Chapter II of Decree 2343, 1996). In 2002 a curriculum document called *Curricular Standards for Mathematics and Language* was issued by the Ministry of Education. This document (see in Appendix 1.1) lists of outcomes (or Attainment indicators), set for the different grades across the 11-year school cycle, which are organised in relation to what the document states are "the components of the mathematics curriculum" (p. 14). The different components are equated to five different types of thinking that the teaching of mathematics is to develop in the learner. These are: numerical thinking, metric thinking, spatial thinking, aleatory and statistical thinking, and variational and algebraic thinking. Apart from this new organisation of 'Attainment targets', which are referred to in the document as "Curricular" Standards, there is no consideration or discussion of the relation of this *content* organisation to classroom practices. Further, there is no policy related to the provision of professional learning experiences of teachers.

As argued earlier, the development of the curriculum is understood, basically, as the adoption of imported ideas[5] in relation to mathematics content. The evidence provided both by some mathematics educators in Colombia and from the official in charge of the design and issuing of the *Curricular Standards for Mathematics* show that this Curriculum document —in the same way that the curricular reform of 1986 (see Agudelo-Valderrama, 1996)— took place without any analysis of the specific conditions and needs of the communities in the country, without the necessary critical adaptation and without the engagement of teachers, teacher educators and community leaders in the process of construction and participation that such decision making would need.

Mathematics teacher education

Students who want to become primary school teachers in Colombia have three options. The first one is to do the twelve-year cycle offered by '*Normal*' schools which traditionally have been schools for the training of primary school teachers. The second one is to do two years of vocational education at a school that offers *pedagogical baccalaureate*, after completion of the nine-year basic cycle of education. The third option is to do a Bachelor of Education in primary as, over the past ten years, many Faculties of Education offer a four-year course for primary teachers. Secondary school mathematics teachers must follow a four-year course at Faculties of Education to obtain a Bachelor of Education or "*Licenciatura*" with a focus on mathematics. Table 1.3 presents an outline of the existing academic requirements for teachers who teach mathematics both in the primary and secondary school levels in Colombia.

[5] The evidence from my inquiry about the process followed in the production of the *Curricular Standards for Mathematics*, through personal conversation and through subsequent e-mail communication with the director of the team of the Ministry of Education in charge of the Curricular Standards, shows that decisions about the inclusion and organisation of the list of 'attainment indicators' or outcomes along the eleven grades of school were made by following the structure of "the outcomes statements of curricula from other countries available on the Internet, mainly from Australia..."

Table 1.3 Outline of the qualification requirements of teachers responsible for the teaching of mathematics in the Colombian school system

School level	*Qualification*	*Required secondary education*	*Required tertiary education*
Primary	'Normalista'	A 12-year cycle offered by 'Normal Schools' that includes 9 years of basic education and 2 years of training for teaching in primary	
	Pedagogical baccalaureate	A 11-year secondary cycle that includes 2 years of 'pedagogical baccalaureate' as vocational education	
	BA in Primary Education	11 years of secondary education as basic requirement	4 years
Secondary	BA in Education (e.g., mathematics & physics)	11 years secondary education as a basic requirement	4 years

1.2 Rationale of the study

As a Colombian school mathematics teacher during the 1970s and part of the 1980s, I was always concerned with what I saw as a lack of relevance of school mathematics for the everyday life of the great majority of children. This concern grew stronger as I became involved in mathematics teacher education programs in Colombia, where I found strong evidence of the prevalence of 'transmissionist' and 'instrumentalist' approaches in Faculties of Education, and of the fact that student teachers were not encouraged to problematise existing teaching approaches but to adapt to what takes place in schools.

My observation that student teachers spent the majority of their preparation time learning about abstract topics of mathematics with very little meaning and insight into the teaching of the subject is corroborated by Niño (1998) who contends that in many Faculties of Education in Colombia, teacher education programs centre on the coverage of a list of mathematics content plus some theories of education, with very little space for questioning what goes on in mathematics teaching and learning. Carrillo (1987) and Gómez-Ocampo (1998) also pointed out that the curriculum of teacher education in the majority of Faculties of Education in Colombia follows a traditional approach for the teaching of mathematics.

I am concerned with the impact that a mathematics education that is focused on the academic requirements of the few moving on to higher education, as in the Colombian educational system, has on the lives and possibilities of the majority of Colombian children. This concern and my awareness of the need to create informed strategies for the provision of professional development for mathematics teachers represented the driving force for undertaking this study.

1.3 Significance of the study

In the specific case of school algebra, although at the international level there is a considerable amount of research on children's learning difficulties, there is very little investigation into the ways that teachers teach algebra, or into teachers' conceptions of the nature of algebra knowledge and the learning by their pupils. The extremely limited evidence that we have [e.g., the work of Stacey & MacGregor (1999), Haimes (1996) and Agudelo-Valderrama (2000)] shows that in teaching algebra, teachers follow what is posed in textbooks. For example, in the Handbook of Research on the Teaching and Learning of Mathematics, Kieran (1992) points out that there is a grave scarcity of literature dealing with the conceptions of algebra teachers. The same situation has been identified in the 2001 ICMI study: The Future of the Teaching and Learning of Algebra (ICMI12 Study[6]. As is noted in Chapter 2 and in Agudelo-Valderrama (2003), at the international mathematics community level, a great deal of debate and attention has been given to the issues of the teaching and the learning of algebra during the last two decades, and several approaches to the introduction of school algebra have been put forward. However, while the debates and conceptualisations shed light on teaching issues, they do not tell us anything about the actual teaching of algebra or about teachers' conceptions of school algebra and its teaching.

[6] Refer to the ICMI12 Website: http:// www.edfac.unimelb.edu.au/DSME/icmi-algebra/. See Working Group: Teachers' knowledge and practice and the teaching algebra.

This study represents a contribution to our knowledge about the teaching of (beginning) algebra in the Colombian context. Further, as highlighted earlier, it focused on the perspectives of teachers and their understandings of their *own* teaching practices, offering insights into their conceptions of the social and institutional factors of teaching and the impact of these conceptions on their attitudes to change. A significant contribution of this study is a typology of teachers' conceptions of their own practices which provides key insights to inform the provision of professional development and teacher education programs in Colombia. Although this contribution is particularly relevant to our understanding of the phenomenon of the 'stability' of 'instrumental' and 'transmissive' mathematics teaching approaches in the Colombian context, it can represent useful insights for a range of other country contexts.

Another significant contribution of the study lies in its theoretical implications for the development of research into teachers' conceptions of mathematics and its teaching. The new model of *mathematics teachers' thought structures* that emerged includes teachers' *social 'knowledge, beliefs and attitudes'* as an integral dimension of teachers' thinking.

1.4 Research Questions and structure of the thesis

This study was conceived as a way of contributing towards the process of understanding the difficulties in the teaching of school algebra in Colombia. The purpose of this study was to describe and analyse the teachers' conceptions of beginning algebra, the teachers' conceptions of their own teaching practices, and how those conceptions relate to their conceptions of change in their practices. The basic Research Questions which arose from the review of relevant research literature in Chapter 2, and my experiential knowledge from my work with teachers are:

Research Question 1: What are the teachers' conceptions of beginning algebra?

Research Question 2: What are the teachers' conceptions of their own teaching practices of beginning algebra?

Research Question 3: What is the relationship between the teachers' conceptions of beginning algebra and their conceptions of their own teaching practices?

The Research Sub questions that guided the collection of data in relation to Research Question 2, which emerged from the review of relevant literature in Chapter 2 and from my previous research in Colombia, are presented in Subsection 2.3.5.5.

This chapter provides an overview of the nature and purpose of the study, and its origins in the author's professional context. It also provides relevant background information about the Colombian educational system. Chapter 2 examines the relevant research literature as suggested by the focus of this study, namely, research on (i) teachers' conceptions of mathematics, (ii) teachers' practices and (iii) teachers' change of practices. My reflections both on the findings and the methodologies used in the studies examined highlight the need both to focus on teachers' perspectives and to integrate affective and cognitive dimensions of teachers' conceptions in this study which is concerned with the identification of possibilities of teacher change. A proposed model of teachers' thought structures, built up as a thinking tool for the research design of the study is also presented in this chapter.

The need for a naturalistic approach and for the *case study* as the research strategy to gain insight into teachers' understandings of their own teaching practices and contextual situations is argued for in Chapter 3. A two-phase data collection strategy (Phase 1 and Phase 2) where the conceptions of an initial group of teachers were to be screened in order to best identify case studies is devised in this chapter. The chapter then reports on the pilot study work. Chapter 4 describes the main study.

The findings from the initial group of teachers in Phase 1 are reported in Chapter 5. Chapter 6 presents the case studies of five teachers who were selected (from a group of nine case study teachers) as representatives of a variety of conceptions of teachers' own practices.

Chapter 7 compares the five case study teachers' conceptions of their teaching practices of beginning algebra. This chapter becomes the heart of explanations for the teachers' attitudes to change in their practices. Chapter 8 synthesises the findings of the study from which clear implications for the provision of mathematics teachers' professional development and the creation of *possibilities of change* in mathematics teaching in Colombia are drawn. With an emphasis on the crucial need to engage mathematics teachers in professional development experiences, as informed by the findings of this study, the chapter concludes with a call for *coherence* in educational policy in Colombia and for action strategies *consistent* with policy, in order to create opportunities for the actual empowerment of Colombian teachers and for the improvement of mathematics education.

Chapter 2: Research context

2.1 Introduction

The focus of this study is on mathematics teachers' conceptions of *beginning algebra*, their conceptions of their own teaching practices of beginning algebra, and how those conceptions relate to their conceptions of change in their practices. This chapter presents the theoretical constructs, the research studies and the findings which were considered particularly relevant for the development of this study.

After a brief description of the scarce research literature on the teaching of school algebra, presented in Section 2.2, the research literature on the teaching of mathematics, which was found central to the specific focus of this study, is reviewed and discussed in Section 2.3. This section has been divided into three main subsections: research on teachers' conceptions of mathematics and mathematics teaching; research on teachers' practices, and research on teachers' change of practices. The discussions and the research questions that arose from the study of the research literature are included in this section. Section 2.4 reviews some available theoretical models of teachers' cognitions, which I found useful for the construction of a specific model that was used as a thinking tool in the design of this study. This model is presented in Section 2.5.

2.2 Research on the teaching of school algebra

At the international level, a great deal of research on pupils' learning difficulties in algebra has been carried out. From a large number of studies (e.g., Bonilla, 1994; Collis, 1975; Kieran, 1992; Küchemann, 1981; Booth 1984; Filloy & Rojano, 1989; Herscovicks, 1984, Lee & Wheeler, 1989; Linchevski & Herscovics, 1996) we know of pupils' common difficulties in the learning of school algebra; for example, the difficulty in the 'acceptance of lack of closure', the interpretation and discrimination of the various uses of the letters in algebra or the difficulty in establishing connections between the work students do with numbers and what they do with letters in algebra. However, we know little about the teaching experiences to which the students studied were exposed. In general terms, there is very little investigation that focuses on the way the teaching of school algebra takes place.

From the evidence of the extremely limited research that has been carried out on the teaching of algebra or on teachers' conceptions of algebra we simply know that teachers follow the textbook. A grave scarcity of literature dealing with the conceptions of algebra teachers was pointed out by Kieran (1992) in the Handbook of research on the teaching and learning of mathematics. The same situation has been identified in the 2001 ICMI study: The Future of the Teaching and Learning of Algebra (ICMI12 - Working Group: Teachers' knowledge for teaching algebra[1]). Haimes (1996) who studied the teaching patterns of a teacher in Australia, reports that although in the teacher's lessons there was a desire to teach for understanding —as promoted by reformed policy— her emphasis was placed on the learning of procedures to apply to a set of exercises. He also explains that the teacher saw the curriculum as a list of content objectives. In another research context in Australia, Stacey and McGregor (1999) found that algebraic thinking was not being promoted in algebra lessons, as the teachers just followed what was posed in some textbooks, which emphasised the use of *backtracking* to solve word problems.

The research that has focused on mathematics teachers' knowledge of the teaching of school algebra or of specific topics like 'algebraic expressions', 'function' or 'slope' has highlighted the inadequacies of the teachers' knowledge for the teaching of such concepts. Examples of such research are: Even and Tirosh and Robinson's (1993) study that focused on teachers' awareness of the pupils' tendency to conjoin "open expressions"; Even's (1990) study that researched teachers' understanding of the concept of function; Stump's (1997) study, which focused on teachers' knowledge of the concept of slope, and Menzel's (2001) study, which researched the understandings of algebra of five school algebra teachers in Australia.

Some researchers, as a consequence of the identification of children's difficulties in assigning meaning to the use of letters in algebra, have studied the effect that specific

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[1] Refer to http:// www.edfac.unimelb.edu.au/DSME/icmi-algebra/. A publication that emerged from this study is also available. See *The future of the teaching and learning of algebra–The 12th ICMI Study*. Edited by K. Stacey, H. Chick and M. Kendal, and published by Kluwer Academic Publishers in 2004.

teaching approaches that are based on the use of technology have on children's learning of the concept of variable. Askew and Williams (1995), Mevarech and Kapa (1996), Noss (1986) and Sutherland (1991) have reported that the use of computer software, like Logo and spreadsheets have a substantial positive impact on pupils' achievements in school algebra. The same results have been reported by Kieran, Boileau and Garacon (1996) with the use of their 'CARAPACE' program. The literature also reports on teacher professional development programs engaging teachers in the use of technology to teach algebra concepts[2] or promoting the use of manipulatives for the teaching of simple equations (see for example, Raymond & Leinenbach, 2000). However, while this research sheds light on teaching issues, it does not tell us much about teachers' conceptions of school algebra and its teaching.

The PROMECA study described in Chapter 1, from where this study arose, focused on the teaching patterns followed by seven teachers who were teaching in both Grades 7 and 8 because, according to them "algebra starts in Grade 8". Therefore, I wanted to learn about their work in Grade 8 where 'algebra work starts' and the work that preceded algebra (i.e., their work in Grade 7). Since my purpose in this current study was to learn more about Colombian teachers' understandings of school algebra and of what they did when they 'began algebra' teaching, I chose the term *beginning algebra* to identify the mathematical focus of this study. I shall now describe my perspectives in relation to the notion of 'beginning algebra'.

What is beginning algebra in the context this study?

Traditionally school mathematics teaching has centred in elementary schools on teaching arithmetic procedures and the mastery of computation, followed by a largely procedural approach to algebra from middle grades onward (Kaput & Blanton, 2001). This approach to the teaching of school mathematics, which Kaput and Blanton state is followed in the United States, is consistent with the patterns —of presenting pupils with *prefabricated* (Mason, Graham, Pimm & Gowar, 1985) algebraic expressions, and then focusing on the manipulations of the expressions— that were described by the Colombian teachers of the PROMECA study (see Agudelo-Valderrama, 2000 and Gonzáles & Pedroza, 1999). In this approach 'beginning algebra' was seen as starting a

[2] See, for example, *Hawaii Algebra Learning Project*. Retrieved 12 January, 2002 from The National Staff Development Council website: http://www.nsdc.org/

(new) course in secondary school, separate from other mathematics and from the knowledge and ideas that the students brought into their classrooms. Students learnt that "the letters are called variables" but they showed great difficulty assigning meaning to the work they were doing with the symbolic expressions, showing the same difficulties that Küchemann (1981) and Booth (1984) identified in their studies with large numbers of British children.

Following the call in mathematics education for a teaching for *understanding* and *meaning* —in which the Cockcroft report (1982) played an important role— the need to provide students with environments embedded in their cultural —everyday life— experiences, and to take account of their own knowledge and ideas has been greatly emphasised by mathematics educators, researchers and teachers (e.g., Bishop, 1988; D'Ambrosio, 1990; Lave, 1988; Mason, Graham, Pimm & Gowar, 1985; NCTM, 1989; Nunes, Schliemann & Carraher, 1993; Mellin-Olsen, 1987; Sierspinska, 1994). Mathematics as a continually expanding field of human inquiry (Ernest, 1989) —arising from people's cultural practices (Bishop, 1988), that is, in learners' near environments— is to be emphasised, in contrast to the approach of school mathematics as the mastery of a static body of abstract definitions and procedural rules.

To make sense of mathematics, students need opportunities to create connections to their "social, historical or personal contexts as well as to other subject matters. Mathematics is connected to virtually everything" (Steen, 1999, p. 49) and therefore *algebra*, which has been described by some as "the language through which most mathematics is communicated" (NTCM, 1989, p.150) is connected to virtually everything. Algebra then is not separated from other mathematics, that is, from arithmetic and notions of geometry which Kaput and Blanton (2001) and Kieran (2004) report make up the focus of (traditional) curriculum in the primary school levels, for it is in this mathematics where the roots of algebra can be found (Mason, 1999; Mason, et al., 1985).

'Beginning algebra' in the context of a 'teaching for understanding and meaning' approach is not concerned with the presentation of 'prefabricated' symbolisations called *algebraic expressions* but with the engagement of learners in mathematical processes from which algebraic thinking may emerge. Wheeler (1996) notes that "there is no

consensus on the attempt to differentiate algebraic thinking from mathematical thinking in general, or on the attempt to reduce the essential content of algebraic thinking to a set of very elementary operations" (p. 322). However, according to Kieran (2004), "algebraic thinking"

> for the early stages of school algebra... involves the development of ways of thinking (within activities) ... such as analyzing relationships between quantities, noticing structure, studying change, generalising, problem solving, modelling, justifying, proving, and predicting (p. 149).

Kieran's characterisation of algebraic thinking is consistent with the types of thinking that Mason (1999) and Mason, Burton and Stacey (1982) have emphasised to be at the heart of mathematical thinking. *'Beginning algebra'* then is concerned with the creation (and awareness) of opportunities to integrate and cultivate habits of mind that attend to the deeper underlying structure of mathematics (Kaput, 1999) in regular mathematics lessons from the early stages of school. The different characterisations of algebra (see for example, Usiskin, 1988, 1999), and approaches to the introduction of school algebra (i.e., generalisation, problem solving, modelling and functions. See Bednarz, Kieran & Lee, 1996) which have been at the forefront of discussion and exploration within the mathematics education international community during the last decade, point to a variety of classroom activities and emphases that can promote in children what I see as the initial stages in their construction of the concept of *variable*. In other words, *beginning algebra* in a 'teaching for understanding and meaning' approach is concerned with the provision of classroom environments that help children engage in the generational processes of the (multifaceted) concept of variable, which I see as being at the centre of algebraic thinking.

2.3 Research on the teaching of mathematics

Due to the scarcity of literature on the specific area of the teaching of school algebra, the research that has looked at the teaching of mathematics in general, was used in this study to inform ideas about the teaching of beginning algebra, which is the area on which the work focused. The previous work carried out in the Colombian context by the researcher was used to provide a contextual reference point.

I have organised the review of the research literature on the teaching of mathematics into three main sections, in accordance with the focus of this study. That is, research on teachers' conceptions of mathematics and its teaching, research on teachers' practices, and research on teachers' change of practices. After the descriptions of the studies included in each of these sections, I present my reflections both on the studies' findings and on the methodologies used. The research questions arising from these reflections are also included in each section. I would like to begin this section, however, with a brief summary of the developments that have taken place in the research of the teaching of mathematics.

2.3.1 Developments that have taken place in researching the teaching of mathematics

According to Ball (1991), much initial research on the teaching of mathematics was carried out focussing on teachers' behaviour in order to identify 'effective teaching'. The characteristics of good teachers were identified on the basis of pupils' assessments of their teachers. Claims from these studies were found weak because in them, the influence of 'good' teachers' characteristics were not tested on what teachers did or on what students learnt.

Recognising the weaknesses of the findings, effective teaching was then conceptualised as the positive results of teaching measured in terms of students' achievements. However, efforts to identify teachers' characteristics on the basis of the number of college credits did not throw light on teachers' characteristics that could be associated with effective teaching (Fennema & Franke, 1992). Fennema and Franke note that this phase of research on effective teaching can be understood in terms of the assumptions about what it means to know mathematics, and the models of teaching and learning that were operating in those studies.

The realisation that what teachers do is affected by what they think (see for example Thompson, 1984) initiated a significant shift in research on the teaching of mathematics at the beginning of the 1980s. Researchers increasingly turned away from their focus on the teacher's behaviour and began examining teachers' thoughts and decisions. "It was in studying teachers' thinking and decision-making that teachers' knowledge and beliefs began to reappear as potentially significant variables" (Ball, 1991, p. 5) in the study of mathematics teaching. With Shulman's (1986) call for attention to teachers' subject-matter knowledge, which he referred to as the "missing paradigm" in the study of mathematics teaching, the need to study teachers' *content* and *pedagogical content* knowledge began to receive attention. Since then teachers' 'beliefs' and 'conceptions' about mathematics and mathematics teaching has sat at the heart of much research undertaken during the last twenty years.

2.3.2 Research on teachers' conceptions of mathematics and mathematics teaching

Research on teachers' conceptions of mathematics and its teaching has been carried out using both statistical designs (i.e., surveys) and qualitative case studies. There is a substantial body of research in the form of surveys using questionnaires and studying the relationship between teachers' conceptions of mathematics and their conceptions of mathematics teaching. The findings from the survey studies reviewed (e.g. Andrews & Hatch, 1999; Middleton, 1990; Philippou & Christou, 1999; Pehkonen, 1997), which will be summarised in the following headings show consistency between teachers' conceptions of mathematics and their professed approach to the teaching of the subject. One particular study of Stipek et al., (2001), which apart from the application of a survey questionnaire, included observation of classroom practice, found consistency not only between teachers' professed conceptions of mathematics and mathematics teaching, but also between teachers' conceptions and their practices.

Survey studies

A summary of the important features of the studies mentioned above, which were chosen here because they have involved large numbers of teachers, and have been carried out in different international contexts, is included in the following paragraphs.

Andrews and Hatch's (1999) study in Britain

With the purpose of exploring the veracity of the perception that teachers of secondary school mathematics have differing conceptions of mathematics, which influence the way in which they work, Andrews and Hatch (1999) carried out a statistical study in Britain, involving teachers from 200 schools. Results based on 577 teachers' responses to a questionnaire, show that although teachers simultaneously hold a variety of not necessarily consistent conceptions of mathematics and its teaching, for most, there are dominant conceptions of mathematics which are manifested in commensurate beliefs about teaching.

Middleton's (1990) study in USA

Middleton (1990) conducted a survey in which 490 secondary school teachers from 11 urban sites around the United States of America were involved. Eighty eight per cent of the surveyed teachers were frequent participants in the Urban Mathematics Collaborative project (UMC) which was initiated in 1984 with the aims of improving mathematics education in urban schools, and of identifying new models for meeting the professional needs of high school teachers. Underlying these purposes was the assumption that teachers are the key to educational reform, advancement and quality. Teachers participating in the UMC were exposed to new approaches in the field of mathematics' teaching –following recommendations from central policy– and a sense of support from university mathematicians and from other mathematics teachers was fostered.

Responses to the questionnaire indicated that teachers viewed mathematics primarily as thinking in a logical, scientific, inquisitive manner, and as being used to develop understanding. They wanted their students to learn critically and to understand and use mathematics effectively. Most teachers seemed to hold an eclectic view of mathematics, although one group viewed mathematics as dynamic and changing, and another group saw it more as a body of skills and rules. "These conceptions of the nature of mathematics were found to be related to teachers' conceptions of mathematics teaching, recommended change, mathematics education and schooling" (ibid, 1990, p. xv). Teachers who were regular attendants in the UMC project had more favourable views towards the recommended changes in mathematics education than others.

The fact that majority of teachers who were surveyed in Middleton's study were regular attendants to their UMC project needs to be taken into account for the interpretation of the results.

Pehkonen's (1997) study in Finland

The consistency between teachers' conceptions of mathematics and their approach to the teaching of the subject was established in Pehkonen's (1997) study carried out in Finland. This study involved 44 teachers of Year-9, and data were collected through a questionnaire and a follow-up interview.

Philippou and Christou's (1999) study, carried out in the context of TIMSS[3]

Drawing on empirical data belonging to twelve countries from TIMSS (which was carried out in 41 countries during the period of 1991-1995), Philippou and Christou (1999) "investigated teachers' conceptions of mathematics and the process of mathematics teaching and learning in an international context". Teachers' responses to selected items of a questionnaire from TIMSS were analysed to investigate the relationship between teachers' conceptions, cultural factors and students' achievement. Within the twelve countries chosen for this analysis are the four East Asian countries whose students' achievement in the TIMSS test figured at the top of the achievement list of participant countries, and the four European countries with average achievement.

In analysing differences in conceptions the authors point to the contrast observed between the East Asian teachers' conceptions, who adhered very strongly to the algorithmic[4] view of mathematics, and that of the European teachers, who adopted the coherent-conceptual approach. Philippou and Christou argue that much of the contrast in teachers' instructional emphases may be explained by differences in their conceptions, and that differences in teachers' conceptions seem to support the claim that pupils' achievement and teachers' conceptions are directly related.

> At first glance, teachers' conceptions seem to constitute a major factor explaining students' performance in mathematics (p. 395).

The results of this study suggest that there is consistency between teachers' conceptions of mathematics and teachers' conceptions about the teaching of mathematics. The consistency was established through analysis of data from the teachers' questionnaire, and was then reinforced through analysis of data from the pupils' test performances. It is important to add that according to Philippou and Christou "the tasks on the test were

[3] Third International Study in Mathematics and Science.

[4] Philippou and Christou (1999) interpreted teachers' conceptions of mathematics in terms of the two ends of a hypothetical linear continuum as either *procedural and algorithmic* or *conceptual and coherent*.

mostly multiple choice items, requiring memorisation, procedure execution, or pattern recognition (e.g. definition of parallelogram, division of fractions and decimals, recognition of similar triangles" (p. 395).

Stipek's et al. (2001) study in USA

Stipek and colleagues carried out a study in the United States of America in which a questionnaire was applied to twenty one teachers, and the teaching practices of a subgroup of teachers were observed in order to establish the links between "beliefs and practice". The researchers report "substantial coherence" among teachers' beliefs, and consistent associations between their beliefs and their practices, at the beginning and at the end of the school year.

Summary of findings of survey studies on teachers' conceptions of mathematics and its teaching

Results from the survey studies reviewed above show that teachers simultaneously hold a variety of conceptions of mathematics but emphasise a privileged perspective. There is a general agreement that teachers' dominant pedagogical beliefs are consistent with their dominant perspectives on mathematics (Middleton, 1990; Pehkonen, 1997; Andrews and Hatch, 1999; Philippou and Christou, 1999). Stipek and her colleagues, who apart from using a questionnaire, researched teachers' classroom practices found that teachers' conceptions of mathematics and mathematics teaching are consistent with their practices.

Case studies

In contrast to the findings of the quantitative research described in the previous section, qualitative studies that have been carried out focusing on teachers' actual practices show a rather different picture. They have identified not only inconsistencies between teachers' conceptions of mathematics and their conceptions of the teaching of mathematics but also inconsistencies between teachers' conceptions of mathematics teaching and their practices. In the first place, I discuss the studies of Thompson (1984) and Raymond (1997), in which teachers' practices were studied through classroom observation. A study carried out by Cooney (1985) will also be referred to in later paragraphs.

Thompson's 1984 study carried out in America

Thompson studied three secondary school mathematics teachers looking at the content of their conceptions and the relation to their instructional practice. She found that from the three teachers she studied only Kay, who had formed her beliefs of teaching through reflection on her actions and their effect on students, was found to maintain consistency between her classroom practice and her beliefs about mathematics and its teaching. Kay viewed mathematics as a stimulating, dynamic subject that provided opportunities for high level mental work, allowing for the discovery of properties and relationships through personal enquiry, and her views were found to be consistent with her actions in the classroom. In contrast, in the case of the two other teachers, whose professed conceptions about teaching did not match their actions in the classroom, "many of their beliefs seemed to be manifestations of unconsciously held views or expressions of verbal commitment to abstract ideas that may be thought of as a part of a general ideology of teaching" (p. 124).

Raymond's (1997) study in America

In the context of a multi-case study carried out by Raymond in which six teachers participated, teachers were asked to identify reasons for the inconsistency found between their practices in their classrooms and their professed beliefs about the teaching of the subject. It is important to note here that teaching practice was characterised by information obtained by the researcher through her observations of classroom practice. The mismatch found by the researcher between Joanna's beliefs and her practice was explained by Joanna as a consequence of factors that influenced her actions such as the topic at hand and the students' behaviour. Joanna had a *traditional*[5] view of mathematics and a *primarily non traditional* view of teaching and learning of mathematics, but her practice was categorised by the researcher as *primarily traditional*. "When Joanna was asked to draw a model (a concept map) showing the relationships between her beliefs and her practices, she identified as potential causes of inconsistency, aspects like *time constraints, scarcity of resources, concerns over standardised testing and students' behaviour*" (p. 566). Raymond concluded that Joanna's practice was more in line with her conceptions about mathematics than with her conceptions about the teaching of the subject.

[5] Raymond's categories of beliefs and practice were based on a five-level scale ranging in order from traditional, primarily traditional, even mix of traditional and non traditional, primarily non-traditional, to non traditional.

Ernest's (1991) explanations for these inconsistencies are found in the role of constraints that the social context plays on teaching, much the same as Joanna explained her situation. However, we do not have sufficient information to explain how is it that, according to the teacher, certain factors of the social context of teaching act to become barriers for implementation of their professed conceptions. "Joanna was not aware of the many inconsistencies between her professed beliefs and her actual practice" (Raymond, 1997, p.568). In the light of this disjunction seen by the researcher between Joanna's beliefs and practice, I will now look at the case of Fred, a young teacher whose teaching of mathematics was studied by Cooney (1985).

Cooney's (1985) study in USA

Cooney's study focussed on a novice teacher, Fred, who professed to have a problem-solving conception of mathematics teaching. However, Fred's professed conceptions did not really accord with what he did in practice. According to Cooney, Fred was very enthusiastic about initiating classroom work by posing what he called "recreational problems", as he thought they could be of interest for the pupils, but when it came to finding solutions for the problems, Fred taught his pupils an established procedure in a very traditional way (i.e., by following a set routine of steps). Fred attributed his difficulty to teach according to his professed problem-solving conception of mathematics teaching, to the fact that a problem-solving teaching approach was not in agreement with the expectation of the students.

Cooney explains that it appeared that for Fred there were two ways of teaching mathematics: the authoritarian way which was based on the textbook approach, or his recreational problem-solving (a more open way) which, according to the researcher, was not integrated with mathematics content (i.e., with the school curriculum), a fact of which Fred was not aware. According to Cooney, Fred's view of problem solving was restricted to extracurricular problems; "Fred appeared to view problem solving as a layer of a cake rather than as an ingredient of it" (p. 335).

Ernest (1991) has pointed out the phenomenon of teachers who espoused a problem-solving approach to mathematics teaching, but whose practices revolved around an expository transmission model of teaching, enriched by the addition of problems. As we can see, neither Joanna nor Fred nor the two teachers in Thompson's study (i.e., Lynn &

Jeanne) were aware of the inconsistencies between their professed conceptions and their actual practice. We need to ask here:

- Was Fred aware of what a problem-solving approach —as regarded by a constructivist perspective (NCTM Standards) or by the researcher's perspective— could mean and could entail in terms of teaching and learning?\
- Was Joanna aware of what a *non-traditional* approach —as regarded by the researcher's perspective— could mean and could entail in terms of teaching and learning?

In my experience, it seems that when some teachers are asked about their teaching of mathematics, they tend to talk about it using the vocabulary that appears in the latest curriculum guidelines or in new trends posed in centrally published documents or in the educational literature. They talk of a preferred situation that has been made from statements from the latest policy, be it the school policy (which has normally been written by the head of the department) or from general central policies which are trying to promote specific views (i.e., the need to embrace a problem-solving approach in the teaching of mathematics). Teachers talk about a teaching that enables the pupils to *participate* in their learning so that they can *construct* their own mathematical knowledge, despite the fact that their teaching patterns are very traditional. This observation is consistent with Thompson's (1992) claim that teachers' verbal responses to questions about their general conceptions of mathematics teaching "are manifestations of a verbal commitment to abstract ideas rather than to their operative theory".

Ball (1991) and Cooney (2001) assert that studies of teachers' conceptions cannot rely on teachers' verbal expressions alone. This observation lead us to consider the shortcomings of some results of surveys when, on the one hand, questions are placed at an abstract (general) level, and on the other hand, the studies rely on teachers' verbal expressions as the only evidence to account for teachers' conceptions. At the same time, it poses methodological implications for this study, as conceptions cannot be studied by means of teachers' verbal expressions and explanations about non-contextual situations alone. It is necessary to elicit information through questions which are based on the actual teaching of a specific topic; in this way questions posed on specific tasks, requiring ideas about how children learn, about their difficulties and about the reaction of teachers to children's difficulties can provide, at the same time, the opportunity to look at teachers' subject-matter knowledge.

Summary of findings of case studies on teachers' conceptions of mathematics and its teaching

The findings of these case studies show that although teachers' professed conceptions about mathematics teaching are not always consistent with their classroom practices, teachers' practices often reflect their views about mathematics (Thompson, 1984, Cooney, 1985; 1992; Raymond, 1997). The specific observations from these studies are:

- A teacher's conceptions of mathematics and its teaching were consistent with her classroom practice because the teacher's conceptions had been formed through reflection on her actions and their effect on the pupils.

- Teachers whose professed conceptions about the teaching of mathematics were inconsistent with their practices were not aware of the inconsistencies. In many cases their professed conceptions about the teaching of mathematics seemed to be manifestations of unconsciously held beliefs or expressions of verbal commitment to abstract ideas which had no effect on their practice.

- Teachers explain the inconsistency between their professed conceptions and their practices by the roles of contextual factors like 'pupils' behaviour' and resources (including time).

2.3.2.1 Reflection on the findings of the surveys and case studies reviewed

Research has looked at the relationship between teachers' professed conceptions of mathematics teaching and their actual practices, but has not focused on the teachers' understandings of their own teaching practices. In the specific case of Raymond's (1997) study, one of the participating teachers, Joanna, was encouraged "to construct her own understanding of the relationship between her beliefs and her practice" (p. 6), but we do not hear Joanna's voice explaining her understanding of her own teaching practice. Therefore, we do not know anything about Joanna's understanding of the inconsistencies pointed out by the researcher. The same applies to the studies of Thompson (1984) and Cooney (1985). Further, in Raymond's study, which intended to help the teachers construct understanding, the teacher's reflection on the relationship between her professed conceptions and her practice did not extend to the aspect of *how* 'the pupils behaviour' and 'the lack of resources' affected her conceptions both of mathematics and her teaching of mathematics. Little is known about teachers' conceptions of the relationship between their teaching practices and contextual factors of teaching.

In thinking about possibilities of change of educational practice, it is crucial to understand not only what factors teachers believe affect the level of consistency between their 'stated beliefs' and their practices, but also *why* the teachers believe that those factors affect their practices, and how those factors relate to their conceptions of mathematics and its teaching. It is important to gain some insight into the teachers' understandings of their own teaching situations, as it is only the teachers who can change what takes place in their classrooms (Stenhouse 1975). This means that in order to think about the possibilities for change of teachers' practices we need to study their teaching situations in the actual institutional context, from the point of view of the *possible* implementers of change (Havelock, 1973; Havelock and Huberman, 1978; Fullan, 1991), namely the teachers.

2.3.2.2 Research Questions arising from these findings

It seems that teachers are not aware of the inconsistencies between their professed conceptions and their actual practices, and that when the inconsistencies are pointed out by researchers, they give reasons related to aspects of the social context of teaching, like 'pupils' behaviour'. At this point, it is necessary to remember that the focus of this study is the teaching of 'beginning algebra', and that due to the lack of research literature on algebra teaching, the literature on the teaching of mathematics in general was used to guide this study. Therefore, the questions that arise from the findings of the research reviewed need to be posed in respect to the focus of this study, which is beginning algebra. These questions are:

Research Question 1: What are the teachers' conceptions of beginning algebra?

Research Question 2: What are the teachers' conceptions (or understandings) of their own teaching practices of beginning algebra?

- How do teachers perceive the relationship between their conceptions of beginning algebra and their conceptions of their own practices?
- If there is consistency between their conceptions of beginning algebra and their conceptions of their practices,

what aspects of their approach to teaching can account for their consistency?

- If there is inconsistency, do they see their inconsistencies?

  (i) What reasons do they give for their inconsistencies?
  (ii) Are teachers aware of what their stated ideas imply in practice? For example, if they talk about a problem-solving approach, what do they mean by that? How do they characterise a problem-solving approach in terms of the learning and the teaching?

The previous Sub questions, related to Research Question 2, have not been numbered at this point. As we shall see in Subsections 2.3.4.2 and 2.3.5.5, there are several Research Sub questions arising from the examination of the literature which are related to Research Question 2. The ordering (i.e., numbering) of these sub questions starts in Subsection 2.3.4.2 as these sub questions are organised starting with the factors *external* to the teacher, as they emerge from the review of the research literature that is presented in Subsection 2.3.3. The classification of factors that surround and influence the teaching of mathematics into 'external' and 'internal' was adopted from Bishop and Nickson's (1983) ideas, which are discussed in the following subsection.

2.3.2.3 External and internal determinants of teachers' practices

The fact that teachers attribute the inconsistency between their professed conceptions and their practices to factors like *pupils' behaviour*, *lack of resources* or *time*, make us think of the classification proposed by Bishop and Nickson (1983) of *external* and *internal* constraints which surround the teaching of mathematics. They refer to the external constraints as those factors surrounding the teaching situation, which are *external* to the teacher, provided by the institutional organisation. They are external to the teacher because they are determined by other people, for example, by school administrators, pupils, parents and by the teaching profession. The *internal* constraints relate to the teachers' own knowledge, attitudes and the role of initial and in-service education (ibid. p. vii). Teachers' knowledge, "...beliefs, meanings, rules, mental images and preferences concerning the discipline of mathematics" (Thompson, 1992, p. 132) constitute an important part[6] of the *internal* determinants of their teaching practices as they "are brought by the teacher[s] to the teaching situation" (Bishop and

[6] Other aspects like teachers' personality, attitudes to teaching and teachers' life outside schools have been identified as playing a role in the practice of teachers (Bishop & Nickson, 1983; Raymond, 1997).

Nickson, 1983, p. 37). The teachers' conceptions constitute a vital constraint in the complex classroom situation, interacting and affecting other teacher characteristics.

Although it is a fact that within the organisational setting of the school there are factors which are *external* to the teacher, like the fact that head teacher or the head of department may decide on teachers' timetables or on schemes to adopt, this study is concerned with the teachers' conceptions of the role of those external factors in their teaching practices. Therefore, it is useful to keep in mind the classification of *internal* and *external* factors, as this provides the language to talk about the constraints of teachers' practices, as perceived by themselves.

In the following two Subsections, 2.3.3 and 2.3.4, relevant studies on teachers' practices and on teachers' change of practice are reviewed in order to identify key aspects that emerge as determinants of teachers' classroom practices, which may be significant parts of teachers' conceptions.

2.3.3 Research on teachers' practices

In the study of Raymond (1997), a teacher, Joanna, explained that her actions in the classroom did not mirror her conceptions about the teaching of mathematics because of the cumulative effect of factors such as pupils' behaviour, time constraints, scarcity of resources and concerns over standardised tests. These aspects identified by Joanna are part of her conceptions of the *external* determinants of her teaching practice, which are found in the social context of teaching previously mentioned. In the following paragraphs I discuss issues emerging from research in relation to the influence of the pupils and the textbook on teachers' teaching practices, as perceived in some cases by the researchers and in some cases by the teachers themselves.

The pupils

The actions and reactions of pupils to what is proposed in the classroom has been identified —by the teachers themselves— as one of the most important determinants of their practice in several studies (see Cooney, 1985; Pehkonen, 1995; Tomazos, 1997; Reid, 1997, Raymond, 1997, Agudelo-Valderrama, 2000). This fact is explained by

Bishop and Nickson (1983) when they state that the influence that pupils exert on teachers

> must be identified not only in terms of what they, as individuals, bring to the mathematical learning situation, in terms of intellectual development, capacity for learning and the past mathematical experience, but also in the light of the part they play in the social arena of the classrooms (p. 15).

In some cases the social factors of teaching are perceived by teachers as barriers to their change, and in other cases they are perceived as enablers as can be seen in the following, in relation to the case of the pupils. In the case of Joanna (in Raymond's, 1997 study), it seems that she perceived the pupils' behaviour as preventing her from putting into action her ideas about the teaching of mathematics. In the particular case of the PROMECA project teachers in Colombia, who became engaged in changing their teaching practices (see Agudelo-Valderrama, 2000), they saw the pupils as the enablers of change. The personal meaning pupils found in their mathematical work, as designed by the teachers in order to promote the creation of meaning for the use of letters in algebra, became the driving force that kept the teachers working in a very demanding project they undertook during the 20-month Development Program. According to the teachers, "pupils showed a motivation for their mathematical work that they have never seen before" (ibid., p. 5).

The contextual determinants of what mathematics is actually taught can affect views of the nature of school mathematics (Middleton, 1990). In the words of Ernest (1991), "the social context leads the teacher to internalise a powerful set of constraints that affect the enactment of their models of teaching and learning in mathematics" (p. 291). Furthermore, Ernest (1991) and Santos (1995) argue that beliefs and practices are part of an interactive system as classroom practices feed back and influence the views of mathematics and its teaching.

The textbook

Researchers have pointed to the role played by the textbook in the teaching of mathematics, as can be seen in the following.

In a study carried out in Australia that focused on the teaching patterns of a Year 9-algebra teacher, Haimes (1996) illustrates the role played by the textbook in this teacher's teaching of the subject. Given the fact that "in Western Australia, a new curriculum for the teaching of mathematics was introduced in 1990, and that teachers had been introduced to the new ideas incorporated in the new curriculum, especially to those related to pedagogical practices"(ibid. p.584), the main purpose of the study was to examine the interactions between the intended curriculum, the teacher's cognitions and the teacher's actions. Haimes found that although the teacher professed that she was aware of the difference between the old curriculum and the new intended curriculum, and that she was using a problem-solving approach as a basis for her teaching of the subject as recommended by the new curriculum framework, the intentions and recommendations of the intended curriculum had been adapted by the teacher to her conceptions that were in line with the old (traditional) curriculum. In this situation, as was found by Haimes, the textbook played an important role in the teachers' decisions and actions, as he argues that the textbook was at the base of this teacher's classroom decisions and work.

In another study on the teaching and learning of school algebra, also carried out in Australia (see Stacey & MacGregor, 1999), the role of the textbook in the teachers' teaching and the pupils' learning is exemplified. The research was carried out from 1991 to 1996, with classes of Years 8, 9 and 10, in twelve schools, studying students' understanding of algebra. Data were also collected on the curriculum being delivered in the schools through lesson observations and teachers' descriptions of units of work being delivered. The researchers highlight their observations that, with few exceptions, the content of the textbooks was a reasonable guide to the curriculum as delivered in schools. They explain that many of today's books use real situations as a context for teaching algebra, an aspect which is welcomed, but that the simplicity of the situations does not justify the use of algebra and, consequently, pupils solve those problems using arithmetic. In conjunction with this —they explain— "backtracking" as a method for solving equations[7] has come to be emphasised throughout some mainstream textbooks, even to the exclusion of other methods. They further contend that in the textbooks there are very few problems that lead to equations with the unknown on both sides, and that

[7] According to the authors, backtracking can be done by using the following reasoning. For example, if the expression $3\ (2x + 5)$ can be interpreted as "start with x, multiply it by 2, add 5 to the result, and multiply that answer by 3"; to solve the equation $3\ (2x + 5) = 21$ one must only recognise that the final answer is 21, and that operations need to be undone in the reverse order to that in which they are done to arrive at 21.

since backtracking can only be used for equations that have one occurrence of the unknown, pupils solve problems using arithmetic methods.

> We therefore, find it very disturbing that three of the seven Year 10 textbooks surveyed promote backtracking up to and including Year 10. (p. 33)

In the studies presented above, we can see that the researchers conclude that the textbook can define what is taught and how it is taught. What is more important for this study, however, is how the teachers perceive the role of the textbook in their teaching. According to the evidence from my previous work with teachers, once the teachers —who were interested in developing their practices— were clear about the goals of their teaching, they worked hard to design activities that they thought would engage the pupils in their work as they progressed through their set agenda, and so the textbook became for them a secondary source of classroom work. These teachers came to identify that the textbook was representing *a constraint* for them (see González & Pedroza, 1999).

It would appear that what is important in helping teachers change their practices is the provision of opportunities for them to question their conceptions of mathematics, for deepening their understanding of the concepts they teach and for reflecting on what takes place in their classrooms. We should see what the literature on teachers' change of practices says about this issue. In doing so, other factors of the social context of teaching are identified as playing a role in teachers' change.

2.3.4 Research on teachers' change of practices

Most studies found in the literature in relation to teachers' change of "conceptions" and "practices" in mathematics have been carried out with the purpose of identifying the extent to which practices in the classroom are in line with policy recommendations in the different contexts. It is important to notice here that these studies have used the criteria for improvement of educational practice posed in policy recommendations to determine if change is happening in teachers' classroom practices.

This study is concerned, in the first place, with the identification of whether teachers think they have changed or are changing any aspect(s) of their teaching and, secondly, with whether they consider the change an improvement of their teaching practices. If teachers identify changes that represent improvement for them, it is necessary to identify what their criteria for identifying improvements are. This suggests the need to be aware of two levels of attention both when eliciting information about teachers' changing their practice, and when analysing the information. Some of the research studies on mathematics teachers' change of practices have started from the assumption that for teachers to implement the recommendations of reform[8], which, according to the researchers, follow constructivist principles, teachers need a far deeper conceptual understanding of mathematics than the one they have when a traditional approach is followed (Middleton, 1990; Schifter & Fosnot, 1993; Ball, 2000). This initial assumption in these studies mentioned, constituted one of the main findings of Agudelo-Valderrama (2000) as has been declared several times before. Based on this assumption, studies have been designed to provide teachers —and prospective teachers— with learning experiences and opportunities to challenge or question their conceptions about mathematics. The studies reviewed, all having the already-mentioned general purpose, differ in the methods used and in the context in which the research took place, as will be seen in the three following subheadings.

Studies on change of conceptions taking place in the context of prospective teachers' course work.

These studies have been carried out in the context of a course for student teachers, where the teacher educator has taken the role of a participant observant in their own teaching classroom. Timmerman (1999) worked with twelve prospective teachers in a 16-week course which focused on the teachers' learning of mathematics using technology resources. With the aim of identifying the student teachers' perceptions of their learning within a technology-enriched environment, data were collected mainly through interviews and students' work. Timmerman concluded that a change in mathematics teaching led to changes in teachers' conceptions of mathematics, and that personal learning preferences and style (e.g., teachers who learnt to work without sharing and discussing their meanings and constructions with peers) influenced the process of student teachers' learning within a technology-enriched environment.

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[8] This refers to, for example, the ones contained in *The NCTM Standards* in America or in *A National Statement on Mathematics for Australian Schools* –Australian Education Council, 1991, or in the National Curriculum for Mathematics in England and Wales –Department for Education, 1995.

In a similar way other studies (e.g., Schram, et al., 1988, cited in Thompson, 1992; Steel & Widman, 1997) examined the effects of courses on pre-service teachers of mathematics which aimed at engaging them actively in constructing their own knowledge. The courses emphasised conceptual development, group work and problem-solving activity. The researchers report that along with change in conceptions of the nature of mathematics came a new image of what it is to teach mathematics.

The common indication given by these three studies is that changes in mathematics teaching, to which prospective teachers are exposed, can lead to changes in the teachers' conceptions of the nature of mathematics. What is more important to establish though is whether once prospective teachers have been provided with the opportunities to see a different perspective in the teaching of mathematics, in this case in terms of the perspectives put forward by reform, they will be willing to work towards making their teaching practices a reflection of their new conceptions of the nature of mathematics. Knowing mathematics differently may not translate into teaching it differently. The findings of studies which have focused on the teachers' actual practices may shed light on whether professional development affects their teaching practices.

Studies on teachers' change of practices taking place in the context of professional development programs

These are studies carried out in the context of professional development programs, where apart from providing teachers with opportunities to explore their mathematical ideas and to find meaning in what they teach, they are also provided with support in their own classrooms while trying to put their new insights into practice.

Two examples of this type of study are the *SummerMath Program (*Schifter & Fosnot, 1993) and the *Cognitive Guided Instruction (CGI)* (Fennema et al., 1996). In these studies the units of analysis were defined in the context of the teachers' classroom activity. The findings of the *CGI* study suggest that when teachers are given the opportunities to revisit and reconstruct their mathematical understanding and to develop their understanding of children's mathematical thinking, they are more able to make fundamental changes in their teaching practices, as recommended in current educational policy in the United States of America. However, Schifter and Fosnot, who report work carried during a period of five years, show a different outcome. Apart from presenting evidence and illustrations of the processes followed by teachers who were able to change their practices, these researchers draw attention to the fact that many teachers are unable to overcome "what they see as insuperable barriers to change" (p. 185). They make clear that the few teachers whose work is described in their book are probably not typical of the many thousands who are confronting the challenge of reform in the United States. In relation to the outcome of their 5-year project, they explain:

> Those few teachers who were able to confront their habits and assumptions about the teaching of mathematics are not even typical of the 500 or more who have participated in the SummerMath for teachers. (p. 183)

These studies tell us that even the provision of opportunities for teachers to become engaged in work that leads them to construct new understandings of their mathematics classroom practice does not represent for them a reason to embrace change. We should note that in these studies there is no mention of the teachers identifying factors that could act as *external* constraints for them. What can other types of development programs with teachers tell us about the possibilities of change in the teaching of mathematics?

Studies on teachers' change of practices taking place in a collaborative approach between teachers and researchers

Within this type of study we can mention studies carried out as collaborative research projects (e.g. Cobb & McClain, 2001) or as collaborative *action-research* projects (e.g. Perry, Gómez, Valero, Castro & Agudelo, 1996; Jaworski, 1998; Raymond & Leinenbach, 2000) taking place in team work between researchers from higher education institutions and school teachers who are interested in the development of their teaching practices. For our purpose here, a study that focused on the teaching of school algebra needs to be mentioned first as it is the case of a teacher who was very committed to changing her classroom approaches, but found the role of colleagues an important barrier to change.

In Raymond and Leinenbach (2000), Marylin, a teacher interested in working towards her professional development in order to implement curricular recommendations posed by the NTCM in America, followed the action-research approach. Marylin was interested in evaluating the use of manipulatives in the teaching of simple equations. Raymond argues that when questions are raised by teachers, the teachers need to systematically investigate the situation concerned so that they can sufficiently answer their questions, and that through such investigations come reflection and change.

The work developed and organised by Marylin for the teaching of equations was shown to be fruitful for the students who gained in understanding as well as in confidence and motivation for their mathematical work. However, although after a year of inquiry and reflection taking place in her classroom, Marylin found answers to her questions in relation to her beliefs about the teaching of basic concepts of algebra, it was evident to her that the innovative work was not viable for the students as it was not shared by subsequent teachers in the higher grades of school. Teachers of high school levels considered pupils' learning through the use of manipulatives as just playing, and when pupils from the feeder middle school where Marylin worked moved to higher levels they had to re-start their learning of equations using the traditional approach. Marylin was frustrated as she felt that she was setting up her students for future disappointment.

The failure of attending to the institutional setting of the school when working for teacher change is one of the main points discussed by Cobb and McClain (2001) when they critique their prior work with teachers. They note that their focus was almost exclusively on changes in the teaching and learning of mathematics in individual classrooms, and explain that their prior collaboration with teachers is atypical in that the teachers have sustained their practices for over ten years because the teachers initiated the creation of institutional conditions that were aligned with their focus on students' understanding. The explanation for these happenings, they add, lies in the fact that when some members of the Government School Board objected to the changes, the teachers were able to justify their views, as a consequence of their first year of work in collaboration with the researchers.

The role of colleagues and the role of school administrators as determinants of teaching practice

The studies reviewed in the previous paragraphs have pointed to teachers' conceptions of the role of colleagues and the role of the school administrators in their teaching practices. Clearly the teachers' conceptions of the role of colleagues and school administrators can become a barrier to change in their practices, and we can see how this was the case for Marylin as she was working in isolation. In this case we can say that, on the one hand, as Cobb and McClain (2001) point out, working for change needs to take on board the issue of working with communities rather than with individuals. But, on the other hand, one wonders about the commitment of Marylin to her change, as after a year of collaborative work with a researcher, despite her observations of what pupils had gained from their work, and despite the fact that those pupils —while doing their mathematical work in their following school grade— attributed their good performance to the understanding they had attained through the use of manipulatives (with Marylin), felt frustrated because other teachers did not recognise her new approach. Could Marylin not have tried to carry on doing something about the change she thought was needed and find alliances for her work?

In the case reported by Cobb and McClain, after a year of work in collaboration with the University consultants, teachers "were more able to justify their new view of mathematical activity in school, and their new instructional practices in terms of the quality of their students' learning" (p. 211). Therefore, they did not allow the role of the school administrators to become a barrier to their change of practice. Why do other teachers give up in the face of perceived constraints like the role of administrators?

The findings from my research in Colombia show that the two teachers (out of the initial group of four) who engaged in the Development Program, and worked for a period of more than 20 months, did not identify the role of administrators as reasons for the difficulty of changing their practices. The reasons they identified were their lack of deep knowledge of mathematics and the fact that a traditional approach had been the only one they had had experience of (see González & Pedroza, 1999). However, the other two teachers, despite the support and facilities provided by the school administrators to participate and engage in the Development Program,[9] withdrew from the Program after four months participation. They gave several explanations for their withdrawal, especially in relation to the pupils and their difficulties in their learning, and to the amount of work that their participation in the program would demand from them. Why did these two teachers see constraints that the other two did not see? Why is it that some teachers are not interested in changing their teaching practices?

[9] Teachers were given one day free per week, and a reduced workload with special timetables, so that they could have time available for their work in the Program.

Summary of findings of research on teachers' practice and teachers' change of practice

The findings of research on teachers' practices (Subsection 2.3.3) and on teachers' change of practices (Subsection 2.3.4) can be summarised as follows:

- Important factors from the school context perceived by teachers as influencing their practice in the form of *external* constraints are: pupils' behaviour, standardised external assessment, lack of resources including time, and the role of colleagues and administrators.
- The textbook was identified by teachers who participated in a development program as a constraint in their teaching, only until their practice started to change.
- Teachers who were interested in developing their teaching practices engaged in processes of enquiry and reflection and changed their practices. They became aware of the impact of their actions on their pupils' learning. These teachers did not see *external* factors of teaching as the main determinants of their practices. They saw their knowledge of mathematics content and their pedagogical content knowledge as the main determinants of their practices.
- Other teachers who were not able to change their teaching practices continued to see *external* factors as barriers to change.

2.3.4.1 Reflection on the findings of research on teachers' practices and change of practices

The three studies on teachers' change of practice in a collaborative approach between teachers and researchers, reviewed in the previous subsection, provided grounds for thinking not only about the teachers' conceptions of the role of others as sources of influence on their practices, acting as 'external' constraints or enablers of change, but also about the teachers' conceptions of their own roles as teachers of mathematics. We can see differences in the teachers' attitudes to their teaching practices, which in Subsection 2.3.2.3 were conceptualised as part of the internal constraints or enablers of teachers' practices. This means that teachers' attitudes to beginning algebra (and mathematics in general) and to its teaching need to be explored in this study that is concerned with identifying possibilities for teacher change.

When teachers are willing to engage in reflection on their practices, they come to see more clearly that their conceptions of mathematics and mathematics teaching are crucial determinants of their practices. Teachers who do not want to reflect on their practices seem to identify their barriers as related to external factors. As pointed out in Subsection

2.3.2.1, we know that they identify external factors to explain the impossibility of change, but we do not know, however, *why* the teachers believe the 'external' factors they mention affect their practices, and *how* those factors affect their practices. These are crucial questions that need to be looked into if we are to gain some understanding of the difficulties of teacher change.

One common feature of the studies reviewed, related to the aspect of methodology, is the fact that teachers have been studied as *one only teacher* from one school, or representing one school. Schifter and Fosnot (1993), Crawford and Adler (1996), to name some authors, have pointed out that teachers work in isolation. In the context of Colombian teachers, according my observation and experience, when teachers decide to participate in projects or to cooperate with a researcher in a specific study, they like to work in pairs, if not in groups. This made me realise that having at least two teachers from the same school —for this study— was to provide the opportunity to collect information about the same institutional context from more than one informant, which would allow triangulation of certain sources of data.

2.3.4.2 Research Questions arising from these findings

The previous reflections lead me to ask the following questions, which are related to the general 'Research Question 2', previously posed in Subsection 2.3.2.2. The reader is reminded that although specific Research Sub questions had emerged in Subsection 2.3.2.2, the numbering of these sub questions starts at this point. The previous sub questions are included in the summary of research question sin Subsection 2.3.5.5.

Research Question 2: What are the teachers' conceptions of their own teaching practices?

2.1: What are the teachers' conceptions of the roles of (external) institutional factors in their teaching?

- What specific institutional factors do teachers identify as playing important roles in their teaching?
- In what ways do those factors affect the teachers' teaching decisions, and their conceptions of their teaching of beginning algebra (and of mathematics in general)?

Asking teachers about their conceptions of the role of institutional factors in their teaching poses the need to ask the following questions:

2.2: What are the teachers' conceptions of their role as teachers of beginning algebra?

- Do teachers perceive their knowledge of mathematics a being an important determinant of their practice?

- Do they perceive their knowledge about how their pupils learn as being an important factor in their teaching?

The question of teachers' conceptions of their roles as teachers does not relate to any of the *external* (institutional) factors of teaching addressed in the previous sections, but to the *internal* constraints or enablers of teachers' teaching practices, as discussed in Subsection 2.3.2.3.

2.3.5 Internal determinants of teachers' practices

Although this study is concerned with teachers' conceptions which is something internal to individuals, the separation between the 'internal' and the 'external' constraints (or enablers) of teachers' practices, pointed out in Subsection 2.3.2.3, was found useful because according to research results, teachers identify (external) factors from the social/institutional context as reasons for the inconsistency between their stated 'beliefs' or 'conceptions' and their practices. Therefore, I considered it necessary to give importance to the teachers' conceptions of the institutional factors which they may see as influencing their teaching practices. As noted in Subsection 2.3.2.3, the internal is related to the teachers' knowledge and attitudes, and as Thompson (1992) suggested, "...beliefs, concepts, meanings, rules, mental images and preferences concerning the discipline of mathematics" (p. 132) that they bring to the teaching situation. I would now like to focus on what research tells us about the *internal* constraints of teachers' teaching practices. I have chosen to focus on teachers' knowledge, beliefs and attitudes because as will be seen in Section 2.5 these are the components for identifying teachers' conceptions in this study.

2.3.5.1 Teachers' knowledge and beliefs

Shulman's (1986, 1987) model of teachers' "pedagogical content knowledge" and "content knowledge" has been widely used as a framework for studying teachers' knowledge for teaching in the various areas of the school curriculum (see for example, Grossman, 1990 who studied the teaching of English).

In talking about teachers' knowledge, Ball (1991), Fennema et al. (1996), and Lloyd and Wilson (1998) have shown that teachers' specific "subject-matter" knowledge plays a critical role in their teaching practices. However, in teaching, 'subject matter knowledge' does not exist separately (in the teacher's mind) from pedagogical content knowledge. Subject matter knowledge is considered by Ball (1991) as "a term in the pedagogical equation" in the teaching of mathematics, as "it is a critical part of the resources available which comprise the realm of pedagogical possibility" (p. 38). While Ball (1991) argues that teachers' knowledge interacts with their beliefs and therefore beliefs and knowledge are inextricably linked, Da Ponte (1994) asserts that beliefs are part of knowledge.

Because of the close conceptual connections between beliefs and knowledge, Thompson (1992) has advised that it is not useful for researchers to distinguish between teacher's knowledge and teachers' beliefs. She suggests to search for whether and how teachers' beliefs —or what they take to be knowledge— relates to their experience, and therefore, to focus on teachers' *conceptions*, that is, "on mental structures encompassing beliefs, concepts, meanings, rules, mental images and preferences concerning the discipline of mathematics" (p.132). Beliefs are distinguished from knowledge in that they are not consensual. Beliefs, as distinct from knowledge, carry the connotation of disputability [i.e., the believer is aware that others may think differently (Thompson, 1992)]. Although in some sense beliefs and knowledge are intertwined, knowledge assumes certain evidence that beliefs do not (Cooney, 2001). Due to this differentiation, beliefs are considered as having a personal *affective* character.

Although there are no simple boundaries to be drawn between some of the components of teachers' knowledge or between beliefs and knowledge, it is necessary to find a way to organise the phenomenon one is studying and, in this case, the components of the mental structures I intend to study. Therefore, I think that it is necessary and useful to discuss both beliefs and knowledge, as knowledge alone does not account for different practices across mathematics teachers (Ernest, 1989; Ball, 1991). Two teachers who have similar understandings of, for example, place value or of the use of letters in algebra, may teach differently, based on their assumptions about the teacher's role. These differences are a function of different assumptions (beliefs) about the teaching and the learning of mathematics (Ball, 1991). Mathematical knowledge (and knowledge in general) has been considered as socially constructed knowledge that has been publicly justified and accepted (Ernest, 1991), or negotiated and institutionalised belief (Peterson, Fennema & Carpenter, 1991), or belief that can be supported (Cooney, 2001). For example, if teachers think that children learn mathematics by routine practice, this thinking can be categorised as part of teachers' beliefs.

2.3.5.2 Teachers' beliefs and attitudes

In the literature on teachers' change of practices there were teachers who, despite the opportunities and support offered by professional developers and school administrators to participate in professional development, continued to see impossibilities for change in their teaching practices. These teachers' decisions may have strong links with their beliefs about mathematics and its teaching, and about *themselves* as teachers of mathematics and the corresponding *attitudes* associated with their beliefs.

Research has found a significant correlation between teachers' attitudes and pupils' achievement (Cockcroft report, 1982; Bishop & Nickson, 1983). Further, Ruffell, Mason and Allen (1998) argue that teachers' attitudes towards mathematics act as a dominant factor in children's attitudes towards mathematics. The need to focus on affective components of teachers' thought structures in this study which is concerned with the identification of possibilities for change in the teaching of mathematics is clear. According to McLeod (1992), who has argued for the need to incorporate affective components in cognitive studies of mathematics learning and teaching, three variables represent central components of the affective domain in the learning and the teaching of mathematics: *beliefs*, *attitudes* and *emotions*. McLeod's (1989) conceptualisation of these variables relates to the degree of affective and cognitive involvement present in them. Beliefs, attitudes and emotions represent constructs with increasing affective involvement and decreasing cognitive involvement, accordingly. These constructs are classified, also, in relation to the degree of intensity and stability. *Emotions* are affective constructs of short-term duration while *attitudes* are constructs of long term duration. The volitional tendency to act in certain ways is due to held *beliefs*. Beliefs provide an important part of the context within which responses (or attitudes) to mathematics and mathematics teaching develops (Mandler, 1989). Therefore, teachers' 'beliefs' and 'attitudes' in relation to beginning algebra and its teaching were investigated in this present study.

Since, teachers' beliefs about themselves and their roles as teachers of mathematics can have a powerful impact on their make-up and approach, teachers' *beliefs* about *themselves* (i.e., beliefs about their knowledge) or teachers' self-concept was an important aspect of teachers' conceptions on which this present study focused, as teachers' self-concept was thought to be crucial for teachers' improvement of their practices. Could the mathematics teachers' resistance to change observed by Gregg (1995) be related to teachers' conceptions of themselves, their role as teachers of mathematics, their conceptions of the role of mathematics as a subject to teach and to learn? Could teachers' resistance to change be related to their conceptions of the nature of teaching?

Bishop (1998a) asserts that ideas about both beliefs and attitudes relate to *values* held by teachers. He contends that it is necessary to focus on *values* rather than *beliefs* in order to establish the deeper affective qualities that underpin teachers' preferred decisions and actions. Since, the issue of values is very rarely acknowledged in the Colombian context, I considered it important to elicit some information in relation to teachers' awareness of the values they teach. In the following paragraphs some important constructs are considered in order to help us clarify some aspects of the nature of teaching.

2.3.5.3 Teachers' self-concept – The teacher as a learner

In Chapter 1 it was said that, due to the effects of the long standing instructional design model in education, in the Colombian educational system the teaching of mathematics was viewed as a mechanistic act. In many Faculties of education, teacher education programs centre on the coverage of a list of mathematics content plus some theories of education, with very little space for questioning what goes on in mathematics teaching and learning (Niño, 1998). Therefore, since student-teachers are not encouraged to problematise existing situations but to adapt to what takes place in schools, change is not considered as needed in seeking a high quality of education because the concern is in the acquisition of classroom routines (Agudelo-Valderrama, 1993). Further, there is no acknowledgment of the values implicit in this approach, despite the fact that "the development of the individual's capacities to continue to learn throughout his/her life" is a stated central goal of the Colombian educational system (see the General Law of Education issued in 1994).

A mechanistic approach is not in the nature of the actions of professionals (Stenhouse, 1975). Since the realities of every day classroom practice constitute complex situations that pose different demands on the teacher, teaching cannot take place by applying general prescriptions that have been posed by specialists outside the classroom. Therefore, teachers need to improve their capacity to generate professional knowledge in the classroom in order to attend to the different changing demands and to pupils' educational needs. In other words, teaching —as any other profession— is an activity in which teachers are in a continuous process of learning. This learning should emerge from the teacher's reflection on her/his own practice with the purpose of improving what is provided for the learners. Consequently, change of teaching practice is an outcome of teachers' learning. In this context change then is understood as a consequence of learning. Teachers are responsible for pupils' change through learning (Crawford & Adler, 1996), and teachers should be fully aware of this relationship through their everyday experiences with students' learning.

In a culture where teachers see themselves as experts in their transmission model approach (Carrillo, 1987), the idea of being a learner may sound unattractive. Even in the case of teachers who see the need to continue learning, we still need to consider what kind of learners those teachers want to be. My experience and that of others (see, for example, Irwin & Britt, 1999; Crawford & Adler, 1996) tell us that many teachers conceptualise learning in terms of their own experience of being told. We need to ask those teachers (respectfully) if they conceptualise themselves as learners, and what kind of learners they would imagine themselves to be. We need to ask them what changes have occurred in their teaching, what reasons motivated their change, what are their ideas about change, and whether they see the need for change in their teaching. A study that is concerned with the identification of possibilities of teacher change needs to pay attention to how the teachers understand their own teaching practices. Teachers' conceptions of their learning constituted a key aspect that was looked at in this study that focused on teachers' conceptions of their own practices. Insights into teachers' conceptions of their learning shed light on the possibilities of teacher change.

2.3.5.4 Research Questions arising from the previous discussion

From the previous discussion on *internal* determinates of teachers' practices emerge the following sub questions which belong to the more general question previously posed as Research Question 2:

Research Question 2: What are the teachers' perceptions of their own teaching practices of beginning algebra?

2.3: Do teachers conceptualise themselves as learners?

- If they conceptualise themselves as learners, what kind of learner do they imagine themselves to be?
- If they do not conceptualise themselves as learners, why not?
- Do they conceptualise mathematical content knowledge and/or pedagogical content knowledge as playing an important role in their teaching?

2.4: What are the teachers' conceptions of changes in their practices?

- What changes have taken place in their teaching, and for what reasons?
- Are the changes equated with improvement, and if so, what are the teachers' criteria to define improvement?
- Will they be interested in changing any aspect of their teaching, and for what reasons will they be interested in the change?

2.3.5.5 Summary of Research Questions

Summarising the questions that have emerged from the three sections on previous research on teachers' conceptions, and from the insight of my previous work in Colombia, the questions this study will try to answer are as follows:

Research Question 1: What are the teachers' conceptions of beginning algebra?

Research Question 2: What are the teachers' conceptions of their own teaching practices of beginning algebra?

2.1: What are the teachers' conceptions of the roles of (external) institutional factors in their teaching?

- What specific institutional factors do teachers identify as playing important roles in their teaching?
- In what ways do those factors affect their teaching, and their conceptions of beginning algebra (and mathematics)?

2.2: What are the teachers' conceptions of their roles as teachers of beginning algebra?

- Do teachers perceive their knowledge of mathematics as being an important determinant of their practices?
- Do they perceive their knowledge about how their pupils learn as being an important factor in their teaching?

2.3: Do teachers conceptualise themselves as learners?

- If they conceptualise themselves as learners, what kind of learner do they want to be?
- If they don't conceptualise themselves as learners, why not?
- Do they conceptualise subject matter-knowledge and/or pedagogical content knowledge as playing an important role in their teaching?

2.4: What are the teachers' conceptions of change in their practices?

- What changes have taken place in their teaching, and for what reasons?
- Are the changes equated with improvement, and if so, what are the teachers' criteria to define improvement?
- Will they be interested in changing any aspect of their teaching, and for what reasons will they be interested in the change?

2.5: How do teachers perceive the relationship between their conceptions of beginning algebra and their conceptions of their own practices?

- If there is consistency between their conceptions of beginning algebra and their conceptions of their practices, what aspects of their approach to teaching can account for their consistency?
- If there is inconsistency, do they see their inconsistencies?

(i) What reasons do they give for their inconsistencies?

(ii) Are teachers aware of what their stated ideas imply in practice? For example, if they talk of a problem-solving approach, what do they mean by that? How do they characterise a problem-solving approach in terms of the learning and the teaching?

Question 3: What is the relationship between the teachers' conceptions of beginning algebra and their conceptions of their own teaching practices?

In general terms it can be said that the research reviewed has focused on the relationships between teachers' conceptions of mathematics, teachers' conceptions of mathematics teaching and teachers' classroom practices. This current study was concerned with teachers' conceptions of their own teaching practices of beginning algebra; that is, it focused on the relationship between teachers' conceptions of beginning algebra and their conceptions of the social/institutional factors of teaching. Figures 2.1 and 2.2 show the difference between the focus of the previous research reviewed and the focus of this study.

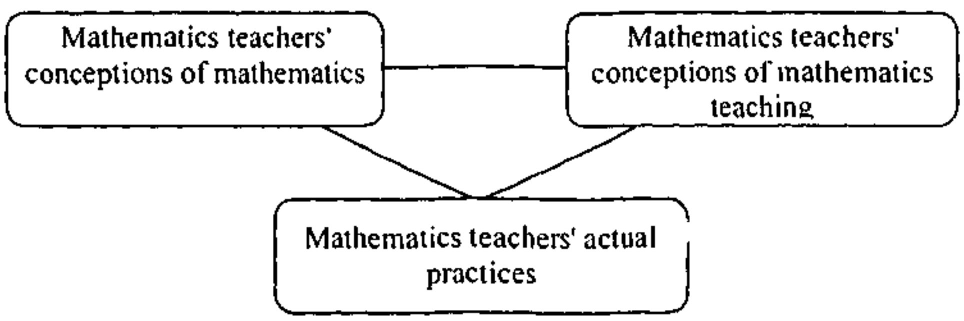

Figure 2.1. The areas of the teaching of mathematics on which the research reviewed has focused

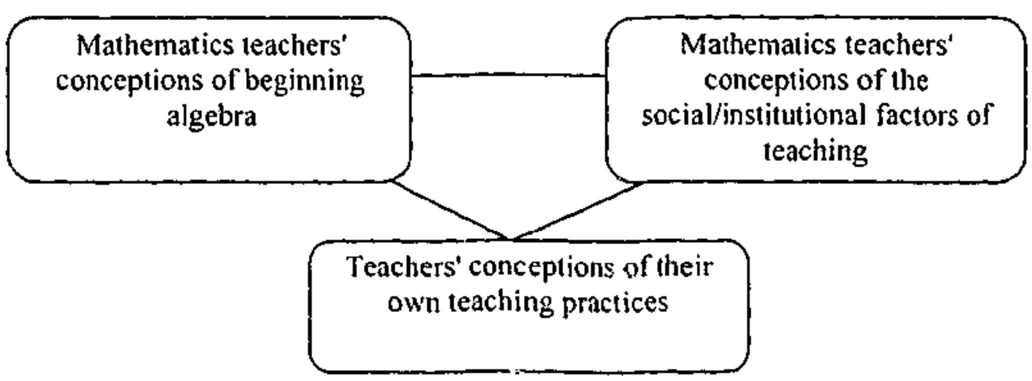

Figure 2.2. The three areas on which this study is focused

2.4 Cognitive models for the study of mathematics teachers' thought structures

In thinking about the components of teachers' cognitions, several models were proposed, starting with Shulman's (1986) model, which is explained in the following paragraphs.

Shulman (1986) proposed a framework that describes the components of teachers' knowledge as *subject matter content knowledge, pedagogical content knowledge and curricular knowledge*. For the category of 'subject matter content knowledge' he refers to "the amount and organisation of knowledge *per se* in the mind of the teacher" and adds that "to think properly about content knowledge requires going beyond knowledge of the facts or concepts of a domain" (p. 9). Within the category of 'pedagogical content knowledge', Shulman includes the most useful forms of representations for the most regularly taught topics of the subject, the most powerful analogies, illustrations examples and explanations. He also includes here "an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions —that very frequently are misconceptions— that students bring with them to the most frequently topics and lessons". *Curricular knowledge*, represents the "*materia medica*" of pedagogy. The curriculum is represented by the full range of programs designed for the teaching of a subject, so the curriculum and its associated materials form the base from where the teacher draws the tools of teaching.

Drawing on Shulman's identification of 'subject matter knowledge' as the "missing paradigm" in the study of teaching, Ernest (1989) suggested an analytical model of the different types of knowledge needed in teaching, adding to Shulman's model of teachers' knowledge, the dimensions *of teachers' beliefs and attitudes* concerning subject matter and its teaching. Ernest makes a distinction between teachers' *thought processes* and teachers' *thought structures*. Planning, interactive decision-making and reflection are the basic psychological aspects the author includes in the former, and in the latter, he considers "the knowledge, beliefs and attitudes stored as schemas in the mind of the teacher" (p. 13). It is for the teacher's *thought structures* that he has proposed his model of knowledge, beliefs and attitudes. Ernest refers to *beliefs* as consisting of the conceptions, values and ideology which account for the differences between mathematics teachers. Beliefs have a powerful influence on teaching through such processes as the selection of content, emphasis and styles of teaching. Ernest asserts that a teacher's conception of mathematics is a belief system, which forms the basis of a philosophy of mathematics, though not necessarily articulated.

Ernest's model is not inconsistent with the ideas of Shulman. It broadens Shulman's (1986) model by subdividing each category of teachers' knowledge into more specific components (e.g., knowledge of the teaching of mathematics includes knowledge of pedagogy and of the curriculum), and by adding to it the dimensions of beliefs and attitudes as key elements that influence the teaching of mathematics. The model was proposed as a foundation on which to base research on teaching and mathematics, teacher education, and as a foundation for understanding the psychological factors underpinning the impact on teachers of curricular innovation derived in the centre.

Fennema and Franke (1992) put forward a model for the study of teachers' knowledge, which identifies *teachers' knowledge of the context of the classroom* as the centre of the model. For them, teachers' knowledge and beliefs are situated in the environment of the school: the context is the structure that defines the components of knowledge and beliefs that come into play. They refer to teachers being able to identify the needs of their students in a specific situation and to adapt their teaching strategies to those specific contextual situations. This component is termed by them as *knowledge of students' cognitions*, and I see it as an component of the *knowledge of the context of teaching* in Ernest's model, and to what Shulman' (1986) considers as 'pedagogical content knowledge', which he refers to as 'knowledge of the educational situations' in his (1987) publication.

In the model that Fennema and Franke propose for research of teachers' knowledge, they consider *knowledge of mathematics*, *knowledge of pedagogy*, *knowledge of students cognitions* and *beliefs* as the components of teachers' knowledge. They emphasise that teachers' knowledge develops in context and that, therefore, it cannot be studied out of context or in isolation, adding that "not only must components be studied in their interrelationships, the dynamic nature of teachers' knowledge must be considered" (p. 162).

2.5 A proposed model of teachers' thought structures to guide this study

I have constructed a model that represents a proposed way to think about the components of teachers' cognitions, or mental structures, their relationships, and how they might relate to their teaching practice (see Figure 2.3). This model has been constructed by taking the common components of teachers' knowledge (e.g., pedagogical content knowledge and mathematical content knowledge) found in the three models described above.

I have chosen to keep subject matter or 'content knowledge' separate from 'pedagogical content knowledge' because teachers' knowledge of mathematics has been found to be one important determinant of teachers' change of practices, not only by researchers (e.g. Ball, 1991; Schifter & Fosnot, 1993; Irwin & Britt, 1999; Agudelo-Valderrama, 2000) but by teachers as well (see González & Pedroza, 1999). I would like to argue that teachers' development of 'pedagogical content knowledge' can only be based on a deeper understanding of the mathematical concepts to which it relates.

Curricular knowledge has been included within 'pedagogical content knowledge' in the model of Figure 2.3. I consider 'curricular knowledge' —which Shulman considers as a separate category from pedagogical content knowledge— to be a crucial component of teachers' pedagogical content knowledge, and I want to highlight the importance of teachers' knowledge of the concept of curriculum and their awareness of the curriculum that operates in their classrooms. In an earlier study of mine, the teachers who were attempting to create innovations for their classroom found it difficult to move from their traditional practices until they gained some insight into the dynamic interaction that exists between the elements of the curriculum present in their teaching (i.e., between the *teaching approaches*, the teachers' *goals* and the *assessment* practices for the *content* at hand). It was the teachers' awareness of those interconnections between the basic elements that structure their classroom practice, and their relation to the general aims of (mathematics) education which allowed them to visualise their new classroom practices (see Agudelo-Valderrama, 2000).

Knowledge of the curriculum at the classroom level [i.e., the *micro level* of the curriculum (Rico, 1996)] is crucial for teachers as it enables them to be involved in the construction of innovation and change of their practices. Further, curricular knowledge is crucial for teachers to be able to evaluate curricular materials *critically*. This knowledge is not considered within the 'curricular knowledge' proposed by Shulman's (1986, 1987) model or by Ernest's (1989) model. By 'curricular knowledge' these authors refer to knowledge of curricular programs and existing teaching materials, which although important, might not necessarily include the concept previously highlighted. So curricular knowledge, which necessarily relates to both the *micro* and *macro* levels[10], is considered in my proposed model of Figure 2.3.

The *practice* dimension has been included in the proposed model, "as it is the end to which the knowledge, beliefs and attitudes are directed" (Ernest, 1989), and it is in this sense that it is shown in Figure 2.3, not to imply that the relationship between, for example, teachers' beliefs and actual practices is a simple one. It is also important to note that the 'practice' dimension has to be included in the model, for if teachers' conceptions refer to teachers' knowledge, beliefs and attitudes, as they do in this study, (see next heading), evidence from practice is crucial. In other words, teachers' conceptions cannot be studied on the sole basis of their verbal statements.

Teachers' conceptions in this study

The term *conceptions* has been used in research in an undefined way. Some authors talk of teachers '*conceptions* referring to their mathematical knowledge, some referring to their beliefs about the nature of mathematical knowledge, and some to their subject matter knowledge and beliefs about the teaching of mathematics. The term 'conceptions' refers to "teachers' mental structures" (Thompson, 1992) or the teachers' semipermanent thought structures stored in the mind as schemas (Ernest, 1989). According to the clarifications previously made of the imperative need to focus on the affective components of teachers' thoughts, in this study that is concerned with the identification of possibilities for teacher change, the term 'conceptions' was defined to encompass teachers' *knowledge, beliefs* and *attitudes* as shown in Figure 2.3.

[10] i.e., it is not possible to work at the *micro* level without a good grasp of the curriculum at the *macro* level.

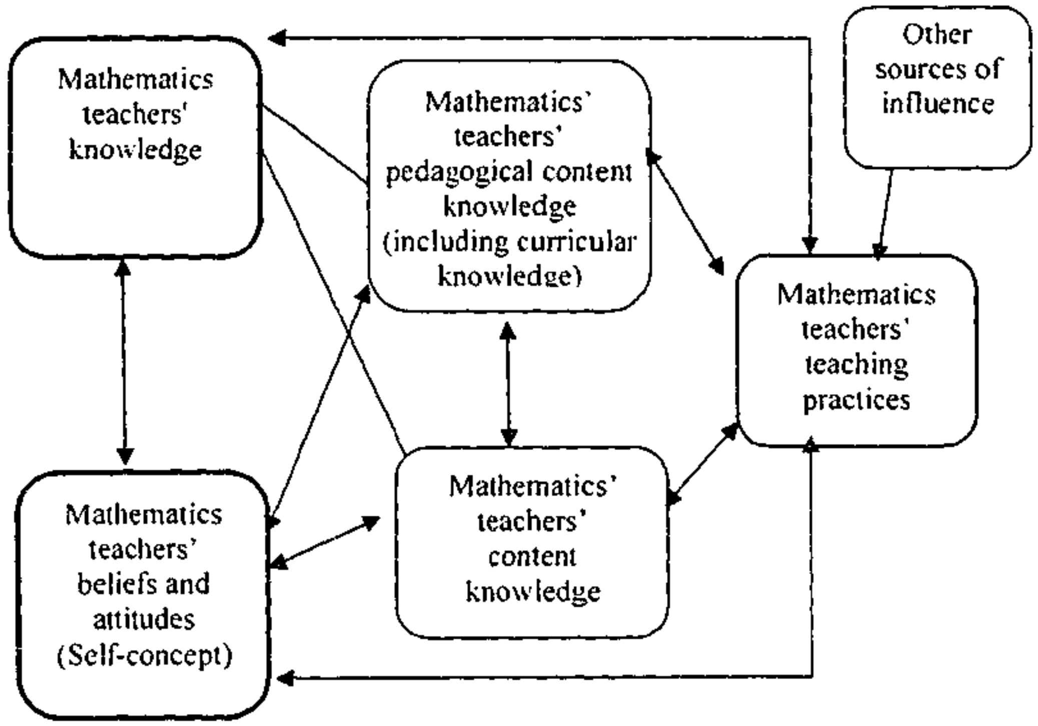

Figure 2.3 A proposed model of the dimensions to take into account for the study of mathematics teachers' conceptions (or thought structures) of their own teaching practices of beginning algebra

In the diagram, the arrows indicate 'influences'.

Mathematics teachers' knowledge refers to teachers' pedagogical content knowledge and teachers' mathematical content knowledge as it is shown in the diagram.

Teachers' pedagogical content knowledge refers to knowledge for the teaching and learning of mathematics, differentiating the following main bodies of knowledge:

- *Curricular knowledge* both at the *micro* and *macro* levels is crucial for the planning of classroom work as well as for understanding and guiding pupils' responses.
- *Knowledge of the context of teaching*. This includes knowledge of how students learn the mathematics content being addressed (i.e., an understanding of the processes students follow in their learning, with special attention to their difficulties), knowledge of the organisation of teaching (i.e., classroom management) and knowledge of the school context.
- *knowledge of other subject matter areas* (e.g., physics, geography, etc, where contexts and connections for the teaching and learning of mathematics are important)
- *Knowledge of education*. This refers to theories of education, in general, and of mathematics education, in particular. It also refers to knowledge of principles of education, and of the general ends of education (i.e., the general aims of the educational system in the particular context).

Teachers' mathematical content knowledge refers to knowledge of the concepts, procedures and problem-solving processes within the domain teachers teach, as well as in related content domains; that is, knowledge of the inter-relatedness of mathematical concepts. In other words, it includes procedural and conceptual knowledge of the specific topics addressed, as well as the knowledge structure of the subject and its unifying concepts.

Mathematics teachers' beliefs and attitudes: *Beliefs* refers to teachers' views of the nature of mathematics, and of the teaching and learning of mathematics. Within *attitudes* there are attitudes towards mathematics itself and towards the teaching of mathematics. These include liking, enjoyment and interest in mathematics and the teaching of mathematics (or the opposites) and their valuing of mathematics (Ernest, 1989). Within attitudes there are also the teachers' beliefs about their knowledge and their confidence in their own mathematical abilities and their teaching abilities; that is, the teachers' self-concept.

Teachers' practice refers to teachers' actions, teachers' decisions related to general curriculum strategies which guide specific classroom work (e.g., content choices, tasks and activities to work on, approaches to used in each activity, forms of pupils assessment), and to general aspects like attitudes to mathematics, to mathematics teaching and to pupils, awareness of the values modelled, etc.

Other sources of influence refers to aspects of the contextual situation of teaching (e.g., the number of pupils, the school setting, the school curriculum and ethos, timetables, etc.).

The model above described represented a thinking tool useful during the planning and the data collection stages of the study. The following chapter focuses on the methodology, and research design constructed for this study.

Chapter 3: Methodology

3.1 Introduction

Research is a process of exploration, planning, refinement and further exploration, a sequence which is reflected in the way this study was conducted. In order to best represent the process of development of my study I first argue the epistemological and methodological principles on which I based the research design, and explain how the study was initially planned to proceed. This, together with a description of the pilot study, forms the focus of this chapter. Chapter 4 continues with the account of the way the main study was carried out, responsive as my plans were to circumstances in the fieldwork context and to issues arising from the interactions during the data collection.

In the previous chapter I constructed a conceptual framework by drawing from the available research literature on teachers' conceptions of mathematics, and from my observations of recent work in Colombia. This framework provided important theoretical tools to identify not only the questions that we needed to ask the teachers about their teaching of beginning algebra but also for the way these questions are to be asked, and for the general strategy of data collection. A theoretical model of the components of teachers' thought structures and their relation to teaching practice was constructed, where the need to include the affective dimension as an important component of teachers' conceptions was emphasised. The term *conceptions*, for the purpose of this study, was defined to encompass teachers' *knowledge, beliefs* and *attitudes*.

I argued that the large number of quantitative research studies about mathematics teachers' "conceptions" and "beliefs" which have focused on consistency or inconsistency between teachers' beliefs about mathematics and mathematics teaching does not tell us much about the difficulties of teacher change, because much of this research has been conducted out of the actual context of teachers' classroom practice. The few qualitative studies that have looked at the relationship between teachers' professed conceptions of mathematics and mathematics teaching and their actual practice have also focused on consistency between professed conceptions and actual practices. From these studies, and from studies on teacher change we learnt that teachers identify factors belonging to the social context of teaching to account for the inconsistencies or for their resistance to change, but we have not learnt about the teachers' perspectives as to *why* and *how* these factors influence their practices.

I also argued that in order to gain some understanding of the phenomenon of mathematics teachers' resistance to change or to the 'stability of mathematics teaching practices' in the Colombian context, as in any other context, I needed to study not just the teachers' conceptions of mathematics and its teaching but also their conceptions of their own teaching practices, which must take into account their understanding of what they see as barriers to change in their specific contextual situation. In other words, I needed to learn not just about the factors teachers name to explain the inconsistencies but also, about *why* and *how* those factors impact on their conceptions of mathematics and their conceptions of their own teaching practices. We have very little understanding about mathematics teachers' conceptions of their own practices. What are teachers' conceptions of, for example, their roles as mathematics teachers, the role of contextual factors in their practices, and change in their teaching? How do teachers see the relationship between their conceptions of mathematics and their conceptions of their own practices —as opposed to how researchers seem to see it?

These research questions call for an overall naturalistic and constructivist line of inquiry (Guba & Lincoln, 1981, 1994; Denzin & Lincoln, 1994), as it is necessary to provide a means of exploring teachers' thinking and understandings of their experiences in some depth. It is necessary to follow a line of inquiry that allows the researcher and the participants to "delve in depth into complexities and processes" (Marshall & Rossman, 1999, p. 57). Since teachers' decisions and actions in their classrooms and the meanings they attach to those actions are significantly influenced by the context in which they occur, teachers' conceptions, as defined in this study (i.e., teachers' knowledge, beliefs and attitudes), need to be studied when focusing on real-life teaching situations. Further, a research design is needed that allows the participating teachers not only to express what they think but also to reflect on what they think about their practices in a particular setting.

Having outlined the methodological stance of this study, this chapter describes in Section 3.2 the research strategy initially constructed in order to shed light on the research questions. The methods chosen, the instruments devised for data collection and the ethics procedures are also described in this section. Section 3.3 presents a report of the pilot study, and the time line initially planned for the data collection is presented in Section 3.4.

3.2 Research strategy

Individuals construct their conceptions within the context of real life; therefore a study of teachers' conceptions requires a research strategy that focuses attention on the frame of reference that the real-life context represents in their conceptions. Because of this understanding I chose the case study as the research strategy for this study. A case study is "an empirical inquiry that investigates a contemporary phenomenon within its real-life context, especially when the boundaries between phenomenon and context are not clearly evident" (Yin, 2003, p.13). As a research strategy, the case study

> comprises an all-encompassing method —covering the logic of design, data collection techniques, and specific approaches to data analysis. In this sense the case study is not either a data collection tactic or merely a design feature alone but a comprehensive research strategy. (ibid, p.14)

What I learnt from my own observations throughout my previous work with teachers has prompted me to join Ball (1991), Thompson (1992) and Cooney (2001) to argue, in Chapter 2, that studies of teachers' conceptions cannot rely on teachers' verbal expressions alone. I take this argument to be even more crucial in this study in which teachers' *conceptions* have been defined to encompass teachers' *knowledge, beliefs* and *attitudes*. Teachers' 'knowledge', 'beliefs' and 'attitudes' need to be studied in the teachers' natural settings, in the course of their teaching practices, and through different modes of data collection. The ability to investigate the context through questionnaires, for example, is extremely limited (Yin, 2003). It also needs to be recognised that while interviews can provide us with data that allow important insights into the teachers' conceptions, a study that places attention on the real-life context cannot rely exclusively on verbal information. Other sources of evidence such as the recording of actual behaviour and documentary evidence (e.g., curriculum materials, in this study) provide relevant data necessary for triangulation. Thus the findings or conclusions in a case study "are based on several different sources of information, following a corroboratory mode" (Yin, 1989, p. 97). Furthermore, the dynamic nature of teachers' knowledge suggests that data collection at more than one point in time is necessary (Fennema & Franke, 1992). Therefore, in order to gain insight into the teachers' conceptions of and their own teaching practices of beginning algebra, this study was designed to draw on data collected over a period of six months and through a variety of data collection activities and instruments.

As a research strategy, case studies can be conducted and written for the purpose not only of identifying and describing individual cases but also to arrive at broad generalisations that expand theory (Evans & Gruba, 2002; Yin, 1989, 2003) and are of practical use (Patton, 1990). In other words, rather than generalising to a universe or population, case studies are used to expand our understanding of theoretical propositions in those situations where the context is important and the events cannot be manipulated, which are common situations in educational research (Yin, 1993).

3.2.1 Research design

I decided to conduct a multi-case study of teachers' conceptions of their own teaching of beginning algebra, from a small group of teachers after collecting and screening information from a larger group of preliminary candidates. Therefore, the data collection was to take place in two phases:

Phase 1: Investigation of the conceptions of the initial group of teachers with a range of teaching experiences and from different school settings.

Phase 2: Further enquiry into the conceptions of a subgroup of teachers selected to represent a range of conceptions as well as a range of teaching experiences.

Phase 1: Investigation of the conceptions of the initial group of teachers

The purpose of Phase 1 was to identify basic data in relation to the teachers' conceptions of beginning algebra and their interest in problem-based teaching approaches. In this phase I planned to study the conceptions of an initial group of eighteen teachers. Data were to be collected through two questionnaires and an interview, applied in the following order: Questionnaire 1, Questionnaire 1 follow up-interview, and Questionnaire 2.

Phase 2: Further inquiry into the conceptions of a subgroup of teachers

The purpose of the work of this phase was to gain a deeper understanding of the teachers' conceptions of beginning algebra, and of the way they made sense of their teaching practices in the specific contextual situation. To attain this purpose I planned to collect data through the observation of a sequence of lessons in Grade 8, followed by an interview for each teacher. A concept map activity where the teachers would be asked to identify the determinants of their teaching practices of beginning algebra was to be carried out at some point, which could only be identified during the actual data collection process. Data were also to be collected through a Focus Group session, and through the examination of curriculum materials (e.g., textbooks, teachers work plans, pupils' work). Table 3.1 outlines the research design.

Table 3.1 Research design

	Data collection stages	*Research methods*
Phase 1	Investigations of the conceptions of an initial group of 18 teachers	• Questionnaire 1 and follow up-interview • Questionnaire 2
Phase 2	Further investigation of the conceptions of a subgroup of teachers representing a variety of conceptions	• Classroom observation of five consecutive lessons plus follow up-interview (Exam. of c. materials) • Concept map activity • Focus Group session

The participants

Teachers to participate in the study were to be recruited from different schools in different settings of Bogotá. As explained in Chapter 2, teachers in Colombia normally like to work with at least one other colleague. Thus at least two teachers were to be selected from each school to participate throughout the study, a fact that would represent an opportunity for triangulating the sources of information. This group of teachers would constitute a "purposeful" group (Patton, 1990) as they were to be selected according to the following criteria:

- Participants would be volunteer teachers, teaching in Grade *eight*, where, according to the established curriculum, the study of basic concepts of algebra takes place in the Colombian context

- They were to be qualified teachers who had had no previous relationship with the researcher

- Teachers were to be recruited to form a group with a variety of teaching experiences in secondary school mathematics.

It was deemed important to study the conceptions of teachers who have much experience in the teaching of Grade 8-algebra as well as those of novice teachers, in order to identify differences (or similarities) in the teachers' conceptions of their teaching practices.

3.2.1.1 The methods of data collection

As shown in Table 3.1, a variety of data collection strategies was planned. Although the data collection process would start with the use of a questionnaire, an essential form of collecting data throughout the data collection process was to be the interview. The reasons for the choice of data collection strategies are explained in the following paragraphs.

Questionnaires

A questionnaire (Questionnaire 1) was to be used as the starting form of data collection with the initial group of teachers, in order to ensure "a structured or systematic set of data" (de Vaus, 1995) in relation to the specific aspects of the teachers' conceptions to be explored (see more specific information in relation to this in Subsection 3.2.1.2). A second questionnaire (Questionnaire 2) was designed to crosscheck responses in Questionnaire 1 (and the follow up interview), and to obtain indications of the respondents' interest in a problem-based teaching approach.

Questionnaires were considered to be a necessary form of data collection in the screening process of the larger preliminary group of teachers because the structure in the data collected would facilitate comparison of responses across cases. Although the questionnaires provided a specific structure in the questions asked, the design of the questions allowed for flexibility in the respondents' answers and sought the respondents' own input, as can be seen in the examples provided in Figures 3.1 and 3.2, and in the copies of the questionnaires in Appendix 3.1. It can also be observed that different sections in the questionnaires were designed to crosscheck responses across sections.

Interviews

Interviews were seen as an essential method of data collection in this study because interviews provide a method for collecting data embedded in the interpretations, perceptions and experiences of the respondents (Drew, Hardman & Hart, 1996). In Phase 1, interviews would provide the opportunity to seek illustrations and explanations to specific given responses to the originally set questions of Questionnaire 1. By interviewing the teachers to seek their explanations of their responses in Questionnaire 1, I intended to establish some rapport with the initial group of teachers from where the teachers of the follow-up study were to be selected. In a similar way, the meanings that the teachers attached to specific aspects of their teaching (e.g., the purpose of assessment) and to (critical) incidents identified during the sequence of observed lessons needed to be explored through interviews in Phase 2.

As interviews are human interactions, and the interviewer can affect the quality and quantity of data gathered, it is necessary for the researcher to be aware of the perspectives she brings into the study. So as the interviewer, I needed to be aware that my own conceptions, experience and interest could introduce bias through my interactions, as I made explicit in Chapter 1.

