# Advances on $K$ Nearest Neighbour Search in Spatial Databases 

by

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Thesis
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To My Supportive Family, Friends and Academics

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# Advances on $K$ Nearest Neighbour Search in Spatial Databases 

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#### Abstract

A spatial database is a database that stores data and makes queries which are related to objects in space, including points, lines and polygons. The spatial database is designed to process the spatial data type which cannot be processed by typical databases. $k$ Nearest Neighbor search is a type of query to classify objects based on closest distances in the feature space, as a result, a large portion of $k$ nearest neighbor search queries are based on road network. Consequently, the most popular method that has been fully discussed is called Network Expansion. By processing more and more complex queries, network expansion methods show a significant drawback which is poor performance because the underlay road network is crossed and connected. Do expansion for each intersection points in road network is unrealistic. The other problem in $k$ Nearest Neighbor ( $k \mathrm{NN}$ ) query processing is that most of the existing $k$ nearest neighbor searches are concentrated on points. In other words, the discrete points are the input and output of $k$ Nearest Neighbor search query. While in reality, points, lines as well as polygons are three types of spatial elements. How to utilize Lines/Route in spatial query becomes another question for our researchers. Motivated by these two points, the following paragraph summarizes the contribution of this thesis.


The first main contribution of this thesis is called Voronoi Based $k$ Nearest Neighbor search query. Compared to the Network Expansion method, the Voronoi
based method aggregates the road segments using a Voronoi Diagram. Instead of expanding from each intersection in road network, the Voronoi Diagram divides the map into polygons by treating the points of interest as generators. In this part, we have proposed 2 algorithms which use the Voronoi Diagram to improve the performance for Multiple types of $k$ Nearest Neighbor Search query and Continuous $k$ Nearest Neighbor Search query respectively. Our experiments have shown the proposed algorithms can improve the performance significantly either from a cost and a processing time point of view compared to the traditional Network Expansion method.

The second main contribution of this thesis is called Route and Path related kNN Queries. The aim of this part in the thesis is to bring lines/routes into the input and(or) output of $k$ Nearest Neighbor search. As a result, users can use lines to find points, use points to find routes, or even use route to find route. Correspondingly, there are three novel approaches discussed in this part, namely, Path Based $k$ NN Search Queries, Path Branch Point based $k$ NN Search Queries and Time Constraint Route Search over Multiple Locations. By proposing these three approaches, the $k$ Nearest Neighbor Search has been enriched and can satisfy various types of user queries.

To sum up, the thesis is concentrated on two main category of query processing: i) Voronoi based $k$ Nearest Neighbor Search which is aiming at improving the Network Expansion Method which is the most popular technique used by existing $k$ Nearest Neighbor Search approaches. ii) Route and Path related $k N N$ Query which brings route and path into the input or/and output of $k \mathrm{NN}$ queries and which fills up the blanket area in $k$ NN query that only concrete points are utilized in spatial queries.

# Advances on $K$ Nearest Neighbour Search in Spatial Databases 

## Declaration

I declare that this thesis is my own work and has not been submitted in any form for another degree or diploma at any university or other institute of tertiary education. Information derived from the published and unpublished work of others has been acknowledged in the text and a list of references is given.

Geng Zhao
March 18, 2013

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## Chapter 1

## Introduction

### 1.1 Overview

Due to heavy traffic load and complex road connections, more and more users need an application to help them navigate crowded roads, guide them to the best route and even give answers to users' queries. With the development of personal computing devices and wireless networks, mobile devices using inexpensive wireless networks provide unlimited convenience to mobile users WST05a. A spatial database is a database that stores data and makes queries which are related to objects in space, including points, lines, polygons and paths. The spatial database is designed to process the spatial data type which can not be processed by typical databases. The application of spatial databases include Geographic Information Systems (GIS), Computer Aided Design (CAD), Very-Large-Scale Integration (VLSI) designs, Multimedia Information System (MMIS) and medicine and biological research. Spatial related query processing has played a more and more important role in our daily life with the decreasing cost of wireless network access, upgrading mobile device's processing ability and widening internet coverage.

With the developing processing efficiency and the growing complexity of road connection, nowadays users are sending out diverse spatial queries based on objects locations. A number of queries can be listed here as samples, e.g. navigating from


Figure 1.1: Example of road navigation


Figure 1.2: Example of route search


Figure 1.4: Example of range matching
$A$ to $B$, finding the nearest neighbor from current locations, searching all objects within given range and so on. We summarize them using different points of view.

From the query point of view, these objects of interest can be a snapshot or continuously moving objects.

In a snapshot query, the results of the query are to be computed only once. In contrast to the snapshot queries, a continuous query requires the results to be continuously updated as the underlying data is updated.

Example 1.1.1. Snapshot query: a user may want to find the nearest 5 petrol stations from Monash University. He may issue a snapshot $k$ Nearest Neighbor query with a query location set as Monash University.

Example 1.1.2. Continuous query: For instance, a person driving a car may want to find the nearest 5 petrol stations of his current location. Since the car is continuously moving, the results are required to be updated continuously. He may issue a continuous $k$ Nearest Neighbor query with the query location set as the location of car..

Example 1.1.3. Continuous moving query: While in the above example only the query is moving, in many applications, all of the query objects and data objects may be continuously moving. For instance, a taxi driver might want to continuously monitor his nearest walking customers of his location. In this example, the query and the data objects all are continuously moving.

From the data accuracy point of view, points of interest can be certain or uncertain data.

Usually, it is assumed that the objects (e.g., locations) have accurate value and the spatial queries to use these locations. However, uncertain data is inherent most of the time such as sensor databases, moving object databases, market analysis, and quantitative economic research. The reason of the inaccuracy might be the limitation of measuring equipment, delayed data updates, incompleteness or data anonymization to preserve privacy. In such applications, the spatial queries are issued on the uncertain data and probabilistic results are returned.

Example 1.1.4. Certain Data:Find the nearest restaurant of point $q$ ( $X=$ $\left.116^{\circ} 23^{\prime} 17^{\prime \prime}, Y=39^{\circ} 54^{\prime} 27^{\prime \prime}\right)$. The query point (q) has accurate location.

Example 1.1.5. Uncertain Data:Find the nearest restaurant of point q $(116<$ $X<117,38<Y<39$ ). The query point (q) has uncertain value.

From the result's type point of view, we can summarize the spatial query into the following categories: route search, object finding and range matching.

Example 1.1.6. Route Search: Find the best road that satisfied users' diverse conditions. For example, the route navigation we use everyday: find the shortest
road starting from point $A$ to destination point $B$ (Fig.1.1 as an example). There might be some other road navigation variants: finding the route with shortest travel time or less traffic lights and so on. In addition, there are some other route search queries. Fig 1.2 gives another example of route search query. In this example, 16 famous tourist places have been chosen by the traveler. The query is to find the route that can visit 4 tourist places with the shortest distance.

Example 1.1.7. Objects Finding: Find the objects that fulfill users' requirements ( $k$ nearest neighbor or within the range and so on). Fig. 1.3 shows an example of objects finding: show all pharmacies in my local suburb.

Example 1.1.8. Range Matching: Find the range or area that meets users' specification. Fig 1.4 shows an example of range matching: find the area that are close (within $2 k m$ walking distance) to shops, schools and train stations.

The following sections emphasis the major problems of the existing methods and the contributions in this thesis in detail.

### 1.2 Major Issues

Although $k$ Nearest Neighbor query search has been fully investigated over the recent decades, there are still some significant problems or, in other words, there are still some gaps/blank zones which have not been explored. Moreover, even the existing methods which can solve the problems perform poorly under some circumstance. After summarizing various spatial queries, we draw several conclusions:

### 1.2.1 Problem 1: Poor performance of Network Expansion

All of the approaches are constructed based on the underlying road connection between objects. Within the road map, roads are connected and joined by thousands of intersection nodes which break the roads into small segments. The total distance of the road is calculated by summing up the component segments distances. As a


Figure 1.5: Network Expansion vs. the Voronoi Diagram
result, network expansion is the technique which has been widely used in existing methods. Network expansion is processed as follows: when encountering any intersection node, the traverse expansion takes every possible directions. In other words, if we suppose every node has four possible directions to go, then the expansion would be $4^{n}$, where $n$ is the number of expansion nodes. From this calculation, we can infer that the performance cost will behave like a parabola with the increasing number of intersection nodes. The poor performance is inevitable because the complex road connection will result in large number of intersection nodes. Consequently, how to merge the intersection nodes or how to avoid the expansion becomes a new topic to the researchers who are engaged in spatial query processing. This is the first main chapter of the thesis.

Fig 1.5(a) demonstrates the expansion directions for each intersection node. The expansion should be invoked for every possible moving direction when the query point is located at this intersection node. Fig 1.5 shows the illustration of a Network Voronoi Diagram which merges lots of segments into polygons.

### 1.2.2 Problem 2: Discrete Points are the input and output of Spatial Queries

The second main chapter of the thesis is to bring line/route into spatial queries. In spatial databases, there are three types of spatial data elements: i) Point ii)Line and route iii)Region and polygon.