Classroom observation

Seeing in action the teaching that the teachers portrayed through the data collection activities of Phase 1 would constitute an important form of gaining understanding of their conceptions of beginning algebra and their own teaching practices. While classroom observations would provide an opportunity to see in action what the teachers have described in Phase 1, classroom incidents (and other aspects) would only facilitate insight into the teachers' conceptions at the follow-up interview. I planned to observe a sequence of lessons, and the teachers' explanations and interpretations of specific aspects and incidents, identified during the observations, would be the centre of discussion at the follow-up interview.

The concept map activity

I decided to use a concept map activity in one of the individual interviews to elicit rich information about the teachers' conceptions of the determinants of their teaching practices. Raymond (1997) conducted a concept map activity in her research with teachers about the relationship between mathematics beliefs and teaching practice, and found it a fruitful way to tap into teachers' thinking. However, as explained in Chapter 2, the teachers in her study were not encouraged to reflect on 'why' and 'how' the external factors identified as constraints affected their beliefs. In my study the activity was to provide opportunities for the teachers to identify and to express through the map the relationships between the factors (the teachers saw as determinants of their teaching practices) and their actual teaching practices in Grade 8. I hoped that if the teachers identified, for example, 'the pupils' motivation' as one of factors influencing their practices, I would encourage them to think about questions like 'why does 'the pupils' motivation' influence your practice? Or 'in what way does 'the pupils' motivation' constrain your preferred teaching practice?

Focus group

The purpose of the planned Focus Group was to collect high quality data in a social context where the teachers can consider their own views in the context of the views of others (Patton, 1990). A Focus Group session —where the moderator would not be the researcher— was to be organised after individual interviews and classroom observation had taken place so that the researcher had a good grasp of the teachers' teaching perspectives and priorities. At the planning stage I expected that the moderator could be any of the mathematics educators that I know in Bogotá, who have shown an interest in research in the area of the teaching and learning of school algebra. The Focus Group session would provide a different form of data collection as by bringing teachers together to share their own experiences it would encourage them to talk openly about their own teaching. I also hoped they would be stimulated to identify and discuss possible difficulties they saw in their teaching situations, which would provide opportunities for the collection of data that would represent "member checks" (Guba & Lincoln, 1981, Patton, 1990). I considered the Focus Group session an important form of interview that was likely to be instrumental in clarifying and highlighting the teachers' differences (or similarities) in their conceptions.

Examination of curriculum materials

Data were to be obtained by close examination of curriculum materials like textbooks used by pupils and teachers, the Mathematics department policy, the teachers' classroom and year-plans and the pupils' notebooks. In the pupils' note books I would identify, for example, the type of questions they would be working on during the lessons and the ones belonging to their homework.

3.2.1.2 The data collection instruments and activities

Questionnaire 1

The purpose of Questionnaire 1 was to try to uncover the teachers' priorities in relation to:

- the purpose of the teaching of school algebra; in other words, in relation to the *why* of the inclusion of school algebra in the curriculum of basic education, and
- the methodological orientations of the teaching-learning process; that is, the *how* of teaching which includes the assessment of pupils' work.

The data related to the content (i.e., the *what*) of the teaching of school algebra were to be collected at the follow-up interview. A copy of Questionnaire 1 is provided in Appendix 3.1.

Questionnaire 1 which was developed using the format of the 'Values and Mathematics Project'[1], is composed of three sections. Section A contains items related to the purpose of teaching algebra in schools, teaching style, type of classroom pupils' work and forms of pupils' assessment. Figure 3.1 shows an example of a question in this section. My own

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[1] Values in Mathematics and Science Education. (n.d.). Retrieved 12th November, 2004, from Monash University Faculty of Education Website:
http://wwww.education.monash.edu.au/centres/sciencemtc/vamp.html

experience and that of other researchers (see Thompson, 1992; Ernest, 1991) indicated that when teachers are asked about the teaching of mathematics they tend to talk about a preferred situation rather that about the actual work that takes place in their classrooms. This aspect was to be taken into account in the design of the questions and, therefore, in this section of the questionnaire, with the exception of question A1 (related to the purpose of algebra), questions were organised so that respondents would make a distinction between their preferred practices and what they actually do in their classrooms (i.e., their actual practices).

Section B was aimed at measuring the teachers' strength of agreement with 20 items about teaching aspects (e.g. algebra knowledge, teaching and learning algebra, pupils' abilities, value-teaching in mathematics) where respondents had to rate their choices according to a given Likert-type scale ranging from 'Strongly agree' to 'Strongly disagree'.

Section C was designed with the intention of crosschecking responses given in Sections A and B and, at the same time, of collecting some information on teachers' knowledge about pupils' difficulties and teaching strategies that could assist the pupils. It consists of a set of items with contextualised teaching-learning situations, where respondents had to state what they think of a pupil's answer to specific questions, and how they would respond to the pupil.

Descriptors and statements used in Sections A and B were developed assuming that teachers' conceptions of the nature of mathematics might be very different, and might range from a perspective of a collection of unrelated facts, rules and skills as in the "absolutist" (Lerman, 1990), "instrumentalist" Ernest (1989), "traditional" (Cooney, 2001; Gregg, 1995) views, to one which emphasises problem-solving and understanding as central in mathematical activity (e.g., Cockcroft Report, 1982; Ernest, 1989, 1991). The basics criteria used for the identification and creation of descriptors to be included in Questionnaires 1 and 2 are outlined in Appendix 3.1.

Section A
For each item in this section, some of the different ways in which the statement may be completed are given. Rank these statements accordingly in the accompanying boxes, where '1' indicates your first choice, '2' your second choice, '3' your third choice, etc. Please do not leave any empty boxes.

Note that the same ranking value can be given to more than one statement. Some spaces have been left at the end of each question in case you have other personal views or approaches. If this is the case, please write them with the corresponding ranking in the spaces provided.

A4. The types of classroom activity that I would **prefer** to see in my Grade-8 classroom are:

Ranking

- Pupils developing efficiency in algorithm-routine practice. ☐
- Pupils working at the board, especially when they have difficulties in applying algorithms. ☐
- Pupils engaged in the creation of algorithms and formulae. ☐
- Pupils discussing ideas and working systematically. ☐
- Pupils solving closed word problems. ☐
- Pupils using calculators to assist their learning and to use their working time more efficiently. ☐
- Pupils posing open problems and working on developing ways to solve them. ☐
- -- ☐
- -- ☐
- -- ☐

A5. The types of classroom activity that **actually** take place in my classroom are:

Ranking

- Pupils developing efficiency in algorithm-routine practice. ☐
- Pupils working at the board, especially when they have difficulties in applying algorithms. ☐
- Pupils engaged in the creation of algorithms and formulae. ☐
- Pupils discussing ideas and working systematically. ☐
- Pupils solving closed word problems. ☐
- Pupils using calculators to assist their learning and to use their working time more efficiently. ☐
- Pupils posing open problems and working on developing ways to solve them. ☐
- -- ☐
- -- ☐
- -- ☐

If the choices you made for question A4 are different from those of question A5, please express the reasons why they are different. --

Figure 3.1. An example of a question belonging to Section A of Questionnaire 1

Questionnaire 1 follow-up interview

The purposes of this interview were to explore the teachers' reasons for their answers in Questionnaire 1 and to collect information in relation to the *content* on which their teaching of mathematics in Grade 8 focuses. As tapping the thinking behind the teachers' answers to Questionnaire 1 requires, at the same time, identifying their interpretations of the different descriptors given in the questionnaire, the questions that were to be asked in the interview could not be spelt out before the actual data were available. Nevertheless, in the following, I provide some examples of possible basic questions that could be asked:

> In the questionnaire you responded (. . .). Why did you choose (...) as your first priority?
>
> What do you mean by (...)?
>
> Why did you not find (...) relevant for your teaching?
>
> In question (...) you responded that (...). Could you provide an example of (...)?
>
> In question (...) you responded that (...). How does that relate to what you responded in question (...)?
>
> Could you describe how you started classroom work with this group of pupils, this year? ...

Questionnaire 2

Data collection through Questionnaire 2 had a double purpose. The first was to obtain complementary information (to that collected through Questionnaire 1 and the follow up interview) about the teachers' conceptions of beginning algebra and its teaching. The second was to gain some insight into what 'a problem-solving approach' might mean for the teachers, and to obtain some ideas about their interest in introducing a problem–solving approach in their teaching of school algebra. Questionnaire 2 can be seen in Appendix 3.1.

Questionnaire 2 is composed of two sections, Section A and Section B. Section A contains the descriptions of the classroom approaches followed by the accounts of two different teachers —Teacher A and Teacher B— introducing the pupils to the concept of variable. In constructing the description of *Teacher A*, I used data from my previous research in Colombia which focused on teachers' conceptualisations of the transition from arithmetic to algebra work, showing a traditional, instrumentalist (Ernest, 1989) approach. For the description of *Teacher B*'s work, whose intention is to follow a problem-solving teaching approach, I drew heavily on Kieran, Boileau and Garacon's (1996) description of their work with beginning algebra learners through the use of a computer environment which they call CARAPACE.. The description of each teacher's classroom work and teaching approach is followed by a set of 7 questions that use what

Leder and Forgasz, (2002) refer to as "projective techniques" in which respondents are asked to react (to the given description) and explain. In each of the 7 questions asking the respondents' opinions about the adequacy of the described classroom approach (in relation to specific teaching-learning aspects), respondents are asked to rate their opinions from a given 5 point- scale that ranges from 'Excellent' to 'Very poor'. In addition to the rankings, respondents are requested to provide explanations for their choices, and to indicate how important the specific teaching-learning aspect being considered in the question is for them. I constructed the questions, assisted by the expert opinion of my supervisor. Figure 3.2 shows an example of one of the seven questions of Section A - Part 2.

Section A **Part 2**

Each of the following questions is composed of two parts. Please complete first given expression the by ticking one of the 5 alternatives, and by writing in the space provided the explanation asked for your specific choice. To complete the second expression, mark a point in the scale given to indicate your view.

A3

For providing opportunities for pupils to develop their communications skills, I consider *Teacher B's* approach to be

☐ Excellent ☐ Good ☐ Fair ☐ Poor ☐ Very poor

because ....

....

....

Providing opportunities for pupils to develop their communications skills is an aspect which, for my teaching of first algebra concepts, is:

Very important — Not very important

1 2 3 4 5

Figure 3.2. An example of a question belonging to Section A - Part 2 of Questionnaire 2

Section B is composed of two main questions. Question B1 is intended to identify the teachers' meanings of the term 'problem-solving teaching approach'. Further insight into what a problem-solving approach might mean to the teachers, and statements about their interest in the incorporation of a problem-solving approach in their teaching is sought through Question B2.

3.2.1.3 Data analysis

The following were the plans in relation to the analysis of data.

Phase 1

Data from Questionnaire 1, from each participant, were to be summarised in order to identify the questions to be asked at the follow-up interview. Data from the follow-up interview were to be screened together with the data from the other sources in order to form a basic initial picture of each participant's (and the group's) conceptions.

Phase 2

The data obtained through individual interviews, the Focus Group, field notes and the rest of data sources were to be coded using categories suggested by the framework established in Chapter 2 (i.e., categories were to be developed to describe approaches that could range from an "instrumentalist" view of mathematics to one which emphasises problem-solving and understanding as central in mathematical activity as in or The Cockcroft Report (1982) and Ernest (1989)). But generation of new categories was also to be sought; that is, codes would emerge from the participants' statements, and descriptions of their teaching practice and from their explanations of specific observed events an episodes, following the grounded theory approach of Glaser and Strauss (1967).

Field notes were to be read not only for regularly occurring phrases but also for "surprising counterintuitive phrases that might need to be clarified elsewhere in the notes or in the field" (Miles & Huberman, 1994), or that might appear not to fit comfortably into emerging categories for analysis. case descriptions were to be produced in order to have a clear understanding of each case teacher before moving into the cross case analysis.

3.2.2 The ethics procedures

This research was designed and, as will be seen in Chapter 4, conducted according to the requirements of the Australian National Statement on Ethical Conduct in Research Involving Humans and the practice of Monash University Standing Committee on Ethics in Research Involving Humans. I submitted an application to the Standing Committee on Ethics in Research of Monash University, which contained clear statements of the purpose, the basic general questions to be asked and the approach to be followed in the study. Attached to this application were:

- The "Explanatory Statement" and the 'Letter of consent' to be given to the possible participants during the recruitment period
- Copies of the English version of the questionnaires as well as details of the general purposes, basic questions and procedures to be followed during Questionnaire 1 follow up interview.
- The rationale and basic procedures for data collection during classroom observation.

The fieldwork started after the letter of approval from the Ethics Committee was received. Copies of each of the documents above listed, including the Research Committee's letter of approval for the research, can be seen in Appendix 3.2.

3.3 The pilot study

This section presents a report of the work that took place in order to test and develop the data collection instruments and activities. This work provided crucial feedback and points for reflection and development not only of the data collection instruments but also for the refinement of the logistics of data collection and the sequencing of activities. It also assisted me in developing a more relevant way of asking questions at the interviews.

Recruiting participants for the pilot study did not require much effort from the researcher. Four out the nine teachers who participated in the pilot study were recruited from a group of school mathematics teachers who were attending a one-week course (from 15 to 19 April 2002) at *Universidad de los Andes*. The other five teachers were contacted by phone, directly by the researcher, and their contact details were obtained from the data base of '*una empresa docente*[2]', *Universidad los Andes*. A copy of the research Explanatory statement and a letter requesting permission to carry out the classroom observation was sent to the school head teachers.

3.3.1 Piloting of Questionnaire 1 and the follow-up interview

Questionnaire 1 had two pilot testings, one in Australia and one in Colombia. The initial version of Questionnaire 1, which was written in English, was piloted with ten people attending the Mathematics Education Colloquium at the Faculty of Education of Monash University. Within the people attending the colloquium there were school mathematics teachers from different countries (e.g., Denmark, Singapore, Australia, Kenya) as well as active mathematics educators who were teaching in teacher education programs in Australia.

Once the English version of Questionnaire 1 was revised according to the identified needs for development that arose from the first piloting, it was translated into Spanish by the researcher and given to colleagues in Colombia for the purpose of checking the clarity of the language and of the interpretation of questions. The Spanish version of the questionnaire and the follow-up interview were piloted with the participation of nine mathematics teachers who were teaching in Grade 8 in three different schools in Bogotá. The nine teachers offered to participate in the piloting of the whole process of data collection as it was described to them at that point (e.g., they were told that the last activity to be piloted was a Focus Group session). The follow-up interview to Questionnaire 1 was conducted in each teacher's school.

3.3.2 Piloting of Questionnaire 2

Questionnaire 2 was also read by colleagues in Colombia with the aim of checking the clarity of the text in the descriptions and the questions, as well as the interpretation of

[2] "una empresa docente" is the name of the Centre for research in mathematics education of Universidad de los Andes, Bogotá, were I worked from 1995 to 1996.

the questions. This questionnaire was distributed to each of the nine teachers (who had returned Questionnaire 1) on completion of Interview 1. Completed questionnaires were collected by the researcher at the teachers' school, as agreed. However, only five teachers completed Questionnaire 2. Three teachers had not completed the questionnaire 'due to lack of time[3]', and one of them did not complete it because he did "not like the type of questions that [were] asked there" (e.g., he argued, "why do we have to ask for opinions of each teacher's approach? If they are mathematics teachers, they know what they are doing..."). Feedback related to the clarity and adequacy of the descriptions or to possible difficulties understanding the directions for completing the questionnaire was also sought from the pilot study teachers. Apart from this, two teachers, who found Questionnaire 2 'a very interesting one' suggested that the questions belonging to the approach of Teacher A should be placed immediately after the corresponding description (rather than at the end of the descriptions of the two teachers).

3.3.3 Piloting of the data collection activities of Phase 2

3.3.3.1 Observation of a sequence of lessons and the follow-up interview

The piloting of the sequence of lessons' observation (which for the pilot study was composed of four lessons), and the follow-up interview were carried out with the participation of four (out of the five) teachers who completed Questionnaire 2. It cannot be said that the four teachers who were chosen were representatives of a variety of conceptions of beginning algebra, for the five, who remained committed to participating in the whole process of data collection, all showed instrumentalist views of beginning algebra but showed interest in learning about a problem-solving approach, though not with the same commitment.

The piloting of the classroom observation took place while the data collection of Phase 1 of the main study with the thirteen teachers was in action, as can be seen from Table 3.2, in the following, and from Table 4.2 in Section 4.3.2 of Chapter 4.

[3] A week later these teachers provided the same explanation for the fact that they had not completed the questionnaire.

Table 3.2 The pilot study schedule

Piloting of instruments/activities	*No. of teachers with whom instruments/activities were trialled*	*Dates (2002)*
Questionnaire 1 (English version)	10	11 to 15 March
Questionnaire 1 (Spanish version) plus follow-up interview	9	16 to 23 April
Questionnaire 2	5	22 to 27 April
Observation of a sequence of lessons plus Follow-up interview (included the concept map activity)	4	2 to 15 July

The purpose of the piloting of these data collection activities was to test and refine my focus on aspects and incidents of the teachers' classrooms that could (or might not) be illustrative of the teaching they had portrayed during Phase 1. For example, in the classroom of a teacher who professed that her teaching drew "on a problem-solving approach", I was looking for instances which would illustrate (or not) such an approach. These included aspects like classroom and homework tasks, the teachers' and students' types of interaction in the classroom, the content emphasised, the assessment of the pupils' work, the classroom organisation. Once I had observed specific aspects within the classroom scenario throughout the sequence of lessons, I needed to seek the teachers' explanations at the follow-up interview. In my questions I was trying to clarify what the teacher's perspectives were in relation to the 'why', the 'what' and the 'how' of Grade 8-algebra.

When the teachers were explaining the incidents and aspects that I brought in for discussion at the follow-up interview, the teachers named factors that accounted for what took place in their classroom, just as they had been doing during the interview in Phase 1, when they were asked to give reasons for the difference in their rankings in Questionnaire 1. This indicated the right point to engage the teachers in the building of the concept map of the determinants of their practices. Consequently, the concept map activity became the second part of the observation follow –up interview.

3.3.3.2 The concept map activity

The data collected through the pilot study offered ideas as to how to conduct the concept map activity and to the point of the data collection process where it was adequate to introduce this activity.

The information the teachers had provided in explaining differences in their rankings in Questionnaire 1, as well as in their explanations for specific incidents in their classrooms, showed a list of common reasons (or factors, e.g., 'the pupils' motivation', 'the set program for Grade 8' and 'time') for their teaching decisions and actions in their classrooms. I could have taken this common list of reasons (or factors) that the teachers were saying influenced or determined their teaching practices, and ask them to use these to build a concept map of the determinants of their practices. However, I felt that the key to data collection through the concept map activity was to engage each teacher in building her/his concept map from scratch, identifying the components (i.e., the factors determining their practices), as well as the interrelationships between these components and their practices. Thus, as a consequence of my interaction with the pilot study teachers during the construction of their concept maps, the following steps were identified as necessary to follow with the teachers of the main study in this activity of data collection:

1. Having discussed and clarified the meaning of "My teaching practice in Grade 8", the teachers would be asked to think of the factors that they think influence[4] or determined 'their teaching practice in Grade 8', and to write them down.
2. The teachers are presented then with examples of simple concept maps, in order to discuss how to build a concept map (if necessary) or how the presented maps may have been constructed.
3. Finally they are provided with yellow 'Post-it' labels to write on them the names of the components of the concept map. With the intention of saving time, a sheet of paper where the central concept of the map has been written is to be used.

It was not possible to pilot the Focus Group as it was difficult to fix a date and a time that suited both the teachers and the moderator.

[4] The meaning of influence, determine, affect or whatever word the teachers use would need to be discussed and clarified in order to have a common referent in the language used with each teacher.

3.4 The planned timetable for the fieldwork

I elaborated an estimated timetable for the data collection in Colombia, which is shown in Table 3.3. The examination of curriculum materials was to take place during the period of time dedicated to classroom observation. Chapter 4 describes what took place in the data collection of the main study.

Table 3.3 Estimated timetable for data collection

Dates (2002)	*Activity*
	Phase 1: Collection of data from a group of 18 teachers
16 - 25 April	Piloting of the Spanish version of Questionnaire 1, the follow-up interview, and Questionnaire 2
20 April – 17 May	Participant recruitment and setting up of the study Distribution (and collection) of Questionnaire 1. Analysis of data from Questionnaire
24 - 31 May	Follow up interviews. Distribution (and collection) of Questionnaire 2
3 June - 28 June	Analysis of data. Deciding and confirming participation of the subgroup of teachers for the second Phase of the study
	Phase 2: Further collection of data from a selected group of teachers
1 - 19 July	Arranging timetables for data collection in the next stage of the fieldwork
22 July-31 August	Classroom observation plus following interviews
1 - 7 September	Data organisation and initial analysis of case studies data Focus Group session
9 - 20 September	Analysis of data from Focus Group and revision of fieldwork data

Chapter 4: The main study

4.1 Introduction

In Chapter 3 I argued that the case study strategy, where multiple sources of data as well as forms of data collection are used, was necessary in order to gather evidence that shed light on the research questions of this study. While Chapter 3 outlined the research strategy and the pilot study, this chapter reports on the process of data collection that took place for the main study. It starts with a brief description of the procedures followed for the recruitment of the participants and the ethics procedures in Section 4.2 and continues in Section 4.3 with a report of the sequence of activities carried out in the data collection process. Section 4.4 presents descriptions and explanations of the data analysis procedures. A brief account of the role of the researcher is the focus of Section 4.5. The chapter ends with a discussion of issues related to the authenticity and quality of the research in Section 4.6.

4.2 Participants recruitment and ethics procedures

While finding participants for the pilot study was an easy task, finding participants for the main study was not. From the twenty nine teachers whom I contacted[1] by phone, twenty five declared themselves to be interested in participating in the study, and requested that the research Explanatory Statement be faxed to them. The other four said that they thought that their head teachers would not give permission to do classroom observation. The Explanatory Statement together with a letter of invitation were either faxed or delivered to the schools of the twenty five interested teachers (See the Explanatory statement and copies of the letters of invitation in Appendix 3.2). However, when I contacted them again to confirm their interest in participating, only twenty one declared that they wanted to go ahead. As will be seen in Subsection 4.3.1, after the twenty one teachers received Questionnaire 1 and the letter of consent, only thirteen

[1] The contact details of these teachers were obtained from databases of '*una empresa docente*', *Universidad de los Andes* and the 'Mathematics education postgraduate program' at *Universidad Distrital* in Bogotá

teachers returned the completed questionnaire together with the signed letter of consent, reconfirming their willingness to carry on being part of the study. I removed the page in Questionnaire 1 containing details of the participants and assigned a number to each participant. These numbers were also used to identify the transcripts of the interviews in order to guarantee the teachers' anonymity for all but the researcher. The names used in this thesis are pseudonyms.

4.3 The sequence of data collection activities

The data collection (in Colombia) required more activities than the ones envisaged at the planning stage (refer to Table 3.1 in Chapter 3). An additional interview 'Interview 3' was included as the final data collection activity as can be seen in Table 4.1 which summarises the sequence of data collection activities for the main study and their specific purposes.

Table 4.1 The sequence data collection activities

Phase 1		*Phase 2*	
Research instrument/activity	Purpose	Research instrument/activity	Purpose
Questionnaire 1	Collect data about the teachers' conceptions of *the why, the what* and *the how* of the teaching of G 8-algebra	Observation of five consecutive lessons (E. of c. materials)	Collect data to further understand the teachers' portrayed practices
Interview 1 (Questionnaire 1 follow-up interview)	Explore the teachers' reasons for their answers in Questionnaire 1	Interview 2 (Observation follow-up interview plus Concept map activity)	Obtain the teachers' explanations of classroom incidents & engage them in the construction of a concept map of the determinants of their teaching of Grade 8-algebra
Questionnaire 2	Identify the teachers' interest in a problem-based teaching approach	Focus Group	Explore further the teachers' conceptions of their teaching of beginning algebra
		Interview 3	Probe key ideas, not properly explored during the Focus Group & revise concept map

I considered the addition of Interview 3 to be important for the following reasons:

- the teachers showed a willingness to continue discussing issues that arose at the Focus Group (e.g., the concerns about teaching the formalisations of mathematics), and
- the teachers' revisions of their concept maps (of the determinants of their own practices, developed at Interview 2) were necessary.

Figure 4.1 illustrates how the research design was devised to develop a convergent line of inquiry; that is, the same questions were basically being asked in each of the data collection instruments and activities. The converging line of enquiry was based on a process of continuous triangulation of "data sources" (Patton, 1990, Yin 2003) and/or of "data collection sources" (i.e., forms, methods) which was seen as necessary to produce well documented descriptions which served as the case study data record (Patton, 1990; Yin 2003). This approach was based on my commitment to "an *emic*, ideographic case-based position [italics added]" (Denzin & Lincoln, 1998, p. 10) which directed my attention to the specifics of particular cases —as opposed to "an *etic* science-based on probabilities derived from the study of large numbers of randomly selected cases". I was highly concerned with the *process* of the research, with the *how* of eliciting the teacher's perspectives on their teaching of beginning algebra.

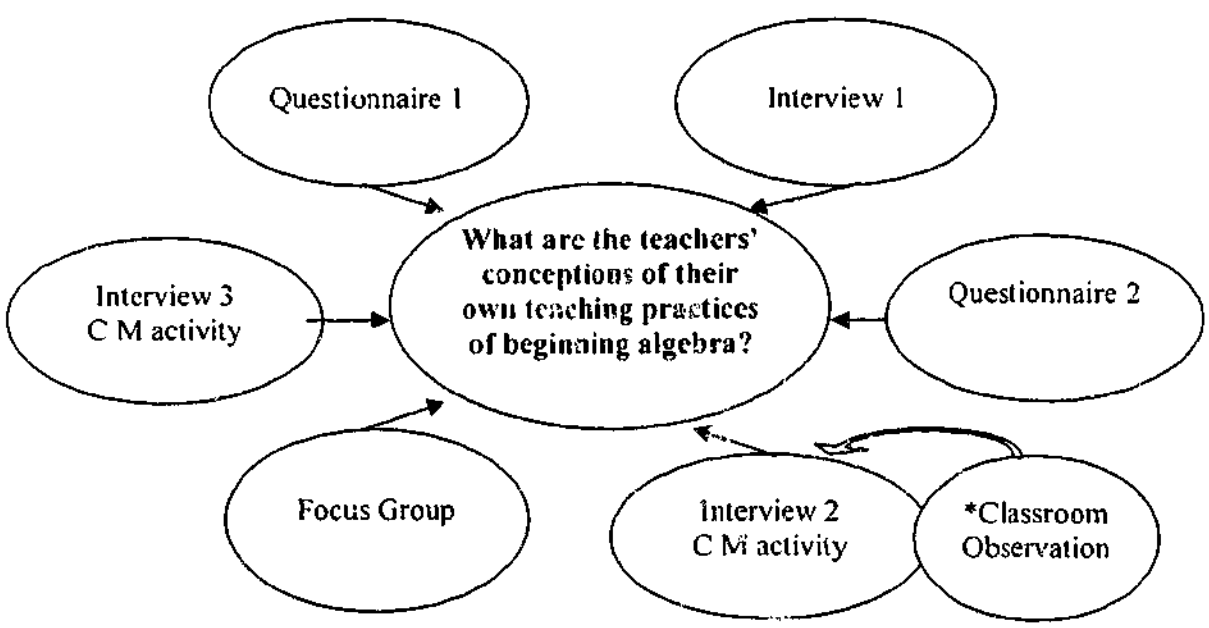

Figure 4.1. *Design of data collection through a convergent line of inquiry*

As can be seen in the diagram, 'Classroom observation' is marked with an asterisk, to emphasise that the incidents I selected during the observation were brought in to Interview 2 in order to obtain the teachers' views and explanations of the incidents. It was through the teachers' eyes that I wanted to see the incidents.

In a sense, I could say that the central question being asked of the teachers in this study was:

> What are your conceptions of your teaching practices of beginning algebra?

However in asking this question, I was also asking them, "What are your conceptions of beginning algebra?" The consideration of the questions as two different questions was necessary, as understanding the teachers' explanations about their own practices required an understanding of what 'beginning algebra' meant for them, hence it is emphasised in the following chapters that the basic Research Questions of this study are:

> Question 1: What are the teachers' conceptions of beginning algebra?
>
> Question 2: What are the teachers' conceptions of their own teaching practices of beginning algebra?

4.3.1 Data collection procedures in Phase 1

Questionnaire 1 together with the letter of consent were delivered by the researcher to the schools of the twenty one teachers[2] who, when contacted by telephone by the researcher, confirmed their interest in participating in the study. However, only thirteen returned the questionnaire, in which they reconfirmed their willingness to carry on being part of the study, together with the signed letters of consent. Two teachers who did not return the questionnaire explained (on the phone[3]) that they did not think that their head teachers would give permission to do classroom observation. Two other

[2] The twenty one teachers belonged to ten different schools. In most schools the questionnaires plus the letters of consent (in sealed envelops) were handed to the coordinator of the mathematics department.
[3] Explanations were given by the teachers on the phone as I had to contact them to agree a specific place and date when I should collect the envelopes.

teachers explained that they were too busy. I could not obtain explanations from the rest of the teachers as it was not possible to contact them anymore.

Questionnaire 1

The data from Questionnaire 1, corresponding to each individual teacher, were summarised in tables from where specific aspects that needed further explanation and probing were identified as the basis for Interview 1. The format of these summary tables as well as a copy of one of the actual summaries is provided Appendix 4.1.

Interview 1

The follow-up interview to Questionnaire 1 (Interview 1) was carried out with the thirteen teachers. The interviews were conducted at the school where each teacher worked, except in the case of Luis who found it preferable to do the interviews at the university where he was teaching mathematics in the mornings. Each interview lasted a minimum of fifty minutes.

In relation to the type of questions used at Interview 1, it can be said that the basic format of questions shown in Chapter 3 (p. 69) was generally used throughout out the interview. I normally started with the teachers' responses to the question about 'the purpose of the teaching of school algebra' of Section A of the questionnaire. I had in mind my general purpose of tapping into the teachers' thinking about the *why*, the *what*, and the *how* of the teaching of algebra in Grade 8. However, there was no specific order of questions in the elicitation of data as the interview took place in the form of a conversation. One particular question moved the conversation to different points of the teachers' responses in the questionnaire (i.e., from their responses to questions of Section A to their responses to questions of Section C and sometimes of Section B), and also to the question of the mathematical *content* they taught in Grade 8. The interview provided an enormous amount of information that spoke about the teachers' conceptions, not only of beginning algebra but also of their conceptions of their teaching practices.

At this interview the teachers provided the first indications of their conceptions of the role of the social and institutional aspects in their teaching practices as will be seen in Chapter 5. The audio recordings of the interviews were transcribed, and a copy of the corresponding *verbatim* transcription was given to each teacher for reviewing and for further clarification of their views, if they found it necessary.

Questionnaire 2

On completion of Interview 1, each teacher received a copy of Questionnaire 2, and it was agreed that the completed questionnaire would be collected by the researcher one week later. The data collected through Questionnaire 2, were also summarised in tables for each participant. The format of these summary tables as well as a copy of one of the actual summaries is provided Appendix 4.1.

4.3.2 Data collection procedures in Phase 2

After a basic analysis of every teacher's data summaries (see a copy of one of this summaries in Appendix 4.1), which focused on identifying their conceptions of Grade 8-algebra, the reasons for any difficulties they identified in putting into action their preferred practices, and their interest in a problem-based teaching approach, nine teachers were selected for the follow-up study. A full description of the criteria for the selection of the case study teachers as well as the list of selected teachers is presented in Chapter 5, Subsection 5.6.1. As will be seen in the next subsection, only seven teachers completed the entire sequence of data collection activities.

The sequence of lessons observed and the follow-up interview

It was agreed with the teachers that a sequence of five lessons, which for the majority of teachers was the number of lessons per week assigned by the school timetable, would be observed. For most participants, the observation of the sequenced lessons took place according to the original agreed timetable, but in the cases of Pablo and Alex there were some changes. For Alex the agreed lesson observations had to be extended as the second timetabled lesson was cancelled due to an unadvised modification of the school-day timetable for extracurricular activities. In the case of Pablo, only four (consecutive) lessons were observed for the same reason as for Alex. Moreover, it was not possible to complete the five consecutive-lesson observation plan in Pablo's Grade 8 classroom as it interfered with the set timetables of other teachers.

Interview 2

The original planned times for the follow-up interview to the classroom observation (Interview 2) needed to be extended as the majority of teachers suggested setting another appointment to continue with the building of their concept maps, which took place in the second half of Interview 2. The first part of the interview lasted approximately half an hour with each teacher, and was dedicated to eliciting the teachers' explanations about classroom incidents and about general aspects of their teaching like planning, examples of assessment tasks, curriculum materials.

The concept map activity. The general guidelines for the procedures used during the concept map activity were presented in Subsection 3.3.3.2, and more detailed descriptions of the sequence of steps that were followed during the concept map activity with each teacher are presented in the case descriptions in Chapter 6.

The Focus Group session

The purpose of the Focus Group session was to explore further the teachers' concerns in relation to their teaching of Grade 8-algebra and to obtain their opinions of the problem-based oriented classroom approaches that (as I presented them to the teachers) other teachers from different school contexts in Colombia followed in the introduction of the concept of variable. The Focus Group moderator was chosen from three possible candidates on the basis of the two following criteria:

- The person was an active mathematics teacher educator, not known by any of the participating teachers
- I had come into contact with this person due to his interest in research in the teaching of school algebra.

Planning for the Focus Group session. During August 2002, I met with the chosen moderator of the Focus Group three times, for one hour each time. In accordance with the purpose of the Focus Group discussion, we planned and discussed the (moderator's) guide for his role of encouraging and leading the discussion during the session. Table 4.2 shows a basic outline of the program to be followed during the Focus Group session, to take place on Tuesday, 3 September, 2002, from 4:00 to 6:00 pm. However, the teachers were asked to be at the venue at 3:45 pm.

We agreed that I would be the presenter of the descriptions of the teaching approaches that three different teachers followed when introducing the concept of variable to their pupils. The text presented in transparencies and other materials used during the presentation are shown in Appendix 4.2.

Table 4.2 Moderator's plan for the Focus Group session (3 Sept, 2002)

	Activity	*Time (min.)*
1.	Introduction: Welcome to participants and moderator's self-introduction. Statement of the purpose of the session.	3
2.	Reminder (and discussion, if necessary) of aspects to keep in mind for the Focus Group session (e.g., Confidentiality, Free participation, and Audio recording of the session)	5
3.	Participants' presentation+ with their answers to a given Introductory Question*, aimed at identifying either the teachers' concerns with heir teaching in Grade 8 or any aspect they considered achievements*	15
4.	Presentation of the classroom approaches followed by three different teachers in the introduction of the concept of variable	25
5.	Discussion based on questions related to the three classroom approaches presented and their relevance in addressing any concerns put forward by the participants	65
6.	Session closure. Thanks to participants for their attendance.	5

+ The participants were to be told that they could choose not to identify the names of the schools were they were working, or to give their full names if they preferred it

* The given introductory question, which was to be shown in a transparency, was as follows:

Is there any aspect of my teaching of Grade 8-algebra that I would like to share?
Note: The reason(s) for sharing whatever aspect you want to share are not specified. It could be either because you are concerned about a specific aspect or because you feel satisfied with it. It can also be that you have a question about a specific aspect of your teaching that you would like to ask your colleagues.

The actual Focus Group session. The Focus Group session took place on September 3, 2002, at the Faculty of Education of the university where the moderator worked as the Director of one of the Postgraduate courses in Mathematics Education. From the two possible venues available to carry out the Focus Group session, most teachers chose this place because it was 'in easy reach'. Very importantly, it was located one block away from the school[4] where the two teachers who, at the end of Interview 2, showed low interest in participating in the Focus Group, naming 'time needed, increased by having to travel' to the venue initially chosen (which would have taken them a maximum of 15 minutes by public transport). I arranged the Focus Group session at a place where distance would not be a reason for no attendance[5] for these two teachers.

The participants arrived punctually at 3:45 pm as had been agreed with them. During the first fifteen minutes (i.e., from 3:45 to 4:00) they engaged in informal chat. I had told them at the end of Interview 2 that they did not need to identify the names of their school if did not want to. However, they all found 'no reason for doing that'.

Apart from the fact that I presented the descriptions of the classroom approaches followed by three different teachers in the teaching of the first notions of the concept of variable, my role during the activity was that of an observer, as the moderator and his assistant were totally in charge of the activity, according to the planning. An outline of specific aspects of teaching that I considered essential in describing the approaches of each of the three teachers mentioned is presented in Appendix 4.2.

Interview 3

The follow-up interview to the Focus Group, Interview 3, which was carried out at the teachers' schools, during the times they set as adequate, lasted a minimum of 45 minutes for each teacher. I had three specific purposes for this interview, which divided the interview into three parts: The first part was dedicated to obtaining the teachers' comments about the Focus Group experience, and their explanations about specific

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[4] These teachers' school is located within Bogotá central area.

[5] The initial reason that these two teachers offered for "the difficulty in attending" was the length of time needed of them, as they would need to take public transport to reach the initial venue that was considered for the Focus Group.

issues that became the centre of attention during the session, especially 'the need to teach the formalisations in Grade 8-algebra'. The second part was dedicated to the revision of the concept maps (of the determinants of their teaching practices), which they had developed in Interview 2. During the third part I elicited information about the teachers' interest and commitment to becoming engaged in professional development. To do this I presented to them a three-option proposal. One of the options represented a genuine opportunity the teachers had if they fulfilled the requirements to be accepted in a professional Development Programs offered by the Secretary of Education[6] in 2002 or 2003. The other two options were imagined options, one of them was "the adoption of a textbook for Grade 8 mathematics, which had been developed on the basis of research results on the teaching and learning of Grade 8-algebra". An outline of the three-option proposal that was present to the teachers at Interview 3 is shown in Appendix 4.3.

4.3.3 The full data collection schedule

The planned schedule for the data collection was shown in Table 3.3 of Section 3.4. Table 4.3 shows the actual full schedule of the data collection. Note that several activities were being developed simultaneously as the dates indicate.

[6] The Secretary of Education was offering financial assistance to the three top graded classroom-based research proposals submitted by school teachers (of any area of the school curriculum). This program was offered through the Institute of Educational Research and Pedagogical Development (IDEP).

Table 4.3 The data collection schedule

Dates (2002)	*Activity*
	Phase 1: Collection of data from a group of 18 teachers
17 – 30 April	•Participant recruitment and setting up of the study •Distribution of Questionnaire 1.
30 April - 31 May	•Collection of Questionnaire 1 and analysis of data •Questionnaire 1 follow-up interviews. •Distribution (and collection) of Questionnaire 2
10 May- 28 June	•Analysis of data. •Deciding and confirming participation of the nine teachers for the second Phase of the study
2 - 19 July	•Arranging timetables for data collection in the next stage of the fieldwork
	Phase 2: Further collection of data from a selected group of teachers
22 July-31 August	•Classroom observation plus following interviews (6 teachers in the afternoon session and 3 in the morning session, distributed in 5 schools) •Classroom observation follow-up interview (Interview 2)
1 - 7 September	•Data organisation and initial analysis of case studies data •Focus Group session
9 - 20 September	•Analysis of data from Focus Group and revision of fieldwork data

4.4 Data analysis procedures

It was argued in Chapter 3 that the case study is a comprehensive research strategy, which covers the logic of design, the data collection techniques and the approaches to data analysis. The logic of doing a two-phase data collection process where the data from a preliminary group of teachers were screened in order to select the case studies, set up specific stages of data analysis. Three stages of data analysis can be distinguished in this study, in which the level of analysis was either "descriptive" or "interpretive", according to Patton's (1990) differentiation. The first stage corresponds to the analysis carried out on the data collected in Phase 1 from where the conceptions of each individual teacher as well as of the group of thirteen teachers, as a whole, were found.

The second stage corresponds to the screening process performed on the voluminous data collected in Phase 2, jointly with the data from Phase 1, in order to construct the descriptions of the case study teachers. The third stage consisted of the comparisons of the case study teachers' conceptions, presented in Chapter 7, where commonalities (or differences) were sought and a theory emerged. The data analysis was carried out in Spanish, the language of the data. However, my thinking switched to the English language in the final stages of the interpretive analysis; that is, when a theory started to be constructed.

4.4.1 The first stage of the analysis

In accordance with my purpose of identifying the conceptions of the initial group of teachers, in order to be able to select a subgroup to be studied further, I first organised summary tables of specific data that I considered to be key for guiding the general and basic accounts (developed later) of the conceptions of each of the thirteen teachers. The initial backbone for the elaboration of each account was represented in what the teachers emphasised as their Grade 8 –algebra teaching top priorities in Questionnaire 1. An example of each of the tables in which data from each of the thirteen teachers, collected through Questionnaire 1, were summarised can be seen in Appendix 4.1.

The transcripts of the follow-up interview (Interview 1) contain the illustrations, the explanations and the images provided by the teachers, which helped me see more clearly what the teachers were talking about with their responses to Questionnaire 1, for the questionnaire had been written using my own language, and not 'the teachers' language'. I began the screening process of the data collected through Interview 1 by reading the *verbatim* transcriptions, and writing labels in the margins of the page, in order to classify the content of the data. The labels used were expressions related both to specific issues put forward by the teachers and to broad aspects of the focus of the study. For example, the label 'problem with children of the south' was a label I used in the transcripts (not only of Interview 1 but also of Interview 2 and 3) for a teacher who, very frequently, explained the difficulties of his teaching by this fact. This label, in the end, became the descriptor I used to refer to his case in Chapter 6. Some examples of the labels I referred to, previously, as relating to broad aspects of the focus of the study are: 'Developing communication skills', 'Learning G 8-algebra', 'The variable as an unknown', Teaching G 8–algebra, 'Teaching factorisation', Homework', 'External factors', 'assessing pupils' work'.

This initial labelling was useful for identifying blocks of data sought during the analysis in Phase 1 and also for later readings as transcripts were read several times, at different points of the analysis, for in each of the blocks there were data revealing aspects related to other questions (e.g., a piece of narrative that said something about the assessment of pupils' work could also say something about the purpose of the teaching of a topic and, most possibly, about 'the how' to teach). The general constructed guidelines for the merging of initial categorisations into the broad 'instrumentalist' and "Non-instrumentalist' categories can be seen in Appendix 3.1.

Although triangulation of data from Questionnaire 1 and Interview 1 was done, as for example Sections A and C of Questionnaire 1 were designed for this purpose, the data collected through Questionnaire 2 offered further opportunity to triangulate the teachers' responses. The findings of Phase 1 of the study which represent the teachers' first indications of their conceptions of beginning algebra and their teaching practices are presented in Chapter 5.

4.4.2 The second stage of analysis

The case study approach to qualitative analysis is a specific way of collecting, organising and analysing data, and the starting point for case study analysis is making sure that the information for each case is as complete as possible (Patton, 1990; Yin 1989, 2003). In order to build a comprehensive picture of the conceptions of each case study, the teacher's answers, explanations of situations and classroom episodes, arguments and concept maps produced throughout the whole process of data collection, as a consequence of my questions, are brought together in the case descriptions presented in Chapter 6.

The descriptions were constructed by merging raw data, from the multiple sources of evidence, which belonged to the specific descriptive themes suggested by the study's basic research questions and the overall aim of my research. In developing the case descriptions I built on items of information already known, (i.e., the individual tables and accounts of each individual teacher constructed from data collected in Phase 1), making connections among different items and blocks of data, and looking for information that completed the picture being described, and then verifying the existence of data that offered possibilities of triangulation. Identifying the blocks of data from the transcripts belonging to specific headings in the case descriptions was a time consuming and complex task. Although the data collected through the multiple forms (as shown in Figure 4. 1) were convergent, they were convergent only in the sense that they illuminated the same questions. However, in several cases the teachers' responses represented, to me, contradictions and inconsistencies which seemed to emanate from the values underpinning their views of their worlds. What I saw as inconsistencies and contradictions, the teachers perhaps rationalised as being consistent, and my job as a researcher was to understand the meanings conveyed by the teachers in their explanations. Therefore, these contradictions and inconsistencies needed to be resolved and revealed and so were the convergent ideas, in an attempt to allow the reader to understand the case as a unique, wholistic entity.

4.4.3 The third stage of analysis

The ultimate purpose of this study was to identify the relationship between the teachers' conceptions of beginning algebra and their conceptions of their teaching practices with a view to unravelling their conception of change. The identification of the teachers' conceptions of change in their teaching (i.e., their conceptions of their learning) represented a key task of this study as the identification of possibilities for change pointed out by the description of the problem in Chapter 1 was the main reason for undertaking this study.

With the purpose of illuminating the relationship between the teachers' conceptions of beginning algebra and their conceptions of their teaching practices, inductive cross-case analysis of the teachers' explanations for their teaching practices was carried out in Chapter 7 through the lenses represented by the following two questions:

- Why do the teachers do what they do in their teaching of Grade 8- algebra?
- Why are they (are they not) willing to incorporate a problem-solving approach into their teaching of Grade 8- algebra?

In identifying the teachers' conceptions of beginning algebra, the focus of the analysis was placed on data related to the fundamental components of teaching (i.e., answers to the *why*, *what* and *how* of the teaching of beginning algebra). Closure of the process of analysis took place when all sources of information had been exhausted, the set of categories had been saturated and the clear regularities that emerged felt integrated, as Patton (1990) explains it.

4.5 The researcher's role

As a Colombian mathematics educator with experience in the teaching of secondary school mathematics in Colombia, I introduced myself to the participating teachers of this study as a colleague who was interested in learning about their experiences as teachers of school algebra, especially in Grade 8. As a researcher interested in gaining some understanding of the views of the world held by the participating teachers, my main role during the data collection was that of listener and learner. My perspective in this research was that of an insider, as I wanted to collaborate with the teachers in bringing forth their understandings of their teaching experiences and in representing their views of their worlds. As a naturalistic researcher I was researching *with* the teachers, which means that they were not just providers of data but collaborators with the researcher as they participated in decision making about the data collection activities. For example, some of them suggested an extension of the concept map activity interview (for which another interview appointment was made) in order develop their concept maps and discuss them further. In the same way, they declared (to the Focus Group moderator) their interest in discussing further (with me) some of the issues that arose during the Focus Group session.

Despite the fact that not all the participating teachers showed the same level of enthusiasm during the last part of Phase 2 of the data collection, as will be explained in Chapter 7 for the cases of Nacho and Loren, all of them opened their classrooms and shared their teaching experiences and concerns with me, and this is evidence that they trusted me as an educator and colleague. Through the learning that the teachers enabled me to achieve, I feel better informed to continue pursuing my duties as a Colombian mathematics educator, citizen and researcher, for my role as a researcher cannot be just to construct the teachers' stories. I feel I have the moral obligation of working not just in the design and delivery of the professional development programs that are envisioned in Chapter 8, but also of working towards addressing what I believe the participating teachers asked me to voice for them. In this way I hope to conduct research that is "committed to participants" (Wright & Wright, 2002) as well as to the broader educational context.

4.6 Authenticity of the research study

The issues of authenticity of naturalistic research have represented a topic of continuing debate and reconsideration for researchers in the social field. Guba and Lincoln (1981, 1989, 1999), for example, have proposed criteria intended to parallel the "rigor criteria" used in the conventional paradigm; that is, counterparts to standards of internal and external validity, reliability and objectivity. The analogous terms proposed are credibility, transferability, dependability, and confirmability (respectively).

> Credibility is seen as a check on the isomorphism between the enquirer's data and the interpretations of the multiple realities in the minds of informants. Transferability is the equivalent of generalizability to the extent that there are similarities between sending and receiving contexts. Dependability includes the instability factors typically indicated by the term 'unreliability' but makes allowances for emergent designs, developing theory, and the like... Confirmability shifts the emphasis from the certifiability of the enquirer to the confirmability of the data (Guba & Lincoln, 1999, p.147)

Although the terms *credibility* and *dependability* were chosen by these authors in order to identify the analogies for *validity* and *reliability*, which in the conventional paradigm are separate, in my study I see them as inseparable. The following features, which by ensuring 'credibility' also provided conditions for 'dependability', are representative of my study:

- The persistent observation of actors through prolonged engagement at the field site in order to provide time to identify salient characteristics as well as to appreciate atypical but meaningful features.

- "Triangulation, whereby a variety of data sources ... different perspective's (theories), and different methods are pitted against one another" (ibid, p.147). There was a continual scrutiny of data by cross-checking of inferences with related interview material and data drawn from other sources (e.g., curriculum materials).

- "Peer debriefing to test growing insights and receive counsel about the evolving design, discharge personal feelings and anxieties" (ibid, p.147), which were represented in my interaction with and advice from my supervisors in relation to the evolving design of my study. There was a continual process of vigilance of the research process on my part, through questioning and evaluation of each phase of the fieldwork.

Furthermore, as described in previous sections, the case descriptions were constructed through the presentation of primary or raw data that were collected through the use of multiple data collection sources, where basically the same questions were being asked. Therefore, evidence was obtained through a continuous process of triangulation, a type of triangulation that involved the convergence not only of data collection sources but also of data sources (Yin, 1989, 2003; Denzin & Lincoln, 1998; Patton, 1990). The case study descriptions and the evidence from the preliminary group of teachers became what Patton (1990) refers to as "the case data record" which was used for further analysis (i.e., interpretation). This allows the external observer, or reader, to follow the derivation of any conclusion. The reader is then able to trace the steps in either direction; that is, from conclusions back to initial research questions, or vice versa. This, to me, ensured the conditions for the "dependability audit" represented in a "chain of evidence" (Yin, 2003) which cannot be achieved in the absence of credibility of the findings above discussed.

With respect to the transferability, the search for a purposive group of teachers maximised the range of information collected. The provision of "stringent conditions for theory grounding" (Guba & Lincoln, 1999, p.147) represented in the availability of data to screen the teachers' explanations for the two different questions described in Subsection 4.5.3, and the comprehensive descriptions and information about the context, facilitates judgements about the extent to which the "working hypothesis" (Stenhouse, 1975) from that context might be transferable to other contexts.

However, recent emphasis is placed on determining measures of rigor appropriate to the naturalistic or realism paradigm which rely on multiple perceptions about reality

(Denzin & Lincoln, 1998; Healy & Perry, 2000). Winter (2000) claims that many allegations of invalidity, in relation to the findings of a specific research, can be attributed to the failure to recognise the purposes to which the methodology was suited. My purpose in this study was to capture the perspectives of the teachers and to understand more about the difficulties teachers face in the teaching of beginning algebra in the Colombian context. Therefore, the quality of this research resides with the representation of the teachers' perspectives and concerns, and the appropriateness of the processes involved in the study to elicit their perspectives; while every effort was made to bring forth the perspectives of the teachers, in their own and unique language, the findings and evaluations also bear my own philosophical perspectives and values.

Furthermore, in discussing the quality of case study research Lincoln and Guba (2002) have included "the ability of a case study to evoke and facilitate action on the part of the readers" (p. 211) as one of their *empowerment* criteria. I would like to argue that part of the empowerment criteria, for this case study, implicitly calls me as the researcher to work towards making the stories of the teachers count not only in the wider mathematics education research community but, most importantly, with policy makers and educational authorities in Colombia. The difficulty of this form of 'empowerment' is related to the position and power of policy makers in Colombia. To empower the voices of teachers in Colombia requires an authoritative voice that would be heard by policy and decision makers, and the achievement of such a voice is often a matter of politics than of the commitment of the researcher.

Chapter 5 presents the findings of Phase 1 of the study, related to the conceptions of the thirteen teachers as a group.

Chapter 5: Findings from the initial group of teachers in Phase 1

5.1 Introduction

It was explained in Chapter 4 that the data collection of this study took place in two different stages: Phase 1 and Phase 2. In Phase 1, whose aim was to identify a variety of conceptions from an initial group of teachers in order to select case studies, data were collected through the use of two questionnaires and an interview that took place in the following sequence:

Questionnaire 1,
Interview 1 (Questionnaire 1 follow-up interview) and
Questionnaire 2

Thirteen teachers, who in 2002 were teaching Grade 8-algebra in the different settings of six schools in Bogotá, participated in the data collection in this phase. In Phase 2, with the participation of nine teachers, selected from the initial group of thirteen, data were collected through classroom observation, follow up interviews, examination of curricular materials and a Focus Group session (see Section 4.3, page 79).

In this chapter the findings of Phase 1 are presented in four main sections. Section 5.2 presents the profile of the participating teachers. The information related to the teachers' conceptions of Grade-8 algebra, collected through Questionnaire 1 and the follow-up interview are outlined in Section 5.3. Section 5.4 presents the findings of the survey on beliefs about Grade 8-algebra and its teaching that were elicited in Section B of Questionnaire 1. The findings of Questionnaire 2 are the focus of Section 5.5. The chapter ends in Section 5.6 with a summary of the findings of Phase 1, and the categorisations of the teachers' conceptions of beginning algebra which varied along a continuum from instrumental to problem solving conceptions.

5.2 Profile of the teachers in the study

From the thirteen teachers who participated in Phase 1 of the study, ten were teaching in state schools and three in private schools in Bogotá. All teachers held a Bachelor of Education degree (for secondary) with a major either in mathematics or in physics. Their teaching experience, which for some included teaching in primary, ranged from two years to thirty-three years, with a maximum of twenty-eight years experience in teaching mathematics in secondary school. Their teaching experience in Grade 8-algebra ranged from one to twenty five years. A summary of data related to the teachers' qualifications and teaching experience is shown in Table 5.1.

Table 5.1 Profile of the study's participating teachers

Teacher	No. of years teaching mathematics in		Total No. of years teaching	Secondary education and basic teaching qualifications		Other qualifications	Participation in Prof. Develop. Programs based on own teaching	Working at (School type)
	G8	Second.		Secondary	University			
Maria	25	28	28	Classic B*	Bachelor of Education (Mathematics & Physics)	Dip in Computing for education	Yes	State S (S)
Gladys	24	26	35	Normal S. Diploma + Classic B.	Bachelor of Education (Physics & Mathematics)	Dip in Mathematics Education	No	S
Juan	18	23	33 (10 p)	Normal S. Diploma	Bachelor of Education (Maths and Physics)	Civil Engineer	No	S
Nacho	16	22	22	Classic B	Bachelor of Education (Maths and Physics)	Dip. in Information T. for teaching	No	S
Loren+	13	25	26 (1 p)	Classic B	Bachelor Education (Maths and Physics)	Civil Engineer	No	S
Alfi	14	15	25 (10 p)	Normal S. Diploma	Bachelor of Education (Physics & Maths)	Doing a *Megatronics* Engineering course	No	S
Mario	10	17	17 (5 p)	Classic B	Bachelor of Education (Physics & Maths)	Dip. In Computing for teaching	No	S
Nora	7	17	17	Classic B	Bachelor of Education (Maths and Physics)	Dip. in Mathematics Education	Yes	S
José	12	13	16 (3 p)	Classic B	Bachelor of Education (Psychology of Education)	Four semesters studying Systems Engineering	No	Private (P)
Stella	6	14	14	Classic B	Bachelor of Education (Maths and Physics)		Yes	S
Luis+	5	8	8	Classic B	Bachelor of Education (Maths and Physics)	Half way through a Masters in Evaluation in education	No	S
Alex	3	4	4	Classic B	Bachelor of Education (Physics & Maths)		No	P
Pablo	1	2		Classic B	Bachelor of Education (Maths and I. T.)		No	P

p: The letter p indicates teaching experience in the' primary' school. + Loren and Luis had been teaching mathematics at University levels

Classic B* (Classic Baccalaureate): A 6-year cycle of secondary education required for higher education at universities.

Normal S. Diploma: A 7-year cycle of education for primary school teachers, where the two last years are dedicated to education for teaching in primary. At present, primary school teachers are expected to hold a Bachelor of Education in primary. A Bachelor of Education (for secondary) normally has a major subject and a subsidiary subject.

5.3 The initial group of teachers' conceptions of beginning algebra

I would like to start this section by reminding the reader that although the focus of the study was the teachers' conceptions of their teaching of algebra in Grade 8 —as according to the PROMECA teachers (see page 19) and to the focus of school mathematics in the Colombia educational system, described on page 9, 'algebra starts in Grade 8'— the goal of the data collection was to identify the teachers' conceptions of beginning algebra (i.e., *when* and *how* algebra work begins in school mathematics, as discussed on page 19). The data collection in Phase 1 focussed on four basic aspects of the teaching of beginning algebra. Those aspects of teaching are:

- The *why* of (Grade 8) algebra (i.e., the purpose)
- The *what* (i.e., the mathematical content to be taught)
- The *how* (i.e., the teaching method and the assessment patterns)

As explained in Chapter 3 (Subsection 3.2.1.2), Questionnaire 1 was devised to collect data on the teachers' conceptions of the *why* —the purpose— of the teaching of school algebra, and the *how* —the teaching-learning styles and the assessment of pupils. The teachers' conceptions of the *what* —the content to be taught— as well as further explanations for their answers in the questionnaire were explored at Interview 1.

To facilitate the reading and contrasting of the teachers' responses in the questionnaire and the interview, regarding each of the aspects listed above, this section is divided into three main subsections. Subsection 5.3.1 presents data collected through Questionnaire 1 and Interview 1 in relation to the *purpose* of the teaching of Grade 8-algebra. The data collected at Interview 1 in relation to the *content* to be taught in Grade 8 is the focus of Subsection 5.3.2. Note that Questionnaire 1 did not include questions regarding the content of Grade 8-algebra. Subsection 5.3.3 presents the data related to the *how* of the teaching of Grade 8-algebra. This subsection includes data on 'Teaching style', 'Pupils' work' and 'assessment practices' which were collected through Questionnaire 1 and Interview 1. A copy of Questionnaire 1 can be seen in Appendix 3.1, and basic guidelines and questions on which the Interview 1 focused were presented in Chapter 3 (p. 69) and in Chapter 4 (p. 82).

5.3.1 The *why* of the teaching of algebra in Grade 8

Data collected through Questionnaire 1

This Subsection as well as Subsection 5.3.3 contains data collected through Sections A and C of Questionnaire 1, as one of the intentions of Section C was to crosscheck data collected in Section A.

Data from Section A

In their responses to Question A1, most teachers chose two of the given descriptors to show their first and second priorities in relation to the purpose of the teaching of algebra in school, as can be seen in Table 5.2.

Table 5.2 Summary of the *group* of teachers' responses to Question A1: The purpose of the teaching of school algebra

	No of teachers for whom a given descriptor was a	
	1st choice	*2nd choice*
The teaching/learning of algebra is important because:		
knowledge of algebra is needed in higher levels of mathematics	6	2
algebraic knowledge provides individuals with opportunities to develop the critical thinking needed by every citizen.	4	4
algebra represents an important tool for solving real world problems, needed by every individual.	3	2
the use of technology poses increasing demands on algebraic knowledge for every individual.	0	4
the teaching of algebra is important, but not for all pupils because knowledge of algebra is not needed by every citizen	0	0
algebra provides the intellectual challenge pupils like in mathematics.	0	1
Total number of respondents	13	13

In each question in Section A, space was provided for the teachers to add their own ideas and preferences. Only one teacher used the space provided. She added as one of her first priorities: "Algebra is important because it develops pupils' abstraction capacities".

Data from Section C

The teachers' responses to Question C1, where in an imagined situation a pupil asked the teacher 'why they have to study algebra', were as follows:

Six teachers: because algebra develops logical and formal thinking.
Four teachers: because of the need for algebra knowledge when solving problems.
Two teachers: because we have to; that is the program of study for this year.
One teacher: because algebra complements arithmetical knowledge.

All the teachers made it clear that the responses they would give to pupils in relation to the purpose of their learning of algebra had nothing to do with any contextual factors of the school or the learners as those would be their answer in whatever school context or situation they were teaching. The general idea presented by these responses was that the logical, abstract and formal thinking required in algebra is necessary for next levels of mathematics and for solving word problems.

Data collected through Interview 1

The explanations given by the teachers, at Interview 1, for their choices when responding to the questions on the purpose of the teaching of algebra in school, in Questionnaire 1, are presented in the following paragraphs.

The six teachers who chose 'algebra is important because knowledge of algebra is needed in higher levels on mathematics' (See Table 5.2), as their first reason for the teaching of algebra, explained their choices as follows:

i) Five teachers: algebra develops the logical and abstract thinking needed in the following levels of mathematics. They referred to the thinking needed to solve problems in Grade 9, in trigonometry in Grade 10, and in work in Physics in Grades 10 and 11.

ii) The other teacher said that "since mathematics is a tool for understanding functions and life situations and objects, algebraic knowledge is crucial to achieve this understanding".

From the four teachers who chose 'algebra is important for the development of critical thinking':

i) Three clarified that by "critical" they referred to the analytical thinking skills needed for problem solving.

ii) The other teacher said, "Critical thinking is logical thinking". An example of this is "the thinking needed in solving an equation is logical thinking".

From the three teachers who chose 'algebra represents an important tool for solving real world problems':

i) Two justified their choice with the argument that algebra represents useful knowledge for the solution of mathematical problems. One teacher asserted that "the mathematical problems you solve in higher levels of mathematics can be real world problems..." (Luis)

ii) The other teacher: claimed that "algebra knowledge is important knowledge for understanding real-life situations..." He further explained that

> pupils need to learn algebra because it is knowledge useful for problem solving... If when studying algebra pupils don't find connections with other mathematical concepts and with the world, then it has no meaning for them. One finds no reason to study it... (Pablo)

The descriptor 'The teaching of algebra is important, but not for all pupils because knowledge of algebra is not needed by every citizen' was found irrelevant to all teachers because, for all them, 'algebra is for all pupils'. The great majority of teachers, except Pablo, found the descriptor 'The study of algebra is important because algebra provides the intellectual challenge pupils like of mathematics' to be irrelevant because for them "pupils, in general, do not like algebra".

5.3.2 The *what* of the teaching of algebra in Grade 8

As pointed out in the introduction of Section 5.3, data about the mathematical content to be taught in Grade 8 was collected at the follow-up interview (Interview 1). The teachers' descriptions of the work they organised in their lessons in Grade 8, the content emphasised and the sequence of topics followed showed a common teaching-learning sequence that can be summarised as follows: once the units on Rational, Irrational and Real numbers has been covered, algebra work starts with the presentation of algebraic expressions and the definition of algebraic expression. After focusing on parts of an algebraic expression (i.e., coefficient and literal part), types of algebraic expressions and ordering algebraic expressions, work moves on to collecting like terms and operating with expressions. The sequence continues with 'Factorisation, 'Linear equations and systems of equations' and then the unit on word problems.

Three teachers reported that they placed emphasis on the learning of formal "definitions", for example, "the definition of rational number" or "of algebraic expression", as they were 'presented in textbooks'. They also noted that questions like "What is an algebraic expression?" What is a rational number?" were given to pupils in quizzes or in tests. Three other teachers said that they liked to include tasks of application of algorithms at the end of each topic. They mentioned areas and perimeters for addition and multiplication of expressions. One teacher reported including in his teaching a weekly activity of tasks connected to the topics being studied in algebra and geometry, which was called "The mathematical calendar". He explained: "In this activity the pupils work in groups, discussing their work and some of their work is discussed with the whole class".

5.3.3 The *how* of the teaching of algebra in Grade 8

5.3.3.1 Teaching style

Data collected through Questionnaire 1

Data from Section A

As can be seen in Questionnaire 1 in Appendix 3.1, the questions related to 'Teaching style' (as well as to 'Pupils' work' and 'Assessment practices') were organised so that respondents made a distinction between their preferred style and what they actually do in their classrooms (e.g., Question A2 was about preferred teaching style and question A3 about actual teaching style). The teachers' rankings of descriptors belonging to pairs of questions (e.g., A2 and A3) were organised in tables, and mean rankings for each question were calculated. The highest mean rankings were taken to be *the group* of respondents' first priorities for the different aspects addressed by the particular questions in the questionnaire. Table 5.3 presents a summary of 'the group' of teachers' priorities for 'preferred' and 'actual' practices for 'Teaching style'.

Table 5.3 Summary of the *group* of teachers' responses to Questions A2 and A3 on 'Teaching style'

Priority	*Preferred practices*	*Priority*	*Actual practices*
1	Giving clear explanations of procedures to follow in assigned tasks	1	Giving clear explanations of procedures to follow in assigned tasks
2	Developing communication skills	2	Developing communication skills
3	Providing work that allows for differentiation	3	Promoting connections between concepts
4	Promoting connections between concepts	4	Providing work that allows differentiation
5	Organising problem-based activities to promote discussion & systematic work	5	Organising problem-based activities to promote discussion & systematic work
6	Assigning repetitive exercises	6	Assigning repetitive exercises
7	Testing pupils at the end of each activity or topic	7	Testing pupils at the end of each activity or topic

As we can see in Table 5.3, the teachers' responses to Questions A2 and A3 showed very similar rankings for preferred and actual practices, and a strong agreement of the respondents on priorities, with the descriptor 'Giving clear explanations of procedures to follow", considered typical of an instrumentalist approach, as the group's first priority. The other two descriptors portraying a traditional, instrumentalist approach (i.e., 'Assigning repetitive exercises and 'Testing pupils at the end of each activity') were, however, ranked as the least relevant of the seven given descriptors. The data from Section C corroborates the group's top priority of 'giving explanations' and telling as will be seen in the following.

Data from Section C

In Questions C3, C4 and C5 the teachers were presented with contextualised teaching-learning situations where pupils showed misconceptions. They were asked to explain what they thought of a pupil's answer to specific questions, and how they would respond to the pupil. Figure 5.1 shows Question C3.

The most common answers to part a) of Questions C3, C4 and C5, were that "pupils don't know the specific concepts" at hand. The most common responses to part b) were of the style: "I would show them how to do it" or "I would explain the topic again..." For example, for Question C3, shown in Figure 5.1, seven teachers answered: 'the pupils don't know what like terms are' or 'they don't understand how to simplify like terms'. From these seven teachers, six answered that they would have to explain the topic again.

C3. Towards the end of the academic year, a teacher asks his Grade eight pupils to find the area of a rectangle whose sides are 5 and $2 + e$. Many pupils, in a group of 29, answered the following:

$$A = 5\,(2 + e) = 10 + 5e = 15e$$
$$A = 15e$$

a) What do you think of this answer?.

....

....

b) How would you respond to those pupils?

....

....

C4. Imagine you ask your Grade 9 pupils to write an equation, using N for the number of nurses and D for the number of doctors, for the statement: *In Central Hospital there are five times as many nurses as doctors*:

Several pupils write as an answer: $5N = D$

a) What do you think of this answer?.

....

b) How would you respond to those pupils?

....

Figure 5.1. Question C3 – An item from Section C of Questionnaire 1

In Question C4, five teachers would tell the pupils that they had to revise the topic of proportionality again, and two teachers, that the pupils needed more practice. Summaries of the teachers' responses to these questions can be found in Appendix 5.1

Data collected through Interview 1

Asking teachers to explain their reasons for their rankings and what the given descriptors meant for them provided information that clarified the initial picture of their priorities in relation to the aspects addressed in each question. In the next paragraphs a small account of the meanings the teachers assigned to some descriptors is presented.

This account is followed by their explanations for some of their choices in each of the questions of Section A of Questionnaire 1.

The teachers' assigned meanings to given descriptors

Certain descriptors had specific meanings for different teachers. Nine teachers associated 'Open problems' with the set of word problems corresponding to the last units of their program of study (i.e., the problems of the "Linear equations and Simultaneous equations" units set in textbooks) or with specific tasks to apply learnt algorithms of procedure (e.g., find the area of a rectangle whose sides are $5 + a$ and $x + y$).

All teachers took 'Providing work that allows differentiation' to mean "decreasing speed of work for the whole group". 'Promoting connections between concepts' meant "linking yesterday's work with today's work" for most, but four teachers referred to connections between some algebraic work and some geometric concepts.

For nine teachers, 'Developing communication skills' meant "having pupils verbalise the procedures that they are applying"; the other four teachers referred to the capacity to "explain and justify what they did". Problem-based activities' was associated by six teachers with what they called "*las guías*" which was, normally, a list of questions and exercises (or closed word problems, if this was to take place at the end of the school year) "to revise" a certain topic or set of topics studied. According to the great majority, this work was done during the lessons with pupils working in pairs "so that they discuss about the work they are doing".

The teachers associated 'assigning repetitive exercises' with asking the pupils to repeat an exercise or to apply a rule to a set of exercises that have the same "identical" structure (e.g., find the numerical value of the expressions 2x + y; 2a + b; 2b + y, when a = 2, b = ...), and not, for example, with applying the same algorithm to a set of given exercises. For this reason the descriptor "assigning repetitive exercises" given in Questions A2 and A3 (about teaching style) had, for the teachers, a different meaning from their meaning of "algorithm routine practice", given in Questions A4 and A5 (about pupils' work).

The teachers' explanations for their rankings of preferred and actual 'Teaching styles'

Table 5.4 shows the top or "Number 1" teaching style priority each teacher indicated in Questionnaire 1. The great majority of teachers did not provide written reasons to explain any differences in their rankings of given descriptors, but they gave their reasons at Interview 1. Some of the explanations they gave either for the differences in rankings or for their top and last choices in these specific questions are presented in the following paragraphs.

Table 5.4 Individual teachers' top 'Teaching style' priorities

	Preferred practices	*Actual practices*
Gladys	Giving clear explanations of rules to follow	Giving clear explanations of rules to follow
Nacho	"	"
Juan	"	"
Loren	"	"
Alfi	"	"
Mario	"	"
José	"	"
Stella	Promoting the developing communication skills	"
María	"	"
Alex	"	Promoting the developing communication skills
Luis	Promoting connections between mathematical concepts	Promoting connections between mathematical concepts
Pablo	"	"
Nora	Organising problem-based activities	Giving clear explanations of rules to follow

'Giving clear explanations of procedures to follow in assigned tasks' as the group of teachers' top teaching style priority. The seven teachers who ranked 'Giving clear explanations of procedures to follow in assigned tasks' as the top or first priority both for 'preferred' and 'actual' practices justified their choices, mostly by their beliefs about the way pupils learn algebra, as can be seen in the following quotes:

> They follow a kind of sequence of steps in their learning: first they learn the algorithms, how to add expressions for example, and then how to solve equations.
>
> The problem is that they don't like to do the exercises where they have to apply the algorithms [i.e., the word problems] (Alfi).

Another teacher said:

> First pupils need to learn what an algebraic expression is, and then how to operate with algebraic expressions. It is after they have seen first grade equations that you start thinking about the problems of application. (Loren)

There were three teachers for whom 'Promoting the developing communication skills' was their preferred *top* priority, but for two of these teachers, 'Giving clear explanations of procedures to follow in assigned tasks', was their top actual priority. These two teachers explained that 'the ideal would be that the pupils engaged in the given tasks and then they themselves presented their work to the whole class for discussion', but that "that [was] just an ideal". They explained:

> I have to explain everything because the pupils do not have the basic mathematical knowledge they are supposed to have when they start in Grade 8. (María)

> You give them questions or tasks so that they work in groups and discuss and, then, come up with something to present to the whole class, but it is a waste of time. (...). Three or four pupils produce something. Many don't have the pre-requisites, the basic concepts needed in Grade 8. You can be there for two hours trying to help them in the task and nothing really happens, and time is short! ... (Stella)

The only teacher whose top preferred priority was 'Organising problem-based activities for the pupils to work in small groups' but who chose 'Giving clear explanations of procedures to follow in assigned tasks' as the top for her actual practice, explained why her "ideal that pupils participated in constructing their mathematical ideas" was not possible:

> Giving situations to pupils to work on, so that they produce their own ideas is very difficult... The pupils struggle and struggle, and I end up telling and explaining everything to get them out of the struggle... besides that, there is no time! ... Any way, Grade 8-algebra is about routine practice; ...it is learning algorithms to apply them in Grade 9 problems... (Nora)

There were two teachers whose priority 'Number 1' for preferred and actual practices was 'Promoting connections between mathematical concepts'. One of them explained that the connections between algebraic expressions could be seen for example in the kind of work the student teachers were doing with his Grade 8 group:

> ... it is like a kind of treatment to help pupils use geometry to see algebraic expressions ... I can not give you details about that work because it is the teaching practice coordinator who is in charge of that work...(Luis)

The other teacher argued that his central aim for the teaching of Grade 8 of "helping pupils see the functionality of what they are learning" could not be achieved by following the textbook approach because

> textbooks bring just a list of exercises after a definition... To give the pupils a list of exercises telling them 'follow this rule' is to teach mathematics as dead mathematics. ... Pupils need to be asking themselves 'why am I doing this?' why am I adding these polynomials? (Pablo)

'Testing pupils at the end of each activity or topic' as the group of teachers' last teaching style choice. Asking why 'testing pupils at the end of each topic' was the least relevant option for preferred and actual practices in questions A2 and A3, took the conversation, in most interviews, into some aspects of assessment practices. The majority of teachers explained that they do a test to assess the pupils at the end of each term or sometimes at the end of each topic. Most of them said that they do frequent quizzes to assess pupils because they need to give grades each term and they need to have support for the given grades. They said that their preferred way of assessing pupils was by examining their folders or notebooks to see the sequence of work they had done in class and as homework, and to see if they had done the corrections of their work.

Eight teachers declared that examining the pupils' folders or notebooks was something that became an ideal because (using the words of a teacher to describe the overall idea given by the eight teachers) "pupils don't have habits of work as students" (Stella). "...their motivation for the study of algebra is very low" (Loren). "Many don't do any homework, and some don't even have a notebook because they lost them!" (Nora). So in order "to push the pupils to do their work, they are given frequent quizzes (Nacho).

Three teachers added to these explanations that they need to have written evidence of their pupils' difficulties to be able to support the grades they are given. The other three teachers whose first preference for assessing pupils was not checking their folders but asking them to come to the board to do exercises from the homework, complained about the pupils, showing dissatisfaction because 'the pupils don't care if they fail, as according to the new government policy, 95% of the pupils in a year group has to be promoted!' (Nacho, Loren, Juan).

Another teacher said that "quizzes [were] not the best way to see if pupils understand a topic because they make mistakes in the quiz and yet you see them working very well during the lessons..." (Pablo). There was a general consensus that the initial part of the lessons were usually dedicated to correcting the homework, as Stella argued, "in order to see if pupils had understood the topic, and decide what to do next".

Having gained some insight into what teachers meant by the given descriptors of Questionnaire 1, and their teaching priorities, as expressed in their *own* words at Interview 1, a new list of teaching style descriptions and a different table of preferred and actual teaching priorities from the *group* emerged. Table 5.5 shows the priorities of the 'group' of teachers as expressed in their own words.

Table 5.5 Summary of the *group* of teachers' 'Teaching style' priorities, as described in their *own* words at Interview 1

Priority	*Preferred practices*	*Priority*	*Actual practices*
1	Giving clear explanations of procedures to follow in assigned tasks	1	Giving clear explanations of procedures to follow in assigned tasks
2	Assigning algorithm-routine practice	2	Assigning algorithm-routine practice
3	Encouraging pupils to work together and discuss their work	3	Correcting exercises at the board
4	Correcting exercises at the board	4	Giving frequent quizzes
5	Examining pupils' folders and note books to assess pupils	5	Encouraging pupils to work together and discuss their work
6	Solving closed word problems where work with algebraic expression is applied	6	Solving closed word problems where work with algebraic expression is applied
7	Giving frequent quizzes	7	Examining pupils' folders and note books to assess pupils
8	Working on tasks and organised activities that promote establishment of connections between concepts	8	Working on tasks and organised activities that promote establishment of connections between concepts

Note that the numbers indicating the group of teachers' *priorities* in this table have been established by identifying the number of teachers that put forward a specific descriptor to describe their preferred and their actual practices. So, for example, while for 7 teachers, 'Giving clear explanations of procedures to follow in assigned tasks' was their top priority or *Priority 1* for preferred practices, 10 teachers ranked this descriptor as their 'Priority 1' for actual practices. *Priority 8* means that 2 teachers used the descriptor 'Working on tasks and organised activities that promote establishment of connections between concepts' for preferred practices, but only 1 teacher used this descriptor to portray his actual practice.

5.3.3.2 Pupils' work

Data collected through Questionnaire 1

Data from Section A

The data related to the teachers' priorities for their pupils' work, which is presented in Table 5.6, showed some interesting contrasts not only between their preferred and actual practices, but also in relation to their answers about teaching styles (see Table 5.3). While the group of teachers' first preference for pupils' work was 'open problems', their first preferred teaching style was 'to give clear explanations of procedures to follow in assigned tasks'.

Table 5.6 Summary of the *group* of teachers' responses to Questions A4 and A5 on 'Pupils' work'

Priority	*Preferred practices*	*Priority*	*Actual practices*
1	Open problems	1	Algorithm-routine practice
2	Discussion & systematic work	2	Correction of exercises at the board
3	Correction of exercises at the board	3	Discussion & systematic work
4	Algorithm-routine practice	4	Open problems
5	Formulae construction	5	Closed problems
6	Closed problems	6	Formulae construction
7	Use of calculators	7	Use of calculators

Equally interesting were their rankings of 'Algorithm-routine practice' as a fourth preferred activity for pupils, while in the previous tables belonging to 'teaching style', 'Assignment of repetitive exercises' was considered within the last two options. Why would the teachers top priority for actual practice be 'algorithm-routine practice' when their preferred type of work for the pupils was 'open problems'? The data from Section C also point to their top choices of doing 'algorithm-routine practice'. The teachers' explanations for differences in their rankings were provided at Interview 1 (see Table 5.7).

Data from Section C

In Question C2, where two pupils asked the teacher if they could work together, nine teachers responded positively to the pupils' request, and added that interaction between pupils is beneficial for their learning. The other four teachers stated that they would answer to the pupils that they needed to work individually because that's how they would really learn.

As described previously, the most common response to pupils in Questions C3, C4 and C5 was that "they [the pupils] had to revise the concepts and do more exercises". Two teachers, however, answered that they would ask the pupils to give values to the letter and check what happens, and one teacher would first ask the pupils to explain to him what they did in order to identify how he had to respond to them.

Data collected through Interview 1

We saw in Table 5.3 that the great majority of teachers' top preferred 'teaching styles' as put forward in their responses to Questionnaire 1 did not differ from what they said (at Interview 1) they actually did in their teaching. However, their responses regarding the 'pupils' work' showed some differences between their preferred and their actual practices. Table 5.7 shows a summary of the teachers' responses to questions related to the pupils work (in the Questionnaire) and the reasons for the differences in the rankings they provided at Interview 1.

The teachers' top priorities for *preferred* and *actual* practices divided them into three groups. One big group whose top priority for 'pupils' work' was 'Open problems', but for some of this group their actual practice differed from 'Open problems' and, for some it did not differ (see Groups 1 & 2 in Table 5.7). These teachers provided reasons for the differences in rankings, and when there was no difference between rankings, the teachers made specific comments and gave explanations about the results of their teaching, as they normally felt that their pupils' performance was not good.

Table 5.7 Individual teachers' top priorities for "Pupils work'

	Preferred practices	*Actual practices*	
Group 1			Reasons for differences
María	Open problems	Algorithm-routine practice	Pupils' motivation
		"	Pupils' preferences &
Alfi	"		My knowledge
Nora	"	"	Time constraints
Stella	"	"	Pupils' motivation
Luis	"	"	My knowledge for teaching
			Teachers' specific explanations
José	Open problems	Open problems	My mathematics knowledge
Mario	"	"	Time constraints for teaching
Alvaro	"	"	Pupils' self concept
Group 2			
Gladys	Correction of exercises at the board	Correction of exercises at the board	Pupils' abilities
Juan	"	"	Pupils ways of learning
Nacho	"	"	Pupils ways of learning
Mario	"	"	Pupils ways of learning
Stella*	"	"	Pupils' motivation
Group 3			
Loren	Discussion &systematic work	Discussion &systematic work	Pupils' motivation
Pablo	Discussion &systematic work	Discussion &systematic work	My knowledge for teaching

*Respondents had the opportunity to give equal rankings to given descriptors

Another big group was comprised by the teachers for whom "Correction of exercises at the board' represented their top preferred and actual practices. These teachers also talked about the results of their teaching practices, naming specific factors that accounted for the situation. The third (smaller) group was represented by two teachers (Loren and Pablo) whose top priorities for the pupils' work was 'Discussion and systematic work', but gave contrasting explanations for what they thought of the results of their teaching as can be seen in Table 5.7.

The teachers were asked to give explanations for their top and their last priorities regarding pupils' work. A summary of their explanations is provided in the following paragraphs.

'Pupils working on open problems' as the group of teachers' top priority. The explanations from the eight teachers who ranked 'pupils working on open problems' as their first preferred practice[1] can be summarised in the following way: their goal for Grade-8 algebra was to see their pupil not just operating with given algebraic expressions but also being able to apply the algorithms learnt in tasks or exercises. These tasks were called by most teachers "Open tasks" and by some other teachers "Open problems". One teacher said that question C3 of the questionnaire (see Figure 5.1) was an example of "Open tasks". For other teachers "Open problems" were the closed word problems of application for the units on linear equations and simultaneous equations. They pointed out that a major difficulty in their teaching of Grade 8 was the fact that they had to spend most of the academic year in "operations with polynomials and solving equations", and very often did not manage to see the word problems of the last units of the program of study.

> Planning is thinking of the ideal because I cannot do what I plan. For example, with this group of pupils, I will just manage to start the topic of equations. We are stuck all year in operations with polynomials *Special products, special quotients* and factorisation, all because the pupils don't wish to learn algebra. They don't like algebra! (María)

> I'll like to see all my pupils working in the problems, but the majority just manage to learn some of the cases of factorisation and not all can handle isolating the variable. They move on to Grade 9 and they (the teachers of Grade 9) will say that I didn't do my job (i.e., I didn't teach). (Stella)

'The use of calculators' as the group of teachers' last priority. There was only one teacher (Pablo) who thought that work with calculators was useful for Grade-8 work.

[1] Note that Stella and Mario ranked two of their given options as their 'Number 1' priorities.

The rest said that they did not really need calculators to teach Grade-8 algebra. Three teachers said that they would not allow the pupils to use calculators because some pupils didn't even know their timetables. Table 5.8 shows the group of teachers' priorities in relation to the type of classroom work and activities for their pupils as described at Interview 1.

Table 5.8 Summary of the *group* of teachers' priorities for 'Pupils' work' according to data collected at Interview 1

Priority	*Preferred practices*	*Priority*	*Actual practices*
1	Solving tasks and word problems to apply knowledge of algebraic expressions	1	Algorithm-routine practice
2	Correction of homework at the board	2	Correction of homework at the board
3	Algorithm-routine practice	3	Interacting with their peers
4	Interacting with their peers	4	Solving tasks to apply knowledge of algebraic expressions
5	Use of calculators*	5	Use of calculators

*The use of calculators was considered an important part of pupils' work only by one teacher (Pablo)

5.3.3.3 Assessment practices

A summary of the teachers' responses to questions A6 and A7 on assessment practices is shown in Table 5.9. In their responses to these questions from Questionnaire 1, the teachers did not provide any reasons for the differences in rankings as requested; however, they provided explanations for these answers at Interview 1 as will be seen in the following paragraphs.

Table 5.9 Summary of the *group* of teachers' responses to Questions A6 and A7 on 'Assessment practices'

Priority	*Preferred practices*	*Priority*	*Actual practices*
1	Pupils' folders	1	Frequent quizzes
2	Diagnostic assessment	2	Diagnostic assessment
3	Frequent quizzes	3	Pupils' folders
4	Pupils' self assessment	4	Daily homework
5	Daily homework	5	Pupils' self assessment
6	Oral questions	6	Oral questions

Data collected through Interview 1

It was already explained in Subsection 5.3.3.1 (see subheading: *'Testing pupils at the end of each activity or topic'*), why while the group of teachers' top 'preferred' form of assessment according to their answers to Questionnaire 1 was 'Pupils' folders', their top 'actual' assessment practice was 'Frequent quizzes'. In relation to 'Diagnostic assessment', as the descriptor coming second in the list of priorities of the group, eight teachers explained at interview that what they did (i.e., quizzes) was a diagnostic form of assessment because the intention was to see if the pupils had learnt the topic being taught. However, all the eight asserted that the pupils were given marks for every quiz. One teacher said that he gave frequent quizzes to his pupils because he needed to know where his pupils were. However, his explanations of what questions he asked in the quizzes and what he did with the results was very much in line with that of the other seven teachers who explained that when pupils made mistakes, for example, when collecting like terms or solving an equation, it was due to the fact that the pupils did not know the rules to be applied or to lack of practice. To the question of what they did if the results showed that pupils did not know the rules, these teachers declared that they would give the pupils more exercises of the same type and that they had to give marks for every quiz given to the pupils. Two teachers explained that if pupils were not given marks they would not do any work at all.

It became clear at Interview 1 that the second most common form of assessment both as preferred and actual practices was "pupils' correcting homework at the board" which the majority of teachers considered another form of diagnostic assessment. Most teachers also named "pupils' participation in class and their responsibility" as aspects that were taken into account in the assessment pupils. The group of teachers' assessment priorities, as they described them at Interview 1, are shown in Table 5.10.

Table 5.10 Summary of the *group* of teachers' priorities for their 'Assessment practices' as described at Interview 1

Priority	*Preferred practices*	*Priority*	*Actual practices*
1	Pupils' folders or notebooks	1	Quizzes
2	Correction of homework at the board	2	Correction of homework at the board
3	Pupils' interest and participation in class	3	Pupils' interest and participation in class
4	Programmed tests	4	Programmed tests
5	Quizzes	5	Pupils' folders and notebooks
6	Pupils' diaries	6	Pupils' diaries

5.4 Data collected through the survey on teachers' beliefs

Section B of Questionnaire 1 was intended to measure teachers' strength of agreement with 20 items about specific aspects of the teaching of algebra and mathematics in general[2] where respondents had to rate their choices according to a 5 point Likert-type scale that ranged from 'strongly agree' to 'strongly disagree'. The summary of responses to the survey, presented in Tables 5.11, 5.12 and 5.13, is organised in terms of the teachers' general agreement (i.e., *Strongly agree* or *Agree* responses) or disagreement (i.e., *Strongly disagree* or *Disagree* responses) to the given statements. Statements categorised as related to a 'problem-solving' approach (Ernest, 1989), as opposed to an 'instrumentalist' one are marked with a (+) sign.

5.4.1 Beliefs about school algebra and its teaching

The summary of responses on the left hand side of Table 5.11 shows that for the statements related to school algebra (and mathematics in general), the great majority of

[2] e.g. algebra knowledge, teaching and learning algebra, pupils' abilities, values teaching in mathematics

the teachers adhered to statements associated with non instrumentalist conceptions of mathematics, supporting ideas of, for example, creative thinking and rejecting the separation between different areas of mathematics in its teaching. These data show great contrast with those of Section C of Questionnaire 1 and with those collected at Interview 1. A few teachers gave scores that suggest some inconsistency on their part.

Table 5.11 Summary of the *group* of teachers' responses to statement related to school algebra (and mathematics in general) and its teaching

Beliefs about school algebra (mathematics)	*SA or A*	*SD or D*	*N*	*Beliefs about school algebra (mathematics) teaching*	*SA or A*	*SD or D*	*N*
(+)B10. In mathematics you can be creative and construct your own mathematical ideas.	11	2		(+) B18. I have not found a textbook that is adequate for my pupils' needs and for what I want my pupils to learn in algebra. So I often produce some classroom materials.	10	2	1
(+)B12. Algebraic thinking should be promoted in the teaching of primary school mathematics.	13	0		(+) B6. I enjoy teaching algebra	12	1	
(-)B1. Algebra should be taught as a separate area in mathematics (i.e., from arithmetic and geometry content).	1	12		(-) B2. For the planning and teaching of Grade 8-algebra, I normally follow the sequencing given by the pupils' textbook.	2	11	
(-)B5. Mathematics involves mostly facts and procedures that have to be learned.	3	10		(-) B5. When I teach algebra I often feel unmotivated by the fact that many pupils don't understand the basic concepts and procedures	4	8	1

Key: A = Agree; SA = Strongly agree; D = Disagree; SD = Strongly disagree; N = Neutral

The teachers' responses related to the teaching of algebra, appearing on the right hand side of Table 5.11 show that a high number of teachers supported statements which are descriptors of a progressive approach in accordance with the beliefs put forward in the previous set of statements but, again, a few teachers showed contradictory responses. One teacher, however, chose the neutral position for two of the four given statements, (B5 and B18) related to beliefs about the adequacy of textbooks available and feelings of low motivation when pupils do not understand algebra concepts and procedures. It is interesting that this particular teacher, Pablo, at Interview 1 was the only teacher from the group who claimed to design classroom activities due to his unhappiness with textbook approaches, and did not identify the pupils' behaviour as the basic reasons for his actual practice.

5.4.2 Beliefs about the learning of school algebra and pupils' abilities

Table 5.12 presents a summary of the responses to the given statements related to school algebra and pupils' abilities. It is interesting to see that in the set of items related to the learning of algebra, while all thirteen teachers responded positively to statements which emphasised the need for understanding rather than practice to get the right answers, eight of them wanted pupils to do more practice if they were not getting the right answer, they gave contrasting scores in statement B3. The repetition aspect was emphasised by four of the respondents in statement B9.

The contrasting responses were, again, observed in the teachers' responses to items B17 and B20. Why do teachers on the one hand agree that the 'pupils will enjoy their classroom work even if it is not graded' (Statement B17) and, on the other hand, that 'giving rewards is a good strategy for getting pupils to complete their assignments (Statement B20). As will be seen in Chapter 6, the teachers expected the pupils to learn the routines to manipulate algebraic expressions because it was "useful for their lives". With this they meant, 'providing the requirements for the next level of algebra' or 'for good performance at the External Examination' and move up to university. In their responses to Statement B14 all the teachers reconfirmed their statement in Question A1 that 'algebra is for all pupils'.

Table 5.12 Summary of the *group* of teachers' responses to statements related to 'school algebra learning' and 'pupils' abilities'

Beliefs about the learning of school algebra	*SA or A*	*SD or D*	*N*	*Beliefs about pupils' abilities*	*SA or A*	*SD or D*	*N*
(+) B11. When pupils are having difficulties with the learning of algebra, I have to revise the teaching-learning situation and organise alternative classroom work that could suit the pupils better.	13	0		(+) B17. Pupils will enjoy and work hard in mathematics if they find classroom work meaningful and challenging, whether or not their work is graded.	12	0	1
(+) B13. More important than getting the right answer is pupils' understanding of the main concepts inherent in a problem.	13	0		(+) B14. All my pupils would be good at mathematics if they worked hard at it.	13	0	
(-) B3. Pupils who aren't getting the right answers need to practice on more problems.	8	4		(-). B8. Mathematical ability is something that remains relatively fixed throughout a person's life.	2	11	
(-) B9. If pupils fail to learn the algebra content taught during a specific term, they have to work by themselves and repeat the corresponding test.	4	6	3	(-) B20. Giving rewards is a good strategy for getting pupils to complete mathematics assignments.	7	4	2

5.4.3 The teachers' awareness of the values they teach

As Table 5.13 shows, all thirteen teachers were in agreement that their teaching of mathematics was concerned not only with mathematics content goals but also with broader educational aims (see Statement B19). I saw these responses as contradictory to the responses some of them gave to Statement B7. However, the teachers may not have seen any contradictions in these two responses for, as it is illustrated in next page, what the meant with the expression 'broad educational aims' was different to what it means to me. It will also be seen in Subsection 5.5.2 that the term 'a problem-solving teaching approach' had different meanings for different teachers.

Table 5.13 Summary of the *group* of teachers' responses to statements that aimed at identifying teachers' awareness of value- teaching in mathematics

Values in the teaching of mathematics	*SA or A*	*SD or D*	*N*
(+) B19. My teaching practice in mathematics is concerned with broad educational goals, and not just with mathematics content goals.	13	0	
(+) B4. Curriculum resources (e.g., curriculum guidelines, textbooks and other teaching materials, etc.) portray values.	7	3	3
(-) B16. The importance of mathematics is as a value free subject.	1	12	
(-) B7. The learning outcomes in my actual mathematics curriculum reflect cognitive learning concerns only.	5	8	

Bishop (1998a) contends that "teachers are rarely aware of teaching values either explicitly or implicitly, yet values teaching clearly does take place..." (p. 1). The data from Table 5.13 does not tell us much about the teachers' awareness of the values they teach as further exploration of their thinking was needed. In relation to their responses shown in Table 5.13, Item B19 was one of the few items whose responses from *all* the teachers were explored further at Interview 1. All the teachers were asked to explain what they referred to by the term 'broad educational goals and not just mathematical content goals'. For ten teachers, the term 'broad educational goals' was related to 'educating the whole child' which required them to 'pay attention to aspects related to the pupils' attitudes', especially to their responsibilities as students. 'Educating the whole child' for some teachers required that they payed attention "to whether the pupils do their duties as students", "to whether they do their homework and assignments". Some teachers said, "[we] need to see whether they pay attention and show interest in classroom work". Two teachers placed emphasis on "whether pupils show respect for their peers and their opinions". For one specific teacher, 'to be concerned with broad educational aims' meant "that they learn more mathematics, more content...". This teacher explained further:

> They need to understand that knowing mathematics is of great value for their future life. If they don't know Grade 8-algebra, they cannot move up to Grade 9, so at that point their opportunities in their lives are truncated, and that was it for them. ... (Nacho)

Although the teachers believed that their teaching was concerned with broad educational aims and not just with subject matter content, they seemed to be totally unaware of the values they were teaching. For example, the majority explained that by having the pupils verbalise the steps of the procedural routines followed in the exercises they were having opportunities to develop their communication skills. As will be seen in Chapter 6, for the majority of the case study teachers, the pupils' responsibilities were defined exclusively in terms their punctuality with homework. Some teachers believed that if pupils did not understand how to manipulate the given algebraic expressions it was because they had not done the homework. These teachers explained that they set speed competitions during the lessons (i.e., the first five pupils who do a given exercise are given a good mark) because that motivated the pupils to pay attention and learn.

The values promoted by the great majority were related to the value of "control" (Bishop 1988), emphasising impersonal learning. There was no evidence in their descriptions of what they did in their Grade 8 classroom that the values the teachers taught, implicitly or explicitly, were of openness, discussion, participation, conjecture and connection to real life situations, but of mathematics as knowledge created by external authorities, and knowledge useful for passing the External Examination at the end of secondary education.

5.5 Data collected through Questionnaire 2

As explained in Chapter 3, the collection of data through Questionnaire 2 had a double purpose. The first was to obtain complementary information about the teachers' conceptions of beginning algebra and its teaching. The second was to gain some insight into what 'a problem-solving approach' meant for the teachers, and to obtain some ideas about their interest in introducing a problem-solving approach in their teaching. Questionnaire 2 can be seen in Appendix 3.1.

5.5.1 The teachers' opinions of the teaching approaches of beginning algebra portrayed by Teachers A and B

Teachers A and B are the two teachers whose classroom approaches when introducing pupils to the concept of variable were described in Questionnaire 2. In this questionnaire, the initial group of teachers were asked to give their opinions (by ranking given statements) of specific aspects of the teaching approaches of Teachers A and B. They were also asked to explain what a problem-solving teaching approach meant for them, and whether or not they thought that they followed a problem-solving approach in their teaching of Grade 8. In the given descriptions, *Teacher A* was portrayed as emphasising an "instrumentalist" approach and Teacher B's emphasis was considered to be similar to the emphasis made in "problem-solving" (Ernest, 1989) teaching approach. A copy of Questionnaire 2 and of the general guidelines for the construction of the descriptions of the two teachers' classroom approaches can be seen in Appendix 3.1

5.5.1.1 The teachers' ratings of Teacher A's work

As shown in the summary of the teachers' responses to Section A Part 1 of Questionnaire 2 which was related to Teacher A (see Table 5.14), four out of the thirteen respondents gave positive opinions about the work and approach followed by Teacher A. Three teachers ranked Teacher A's work as "Good" because he/she emphasised the regular algorithm and formal language of mathematics. The other teacher ranked all aspects as "Excellent' because

> She/he follows the logical order while the other teacher gives opportunity to wander and give solutions even if they don't make sense (Nacho).