Points: A spatial point is a primitive notion upon which other concepts may be defined. In general, points are zero-dimensional; i.e., they do not have volume, area, length or any other higher-dimensional analogue. In branches of mathematics dealing with set theory, an element is sometimes referred to as a point. In spatial query processing, points of interest belong to this type. They are distributed discretely over the map. The discrete location on the surface of the planet, represented by an x and y coordinate pairs. Each point on the map is created by latitude and longitude coordinates, and is stored as an individual record in the shape file, see Fig 1.6(a),

Lines: As an extension of a point, an elongated mark, is the connection between two points, Lines are the paths through networks, which may have line feature, such as a street, highway, river or pipe. Lines are formed by connecting two data points. The computer reads this line as straight, and renders the line as a vector connecting two x -y coordinates $(\mathrm{X}=$ longitude, $\mathrm{Y}=$ latitude $)$. The more points used to create the line, the greater the detail. FPA requires that the line and polygon features include topology. For lines, this means that the system stores one end of the line as the starting point and the other as the end point, giving the line direction, see Fig.1.6(b).

Polygons: An area fully encompassed by a series of connected lines. Because lines have direction, the system can determine the area that falls within the lines comprising the polygon. Polygons are often of an irregular shape. Each polygon contains one type of data (e.g., vegetation, streets, and dispatch locations would be different polygons). All of the data points that form the perimeter of the polygon must connect to form an unbroken line. When preparing files, the polygons are verified as closed area, see Fig 1.6(c),


Figure 1.6: Spatial Element Types


Figure 1.7: Traditional $k$ NN input and output are discrete points

Nearly all spatial queries are objects related which means both the input(fig 1.7(a)) and output (fig. $1.7(\mathrm{~b})$ ) of the queries are discrete points, for example, traditional $k$ nearest neighbor search tries to find the nearest neighbors (points) of query points. But lines have not been fully introduced into the query processing. Motivated by this point, the second main chapter of my thesis is called Route and Path related $k N N$ Queries.

While in reality, path/route is another important element in spatial space, the user might want to input a path to find a set of points, or input a set of points to create an optimal path, or even input a query path and output a result path at the same time.

### 1.3 Contributions

Based on the problems and possible extensions of existing works listed in section 1.2, we list our contributions generally in this section. Altogether there are 2 main categories of contribution which includes 5 topics.

### 1.3.1 Contribution 1: Using the Voronoi Diagram to enhance the performance

As stated in section 1.2, network expansion is the widely used methodology to process the spatial queries based on underlying road network. The performance has become a significant drawback because each intersection node will perform traverse expansion to different directions. As a result, our first contribution of the thesis is using a Voronoi Diagram to merge the road segment although it requires precalculation. In this chapter, we have proposed 2 approaches which use the Network Voronoi Diagram as the methodology.

## - Query Optimization on Continuous $k$ NN Query Search

In Section 3.2, we propose an alternative approach for Continuous $k$ Nearest Neighbor query processing, which is based on a Network Voronoi Diagram (we call our proposed method $\mathrm{VC} k \mathrm{NN}$, for Voronoi $\mathrm{C} k \mathrm{NN}$ ).

This approach avoids the weakness of existing work GR03, GR99 and improves the performance by utilizing a Voronoi diagram. VCkNN ignores intersections on the query path; instead, it uses Voronoi polygons to subdivide the path. In section 3.2, the Voronoi diagram, which originates in computational geometry and has been used successfully in other areas, such as industrial electronic area VS08, and will demonstrate its effectiveness in a mobile environment.

Our proposed VCkNN approach is based on the attributes of the Voronoi diagram itself and using a piecewise continuous function to express the distance
change of each border point. At the same time, we use Dijkstra's algorithm to expand the road network within the Voronoi polygon.

VCkNN, DAR SE06 and IE KS05 are all approaches for C $k$ NN queries. But VCkNN is different from DAR and IE in most aspects. Therefore before introducing our VCkNN algorithm, we would like to highlight the main differences between VCkNN and DAR and IE: a) Path division mechanism, b) $k$ NN processing, c) Sequence finding of split nodes, d)Processing split nodes. These are discussed in detail in section 3.2.

## - Query Extension on Multiple Objects Types

Current approaches of $k$ nearest neighbor search focus on one object type, which narrows down the mobile query scope. For example, find the nearest 3 hospitals from my current location. In some cases, users may want to get $k N N$ of different object types (multiple object types), as well as to obtain the shortest routes. Motivated by these, section 3.3 proposes new approaches on three different queries involving multiple object types using a network Voronoi Diagram. In these queries, more than one object type is considered and the query result is highly related with the object types. Every object belongs to one of the category and there is no overlap between categories. That is the basic property of a multiple-object-type query.

Section 3.3 focuses on three different types of $k$ NN mobile queries, including: a) query to find nearest neighbor for multiple types of interest point (or 1NN for each object type), b) query to give the shortest path to cover multiple-object-types in a pre-defined sequence, and c) query to find an optimum path for multiple object types that gives the shortest path that covers the required interest objects in a random sequence.


Figure 1.8: Comparison of $k \mathrm{NN}$ and path $k \mathrm{NN}$ result

### 1.3.2 Contribution 2: Bringing route into $k$ NN spatial queries

As stated in section 1.2, most of the existing queries put discrete points as input and output. Consequently, Chapter (4) concentrated on bringing route/path into the input or output or both of the queries.

## - Path based $k$ NN Search Query

A possible query that a user may invoke is as follows: A market researcher wants to do a survey on restaurants and the sample size should be 10. The question is to find the shortest path for the user to visit all the 10 restaurants one by one. Range search cannot be used as there is no fix range. $k$ NN search cannot be used either because after we visit the first interest point, the user may not want to return to query point and go to the second one. In this case, the user wants to continue to go to the second location from the first, and so on. This is a typical path based $k$ nearest neighbor query ( $\mathrm{p} k \mathrm{NN}$ ) and we propose a corresponding method in section 4.2 to process this type of query efficiently.

Fig $1.8(\mathrm{~b})$ shows the aim result of the Path based $k$ nearest neighbor queries. Unlike fig $1.8(\mathrm{a})$ which considers all objects as discrete points, Path bases $k$ nearest neighbor search is to find the shortest path which goes through $k$


Figure 1.9: An example of Path Branch Point based $k$ NN Search Queries
objects. In general, the overall distance of the path becomes the selection criteria.

## - Path Branch Point based $k$ NN Search Queries

In section. 4.3, we bring a novel query which is called path branch point (PBP). PBP can be defined as follows: given a set of candidate interest objects and a pre-defined path which starts at $S$ and end at $E$, find a path which starts at $S$, via an interest point $P$ and ends at $E$. This path should overlap with the pre-defined path as much as possible with acceptable distance increment. This is a novel query which is motivated by users' common requirements because most users have ad hoc paths in their daily travel and they can tolerate a longer driving distance to some extent if they can drive on a familiar path when they want to visit a certain type of object on the way. In this proposed approach, an Adjust Score is calculated for each path which is determined by overlapping distance and increased distance cost. The following example explains the query.

Fig. 1.9 shows an example of Path Branch Point based $k$ NN Search Query. The path from $S$ through red line to $E$ is the query path. User query is to find an
alternative path which starts from $S$ and ends at $E$ as well as on the way, an object $(p)$ should be visited. So yellow (via object $p_{2}$ ) and purple lines (via object $p_{1}$ ) are two candidate results. We will calculate the Adjust Score for each candidate path and decide which path is optimal.

## - Time Constraint Route Search over Multiple Locations

Conventional route search queries aim at finding the shortest distance which is certainly useful, but often impractical, due to these following reasons: (i) each location or place, which are normally a spatial business entity (e.g. bank, dry cleaner, supermarket) has the opening hours - this implies that when this place is visited, it must be during their business hours; and (ii) the traveling time from one location to another needs to be considered, as in many cases, traveling time is more useful than the distance alone. Hence, in order to make route planning over visiting locations, we must take these two constraints into account. Section 4.4 defined these constraints as Time Constraints. Therefore, we proposed an approach called route search over multiple locations which takes time constraints into consideration (see fig. 1.10).

It is therefore imperative to assume that the route or path that arrives on the location outside the operation hours is considered as an invalid path. This problem exists in daily life, whereby we sometime have to choose a longer path to go back and forth to places just to meet the business hours of one location before its closing time. Hence, we need to draw time constraints into our proposed methods.

In section 4.4, we focus on two problems of route search over multiple heterogenous locations: one for fixed locations, and the other for flexible locations. Fixed locations refer to predetermined locations by the user, such as Citibank on a specific location, Pharmore pharmacy on a specific location, etc. In this case, not only a specific business entity is specified, such as Citibank and not any bank, or Pharmore pharmacy and not any pharmacy, but also


Figure 1.10: Motivation of time constraint route search over multiple locations the specific location, such as Citibank on 180 High Street, or Pharmore pharmacy on 25 Cure Road, etc. Hence, a Route Search over Fixed Locations (our proposed algorithm is then called RFix) finds the most efficient route to visit the user-defined fixed locations in a non-predefined order (see fig.1.10(a)).

Flexible locations, on other hand, refer to predetermined location types that are not the exact location itself. For example, if user wants to visit a pharmacy, which can be the pharmacy anywhere; or to visit Citibank, but can be in any branch. So, a route search over flexible locations for example is to find the most efficient route to visit Citibank, a pharmacy, etc, in a non-predefined order. Our proposed algorithm for Route Search over Flexible Locations is abbreviated as RFlex, (see fig $1.10(\mathrm{~b})$ ).