This teacher, however, ranked teacher B's work as Good in all aspects contradicting his written explanations in Question B1. He showed no willingness to explain further his answers in Section A of questionnaire; however, he offered comments about the inadequacy of teacher B's approach for the teaching of Grade 8-algebra. The rest of the teachers graded Teacher A's work as "Poor or "Just satisfactory". Most of these teachers stated that Teacher A's approach was mechanistic.

5.5.1.2 The teachers' ratings of Teacher B's work

Eleven teachers rated teacher B's work either as "Excellent" or "Good" in all questions of Section A-Part 2. Two teachers rated most of Teacher B's described aspects as "Just satisfactory" or "Poor" explaining that, on the whole, Teacher B's work was not good because she/he did not emphasise algorithm-routine or formal definitions of concepts.

Table 5.14 The teachers' answers to Questionnaire 2

	Ratings given to specific aspects of Teachers' A & B' work (Section A)		*Teachers conceptions of problem-solving approach (See Section B - Question B1)*	*What possibilities do you see for incorporating a P-S approach in your teaching of G8? (Question B2)*
	Teacher A	*Teacher B*		
Maria	Most 'Poor'	All 'Excellent'	When a problem situation is used to teach certain concepts, helping pupils create meaning.	It's not easy to develop the type of work Teacher B does.
Gladys	Even mix of 'Satisfactory' & 'Good'	Even mix of Poor & 'Good'	None of the teachers follow the P-S approach. P-S means giving a problem to find a specific answer	I have confirmed that pupils do not learn by giving them the problems.
Juan	Even mix of 'Poor' & 'Satisfactory'	All 'Good'	Both teachers use P-S, but Teacher B places more emphasis on it.	I do work like Teacher B because I do 'open market' when I introduce algebraic expressions
Nacho	Most 'Excellent' & a few 'Good'	All 'Good'	Teacher A is mechanistic. Teacher B gives opportunity for pupils to wander about even if …	NO DATA
Loren	Even Mix of 'Good' & 'Satisfactory'	Even mix of 'Good' & 'Excellent'	Teacher B follows the P-S approach because he uses a variety of work and strategies.	I use the two approaches
Alfi	Even mix of 'Poor' & 'Satisfactory'	All 'Good'	Teacher B emphasis the P-S approach	Very rarely I do this because I don't know how.
Mario	Even mix of 'Poor' & 'Satisfactory'	All 'Excellent'	Teacher B provides the basis for a P-S approach.	I am already following the two approaches.
Nora	'Very poor', 'Poor' & 'Satisfactory'	All 'Excellent'	Teacher B emphasis the P-S approach as he/she takes pupils to work on ideas.	I would like to follow the P-S approach in all levels of mathematics. … In Grade 8, especially, you cannot neglect the type of work that Teacher A does…
Stella	Most 'Good'	Even mix of Poor & 'Good'	Teacher A is good because s/he helps the pupils with the understanding of applications of procedures.	Teacher B's work is 'Poor'. It doesn't allow pupils to grasp concepts formally.
José	All 'Poor'	Even mix of 'Good' & 'Excellent'	Teacher A uses P-S at the arithmetical level; Teacher B at the intuitive level.	NO DATA
Luis	All 'Satisfactory'	All 'Good'	Teacher B emphasises P-S approach	I do what Teacher A does. Teacher B's work is difficult to do due to the pupils' knowledge & family context.
Alex	All 'Satisfactory'	All 'Good'	Teacher B emphasises a P-S approach.	I try to follow Teacher B's approach at times.
Pablo	'Poor' & 'Satisfactory'	Even mix of 'Good' & 'Excellent'	Teacher B emphasises P-S approach. I am trying to combine the two approaches, especially when introducing topics. …	What B does is what I would like to do… I am learning and I still have a long way to go.

Note: P-S: Problem-solving

Although, in their responses to Section B of the questionnaire, nine teachers stated that the work of Teacher B emphasised a problem-solving approach, only five of them (Luis, Pablo, María, Alfi and Nora) explained, as requested, why they thought Teacher B emphasised a problem–solving approach. They provided statements suggesting that Teacher B did the opposite of Teacher A because instead of telling and explaining the concepts and procedures to the class, Teacher B provided a problem-situation in order to help the pupils develop the concepts. One of these five teachers wrote, however, that a problem solving could not be used in Grade 8-algebra because

> I would like to follow the problem-solving approach in the mathematics of all grades. However, it is not easy due to diverse reasons, especially lack of time. In Grade 8-algebra especially, you cannot neglect the type of work that Teacher A does because some students require explanations and reinforcement work. Also they need to learn the mathematical language to be able to continue in their school work. (Nora)

Another teacher provided explanations for the difference in the two approaches as follows:

> Teacher B follows a problem solving approach because the teacher:
>
> • poses a contextual, real situation in the form of a problem,
>
> •promotes discussion and interaction between pupils, trying to direct the discussion toward answering the proposed questions,
>
> •considers the different possibilities that pupils see; after a process of socialisation they arrive at a consensual academic [i.e. mathematical] idea because an algorithm was constructed from the table of values. Also,
>
> •special software was used to complement and reinforce previous ideas worked by the pupils.
>
> •The activity has been designed so that it can be adapted and changed to fit particular contexts, rhythms of learning and unforseen situations. Assessment was designed not only to verify and look at results and to comply with a legal requirement [i.e., school reporting] but, also, to identify difficulties and achievement along the process.
>
> But I do what Teacher A does:
>
> •Terminal and summative assessment (to fulfil the legal requirements, also due to the particularities of the group of pupils —their pre- algebra concepts, discipline and family context).
>
> •Follow textbooks
>
> •Schema of classroom work as: Definitions, Examples, and Exercises.

Three teachers answered that they were already using a problem-solving approach. For these teachers, problem-solving meant adding word problems for application of the procedures that had been taught.

5.5.2 The teachers' interest in the incorporation of a problem-solving teaching approach in their teaching of Grade 8-algebra

From the eleven teachers who, in Section A of the questionnaire, rated Teacher B's work as 'Excellent' or 'Good', three stated, in Section B, that 'to include Teacher B's approach in their teaching was difficult'; for two of them, "because they were unfamiliar with it" (Maria & Alfi); and for the other teacher "because of the pupils' family context" (Luis). However, Alfi and Luis asserted that they would like to learn about a problem-solving teaching approach. Three teachers answered that they were already including a problem-solving approach in their teaching of Grade 8, and one teacher noted that "[he] would like to learn more about it".

> ...I am trying to combine the two approaches... What Teacher B does is what I would like to do... I am learning and I still have a long way to go. (Pablo)

Three teachers were not convinced that the pupils would learn Grade 8-algebra through a problem-solving approach. The other two teachers did not answer the corresponding questions.

5.6 Summarising the findings of Phase 1 and selecting the teachers to take part in Phase 2 of the study

The purpose of this phase of the data collection was to identify a variety of conceptions in order to study further the conceptions of a group selected to represent a range of conceptions. In order to classify the conceptions of the initial group of teachers, the data were categorised drawing on Ernest's (1989) categories of teachers' conceptions of the nature of mathematics, along a continuum that goes from 'Instrumentalist conceptions' to 'Problem-solving conceptions'. However, taking into account the indications of conceptions shown by the data in this particular study, I will refer to a continuum from 'Instrumentalist conceptions' to 'Non-instrumentalist conceptions'.

To assign a category to the data about the teachers' conceptions of beginning algebra (i.e., their conceptions of the *why*, the *what* and the *how* of beginning algebra), their responses to Questionnaire 1 were looked at together with their clarifications, in relation to these responses, obtained through Interview 1 and, when necessary, through Questionnaire 2. Further, the teachers' responses to specific questions, for example, the question of the *why* of the teaching of algebra in Questionnaire 1, cannot be interpreted in isolation from their responses to the *what* and the *how*, for these elements of the curriculum that operates in a teacher's act of teaching are dynamically interrelated. For example, in relation to a teacher's conception of the 'why' of the teaching of school algebra, Gladys, in Questionnaire 1 ranked as her 'Number 1' priority, the given option: "because algebra provides individuals with the opportunities to develop the critical thinking needed by every citizen". However, at Interview 1 this teacher explained that

> ...in the lessons the pupils have the opportunity to develop critical thinking because they are developing the logical thinking that is needed, for example, to solve a given equation; and this logical thinking prepares them form life ... if pupils develop this logical thinking they will be critical thinkers".

The data collected from Gladys in relation to the various aspects of her teaching through the three research instruments or activities used in Phase 1 need to be seen wholistically. In Questionnaire 2, Gladys responded that "Neither Teacher A nor Teacher B followe[d] a problem-solving approach because a problem to find the answer was not given to the pupils". This answer provides further insight into her conception of the 'why' of the teaching of beginning algebra, and therefore on what it means for her 'to be critical'.

The summaries of data collected during Phase 1, belonging to each of the thirteen teachers were put together in tables. The data were categorised according to the broad categories described in Appendix 3.1 in order to identify the teachers who would be selected for participation in Phase 2 of the study (see Appendix 5.1 for an example of the data-summary tables of individual teachers). The categorisations allowed the location of the teachers' conceptions along a continuum from 'Instrumentalist' to 'Non-instrumentalist' conceptions. Figure 5.2 shows how the categorisations of the data belonging to each teacher are placed along a continuum of conceptions.

	Instrumentalist conceptions				Non-instrumentalist conceptions
Why is it important to include algebra in the curriculum of basic education?		Nacho Stella, Alfi, Gladys, Juan, Loren Alex, Maria, , Nora, Jose	Mario Luis	Pablo	
What content do I teach as beginning algebra	Nacho	Gladys, Loren, Stella, Juan Alex, Maria, Nora, Jose, Alfi, Luis, Mario		Pablo	
How do I teach beginning algebra (what is my role and the role of the learner)?	Nacho	Gladys Juan, Stella	Maria, Alfi, Nora, Loren Jose, Alex, Mario, Luis	Pablo	
What aspects of the pupils' work do I assess, and how?	Nacho	Stella, Loren Gladys, Alex, Maria, Nora, Luis	Juan, Jose Mario, Alfi	Pablo	
What reasons do I identify to account for the results of my teaching?		Nacho, Nora, Mario, Juan, Stella, Loren, Gladys, Alex, Maria		Alfi, Luis	José Pablo
Am I giving any hints of my interest in other teaching approaches?	Nacho	Stella Gladys, Alex, Nora Loren, Maria, Juan, Mario	José , Alfi		Luis Pablo

Figure 5.2. An approximate [illegible] showing the location of the teachers' conceptions of beginning algebra and their own practices, based on data from Phase 1

5.6.1 Selecting the teachers to take part in Phase 2 of the study

All the thirteen teachers indicated their willingness to participate in the follow up study. However, during the mid-year break, Alfi learnt that he had to be on leave for the whole of August due to personal circumstances. The teachers to be followed in Phase 2 of the study were selected according to the following criteria:

1) Conceptions of beginning algebra
2) Type of reason given to explain their described teaching practices (i.e., internal or external), and attitudes to change
3) Teaching experience

Figure 5.2 shows an approximate location of the teachers' conceptions along a continuum from Instrumentalist to Non-instrumentalist conceptions that takes into account the two first criteria listed above. The categorisation of the teachers according to the third criterion, that is, the teachers' experience is presented in Table 5.15.

Table 5.15 Categorisation of the initial group of teachers according to their teaching experience

Categories	*Teachers*	*No. of years teaching*
Very experienced	María*, Gladys, Juan, Nacho, Loren, Alfi	More than 15
Experienced	Mario, Nora*, Stella*	Between 10 & 15
Not experienced-Not novice	Luis, José	Between 5 & 9
Novice	Alex, Pablo	Less than 5

* Teachers who had participated in classroom based Professional Development

Combining the categories of Figure 5.2 with those of Table 5.15 a new table of the combined categories was obtained as shown in Table 5.16.

Table 5.16 Table of categorisations of the initial group of teachers according to the three basic criteria

		Instrumentalist conceptions		Non-instrumentalist conceptions
Very experienced	Nacho	Juan Loren Gladys María*	Alfi	
Experienced		Mario Nora* Stella*	José	
Not experienced-Not novice			Luis	
Novice		Alex		Pablo

Table 5.16 shows that within the instrumentalist conceptions, three subgroups of teachers can be identified, according to the emphasis they placed on a fixed teaching approach, the reasons for the results of their teaching and the interest they professed to have on a different teaching approach. Alfi, José and Luis declared to need to learn more about the teaching of algebra and to be interested in a problem-solving approach.

On the basis of the categorisations shown in Table 5.16, the following eight teachers were selected as key representatives of the different conceptions identified. However, as will be explained in the next page, a reason arose for including another teacher (Loren) in the group of teachers to be studied further in Phase 2.

From the subgroup of 'very experienced':	Nacho, Juan and Alfi
From the subgroup of 'experienced':	Nora and José
From the subgroup of 'Not experienced-Not novice':	Luis
From the subgroup of novice teachers:	Alex and Pablo

Nacho, Juan and Alfi represented each a different subgroup of the 'very experienced' category. Nacho and Pablo were two key teachers to be selected as they showed the most contrasting conceptions. From the subgroup of 'experienced' teachers, Nora was selected because she had participated in classroom-based Professional Development programs. José represented the other subgroup of 'experienced' teachers. Luis was the only teacher representing the 'not experienced-not novice' category, indicating instrumental conceptions and interest in change. Alex, as a novice teacher, was also selected as he showed contrasting conceptions to those of Pablo.

In selecting the case study teachers, however, a factor that was mentioned in Chapter 3 needed to be taken into account. Some teachers wanted to participate along with at least another colleague from the school. This factor was recognised to be of advantage for comparing the influence of school factors on the teachers' conceptions of their practices. As Alfi (from Nora's school) could not participate in the follow-up study for reasons that were explained, Stella was selected instead because she had also participated in professional development projects. José and Alex' worked at the same school and represented different subgroups. Loren was included in the group of 'very experienced' due to Juan's and Nacho's request that their colleague Loren were included in the second Phase in the study. It was considered important to grant their request as these teachers were participating in the study because they wanted "to collaborate with the research study".

After these reconsiderations, the subgroup selected for the follow-up study of Phase 2 had *nine* teachers, as shown in the following:

From the subgroup of 'very experienced':	Nacho, Juan and Loren
From the subgroup of 'experienced':	Nora, Stella and José
From the subgroup of 'Not experienced-Not novice':	Luis
From the subgroup of novice teachers:	Alex and Pablo

These teachers represented a variety of teaching experiences (see Table 5.16), and had provided a variety of explanations of their teaching of algebra in Grade 8 (see Figure 5.2) during Phase 1 of the data collection.

5.6.2 Concluding summary

The findings of Phase 1 of the study, which aimed to identify a variety of conceptions of beginning algebra and its teaching from the initial group of thirteen teachers, has been described in this chapter.

The data show that the great majority of teachers conceptualised school mathematics as a set of unconnected topics to be studied in a strictly sequenced order, where algebra is the block that is studied in Grades 8 and 9. Their portrayals of their teaching practices presented an image of the delivery of each topic, in which the teacher starts by presenting formal definitions and detailed explanations of procedures to follow in a set of exercises and then the pupils repeated what had been explained to a list of similar exercises. These portrayals of the teaching of beginning algebra were consistent with the ones made by other teachers from a different area of Colombia, who participated in previous projects (see Agudelo-Valderrama, 2000; González & Pedroza, 1999, and Perry et al., 1996).

Assessment was conceptualised by the great majority of teachers as a practice to give marks to pupils after teaching is done, and not to try to identify the pupils' thinking; for example, whether or not the pupils had assigned meaning to the letters used, in order to inform the design of further teaching. The purpose of the teaching of Grade 8-algebra put forward by the great majority of the participating teachers was to prepare pupils in the routines of manipulating algebraic expressions that they will need in the following school grades, and later in the External Examination that all school leavers need to sit.

These conceptions are very similar to the "instrumentalist" (Skemp, 1986; Ernest, 1989) and "absolutist" (Lerman, 1990) conceptions of mathematics where emphasis on facts, rules and skills is made without regard for meaning and understanding of the basic concepts involved. Although the professed conceptions of the great majority of teachers are in line with the instrumentalist view, individuals placed different degrees of emphasis on specific aspects of their teaching of Grade-8 algebra as shown in their explanations for the differences in their priorities or for the "unsatisfactory" results of their teaching. Although four teachers declared that they needed to improve their knowledge of the teaching of Grade 8-algebra, when establishing reasons for the pupils' low results, the great majority blamed the pupils for their inadequate prerequisite knowledge or for their lack of motivation for the learning of the subject.

The data collected through the survey on beliefs about school algebra and its teaching (Section B of Questionnaire) show, in general, a contrasting perspective on the part of the teachers from that obtained through Sections A and C of Questionnaire 1 and from the follow-up interview. There was, however, consistency between their answers in the survey and their explanations at interview, in relation to "the need for repeated practice when pupils were not getting the right answers".

Although the conceptions of beginning algebra of the great majority of teachers were very similar, there was variation in the way they explained their own teaching practices. In order to study further the teachers' conceptions of their own teaching practices, the case studies of five of the nine teachers that were followed in Phase 2, and who were selected as representatives of a variety of conceptions of beginning algebra and their own teaching practices, will be the focus of Chapter 6.

Chapter 6: Insights from the study of individual teachers' conceptions

6.1 Introduction

The overall purpose of this study was to investigate the relationship between Colombian mathematics teachers' conceptions of beginning algebra and their conceptions of their own teaching practices with a view to identifying their conceptions of change in their teaching. With this aim in mind, the study initially focused on answering the Research Questions:

Question 1: What are the teachers' conceptions of beginning algebra?

Question 2: What are the teachers' conceptions of their own teaching practices of beginning algebra?

In Chapter 5, preliminary indications of the conceptions of the initial group of thirteen teachers obtained from data collected in Phase 1 were presented. These preliminary indications were identified by examining the teachers' meanings and explanations for their responses to Questionnaire 1, which they provided at Interview 1, and to their responses to Questionnaire 2, and by looking for patterns in the priorities of their teaching of Grade 8-algebra.

This chapter, continuing the search for answers to the Research Questions highlighted above, investigates further the conceptions of five teachers —Pablo, Nora, Luis, Alex and Nacho— who were chosen from the group of nine case study teachers to represent a variety of conceptions, as established on the basis of data from Phase 1 (see Figure 5.1). As explained in Chapter 4 and 5, a group of nine teachers were followed in Phase 2 of the study, where data were collected through the observation of five consecutive lessons with a follow-up interview, a Focus Group and a follow- up interview. The selection of five case study teachers from the group of *nine* (see Chapter 5, p. 129) was made on the basis of the variation of their conceptions as well as of their teaching experiences. Pablo and Nacho were selected because they showed great differences not only in their conceptions of both beginning algebra and their own teaching practices, but also in their teaching experiences. Nora and Alex had similar conceptions, but they differed greatly in teaching experiences, Alex being the novice. Alex was included in the group of five as it was considered necessary to contrast his conceptions with those of Pablo, the other novice teacher. Luis was selected as the one who not being an experienced or a novice teacher, explained his own teaching practice and situation in a very different way from the rest of teachers.

Having selected the case teachers according to the criteria mentioned, a variation of school contexts was obtained. Data collected in both Phase 1 and Phase 2 of the study were looked at in a wholistic way, in order to construct the individual case descriptions of these five teachers, paying special attention to how they conceptualised their own teaching practices in order to identify their conceptions of change in their teaching. The chapter is divided into six main sections. Sections 6.2, 6.3 and 6.4 present the case descriptions of Pablo, Nora and Nacho. Due to restriction of space, only the summaries of the cases of Luis and Alex are presented in Sections 6.5 and 6.6 respectively. The corresponding full descriptions of their cases are included in Appendix 6.1. The chapter ends in Section 6.7 with my reflections on the insights I gained from the case study teachers.

The case study descriptions. Each case begins with a brief introductory background about the teacher's experience and the school context. Data pertaining to each teacher's conceptions are presented in two main subsections corresponding to the two basic Research Questions highlighted above. Addressing the question of the teachers' conceptions of beginning algebra, the first subsection has been organised into two main headings: 'Learning beginning algebra' and 'Teaching beginning algebra'. The second subsection describes the teachers' conceptualisations of their own teaching practices. This subsection is divided into three main headings: 'The teachers' conceptions of the determinants of their teaching practices', 'The teachers' self concepts and attitudes to beginning algebra', and 'The teachers' knowledge of the teaching of beginning algebra'. The heading 'The teachers' conceptions of the determinants of their teaching practices' emerged from the emphasis made by the teachers when explaining their teaching practices. The second and third headings contain data related to the components to take into account (i.e., knowledge, beliefs and attitudes) when studying a teacher's *conceptions*, as defined in the research design. Each case description ends with a summary.

6.2 Pablo – The textbook rejecter

Pablo was teaching in a private school that caters for children of a middle socioeconomic background, and he was in his second year of teaching. The school is a day school and Pablo was "completely dedicated" to his job in that school. Pablo did his teaching practice for his Bachelor of Education degree (in mathematics) during 3 semesters, in Grades 6, 8 and 9. He had taught for a year (in 2001) in Grades 5, 6 and 7. Consequently, the year 2002 when he participated in this study, was his first time as a teacher of Grade 8. Pablo was very enthusiastic about him participating in the study and spoke confidently during the interviews

6.2.1 Pablo's conception of beginning algebra

Pablo stated that "mathematics is a body of knowledge that helps us understand real life situations and things and objects which are present in the universe".

> Algebra knowledge represents an important step for the achievement of this understanding of the world because algebra helps us to understand even simple things of everyday life. (Int. 1)

So the main reason why pupils need to learn algebra is "because it is knowledge useful for problem solving, and this knowledge is also needed for the next mathematics levels and, later, for pupils' career opportunities".

Pablo strongly disagreed with the statement, given in Questionnaire 1, that 'algebra of Grade 8 should be taught in isolation from other areas of mathematics (i.e., from arithmetic and geometry content) or from "pupils' daily life experiences". Explaining his disagreement with this statement, at Interview 1, he claimed that "if when studying mathematics, one does not find connections with the world then it does not have meaning; one finds no reason to study it." His teaching goal for Grade-8 algebra was "to help pupils see the functionality of what they are learning… They need to be asking, for example: What is the function of this formula? Why do I need to add polynomials?"
Pablo referred to algebraic thinking as "thinking of the general", and believed that algebraic thinking could be promoted in the primary school, for example, helping children to make generalisations about their working methods.

When children buy sweets, they can be encouraged to think about the formula for calculating the cost of any number of sweets, having the price of a sweet; that is without mentioning the word formula, but just saying it as a primitive algebra (writes and says, 'number of sweets × price of one sweet = cost').

6.2.1.1 Learning beginning algebra

At Interview 1, Pablo said that "to learn mathematics, pupils need to be provided with activities that help them see where things, that is, mathematical ideas, can come from". He declared that he did not like the pupils' textbook "because it is just a list of exercises for each topic, and if you don't do activities that help them see the application of topics or the connection that certain concepts have with other concepts or with real life situations, then they don't see any point in their work".

Of the six mathematics lessons assigned to Grade 8, in the weekly school timetable, four were assigned to algebra, one to geometry and one to work on the Mathematical Calendar[1]. Pablo explained that pupils had to "work by themselves on these tasks, continuously, during the week", and then they used "one lesson to discuss their work, their questions and solutions of specific tasks" (see examples of the 'mathematical calendar' in Appendix 6.2).

In his responses to Questionnaire 1. Pablo's two '*Number 1*' priorities for pupils' work were 'discussion and systematic work' and 'algorithm-routine exercises'. In relation to these choices he said at Interview 1 that

it is important that pupils learn to apply algorithms efficiently, but more than to become efficient in manipulation of expressions, I want the pupils to be thinking of 'what I am doing this work for' and 'why I do this in this way'.

One of the examples he provided for illustration of this intention was related to "the construction of an algebraic expression where pupils can see that letters represent variables". Pablo argued: "I don't pay attention to the definitions that come in the textbook" that an algebraic expression is a combination of numbers letters and operations!

When we started to talk about letters or simple expressions we did it because we were talking about area and perimeter of rectangles or triangles. Later, when we

1 The Mathematical calendar was the name of a series of worksheets that contained practical tasks for different year levels, and were generally based on basic geometry topics.

needed expressions with different letters, what I did was to draw on an activity that they were doing in PE with long jump where they were allowed to do 13 running steps for impulse, and then the jump. We started talking about the distances that Martha and Juan would have travelled during their 13 running steps... They concluded that even if each pupil run 13 steps, the distance travelled was different for each pupil, and that the same applied for the distance travelled in the jump. We said: 'let say that the distance travelled when they take impulse is x and the distance travelled in the jump is y. How can we express the total distance travelled by any pupil? They saw that in the expression $x + y$, x and y were variables. (Int. 2)

6.2.1.2 Teaching beginning algebra

In this subsection data is presented according to the order of priority of *preferred* practices established by the teachers in their responses to questions on teaching styles in Questionnaire 1. I have chosen to organise the data in this order because this is where it is more clearly spelt out. However, the data collected through other means during the whole process of data collection will be used to contrast and confirm the claims. Pablo's priorities in relation to his teaching styles are shown in Table 6.1.

Table 6.1 Pablo's teaching style priorities

Preferred	*Teaching style descriptors in Questionnaire 1*	*Actual*
1	Designing classroom work that promotes connections between different mathematical topics studied	1
1	Organising problem-based activities for the pupils to work in small groups where they can present their ideas to the whole class for discussion	5
2	Providing opportunities for pupils to develop their communication skills so that they can express their mathematical ideas with confidence	2
3	Giving clear explanations of definitions and procedures to follow in different exercises and problems of application, in the topics studied	2
3	Giving pupils lots of exercises for algorithm application as homework	3
4	Designing activities that provide space for pupils' self-paced learning	5
6	Testing pupils at the end of each activity or topic, in order to have sufficient marks for assessment in each Attainment target	4

Keeping in mind Pablo's responses to questions of 'teaching style' in Questionnaire 1, we shall move on to the examination of explanations and illustrations in relation to his first two preferred teaching styles in order to help us understand the reasons for his priorities. Note that Pablo's system of numbering his priorities is different from the conventional system.

Priority 1 for both 'preferred' and 'actual' practices: Designing classroom work that promotes connections between concepts

Pablo explained at interview that, in his teaching, designing classroom work that promotes connections between different topics was what he tried to do. He emphasised the word "tried" here, because he was "just starting to see how [his] ideas for teaching work". Before providing examples of the type of work mentioned, he wanted to clarify that "that work was not to be taken as illustrative of a problem solving approach" because they were "simple tasks" that he had devised to introduce topics like algebraic expressions.

Figure 6.1 shows two examples of the "simple tasks" Pablo used "to help pupils construct algebraic expressions with the same letter". The first task, marked 'Situation 1' was explained at Interview 1. 'Situation 2' has been taken from one of the observed

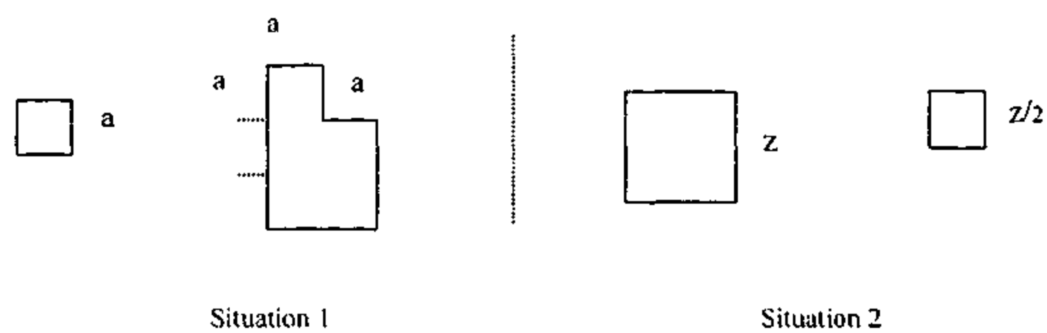


Figure 6.1. Pablo's "simple tasks" for introducing pupils to algebraic expressions

lessons[2]. In both situations, after the figures were drawn on the board, pupils were asked to draw them in their books and then asked for the measure of the sides of their drawings [the measure expressed by using the small squares of their squared paper], because he "wanted them to see that *a* can represent a different number for each pupil, and that the areas of the figures depend on the value of *a*.

In situation 2, however, apart from finding the perimeter and the area of the two given figures, pupils were asked to add the two areas. Pupils worked in pairs and sometimes discussed their ideas with other groups showing great enthusiasm about their work. Most pupils completed the task successfully. When Pablo brought the class together, and pupils were invited to show their work "to explain the method used", three pupils

[2] According to Pablo, this (observed) lesson took place one week after the lesson when Situation 1 was worked out with the class.

said that they had difficulties adding fractions and others had difficulty identifying the coefficient of Z^2 [when adding Z^2 to $Z^2/4$]. When the teacher asked them to think of equivalent fractions, two pupils suggested that that was too long and more difficult than cross-multiplying, according to the rule. Pablo asked what the rule was. A pupil went to the board and gave an example explaining the rule to add fractions with different denominator. The two pupils who were asking how to add fractions also said that the rule was better. Without any more comments about this, Pablo continued with the lesson.

At interview 2, after a sequence of three lessons was observed, Pablo provided explanations about some aspects of this specific lesson. It is important to note here that only three consecutive lessons (two double-lessons and one single) from the five programmed with Pablo were observed, as two of the lessons were cancelled because of the school's extracurricular activities[3]. As will be seen later, this was a contextual aspect identified by Pablo as an important constraint in his teaching. To my questions about pupils' difficulties in identifying the coefficient of Z, and about the few pupils saying that $Z + Z = Z^2$, Pablo said:

> I am aware that some pupils don't understand. I have to design better activities but it is difficult to design activities for all of them. ... My main concern has been to give some more work to the fastest pupils because they get bored if they have finished and I carry on working with the ones that haven't understood. Sometimes I have had to improvise with the fastest because they start disrupting if they have nothing to do.

In relation to the aspect of working by the rule for addition of fractions Pablo explained that "some pupils like to learn rules by heart":

> They want just to be given simple rules. I have to rush because I am behind with the program and many pupils are anxious because they know we are behind. They think that we haven't started algebra because I am not teaching the ten case of factorisation that their friends from other schools already know! They keep on asking, 'when we are going to start algebra?'

Learning to factorise according to the ten cases presented in some popular textbooks was not learning for Pablo. At Interview 3, he argued that learning to apply the rule for each case was "just manipulating specific expressions.

3 It was not possible to continue observing Pablo's work in the following week as the observation timetable with teachers from other schools had been already agreed.

> If I really learn, for example, how to factorise $a^2 - b^2$, and why I factorise it in a specific way, I should be able to factorise the other types of polynomials or the other cases. And why do I have to teach factorisation as a separate topic if factorising is a way to know if I understand multiplication? If when knowing the sides of the figure they can find the area, then given the area, we find the sides. Pupils need to be asking themselves 'why am I doing this?' For me, mathematics is analysis, and I don't have to be explaining much because, as I have seen it, pupils answer each others' questions.

Pablo was aware of the difficulty that beginners have in accepting "lack of closure", or their tendency to "conjoin the expressions" (Booth, 1984), and of the inadequacy of the "fruit salad approach" (see MacGregor, 1991; Mason et al., 1985) to try to help the pupils accept lack of closure, as he pointed it out to his colleagues during the Focus group session. Because of this awareness, he explained at Interview 1, that he was designing an activity, "which is a game and not a real-life situation about buying and selling things, using notes that have not got specific values" to try to get the pupils into seeing that unlike terms cannot be converted into a single expression. Pablo said:

> I know that pupils convert expressions like $2a + b$ into something like $3ab$. So the idea is that they do something that get them into seeing that you cannot add the *as* with the *b*s, that the *as* and the *b*s are not like terms. ... I would like them to see by themselves that they can't add, for example, 5 $G notes with 5 [illegible] notes.

Stating to the learners, however, that unlike terms cannot be added was an expression that Pablo, and all the participating teachers, used and neither Pablo nor the other twelve teachers showed awareness of the possible difficulty this was could represent for the pupils.

The initial part of this activity took place during the third observed lesson. As explained before, due to the fact that pupils were busy in a PE competition organised by the school, the fourth and fifth programmed lessons could not be observed. For more information about the initial part of this activity, and Pablo's descriptions of what happened during the subsequent lessons, refer to Appendix 6.2.

Priority 1 for preferred practice: Organising problem-based activities

Pablo pointed out that his "ideal practice" of organising problem-based activities for the pupils to work on was difficult for him to implement, as can be seen from the actual rating. At Interview 1, Pablo spoke of problem-based activities as more complex tasks than the ones he had been working on in the lessons, where pupils had to apply the knowledge of various topics studied. At Interview 2, he said that "what Teacher B of Questionnaire 2 did was an excellent example of a problem-solving approach because classroom work for the teaching of certain concepts was started by discussing and solving a "problem-situation" that has been specifically designed so that the pupils can use what they know to construct the concepts for which the activity was designed". He further added, "what Teacher B does is what I am trying to do but there are topics like the basic laws of operations with rational numbers that can take me to follow a trasmissionist approach. There are no practical situations to teach this topic, or at least I don't know..."

Priority 2 for both preferred and actual practices: Providing opportunities for pupils to develop their communication skills

Pablo's reasons for the his teaching style priorities described above explained his professed difficulty with 'Designing activities that provide space for pupils' self-paced learning', and why "sometimes" 'Giving clear explanations of definitions and procedures to follow in giving exercises' was a higher feature in his teaching despite his preference for not telling. These explanations were consistent with his rankings in Questionnaire 1 (see Table 6.1). 'Providing opportunities for pupils to develop their communication skills so that they can express their mathematical ideas with confidence' was ranked second both for preferred and actual practices because, for Pablo, this provision was represented in his emphasis on "the importance of having pupils working and discussing their work in pairs and, sometimes, presenting it to the whole class". As we will see in the following section, although "pupils' development of communication skills" was considered "important" by Pablo, it only took place at the level of "pupils talking and explaining their work verbally", and not in writing. The communication skills of pupils did not represent an aspect to pay attention to in the assessment of their work as the teacher had to "just provide a grade for the pupils' school report".

Assessing pupils' work

Pablo's responses, in Questionnaire 1, to questions about preferred assessment practices identified 'Pupils' folders and assessment records, showing evidence of several aspects of the process followed throughout a term or a set of terms' as one of his three equally top priorities (i.e., Number 1). The other two, which he added to the given alternatives, were:

- Monitoring continuously pupils' work in class, in order to observe their learning process and their commitment
- Written tests where pupils can apply what they have learnt during a period of time, without partitioning the topics

In relation to these three 'Number 1' preferred practices, Pablo explained at Interview 1 that although to follow each pupil individually was his "ideal" (i.e., his preferred practice), he had found that "it was very difficult to do it in practice, due to the number of pupils and to time pressures, as he had to hand in the term grades at specified dates". So in practice he managed "to monitor pupils' work by collecting the pupils' notebooks, with some frequency, to see what they [had] done during the lessons and, obviously, in their homework". But he did "not manage to do written tests at the end of the term without partitioning the topics"; instead, he did "a test after each topic was taught, as [he had] indicated it in his rankings of his actual practice".

Without being asked, Pablo explained that he was "aware that tests [were] not the best way to identify pupils' difficulties. ... You see them working very well during the lessons, and yet they make mistakes in the tests.

> Immediately after they hand in their tests they say, 'Oh! I really got confused with this or with that!' Look at what I had done in my homework and look at what I did in the test! . . . The work that really tells you if pupils understand is their daily work in the lessons. (Int. 2)

So why did Pablo give the tests at all? He had to give a test after each topic was taught

> because of the deadlines to hand in the pupils' grades; and following each pupil in a personalised fashion is just not possible; so I have to do something that can be manageable for everybody, in a simple way. ... I have to hand in, to the Academic Coordinator, the grades in relation to the content objectives stated in the Grade 8 program for that specific term. ... Those objectives represent the 'assessment indicators' for which I have to provide pupils' grades for the school assessment report.

To the question of whether and how the development of communication skills was monitored or assessed, Pablo replied that that aspect did not count in pupils' assessment "because what counts is how they do in the specific attainment indicators that you have to report for that term. ..."

> We also report on aspects like the pupils' responsibility and interest, and pupils keep a register of that. Pupils who do well have an *E* in their reports", and according to the school's reports' format, when a pupil gets an *E* [Excellent], you don't have to give any comments. But if a pupils gets *I* [i.e., *Insuficiente* -which is equivalent Poor], the teacher needs to give two comments which are standard, that is, the same comments for everybody that gets that grade...

Pablo did not question these assessment practices. When he was asked what he thought of the way pupils' reports were being done, given his emphasis on teaching and learning for understanding, and his preferred assessment practice of continuously monitoring the pupils' work in order to observe their learning process and their commitment, he said:

> To do that would require a lot of time. That would be more difficult. If you had a small group of pupils, one could think of that but with groups of thirty!

6.2.2 Pablo's conception of his own teaching of beginning algebra

This subsection contains information about Pablo's evaluations of what was taking place in teaching of Grade 8-algebra and about the interplay of internal and external factors he saw as determinants of his teaching practice during the period of data collection (April to September 2002). As already mentioned, data related to Pablo's knowledge for the teaching of beginning algebra, and to Pablo's self-concept is also included in this subsection. The description starts with Pablo's identification of reasons for differences between his *preferred* and *actual* practice in Questionnaire 1, which were further explained at Interview 1. Additional information related to these reasons was provided at Interview 2, when Pablo talked about specific incidents taken from the series of lessons observed. It continues with the presentation of data collected during Interviews 2 and 3, where Pablo was invited to construct a concept map showing the determinants of his teaching practice.

6.2.2.1 Pablo's conception of the determinants of his teaching practice

At Interview 1, Pablo gave more details about the reasons he had listed, in Questionnaire 1, for the difference in rankings between his preferred and his actual practice, and at Interview 2 he provided further information that reinforced what he had said on previous occasions. Pablo attributed the differences between his preferred and his actual teaching practice to the following factors:

The difficulties in teaching according to his professed conception of learning

Pablo openly admitted that "sometimes [he] just gave the formula out or told the pupils how to do it because [he] didn't know yet how to find activities or ways to work so that [he] is not the one that tells how to do it". But he pointed out that, on other occasions, he gave the formula out because of other important "stronger" factors that had discouraged him from insisting that "the pupils explain and justify their reasoning". Those factors were:

The pressures exerted by the pupils and their parents

Some pupils want "just to have a given rule to solve tasks and problems"; and several pupils and their parents put pressure "to cover the set program" at the speed pupils from other schools are doing it:

> There are several pupils who expect the teacher to give rules and explain how to do everything. For example when operating with fractions they don't want to know about finding equivalent fractions, they complain that it is more practical to apply the rule of cross-multiplying. They keep on asking: Pablo[4], when are we going to start algebra? and say that their friends have already seen the 10 cases of factorisation. They think that algebra is that. And some parents have come to complain to the Academic Coordinator that their children are behind in mathematics. (Int. 1)

> Parents are very powerful because they pay high fees, and that's how the school functions. If parents and pupils complain that my teaching is not good, nobody is going to say: 'Oh! that's because that teacher wants the pupils to understand what they are doing'. Not, especially, when they know that I am just starting. (Int. 2)

Pablo's status in the school

The last statements of the previous quotation talk about Pablo's concerns in relation to the consequences he could face if he did not cover the list of topics he was expected to cover. He felt this created a conflict for his teaching due to the fact that as a novice teacher he could expect no support. When asked if there was no support from the mathematics department he replied: "That's the problem! We don't work as a group; it is everyone his/her own side ... Our mathematics department meetings have, mainly, administrative purposes; and I already told you about how some teachers see their job (refer to the heading Pablo's self-concept in Subsection 6.2.2.2).

4 In Colombia in the private schools that cater for socially advantaged pupils, pupils call teachers by their names. In state schools where the majority of pupils belong to less advantaged communities, pupils address their teachers with the title of *profesor(a)*.

Time available for teaching

Pablo stated twice, at Interviews 1 and 2, that the time available for teaching was "becoming a worry" for him. He argued that "the school [was] always organising extracurricular activities without taking into account the set timetables" (Int. 1). That the school placed "high importance on extracurricular activities decreasing teaching time" was a factor that Pablo mentioned again, at Interview 2:

> We see, very frequently, that from the four weeks one thinks one has available for teaching in a month, there are only three, or, sometimes, we even have only two available. Working with activities, as we [my pupils and I] have done sometimes, takes longer than just explaining and telling everything. So the goal of helping them to produce their ideas sometimes loses importance when one has to work under all these limitations and pressures...

We shall now see which factors Pablo identified as the determinants of his teaching practice in the concept map activity, which was specifically designed to encourage the teachers to make explicit and to explain the ways they saw their own teaching situations. We need to remember that, in the second part of Interview 2, the teachers were asked to think of the aspects or factors they thought influenced their teaching of Grade-8 algebra, and then to construct a concept map showing the strength with which the factors impacted on their teaching practice. The teachers built an initial concept map that was done in two stages or parts, at Interview 2. Their initial map was reviewed at Interview 3. This map is called the final concept map. In the following subsections the different maps that were constructed by Pablo as he considered the different factors which affected his teaching practice, in different moments of the data collection process are described.

Pablo's initial concept map – Part 1.

When Pablo was asked to make the list of the factors, he wrote:

- Pupils' dispositions
- Time
- Pressure from parents to cover a list of topics
- Pupils' prerequisites (i.e., mathematical knowledge expected to have when they finish a year level)
- School environment

Before Pablo was invited to build a concept of the determinants of his teaching practice on the basis of his list of factors, an example of a very simple concept map was presented to Pablo. The necessary discussion and clarifications about the construction of the concept map were made according to the guidelines described in Chapter 3 Subsection 3.3.2. The meaning of 'My teaching practice' was discussed, and it was made clear by the interviewer that 'teaching practice', for the purpose of the concept map, included aspects like: the classroom work that was organised, the emphasis the teacher made on specific aspects of classroom work and content taught, the forms and content of pupils' assessment and the environment which was created in the classroom. These aspects had been written down on the sheet of paper provided for teachers to construct their concept map. It was equally discussed that the concept was to be constructed referring to what takes place in the classroom, rather than to the preferred or ideal situation.

When Pablo saw the descriptors[5] of 'My teaching practice' (written down of the sheet to be used for his concept map), he immediately said: "It all depends on the dispositions of the teacher. An important factor is the teacher's dispositions", which for him meant "something like the teacher's philosophy of the teaching of mathematics":

> By the teacher's dispositions I mean (i) The time dedicated to prepare classroom work. For example, there may be some [teachers] who just repeat the same set of questions every year. (ii) The desire to improve what one does and the interest in increasing what one knows. There are some colleagues that want to stay in grades 6 and 7 all the time because 'Oh no! I don't want to teach in grades 10 or 11 because I haven't taught in those grades for a long time'. (iii) The enjoyment of what one does. Do I do this because I have to, or because I want to? Some teachers don't want to be more than just the *repetitors* of a routine. In one word, it is the philosophy that one has about the teaching of mathematics

Figure 6.2 shows the first part of Pablo's initial concept map. The arrows mean "influence(s)", "affect(s)" or "determine(s)" as it was agreed with teachers during the activity. Pablo's thinking aloud during the construction of map, and the clarifications he provided as requested by the interviewer are provided after each concept map.

5 My teaching practice: the type of work I organise, the content I emphasise, the assessment patterns, and the environment that is created in my classroom.

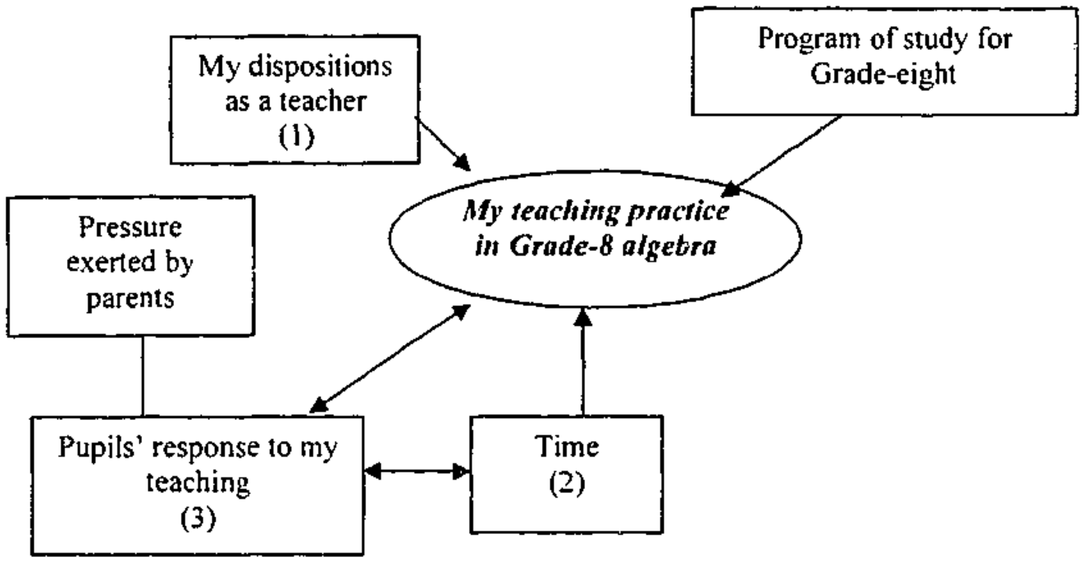


Figure 6.2. Pablo's initial concept map - Part 1

While organising the labels and drawing the connecting lines, Pablo said:

> The program of study influences my teaching practice because it gives us the guidelines for what we have to teach. And this is something that has been established by the mathematics department and has to follow the general curriculum guidelines of the Ministry of Education. 'The pupils' response' determines the 'Time' spent in their work and, sometimes, the 'Time' available affects 'Pupils' response'. Both the 'Pupils' responses' and 'Time' influence 'My teaching practice'. The 'Pressure of the parents' is related to the way 'Pupils respond' and affect my work because I feel the pressure to hurry up. And 'My dispositions as a teacher' are the most important because what I propose, or what I do depends on 'My dispositions as a teacher'. It depends on what I feel like doing, '*las ganas*' (i.e., the desire and commitment) that I have, my motivation and what I think teaching is.

Pablo was asked to give numbers to factors, if necessary, in order to determine the strength of influence they played in his teaching. 'Pressure from parents' was not given a number because "it is mainly done through the pupils; they are the ones with whom I am always in contact. The same applied to 'Pupils' prerequisites and the school environment' (from his initial list) "because they affect my teaching but in a lesser way than the factors" appearing in the map.

Pablo's initial concept map – Part 2.

The second part of the initial map resulted from the interviewer questioning Pablo, in order to see if, he thought his knowledge influenced his teaching. Although in the first part of map Pablo had identified his dispositions as the most important factor in his teaching practice, which included his knowledge, he had not explicitly listed his *knowledge* within the factors. He had emphasised the role of his knowledge when he was explaining specific actions and decisions in his classroom but in his concept map, he was not making explicit mention of the role of his knowledge. As identifying how the teachers conceptualised the role of internal factors like their mathematical knowledge and their pedagogical content knowledge (Shulman, 1986) was one of the Research Sub-questions, after Pablo constructed his initial map, he was asked:

> Suppose that you have to go on leave and a primary school teacher comes to replace you for sometime. How would you think classroom work with your Grade 8-pupils would go?

Pablo answered, "Not very well because they don't not even know the mathematics to teach in primary...". This answer led him into adding 'My knowledge', which he claimed was as a "very important factor" influencing his/her teaching. Through discussion about the teacher's types of knowledge, Pablo decided that two important types of knowledge were to be included in the concept map: 'knowledge of mathematics' and 'knowledge of the teaching of mathematics'. Figure 6.3 shows Part 2 of Pablo's initial concept map, where the two labels belonging to "knowledge" were included.

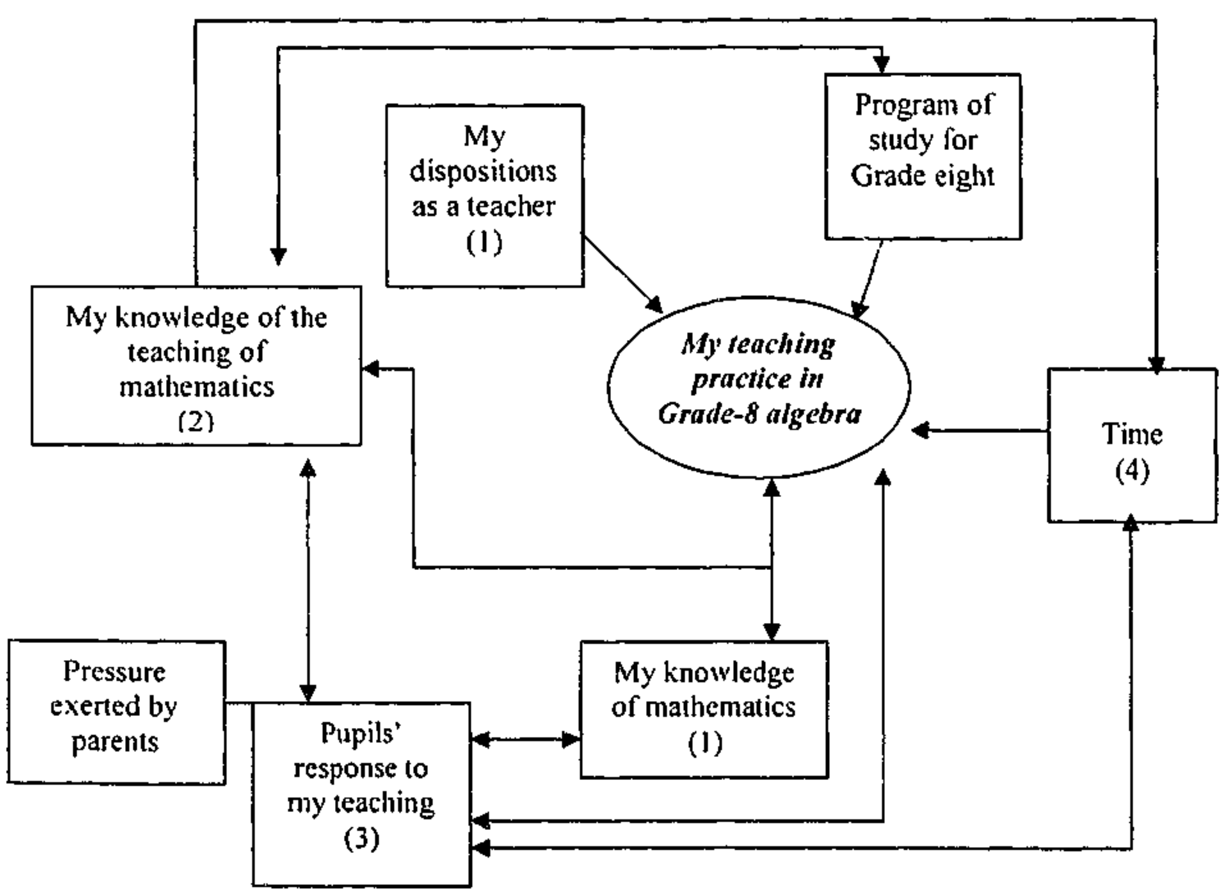


Figure 6.3. Pablo's initial concept map – Part 2

The following is what Pablo said when he was rearranging the labels:

> 'My knowledge of the teaching of mathematics' allows me to express 'My mathematical knowledge' in an efficient way, according to the pupils' level. At the same time, it determines the 'Time' because it allows me to design effective activities, which take less time. 'The pupils' response' also determines the 'Time' because you can teach a topic faster according to the 'Pupils' response'. 'The Program of study' is the guideline. You cannot talk about 'knowledge of the teaching of mathematics' if you don't have 'knowledge of mathematics'. Of 'My knowledge of mathematics' depends what I organise or propose for my teaching, so it determines 'My practice' and the 'Pupils' response'.

Pablo then gave numbers to the labels to show the strength with which he thought the factors influenced his teaching.

Pablo's final concept map

At Interview 3, after some specific points related to the classroom activities discussed at the Focus Group were reconsidered, Pablo was presented with the factors of his initial concept map (i.e., the boxes without the connecting arrows he had drawn in his initial map at Interview 2). He was asked to think if the factors he had identify in his previous concept map were still relevant, to draw the connecting arrows or lines, and to identify the strength with which the factors influenced his teaching, once again. This time Pablo identified the "use of teaching time" a number one factor and emphasised the primacy of the role of this factor in his teaching, as a consequence of his perception of the way the school functioned and the pressure to cover the set program put by his pupils and their parents. Figure 6.4 shows Pablo's final concept map.

As Pablo started to draw the connecting lines, he said to himself, "My teaching practice: what I propose, what I do, what takes place in my classroom". The following transcript[6] shows what Pablo explained about some of the lines and arrows connecting the factors in his final concept map. (The letter P means 'Pablo', and the letter 'I' means 'Interviewer'):

> P: I am going to change Program of study of Grade-8, for 'Curricular guidelines' because the program is based on those guidelines. These guidelines enrich my teaching but they don't have much importance in what I do, so the influence is through knowledge for the teaching of mathematics. I had already explained why my

[6] The initial letters of the teacher's name and the letter 'I' representing 'Interviewer' are used in all the transcripts presented in this thesis. When in a transcript a slash (/) is used, it indicates a sudden cut or incomplete sentence.

dispositions as a teacher are important in my teaching practice, and according to the dispositions comes the interest to be up to date in the knowledge for the teaching of mathematics.

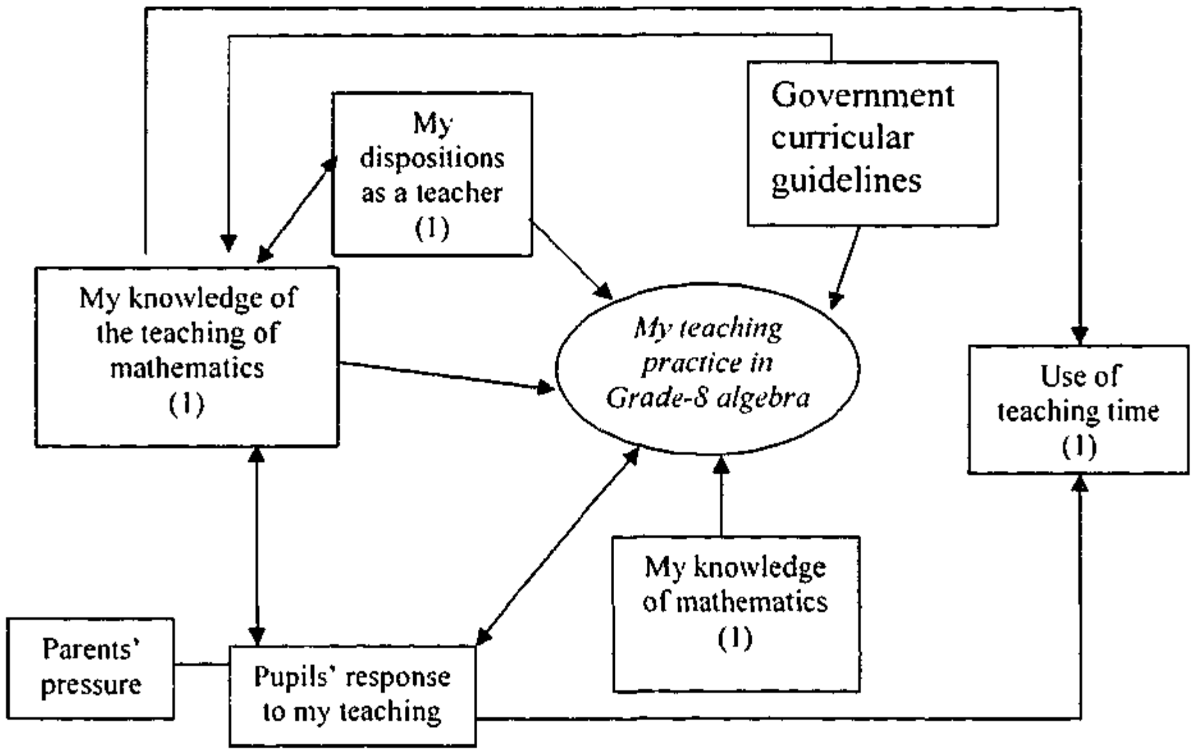


Figure 6.4. Pablo's final concept map

I: Why the double arrow from 'My dispositions' to 'My knowledge of the teaching of mathematics'?

P: Because if I have motivation for my teaching, I do something to improve my teaching, and if I have more knowledge of how to teach, then I feel more motivated.

I: And the arrow from 'Pupils' response' to 'Time'?

P: Pupils' responses' affect 'Time'. And, actually, it is not time; it is the use that I make of the time available. So it's 'Use of teaching time', not time. And the way I make use of teaching time affects what takes place in the classroom because for example if I do activities that could take two weeks, it doesn't work very well because if there are interruptions [i.e., the school extracurricular activities], the effect I was trying to see is lost. So the activities need to be effective but shorter because time is short.

Asked to identify the strength with which the factors influenced his teaching, he wrote the number "1" in the 'Use of teaching time', 'My dispositions' and 'My knowledge' boxes, and said, "Now that I know how this school works, I know that the use of time is my main priority".

It is important to notice that before Pablo did his final concept map, he had stressed the concern he had with the constraint represented by the shortage of teaching time. Discussing about the classroom work that had been presented at the Focus Group, he said that he had been "impressed, particularly, by the use of spreadsheets to help pupils familiarise with the concept of variable..." when he was asked what he would do to address the concern he had with the teaching time available, as he had pointed it out at the Focus Group. He made a differentiation between what 'improving' his teaching meant for him, and what he would have to do in order to fulfil what he perceived to be the requirements of his job:

> I: So in terms of the activity itself, the type of work that is proposed to the pupils would you do any changes? For example, if you were going to teach Grade 8-algebra next year?
>
> P: It is not change because the approach would be the same. The adequate word is improvement or adaptation of the activities in order to help all pupils understand. But I don't know because I, sometimes, due to the factor time do things like Teacher A [i.e., telling].
>
> I: So what do you think you would do to improve the activities?
>
> P: I don't know yet. For example, the activities of Teacher B [of Questionnaire 2] or the ones we saw in the Focus Group are excellent to help pupils make sense of the work, but those require enough time to work with the pupils. ... If I were totally free one could work in that way in that sort of activity but one is always behind with the program, so I have to hurry up. There is the time factor and other external factors that you have to hurry up.
>
> I: Other external factors like?
>
> P: Knowing how this school works/ you have to plan knowing that there are lots of interruptions and missing lessons, but you have to cover the program because the pupils are the first ones that are pushing, and the parents complaining, then improving has to be doing activities of this sort but much shorter because I have to hurry up!

6.2.2.2 Pablo's self-concept and attitude to beginning algebra and its teaching

The findings related to Pablo's conceptions of beginning algebra and the determinants of his teaching allowed us to get some insight into his beliefs about his knowledge for the teaching of beginning algebra and about his attitudes to his pupils. This section provides us with more information about Pablo's beliefs of his knowledge of beginning algebra and its teaching, the contextual factors of his teaching and his learning.

Pablo spoke of his positive feelings about his knowledge of mathematics. He noted that he "had always liked mathematics and it had always been easy" for him. He "did not choose to be a mathematics teacher but studying mathematics, even if it [was] for teaching, [was] good because mathematics opens many doors for you". However, Pablo made it clear, twice (at Interviews 1 and 3), that he was "not satisfied with the financial side" of the teaching profession:

> Once you are studying (a Bachelor of Education), you feel you are there, in the vocation to teach and to be in contact with the children who are learning, though one is learning too. I don't regret to have chosen to be a teacher because I like it, but the economic side of it is not attractive, so I am thinking of doing a Maters or another degree that does not have to do with education.

He enjoyed teaching algebra, particularly because he felt confident about being able to teach the beginning algebra concepts without adhering to what textbooks suggested.

> Textbooks bring just a list of exercises after a definition or an algorithm has been given. If one portrays mathematics just a set of exercises that you have to do this way because that's the way the textbook does it, without seeing something of where things could come from, then pupils don't see the point. For example, I asked them to find whatever information about what algebra is and why algebra emerged, and I said 'we *all* are going to get into Internet to see what we can find there'. What they found was that algebra is the utilisation of letters, and some said that they had found that an algebraic expression is a combination of number letters and operations! ...

Although he was motivated about his teaching approach, which he felt was different from the one posed in textbooks, the fact that he was 'teaching by telling" some of the topics, especially the ones preceding the work with algebraic expressions, lowered his motivation for the teaching of Grade-8 algebra. Explaining his response to the statement "I enjoy teaching algebra" given in Questionnaire 1, he said:

> I put just Agree [and not Strongly agree] because there are things which are just theory..., like the properties of operations in the sets of rational or irrational numbers. There are things that cannot be applied, there are no activities to find the application, or at least I don't know. Twenty pupils understand and the rest just copy from their peers ...

Pablo's conceptions of his learning

Although Pablo felt confident about his mathematical knowledge, he acknowledged that the activity of teaching provides the teacher with opportunities to learn not only about how to teach mathematics but also about mathematics itself.

> One's mathematical knowledge is enriched when one is teaching. ... Mathematics can be learnt even from the pupils. For example, a pupil brought a solution to a problem with ideas that had never crossed my mind!

Learning about teaching "could take place by interacting with other colleagues who are ahead, have more knowledge and experience, or by doing courses, learning to do research..." His desire to learn was "the reason why [he] wanted to participate in this [present] study". On more that one occasion, Pablo put forward the idea that teaching entails a learning process which is continual. He spoke of experienced teachers he knew, "who display models of teaching when they talk about their teaching". Referring to a teacher who participated in the Focus Group session, and to one of his colleagues in the school, he said:

> He makes explicit in our department meetings that he doesn't want to take groups other than Grades 6 or 7 because he has been teaching in those groups for the last six years, and he does not want to have to dedicate more time to planing teaching. That gives me the idea that he sees his teaching as repeating the same lists of tasks every year. It is the mentality that because he has done it for five or six years, then he knows how to do it! (Int. 3)

Pablo argued that "the teacher who claims that "he/she knows how to do it is the one that portrays mathematics as a dead subject." A dead subject means: "here is this rule and these exercises; do the exercises applying the given rule". He was strongly opposed to teaching by the textbook. He also argued that teaching mathematics is not putting into action something that a textbook has designed for you.

> Yes, I have found ideas in textbooks that I have used to think of what I am going to do. But when I don't find anything of the sort of things I consider important, I create what I want to do. The problem is that creating activities that work for all pupils is very difficult, or at least I don't know ho to do it yet.

Pablo totally refused the idea of adopting a textbook, even if it were a new, innovative, 'promoting-understanding book', that I described to him as one of the three-option proposal I presented to him in relation to his teaching of Grade 8 in the next academic year. A description of the three-option proposal is presented in Appendix 4.3, and Figure 6.5 shows a basic outline of these three options.

In relation to Option 1, Pablo said:

> To just pinch the ideas of a textbook? Okay! But I would like to find books that talk about how to find or how to create activities. I don't want books telling me, 'this is the activity!'

Option 1: To adopt a new textbook that has been produced by drawing on results of research on the teaching and learning of first algebra notions. According to the authors, the textbook emphasises a teaching for understanding and meaning approach.

Option 2: To participate in an 8-month professional development program that focuses on the use of technology in the teaching of school algebra. It requires attendance to weekly-workshops during the first two months of the program. This project is financially supported by '*Fundación Compartir*'.

Option 3: To engage in a 14-month classroom-based research professional development project that requires at least 5 hours of daily work throughout the first 11 months.... That means that participants will need the school's support and cooperation for their participation in the project. This project is financially supported by Bogotá District Secretary of Education, and is offered through IDEP.

Figure 6.5. Basic outline of the three-options proposal presented to the case study teachers, at Interview 3, when discussing how to address their teaching concerns

Pablo chose Option 3 and showed great enthusiasm when he was asked if he would like to engage in a project where he would work in a team researching his own teaching of beginning algebra. He said:

> I would like to participate in the research project of Number 3 but I don't think I could do it if I am working in this school.

After I explained to Pablo that there was a possibility that the successful applicants were offered the possibility to be placed in one of Bogotá District schools that supported the introduction of innovation in mathematics education, he answered:

> I will definitely choose Option 3. I am prepared to give up my job if I need to... Of course, if they offer me some kind of income to survive. I'd like to work in a team. [pause] I have heard about the projects of the IDEP...

6.2.2.3 Pablo's knowledge of the teaching of beginning algebra

This section contains a summary of the data collected on Pablo's knowledge and beliefs about the teaching of beginning algebra concepts, specifically of the concept of variable. The data were collected through Section C of Questionnaire 1, Questionnaire 2, and through the rest of data collection activities in which the teachers offered explanations for their decisions and actions in their classrooms.

Knowledge of the teaching of the concept of variable

Pablo provided evidence of his knowledge of the "multifaceted character" of the variable (Trigueros & Ursini, 2001) and of some of the common difficulties children have in the learning of this concept. We saw, in Subsections 6.2.1.1 and 6.2.1.2, the emphasis he made in starting algebraic work by helping pupils see the variable to represent quantities whose value varies in a relationship. He identified the task given in Question C4 of Questionnaire 1 as a good situation to help Grade 8-pupils become familiar with the concept of functional relationships. When asked how he would respond to the pupils who wrote 5N = D to represent an equation for the statement '*In Central Hospital there are five times as many nurses as doctors*', he explained that he "would encourage the pupils to see by themselves how the variables change for specific values", and then he would help them to establish the relationship between the two variables.

> P: The idea is to help them find the relationship between the doctors[7] and the nurses, according to the context of the situation I would ask them: how many nurses if there is one doctor? Then, we can carry on finding/ we can do a table.
>
> I: How would that table be done?
>
> P: [while writing the first two columns of a table, shown in Figure 6.2] We are being told that for each doctor there are 5 nurses. For 2 doctors there are 10 nurses . . ., etc. So, what do we have to do to the doctors to find the nurses? I would ask questions to help them see that you have multiplied the doctors by 5 to get the nurses. [At this point he starts writing the third column of the table, without the name] We multiply 1 by 5 to get 5; 2 by 5 to get 10. ... It would be a construction by asking them, 'how do we do?' or 'what is it that we did here?' ... and hopefully to arrive a this [writes D × 5 = N, the arrow and then the expression D . 5 = N in the third column of Figure 6.6)

D	N	D . 5 = N
1	5	1 × 5 = 5
2	10	2 × 5 = 10
3	15	3 × 5 = 15
4	20	4 × 5 0 20
.		
.		
		$\boxed{D \times 5 = N}$

Figure 6.6. Pablo's table in his explanation to Question C4 of Questionnaire 1

[7] A language problem was observed, though, in the way Pablo portrayed the meaning of the letters used to represent the variables.

Pablo did not pay attention to the words he was using to convey the meaning of the letters representing the variables:

> I: But when you write this D, do you say doctors?
>
> P: The quantity of doctors.

It is interesting to see that in this specific situation Pablo did not pay attention to the possible risk that his way of expression could represent for the learners (i.e., D for doctors), as he showed awareness of the inadequacy of the use of the 'fruit salad' approach (MacGregor, 1991) and of what the letters may mean or stand for beginners. Although question of the doctors and nurses he was thinking of the "quantity" (i.e., number) of doctors, in subsequent discussion he continued to use the word "doctors" without paying attention to the particular question posed by the interviewer[8]:

> I: So do you think that these pupils were seeing D as the quantity of doctors or as doctors?
>
> P: The problems is that they are not analysing their answer; the type of variable that D and N are; they are not asking [themselves] 'which one is the independent variable, the doctors or the [illegible]?'

Pablo would use a situation like this in Grade 8 to teach the concept of variable, "It would be a good situation to take encourage them to become familiar with the concept of functional relations; and the tabular representation is useful for them to analyse the type of variables [dependent and independent], how one varies depending on the value of the other".

In relation to the use of the variable to represent the unknown number, Pablo anticipated that the pupils would encounter specific tasks and situations, especially in their textbook where the variable was used:

> I want them to first see that the letters there [in the tasks with rectangles. See Figure 6.1] are representing any number, according to the particular situation we are talking about. Of course they will work later, with the rectangles, where the letter is a specific unknown. All the textbook exercises are about that and they have done that in Grade 7". ... In the work with areas and perimeters they are generalising, it [illegible] like showing a formula. (Int.2)

[8] There is no data about Pablo's awareness of the need to help the learners clarify what the variable represents, for example, when using the term "Cost" (see his example in Subsection 6.2.1).

But although in using letters to show a formula, letters were used "to generalise", it is not clear whether Pablo saw the variable as an arithmetic pattern generaliser, as at Interview I Pablo stated that when teaching the properties of operation with rational numbers, he had to just tell because that was theory and he could not find activities to help the pupils make sense of that work.

Pablo was aware of the need to cater for pupils' differences in abilities and self-paced learning, and considered it important to give attention to this aspect when teaching and designing activities. He also saw the importance of the pupils' contact with different resources that could assist them in their learning, and acknowledged that he needed to learn more about how to use some computer facilities. In relation to the use of spreadsheets in the introduction of the concept of variable he said during the Focus Group session: "I never thought that spreadsheets could be used in such a way to help children construct the concept of variable. "We see that we have the resources there, waiting to be used but I have to learn how to use them more efficiently".

Knowledge of the interrelatedness of mathematical concepts and of different representation systems

Pablo provided evidence of his knowledge of the inter-relatedness of some mathematical concepts, and of the role of different systems of representation in the learning of the concept of variable. In Question C3, where a pupil wrote $A = 5\ (2 + e) = 10 + 5e = 15e$, for the area of a rectangle of sides 5 and $2 + e$, he explained that he would "ask the pupils to draw the rectangle and see how he was calculating the area of each part" of the big rectangle. This response and his design of tasks in the observed lesson showed Pablo's awareness of the use of two types of representations to help the pupils make connections. However, it is not clear how this would work in the teaching-learning situation as we saw that, during the lessons, pupils' thinking was not explored to try to understand why pupils were not making the connections he expected.

Knowledge of curricular materials and of the curriculum

Pablo was aware of the dynamic interaction between his teaching goal of teaching for understanding and the teaching method he argued for. He was critical of the textbook approaches which only gave a rule and a list of exercises saying "Do this in this way". However, his awareness of the need to monitor pupils' work continuously, and of the fact that "tests do not tell you what pupils know", did not lead to his questioning of the school's assessment system. According to Pablo, "reporting on pupils' work and progress in a different format from the one established by the school [would] require more time that was not available".

Knowledge of the context of teaching beginning algebra

Pablo provided evidence of his knowledge of the context of teaching. He used his knowledge about his pupils' activities from other school subjects like Physical Education from where to draw when looking for relevant context for the classroom activity. He also knew that to learn with meaning, pupils needed to be provided with work that allowed them to make connections between familiar situations or mathematical concepts and the concept being explored. They needed to interact and discuss their work, "hopefully each making an input". However, there was no evidence of Pablo's awareness of the need to inquire into the pupil's mathematical thinking and, therefore, of his knowledge of how to make their thinking the basis of the teaching for understanding he was trying to pursue.

As already pointed out, Pablo also acquired knowledge of how the school functioned, and felt that he had to fulfil the expectations of the pupils and their parents who were "very powerful" in the school.

6.2.3 Summary of Pablo's case

Pablo conceptualised beginning algebra as a collection of interrelated topics, concepts and procedures whose application to real life situations made it worth studying. In order "to help pupils see the application of algebraic ideas and the connection between concepts", he wanted to design tasks and activities where they could use their already learnt mathematical concepts. However, having to teach "topics like Rational, Irrational and Real numbers" where he could not see any way of providing connection to contextual situations affected his level of motivation for the teaching of Grade 8-algebra. He saw that since those units (topics) contained "the laws and properties of operations for each set of numbers", which, at the same time, "govern operations with algebraic expressions", they had to be taught prior to the introduction of algebraic expressions.

Pablo strongly disagreed with teaching beginning algebra by the textbook because "textbooks have just a list of exercises to apply a given algorithm", and he wanted the pupils to "construct", at least some of the algorithms of procedure to operate with the expressions. To do this he used the concepts of area and perimeter of rectangles, and other activities designed by him to help pupils to construct algebraic expressions and to establish some basic procedures of operation by themselves.

Of his two preferred forms of assessing pupils work, Pablo only managed "to monitor pupils' work by collecting their notebooks" but he did not manage "to do tests where the pupils could apply what they had learnt during a period of time without portioning the topics". This was due to the fact that he had to hand in the pupils' grades, according to the content objectives specified in the program for each term, at specified dates.

Pablo emphasised that one of his teaching goals for Grade-8 algebra was to help the pupils see the *raison d'être* of what they were learning, but found that his activities did not produce, for some pupils, the results he was expecting to have. He explained that this was due to the inadequacy of the activities he designed, as he needed to improve his knowledge for teaching. Throughout the first five months of the data collection process Pablo maintained that the crucial factors influencing his teaching were his knowledge and his dispositions as a teacher. However, as the end of school year approached (September/02), he started to put more emphasis on "the great limitations that time represent[ed] for his teaching". Despite the fact that he pointed out that the activities of Teacher B (of Questionnaire 2) or the ones discussed at the Focus Group were excellent to help pupils make sense of the work, he argued that that type of activity required much time in the classroom that he could not afford. He felt pressured by the expectations of his pupils and their parents who were "very powerful" people in the school. While improving his teaching meant designing activities that were motivating and helped all pupils understand, they had to be shorter than the ones he had used because time was short and he was behind with the program, so he had to "hurry up" when teaching.

Pablo declared that he saw himself as a teacher who has to keep learning all the time, not by being told, and he stated that rather than adopting an innovative textbook, he was prepared to give up his job in order to engage himself completely (full time) on a proposed research (learning) project, provided he had "some income to survive". He showed great enthusiasm about working as part of a research team or about team teaching.

6.3 Nora – The mathematics language translator

During the last two years Nora has been the head of the Mathematics department for the afternoon session of a state school that caters for economically disadvantaged children in Bogotá. Nora was teaching "only twelve hours per week" (and not 20 as her full workload would require) because, as the head of department, she had "two specific tasks: i) to attend regular meetings with the school's area representatives of local educational authorities, and ii) to provide leadership for curriculum development for [her] colleagues". Nora was also "teaching mathematics in a private school during the mornings". She had been teaching secondary school mathematics for 17 years, with 7 years of teaching in Grade 8. She had done "a one-year postgraduate course in mathematics education and, together with [her] colleagues, had been working "on two classroom centred action research projects in previous years". Nora participated with great enthusiasm in this study, dedicating more time to the interviews than initially asked, and suggesting extensions to the data collection activities.

6.3.1 Nora's conception of beginning algebra

According to Nora's responses to Questionnaire 1, the '*Number 1*' reason for the teaching of algebra in school was "because algebra represents a key tool, needed by every individual, for solving real world problems". However, her main aim for the teaching of algebra in Grade 8, as stated at Interview 1, was "that pupils be able to operate with algebraic expressions". This is because "in Grade 8-algebra, pupils need to acquire fluidity in operating with algebraic expressions before they move on to the solution of word problems". Nora emphasised what the nature of Grade 8-algebra is, on more that one occasion. When talking about her preference for Grade 9-algebra to that of Grade 8, she noted:

> The algorithms of operation are what Grade-8 algebra is about. The problems of application will come later in Grade 9 or in higher levels of mathematics. That's why I don't like much teaching in Grade 8 because it is a repetitive work. It is monotonous. (Int. 1)

Algebraic thinking "starts when they start to use letters or symbols to represent numbers". Because "in Grade 8 the pupils have great difficulty with algebraic expressions", Nora decided to give the pupils in Grades 6 and 7

> things like 7. (b + 5) = 21, where they use the letter to represent unknowns, or I give them things like doing translations —that is, writing with letters, expressions given in Spanish.

But although this type of exercise can be started in Grade 6, what Nora calls *algebra* is what is done in Grade 8:

> I: Is that Grade-6 algebra?
>
> N: That is not Grade 6- algebra; it's some exercises to provide pupils with that type of language, so that they will be familiar with it when they arrive at algebra.

Teaching Grade-8 algebra did not need new decision making or reworking of initial plans as what the teacher had to do in the classroom had been decided well in advance, at the beginning of the school year. In the same way that two of Nora's colleagues, who participated in this study explained, Nora declared that she did not need to do lesson planning:

> The work for each grade is planned at the beginning of the school year. We establish, at the start of the academic year, the content objectives and the assessment indicators for all the Grade 8 groups, and individual lesson plans are not done because time is short. (Int. 1)

In describing what things she thought about before she taught a particular lesson, she repeated her view of *algebra* but, this time, she provided information that said something about the power of her conceptions of *assessment* in her conceptions of algebra teaching:

> I think of the subject contents, the resources we have, and how I am going to assess pupils in that lesson. For example, now that we are in 'Factorisation', I give them a quiz each day. Why? Because factorisation is something mechanical, something that they learn in the lesson but they forget very easily. So each day I teach one case, and they have a quiz; but each quiz includes the previous cases that we had seen before. (Int. 2)

For Nora, real algebra was composed of the traditional exercises where pupils are given some algebraic expressions to manipulate. At Interview 3, she was asked to explain what she (and other teachers) were referring to, at the Focus Group session, when they used the word *formalisations.* while they were discussing the activity of 'Pedro the builder[9]:

> I: I was curious to know what you, and the other teachers, were referring to when you were asking, how the pupils moved on to the formalisations.
>
> N: What mathematics is seeking of students is to handle the formal part. From the activities that we have seen from these different teachers [in Questionnaire 2 and the Focus Group], I have concluded that through the method that we use in class we have to make the pupils get in love with mathematics helping him [sic] to see the application of mathematics. So when the pupils arrive at the algebraic expression, they should work in the opposite direction: 'Here is the expression, now you have to operate with it'. Pupils need to do this type of work. They need to spend time reinforcing it in order to be well prepared for the following school year.

Nora commented that she had been working in an action research project that had provided her with evidence of the fact that

> children do not learn mathematics by engaging in activities where what they do is at the level of *la parte lúdica* (i.e., play and exploration). The pupils became motivated and enjoyed what they were doing in class but they did not learn what they had to learn. For example, they could not operate properly with rational or with irrational num';ers. They did not know how to do for example additions or multiplications with certain fractions, and assessment had to focus on the pupils' abilities in mathematics... (Int. 1)

6.3.1.1 Learning beginning alg?bra

Nora's conceptions of how pupils learn beginning algebra were explored on different occasions and in different ways. While her responses to sections A and B of Questionnaire 1 —where they had to either rank the given descriptors about the teaching of school algebra, or provide their own ideas— showed a progressive (i.e., child centred) view, her explanations for these responses, at Interview 1, showed an approach based on transmission. In Questionnaire 1 Nora ranked '*Pupils developing efficiency in applying algorithms and formulas*' as her last of the seven given options. However, at Interview 1 she argued for "pupils' acquisition of fluidity with the manipulations of algebraic expressions" as the most important achievement of pupils. She said:

> What I most want my pupils to learn in Grade 8 is to acquire fluidity with the expressions because that is what they need in the following year.

[9] This was the name of an activity, described during the Focus Group session, whose intention was to provide the learners with a simple real-life context to help them construct generalisations, expressing the number of tiles Pedro neede:l for specifics arrangements to pave houses doorways. (see Appendix 4.2).

It is interesting to see that Nora's responses to Section C, where the questions were related to a specific teaching learning situation, showed again the transmissive and instrumentalist approach she had described when explaining her responses to Section A and B of the Questionnaire. As will be seen later in the subheading *'Providing space for pupils' self-paced learning'*, Nora believed that pupils learn by working individually "because that is the only way they can see if they have learned". She also believed that although you need algebra to solve problems, "for example in geometry", algebra algorithms are learned first, and then they are applied in geometry. When describing how she taught "*Productos notables*" [i.e., identities like $(a + b)^2 = \ldots$], which in Colombian Grade 8-textbooks are presented as "special products', she explained that

> We don't make use of geometry to teach concepts in algebra, we do the opposite: they learn the algebraic algorithms so that they later apply them in the solution of geometrical problems.

6.3.1.2 Teaching beginning algebra

Without adding any other options of her own[10] to the descriptors provided in Questionnaire 1 for teaching styles, Nora organised the given descriptors for her *preferred* practices, from 1 to 7. She gave similar rankings for her *actual* practice, though not for her two-first preferred choices, as can be seen in Table 6.4. Her explanations for the differences in rankings and her conceptualisation of her teaching practice are presented in the headings related to her three first 'preferred' teaching styles, in the following subsections.

[10] In Questionnaire 1, respondents were provided with space to add their own ideas and comments about each question.

Table 6.2 Nora's teaching style priorities

Preferred	*Teaching style descriptors in Questionnaire 1*	*Actual*
1	Organising problem-based activities for the pupils to work in small groups where they can present their ideas to the whole class for discussion	2
2	Giving clear explanations of definitions and procedures to follow in different exercises and problems of application, in the topics studied	1
3	Designing activities that provide space for pupils' self-paced learning	3
4	Providing opportunities for pupils to develop their communication skills so that they can express their mathematical ideas with confidence	4
5	Designing classroom work that promotes connections between different mathematical topics studied	5
6	Giving pupils lots of exercises for algorithm application as homework	6
7	Testing pupils at the end of each activity or topic, in order to have sufficient marks for assessment in each Attainment target	7

Priorities 1 and 2 for preferred practices: Organising problem-based activities' and 'giving clear explanations

At Interview 1, Nora explained why her 'Number 1' *preferred* practice *of 'organising problem-based activities, where pupils could discuss their ideas*' became a 'Number 2' for her *actual* practice. In doing so, she justified why '*giving clear explanations*' was *her* 'Number 1' actual teaching style. An initial reason was, because of *time* constraints and the fact that "some *pupils* work faster than others", but she later clarified that the reason was the nature of Grade 8-algebra, as can be seen in the following explanations:

> I would like to start [classroom work] by that type of activity where one gives the problem and the pupils arrive at the idea, but that is not easy; that is quite hard. Above all, one does not have the necessary time. There are some pupils that work fast and others who work at a very slow pace. Many times I end up changing [what I had in mind]; I cannot hold back and end up giving them all the explanations that I didn't want to give, to help them end the struggle in which they are.

But thinking again, 'time' was not really a strong reason for the actual teaching 'by telling' style. "A more important reason [was] that many topics of Grade-8 algebra do not lend themselves to a different teaching approach". At a later point during Interview 1, Nora clarified:

I: But is it because you don't want to see the pupils struggling with the work or because the pressures of time make you think that letting the pupils use whatever time they may need is not possible?

N: No no! it is not to save time! It is that to work that way is not easy, especially in the topics of Grade 8. There are other topics, for example, in geometry. An example of topics where the teacher doesn't' have to tell, in geometry. In the properties of a triangle, for example, I tell them 'draw the triangle, measure it, use the protractor. Cut the triangle out and fold it, and see in what way you add the internal angles'...

Further questioning in relation to this explanation provided more evidence of Nora's beliefs about the nature of algebra knowledge. Once again, she described algebra as procedural knowledge to manipulate given symbolic representations, which is learned in isolation from other concepts and from contextual situations, in a mechanistic and impersonal manner.

I: Are you saying that when you use situations brought out of geometry, and leave them to explore those situations, then, it is easier for them to arrive at some algebraic ideas, or to some of the algorithms you were thinking of?

N: The algorithms? The ones specific to algebra? In algebra no. In algebra that part takes place in the opposite way. First they have the explanations and practice and then they apply that in the solution of problems in geometry. Because I think that, in algebra, the boy [sic] has to/ initially, he has to learn a lot of the algorithmic part, to do some drill, and later they will have the problems where they apply that.

Nora continued explaining that "solving problems is the most difficult part of algebra; that is, to set up the equations". For this reason in Grade 8 she gave her pupils "the translation exercises", where "the pupils have to write in mathematics, the specific expressions that are given to them in Spanish". The translation exercise was the type of work to which the sequence of five lessons observed was dedicated. In order to provide an illustration of what Nora referred to as "the translation exercises", a description of the activity developed in the classroom work observed is presented in Appendix 6.2. Figure 6.7 shows the first five questions (of the 25) of the "*guía*"[11] she gave her class during the first lesson observed. This *guía* was "to be used by each of the five Grade 8 teachers" in the school, as was observed in the classroom work of Stella, the other teacher from Nora's school who was selected to form the group of nine teachers who participated for the follow-up study in Phase 2.

[11] The "guías" for Nora and her colleague Stella were a list of exercises given to pupils to work in pairs or in bigger groups.

Write an algebraic expression to describe each of the following concepts or situations:

1) The perimeter of a square ____________________

2) The perimeter of a rectangle ____________________

3) The volume of a cube ____________________

4) The distance travelled by a car in 2 hours __________

5) The sum of three consecutive even numbers ________

Figure 6.7. The first five questions of Nora's "guía"

When the second lesson (a double lesson) finished, Nora commented, again, that doing the work on translations was very important; "it prepare[d] the pupils for the setting up of equations when they arrive to the [word] problems", but she seemed not to pay attention to whether the pupils were assigning meaning to the letters used, and if so, what meaning. I asked her if she thought that when the pupils wrote $x + x + x + x$ or $4x$ as the expressions representing the perimeter of a square, in Question 1 of "the guía", the pupils were using the letters to represent variables. She replied:

> as they wrote x for the side of the square, they know it is for any square, so it is a variable.

It seemed that Nora did not pay much attention to the issue of whether the pupils could have been puzzled by the different meanings or uses of the variable, the questions she was giving them required.

> I: Yes, in the lessons they referred to x as the variable. But what do you think they thought about the x, the variable, in the following questions given in the lesson, for example, this: 'The sum of two consecutive even numbers is 38. Find the numbers'?
>
> N: Then the letter takes one value because there is a restriction.
>
> I: Why do you think they found the questions of the *guía* so difficult?
>
> N: I knew that those types of situations were going to be difficult for them because that is what we have found in our practices; but those are the situations that one should give them, despite the fact that they find them so difficult.

At Interview 2, after the sequence of five lessons had been observed, Nora explained that the purpose of the *guía* was "to take the pupils little by little in the translations" because to solve a problem they need two things: one is to set up the equations and the other one is to solve the equations. Setting up the equations is what is most difficult for them. If the boy [sic] manages to translate the information, the rest is easy.

According to Nora this lesson was, "in a way, a successful lesson because despite the fact that it was difficult, *the group*[12] showed interest and started giving values...for example, in the question of speed, and that was a rich experience for them".

A successful lesson is the one in which pupils show interest in the work that is given. The success of a lesson depends in part on the motivation of the pupils and the acceptance I have on the part of the pupils.

Despite the fact that some pupils started giving values, Nora's focus in the classroom work observed was on the *translations* and not, for example, on exploring concepts like *change* or *variables* to express relationships between quantities, as that work was "the subject of Grade 9" algebra:

I: Do you think that situations like the ones they had in that *guía*, for example the one of Question 4 of the distance travelled by a car, could be used to explore with them, concepts like change, the concept of variable to express a relationship between quantities?

N: Well, of course; but that would be when they see relationships and functions in Grade 9, but here we were doing just the translations.

Without acknowledging it, Nora made it clear that her teaching of "Grade 8-algebra" followed the sequence laid down by pupils' textbooks.

Priority 3 for preferred and actual practices: Providing space for pupils' self-paced learning

Nora explained at Interview 1 that she had identified the descriptor '*Designing activities that provide space for pupils' self-paced learning*' as her priority 'Number 3' because her pupils were given space to ask any questions related to the exercises or the homework. In explaining this she clarified that pupils learn by working individually, doing repeated practice.

Our [the group of teachers] way of dealing with pupils is by asking them, 'what doubts do you have about the work you did at home or about the work we did in the previous lesson?' for they each have many questions about the homework... They themselves go to the board, and their peers explain or correct them and, as a

[12] See Section 6.3.1.2 - 'Nora's additional concept map' for Nora's further explanation in relation to what she meant by 'the group'.

final resource, if the majority have the same difficulty, I do it. I insist a lot in pupils' individual work because that is the only way they can know if they have learnt. (Int. I)

However, if the difficulties persisted it was due to the fact that the pupils did not do their homework, and there were specific days for pupils to be able to ask questions. Consider Nora's explanations for the cases of Alfonso and other pupils, who were called to the board during one of the lessons observed, and could not do the exercises but were not encouraged to ask questions or to explain their difficulties. See Appendix 6.2 for a description of this classroom incident.

I: But there are specific lessons or moments for pupils to ask questions in relations to their homework? For example, during the first lesson I was observing, when Alfonso and other pupils were doing the exercises on areas (Nora interrupts).

N: That day was no day to do that because we had clarified multiplication with expressions in previous lessons, but as they don't do the homework then/ I have told them" 'I am not going to argue with you because you don't do your homework, you hand in you work, I mark it and give it back to you.

Nora valued pupils' working speed when doing the exercises. Further, she spoke of what her fastest pupils (which were a small number) did as if they were the whole class (see Section 6.3.2.1 –'Nora's additional concept map). When asked why she gave an exercise for the first five:

They care about competing. They like it; although I pay more attention to their reflections [pause]. The girls work slowly, they are not very interested in the work.

Assessing pupils' work

According to Nora's responses to Questionnaire 1, the two first *preferred* forms of assessing her pupils' work and learning were, *i*) 'Pupils' folders and assessment records, showing evidence of several aspects of the process followed throughout a term or a set of terms' and *ii*) 'Pupils' own reports about their progress and their difficulties'. However, at Interview 1, Nora explained that although she would prefer that pupils' folders were the way to assess pupils, it could not be the first way to assess them because a first requirement was to provide evidence that the pupils were able to do the mathematical tasks. So "frequent quizzes", her first *actual* form of assessment, was necessary:

One would like to just ask the pupils to keep all their work in their folders and have there all the evidence of what has been done, but when the time to decide whether he[sic] is promoted or not, if he reached the attainment targets or not/ and very

> frequently we have the student asking '*profe*, why did I fail? So one has, necessarily, to resort to that [to frequent quizzes] because one needs the support 'look there is evidence of your difficultly'. I can have a lot of participation and interest [during the class], but one cannot be that subjective to give them a mark just by their interest. I need evidence that he can deal with a mathematical situation. And because we have to comply with the requirement of giving marks that (i.e., quizzes] has to be done.

In relation to her second choice of '*Pupils' own reports about their progress and their difficulties*', Nora explained that "pupils keep a folder with their quizzes and the *guías*, and a record of their own assessment... Their attitude and their responsibility are the focus of pupils' self-assessment":

> In pupils' self-assessment, they have to assess their attitudes and their responsibility. When we are doing the assessment, I tell him [sic] 'how has your responsibility been?' He says, 'No, I don't understand'. We make a lot of emphasis in their responsibility. So 'If you haven't given yourself those opportunities to work yourself [doing the homework] then you cannot ask me why you failed'. (Int. 1)

The assumption that "pupils do not understand algebra because they don't do their homework" was not only Nora's assumption. Two more teachers from Nora's school complained, at Interview 1, about the fact that 'the bad results [were] due to the fact that pupils, in this school, don't do their homework'. We saw in Chapter 5 that the majority of teachers identified the pupils' lack of motivation or the pupils' lack of pre-requisite knowledge as a factor that explained the teachers' dissatisfaction with the results of their teaching.

6.3.2 Nora's conception of her own teaching of beginning algebra

In the previous section we saw that Nora's explanations for the differences in her rankings of *preferred* and *actual* practices, and for specific classrooms incidents were related either to the pupils' lack of work habits or to the nature of Grade 8-algebra. In this section we will learn about the factors she identified as the main determinants of her teaching practice through the three-phase concept map activity. In *Part 1* of her of initial map, Nora identified "the pupil's prerequisite knowledge and their attitudes and motivation" as the crucial factor influencing her teaching practice. Prompted to think of the reasons for the pupils' lack of motivation, she decided to emphasise the primacy of "the teacher's vision and knowledge" as an aspect that impinged on "the teacher's practice" as will be seen from the findings of the concept map activity in the following sections.

6.3.2.1 Nora's conception of the determinants of her teaching practice

Nora produced the following list, when she was invited at Interview 2 to think of the factors that influenced her teaching of Grade-8 algebra, and to write them down (refer to Subsection 3.4.3.2 for guidelines for the concept map activity):

- Time
- The pupils
- Curricular guidelines
- The programs
- Pre-requisite knowledge and attitude of the pupils

After she wrote the last item of the list, she immediately started explaining:

> *Curricular guidelines and the programs.* One starts by the curricular guidelines for which there is certain time available. We have our [assessment] indicators for each school term and for the whole year. *The pupils.* The pupils' prerequisite knowledge and attitudes influence [my practice]. So I would say that yes, the pupils' attitudes influence my practice.

Nora's initial concept map – Part 1

Nora was asked to start building the concept map, drawing any connecting lines. After she had built the map shown in Figure 6.8, she kept on looking at the map she was building and there was a long silence.

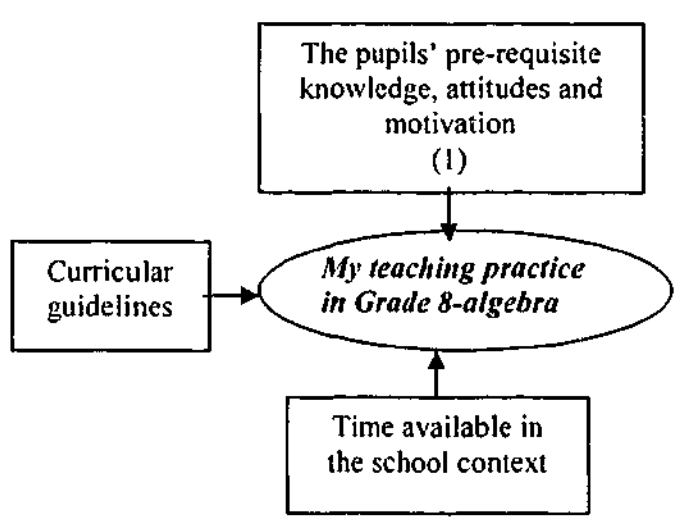


Figure 6.8. Nora's initial concept map – Part 1

She identified "the pupils pre-requisite knowledge, their attitudes and motivation" as the "Number 1 factor" influencing her teaching practice. This is what she said when asked if each of the three factors named influenced her teaching with the same level of strength:

> The 'Number 1' is the pupils' attitudes and motivation. It is not the same to work with a group of pupils who are willing to learn, highly interested, highly motivated, to another to which you have to inject some energy each day (silence).

But when I asked her about the possible reasons why pupils become motivated in their mathematical work, she started talking about the role of the teacher's vision in her/his practice. In talking about the teacher's vision Nora offered more information about her conceptions of beginning algebra knowledge.

I: On what, do you think, the pupils' motivation depends?

N: That depends (pause) on the type of work that is proposed to them.

I: And on what does the type of work that is proposed to them depend?

N: It depends on the teacher's vision.

I: What do you mean by the teacher's vision?

N: Independently of the time and the goals, the teacher must have a consciousness of what mathematics is for him [sic] and what he wants to achieve with his pupils. What I propose to the student/ right now, with these Curricular Standards[13], we have the task of laying out our work again. The important thing is the approach that I give to the content.

I: The approach that you give to the content. Could you explain what that means?

N: We received the Curricular Standards document from the Ministry, and we have the task of re-organising our work plan, but that [The Standards] is just a list of contents, that is not the Curricular Standards. That doesn't bring anything that helps us reflect on our practice; it contains just the same list of contents that we know of.

I: Is it the same list of contents, for example, for algebra?

N: Well, for Grade 8 it is practically the same list, and as I said the other day, the work with the variable doesn't start in Grade 8, it can be initiated in previous grades but according to The Standards it starts in primary!

At this point Nora was asked about the role of a teacher's knowledge in his/her teaching of beginning algebra. She declared, at this specific point, that a teachers' knowledge was the factor 'Number 1', and this prompted her to change her map, providing *Part 2* of her initial map.

[13] As explained in Chapter 1, the 'Curricular Standards for Mathematics' released in 2002 (see Appendix 1.1) sets "*variational* and algebraic thinking" as "one of the components of the mathematics curriculum" across the 11 year school cycle.

I: How do you think the work with the concept of variable could be started in primary?

N: I imagine that in primary they can take simple things like the ones I told you I was doing in Grade 6, but simpler than those ones. But that's why I say that it just brings a list of contents. It doesn't bring (silence).

I: So the work that the primary teacher proposes depends on . . . ?

N: Her/his knowledge. No! of course! It depends on the teacher' knowledge!!! The teacher will have poor ideas if she has a poor knowledge.

I: The teacher's knowledge. Where would you put the teacher's knowledge in your map and what priority would it have for you.

N: It is the number 1!!! Ha ha ha! [Nora laughed and her laughter was made louder by the interviewer's laugh]. And the teacher that needs to be more knowledgeable is the primary school teacher. The path to arrive at algebra has to be started in primary. My knowledge is the central point! My knowledge is the central point; from it everything derives! I start again [building her map].

Nora's initial concept map – Part 2

In constructing *Part 2* of her initial map, Nora eliminated the "The pupils' pre-requisite knowledge, attitude and motivation" box replacing it with "The teacher's knowledge". When asked to specify this, she named it as "The teacher's knowledge of mathematics". Nora tended to talk of "the teacher's knowledge" rather than of her own knowledge[14]. In further discussion, she identified "The teacher's knowledge of mathematics" and "the teacher's knowledge for mathematics teaching", which she preferred to call "The teacher's pedagogical knowledge":

I: But what specific knowledge are you thinking of, because here we are talking about 'My teaching practice of Grade 8-algebra'?

N: The teacher's knowledge of mathematics. Because algebra is one part of mathematics but to teach algebra you need knowledge of arithmetic, of geometry and, obviously, of algebra; and that's why I said that the primary school teacher is the one that needs to have more knowledge because they have to teach everything! And the *Standards* bring now data handling! so (silence).

I: Okay. The teacher's knowledge of mathematics.

N: The teacher's knowledge of mathematics has to do with the intellectual preparation, with the teacher's constant training. We teachers, who have to deal with persons, need to be constantly looking for new possibilities because the young are changing.

I: Apart from 'knowledge of mathematics' to teach algebra, does the teacher need other types of knowledge?

N: Of course, the teacher needs knowledge of psychology, of pedagogical currents; knowledge of pedagogy. ... I would call it pedagogical knowledge. ... The teacher's mathematical knowledge and the teacher's pedagogical knowledge are different. An

[14] Nora tended to talk of "the teacher's knowledge"; she only said "My knowledge" when the interviewer remainder her of the fact that "we [were] talking about "My teaching practice", and not about the practice of any teacher. I felt it was not comfortable for Nora to talk about her own knowledge (i.e., to use the expression My knowledge, so I stopped insisting on that).

> engineer can have good knowledge of mathematics but not knowledge of how to teach…!

It will be seen in the following subsections that Nora did not talk about her own knowledge, as other teachers did. Eliciting teachers' information about their own cases and about their beliefs of their own knowledge is not an easy task. There were tensions for me between gathering information about the teachers' beliefs about their own knowledge, and not placing them in uncomfortable situations. So if, at a specific point, the teacher was talking about "the teacher", and not about their own case, that had to be accepted because I was aware it was a sensitive issue (see further discussion of this point in Chapter 8, Subsection 8.4.3).

Figure 6.9 shows the map that Nora completed with the numbers she wrote when asked to show the level of influence she thought the different factors played in her teaching.

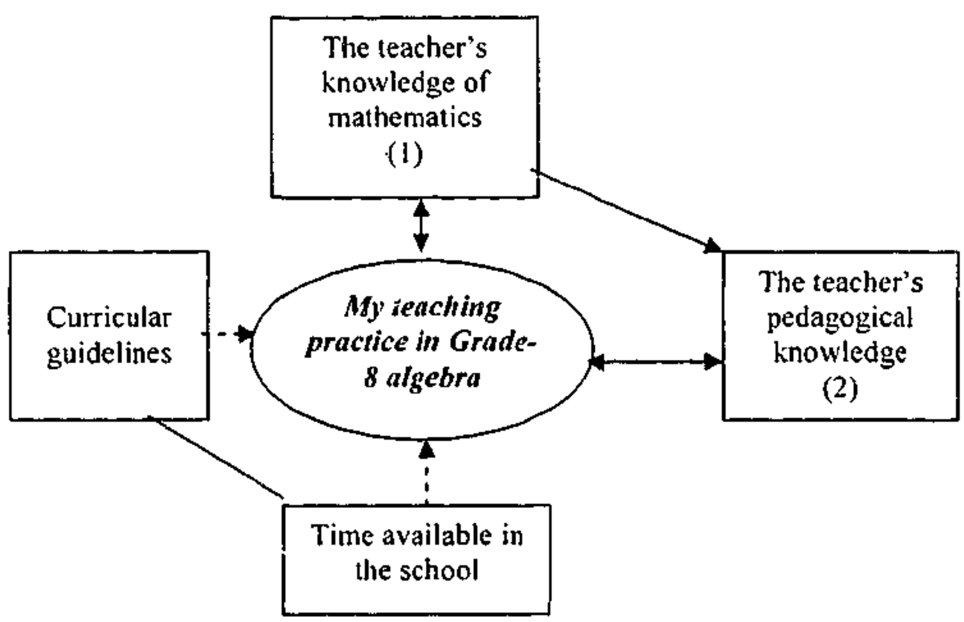


Figure 6.9. Nora's initial concept map – Part 2

As can be seen in her second part of her initial concept map (Figure 6.9) and in her explanations about this specific map, Nora did not mention the pupils' attitudes as a factor that influenced her teaching.

> N: I can have much knowledge of different aspects and resources for the teaching of mathematics but if I don't have knowledge of mathematics/ definitely, 'My knowledge of mathematics' gives me that vision of its teaching. In the measure that 'My practice' is rich it generates new possibilities. I have to connect the two [types of knowledge], and the two influence 'My practice.' Does 'Time' determine what I can do in my classroom? Yes, but it is not the excuse that because of *time* I cannot do an

effective work/ or to say that it is the cause all the difficulties. So I put this [connected] with a broken line. Curricular guidelines do not affect my knowledge, maybe they create, amplify my vision.

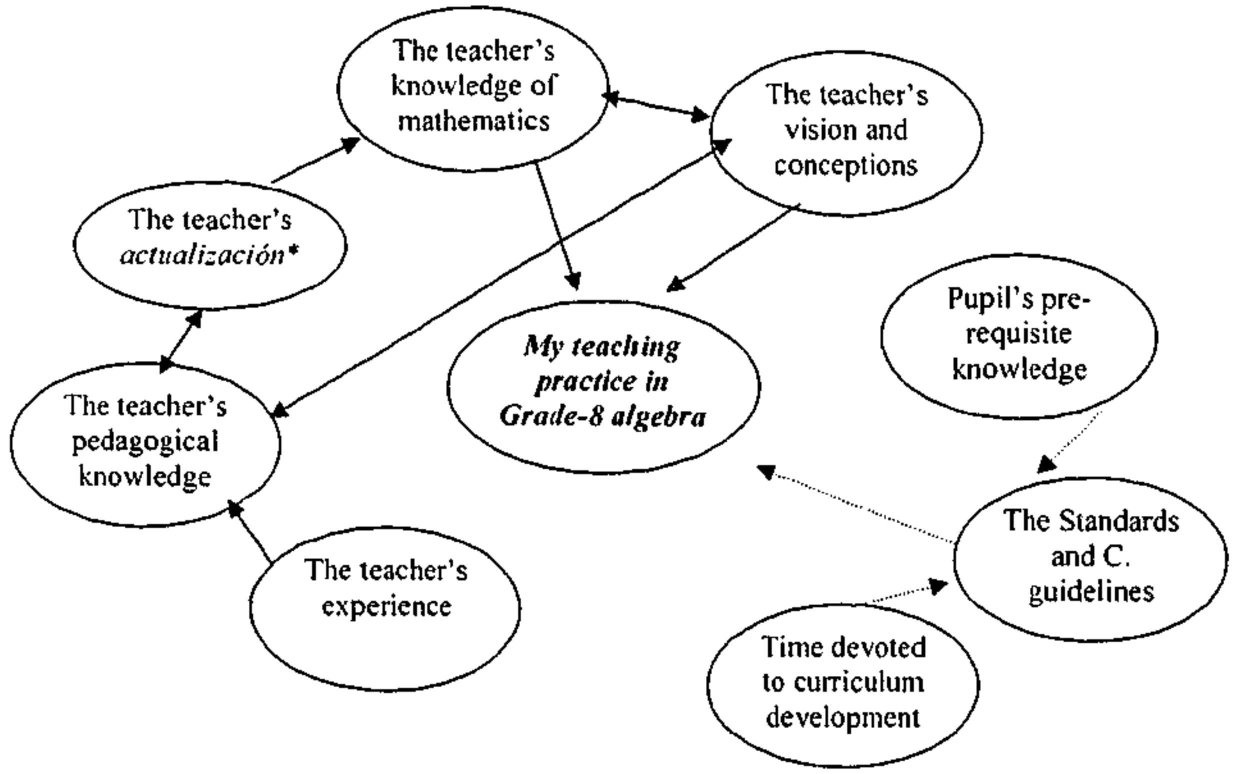
I: So pupils' attitudes don't go in this map?

N: They have to be considered, but the teacher's attitude would have to be considered as well. Pupils are changing. This world is changing all the time.

When the time available for the interview was up Nora said that she wanted to dedicate more time at home to review her concept map, and that she would bring it in next day.

Nora's additional concept map

The map that Nora drew at home and brought in next day, as agreed, is shown in Figure 6.10. In this map Nora included more factors belonging to the teacher (i.e., internal factors). This map had been made to show a stronger influence on her teaching from factors that belonged to "the teacher" than to those that were external. When asked to explain this map, Nora made emphasis on the teacher' knowledge and vision, providing very similar explanations to those of her colleague "with whom [she] had been working as a team".



*The equivalent in English of the Spanish word *"actulización"* is 'keeping up to date'. Note that Alex, one of the five case study teachers also referred to the teacher's *"actulización"* during the concept map activity.

Figure 6.10. Nora's additional concept map

The following are the initial explanations Nora provided for this new map:

> The broken lines show aspects that one takes into account; they influence [my practice] but not in a decisive way. These two: ['The teacher's knowledge of mathematics' and 'The teacher's pedagogical knowledge] are fundamental. To me, having knowledge of mathematics is fundamental but this knowledge can be strengthened in the measure that I integrate various elements of my experience. I can know much mathematics but if I have not been in contact with the students and see what the situation that I have to dealt with, the difficulties, then it is going to be difficult to have knowledge for teaching.

These are Nora's explanation for what she named "the teacher's "*actualización*":

> If the teacher has the opportunity of working as a team, to enrich himself/herself as a department/ one can have richer experiences, learn more by working in a team than by doing a one year specialisation. Furthermore, the opportunity to increase one's knowledge, to learn, is even greater if one works in a project about your teaching practice in order to look for other alternatives.

The teacher's vision – What did it mean for Nora?

Nora was asked, twice, to explain what she meant with the teacher's vision and conceptions. In the first opportunity, she drew from the cases of the two teachers described in Questionnaire 2, offering once again evidence of her teaching style priorities in Grade 8-algebra:

> I: What is the difference between your knowledge of mathematics and your vision of mathematics?
>
> N: Basically, two people just graduated, you could say that their knowledge of mathematics can be the same but when they teach, the lesson, what is important to me? Let's see the case of the Teacher A and teacher B [of Questionnaire 2]. They give importance to different things: one teacher gives importance to do 10 exercises from the textbook. For the other teacher it is more valuable that the pupils did one problem and arrived at a solution.
>
> I: What of the two do you find more valuable?
>
> N: Well I cannot say that teacher's B approach is not valuable for me, and although Teacher's B approach is the ideal to initiate the introduction of algebraic expressions, the type of work Teacher A does is necessary because some students need reinforcement, and they should handle the mathematical language to be able to carry on in their following grades.

In the second opportunity, Nora was asked to draw on the work of the *guía* used in the observed lessons (refer to Section 6.3.1.2 if necessary) to explain what she meant with the teacher's vision".

> I: Could you explain the meaning of 'the teacher's vision' with, for example, the classroom work I was allowed to observe, the work with the *guía*?

N: In those lessons that you saw, I was seeking that the pupils arrived a the representations of what we normally say in Spanish... In the case of the speed situation, they took numerical values. That very process is already 'a riches'. So in that example I see my vision. My vision is that it doesn't matter that the student has that difficulty; the important thing was that he [sic] questioned himself about that. ... They took those numerical values and that, for me, was a rich experience for *the group.*

Nora always referred to the work of the fastest pupils as the work of "the group". In order to try to identify if she was aware of the fact that there were only about 5 or 6 pupils[15] from a group of 35 who had started doing something with the tasks of the "guia", I asked her what she meant with "the group" for that particular lesson. In doing so she clarified that all the class had to be given a mark for what they had done during the lesson, even if the classroom experience had been a rich one only for some pupils:

I: When you refer to 'the group's work', in the context of the lesson that we are talking about now, are you talking about the whole class or about the majority of the class or about a group of pupils?

N: Well, it was not for the majority; for some pupils.

I: But will they all be given a mark for what they did in that *guia*?

N: We have concluded that we have to mark what we collect, because if one doesn't mark what they do, then, the next time they don't do any work...

These explanations did not show consistency to me with what she had said at Interview 1 about her priority of 'Providing space for pupils' self-paced learning' (refer to Section 6.3.1.2). Therefore, I asked her again about her provision for pupils' differences in their learning processes, but this time Nora declared that she was not doing what she could do.

I: How do you deal with the ones that need more time or show difficulties?

N: For the ones that need more time or have difficulties, I would have to give them a different type of work. I am not doing that with this group, but I could do that. We have good support at this school for introducing innovations. Further, no one can stop a teacher from doing what he wants in his classroom.

Nora had said when explaining her additional concept map that the teacher had the opportunity to learn when s/he worked in a team and when s/he reflected on her/his

[15] This information was obtained through constant observation of the pupils work as Nora suggested that I could walk around the classroom and talk to the pupils if I wanted to.

teaching practice. I asked her about any learning emerging from the lessons she had invited me to see:

> I: Would you think that this classroom work that we are talking about provided some opportunity for learning? Something that would like to comment about?
>
> N: I learned that I could've centred the work on one or two questions and spent much more time, for example, using numerical values, comparing, finding the table of values. There, he [sic] can see the relationships between the ideas. I concluded that the *guia* was very long. ... This type of conclusion is what we take to our meetings of the department to discuss and see what we have to do. Why things don't work.

Nora's observations showed contrast with those of her colleagues [who participated in the study]. They all expressed their wishes for some form of discussion and evaluation of what they were doing at the level of their mathematics department group. These are examples of comments made by some of Nora's colleagues:

> The problem is that we leave everything just as simple comments but we never do any serious evaluation of the work we do. For example, the *guias/* we don't look at them to see wether they can be adapted or improved... (Stella, Int. 1).

> There is no time to think of how to plan a lesson in a different way... There is never time or attention to these things. On the contrary, there is lot of pressure if you do things differently... For example one of the teachers who teach in Grade 6, whose teaching is based on playing and activities, and whose pupils never fail, has difficulties and lost of opposition... " (Gladys, Int. 1).

Nora did not identify the textbook as a factor that influenced her teaching of Grade 8 algebra —nor did any of her school colleagues— despite the fact that they all declared to use textbooks available for the organisation of the year plan and for "the *guias*".

> I: Other teachers have named the textbook as a factor that influences their teaching. Do you think that the textbook or available textbooks play a role in your teaching?
>
> N: We don't use textbooks in this school. We design our own materials. Some pupils bring what they have got for example the algebra of *Baldor*[16], and I get some exercises from that book, sometimes, I take exercises from other books, but I would not say that textbooks influence my teaching, because we don't follow the order that textbooks have.

[16] The Baldor's algebra book is an old textbook that has been known in Colombian algebra classrooms since the first half of the Twentieth century.

Nora's final concept map

In Figure 6.10 we can see the map that Nora completed at Interview 3, when she was presented with the boxes of her 'additional concept map' (see Figure 6.10) which she said represented her "thing". She was asked to check if the factors she had considered in that map were still relevant. After she said that they were "the same' because that was the way she saw her "case", she was invited to show how the factors in the boxes connected with the central concept. While she was drawing the connecting lines and arrows (see Figure 6.11), she explained:

> I still think that these two things ('The teacher's knowledge of mathematics' and 'The teacher's vision and conceptions') cannot be separated they go hand in hand. I need to have clear convictions to know what type of work I want my pupils to do. Definitely, these two determine 'My practice' in a direct way. They determine the way I teach.
>
> *The teacher's 'actualización'*. One can have very good intentions but one needs to have training, be up to date. I can be researching my classroom but I need to enrich myself with the work of other people, with their experience. This '*actualización*' is going to be reflected in 'My practice'.

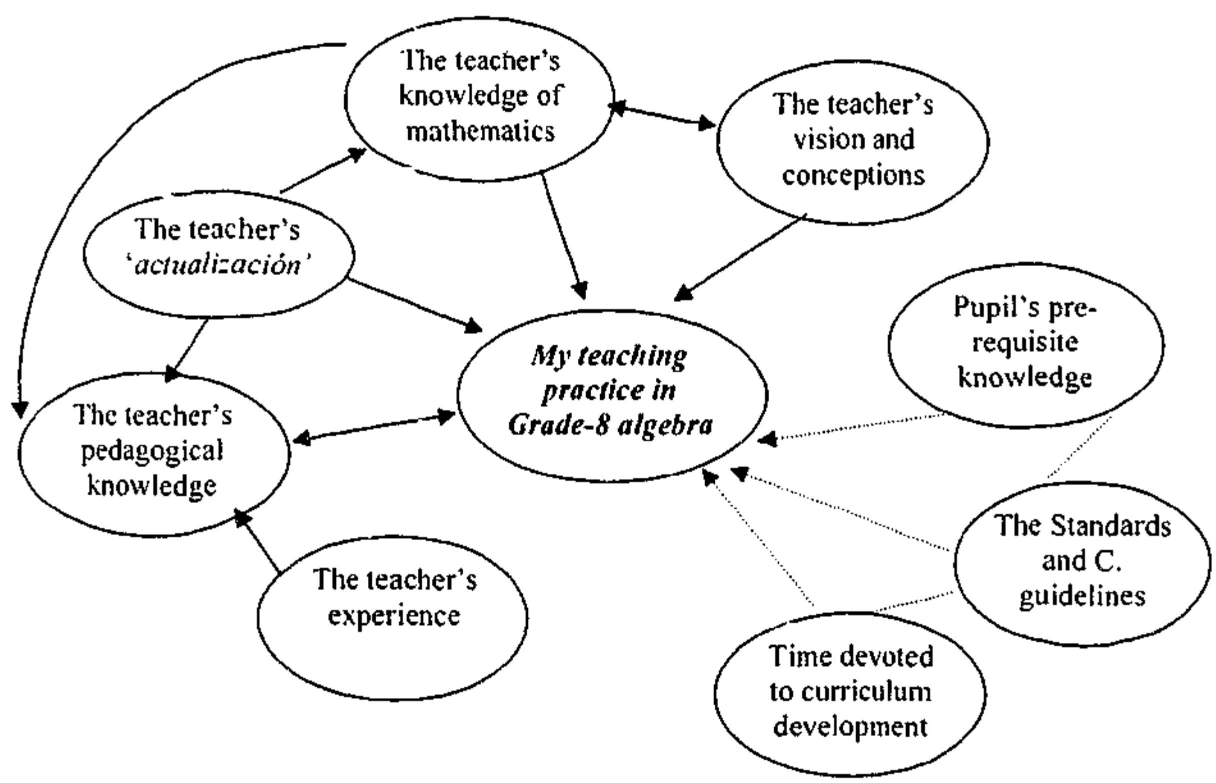


Figure 6.11. Nora's final concept map

> I put these three isolated [i.e., 'Pupils' pre-requisite knowledge', 'The Standards' and 'Time devoted to curriculum development']. Last time I was not sure of what type of line connected them because I think that we sometimes deceive ourselves; we kind of look for excuses. 'Oh! no, time is not sufficient, thus I don't do anything! because it all depends on my vision and my knowledge of mathematics.

My knowledge of mathematics is the basis for my pedagogical knowledge. There is always something that one can do! (pause). But anyway, yes, 'Time' determines the type of things that I can do in the classroom; and the 'number of pupils' and 'their pre-requisite knowledge' because you cannot do what you need to do if the pupil starts in zeros.

Nora identified "time" as the crucial determinant of her teaching, soon after she had provided this explanation in her final concept map. In her statements about what she could do to gain the motivation of the pupils for the learning of algebra, she disclosed again the emphasis she wanted to make on procedural knowledge.

I: You said that in order to address the problem of the pupils' lack of motivation, you were going to work with classroom activities [i.e., contextual situations]. How would you do to identify or to design the activities?

N: I am sure now that I need and would like to change my teaching method and do something like the approach of Teacher B or the type of activities of the teachers we saw in the Focus Group session. I confirm it, and that worries me. It worries me because I know that we need a lot of time. I have to sit down and work, investigate, enquire with other teachers and with university teachers. And that requires time that I don't have now, and we are going to even shorter of time from now onwards because, with the new legislation, the government has shortened our time for this type of work... But one has to be careful when working with activities because the work cannot be left at the level of '*la parte lúdica*' [i.e., play]. They need to learn all what they would be required in Grade 9.

As will be seen in the next section, 'time' was also Nora's reason for choosing the adoption of a textbook rather than participating in team work research project presented to her at Interview 3. She explained then how the pressures of time were increased by the "unfair" working condition of teachers when she was asked to choose from the three options I offered her at Interview 3, in relation to her subsequent teaching of algebra in Grade 8 (refer to Figure 6.5 for information about this interview question).

6.3.2.2 Nora's self-concept and attitude to beginning algebra and its teaching

In previous sections we learnt about Nora's conceptions of the role of factors like the pupils' attitudes and motivation, and the teacher's knowledge in her teaching of Grade 8-algebra. This section presents more information on Nora's beliefs about her knowledge of algebra and its teaching, about the role of contextual factors in her teaching, and about her learning.

Nora responded positively to the invitation to participate in the study, expressing her concern for the problem that she saw with the teaching of algebra (and of mathematics in general) in her school:

> We, the group of teachers who are teaching in the Grade 8-groups[17] want to participate in the study because we have many difficulties with the pupils. They don't have aspirations due to the situation of the country. They don't have any motivation for their school work. They themselves say, '*profe*, we have to learn this [i.e., Grade 8-algebra] and what are we going to do with this? We cannot carry on studying [i.e., doing a higher education course]'. (Fieldwork notes)

Nora believed that her participation in the study represented a benefit not only for her as a teacher but that it, also, added to her role as the head of the department. When asked, at Interview 3, why she had decided to participate in the study, she replied:

> I was attracted to participate in the study for two reasons: one was the fact that as the head of the department, my participation sets up an example for the teachers. The other one, by participating in this type of work one has the opportunity of questioning and reflecting on our practices. What we have discussed in this study has made me think about many aspects of my work. I have to improve. I have to change. I have to be willing [to change]. This [study] came at the right time because with so many changes we are facing, we need to soak ourselves in new ideas.

Nora provided information that revealed her confidence in her knowledge of algebra and her knowledge of its teaching. Although she emphasised the importance of "the teacher's knowledge of mathematics for teaching", she did not *explicitly* say anything about what she thought of her knowledge of algebra, as Pablo, Luis, Alex or Nacho did. What Nora was saying to me was that the pupils did not show motivation for their (algebra) mathematical work and, in general, because they did not see its relevance for themselves due to the conflictive situation of the country. And due to this situation she and her colleagues needed help to be able to gain their motivation.

Nora's conception of her learning

Nora's explanations about changes in her teaching of Grade-8 algebra provided a good opportunity to explore, once again, her conceptions of the nature of her teaching and her learning. When asked about any changes in her teaching of Grade 8-algebra since she had started teaching, she said:

[17] There were eleven Grade 8 (or Year 8) groups in Nora's school.

Changes in the previous years? Yes; the first time one teaches the topics that are in a text book as they are put there. Then I was discovering that one has to make an emphasis on something. So I made emphasis in the identities (*"productos notables"*). But as the time of the truth comes, the boy[sic] has to be a reflexive person. If he has the concept of multiplication he can be creative to develop an expression without the need to resort to memory or to a rule or to the name of that identity [i.e., *notable product*]. ... And now I want that my pupils learn to solve problems, to work in activities. Here, in this [current] study I have not seen the work in relation to mathematics as such, but at a more general level. More in terms of the fostering of values, that pupils have ample goals. So I see that our labour is not confined to the space of mathematics.

About the changes she said she was envisioning, she gave explanations at Interview 3:

I: What is the reason for working with activities now?

N: Because they have difficulties in seeing mathematics useful for their lives. So we have to make them feel in love with mathematics by giving them contextual situations/ through a different methodology because supposedly it is difficult for them if you given them the letters. So I need to change my methodology... But as I said before, time available is a problem because one could use all the teaching time working on those activities, and they need to learn what they need in the following grade...

As we saw above, Nora wanted to find a way to improve her teaching of beginning algebra. She thought that she needed to do something to see if the pupils became motivated in their learning. However, she was convinced that working in contextual activities where pupils did not focus on the textbooks formalisations was just play, not real algebra work. This is something that Nora continuously emphasised during the data collection process. The *formalisations* of mathematics were the most important part of mathematics learning because "they represent mathematics knowledge organised in an efficient way".

The language of mathematics is universal, and pupils need to acquire fluidity in the manipulation of that language. To teach hat is to prepare them for life, for their studies at university... (Focus Group)

I do not agree with the comment, that Marcos[18] made after the Focus Group session, that 'mathematical formalisations are of no use to students' because that is the core of mathematics, that is the language with which mathematics is known universally. (Field notes)

[18] Marcos was the name of Focus Group moderator.

When Nora was asked to choose from the three options given (see Figure 6.5), she emphasised that her first choice would be the 14-month research project. "It would be a valuable experience" for her and she "would not just do it because of the teachers' scale points", but because she was convinced that that was the type of work from where she would learn the most. However, that was not possible for her as she had two jobs:

> The only alternative would be to work during the weekends because I cannot afford to lose my other job in the morning, and we too have families that we need to attend. I could, for example, try to stop working for a year in the private school but that means, I could definitely loose that job... So if you ask me to choose from the three options, I would have to choose adopting a textbook that has been designed according to some research results.

Nora felt that "the teachers' situation, especially in the light of new legislation" that was passed in August 2002 while the data collection was taking place, was "very unfair". Showing disappointment, she declared that she may need to give up her job in the morning session.

> With the increase of teaching time for the school session I may have to give up my job in the morning session... Besides that, the government's policies are contradictory. There are clear contradictions; on the one hand, the government wants the teacher to construct and develop the curriculum and, on the other hand, they are cutting the teacher's time for doing it!... The teacher is expected to dedicate more time to his/her job but, at the same time, has to look for other means of income [i.e., find work in the two working sessions] to be able to cope with the cost of living... Is the government interested in quality of education? (Nora, Int. 3)

After the Interview 3, Nora declared again that she was worried about her situation as a teacher:

> ...now with the new legislation, the one that was passed on last week, we need to teach longer lessons so, for example in my case I will have to give up my job in the morning session!!!... (Field work notes)

The problem of teachers' low salaries and unfair working conditions was a complaint expressed by the participating teachers, who worked in two sessions, except for the case of Luis who worked at a university in the morning.

In summary, although through the concept map activity, Nora pointed out the role of factors like the teacher's knowledge as the crucial determinant of "a teacher's practice", in her explanations about her teaching, she constantly emphasised the 'time' factor and the nature of Grade 8-algebra. She emphasised, at Interview 3, how the pressures of time were increased by the inadequate working conditions of teachers, when she was asked to think about ways of addressing her concerns in relation her teaching of algebra.

6.3.2.3 Nora's knowledge of the teaching of beginning algebra

Knowledge of the teaching of the concept of variable

According to Nora's description of the type of work she did "in Grade 6, before [she] moved on to algebra", the emphasis was placed on the notion of the letters to represent an unknown number and on just finding the specific unknown value. Although she claimed that the word *variable* was used when referring to the letters used, because the values the letter takes vary, Nora did not show interest in using the opportunities that the contexts of the very questions she gave to her pupils represented for thinking about quantities that vary in a relationship:

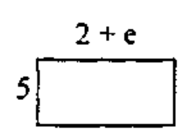
> N: I start by giving them translation exercises to start using letters. I give them expressions like 'the double of a number', 'a number augmented by three', etc. so that they represent those in mathematics.
>
> I: And what do they do? How do they represent them?
>
> N: In Grade 6, I normally tell them that they don't have to use a letter; they can use a box or brackets. So we write things like
>
> $$2(\ )\ \text{or}\quad 2\square;\quad \square+3$$
>
> In subsequent lessons we clarify that in algebra we use letters, so they then write the expressions with letters (writes $2x$). After they have done that I give them equations. For example, this morning we worked on expressions like this: 7. (b + 5) = 21. This is difficult for them. I told them: Okay; then start by finding a number that multiplied by 7 gives me 21 (Nora writes):
>
> $$7.\square = 21$$
>
> And they said, 'Oh! 3'. So they know that this expression, 'b + 5', should be 3. So having done that, practically the whole equation is solved…

As was shown in Section 6.3.1, this represented some "kind of exercises to familiarise them with the use of letters before they arrived at algebra". In Grade 8 pupils were given the same kind of exercise, "the translation exercise" where they did the translation bit by bit with more sophisticated situations as we saw in the work of the *guía*. For example after pupils managed to write 4x for the perimeter of a rectangle, or x + x + 1 as the sum of two consecutive numbers, they immediately were given word problems where they did the translation, that is "the setting up an equation, which is the most difficult part of algebra" (refer to Section 6.3.1.2). As will be seen again in the following section, the focus was on the 'translation' without regard for the meanings (or lack of meaning) letters had for the pupils.

Knowledge of the interrelatedness of mathematical concepts and of different representation systems

Nora made it clear that she "did not believe that the pupils [could] construct some generalisations" to describe their working methods when operating with simple expressions like the ones given in Question C3 of Questionnaire 1. She showed little interest in helping the pupils use other forms of representation, different from the symbolic form, which she referred to as " the translations into mathematical language". When elaborating on her response to Question C3, of "telling the pupils that they cannot add the terms because they are not like terms", she insisted that pupils would not think of using the drawings of the rectangles to help themselves clarify or explain the result of the area:

> I: Do you think that when the pupils were doing this they thought of the rectangle whose area was being represented by the expression 5(2 + e)?
>
> N: No. I don't think that thought of the diagram (pause). Maybe, if you asked them to draw the rectangle, they'd do it and put the expressions of the sides like this:



> I: But if they were given the opportunities to connect their ideas of area with questions like this, would they be able to arrive at a generalisation, for example, about how to establish a procedure to follow when adding these terms [pointing at '10 + 5e']?
>
> N: By looking at the drawing he does 5(2 + e) and this bit: '10 + 5e'. The rest he [sic] will only do with their knowledge of the distributive law, but not looking at the diagram. They struggle with this. They struggle a lot! It would not come to his [sic] mind to think of the situation in terms of the area- diagram. That's why I end up telling them to help them with the hard bits. They will do it with the distributive law, but it would never occur to them to do such a thing!

She did not provide evidence that she paid attention to what the children were thinking when they were writing expressions like the ones given in Question C3, or to the analysis of relationships, but just to the operations of the values given. In the following a section of Nora's elaborations on how she would respond to pupils who wrote 5N =D to represent the expression, *'In Central Hospital there are five times as many nurses as doctors'* of Question C4 in Questionnaire 1.

> N: I will show them with the two tables: one for the expression $5N = D$, and for the one for $N = 5D$. So 'let's compare. Here I give several numerical values... Furthermore, I will do the tables for them in the same order of N and D so that he [sic] can compare more easily. So he sees what expression corresponds to the situation given {draws the tables].

E = 5N

N	D
5	1
10	2
15	3

D = 5N

D	N
5	1
10	2
15	3

> I: What do you think the child thinks N stands for when he/she writes 5N = D?
>
> N: Yes (pause). Apparently, he is not doing the right reading. Is he? He reads, *In Central Hospital there are five times as many nurses as doctors* (pause). So, (silence). I think that he (pause) how is he thinking there? (silence).
>
> I: What I am asking is: When he/she wrote 5N = D, what do you think the letter N was representing for him/her? You said that the relationship was not well expressed.
>
> N: When he/she wrote 5N = D (silence).

Knowledge of the context of teaching beginning algebra

As we saw in section 6.3.1, Nora did not provide evidence of how algebraic thinking could be promoted in the primary levels. Further, she seemed to not be aware of the difficulties pupils experience when using letters to represent variables. She assumed that "pupils know that letters are used to represent any number", and that in the case of specific word problems "letters represent specific unknowns because there is a given restriction". The data show that Nora had no awareness or curiosity in her pupils' thinking when they were required to use letters, as it was seen in the case of "the translations" or in her responses to questions of Section C of Questionnaire 1.

When Nora was describing her responses to pupils who had difficulties adding unlike terms, she did not show awareness of the possible misconception that could be introduced by the analogy of adding *terms* with adding objects. When elaborating on the answers she had given to part C of Questionnaire 1, Nora repeatedly pointed out that she would "tell them how to do it" because otherwise it would be very difficult for the pupils. In question[19] C3, she would use what some authors have termed the *fruit salad* approach:

> I would clarify to them that they cannot add those two terms because they are not like terms. Those are not the same types of objects, 'you are going to tidy up the fridge... You put the same type of fruit in one bag and the other in another bag'. So you don't add bananas and onions, for example... When they are learning to reduce like terms, I tell them 'I have got one *balloon* plus another *balloon*, how many balloons have I got? 'Oh! 2 of the same type' [i.e., balloons]. So they start to realise that *like terms* means the same kind of objects. (Int. 2)

6.3.3 Summary of Nora's case

For Nora Grade 8-algebra needed to be studied "because of its importance in the solution of problems", but before considering work with word problems, pupils had to "acquire fluidity with the expressions". They need to acquire fluidity when operating with expressions through application of the algorithms to a given set of exercises. As the most difficult task for Grade 8-pupils was "the setting of the equations for a given word problem", they should be trained, "little by little", in doing "translations of expressions from Spanish into mathematics". If the pupils did not learn by this way of teaching, it was because they had not done the exercises of their homework. They had not done enough individual work; therefore, they were being irresponsible.

Nora did not like teaching Grade-8 algebra because it was "mechanical", it was "routine procedures". It was the rules to operate with algebraic expressions. Once pupils learnt algebra they could apply it to geometry topics like area and perimeter of a rectangle. Starting classroom work from "problem-situations", like the example used by Teacher B of Questionnaire 2 was "good, interesting", but it was "not always possible to follow such approach in Grade 8" because there were time constraints, and pupils needed "to dedicate sufficient time to work on the formal part of mathematics", the algorithms. Furthermore, she had learnt from her previous action research projects that working on activities like those described at the Focus Group session could motivate the pupils, but they belonged to "*la parte lúdica*" (i.e., play), and through them pupils did not learn real mathematics.

[19] Question C3: Find the area of a rectangle whose sides are 5 and $2 + e$. A pupil wrote: $A = 5(2 + e) = 10 + 5e = 15e$. .... How would you respond to this pupil?

Nora was concerned that her pupils and all pupils from the school, in general, had many difficulties in algebra and had no motivation for their classroom work. By participating in this current study, she would have "the opportunity to reflect" on her teaching and, at the same time, she would set a good example for her colleagues, as she was the head of the department. In explaining how she saw her own teaching practice, after she had emphasised that her pupils' attitudes and motivation represented the crucial determinant of her teaching, she came to emphasise the role of "the teacher's knowledge" in her/his teaching, when she was prompted to think about the possible factors influencing the pupils' motivation. Thus in her final concept map she identified "the teacher's *knowledge of mathematics, vision of mathematics,* and *pedagogical knowledge mathematics"* as the most important factors determining "the teacher's practice". However, when explaining her teaching situation and her classroom decisions she constantly emphasised the *time* factor and 'what algebra knowledge is'.

After the Focus Group session took place, Nora declared that she wanted to change and improve her teaching method because she wanted quality in her teaching. She wanted to work "with situations or activities, like the ones described at the Focus Group, "so that pupils get in love with mathematics", but nevertheless, she would have to "be careful with this new type of work because the activities [could] not be left at the level of *la parte lúdica"*. The pupils should work on the formalisations, the universal language of mathematics, and acquire fluidity with the manipulations so that they would "be well prepared for Grade 9, for life, that is, for the university".

Would Nora want to participate in a teamwork- classroom based research project that focused on the introduction of school algebra? She "would love to do it as [she] had done it twice before, giving out her time during the weekends. ... It would be a valuable experience" for her, and she "would not just do it because of for the teachers' scale points", but because she was convinced that that was the type of work from where she would learn the most. However, she could not do it because of her family commitments and the fact that she could not afford to loose her other job in the morning session. As this option was not possible, the alternative would to adopt a new research-based textbook for her teaching of Grade 8-algebra.

Nora expressed her dissatisfaction and her discouragement with the unfairness of the teachers' situation because "on the one hand, teachers are expected to construct the curriculum which requires time but, on the other hand, the new legislation is shortening the time to devote to curriculum development". Because "teachers have very low salaries, they have to find jobs in the two sessions [morning and afternoon] in order to cope with the cost of living". Furthermore, since according the recent legislation, the number of hours for a session increased, she would have to "give up" her job in the morning session.

6.4 Nacho – I know it all

Nacho is the Head of the Mathematics Department of the afternoon session in a state school that caters for children of low-income families who live in different areas of Bogotá. The school provides facilities for three working sessions a day[20]: the morning, the afternoon, and the evening sessions. Nacho and the other two teachers from his school, who participated in the study, worked in the afternoon session.

Nacho had been teaching mathematics for twenty years, and Grade-8 algebra for sixteen years. In 1997 he "took a one-year Postgraduate Diploma in Information Technology for Teaching". As the Head of department, his teaching workload had been lowered because he had "to attend meetings with the principal and the academic coordinator for administrative reasons". Nacho decided to participate in the study because he wanted "to cooperate with the researcher's project". During Phase 1 of the data collection and during the classroom observation period, he showed enthusiasm in participating in the study; however, after Interview 2 he showed less interest.

6.4.1 Nacho's conception of beginning algebra

According to Nacho's responses to Questionnaire 1, the main reason for the inclusion of algebra in the school curriculum was to provide pupils with the knowledge needed in higher levels of school mathematics. Nacho's justification for this answer was:

> School mathematics is a well-organised collection of topics related to techniques to operate with numbers and, in Grade-8 algebra we see the rules of procedure which are established in arithmetic and brought in to operations with algebraic expressions (Int. 1)

This collection of algorithms "is hierarchically and strictly ordered". Thus, the goal of his teaching of Grade-8 algebra was "to teach the pupils these rules of procedure that they will need for Grade 9".

Nacho, repeatedly, made explicit that Grade-8 algebra involved mostly facts and procedures. He "strongly disagree[ed] that in school algebra, or in school mathematics,

[20] Morning session: from 7:00 a.m. to 12:30 p.m. Afternoon session: from 12:30 p.m. to 6 p.m. Evening session: from 6:00 p.m to 10:00 p.m.

pupils can be creative" (Questionnaire 1). At Interview 2, he declared that "mathematics is the knowledge produced by great mathematicians and experts", which can be found in books. "Mathematics can be knowledge that develops and expands but this does not take place at the level of students, let alone of secondary education, it is developed by great mathematicians".

Nacho's specific teaching goal for Grade-8 algebra connected very well with his statements: "mathematics has a high level of precision; you cannot make cuts when you are teaching". Algebraic knowledge has to be "strictly organised and meted out so that the pupils do not get lost in their learning". This pre-established ordering of topics coincided with the one presented in an old algebra textbook which he often referred to as the basis for his teaching.

For Nacho, "Algebraic thinking is an operational abstraction".

> It is something more confusing to pupils than arithmetical thinking because it is something new for them. It is like when we move from Geometry to Physics. They get confused because Physics is something totally new.

Nacho put forward the idea that algebra is the block of mathematics that you start to teach in Grade 8, and that its teaching cannot be initiated until after pupils have covered the arithmetic program. He believed that "if the pupils [knew] well how to operate with numbers then operating with algebraic expressions [was] easy for them". This conviction took him into making a proposal of increasing the number of school years dedicated to the teaching of arithmetic:

> Algebraic thinking cannot be promoted in the primary levels because you cannot teach algebra if pupils do not know their arithmetic. Some years ago, I had two proposals but my colleagues were not in agreement with them. I don't know why. Two things I proposed. One: that the pupils could not be promoted to Grade 8 until they had understood arithmetic. Two: that we increased the number of years to 3 (rather than 2) for pupils to study arithmetic in secondary, and then we taught the algebra of Grades 8 and 9 in one year.

6.4.1.1 Learning beginning algebra

Nacho's constant references to having the pupils working on repeated practice revealed further his conception of the learning of beginning algebra concepts. At Interview 1, when giving reasons for his two '*Number 1*' priorities for pupil's work (i.e., 'algorithm routine exercises' and ' correcting exercises at the board'), he explained that pupils learn by 'first paying attention to [his] explanations and then repeating what has just been explained".

In accordance with his stated reason for the teaching of Grade-8 algebra (in Questionnaire 1) of providing pupils with the knowledge needed in higher levels of mathematics, Nacho declared that "while learning to operate with polynomials takes place, pupils don't need to see the application of what they are learning". The use and importance of learning the topics of Grade 8 was represented in the pupils' mastery of the topics, due to the requirements of Grade 9:

> Initially it is something mechanical; something that doesn't have any applicability, but something that I explain how to do. They may find the applicability of it in Grade 9; for example, with equations. They see first grade equations in Grade 8, and in Grade 9 they will see second grade equations. There is where they are going to see the applicability. (Int. 1)

Nacho, openly, made clear his valuing of the mechanistic aspect in the process of learning and of keeping what pupils did very controlled and defined. At interview 2, after the sequence of lessons had been observed, he explained that he liked "to have all pupils working at the same pace on the same tasks." For this reason he "[had] the best five pupils helping [him] to teach the algorithms to the rest of the class". The rote-learning predilection was further identified in his strong agreement with statements describing routine practice, in Questionnaire 1, and in his responses o Questionnaire 2. In relation to the approaches followed by the two teachers, as described in Questionnaire 2 (i.e., Teacher A, who emphasised an instrumentalist approach and Teacher B, who emphasised a problem-solving approach), he commented:

> Teacher A advances faster than teacher B. Teacher B poses situations for classroom work, even if they make no sense, opening the opportunity for pupils to wander, and s(he) is departing too much from the pre-established order (the sequence of topics to cover) and there is when pupils can get lost. If we give the pupils small doses of this content, in an orderly fashion, it can work out better (Int. 2).

But giving "small doses" to pupils meant that the path "to arrive to the last topic of the set program of study could take a long time", so in order to push pupils "to move at a good pace" he gave his pupils, very frequently (as shown in the observed lessons), similar exercises to the ones he had just explained, offering a good mark to the first five who completed the exercise. When asked if he had set the exercises for the first five because he thought that they had already learned what he wanted then to learn, he explained:

> They like to be given these exercises for the first 5 or the first 10. It motivates not only the ones that managed to do it and to hand it in, in time, but also the rest of the class. If they know that I am giving these exercises, they are more attentive. ([illegible]. 2)

This type of exercise took place during two of the five observed lessons. Although Nacho referred to it as if the majority of pupils showed interest in the competition, there was one occasion when only three pupils responded positively to Nacho's proposed exercise (i.e., got up and handed in their books). Nacho explicitly made clear to the class (on more than one occasion) that repeated practice was the way to learn algebra. He said to his class:

> Those who have learnt mathematics have done it by doing exercises, repeating, and reading.

6.4.1.2 Teaching beginning algebra

As in the other case descriptions, data in this section is organised following the order of priority established by Nacho in his response to the question on 'Teaching styles' of Questionnaire 1. Although the discussions were always focused on the teaching and learning of Grade 8-algebra, when explaining his thinking Nacho always spoke of his teaching of Grade 8-mathematics. Nacho's responses to questions about teaching style were as shown in Table 6.3. Note that Nacho assigned the number "1" to four of the given descriptors for both *preferred* and *actual* practices.

Table 6.3 Nacho's teaching style priorities

Preferred	*Teaching style descriptors in Questionnaire 1*	*Actual*
1	Giving clear explanations of definitions and procedures to follow in different exercises and problems of application, in the topics studied	1
1	Giving pupils lots of exercises for algorithm application as homework	1
1	Designing classroom work that promotes connections between different mathematical topics studied	1
1	Providing opportunities for pupils to develop their communication skills so that they can express their mathematical ideas with confidence	1
2	Organising problem-based activities for the pupils to work in small groups where they can present their ideas to the whole class for discussion	2
2	Designing activities that provide space for pupils' self-paced learning	2
3	Testing pupils at the end of each activity or topic, in order to have sufficient marks for assessment in each Attainment target	3

Two of Nacho's 'Number 1' priorities for both 'preferred' and 'actual' practices: 'Giving clear explanations of procedures to follow' and 'Giving pupils lots of exercise for algorithm application'

For Nacho, there was no point in considering preferred and actual practices when talking about his teaching. He declared at Interviews 1 and 2: "Everything I want to do, I do". This represented a confirmation of what he had meant by giving the same rankings to the given descriptors for both preferred and actual practices, as seen in Table 6.3. These priorities were clearly illustrated in the classroom work structure of the sequence of observed lessons, where he asked pupils to learn "ten cases of factorisation" in a specific order and with the specific names with which they appeared in the textbook of Baldor[21] . During the whole of the first lesson, he had the class listening to his explanations on "how to identify trinomials that belong to the fifth and the sixth cases of factorisation", and then on "how to find the factors needed". In order to present some illustration of Nacho's preferred teaching style, a scenario from one of the five consecutive lesson observed is described in the following paragraphs. Nacho's views of specific incidents are presented after this description.

[21] Baldor is the author's name of a school algebra textbook which has been used for the teaching and learning of school algebra since the first half of the Twentieth century.

A scenario from one of Nacho's lessons

After Nacho asked the class for the names of the cases of factorisation, from the first to the sixth, he presented the seventh case (i.e., Trinomial of the form $ax^2 + bx + c$), and explained, in a very detailed manner, the steps to be followed when looking for the factors of a trinomial that belonged to this case. He did this by solving three exercises on the board, but asked the pupils to "copy only the first exercise so that the mechanics to follow could be seen". When he finished explaining the three examples, he wrote a different exercise on the board and said to the class:

> This exercise is for those who already understood.

Minutes later, Carlos, one of the three pupils who had put their hand up to show that they had already done the exercise, went to the board to show his working, but showed difficulties finding the factors. The transcript that follows describes the teacher and the pupils' interaction, from the moment when Carlos' steps to factorise the trinomial $2x^2 + 11x + 5$, after he had arrived at the expression $2(x + 5)(2x + 1)$.

At the board, Carlos had arrived at the expression (but got stuck there):

$$2(x + 5)(2x + 1)$$

Teacher:	Now you have to divide by 2 the whole expression.
Carlos:	[Keeps silent -does not do anything].
Teacher:	Draw a line under the expression and put 2 under the line!
Carlos:	[draws the line and writes 2 under the line as shown]

$$\frac{2(x+5)(2x+1)}{2}$$

Teacher:	Now we cross the 2s out because one is multiplying and the other one is dividing.
Carlos:	But [teacher interrupts]
Teacher:	I said cross the 2s out!
Carlos:	[crosses the 2s out].
Teacher:	Now multiply the factors to check that they produce the given trinomial.
Carlos:	x multiplied by $2x$ is (there is silence for a few seconds).
Another pupil:	$3x$.
Teacher:	[raising the tone as if the one who said it needed to be reprimanded]: Who said 3x?

Nacho's explanations of this scenario

In relation to this classroom scenario, at Interview 2, Nacho provided further evidence of his beliefs about how pupils learn and, consequently, of his role as a teacher. The beliefs that " at the beginning it is something mechanical" and that "pupils first have to pay attention to his detailed explanations of the procedures to follow" were exposed again when he explained some aspects of this lesson. In relation to my query about why when pupils were at the board, they were not encouraged to ask questions, he said:

> At the beginning they find it difficult but they, later, they become confident when they see that they can do things as I teach them to do them.

Nacho then explained why, during the lessons, the pupils did not ask any questions while he was explaining, despite the fact that the great majority showed not to know what to do when he gave the exercises:

> They know they are not allowed to interrupt the flow of explanations because they have to first understand what I am explaining... That is the normal course of a lesson! That case (Case VII of factorisation: Trinomial of the form 'a x^2 + bx + c') was a new case for them in that lesson.

In relation to the why of giving "an exercise for those who already understood", he said:

> With *those who already understood*, I was referring to those of more ability. Those who understand first, who are, obviously, the ones who have interest and are always paying attention, and not looking through the windows, for example. There are a few pupils who are practically the *monitors*[22]. They help me with the rest of the class. Each of them takes a group of four of five pupils, and explains to them how to do it; they ask me: how do I explain this? How do I do this here?

The other two 'Number 1' priorities for preferred and actual practices: "Designing classroom work that promotes connections between concepts' and 'Promoting the development of communication skills'

The reason Nacho gave, at Interview 1, for his rankings of the descriptor *Designing classroom work that promotes connections between different mathematical topics studied,* as one of his 'Number 1' priorities was related to what he saw as "the connection between algebra and arithmetic". He claimed that

> the algebraic algorithm can only be constructed on the basis of the arithmetical case. To me doing algebra is reinforcing an arithmetical knowledge because algebra is a generality of arithmetic. There has to be a connection with arithmetic when you teach algebra. If we start talking about *a,* and teaching rules to operate with the letters in a *memory-based* way, one can never get any learning from the students. They are not going to get to any point if I just centre my teaching on the algebraic matter. I always try to keep this connection.

---

[22] The monitors for Nacho were the pupils in charge of explaining to specific groups of pupils how to do the exercises.

However, despite his professed intentions of showing the pupils that "what one does in arithmetic one carries on doing with the letters in algebra", he found "a difficulty in the case of *division* because what one does when one divides, for example, $a^2 + 2ab + b2$ by $a + b$, cannot be done when one divides numbers, for example the number 724:

> 724 cannot be written as an arithmetical polynomial divided by another polynomial because you cannot write 7 + 2 + 4 divided, for example, by 5, in order to apply the algorithm that we are talking about in the algebra case. So we [my class and I] talk of that difference between what you do in arithmetic and what you do in algebra for division (refer to section 6.4.2.2 for more explanation of this quote).

Another part of what was considered by Nacho as the connection between the arithmetical and the algebraic was the way in which the expressions were organised to apply the procedures. When explaining what he did when he was teaching multiplication of expressions, he referred to "doing the multiplication *(a+ b) (a + b)*, horizontally, as in algebra, and vertically, as in arithmetic" and wrote as he was saying it:

$$(a+ b)\,(a + b) = \qquad \begin{array}{c} a + b \\ a + b \\ \hline \end{array}$$

'Promoting the development of communication skills', was another 'Number 1' priority for Nacho, that was represented in the act of asking the pupils to verbalise the steps followed when applying rules of procedure to operate with algebraic expressions. This took place when pupils were asked at the board to solve classroom work or to correct the homework, which was the main way of assessing pupils.

> I: You ranked as a 'Number 1' descriptor of your preferred and your actual practices, 'Providing opportunities for the development of communication skills'. Could you explain why this is a number one priority?
>
> N: Pupils develop communication skills by doing the set exercises, and they feel more confident as they mechanise the procedures and are able to explain the steps they follow.
>
> I: When do they have the opportunity to explain?
>
> N: Normally when they are called at the board.

Assessing pupils' work

Nacho's two first priorities for assessment practices, in his responses to Questionnaire 1, were 'pupils' folders' and 'frequent quizzes'. However, at Interview 1, he clarified that he had decided not to use pupils' work in their folders or notebooks because he had found that they copy from their peers:

> They have the answer to the exercise, but don't know how to do the exercise. So pupils are assessed through quizzes, but more importantly when they are asked at the board to do the exercises and to correct homework. For example, as soon as we finish with the ten cases of factorisation, I am going to start doing assessment. It is going to takes about two weeks assessing the topic because I have to assess case by case.

6.4.2 Nacho's conception of his own teaching of beginning algebra

During Phase 1 of the data collection, Nacho declared to be satisfied with his teaching of Grade 8-algebra. As can be seen in the following transcript taken from Interview 1, the idea that he conveyed was that his pupils liked algebra because of the way he taught them.

> I: Do pupils in your school like algebra?
> N: When we know how to teach it.
> I: Do your Grade 8-pupils like it?
> N: They are fascinated by the way I teach them.
> I: And how are they doing in algebra?
> N: The great majority have good results.

However, at the concept map activity the picture changed. We shall now see how Nacho explained his teaching situation at the concept map activity during Interview 2.

6.4.2.1 Nacho's conception of the determinants of his teaching practice

The concept map activity, which took place in the second half of Interview 2, started by the interviewer reminding Nacho's of a previous declaration he had made: "You have said that in your teaching what you want to do, you do". Nacho, immediately added to this: "What I want to do is what I do. The problem is the results". At this point he was asked what he thought determined what he did in his teaching, and the dialogue developed as shown in the following:

> I: On what do you think the type of work that you propose or that you organise for your pupils depend?
> N: That depends on the interest of the pupils.
> I: Could we talk now about the aspects or factors you think influence your teaching practice?

At this point the meaning of "teaching practice" was discussed, and it was made clear to Nacho that that "teaching practice", for the purpose of the discussion, included aspects like: the classroom work that was organised, the emphasis the teacher made on specific aspects of classroom work and content taught, the forms and content of pupils' assessment and the environment which was created in the classroom. The dialogue continued:

> I: So what factors do you think influence your teaching practice?
>
> N: I have to give the pupils some work, deliver the contents, assess them and get some conclusions. I have to do what I have to do but I am not satisfied with the results.
>
> I: The results?
>
> N: Yes; because I do all what I have to do but the pupils do not do what they have to do! (i.e., the given homework and work done during the lessons). I am not satisfied with the results. We, the mathematics teachers, try everything we can, we are the ones who have new proposals for work, do things in the school, but the pupils' performance is not the expected!
>
> I: Could we try to identify the reasons for the situation you are describing here?

Nacho wrote the following list, saying with emphasis, "These are the reasons why I don't obtain good results':

a A permissive legislation by which only 5% of the pupils can fail

b The pupils' lack of interest for learning

c The pupils' lack of prerequisite knowledge for algebra (i.e., inadequate knowledge of arithmetic)

d The lack of cooperation on the part of the pupils' parents

e The government's lack of interest in their peoples' education

At this point an example of a very simple concept map was given to Nacho. After looking at the given example, some explanation and discussion about how to build a concept map took place, and he was invited to build a concept map showing the factors he had listed affecting his teaching practice (detailed information about the guidelines for the concept map activity are shown in Chapter 3 Subsection).

> I: Could we try to make a diagram or a concept map with those reasons in order to explain how they affect or influence your teaching? That is, to explain the situation you have described?
>
> N: I cannot do a concept map, and I have already explained why the bad results of my teaching.

Nacho wrote, "Why I do not get good results in mathematics (algebra of 1st grade) [i.e., Grade 8-algebra]" as the heading of the list of reasons he had provided, as shown in Figure 6.12 (Nacho's writing can be found in Appendix 6.2).

Why I do not get good results in mathematics (algebra of 1st grade)

a. A permissive legislation by which only 5% of the pupils can fail.
b. The pupils' lack of interest for learning.
c. The pupils' lack of prerequisite knowledge for algebra (i.e., inadequate knowledge of arithmetic).
d. The lack of cooperation on the part of the pupils' parents.
e. The government's lack of interest in their peoples' education.

Figure 6.12. Nacho's list of factors that constrained his teaching

There was an expression of anger and frustration in Nacho's words and face, when he talked about his teaching situation. But he wanted to talk about this situation. Before Interview 1 started, he had said to me: "There are things of the teaching of mathematics that I want to talk about to you. There are things that I find are absurd. You can write about that and publish it if you want".

I: Would it be possible to identify if any of these aspects that you have listed here, is acting in the situation, in a more powerful way than the others?

N: I cannot establish priorities because everything works as a whole. It is a whole that depends on the interest of the pupils. I do everything I can. I try whatever I can. But (pointing at the list he had just written) this is why we the mathematics teachers don't get good results.

I: But thinking about what is done in your Grade-8 classroom, could you think of some factors that can determine what you do in your teaching?

N: Nobody and nothing determines what I do! I already explained that the problem is the results from the pupils.

I: The results, you mean?

N: Once again, the pupils' lack of interest in the classroom work; they don't do their homework, and the low scores they got in the quizzes. They have another opportunity for recuperation at the end of the period, but don't do anything; Yet, I have to pass them! They and their parents know this!

Nacho's views of the role of other school factors in his teaching

At this point I presented a list of factors to Nacho, some of which had been identified by other teachers from the pilot study[23] as the most important for their practice. I asked him if any of those factors were relevant to his case, and he sounded surprised that I was asking those questions. The way he answered these questions gave me the impression that he perceived the questions as nonsense questions. These are Nacho's responses. The responses are preceded by my question.

Interview question:

> Some teachers, who are participating in this study, have identified these factors as the ones that determine or influence their practice the most:
>
> •Curricular guidelines
> •Time
> •The textbook
> •Pupils' behaviour.
>
> Do you think that any of these factors determine or influence your teaching of Grade 8-algebra?

Nacho's views in relation to the role of each factor named are presented in the following.

Curricular guidelines. Nacho said that he agreed with Curricular guidelines influencing his teaching because "one has to follow what is there".

Time and the textbook. For Nacho neither 'time' nor 'the textbook' influenced his teaching. The following presents the discussion that took place when considering these two factors:

> N: Time? No! To me, the pupils. When the pupil wants to learn, s/he learns, shows a good behaviour. To me, time? No!! If I have pupils with good performances, I have more than enough time! But what I have had to do is to limit my teachings to what the pupils do! They are the ones who haven't got enough time. It is just obvious that the more I teach in a lesson, the more time left I will have.
>
> I: Why do you think pupils are short of time?
>
> N: Because they never have time! They use their time in other things that are not mathematics.
>
> I: So time available does not influence your teaching practice?
>
> N: No! In what I do, no! in the pupils' performance, yes.
>
> I: And the text book?
>
> N: No, no, no!!! I don't care about the textbook! I can have one, I cannot have one.
>
> I: And, actually, pupils in this school don't have a textbook; am I right?
>
> N: I always use a textbook and try not to change it in case I have the same group in their next grade, so that they will have one way of seeing things... As, by law, we are forbidden to ask pupils to have a textbook, I tell them to photocopy some of the pages of *Baldor*, if they can, or to find a book that bring lots of exercises. But having a

[23] The textbook, for example, was not considered by any of the teachers as a factor influencing their practice, but I purposely included it in this list.

textbook or not having it is not important for the teacher. Those who have had an academic training for teaching know what they are doing.

Pupils' behaviour. When asked about pupils' behaviour, Nacho responded referring specifically to discipline.

N: Pupils' behaviour? No. It is their desire and their want of learning. Nowadays, I cannot ask the pupils to do 50 exercises, as we had to do when we were in school. If I give a 5 to a pupil (i.e., 5 out of 10 as a mark), I will have his/her father here, tomorrow, hitting me or taking me to court! Pupils don't do their work or their assignments and no one can force him/her to do them!

I: How can we explain that the pupils do not want to do their assignments?

N: Well! because they are not interested in mathematics! They are interested in other things: playing cards, billiards, or dice, smoking, drinking; and the parents don't oppose anything! All what is to be done in education has been left in the hands of one person, the teacher! I cannot punish any of those pupils because they can take me to court! A judge, who does not know anything of education, comes to tell me what to do!!! Full stop. Everything is being made, in this country, in such a way, just so that the country can make some progress!

A concept map representing Nacho's explanations

These explanations, together with the list of factors above listed and the corresponding justifications provided by Nacho were taken as the bases to draw the concept map shown in Figure 6.13. It was considered useful to draw the map using Nacho's explanations about his teaching situation as it can help us to see, in a simple and general way, his conceptions of his teaching practice and to compare them with those of the other teachers.

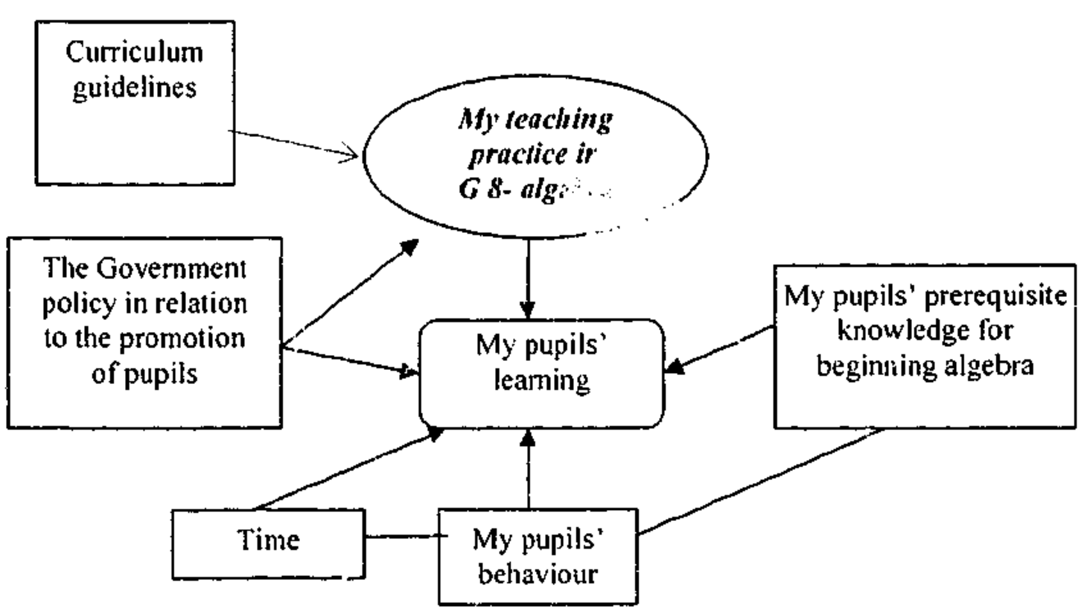


Figure 6.13. A concept map (constructed by the author) based on the interpretations of Nacho's explanations for factors belonging both to his given list and other factors given by the interviewer.

6.4.2.2 Nacho's self-concept and attitude to beginning algebra

For Nacho having a Bachelor of Education degree in mathematics, together with his twenty two years of experience represented more than sufficient proof that a teacher had the necessary knowledge for his/her teaching. When discussing an imagined situation, presented to him at Interview 2, of a primary school teacher replacing him for a while in his Grade 8-class while he was on leave, he identified the teacher's *knowledge* as a key factor of her/his teaching. He said: "If a primary school teacher came to replace me, classroom work wouldn't go well because a primary school teacher does not have the knowledge to teach in secondary". He added:

> What mathematics can a teacher, who doesn't know mathematics, teach? If a teacher has a Bachelor of Education, and has a job with the State, it is because that teacher is professionally competent. Pedagogical knowledge can only be constructed with experience. My mathematical knowledge would have nothing to do with the learning of the pupils. I could be a genius, but if the pupils don't want to learn, they don't learn. They wouldn't learn, even if Newton came and taught them.

Nacho seemed to be very fond of his teaching style because he believed that good teaching was making a display of his knowledge to manipulate algebraic expressions, and then having pupils repeat what he did. When asked, at Interview 2, how he saw his pupils' learning during the observed lessons, he once again claimed that the pupils were fascinated by his way of teaching:

> I love explaining to them every single step to be followed as if they were babies, and they are fascinated by this way of teaching. When they don't understand something, I explain it again. I explain it as many times as it is needed... They learn by listening to my explanations and by looking at how I can play with *something* called algebra.

When he gave an exercise for "the first five" pupils (refer to heading *Learning beginning algebra* in Section 6.4.1.1, in the fourth lesson observed [he had given this type of exercise in the third lesson], he looked satisfied when refusing to receive the pupils' notebook as they did not have the right answer. He said to each of them, "Yours is out! Yours is out!" (i.e., you have got it wrong!), and rubbing his hands (which in Colombia, in certain situations means *being enthusiastic about something*), he said to the class, "How nice to participate like this!"

Nacho saw this practice as a way of motivating the pupils to engage in doing the exercises. He saw no problem in this practice; he did not consider the possible effects this practice could have on the pupils who never managed to participate in the regular competition established during the lessons because he thought that it was a case of the pupils not wanting to learn. About this situation, he said with certainty:

> All of them can do it, but if they don't want to learn, there is not much I can do about it.

Nacho's conceptions of his learning

Nacho showed a high level of confidence in his knowledge of the teaching of beginning algebra. He firmly believed that knowing mathematics was knowing Baldor's book by heart, and this book's setting and structure represented the sequence and the style of work that had to be done in the classroom. He chose to make public what he thought about his teaching, by saying it aloud, [in front of a large number of teachers in the staff room] what he had said to me at interview. He said:

> I have done it for 22 years, so I know *Baldor* by heart! What would I need to plan for my teaching of algebra?

It is relevant to note here that, in contrast to the rest of the participating teachers, Nacho did not show interest in attending the Focus Group session. Although he was aware that several teachers from different schools would attend the Focus Group session[24] to share experiences of their teaching la Grade 8-algebra, he was not able to attend it although all other participating teachers were able to arrange it.

6.4.2.3 Nacho's knowledge of the teaching of beginning algebra

Knowledge of the teaching of the concept of variable

Nacho's thinking of Grade-8 algebra content was exclusively focused on formalistic procedural knowledge; that is, on the symbolic and conventional language of rules of procedure to operate with given algebraic expressions. Although Nacho spoke of variables representing the *general number* when establishing procedures, he showed himself to be using the letters as representing arbitrary symbols or marks on a paper (Mason et al., 1985; Usiskin, 1988). He was operating on the letters without going to the numbers that the letters were representing. As we saw in Subsection 6.4.1.2, he wanted

[24] Nacho knew that the Focus Group was going to held at a place that was a two minutes walk from his school.

the pupils to see that "what you do in arithmetic is the same that you do in algebra". However, he explained that this did not apply to the case of division because what you do in algebra:

> ...for example, when you divide $a^2 + 2ab + b^2$ by $a + b$, you cannot do when you divide 724 by 5, for example... 724 cannot be written as an arithmetical polynomial divided by another polynomial. In other words, you cannot write 7+2+4 divided, for example, by 5, in order to apply the same algorithm that we applied in the given algebra case. So we talk of that difference. ... The principles of division in algebra are not understood even by the book's author!

This use of the letter, to represent an arbitrary number, continued to be promoted throughout his teaching of factorisation. He expected the pupils to learn the ten cases by heart, identifying the structure of a given expression, associating that structure with one of the structures of the ten given cases and, then, find the factors by following the rule given for each case. If it was to be proved that the factors found were the correct ones, a multiplication was done to check that they gave the original expression without the use of numbers as referents. Doing this becomes a routine manipulation of expressions as it is by multiplying the binomials that students found the factors. Usiskin (1988) argues that in this type of practice the variable becomes an arbitrary object in a structure related by certain properties, and that it is the view of variable found in abstract algebra.

Nacho found it difficult to understand what was being asked in Question C4 of Questionnaire 1, where he was asked how he would respond to a pupil who wrote $5N = D$ to represent an equation for the statement *In Central Hospital there are five times as many nurses as doctors*. He answered that the pupil's answer was incorrect and that he would tell them: "We have to find an expression that generalises the situation". The question itself seemed to have challenged him as, at Interview 1, it took him more that ten minutes to actually broach the issues of what he had meant with what he wrote (in Questionnaire 1) and of how he would respond to the pupil who answered $5N = D$. The following transcript shows the dialogue that developed when Nacho started to refer to whether the expression 5N = D represented the relationship between the number of nurses and the number of doctors:

> N: The expression 5N = D is too open, it is not clear what is being asked because how is the pupil going to calculate how many doctors there are or how many nurses. This is a very particular situation, we need more data. Furthermore, that is not an algebraic

reading because the first thing you teach in algebra is that you don't use capital letters. Capital letters are for the names of sets.

I: Let's read the question again. (Interviewer reads the given question making emphasis on *the number of nurses).*

N: Oh! the number of nurses! then there is no N! That N can be any number of nurses, which is being indicated by 5 in this case, but then this is going to change completely, because if the letter is going to take any value, all the numerical data are going to be completely different. If I can change N, and I say that N is 3, then, 5 times 3 is 15 doctors. Then we don't have the relation 5 to 1 anymore. It changes as we give different values to N. This is a situation to confuse the pupils, even for me it is confusing! When it comes down to it, can N change or not?

I: What would you write for this question if the question were given to you?

N: If I were the pupil, I don't know what sort of nonsense I would write!

In his response to Question C3, Nacho illustrated, at Interview 1, his approach of *learning by being told,* as Nacho repeatedly emphasised that pupils clarify when they are asked at the board:

I: If a pupil writes A = 10 + 5e = 15e. What happens when he goes to the board?

N: They go to the board and repeat what they have done wrong.

I: What would you say to this pupil?

N: I'd say: 10 + 5e are not like terms.

I: And so they are clear with that explanation?

N: They clarify with that.

I: They don't make that mistake anymore.

N: No! They make some even bigger mistakes! What happens is that this type of mistake is minimised. They will know that when a number is multiplied by another number, they obtain a number. When a number is multiplied by a letter, they obtain a combination because the number and the letter cannot disappear, but cannot be added the one with the other.

Nacho was not interested in the formation of the relationship between conceptual knowledge and procedural knowledge. He showed not concern for the promotion of conceptual understanding. He declared at Interview 1 that "pupils don't need to do much reasoning when beginning algebra. To understand mathematics, maybe, they need to understand the mechanics of it and when they have the mechanics then they can start to see situations where they can apply that".

Knowledge of the interrelatedness of mathematical concepts and of different representational systems

There is evidence that suggests that although Nacho spoke of the connection between arithmetic and algebra, he isolated algebraic thinking to the symbolic manipulative category as if it could originate and be held in isolation (i.e., no need to be grounded in a context from where the learner could find meaning to the generalisations). There was no evidence that Nacho used or considered the geometric representation in his teaching of beginning algebra. He explained that geometry was taught by a different teacher because if the two subjects were given to the same teacher, pupils are taught algebra but not geometry.

Knowledge of the context of teaching beginning algebra

Nacho firmly believed that "pupils learn mathematics by being told how to do exercises as many times as necessary". He believed that "when pupils don't do well it is due to them not wanting to work" and lack of motivation for their own learning. When asked to explain why he had 'strongly agreed' to the statement of Questionnaire 1: *Pupils will enjoy and work hard in mathematics if they find classroom work meaningful and challenging whether or not their work is graded,* he explained that "by *meaningful* in that expression", he was referring to mathematics being considered important by pupils for their future.

Nacho's knowledge of the curriculum was limited to knowledge of the techniques curriculum (see Bishop, 1988) and was based on the approach followed by a specific traditional textbook.

6.4.3 Summary of Nacho's case

According to Nacho, "mathematics knowledge has been created by the experts and, therefore, it cannot be developed by students, least of all those in secondary education". Grade-8 algebra represented the generalisation of the procedures used in arithmetic operations, and the purpose of its teaching was to provide pupils with the rules to operate with given algebraic expressions, as a pre-requisite knowledge for the following school grade.

As algebra knowledge is static knowledge, to be transmitted from generation to generation in a mechanical way, learning the lists of algorithms required the learner to first pay attention to the steps followed by the teacher, and then acquire him/herself the skills to perform the routines on a given list of exercises until the steps to be followed were internalised and mastered. The role of the teacher then, was to distil drop by drop each step to be followed in each of the rules. Pupils did not need to find meaning in what they were doing (i.e., the use of the letters); "they had just to learn the mechanics of each operation and each case of factorisation, and then later they will find the application of the work"; for example, they will see that they needed knowledge of first grade equations, in Grade 9, when they see quadratic equations.

Consequently, as Nacho had taught the same routine for twenty two years, he was at a stage where "there was nothing of his teaching to be planned", his experience had provided him with the necessary knowledge for his teaching of Grade –8 algebra because "he knew the textbook that guided his learning, by heart". Further, if he was a secondary school teacher, he was a teacher "full of knowledge ... more so, after his twenty two years of experience!"

Nacho seemed to be proud of being at that stage, and he felt very confident teaching in Grade eight. The pupils who could not learn according to the type of work provided simply "did not want to learn", and they had to fail (which means they could not be promoted to the next grade). He "had learnt that way, and previous groups of pupils had learnt that way". "Nobody need[ed] theories of Psychology or of Philosophy, talking "about meaning", for example, "to learn mathematics". However, the new legislation in relation to the number of pupils that needed to be promoted from each year group was disrupting the patterns of his teaching practice. That legislation was "another element evidencing the corruption and inadequacy of the government, for the interest of the government was not in the education of the people. "The government [was] saying to the pupils: Don't worry, you may know nothing but you will be passed". And "the pupils and the parents [knew] this!"

Nacho was feeling frustrated with his teaching job. He showed anger "because by prohibiting the teacher to have more than 5% of [the number of] pupils failing in mathematics —given the fact that they were not interested in their learning— the State was telling [him] that he was a failure". He felt that "in the end, after twenty two years of service", what the government was saying to him was "that [he] was not a useful teacher".

6.5 Luis – I am constrained by "my pupils' social background": Summary of Luis' case

Luis conceptualised beginning algebra as a collection of topics and procedures to manipulate algebraic expressions that constituted the content of "mathematics for Grade 8". Teaching Grade 8-algebra was important because "it develops the formal thinking that is needed in the solution of mathematical problems of higher levels of mathematics; for example in the Calculus that he was teaching at a tertiary level". In Phase 1 of the study, Luis declared that he "just [taught] in the traditional way, giving his class a list of exercises", and that "that style of teaching [was] the reason why the majority of pupils did not like mathematics". He argued that it was important to organise problem-based activities for the classroom, so that the pupils could engage in the solution of problems and have the opportunity to discuss what they were doing.

Luis believed that algebraic thinking could not be promoted until after pupils knew their arithmetic content, "including the topics of rational and irrational numbers". He liked teaching Grade-8 algebra "but when the pupils [had] already overcome certain difficulties in their learning of Grade 8- algebra; when there [was] no problem in the pupil; when they [knew] how to manipulate algebraic expressions and [had] no difficulty understanding the instructions given". Teaching his actual Grade 8-group was particularly difficult because besides the pupils' deficiencies in their knowledge, these pupils had acute social problems. They belonged to a poor community from the south of the city, so they had not only "academic problems but also emotional and family problems", and the social context of the pupils represented one of the most important factors determining Luis' teaching practice of Grade 8-algebra.

Despite the number of factors Luis identified to explain the fact that the pupils did not like algebra (i.e., "*la problemática* of school mathematics"), in which the teacher's knowledge and commitment was included, and of his declarations that he was not clear about the uses of the variable, his current pupils' behaviour, which was due to their social background, was always identified as a key factor influencing his teaching practice. His knowledge of mathematics was important for what he planned, but for his practice —in the classroom— his "pupils' behaviour" was the 'Number 1' factor. The pupils made him "sacrifice the classroom work" he had prepared, which affected "the rhythm of [his] practice", and that was not the case with pupils of the private schools of the North (of the city) because they did not have problems in the academic sense.

The need to design the problem-based activities that he had initially argued for at Interview 1, lost importance after Luis participated in the Focus Group, as besides the fact that designing activities required "a conceptual clarity of mathematics on the part of the teacher", it also required time and effort. Furthermore, he was not sure about the activities described and discussed by his colleagues during the Focus Group session because those were "not consistent" with what the pupils had to learn. After all, in those activities the pupils did not work in the *formalisations* of mathematics. They did not do "the historical exercise, like *this is an equation, solve it*". If they did not do that, they were going to have problems later and at the university.

The need to design "problem-based" activities completely vanished when he was asked how he was thinking of addressing the concerns he had raised regarding the pupils' lack of interest for the study of Grade 8-algebra. His current pupils' behavior and lack of prerequisite knowledge was going to remain "the main problem" in his teaching practice because if the pupils did not respond when they were told everything (i.e., how to do the exercises), "how [were] they going to respond in the case that he gave them the problems?" Further, even if his pupils were not going to go to the university, they needed to learn all the list of topics and the historical exercises because everybody should learn the same mathematics and, "anyway, they [were] going to be evaluated through the External Examination of ICFES".

Luis would like to participate in the 14-month, classroom-based research project presented to him at Interview 3 because he wanted to learn how to do research. Why? "Because I want to become a well known person in academia".

6.6 Alex – The *mathephobia* battler: Summary of Alex's case

For Alex school algebra is a collection of formal definitions and procedures that must be acquired one by one in a logical order. Thus, the students need to learn first the definition of algebraic expression ("that comes in textbooks") and the rest of formal conventions about algebraic expressions, to then move on to the basic operations with polynomials. This is a logical order in the teaching of beginning algebra "because they first have to know what an algebraic expression is to then learn how to operate with the expressions and how to use them". This collection of disconnected skills constitutes "the tools" for the solution of the mathematical word problems that appear in textbooks.

Teaching children to write natural language expressions by using conventional symbols used in mathematics, is "preparing them for [Grade 8] algebra", and Grade 8-algebra is important because it develops the critical thinking needed by every citizen. "An individual has critical thinking when he/she is able to give his/her opinion, and this is what the students do when they solve the given exercises".

Out of the given statements, in Questionnaire 1, to describe 'Teaching styles in Grade 8-algebra, the one that describes Alex's first teaching priority is 'Providing opportunities for pupils to develop their communication skills' because "in the lessons the students have many opportunities and space to talk and discuss about their work". They talk about how to solve the given exercises or why an exercise had been done wrongly.

Alex likes teaching algebra but explains that it has been a difficult experience because of the pupils' phobia to mathematics. "The students' aversion to mathematics is a great problem" and it is a crucial determinant of Alex's teaching practice because it is due to their fear of mathematics that they lack motivation for the learning of the subject., Alex also emphasises the impact of the school environment and "the curriculum that the Ministry imposes", when explaining how he sees his own teaching. The textbook does not influence Alex's teaching practice because he uses several textbooks when designing "the *guías* ". However, introducing a teaching approach like the one portrayed by Teacher B of Questionnaire 2 or by the work of the teachers described at the Focus Group "is very difficult because in textbooks everything (i.e., questions and exercises) comes decontextulaised". It is difficult to teach in a different way from the mechanical, "tied-to-contents approach of textbooks because that is part of the [Colombian] culture". Nevertheless, when Alex explains his teaching situation through the concept map he identifies the *students' motivation* as the *crucial* factor in his teaching practice because of the students' fear of mathematics.

Alex "may be interested" in learning about the use of technology in the teaching of mathematics but he would not want to give any commitments of that sort, for the moment. Adopting an innovative textbook "would be another option" that may interest him.

6.7 Reflections on the insights gained from the case studies

The descriptions of the five teachers' cases presented in this chapter have provided us with rich data that has afforded insights into each teacher's conceptions of beginning algebra and his/her own teaching practice. At the same time, the descriptions are testimony to the uniqueness and complexities involved in each teacher's way of conceptualising his/her own teaching situation.

By exploring the meaning each teacher assigned to descriptors regarding specific dimensions of the teaching of Grade 8-algebra, and by eliciting their explanations for specific aspects and episodes observed in their actual classrooms, a window into their conceptions of the nature of algebra knowledge was opened. The window opened wider when the teachers provided their opinions and evaluations of the alternative classroom work that other teachers organised for the introduction of the concept of variable. Important insights into the way each teacher saw his/her own teaching practice in Grade 8 were also gained, including their beliefs about their own knowledge of the teaching of beginning algebra. We observe striking differences between Pablo's conceptions of beginning algebra and those of the other four teachers, but we also see all the teachers coming together in emphasising the primacy of the social/institutional factors to explain the difficulties of incorporating a problem-solving approach in their teaching of beginning algebra. However, the identification and understanding of the dynamics of each teacher's conceptions of beginning algebra and his/her own teaching practices remains the key question to be answered. We need to understand, for example:

- Why did Pablo, despite his awareness of the impact of his "knowledge and dispositions" on his teaching, an aspect that he emphasised throughout the first five months of the data collection, come to identify 'time' as the crucial determinant of his teaching at the end of the data collection period? Why was Pablo, despite his awareness of the inadequacy of his pupils' and their parents' expectations —that the teacher provide 'knowledge' via presentation of a series of discrete topics— prepared to meet these very expectations of 'traditional teaching', which he himself was opposed to?

- Why did Luis and Alex —while declaring the limitations of their knowledge of the teaching of beginning algebra— identify aspects of the pupils' behaviour as the crucial determinants of their teaching practice?

- Why did Nora, being so keen on doing something to improve her teaching and gaining her pupils' motivation for their learning of algebra, anticipate that she needed to be cautious if working with classroom activities like the ones she herself described as "Excellent and interesting"?

- Why did Nacho, having declared —even publicly— that he was a knowledgeable and experienced teacher and that he had such important issues to discuss about the teaching of Grade 8-algebra, and of school mathematics in general, show such little interest in interacting with his colleagues from other schools?

In general, though, we seek to answer the question:

- What is the relationship between a teacher's conception of beginning algebra and her/his conception of her/his own teaching practice?

One could try to develop a causal network explaining interrelations between the conceptions of an individual case teacher, but how does one establish the relevance of the particularities of one case to the other cases? A more powerful way of identifying, developing and relating concepts, in order to deepen our understanding of the phenomenon we are studying is by making comparisons in the data (Strauss & Corbin, 1998); "by reconciling an individual case's uniqueness with the need for a more general understanding of generic processes that occur across cases" (Miles & Huberman, 1994, p. 173); in other words, by following a case replication strategy (Yin, 1989, 2003). This will be the aim of the following chapter.

Chapter 7: Analysis of the teachers' conceptions of their own teaching practices

7.1 Introduction

As stated in Chapters 5 and 6, the collection of data was guided by the two basic research questions:

Question 1:	What are the teachers' conceptions of beginning algebra?
Question 2:	What are the teachers' conceptions of their own teaching practices of beginning algebra?

Chapter 5, which drew on data from Phase 1 of the study, provided us with an initial picture of the teachers' conceptions of beginning algebra and with some first indications of the reasons they believed explained their portrayed teaching practices. In Chapter 6 the cases of five teachers were studied in depth by drawing on data collected in both Phase 1 and Phase 2. Each individual case consisted of a whole study (Yin, 2003) in which convergent evidence was sought regarding each teacher's conception of beginning algebra and his/her conception of his/her own teaching practice. We observed the five teachers emphasising the primacy of the social/institutional factors of teaching when explaining their teaching situations. However, we need to identify and understand the dynamics of the teachers' conceptions of beginning algebra and their conceptions of their own teaching practices. In order to establish the pattern of interaction between the teachers' conceptions of beginning algebra and their conceptions of their own teaching practices, where all dimensions of their conceptions (i.e., knowledge, beliefs and attitudes) are included, it is necessary to identify the common patterns across the five case study teachers' conceptions.

The aim of this chapter, then, is to establish the relationship between the teachers' conceptions of beginning algebra and their conceptions of their own teaching practices. In doing so we will be able to establish more robust answers to the two basic Research

Questions of the study, listed above, which were already addressed for each case study teacher in the previous chapter.

The uniqueness and the complexities observed in the way each teacher explained his/her own teaching situation suggested that comparison across the teachers' conceptions was not an easy task, as by focussing on specific concepts we can fall into the trap of treating the teachers' responses as fragments removed from the social and psychological contexts (see Miles & Huberman, 1994). Two methodological strategies were used to deal with this complexity, allowing thus the identification of structural aspects of the teachers' conceptions, which facilitated comparisons:

- First

To identify the teachers' conceptions of beginning algebra, the focus of the analysis was placed on data pertaining to the teachers' conceptions of the fundamental components of their teaching; that is, answers to the *why*, the *what* and the *how* of the teaching of beginning algebra were sought. It is important to note that the teachers' conceptions of pupils' assessment was a key point in trying to identify their conceptions of the 'how' of their teaching.

- Second

To identify the teachers' conceptions of their own teaching practices, their conceptions of the *why*, the *what* and the *how* of beginning algebra was kept in perspective, while focusing the analysis, at one point, on *why the teachers taught Grade 8-algebra in the way they did* and, at another point, *on why they would (or would not) be willing to consider the incorporation of a problem solving approach in their teaching of Grade 8-algebra.*

The chapter is organised into five main sections. The first two sections correspond to the two basic Research Questions, in the following way: In Section 7.2 the case study teachers' conceptions of beginning algebra are compared. This section is divided into subsections that align with the teachers' conceptions of the fundamental components or elements of the teaching of beginning algebra, namely *the Why* (the purpose), *the What* (the content), and *the How* (the method). A comparison of the case study teachers' conceptions of their own teaching practices is the subject of Section 7.3, and this section is divided into two subsections: Subsection 7.3.1 presents the teachers' explanations for 'why they taught Grade 8-algebra in the way they did', and Subsection 7.3.2 presents the teachers' explanations for 'why they would (or would not) be willing to consider the incorporation of a problem solving approach in their teaching of Grade 8-algebra'. Some references to the identified conceptions of the larger initial group of thirteen teachers will also be made, when relevant.

Having identified the patterns in the case study teachers' conceptions of beginning algebra and their own teaching practices, Section 7.4 attempts to clarify the relationship between the teachers' conceptions of beginning algebra and their conceptions of their own teaching practices, and Subsection 7.4.1 provides a general model that emerged from the analysis of the teachers' conceptions, explaining a teacher's attitudes to change in her/his teaching. A theoretical continuum of teachers' conceptions of their own practices, depicted by the model that emerged is clarified in Subsection 7.4.2 where a typology of teachers is described. The characteristics of this typology are presented in Subsection 7.4.3. The chapter ends with a conclusion in Section 7.5.

7.2 The case study teachers' conceptions of beginning algebra

A significant relationship between the teachers' knowledge and beliefs about the nature of algebra knowledge, and the main purpose for the inclusion of algebra in the school curriculum, as well as their topic-specific teaching goals was identified. Reinforcing Eraut, Goad and Smith (1975) and Grossman's (1989) observations that teachers' beliefs about the goals for teaching function as an organisational framework or conceptual map for teaching, the data further show that the teachers' goals for teaching were born out of their knowledge and beliefs about the nature of algebra knowledge.

7.2.1 What is beginning algebra?

Although the five teachers' teaching was based on a common list of specific subject matter content, a clear distinction between Pablo's conceptions of the nature of beginning algebra knowledge and those of the rest of the teachers was identified. While ample and strong evidence of the teachers' conceptions of beginning algebra was found in their responses to questions related to the fundamental components of teaching (i.e., the *why*, *what* and *how*), their beliefs about the possibilities of promoting algebraic thinking in the primary school provided additional evidence for the identification of their conceptions. The teachers' views about the possibilities of promoting algebraic thinking in the primary school will be focused on first, in the following paragraphs, and then their conceptions of 'the why', 'the what' and 'the how' of the teaching of Grade 8-algebra will be presented.

Promoting algebraic thinking in the primary school

While Nacho and Luis believed that the promotion of algebraic thinking should be delayed "until after the pupils have mastered the arithmetical work that is the focus of the first seven years" of the basic cycle of education (i.e., it should start in Grade 8), Pablo, Nora and Alex adhered to the idea that algebraic thinking could be promoted in the primary levels of schooling. However, the examples that Pablo, Nora and Alex provided of how the promotion of algebraic thinking could take place showed contrasting meanings of 'algebraic thinking'. Pablo considered that children's generalisations about their working methods, using natural language, represented algebraic thinking:

> when children buy sweets, they can be encouraged to think about the formula, for calculating the cost of any numbers of sweets, having the price of a sweet; that is without mentioning the word formula, but just saying it as a primitive algebra; (says and writes, 'Number of sweets × Price of one sweet = Cost').

Nora and Alex, on the other hand, located the initiation of algebraic thinking in the use of formal symbolisation to translate natural language expressions "into algebra". Nora would give primary-level children similar questions and tasks to the ones she gave her Grade 6 pupils, where they used "symbols to represent numbers so that they start to become familiar with the language of algebra". She would ask, "for example, what is the number that multiplied by 7 gives us 21? and then ask them to express this mathematically". Alex believed that algebraic thinking could be initiated "when normal expressions are written using mathematical symbols". For example,

> Mercury is a planet, can be written: 'M ∈ P'.

7.2.1.1 Why teach beginning algebra and what to teach

The teachers' beliefs about the central reasons for the teaching of Grade 8 algebra, and the sequencing of topics were related to the importance they gave to the textbook approach in their teaching. While Pablo believed that the central reason why pupils need to learn algebra is "because it is important knowledge for understanding real-life situations, and without the pupil's identification of this they would see no point in learning it", Nora, Luis, Alex and Nacho believed that pupils need to learn algebra because it is prerequisite knowledge needed in the next school levels and in higher education. Nora's, Luis', Alex's and Nacho's descriptions of the sequencing of topics coincided with the sequencing of topics observed in popular textbooks (that is, starting with a definition of 'algebraic expression', moving on to 'operating with expressions', 'special products,' 'factorisation', 'equations' and 'problems of application'). However, Pablo —who was opposed to teaching by the textbook, and did not want to pay attention to the definitions of algebraic expression given in the textbooks— included "the Pythagoras theorem at the start of the work with algebraic expressions because [he] wanted to use the concepts of area and perimeter" of basic geometric shapes to teach algebraic expressions". He wanted to integrate his teaching of algebra with topics of geometry and have pupils working on the *mathematical calendar*[1] in a continuous way.

In discussion about what to teach, the issue of the need to teach algebra concepts as separate from other mathematical concepts was emphasised by the majority of teachers. As we saw in the case descriptions, Nora, Alex and Nacho saw the study of geometry as an area where algebra knowledge could be applied and, therefore, the algebra concepts had to be taught before the geometry. Further, Nora and Nacho, who were heads of the mathematics department in their schools, commented that they had agreed that geometry was taught by a different teacher (from the one teaching algebra) because otherwise they would dedicate the whole school year to teaching the algebra program and they did not get to teach any geometry. This view of taking basic geometrical concepts like area and perimeters of simple shapes as topics in which to apply what was learnt in algebra was common within the initial group of thirteen teachers. Five more teachers (out of the initial group of thirteen) explained that they had concluded that

[1] It was explained in Pablo's case description in Chapter 6 that the mathematical calendar was a series of worksheets containing a variety of tasks and problems relating geometry and algebra concepts. See examples of these in Appendix 6.2

'geometry in Grade 8 was taught by a different teacher from the one teaching algebra because when one teacher was in charge of both algebra and geometry programs, he/she would teach the algebra and not the geometry'.

Luis who claimed at Interview 1 that "connections of algebraic expressions with other mathematical concepts like area and perimeter [were] very important" for him, openly declared at Interview 2 that

> the student teachers are doing that type of work with areas and perimeters of geometrical shapes but I don't know what specific work it is.

A bigger window into the teachers' conceptions of beginning algebra was provided by their descriptions of how to teach specific topics, an aspect to which the next subsection is dedicated.

7.2.1.2 How to teach beginning algebra

Having outlined the teachers' descriptions of the sequences they followed in their teaching of Grade 8-algebra, the questions of how specific topics and concepts were taught became an important (though not the only) focus of inquiry during Interviews 1 and 2. The teachers' descriptions of how algebra work was initiated, together with their explanations of specific classroom incidents in the observed lessons provided insightful evidence of their conceptions of beginning algebra. Another data point was represented in their explanations about the teaching of factorisation, a topic that was the subject of Nacho's and Luis' observed lessons. Table 7.1 contains the teachers' specific descriptions and arguments for the introduction of algebra work in Grade 8, and about their teaching of factorisation[2]. A striking contrast between Pablo's and the rest of the teachers' explanations of their teaching of these two topics is evident.

2 The teaching of factorisation was a topic that emerged spontaneously in the conversation of Nora and Pablo, both at Interview 2 and Interview 3.

Table 7.1 The teachers' descriptions and explanations about the way they introduced algebra work to pupils and the way they taught factorisation

Introductory work with algebraic expressions	*The teaching of 'factorisation'*	
I don't pay attention to the definitions that come in the textbook; that an algebraic expression is a combination of numbers ... When we started to use letters or simple expressions we did it because we were talking about area and perimeter of rectangles or triangles! ... I know that pupils convert an expression like 2a + b into something like 3ab. [for that reason] I am designing an activity which is a game and not a real life situation... the 'buying and selling' activity ... (Int. 1)	If I really learn, for example, how to factorise $a^2 - b^2$, and why I factorise it in a specific way, I should be able to factorise the other types of polynomials or the other cases. And why do I have to teach factorisation as a separate topic if factorising is a way to know if I understand multiplication? ... Pupils need to be asking themselves 'why am I doing this?' For me mathematics is analysis, ... (Int. 2). ... if the pupils learn the formulas for each case of factorisation, by heart, they are not learning they are just manipulating the expressions (Int. 3).	Pablo
Because in Grade 8 pupils have great difficulty with algebraic expressions, in Grades 6 & 7 I give pupils things like 'the double of a number', that is represented 2 ( ) or... When pupils have difficulties with like terms I send them to tidy the fridge. You cannot mix oranges with pears... (Int. 1)	To decide what I am going to do in a lesson, I think of the subject contents, the resources we have, and how I am going to assess pupils in that lesson. For example, now that we are in factorisation I give them a quiz each day.... Why? Because factorisation is something mechanical, something that they learn in the lesson but they forget very easily. So each day I teach one case and they have a quiz. (Int. 3)	Nora
Algebraic work commences with algebraic expressions and all that is related to that: coefficient, literal part, type of algebraic expressions, etc., and then we move on to addition and the rest of the operations with polynomials. (Int. 1). They have difficulty in seeing that 2a and b cannot be added. It doesn't matter how many times you explain it to them... (Int. 2)	They learn each case of factorisation by paying attention to what the teacher explains... They follow some levels to learn each case of factorisation: In level 1, pupils identify or establish the form of the [given] expression. In level 2 they practice finding the factors of given expressions [of this form], in order to mechanise the procedure. In level 3 they can apply this procedure to an artificial problem... (Int. 2)	Luis
We start by defining an algebraic expression, that it is a set of letters... I explain what a term is, what binomials and trinomials are, like terms, etc., in order to start doing addition and subtraction with those. We then move on to all the operations with polynomials (Int. 1). Working the topics in this logical order is very important for me because it helps the students understand algebra. (Q. 2)	We saw all the cases of factorisation, paying attention to those they need most for the solution of quadratic equations. ... when they [the pupils] need to factorise a given expression, they identify to which case it belongs, to be able to decide how to factorise it. (Int. 2)	Alex
When we start with algebraic expressions I take time making sure they understand that a letter represents any number. We see first algebraic expressions and numerical values; then operations with algebraic expressions ... showing them that what one does in arithmetic ones carries on doing in algebra. ... Initially it is something mechanical; something that does not have any applicability but that I show how to do. (Int. 1)	The Case VII of factorisation, Trinomial of the form '$a x^2 + bx + c$', was a new case for them in that lesson. We saw the first six cases in previous lessons. At the beginning they find it difficult but, later, they become confident when they see that they can do things as I teach them to do them. ... They know they are not allowed to interrupt the flow of explanations because they have to first understand what I am explaining. That is the normal course of a lesson! (Int. 2)	Nacho

Examination of specific data collected through Questionnaire 1 and Interview 1, representing elicited information about their teaching method, showed again a contrast between Pablo's and the rest of the teachers' approaches. Table 7.2 shows the teachers' responses to questions C3 and C4 of Questionnaire 1, where they had to explain how they would respond to pupils who showed specific misconceptions when operating with algebraic expressions. While Pablo showed a concern with engaging the pupils in the identification of the relationship connecting the given variables in Question C4, or of encouraging the pupils to see the connections between the expression and the corresponding geometric representation in Question C3, the rest of the teachers would either explain the rule and the procedure to follow or refer the pupils to "the given indications".