Both RFix and RFlex use the travel time network to estimate the travel time between any two locations, as well as using the time constraints imposed by not only the operating hours of each location, but also the traveling time itself.

### 1.4 Thesis Organization

This thesis is organized as follows (Refer to Fig. 1.11).

- Chapter 1 gives a brief introduction of spatial database, problems and contributions.


Chapter 2: Preliminary and Related Work
Chapter 3: Voronoi Based $k N N$ Query
Section 1: Voronoi-based Continuous $k N N$ Search Queries
Section 2: Multiple object types $k$ NN Search
Chapter 4: Route/Path Based $k N N$ Search
Section 1: Path based kNN Search Queries
Section 2: Path Branch Point based kNN Search Queries
Section 3: Time Constraint Route Search over Multiple Locations

## Chapter 5: Final Remarks

Figure 1.11: Thesis structure

- Chapter 2 provides a survey of the related works.
- Chapter 3 is the first main part of the thesis, which includes 2 approaches of Voronoi based $k$ Nearest neighbor query. More specific descriptions are:
- Section 3.2 optimizes the existing continuous $k$ nearest neighbor search query by using a Network Voronoi Diagram. It improves the efficiency of the CkNN methods.
- Section 3.3 presents a $k$ nearest neighbor search query with multiple types of objects and uses a Voronoi Diagram to find $k$ Nearest Neighbor which has been proven to outperform existing methods.
- Chapter 4 is the second main chapter, which includes 3 approaches of path/route based $k$ Nearest neighbor search. More specific descriptions are:
- Section 4.2 proposes a query that is called path based $k$ nearest neighbor search. It aims at providing a path that visits $k$ objects and the length of the path is the shortest.
- Section 4.3 explains a query which is called path branch point route search. By given the query path and an object type, path branch point route search retrieves the optimal path that balances the overlap ratio of query path and the length of result path.
- Section 4.4 describes a novel route research which adds time constraint into the search. In addition, a user may define the objects visiting sequence as sequential or random.
- Chapter 5 concludes our research, describes some of the open problems and provides several possible directions for future work.


## Chapter 2

## Preliminary and Related Work

### 2.1 Introduction

In this chapter, we consider the literature review on the work related to spatial query processing which requires the database system to access the objects with spatial features in the database. We briefly describe the queries which process the queries more accurately and efficiently. To sum up, in most cases, the distances between the objects are the merits in the results of these queries. The distance is calculated using Euclidean distance or network distance relying on the underlying road networks. The special purpose spatial index and query specific properties are used in order to reduce the system cost. As a result, the spatial objects are treated as points in the space throughout this thesis.

This chapter is constructed as follows:
Firstly, section 2.2 is the literature review with the motivation that originated this thesis. After introducing the basic concepts of spatial query processing elements, the following 5 subsections survey the related work on $k$ NN queries summarized by Fig. 2.4 .

- Section 2.2.1, we review the basic concepts of spatial queries elements and the features of the Voronoi Diagram and the Network Voronoi Diagram.
- Section 2.2 .2 describes the typical $k$ nearest neighbor approaches including $I E R, I N E$ and $V N^{3}$. IER, INE are using Network Expansion as the technique and $V N^{3}$ is using the Voronoi Diagram as its metrology. These two methods are the most popular methodology in spatial query processing named as Network Expansion and Voronoi Diagram. There is a brief demonstration of these methods illustrated and the differences are analyzes as well. The conclusion is drawn that under most of the case, the Voronoi Diagram outperforms Network Expansion. That is the motivation of the first main chapter of my thesis.
- Section 2.2 .3 focuses on the continuous $k$ nearest neighbor queries including DAR/eDAR and IE. Both DAR/eDAR and IE are using Network Expansion whereas we proposed another approach using Voronoi Diagram to merge the road segments into polygon. The performance Evaluation has proven that our new method can significantly improve the efficiency.
- Section 2.2.4 discusses the existing route search queries although it is still new to spatial queries. The second main chapter of the thesis is to enrich the route search queries.
- Section 2.2.5 introduces other $k$ nearest neighbor queries including Reverse Nearest Neighbor Queries and Mutual $k$ Nearest Neighbor Queries search.

Secondly, section 2.3 points out the outstanding problems after reviewing the existing works and formally defines the problems.

Finally, before proposing new approaches, section 2.4 concludes this chapter by using fig 2.13 to compare the contribution with the existing works in order to highlight the principle points of this thesis.

### 2.2 Related Work

### 2.2.1 Preliminaries

## Spatial Queries Elements

Definition 2.2.1. (Road networks) ( $R$ ) is a weighted graph $G=\{V, E\}$, where $V$ is a set of vertices $\left\{v_{1}, v_{2}, \ldots v_{n}\right\}$, and $E$ is a set of edges $\left\{e_{1}, e_{2}, \ldots e_{m}\right\}$ and $\forall e_{i} \in E$, weight $\left(e_{i}\right) \in R^{+}$.

In Def. 2.2.1, the underlying road network is constructed by choosing the layer of the map. In the road network diagram, the objects are called vertices while the connections between vertices are called edges, in other word, road segments.

Definition 2.2.2. (A vertex) $v_{i} \in\left\{n_{1}, n_{2}, \ldots n_{j}\right\} \cup\left\{p_{1}, p_{2}, \ldots p_{k}\right\}$, where $n$ is an intersection node, and $p$ is an interest point.

Definition 2.2.3. (Vertex Scope) Let $N=\left\{n_{1}, n_{2}, \ldots n_{j}\right\}$ be a set of intersection nodes, and $P=\left\{p_{1}, p_{2}, \ldots p_{k}\right\}$ be a set of intersection points, then a vertex $v_{i} \in\{N \cup P\}$.

Def. 2.2.2 and Def. 2.2 .3 defines the scopes of vertices, which includes intersection nodes and interest objects. $N$ represents the set of intersection nodes while $P$ represents the set of interest objects.

Definition 2.2.4. (Weight) $\forall e_{i}=\left(v_{i}, v_{j}\right) \in E$, weight $\left(e_{i}\right)=d_{\text {net }}\left(v_{i}, v_{j}\right)$, where $d_{n e t}$ is the network distance between $v_{j}$ and $v_{k}$.

The weight of the edges is determined by the measure of the query (def.2.2.4. If the optimal result is based on the travel time, then the weight of the edges is the cost of travel time, while in this chapter, the shortest network distance determines the weight because the query result is defined as the shortest path.

Fig 2.1 is an example of road networks, in which road network intersections $n_{1}-$ $n_{10}$ (white points), and interest points $p_{1}-p_{3}$ (black points) are vertices and the solid lines connecting these vertices are edges. The number on each edge represents the


Figure 2.1: An example of road networks
shortest distance, in other words, the weight of the edge. Most of the spatial query is based on Euclidean distance. In reality, the distance between objects should not be determined by the length of the direct line linked objects. The network distance between objects suits the spatial query the most.

## Voronoi Diagram based on Euclidean Distance

Voronoi Diagram is a special kind of decomposition of a metric space determined by distances to a specified discrete set of objects in the space OBSC00. Given a set of points S , the corresponding Voronoi diagram will be generated. Each point $s$ has its own a Voronoi cell $\mathrm{V}(\mathrm{s})$, which consists of all points closer to s than to any other points. The border points between polygons are the collection of the points with equation of distance to shared generators. Fig 2.2 gives an example of Voronoi Diagram based on Euclidean distance. Pi represents the interest points and the lines are the shared border edge between polygons.

There are some basic properties associated with Voronoi Diagram, which have been well presented by Okabe, et al Saf05. We will list some of the relevant properties below:

- Property 1: The Voronoi diagram of a point set $P, V(P)$, is unique.


Figure 2.2: The Voronoi Diagram

- Property 2: The nearest generator point of $p_{i}$ (e.g. $p_{j}$ ) is among the generator points whose Voronoi polygons share similar Voronoi edges with $V\left(p_{i}\right)$.
- Property 3: Let $n$ and ne be the number of generator points and Voronoi edges, respectively, then $n e \leq 3 n-6$.
- Property 4: From property 3, and the fact that every Voronoi edge is shared by exactly two Voronoi polygons, we notice that the average number of Voronoi edges per Voronoi polygon is at most 6 , i.e., $2(3 n-6) / n=6-12 / n \leq 6$. This means that on average, each generator has 6 adjacent generators.

Using Voronoi Diagram to find nearest neighbor will let the algorithm perform more efficiently as all distance between borders and generators can be pre-calculated and stored. $V N^{3}$ and PINE utilize Voronoi diagram efficiently to find $k N N$. While currently there is no CkNN approach using Voronoi diagram to ignore the real network connection within the polygon, this point becomes our motivation of this chapter, Voronoi-based CkNN.