As can be observed in Tables 7.1 and 7.2, the teachers' responses to the specific questions that asked them to account for their teaching approaches, seen in conjunction with their purposes for the teaching of these specific topics showed a striking contrast between Pablo's and the rest of the teachers' conceptualisations of the teaching of beginning algebra that exposed, at the same time, their beliefs about how pupils learn. While Pablo believed that pupils learn algebra if they are provided with activities that help them establish the algorithms of procedure to operate with algebraic expressions, the other four teachers believed that pupils needed 'to be told because it was very difficult for them to construct their own ideas'.

Among the teachers with an instrumental approach, Nacho was identified as the one who placed more emphasis on a 'spoon-feeding' approach and who followed a specific textbook sequence in a predetermined 'step-by-step' fashion. These findings represented some grounds for a first attempt to categorise the teachers' conceptions in relation to the instrumentalist vs. problem-solving views of Ernest (1989, 1991), where Pablo and Nacho were located at the opposite ends of a continuum, with Nora, Luis and Alex being in the middle as shown in Tables 7.1 and 7.2. However, this initial organisation of the teachers' conceptions within a continuum was considered as only a temporary categorisation and, therefore, needed to be tested throughout the process of analysis.

Table 7.2 The teachers' responses to Questions C3 and C4 of Questionnaire 1

Question C3 *Towards the end of the academic year, a teacher asks his Grade eight pupils to find the area of a rectangle whose sides are 5 and 2 + e. Many pupils, in a group of 29, answered the following:* $A = 5\,(2 + e) = 10 + 5e = 15e$ $A = 15e$ *a) What do you think of this answer?* *b) How would you respond to these pupils?*	*Question C4* *Imagine you ask a grade-nine pupils to write an equation, using N for the number of nurses and D for the number of doctors, for the statement: In Central Hospital there are five times as many nurses as doctors. Several pupils write as an answer: 5N = D* *a) What do you think of this answer?* *b) How would you respond to these pupils?*	
a) No clarity in structure of a polynomial. ... b) I would refer them to the activities where they have worked to see if they can connect this with something from the activities, and see if that is clear. (Q-1). I know pupils have the tendency to do this. They convert expressions like 2a +b into... (Int.1)	a) They are not analysing how the variables vary. b) Let's check what you did and see if according to the given statement, E is dependent or independent. (Q-1). I would encourage them to see by themselves how the variables change for specific values... (Int. 1)	Pablo
No clarity with like terms. I will explain again... (Q-1). I will clarify to them that those two terms are not like terms. Those are not the same type of objects... When they are learning to reduce terms I tell them if I have got one balloon plus another balloon... It would not come to his mind to think of the situation in terms of the area-diagram, that's why I end up telling them in order to help them with the hard bits... (Int. 1)	a) No clarity in the relation of N and D. b) I'll show them, with examples, that the expression is not correct (Q-1) I will show them the two tables one for the expression D =5N and another one for N = 5D... I will do the tables in the same order of N and D so that they can compare more easily... (Int. 1)	Nora
a) No clarity with unlike terms. b) Replace by different numbers and see what happens. (Q-1) See if 15e is always the same. I'd like to present an activity where they can see the parts of an expression. (Int. 1)	a) There was only mental calculations in the posing of their expression. b) Do a tally and write the proportions. (Q-1). We'd do a table and then write the representative fraction or ratio to, then, write the proportion. ...But the situation is not even clear for me! (Int. 1)	Luis
a). No clarity . b) I will explain to them again if necessary. (Q-1). I always give them the indications to simplify like terms when the topic is introduced .. I'd recommend them to revise the indications... (Int. 1)	a) Their construction of the expression is literal. b) Check your equation. Give several examples replace and verify (Q-1)... I would ask them to give numerical values. In that way they realise that the expression is not correct... They are using the N to represent the nurses but the setting of the equation is very literal... (int. 1)	Alex
a) No clarity with like terms. b) I will explain again that those are not like terms. ... They go to the board and repeat what they have done wrong. They clarify with that [explanation]...	a) It is wrong b) Find a expression that generalises the situation (Q-1). It is wrong. It is a very particular case (5N). The expression 5N = D is too open. It is not clear what is being asked... (Int. 1)	Nacho

How to teach beginning algebra – Describing *preferred* teaching styles

Throughout the data collection process the teachers provided descriptions of how they taught and how their pupils learnt. These descriptions were initiated with their *Number one* choices of preferred and actual teaching styles. The comparisons of data related to their teaching style, collected from several sources, clearly showed that Pablo and Nacho could be distinguished within the five case study teachers —and, indeed, within the group of nine case studies— as two whose teaching styles as portrayed in Questionnaire 1 remained consistent throughout the whole period of data collection. The consistency of Pablo's and Nacho's ways of describing their teaching is shown in Table 7.3 which contains data collected through Questionnaire 1, Interview 1, Questionnaire 2, Interview 2, and Interview 3. While Pablo was concerned with a progressive view of teaching, Nacho put forward a purely transmissive view. Nora and Alex, who in their overall rankings of teaching style descriptors for their preferred and actual practices, in Questionnaire 1, did not privilege a transmissive approach, found themselves —during the rest of the data collection activities— providing explanations and arguments for the centrality of a teaching by telling and a procedure-based approach in their teaching of beginning algebra. Luis presented himself, in Phase 1, as a rejecter of the "traditional… teaching by giving the pupils a list of exercises" approach, but he became a defender of this very approach during Phase 2 of the data collection (see Table 7.3).

Table 7.3 Teaching approaches put forward by the five teachers throughout the data collection process

		Pablo	*Nora*	*Luis*	*Alex*	*Nacho*
Quest. 1	Prefer.	Promoting connections between concepts	Organising problem-based activities	Promoting connections between concepts	Developing communication skills	Giving clear explanations of procedures to follow
	Actual	Promoting connections between concepts	Giving clear explanations	Promoting connections between concepts	Developing communication skills	Giving clear explanations of procedures to follow
Int. 1		Pupils need to see where the expression come from, the reasons why they are adding polynomials, so I start with simple situations of area and perimeter …	Before the work on problems, pupils learn to operate with algebraic expressions and that is what the work of G 8-algebra is about. It is routines.	Promoting connections is very important for me. That work is being done by the student teachers, and I don't know the details of the work they are doing.	I give the definitions of algebraic expressions, a term, etc. then I explain how to do operations with polynomials… They have plenty of space to talk about what their work.	I like to explain as if they were babies. Initially it is something mechanical; it doesn't have any applicability, it's something that I explain how to do…
Quest. 2		*Teacher B follows a problem-solving approach.* I don't think s/he follows a textbook, given the type of work and the method she/he uses.	*It is necessary to pay attention* to Teacher A's work because some pupils struggle, they need telling and they need to learn the language of mathematics.	Teacher B *follows a problem-solving* approach, but I teach like Teacher A does.	*I give attention to both problem-*solving (Teacher B's work) and Teacher A's type of work… Teacher A's work can be good because you save time.	Teacher A's *approach is* excellent, and he/she advances faster than teacher B. Teacher B is departing too much from the established order.
Int. 2		I design the activities on the basis of what connects for me, but they have to connect for the pupils…	In order to prepare them for the solution of problems I give them the translation exercises, where they write in mathematics what is said in Spanish..	Pupils learn at three levels: 1) pupils identify the form or structure of the given expression. 2) they practice in order to mechanise the procedure. 3) they can apply this procedure to a problem.	I frequently explain the procedure to follow in a set of exercises so that they don't make mistakes…	They learn by paying attention to my explanations without interrupting me.
Int. 3		All three teachers discussed in the FG followed a problem solving approach. They designed activities according to clear purposes.	I would like to teach my pupils to solve problems but I have to be careful not to use too much time in that as the algorithms need attention too.	I want to know if those activities discussed in the FG are consistent with what one has to teach: …the formalisation (i.e., the historical exercise)	The activities discussed at the FG are a good way to help them see the applications of maths… But all that one finds in the textbooks is tied to a list of contents… Everything comes...	N/A

Assessing pupils' work

The teachers' descriptions and explanations about their assessment practices provided further evidence of their conceptions of the nature of algebra knowledge. The four teachers that focused on a purely transmissive approach justified their central practices —identified in Chapter 5 as their top priorities— of "giving pupils frequent quizzes" and "having pupils at the board correcting the homework" with reasons similar to Nora's, who did not like "teaching Grade-8 algebra":

> The algorithms of operation are what Grade-8 algebra is about... That's why I don't much like teaching in Grade 8 because it is a repetitive work. It is monotonous (Int. 1). ... Factorisation is something mechanical, something that they learn in the lesson but they forget very easily. So each day I teach one case, and they have a quiz. (Int. 3)

Alex thought that learning to solve equations was a matter of memory:

> The lessons normally start by correcting the given homework, and the quizzes are normally given after we correct the homework (Int. 2)... We correct the quizzes as soon as they finish. I explain the steps to follow several times because they forget them. ... [I] strongly agree that giving pupils marks and rewards are good strategies for getting them to complete their assignments (Questionnaire 1), because, unfortunately, if you don't give them marks for their work they don't do the work! ... When they fail in the term final tests, they know they owe me an attainment target, so they have to revise the topic and do the test again. (Int. 1)

Luis who claimed, at Interview 1, that the assessment of pupils' work should inform decisions about the design of classroom activities, argued at Interview 2 that he did quizzes after each topic was taught, because he was required to provide pupils' marks each term, and to report "that some pupils [had] not made any progress". He argued:

> I would say that one should collect pupils' notebooks, not just to check if they have a record of all the work done in the classroom, but to look at how they are thinking... But that is difficult to do when you have to teach groups of 40 children... (Int. 1). ... when I collect the notebooks to see their homework, I just put a tick or a signature because there is no time. (Field notes)
>
> I do frequent quizzes because, first of all, by legal requirements you have to report pupils' grades every term; but, secondly, because some pupils did not make any progress in the process; they did not move forward in the process, so one has to do assessment through closed tasks. For example when they ask for 'a participation[3]', I do it because they need that mark. (Int. 2)

[3] Remember that 'a participation' for Luis and his pupils was the speed exercise for the first 5 or 10 to hand in.

And Nacho's preferred practice was to assess pupils when he asked them at the board "to correct the homework" exercises.

> Pupils are assessed through quizzes after a topic is taught, but especially when they are asked to correct the exercises of the homework, at the board (Int. 1). ... Negative results from pupils' quizzes means that they are not doing the homework. (Int. 2)

Assessment tasks in Nora's, Luis', Alex's and Nacho's classrooms were given to pupils with the purpose of checking if they had followed the procedures taught to obtain a right answer, as also reported by Yackel, Cobb & Wood (1992). Unsatisfactory results meant to the teacher that the pupils were not doing the list of homework exercises and therefore had low motivation for their learning. How was assessment of the pupils' work conceptualised by Pablo? Pablo had "two preferred forms of assessing his pupils (as he wrote in his responses to Questionnaire 1, and explained at Interview 1): i) "monitoring, continuously, the pupils' work, which [he did] by collecting the pupils' notebooks, to see what they [had] done during the lessons and, obviously, in their homework" and ii) "through written tests, where pupils can apply what they have learnt during a period of time without partitioning the topics". But giving pupils tests where they could apply what they had learnt without partitioning the topics could not be managed as he "had to do tests after each topic". Without being asked, Pablo explained that he was aware that tests were "not the best way to identify pupils' difficulties ... you see them working very well in class and yet they make mistakes in the tests". So why did Pablo give tests at all? He had "to give a test after each topic" was taught

> because of the deadlines to hand in the pupils' grades; and following each pupil in a personalised fashion is just not possible; so I have to do something that can be manageable for everybody, in a simple way. ... I have to hand in, to the Academic Coordinator, the grades in relation to the content objectives stated in the Grade 8 program for that specific term. ... Those objectives represent the 'assessment indicators' for which I have to provide pupils' grades for the school assessment report.

Assessment tasks in Pablo's classroom were given to pupils to establish whether they had made the connections. However, despite his disagreement with the fact that he had to "do tests partitioning the topics", Pablo did not question the school established assessment patterns "because that's how the school works".

7.2.2 The teachers' conceptions of how pupils learn

The evidence on which the analysis has focused so far shows that although the five teachers' teaching was guided by the same list of topics which reflected the sequences followed in the available textbooks, Pablo's emphasis on the establishment of connections between mathematical concepts and on helping pupils to make sense of what they were doing was in high contrast with the emphasis made by his colleagues. Pablo was opposed to presenting the pupils with textbook definitions and algorithms, but especially to Nacho's approach of learning without seeing any applications, but just "mechanically".

The five teachers' learning models, as identified at this point of the analysis, will be kept in perspective in the following section where the data will be screened in order to establish the teachers' conceptions of their own teaching practices and identify answers to Research Question 2.

An observation that deserves attention is that all teachers, regardless of their conceptions of how pupils learn beginning algebra believed that the development of communication skills of their pupils was an aspect that they gave attention to in their teaching of beginning algebra. As shown in Chapter 5, the descriptor, "Providing opportunities for pupils to develop their communication skills so that they can express their mathematical ideas with confidence" was ranked by the initial group of thirteen teachers as their second top priority. Pablo provided some evidence of his awareness of the need to organise classroom activities to promote interaction between pupils, and "comparisons of individual's understandings" (Cooper & McIntyre, 1996) with significant others (e.g., teachers and fellow pupils). However, there was no report on the part of the teachers of their concern for giving specific attention to the development of their pupils' communication skills (i.e., how pupils articulated their thinking, their meanings and/or difficulties when presenting them to the teacher or to their peers). Neither did they report that they encouraged the pupils to record their ideas when they were working individually or in groups. The teachers' beliefs about what algebra knowledge is, seemed to define the type of communication skill the teacher thought needed to be developed. These beliefs represented the frame of reference of the assessment requirements and, therefore, for the communication skills necessary. For example, Nacho believed that a pupil who was able to verbalise the steps followed in a routine (defined by the teacher) had developed the necessary communications skills in beginning algebra, which he declared were assessed when calling pupils at the board to do the given exercises. Luis and Nora believed that discussion about the pupils' own ideas, their difficulties and/or interest were outside the domain of the context of mathematics. Pablo explained that he did not monitor the pupils' development of communication skills because that aspect was not on the individual pupils' school report.

If, as the thirteen teachers stated in Questionnaire 1,"[their] mathematics teaching was concerned not just with mathematical content goals but with broader educational goals", then the development of the pupils' communication skills needs to be a central goal of mathematics education. Further, effective mathematics teaching and learning is dependent of effective communication and cooperation between teachers and pupils, where teachers' decisions are informed by their knowledge of pupils' thinking (Carpenter, Fennema, Peterson & Carey, 1988; Fennema, Carpenter, Frank, Levi, Jacobs & Empson, 1996; Fennema & Franke, 1992; Foster, 2003; Schifter & Fosnot, 1993; Yackel, Cobb & Wood, 1992). Teachers can take teaching decisions which are informed by their knowledge of the pupils' mathematical thinking *if the pupils' skills of communicating about this thinking,* (rather than reiterating terms or repeating algorithms) *are given the necessary attention in the mathematics classroom.*

7.3 The case study teachers' conceptions of their own teaching of beginning algebra

As the ultimate purpose of this study is to identify the teachers' conceptions of their own teaching of beginning algebra and their relationship to the conceptions of change in their teaching, the teachers' conceptions of their teaching of Grade 8-algebra were studied through a process of inquiry that distinguishes two key research points. At one point, the teachers explained *why they taught Grade 8-algebra in the way they did* and, at another point, *why they would or would not consider the incorporation of a problem-solving approach in their teaching.* Since the teachers' explanations of 'why they did what they did' —or of why they would or would not do it differently— contained information about their conceptions of beginning algebra, the findings presented in the previous section are to be compared with those in this section. Subsection 7.3.1 discusses the findings related to the first point, and Subsection 7.3.2 the findings related to the second point.

7.3.1 Why the teachers taught Grade 8 algebra in the ways they did

As explained in the previous section, although specific opportunities for the teachers to talk about the way they saw their teaching and why they did what they did were provided in Questionnaire 1 and the Concept map Activity, the data collected through each instrument or research activity contained evidence to answer not only Research Questions 2 but Research Question 1 as well. Therefore the teachers' conceptions of their own teaching needed to be drawn out by continual contrasting or cross-referencing between different sets of data collected in different moments and through different means. In examining and organising the teachers' accounts of what they did and why they did it, I was confronted with patterns of data as strong evidence that spoke about the teachers' conceptions of their roles as teachers of beginning algebra as well as of their conceptions of the crucial determinants of their teaching and of their beliefs about themselves. These specific themes, which were initially (i.e., at the research design stage) identified as the key aspects or Research Sub-questions to focus on when identifying the teachers' conceptions of their own teaching practices, are the focus of the next subsections. The Research Sub-questions mentioned, as specified in Chapter 1 are:

- What are the teachers' conceptions of their roles as teachers of Grade 8 algebra?
- What are the teachers' conceptions of the roles of social/institutional factors in their own teaching practices?

The evidence to answer these specific Research Sub-questions was found by following the threads in the fabric of the teachers' conceptions of their central teaching styles as can be seen in the following subsections.

7.3.1.1 The teachers' conceptions of their roles as teachers of beginning algebra

As we saw in the case descriptions, at Interview 1 the teachers provided explanations for their top teaching style priorities (established in Questionnaire 1) and described the sequences of work they followed in their teaching of Grade 8-algebra. They justified their teaching-style top rankings by explaining what they did in the teaching of specific topics and, in all cases, they offered their evaluations of the results of their teaching. Table 7.4 shows the teaching style each teacher described as the one that took place in their teaching of Grade 8, and the corresponding evaluations of the situation. As we can see, the teachers provided indications of what they saw as determinants of their teaching.

Table 7.4 The teachers' central teaching style and their evaluation of the results of their teaching

	Pablo	*Nora*	*Luis*	*Alex*	*Nacho*
Central teaching style	Promoting connections between concepts through the organisation of problem-based activities	Giving clear explanations of the rules of manipulation needed in the solution of textbook problems	Explaining the steps to follow in a list of exercises which represent the traditional mathematical tasks	Telling the rules of manipulation that they need to solve mathematical word problems	Giving clear explanations, with the greatest of detail of procedures to follow
Evaluation of results	My activities work for the majority of pupils but not for all, and sometimes I have to tell because I don't know how to design activities for all topics	The results are not good because of the pupils' lack of motivation	The pupils' performance is not good due to their [inadequate] prerequisite knowledge and to their social background	The results are just average due to the students fear of mathematics and their self concept	My pupils are fascinated with my teaching but the results are not good due to their lack of prerequisite knowledge

The teachers' conceptions of their central roles - Reasons for unsatisfactory results

Further explanation and evaluation of the results of their teaching was provided by the teachers at Interview 2, when they evaluated the results of classroom work observed against their central teaching intentions. They offered further evidence of their conceptions of their roles as teachers of beginning algebra as well as attributions, in the locus of control dimension (Weiner, 1980) for their teaching situation. When they were given questions about specific aspects or incidents brought in from classroom observation, which aimed at clarifying or contrasting the data already obtained, Nora, Luis, Alex and Nacho always continued expressing their dissatisfaction with the pupils' behaviour and justified the low results of their teaching by aspects related to the pupils. Pablo, in contrast, spoke of the limitations of the type of work he designed "so that all the pupils [could] understand", and attributed the problem to his limited knowledge of the teaching of mathematics.

Contrasting and corroboration of data prompted the identification of connections between the teachers' conceptions of their central role in their teaching and the dimension of attribution. A teacher with a teaching style that focused on 'giving clear explanations' attributed the unsatisfactory result of his/her teaching to external factors mainly related to the pupils, while Pablo, whose concern was "not to tell" but to promote pupils' creation of the mathematical ideas, attributed his pupils' lack of success in establishing the connections to his inadequate knowledge for the teaching of beginning algebra (see Table 7.5). The teachers' professed purposes for the teaching of specific topics, and their beliefs about how pupils learn represented the basis for their decisions not only about the types of classroom work to be provided in the classroom but also about their roles as teachers. Once again, a striking difference between Pablo's explanations of his teaching situation and those of the other four teachers was observed.

Table 7.5 The teachers' conceptions of their central role and the reasons for unsatisfactory results

	Pablo	*Nora*	*Luis*	*Alex*	*Nacho*
Teachers' central role	Not telling	Telling	Telling	Telling	Telling
Teachers' reasons for unsatisfactory results	My knowledge for teaching	The pupils' behaviour	The pupils' behaviour and knowledge	The pupils' behaviour	The pupils' behaviour
Locus of control	Internal	External	External	External	External

The teachers' conceptions of how pupils learn beginning algebra concepts - Further evidence

These data further show that the teachers' conceptions of their central roles were based on their beliefs about the nature of beginning algebra knowledge, their pupils and how they learn beginning algebra. Although the end products of classroom work were represented in the algorithms for all five teachers, the difference between Pablo's conceptions and those of the other teachers was on how the pupils came to know "the given mathematical entities … absolutes that students had to acquire … as reflected in curriculum statements" (Jaworski, 1999, p. 162). While for Pablo, the pupils needed to draw meaning for the mathematical algorithms, through engagement in activities that he designed, and the success of a lesson depended on the activity, for Nora, Luis, Alex and Nacho the pupils learned by being told how to follow each step of a given procedure. The success of a lesson for these teachers was represented in the pupils' motivation to engage in repeated practice of the procedures shown to them.

7.3.1.2 The teachers' conceptions of the crucial determinants of their teaching practices

The teachers' reasons for the unsatisfactory results that were highlighted in Table 7.5 pointed to the type of factors (i.e., internal or external) they identified to account for what was taking place in their teaching. We need to remember here that the teachers were engaged in a concept map activity where they were invited to identify the main factors, which they saw as determinants of their teaching of Grade 8-algebra. This activity took place in three stages. During the first two stages the teachers' built their initial concept maps, drawing on their current teaching practices. In the third stage they reviewed their initial map, after they had had the opportunity of discussing alternative approaches for the teaching of beginning algebra.

The results from the two first stages of the concept map activity (see Table. 7.6) showed that Pablo and Nacho —the two teachers that on previous occasions had been categorised at the ends of the continuum— maintained their initial *Number one* factors constant throughout the data collection process. While Pablo and Nacho maintained consistency in the identification of the determinants of their teaching, both when they provided explanations for specific classroom episodes and during the initial stage of the Concept map Activity, the other three teachers, Nora, Luis and Alex, identified different factors as their 'Number one' determinant of their practice at different points of the data collection process. Furthermore, in the second stage of the concept map activity, these three teachers, pointed at one specific factor as the crucial determinant of their teaching (e.g., the teacher's knowledge in the cases of Luis and Nora, and "the motivation" created by the "Student-teacher relationship", in Alex's case) while in their talk and explanations of situations, they identified factors related the pupils' abilities and behaviour. Table 7.6 shows a summary of the factors the teachers identified as important, both when explaining classroom situations and during the two first stages of the concept map.

While Pablo emphasised the role of internal factors in his teaching practice, Nacho attributed the unsatisfactory results exclusively to external factors. Nora, Luis and Alex, despite their acknowledgement of the importance of the teacher's knowledge, identified external factors to justify what took place in their classrooms.

The findings already presented show us that when the teachers explained what was taking place in their classrooms, they did it in terms of aspects related either to their conceptions of the nature of beginning algebra knowledge, or to the role of factors belonging to the social context of their teaching. However, only Pablo was aware of the influence of his conceptions of beginning algebra on his teaching; when explaining what he did in his teaching he identified mainly internal factors as the crucial determinants of his practice. Nora, Luis and Alex, despite their acknowledgement of the importance of "a teacher's knowledge", when asked about it in the second stage of the concept map, in their explanations of what took place in their classrooms, they pointed to the primacy of external factors. Nacho, on the other hand, refused completely to consider "a teacher's knowledge" as a possible factor influencing his teaching because "if a teacher is given a teaching job, especially by the government it is because that teacher is well qualified to do the job".

Table 7.6 Summary of factors identified by the teachers as the crucial determinants of their *current* teaching of Grade-8 algebra (Why they did what they did)

		Pablo	*Nora*	*Luis*	*Alex*	*Nacho*
Identifying factors that determined their teaching practice	Explaining reasons for differences between preferred and actual practices *(Int.1)*	My knowledge for the teaching of mathematics	Pupil's attitudes Nature of G 8-algebra	Pupils' motivation and social background The teacher's knowledge	Pupils' fear of mathematics	The pupils' lack of prerequisite knowledge
	Explaining classroom episodes *(Int. 2 - Part 1)*	The pressure from pupils & parents to cover the set program My knowledge for teaching	The pupils' attitudes	The pupils' behaviour	The pupils' motivation My knowledge for the teaching of mathematics	The pupils' lack of interest for learning, and prerequisite knowledge
Initial concept map (Int. 2 – Part 2)	*Stage 1:* Identifying the crucial factors that determined their teaching practice	My dispositions (which include My knowledge)	Pupils attitudes	Pupils' behaviour and prerequisite knowledge	Pupils' motivation	A permissive legislation… The pupils' lack of interest for learning & prerequisite knowledge The lack of cooperation of parents… The government's lack of interest in…
	Stage 2: Discussing the role of the teacher's knowledge	My knowledge and dispositions	The teacher's knowledge and vision	Pupils' behaviour and prerequisite knowledge	The pupils' motivation & knowledge	

Table 7.7 shows the categorisation into 'internal' or 'external' of the factors identified by the teachers as crucial, during the data collection activities that focused on the question 'why they did what they did in their teaching of Grade-8 algebra'.

Table 7.7 Categorisations of the teachers' conceptions of the crucial determinants (in the context of why they did what they did)

Pablo	*Nora*	*Luis*	*Alex*	*Nacho*
Mainly Internal	Mainly External	Mainly External	Mainly External	Totally External

At this point of the analysis, the location of Pablo and Nacho at the two ends of the continuum, and the other three teachers in the middle was confirmed. Table 7.7 shows a continuum where the teachers' conceptions of the determinants of their teaching practice were in direct relation to their conceptions of the nature of beginning algebra knowledge which, at the same time, determined the teachers' learning models. It is observed that the more emphasis a teacher placed on a traditional model of learning, the more he/she emphasised the impact of external factors in his/her teaching. Here we see the teachers explaining the determinants of their teaching practice in terms of internal or external factors (to them), in accordance with the categories of the components of the model that was originally constructed as a thinking tool for this research in Chapter 2. We shall now see how these findings relate to the teachers' self concepts.

7.3.1.3 The teachers' self-concepts, self-efficacy, and attitudes to change in their teaching

It was thought at the planning stage of this study that teachers' self-concepts, and more specifically teachers' beliefs about their knowledge, would relate to their attitudes to change in their teaching. However, as will be seen in this subsection, the findings show that the teachers' beliefs about their knowledge of beginning algebra and its teaching is not an indicator of attitude to change. The five teachers' "self-efficacy beliefs" (Bandura, 1986, 1997) were found to be directly related to their attitudes to change.

Self-concept has been defined as a person's overall evaluation of his/her traits and abilities, which is presumed to be formed through experience with the environment and the reinforcement of significant others (Bandura, 1986, 1997; Friedman & Farber, 1992). Therefore self-concept can be studied in relation to multiple dimensions of the self, depending on the specific area of concern one is studying. In studying teachers' self-concept, Friedman and Farber (1992) distinguished professional competence, professional satisfaction and personal competence as three different dimensions of self-concept. Although it was not an explicit intention of this study to collect data about teachers' professional satisfaction, the study provided information in relation to this dimension of the teachers' self-concept, as considered by Friedman and Farber. This aspect will be focussed on in Chapter 8. This study set out to examine teachers' beliefs about their knowledge and their learning for teaching in the particular domain of beginning algebra. How a teacher's beliefs about his/her knowledge of beginning algebra related to his/her attitude to change in his/her teaching.

Data on teachers' beliefs about their knowledge and about their attitudes to beginning algebra were not collected through direct questioning (see Ruffell, Mason & Allen, 1998) but through the teachers' spontaneous declarations and arguments about their teaching. Although specific questions intended to elicit information about the teachers' beliefs and attitudes to beginning algebra teaching were given at specific moments (e. g., in Questionnaire 1: asking for the reasons for differences in rankings between preferred and actual practices, the beliefs survey and the concept map activity), most teachers in their talk provided information about their self-concept. Nacho declared that he was full of knowledge, and argued that he knew Baldor's textbook by heart, and that therefore he had nothing to plan for his teaching. Alex and Luis showed certainty of what beginning algebra was about, despite the fact that they explicitly said that their knowledge of its teaching was not adequate. Despite the fact that Nora did not explicitly say anything about her own knowledge of algebra and its teaching, she showed high certainty about what Grade 8-algebra was composed of, and asserted at Interview 1 and Interview 3 that the children did not learn by engaging in activities (like the ones described at the Focus Group session or in Questionnaire 2) as her previous experiences in action research projects had shown her. Pablo, who showed confidence and certainty in his knowledge of beginning algebra, declared that he had learnt mathematics from his pupils. Despite his acknowledgment of his limited knowledge of the teaching of Grade 8-algebra, Pablo showed high confidence in his capacity to design and organise working environments for his pupils and refused to adopt a text book. The categorisations of the teachers' beliefs about their knowledge of Grade 8-algebra, together with their beliefs about the reasons for introducing change in their teaching are shown in Table 7.8. Further data about the teachers' beliefs about their knowledge were collected at Interview 3 in the context of discussions on incorporating change, as will be seen in Section 7.3.2.

Table 7.8 Summary of findings on the teachers' self-concept and attitudes to the teaching of beginning algebra

	Pablo	*Luis*	*Alex*	*Nora*	*Nacho*
Beliefs about their knowledge of G 8-algebra	Certain	Certain	Certain	Highly certain	Highly certain
Beliefs about their knowledge for the teaching of G 8 algebra	Not very certain	Not certain	Not certain	Highly certain	Highly certain
Reason for any changes introduced/ to be introduced in their teaching	Internal	External	External	External	Opposed to change
Attitude to introducing changes in their teaching	Positive	Negative	Negative	Negative	Totally negative

We can see from Table 7.8 that the teachers' degree of certainty about their knowledge of algebra or of its teaching was not an indicator of attitudes to change across the five teachers. Nora and Nacho, who were very certain (i.e., had highly positive beliefs) about their knowledge of Grade 8-algebra and its teaching, showed negative attitudes to change (though in different degrees), but so did Luis and Alex who declared they did not feel confident about their knowledge of the teaching of Grade 8-algebra. Pablo, who while showing confidence about his knowledge of beginning algebra, declared that he had "learnt mathematics from [his] pupils", and acknowledged the inadequacy of his knowledge for the teaching of Grade 8-algebra, was engaged in designing tasks and activities that could help all his pupils learn. If pupils did not learn it was due to the inadequacy of the work he provided so he had to continuously change and improve his teaching. He showed enthusiasm about learning from other teachers' because he believed that teaching was a continual learning process. In contrast, for Nora, Luis and

Alex, change had to do with new regulations or requirements from ICFES[4] of Ministry of Education. Nacho, on the other hand, was totally opposed to change in his teaching. He opposed even the change introduced by the Ministry's regulations. In fact it was the regulation that only 5% of a year's group could fail which was the main reason for Nacho's frustration with his current teaching situation. For Nacho, the pupils' lack of motivation for the learning of algebra was exacerbated by their knowledge about "the 5% policy", and there was nothing he could do.

What were the other teachers' positions in relation to their teaching situations? Nacho, Nora, Luis and Alex attributed the unsatisfactory results of their teaching to the pupils' lack of motivation, or to their deficient prerequisite knowledge, and to address the situation was very difficult for them. Pablo, in contrast had positive beliefs in his capacity to undertake courses of action in order "to improve" his activities. This shows that the teachers' beliefs about their knowledge were not indicators of their attitudes to change; rather it was their beliefs in their capabilities to take actions and their desire to learn in order to help pupils become motivated and involved in their classroom work that showed a strong relationship with their attitudes to change. This phenomenon is better explained in terms of Bandura's (1986, 1997) concept of self-efficacy.

The teachers' self-efficacy beliefs

Bandura (1997) defines perceived self-efficacy as "people's beliefs in their capabilities to produce desired effects by their actions" (Preface p. vii). He argues that people guide their lives by their beliefs of personal efficacy.

> Perceived self-efficacy refers to beliefs in one's capabilities to organize and execute the courses of action required to produce given attainments. (p. 3)

Acknowledging the fact that teachers' sense of efficacy in teaching is becoming a focus of research in teacher education, Philoppou and Christou (2002) have taken Bandura's self efficacy concept to define efficacy with respect to teaching any subject as "one's confidence in one's capabilities to organize and orchestrate effective learning environments" (p. 216).

4 ICFES is the Colombian Institute for the Promotion of Higher Education which is in charge of the National External examination that takes place at the end of the secondary school cycle.

Bandura (1997) argues that self-concept is not a useful construct to focus on when trying to predict people's behaviour because self-concept or self-esteem is concerned with judgments of self-worth whereas self-efficacy is concerned with judgments of personal capabilities.

> Individuals may judge themselves hopelessly inefficacious in a given activity without suffering any loss of self-esteem whatsoever, because they do not invest their self-worth in that activity. (p. 11)

"People need much more than high self self-esteem to do well in given pursuits (ibid., p. 11). Self-concept judgements are more global and less context specific than self-efficacy judgements (Philoppou & Christou, 2002, p. 217). Therefore teachers' self-efficacy in the context of this study is concerned with the teachers' beliefs about their capabilities to accomplish the specific tasks of motivating their pupils and engaging them in learning beginning algebra.

Throughout the whole process of data collection the teachers provided compelling evidence of their self-efficacy beliefs regarding their capabilities to produce desired effects by their actions. Nora, Luis, Alex and Nacho blamed the pupils for the unsatisfactory results of their teaching practices. But we need to see here that the reason why they blamed the pupils for the situation was their beliefs about the nature of beginning algebra and therefore how beginning algebra was learnt. As Pablo had contrasting beliefs about how pupils learn —and a different way of knowing beginning algebra concepts— he did not blame the pupils. He was aware of the limitations of his knowledge but had high perceptions of his capacity to learn and improve his activities. In other words, he considered change an integral part of his teaching, unlike Nora, Luis and Alex, who viewed change as due to the need to comply with new central policies, and in contrast to Nacho who was totally opposed to change. It can be said that Nacho's self-efficacy beliefs were lower than those of Nora, Alex and Luis, for he did not want to deviate from his well known teaching path as his pupils could get lost. Further,

> People with high self-efficacy trust their own capabilities to master different types of environmental demands... [which they] interpret more as challenges than as threats... In contrast individuals who are characterised by low self-efficacy are prone to self-doubts... and have weak task-specific competence expectations... (Jerusalem & Mittag, 1995, p.177)

Could this be the case of Nacho? From the nine case study teachers he was the only one who did not want to attempt the construction of a concept map, and he chose not to participate in the Focus Group session.

The categorisations of the five case study teachers' beliefs about their capacities to organise teaching environments that motivated and engaged their pupils (i.e., their self-efficacy), and the relations of these beliefs with the teachers' attitudes to change are shown in Table 7.9.

Table 7.9 The teachers' self efficacy beliefs and their relation to their attitudes to change in their teaching

	Pablo	*Alex*	*Luis*	*Nora*	*Nacho*
Reason for any changes introduced/to be introduced in their teaching	Internal	External	External	External	Opposed to change
Self-efficacy beliefs	High	Low	Low	Low	Low*
Attitude to change	Positive	Negative	Negative	Negative	Totally Negative

* Nacho's self-efficacy beliefs were lower than those of Alex, Luis and Nora as explained above.

We can observe from the information presented in Table 7.9, that self-efficacy beliefs are related to individuals' attitudes to change since positive beliefs in one's capacity to take courses of action in order to achieve given attainments is based on the belief in one's capacity to learn; that is, to change (Crawford & Adler, 1996). Bandura (1997) argues that teachers who have positive self-efficacy beliefs are able to organise learning environments that promote students' learning.

> Thus teachers who believe strongly in their ability to promote learning create mastery experiences for their students, but those beset by self doubts about their instructional efficacy construct classroom environments that are likely to undermine students' judgments of their abilities and their cognitive development". (p. 241)

The evidence shows that a teacher's capacity to design and organise classroom environments that promote learning, and high self-efficacy beliefs are connected to the teacher's knowledge base. In the case of Pablo, the only teacher who had high self-efficacy, his good dispositions were related to his conceptions of the nature of algebra knowledge and of how pupils learn and on his awareness of the impact of his knowledge on his teaching. The other four teachers had low self-efficacy beliefs because they conceptualised beginning algebra as a static body of formalisms, created by external authorities, that their pupils learnt by a fixed transmissive approach. In this context, unexpected situations like pupils' low engagement in the repetitive practice or the introduction of a policy that specifies the number of pupils that need to be promoted to the next grade represented extraordinary tasks for the teachers because they did not see themselves as creators of knowledge. In other words, the teachers' self-efficacy beliefs were related to their knowledge of beginning algebra and its pedagogy. These findings are consistent with Cakiroglu and Boone's (2001) explorations of the relationship between 79 Science pre-service teachers' knowledge of the concept of photosynthesis and "personal Science teaching efficacy beliefs". They concluded that the holding of misconceptions regarding photosynthesis was associated with those participants who had low personal science teaching efficacy.

In the previous sections the teachers' accounts of what they did in their teaching of Grade 8-algebra and their explanations of why they did what they did have been compared. The patterns observed reconfirmed that the teachers' learning models were borne out of their conceptions of the nature of beginning algebra knowledge which, at the same time, were indicative of their conceptions of the crucial determinants of their teaching. A teacher's self-efficacy was related to his/her knowledge of the nature of beginning algebra and his/her beliefs of how pupils learn.

7.3.2 Why the teachers would (or would not) consider the incorporation of a problem solving approach in their teaching of Grade 8-algebra

The task of studying the teachers' conceptions of their own teaching of beginning algebra in the context of considering possibilities for change was achieved by focusing on the teachers' arguments during two different research situations:

- In giving their opinions and evaluations of the work of other teachers who followed a problem solving approach in the introduction of the concept of variable (as it was presented to them by the researcher, in the research instruments), and about the possibilities for the incorporation of such an approach in their teaching of beginning algebra

- In revising their initial concept map of the determinants of their teaching practices

Three sources of data were used for this purpose:

- Questionnaire 2, where the teachers were presented with ample descriptions of the teaching approaches followed by two different teachers, one of whom followed a problem based approach
- The Focus Group, where discussion of the approaches followed by three different teachers when introducing pupils to the concept of variable took place
- Interview 3, where some of the ideas and opinions that the teachers put forward during the Focus Group were further probed, and the concept map of the determinants of their teaching that had been built at Interview 2 was reviewed

By focusing on the teachers' evaluations of the different examples of the work of other teachers, and on whether —and why— they would or would not be willing to consider the incorporation of alternative approaches in their own teaching, further evidence of their conceptions of the nature of beginning algebra knowledge and the determinants of their teaching practice was obtained, offering, at the same time, further evidence of their beliefs about their knowledge for algebra teaching. These findings are presented in the following subsections under corresponding headings for each area previously named.

7.3.2.1 Further insights into the teachers' conceptions of beginning algebra and the crucial determinants of their teaching

In their responses to specific questions in Questionnaire 2, which were intended to explore what the teachers meant by 'a problem solving approach', Pablo, Nora and Luis provided statements that suggested that within a problem solving orientation, the teacher did not just tell but organised classroom situations in order to encourage the learner to engage in the production of their mathematical ideas (see Table 7.10). However, only Pablo found Teacher B's classroom work "an example of a problem solving approach ... the type of work [he] was intending to do..." Nora, Luis and Alex found their pupils' abilities and ways of learning as the basic barriers to the incorporation of a problem solving approach in their teaching.

Table 7.10 The teachers' responses to two specific questions of Questionnaire 2 intended to elicit information about their conception of the nature of algebra knowledge and of the determinants of their teaching (Phase 1 of the study)

	Question *The mathematics teacher of grades 8 and 9 of School "N" states that he/she emphasises a problem solving(p-s) approach in his teaching of beginning algebra. Would you think that any of the approaches described in this questionnaire (i.e., Teacher A's and Teacher B's) emphasises a problem solving approach? Please explain your answer fully.*	*Question* *Is there any aspect of any of the teachers' work described that is not part of your teaching and that you would like to incorporate in your teaching?*	*The teacher's identified factor influencing his/her teaching*
Pablo	Teacher B emphasises a p-s approach. His/her classroom methodology shows it. A simple problem situation was designed, according to the teacher's specific purpose for the situation described.	What Teacher B does is what I am trying to do. But there are topics like the basic laws of operations with rational numbers that can make me follow a trasmissionist approach. There are no practical situations to teach this topic or at least I don't know any. . .	My knowledge for teaching
Nora	Teacher B follows a p-s approach because starting from simple problem situation he [sic] managed to develop other algebraic ideas like algebraic expressions and the need to express ideas.	I would like to do my teaching as Teacher B does. However, in Grade 8 algebra you cannot neglect the type of work that Teacher A does... Further, pupils find it difficult and they need to be told.	Pupils' abilities
Luis	Teacher B is the one that follows a p-s approach because he/she poses a contextual real situation in the form of a problem; promotes discussion and interaction between pupils, trying to direct the discussion toward answering the proposed questions; considers the different possibilities that pupils see; after a process of socialisation they arrive at a consensual academic [i.e. mathematical] idea because an algorithm was constructed from the table of values. . .	I follow Teacher A's approach because my assessment practices are done according to a legal requirement; I teach according to the textbook orientation; I structure the lessons by definitions, examples and exercises... I do this because of the difficulty with pupils' prerequisite knowledge, discipline and problematic family context...	Pupils behaviour and social background
Alex	Both teacher A and Teacher B follow a p-s approach. Teacher B starts by a problem and Teacher A adds the problems after the topic has been taught.	I am trying to do what Teacher B does but it is difficult because there are no text books that bring situations like those. Everything comes decontextualised.	The textbook
Nacho	Teacher B is departing too much from the established structure of topics. He/she gives more opportunity for the pupils to wander and to pose solutions even if they make no sense.	In Teacher A's approach pupils learn about the basics elements of algebra while in teacher B there is more space to wander and get lost. Pupils learn better if they are given small doses following the logical order.	Pupils' ways of learning

That Nora and Luis were not convinced that their pupils could learn that way was evident in their reasons for the low possibilities of incorporating the approach into their teaching. They had to teach following Teacher A's approach (i.e., telling) because of their pupils' abilities and motivation (See Table 7.10). Alex who thought that following a problem-solving approach was adding a problem of application to the topic (or topics) at hand (cf. Ernest, 1991), found the classroom work and activities very interesting but argued that it was difficult for him to teach in that way because in the text "everything comes decontextualised". As far as Nacho was concerned, "Teacher B was departing too much from the established topics ... and pupils learn better if they were given small doses following the logical order".

This further evidence of the teachers' conceptions of the nature of beginning algebra knowledge and of the determinants of their teaching offered by their responses in Questionnaire 2 was contrasted with the findings from the Focus Group session and Interview 3, where they reconfirmed their attitudes to beginning algebra and to change in their teaching (see Table 7.11).

The teachers' main concerns - Insight from the Focus Group

At the Focus Group session —which Nacho chose[5] not to attend— the teachers were invited to describe an aspect that represented a concern, or an important insight to share or an aspect they thought deserved reconsideration in their teaching of Grade 8-algebra. Apart from Pablo, who emphasised the great limitation that teaching time available was representing for him, the rest of the participants expressed their concern for the pupils' lack of motivation in their learning. Alex spoke of "the pupils' phobia to mathematics and especially to algebra" and called attention to the fact that the pupils did not see the practical applications of algebra. The teachers also manifested their desire to do something about the situation.

Their first opinions about the teaching approaches and classroom work described were all positive: "...the type work that can build the path to algebra" (Nora); "good examples of the practical applications of algebra" (Alex); "interesting activities related

[5] The Focus Group's venue was a 2–or-3-minute walk from Nacho's school. It was organised during one of Nacho's free periods. Although he was aware that several teachers from different schools would attend the Focus Group session to share experiences of their teaching in Grade 8-algebra, he chose not to attend it, although all other participating teachers were able to make arrangements for their attendance.

to real life…" (other participating teachers). However, when asked whether there was any aspect in any of the approaches described that could be found relevant for addressing their professed concerns, apart from Pablo, the rest of the teachers were preoccupied with the fact that the pupils had not worked on the formalisations. By the 'formalisations', they referred to:

> The language of mathematics which is universal; and pupils need to acquire fluidity in the manipulation of that language. That is to prepare them for life, for their studies at university because they need to progress intellectually (Nora, F G).
>
> The historical exercise, for example when you give them an equation and ask them to solve it (Luis, Int. 3)
>
> The definition of a concept… the ones that come in the textbook (Alex, Int. 3)

Pablo, who found "the activities [as] excellent examples of activities to help pupils make sense of the work" declared that he was impressed particularly with the use of spreadsheets to help the pupils become familiar with the concept of variable. He asserted:

> We see that the technological resources are there waiting to be used but I need to learn more about how to use them effectively. I know how to use spreadsheets to work with equations and graphics but I never thought that spreadsheets could be used in that way to introduce the concept of variable…

The difference of concerns between Pablo and the rest of the teachers was reconfirmed at Interview 3 when the teachers' conceptions of change in their teaching was explored, once again, identifying what improving in their teaching practice meant for each teacher.

Focussing on expressed concerns – Exploring conceptions of change and improvement

The teachers' comments and arguments about how to address their expressed concerns in their teaching of Grade 8-algebra, which they expounded in the Focus Group, were related to the teachers' professed goals for their teaching. Once again, their teaching goals were directly related to their conceptions of the nature of algebra knowledge and their learning models. Their arguments also contained statements about their conceptions of the determinants of their teaching as can be seen in Table 7.11. For Pablo, whose main goal was "to help pupils see the *raison d'être* of what they are learning … the activities described in the Focus Group, or the ones of Teacher B [were] excellent…, but those require enough time to work with the pupils…" Improving involved adapting his activities so that all pupils could learn but

> I don't know because due to the time factor, I sometimes have to do things like Teacher A (i.e., telling) because time is short. … If I were totally free I would be able to dedicate more time in the classroom to work in that sort of activity (i.e., the one of Teacher B) but one is always behind with the program, so I have to hurry up! … (Int. 3)

For Nora and Luis the activities were motivating, but they needed to give attention and time to 'the formalisations'.

> I do not agree with the comment that [the focus Group moderator] made after the session, that 'mathematical formalisations are of no use to students' because that is the core of mathematics…(Nora, Field notes) … But I have to be careful not to spend too much time in that sort of activity because we cannot stay in *la parte lúdica*. They need to learn what they need in Grade 9… (Nora, Int. 3)

> Pupils need to learn the historical exercise; they arrive at university and are going to have problems. … If they don't learn when one tells them everything, how are they going to learn if I give them the problems? (Luis, Int. 3)

For Alex, "working in that sort of activity [could] be good" but it was difficult because the textbooks brought "everything decontextualised". Further,

> One is put in a dilemma; one would like to separate one's teaching from the list of subject contents and see other things different of mathematics; that is to use mathematics for something which is very interesting and, on the other hand, I need of that list contents to be able to guide the pupils; but furthermore, I have to cover a program that the Ministry imposes, and follow what the ICFES underlines.

And Nacho disagreed with that type of work because pupils needed to be given "small doses in an orderly fashion".

> Teacher A advances faster than teacher B. Teacher B … is departing too much from the pre-established order and pupils can get lost… (Questionnaire 2 & Int. 2)

Apart from corroborating their previously manifested conceptions of beginning algebra, their learning models and, therefore, their conceptions of their roles as teachers, the teachers' declarations contained clear statements about their conceptions of the determinants of their teaching. These statements need to be seen in conjunction with what they explained when revising their concept map of the determinants of their teaching at Interview 3 which we can see in the next section.

Table 7.11 The teachers' expressed concerns in their teaching and their decisions in addressing these concerns

	The teachers' professed goals for their G 8-algebra teaching	*The teachers' expressed concerns during the Focus Group*	*Addressing the expressed concerns*	
			During the Focus Group	*At Interview 3*
Pablo	Help them see the functionality of what they are leaning	The pupils' motivation is not the problem for me. It is the teaching time available	I found the work with the spreadsheets to introduce the concept of variable very impressive. … the resources are there waiting to be used but I just need to learn more about their use.	Improving… is adapting the activities so that all pupils can learn. But… due to the 'time' factor, I have done things like Teacher A (i.e., telling)…
Nora	That they acquire fluidity in the manipulation of algebraic expressions	Pupils' lack of motivation for the learning of algebra and of mathematics in general.	That type of activity is a way to help them build the path to arrive at algebra…	We have to make them feel in love with mathematics… through a different methodology because it is supposedly difficult for the pupils if you give them the letters… But I have to be careful not to expend too much time in that sort of activity…
Luis	That pupils learn the elements of algebra that they need in subsequent levels	I would like to ask whether classroom activities like the one of Teacher B are consistent with what pupils need to learn. ... I want to ask what do those activities have to do with what the pupils have to learn…?	I see that the teachers are trying to help the pupils see the variable but I don't see clearly how they move on to the formalisation. I have still many questions in relations to those activities they look more like experiments than a way of teaching. There is uncertainty.	With those activities, yes, we can motivate the pupils… but I don't think that the pupils can learn in that way... (If I wanted a change I'd choose the 1st PD option so that I become a well known person in the academic field.)
Alex	Learn to solve equations and solve problems	Pupils phobia to mathematics	Those teachers are working in the applications of mathematics rather than on the formalisations. Doing that can be a good way to motivate them and help them see the usefulness of mathematics.	One is put in a dilemma,…one would like to separate one's teaching from the list of subject contents... but I need of that list of contents to be able to guide the pupils. Further, I have to cover a program that the Ministry imposes, and pay attention to what the ICFES requires.
Nacho	That pupils learn what is required for Grade 9-algebra	[Nacho chose not to attend the Focus	Group, therefore he did not do Interview 3..	However, see his comments of Teacher B's work]

The teachers' conceptions of the determinants of their practices – The point of convergence observed in the final concept map

The teachers' revision of their concept map, at this point of the data collection, offered key information about their conceptions of the determinants of their teaching of Grade 8-algebra. It was key information in clarifying the teachers' conceptions of the crucial determinants of their teaching because at this point of the data collection process, the teachers had had the opportunity to reflect on their own teaching approaches and those of other teachers, and had been thinking of how to address their expressed concerns in their teaching of Grade 8-algebra. When revising their initial concept map of the determinants of their teaching a strong point of convergence in the identification of the crucial determinants was observed. All five teachers identified external factors as the crucial determinants of their teaching of Grade 8 algebra. Pablo, who during the first five months of the data had been emphasising the primacy of internal factors in his teaching, came to the conclusion that "the use of time" was a *number one* factor too. He had pointed to "the great limitations" that time was imposing on his teaching, during the Focus Group session, and during the revision of his initial concept map, he declared that at this point of his teaching, when he knew how the school worked " the use of time" was his "main priority".

The data show that although the concept map activity represented an important opportunity for the teachers to think and reflect about the way they saw their teaching situation, only the concept map constructed by Pablo was representative of the way he saw his teaching. We need to remember that Nacho did not build a concept map but provided clear written statements that enabled depiction of a concept map. This (delineated) map was consistent with the way he explained his teaching practice throughout the data collection period in which he participated. Pablo and Nacho, the two teachers at the ends of the identified continuum maintained consistency in their explanations throughout the whole process of data collection. As we saw in the case descriptions, when building their initial concept map, Nora, Luis and Alex pointed to the importance of internal factors like the teacher's knowledge, when asked about it, but in their evaluations of their teaching practices and in their explanations about specific classroom situations, they kept on stressing external factors as the most important. The same phenomenon was observed when they were invited to revise their concept maps at Interview 3. Table 7.12 shows a summary of the teachers' identified *crucial* factors both in their continual explanations and when revising their initial concept map. Table 7.13 shows the categorisations of these factors.

Table 7.12 The teachers' identified crucial determinants of their teaching (in the context of considering alternative teaching approaches)

	Pablo	*Nora*	*Luis*	*Alex*	*Nacho*
As identified in their revised concept map	Teaching time My knowledge & dispositions	The teacher's knowledge & vision *	My knowledge Pupil's abilities & social background+	The motivation in the S-T relationship#	N/A
As emphasised in arguments at the F G and at Int. 3	Teaching time	Time	Pupils' ways of learning &social background	Textbooks Time Program	N/A

* Nora identified "The teacher's knowledge and vision" in her final concept map as the 'Number 1' factor but pointed to the crucial role of 'Time' when explaining the difficulties of her teaching in G- 8.
+ Luis explained that "My knowledge is important for what I plan but the pupils are the stronger factor because they make me loose the rhythm of my teaching ..."
# Alex identified 'The motivation in the student- teacher relationship' as the 'Number 1' determinant of his teaching, but kept on emphasising the fact that "in the textbooks everything comes decontextualised" .

Table 7.13 Categorisations of the teachers' identified crucial determinants of their teaching (in the context of considering alternative teaching approaches)

Pablo	*Nora*	*Luis*	*Alex*	*Nacho*
Internal & External	Mainly external	Mainly External	Mainly External	Totally external

Pablo made a differentiation between what improving in his teaching meant to him and what restructuring his teaching meant, to comply with what he perceived as the demands of his job. He did not sound happy with the decision to find shorter activities and "hurry up", and when asked to choose from the three working-plan options offered to them by the interviewer, he showed great enthusiasm about the possibility of engaging full time in a classroom research project. In contrast, Nora, Luis and Alex were doubtful and showed negative attitudes both to incorporating a problem solving approach into their teaching and to engaging in the projects offered to them. Further, they explained why changing their teaching approaches was not a high possibility. Nacho did not even want to attend the Focus Group session, and we can only surmise that he would also not be interested in engaging in researching his own teaching.

When the teachers considered and explained the possibilities and impossibilities for introducing change, they exposed further their beliefs about their knowledge of beginning algebra and its teaching, their learning and their attitudes to the pupils and to teaching, providing further evidence of their beliefs about their knowledge and their self-efficacy.

7.3.2.2 Further insights into the teachers' self-concepts and self-efficacy

As shown above, once again, the teachers' beliefs about their knowledge for introducing a problem-solving approach in their teaching did not interrelate with their attitude to change as can be seen in Table 7.14. However, their self-efficacy beliefs once again were directly related to their attitudes to change as can be observed in Table 7.15.

Table 7.14 Summary of findings on the teachers' self-concept and attitudes to the teaching of beginning algebra, when considering the incorporation of change

	Pablo	*Nora*	*Luis*	*Alex*	*Nacho*
Beliefs about their knowledge of G 8-algebra	Certain	Highly certain	Not certain	Not certain	Highly certain
Beliefs about their knowledge for the introduction of a p-s approaches	Not very certain but eager to learn	Not certain & Not confident	Not certain & Not confident	Not certain & Not confident	Not interested
Reason for change to be introduced in their teaching	Internal	External	External	External	Opposed to change
Attitudes to introducing alternative approaches	Positive & enthusiastic	Negative	Negative	Negative	Totally Negative

Table 7.15 The teachers' self-efficacy beliefs and their relation to their attitudes to change in their teaching

	Pablo	*Alex*	*Luis*	*Nora*	*Nacho*
Reason for change to be introduced in their teaching of G-8 algebra	Internal	External	External	External	Opposed to change
Self-efficacy	High	Low	Low	Low	Low*
Attitude to change	Positive	Negative	Negative	Negative	Totally Negative

* Nacho's self-efficacy beliefs were, once again, lower than those of Alex, Luis and Nora.

Having focused on the teachers' self-efficacy in both the context of explaining their current teaching and the context of considering the incorporation of a problem-based approach, we can see a relationship between the teachers' self-efficacy beliefs, their loci of causality and their attitudes to change. This relationship is explained by the teachers' beliefs about the nature of beginning algebra knowledge, which are at the base of their convictions of how pupils learn. Table 7.16 shows a summary of the categorisations of the teachers' conceptions of the nature of beginning algebra, the crucial determinants of their teaching (Loci of control), their self-efficacy and their attitudes to change.

Table 7.16 Categorisation of the teachers' conceptions of the nature of beginning algebra, the crucial determinants of their teaching, their self-efficacy and their attitude to change

	Pablo	*Alex*	*Luis*	*Nora*	*Nacho*
Conceptions of the nature of algebraic knowledge	Internal	External	External	External	External
Conceptions of the crucial determinants of their teaching practice	Mainly Internal	Mainly external	Mainly external	Mainly external	External
Self-efficacy	High	Low	Low	Low	Low*
Attitude to change	Positive	Negat've	Negative	Negative	Totally Negative

The summary of categorisations of the teachers' conceptions of their own teaching of beginning algebra presented in table 7.16 shows that while there was a dichotomy in the teachers' conceptions of the nature of algebra knowledge (i.e., External vs. Internal), their conceptions of their own teaching varied according to the emphasis they placed on the role of external factors in their teaching from where a dimension in their conceptions of the crucial determinants of their teaching was identified. This dimension was defined by Pablo's and Nacho's conceptions at the two ends, with Nora's, Luis' and Alex's conceptions in the middle. A positive relationship between the teachers' conceptions of beginning algebra and their conceptions of their own teaching practices was identified. We shall focus on this identified relationship and the interactions of these conceptions with their attitudes to change in Section 7.4.

7.4 The relationship between the teachers' conceptions of beginning algebra and their conceptions of their own teaching practices

A significant finding of this study is that the teachers' conceptions of their own teaching practices stemmed from their conceptions of the nature of algebra knowledge. The teachers' conceptions of the nature of beginning algebra underpinned their conceptions of their own teaching practices, dividing them into two basic groups: the four teachers' for whom algebra knowledge is produced externally, and Pablo for whom knowledge was produced internally. For Nora, Luis, Alex and Nacho, knowledge was externally produced and passed on from teachers or books to learners. If pupils did not learn it was due to their lack of motivation or prerequisite knowledge and these teachers could not do much about the situation under these circumstances (i.e., low self-efficacy); therefore, the crucial determinants of their teaching were related to the pupils' behaviour (i.e., external loci of control). In contrast, for Pablo, the pupils needed to create meaning in their algebra work, and he was capable of designing and organising classroom situations and activities for this purpose (i.e., high self-efficacy). He believed that his knowledge and dispositions were crucial determinants of the teaching he was trying to enact (i.e., internal locus of control). A direct association between the teachers' knowledge bases or "ways of knowing" (Bishop, 1988; Cooney, 1999) beginning algebra and their attitude to change was evident, for the way of knowing determined the consideration (or no consideration) of change in the nature of beginning algebra or in their 'knowing' of teaching as already illustrated. An emergent basic model that explains a teacher's attitude to change in presented in Subsection 7.4.1. The emerging theoretical continuum of teachers' conceptions of their own practices, depicted by the basic model, and the characterisations of four types of teachers are presented in Subsections 7.4.2 and 7.4.3 respectively.

7.4.1 Explaining teachers' attitudes to change: A model

A teacher's learning model contained statements about the teacher's role and responsibilities and revealed the nature (i.e., internal or external) of the crucial determinants of a teacher's teaching. The data show that every teacher's learning model also applied to their own learning, describing two models: if learning was seen as an act of passive reception (of someone else's knowledge, mainly the textbook as was seen in Subsection 7.3.2), then invitations or requests to break the established pattern (of telling) were met with uncertainty and low self-efficacy beliefs. In the specific case of Nacho there was total rejection of change. On the contrary, if learning was understood as a continual process of change and "adaptation to changing circumstances" (Begg, 2003, p. 146), where the learner needed to make his/her own input, then this learner had high beliefs in his/her capacity to undertake courses of action to meet his learning goals or to meet the demands of the new classroom. The two models of learning were clearly illustrated in Pablo's and Nacho's cases. Nora's, Alex's and Luis' cases provided further illustration of the differences between the two models of learning. Every time I studied the data of the case study teachers further, I learnt more about their understandings of their own teaching practices, and the connection between their attitudes to change and their self-efficacy beliefs became more evident. Figure 7.1 depicts the interrelationships between a teacher's conception of beginning algebra knowledge and her/his conception of her/his own teaching of beginning algebra, as identified through the insights gained from the five case study teachers.

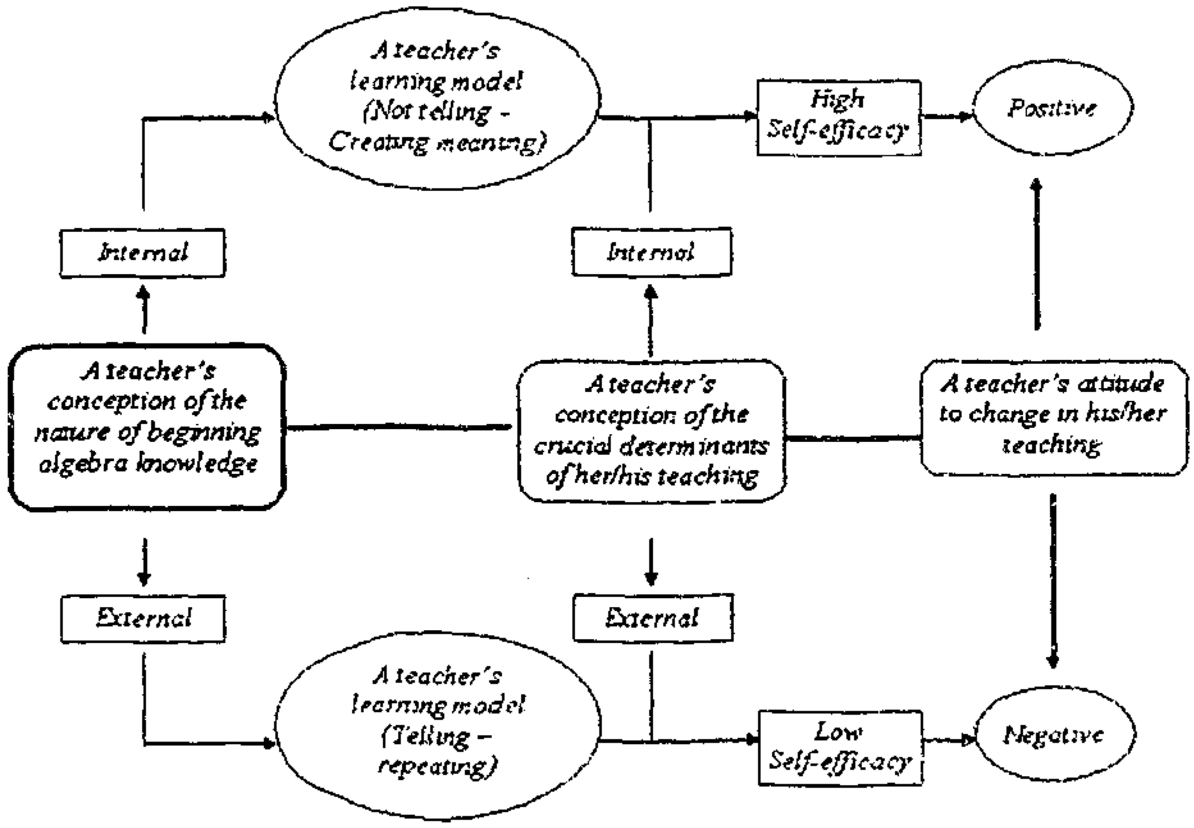


Figure 7.1. A basic model that explains the relationship between a teacher's conceptions of beginning algebra, his/her own teaching practices and his/her attitude to change

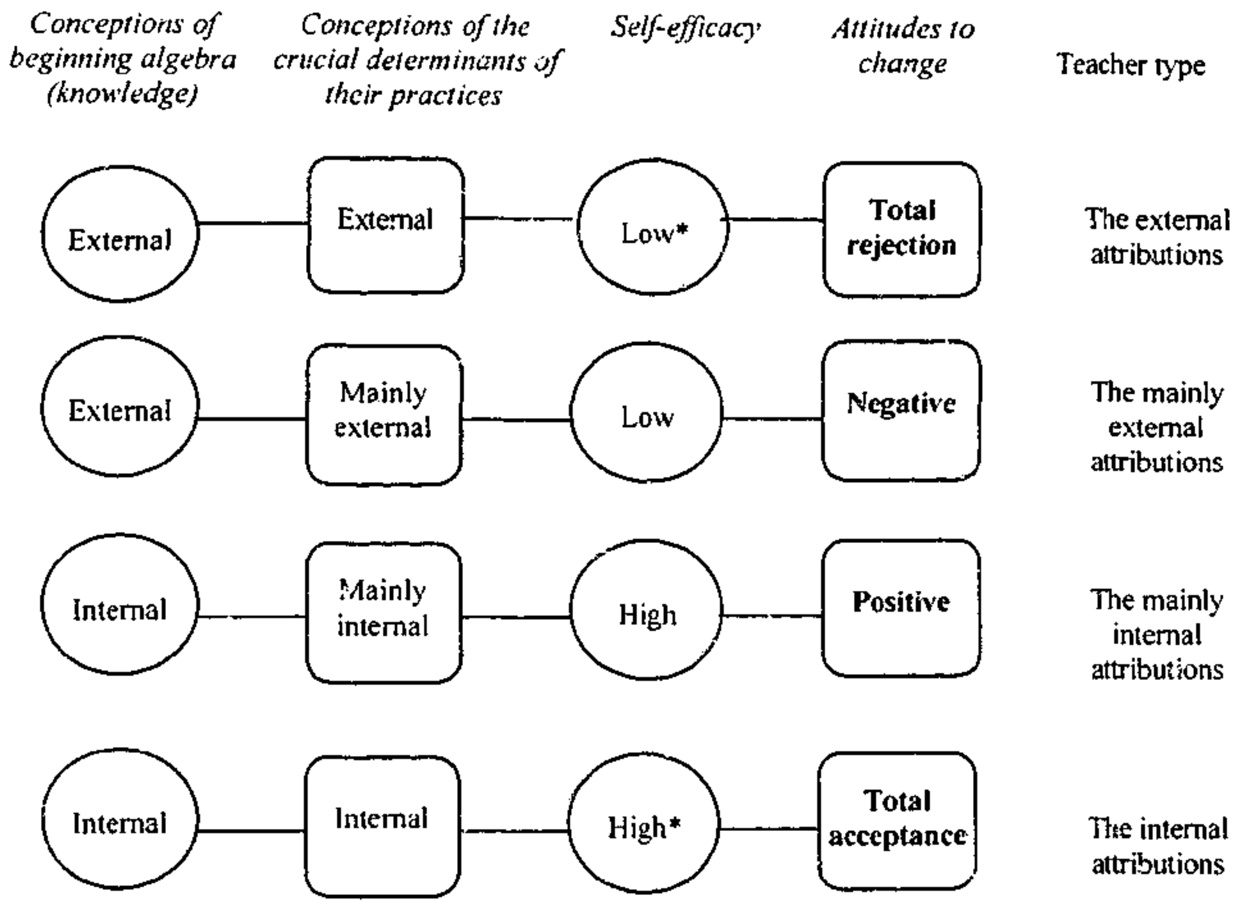
7.4.2 Theoretical continuum of teachers' conceptions of their own teaching practices

As shown in the general model of Figure 7.1, a teacher's conception of his/her own teaching practice is dependent on his/her conception of the nature of beginning algebra knowledge. The model, which as already said, was based on the identified patterns of interaction between the teachers' conceptions of beginning algebra and their conceptions of their own teaching practices, suggests or depicts a continuum of teachers' conceptions of the crucial determinants of their practices that goes from [fully] external to [fully] internal. This continuum shows four types of teachers that, as shown in Figure 7.2, are located according to the degree of influence that the social/institutional context of teaching (external factors) played on their practices.

The two types for whom beginning algebra knowledge is 'externally' created but who put different emphases on the role of *external* factors in their practices are the [fully] 'external attributions' teacher and the 'mainly external attributions' teacher.

The two types for whom beginning algebra knowledge is internally created but who put different emphases on the role of *external* factors in their practices are the 'mainly internal attributions' teacher and the [fully] 'internal attributions' teacher.

The categorisations of the basic concepts analysed to identify each type of teacher are shown in Figure 7.2, and the characterisations of the four types of teachers follow. In presenting these categorisations I would like to provide an explanation about the order of presentation of the case study teachers representing each type form this point onwards. The case descriptions, in Chapter 6, started with Pablo's case and ended with Nacho's. That order of presentation of the case study teachers' conceptions was kept in this chapter, when data for each teacher suggested it. However, the order of presentation of the categorisations of the four types of teachers has been reversed at this point (i.e., it starts with the 'external attributions' teacher of which Nacho is a representative) because it is the order that best reveals the representativeness of the participating teachers in terms of the identified continuum.



* The category 'Low*' means lower than 'Low', and consequently, 'High*' means higher than 'High'.

Figure 7.2. A continuum - Four types of [teachers'] conceptions of their own teaching practices

7.4.3 Four types of teachers' conceptions of their own teaching practices

From the data, the teachers that we have are representatives of three types of teachers, namely the 'external', the 'mainly external' and the 'mainly internal attributions' teachers. There was no representative of the 'internal attributions' teacher. Nevertheless, what I learnt from each teacher helps me imagine the characteristics of the 'internal attributions' teacher. On the basis of my understanding of each teacher's way of explaining his/her own teaching situation and practice, and the general patterns of interactions between the teachers' conceptions of beginning algebra and their conceptions of their own practices, the characterisation of the 'internal attributions' teacher can be inferred.

7.4.3.1 The 'external attributions' teacher

As already stated, the 'external attributions' teacher represents a subgroup of those who see knowledge as externally created. This teacher promotes an instrumentalist (Ernest, 1989) view of mathematics. The 'external attributions' teacher has an extremist position within the larger group of instrumentalists, as she believes that her teaching is dictated by one specific textbook which she considers to be the best, for it takes the learner through every single step and follows a predetermined, hierarchically ordered list of topics. This teacher attributes what takes place in her classroom exclusively to external factors.

Represented by the case of Nacho, this type of teacher believes that algebra knowledge and mathematical knowledge, in general, is produced only by the great mathematicians. It is written in textbooks, and "if the learners don't follow the textbook which is written in a logical order the pupils may get lost". Beginning algebra does not have to make sense to the learner (i.e., the learner does not need to see a reason for working with letters, and/or for what interpretation of the variable is being used in a specific situation because "they will see the relevance in the topics that follow in the next grade"). This teacher believes that "at the beginning, algebra is something mechanical, something that you just do". The role of the teacher is to explain the pre-established procedural rules in a clear as possible a way. Eliciting pupils' thinking is not permitted because they cannot challenge the authority (of the teacher).

The position of this type of teacher is similar to Cooney, Shealy and Arvold's (1998) "isolationist" in that s/he tends to reject the beliefs of others when it comes to her/his own teaching situation. Pedagogical knowledge is obtained by experience, by getting better at explaining the mechanical rules which do not need to make sense because the important thing is to get the right answer which have "external confirmation" (Mason & Spence, 1999) (i.e., it is normally in the textbook). This type of teacher believes that due to the long experience in her/his defined model of teaching s/he knows the right way to teach as there is only one way to know (her/his way). This teacher is highly certain about her/his knowledge of beginning algebra and its teaching, therefore, any requirements or suggestions of change to the established routines are opposed.

In contrast to the 'mainly internal attributions' teacher, this teacher attributes the failure or success of his pupils' learning exclusively to their motivation for learning and rejects the idea that factors related to the teacher's knowledge of disposition may influence her/his teaching. The determinants of her/his teaching practice are only "external, stable and uncontrollable" (Weiner, 1979, 1980) by the teacher. They are external and stable because everything depends on the pupils' desire to learn what the teacher teaches them. They are uncontrollable because everything depends on external people like parents and/or the government regulations (e.g., "there is a government law that prohibits the teacher to fail the children" who do not want to learn). Improvement of her/his teaching takes place only when the external factors are removed (i.e., when this law is suppressed).

This teacher's self-efficacy is low; and it is lower than that of the 'mainly external attributions' teacher; that is, her/his self-doubts in her/his capacity to change (i.e., to learn) are greater than those of the 'mainly external attributions' teacher. S/he sees no opportunity to learn from someone who is at a lower rank than her/his own one (e.g., the pupils), and s/he is not aware of the impact of her/his conceptions on her/his teaching practice.

7.4.3.2 The 'mainly external attributions' teacher

The 'mainly external attributions' teacher also believes that algebra knowledge is produced by mathematical authorities, contained in textbooks and possessed by teachers. Mathematics is a set of hierarchically organised topics, where algebra is the block of topics to be taught in Grades 8 and 9. Algebra knowledge is composed of formal definitions of concepts and procedural rules which are written in textbooks. The pupils know algebra when they know the formal definitions. This teacher is not aware of the processes from which algebraic thinking can emerge and believes that learning algebra takes place by telling and repetitive practice of procedures to manipulate given algebraic expressions. Unlike the 'external attributions' teacher, s/he agrees that the pupils could participate in the creation of some mathematical ideas but believes that it is very difficult for them; therefore they need to be told.

This case is similar to the "technological pragmatist" ideology of Ernest's (1991) in that school mathematics is seen to have two parts. The first is the pure mathematical skills, procedures, facts and knowledge,"...the dry bones of the subject, which are simply tools to be mastered" (p. 162). The second part is the applications and uses of mathematics. Illustrated in the cases of Nora, Luis and Alex, the 'mainly external attributions' teacher believes that as beginning algebra is mechanistic knowledge that has been produced externally, the failures or successes of the pupils in their learning are to be attributed to their motivation to engage in the assigned repetitive work. Another cause for the pupils' lack of success is their inadequate pre-requisite knowledge for the study of beginning algebra. Consequently, the crucial determinants of this teacher's teaching practice are external, stable, and mainly represented by their pupils' behaviour, which they are unable to control. These crucial determinants are stable because teaching is predetermined and fixed. The role of the teacher is to pass on to the pupils the fixed body of "formalisations" which represent their prerequisite knowledge to be able to move up to the next school level. If pupils do not learn what they are taught it is because they don't want to see the importance of mathematics in their lives. For this teacher, the teacher's knowledge is considered important to plan lessons but what takes place in the classrooms depends solely on the pupils' motivation. Therefore, the teacher cannot do much about the situation. Unlike the 'external attributions' teacher, this teacher shows uncertainty about his/her knowledge of the teaching of beginning algebra, especially when a need for change of teaching approach is suggested to them, revealing his/her low self-efficacy beliefs, and therefore showing a negative attitude to introducing change in teaching. However, this teacher is looking for someone who tells him/her how to motivate the pupils to learn algebra because s/he is concerned that the pupils will have problems at the next mathematics levels, and ultimately in the External Examination.

This teacher is not aware of the impact of her/his conception of beginning algebra on the conceptions of her/his teaching practice.

7.4.3.3 The 'mainly internal attributions' teacher

In contrast to the 'mainly external attributions' teacher, this teacher knows that, historically, algebra emerged from the work of humans in their search for solutions to specific problems and needs and, therefore, emphasises connections between the topics to be taught and the learner's environment and life's activity. Such a teacher is aware of the processes from which algebraic thinking emerges, and emphasises the learner's need to see where the ideas come from and "why they are doing what they are doing" (e.g., why the letters are being used). The learner needs to learn the procedural knowledge, specified in the School curriculum, but through meaning making. In the terms of Ernest's (1991) ideologies of mathematics education, this teacher is a "progressive absolutist" because "great value is attached to the individual in coming to know the mathematical truths" (p. 182). Since this teacher's "way of knowing" (Bishop, 1988; Cooney, 1999) algebra is based on connections between mathematical concepts and real life situations, the approaches portrayed in textbooks, especially those that suggest a traditional approach of presenting pupils with formal definitions of concepts and rules of procedure to follow are rejected. Instead, this teacher wants to learn from listening to other's voices including the voices of the learners. Since the teacher needs to attend to what pupils are doing and thinking in order to take decisions, continual learning and change are at the centre of the teaching activity.

Represented by the case of Pablo, the 'mainly internal attributions' teacher is a novice teacher who starts his teaching experience emphasising the role of internal factors in his teaching practice. However, as his knowledge of the social and institutional factors of teaching increases, he starts to attribute great importance to these factors which are external in his teaching.

Consistent with his epistemology of beginning algebra, this teacher attributes the failures or successes of the pupils to the inadequacy or adequacy of the work that the teacher provides for them. Therefore, the crucial determinants of his own teaching practice are his "knowledge and dispositions"; that is, internal, unstable and controllable factors (Weiner, 1979). They are unstable because what the teacher does and organises for the classroom needs to keep changing and are controllable because this teacher believes he has the necessary understanding of the concepts he wants to teach to be able to create his own teaching activities. Although there is no certainty about his knowledge of the teaching of beginning algebra, this teacher feels very positive about his capacity to learn from his teaching and shows enthusiasm about working in a team, for he conceptualises teaching as a continual learning process. Therefore, this teacher has high self-efficacy beliefs.

He is aware of his teaching philosophy and "the models of traditional teachers", which he finds inadequate. Improving practice requires reflecting on what happens in the classroom and giving attention to the needs of all pupils. However, as this teacher's knowledge about the contextual/institutional factors of teaching increases (e.g., learns about the school reporting scheme or the pupils' and parents' expectations to cover the set program), he starts to restructure his teaching in order to align it with the institution (i.e., starts to do the teaching by the telling which he is opposed to). He makes a differentiation between what improving his teaching means for him, and what 'improving' means for the school system and the parents, but he chooses to comply with the school requirements and expectations of parents because "they are powerful people".

7.4.3.4 The 'internal attributions' teacher

Like the 'mainly internal attributions' teacher, the 'internal attributions' teacher is a teacher who is aware that beginning algebra, and school mathematics in general, should engage the learners not only in the identification of connections between mathematical concepts but also of the interconnections of mathematical knowledge and the learners' everyday activities. This teacher also rejects traditional textbooks because s/he wants the learner to learn with meaning. However, unlike the 'mainly internal attributions' teacher who teaches according to the requirements of an institutionalised traditional curriculum (i.e., a formalistic curriculum whose aims are the study of mathematical truths for its own sake), the 'internal attributions' teacher questions the relevance of such curriculum. This teacher is aware of the role that such curriculum has played in the reproduction of social inequalities in Colombia. S/he is aware of the implications for classroom practice of a mathematics educations whose aim is "to develop the learners' critical mind required for citizenship in a democratic society…" as the General Law of Education (1994, Articles 5 & 22) states. This teacher is aware of the implications for classroom practice of the basic aims of the Colombian mathematics education system which have been the flag of the educational system for decades:

> to recognise the value and function of mathematics in the betterment of the individual's life conditions... (Ministry of Education, 1978, 1990; General Law of Education, 1994, Articles 21 & 22)

In contrast to the 'external attributions' teacher, who thinks that beginning algebra (and mathematics) knowledge is limited to the formal definitions and rules contained in a textbook, this teacher takes a critical epistemological perspective, viewing mathematical knowledge as culture-bound and value-laden knowledge (Bishop 1988), which is the key to action and power and not separated from the learners' reality (Freire, 1976; Mellin-Olsen, 1987; Ernest, 1991). As in Ernest's (1991) public educator ideology, the goal of this teacher is the fulfilment of the individual's potential within the context of society.

Like Pablo, this teacher conceptualises teaching as a continual learning process that should focus on the learners' identifications of connections between the mathematical concepts at hand but, unlike Pablo, the *continual change* for this teacher goes further as s/he is concerned with the relationship between mathematics and society and the need for continual social change. Therefore classroom work needs to be embedded in the learners' near environment in order to foster meaningful learning and the development of the learner's awareness of the context in which they live and the role of mathematical knowledge in society.

As in Nacho's case, there is a consistency between his conceptions of the nature of beginning algebra and his conceptions of the crucial determinants of his teaching, but while for Nacho the consistency is externally related, for this teacher it will be internally related. The crucial determinants of her/his teaching practice are internal, represented in her/his knowledge and dispositions and her/his commitment to the empowerment of the learners with the hope to help them to become critical citizens. Like Pablo, this teacher will have a positive attitude to working in a team but, unlike Pablo, this teacher will want to adopt the role of a change agent and therefore will act upon the constraints posed by the pupils and their parents' expectations based on his pedagogical purposes and as a consequence of her/his strong believes in her/his capabilities to produce desired effects by her/his actions (Bandura (1997). Therefore, this teacher's self-efficacy beliefs will be higher than those of Pablo. Further, s/he will want to project her/his change agent role outside the school boundaries into the school community, wanting to make alliances with other colleagues and leaders of the broader community to try to build a broader community of practice.

7.4.4 Comparison of the typology of teachers' conceptions of their own teaching practices with the conceptions of the larger group of teachers

We need to remember that the five case study teachers are representatives of groups of teachers as they were selected through a filtering process, from the group of nine who were followed through in Phase 2 of data collection process. The nine teachers had been selected from the initial group of thirteen participating teachers in Phase 1. Therefore, we need to compare the four emergent types of teachers with the conceptions of the larger group of participating teachers in order to identify any relation of the patterns observed in the case study teachers with those of the rest of teachers who participated in this study.

The teachers' conceptions of beginning algebra and their own teaching practices are compared by applying to their data the categorisations of the typology of the case study teachers. The conceptions of the other four teachers — from the group of nine who were followed in Phase 2 of the data collection process— will be compared with those of the emergent types of teachers in Subsection 7.4.4.1. In Subsection 7.4.4.2 the findings of Phase 1, which were qualified as first indications of the conceptions of the initial group of teachers will be presented in terms of the categorisations of the basic elements of the emergent typology in order to compare them.

7.4.4.1 Comparison of the typology with the conceptions of the other four teachers from the group of nine case study teachers

The other four teachers (from the group of nine) who were followed in Phase 2 of the data collection were Juan, Loren, Stella and José as shown in Table 7.17. In order to compare the conceptions of these four teachers, the data belonging to their conceptions of beginning algebra and the crucial determinants of their practices have been categorised in Table 7.17, in terms of the categories used in the four emergent types of teachers of Figure 7.2. In identifying the conceptions of these four teachers, the data presented in Chapter 5 (see Figure 5.2 on page 126) were considered as first indications.

Table 7.17 excludes the categorisations of self-efficacy beliefs as these emerged from the study of the five case teachers presented in this thesis.

Table 7.17 Categorisations of the conceptions of the other four teachers who were studied in Phase 2 of the study

	Conceptions of beginning algebra knowledge	*Conceptions of the crucial determinants of their practices*	*Attitude to change*
José	External	Mainly internal	Negative*
Juan	External	Mainly External	Negative
Stella	External	Mainly external	Negative*
Loren	External	External	Negative

* Negative attitude to incorporating a problem-solving approach but interested in learning about the teaching of Grade 8-algebra

Table 7.17 shows that all four teachers conceptualised beginning algebra as externally produced knowledge, and had negative attitudes to change in their teaching. The case of José looks different to the patterns identified in the case study teachers. Why did José identify mainly internal factors as the crucial determinants of his teaching but had a negative attitude to change? José, who identified his knowledge of algebra as the crucial determinant of his practice, asserted several times that the reason why he was dissatisfied with his teaching was because he could not teach "like a proper mathematics teacher does" because of his limited knowledge of algebra.

> I need to learn more algebra because despite the fact that I had been teaching it for 12 years, I don't have a degree in mathematics to feel myself a proper mathematics teacher[6]. I need to learn more algebra, not necessarily to design activities or to do things in the way the teachers described at the Focus Group session did, but to be able to tell the pupils better. I would feel better if I were able to explain to them in a better way... Why? Because that is how one was taught. ... (Int. 3).

The case of a teacher pointing to internal factors as the crucial determinants of teaching practice, and having negative attitudes to change was also evident in the case of Luis, one of the case study teachers. During Phase 1, Luis emphasised that "the teacher's knowledge" was the reason why a teacher could not move away from traditional teaching. In Phase 2, however, he clarified that although his knowledge "of the

[6] José had a Bachelor of education in Psychology of education, and had done four semesters in Engineering.

interpretations of the variable" was not adequate, the crucial determinant of his teaching was his pupils' low abilities to learn due to their social background. Like Luis, José acknowledged the importance of his knowledge in his teaching. However, these two teachers were interested in learning about the teaching of beginning algebra, not because they wanted to change their instrumental teaching approach but because they wanted to be better equipped to enact it. These clarifications in relation to their conceptions of the crucial determinants of their teaching reinforce the pattern of interactions observed in the case study teachers' conceptions depicted in Figure 7.1.

In terms of the four emergent teacher types, the cases of Juan and Stella look similar to the *Mainly external attributions* teacher. Although both Juan and Stella showed a negative attitude to the incorporation of a problem-solving approach in their teaching of beginning algebra, there was a difference between the two teachers in their attitude to their teaching. While Stella showed high interest in continuing to learn about ways of motivating the pupils, Juan was not prepared to invest "a minute" in doing anything different in order to address the difficulties he observed in his pupils' learning (for more explanations, see under next heading).

Stella, who believed that pupils do not learn if they are not told, did not see the work of Teacher B (in Questionnaire 2) as portraying good teaching because the teacher did not use the formal language of algebra. When discussing the classroom work of other teachers, described at the Focus Group session, she declared that

> The spreadsheets work is a possible way to motivate the pupils because most of them are now very interested in computers. They prefer to spend the whole day at the computer games shop at the corner than to come to school... (Int. 3)

Stella showed she would be determined to arrange her time if she received a serious offer to participate in the classroom centred research project proposals that were presented to her by the interviewer at Interview 3.

Loren's conceptions were similar to Nacho's conceptions, though Loren was not as emphatic as Nacho in the way he related to the pupils in the classroom (e.g., pupils could ask questions while the teacher was explaining). Like Nacho, Loren chose not to attend the Focus Group session. Both Loren and Juan, who worked with Nacho, complained about the fact that they had to pass the pupils, even if the pupils did not know what they had to know, but because of the regulation that 95% of the pupils needed to be promoted.

The role of experience in an instrumentalist conception of teaching

All four teachers described in Table 7.17 had instrumentalist conceptions of beginning algebra and negative attitudes to change in their teaching. However, it is important to highlight the differences between the attitudes to continued learning of Juan and Loren, the most experienced of the four, with those of Stella and José. While Stella and José were looking for something that helped them to gain the motivation of the pupils to learn algebra, even if that was within a learning-by-being-told model, Juan and Loren were not prepared to spend time doing anything themselves to address the unsatisfactory results of their teaching. As in Nacho's case, for Juan and Loren, the poor results had nothing to do with their teaching practices. The problem was that the new regulation regarding the percentage of children who needed to be promoted was disrupting their established teaching routines. Nacho and Loren commented that both the pupils and their parents knew that the pupils would be promoted even if they did nothing to learn, and that therefore, they had nothing to worry about.

Juan, who declared with enthusiasm at the Focus Group session that

> Nobody in my long life as a teacher had shown me such attractive and useful activities as the ones we just have seen, where the pupils can see the applications of algebra...We normally have a lecturer each year that comes to school and talks about constructivism and this and that, but none of them have told us how to do it in the classroom...

clarified later, at Interview 3:

> I committed myself to collaborate in your study when I signed the letter of consent, and I am pleased that I did it because at the Focus Group session I saw very interesting things that makes one think about ways of helping the pupils see the applicability of mathematics... But do you think that a teacher, working within the conditions that we have in this country, where we are treated as the rubbish of the country, where our opinions and work is not recognised has got any motivation to spend a minute investigating or doing more work? No! ... My goal is to get my pension and disappear from the world of teaching! ...

Juan was so disappointed with being a teacher, with the salary and the school conditions (e.g., the fact that "the headmaster does not recognise my work, he treats us as if we were some *jornaleros*[7]), that he would not recommend anyone to get into the profession:

> I have met some of my previous students who are already at university, and some of them have are doing a Bachelor of Education in mathematics. I have told them, 'Are you crazy? Are you thinking of getting into the teaching profession?' ... Don't you see that in order to survive, the majority of teachers need to work at least in two sessions? ... (Int. 3)

Loren, whose interest as a participant of the study lowered after he returned Questionnaire 2, believed that he followed "both Teacher A's approach and Teacher B's approach" (Questionnaire 2) in his teaching, but the problem was that the pupils did not want to learn algebra.

> Pupils will learn when they decide that they want to learn... It doesn't matter what the teacher does, if the pupils are not interested in learning, if they are not concerned with their future, no teaching approach will work for them... (Int. 2)

The school environment also seemed to play a role in Nacho's, Juan's and Loren's attitudes to their teaching of Grade 8-algebra and their dissatisfaction with "the current situation of teachers". Although it cannot be said that Juan's or Loren's conceptions of their own teaching of beginning algebra fit Nacho's model perfectly, their emphasis and attitudes were very similar to Nacho's conceptions (i.e., The 'External attributions' teacher). These attitudes were also observed in two experienced teachers who offered to participate in the pilot study, but after they were given Questionnaire 2 they expressed their dissatisfaction, on the one hand, with what was asked there and, on the other hand, with the type of problem that was the subject of Teacher B's work. They gave excuses for the fact that they could not carry on participating in the pilot study.

We now need to compare the findings from the initial group of thirteen teachers, in Phase 1 of the study, with the emergent typology of teachers shown in Figure 7.2.

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[7] The word *jornalero*, in Colombia is applicable to people who work in the informal sector, especially in farms and the building industry, and are paid by the hour. Juan referred to the fact that in his school, there was a pupil, in each classroom, in charge of a control sheet that the teacher had to sign in order to have evidence of each teacher's starting time.

7.4.4.2 Comparisons of the typology with the conceptions of the initial group of thirteen teachers

To compare the conceptions of the initial group of thirteen teachers with the categorisations of the case study teachers' conceptions of Table 7.16, the findings of Phase 1 from each participant, have been categorised in terms of the 'external' and 'internal' terms used in the summary of categorisations obtained from the five case study teachers. Table 7.18 shows the categorisations of each of the data belonging to each of the thirteen teachers. The data belonging to each individual teacher has been drawn from Questionnaire 1, Interview 1 and Questionnaire 2 (e.g., Tables 5.4, 5.7 and 5.14, and the summaries shown in Appendix 4.1). Field notes were used in specific cases. For example, Nacho and José did not specifically answer Question B2 of Questionnaire 2, but offered comments about the possibility of incorporating a problem-solving approach in their teaching of Grade 8.

Table 7.18 Categorisations of the conceptions of the initial group of thirteen teachers as studied in Phase 1 of the study

	No. of years teaching		*Conceptions of beginning algebra*	*Conceptions of the crucial determinants of their practices*	*Interest in a p-s teaching approach*
	G 8-algebra	*Mathematics*			
Maria	25	28	External	External	No data
Gladys	24	26	External	External	No
Nacho	18	23	External	External	No!
Juan	16	22	External	External	(I already follow it*)
Lorn	13	25	External	External	No
Alfi	14	15	External	External & internal	Yes[+]
Mario	10	17	External	External	(I already follow it*)
Nora	7	17	External	External	No
José	12	14	External	Mainly internal	Yes[#]
Stella	12	13	External	External	No
Luis	5	8	External	Internal & external	Yes**
Alex	3	4	External	External	(I already follow it) Yes*
Pablo	1	2	Internal	Mainly internal	Yes!

* problem-solving for these teacher meant adding a word problem to apply what has been taught.
[+] but I don't know how.
**but it is difficult due to pupils' family context.
José clarified in Phase 2 that he wanted to learn in order to be able to do a better 'teaching by telling'.

Note that the teachers' answers to Part B of Questionnaire 2 (especially to Question B2) have been treated as first indications of their interest in the incorporation of a problem-solving teaching approach in their teaching, hence a 'Yes' or a 'No' has been used to indicate their interest.

Alfi commented that he had been working at an Institute where pupils from all around the country received "preparation" for the external examination. He asserted that he wanted to participate in a Professional Development program where he could "find explanations for the eternal problem he had observed in the pupils' learning of school mathematics, especially in algebra":

> For the last ten years I have registered myself, each year for the ICFES examination. I have myself sit the examination. I have done it because I need to find out what it is that they ask the students because I work at an institution that offers preparation for the ICFES examination... I have been wondering why it is that all the students make the same mistakes! ... I have concluded that we 'mathematics teachers' are doing something wrong! Because it is not possible that students that come from seventy different schools make the same mistake; why is it that they all have the same difficulties?!...

Apart from Alfi, who was one of the selected teachers for the follow-up study, but due to his personal circumstances[8] had to be excluded, the teachers who had at least 13 years of experience, teaching Grade 8-algebra, indicated a low interest in a problem-solving approach. We observe in the initial group of the thirteen teachers a predominance of instrumentalist conceptions of beginning algebra and the identification of social or institutional factors (external factors) as the reasons for their teaching practices. These ways of explaining practices accord with the 'Mainly external attributions' teacher's ways of thinking about her/his practice (refer to Figure 7.1).

According to the data from Phase 1, we can say that the emergent typology of teachers is a reflection of the pattern of first indications of the teachers' conceptions of their own practices, where the 'internal attributions' teacher does not exist. As pointed out earlier, the cases of Luis and José called attention to the need to explore and study a teacher's conceptions not only of beginning algebra but also of their own practices [in this case] by focusing on their explanations of actual classroom practice, and at various points in

[8] Alfi was on leave for a month, after the school holiday in June July due to medical conditions.

time. It is in the context of discussing teachers' actual practices that "the dynamic nature of teachers' knowledge" (Fennema & Franke, 1992) become evident.

Reflections on the emergent typology of the teachers' conceptions of their teaching practices of beginning algebra

The four teacher-types representing the teachers' understandings of their own teaching of beginning algebra have emerged from the study of a specific group of thirteen teachers with a variety of teaching experiences who were working in a variety of school contexts. The characterisations of the teachers' conceptions of their own teaching practices show a direct association between the teachers' knowledge bases or ways of knowing beginning algebra and their conceptions of the crucial determinants of their teaching practices from where attitudes to change in their teaching developed. The majority of teachers conceptualised their teaching practices in a way consistent with the *Mainly external attributions* model where the teachers showed no awareness of the impact of their conceptions of beginning algebra on their teaching practices, despite their acknowledgment of the importance of the teacher's knowledge. A smaller number of teachers were representatives of the *External attributions* teacher-type. These teachers rejected the idea that the teacher's knowledge of the teaching of beginning algebra could influence the teacher's practice. There was only one teacher, the most novice of the group, representing the *Mainly internal attributions* type, who showed high awareness of the impact of his conception of beginning algebra on his teaching. It cannot be said that this teacher was a representative of the participating novice teachers however, as Alex, the other novice teacher, belonged to the 'Mainly external attributions' type as we saw in the case studies.