Figure 2.3: Network Voronoi Diagram

## Network Voronoi Diagram

Voronoi diagram mentioned previously is the Voronoi diagram based on Euclidean distance. In the real world, when we want to search nearest neighbor or to generate the appropriate moving path, we use network distance, and not Euclidean distance. Network Voronoi Diagram is the Voronoi diagram, which uses network distance to generate the diagram, instead of Euclidean distance XZTS08, Saf05. In a typical Voronoi diagram, the shared borderline is the mid perpendicular of the links connected with two corresponding generators. However, in Network Voronoi Diagram, the borderline consists of discrete points, which are the middle points of network roads connected with two corresponding generators. A polygon in a network is the set of nodes and edges, which are closer to one generator than to any other. This is the principal difference between Voronoi Diagram and Network Voronoi Diagram. Network Voronoi Diagram will be used in our proposed method. The most basic property is the generators with shared border points have equal network distance to the same border point they shared. In fig 2.3 , the different colors represent different polygons and the border points of the Network Voronoi Diagram are the discrete points on the roads.

| Category | Query | POI | Remarks | Existing Works | Techniques |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Typical $k \mathrm{NN}$ | Static | Static | $k$ near neighbor of $q$ | Incremental Euclidean Restriction (IER) | Network Expansion |
|  |  |  |  | Incremental Network Expansion(INE) |  |
|  |  |  |  | Voronoi-based k nearest neighbor search (VN3) | Voronoi Diagram |
| Continuous $k \mathrm{NN}$ | Moving | Static | Find $k$ near neighbor of a moving $q$ | DAR/eDAR | Segment Split / <br> Network Expansion |
|  |  |  |  | Intersection Examination (IE) |  |
| Route <br> Search | Route | Static | Find Route for a query $q$ | Efficient Orienteering Route Search over Uncertain Data | Network <br> Expansion/ <br> Pruning\&Fileting |
|  |  |  |  | Incremental Route Search Query |  |
| $k \mathrm{NN}$ <br> Variants | Static | Static | Find POIs consider $q$ as $k \mathrm{NN}$ | Reverse Nearest Neighbor Queries | $R$ tree |
|  | Static | Static | Find mutual $k$ NN of dataset | Mutual k Nearest Neighbor Search | R tree, Pruning... |

Figure 2.4: Related Work Summary Chart

In the following 4 sections, we are going to categorize existing works which are highly related to this thesis. The following chart, fig 2.4, shows the general categorized criterion as well as the techniques that are used in these existing works.

### 2.2.2 Typical $k$ NN Queries

The existing methods for static $k$ NN cover Incremental Euclidean Restriction (IER), Incremental Network Expansion (INE) PZMT03 and Voronoi Based Network Nearest Neighbor $\left(V N^{3}\right)$.

## Incremental Euclidean Restriction (IER)

Incremental Euclidean Restriction (IER) was proposed in 2004 PZMT03. Firstly, IER uses the entity R-tree to retrieve the $k^{t h}$ node's Euclidean distance. Secondly, IER restricts the interest point by this distance and calculates every point's network distance to a query point that is within the range of $k^{\text {th }}$ node's Euclidean distance and then sorts them in ascending order of network distance to query point. Then set the $k^{\text {th }}$ node's network distance as $d_{\text {max }}$. Finally, for the following interest points in Euclidean distance sequence continue to calculate their network distance to query point. If it is smaller than $d_{\text {max }}$, insert in into network distance queue and update
$d_{\text {max }}$. These operations will terminate until next node's Euclidean distance is larger than $d_{\text {max }}$.

IER algorithm performs well when there are not many false interest points which means its Euclidean distance falls into the restrict zone while its network distance is far from the query point. Too many false points will reduce the performance sharply and if the density of the interest points is high, the performance of IER leaves much to be desired.

For example, there are 10 interest points and their Euclidean distances to query point are $\left\{p_{1}, p_{2}, p_{3}, p_{4}, p_{5}, p_{6}, p_{7}, p_{8}, p_{9}, p_{10}\right\}=\{1,2,3,4,5,6,7,8,9,10\}$. The query is 3 NN .

Firstly, the $3^{r d}$ node according to its Euclidean distance will be $p_{3}$.
Secondly, calculate $\left\{p_{1}, p_{2}, p_{3}\right\}$ network distance to query point. Suppose $\left\{D N\left(p_{1}\right)\right.$, $\left.D N\left(p_{2}\right), D N\left(p_{3}\right)\right\}=\{3,7,12\} . d_{\max }=12$.

Then as next node is $p_{4}$, calculate its network distance $D N\left(p_{4}\right)=4$. Then insert $p_{4}$ into the queue and $d_{\max }=7$. Continue to do $p_{5}, D N\left(p_{5}\right)=5$. Then $d_{\max }=5$. Queue will become $\left\{p_{1}, p_{4}, p_{5}\right\}$.

Finally, next node will be $p_{6}$, because its Euclidean distance (6) is larger than $d_{\max }(5)$. This algorithm terminates.

## Incremental Network Expansion(INE)

Incremental Network Expansion (INE) is an approach of $k$ nearest neighbor query that was proposed in the same paper of IER. The INE algorithm is based on Dijkstra's algorithm. The basic idea of INE is network expansion.

INE firstly locates the query point to find which segment includes the query point. It records the start and end nodes of this segment with their distance to query point, puts them in $R S$ and sorts them in ascending sequence. Then checks whether there is any interest point on this segment and add these points into the result list. Expand the top node in $R S$, add its adjacent nodes into $R S$ and loop these operations until $k$ NN has been found. This distance should be record as $d_{\text {max }}$.


Figure 2.5: An example of INE query

Continue to expand the other nodes in $R S$, once the $k \mathrm{NN}$ has been found, update $d_{\max }$ if the new distance is smaller than origin $d_{\max }$. Terminate the algorithm if all other nodes in $R S$ have a larger distance to $q$ than $d_{\text {max }}$.

Take Fig 2.5 as an example. The query is defined as finding 2NN from query point $q$.

Firstly, locate $q$ to find the segment which covers $q$, the result is $F B$. Then check whether any object exists in this segment.

Secondly, add $F$ and $B$ into $S$ set. $S=\{(F, 1),(B, 3)\}$. Sort the nodes in $S$ by their distance to $R$ in ascending order. Then add $F$ in the top of $S$, expand $F$ and check whether any object is on $F A$ or $F G$ or $F E$. We find $p_{1}$ as first NN. Then add all $F$ 's adjacent node into $S$ set and sort again. $S=\{(B, 3),(G, 4),(E, 4),(A, 8)\}$.

Thirdly, expand $B$ and check whether any object is on $B A$. Then we find $p_{4}$ as the second NN, and $d_{\max }=7$. Check whether any object is on $B C$. If $p_{2}$ is nearer than $p_{4}$, update $d_{\max }=5$ and $S=\{(G, 4),(E, 4),(C, 6),(A, 8),(D, 8),(A, 11)\}$.

Finally after expanding $G$ and $E$, the distance is over $d_{\max }$, INE terminates.
The main advantage of INE is that its architecture can be used in other query solutions, although compared to PINE its performance is not the best Saf05.

## Voronoi-based $k$ nearest neighbor search $\left(V N^{3}\right)$

Voronoi-based $k$ nearest neighbor search $\left(V N^{3}\right)$ was proposed KS04 in 2004. $V N^{3}$ is based on the properties of the Network Voronoi diagrams and also localized precomputation of the network distances for a very small percentage of neighboring nodes in the network. In general, it keeps the result in ascending order, adopts a filter and makes refinement steps to generate and filter candidate results, it also uses localized pre-computed network distances to save response time.

To talk in detail, the first nearest neighbor query point can be told directly by intuition via the Voronoi diagram. The polygon that contains the query point will be the first nearest neighbor. Subsequently, $1^{\text {st }} N N$ 's adjacency information can be utilized to provide a candidate set for other nearest neighbors of $q$. Finally, the actual network distances from $q$ to the generators in the candidate set can be precomputed and this step will refine the set. The filter/refinement process in $V N^{3}$ is iterative: at each step, firstly, a new set of candidates is generated from the NVP $s$ of the generators that are already selected as the nearest neighbors of q , then the pre-computed distances are used to select only the next nearest neighbor of q . Hence, the filter/refinement step must be invoked $k$ times to find the first $k$ nearest neighbors of $q$ ( KS04 $)$.

The following figure 2.6 shows the example of $V N^{3}$. Suppose the query is 3NN.
First of all, we can tell $1^{s t} \mathrm{NN}$ of query point is $p_{1}$. Then candidate set $C S$ updates as $C S=\left\{p_{2}, p_{3}, p_{4}, p_{5}, p_{6}, p_{7}, p_{8}\right\}$.

Secondly, calculate the distance from candidate interest points in $C S$ to query point and select the $2^{n d} \mathrm{NN}$. Suppose it is $p_{6}$. Then update $C S$ by pop out $2^{\text {nd }} \mathrm{NN}$ and add all its adjacent interest points into $C S$.
$C S=\left\{p_{2}, p_{3}, p_{4}, p_{5}, p_{6}, p_{7}, p_{8}, p_{18}, p_{17}, p_{16}, p_{15}\right\}$.
Subsequently, repeat the previous step and suppose $p_{5}$ is the $3^{r d} \mathrm{NN}$. As $k=3$, which means all interest points have been found, the algorithm terminates.

In summary, $V N^{3}$ performs well if we are only concerned with a static $k \mathrm{NN}$ query.


Figure 2.6: An example of $V N^{3}$ query

### 2.2.3 Continuous $k$ NN Queries

If the query point is moving, it is infeasible to apply $k \mathrm{NN}$ at every point of the line, because it will generate a large number of queries and a large overhead. So the objective of a moving or continuous query is to efficiently find the location of the split node(s) on the path, in other words, where $k \mathrm{NN}$ changes. There are two important existing works on continuous $k$ nearest neighbor (CkNN) based on network distance. The first one is DAR and $e$ DAR based on PINE, proposed by Safar and Ebrahimi Saf06. Another C $k$ NN work is Intersection Examination (IE) based on $V N^{3}$ proposed by Kolahdouzan and Shahabi [KS05]. Hence, the following section will discuss these two works and analyze their strengths and weaknesses.

## DAR/eDAR

DAR/eDAR was proposed by Safar and Ebrahimi Saf06. These are based on PINE, which uses road networks as the underlying map. These two algorithms start by dividing the query path into segments, each of which is separated by a network


Figure 2.7: An example of DAR: step one
intersection node. Then they find $k N N$ tables for two adjacent nodes, compare the two tables, and swap the position to make these two the same. Every swap would incur a split node, and when the two tables are exactly same, all split nodes have been found. Then split nodes' position and $k$ NN tables are the result of the query. In order to illustrate this clearly, the following shows an example of the process.

Firstly, we divide the query path into segments using the intersect nodes on the path as shown in fig.2.7. In this example, the query starts from $S$ and ends in $E$. The path from $S$ to $E$ has a number of intersections, and the path separated by an intersection is a segment. In this example, the path from $S$ to $A$ is one segment, and from $A$ to $B$ is another, and so on.

Secondly, for every segment (e.g. like $A D$ in fig 2.8), we find the $k N N$ of the two ending points ( $A$ and $D$ ), from which we generate two $k \mathrm{NN}$ lists for both ending points (see fig.2.9, assume the query is 2NN). Then we aggregate these lists to form one complete list (see fig, 2.9).

Then for every adjacent interest points, calculate $\lambda$ according to the following formula (note that $I$ is the distance column in the ready queue $R Q$ for a particular interest point, and Dist is a distance function).

$$
\lambda_{i, i+1}=\frac{\operatorname{Dist}(A, D)+\operatorname{Dist}\left(D, I_{i}^{\prime}\right)-\operatorname{Dist}\left(A, I_{i}^{\prime}\right)}{2}
$$

Then apply the same operation between the last interest point and every point in $R Q$. The smallest $\lambda$ will be the moving direction of query point. Swap the list to find another split until the two lists are the same.


Figure 2.8: An example of DAR: Step 2
A's list will be

| Interest point | Distance list will be | Interest point | Distance |
| :--- | :--- | :--- | :--- |
| $P_{4}$ | 4 | $P_{2}$ | 3 |
| $P_{3}$ | 5 | $P_{1}$ | 7 |

Then create a complete list for A and D

| A | Interest point | Distance | D | Interest point | Distance |
| :---: | :---: | :---: | :---: | :---: | :---: |
| PQ | $P_{4}$ | 4 | PQ | $P_{2}$ | 3 |
|  | $P_{3}$ | 5 |  | $P_{1}$ | 7 |
| $R Q$ | $P_{3}$ | 9 | $R Q$ | $P_{1}$ | 7 |
|  | $P_{3}$ | 9 |  | $P_{2}$ | 11 |

Figure 2.9: An example of DAR: Step 3

It is an undeniable fact that DAR and $e \mathrm{DAR}$ perform well for a $\mathrm{C} k \mathrm{NN}$ query, except that they divide the query path into segments. This will let the performance go worse as the number of intersections increases. Also a large number of overheads will be incurred even if there is no split node in some segments. Nevertheless, we need to do $k \mathrm{NN}$ for every segment although we find no split node. In view of the above mentioned reasons, an approach should be proposed which does not take intersections into account.

## Intersection Examination (IE)

The second approach of CkNN is Intersection Examination (IE) which is based on $V N^{3}$. In general, similar to $e \mathrm{DAR}$, IE separates the query path into segments. IE then tries to find the split nodes by defining the trend for each interest point in the current $k \mathrm{NN}$ result list and sorts them in an ascending order. When there is any change in the position of the interest point, it becomes a split node.

To be specific, if the query is to find continuous 1 NN , it can simply find all nodes that intersect with the border of the Voronoi diagram. The IE algorithm divides the query path into smaller segments using the intersection nodes on the path. From every segment, IE uses $V N^{3}$ to find $k \mathrm{NN}$ for the two terminating nodes. The $k \mathrm{NN}$ results of every segment should be within the combination congregation of the $k \mathrm{NN}$ result of the two terminating nodes. We can obtain the trend of every interest point at the start point's $k \mathrm{NN}$ results, and then find the point where two adjacent nodes have the same distance to the query point, that is the split node.

Similar to DAR and $e \mathrm{DAR}$, IE indeed is an alternative approach to a CkNN query, except that it also needs to divide the query path into segments. Using IE, the trend of interest points can be monitored either moving closer or away from the current position of the query. Our approach of Voronoi C $k N N$ will provide a more comprehensible way to let the user read $k \mathrm{NN}$ results for any node on the query path.

### 2.2.4 Route Search query

Route search has many important applications in various fields such as commerce, transportation, tourism, security and health-care services. In such applications, a route search should be efficient, intuitive and expressive, allowing a user to specify complex search queries and receive an immediate answer. However, current routesearch applications on the Web are limited to a point-to-point search. When computing a route, different goals and constraints can be defined, such as minimizing the traveling length, limiting the route to be over roads of a certain type, etc.

In this section, we are going to review some popular route search query although the area has not been widely discussed followed by couple of algorithms discussed in detail because they are close to the approaches I proposed in the thesis. More specifically, we will present the related work on efficient orienteering-route search over uncertain spatial data sets and Incremental Route Search Query.

## Efficient Orienteering-Route Search over Uncertain Spatial Data sets

Paper DKD08 considers route search over uncertain data sets. Spatial data might be instinctively uncertain due to various reasons such as its acquisition process, imprecise modeling and manipulation. An uncertain data set can contain correct and incorrect objects. The uncertainty of the data represents a confidence value indicating its probability to be correct. When it is a real-world entity, the object is considered as correct, it is considered as incorrect otherwise. A user may be able to test the correctness of an object by visiting the entity at the location of that object. In this paper, the author defined a problem called a generalization of the Orienteering Problem (OP). OP considers a route search where the aim is finding a route that starts at a given location and traverses through as many correct objects as possible without exceeding a given distance. Finding a solution to OP is a problem that cannot be computed efficiently because OP is a generalization of TSP (Traveling Salesman Problem); hence, it is an NP-hard (nondeterministic polynomial-time hard) problem that is unlikely to have a polynomial-time algorithm. This paper presents heuristics to OP that are efficient and scalable.

The Greedy Algorithm: The first algorithm proposed in paper [DKD08] is called the greedy algorithm. The greedy algorithm constructs a route iteratively by making the most profitable increase in each step. Suppose $P_{i}$ is the path constructed in step $i$ and let $o_{i}$ is the last object of $P_{i}$. $P_{i}$ will be considered as the starting point $s$ in step 0 . For each step $i$, the algorithm checks the set $N$ of objects that are in $D$ and are not already in $P_{i}$. In each step, the object $o^{\prime}$ is retrieved from $N$ if distance $\left(o_{i}, o^{\prime}\right) /$ confidence $\left(o^{\prime}\right) \leqslant$ distance $\left(o_{i}, o^{\prime \prime}\right) /$ confidence $\left(o^{\prime \prime}\right)$ for any object
$o^{\prime \prime}$ in $N$. If length $\left(P_{i}\right)+\operatorname{distance}\left(o_{i}, o^{\prime}\right) \leqslant L_{\text {max }}$, it adds the $\operatorname{edge}\left(o_{i}, o^{\prime}\right)$ to $P_{i}$ and continues to step $i+1$. Else, it returns $P_{i}$. The performance evaluation illustrated that the greedy algorithm is simple and relatively efficient as it does not require any preprocessing and its time complexity is $O\left(|D|^{2}\right)$ where $|D|$ is the size of $D$. The greedy algorithm is effective when the objects of $D$ are uniformly distributed, i.e., the data set is uniform in all directions and their confidence values have a small variance, i.e., when all the confidence values are approximately equal. In other word, the greedy algorithm for any direction performs as well as in any other direction, and the produced route will have an expected prize value close to the optimal. However, when the data set is not uniform, the greedy algorithm may not provide good results.

The Double-Greedy Algorithm: The Double-Greedy Algorithm (DG) in paper DKD08 is an improvement of the Greedy Algorithm. The Double-Greedy Algorithm (DG) intuitively examines pairs of edges for deciding which node to add. Formally, in step $i$, the algorithm extends $P_{i}$ by adding the object $o^{\prime}$ such that there exists $o^{\prime \prime}$ for which confidence $\left(o^{\prime}\right) /$ distance $\left(o_{i}, o^{\prime}\right)+$ confidence $\left(o^{\prime \prime}\right) /$ distance $\left(o^{\prime}, o^{\prime \prime}\right)$ $\geqslant$ confidence $(o *) /$ distance $\left(o_{i}, o *\right)+$ confidence $(o * *) /$ distance $\left(o^{\prime}, o * *\right)$ for any $o *$ and $o * *$ that are in $D$ and are not in $P_{i}$ (Note that also $o^{\prime}$ and $o^{\prime \prime}$ are in $D$ and are not in $P_{i}$ ). Algorithm DG has time complexity $O\left(|D|^{3}\right)$. In order to increase efficiency, DG checks a pair of edges only when the following condition holds: $\alpha$ * $\operatorname{distance}\left(o_{i-1}, o_{i}\right) \geqslant \operatorname{distance}\left(o_{i}, o^{\prime}\right)$, where $\alpha \geqslant 1$ is a fixed factor. Intuitively this condition is satisfied when the next edge we consider to add to the route is much longer than its preceding one. The factor $\alpha$ is to detect when the route leaves a cluster and we want to direct the route to a new cluster.