For the great majority of teachers, that is the 'Mainly external' and 'External' attributors, beginning algebra was the set of formalisations contained in Grade 8-algebra textbooks which pupils needed to reproduce in assessment tasks and in the external examination. These teachers showed great awareness of their responsibilities to deliver the school's set program for Grade 8 as it represented the knowledge the pupils would need at the following school levels and consequently in the external examination which determines entrance to higher education as well as evidence of the academic quality of schools (Díaz, Solarte & Arce, 1997). The 'Mainly internal attributions' teacher who made a differentiation between what beginning algebra was and the (school) algebra that was being portrayed in textbooks and in other teachers' teaching models, was aware of the inadequacies of the school curriculum and of the inadequacy of the pupils' and their parents' expectations. However, this teacher did not act upon these inadequacies to stop them from constraining his preferred teaching practice. Rather, he started to restructure his teaching in order to align with the institution due to his conceptions of his status and role in the school system and in the broader educational system.

In contrast to common findings of research, at the international level, of experienced teachers being more knowledgeable (see for example, Even, Tirosh & Robinson, 1993; Smith, 2001), this study has found that in the Colombian context the most knowledgeable was one of the participating novice teachers. However, we saw that as this novice teacher started to feel constrained by the social and institutional factors of teaching, he started to give increasing priority to external factors when restructuring his teaching, despite his awareness of what improving his teaching meant for him. The question to be asked at this juncture is, *would this novice teacher give up his ideals and align with the institution to join the group of the 'Mainly external attributions' teachers?*

7.5 Conclusion

The picture from Chapter 2, where a theoretical model of the components of teachers' thought structures was constructed in order to guide this research, pointed to mathematics teachers' *content* and *pedagogical content knowledge* as the decisive factor influencing their teaching practice. This study has shown that the great majority of participating teachers did not see their knowledge of beginning algebra and its teaching as the decisive factor when explaining *why* they did what they did in their teaching. Instead, they saw their pupils' behaviour and pre-requisite knowledge as the crucial reasons for the unsatisfactory results of their teaching. Furthermore, even Pablo, the only teacher who showed great awareness of the impact of his knowledge and dispositions on his teaching practice, emphasised the role of external factors to explain the impossibilities of incorporating a problem-based approach in his teaching.

All the teachers, including Pablo, emphasised the role of the factor 'time' when explaining the impossibilities of implementing a problem-based approach. Time available for teaching and the pupils' behaviour were the crucial constraints identified by the great majority of teachers for the implementation of their perceived duties of covering the set list of subject matter contents set out for Grade 8. Some of the teachers explained their difficulties as due to the pupils' abilities, which were dependent on their social background. An important factor playing a strong role in the teachers' conceptions of their own practices was represented in their conceptions of the role of assessment, as the itemised list of topics represented for them the assessment indicators for which they felt accountable. Apart from Pablo, the teachers who declared to have incorporated some change in their teaching, explained it as a consequence of the legal requirements set by the External Examinations authority (ICFES).

These findings suggest that a teacher's knowledge for the teaching of beginning algebra is more complex than suggested by the list of categories included within the *content knowledge* and *pedagogical content knowledge* classifications established in the theoretical model of Chapter 2, and that we need to pay attention to teachers' practical knowledge (Richardson-Koehler & Fenstermacher, 1988). As Gates (2001) has pointed out, teachers' knowledge for teaching includes teachers' social knowledge, and therefore the theoretical model built in Chapter 2 needs to be revised. The revision of this theoretical model is the subject of Subsection 8.4.2 in next chapter.

The evidence shows that while the teachers' conceptions of beginning algebra played a strong role, their conceptions of the social/institutional factors played a stronger role in their conceptions of their teaching practices. A positive relationship between the teachers' knowledge bases, or ways of knowing beginning algebra, and their attitudes to change was identified. The identified patterns of interaction between the teachers' conceptions of beginning algebra and their conceptions of their practices, where self-efficacy beliefs and attitudes to change were explored, depicted a basic model of the relationship between a teacher's conception of beginning algebra and her/his conception of her/his own teaching that explained attitudes to change. The identified basic model depicted a continuum of teachers' conceptions of the crucial determinants of their own practices that goes from (fully) external to (fully) internal, showing thus four types of teachers:

- the 'external attributions' teacher
- the "mainly external attributions' teacher

- the "mainly internal attributions' teacher, and
- the "internal attributions' teacher

The characterisations of each type of teacher provides a key to inform the design of professional development programs in Colombia as will be seen in Subsection 8.3.1 of Chapter 8.

The findings also show that none of the teachers conceptualised beginning algebra as a teaching and learning activity through which the critical mind of the individual is developed. As discussed in Chapter 2 within a problem-solving teaching approach, mathematics knowledge is seen as dynamic (Ernest, 1991) and embedded in the learners' physical, historical and social environments, whose learning develops an understanding of one's social situation and reality (Freire, 1976; Mellin-Olsen, 1987; Bishop, 1988). The findings show a lack of awareness on the part of all the participating teachers of the fact that their focus in the teaching of beginning algebra is to enculturate pupils into the social reproduction that the Colombian "Pure mathematics model" (Robitaille & Dirks, 1982), designed to serve the needs of the few moving on to higher education —as established by the educational system— has served (Agudelo-Valderrama, 1996). In other words, the 'internal attributions' teacher-type sought was not found, and it is unlikely to find this type of teacher in Colombia, given the context described.

How can we help teachers to question their conceptions of the nature of beginning algebra knowledge? How can we help them become aware of the role of their conceptions of mathematics (and general) knowledge in the reproduction of the structure of society in Colombia where exclusion and conflict have become aspects of every day life which are not problematised in the educational contexts, as this study has shown?

The practical implications of these findings for teacher professional development, educational institutions and policy makers in Colombia, as well as the theoretical implications for research on teachers' conceptions of mathematics and its teaching will be the focus of Chapter 8.

Chapter 8: Conclusions and implications

8.1 Introduction

The problem that inspired this research project was the observed lack of motivation on the part of Colombian Grade 8-algebra teachers to engage in professional development programs, despite their awareness of the high rates of student failure in the subject, their acknowledgement of their pupils' great learning difficulties and their own difficulties in the teaching of the subject. The teachers who engaged fully in a 20-month Professional Development program that focused on their teaching of beginning algebra, mentioned in Chapter 1, emphasised the role that their deeper understanding of the mathematical concepts taught and their questioning of the role of the teacher had played in their capacity to change their teaching practices (see González & Pedroza, 1999). However, the observation that these teachers had the motivation to engage in the program's work and to follow it through, in contrast to the teachers who dropped out, suggested differences in their attitudes to their teaching. The need was established to include the affective dimension as a key component of a teacher's "thought structures" (Ernest, 1989) in this present study which is concerned with the identification of possibilities for teacher change. Therefore, the term 'conceptions' was defined to encompass teachers' *knowledge*, *beliefs* and *attitudes*.

The review of the literature showed that our knowledge about how school algebra is taught, and about mathematics teachers' conceptions of school algebra and its teaching is very limited. Therefore the research on teachers' conceptions of mathematics and its teaching, in general, was used to guide this study. Before this study was done we had learnt that:

- Mathematics teachers' professed conceptions of mathematics are consistent with their professed conceptions of mathematics teaching (see for example, Andrews & Hatch, 1999; Middleton, 1990; Pehkonen, 1997; Philippou & Christou, 1999).
- Teachers whose professed conceptions about the teaching of mathematics were inconsistent with their practices were not aware of the inconsistencies, and named factors belonging to the social context of teaching (i.e., external factors) to account for the inconsistencies (see Thompson 1984; Cooney, 1985; Raymond, 1997).

- Some teachers who were interested in developing their practices engaged in processes of enquiry and reflection on their practices and changed their conceptions. Other teachers who were not interested in change identified *external* factors like pupils' behaviour or the expectations of colleagues as barriers for change in their teaching.

However, we had not learnt *why* the teachers believe those factors affect their teaching practices and *how* those factors impact on their conceptions of school mathematics and their own teaching practices. Can the perspectives of the *implementers* of proposed changes in the mathematics classroom provide us with explanations for the persistence or 'the stability' of the school mathematics tradition reported by some researchers (e.g., Gregg, 1995; Smith III, 1996; Price & Ball, 2001)?

I argued that in order to gain some understanding of the phenomenon of the stability of mathematics teaching practices in the Colombian context —or in any other context— we need to study not just the teachers' conceptions of mathematics and mathematics teaching but also their conceptions of their *own* teaching practices. In other words, we need to focus not just on what factors the teachers name to explain what takes place in their classrooms but also on *why* and *how* those factors impact on their conceptions of their teaching practices. Do teachers see their conceptions of mathematics as the decisive factor or crucial determinant of their teaching practices, as researchers apparently see it, or do they see a strong relationship between their conceptions of the role of social/institutional factors of the context of their teaching and their practices?

This study was designed to shed light on these questions. In order to research teachers' conceptions of their own teaching practices (i.e., their knowledge, beliefs and attitudes), the inquiry focused on the specific area of beginning algebra as it was defined in Chapter 2. The overall purpose of this study was to explore the relationship between the teachers' conceptions of beginning algebra and their conceptions of their own teaching practices with a view to unravelling their conceptions of change.

A major contribution to our knowledge from this study is that although the teachers' conceptions of beginning algebra knowledge directly influenced their teaching practices, the great majority of teachers did not see their conceptions of beginning algebra as the decisive determinant of their teaching. They saw, instead, external factors of the social/institutional context of teaching as the crucial determinants of their teaching, and their beliefs about the role of these factors influenced their teaching practices and their conceptions of their practices. A positive relationship between a teacher's conception of the nature of knowledge and her/his conception of the crucial determinants of her/his teaching practice was identified. A model explaining a teacher's attitude to change emerged from the identified patterns of interaction between the teachers' conceptions of the nature of knowledge and their conceptions of the crucial determinants of their practices, depicting a continuum of teachers' conceptions of their own teaching practices.

This chapter discusses the implications of the significant findings of the study, which shed light on the basic Research Questions and led to the achievement of the overall aim of identifying the relationship between the teachers' conceptions of beginning algebra and their conceptions of their own teaching practices. It also discusses issues arising from the research process in order to consider its limitations as well as implications for further research. The significant findings are presented in Section 8.2 as they emerged from Chapters 5, 6 and 7. The implications of the findings for the provision of professional development and for the creation of possibilities of change in Colombia are presented in Section 8.3. Section 8.4 focuses on the implications of the study for the development of the research on teachers' conceptions of mathematics and its teaching, presenting a new model of mathematics teachers' thought structures as part of the theoretical implications of this study. The contributions and limitations of the findings and methodological issues are also included in this section. The recommendations for further research and some action steps suggested by this study are the subject of Section 8.5, and Section 8.6 presents my concluding remarks.

8.2 Significant findings

Significant findings have thrown light on the following basic research questions:

- What are the teachers' conceptions of beginning algebra?
- What are the teachers' conceptions of their own teaching practices of beginning algebra?

And most importantly on the question,

- What is the relationship between the teachers' conceptions of beginning algebra and their conceptions of their own teaching practices?

These findings are described in the following paragraphs as they emerged from Chapters 5, 6 and 7. The research Sub-questions can be seen in Chapter 1, Section 1.4.

From the initial group of thirteen teachers, we learnt in Chapter 5 that the great majority conceptualised their teaching responsibilities as the enculturation of pupils into a mechanical reproduction of rules to manipulate given algebraic expressions; their declared aim for the teaching of Grade 8 algebra was to prepare the pupils for what they would be required to do in the next school Grade. Assessment of pupils' work was conceptualised as a practice that takes place after the teaching of a specific topic is completed, in order to check if the pupils had learnt the algorithms taught, rather than a practice through which learning takes place about the pupils' thinking to inform further teaching. The great majority of teachers reported dissatisfaction with the pupils' performance and attributed the situation to external factors, mainly to the pupils themselves.

The findings from the initial group of teachers in Phase 1, taken as indications of the teachers' conceptions of beginning algebra, and of their interest in alternative teaching approaches were further illustrated by the case studies in Chapter 6. The case study descriptions provided important insights into the way each teacher saw his/her own teaching practice in Grade 8. Four out of the five case study teachers presented in this thesis (and out of the nine who participated in Phase 2 of the study) identified their responsibilities, exclusively, as delivering the list of subject matter contents as they were presented in textbooks. There was no evidence from these four case study teachers (or from the other four) of their valuing of pupils' knowledge or thinking. However, all of them believed that they were promoting, for example, the development of communication skills, and that they were pursuing broad educational goals in their teaching of beginning algebra. From the case descriptions we observe striking differences between Pablo's conceptions of beginning algebra and those of the other four teachers, but we also see *all* the teachers coming together in emphasising the primacy of the social/institutional factors to explain the difficulties of incorporating a problem-solving approach in their teaching of beginning algebra.

The comparative analysis of the five case study teachers' conceptions of their teaching practices of beginning algebra, carried out in Chapter 7, showed that while their conceptions of beginning algebra played a strong role, their conceptions of the social/institutional factors played a stronger role in their conceptions of their teaching practices. A positive relationship between the teachers' knowledge bases or *ways of knowing* beginning algebra and their attitude to change was also identified. The identified patterns of interaction between the teachers' conceptions of beginning algebra and their conceptions of their practices, where their self-efficacy beliefs and attitudes to change were related, depicted a basic model that explained 'a teacher's attitude to change'. A teacher's negative attitude to change was directly related to a conception of beginning algebra as knowledge *externally* produced (i.e., by textbook authors or great mathematicians). In the same way, for this teacher, the crucial determinants of her/his teaching practice were *external*. In contrast, a teacher's positive attitude to change was directly related to a conception of beginning algebra as knowledge *internally* produced. For this teacher, *change* was an integral aspect of teaching, and the teacher's conceptions (i.e., "the knowledge and dispositions") were conceptualised as crucial determinants of his/her teaching.

The identified basic model depicted a continuum of teachers' conceptions of the crucial determinants of their own practices that goes from (fully) external to (fully) internal, showing thus four types of teachers. The characterisations of each type of teacher offered direct implications for the provision of professional development programs in Colombia which are discussed in the following section.

As shown in Chapter 5 (Table 5.1) and in Chapter 6, this study was carried out with the participation of a group of thirteen teachers who had a wide range of teaching experiences and who were teaching in a variety of school contexts. Some of them were young beginner teachers, some were experienced teachers who had started their teaching at the primary level and some others, beside their secondary school job, were teaching at universities. Some of the teachers had done postgraduate courses in Education, and some had participated in professional development programs that had focused on classroom practice. One of the teachers reported working for ten years at an institution whose mission was to prepare secondary school leavers for successful performance in the External Examination.

Despite the variety of teaching experiences and school contexts where the participating teachers worked, the evidence shows that the 'internal attributions' teacher or the 'problem–solving' teacher was not there. These findings are consistent with the evidence from previous work with teachers from four schools in a different area of the country (see Agudelo-Valderrama, 2000) and with mathematics teachers from fifteen schools in Bogotá (see Perry et al., 1996) which showed that the teaching patterns of school algebra portrayed by all the participating teachers in these projects were instrumental. This suggests that the 'problem-solving' teacher may not exist —either in theory or in practice— in Colombia. The results of this study have overwhelming and immediate implications for practising teachers' professional development, which are discussed in the next section.

8.3 Implications of the findings for the creation of possibilities of change in the teaching of beginning algebra in Colombia

The problem of Colombian school students' high rates of failure, school drop-out and dislike of mathematics, especially of algebra, highlighted in Chapter 1, calls for the provision of a mathematical education that offers students the opportunity to engage in meaningful and critical understanding of the subject. The mathematics classroom envisioned, where learners are provided with learning environments that help them develop their potentials and the analytical thinking required to be able to participate in a democratic society looks very much like the one portrayed by the 'Internal attributions' teacher-type, the problem-solving/public educator of Ernest (1989, 1991), described in Chapter 7.

The fact that the sought 'problem-solving' teacher was not found in this study, and the identification of a positive relationship between the teachers' knowledge bases and their conceptions of their own practices have direct implications for the provision of professional development. However, it is important to emphasise, at this point, that while the development of a strategy for the provision of professional development programs represents the key area of work, when thinking about possibilities for change in the Colombian context, the specific institutional factors of the school context as well as the systemic factors that were pointed out by the teachers, when they explained *why* they taught Grade 8- algebra in the way they did, need to be addressed in a parallel way, as it is necessary to create the conditions for change. The data show that the teachers did not see themselves as the curriculum innovators or "curriculum constructors" that the educational policy advocates.

The evidence shows that even Pablo, the 'mainly internal attributions' teacher, who was aware of the impact of his knowledge and dispositions on his teaching practice, came to identify factors of the institutional context of teaching as the reasons for his difficulties in the implementation of his preferred teaching practice. The other case study teachers acknowledged the importance of the teacher's knowledge but in explaining what they did in their teaching, and why they could not incorporate a problem-solving approach, their knowledge was not identified as the crucial determinant. These teachers felt responsible for delivering the list of topics belonging to the *Curricular Statements* of the *National Curriculum* document that was in force before the current educational policy was issued (i.e., before 1994). They identified 'the pupils' behaviour' and 'time' as the main constraints in teaching the program needing to be covered in Grade 8, as the mastering of the set list of topics was a prerequisite for pupils to move to the next school level. They felt responsible for preparing pupils to perform well in the External Examination. All teachers (including Pablo) felt they had to fulfil the requirements of the schools' reporting schemes.

The implications for the creation of possibilities for change in Colombia drawn out of these findings are presented in the following order. Subsection 8.3.1 discusses the 'why' of the professional development and the specific lessons implied in the findings of this study. The implications for curricular guidelines and materials, and the External Examination institution are described in Subsection 8.3.2. Subsection 8.3.3 presents the implications of the findings for the institutional context of the school, and reference to the implications for teacher education is made in Subsection 8.3.4. Section 8.3.5 discusses issues related to teachers' professional satisfaction that arose from the findings of this study.

8.3.1 Implications for professional development programs

At the heart of the participating teachers' conceptions of their own teaching practices and attitudes to change in their teaching, and the values of authority and control that were implicitly taught, were their conceptions of the nature of mathematics knowledge. The purpose of beginning algebra and of school mathematics was to enculturate the learners into the reproduction of a fixed list of abstract formalisations; for the majority of teachers, preparing the pupils for real life meant preparing them for the External Examination requirements. The teachers were not aware of the tension between the curriculum that operates in their classrooms and their espoused aims of preparing pupils for life, or of promoting the development of the pupils' critical mind. Further, as shown in Chapter 5, they believed that in their teaching of Grade 8, they were paying attention to the education of the whole child. They believed that they were addressing broad educational aims and not just mathematical content objectives. They did not show awareness of the values they were teaching and, therefore, of the impact of those taught values on their pupils. These findings have overwhelming implications for Colombian mathematics teachers' professional learning, and set out clear purposes for professional development programs.

8.3.1.1 The *why* of professional development implied by this study

A central purpose of the professional development programs envisioned should be to help the teachers see the tension between their conceptions of their teaching practices and their beliefs that they are providing pupils with relevant education for life, or for the development of the individual's critical mind needed for democracy and for citizenship as the central policy advocates. Teachers need to be provided with opportunities and learning environments that help them question their existing conceptions of the nature of knowledge and, consequently, of the nature of teaching. In other words, the purpose of the professional development that this study sets forth is to help the existing teachers develop the critical thinking of the 'Internal attributions' teacher described in Chapter 7, who is aware of what a teaching of mathematics for understanding and for the development of the learner's critical mind entails. But how can the existing teachers be helped to shift their conceptions of their own teaching practices of beginning algebra, and of school mathematics in general, towards those of the 'Internal attributions' teacher? What should be the form and focus of these professional development programs? It is not possible, not desirable, —and not within the scope of this thesis— to determine the parameters of the Curriculum of the envisioned Professional Development project. However, this study has provided important insights into the teachers' conceptions of mathematics knowledge and the nature of teaching which, as noted above, are in contrast with the basic aim of helping children develop their potential within the context of society. These are the reasons '*why*' the teachers need to be provided with professional development opportunities urgently. The findings of this study have also provided important lessons in relation to the '*what*' and the '*how*' of the professional learning experiences that the teachers need to be provided with. The crucial lessons offered by the findings of this study are highlighted in the following subsection.

8.3.1.2 Lessons provided by this study in relation to the *what* and the *how* of the professional development envisaged

The need for differentiation of the professional learning experiences offered to different types of teachers

We learnt from the case study teachers that a teacher's way of knowing beginning algebra represented the basis for the teacher's pedagogical purpose behind her/his preferred teaching practice. A teacher's way of knowing carried an attitude to beginning algebra, to the learner, and to the introduction of change in her/his teaching. As we saw in the described continuum of conceptions of teaching practices, the more a teacher's way of knowing beginning algebra centred on a fixed sequence of itemised tasks for the manipulation of given symbolic expressions, the lower her/his self-efficacy beliefs were, and the less openness to the consideration of alternative approaches was observed. Figure 8.1 reminds us of the four types of teachers whose characterisations were based on the identified continuum of teachers' conceptions of their own teaching practices. The teachers' conceptions of their practices were named in terms of the categorisations of the crucial determinants of their practices.

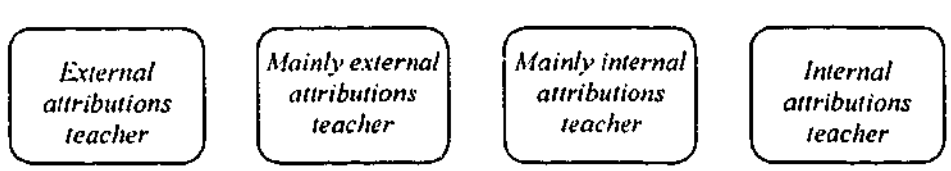

Figure 8.1. The continuum of types of teachers, according to the categorisations of the crucial determinants of their practices

The identified relationship between a teacher's way of knowing beginning algebra and her/his attitude to change suggests, on the one hand, that a prime focus of the professional development programs should be the organisation of mathematical work to help teachers enhance their conceptual understanding of beginning algebra (i.e., of mathematics) concepts and its teaching and, on the other hand, that different types of teachers need different learning experiences. For example, the 'mainly internal attributions' teacher's (represented by Pablo) way of knowing beginning algebra, as connected knowledge, contained the seeds of his pedagogical purposes. He wanted his pupils to construct connected knowledge of the concept of variable. Although he did not yet know how to make his pupils' thinking the centre of his teaching, he valued the pupils' thinking. He believed that all pupils were able to construct their mathematical ideas and, therefore, he was concerned with the learning of all his pupils. He qualified the type of work done by the problem-solving oriented teachers (described at the Focus Group and Questionnaire 2) as excellent examples to help the pupils assign meaning to their beginning algebra work. He conceptualised change as an integral part of his teaching. In contrast, Nacho and Loren, whose patterns of thinking were associated with those of the 'external attributions' teacher, strongly believed that the pupils did not learn because they did not want to learn. These two teachers did not see any need to introduce changes in their teaching approach. This suggests that the two types of teachers mentioned need to be offered different professional learning environments and need to be approached (by the professional developer) differently.

The direction of steps demarcated by the continuum of teachers' conceptions of their own practices, which is shown in Figure 8.2, suggests that the goal of an initial professional development program designed with 'external attributions' teachers in mind would be 'to help them shift their conceptions towards the ones of the *mainly external attributions*'. In the same way an initial goal for the work offered to those teachers whose conceptions show patterns similar to those of the 'mainly external attributions' teacher would be 'to help them shift their conceptions towards the ones of the *mainly internal attributions*' teacher, and so on. The characterisation of each type of teacher provides us with key information to guide the design and provision of professional development programs to cater for the needs of specific types of teachers. They provide us with important insights into why some teachers are not prepared to learn from other peers or why they are reluctant to engage in professional development, a situation that was pointed out in Chapter 1.

Figure 8.2 provides an idea of the suggested sequence of steps needed to help any particular type of teacher reach the conceptions of the 'internal attributions' teacher sought. The continuum helps us visualise the goals to work towards in the initial programs (or steps) offered to specific types of teachers, within the broader process whose ultimate aim is to help them shift their conceptions towards those of the 'internal attributions' teacher.

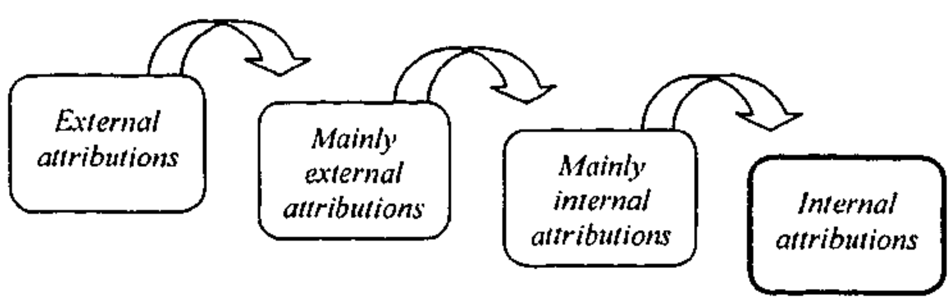

Figure 8.2. Suggested sequence of steps needed to help different types of teachers to shift their conceptions of their own teaching practices towards those of the' internal attributions' teacher

A question that bears consideration at this point is, 'how do we convince the 'external attributions' teacher of the need to engage in professional development programs like the ones envisaged in this study? Owen, Johnson, Clarke, Lovitt and Morony (1988) when considering the "appetite for in-service education" of the "entrenched" type of teacher who, according to Joyce and McKibbin's (1982) typology[1], exhibited similar attitudes to change to those of the 'external attributions' teacher, advise that "they require a tremendous amount of outside energy if they are to become involved [in professional development]" (Owen et. al., p.11). As a speculative implication of this observation, Owen and colleagues suggest that when thinking about initial activities associated with a change proposal, "in terms of cost-benefit ratio", it may be wise to select only the types of teachers who either are seeking opportunities for personal growth or easily become involved in professional learning (i.e., the *Omnivores* and *Active Consumers* in Joyce and McKibbin's categories).

[1] See Joyce, B. & McKibbin, M. (1982). Teacher growth states and school environments. *Educational Leadership*, 40(2), 36-41.

> This could be justified in terms of greater possibility of 'take-up', and a consequent increased likelihood of teacher models being available to 'energise' others. (Owen et. al., 1988, p. 12)

In a context with limited human and financial resources available for the provision of professional development of teachers, like the Colombian context, there may be the need to prioritise when taking decisions about what the most urgent cases are. Would Owen and his colleagues' suggestion be useful for the Colombian case where the 'mainly internal attributions' parallels the attitudes of the *Omnivores*, and the 'mainly external attributions' teachers show some interest in learning about the teaching of beginning algebra?

There is no evidence from the Focus Group that interaction between teachers with different conceptions could have positive effects on the teachers. For example, none of the teachers seemed to have paid attention to Pablo's arguments during the Focus Group session, and none of them reported, at Interview 3, to have been struck by Pablo's views. I give no support for working with mixed groups as it is possible that experienced teachers with strong 'instrumentalists' conceptions of mathematics and 'external attributions' may influence other types of teachers.

The centrality of promoting a deeper understanding of mathematical concepts

Nora told us that her engagement in two classroom based action-research projects had led her to reaffirm her belief that pupils do not learn if they are not told "the mathematical formalisations". The same argument was offered by Stella, (Nora's colleague from her school), and by Luis who apart from being a teacher of Grade 8-algebra was teaching mathematics at a university. Nora, Stella and Luis were concerned that by engaging pupils in contextual classroom activities (i.e., the ones "suggested" by Nora's "project consultant" or the ones described in Questionnaire 2 and the Focus Group) and not focusing on the formal definitions, the pupils would not learn "the language of mathematics" or "the historical exercise". Working on these contextual situations, for Nora, was focusing just on "*la parte lúdica*" (i.e., on playing), and for Luis, "that type of work [had] nothing to do with what pupils [had] to learn". The evidence does not show instances where Nora's or Luis' teaching of beginning algebra used, for example, the pupils' arithmetic work to help them construct the concept of variable through connections with other concepts like functional relationships or basic geometry. Their teaching of Grade 8-algebra focused on the manipulation of given symbolic expressions, which means that the emphasis was on the use of one form of representation.

As shown in Chapters 5, 6 and 7, for Nora, Stella and Luis, and for the great majority of the teachers (i.e., twelve out of thirteen), beginning "algebra" meant moving from the arithmetic program to the new list of topics set out for Grade 8. This would normally start by giving examples and a definition of algebraic expressions, moving on to the manipulation of the given expressions without any concern for the reasons why the shift of the work with numbers to the one with letters is made or for what the letters used represent. There was no evidence, from the great majority of teachers, of their awareness of the mathematical processes that could lead children into algebraic thinking, or of their awareness that a conceptual approach exists. These findings indicate that beginning algebra teachers need a fuller and deeper understanding of the multifaceted nature of the concept of variable and of the mathematical (classroom) process from which this concept can be built.

The importance of helping teachers to rethink their understanding of mathematical concepts and of the nature of mathematics knowledge, and their ways of knowing mathematics cannot be overemphasised. If teachers are not provided with the opportunity to question their assumptions about the nature of mathematics knowledge, and about its connection with the learner's social and natural worlds, it is likely that they, like Nora, Stella and Luis, continue to ignore the importance of learning about their pupils' thinking. It is likely that they continue to ignore the value of using pupils' situational knowledge to explore and analyse relevant real-life situations from where important mathematical knowledge that leads into "Activity" (Mellin-Olsen, 1987) can emerge and, instead, continue to see the role of mathematics education as the preparation of students for an external examination. If teachers are not provided with the opportunities to reflect on the nature of knowledge, and if they do not see themselves as creators of knowledge, how can we ask them to carry out classroom action-research projects? On what are they supposed to reflect?

This suggests that, in the same way that school learners need to be provided with learning environments that suit their needs and promote reflective learning, the professional learning opportunities for teachers can be created if the programs offered to them are designed on the basis of this principle. The professional development programs envisioned would need to be based on knowledge of the teachers' conceptions of their current classroom practices and their perceived needs, rather than by offering them, for example, courses that have been thought to be useful for the teaching of mathematics in general. For example, Luis openly declared that he and his colleagues did not have the necessary understanding of the concept of variable, and Alex and José continuously emphasised that their knowledge of the teaching of algebra was inadequate. A deeper understanding of the nature of mathematics on the part of the teachers will provide them with a different conception not only of the 'what' and the 'how' but especially of the 'why' of school mathematics.

The crucial role that teachers' conceptions of assessment played in their conceptions of their practices

We also learnt from the teachers that they take the lists of itemised topics of textbooks as the sequences to follow in their teaching, which automatically become the assessment indicators on which to base the tests through which they assess their pupils' work. This was not a surprising finding since this was the logic indicated by the *Statements of the National Curriculum* that was in force for decades (i.e., before the issuing of the General Law of Education in 1994, see Agudelo-Valderrama 1996) where the program of study for each Grade came organised into units and subtopics with specific teaching objectives and assessment indicators and number of hours to be spent in each unit. As we saw in Chapters 6 and 7, Pablo, the only (novice) teacher, who rejected the textbook approach and argued that tests were not the best way to assess pupil's work, also justified the fact that he had to give tests to his pupils by his urgency to comply with the school report system. From Richardson and Placier (2001) we learnt that the phenomenon of novice teachers giving up their ideals in order to adapt to organisational realities is not only typical of the Colombian context. How can Faculties of Education build close working relations with schools in order to provide support for novice teachers like Pablo? How can teachers' professional development experiences help them conceptualise assessment as an integral aspect of their teaching practices where further learning for teaching can take place?

For example, the evidence from my previous work with teachers of the PROMECA project, who were interested in changing their teaching of beginning algebra (see Agudelo-Valderrama, 2000) shows that their engagement in *relearning* the mathematical concepts they were teaching took them to identify a new '*why*' of their teaching of these concepts. They drew on their own 'relearning' process in order to identify specific areas on which they wanted to focus their classroom projects, and to visualise the '*how*' of their *new* teaching practices. A new 'how' of teaching that was based on '*their new*' learning processes led them to focus their attention on their pupils' thinking processes which, consequently, changed the 'what' and the 'how' of their assessment practices.

In another professional development context, Pegg and Panizzon (2004) tell us that by focusing on assessment practices, through the use of a theoretical model for identifying the structure of students' understanding, a group of secondary mathematics teachers in New South Wales (Australia) changed their teaching approaches. The common observations in these two Professional Development contexts are that

- the participating teachers became critical of their own practices realising that asking questions that required of pupils, simply, to apply standard algorithms of procedure promoted rote learning, and did not provide insight into pupils' difficulties or understandings, and

- assessment was seen as an integral part of the curriculum; that is, of the acts of teaching rather than as a practice that takes place after teaching

It is necessary to highlight here that the professional development programs described above were long term projects and that, in the Colombian case, the work of the teachers in the project had the *keen* support of the school head teacher and the academic coordinator. As noted in the corresponding Research Reports[2] of the PROMECA Project, this head teacher was interested in learning from the experiences of the PROMECA teachers about how to improve his capacity to provide leadership in the area of curriculum development in his school. These observations point to the need not

[2] (i) Research Report presented to IIFA, Universidad Pedagógica y Tecnológica de Colombia (Dec., 1999). *Trabajando con Profesores de Matemáticas en la 'Promoción de una enseñanza 'basada en la comprensión', en álgebra elemental' (PROMECA)*. Project No: 9002.048. (ii) Research Report presented to Fundación par al Inveatigación y la Tecnología, Banco de la República (Oct., 2000). PROMECA Project No: 951.

only of creating conditions for change at the school level, in order to support the initiatives of teachers who work towards the enactment of innovation in their classrooms, but also of conceptualising schools as "professional learning communities" (Fullan, 2001). These crucial needs are further highlighted in Subsection 8.3.3.

Since the ultimate aim of the proposed professional development strategy should be to help the teachers shift their conceptions of their own teaching practices towards those of the 'internal attributions' teacher, who sees her/himself not only as a learner but also as a change agent, the empowerment of the teachers to become change agents is an implicit task of the professional development envisaged.

8.3.2 Implications for curricular guidelines and materials, and for the External Examination institution

This study has provided evidence of the powerful role that curricular guidelines and materials play in the teaching and learning of school mathematics in the specific context of Colombia. The evidence shows that all the participating teachers saw the list of topics contained in textbooks as mandatory statements for their teaching of beginning algebra, and that the pupils' textbooks were the only curricular materials that the teachers had access to. The great majority of participating teachers' teaching of beginning algebra relied heavily on the textbooks. Pablo was reluctant to follow the textbook's suggested sequence, and showed great initiative in deciding his own classroom work approach, but he also felt compelled by the school's assessment system which was related to the Mathematics department's list of topics representing the program for Grade 8.

In the contrasting situation of China, Ma (1999) tells us that teachers see curriculum guidelines and curricular materials, in which a teacher's guide is included, as sources of inspiration for their learning of mathematics and for the teaching of the subject. She also tells us that the curricular materials are written by experienced teachers. The identified contrasting situation in Colombia, where the only curricular materials available for teachers were the pupils' textbooks makes us ask: How realistic is it, on the part of educational authorities and policy makers, to expect teachers to become "constructors" of an innovatory curricula if, on the one hand, the only resource teachers can draw on is their own experience of a traditional prescriptive curriculum and, on the other hand, they feel accountable for the preparation of pupils to succeed in the standardised tasks in the External Examination? Perhaps the educational system is not providing the means and space for teachers to challenge their conceptions and to reconsider their roles as teachers, despite the policy emphasis on teachers as curriculum constructors or developers. How does the case of China described by Ma, where curricular guidelines and materials are designed by experienced teachers to encourage and promote teachers' learning, help us see the possibilities for creating the guidance and support that teachers in Colombia need to change their practices of beginning algebra, and of school mathematics in general?

As we saw in Chapter 6, the teachers asserted that the new *Curriculum Standards for Mathematics* (released in 2002 when the data collection took place) were just 'a list of contents'. Apart from the case of Pablo, who provided evidence of his knowledge of some of the classroom processes from which algebraic thinking could emerge, the teachers showed no confidence when considering how to promote algebraic thinking in the primary school levels of mathematics, as one of the requirements implicit in the set of topics that comprised the Curriculum Standards document (see Appendix 8.2). The contrast between the Colombian context and the case of China needs to be pointed out again. The most experienced teachers who participated in this study did not show any confidence in reorganising their curriculum, when the "list of topics" given in the Curriculum Standards was not accompanied by the corresponding prescriptions of 'how' to teach it. To what extent is it possible to design curriculum guidelines and standards that, while providing guidance and support for teachers, encourage the flexibility and creativity needed for different classroom and school contexts?

I believe that curricular guidelines and curriculum standards, more than just being composed of a list of contents, would need to provide a comprehensive framework for the national education system and for mathematics education in particular where the dynamic interrelationship between the educational aims and principles, the teaching approaches and the content is highlighted. Further, curricular guidelines and standards need to be constructed by a nation's education community through a process of discussion and negotiation in relation to the specific needs and values of a society, in a specific historical moment, speaking the community's language rather than being a set of curriculum statements imported from other systems, as has been the case in the

Colombian context. If teachers are not provided with the opportunities to become active participants in the process of construction of a nation's curricular guidelines and educational standards it is not surprising that they see themselves as transmitters of the knowledge that external authorities create, or as the implementers of orders imposed on them:

> ...there is always a strong aversion for mathematics. ... Maybe, it has to do with what is being taught and how it is taught ... what is taught depends on the curricular guidelines ... There are two levels of curricular guidelines: the ones that the Ministry imposes, and the ones we have at school. Obviously, the one at the school are based on the ones of the Ministry... The Curricular guidelines from the Ministry are the ones in which the topic-objectives, the approaches and the assessment indicators are defined; the ones of Vasco[3]... (Alex, Int. 2, initial concept map)

The role of the External examination institution

From the group of nine teachers who participated in Phase 2 of the study, eight of them argued that if they did not cover the set Program for Grade 8, their pupils will not be prepared for the requirements of the next levels, a fact that will ultimately affect their performance at the ICFES examination. As explained in Chapter 7 (Subsection 7.3.1.3) the teachers' conceptions of the requirements of the External Examination influenced their conceptions of beginning algebra. They emphasised the fact that pupils needed to master the formalisations of mathematics which prepared them for moving on to higher education. Consider the explanation given by Juan when he was asked why his 'Number 1' reason for the teaching of algebra in schools was, 'because knowledge of algebra is key for pupils' access to higher levels of school mathematics (e.g. trigonometry and calculus)':

> ... the teacher has to aim at having his/her pupils prepared in the best possible way for the ICFES examination. So you know you have to try to cover as much as you can from the 'Program'. You cannot stay too long in one topic because, even if these pupils are not moving on to higher education, they themselves, their parents, the Ministry, everybody! measures the school and the teachers by the ICFES results. (Juan, Int. 1)

Despite the teachers' declarations that their teaching practices were driven by the requirements of the External Examination, in their concept maps of the determinants of their teaching practices, they did not identify 'the External Examination' as one of the

[3] Vasco is the name of Curriculum Development Office Advisor of the Ministry of Education.

determinants. Nor did they identify the textbook, despite the fact that the majority of teachers acknowledged their teaching being guided by the approaches presented in the textbooks, and in some cases (see the case of Alex) being constrained by the fact that in the textbook "everything comes decontextualised". As shown in Chapter 7, this is because, for the majority of teachers, the essence of school mathematics knowledge was (externally) determined (i.e., by textbooks), and teaching duties were conceptualised as being prescribed by the requirements of the External Examination. Therefore, 'the External Examination' and 'the textbook' were not identified as factors that influenced their teaching practices because those two institutional elements represented the 'knowledge authorities' which showed to them what they felt accountable for. Examination of the pupils' textbooks which, as explained before, are the only curricular materials available for teachers, showed that the editors, in the textbooks' presentation, underline the fact that the books have been written according to the ICFES requirements.

It is not surprising that the teachers showed no awareness of the tension between their espoused teaching goals of educating the whole child, and their assessment practices which focused on pupils' capacities to reproduce 'the mathematical formalisations' and to manipulate symbolic expressions. The same tension seems evident in the broader context represented by the way the educational system functions for, on the one hand, the central policy proclaims that schools need to construct flexible curricula in order to attend to the needs of the different communities they serve but, on the other hand, all pupils regardless of which community they belong to or of which specific educational needs they may have, are obliged to sit the *one* External Examination at the end of secondary school. Furthermore, as noted in Chapter 1, the results of this examination are not only used to control the students' admission to higher education but also to measure the success of schools and teachers.

Is it possible that the body in charge of the External Examination takes into account the need to provide flexibility and variability in the examination, in order to maintain coherence with the new approaches highlighted by the General Law of Education? Is it possible that this body extends its role to the promotion and support of the implementation of the proclaimed educational approaches?

8.3.3 Implications for the school institutional context

In Section 8.3.1, and in the whole thesis, the emphasis has been on the teachers' attitudes to change as being dependent on their own individual conceptions of the nature of knowledge and, therefore, on their conceptions of the nature of the teaching activity. This speaks of teachers' conceptions as something *internal* to the individual. However individual this phenomenon may be, the evidence from this study also shows that similarities in the ways teachers conceptualised their teaching practices and situations were observed in teachers who belonged to the same school. Therefore, the 'internal, individual' conceptions need to be looked at as being construed within a school community.

Juan and Loren, though not fitting exactly the teacher type that Nacho portrayed, had similar ways of explaining their practices and showed similar attitudes to change. These teachers, who belonged to the same school, spoke of the rigid school environment and the unmotivating role of the head teacher, declaring their low satisfaction with their jobs as teachers. Nora and colleagues, on the other hand, emphasised that they had the support and encouragement of the head teacher to participate in this study and to introduce innovations in their teaching. The same was observed in the cases of Alex and José. They reported that they had been encouraged to participate in this current study by the school principal and the mathematics coordinator. As highlighted earlier, this makes us think of the need to conceptualise *teaching* and *learning* as a phenomenon that belongs to communities of practitioners, where the head teacher and the head of the department play crucial roles in the development of the school's curriculum and the teachers' attitudes to teaching. But how supportive of this trend is the evidence from the head teachers and heads of departments in this study?

Although (apart from Nacho and colleagues) the participating teachers reported feeling supported by the head teacher in adopting classroom innovations, none of them reported having the benefits of the leadership of their head teachers in the school's curriculum development. Nor did they report on the leadership provided by the head of the mathematics department, as we saw in Chapter 6. What could be the reasons why the teachers failed to report on this? Could it be because they were not provided with that leadership or because they thought that it was not significant for them? Through my informal conversations with two of the head teachers, about the introduction of innovations in the classroom, I learnt that these head teachers were feeling constrained in their roles at school as they described their 'new responsibilities' assigned to them by the new legislation. One of the head teachers commented that with the introduction of the new legislation, head teachers' responsibilities had "enormously increased" as they had been given not only the "new task of being the school's budget administrator but also of being the head teacher in the other school session[4]". This is a topic that requires further inquiry. However, this information is consistent with the findings from fifteen school head teachers in Bogotá (see Perry et al, 1996) who declared that they were overwhelmed by the amount of time that their new roles of 'budget administrators' was requiring of them, as they had found themselves using a significant amount of time at the local education authorities offices in order to comply with the introduced bureaucratic procedures.

The two heads of department (Nora and Nacho) who participated in the study also reported using part of their time being involved in administrative activities both outside (with the local education committees) and inside the school. Nora who showed great enthusiasm for participating in this study and in "setting an example" for her colleagues, explained the difficulties in being able to participate in a professional development project offered to her by the interviewer, due to the constraining work conditions of having to teach in two different schools (i.e., sessions) in order to make a living. As did the majority of the participating teachers, Nora pointed out "the unfair working conditions of teachers" in Colombia. As we saw in Chapter 6, after discussing what to do in order to address the problematic situation of the pupils' lack of motivation for their learning la algebra, Nora explained why she could not choose to participate in a Professional Development Project that I offered to her. This is what Nora said:

> My first choice would be to participate in the 14-month classroom based research project... I would not do it just because of the teachers' scale points but because I believe that that is the type of experience from where I would learn the most. ... But the only way I could do it would be during the weekends because I cannot afford to lose my other job in the morning...; and we too have families that we need to care for! I could, for example, try to stop working for a year in the private school in the morning, but that means I would definitely lose that job because... So

[4] State schools in Bogotá, and in most areas of the country, provide two school sessions a day: the morning session and the afternoon session. With current regulations, the previous arrangement of having a head teacher in charge of each school session was changed, and the head teacher is responsible for both the morning and the afternoon sessions.

> if you ask me to choose from the three options, I would have to choose adopting a textbook ... The teachers' situation, especially in the light of new legislation is very unfair! ... Besides that, the government's policies are contradictory. There are clear contradictions; on the one hand, the government wants the teacher to construct and develop the curriculum and, on the other hand, they are cutting the teacher's time for doing it!... The teacher is expected to dedicate more time to his/her job but, at the same time, has to look for other means of income (i.e., find work in the two working sessions) to be able to cope with the cost of living... Is the government interested in quality of education?

What Nora pointed out in relation to 'the teachers working conditions' seems to coincide with the reasons given by the head teachers when they talked about their increased responsibilities. They were pointing out that they were being asked by the reform to do more with less time (as a resource). But the implementation of the change advocated in the current policies in Colombia requires of teachers, head of departments and head teachers, not only additional resources but also access to expertise (Fullan, 2001). Head of departments, like Nora, and especially head teachers require specific qualities, skills and expertise to be able to play *leadership* roles not only as school curriculum developers but also as organisational managers. I argued elsewhere that educational change in Colombia will not take place because there is a new regulation that *decrees* change (Agudelo-Valderrama, 1993, 1996, 2000). Educational governments, since 1994 when the General Law of Education was issued, have been claiming that Colombia has embarked on an *educational revolution*[5]. The findings of this study do not support the government claims. How can change policies become *action* in the schools when the policies have been drawn in total disconnection from the realities of those who are to be the implementers of the advocated change?

A clear implication of these findings for policy makers and educational authorities, which I pointed out earlier (see Agudelo-Valderrama, 2002), is that possibilities of change can only be considered when policies include a systemic change strategy; that is, the creation of an infrastructure that support specific *actions* for the implementation of the proposed change. The systemic change strategy needs to attend to the *dynamic* interrelations that exist between the various components of the educational system, which are addressed in different subsections of Section 8.3 in this chapter (i.e., teacher professional development programs, curricular guidelines and materials and the

[5] '*La revolución educativa*' (n.d.).Retrieved 23/11/2004 from the Colombian Ministry of Education website: http://www.mineducation.gov.co/estandares/solofam.asp

External Examination, teacher education programs). I see the dynamic interconnection between these components as being born by the fact that their ultimate purposes and/or activity are directed towards the education that is provided by schools. A pivotal part of this systemic strategy is represented in the provision of professional development both for teachers and school administrators, which as emphasised in Subsection 8.3.1, needs to the based on a reconceptualisation (in Colombia) of schools as communities of practitioners which need to come together as *continuos* learners in their professional activity, extending this learning to the broader community of parents. Is it possible that policy makers, central and local educational authorities include in their political agendas the creation of change implementation strategies as a first sign that there is some interest in an 'educational revolution'?

8.3.4 Implications for teacher education programs

As pointed out in Chapter 1, what I learnt through my three years of experience as a mathematics educator and the results of the Ministry of Education inquiry into teacher education programs (see Niño, 1998) showed that in the great majority of Faculties of Education, teacher education programs centre on the coverage of a list of (the area specific) subject matter content plus some theories of education, with very little space to question what goes on in the teaching and learning of mathematics. However, Pablo told us that his teaching approach had been constructed during his teacher education program, specifically during his teaching practicum. This suggest that his teacher education experience had been influential in helping Pablo question the traditional 'instrumentalist' teaching approach, and provides evidence of the power that teacher education programs can have on prospective teachers.

The lessons that we have learnt from this study in relation to the professional development of practising teachers provide valuable information for teacher education programs if the goal is to prepare teachers to construct curricula that suit the needs of the different school communities, as central policy emphasises. However, Pablo also told us that he is prepared to give up his preferred teaching approach since he had to comply with the school's and the parents' expectations of his teaching. Once again, as highlighted in the previous heading and in the first paragraphs of heading 8.3, we return to the issue of the need to address the institutional and systemic factors, ensuring the creation of possibilities of change. What strategies can Faculties of Education and Local Education Authorities build in order to provide support for novice teachers to develop their often new ideas?

8.3.5 Teachers' professional satisfaction

As explained in Chapter 7 one finding of this study was the dissatisfaction and low morale declared by all the participating teachers. Some teachers complained about the restrictive role of their head teacher, and all the teachers felt they were negatively affected by the low salary and the increase in teaching time[6] decreed by the new legislation. According to Friedman and Farber's (1992) study carried out in Israel, from several dimensions of teachers' professional self-concept, "professional satisfaction" —how teachers feel about the gratification they receive from teaching— "showed the strongest correlation with teachers' burnout" (p. 33). We remember that, for example, Juan declared very low motivation for his work as a teacher, which he explained as due to the treatment he received from the head teacher and to the low salary of teachers. We remember that he stated that he was giving strong advice to his old students not to become teachers due to the difficult conditions. How can teachers and head teachers in Colombia be encouraged to work in a collegial fashion with a view to enhancing their self-esteem and their professional satisfaction?

In other countries teachers themselves create professional subject associations with the aim of providing continuous leadership in the improvement of education related to their specific subject. For example, in the area mathematics education such a role is played by the *National Council of Teachers of mathematics* (NTCM) in the United States, the *Association of Teachers of Mathematics* (ATM) in Britain, and the *Australian Association of Mathematics Teachers* (AAMT) in Australia. Clarke (2004) claims that the construction of the *Professional Teaching Standards* in Australia constitutes a way of increasing public esteem of the profession. She describes the involvement of the Australian Association of Mathematics Teachers in the development of 'Professional Teaching Standards' as a tool for individual and systemic evaluation of teachers of

[6] In August 2002 the government issued a new policy increasing the length of public schools sessions. Nora and colleagues explained that teachers who work in two sessions, like all of them, would have difficulties meeting the demands of their two jobs.

mathematics, a requirement that is fundamental to increase public esteem of the profession. The active involvement of mathematics teachers in the construction, implementation and continuous development of professional teaching standards, it is argued by Bishop, Clarke and Bennett (2000), is a crucial requirement for the attainment of a system of professional recognition amongst teachers. "Moves by education systems to establish teaching standards will inevitably focus on generic and/or beginning general competencies...[which] do not take account of the particular knowledge and skills (in mathematics and the teaching and learning of mathematics) that contribute to being an effective teacher of mathematics" (p. 2)

A project for the development of professional standards for excellence in the teaching of mathematics in Colombia, similar to the one described by Clarke, cannot take place without educational leaders or the work of active Associations of teachers like the ones mentioned above. How can committed teachers and educators in Colombia draw on the examples of Australia and other countries to work together towards the creation and implementation, by teachers and for teachers, of professional standards in the teaching of mathematics? I believe that in addition to providing the platform needed for the improvement of mathematics education, the development and implementation of professional standards represents a valuable scheme for increasing public recognition of the profession in Colombia.

As a summary of this section which has focused on the implications of the findings for the creation of possibilities of change in the teaching of beginning algebra and mathematics in general in Colombia, I would like to highlight the following points.

The need to provide mathematics teachers with professional development has emerged as the most important implication of the findings of this study. The central purpose of the professional development programs envisioned in this study should be to help teachers shift their conceptions of their own teaching practices of mathematics towards those of the 'internal attribution teacher' described in Chapter 7 (Subsection 7.4.3.4). However, the implications for the institutional context belonging both to school level and the wider educational system that were drawn from the findings of this study, and discussed in this section, need to be addressed simultaneously, attending to the dynamic interrelationships between them, in order to create the conditions for change that support the "epistemological empowerment" (Ernest, 2000) of teachers. The inclusion of systemic action strategies both by central and local educational governments for the implementation of reform —that take account of the realities of teachers and schools—was highlighted as an imperative. Implications were also drawn for:

- Curricular guidelines and materials and for the External Examination institution
- The institutional context of the school
- Teacher education programs

The need for teachers to work collegially and to engage in the creation of a professional association geared towards the setting of professional standards for teaching of mathematics, and the evaluation of teachers' professional performance was also highlighted.

8.4 Implications of the study for developing the research into teachers' conceptions of mathematics and its teaching

Wilson and Cooney (2002) pointed out that in order to gain some understanding about the nature of teachers' thinking and what provides the foundation for teacher change, it is important that researchers attend to both teachers' knowledge and beliefs. I argued in Chapters 1 and 2, however, that in order to identify possibilities for teacher change we need to focus our attention not only on teachers' knowledge and beliefs but also on the "attitudes associated with [their] beliefs" (Ernest, 1989). Research on mathematics teachers' thinking (e.g., research found under the categories of teachers' 'conceptions' or 'beliefs' about mathematics and its teaching) has generally tended to focus only on one component of teachers' thought structures at a time. That is, studies have focused either on cognitive components or on affective components, limiting the possibilities of understanding the relationship between the cognitive and the affective components of teachers' thinking and its relevance for the improvement of professional development programs. Furthermore, a premise of this study was that in order to gain insight into the possibilities of teacher change I needed to focus on the perspectives of teachers about their own teaching practices. Focusing on the perspectives of teachers afforded opportunities that exposed the powerful role that the teachers' conceptions of social/institutional factors of teaching played in their conceptions of their practices. These insights have theoretical implications related not only to the theme of *teachers' thought structures* but also to the theme of *teachers' pedagogical content knowledge*, which I discus in the following subsections.

8.4.1 Theoretical implications - Models of teachers' thought structures

As shown in Chapter 2, the journey into researching the teachers' conceptions of their teaching of beginning algebra began with the construction of a model representing a proposed way to think about the teachers' thought structures, "the knowledge, beliefs and attitudes stored as schemas in the mind of the teacher" (Ernest, 1989, p.13), and their relation to their practice. The model, to which I shall refer as 'the initial theoretical model of teachers' thought structures', was constructed by using the ideas related to the components of teachers' knowledge put forward by several researchers, and by acknowledging the importance of including teachers' beliefs and attitudes as components of a teacher's thought (or mental) structures. However, as pointed out in Chapter 7, the findings of this research have provided a different model offered by the participating teachers, which I shall call 'the teachers' model'. In the following subsections, I will first remind the reader of the basic details of the 'initial theoretical model of teachers' thought structures' and then I shall focus on 'the teachers' model'.

8.4.1.1 The initial theoretical model of teachers' thought structures revisited

The initial theoretical model of teachers' thought structures, shown in Figure 8.3, was built by taking Shulman's (1986, 1987) concepts of content knowledge and pedagogical content knowledge whose subcomponents were common to the models put forward by Ernest (1989) and Fennema and Franke (1992). Teachers' conceptions were defined to encompass teachers' knowledge, beliefs and attitudes following Ernest's (1989) model. The beliefs and attitudes components were considered important parts of teachers' conceptions in this study because of their "powerful impact on teachers' make-up and approach" (Ernest, 1989, p. 25) and, therefore, on teachers' conceptions of change in their teaching.

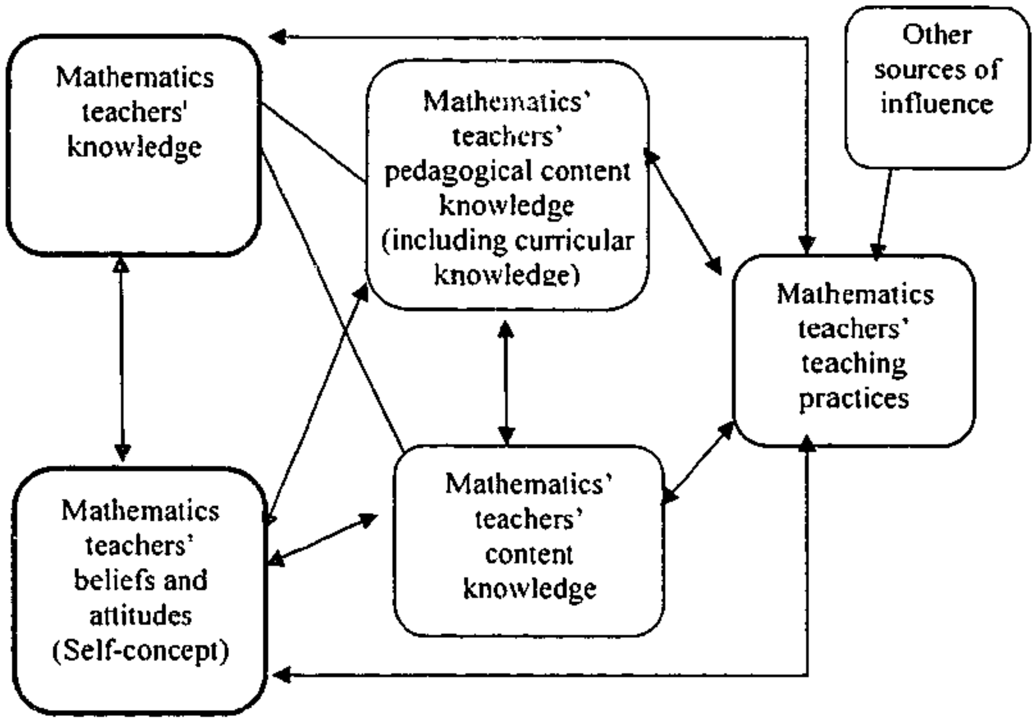

Figure 8.3. The initial theoretical model of mathematics teachers' thought structures (or mental structures) and their relations to practice

Although in constructing the model there was acknowledgment of Ball's (1991) claim that "content knowledge is a term in the pedagogical equation" (p. 38), I chose to keep 'content knowledge' and 'pedagogical content knowledge' as separate terms in the model, with the intention of looking for evidence of how each type of knowledge would relate to the teachers' conceptions of their practices. An important assumption that was implicit in the structure of the initial model was that although the contextual factors of teaching can influence what takes place in teachers' teaching practices, the teachers' knowledge of the concepts they teach (i.e., content knowledge), and their knowledge for teaching these concepts (i.e., pedagogical content knowledge and the associated beliefs and attitudes) were the most important determinants of teachers' teaching practices. The simplification of this initial theoretical model that is shown in Figure 8.4 facilitates the observation of this implicit assumption and, at the same time, encourages the comparison with the obtained model from the teachers which is shown in Figure 8.5.

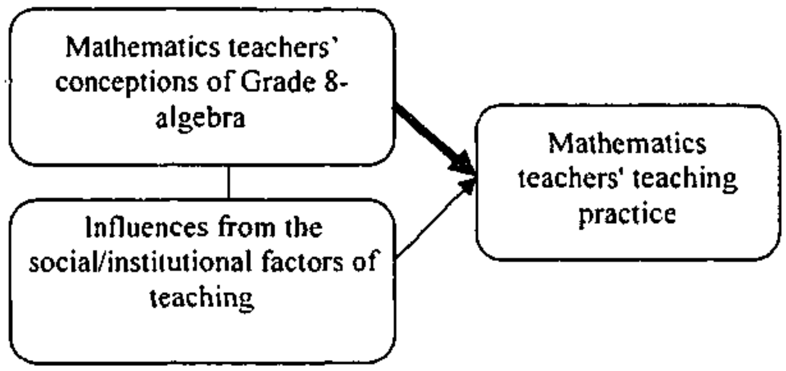

Figure 8.4. The initial theoretical model - A simplified version

8.4.1.2 The teachers' model

Although I saw in the case descriptions that every teacher had a unique individual representational model to describe their conceptualisations of their teaching of Grade 8-algebra, their primacy of external factors as the crucial determinants of their own teaching practices provides a different, distinct model of their conceptions of their teaching, which is in contrast with the initial theoretical model shown in Figure 8.3. Nora, Luis and Alex, when asked about the role of a teacher's knowledge in his/her teaching, acknowledged its importance but continuously emphasised the primacy of external factors, especially the pupils' behaviour, throughout the different activities of data collection. In Nacho's portrayed model the crucial and unique determinants of his teaching practice were external.

Pablo, the only teacher from the five cases included in this thesis and from the nine teachers who were followed in Phase 2, showed awareness of the impact of his "knowledge and dispositions" on his teaching, but also came to identify the primacy of the "time factor" as a crucial determinant of his teaching, as he gained more knowledge about the social factors of his teaching. The model of *teachers' thought structures* that emerged from studying the teachers' conceptions of their own teaching practices is different from *the initial theoretical model* with which the journey of this study started. While 'the initial theoretical model' emphasised the teachers' conceptions of mathematics as the *crucial* determinant of their teaching practices, 'the teachers' model' points to the teachers' conceptions of the social/institutional factors of teaching as the crucial determinant of their teaching practices. The contrast between the two models is indicated in Figure 8.5 by the difference in the width of the arrows.

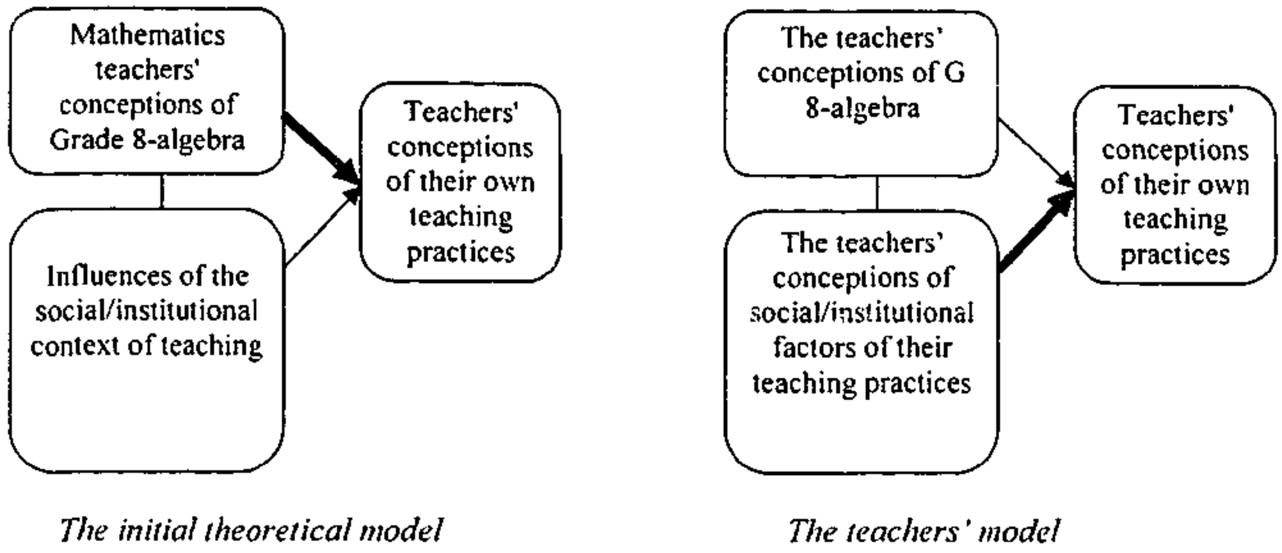

Figure 8.5. Contrast between 'the initial theoretical model' of teachers' thought structures and 'the teachers' model'

8.4.1.3 A new model of teachers' thought structures

Figure 8.6 shows a revised model of the dimensions to take into account when studying the components of mathematics teachers' thought structures 'or conceptions' of their own teaching practices. According to the findings, the teachers' conceptions of beginning algebra directly influenced their conceptions of their own teaching practices; that is, their conceptions of their roles and the role of the learner which, in turn, related to their self-efficacy beliefs and their conceptions of the crucial determinants of their teaching practices. The teachers' conceptions of the nature of knowledge influenced their conceptions of the role of social/institutional factors in their teaching, and these played back on their conceptions of the nature of beginning algebra knowledge.

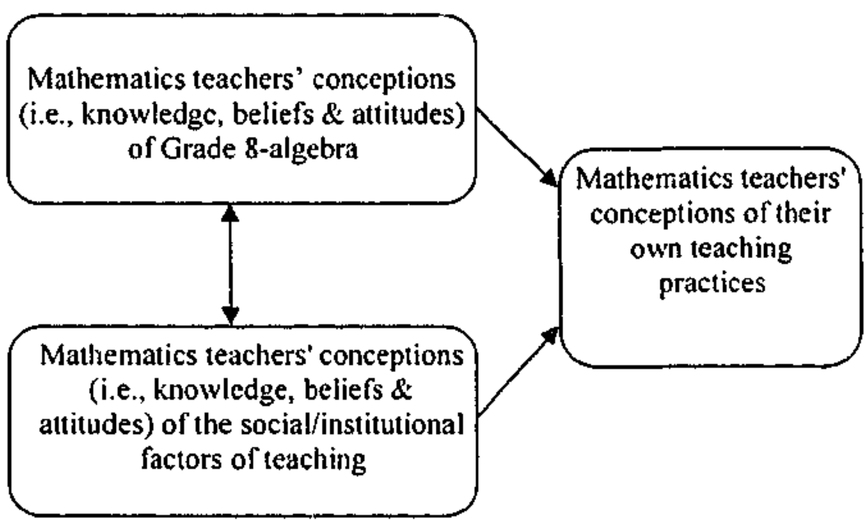

Figure 8.6. A new model of the dimensions to take into account for the study of mathematics teachers' conceptions of their own teaching practices

8.4.1.4 Pedagogical content knowledge and the teachers' social knowledge - the missing dimension

As already explained, in the initial theoretical model the categories of teachers' *mathematical content knowledge* and *pedagogical content knowledge* were kept separate in order to try to identify evidence of each type of knowledge and their relation to the teachers' conceptions of their own teaching practices. With this in mind, an effort was made to identify instances of the two types of knowledge. In analysing Pablo's evidence I found it difficult to differentiate between them. Pablo's work, organised with the purpose of helping the pupils establish connections between different mathematics concepts, represented evidence of his pedagogical content knowledge, but this knowledge was also mathematical (i.e., content knowledge). In Nacho's case, his pedagogy of presenting pupils with items of procedures to manipulate given (symbolic) expressions was a consequence of his way of knowing beginning algebra. This prompted the question: Is it useful to differentiate between 'content knowledge' and 'pedagogical content knowledge'?

As was shown in Chapter 7, the teachers' conceptions of the nature of algebra knowledge contained the elements of their pedagogical approaches, that is, the 'why', the 'what' and the 'how' of their teaching acts. Therefore the teachers' *ways of knowing* beginning algebra underpinned their conceptions of their own teaching practices —and not the different categories of knowledge indicated in the initial theoretical model. This represents a significant finding of this study, obtained by focussing on teachers' actual practices.

As shown in 'the teachers' model' of Figure 8.6, the teachers' conceptions of the social/institutional factors of their teaching represent a key component of their thought structures. The teachers' social knowledge, beliefs and attitudes impacted on their pedagogical decisions, a fact that supports McEwan and Bull's (1991) claim that all knowledge is pedagogical in varying ways. I argued elsewhere (Agudelo-Valderrama, 2004) that studying teachers' conceptions of their teaching of mathematics requires us to consider teachers' conceptions of social aspects of their teaching or their "social perspectives" (Gates, 2001), if we are to gain some understanding of the barriers and possibilities of teacher change. In other words, we need to start addressing what Gates

(2001) has termed as "the missing dimension" in research on mathematics teachers' beliefs and conceptions.

8.4.2 The contributions and limitations of the findings of this study

This study has provided important contributions, first of all in relation to the identified gap in research in the teaching of school algebra that was pointed out in Chapter 1 and, secondly, in relation to our understanding of the phenomenon of the stability of traditional mathematics teaching approaches in Colombia. One of the significant contributions of this study for the development of the research on teachers' conceptions of mathematics and its teaching is represented in a new model of teachers' thought structures. This new model has added to previous models (i.e., the ones on which the initial theoretical model constructed in Chapter 1 was based) the key dimension of teachers' social *knowledge*, *beliefs* and *attitudes* (that is the teachers' conceptions of the social/institutional aspects of teaching).

One implication for research that is concerned with possibilities and/or difficulties of change from this new model is that it needs to focus not just on teachers' *knowledge* of mathematics and mathematics teaching, for this study has shown that teachers' thinking about their teaching is not exclusively focused on the list of categories of knowledge which have been included by some researchers as *pedagogical content knowledge* (see for example Shulman, 1986, 1987; Grossman, 1990; Turner-Bisset, 1999). This study has shown that the teachers' ways of knowing beginning algebra carry an attitude —an affective component— that cannot be separated from the cognitive.

Another significant contribution of this study is the typology of teacher's conceptions of their own practices which provides key insights to inform the provision of professional development and teacher education programs in Colombia. Although this typology is most relevant to the Colombian context it is likely that it has implications for a range of other contexts. Even if the findings of case studies are not *transferable* from one context to another context, a case study may facilitate "the drawing of inferences by the reader that may have applicability in her/his own context or situation (Lincoln & Guba, 2002, p. 211). The experience described in this case study might provide the reader with "a sense of vicarious, *'deja vu'* experience", as Lincoln and Guba refer to it, and s/he can learn from it, and "as is the case with all learnings, make applications even in situations that do not appear on the faces to be similar".

> If one is a principal in a school and is reassigned to another school, however different from the first, the first experience 'stands one in good stead' (ibid, p. 212).

Nevertheless, we need more naturalistic studies focusing on mathematics teachers' conceptions of their teaching practices of mathematics which are carried out in a diversity of cultural and social contexts with teachers with varying degrees of experience and expertise, which include the case of primary school teachers. We need to know more about the variations in teachers' conceptions of their own teaching practices of school algebra, and mathematics in general, across different classrooms in different countries. Do mathematics teachers from other countries see the social/institutional factors of teaching as the crucial determinants of their teaching practices?

8.4.3 Methodological issues

Researching teachers' beliefs about their own knowledge of beginning algebra and its teaching was not an easy task

Researching teachers' beliefs about their knowledge of beginning algebra and its teaching was not always an easy task in this study. While gaining insight into the teachers' ways of knowing beginning algebra was accomplished through an indirect form of questioning, (e.g., by asking questions about specific teaching-learning situations that had been included in the questionaries or that were selected from the classroom observation), researching the teachers' beliefs about their own knowledge proved to be a difficult task in specific cases. The least experienced teachers and some of the experienced teachers spontaneously and openly declared the inadequacy (or the adequacy) of their knowledge for the teaching of Grade 8-algebra when they were talking about what they did in their teaching. However, not all teachers showed the same spontaneity and openness to make comments about their knowledge when talking about their teaching practices as could be seen in Chapter 6 in the particular case of Nora, and to a certain extent in the case of Alex (see Appendix 6.1).

Borko and Putnam (1996) in their review of research on teachers' knowledge noted that researchers do not want to ask questions that can make the teachers feel inadequate and put them in a negative light. Had I been making efforts to tempt Nora into revealing her beliefs about her knowledge, I would have affected the nature of the communication that I saw in the interviews as an opportunity to enquire, *collaboratively* with her, into her lived experience of her teaching of beginning algebra. It was perhaps due to the teachers' perceptions that I was not researching *on* them but researching *with* them, that Nora and some other teachers suggested new opportunities to continue the conversations and the interaction for data collection, showing enthusiasm about doing it. The prolonged engagement with the teachers during the data collection, the use of a multiplicity of data collection sources and the fact that the teachers were being asked the same question in different data collection contexts (e.g., in the context of explaining why they did what they did in their teaching, and why they would, or would not, do it differently) proved to be useful in collecting data about the teachers' beliefs of their knowledge, especially in cases where they did not talk about them spontaneously, as in the case of Nora.

The usefulness of 'the beliefs survey' included in Questionnaire 1

It was noted in Chapter 3 that one of the observations of my previous work with mathematics teachers was that when they were asked about the teaching of mathematics without focusing on the specific actual context of their classroom, they tended to talk about a teaching approach by using the language of policy statements or of the latest education literature, despite the fact that their classroom practices showed, *to me,* contrasting approaches. While the 'beliefs survey' was included in Questionnaire 1 (as Section B) to provide one more way of eliciting information from the teachers about their conceptions of the teaching of school algebra, it also provided an opportunity for exploring my conjecture that teachers' responses to questions that are not related to a specific actual teaching situation do not tell us much about their conceptions of their own teaching. I also hoped to be able to obtain their explanations for the possible observed differences already mentioned.

As shown in Chapter 5, the data collected through the beliefs survey showed that the majority of teachers adhered to statements categorised as related to a problem-solving conception of mathematics, which showed great contrast with their responses to Section C of Questionnaire 1, and with the conceptions they put forward throughout the rest of data collection activities. Through the probes or explorations that the time available allowed me to do of their agreements (or disagreements with a few of the statements of the survey), I learnt that the reasons why I saw contrasting responses was because they assigned different meanings from the ones I, —the producer of the statements— assigned to the statements, as was illustrated in Chapter 5. It would have been desirable to have been able to explore the teachers' thinking in relation to some more of their responses in the beliefs survey. However, it was not possible due to time constraints. In any case, this highlights the crucial need to always give the necessary attention to enquiry into the respondents' meanings and understandings, as was done with their responses to the rest of the questionnaires.

8.4.4 Further research and action steps suggested by this study

As noted in Subsection 8.4.2 we need more research on how mathematics teachers with varying degrees of expertise, in a diversity of cultural and social contexts, conceptualise or understand their own teaching practices of school mathematics. If these studies are to illuminate the possibilities for teacher change, they need to be collaborative, with teachers as professional partners in the acquisition of understanding about their conceptions of their teaching practices.

An immediate area of research that arises from this study for the Colombian context —and that I would like to undertake— is related to the design and evaluation of the professional development programs envisioned, which will take into account the insights gained through this study. The selection of a group of novice teachers to undertake a longitudinal study which illuminates the processes of change in their conceptions, or their difficulties of change, is of critical importance. Pablo showed us the reasons why as a novice teacher with problem-solving-like oriented conceptions, he started to restructure his preferred teaching practice, providing strong explanations for the difficulties of change in Colombia, and the implications of the findings for the creation of possibilities for change were discussed in Section 8.3. We need more research that helps us understand the following questions:

- What cognitive —and the corresponding affective— processes does a novice teacher like Pablo, who represented the 'mainly internal attributions' teacher, need to go through to shift her/his conceptions towards those of the 'internal attribution teacher'?
- What cognitive —and the corresponding affective— processes does a novice teacher like Alex who, having instrumental conceptions of beginning algebra and representing the 'mainly external attributions' teacher, need to go through to shift her/his conceptions towards those of the 'mainly internal attributions' teacher, and then towards those of the 'internal attribution teacher"?

By mentioning the need to work with novice teachers I am not suggesting that research and provisions of professional development for experienced teachers is not a case for urgent action in Colombia. Rather I believe that the two cases need simultaneous attention. However, in thinking about priorities in the specific case of Colombia, it seems that the case of novice teachers needs to be considered a 'number one' priority as Pablo, Alex and Luis showed us the impact that traditional institutional practices can have on novice teachers. I have not mentioned the need for researching primary school teachers' conceptions of their own teaching of beginning algebra which represent another area deserving urgent attention in Colombia, when thinking about the possibilities of change in the teaching of beginning algebra and mathematics in general.

8.5 Concluding remarks

This study has investigated the phenomenon of the stability of mathematics teaching approaches in Colombia. With the aim of identifying possibilities of change in the teaching of mathematics, it has researched the teachers' conceptions of their own practices of teaching beginning algebra. The teachers described their practices (i.e., their duties and their roles) as determined by external authorities, with special reference to the requirements of the External "ICFES Examination" and the National Curriculum Statements that were in force for decades and were converted into textbooks for each school grade. According to the claims of *past* and *current* central educational authorities, Colombia embarked on a period of *educational revolution* as of 1994, when the General Law of Education was issued. However, this study has shown that the teachers' understandings of their practices are in great contrast with the tenets of central educational legislation which declared the curricular *autonomy* of schools, and urged teachers to become *continuous* constructors of the curriculum.

A great tension is identified at the Colombian systemic level of education when, on the one hand, schools are to attend *the different educational needs of the communities they serve* but, on the other hand, all school leavers are required to sit the standard External Examination that controls students' access to higher education and whose results are used to judge the academic quality of schools. As explained in Chapter 1, the academic quality is based on the requirements of a formalistic curriculum, designed according to the established requirements of the few moving on to higher education. This great tension is further increased when according to the educational policy in force, the central purpose of schools is to provide students with (mathematics) education that develops the individual's critical mind required for *democracy* and *citizenship* but, on the other hand, teachers are not provided with the necessary conditions and opportunities to become critical citizens.

Would these tensions and contradictions be taken into account by policy makers and government officials when looking for explanations for the stability of mathematics teaching and the high rates of school drop out in the Colombian context? Would they be prepared to take on board the crucial need, identified in this study, of providing opportunities for teachers to increase their professional competence rather than just changing what goes in official documents and legislation in the form of new decrees stating what is required of schools and teachers? Clearly some *coherence* is *urgently* needed in the educational system, not only at the policy drawing level but also at the decision making and implementation levels. Calls for change will stay at the level of *calls* —in documents— if the issue of provision of professional development experiences for teachers and schools administrators is not placed as a *first priority* on the government's agenda. Calls for change in teaching approaches *necessarily* mean calls for a re-examination of external examination practices. Would teachers and school administrators engage in change if they feel that the quality of their professional activity is to be measured by *one* standardised External Examination?

I have already explained, in Chapter 4, the difficulties for teachers like the participants of this study (or myself), to have their voices heard in the Colombian political context.

Notwithstanding this I see my role as a researcher and mathematics educator as not just trying to make the stories of these teachers known within the broader mathematics educational community but, more importantly, in raising the awareness of both teachers and leaders of educational communities in Colombia of the need to focus on the relevance of current school mathematics, and to take action in this crucial area of education that concerns the lives and possibilities of the Colombian people. While, according to the government officials, the country is going through an *educational revolution*, teachers struggle to fulfil what they have come to understand to be their duties through their experiences as teachers in the Colombian educational system. The voices of teachers should not be silenced:

> ... the teacher has to aim at having his/her pupils prepared in the best possible way for the ICFES examination. So you know you have to try to cover as much as you can from the 'Program'. You cannot stay too long in one topic because, even if these pupils are not moving on to higher education, they themselves, their parents, the Ministry, everybody! measures the school and the teachers by the ICFES results. (Juan, Int. 1)

> If I were totally free I would be able to dedicate more time in the classroom to work in that sort of activity, but one is always behind with the program, so I have to hurry up! ... due to the *time* factor, I sometimes have to do things like Teacher A (i.e., telling). ... If I were going to teach in Grade 8 next year, now that I know how this school works, I would have to do activities that are much shorter because time is short and you have to cover the program. (Pablo, Int. 3)

> ...if you ask me to choose from the three options, I would have to choose adopting a textbook ... The teachers' situation, especially in the light of new legislation is very unfair! ... Besides that, the government's policies are contradictory. There are clear contradictions; on the one hand, the government wants the teacher to construct and develop the curriculum and, on the other hand, they are cutting the teacher's time for doing it! ... The teacher is expected to dedicate more time to his/her job but, at the same time, has to look for other means of income (i.e., find work in the two working sessions) to be able to cope with the cost of living... Is the government interested in quality of education? (Nora, Int. 3)

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Appendices

Appendix 1.1

Copies of the introductory pages, and the pages related to Grades 7 and 8-mathematics of the Colombian 'Curricular Standards' document for school Mathematics, released in 2002

PRINCIPIOS GENERALES

Naturaleza de las matemáticas

En términos muy generales, la matemática es el estudio de los números y el espacio. Más precisamente, es la búsqueda de patrones y relaciones. Esta búsqueda se lleva a cabo mediante conocimientos y destrezas que es necesario adquirir, puesto que llevan al desarrollo de conceptos y generalizaciones utilizadas en la resolución de problemas de diversa índole, con el fin de obtener una mejor comprensión del mundo que nos rodea y contribuir a la solución de necesidades específicas de las personas.

La matemática es una manera de pensar caracterizada por procesos tales como la exploración, el descubrimiento, la clasificación, la abstracción, la estimación, el cálculo, la predicción, la descripción, la deducción y la medición, entre otros.

Además, la matemática constituye un poderoso medio de comunicación que sirve para representar, interpretar, modelar, explicar y predecir.

La matemática es parte de nuestra cultura y ha sido una actividad humana desde los primeros tiempos. La matemática, por tanto, permite a los estudiantes apreciar mejor su legado cultural al suministrarles una amplia perspectiva de muchos de los logros culturales de la humanidad.

Naturaleza del aprendizaje de las matemáticas

El aprendizaje de las matemáticas, al igual que el de otras áreas, es más efectivo cuando el estudiante está motivado. Por ello resulta fundamental que las actividades de aprendizaje despierten su curiosidad y correspondan a la etapa de desarrollo en la que se encuentra. Además, es importante que esas actividades tengan suficiente relación con experiencias de su vida cotidiana. Para alimentar su motivación, el estudiante debe experimentar con frecuencia el éxito en una actividad matemática. El énfasis en dicho éxito desarrolla en los estudiantes una actitud positiva hacia la matemática y hacia ellos mismos.

Es importante reconocer que los estudiantes aprenden matemáticas interactuando con el entorno físico y social, lo cual lleva a la abstracción de las ideas matemáticas. Puesto que los estudiantes también aprenden investigando, se les debe dar oportunidades para descubrir y crear patrones, así como para explicar, describir y representar las relaciones presentes en esos patrones.

Propósitos generales del currículo de matemáticas

Cualquiera sea el currículo que adopte la institución dentro de su plan de estudios, así como los mecanismos que opte para implementarlo, la enseñanza de las matemáticas debe cumplir los propósitos generales siguientes:

- Generar en todos los estudiantes una actitud favorable hacia las matemáticas y estimular en ellos el interés por su estudio.
- Desarrollar en los estudiantes una sólida comprensión de los conceptos, procesos y estrategias básicas de la matemática e, igualmente, la capacidad de utilizar todo ello en la solución de problemas.
- Desarrollar en los estudiantes la habilidad para reconocer la presencia de las matemáticas en diversas situaciones de la vida real.
- Suministrar a los estudiantes el lenguaje apropiado que les permita comunicar de manera eficaz sus ideas y experiencias matemáticas.
- Estimular en los estudiantes el uso creativo de las matemáticas para expresar nuevas ideas y descubrimientos, así como para reconocer los elementos matemáticos presentes en otras actividades creativas.
- Retar a los estudiantes a lograr un nivel de excelencia que corresponda a su etapa de desarrollo.

Componentes del currículo de matemáticas

Tal como quedó planteado en el documento *Matemáticas - Lineamientos curriculares*, el currículo de matemáticas a lo largo de la educación básica y media se compone de los siguientes elementos:

■ ***Pensamiento numérico y sistemas numéricos***

Este componente del currículo procura que los estudiantes adquieran una comprensión sólida tanto de los números, las relaciones y operaciones que existen entre ellos, como de las diferentes maneras de representarlos.

■ ***Pensamiento espacial y sistemas geométricos***

El componente geométrico del currículo deberá permitir a los estudiantes examinar y analizar las propiedades de los espacios bidimensional y tridimensional, así como las formas y figuras geométricas que se hallan en ellos. De la misma manera, debe proveerles herramientas tales como el uso de transformaciones, traslaciones y simetrías para analizar situaciones matemáticas. Los estudiantes deberán desarrollar la capacidad de presentar argumentos matemáticos acerca de relaciones geométricas, además de utilizar la visualización, el razonamiento espacial y la modelación geométrica para resolver problemas.

■ ***Pensamiento métrico y sistemas de medidas***

El desarrollo de este componente del currículo debe dar como resultado la comprensión, por parte del estudiante, de los atributos mensurables de los objetos y del tiempo. Así mismo, debe procurar la comprensión de los diversos sistemas, unidades y procesos de la medición.

■ ***Pensamiento aleatorio y sistemas de datos***

El currículo de matemáticas debe garantizar que los estudiantes sean capaces de plantear situaciones susceptibles de ser analizadas mediante la recolección sistemática y organizada de datos. Los estudiantes, además, deben estar en capacidad de ordenar y presentar estos datos y, en grados posteriores, seleccionar y utilizar métodos estadísticos para analizarlos y desarrollar y evaluar inferencias y predicciones a partir de ellos.

De igual manera, los estudiantes desarrollarán un comprensión progresiva de los conceptos fundamentales de la probabilidad.

■ ***Pensamiento variacional y sistemas algebraicos y analíticos***

Este componente del currículo tiene en cuenta una de las aplicaciones más importantes de la matemática, cual es la formulación de modelos matemáticos para diversos fenómenos. Por ello, este currículo debe permitir que los estudiantes adquieran progresivamente una comprensión de patrones, relaciones y funciones, así como desarrollar su capacidad de representar y analizar situaciones y estructuras matemáticas mediante símbolos algebraicos y gráficas apropiadas. Así mismo, debe desarrollar en ellos la capacidad de analizar el cambio en varios contextos y de utilizar modelos matemáticos para entender y representar relaciones cuantitativas.

Procesos matemáticos

a. Planteamiento y resolución de problemas

La capacidad para plantear y resolver problemas debe ser una de las prioridades del currículo de matemáticas. Los planes de estudio deben garantizar que los estudiantes desarrollen herramientas y estrategias para resolver problemas de carácter matemático, bien sea en el campo mismo de las matemáticas o en otros ámbitos relacionados con ellas. También es importante desarrollar un espíritu reflexivo acerca del proceso que ocurre cuando se resuelve un problema o se toma una decisión.

b. Razonamiento matemático

El currículo de matemáticas de cualquier institución debe reconocer que el razonamiento, la argumentación y la demostración constituyen piezas fundamentales de la actividad matemática. Además de estimular estos procesos en los estudiantes, es necesario que se ejerciten en la formulación e investigación de conjeturas y que aprendan a evaluar argumentos y demostraciones matemáticas. Para ello deben conocer y ser capaces de identificar diversas formas de razonamiento y métodos de demostración.

c. Comunicación matemática

Mediante la comunicación de ideas, sean de índole matemática o no, los estudiantes consolidan su manera de pensar. Para ello, el currículo deberá incluir actividades que les permitan comunicar a los demás sus ideas matemáticas de forma coherente, clara y precisa.

Los estándares curriculares para matemáticas están formulados para cada grado, desde el grado primero hasta el grado undécimo, y contienen orientaciones generales para el grado obligatorio de preescolar.

ESTÁNDARES CURRICULARES PARA MATEMÁTICAS

Orientaciones para el grado obligatorio de preescolar

p

Los niños y las niñas llegan a la educación preescolar, no importa cuándo se inicia, con amplios conocimientos acerca de su entorno, del espacio y de los objetos que se hallan en él. No es, pues, la educación preescolar el inicio de su educación sino, por el contrario, la oportunidad para recoger todo lo que los pequeños conocen y saben hacer, para consolidarlo y ampliarlo. Al terminar el grado de transición se puede esperar que realicen de manera natural cada una de las siguientes acciones:

- Señalar entre dos grupos o colecciones de objetos semejantes, el que contiene más elementos, el que contiene menos, o establecer si en ambos hay la misma cantidad.
- Comparar objetos de acuerdo con su tamaño o peso.
- Agrupar objetos de acuerdo con diferentes atributos, tales como el color, la forma, su uso, etc.
- Ubicar en el tiempo eventos mediante frases como "antes de", "después de", "ayer", "hoy", "hace mucho", etc.
- Reconocer algunas figuras y sólidos geométricos con círculos, triángulos, cuadrados, esferas y cubos.
- Usar los números cardinales y ordinales para contar objetos y ordenar secuencias.
- Describir caminos y trayectorias.
- Representar gráficamente colecciones de objetos, además de nombrarlas, describirlas, contarlas y compararlas.

Estándares para el grado primero

Al terminar el primer grado, el programa de matemáticas que los estudiantes hayan completado de acuerdo con el currículo implementado en cada institución, deberá garantizar, como mínimo, los siguientes estándares para cada componente.

Pensamiento numérico y sistemas numéricos

- Clasifica conjuntos de acuerdo con el número de objetos que se encuentren en ellos.
- Representa conjuntos de hasta 999 objetos, utilizando materiales concretos.
- Lee, escribe y ordena números hasta 999.
- Reconoce los valores posicionales de los dígitos en un número de hasta tres dígitos.
- Comprende el significado de la adición, reuniendo dos conjuntos de objetos.
- Lleva a cabo la operación de la adición (con o sin reagrupación) de dos o más números de hasta tres dígitos.
- Comprende el significado de la sustracción, retirando uno o varios objetos de un conjunto de ellos.
- Lleva a cabo la operación de la sustracción (con o sin desagrupación), utilizando números de hasta tres dígitos.
- Comprende la relación que hay entre la adición y la sustracción.
- Modela, discute y resuelve problemas que involucran la adición y la sustracción, tanto por separado como simultáneamente.

Pensamiento espacial y sistemas geométricos

- Describe y argumenta matemáticamente acerca de figuras, formas y patrones que pueden ser vistos o visualizados.
- Clasifica figuras y formas de acuerdo con criterios matemáticos.
- Reconoce algunas figuras y formas geométricas tales como puntos, líneas rectas y curvas, ángulos, círculos, rectángulos, incluidos cuadrados, esferas y algunas de sus partes y características (lados, vértices, superficie, etc.).
- Se ubica en el espacio y da direcciones de manera precisa.
- Reconoce y aplica traslaciones a objetos y figuras y los representa mediante objetos.

grado 7

Pensamiento aleatorio y sistemas de datos

- Identifica el término "probabilidad" como un número entre cero y uno que indica qué tan probable es que un evento ocurra.
- Calcula la probabilidad de algunos eventos sencillos.
- Hace inferencias significativas a partir de la moda, la mediana y la media de una colección de datos.

Pensamiento variacional y sistemas algebraicos y analíticos

- Conoce las propiedades de una serie de razones iguales o proporciones.
- Encuentra un elemento desconocido en una proporción.
- Distingue entre magnitudes directamente proporcionales e inversamente proporcionales, y resuelve problemas relacionados con éstas.
- Representa en el plano cartesiano la relación entre dos variables.
- Conoce las reglas de tres simple y compuesta y las utiliza para resolver problemas pertinentes.

Procesos matemáticos

a. Planteamiento y resolución de problemas

- Formula problemas matemáticos en el contexto de otras disciplinas y los resuelve con los conocimientos y herramientas adquiridas.

b. Razonamiento matemático

- Reconoce una proposición condicional y sus componentes (hipótesis y conclusión), da ejemplos de ellas e identifica las condiciones necesarias y suficientes para que una proposición condicional sea verdadera o falsa.
- Argumenta en forma convincente a favor o en contra de alguna proposición matemática.

c. Comunicación matemática

- Utiliza lenguaje, notación y símbolos matemáticos para presentar, modelar y analizar alguna situación problemática.

Estándares para el grado octavo

Al terminar el octavo grado, el programa de matemáticas que los estudiantes hayan completado de acuerdo con el currículo implementado en cada institución, deberá garantizar, como mínimo, los siguientes estándares para cada componente.

Pensamiento numérico y sistemas numéricos

- Reconoce las propiedades de los números irracionales.
- Comprende el significado y las propiedades de la recta real.

Pensamiento espacial y sistemas geométricos

- Reconoce e identifica las propiedades de conos, prismas y pirámides.
- Reconoce ángulos adyacentes, complementarios, suplementarios y verticales, y comprende y aplica sus propiedades.
- Comprende el concepto de congruencia de dos o más figuras geométricas, así como las propiedades reflexiva, simétrica y transitiva de la congruencia.
- Conoce los teoremas acerca de líneas paralelas y líneas transversales a éstas.
- Conoce y demuestra las propiedades de un triángulo isósceles.
- Reconoce la simetría rotacional, sus componentes y propiedades.
- Identifica y clasifica los polígonos y sus partes, y deduce sus propiedades fundamentales.
- Conoce, demuestra y aplica las condiciones para que dos triángulos sean congruentes o similares.
- Reconoce un grafo (o red) como un conjunto de puntos (o vértices o nodos) algunos de los cuales (o todos) están unidos por líneas (o arcos).
- Modela situaciones de la vida real mediante grafos (relaciones de amistad, parentescos, rutas de transporte, etc.), y deduce propiedades del modelo.
- Comprende el concepto de "grafo atravesable", y conoce y demuestra informalmente el teorema de Euler para determinar si un grafo es atravesable o no.

grado 8

Pensamiento métrico y sistemas de medidas

- Deduce y aplica las fórmulas para el área de superficie y el volumen de conos, prismas y pirámides.
- Deduce y aplica la fórmula para la distancia entre dos puntos del plano cartesiano.

Pensamiento aleatorio y sistemas de datos

- Encuentra el mínimo, máximo, rango y rango intercuartil de una colección de datos y deduce inferencias significativas de esta información.
- Identifica el espacio muestral de un experimento sencillo y calcula la probabilidad de eventos sencillos.

Pensamiento variacional y sistemas algebraicos y analíticos

- Reconoce una expresión algebraica, las variables y términos que la componen.
- Distingue entre las diferentes clases de expresiones algebraicas (racionales, irracionales, enteras, fraccionarias, etc.).
- Dados valores para las variables de una expresión algebraica, halla el valor de ésta.
- Reconoce un monomio y el grado de éste.
- Halla sumas, diferencias, productos, cocientes y potencias de un monomio.
- Reconoce un polinomio y sus partes.
- Halla la suma y diferencia de dos polinomios, y conoce y comprende las propiedades de la adición y la sustracción de polinomios.
- Halla el producto de dos polinomios y recuerda con facilidad los productos notables.
- Construye y utiliza el triángulo de Pascal para calcular las potencias de un binomio cualquiera.
- Halla el cociente de dos polinomios y recuerda y aplica los cocientes notables.
- Conoce, comprueba y aplica el teorema del residuo.
- Desarrolla técnicas para factorizar polinomios, en particular, la diferencia de dos cuadrados, la suma y diferencia de potencias impares, los trinomios cuadrados perfectos y otros trinomios factorizables.

(continúa)

Estándares para el grado octavo

grado 8

(continuación)

- Reconoce una fracción algebraica como el cociente indicado de dos polinomios.
- Suma, resta, multiplica, divide y simplifica fracciones algebraicas.
- Distingue entre una ecuación y una identidad algebraica.
- Clasifica las ecuaciones de acuerdo con su grado y número de variables.
- Halla la solución a cualquier ecuación de primer grado en una variable.
- Reconoce una inecuación de primer grado en una variable, halla su solución y la representa en la recta real.
- Encuentra dos o más soluciones de una ecuación de primer grado en dos variables y las utiliza para representar la ecuación en el plano cartesiano mediante un línea recta.
- Encuentra la solución de una inecuación lineal y la representa en la recta real.
- Utiliza una calculadora científica, de manera creativa, para evaluar expresiones algebraicas y fórmulas, resuelve ecuaciones e inecuaciones y, en general, para facilitar el trabajo computacional.

Procesos matemáticos

a. Planteamiento y resolución de problemas

- Traduce problemas del lenguaje común al algebraico y los resuelve satisfactoriamente.
- Idea un plan para resolver un problema y lo lleva a cabo con éxito.

b. Razonamiento matemático

- Presenta demostraciones directas o indirectas de proposiciones matemáticas significativas.

c. Comunicación matemática

- Expone ante una audiencia, de manera convincente y completa, argumentos matemáticos.

Estándares para el grado noveno

grado 9

Al terminar el noveno grado, el programa de matemáticas que los estudiantes hayan completado de acuerdo con el currículo implementado en cada institución, deberá garantizar, como mínimo, los siguientes estándares para cada componente.

Pensamiento numérico y sistemas numéricos

- Reconoce progresiones aritméticas y sus propiedades.
- Deduce fórmulas para un término cualquiera, así como la suma de los términos de una progresión aritmética.
- Reconoce progresiones geométricas y sus propiedades.
- Deduce fórmulas para un término cualquiera, así como la suma de los términos de una progresión geométrica.
- Identifica fenómenos en la física, la ingeniería, la economía u otras ciencias que pueden modelarse mediante progresiones aritméticas y geométricas.

Pensamiento espacial y sistemas geométricos

- Comprende el concepto de escala.
- Interpreta y construye dibujos a escala.
- Reconoce triángulos similares y sus propiedades.
- Deduce y aplica las propiedades especiales de un triángulo con ángulos de 30°, 60° y 90°.
- Conoce y calcula las razones trigonométricas seno, coseno y tangente para los ángulos agudos de un triángulo rectángulo y las utiliza para resolver triángulos.
- Realiza proyecciones planas de algunos sólidos.

Pensamiento métrico y sistemas de medidas

- Conoce y aplica las fórmulas para el área de superficie y el volumen de una esfera.

(continúa)

Estándares para el grado noveno

(continuación)

Pensamiento aleatorio y sistemas de datos

- Interpreta diagramas, encuestas, gráficas y tablas que recojan datos de asuntos cotidianos y hace inferencias y predicciones a partir de éstos.
- Comprende y aplica las medidas de tendencia central en el análisis de datos de diversa índole.

Pensamiento variacional y sistemas algebraicos y analíticos

- Dados dos conjuntos, *A* y *B*, reconoce como una relación entre *A* y *B* a cualquier subconjunto del producto cartesiano de *A* y *B*.
- Reconoce el dominio y rango de una relación.
- Da ejemplos de relaciones entre conjuntos de números y objetos.
- Reconoce cuando una relación entre dos conjuntos es una función.
- Proporciona ejemplos de funciones entre conjuntos de números reales y, si es el caso, las expresa mediante una fórmula.
- Reconoce una función lineal, construye su gráfica en el plano cartesiano y halla sus principales atributos (pendiente, intersecciones con los ejes, etc.).
- Dada una recta en el plano cartesiano, halla su ecuación.
- Dados dos puntos en el plano cartesiano, encuentra la ecuación de la recta que pasa por ellos.
- Dada la pendiente de una recta y un punto que pasa por ella, deduce la ecuación de la recta que pasa por ella.
- Reconoce una función cuadrática, construye su gráfica en el plano cartesiano, describe sus principales características e identifica sus componentes principales.
- Deduce los criterios para determinar si una ecuación cuadrática tiene o no soluciones reales y, en caso afirmativo, los métodos para hallarla(s).
- Reconoce los números complejos como raíces no reales de una función cuadrática, y desarrolla y comprende sus propiedades.
- Identifica fenómenos en la física, la ingeniería, la economía u otras ciencias que pueden modelarse mediante funciones y ecuaciones cuadráticas.

Appendix 3.1

Questionnaire 1

Thank you for your time and participation!

Please note:

- *There are no right or wrong answers to the questions posed in this questionnaire.*
- *Since every teacher is different, it is important that your answers reflect your personal views about what is being asked. Your individual answers are important, and your cooperation is much appreciated.*
- *The confidentiality of individual answers is guaranteed, and they will only be used if there is, beforehand, an agreement between the researcher and the respondent for the use of his/her personal answer(s).*
- *Please do not leave any questions unanswered.*

Section A

For each item in this section, some of the different ways in which the statement may be completed are given. Rank these statements accordingly in the accompanying boxes, where '1' indicates your first choice, '2' your second choice, '3' your third choice, etc. Please do not leave any empty boxes. Note that the same ranking value can be given to more than one statement.

Some spaces have been left at the end of each question in case you have other personal views or approaches. If this is the case, please write them with the corresponding ranking in the spaces provided.

A1 In relation to the inclusion of algebra in the School curriculum for the basic cycle of education, I consider that:

Ranking

- the teaching of algebra is important because knowledge of algebra is key for pupils' access to higher level of school mathematics (e.g. trigonometry and calculus). ☐
- the teaching of algebra is important because algebra represents an important tool for solving real world problems, needed by every individual. ☐
- the teaching of algebra is important, but not for all pupils because knowledge of algebra is not needed by every citizen. ☐
- the teaching of algebra is important because the use of technology poses increasing demands of algebraic knowledge for every individual. ☐
- the study of algebra is important because it provides individuals with equal access to career opportunities. ☐
- the study of algebra is important because it provides individuals with opportunities to develop the critical thinking needed by every citizen. ☐
- the study of algebra is important because algebra provides the intellectual challenge pupils like of mathematics. ☐

- .... ☐
....
- .... ☐
....

A2 My **preferred** teaching style in Grade 8 -algebra involves:

Ranking

- Giving clear explanations of definitions and procedures to follow in different exercises and problems of application, in the topics studied.
- Organising problem-based activities for the pupils to work in small groups, where they can present their ideas to the whole class for discussion.
- Testing pupils at the end of each activity or topic, in order to have sufficient marks for assessment in each Attainment Target.
- Giving pupils lots of exercises for algorithm application as homework.
- Designing activities that provide space for pupils' self-paced learning.
- Providing opportunities for pupils to develop their communication skills so that they can express their mathematical ideas with confidence.
- Designing classroom work that promotes connections between different mathematical topics studied.
- ....
- ....

A3. My **actual** teaching style in Grade 8 -algebra involves:

Ranking

- Giving clear explanations of definitions and procedures to follow in different exercises and problems of application, in the topics studied.
- Organising problem-based activities for the pupils to work in small groups, where they can present their ideas to the whole class for discussion.
- Testing pupils at the end of each activity or topic, in order to have sufficient marks for assessment in each Attainment Target.
- Giving pupils lots of exercises for algorithm application as homework.
- Designing activities that provide space for pupils' self-paced learning.
- Providing opportunities for pupils to develop their communication skills so that they can express their mathematical ideas with confidence.
- Designing classroom work that promotes connections between different mathematical topics studied.
- ....
- ....

If the choices you made for question A2 are different from those of question A3, please express the reasons by which they are different. (Please continue at the back of this page).

A4. The types of classroom activity that I would **prefer** to see in my Grade 8-classroom are:

Ranking

- Pupils developing efficiency in applying algorithms and formulae. ☐
- Pupils working at the board, especially when they have difficulties in applying algorithms. ☐
- Pupils engaged in the creation of algorithms and formulae. ☐
- Pupils discussing ideas and working systematically. ☐
- Pupils solving closed word problems. ☐
- Pupils using calculators to assist their learning and to use their working time more efficiently. ☐
- Pupils posing open problems and working on developing ways to solve them. ☐
- .... ☐
- .... ☐

A5. The types of classroom activity that **actually** take place in my classroom are:

Ranking

- Pupils developing efficiency in applying algorithms and formulae. ☐
- Pupils working at the board, especially when they have difficulties in applying algorithms. ☐
- Pupils engaged in the creation of algorithms and formulae. ☐
- Pupils discussing ideas and working systematically. ☐
- Pupils solving closed word problems. ☐
- Pupils using calculators to assist their learning and to use their working time more efficiently. ☐
- Pupils posing open problems and working on developing ways to solve them. ☐
- .... ☐
- .... ☐

If the choices you made for question A4 are different from those of question A5, please express the reasons by which they are different.

A6. My **preferred** assessment forms in Grade 8 -algebra are:

Ranking

- Oral questions within the lesson ☐
- Pupils' completion of given homework ☐
- Pupils' folders and assessment records, showing evidence of several aspects of the process followed throughout a term or a set of terms ☐
- Pupils' own reports about their progress and their difficulties. ☐
- Pupils' individual marks obtained from quizzes during the teaching of a specific topic ☐
- Written tests at the end of each topic for learning diagnostic purposes ☐
- .... ☐
- .... ☐

A7 My **actual** assessment forms in Grade 8 -algebra are:

Ranking

- Oral questions within the lesson. ☐
- Pupils' completion of given homework. ☐
- Pupils' folders and assessment records, showing evidence of several aspects of the process followed throughout a term or a set of terms. ☐
- Pupils' own reports about their progress and their difficulties. ☐
- Pupils' individual marks obtained from quizzes during the teaching of a specific topic. ☐
- Written tests at the end of each topic for learning diagnostic purposes. ☐
- .... ☐
- .... ☐

If the choices you made for question A6 are different from those of question A7, please express the reasons by which they are different.

....

Section B
Please tick in the appropriate box against each item below to indicate your view.

		Strongly Agree 1	Agree 2	Neutral 3	Disagree 4	Strongly Disagree 5
B1	Algebra should be taught as a separate area in mathematics (i.e., from general arithmetic and geometry content).	☐	☐	☐	☐	☐
B2	For the planning and teaching of Grade 8 algebra, I normally follow the sequencing given by the pupils' text book.	☐	☐	☐	☐	☐
B3	Pupils who aren't getting the right answers need to practice on more problems.	☐	☐	☐	☐	☐
B4	Curriculum resources (e.g., curriculum guidelines, textbooks and other teaching materials, etc.) portray values.	☐	☐	☐	☐	☐
B5	Mathematics involves mostly facts and procedures that have to be learned.	☐	☐	☐	☐	☐
B6	I enjoy teaching algebra.	☐	☐	☐	☐	☐
B7	The learning outcomes in my actual mathematics curriculum reflect cognitive learning concerns only.	☐	☐	☐	☐	☐
B8	Mathematical ability is something that remains relatively fixed throughout a person's life.	☐	☐	☐	☐	☐
B9	If pupils fail to learn the algebra content taught during a specific term, they have to work by themselves and repeat the corresponding test.	☐	☐	☐	☐	☐
B10	In mathematics you can be creative and construct your own mathematical ideas.	☐	☐	☐	☐	☐

		Strongly Agree 1	Agree 2	Neutral 3	Disagree 4	Strongly Disagree 5
B11	When pupils are having difficulties with the learning of algebra, I have to revise the teaching-learning situation and organise alternative classroom work that could suit the pupils better.	☐	☐	☐	☐	☐
B12	Algebraic thinking should be promoted in the teaching of primary school mathematics.	☐	☐	☐	☐	☐
B13	More important than getting the right answers is pupils' understanding of the main concepts inherent in a problem.	☐	☐	☐	☐	☐
B14	All my pupils would be good at mathematics if they worked hard at it.	☐	☐	☐	☐	☐
B15	When I teach algebra I often feel unmotivated by the fact that many pupils don't understand the basic concepts and procedures.	☐	☐	☐	☐	☐
B16	The importance of mathematics is as a value-free subject.	☐	☐	☐	☐	☐
B17	Pupils will enjoy and work hard in mathematics if they find classroom work meaningful and challenging, whether or not their work is graded	☐	☐	☐	☐	☐
B18	I have not found a textbook that is adequate for my pupils' needs and for what I want my pupils to learn in algebra. So I often produce some classroom materials.	☐	☐	☐	☐	☐
B19	My teaching practice in mathematics is concerned with broad educational goals, and not just with mathematics content goals	☐	☐	☐	☐	☐
B20	Giving rewards is a good strategy for getting pupils to complete mathematics assignments.	☐	☐	☐	☐	☐

Feel free to comment (at the back of this page) on any reasons for your choices:

Section C

Please answer the questions posed for each of the following imagined classroom situations.

C1. During one of the lessons in the first week of the academic year, one of the pupils raises his/her hand and asks you why pupils have to study algebra in school.

a) What would you answer to that pupil?

b) Please, explain whether and how your answer is connected to contextual factors. Describe possible contextual factors.

C2. You are starting a lesson and you have just asked the pupils to work individually on a specific task. Two pupils come up to you and ask if you would allow them to work together.

a) What would you answer to those pupils?

b) Please, explain whether and how your answer is connected to contextual factors. Describe possible contextual factors.

C3. Towards the end of the academic year, a teacher asks his Grade eight pupils to find the area of a rectangle whose sides are 5 and $2 + e$. Many pupils, in a group of 29, answered the following:

$$A = 5\,(2 + e) = 10 + 5e = 15e$$
$$A = 15e$$

a) What do you think of this answer?.

b) How would you respond to those pupils?

C4. Imagine you ask a grade-nine pupils to write an equation, using N for the number of nurses and D for the number of doctors, for the statement: *In Central Hospital there are five times as many nurses as doctors.*

Several pupils write as an answer: $5N = D$

a) What do you think of this answer?

b) How would you respond to those pupils?

C5. In Ms Rodríguez' class, during the last term of Grade eight:
After a whole class discussion took place in order to set up an equation to represent a word problem, the equation $\mathbf{5x + 2 = 24}$ was written on the board. In finding the value for x, one of the pupils, Ricardo, wrote: $x = 26/5$, and asserted that he was correct.

a) What do you think of Ricardo's answer?

b) If Ricardo were one of the pupils in your class, how would you respond to him or to those pupils who answered in the same way?

Section D

The information you provide in this section will provide us with valuable background information to your professional work. Thank you for responding to the items.

D1. Your name: ______________________________

D2. Gender: ☐ Male ☐ Female

D3. Teaching qualifications (please include all general academic qualifications):

Year	Academic award	Educational Institution

D4. Teaching experience in mathematics:

Period	Year levels	Other academic responsibilities
current academic year		

D5. Total number of years teaching: ______, of which total number of years teaching mathematics: in the primary level______, in secondary level ______

D6. Name of present school: ______________________________

D7. School type: ☐ State ☐ Private

D8. Would you be interested to receive more information regarding participating in the next stage of the Project, which involves interviews, lesson observations and discussions on different approaches to the introduction of elementary algebra?

☐ Yes, keep me in touch. Your contact details:

Address: ______________________________

Ph: __________ ______________ Fax: ______________

Email: ______________________________

☐ No, thank you.

Thank you again for your time and participation

Questionnaire 2

Thank you for participating in this study

Once again, please note:

- *There are no right or wrong answers to the questions posed in this questionnaire.*
- *Since every teacher is different, it is important that your answers reflect your personal views about what is being asked. Your individual answers are important, and your cooperation is much appreciated.*
- *The confidentiality of individual answers is guaranteed, and they will only be used if there is, beforehand, an agreement between the researcher and the respondent for the use of his/her personal answer(s).*
- *Please do not leave any questions unanswered.*

General instructions
In this questionnaire you will find descriptions of the ways two teachers initiate classroom work when teaching what they call first algebra concepts in Grade eight. The descriptions have been produced using data from classroom observation and from audio taped interviews with the teachers, about the teaching approach followed by each of them. The data collected at the interviews, which were considered more relevant, are presented by using dot points which are preceded by the corresponding interview-question. The teachers' names have been replaced by the terms *Teacher A* and *Teacher B*.

Please answer the questions included as **Section A** and **Section B**. Section A is divided in two parts: "Section A – Part 1", to be answered after reading the information corresponding to Teacher A, and "Section A – Parte 2", to be answered after reading the information corresponding to Teacher B. The last part of the questionnaire (Section B) can be found on page 14.

Teacher A

1. Data from the observation

The planned lesson to be observed was about 'addition of polynomials'. However, that lesson did not take place because the teacher had to give a test to the pupils, that day, as she was asked to hand in the term grades before the initially planed date. Pupils answered the following questions, which were written on the board:

1. What is an algebraic expression? Give examples.

2. Find the numerical values for the expressions:
$$3abc;\ 3a+2b$$
$$5a^2c;\ 2a+b+c$$
$$a+b+c+1, \qquad \text{when: } a=3,\ b=5,\ c=1.$$

3. Classify the expressions given in question 2, according to the number of terms in them.

4. Simplify the following expressions: $5ax+5+x$; a^2-b+2a^2; $x+2y+5x$.

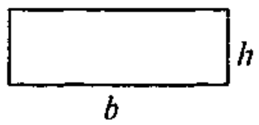
Some pupils finished their test fairly soon (about 15 minutes later) but other pupils spent more that 30 minutes doing the test.

The homework
The teacher said to the pupils that there was not a specific homework set for the next lesson, but that they should make sure that they had finished doing the exercises of this topic which were in the text book.

2. *Data from the interview*

2.1. Could you describe how the work for the introduction of algebra started this year, in this group?

- I started with this Grade 8-group by checking what they knew about rational numbers. It is important for me to do that because if they don't know how to operate with fractions, for example, they may have more difficulties in algebra. So I did some work on rational numbers before starting algebra. I started algebra by presenting them with situations like the perimeter of a rectangle, like this (showing from the textbook what appears in the box).

> **Algebraic expressions**
>
> 
>
>
> The perimeter of this rectangle can be expressed as the double of the base plus the double of the height. That is: 2b + 2h.
>
> $\mathbf{2}b + \mathbf{2}h$ is an algebraic expression
>
> $\vdots$

- I explained to the pupils that instead of writing $2 \times b + 2 \times h$, in algebra we write the letter next to the number (i.e., $2b$) because writing the multiplication sign can be problematic when there are letters like x because . . .

- I also explained that this expression is called *algebraic* because in contains letters that represent numbers, and that in algebra, letters are *variables* because the letters can represent several values. For example, in this case, b represents what the base measures and h represents what the height measures.

- After that, we saw different algebraic expressions like: 3a, 3ab, 2a +3b, etc. in order to see different types of algebraic expressions. e.g., expressions with one term, with two terms, etc.

- Once the pupils knew what a expression is, what a term is, what the coefficient is, and the different types of expressions, classroom work moved on to finding numerical value of an algebraic expression. After they had practiced this type of exercises then we started with simplification of algebraic expressions. For today we had 'addition of algebraic expressions', but I couldn't attar today....

2.2. What do you think they will they be doing after addition of algebraic expressions?

- Once they have learnt how to add and simplify polynomials, then we will continue with the rest of the operations: substraction, multiplication, etc.. We are following this textbook for the program for Grade 8.

The textbook was organised into Units with the following sequence of headings: Rational number,. Algebraic expressions, Operations with algebraic expressions, Factorisation, Equations, … Topics in the text were presented by giving definitions of terms and concepts in a very formal way. The definitions were followed by some examples of application for the topics, and sets of exercises for practice. The answers of the exercises were found at the end of the textbook.

2.3. ***Do you think that there are specific steps that are followed during your lessons that could characterise your teaching style? In other words, is there a pattern of steps that you follow when you teach in Grade 8?***

- I normally start by explaining the topic for all the class. I explain one or two exercises of application, where the pupils ask questions about particular things that they don't understand. Obviously, I too ask them questions in order to see if they understand what I am explaining. Working in pairs or, sometimes, in groups of three or more pupils, they then carry on doing more exercises or problems. Sometimes they work individually.

2.4. ***Is it possible to identify a basic goal of your teaching for this academic year, in this Grade 8-group?***

- My ultimate goal is that the pupils see the use of algebra in the solution of problems, and be able to use it to solve the problems.
-

2.5. ***How does the assessment of pupils' work take place? Could you give me a general idea?***

- They have several quizzes and tests at different moments throughout each academic term. They also have an end-of-term test, and I also mark other aspects like homework or classroom participation. So the ones that participate actively in the classroom have and additional mark because of that.

Section A Part 1

Please complete the first statement by ticking one of the 5 alternatives given, and by writing in the space provided the explanation asked for your specific choice.

1. For engaging all pupils in problem-solving work, I consider *Teacher A's* approach to be

☐ Excellent ☐ Good ☐ Fair ☐ Poor ☐ Very poor

because ....
....
....

2 For giving clear explanations of definitions and procedures to follow in different exercises and problems of application, I consider *Teacher A's* approach to be

☐ Excellent ☐ Good ☐ Fair ☐ Poor ☐ Very poor

because ....
....
....

3 For providing opportunities for pupils to develop their communications skills, I consider *Teacher A's* approach to be

☐ Excellent ☐ Good ☐ Fair ☐ Poor ☐ Very poor

because ....
....
....

4 For focusing classroom work in the development of pupils' abilities to do the lists of exercises and closed tasks posed in the text book, I consider *Teacher A's* approach to be

☐ Excellent ☐ Good ☐ Fair ☐ Poor ☐ Very poor

because ....
....
....

5 For involving pupils in the creation of their own mathematical ideas, I consider *Teacher A's* approach to be

☐ Excellent ☐ Good ☐ Fair ☐ Poor ☐ Very poor

because ....

....

....

6 For promoting pupils' understanding of the mathematical concepts inherent in a problem, I consider *Teacher A's* approach to be

☐ Excellent ☐ Good ☐ Fair ☐ Poor ☐ Very poor

because ....

....

....

7 For encouraging pupils to construct the concept of variable, I consider *Teacher A's* approach to be

☐ Excellent ☐ Good ☐ Fair ☐ Poor ☐ Very poor

because ....

....

....

Teacher B

1. Data from the observation

The teacher's intention for this lesson was "to continue with the preparatory work for the introduction of pupils to special software designed to promote the formation of the concept of variable".

The class was organised in groups of three pupils. The teacher wrote the following problem-situation on the board, for pupils to work on and to discus.

> **Martha works on Sundays selling ice cream in Plaza de Bolívar. She gets paid $2000 as a base salary per day, plus $100 for each ice cream she sells.**
>
> Question 1: How much can Martha earn in a Sunday?
> Question 2: How many ice creams must she sell to earn at least $6000 in a Sunday?

These are some of the instructions she gave to the pupils: "More than your answers to the specific questions, I am interested to know what you think of what Question 2 is asking once you have done the work to answer Question 1. Do not forget to write all the calculations you do, and to think about the names you need to give to numbers and to calculations".

Pupils in most groups engaged in discussion, and the three pupils who were next to me started doing calculations in order to find out how many ice creams Martha needed to sell if she wanted $6000 as her salary. The teacher went to the different groups observing what they were doing and, on occasions, she discussed things with some of the pupils.

After the teacher had been to all the groups, she explained to me that she was interested in seeing how each group approached this situation, whether pupils would decide to produce several examples of earnings to try to answer Question 1 (see box above), and produce tables of values or give a general explanation on how to calculate Martha's salary. She also explained that she hadn't said anything else to the pupils because they had previously worked in situations similar to this but where "there was only a Question 1-type question.

About 20 minutes later, the pupils seemed to have completed the tasks, and one of the groups was asked to explain on the board what he had done. Paula, one of the pupils in the group that went to the board wrote:

If Martha sells 10 ice creams, she earns $3000 because:

$10 \times \$100$ gives $1000

$2000 + $1000 gives $3000

and explained: "In this way we found Martha's salary for different numbers of ice creams. We did a table with the calculations for other numbers of ice creams sold". She copied the table on the board and carried on explaining:

If Martha sells	*10*	ice creams, she earns	*3000*	pesos
	20		*4000*	
	30		*5000*	
	40		*6000*	
	50		*7000*	

"So we thought that in answering Question 1, we also answered Question 2. In Question 1, you start with the number of ice creams sold, and in Question 2 you have to think: if she earns 6000 how many ice creams did she sell? But we didn't have to do that, separately, because we had found that case already".

A pupil from a different group said: "we answered thinking of the two questions at the same time". The teacher then asked the pupils in Paula's group:

> What would you have done if the number 6000 would have not appeared in the second column of you table?

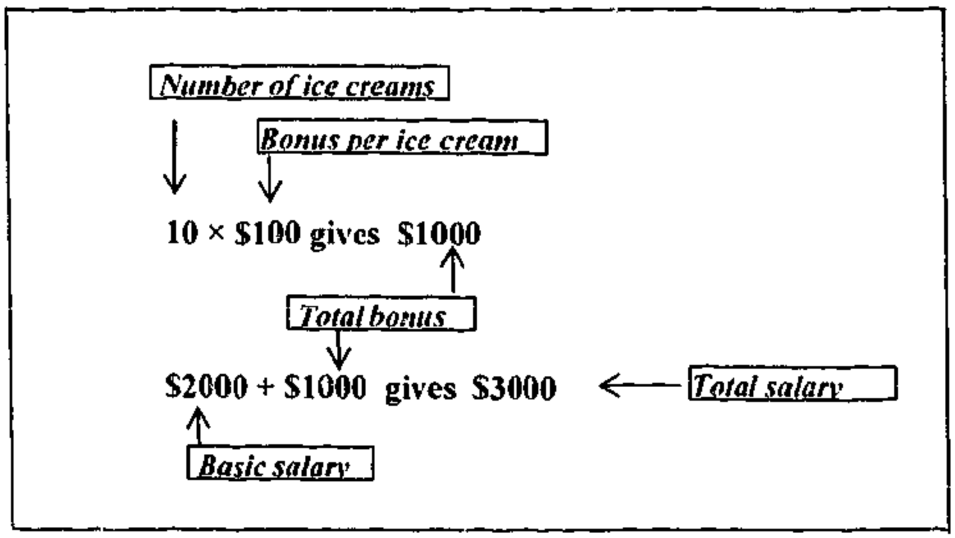
One of the pupils said:

> We would have had to find a number that multiplied by 100 gave 4000, because 4000 plus 2000 gives 6000.

The teacher then asked the class: "What names could we give to each column? The names need to describe what there is in each of them". The following is what was written as the columns' names, after several pupils had given their ideas, and had agreed with the teacher on those names.

Number of ice creams sold	Martha's salary (*Number of pesos*)
10	3000
20	4000
30	5000
40	6000
50	7000

After this table was written, the teacher asked all pupils to write the names of the calculations they had initially done, for example, in the calculation presented by Paula:



At this point the teacher asked the class to organise themselves to work on a computer and to enter the data of Martha's situation into specific boxes of a window that the computers were showing. The teacher also asked the pupils to set up their own questions in relation to the case of Martha (selling ice creams), and to work on those questions. Further explanations about this work was provided by the teacher at interview.

2. Data from the interview

2.1. Could you describe how the work for the introduction of algebra started, this year, in this group?

- During the first eight lessons, they were presented with simple functional situations like this one (i. e., the one of Martha). I gave them questions of this type (i.e., Question 1 on Martha's problem). As the response to that question has to be 'It depends on how many ice creams Martha sells', they can start thinking of the variable aspect of the problem situation, and they have to set up the general functional relation.

- I normally asked them questions like, let's consider this case of Martha: 'If Martha sells 10 ice creams, how much will she earn?' I did this in order to check their understanding of the situation, and I insisted that they had to write all the calculations they performed to answer that question. I intentionally minimise the complexity of the calculations so that they focus their attention on the relationship between the quantities.

- From the 4th lesson onwards I asked the pupils to make up their own questions to go with the problem-situations given. After they had done several calculations I encouraged them to organise their findings into a table.

- Now, once they are clear about the label they need to give to the numbers appearing in the calculations for the table pairs, then the pupils are directed into producing a general description of the calculations... How? For example in this specific case, where 100 and 2000 are fixed values, a general description of the calculations may be like this:

Number of ice creams × \$100 **gives** total bonus.
\$2000 + total bonus **gives** total salary.

Once they have established a way of expressing the procedures I introduced the computer *software*. Let's see the first window:

Request values

Carry out these calculations:

Show values of:

I explain that the data should be entered according to the procedure they had already established ;so 'the form' must be filled in as in the following, for this case:

Request values for:

number of ice creams

Carry out these calculations:

number of ice creams × 100 <u>gives</u> total bonus
2000 + total bonus <u>gives</u> total salary

Show values of:

total salary

The teacher provided me with further explanations about the way the way she uses this software in her teaching:

- During the first contacts with the computer software I do not insist on explaining 'the form' because for most pupils, there are many things to learn at the same time: moving the mouse. the functioning of the word processor by which the pupils can enter their programs. etc. An approximate explanation is therefore sufficient at first.

- The program first asks for input variables then goes through the general calculations in the second section. The following is an example during calculations:

Request values for:
number of ice creams 10
Carry out these calculations:
number of ice creams × 100 gives total bonus 10 × 100 gives 1000
2000 + total bonus gives total salary 2000 + 1000

- The pupils can ask for calculations in the table mod, where intermediary calculations are not shown, as can be seen in the Inputs-Outputs table: and.

INPUTS	OUTPUTS
Number of ice creams	Salary
10	3000
20	4000
30	5000
40	6000
50	7000

- Later they can call upon graphical methods as well... The program also accepts symbolic expressions, such as $ax^2 + bx + c$ gives y.

2.2. *What sort of work do you think they will be doing after they have worked in situations like the case of Martha?*

- I can see two possibilities here: one is to try to move to the use of formal symbols. As you see, we are using natural language for the names of the variables. The other possibility is to leave the transition to the use of formal language for later, and continue working with situations where you get a different type of equations like $ax + x = c$, to give an example. Later we can think of equations that have got the letter on both sides.
- But also, before I can think of which path to follow, from the two I just have mentioned, we could spend some time working with graphs. The decision really depends on how the pupils are responding. It depends of what is happening in the classroom.

2.3. Do you think that there are specific steps that are followed during your lessons that could characterise your teaching style? Put another way, is there a specific pattern of steps that you follow when you teach in Grade 8?

- I normally design problem-situations or activities that I think address specific mathematical content. In terms of the steps that are followed in the lessons, as I told you before, it depends on what is happening in the classroom. Sometimes we start by discussing questions that they have from their previous work. Some times, like today, I consider it important that the whole group focus on the same question, and from there the work develops sometimes into something that I had not anticipated. Sometimes half of the class works on one particular task and the other half works on another task, and then they share and discuss, with the whole class, what they have done. Sometimes it is necessary to work on the computers, and sometimes it is not. Sometimes, while they are working on a particular problem, I call individual pupils to discuss specific points of their work, or of their self-assessment records...

2.4. Is it possible to identify a basic goal of your teaching for this group of students, for this academic year?

- My basic aim is to be able to organise classroom work which help them assign meaning to the algebra language.

2.5. How does the assessment of pupils' work take place? Could you provide me with a general idea?

- As my goal is to engage them in the creation of their mathematical ideas, the assessment of their work is geared towards the identification of their difficulties, and obviously of their progress. Focussing on these aspects helps me, first of all, in seeing whether I need to modify or extend given tasks and, secondly, in identifying activities and tasks that are useful in the teaching of first algebra notions.

- Both the teacher and the pupils are responsible for the assessment records. The pupils keep a kind of journal where they write about what they have done during the lessons or about the work that they think need to do. They are encouraged to write about what progress that have made and why they think they have, or have not, made progress. When the time to write reports comes, if there are discrepancies in their reports and mines, it is necessary to dedicate some time to examine and clarify whatever we think is necessary to clarify.

Section A Part 2

In this part, each question is composed of two statements to be completed. Please complete the first given expression by ticking one of the 5 alternatives, and by writing in the space provided the explanation asked for your specific choice. To complete the second expression, mark a point in the scale given to indicate your view.

A1

- For engaging all pupils in problem-solving work, I consider *Teacher B's* approach to be

☐ Excellent ☐ Good ☐ Fair ☐ Poor ☐ Very poor

because ....
....
....

- Engaging all pupils in problem-solving is an aspect which, for my teaching of first algebra concepts, is:

Very important — Not very important

1 — 2 — 3 — 4 — 5

A2

- For giving clear explanations of definitions and procedures to follow in different exercises and problems of application, I consider *Teacher B's* approach to be

☐ Excellent ☐ Good ☐ Fair ☐ Poor ☐ Very poor

because ....
....
....

- Giving clear explanations of definitions and procedures to follow in different exercises and problems of application is an aspect which, for my teaching of first algebra concepts, is:

Very important — Not very important

1 — 2 — 3 — 4 — 5

A3

- For providing opportunities for pupils to develop their communications skills, I consider *Teacher B's* approach to be

☐ Excellent ☐ Good ☐ Fair ☐ Poor ☐ Very poor

because

- Providing opportunities for pupils to develop their communications skills is an aspect which, for my teaching of first algebra concepts, is:

Very important — Not very important

1 2 3 4 5

A4

- For focusing classroom work in the development of pupils' abilities to do the list of exercises and closed tasks posed in the text book, I consider *Teacher B's* approach to be

☐ Excellent ☐ Good ☐ Fair ☐ Poor ☐ Very poor

because

- Focusing classroom work in the development of pupils' abilities to do the lists of exercises and closed tasks posed in the text book is an aspect which, for my teaching of first algebra concepts, is:

Very important — Not very important

1 2 3 4 5

A5

For involving pupils in the creation of their own mathematical ideas, I consider *Teacher B's* approach to be

☐ Excellent ☐ Good ☐ Fair ☐ Poor ☐ Very poor

because

- Involving pupils in the creation of their own mathematical ideas is an aspect which, for my teaching of first algebra concepts which, is:

Very important				Not very important
1	2	3	4	5

A6

- For promoting pupils' understanding of the mathematical concepts inherent in a problem, I consider *Teacher B's* approach to be

☐ Excellent ☐ Good ☐ Fair ☐ Poor ☐ Very poor

because ....
....
....

- Promoting pupils' understanding of the mathematical concepts inherent in a problem is an aspect which, for my teaching of algebra which is:

Very important				Not very important
1	2	3	4	5

A7

- For encouraging pupils to construct the concept of variable, I consider *Teacher B's* approach to be

☐ Excellent ☐ Good ☐ Fair ☐ Poor ☐ Very poor

because ....
....
....

- Encouraging pupils to construct the concept of variable is an aspect which, for my teaching of algebra concepts, is:

Very important				Not very important
1	2	3	4	5

Section B

Please answer the following questions related to the approaches followed by *Teacher A* and *Teacher B* in the teaching of first algebra concepts.

B1

The mathematics teacher of grades 8 and 9 of School "N" states that he/she emphasises a problem-solving approach in his teaching of beginning algebra. Would you think that any of the approaches described in this questionnaire (i.e., *Teacher A's* and *Teacher B's)* emphasises a problem-solving approach? Please explain your answer fully.

B2

Is (are) there any aspect(s) in any of the approaches described which is (are) part of your teaching style of first algebra concepts?

Yes ☐ No ☐

B2.1. If your answer was **Yes**, please describe the specific aspect(s) in the following.

B2.2. If your answer was **No**, please indicate if there is (are) any aspect(s) in any of the approaches described, which you would like to incorporate in your teaching of first algebra concepts, and what possibilities you see for this to take place in your classroom.

Basic criteria used for the identification and creation of descriptors to be included in the questionnaires

As discussed in Chapters 2 and 3, teaching patterns where the introduction of basic algebraic concepts take place by presenting pupils with definitions and rules of procedures to manipulate given, *prefabricated* (Mason, et al., 1985) algebraic expressions are in line with the "instrumentalist" (Ernest, 1989), "absolutist" (Lerman, 1990) "traditional" (Gregg, 1995, Cooney 2001) conception of mathematics. According to this conception, mathematics is a static body of knowledge, involving a set of rules and procedures that are applied to a set of exercises or world problems to yield one right answer.

i contrast, classroom work designed to engage pupils in problem-solving and discussion, which aims at helping pupils not only to establish links between the new concepts introduced and their existing mathematical ideas, but also to see the connections of mathematics with their own world is indicative of a different conception of mathematics; a conception of mathematics as a fallible (Lerman, 1990) and dynamic, problem-driven field of human enquiry (Ernest, 1989).

As Ernest (1989), who considers three forms of mathematics: instrumentalist, Platonist and problem-solving, I conjecture that teachers' conceptions of mathematics contain different proportions of each type and may place special emphasis in one the three, which I see go from the 'instrumentalist' to the 'problem-solving' views of Ernest. Therefore, in identifying (in the literature) and creating the descriptors used in the questionnaires during data collection in Phase 1, I considered two broad categorisations of mathematics conceptions, which I called 'Instrumentalist' and 'Non instrumentalist'. Descriptors which can be associated with a view of mathematics as a static set of definitions and algorithms, and a teaching for rote learning were included under the 'Instrumentalist' category. Descriptors associated with a view of mathematics as dynamic, arising from human activity, and a teaching for understanding and meaning were included under the 'Non-instrumentalist' category. The following table shows the general criteria that guided the creation of descriptors used in the questionnaires.

Criteria used for the identification and creation of descriptors for Questionnaires 1 and 2

Instrumentalist	***Non-instrumentalist***
Nature of school algebra (mathematics)	
Mathematics is a collection of unrelated facts, rules and skills. Mathematics is unconnected, predetermined. Mathematics is a set of operations.	Mathematical concepts are connected Mathematics is integrated with other areas of knowledge Mathematics id dynamic, problem driven and continually expanding field of enquiry
Importance of school algebra	
Needed for higher level of school Maths Important to have good marks in maths in your C.V. when looking for a job. To do well in your external examinations. Algebra for some pupils only Requirement for access to a career	Valuable thinking tool for problem solving Provides elements for critical thinking Provides opportunities for pleasurable learning Develops pupils' capacities to conjecture, generalise and develop creative thinking, and to understand everyday-life cultural practices
Learning school algebra	
Pupils learn by: * doing lots of exercises to apply a rule; * memorising formal definitions of concepts; * just paying attention to teacher's explanations; *correcting each (homework) exercise at the board; *explaining procedures to them as many times as necessary. Pupils learn better when working individually	Working on: Open tasks and problem-based activities Discussion and systematic work Construction of formulae and algorithms use of calculators or other resources to aide pupils' work
Teaching school algebra	
Teacher instructs from textbook so no need to plan classroom work. Classroom work takes place explicitly according to plan. "I have taught it for so many years that I don't need to plan". There is an established routine in my lessons. Pupils spent most of the time listening to teacher's lecture about procedures during the lesson. Frequent testing to give marks and keep pupils working. Giving clear (step by step) explanations of procedures to follow in exercises. Having control over pupils' work. Having pupils working individually. Topis are treated in isolation. Creating an environment of rigidity and apprehension.	Encouraging discussion and autonomous work. Organising classroom activities that provide opportunity to: * develop pupil's communications skills and self confidence; * work according to level of attainment * promote connections of Maths concepts Organising work that engages pupils in problem-solving and exploration. Creating an environment in which: * pupils feel comfortable when sharing of ideas and discussing mistakes. * respect for pupils' ideas and cooperative work are promoted.
Assessing of pupils' work	
Short tasks to see if a taught procedure has been mastered. Frequent quizzes. Oral questions during lessons (to give marks); Daily homework (to give marks); Focus on answers rather than processes and analysis. Promotion of competition based on working speed	Engaging pupils in the monitoring of their own work and progress Assessing classroom work for decision making of following up pupils' work. Focusing on the mathematical processes followed by pupils and on their analytical arguments

Appendix 3.2

Copies of the Research Ethics documents listed in Chapter 3

MONASH
UNIVERSITY

10 April 2002

Prof. Alan Bishop
Education
Clayton Campus

Ana Cecilia AGUDELO-VALDERRAMA
Education
Clayton Campus

Re: Project 2002/092 - Mathematics teachers' conceptions of their teaching of school algebra

Thank you for the information provided relating to the changes as requested by the Standing Committee on Ethics in Research Involving Humans.

This is to advise that the amendments have been approved and the project may proceed according to the approval as given on 19 March 2002.

[redacted]

Ann Michael
Human Ethics Officer
Standing Committee on Ethics
in Research Involving Humans

RESEARCH GRANTS AND
ETHICS BRANCH
PO Box 3A
Monash University
Victoria 3800, Australia
Telephone: +61 3 9905 3012
Facsimile: +61 3 9905 3831
Email: offres@adm.monash.edu.au

www.monash.edu.au
ABN: 12 377 614 012

MONASH UNIVERSITY

20 March 2002

Prof. Alan Bishop
Education
Clayton Campus

Ana Cecilia AGUDELO-VALDERRAMA
Education
Clayton Campus

Re: Project 2002/092 - Mathematics teachers' conceptions of their teaching of school algebra

The above submission was approved by the Standing Committee on Ethics in Research Involving Humans at meeting A2/2002 on 19 March 2002 provided that the following matters are satisfactorily addressed:

- It is suggested that the research takes place in two stages.
  1. The questionnaire is sent out and returned anonymously. A slip may be provided inviting those who are willing to be available for the observation phase to provide contact details.
  2. A separate letter of invitation is later sent to those who have indicated their availability for the observation and focus group. A copy of this letter of invitation should be provided.

- Explanatory statement should tell prospective participants where the names came from.
- Researcher needs to state that she is studying for a PhD at Monash University including supervisors name in the Explanatory Statement.
- Has permission been obtained from University of Los Andes to use the data base to get teachers names?
- If the classroom is to be observed are schools and parents to be asked permission to do this?
- As this research is taking place in Columbia is there an academic to assist the student locally?

The project is approved as submitted for a three year period and this approval is only valid whilst you hold a position at Monash University. You should notify the Committee immediately of any serious or unexpected adverse effects on participants or unforeseen events that might affect continued ethical acceptability of the project. Changes to the existing protocol require the submission and approval of an amendment. Substantial variations may require a new application. Please quote the project number above in any further correspondence and include it in the complaints clause which may be expressed more formally if appropriate:

You can complain about the study if you don't like something about it. To complain about the study, you need to phone 9905 2052. You can then ask to speak to the secretary of the Human Ethics Committee and tell him or her that the number of the project is ________. You could also write to the secretary. That person's address is:

The Secretary
The Standing Committee on Ethics in Research Involving Humans
PO Box No 3A
Monash University
Victoria 3800
Telephone (03) 9905 2052 Fax (03) 9905 1420
Email: SCERH@adm.monash.edu.au

Continued approval of this project is dependent on the submission of annual progress reports and a termination report. Please ensure that the Committee is provided with a report annually, at the conclusion of the project and if the project is discontinued before the expected date of completion. The report form is available at http://www.monash.edu.au/resgrant/human-ethics/forms-reports/index.html.

RESEARCH GRANTS AND
ETHICS BRANCH
PO Box [illegible]
Monash University
Victoria 3800 Australia
Telephone: +61 3 9905 [illegible]
Facsimile: +61 3 9905 [illegible]
Email: [illegible]
www.monash.edu.au
ABN 12 377 [illegible]

The Chief Investigators of approved projects are responsible for the storage and retention of original data pertaining to a project for a minimum period of five years. You are requested to comply with this requirement.

[redacted]

Ann Michael
Human Ethics Officer
Standing Committee on Ethics In Research Involving Humans

RANTS AND
CS BRANCH

MONASH UNIVERSITY

Faculty of Education
Clayton Campus
Gippsland Campus
Peninsula Campus

20 February 2002

EXPLANATORY STATEMENT

Chief investigator
Professor Alan Bishop
Associate Dean -Faculty of Education
Monash University, Melbourne, Australia

Student Researcher
Cecilia Agudelo-Valderrama

Project Title: Mathematics teachers' conceptions of their teaching of school algebra

The aim of this research is to gain an understanding of teachers' conceptions of their teaching of school mathematics, especially in relation to algebra. A better understanding of how teachers see their teaching situation in the specific school context represents crucial information for the design and provision of teacher development programs and teacher education programs in Maths education.

I would like to request your participation in this study. The activities in which you are invited to participate will be organised at times that are convenient for you. Participating in the first part of the study involves answering a questionnaire, and taking part in a follow up interview that will last from 30 to 45 minutes.

Once this information is analysed, you may be selected to participate in a further exploration phase of the study, which will require observation of five consecutive lessons in your Grade 8 class, followed by an interview that will last about 45 minutes. You will also be asked to participate in a 'focus group' session which will involve a discussion (with five of your colleagues) of proposals for the introduction of different approaches to the teaching of elementary algebra.

Maintaining confidentiality is of great importance if useful information is to be gained from a study. No names will be put into any interview transcripts or written reports of any activity. Names will be removed from the questionnaires and replaced by code numbers. Only my supervisor and I will have access to these data, which will be stored for at least five years as prescribed by the University regulations.

All participants will be provided with a copy of the final report.

Should you have any complaints concerning the manner in which this research is conducted, please do not hesitate to contact the Standing Committee on Ethics in research on Humans at the following address:

The Secretary
The Standing Committee on Ethics in Research Involving Humans
PO Box No 3A, Monash University
Victoria, AUSTRALIA 3800 *Phone: (03) 9905 2052* *Fax: (03) 9905 1420*

Thank you. [redacted]

Cecilia Agudelo Valderrama
Phone: . . . .

PO Box 6
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Victoria 3800, Australia
Telephone: +61 3 9905 2819
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ication
Campus
Campus
Campus

22 March 2002

LETTER OF INVITATION
(for the second phase of the data collection)

Name of teacher
Address

Project Title: Mathematics teachers' conceptions of their teaching of school algebra

Dear X,

Thank you for answering the questionnaire and for your interest in participating in the second phase of the study.

The second phase will involve observation of five consecutive lessons in your Grade-8 class. After the last lesson has been observed, there will be an interview that will last about 45 minutes.

You will also be asked to participate in a focus group session, which will involve a discussion, with five other teachers of Grade-8 algebra, of proposals with different approaches for the introduction of elementary algebra. The focus group session will last about 2 hours.

Maintaining confidentiality is of great importance if useful information is to be gained from a study. No names will be put into any interview transcripts or written reports of any activity. Only my supervisor and I will have access to these data, which will be stored for at least five years as prescribed by the University regulations. When the project is written up, no participant will be identifiable in any way.

Agreeing to participate in the second phase does not compel you to take part in all the activities of the project you have been invited to participate. For example, you are not compelled to attend the focus group session, and you will not be required to give a reason for your decision.

All participants will be provided with a copy of the final report.

You can complain about the study if you don't like something about it. To complain about the study, you need to phone (03) 99052052. You can then ask to speak to the secretary of the Human Ethics Committee and tell him or her that the number of the project is 2002/092. You could also write to the secretary. That person address is:

The Secretary
The Standing Committee on Ethics in Research Involving Humans
PO Box No 3A, Monash University
Victoria, AUSTRALIA 3800

Telephone: (03) 9905 2052 Fax: (03) 9905 1420
Email: SCERH@adm.monash.edu.au

Thank you.

[redacted]

Cecilia Agudelo Valderrama
Telephone: . . . .

PO Box 6
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0, Australia
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ash.edu.au
77 614 012

The follow-up interview with the teachers of the initial group

As the purposes of the questionnaire[1] follow-up interview with the initial group of eighteen teachers are: (i) to tap the thinking behind their answers to the questionnaire given by the teachers, and (ii) to check the teachers' interpretations of the different statements and items in the questionnaire, the questions that will be asked in the interview cannot be spelt out before the actual data are available. Nevertheless, in the following, I am giving some examples of possible basic questions that could be asked:.

- From the alternatives given in Question A3 to describe your **actual** teaching style, you are saying that you design activities that provide space for pupils' self-paced learning. Could you give an example of a specific classroom situation that you have found to provide opportunities for pupils' self-paced learning?

- You also say in Question A7 that you assess pupils by giving them tests during the teaching of a topic. How does this take place? In what ways do you see this form of assessment interacting, for example, with the explained teaching style?

- Another case: If teachers identify specific contextual factors (e.g., time constraints, pupils behaviour, the role of administrators, etc.) as reasons (or causes) for differences between their **preferred** and their **actual** teaching style, a basic question would be:

  Why do you see (for example) pupils' behaviour as a constraint in your teaching?

The teacher will also be asked questions such as:

- Please explain what you mean by this statement.
- Please explain your reasons for the choice you made.
- Please give an actual example from your experience.
- What other possibilities could you imagine in that situation?

[1] The questionnaire was presented to the Ethics Committee as a two-part questionnaire. However, in preparing for the pilot study it was decided that the questionnaire should be divided into two parts, hence the names Questionnaire 1 and Questionnaire 2.

Project Title: Mathematics teachers' conceptions of their teaching of school algebra

I agree to take part in the above Monash University research project. I have had the project explained to me, and I have read the Explanatory Statement, which I keep for my records. I understand that agreeing to take part means that I am willing to:

- complete questionnaires asking me about my teaching of mathematics (especially algebra) in the basic cycle of education;
- be interviewed by the researcher;
- allow the researcher to observe some lessons;
- make myself available for a further interview should that be required;
- allow the interviews to be audiotaped;
- participate in a 'focus group' (a discussion, between myself and five other colleagues who are also teaching algebra in grade eight) about the possibilities of introducing innovative approaches in the teaching of school algebra).
- allow the focus group session to be audiotaped.

I understand that any information I provide is confidential, and that no information that could lead to the identification of any individual will be disclosed in any reports on the project, or to any other party.

I also understand that I will be given a transcript of data (from interviews and observation) concerning me for my approval before it is included in the write up of the research. And I also understand that confidentiality cannot be guaranteed for information which I might disclose in the focus group session.

Name: ....

Signature: ....

Date................................

Classroom observations and interview with case-study teachers

The purpose of the classroom observations that will take place with the teachers in Phase 2 is to collect information to deepen the understanding gained through the collection of information with the initial group of eighteen teachers. Therefore, during the classroom observations the researcher will be looking for information that will help to clarify and/or confirm what has been expressed by the teachers about their teaching of algebra. The researcher will be looking for examples in relation to:

- the content emphasised,
- the teachers' methods,
- the assessment patterns, and
- the teaching goals.

Having identified specific information about these aspects, the researcher would be interested in having the teachers' explanations about those aspects. The type of questions that will be asked in the interview after the lessons would be:

- o What did you want the pupils to achieve in this lesson (or in the last two lessons, etc.)?
- o How do you think the given homework will help the pupils in the achievement of that goal?
- o Would you say that this (…) is an example or an illustration of what you said about (...)?

No criticisms will be implied about the teachers' practice; the task is only to understand more about the teachers' conceptions of their practice.

Focus group (with the case study teachers)

The general theme for discussion in the focus group session will be the relevance and possibilities for adoption, by the teachers, of different classroom approaches (in the introduction of school algebra). The specific choices for the proposals will depend on the data gained from the case studies, but will be guided by previous in-service development activities.

The group discussion will be audiotaped and significant sections transcribed. Individual names will be eradicated from the transcript to preserve anonymity.

……………………………….

Appendix 4.1

Format of the tables used to summarise data from the questionnaires, from each of the 13 teachers of Phase 1 (including copies of actual summaries)

Tables for data summaries of Questionnaire 1

Teaching style Teacher No. ___

Establishing. connections	Problem-based activ..	Prov. for differentiation	Develop. Commun. s..	Frequent testing	Repetitive practice	Giving clear explanations of

Pupils' work:

Open Problems.	Formula construct.	Discussion & sist. work	Use of calculator	Closed tasks	Correcting exercises	Routine practice

Assessment

Pupils' folders	Pupils' reports.	Diagnostic assessment	Oral questions	Daily homework	Frequent quizzes

BELIEFS SURVEY

Algebra knowldge

B10. In mathematics you can be creative and construct your own mathematical ideas.	**B12.** Algebraic thinking should be promoted in the teaching of primary school mathematics	**B1**. Algebra should be taught as a separate area from arithmetic and geometry content	**B5.** Mathematics involves mostly facts and procedures that have to be learned

Teaching algebra

B18. .I have not found a textbook that is adequate for my pupils' needs and for what I want… So I often produce some classroom materia	**B6.** I enjoy teaching algebra.	**B2.** For the planning and teaching of Grade 8 algebra, i normally follow the text book's sequencing	**B15.** I often feel unmotivated by the fact that many pupils don't understand the basic concepts and procedures that are taught

Learning algebra

B11. When pupils are having difficulties with the learning of algebra, I have to revise the teaching-learning situation and organise alternative...	**B13** More important than getting the right answers is pupils' understanding of the main concepts inherent in a problem	**B3.**. Pupils who aren't getting the right answers need to practice on more problems	**B9.** If pupils fail to learn the algebra content taught during a specific term, they have to work by themselves and repeat the corresponding test.

Pupils' capacities

B17. Pupils will enjoy and work hard in mathematics if they find classroom work meaningful… whether or not their work is graded	**B14.** All my pupils would be good at mathematics if they worked hard at it	**B8 .** Mathematical ability is something that remains relatively fixed throughout a person's life	**B20.** Giving rewards is a good strategy for getting pupils to complete mathematics assignments

Awareness of value-teaching

B19. My teaching practice in mathematics is concerned with broad educational goals, and not just with …	**B4** Curriculum resources (e.g., ...) portray values	**B16.** The importance of mathematics is as a value-free subject	**B7.** The learning outcomes in my actual mathematics curriculum reflect cognitive learning concerns only

* What do would like to do and -- Nan ...

Prof 7 (011)

Estilo de enseñanza

Establecim. conexiones	Actividad Discusión	Act. Des. Cap de logro	Desarrollo C. Comuni	Sacar mayor num notas	(4) Tarea ej repetit	(1) Claras Explic
			1			
			1			

Ejemplos?

En clase, los alumnos trabajan en:

Plan. Probl. abiertos	(3) Construc. formulas +	Discus + trab sistematico	Uso calculadora	Soluc. Prob cerrados	(2) Clarif. Pas tablero	(1) Desar Ejer. alg
4			4			2
3	4			5	4	

Evaluación desempeño alumnos:

Folderes	Reporte alumnos	Pruebas diagnosticas	Preguntas orales en el	Revisión tarea	Lecciones escr duran enseñanz
			1		

CREENCIAS (mate y enseñanza)

Conocimiento algebraico

B10. En el aprendizaje de las M podemos ser creativos y construir nuestras propias...	B12. El pensamiento alg debe ser promov en enseñ de las M de ***primaria***	B1 El álgebra se debe enseñar como un area **separada** de los contenidos de la aritmeti y la geometria.	B5 Las matemáticas estan compuestas básicamente de hechos y procedim. que deben ser aprendidos
Comp acuerdo (5)	Acuerdo Rel (4)	Acuerdo Rel. (2)	(4)

Enseñanza (attitudes)

B18 Como no he encontrado un texto guia que se acomode a las necesidades de mis alumnos, y ..., yo diseño mis	B6. Me gusta enseñar algebra	B2. Para planear mi trabajo de álgebra en G8, normalmente sigo la secuencia que plantea el texto.	B15 Cuando enseño álgebra, con frec me siento desmotivada(o) por el hecho de ... alumnos **no entienden** los procedimientos
Acuerdo (4)	Comp Acuerdo (5)	Acuerdo Rel. (2)	(2)

attitudes

Cómo se relacionan?

Aprendizaje de los alumnos

B11 Cuando un grupo de alumnos **muestra dificultades** en su apren del algebra, yo tengo que revisar la situación y....	B13 La **comprensión**, por parte de los alumnos, de los concep matemáticos implicitos en un determ **problema** es mas importante	B3. Los alumnos que no dan con las **respuestas correctas** necesitan mas práctica en el desarrollo de ejercicios y problemas	B9 Cuando los alum **no aprenden** los contenidos matem enseñados, ellos mismos tienen que trabajar por su ... y para la evaluación correspon
Compl. acuerdo (5)	Comp Acuerdo (5)	Des. Rel (4)	Desac. relat (4)

*Podría explicar?

Capacidad de los alumnos

B17 Los estudiantes **disfrutarian** y trabajarian más duro en M si le vieran algún **signif** a su trabajo y ...	B14 **Todos** mis alumnos podrian tener un **mejor** desempeño en M si trabajaran con mayor e y d.	B8 La **habilidad** matematic de una persona es algo que permanece relativamente **constante** a lo largo de su v	B20. Una estrategia para que los alumnos hagan sus trabajos completos está en **la asignación de calificaciones** y ...
Completo acuerdo (5)	(4)	Desacuer. relat (4)	Ac. relat. (2)

Valores

B19 Mi enseñanza de las M se basa en unos **propósitos** educativos **amplios**, y no en una lista de obj de contenidos.	B4 Los **recursos curriculares** como los lineamientos, promueven determinados valores	B16 Las M constituyen una materia de enseñanza en donde no hay que consid la promoción de **valores**	B7 La lista de logros de aprendizaje de mi curriculo de M se centra en **aspectos cognitivos**.
Compl. acuerdo (5)	(4)	Comp Desacuerdo (5)	(4)

Non-trad

Qué tipo de valores

376 A

Teacher No. ____

Tables for data summaries of Questionnaire 2

Section A

	Responses to Section A –Part 1	*Responses to Section A –Part 2*		***Teaching approach put forward***

1. For engaging all pupils in problem-solving work...

Crosscheck responses with Section A4 and				
	Importance of this aspect:			

2. For giving clear explanations of definitions and procedures to follow in different exercises and problems

Crosscheck with responses to A2 and				
	Importance of this aspect:			

3. For providing opportunities for pupils to develop their communications skills

Crosscheck with responses to A2				
	Importance of this aspect:			

4. For focusing classroom work in the development of pupils' abilities to do the lists of exercises and closed tasks posed in the text book

Crosscheck with A2 & A4				
	Importance of this aspect:			

5. For involving pupils in the creation of their own mathematical ideas

Crosscheck with A3 & A4 (B10)				
	Importance of this aspect:			

6. For promoting pupils' understanding of the mathematical concepts inherent in a problem

Crosscheck with B13 ...				
	Importance of this aspect:			

7. For encouraging pupils to construct the concept of variable

	Importance of this aspect:			

Section B

B1

B2

Profesor(a) 04

Sección A

1. Para que los alumnos tomen parte activa en la resolución de problemas

Consist. con A4 (Prims 4 col H. Res.)	Sección A – 1ª parte	Sección A – 2ª parte	Concepción proyectada
No muy clara	[M.P] El estud. no tiene oportunidad de crear o buscar soluciones	[E] (no explicit)	[illegible]
		M.i	

2. Para que los alumnos cuenten con explicaciones claras de las definiciones

Consist con A2	Sección A – 1ª parte	Sección A – 2ª parte	Concepción proyectada
	[P] se involucra para el est. y el prof. tu... todo el trab.	[B] El est. tuvo la oport. de buscar una ... definir concept. crear sus propias ideas	No hablan de lo adecuado de dar explic. claras de los procedim.
		3	

3. Para que los alumnos desarrollen sus capacidades para comunicar

Consistencia con A2 ( )	Sección A – 1ª parte	Sección A – 2ª parte	Concepción proyectada
No (4/4)	[M.P] El est. no se ve enfrentado a la sit. de explicar o debatir sino ...	[E] E equipo, el estud. tuvo oport. exponer sus ideas y llegó a una conclusión.	[illegible]
		1	

4. Para ... desarrollo de sus habilidades para solucionar los ejercicios y ...del texto guía

Cómo se entiende esta pregunta

Consist con A2: *1.*4. A4 *1	Sección A – 1ª parte	Sección A – 2ª parte	Concepción proyectada
Sí ya no	[R] El prof. sigue el libro, el prob. es el libro y los textos no ofrecen probl. para aprovechar todos los probls. de laplom	[B] si el estud. puede resolver este tipo de situac. podrá ser una ... y ...	[illegible]
		3	

[illegible]

5. Para que los alum. participen activamente en la creación de sus ideas matemáticas

Consist con A43 B10	Sección A – 1ª parte	Sección A – 2ª parte	Concepción proyectada
Sí / no	[P] El est. no tiene oport. de crear ni debatir.	[E] Se da la op. de crear y aportar sus ideas	No [illegible]
		1	

6. Para promover comprensión de los conceptos matemáticos implícitos en un problema

Consist con B13	Sección A – 1ª parte	Sección A – 2ª parte	Concepción proyectada
Sí	[M.P] se está limitando a la pte. algorít. nica	[E] El est. fue llegando a una conclusión o solución poco a poco y el análisis de sol. ... particul.	[illegible]
		1	

7. Para que los alumnos construyan el concepto de variable

	Sección A – 1ª parte	Sección A – 2ª parte	Concepción proyectada
	[M.P] No se hace diferencia entre lo q. es una incógnita y una variable	[E] El ... estableció las relaciones entre número de ... y ... total, y ... una expresión ... salario total	[illegible]
		1	

Sección B

B1 Prof-B. hace énfasis en el enfoque de soluciones de probl. porque a partir de una sit. problemática muy sencilla logró desarrollar otras ideas del álgebra como los conceptos de expresión algebraica, y la necesid. de ellas para expresar ideas.

Sí, de los 2 prof. A: no opor. hay necesidad de dar explicaciones y aclarar algunos procedim. básicos.

B2: B: Hay situac. en los q. se puede desarrollar habilid. pr/s de situac. como la presentada x el prof B.

T-EXT-FA 4 Me gustaría ver en el trabajo ... de todos los grados (en) el enf. de soluc. de probl. Δ yo no siempre es fácil por tiempo. Tampoco se debe descuidar otro tipo de trabajo como Prof A: algunos ... ejercicios, ... y también deben ... y manejo del lenguaje matemat. para poder ... de conceptos.

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Appendix 4.2

Examples of text and other materials presented in transparencies (at the Focus Group) in order to describe the classroom work and approaches followed by three different teachers when introducing pupils to the concept of variable

Text used for the descriptions of the classroom approaches followed by three different teachers in the introduction of the concept of variable

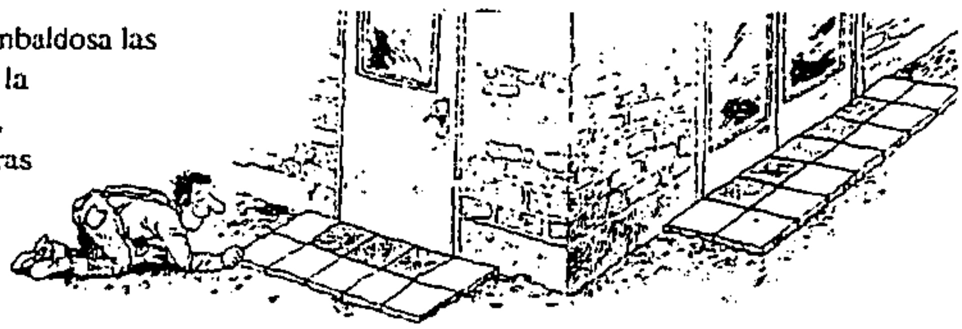
The three teachers	*Rationale*	*Specific teaching goals*	*Classroom approaches*			Examples of classroom tasks and activities presented at the Focus Group session
			Classroom organisation	*Materials*	*Team teaching*	
The teacher from a school in Tunja *(Teacher T)*	According to our experience, the pupil's lack of motivation for algebra work is due to the fact that they do not see the relevance of the use of letters and algebra work in general.	Provide the pupils with classroom experiences that help them see the reason why we use letters in algebra	Classroom tasks are designed taking into account the pupils' knowledge & difficulties. Pupils work in small groups and then, they discuss their work with the whole class. They work individually, in writing and reporting	We (the Grade 8 teachers) have been engaged as a team in the adaptation and production of materials for the introduction of the concept of variable	YES!	(See following pages) •Tiling •Exploring area and perimeter of basic geometric shapes
The middle-grades school mathematics coordinator from a school in Bogotá *Grades 4, 5, 6, 7* *(Teacher C)*	The foundations of algebraic thinking can be constructed from the start of the child learning of mathematics	Provide students with classroom work experiences that help them find connections between mathematical concepts (e.g., for this activity: area, perimeter and variable)	Exploration and analysis of a series of problem-based situations which help them identify relationships between quantities and to see the need for the use of generalisations. Small group discussion Whole group discussion Individual work when necessary	Calculators & other materials available	Yes	•Making a box •At the open market
The teacher from a school in Bogotá *(Teacher M)*	Given the fact that a large number of students do not benefit from covering a set lists of topics, we have chosen (for the middle grades) to work on activities which address what we have thought to be the basic and central algebra concepts	Provide students with classroom work experiences that help them form the concept of variable	Children make sense of their mathematical work when they are given the opportunity to find the *raison d'être* of mathematics & the connections between different mathematical concepts Small group work and discussion Individual work when necessary	Worksheets Software available Calculators	Yes, when possible	•The use of Spreadsheets for the teaching of the concept of variable (Actual demonstration) •The use of spread sheets to work on the problem of 'Martha selling ice creams' (see Teacher B's work in Questionnaire 2)

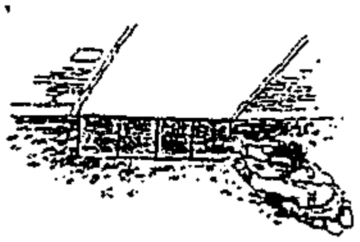
Tiling

(Part One)

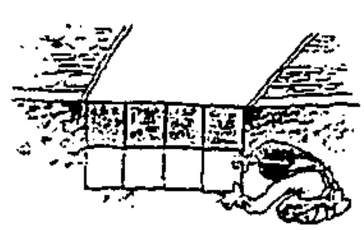
A embaldosar

Pedro es constructor. Embaldosa las entradas de las casas de la urbanización *La Colina*, utilizando baldosas negras y blancas

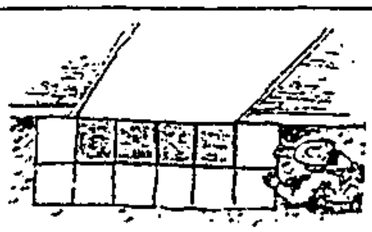




Pedro siempre empieza con una fila de baldosas negras.

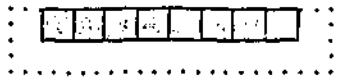


Luego pone una fila de baldosas blancas a continuación de las baldosas negras.



Y luego pone 2 baldosas blancas en cada extremo para finalizar el embaldosado de la entrada.

1 ¿Cuántas baldosas blancas usa Pedro si empieza con una fila de 8 baldosas negras?



2 ¿Cuántas baldosas blancas usa si empieza con:

a) ¿5 baldosas negras?

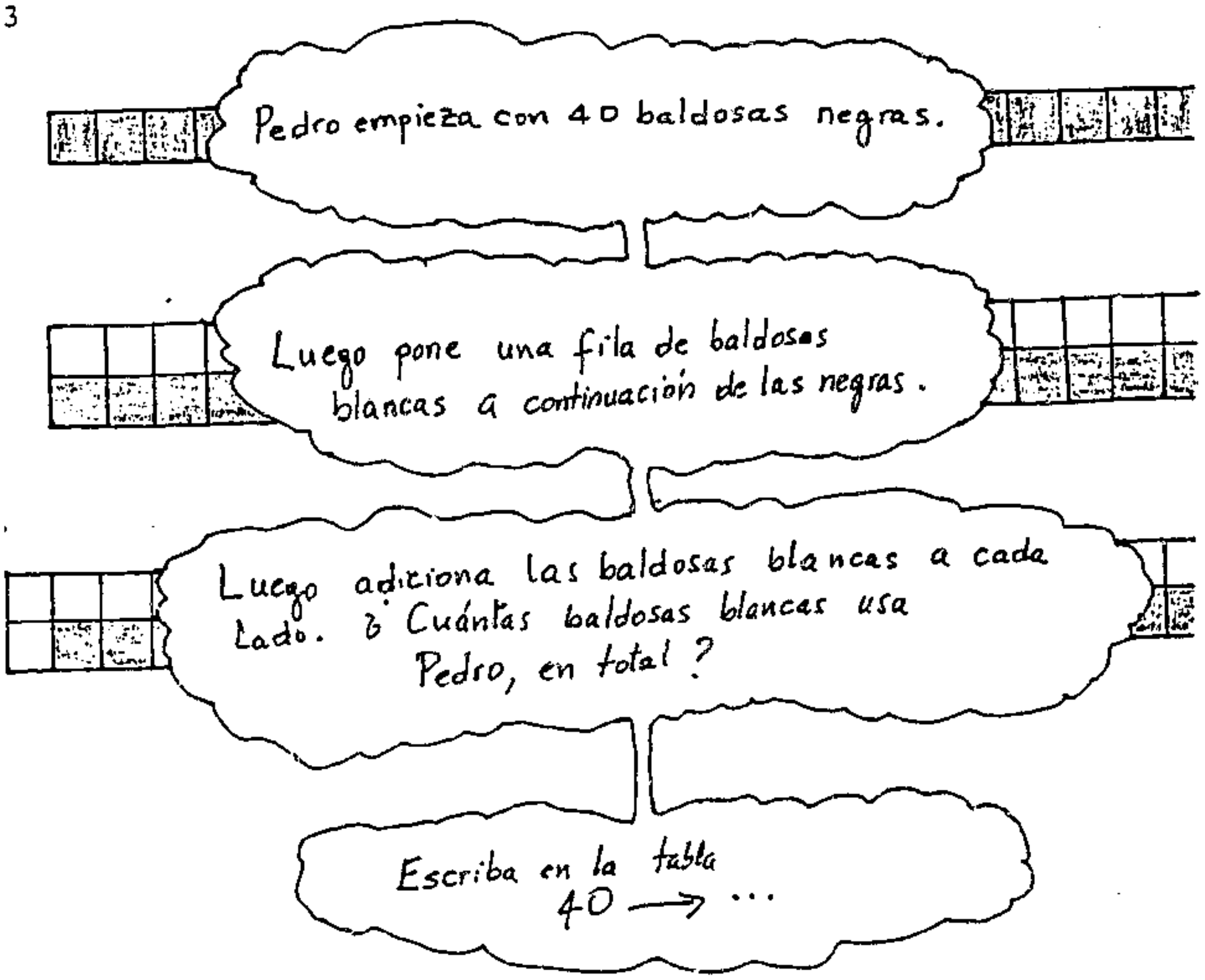
b) ¿9 baldosas negras?

Puede escribir sus respuestas a las preguntas 1 y 2 en una tabla como la siguiente:

Número de baldosas negras	*Número de baldosas blancas*
3 →	. . .
4 →	
5 →	
⋮	
9 →	
⋮	

Deje espacio en la tabla para anotar más números.

3



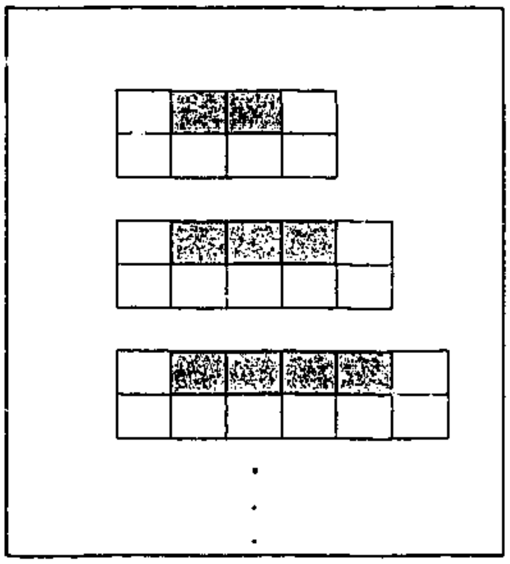
4 Pedro empieza con 100 baldosas negras. ¿Cuántas baldosas blancas necesita? Escriba estos datos en la tabla.

5 ¿Puede encontrar el número de baldosas blancas cuando conoce el número de baldosas negras?

¿Existe alguna regla que usa para encontrar el número de baldosas blancas? Escríbala.

- Para las entradas de las casas de *La Colina*, piense en una forma de hallar el número de baldosas negras cuando se conoce el número de baldosas blancas.
- Idee otra forma de colocar (o combinar) tabletas de dos colores para embaldosar las entradas de las casas, y encuentre las reglas correspondientes para calcular el número de baldosas de cualquiera de los colores que escogió.

Tiling
(Part two)



Number of *white* tiles	Number of *black* tiles
6	2
7	3
8	4
⋮	⋮

Number of white tiles = Number of black tiles + 4

$$W = B + 4$$

where: W is the number of white tiles, and
B is the number of black tiles .

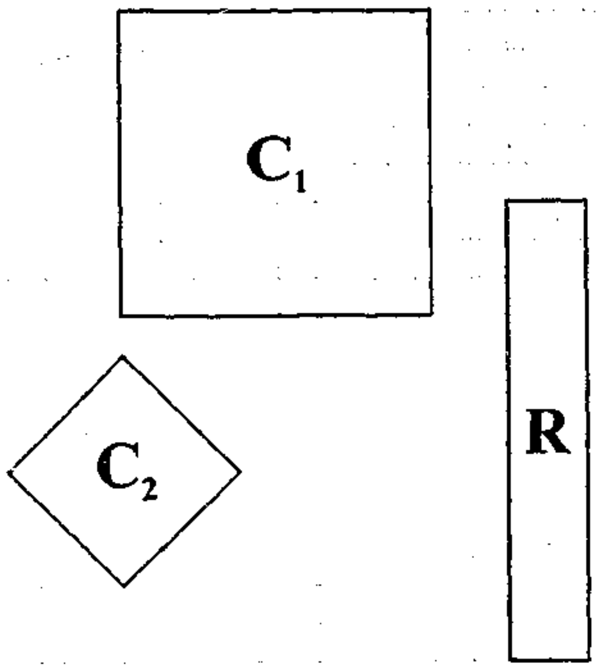
Pedro wants to know how many tiles (in total) he needs for any of the house paths he is asked to tile

Create your own pattern (or patterns) with tiles of two different colours to tile the paths of the houses, and find the corresponding rules to calculate any number of tiles, as in the previous case

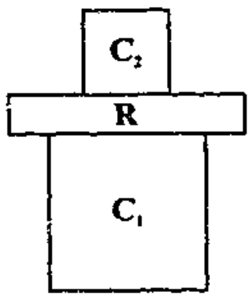
Exploremos los conceptos de área y perímetro

Primera parte

En la cuadrícula de la derecha tenemos dos cuadrados y un rectángulo. Corte las tres figuras (C_1, C_2, R).



Estas figuras se pueden arreglar de diferentes formas, de tal manera que se toquen a lo largo de cualquiera de sus lados. Este es un ejemplo:



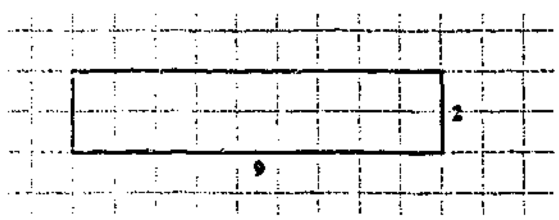
A) Arregle las figuras de tal manera que el perímetro de la figura resultante sea el máximo (el mayor posible).

B) ¿Qué puede decir sobre el valor del área de la figura resultante para cada arreglo que usted hizo?

Dibuje las figuras que va armando. Halle el perímetro de cada figura resultante, indicando cómo lo calcula. También escriba lo que va pensando a medida que va desarrollando el trabajo para responder a las preguntas A y B.

Segunda parte

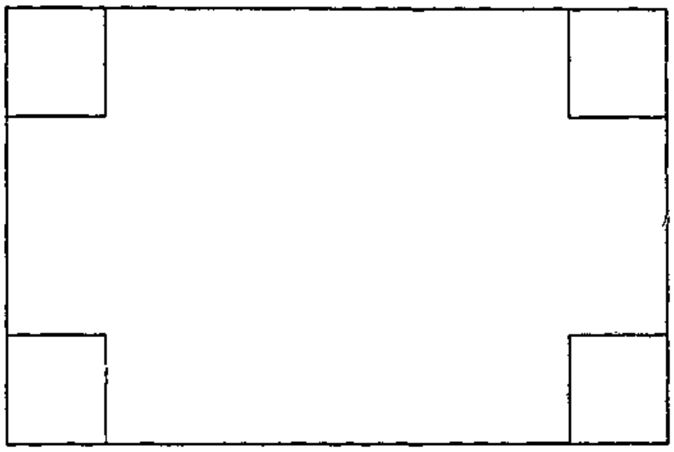
Vamos a trabajar con los valores que representan el área y el perímetro de este rectángulo.



¿Qué podemos hacer a la figura para incrementar (hacer más grande) el perímetro, manteniendo el área? Dibuje dos casos en los cuales muestre que el área se mantiene igual y el perímetro de la figura se aumenta.

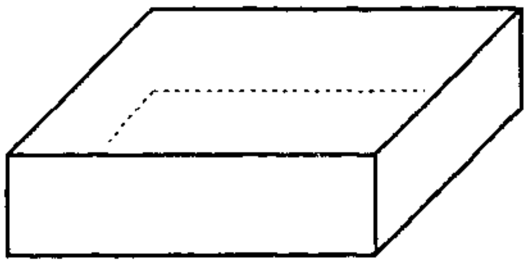
Making a box

A box without a lid can be made from a rectangular sheets of card by cutting out squares at the corners of the sheet, and folding the remaining pieces up, to form the sides of the box.



Investigate what size squares should be removed so that the volume of the box is as big as possible.

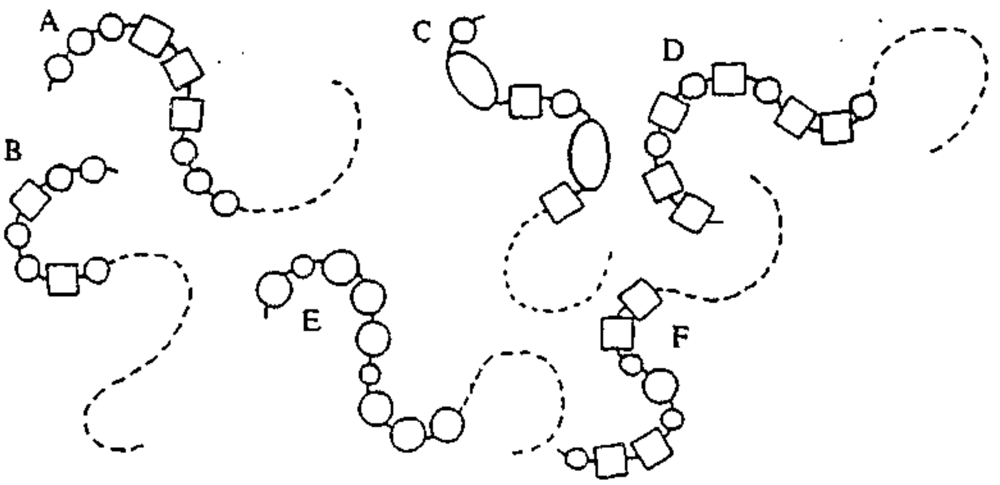
Suggestion: Start with a sheet of card of 24 cm by 20 cm. Record in a table the measurements of each trial as you work.



At the Open Market

La profesora Antonia ha estado trabajando con sus alumnos de 4º elemental en situaciones como las que se presentan a continuación:

José hace manillas y collares para vender en el 'mercado de las pulgas'. Para ello crea *patrones* (arreglos que tienen un orden determinado) insertando en un hilo cuentas de diferentes formas y colores. En cada una de las siguientes figuras identifique el patrón que estableció José. y dibuje al menos las 3 cuentas que siguen.



¿ Diría usted que este tipo de trabajo puede promover, en los niños, la construcción de ideas algebraicas ?

En su explicación refiérase al tipo de trabajo matemático que es considerado como "las raíces del algebra" por los autores del libro 'Raíces del álgebra y rutas hacia el álgebra'.

Appendix 4.3

The three options regarding their future teaching of beginning algebra as presented to the case study teachers at Interview 3

These are three options for teachers who are interested in learning about curricular innovation in school mathematics that focuses on the area of school algebra. Would you be interested in any of these proposals?

Option No 1:

To adopt a textbook that has been designed by drawing on results from research on the teaching and learning of beginning algebra. The textbook makes emphasis a problem-solving teaching approach.

Option No 2

To engage in an 8-month professional development program offered by *Fundación Compartir* and the support of the Texas Instruments Company

Terms of the program

- Two-month training in the use of specific computer software with a view to its use in the teaching of school algebra. Participants then should submit a classroom-based project proposal which should be put into action and monitored for at least during the following 6 months.
- Participants will have the continuos support of an adviser
- Judging by the scope and relevance of each participant's project, a decision will be taken regarding the allocation of technological resources for her/his corresponding school.
- Publications will be posted on the website of the Secretary of Education, and will represent a positive experience that can be included the Curriculum Vitae.

Option No. 3

To engage in a 14-month professional development program offered to secondary mathematics teachers by the Secretary of Education, through 'The Institute for Education and Pedagogical Development' IDEP.

Terms of the program

- This programme is available only for teachers working in Bogotá District schools. However, teachers who are not currently working in District schools can qualify for a position, provided that their proposal has been accepted.
- Participants will engage in a classroom-based research project that requires at least 5 hours of daily work throughout the 14 months. Time is needed for working sessions with the program advisers, the preparation of proposals, monitoring and developing of classroom work, writing up of reports, etc. This time cannot be time belonging to the participant school working session.

- Team work is encouraged in this program. Therefore priority will be given to projects in which more than one teacher is involved in the proposal. The working sessions with the program advisers, the preparation of proposals and time required for the development and writing up of reports

- The total amount of money allocated to each project is Colombian $30 millions, of which up to $12 millions can be assigned to the advisory institution, and a maximum of $10 million to the project proponents.

Appendix 5.1 Summary table of the 13 teachers' responses to Questions C3, C4 and C5 of Questionnaire 1

	C3: … A student wrote 10 + 5e = 15e a)… b) how would you respond to this student?	C4: A student wrote 5N= D to represent the expression No of doctors …. a)… b) how would you respond to this student?	C5. In Ms Rodriguez class …: 5x + 2 = 24. … Ricardo wrote: x = 26/5) .. b) how would you respond…?
Maria	I'd give then concrete cases to help them identify their error. Can you add desks with pencils?	Concept of relation limited to equality. I'd give them examples of a big gathering to establish different relations.	I go back to equality to explore what application he does in transposing terms. Simular the scales
Gladys	I'll explain it again, and more exercises	Read the expression. Give values and discuss the right answer.	It's a normal mistake. I'll ask them to discuss about the truthfulness of Ricardo's answer.
Juan	They need to clarify concepts "invite them to check if their expression is equivalent to the situation. Remember the concepts with practical examples. Ask questions that induce them into identifying their error.	With a given value. Ask is it true that that equality represents the given statement? If not write again.	Take 26/5 replace that value in the equation. If it is not true, analyse and discover the error.
Nacho	No clarity with like terms. I will explain again	It is wrong. It is a very particular case (5E). Find an expression that generalise the situation.	I'll show the situation in the arithmetic case to then generalise: use symbology.
Loren	No clarity with like terms. Need practice. "Revise the concept of like terms and its application.	You should revise the topic of proportionality. Need reading comprehension and practice.	Revise the topic properly and understand the concepts properly. Do variety of exercises.
Alf	We need to work more on 'like terms': pineapples with pineapples, oranges with oranges.	You should read with more attention.	I will work more in solving equations.
Mar	S/he has confusion when adding like terms. 'Revise"..	They have confusion with dependent and independent variables. Let's revise some examples on the topic. I'll explain to them that …	Revise inversive, modulative laws or the transposition method evaluating expression to check.
Nora	I'd make clear to them that the terms are not like. terms	I'll show them with examples and with a table of values that this is not correct.	I'll ask him to give a value to x check if that works find his error.
Stella	I explain again and give more exercises.	Give values to E and discuss answers in order to clarify why the error.	After asking why that answer, I'd explain again with an example.
José	I'd take them to perceive with clarity data given for the rectangle (… replaces by a number)	No data	
Luis	Replace by different numbers and see what happens	There were only mental calculations in the posing of their expression. Do a tally and write the proportions..	According to the scales, if I add 2 to one side … it is not possible … Also, replace 36/5 and …
Alex	No clarity I'll explain the concept. Explain again.	Their construction of the expression is literal. "Check your equation. Give several examples replace and verify.	Check given indications for transposing terms.
Pablo	No clarity in structure of a polynomial. I would refer them to the activities where they have worked and see if that is clear. See if they can connect this with something from the activities.	Lets' check what you did and see if according to the given statement, E is dependent or independent.	Explain to me your procedure so that I can see your questions. What did you do here?

Appendix 5.2 – An example of a summary table with data from Phase 1 (e.g.,. from Teacher 3)

Teaching Aspect / Data coll. activity	*Questionnaire -1*		*Questionnaire 1 -follow up Interview*		*Questionnaire - 2*	
Why algebra (A1)	1). Next Maths levels 2) Use of technology 3) Problem solving.	*Even mix*	The transition Arithmetic to Algebra is a drastic shift.	*Inst*	Describes teacher A as mechanical, leaving no room for pupils to	
Teaching method A2 +A3	Connections, Comm skills, T by telling (1,1) Provide for differentiation, Routine exercises (2,2) . . .(3,3)	*Even mix*	Use of letters to represent objects' names as an algebraic use. Commun. skills go **to board** (key for learning) and say what they are writing.)	*Inst*	participate in the lesson But adds that work needs mechanisation.	*Mainly non Inst.*
Pupils' work A4 + A5	Correct exer. at the board, Discussion + syst work (1,1) Routine exerc, closed tasks, formula construction, open tasks 92,2)	*Even mix*	No calculators. No textbook Every time they see application to a topic they learn, but it is difficult to do that but furthermore, there is no time.	*Inst*	For teacher B: praises the environment created for pupils to discuss and find applicability in what they learn.	Perceives problem solving as solving problems
P' work assessm A6 + A7	Homework (1,1) Frequent quizzes, oral questions, P' self assess (2,2)	Primarily Inst	When I ask them oral questions they get lots of marks. Homework is the most important aspect for assessment.	*Inst*	Asserts that Teacher B makes more emphasis on problem solving and	(i.e. T. A can be an example
B1 (Algebraic know)	19	*Non Inst*	Variable: When it can take any value. e.g. $x^2 = x. x$		explains again that he starts the intro of the use	But T B makes.
B2 (Teaching A)	13	*Even mix*	What I plan is what I do. How ? with love and authority. Connections: yesterday's work with today's work. I use primary: things like 'go shopping'		of letters by going to the open market.	more emphasis
B3 (P' learning)	15	*Even mix*	The only way for the to learn is at the board cause they are focusing on what they are doing.			
B4 (P' abilities)	16	*Mainly N Inst*	Nowadays pupils don't do their work. They don't care because other priorities (women, friends, cinema)			
B5 (T. Values & awareness of them)	15	*Mainly N Inst*	Authority and & love. "You are my friends". If the student adds algebraic fractions I am happy.			
Section C	C1. It is useful to learn to operate with quantities that vary. C5 no clarity in the rules to manipulate terms. Give a value and check if you get an equality or not	*Mainly N-Inst*	C2. There are pupils who just rely on the work of others. I have not arrived at the concept of function yet			

Reason for bad quality of maths teaching (education in general):
Teachers' practice, but this is due to:
Central policy. Low salary). But accepts that Maths work does not have meaning.
Difficulty: no access to computers. One would like to work for meaning (i.e. areas) but no time. One has to run because of ICFES requirements.

Key:

Appendix 6.1

The case descriptions of Luis and Alex, the other two case study techers from the group of five, presented in this thesis

This appendix contains the case study descriptions of Luis and Alex, the two case study teachers whose summaries were presented in Chapter 6, Sections 6.5 and 6.6. The numbering of sections, tables and figures that was set up in Chapter 6 continues in this appendix.

Appendix 6.1 - Table of Contents

6.5 Luis – I am constrained by 'my pupils' social background'

Luis had taught secondary school mathematics for eight years and, in 2002, was teaching Grade-8 algebra for the fourth time. He was also teaching Calculus in a tertiary institution, and "doing a part time Masters course in Educational evaluation". He showed high motivation and enthusiasm about participating in the study. He said that he couldn't say "no" to the invitation, as it sounded "a really attractive study" and he wanted to know what it was about.

The school where Luis was teaching was located at the outskirts of the south of Bogotá, in one of the poorest areas of the city. The school had ample classrooms and green areas for the children, and provided facilities for two working sessions a day —the morning and the afternoon sessions. Luis worked in the afternoon session, which started at 12:30 p.m. and ended at 6 p.m. According to Luis, there was a group of student teachers doing their teaching practicum and teaching in his Grade-8 class, once a week.

6.5.1 Luis' conception of beginning algebra

At Interview 1, Luis stated that "algebra is the set of topics and concepts related to algebraic expressions that are to be learned in order to be able to solve mathematical problems." He further explained that "the set of topics to be studied, or the sequence I follow in the work of Grade 8 is the one that one finds in Grade 8-textbooks".

> After I do the work on rational numbers, algebra work commences with algebraic expressions and all that is related to that: coefficient, literal part, type of algebraic expressions, etc.; and then we move on to addition and the rest of the operations with polynomials.

Including algebra in the school curriculum of the basic cycle of education is important "because it develops formal thinking and it represents key knowledge in the solution of problems that belong to higher level mathematics".

Luis declared at Interview 1 that "algebraic thinking is related to the notion of variability.

> There is algebraic thinking when the student is able to handle the variable; not only the dead variable (i.e., the written symbol) but for example if I have a cube, what happens to the volume if I change the area; what happens to the edges of the figure; the analysis of the variation..."

At Interview 2, after the Curricular Standards in Mathematics document[1] was released, Luis proclaimed that algebra was about "the work with the variable and its different uses". However, he declared not to be clear about the reorganisation of the teaching of variable according to what the Standards stated (refer to the concept map activity in Section 6.5.2.1 for Luis' further explanations on this aspect):

> One can be guided by what *The Standards*, say in terms of the types of thinking: *variational* thinking, numerical thinking, metric thinking, etc. that need to be considered in mathematical work, even in primary. From now onwards, it is not a list of topics that have to be ordered —as we are doing— but the content which gives account of the concept of variable…

6.5.1.1 Learning beginning algebra

Luis' responses to items of Questionnaires 1 and 2, as well as the corresponding explanations at Interview 1, conveyed the idea that he was not in agreement with traditional approaches of just giving pupils a list of exercises. He did not like the statement of Questionnaire 1 that '*algebra provides the intellectual challenge pupils like of mathematics*'. "On the contrary, it is because of algebra that pupils start to dislike mathematics", and that is due to

> the way algebra is taught. I consider it important to organise activities for the classroom, so that the pupils become engaged in the solution of problems and have the opportunity to interact and discuss what they are doing. I have noticed that interesting things and ideas can emerge from pupils' interactions. (Int. 1)

> Teacher B has made an effort in finding a 'problem situation' in his/her search for relating a contextual situation with a symbolic expression of variation… Teacher A shows the letter to represent different values but independently of a problem-situation that could help pupils see the meaning of the letter, and does not give space for pupils to show their differences in abilities... I do what Teacher A does. (Questionnaire 2)

However, at Interview 2, when he was explaining why he had proposed the specific tasks showed in Figure 6.14 for his teaching of factorisation, he proclaimed that pupils learn "by repeated practice", and that they follow a set of levels when they are learning each case of factorisation:

> The teacher explains the methods or procedures to factorise… They learn each case by paying attention to what the teacher explains and generating questions when there is something they do not understand. To learn each case of factorisation they

[1] In June 2002 the Ministry of Education published a document called Curricular Standards in Mathematics See Appendix 1.1.

> follow some levels: In level 1 the pupils identify or establish the form, the structure of the [given] expression. In level 2 they practice finding the factors of given expressions [of this form], in order to drill the procedure. In level 3 they can apply this procedure to an artificial problem and then to a practical problem from their environment.

6.5.1.2 Teaching beginning algebra

As shown in Table 6.4, Luis' rankings of the teaching style' descriptors given in Questionnaire 1, showed 'the promotion of connection between different mathematical concepts', and 'the provision of opportunities for pupils to develop their communication skills' as his first two-top preferred teaching styles. The data related to Luis' interpretations and explanations of each of his top teaching priorities are presented immediately after Table 6.4.

Table 6.4 Luis' teaching style priorities

Preferred	*Teaching style descriptors in Questionnaire 1*	*Actual*
1	Designing classroom work that promotes connections between different mathematical topics studied	1
1	Providing opportunities for pupils to develop their communication skills so that they can express their mathematical ideas with confidence	3
2	Organising problem-based activities for the pupils to work in small groups where they can present their ideas to the whole class for discussion	2
2	Designing activities that provide space for pupils' self-paced learning	2
3	Giving clear explanations of definitions and procedures to follow in different exercises and problems of application in the topics studied	1
4	Giving pupils lots of exercises for algorithm application as homework	2
4	Testing pupils at the end of each activity or topic, in order to have sufficient marks for assessment in each Attainment target	2

Priority 1 for preferred and actual practices: 'Promoting connections between different topics studied'

At Interview 1, Luis stated that "designing classroom work that promotes connections between different mathematical topics studied" was considered by him a very important aspect of the teaching of Grade 8-algebra. He explained that "for the promotion of connections, the student teachers who were doing their teaching practice in his Grade 8 group, [were] doing that type of work" during in a weekly two-hour lesson.

> The student teachers are doing a project that is like a kind of treatment to help the

> pupils use geometry to see algebraic expressions... It's with areas and perimeters of geometric shapes.

When asked about what sort of work that was, Luis said that they are working with areas and perimeters of rectangles, but

> I cannot give you details about that work because it is the teaching practice coordinator (a teacher from a university) who is in charge of that work. Maybe, you can see it when you go to the school[2] for the lesson observation. I am going to try to ask the student teachers if a meeting with you can be organised.

To my request to see the work that pupils were doing for that purpose (as there was nothing of that type of work in the pupils' notebooks), Luis explained that "the pupils' work [was] in the work sheets that the student teachers had collected and that they had not been returned to the pupils yet".

Priority 1 for preferred practices: 'Providing opportunities to develop pupils' communication skills'

Luis' explanations about his responses to questions related to the provision of opportunities for pupils to develop their communication skills posed in Questionnaires 1 and 2 were probed at Interviews 1 and 2. At Interview 1, he said that these opportunities were given "when pupils had to discuss what they [were] doing with their peers". When he was asked why the difference in rankings for preferred and actual practices (see Table 6.3), he talked at length about "*la problemática*" of school mathematics (i.e., the problematic situation of "pupils not liking school mathematics"). He explained that "although the provision of opportunities for pupils to discuss their work with their peers [was] an important preferred practice, its implementation in the classroom was a highly complicated aspect", and that that was "due to two main factors":

- the social context of the pupils and
- the pupils' pre-requisite knowledge for the learning of Grade-8 algebra

In relation to the social context of the pupils, Luis explained that "due to the pupils' socioeconomic background, [his] targets of some mathematics lessons [had] to be given up in order to focus on aspects which [had] little to do with learning mathematics". The pupils at the school have many problems at home.

[2] Luis' interviews where held at the university were he worked, as he said it was "more convenient" for him.

They belong to social strata where parents are not concerned with their children's academic achievement. To manage that the pupils pay attention to the teacher is an achievement! So, the teacher has to work a lot at something that is outside the context of mathematics. It is something that belongs to their upbringing. So on many occasions all the attention has to be given, basically, to their change of attitude, and this is assessed as a move forward in their process, but the academic part is left in a second place.

In relation to the pupils' prerequisite knowledge for Grade 8 algebra, he said that

> due to the automatic promotion[3] regulation [he had] pupils in Grade 8, who [didn't] even know how to read or how to operate with whole numbers and not least with fractions. To work on activities, in these circumstances, requires a lot of time and that's why I don't do it.

Luis added that "due to the pupils' deficient pre-requisite knowledge", in the teaching of factorisation, he was working with different ways of representing the expressions and that he [was] trying to see if that work helped the pupils (see Figure 6.14).

As stated above, the aspect of 'providing opportunities for pupils' development of pupils' communication skills' was discussed again, at Interview 2, after the series of lessons had been observed. In order to illustrate the type of work that was discussed with Luis at Interview 2, a description of the work done during the first lesson (a double lesson) observed is presented in the following paragraphs. Key parts of the discussion related to the classroom work described are presented in the subsequent subsections.

The lesson started with a review of "how to express a given number as a product of its prime factors". The teacher reminded the class what a prime was and then wrote the numbers 18 and 54 on the board. With some input from two of the pupils, these two numbers were expressed on the board as follows:

$$18 = 2\times 3^2;\ \ 54 = 2\times 3^3$$
$$18 + 54 = \ 2\times 3^2 + 2\times 3^3$$

Luis then asked the class "to remember the processes [they] were doing in previous lessons", and started writing a sequence of tasks on the board as shown in Figure 6.14. In the figure, the right column shows what Luis wrote, and the right column what Luis said to the class after he wrote the task.