The Adjacency-Aware Greedy Algorithm: Motivated by the entities's distribution, for example, hotels are usually located near the coast or near tourist sites; restaurants are located in the city center, clusters should be taken into account. Given a data set that contains clusters of objects, a good heuristic for constructing an OP route is to give precedence to objects that are in a cluster over objects that
are not in a cluster. This is defined as The Adjacency-Aware Greedy Algorithm (AAG).

AAG does modeling on the given data set as a directed weighted graph where the objects of the data set are the nodes and the weight of the edge between every pair of nodes is a combination of the distance between the objects and the confidence of the target node. Then, AAG computes for each node the probability of reaching this node in a random walk on the graph. Next, AAG replaces the confidence values on nodes by a combination of the confidence values and the random-walk probabilities. Finally, it applies the greedy algorithm using the new values. AAG outperforms the other algorithms of the Greedy for data sets that have clusters. The AAG improves the Greedy algorithm by giving a higher weight to objects that have many near neighbors, especially if the near neighbors have high confidence values.

The Adjacency-Aware Greedy Algorithm with Buffering: The AdjacencyAware Greedy Algorithm with Buffering starts by a similar computation as in AAG, and for each edge in the route, AAGB builds a buffer. It applies a pre-processing step similar to AAG by the calculation of new weights. In addition, it finds the distance between every pair of objects in $D$, and it computes the mean of these distances, denoted this mean by $\bar{L}$. AAGB constructs the route greedily in the same way as AAG, but uses a buffer to add objects that are near the route constructed by AAG. The buffering is computed as follows. Suppose in step $i$ the last object is $o_{i}$, AAGB increases the route by adding the object $o_{j} . d_{i, j}=\operatorname{distance}\left(o_{i}, o_{j}\right)$ and $b_{i, j}$ is the width of the buffer. We compute the size of $b_{i, j}$ to guarantee that $\Delta L_{i, j}$ $=\operatorname{distance}\left(o_{i}, o^{\prime}\right)+\operatorname{distance}\left(o^{\prime}, o_{i}\right)-d_{i, j} \leqslant \bar{L}$. That is, the added distance by going to some object $o^{\prime}$ in the buffer $\Delta L_{i, j}$ should not exceed the mean distance between objects in the dataset.

Incremental Route Search Query: The following paragraphs are going to summarize some existing works of route search query which have attracted increasing attention nowadays. Li et al. [LLT11] propose a new query called Trip Planning Query (TPQ) in spatial databases, in which each spatial object has a location and
a category, and the objects are indexed by an R-tree. Each Trip Planning Query consists of three components: a start location $s$, an end location $t$, and a set of categories $C$, and it is to find the shortest route that starts at $s$, passes through at least one object from each category in $C$ and ends at $t$. TPQ has been proven that it is a deduction from the Traveling Salesman problem (NP problem). Based on the triangle inequality property of metric space, two approximation algorithms including a greedy algorithm and an integer programming algorithm are proposed.

Compared with TPQ, keyword-aware optimal route query, denoted by KOR, which is to find an optimal route such that it covers a set of user-specified keywords, a specified budget constraint is satisfied and the objective score of the route is optimized. The problem of answering KOR queries is NP Problem. Paper [CCCX12] devises two approximation algorithms, i.e., OSScaling and BucketBound. Results of empirical studies show that all the proposed algorithms are capable of answering KOR queries efficiently, while the algorithms BucketBound and Greedy run faster. We also study the accuracy of approximation algorithms.

Sharifzadeh et al. [SKS08] propose a variant problem of TPQ, called optimal sequenced route query (OSR). A total order of OSR on the categories $C$ is imposed and only the starting location $s$ is specified. Two elegant exact algorithms LLORD and R-LORD are proposed to deal with query OSR. OSR are constructed under the same setting which is indexed by an R-tree. The metric space based pruning strategies are developed in the two exact algorithms.

Chen et al. CKSZ08 define the multi-rule partial sequenced route (MRPSR) query, which is a unified query of TPQ and OSR. Three heuristic algorithms are proposed to answer MRPSR. KOR is different from OSR and MRPSR and their algorithms are not applicable to process KOR.

Kanza et al. KSSD08 brings in a different route search query on the spatial database: the length of the route should be smaller than a specified threshold while the total text relevance of this route is maximized. Greedy algorithm is proposed
without guaranteeing to find a feasible route. Their team develop several heuristic algorithms for answering a similar query in an interactive way KLSS09. The progress is like this: the user provides feedback on whether the object satisfies the query after visiting each object and the feedback is considered when computing the next object to be visited. Another work proposed by this team, LKSS10, developed approximate algorithms to solve OSR with order constraints in an interactive way. Kanza et al. also study the problem of searching optimal sequenced route in probabilistic spatial database KSS09.

Malviya et al. MMB11 is aiming at answering continuous route planning queries over a road network, in other words, to find the shortest path in the presence of updates to the delay estimates.

Roy et al. RDAYY11 consider the problem of interactive trip planning. The query helps the users' itineraries based on the users preferences and time budget.

Yao et al. YTL11 propose the multi-approximate-keyword routing (MARK) query. A MARK query is specified by a starting and an ending location, and a set of (keyword, threshold) value pairs. It searches for the route with the shortest length such that it covers at least one matching object per keyword with the similarity larger than the corresponding threshold value.

### 2.2.5 Other $k$ Nearest Neighbor queries

In this section, the popular variants of nearest neighbor queries are described. More specifically, we present the related work on reverse nearest neighbor queries in section 2.2.5. Then section 2.2 .5 reviews the mutual $k$ nearest neighbor queries.

## Reverse Nearest Neighbor Queries

Reverse nearest neighbor queries search, as one of the most popular variant of nearest neighbor query, focuses on the inverse relation of $k$ nearest neighbor search. The definition of reverse neighbor query (RNN) is to find all the objects that consider $q$ as nearest neighbor, which is formally defined in Definition 2.2.5.


Figure 2.10: An example of reverse nearest neighbor query approach

Definition 2.2.5. Reverse Nearest Neighbor: Given a set of objects $P$ and a query object $q$, a reverse nearest neighbor query $R N N=\left\{p \mid q=N N_{p}, p \in P\right\}$.

The query result set of RNN may contain 0 element or one or more elements. In Ber93, the formal definition of reverse nearest neighbor query and some applications are proposed. For example, when a shopping mall chooses a site to open a new branch, we may use $R \mathrm{NN}$ to find the customers effected by this shopping mall. Moreover, $R \mathrm{NN}$ can also be used to choose the location which maximizes the number of potential customers. That is a bichromatic example. Take another monochromatic example, a RNN query may be issued to find petrol stations that are affected by opening a new petrol station at the new site. In summary, a bichromatic query (the first example) is to find the reverse nearest neighbors within two different types of objects. A monochramtic RNN query (the second example) is to find the reverse nearest neighbors where the data set contains only one type of object Ber93.

There are a lots of existing approaches of reverse nearest neighbors proposed in the past few years SRAE01,SAE00, MVZ02, LNY03, YL01. We will briefly describe some most popular and general algorithms in the following paragraphs.

In paper KM00, a RNN query firstly pre-calculates a circle of each object $p$ that its nearest neighbor lies on the perimeter of the circle as shown in Fig 2.10.


Figure 2.11: An example of reverse nearest neighbor query approach - SAA

Another technique that does not have any preprocessing involved was proposed by Stanoi et al. [SAE00], denoted as SAA. They partitioned the whole space centered at the query point $q$ into six equal regions of 60 degrees each $(b, c, d, e, f$ and $h$ in Fig. 2.11). In each region, only the nearest neighbor to $q$ can be the reverse $k$ nearest neighbor result. So the other point in the same region can be pruned. Take fig 2.11 as an example, in the left-below region, assume $h$ is the nearest neighbor of $q$, they observe that for a nearest neighbor object $h$ of $q$ in this region; either $h$ is the RNN of $q$ or there is no RNN in this region. But we can observe that $h$ is closer to $i$ than $q$. As a result, there is no RNN of $q$ in this left below region and we do not need to consider other objects in this region. To sum up, SAA processes RNN queries in two steps: firstly, find the nearest neighbor for each of the six regions and then form as a candidate list. Secondly, for each point in the candidate list, generate its nearest neighbor query. If the result is $q$, it should be included into $q$ 's $R N N$ result. Otherwise, discard this point. As a result, $a, i$ and $g$ are pruned. When $k$ $\geq 1$, the $\mathrm{R} k \mathrm{NN}$ queries can be solved in a similar way, i.e., in each region, the $k$ th nearest neighbor of $q$ defines the pruned area.

Another reserve $k$ nearest neighbor search approach is proposed by Tao et al TPL04], denoted as Half-Plane Pruning RNN. It brought the idea of perpendicular


Figure 2.12: An example of reverse nearest neighbor query approach - Half-Plane Pruning RNN
bisector into the methods in order to reduce the search space. Firstly we link $d$ with $q$, then find the midpoint of the the link $d q$. After that we form the perpendicular bisector line (line $d: q$ ) with the link $d q$. Line $d: q$ divides the space into two half planes $P L_{q}$ and $P L_{d}$ where $P L_{q}$ contains $q$ and $P L_{d}$ contains $d$. In other words, there will not be any point considering $q$ as nearest neighbor in plane $P L_{d}$ because in this plane, points are closer to $d$ other than $q$. Based on this property, we can use the line to prune the MBRs which completely fall into Plane $P L_{d}$. The same steps are invoked for the rest of the objects until the smallest region is found. Their approach can also be extended to answer $\mathrm{R} k \mathrm{NN}$ queries that is to find all objects for which $q$ is one of their $k$ nearest neighbors (see fig 2.12 ).

## Mutual $k$ Nearest Neighbor Search

Definition 2.2.6. Given a dataset $P$, a query point $q$ and user defined $k$, mutual $k$ Nearest Neighbor search is to find the set of object $S \subseteq P$, that $S=p \mid p \in N N_{k}(q)$ and $q \in N N_{k}(p), \forall p \in S$.
$k \mathrm{NN}$ search is asymmetric. However, MkNN retrieval is symmetric. For example, $\operatorname{M} k \mathrm{NN}\left(p_{1}\right)=\left\{p_{2}\right\}$ indicating that $\operatorname{M} k \mathrm{NN}\left(p_{2}\right)=\left\{p_{1}\right\}$.

The following items list the popular mutual $k$ nearest neighbor approaches GZCL09. SP is very inefficient in terms of I/O overhead and CPU cost, especially for large values of $k$. To overcome this deficiency, the last 4 approaches are proposed to improve the performance of $\mathrm{M} k \mathrm{NN}$ query processing via different optimization techniques.

- Simple processing algorithm (SP)

Simple processing algorithm (SP) is proposed based on the definition of $\mathrm{M} k \mathrm{NN}$ query by GZCL09. It firstly conducts a $k N N$ search to retrieve the candidate set CandidateSet $=k N N_{q}$. Then it verified each candidate $p \in$ CandidateSet. The verification of a candidate $p$ can be conducted again via a $k N N$ search to check whether $q \in k N N_{p}$. If the result is yes, it means that $q$ is among the $k \mathrm{NN}$ of $p$ and hence $p$ is returned as an answer object. Otherwise, $p$ is discarded, i.e., it is a false hit.

- Two-step Algorithm (TS)

As every object included in the candidate set CandidateSet need to be verified in the SP algorithm, the overhead and cost are extremely high. Since reverse $k$ nearest neighbor search can verify the object $p$ as well, so this step can be made to do the reserver $k$ nearest neighbor of query point $q$. This method is defined as two-step algorithm in (GZCL09].

- Reuse Two-heap Algorithm (RTH)

Reuse two-heap algorithm (RTH) is proposed together with SP and TS in [GZCL09] , which attempts to fully use locally available nodes in order to reduce the redundant node accesses. In addition, an early termination condition is developed to be applied in the verification process of the CandidateSet. It is possible that any $p$ in CandidateSet may be terminated earlier without finding all the $k \mathrm{NN}$ of $p$.

- NN search with Pruning (NNP)

NN search with Pruning (NNP) introduces pruning heuristics at two places to improve the search performance. The first pruning is conducted in integrating
with kNN search, handled by an NNP Finding algorithm; and the second pruning is introduced as a self-pruning process. The main target is to remove those candidates that will not have any possibility to be $\mathrm{R} k \mathrm{NN}(q)$.

- RNN search with pruning (RNNP)

When $\operatorname{Size}\left(k N N_{q}\right) \geqslant \operatorname{Size}\left(\operatorname{R} k N N_{q}\right)$, there is a better way to do the reverse $k$ nearest neighbor search of query point first, then verify each object in $\operatorname{RkN} N_{q}$ set. It is a reverse way of the traditional mutual $k$ nearest neighbor, which does the $k N N_{q}$ first and verifies it after that.

To sum up, this chapter reviews the existing works related to typical $k$ nearest neighbor search, continues $k$ nearest neighbor search as well as $k$ nearest neighbor variants. Although the areas of interest have been filled up in the recent decades, there are still some significant problems or, in other words, there are still some gaps/blank zones which have not been explored. Moreover, even the existing methods which can solve the problems, but perform poorly under some circumstances.

### 2.3 Problem Definition

As mentioned above, the following list summarizes the general problems of existing methodologies which are the motivation of this thesis as well. Fig 2.13 compares the our proposed approaches with related works.

## - Poor performance of Network Expansion Methodology

All of the approaches are constructed based on the underlying road connection between objects. Within the road map, roads are connected and joined by thousands of intersection nodes which break the roads into small segments. The total distance of the road is calculated by summing up the component segment distances. As a result, network expansion is the technique which has been widely used in the existing methods. Network expansion is processed as follows: when encountering any intersection node, the traverse expansion is done in every possible direction. In other words, if we suppose every node has
four possible directions to go, then the expansion would be $4^{n}, n$ is the number of expansion nodes. From this calculation, we can infer that the performance cost will behave like a parabola with the increasing number of intersection nodes. This poor performance is inevitable because the complex road connection will result in large number of intersection nodes. Consequently, network expansion methodology has an instinctive drawback which will lead to poor performance of spatial queries. How to merge the intersection nodes or how to avoid the expansion becomes a new topic to researchers who are engaged in spatial query processing.

## - Discrete points as $k \mathrm{NN}$ input and output.

Nearly all spatial queries are objects related which means both the input and output of the queries are discrete points. While in reality, path/route is another important element in spatial space. The second outstanding problem is the unicity of the input/output types. Consider that the user might want to input a path to find a set of points, or input a set of points to create an optimal path, or input a query path and output a result path at the same time, the second contribution can be made is bringing the path into $k \mathrm{NN}$ input and output.

Consequently, the problem and contributions are summarized as follows:

Contribution 1 Network Voronoi Diagram is used to merge the road segment which highly proves the performance of $k$ Nearest Neighbor Search compared to Network Expansion Methodology. Two Voronoi based $k$ Nearest Neighbor search queries are proposed in chapter 3, namely Voronoi-based Continuous $k$ NN Search Queries in section 3.2 and Multiple object types $k$ NN Search in section 3.3 .

Contribution 2 As stated above, most of the existing queries put discrete points as input and output. Consequently, Chapter 4 concentrated on bringing route/path into the input or output or both of the queries. Three route search queries


Incremental Route Search Query
Figure 2.13: Related Work vs. Approaches proposed in this thesis
are proposed in this chapter which are all route related, namely Path based $k N N$ Search Queries in section 4.2, Path Branch Point based $k$ NN Search Queries in section 4.3 and Time Constraint Route Search over Multiple Locations in section 4.4.

### 2.4 Summary

In this chapter, firstly we introduce the divisions of spatial queries from a query point of view, a result point of view as well as the accuracy point of view. Then it outlines the structure of this related work chapter. Secondly, a comprehensive summary of the existing work is explained. Preliminarily it goes first followed by several example and goes through a process of static $k$ Nearest Neighbor Search, continuous $k$ Nearest Neighbor Search, route search queries and other spatial queries.

After reviewing these approaches, two outstanding problems are pointed out which also lead our two main part of contribution in my thesis. Finally, it summarizes this chapter in this conclusion section and illustrates the contribution to existing work in fig. 2.13.

## Chapter 3

## Voronoi Based $k$ Nearest Neighbor Search ${ }^{1}$

### 3.1 Introduction

With the developing wireless devices and booming spatial query searching, nearly all of the approaches are constructed based on the underlying road connection between objects. Within the road map, roads are connected and joined by thousands of intersection nodes which break the roads into small segments. The total distance of the road is calculated by summing up the component segments distances. As a result, network expansion is the technique which has been widely used in the existing methods. Network expansion is processed as follows: when encountering any intersection node, the traverse expansion is made to every possible directions. In other words, if we suppose every node has four possible directions to go, then the expansion would be $4 \mathrm{n}, \mathrm{n}$ is the number of expansion nodes. From this calculation, we can infer that the performance cost will behave like a parabola with the increasing number of intersection nodes. This poor performance is inevitable because the complex road connection will result in a large number of intersection nodes.

[^0]Consequently, Network Voronoi Diagrams are chosen to merge the network segments which obviously improves the performance and reduces the cost. In this chapter, we propose two approaches, namely Voronoi based $k$ Nearest neighbor search and Voronoi based multiple types $k$ Nearest neighbor. In both queries, Voronoi Diagrams are used as the methodology which has been proven to be applicable and efficient. The following paragraphs introduce them in detail.