[3] According to 'the automatic promotion policy' when pupils in the primary school finish a school grade, they should be automatically promoted to the next year level.

a) □ ▲ + □ ¤ =	"Remember if we have these operations with these small shapes, what is the result?"
b) $\dfrac{\square\ \blacktriangle}{\square} =$	
a) □ ▲ + □ ¤ = (□) (▲ + ¤)	[This is what was written on the board after two pupils gave some ideas of what the result was]
b) $\dfrac{\square\ \blacktriangle}{\square} = \blacktriangle$	
Ex. 1: $\square^2\ \blacktriangle + \square^3\ ¤ = (\square^2)(\blacktriangle + \square\ ¤)$	'How would the answer be in this case? [Asking the pupils what was "common in the two terms of the expression", Luis wrote the part, after the equals sign].
Ex. 2: $2\times 3^2 + 2\times 3^3$	"Now we are going to work this process with numbers. Let's take 18 and 54, that we already used, and let's say that we have 18+ 54, which gave us: $2\times 3^2 + 2\times 3^3$.
Ex. 3: $x^2 + x^3y + xy^2 =$	"We are going to do it now only with letters. We follow the same procedure that we had with the small shapes and the numbers. So [writing the expression], What is the variable that is repeated?" [The answer is given by the same two pupils]. "What are we expected to do? To combine; Okay?"...
Ex. 4: $4x^2 - 2x^3y + 6xy^2 =$	"Look, here, apart from the letter, the common factor should have a number..."
Ex. 5: $\frac{4}{15}x^2 - \frac{6}{12}x^3 + \frac{2}{3}x^5 =$	"It continues with the same story, but we can also find exercises with fractions".

Figure 6.14. The sequence of tasks' to teach factorisation' in Luis' first observed lesson.

Luis continued with a list of exercises until the bell rung. He gave the pupils more exercises for homework. Within the homework exercises there were some which had letters as coefficients (e.g., $ax^2 + abx - abx^3$). These became the focus of work during the following lessons.

At Interview 2, Luis provided reasons for his teaching approach during the sequence of lessons observed. His goal was that "the pupils drilled the routines to follow when doing factorisation". For Luis, "there was no promotion of communication skill because the exercises were mute exercises".

I: I found it interesting to see the work with the small shapes that was being done during the lessons. Could I ask, what was the intention of the work with the shapes in the first lesson about factorisation?

L: My goal was that pupils reached the first level; that is, the manipulation of a language. In other words, that they learnt the structure, the form of the given expression.

I: I remember that you considered the creation of opportunities for pupils to develop their communication skills as an important aspect of your teaching. How did you see the work of, for example that lesson, in terms of opportunities to develop pupils' communication skills?

L: Well, it was a mute exercise. I wanted them to overcome the first level; to identify the form or the structure of the given expression; then they practice it, which is the second level, and then they can apply that to an artificial situation or to other situations, which is the following level, the third level.

I: Okay. And what sort of work are you thinking of for the third level?

L: It could be the work with areas, for example.

I: So this is where the work with the student teachers comes in?

L: Yes; that's why I told you that it would be good if you saw what the student teachers are doing in their project. When the bell for the break rings, we could try to see if they are in so you can speak to them.

Luis did not consider the lesson described "really a successful lesson". A successful lesson was "one that creates interest and motivation... when the pupils ask questions, and ask for 'a participation'. The pupils needed more practice in that lesson to be able to say that it was a successful lesson". "A participation" for Luis meant an exercise given to the class, in which the first 5 or 10 pupils who hand in their books, with the right solution, are given a good mark". During the lessons observed there were a few pupils who frequently asked the teacher for "a participation".

'Priority 2 for preferred and actual practices: Organising problem-based activities

On several occasions, Luis declared his strong commitment to changing his practice "to a problem-based approach", but highlighted the reasons for his difficulties in achieving this "ideal". *The teacher's knowledge and commitment* and *the school context* were identified this time, by Luis, as the most important factors preventing him from implementing a problem-based teaching approach.

At Interview 1, Luis openly stated that he was aware of the fact that "the reason why pupils do not like mathematics is the type of work the teacher proposes". He also said that it is necessary to "introduce mathematics work by presenting pupils with activities or problem-situations" but that it was difficult for him:

> I don't do any classroom planning; I just do traditional teaching, following a textbook sequence. I have tried to do problem-based activities but I do it, maybe, once a month. That's not the common way of work. It is difficult.

Without being asked why it was difficult, Luis went on further to explain that the problem was rooted in the teacher's knowledge and commitment:

> Teachers do what they know. All teachers do it! They give the pupils a list of exercises because we have not planned an activity for the lesson... On the other hand, teachers don't have enough commitments because they just adjust to the school environment and do what is easiest... We know it is true... Precisely because they have not prepared the lesson or designed an activity, they end up saying, 'do this list of exercises or do the even numbers of the list of the textbook' to prioritise in the first level: the algorithm.

Was Luis describing his own case here? Asked to explain the situation further, Luis identified another aspect that, combined with the teacher's knowledge and lack of commitment, represented "the basic reasons for '*la problemática*' of school mathematics":

> I: Could you explain more about what you said of the teachers adjusting to the school environment?
>
> L: The context where one works influences the teacher's work because I have worked both in private schools, in the north, and in District [state] schools in the south. And why is it that for the work in the north, I had to plan and do activities, and not just exercises? Because in those schools the pupils themselves are going to ask for different activities... Of course there can be problems in schools of high strata, but not in the academic sense because, in the first place, the kids are well fed and dressed, they have other customs and principles, and their parents have targets for them. In the second place, there is dynamism in the school, the principal's role is different; there is control and more requirements for the teacher... One was more careful! (silence) and at least lessons were not missed out.
>
> I: And (Luis interrupts)
>
> L: In contrast, in public schools, sometimes, the teacher kind of does things the way he/she wants. There isn't a set of criteria to guide the work one does. The school environment is different.
>
> I: Why do you think that is? How would you explain that?
>
> L: There are good human resources in public schools, but the dynamics of the private school is not there. I think it is also the ethics of the teacher (silence). As they have a life-lasting job contract/ it's painful to say it, but there are teachers that work just because of the salary! And I cannot say that I am out of that case...

I: As a teacher would you feel better in a school where you think you don't have to work hard or?

L: I think that one shouldn't feel better in one or the other. One should work as it should be! But, I repeat, the pressure from the school environment is strong! Sometimes one doesn't work as it should be (Silence). I am a bit worried about that.

In Questionnaire 2, Luis rated Teacher B's work as "an excellent example of a problem-solving approach", and Teacher A as "someone who follows the traditional style". He also wrote that although he was trying to learn how to teach following a problem-solving approach, he did what Teacher A did. He wrote:

Teacher B follows a problem solving approach because the teacher:

- poses a contextual real situation in the form of a problem,
- promotes discussion and interaction between pupils, trying to direct the discussion toward answering the proposed questions,
- considers the different possibilities that pupils see; after a process of socialisation they arrive at a consensual academic [i.e. mathematical] idea because an algorithm was constructed from the table of values. Also,
- special software was used to complement and reinforce previous ideas worked by the pupils.
- The activity has been designed so that it can be adapted and changed to fit particular contexts, rhythms of learning and unforseen situations. Assessment was designed not only to verify and look at results and to comply with a legal requirement [i.e., school reporting] but, also, to identify difficulties and achievement along the process.

But I do what Teacher A does:

- Terminal and summative assessment (to fulfil the legal requirements, also due to the particularities of the pupils group —their pre- algebra concepts, discipline and family context).
- Follow textbooks
- Schema of classroom work such as: Definitions, Examples, and Exercises.
- Application of algorithms to a list of exercises individual pupils do and then give marks for their participation in class

Assessing pupils' work

Luis identified two more reasons for the difficulty he had in implementing his preferred teaching practices. This time the reasons were" the number of pupils" and "time". According to Luis' responses to Questionnaire 1, his two first *preferred* forms of assessing his pupils' work and learning were 'Pupils' folders and assessment records, showing evidence of several aspects of the process followed throughout a term or a set of terms' and 'Written tests for learning diagnostic purposes.' But his two first *actual* forms of assessment were 'Pupils' individual marks obtained from frequent quizzes given during the teaching of a specific topic' and 'Oral questions within the lesson'.

One of the reasons for doing 'frequent quizzes' and not his preferred assessment practices was "the number of pupils" in his class:

> I would say that one should not collect pupils' notebooks, not just to check if they have a record of all the work done in the classroom, but to look at how they are thinking. If they are given an activity, for example, one that is about the perimeter of a rectangle, and the children have errors, one has to look at why is it that the children don't handle the concept of perimeter. And then the teacher would have to initiate a process of reflection or investigation, using bibliographic references if necessary, in order to be able to design activities, which are consistent with the situation. But that is difficult to do when you have to teach groups of 40 children... (Int. 1)

Another reason for assessing pupils through frequent quizzes was "the lack of time".

I do frequent quizzes because, first of all, by legal requirements you have to report pupils' grades every term and, secondly, because some pupils did not make any progress in the process; they did not move forward in the process, so one has to do assessment through closed tasks. ... (Int. 2). One is always short of time. ... When one collects the notebooks to see their homework, one just puts a tick or a signature because there is no time (Field notes).

As we saw in Nora's and Nacho's cases, Luis did "the exercise for the first five", which he and his pupils referred to as "a participation". The exercise for the first five was given to the class after the procedure to do a given exercise had been explained and a few exercises to apply the routine had been done. It was given "because the pupils want to show that they are succeeding and/or because they need a good mark to pass [in the term]".

6.5.2 Luis' conception of his own teaching of beginning algebra

As we saw in the previous section, when Luis was giving explanations for his difficulties in putting into practice what he said he would prefer to do in his teaching of Grade 8, he identified a number of factors as the reasons for keeping his "traditional teaching" in place. These factors were *the teacher's knowledge and commitment, the school environment, the social context of the pupils, the number of pupils and time.*

Luis ways of speaking about "*la problemática* of school mathematics" when explaining his teaching made him sound distant from the situation; he spoke of the case of "the teacher" rather than of his own case. The following subsection focuses on Luis' explanations about his own teaching practice and situation through the concept map activity, where he was asked to identify the determinants of his teaching practice

6.5.2.1 Luis' conception of the determinants of his teaching practice

At Interview 2, after Luis had explained, for the second time, why designing activities that provided opportunities to develop the pupils' communication skill was difficult for him, the building of a concept map to explain his teaching situation was suggested by the interviewer. After looking at, and discussing the example of a concept map provided (see Subsection 3.3.3.2), Luis was asked to think of the aspects that influenced or determined his teaching of Grade-8 algebra. Luis did not name any aspects, but asked a question, initiating the following dialogue which took him into identifying two different types of knowledge "a teacher needs for the teaching of mathematics":

L: This map is to explain why I teach in the way I do. Isn't it? Why I do what I do.

I: Yes. What aspects or factors determine what takes place in your teaching of Grade 8.

L: (Luis took some time to think, and then said): Right. The teacher designs activities. (Silence). I think that the teacher must have a conceptual clarity. If I have conceptual clarity I can design classroom work in a coherent way. Um, because I can design activities but they may not be consistent with the topic that I am going to teach.

I: What do you mean by consistent with the topic?

L: Yes. I am talking about a conceptual clarity of mathematics that the teacher needs to have to be able to design activities that are attractive and adequate for the pupils' level.

I: On what does the teacher's design of 'consistent' activities, according to the pupils' levels depend?

L: It depends on the teacher's didactical knowledge of mathematics.

I: Is that knowledge different from conceptual clarity of mathematics?

L: Yes, it is different because some teachers kind of lock themselves in 'my mathematical knowledge' and other teachers pay attention to pedagogy; that is to theories of learning and theories of knowledge. The two are different, and we need both. If I don't connect those theories with the subject matter then it becomes something decontextualised.

I: Would you be talking about a pedagogical knowledge that is specific to the area of mathematics?

L: Exactly. It's the didactics of mathematics.

I: So you are naming two types of knowledge that you think influence your teaching practice?

L: That's right.

I: Could we start building the map with these aspects, or factors, you have said determine what you do in your teaching?

Luis' initial concept map – Part 1

The following paragraphs contain what Luis was saying as he was building the first part of his initial map, which is shown in Figure 6.15.

L: If I have a 'conceptual clarity of mathematics' I can/ it relates directly to my "knowledge of the didactics of mathematics", and these two generate 'my practice'. These two [types of knowledge] relate to each other but, at the same time, this practice connects with those like this (draws arrows connecting the two types of knowledge with the central concept presented to him for the concept map).

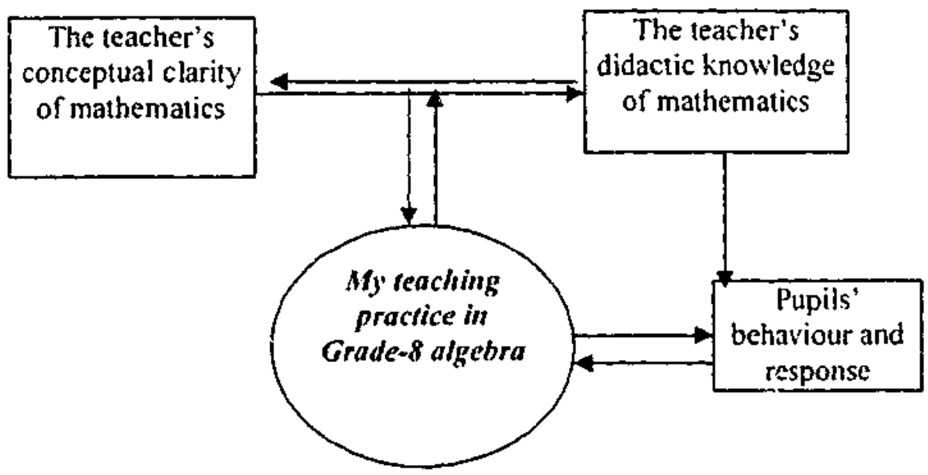


Figure 6.15. Luis' initial concept map –Part 1

These two [types of knowledge] are the factors that most influence my practice, but my 'didactic knowledge' imply to me that I have to look at the pupils' behaviour, because then I can look at the pupils in terms of their attitudinal aspect and at their procedural knowledge. Further, if this knowledge [the didactic] is coherent, I would have to assess not only the pupils but my practice as well. So this arrow goes like this (draws the arrows connecting 'My didactic knowledge' with 'Pupils' behaviour' and, then, the one connecting 'Pupils' behaviour' with 'My teaching practice').

I: With pupils behaviour you are referring to ...?

L: To how they respond in the classroom; their interest in the work and their attitude. And if the activity is consistent, then it is adequate for the pupils, according to their knowledge.

I: Could you explain again why the arrow from 'Pupils' behaviour' to 'My practice'?

L: This arrow means 'Pupils' behaviour' affects 'My practice'. Of course, that's what I was saying.

I: And would that be a one way arrow?

L: This one is a double way arrow because what I do affects the 'Pupils' behaviour'. Okay, I said that my conceptual understanding influences 'My teaching'. And in the measure that I evaluate 'My practice', these two [types of knowledge] revalue, so here there is a double arrow too.

Asked to identify the factor or factors that influenced his teaching in a stronger way, he said: "the teacher's knowledge, especially his/her conceptual clarity of mathematics".

Luis' initial concept map – Part 2

Asking Luis what he meant by the term conceptual clarity of mathematics lead him to the inclusion of more factors and the reorganisation of his map. The reorganised map has been called *Initial concept map- Part 2*, and is shown in Figure 6.16.

I: What do you mean by "conceptual clarity of mathematics"?

L: A clear understanding of/ for example, we are trying to design a module for the teaching of the different interpretations of variable but we are not clear yet on which topics go for each interpretation (see further explanations in Subsection 6.5.2.2.)

I: So could we say that what you mean with clear understanding of mathematics is knowledge of mathematics?

L: Yes it is the same. [it was agreed at this point to talk of knowledge of mathematics].

At this point Luis clarified that to teach Grade 8-algebra, a teacher needed a conceptual understanding of mathematics, and added the "Curricula guidelines" and "Time" boxes to his map.

I: We are talking about the teaching of Grade-8 algebra or school algebra. Would we need to talk about the didactics specific to algebra?

L: Well, but you/ at present we have the *Standards of mathematics* that we have to keep in mind. So we are looking for flexibility within mathematics teaching, and we also need to relate the mathematical work with other disciplines like Science, for example. According to the Standards, we need to work in terms of the types of thinking: variational thinking, numerical thinking, metric thinking; so it is not a list of topics that have to be ordered, as we have always done. That's why I said that the teacher needs a conceptual understanding of mathematics, here [in this concept map], because now we have to think of the topics which give an account of the concept of the variable, and that is for primary too.

I: When you say the topics which give an account of the concept of variable, what topics do you think of?

L: For example, areas of basic geometrical shapes, there you have topics of geometry.

I: So you are thinking too of topics of arithmetic which give an account of the concept of variable, or how are you thinking for the primary school?

L: Yes (silence). It seems that one is always meditating the topics. We [my colleagues and I] are not clear about that ye. , ilence). Again, the teacher needs a conceptual understanding of mathematics to teach the variable as it is in The Standards. The Standards represent a content guideline for the teacher, for my teaching.

I: So the Curricular Standards would be another factor that you would put in this map?

L: Yes. From the Standards I can learn about mathematics. It doesn't change what I know about mathematics but about how to reorganise the teaching.

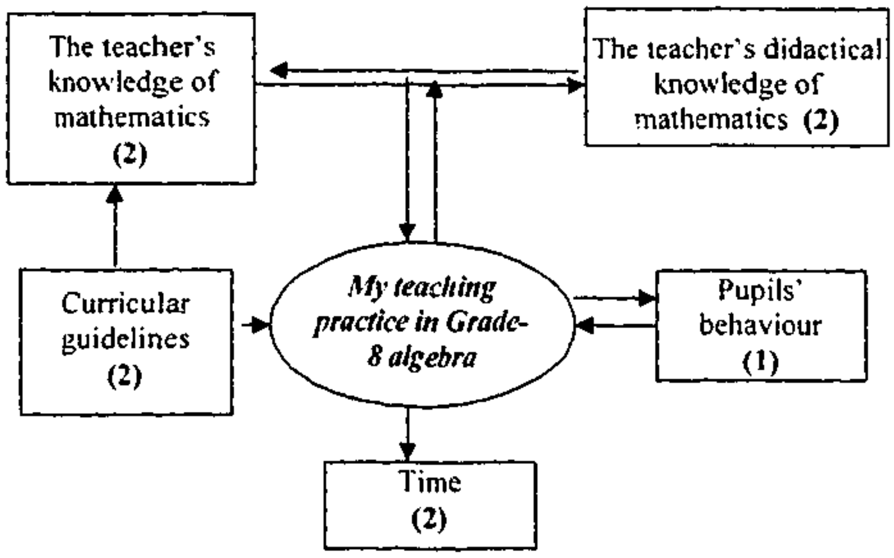


Figure 6.16. Luis' initial concept map – Part 2

Luis added another box with '*Curricular guidelines*' to his map. He also added '*Time*' as a factor influencing his teaching, as the interviewer asked him if 'time', which had been identified by other teachers as a relevant factor in their teaching, was relevant to his case. In relation to "time', he said:

> The time available is obviously our worst enemy. And we shouldn't be restricted by time because it is impossible to know how long one is going to spend. Yes, definitely time affects my teaching.

The textbook was not considered as an element that influenced Luis' teaching despite the fact that he had declared that he just followed the sequence of topics found in textbooks:

> I: Some teachers identified the textbook as an important factor influencing their teaching of Grade 8-algebra. Do you think that the textbook influences you teaching?
>
> L: We are not allowed to ask the pupils to buy textbooks in the school of the south, and that is one of the difficulties because, being extremist, even to copy some exercise on the board requires time…
>
> I: But do you think that the textbook influences you teaching?
>
> L: Well yes I use several textbooks but I don't guide myself by one text…

The factors he had mentioned at Interview 1, (i.e., the teacher's commitment, the school environment), which he had said absorbed the teacher, were not to be included in his map either. When Luis was asked why, he explained:

What happens is that, um yes, one feels that, but what happens is that one has to measure one's honesty; one has to mediate very well the work that one is going to do in those schools because the environment of those kids is very complicated. They have too many problems. I said that because I thought that my colleagues were being unethical because at one point they considered that because those pupils are not going the university, they might not need to study all the topics, and I don't agree with that...

In his Part 2 of the initial map Luis did not identify 'the teacher's knowledge' as the crucial determinants of his/her teaching. When was asked to identify the factors that affected his teaching practice in a more powerful way, he said, this time:

The ones that most influence my practice are, first, 'the pupils' behavior' and second, the 'Curricular guidelines'. The pupils' behavior because the teacher has to make a greater effort when teaching them. The Curricular guidelines because now we have to reorganise the teaching according to what the Standards say.

An important explanation was offered for this differentiation:

I: But you had said that these two types of knowledge were the ones that most influenced your practice?

L: Yes, they are the most important ones for the work that I plan but because of the pupils' behavior I have to sacrifice what I prepare for my work. Their social problem is very acute.

Figure 6.16 shows Luis' map with the assigned numbers to show the level of impact each factor had on his teaching. As we see, His "knowledge of mathematics" was, then, "first, in what he proposed to do or planned for his teaching but not for what [he] did in the classroom"."

Luis' final concept map

When Luis was presented with the boxes of his initial concept map, and was invited to draw the connecting lines, he produced the map shown in Figure 6.17 while saying:

The central axis is 'My teaching practice'. In the measure that I have knowledge of mathematics I will have better ideas for my teaching. The 'Curricular guidelines' are a parameter for what I do and also I can look at my teaching ad get feedback from the 'Curricular guidelines'. 'Pupils' behavior' and many factors of their social context influence 'My practice'. In this map I will change 'School curricular guidelines' to 'the Ministry Curricular guidelines' because the ones from the school have to be according to the Standards now.

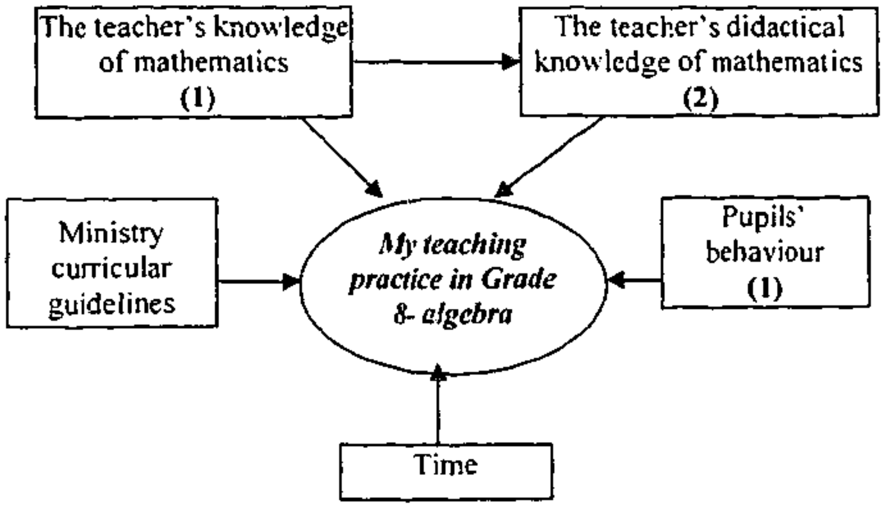


Figure 6.17. *Luis' final concept map*

The strength with which specific factors determined his teaching, as shown by the numbers he wrote in the boxes, was explained as follows:

> First I have to know mathematics to be able to teach algebra, or mathematics, and then I will be able to think about the teaching of mathematics. But pupils' behavior strongly affects what I can do (pause). Yes, I think that what most influences my practice is the pupils' behavior. ... Why? Because they go to school by force and their behavior causes me to lose the rhythm of my practice.

Luis commitment to changing his teaching practice to a problem-based approach

Luis was reconfirming at this point that the pupils' motivation and ways of learning were the crucial determinants of his teaching. He did not provide an answer, during the Focus Group nor at Interview 3, to the question of how he was thinking of addressing his concern about his pupils' lack of interest for the learning of Grade 8-algebra. At Interview 3, before the concept map was reviewed, he raised the same concern he had pointed out during the Focus Group session in relation to the teaching of "the formalisations" and then asserted that his pupils would not learn by working in problem-based activities:

> I would like to ask how *consistent* those activities are with the concept one wants to teach. The activity of 'the tiles' is a kind of unknown step in the process of learning of equations for example. I still have the doubt about the following step [the teacher does]. It seems that these activities are experiments and one does not see how they are going to do the formalisations.

After revising some of the pupils' work, described at the Focus Group, Luis agreed that a certain level of formalisation had taken place but stressed that if pupils do not learn "to do the historical exercise" (i.e., "here is this equation, solve it...") they will have problems at the university. He added:

> The pupils used their time to do an intuitive work, and yes, they did a kind of generalisation here but they did not do exercises of the historical type. ... For example, here are these equations solve them. ... And anyway, if those children/ if they don't learn when one explains everything, step by step, how to do the exercises, how are they going to respond if I give them the problems? (Int. 3)

6.5.2.2 Luis' self concept and attitude to beginning algebra teaching

Throughout the whole process of data collection Luis was conveying the message that he did not have the necessary knowledge for the teaching of beginning algebra. He declared that he did not have "clarity of the different uses of the variable" which, for him, was "one of the basic concepts his pupils of Grade 8 should end up having at the end of the school year":

> I would like my pupils to learn the different interpretations of the variable. One normally uses it as an unknown but we have to show the pupils that there are several uses of the variable; as an object, as an unknown and as generalised number (Int. 2).

Luis pointed out that "[he was] insecure about the use of the variable".

> Actually, here [in the work the children of one of the teachers discussed at the Focus Group], I am not sure how the variable is used, what to say about this when I am going to transmit this to the pupils. We [my colleagues and I] are just trying to elucidate this topic. (Int. 3)

Nevertheless, he disliked teaching algebra for reasons relating to the pupils, specifically, to their inadequate prerequisite knowledge and their problematic social background. Reinforcing his *Agree* response to the statement of Questionnaire 1, '*When I teach algebra, I feel unmotivated by the fact that my pupils don't understand the basic concepts and procedures*', he stated at Interview 2:

> I do not like teaching Grade 8-algebra when pupils have difficulties following the instructions. I like teaching algebra but when the pupils have already overcome certain deficiencies. When there is no problem in the pupil; when they understand the instructions given and know how to manipulate algebraic expressions.

The situation was exacerbated due to the social background of the pupils:

Teaching beginning algebra to this group of pupils is highly complicated and difficult because the school is one of the areas in the south of the city and there are very few pupils that are able to reach a certain level of understanding or to reach certain topics. One cannot even start working with the pre-algebra concepts because they don't manage to work even with whole numbers or have difficulties handling the variable (pause). The work becomes very demanding. It requires time and effort.

Luis "[was] interested in being a teacher but at the university level ... that's why I am doing a masters course." (Field notes). Further, he wanted to become a well known person. When he was asked if he would like to participate in a team work research project, where he would have the same financial situation he was having in his school teaching job, he answered with enthusiasm:

Yes, I would really like to participate in one of those projects with the IDEP[4]. I would like to learn how to do research. ... Why? Because I want to become a well known person in academia...

Luis' conception of his learning

Luis mentioned the word *learning* when he was asked if there was something he had changed during his four times he had been teaching Grade 8 algebra. He seemed to associate his learning of teaching with being informed about change in central educational policy, and explained that the basic reason for introducing change in his teaching was external pressure from the government regulations.

Initially I just transcribed from the work with numbers to the work with expressions, and as there was no restriction in relation to the percentage of pupils that had to be promoted to the next level, one carried on teaching the program with no problem. One didn't worry about how many pupils failed. With the requirement of the 95% promotion, I thought that I had to change something. I started asking myself, 'what is it that the pupil needs in relation to the teaching method? How can I teach this in a different way?' And now, with the Standards, one sees that there is the need to reorganise the teaching. ... Problem-solving is in fashion. Even if you don't want to accept it, there is something that contaminates the environment [is everywhere]. The theories of mathematics education and the Standards (silence). I think that I have made some progress in that sense. (Int. 3)

So what were Luis' reasons for thinking about possible changes in his teaching?

Basically the reasons for change are due to change in [central] regulations, to the pressure that one has to comply with whatever the Ministry or the ICFES require. It is also the way the pupils are responding, but there has been a continuous change of regulation and policy and the teacher has to keep up to date with the regulations.

[4] IDEP: Institute for Pedagogical Development and Educational Research of Bogotá District.

6.5.2.3 Luis' knowledge for teaching beginning algebra

This section contains a summary of the data collected on Luis' knowledge and beliefs about the teaching of beginning algebra concepts, specifically the concept of *variable*. The data were collected through Section C of Questionnaire 1, Questionnaire 2, and through the rest of data collection activities in which the teachers offered explanations for their decisions and actions in their classrooms.

Knowledge of the teaching of the concept of variable

Luis believed that expressions where the letter stands for the name of an object (i.e., $2x$ where x stands for apple) were algebraic expressions where the letter was used as a variable. This belief was made explicit at Interviews 1 and 3. At Interview 1, when explaining how he initiated algebraic work, Luis said:

> I tell them: 'one apple plus one apple is two apples'. That is very simple for them, but if I give the representation that the apple is x (writes $x + x$, as he talks), they suddenly say that it is x^2. Only a few say that it is $2x$ (silence). I know that the term variable is complex; and it becomes more difficult for them when they have to replace numbers with letters. They know that $3 + 5 = 8$ but if I tell them that x is 3 and y is 5. And ask them, what is $x + y$? That is very difficult for them.

At Interview 3, when he was asked what was something basic that he thought the pupils needed to learn in Grade-8 algebra, he said: "I'd like them to learn the different uses of the variable. That is, as an object, as an unknown and a generalised number". And when asked to give examples of the use of the letter as an object, he gave again the examples of the apples, and asserted that this use of the letter was being contemplated for next year's work:

> We [my colleagues and I] are working on a project, designing the modules for Grades 6, 7 and 8. We are still in the process of identifying which topics we are going to include in the algebra module of Grade 8, to teach the use of the letter as an object, as an unknown, and as a general number…

Luis openly acknowledged the limitation of his knowledge of the concept of variable. While showing uncertainty when he was explaining his response to question C4 of Questionnaire 1, at Interview 1, he declared that the situation posed in question C4 was not even clear for him. The following transcript presents the evidence:

I: In Question C4, you said that you would answer to the pupils who wrote 5N = D, "Do a record of numerical values and construct the proportions so that you can deduce the equation". What type of things were you thinking the pupils would do when you said that to them?

L: I was thinking that I would like them to try and do like a table: if there is 1 doctor, then there are 5 nurses, if there are 2 doctors …, etc. (While Luis was talking, he wrote the table shown in Figure 6.18).

I: Suppose they write the table and don't do or say anything else. What would you say to them?

L: I would ask them what is the relationship that we can see there. … I would expect them to do, maybe this (writes the ratios shown at the right of the table):

D		N		Ratio or representatalative fraction
1	———	5	5/1	
2	———	10	10/2	
3	———	15	15/3	

Figure 6.18. *The table that Luis drew while explaining his response to the pupils in Question C4 of Questionnaire 1*

If they did that then I would try to get them into finding the proportion by equivalent fractions. Then we would write:

$$\frac{N}{D} = \frac{5}{1}$$

Then they would have this multiplied by this (pointed at N and 1, and wrote): N = 5D.

Luis' had claimed that "with the expression 5N = D, the pupils were translating from the expression into the algebraic expression" (Questionnaire 1), but when he was working with specific numbers in the new expression he had arrived at this point (i.e., 5D = N), he became confused with the expression 5D = N. It is not clear from the data whether Luis was in this case using the letters to represent the initial of the words 'nurses' and 'doctors' or whether his confusion was due to him not being aware of the rule of correspondence defined in the given question.

L: Anyway. Why did I say find numerical values? Because it gives us the sensation that (sudden silence). I don't know how!!! because one could say Okay, $5 \times 1 = 1$? (pause) No! but if N is 1 ... (silence).

I: Why if N is 1?

L: There is something confusing here that/ even for me the exercise is not clear. I realise that it isn't clear for me.

I: If the pupils arrived at this expression (5D = N), how would you say they were seeing the letters? To represent what?

L: He[sic] is using them (long silence). Here he is using the letter as 5 Doctors (silence).

Asked if he thought that a situation like this [the nurse and doctors] could be used to teach the concept of variable, he said;

> One just emphasises the variable as an unknown. One is always analysing the topics and the situations. I am not sure here how I see the variable so even less when I going to transmit it to the pupils.

Luis said that he was not sure what use of the variable was being promoted in the activity of the 'Pedro the builder' described at the Focus Group when pupils arrived at the expression W = B + 4, where W represented 'Number of White tiles', and B represented 'Number of Black tiles' (see Appendix 4.4).

Knowledge of the interrelatedness of mathematical concepts and of different representational systems

The data show that Luis was not interested in the promoting of links between mathematical concepts in his teaching of Grade 8-algebra. He emphasised his concern for the lack of consistency between the activities described and what the pupils needed to learn. Although he me·.tioned that in the teaching of algebraic expressions, connections could be established with the concepts of areas of rectangles, "that work was being done by the student teachers". The small diagrams he used were" intended to help the pupils to *automatise* the structures of given expression and the routines to follow", Luis did not mentioned any other form of representation as necessary in the teaching of Grade 8.

Knowledge of the context of teaching

The previous section shows that although Luis pointed out his desire to learn how to teach by organising activities for the pupils, he believed that pupils learn by following the routines explained to them. There was no evidence of his awareness of the need to cater for pupils' different abilities or of pupils' common difficulties in the learning. On the contrary, he believed that his teaching difficulties were due to the fact that the pupils made him lose his teaching rhythm.

6.5.3 6.4.3 Summary of Luis' case

Luis conceptualised beginning algebra as a collection of topics and procedures to manipulate algebraic expressions that constituted the content of "mathematics for Grade 8". Teaching Grade 8-algebra was important because "it develops the formal thinking that is needed in the solution of mathematical problems of higher levels of mathematics; for example in the Calculus that he was teaching at a tertiary level". In Phase 1 of the study. Luis declared that he "just [taught] in the traditional way, giving his class a list of exercises", and that "that style of teaching [was] the reason why the majority of pupils did not like mathematics". He argued that it was important to organise problem-based activities for the classroom, so that the pupils could engage in the solution of problems and have the opportunity to discuss what they were doing.

Luis believed that algebraic thinking could not be promoted until after pupils knew their arithmetic content, "including the topic of rational and irrational numbers". He liked teaching Grade-8 algebra "but when the pupils [had] already overcome certain difficulties in their learning of Grade 8- algebra; when there [was] no problem in the pupil; when they [knew] how to manipulate algebraic expressions and [had] no difficulty understanding the instructions given". Teaching his actual Grade- 8 group was particularly difficult because besides the pupils' deficiencies in their knowledge, these pupils had acute social problems. They belonged to a poor community from the south of the city, so they had not only "academic problems but also emotional and family problems", and the social context of the pupils represented one of the most important factors determining Luis' teaching practice of Grade-8 algebra.

Despite the number of factors Luis identified to explain the fact that the pupils did not like algebra [i.e., *la problemática* of school mathematics], in which the teacher's knowledge and commitment was included, and of his declarations that he was not clear about the uses of the variable, his current pupils' behaviour, which was due to their social background, was always identified as a key factor influencing his teaching practice. His knowledge of mathematics was important for what he planned, but for his practice —in the classroom— his pupils' behaviour was the *number one* factor. The pupils "made [him] sacrifice the classroom work he had prepared, which affected the rhythm of his practice", and that was not the case with his pupils of the private school (of the north) because they did not have problems in the academic sense.

The need to design the problem-based activities that he had initially argued for lost importance after Luis participated in the Focus Group as besides the fact that designing activities required "a conceptual clarity of mathematics" on the part of the teacher it also required time and effort. Furthermore, he was not sure about the activities illustrated and discussed by his colleagues during the Focus Group session because those were not consistent with what the pupils had to learn. After all, in those activities the pupils did not work in the formalisations of mathematics. They did not do "the historical exercise, like *this is an equation, solve it*". If they didn't do that they were going to have problems later and at the university.

The need to design problem-based activities completely vanished when he was asked how he was thinking of addressing the concerns he had raised regarding the pupils' lack of interest for the study of Grade 8-algebra. His current pupils' behavior and lack of prerequisite knowledge was going to remain the main problem in his teaching practice because if they did not respond when they were told everything (i.e., how to do the exercises), "how [were] they going to respond in the case that he gave them the problems?" Further, even if his pupils were not going to go to the university, they needed to learn all the list of topics and the historical exercises (i.e., the formalisations) because everybody should learn the same mathematics and, anyway, they were going to be evaluated through the external examination of ICFES.

Luis would like to participate in the 14-month, classroom-based research project presented to him at Interview 3 because he wanted to learn how to do research. Why? "Because I want to become a well known person in academia".

6.6 Alex – The *mathefobia* battler

Alex was teaching mathematics in a private school for girls, which was located in a suburb area in Bogotá that is inhabited by people of a middle socioeconomic background. The school was a day school with a very welcoming atmosphere created by both the academic and administrative staff. According to Alex, all his time, including part of his family time was dedicated to his work in the school.

Alex had taught in the primary school for a year, and 2002 was his third year teaching mathematics in secondary school. Alex declared that, including the teaching practice he did for his degree in Physics and Mathematics Education, in 2002 he was teaching in Grade 8-algebra for the third time. Alex "decided to participate in the study because [he] thought that having the opportunity to interact with other mathematics educators [could] represent a good experience" for him. Alex seemed to be a little tense at Interview 1. He declared that

> being two [participants from his school] with my colleague[5] made me feel more supported.

However, as the number of lessons observed increased, he started to show more enthusiasm for his participation in the rest of the data collection activities.

6.6.1 Alex's conception of beginning algebra

According to Alex's answers in Questionnaire 1, the main purpose for the teaching of school algebra is "to provide individuals with the opportunity to develop the critical thinking needed by every citizen". In relation to this answer, Alex explained at Interview 1 that "an individual has critical thinking when he/she has the capacity to see things and reflect about them and give his/her point of view".

> An algebra classroom provides the opportunity for the pupils to present and discuss their ideas, for example, when they are solving word problems. That type of work helps the girls to organise their minds and to structure their thinking. If in the lesson they have the opportunity to give their ideas and opinions, the space for the development of that type of thinking is provided. (Int. 1).

[5] The other teacher from Alex's school, who participating in the study, was José.

Beginning algebra work had to be with the formal definition of algebraic expressions because "to learn to operate with algebraic expressions you need to know first the concept of algebraic expression" (i.e., the formal definition).

> We start by defining an algebra expression, that it is a set of letters and numbers that are separated by signs and with a determined structure. We then talk about algebraic expressions and algebraic polynomials. I explain what a term is, what binomials and trinomials are, like terms, etc., in order to start doing addition and subtraction with those. We then move on to all the operations with polynomials. (Int. 1)

As the group of teachers in Agudelo-Valderrama (2000), Alex believed that to know a specific mathematical concept was to know the formal definition that is found in the textbook. Insightful evidence was provided by Alex at Interview 3 when he was asked to explain further what he was referring to with the word "formalisations" during the Focus Group:

> A: To formalise for me is to arrive at the conceptual part.
>
> I: Could you illustrate with an example what you mean?
>
> A: For example the concept of function. Teacher B [of Questionnaire 2] worked with an example of the applications of functions but she [sic] has to end up with the concept of function.
>
> I: The concept of function (pause)?
>
> A: The one that comes in the text: 'If A and B are two set of numbers, and x is an element of A…'
>
> I: Oh! the formal definition that one finds in textbooks.
>
> A: Exactly.
>
> I: [After looking at specific classroom work that had been discussed during the Focus Group] So when the children arrived at the expression W = B + 4, when they were working on the activity of the tiles (see Appendix 4.4 for a description of 'Pedro the builder', during which this expression was constructed), were they not formalising?
>
> A: $W = B + 4$ is not a formalisation; the formalisation would be to define a linear function. It would be to construct the concept as we find it in the textbooks.

Alex explained that "algebraic thinking can be promoted in the primary school levels "when we ask the children to reason with letters. There is algebraic thinking when we give definitions using symbols rather than natural language". For example,

> when I express a rational number with letters [writes: a/b], the letters represent any number.

Algebraic thinking "can be promoted in the primary levels", for example:

> When I was teaching in primary, I taught the children to express things using

mathematical language; an example: 'to write M ∈ P instead of writing Mercury is a planet'. Becoming familiar with this mathematical language helps them for when they arrive at real algebra. (Int. 1)

6.6.1.1 Learning beginning algebra

For Alex learning beginning algebra takes places by repeated practice and repeated explanation (on the part of the teacher) of the steps to follow when operating with algebraic expressions. In his responses to Questions C3 and C5 of Questionnaire 1, which presented situations where pupils had made mistakes when simplifying like terms or isolating the variable, he would tell the pupils "to revise the indication given" in relation to each case. He further explained at Interview 1 that he "always [gives] them the indications when the topics are introduced".

Asking pupils to check the given indications or "re-explaining the topic to the class, if necessary" were Alex's ways of responding to pupils' difficulties. He reconfirmed this when he was asked, once again, how he would respond to a group of pupils who responded in the same way Ricardo of Question C5 did:

I: Suppose that fifty per cent of your pupils of Grade 8 make this mistake in one of the quizzes. How do you deal with that situation?

A: I'd tell them what I wrote here [in the questionnaire]: 'I recommend them to revise the indication given for isolating the variable'.

I: And so they do the revision. Do you think that they will clarify with the revision?

A: Some do the revision, the majority don't do it but, anyway, we correct the quizzes afterwards. As a general rule, as soon as they finish the quiz, we do the corrections at the board. At the beginning when we start the work with equations, the necessary indications to isolate the variable are given, that if this number is positive [on one side of the equation], then it goes to the other side as a negative, etc. But they forget it.

Alex commented at Interview 1 that the pupils' errors when isolating the variable was "something normal", it was "a typical mistake... One sees students of higher grades making the same type of mistake". Furthermore, he attributed the pupils' errors, when solving simple equations, to problems with memory. At Interview 2, he explained that he reminded the pupils of the specific steps to be followed, before they set out to solve the given equations "because when they are isolating the variable they forget the steps to follow".

I: Why do you think they forget how to isolate the variable?

A: First of all, memory because the majority of students work mathematics with memory.

I: So the problem here is a problem of memory?

A: It is memory, in part.

The need for repeated explanation was emphasised, once more, when explaining his response to the pupils who did not do well in the quizzes he gave them "with some frequency".

> I look at the difficulty that the majority shows and I explain specific point again. If different pupils have different difficulties, then I go over the whole topic, again, if necessary.

6.6.1.2 Teaching beginning algebra

Without adding any other options of his own to the 'teaching style' descriptors provided in Questionnaire 1, Alex assigned numbers from 1 to 5 to the given descriptors, identifying no difference between *preferred* and *actual* practices as can be seen in Table 6.5.

Table 6.5 Alex's teaching style priorities

Preferred	*Teaching style descriptors in Questionnaire 1*	*Actual*
1	Providing opportunities for pupils to develop their communication skills so that they can express their mathematical ideas with confidence	1
2	Designing activities that provide space for pupils' self-paced learning	2
3	Organising problem-based activities for the pupils to work in small groups where they can present their ideas to the whole class for discussion	3
4	Giving clear explanations of definitions and procedures to follow in different exercises and problems of application in the topics studied	4
5	Designing classroom work that promotes connections between different mathematical topics studied	5
5	Giving pupils lots of exercises for algorithm application as homework	5
5	Testing pupils at the end of each activity or topic, in order to have sufficient marks for assessment in each Attainment target	5

The descriptors to which he assigned the numbers 1, 2 and 3, which he said, at Interview 1, represented his teaching style top three priorities are the focus of attention in the following subsections.

Priority 1 for *preferred* and *actual* practices: Providing opportunities for pupils to develop their communication skills:

The reason why Alex ranked the descriptor 'Providing opportunities for pupils to develop their communication skills so that they can express their mathematical ideas with confidence' as his first priority was because in his Grade 8 class "all pupils [had] many opportunities to participate in the lessons" as he had "a small group of sixteen pupils".

> They have the opportunity to present their questions and ideas, and to discuss with their peers what they are doing during the lessons. (Int. 1)

At Interview 2, Alex provided more information about the opportunities to develop communication skills, when he was asked about the purpose of four (of the five) consecutive lessons observed. Figure 6.16 shows a few examples of the list of 25 tasks on which Alex's class was working during these four consecutive lessons. Alex referred to this list was as the "*guía*" by.

"Guía de matemáticas"
Solve the following equations:
1) $3x + 5 = 1$
9) $2(x + 4) - 1 = 0$
24) $-2/3\ (x - 5) = 3/2\ (x + 1)$

Figure 6.19. Three questions from the list of 25 of Alex's "guia".

The purpose of the work done in the four lessons was "that the pupils were drilled in the operations and the isolation of variables..."

> The mechanical part is fundamental in mathematics; if they don't know how to solve equations they cannot solve the problems.

In relation to the development of communication skills, Alex declared that

> They discussed [during the lessons] about how they did or how they were doing the exercises, but there is more opportunity to discuss their ideas when they are working on open problems. (Int. 2)

Priority 2 for *preferred* and *actual* practices: Providing space for pupils' self-paced learning

Designing activities that provide space for pupils' self-paced learning was Alex's second teaching style top priority. This provision took place "not because [he] design[ed] activities that allow for pupils' different ways of working, but because"

> I give several deadlines for pupils to hand in their work or to present the required test... and because I explain again whatever is necessary to pupils who have more difficulties.

Priority 3 for *preferred* and *actual* practices: Organising problem-based activities

According to Alex's responses to Questionnaire 1, 'Organising problem-based activities for the pupils to work in small groups where they can present their ideas to the whole class for discussion' was one of his three top (i.e., Number 1) priorities. An example of an 'open problem' was:

> Find the volume of a cube whose sides are such and such. This is an open problem because each student can do that task in their own way... They can explain to the class their working processes; how they do it.

The problems are given to the class after they have learned how to manipulate the expressions.

> In algebra, basically when we talk about addition we talk about perimeter. Then we move on to, for example, volume of a cube... They first need to know how to operate with algebraic expressions. If they don't know how to manipulate the expressions they can't solve problems... Working the topics in this *logical order* helps the learners understand algebra.

Although in Part A of Questionnaire 1, Alex' ranked 'Giving clear explanations of definitions and procedures...' as a secondary aspect of his teaching, in his responses to Part C of the questionnaire and in his descriptions of what he did in his teaching, he emphasised the need to "give clear explanations of definitions and procedures to follow in the exercises". His responses in Questionnaire 2 further corroborated his preference for a transmissive and instrumentalist approach:

> Teacher A's approach is 'good' because he/she follows the logical order that allows the pupils to comprehend the topics... Giving clear explanations of definitions and procedures, and focusing classroom work on the development of pupils' abilities to do the list of exercises posed in the textbook is very important in my teaching. (Questionnaire 2)

But Alex believed that in his teaching he followed a problem-solving approach:

> My teaching of Grade 8 contains aspects of both teacher's A and Teacher' B approaches because, on the one hand, I make emphasis on pupils' developing skills in applying rules of procedure to exercises but, on the other hand, I give attention to the solution of word problems. (Questionnaire 2)

One of the concerns that Alex put forward in relation to his teaching of Grade 8 during the Focus Group was related to the fact that the textbooks available [commercially] presented the topics in the traditional way and did not place them in a contextual situation:

> ... we need to give more attention to problem solving; we need to integrate algebra to other areas of knowledge but there's nothing of that available. There are many topics which come decontextualised [in the textbooks].

Alex elaborated on this point at Interview 3, and explained that he had never seen in textbooks anything like what teacher B had done, but emphasised that "the traditional work cannot be put aside..."

> Furthermore, following Teacher A's approach has the advantage that the teacher saves time.

Assessing pupils' work

Alex provided more evidence of his conceptions of Grade 8-algebra as a collection of fragmented procedural techniques for the manipulation of algebraic expressions. In relation to his preferred forms of assessment (i.e., frequent quizzes, and tests at the end of a topic), Alex stated, at Interview 1, that frequent quizzes were his "principal form of assessing" his pupils, "but not in order to have sufficient marks for each pupil at the end of a term", at it was said in Questionnaire 1, but because it was "important for the teacher to know how the students are doing". Alex, added:

> in some sense, however, it is necessary to have a number of marks because I need to have evidence of what the pupils do and know in every topic.

Alex commented at Interview 2 that he normally had to use part of his family time to mark pupils' homework. This was because

> the lessons normally start by correcting the given homework. So everyone gets their notebooks, we go over each exercise, if necessary, and the quizzes are normally given after the homework is corrected. Otherwise we cannot move on the next part of the topic, or on to the next topic. ... Tasks for quizzes are taken from

the set of tasks of the homework, or sometimes they were a bit different.

Alex "strongly agree[d]" that 'Giving pupils marks and rewards are good strategies for getting them to complete mathematics assignments' (Questionnaire 1). When he was asked to explain why he strongly agreed with the statement, he said:

> If you don't give marks for their work they don't do the work. Unfortunately it is like that. We belong to a culture of reward and punishment since we are children in our homes... (Int. 1)

Another way of assessment was through the students' work in what Alex called the "*guías*". As explained before, the *guías* were "a compendium of tasks and word problems that summarise the different exercises and work done for a specific topic or unit or study", and these were normally given to students "when they owe me[6] some attainment targets".

> I give them the guía, they work on those exercises and, then, they do the test.

Another example of "a *guía*" that Alex provided was a list of "20 exercises of factorisation". Alex described how the topic "factorisation" was taught and what the pupils needed to do when they were solving the questions:

> We saw all the cases of factorisation, paying attention to those they need most for the solution of quadratic equations. ... When they [the pupils] need to factorise a given expression, they identify to which each case it belongs, to be able to decide how to factorise it (Int. 2).

6.6.2 Alex's conception of his own teaching of beginning algebra

In the previous section we saw that when Alex responded to Questionnaire 1, he did not see differences between his *preferred* and his *actual* practice. To my comment in relation to his rankings in the questions of Section A of Questionnaire 1, that some teachers had different ranking for 'preferred' and 'actual' practices, Alex responded:

> Well, one does what one plans; the problem is the performance of the students. The students' performance is just average... (Int. 1)

[6] With the expression 'the pupils *owe* me some attainment targets', Alex meant, "they failed in the test, they owe that target", and so "they need to do it again". This was a common expression used by the teachers of my previous research in Colombia.

We will see in the following subsection that through the three-phase concept map activity in which Alex was asked to identify the determinants of his teaching, he emphasised the problem of "the pupils phobia of mathematics", and role of "the pupils' motivation" in the learning of algebra. He emphasised aspect belonging to the pupils, despite his acknowledgment of the role of "the teacher's knowledge" on his/her practice.

6.6.2.1 Alex's conception of the determinants of his teaching practice

Alex wrote down list of factors when he was invited to think of the aspects he thought influenced his teaching of Grade 8:

- The teacher's constant "actualización" (i.e., "keeping up to date"
- The school environment
- The teacher's position,
- The pupils' position

Alex explained why each of the factors influenced his teaching, starting with the pupils' position. The dialogue that developed from this, leading to Alex's building of *Part 1* of his *Initial concept map* is presented in the following paragraphs.

> ***The pupils' position*** relates to their pre-requisite knowledge and the processes that the pupils have because one is obliged to reinforce those processes. One has to be subjected to what the students bring in, and that affects my teaching.

For Alex, "what the students bring in also includes their fear of mathematics" and he had to battle in that.

> I: In what sense does what the students bring in affect your teaching?
>
> A: When I start with a new group I have to adapt myself to the group. If the students see mathematics as "el *coco*[7]," if they are terrified by it, then it is going to be very difficult for the teacher. Some of the girls have a fear of mathematics, and they believe that they are stupid. That's what they say. So I had to battle with them when I first started in this school. Nowadays, things are a bit different, they know me better and they are more *dedicaditas*[8].

> Now, ***the teacher's position*** refers to his/her financial situation. It also refers to his/her position within the school. That has to do with personal relations, which depend on the school environment, and also with whether the teacher is an open person or not,

[7] The equivalent in English of "El coco" is 'the bogey man'.
[8] The word *dedicaditas* Alex meant, "being receptive to being given instructions and applying themselves to the work required".

> whether he/she is an accessible person because this affects the relationship with the girls… A teacher who has no financial worries can dedicate more time to his/her work. He/she can dedicate time to think about doing new things for the classroom; he/she has the *disposition*, while the teacher who has financial difficulties finds himself [sic] in the obligation to have two or three jobs, and the time he can dedicate to his teaching is not the same. So the time available affects the *motivation* for his [sic] work..

I reminded Alex that we were talking about his own case of his teaching of Grade 8-algebra, and not about the case of teachers in general. Asked to clarify if there was as distinction between his 'position', his 'disposition' and his 'motivation', he explained that the right word was *motivation*. Note that Alex when he explained why 'motivation' was the right word, he ignored the request to speak about the topic using the first person, and so I decided that the conversation should proceed referring to the third person if that was the most comfortable way for Alex:

I: Right. You were explaining about *the teacher's position* as one of the factors that you put in your list. When you mentioned the teacher's financial aspect, and time available, you said that those affect the teacher's *disposition*; and now you have said, that the time available for the teacher affects the teacher's *motivation*. Do you see a distinction between the teacher's *disposition*, the teacher's *position*, and his/her *motivation*?

A: I was talking about the teacher' position (pause). Actually, all that affects the teacher's *motivation*. So I am going to leave 'motivation' rather than position. The teacher's motivation depends on all those things that I have mentioned, and the teacher's *motivation* affects the student's *motivation*. So it's *the teacher's motivation* and *the pupils' motivation*, here (pointing at the corresponding items of his list).

Eliciting teachers' information about their own cases is not an easy task. There were tensions for me between gathering information about the teachers' beliefs about their own knowledge, and not placing them in uncomfortable situations. So if, at a specific point, the teacher was talking about "the teacher", and not about their own case, that had to be accepted because I was aware it was a sensitive issue (see further discussion of this point in Chapter 8, Subsection 8.4.3).

Alex's initial concept map- Part 1

At this point Alex was invited to write the names of the factors in the "post it" provided, and to start building the concept map. He placed the '*The teacher's motivation*' and '*The pupils' motivation*', boxes saying: "The motivation is a two way thing", while drawing the double arrow connecting the two. The discussion continued:

I: And you named *time* available because you can have two jobs. Can't you?

A: That is not my case because I dedicate all *my time* to this job. Fortunately, at present, I have the financial means to be able to manage with one job. ... But even so, the *time* that is given for curriculum in the school timetable is not enough. I use a lot of time at home marking homework and preparing the '*guías*', and even so time is always short. One is always worried that one is behind with the program. So the *time* assigned for school work is another factor that affects my teaching. (Alex added "time assigned for school work" to his map).

In relation to **the school environment,** Alex thought that that was "a too large and complex factor to describe. "It relates to things like the kind of relationships established between colleagues; that one feels comfortable in the school.

> For example, if the teacher shares things with other teachers, if the students appreciate what one does; all those things influence how the teacher feels.

The remaining factor to be explained by Alex, from the given list, was *the teacher's constant "actualización"*:

I: And the teacher's constant *actualización*?
A: ***The teacher's constant** 'actualización'*. That, again, depends on the teacher's motivation and vocation. Personally, I think that if someone likes his/her subject, if one has a love for knowledge, one tries to find new things for them [the pupils]. One tries to transmit that knowledge in a way that actually touches them, in such a way that the mathematical knowledge can be preserved. Our ultimate goal as teachers is to preserve the knowledge that has been accumulated throughout many centuries.

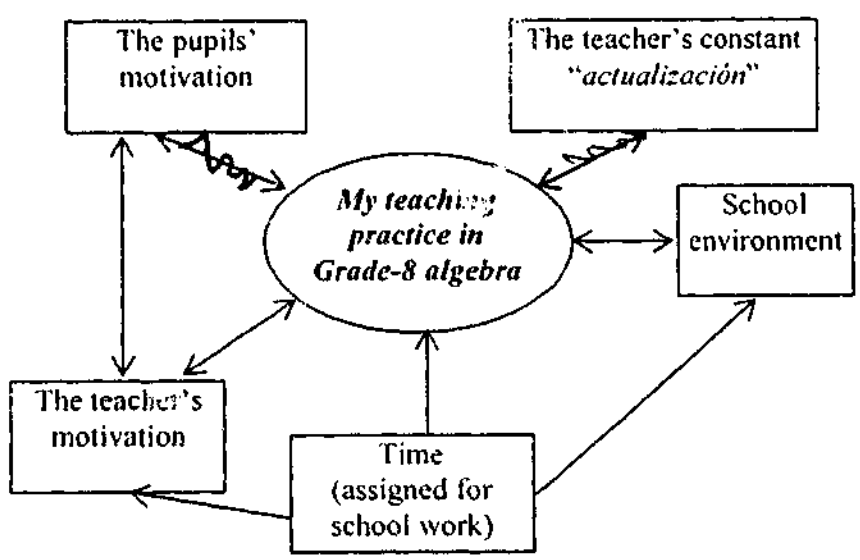


Figure 6.20. Alex's initial concept map – Part 1

Asked to identify the factors that influenced his teaching in a decisive way, and showing any differences in the strength with which each factor influenced his teaching practice, Alex highlighted two arrows as shown in Figure 6.20. He explained:

The pupils' motivation is the strongest because while you are at the university, you are asked [by people], 'what are you studying?' 'I am studying Physics". And what is that stuff? Well, mathematics plus other/ 'Oh, you are a genius'... So when you face the pupils for the first time, there are students for whom mathematics has been very difficult! And the same for their parents, 'I heated mathematics' [they declare].

Alex's initial concept map – Part 2

Having asked Alex to explain further what he had said about "the teacher's constant *actualización*" brought the issue of the teacher's knowledge into the discussion, giving origin to Alex's second part of his initial concept map (See Figure 6.21). The teacher's knowledge was an aspect that Alex thought influenced her/his practice. He identified this factor when he was talking about the case of Teacher B's (of Questionnaire 2) teaching approach when he himself brought the topic into the discussion.

I: You said that the teacher' constant '*actualización*' had to do with the teacher's motivation, with trying to find ways to transmit mathematics knowledge in a way that touches the pupils. What do you mean when you say 'in a way that touches the pupils?'
A: Yes; that is has a relation with their world, with what they like, with something that is concrete for them. For example, in the case of Teacher B. Teacher B used a contextual situation, a problem situation that is accessible for all the pupils. He [sic] has a motivation to show mathematics from a different point of view.
I: And on what, do you think, depends that a teacher be able to organise classroom work that is accessible for the majority of his/her pupils?
A: On his/her knowledge.
I: On his/her knowledge of (small pause), for example, algebra?
A: No; on his knowledge in general.
I: On his knowledge in general, but in the case of the teaching of algebra?
A: Exactly! The teacher needs not only knowledge of mathematics. I call knowledge all tat the teacher can bring in when he/she is teaching, and when one is teaching, the knowledge that I use is not bound to mathematics. I like Chemistry and Physics and all knowledge is connected.

Alex declared that "to teach Grade 8-algebra the teacher needs knowledge of all kinds". Was Alex considering "the teacher's knowledge" as an important determinant of what s/he does in the classroom?

I: Would you consider the teacher's knowledge an important determinant of his/her teaching? Would you include it in your list of factors that influence your teaching?
A: Well, yes. But I wasn't looking at it from that point of view (pause).
I: How were you looking at it?
A: What determines what I do is (long pause) but yes, when I speak of the pupils' pre-requisite knowledge, I should speak of the teacher's knowledge. Then, *knowledge* is of both sides; that is, *the pupils' knowledge* and *the teacher's knowledge*. And *motivation* is of both sides too; *pupils' motivation* and *teacher's motivation*. And that motivation has a direct relation with the teacher's knowledge.

I: The more knowledge the more motivation?

A: No, on the contrary; the more motivation, that is for constant actualisation, the more knowledge. I need to change all these names [in the map]. And the same applies for the pupils.

Alex removed 'The teacher's constant '*actualización*' box, and replace it with 'The teacher's knowledge and 'The pupils' knowledge' boxes, completing his second part of his initial map (See Figure 6.21). Hi did not want to identify categories of knowledge (i.e., different boxes). When I commented that other teachers had categorised the teacher's knowledge into *Mathematical knowledge* and *knowledge for the teaching of mathematics*, he replied:

> For what purpose do we need to separate knowledge into types? There is an interaction between knowledge and practice and between practice and knowledge. And it is the same with the pupils: the knowledge that the pupils have or that the pupils acquire directly influences my practice. Everything is connected.

Asked, again, to show with numbers the level of strength with which the factors influenced his teaching practice, Alex identified this time '*The teacher's motivation*' and '*The pupils' motivation*' as the 'Number 1" factors, but emphasised, once again, that the pupils' motivation was the stronger of the two because

> there is always a strong aversion for mathematics. A situation for which one doesn't find the 'why' or the 'where it comes from' (pause). Maybe, it has to do with what is being taught and how it is taught.

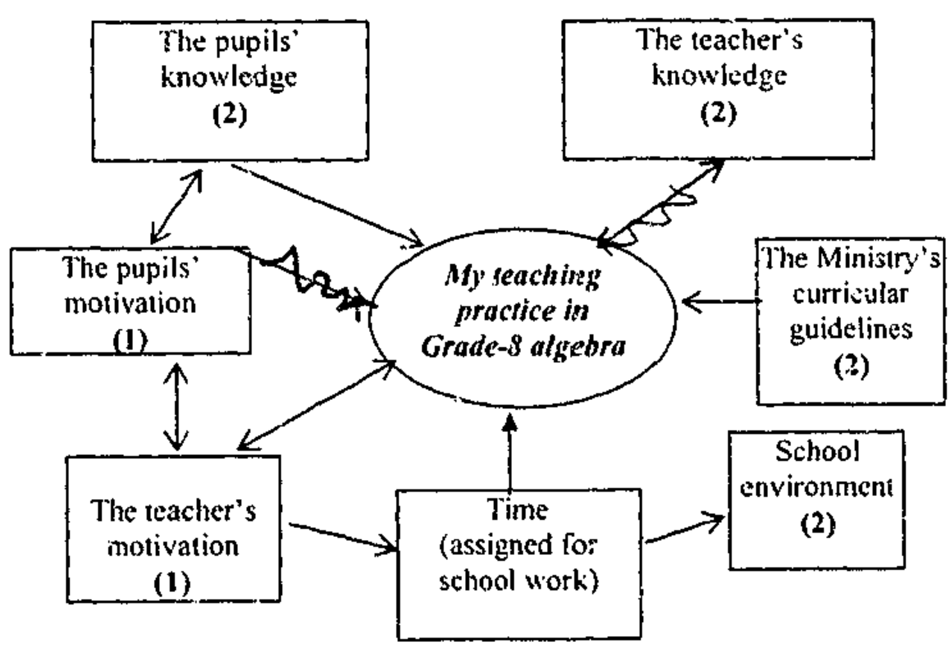


Figure 6.21. Alex's initial concept map – Part 2

Alex mentioning that the pupils' aversion for mathematics may have to do with what is been taught gave origin to his identification of "Curriculum guidelines as another factor influencing his teaching.

> I: Yes. And on what, do you think, depends the 'what is being taught and how it is being taught'?
>
> A: On the curricular guidelines; on the curriculum.
>
> I: The curriculum guidelines. Are you talking about the one of the Ministry or the ones you have here at the school.
>
> A: There are two levels: the ones that the Ministry imposes, and the ones we have at school. Obviously, the one at the school are based on the ones of the Ministry... The curricular guidelines from the Ministry, the ones in which the topic objectives, the approaches and the assessment indicators are defined; the ones of Vasco[9]...
>
> I: And do those determine what you do in your classroom?
>
> A: Those in some way direct me.
>
> I: So the C\curricular guidelines would be another factor that you would put in you map?
>
> A: Yes!

After Alex added 'The Ministry Curricular guidelines' box to his map, putting a number in it, he was asked whether the textbook was a factor that influenced his teaching:

> I: Some of the other participating teachers of this study named *the textbook* or the textbooks available as one the factors that determine their teaching. What do you think of that?
>
> A: I wouldn't say that I guide myself by the pupils' textbook or by a textbook because I look in several textbooks, and I myself design the *guias*. I don't think that the textbook determines my teaching practice.

Alex's final concept map

When reviewing his concept map[10] at Interview 3, Alex clarified: "the motivation is a practical relationship that is difficult to show in the map". Crossing out the 'The' teacher's motivation' and 'The pupils' motivation' boxes, he said:

> Actually as it is a practical relationship between the teacher and the pupil, when I talk about *motivation*, we are talking both of 'the teacher's motivation' and 'the student's motivation'. And further, the students' motivation is related to their knowledge. Knowledge and motivation are bound together. It is difficult to show it in the map because it is like a relation between the pupil and the teacher but is not separated from knowledge. Knowledge and motivation are stuck together... It is like something here between knowledge and practice...

[9] Alex was talking here about the National Curriculum Statements that were in force until 1994 when the General Law of Education was issued.

[10] We need to remember that this time Alex was presented with the drawing of the boxes his initial concept map.

Alex drew the box with "Motivation" as shown in Figure 6.19, and explained that the arrow connecting "The teacher's knowledge and "The pupils' knowledge" was there because

> the knowledge the pupils can have or develop depends on the teacher's knowledge and pupils knowledge can motivate the teacher to increase his knowledge.

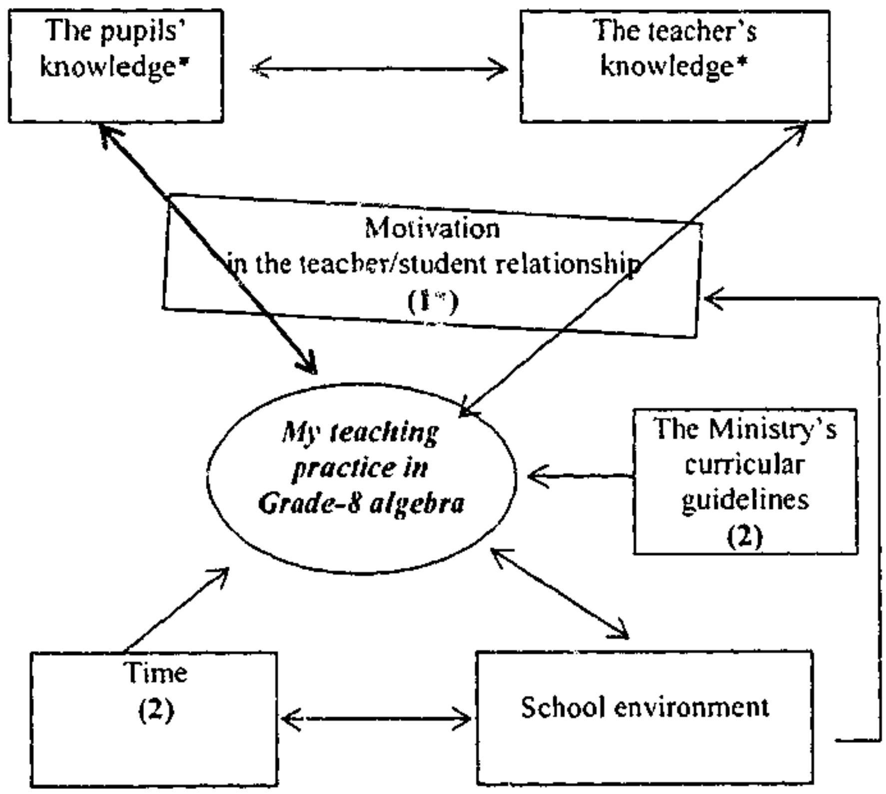


Figure 6.22. *Alex' final concept map*

When identifying the factors that influenced his teaching in a stronger way, by giving numbers to show ant possible ordering, Alex emphasised "the pupils' aversion to mathematics", again, an the 'Number 1' factor influencing his teaching practice. He said:

> The 'Number 1' is all this triangle (pointing at the one delineated by the 'Pupils' and teacher's knowledge and motivation boxes). I put the number 1 here, and it shows that all the three are connected (writes an asterisk (*) in each box to show this), but the pupils' motivation is the strongest (highlights the arrow connecting 'pupil's knowledge and motivation' with 'my teaching practice' (See Figure 6.22). The aversion they have to mathematics is a great problem!

It is interesting that Alex did not identify the text book as a determinant of his teaching, when reviewing his concept map, especially when, at the Focus Group, he had related the difficulty of introducing a Problem-solving approach to the fact that "in textbooks everything comes decontextualised". This problem with the textbooks had been highlighted before the revision of the concept map:

> I; At the Focus Group, you said that your main concern in your teaching of algebra was represented in the pupils' fear of mathematics. What do you think of doing in order to address this situation that you see in your teaching?
>
> A: A good way would be to work in activities that help them see the application of mathematics as some of the teachers discussed at the Focus Group did. But that is a very slow process... One is put in a dilemma; on the one hand, one would like to separate one's teaching from the list of subject contents and see other things different of mathematics, that is to use mathematics for something which is very interesting and, on the other hand, I need of that list contents to be able to guide the pupils. Further, I have to cover a program that the Ministry imposes, and pay attention to what the ICFES requires.

Alex argued, once again, that the textbook was a crucial factor in the introduction of change in his teaching, highlighting that a mechanical approach to mathematics was "part of the culture", where the textbook approach played a crucial role.

> I: But I thought that the Ministry provides just a set of general guidelines, as they are encouraging schools and teachers to work toward the flexibility of the curriculum and the introduction of innovation. I think that is what I read in some document.
>
> A: Yes, but it is very difficult Cecilia! There is a vacuum in the cultural part. All that one finds in the textbooks is tied to a list of contents. It's a mechanical mathematics; one cannot get out of that scheme. We [at this school] are trying to change and focus on understanding; but even Teacher B [of Questionnaire 2] is constrained by that scheme because although he/she got out of the parameters, she is going to end up in the concept of Function [i.e., the definition found in textbooks].

In summary through the concept maps, Alex emphasised the influence of the pupils' knowledge and motivation, the teacher's knowledge and motivation, but in his explanations of what he did and what took place in his classroom, the 'Number 1 factor influencing his teaching was the students fear of mathematics. Although the textbook brought everything deconetxtulaised, it was not a determinant of his teaching because he used several textbooks when planning the '*guías*'.

6.6.2.2 Alex's self-concept and attitude to beginning algebra teaching

The descriptions presented in previous sections have provided some indications of Alex's beliefs about beginning algebra and its teaching and the associated attitudes. This section will take us further into Alex's beliefs about his knowledge of algebra and of the teaching of algebra, the determinants of his teaching practice, and his learning.

Alex declared at Interview 1 that he liked teaching algebra despite the difficulties he had encountered as a beginner teacher. He explained his difficulties, first of all, in terms of "the need to adapt to a different school environment [from the one of the previous school]" and, secondly, to "the [inadequacy] of his knowledge for the teaching" of the subject. In relation to his need to adapt to a different school environment, when explaining his "Strongly agree" response to the statement 'I enjoy teaching algebra' of Questionnaire 1, he said:

> I like teaching algebra because I like mathematics in general, although it has been somewhat difficult in this school because I have had to battle in order to dislodge the phantom of mathematics...

That some girls had a fear of mathematics was a claim that Alex made both repeatedly as we have seen it in previous sections. However, he also observed that some of the students had a type of thinking which was different from his, showing that they had some abilities more developed than his because they lived in a different environment:

> But you find students that can beat you sometimes; for example solving a problem. One sits down for two or three hours trying to solve a problem, and it is gratifying when one has found the solution. Then, one gives it [the problem] to the student, and she solves it in five minutes! 'But why did she solve it in less time than me? (pause). These are things that one thinks about, and that are important. Some of them have some abilities that are more developed than one's own because they are in a different environment... (Int. 2)

With the expression "a different environment" Alex was referring to the socioeconomic background of the girls which was according to the area where the school was located:

> I: When you say they are in a different environment (Alex interrupts).
> A: They belong to wealthy families from this area or from better areas than this one. And my first year of teaching was in a primary school, School X[11]

The issues of "the students' different environment" and their "motivation for the learning of algebra" represented important factors that Alex identified as influencing his teaching. He mentioned "the students' different environment" in each of the interviews as a factor that made his teaching more demanding.

---

[11] This is the name of a private school that has a high reputation for academic standards, in Bogotá, but it is not normally chosen by wealthy families.

> I had to first find out what they understand and don't understand, but also about what they like. For example, they don't like the schema of addition, subtraction and multiplication, etc.. They don't like that one talks to them about abstract things like vectors; they like that I talk about things of their world, for example body weight, the scales... (Int.1)

But the students' lack of motivation, which was explained by their "aversion to mathematics", was the factor that Alex put forward as *crucial* in his teaching. At Interview 2, he talked again about his struggles with "the students' motivation".

> I: When do you consider a lesson to be successful?
>
> A: Successful in relation to everything or in to the teacher- pupil relationship?
>
> I: In relation to whatever you want to consider.
>
> A: A successful lesson is one where one sees good participation from the students, when they show interest in the lesson. When they feel that what is being done in the classroom is important.
>
> I: Would you consider any of the lessons that you allowed me to observe a successful lesson?
>
> A: As they took place? Actually, that is one of the groups where one can work in a good/ in the first place there was discipline. They are already used to the type of work that is given. There was good participation. It is very difficult that the girls participate, and to break up with the schema of the aversion to mathematics. They are more *dedicaditas;* they now know that mathematics is important and that the person who is in front of them knows where he is leading them to.

In relation to his perceptions of his knowledge for the teaching of the subject, Alex gave indications, at different times, of his lack of confidence as a teacher of Grade 8. At Interview 2 he declared that the mathematics he had learnt "in school [was] not adequate for [his] teaching" and at Interview 3 he corroborated this.

> One leaves school thinking that mathematics is all those things that one did mechanically. And when one leaves university one goes around blind folded... When I started here I tried to do the same as I used to do when I was at school because that's the way one was educated. 'How did I use to do this?' by multiplication/ mechanically. One realises that that is not working... (Int. 2)

> I realise that to improve my teaching practice I need to analyse why what I am teaching is important for the pupil. But many times one does not know. For example, last year when I was teaching the Pythagoras' theorem, in geometry, a student asked me, 'why do I have to learn this; is it going to save my life or what?' And I didn't know what to answer/ there are many teachers who don't know why they teach algebra... (Int. 3)

Alex's conceptions of his learning

Alex said that learning about the teaching of mathematics[12] provided opportunity for learning more mathematics.

> When one learns about the teaching of mathematics, one learns more mathematics. One encounters new situations and things every day; even in the process of this study, one has gained a lot because one encounters things and situations that one had not seen before. (Int. 3)

It seems from the data that Alex, in contrast to Pablo, preferred to be a learner by being told. He pointed out that teaching as Teacher B did was difficult because in the textbooks everything came decontextualised, and he continued to identify this factor as one of the reasons for the impossibilities of incorporating change in his teaching, as we saw in previous sections. Alex did not provide an affirmative answer when asked about whether there had been something that he had changed in his teaching of Grade 8-algebra; however, he provided evidence of his attention to what the teachers discussed at the Focus Group were doing when introducing children to the concept of variable. At Interview 3, when asked if throughout the three times he taught Grade 8 algebra, he had made any changes in the way he initiated the algebraic work, he said:

> I start with 'an algebraic expression is this', and give examples of algebraic expressions. But then one looks at one does and thinks, 'if I give them this and she makes those big eyes it means that she did not understand anything'. Then, the language has to be changed according to the language that they know. And so I have to start by giving them an example.

The example that Alex gave was based on the same situation he had mentioned at Interview 1, as an example of how he introduce the pupils to first algebraic concepts, but this time he had taken on board an aspect of language and representation which he had not shown awareness of at Interview 1. Let's see the difference between the two examples Alex provided. This is the example provided at Interview 3:

> A: When I speak of equations: I have got 3 apples. Each apple costs $1000. How can I express that? Then we can organise the data in a table like this (draws a table see Figure 6.23). Two apples cost $2000, three apples...etc. It is explained that C represents the cost of all the apples: C equals 'the number of apples' times 'Price' of each apple (writes $C = P \times a$).

[12] Alex always use the word mathematics to talk about his teaching of Grade 8-algebra, despite my questions about his teaching of Grade 8 algebra.

P	No of apples
1000	1
.	2
.	3
	.

Figure 6.23. *Alex's explanations of how he would start introducing the use of letters(after his participation in the Focus Group session)*

I: So C is the number of something too?

A: Yes, the amount of money that you pay.

Example provided at Interview 1:

I: Do you think that it would be possible to start the promotion of algebraic thinking in primary?

A: Yes, it can be done for example in things like shopping. I had an experience of last year in Grade 8 group, with problems; and it is where one realises that the students do no have the necessary pre-requisite algebraic ideas. I told them 3 pears plus 2 apples cost $2700 [Colombian pesos]… Then the 3 pears and the 2 apples are drawn without the need to put letters (Alex draws the pears and the apples shown in Figure 6. 24). Now I can tell them: 2 pears and 1 apple cost $1600. They can draw the 2 pears and/. In some way we are working with algebra because we are substituting numerical values by symbols. There is a type of algebraic thinking when you substitute numerical values for symbols.

I: What would be the intention when giving this type of situation to the primary level children?

A: Oh, well that they solve the problem by trial and error.

I: And you gave this example to your Grade 8 class last year, what was the intention? Did you ask them the same question or?

A: No; last year it was, simply, how do you represent this? So it started by this (pointing to the drawing of Figure 6.24); and then they had to solve the problem, the system of equations.

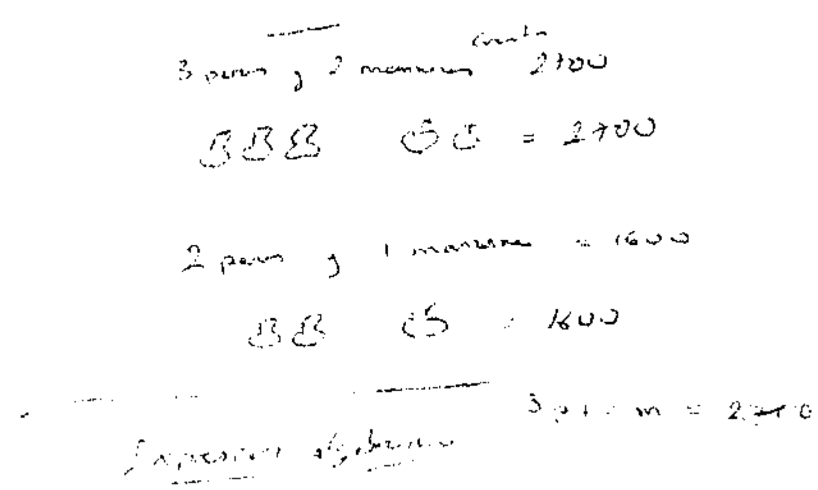


Figure 6.24. *Alex example of what he did when using letters in algebra*

I: So what did they do next, after this representation?

A: Then no more drawing but letters; so for example, three p plus two m equals two thousand seven hundred (writes the expression).

I: The p representing?

A: What happens is that 3 pears and 2 apples cost 2700. It is like the summary.

This way of introducing the use of letters "in algebra" was the same described by Alex's colleague, José, who had been teaching mathematics for twelve years (though he had a Bachelor degree in Psychology of Education).

Alex did not show much interest in engaging in the professional development projects I offered him at Interview 3. When I presented him with the three-option proposal (see Figure 6.5 on page 153, he asked if his answers would represent "any actual commitment" as life was very busy:

> ...I may be interested in the project of the use of technology. That would be of interest to me but, at the moment, I don't want to have any commitments of that sort because life is very busy at the moment... The two research projects look interesting and that would be something to think about for the future. ... The new textbook, as it is described would be an option that looks more feasible...

6.6.2.3 Alex's knowledge of the teaching of beginning algebra

Knowledge of the teaching of the concept of variable

As described in Subsection 6.5.1.2, Alex stated that he introduced algebraic work in Grade 8 by giving the definitions of algebraic expressions, and like terms, then moving on to identifying like terms and simplifying expressions. What conception of the variable (see Usiskin, 1988) was Alex emphasising in his teaching? Consider his response to my question, at Interview 2, of whether he thought that the pupils were using the letters to represent variables, in the task given by Teacher A (of Questionnaire 2): 'Find the numerical values for the expressions: $5ab$; $3abc$; $3a + 2b$, when $a = 1$, $b = 3$, $c = -1$":

A: The letters there represent any number, and they [the pupils] see it when they work with numerical values.

I: That means that when they do this type of task, they go into considering and examining what you just said. That is, they look at for example, how the value of $5ab$ changes for different values of a and b?

A: No. at that point, they replace and find the value. Do what the exercise is asking. Later when we get to the concept of function they see things like that…

Further, was Alex concerned with what letters mean or represent for the pupils? Some evidence can be seen in his explanations (provided at Interview 1) about how to respond to pupils as required in Question C4 of Questionnaire 1, about the answer D = 5N to represent 'number of doctors equal to 5 times the number of nurses':

I: What would you say to the students, or what would you do if they gave this answer?

A: I would ask them to go over the equation first and see if there any coherence with what the text of the problem says. Some may read it and say, 'yes, it is right'. Then I would ask, what happens if there are 2 nurses, how many doctors? And if there are 4 nurses, how many doctors? In that way they realise that the expression is not correct.

I: How do you think they would clarify that the expression is incorrectly written by considering the situation, 'when there are 2 nurses'?

A: Because if they take this expression (5 N = D) and replace N by 2, then they will have 10 doctors, which means that there are 10 doctors and 2 nurses; and that is not what the text is telling us. So it shows them that that expression is not the correct one.

I: But are they using the N to represent what the text is telling us?

A: Yes, they are using the N to represent the nurses but they are not/ the setting of the equations is very literal. Or better said, the shift from the literal part to algebra is not good.

I: But how do we know what the pupils think the N represents?

A: How do we know? (silence).

I: Would you use a situation like this in your teaching of Grade 8?

A: Yes; this situation would be adequate when we are going to see the different types of mathematical relations and the concept of function.

As we can be seen Alex provided no evidence of his attention to what the letters stand for or to the analysis of the relationship between the quantities, but simply to the correct answer or to the rules to follow as he showed in his responses to questions C3 and C5. There was no evidence to support the idea that, for Alex, knowing what pupils think was an aspect to pay attention to.

Apart from Alex knowledge of the list of contents listed in curriculum statements, and in textbooks, the data do not show evidence of Alex's knowledge of the context of the teaching of beginning algebra.

6.6.3 Summary of Alex's case

For Alex school algebra is a collection of formal definitions and procedures that must be acquired one by one in a logical order. Thus, the students need to learn first the definition of algebraic expression ("that comes in textbooks") and the rest of formal conventions about algebraic expressions, to then move on to the basic operations with polynomials. This is a logical order in the teaching of beginning algebra "because they first have to know what an algebraic expression is to then learn how to operate with the expressions and how to use them". This collection of disconnected skills constitutes "the tools" for the solution of the mathematical word problems that appear in textbooks.

Teaching children to write natural language expressions by using conventional symbols used in mathematics, is "preparing them for [Grade 8] algebra", and Grade 8-algebra is important because it develops the critical thinking needed by every citizen. "An individual has critical thinking when he/she is able to give his/her opinion, and this is what the students do when they solve the given exercises".

Out of the given statements, in Questionnaire 1, to describe 'Teaching styles in Grade 8-algebra, the one that describes Alex's first teaching priority is 'Providing opportunities for pupils to develop their communication skills' because "in the lessons the students have many opportunities and space to talk and discuss about their work". They talk about how to solve the given exercises or why an exercise had been done wrongly.

Alex likes teaching algebra but explains that it has been a difficult experience because of the pupils' phobia to mathematics. "The students' aversion to mathematics is a great problem" and it is a crucial determinant of Alex's teaching practice because it is due to their fear of mathematics that they lack motivation for the learning of the subject., Alex also emphasises the impact of the school environment and "the curriculum that the Ministry imposes", when explaining how he sees his own teaching. The textbook does not influence Alex's teaching practice because he uses several textbooks when designing "the *guías*". However, introducing a teaching approach like the one portrayed by Teacher B of Questionnaire 2 or by the work of the teachers described at the Focus Group "is very difficult because in textbooks everything (i.e., questions and exercises) comes decontextulaised". It is difficult to teach in a different way from the mechanical, "tied-to-contents approach of textbooks because that is part of the [Colombian] culture". Nevertheless, when Alex explains his teaching situation through the concept map he identifies the *students' motivation* as the *crucial* factor in his teaching practice because of the students' fear of mathematics.

Alex "may be interested" in learning about the use of technology in the teaching of mathematics but he would not want to give any commitments of that sort, for the moment. Adopting an innovative textbook would be another option that may interest him.

Appendix 6.2

Additional information on Pablo, Nora and Nacho's case descriptions

Further description of Pablo's "Buying and selling" activity

Pablo asked the pupils to bring in things that they thought they or their brothers or sisters did not want anymore. For example, "Music CD, posters, or whatever —you know more than me what you think you can bring in for the next lesson". During the next lesson you are going to sell to your peers something that you don't need anymore here, in the next lesson. Bring also scissors and a piece of 'bond' paper.

During the first 15 minutes each pupil made currency notes that were called H currency notes and G currency notes. Each of them had 5 H notes and 5 G notes, which according to Pablo had "unknown values". The number of notes involved in the selling prices "should be less than 5". Pupils assigned a price to the object they were selling. At this point the lesson was interrupted as pupils were called to go to a PE competition.

Pablo reported that after the activity the selling and buying activity had taken place the pupils had been asked to write an expression to represent the value of the money they each had left. They then checked whether each pupil had written the expression in a correct way.

Note: An example of the *Mathematical calendar* used by Pablo can bee seen on page 442.

Further description of the classroom incident with Alfonso, reported in Nora's case description in Chapter 6

Lesson 1, a double lesson lasting 90 minutes, started by the teacher asking pupils go to the board to correct some exercise from the homework. One pupil was asked to "read Question number 1". The pupil read:

Find the area of a rectangle whose sides are $\frac{x}{2}+1$ and $3x + y$.

The teacher drew a rectangle on the board, wrote the expressions for the sides of the rectangle and asked Alfonso to the board. Alfonso, taking one of the given expressions and without saying a word, wrote:

$$\frac{x}{2}+\frac{1}{1}=\frac{x+2}{2}=\frac{2x}{2}$$

Pointing at the x and the 2, in '$x + 2$', the teacher said: "I cannot group those two expressions". The voice of a girl was heard saying "first, the base by the hight is multiplied". Alfonso rubbed out what he had written and wrote again the given expression:

$$\frac{x}{2}+1$$

Alfonso said: "First I have to do this addition", and then kept quiet. The teacher then wrote on the board:

$$\text{Area} = (\quad)(\quad) =$$

Alfonso followed the teacher's suggestion by writing:

$$\text{Area} = (\frac{x}{2}+1)(3x+y) =$$

Alfonso kept quite for a while, and the teacher called Cesar, who went to the board and wrote:

$$(\frac{x}{2}+1)(3x+y) = \frac{3x^2}{4}+3x+$$

As Cesar stopped there and kept quiet, the teacher called Paola, who had bee putting her hand up for some time, saying: "Let see what Paola says". Paola went to the board and did the exercise correctly. Paola was, later, called again to the board, to do the next exercise as the pupil who had been asked at the board was not successful. Paola, without speaking, did the new exercise correctly again. The teacher said to the class that for the following exercise they had to follow the same process, and then announced the work to be done for the rest of the lesson. She said: "Now you are going to work in this *guia*, you can discuss your work with your next door neighbour, explaining that "what is given there is written in Spanish and then they are going to write each sentence in mathematics".

Each pupil was given one copy of the *guia*, which had 25 questions. The first 5 questions can be seen in Figure 6.12. Pupils worked during the next 45 minutes, but all got stuck in question number 4. This work was left as homework to be handed in next day.

Write an algebraic expression to describe each of the following concepts or situations:

1) The perimeter of a square ____________________

2) The perimeter of a rectangle ____________________

3) The volume of a cube ____________________

4) The distance travelled by a car in 2 hours __________

5) The sum of three consecutive even numbers ________

Etc.

The first five questions of Nora's *guia*

Pupils worked on the four first questions during the rest of the lesson (about 45 minutes) but got stuck in question 4 The questions in general —especially question

4—, were puzzling for many pupils, who came to the observer asking for clarification and for help. Nora moved around the classroom and spoke to almost every pair of pupils. When the bell rung, Nora said to the class that 'as a homework, for the next lesson", they should finish the *guía*, and that this homework was going to be collected for assessment purposes.

Two days later, after collecting the *guía* from each pupil, Nora started the lesson by saying to the class that during the lesson they were going to work individually "solving situation with algebraic expressions". Nora gave one exercise at a time and asked pupils at the board for each exercise. After completing three exercises, which had expressions similar to those of the *guía*, Nora said to the class: "A positive point for those who have good solutions for his question: The sum of two even consecutive numbers is 38. What are the numbers?"

Three pupils handed in their notebooks with their solutions. Nora wrote something in each notebook and gave them back to the pupils. The lesson, that lasted 90 minutes, continued with the same type of activity; that is, Nora giving exercises, the pupils working on each exercise individually and them paying attention to what was done at the board, when Nora called someone to do it. However, there were no exercises related to the case of question 4.

At Interview 2, Nora explained that the purpose of the work of the *guía* was that the pupils used their algebraic knowledge to represent situations

> in order to take them, little by little, because to solve a problem they need two things: one is to set up the equations and the other one is to solve the equations. Setting up the equations is what is most difficult form them. If the boy manages to represent the information, to translate it, the rest is easy.

The list of factors, as written by Nacho, when identifying the factors that influenced his teaching practice and explaining why he did not get good results in his teaching of Grade 8-algebra

T. 6)

Porqué no obtengo logros suficientes en matemáticas (álgebra) 1er grado.

a) Por una legislación permisiva. No puede perder matemáticas más del 5% del curso

b) Por falta de interés del alumno

c) Por falta de colaboración en el círculo familiar

d) Por la carencia de hogar

e) Por el desinterés del estado en la formación del pueblo

Calendario Matemático Febrero Tercer Nivel

COLOMBIA APRENDIENDO
LABORATORIO DE MATEMÁTICAS

LUNES	MARTES	MIÉRCOLES	JUEVES	VIERNES	Problema en Familia

MARTES

Apreciado Colega:

- Protejamos y respetemos los derechos de autor.
- No utilice este material sin la debida autorización.

JUEVES

Muchas veces se arrepiente uno de haber hablado, y ninguna de haber callado.

Simónides

1/2

Put the positive digits, one in each region, in such a way that A+B+C+D = D+E+F+G = G+H+I+ A=21.

3

O	R	E	R	O
R	■	R	■	R
F	E	B	E	F
R	■	R	■	R
O	R	E	R	O

Encuentre la razón entre el número de veces que se puede leer la palabra **FEBRERO** y el número de veces que se puede leer la palabra **ORFEBRE** en el arreglo

4 **DE-MENTE**

¿Cuál es el resultado mas cercano a 63?

$(18+7)+(3\times 11)$ $(7+3)\times(18-11)$

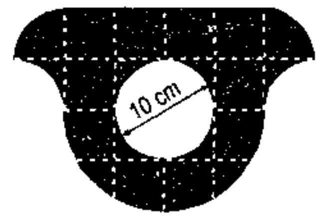
$(11\times 7)-(18+3)$ $7\times 11-\frac{18}{3}$

5 A, B, C, D, E y F son dígitos positivos diferentes.
DAC es una potencia de 2.

$$\frac{ABC}{DEF} = \frac{CAB}{FDE} = \frac{1}{3}$$

A = ? B = ? C = ?
D = ? E = ? F = ?

6



Calcule el área de la región sombreada.

7

	3		7		11		
1	2	5	6	9	10	13	....
		4		8		12	

Si se siguen escribiendo los números en el arreglo, ¿en cuál fila aparecera el 203? ¿en cuál el 2003?

8/9 Un día 9 de febrero nació el matemático inglés Harold Coxeter.

```
  ABC
+ ABD
  ABE
 ----
 CEFD
```

El año de su nacimiento corresponde a la menor suma posible en la criptoaritmética, con A<D, consecutivos. ¡Descúbralo!

10

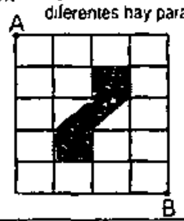
En una familia donde trabajan el padre, la madre y un hijo han ganado en un mes \$1'520.000 entre los tres. El salario del padre corresponde a dos tercios del salario de la madre. El salario del hijo corresponde a dos tercios del salario del padre.
¿Cuánto gana cada uno?

11 **Reconstruya la multiplicación**

```
   ABC
 ×  DE
 -----
  BFFE
 CBFD
 -----
 CGHBE
```

Ni el dígito 0 ni el dígito 6, aparecen en esta multiplicación. H<F, consecutivos.

12 Siguiendo las líneas, ¿cuántos caminos diferentes hay para ir desde A hasta B, si en cada recorrido no está permitido pasar dos veces por el mismo sitio, ni tocar la región sombreada?

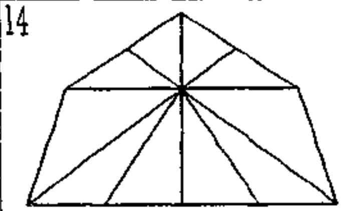


13

$$\sqrt{3+8+15+24+\ldots+K} = 35$$

K = ?

14

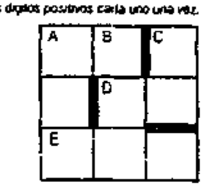


¿Cuántos triángulos?

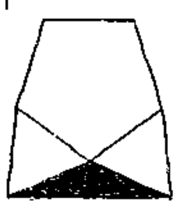
15/16 **Crucinúmero**
Se utilizan los dígitos positivos cada uno una vez.

Horizontales
A. Potencia de 2
D. Múltiplo de 13
E. Múltiplo de 25

Verticales
A. Potencia de 2
B. Múltiplo de 49
C. Número cuadrado



17



La figura se construyó a partir de un pentágono regular y de dos triángulos equiláteros. Halle los ángulos del triángulo sombreado.

18 **AMONG PRIMES**

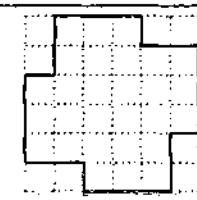
3 5 7 11 = 13
3 5 7 11 = 17
3 5 7 11 = 19

Use signs and brackets, if necessary, to obtain an equality in each case.

19

Se tienen dos dados, uno marcado con los números 0, 1, 2, 3, 4 y 5, y el otro marcado con los números 3, 4, 5, 6, 7 y 8.
Si se lanzan al tiempo, ¿qué es más probable, que la suma de los puntos sea par o que sea impar?

20



Divida la figura en 2 regiones congruentes.
Divida la figura en 4 regiones congruentes.

21 **Alphametic**

```
  PLEASE
+   MAKE
  ------
  OFFERS
```

E+A+R=10

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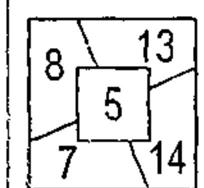
22/23 **Tiro al blanco**



Obtenga exactamente 100 puntos con el menor número posible de disparos.

24 ***Reconstruya la división***

```
 AB'C'D' |BE
-FG       DCH
 --
  HC
 -BE
  --
   CAD
  -CAI
   ---
     D
```

No se utiliza el dígito 9.
E es el doble de D.
G > 8.

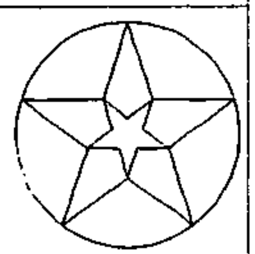
25

Descubra el municipio

Con todas las letras de *"IR LOCA"* se puede formar el nombre de un municipio en el departamento de Córdoba (Colombia).
¿De cuál municipio se trata?

26

Trace la figura sin levantar el lápiz del papel y sin repetir línea.



27 **El personaje**

Hypatia de Alejandría
(370 - 415 d. C)

Fue la primera mujer en hacer una contribución sustancial al desarrollo de matemática.

28

Al 80% de cierta cantidad se le resta la mitad de la misma cantidad.
¿Qué porcentaje de la cantidad original se obtiene?

Harold Coxeter

442