Voronoi based $k$ Nearest neighbor search is an approach to deal with the Continuous $k$ NN (abbreviated as C $k \mathrm{NN}$ ) query. C $k \mathrm{NN}$ SE06, KS05, TPS02a also have attracted other researchers' interest. In order to find split nodes, all existing continuous $k$ NN approaches divide the query path into segments, find $k$ NN results for the two terminate nodes of each segment and then, for each segment, find the split nodes. One segment of the path starts from an intersection and ends at another intersection. For every segment, a $k$ NN process is invoked to find split nodes for each segment. If there are too many intersections on the path, there will be many segments, and consequently, the processing performance will degrade. These are the obvious limitations of the current $\mathrm{C} k \mathrm{NN}$ approaches. As a result, section 3.2 proposes an alternative approach for $\mathrm{C} k \mathrm{NN}$ query processing, which is based on the Network Voronoi Diagram (we call our proposed method VCkNN, for Voronoi CkNN). This approach avoids these weakness mentioned above and improves the performance by utilizing a Voronoi diagram. VCkNN ignores intersections on the query path; instead, it uses Voronoi polygons to subdivide the path. In this chapter, the Voronoi diagram, which originated in the computational geometry GR03, GR99 and has been used successfully in other areas, such as industrial electronic area VS08, will be demonstrated in its effectiveness in a mobile environment.

Current approaches for $k$ NN mainly use network expansion. Network expansion consumes a large amount of processing time because segments invoke functions iteratively. Consequently, a Voronoi diagram is adopted as the most suitable tool to solve $k \mathrm{NN}$ queries because it aggregates lots of segments into polygons. However, current approaches focus on one object type, which narrows down the mobile query
scope. For example, to find the nearest 3 hospitals to a current location. In some cases, users may want to get $k \mathrm{NN}$ of different object types (multiple object types), as well as to obtain the shortest routes. Motivated by these, this chapter proposes new approaches on three different queries involving multiple object types using a network Voronoi Diagram. In these queries, more than one object type is considered and the query result is highly related with the object types. Every object belongs to one category and there is no overlap between categories. That is the basic property of multiple-object-type query. In section 3.3 focuses on three different types of $k \mathrm{NN}$ mobile queries, including: a) query to find nearest neighbor for multiple types of interest point (or 1NN for each object type), b) query to give the shortest path to cover multiple-object-types in a pre-defined sequence, and c) query to find an optimum path for multiple object types that gives the shortest path that covers the required interest objects in a random sequence.

From this point, two methods are proposed in the following sections followed after the performance evaluation.

### 3.2 Approach 1: Voronoi-based Continuous $k$ NN

## Search

Continuous $k$ nearest neighbor search is not a novel type of query in a mobile environment, as it has been well studied in the past. Continuous $k$ nearest neighbor can be defined as given a moving query point, its pre-defined moving path and a set of candidate interest points, to find the point on the way where $k$ nearest neighbor changes. This is a traditional query in mobile navigation. To get the exact point on the road in short response time is not easy. Almost all of the approaches try to find the split nodes, which are the locations where the $k$ NN results are changed. The already existing works on $\mathrm{C} k \mathrm{NN}$ have some limitations as follows.

- Both DAR/eDAR and IE need to divide the pre-defined query path into segments using the intersections on the road. It means that once there is an
intersect road in the path, it becomes a new segment, and we need to check whether there are any split nodes on this segment.
- Using DAR/eDAR and IE, for every segment we should find $k$ NN for the start and end nodes of the segment. It obviously reduces the efficiency of the performance when the number of intersections on the query path becomes large.
- DAR/eDAR uses PINE (based on a Voronoi diagram) to do the $k$ NN for the start and end nodes of each segment. But when doing continuous $k N N$, DAR/eDAR discards the Voronoi diagram and adopts another method to detect split nodes. While in our proposed approach, we use Voronoi diagram all the way through both in the $k \mathrm{NN}$ and $\mathrm{C} k \mathrm{NN}$ stages. Hence, the properties of the Voronoi diagram are used to enhance the $\mathrm{C} k \mathrm{NN}$ process.
- Both DAR/eDAR and IE cannot predict where split nodes will appear. In our proposed Voronoi-C $k \mathrm{NN}(\mathrm{VC} k \mathrm{NN}$ ), it is known even before we reach the point and also it gives us the visibility of which interest point is moving out or into the list and at which position the node will become a split node.

Our proposed VCkNN approach is based on the attributes of the Voronoi diagram itself and using a piecewise continuous function to express the distance change of each border point. At the same time, we use Dijkstra's algorithm to expand the road network within the Voronoi polygon.

## Comparison (VC $k$ NN vs. DAR vs. IE)

$\mathrm{VC} k \mathrm{NN}, \mathrm{DAR}$ and IE are all approaches for $\mathrm{C} k \mathrm{NN}$ queries. But VC $k$ NN is different from DAR and IE in most of aspects. Therefore before introducing our VCkNN algorithm, we would like to highlight the main differences between VCkNN and DAR and IE.

- Path division mechanism

For the same network connection, DAR and IE divide the query path into


Figure 3.1: Segments using DAR and IE


Figure 3.2: Segments using VCkNN
segments as shown in Fig 3.1, whereas VC $k$ NN processes the path as in 3.2. Note that in Fig 3.1, for every intersection in the query path, it becomes a segment. In this example, the query path is divided into 18 segments, as there are as many intersections along the query path. In contrast, using the same query path, our approach has only 5 segments (see 3.2 ). The number of segments is determined by the number of Voronoi polygons. Even though there are many intersections in each Voronoi polygon, our method will process each Voronoi polygon as a unit, and hence, there is no need to check intersection by intersection.

- $k$ NN processing

For each segment, DAR and IE use either PINE or $V N^{3}$ to perform $k$ NN processing for the two terminating nodes (e.g. start and end of the segment). In contrast, $\mathrm{VC} k \mathrm{NN}$ does not need any algorithm to do $k \mathrm{NN}$ on any point on the path. VCkNN finds $k$ NN level by level (from $1^{s t} \mathrm{NN}$, then $2^{\text {nd }} \mathrm{NN}$, then $3^{r d} \mathrm{NN}$, and so on) for the entire query path. Hence, $k \mathrm{NN}$ results can easily be visualized using VCkNN.

- Sequence finding of split nodes

DAR and IE use formulae to calculate the distance between two adjacent split nodes. Subsequently, we find split nodes one by one. This also means that we do not know the $(k+1)^{\text {th }}$ split node until we find $k^{\text {th }}$ split node. In contrast, $\mathrm{VC} k \mathrm{NN}$ locates split nodes using query point moving distance. For each interval, we identify the split nodes directly, which are the nearest distance between the query point and the intersected paths in the Voronoi polygon. Consequently, all split nodes are identified in one go.

- Processing split nodes

DAR and IE compare the $k N N$ results of the two terminate nodes of each segment to find all split nodes within this segment. On the other hand, VCkNN finds all split nodes top down from $1^{s t} \mathrm{NN}$, and then $2^{\text {nd }} \mathrm{NN}$ and so on. The following Tab 3.1 summarizes the differences between DAR, IE and VCkNN.

|  | VCkNN | DAR | IE |
| :--- | :--- | :--- | :--- |
| Query | Continuous k nearest neighbor search |  |  |
| Basic idea | Monitor border <br> points | Swap the position <br> to calculate the split <br> nodes | Monitor candidate <br> POI and using trend <br> to find split nodes |
| Segment | Ignore | Need to check segment by segment |  |
| Voronoi <br> polygon | Expansion polygon <br> by polygon | Ignore | Ignore |
| Split node <br> predicable | Yes | No | No |
| Visible | Yes | No | No |
| Do $k$ NN | No | Yes | Yes |

Table 3.1: VCkNN vs. DAR vs. IE

## VCkNN Algorithm

The benefits offered by the proposed VCkNN processing are supported by the inherent propositions of a Network Voronoi Diagram, which are as follows:

Proposition 1. The generator of the Voronoi polygon that includes the query point must be the nearest neighbor of the query point.

Proof. It is self-evident because the polygon defines the area where any point in this area is closer to the polygon's generator than other generators (refer to Property 2 listed in section 2.2.1.

The split nodes in Network Voronoi Diagram are determined by the following lemmas, which are the basis of our $\mathrm{VC} k \mathrm{NN}$ algorithm. The first lemma is about the split nodes, whereas the second lemma is about $k$ NN results.

Lemma 3.2.1. In Voronoi CkNN, all border points that intersect with the query path and the generator edge are split nodes.

Proof. It is obvious that when the query path reaches the generator edge, the 1stNN will change because the distance to the shared edge generators are the same (refer to Property 2 listed in section 2.2.1).

Axiom 3.2.1. If the query path overlaps with generator edge for a while, the first time when they intersect will be the split node and the last point where they no longer overlap will be the split node too.

Lemma 3.2.2. Suppose $q$ 's $k N N=p_{1}$, , $p_{k}$, the $(k+1)^{\text {th }} N N$ of $q$ should be within the neighbor of $p_{1}, p_{k}$.

Proof. According to the a property of the Voronoi diagram, Let $G=g_{1}, g_{k} \mathrm{P}$ be the set of the first $k$ nearest generators of a location $q$ inside $V\left(g_{1}\right)$, then $g_{k}$ is among the adjacent generators of $G \backslash g_{k}$.

Before the $\mathrm{VC} k \mathrm{NN}$ algorithm is presented in Algorithm.1, we need to define moving interval (ML):

Definition 3.2.1. (Moving interval) (ML) is the interval between two split nodes; in other words, ML is determined by two split nodes.

The location of split node is marked by the query point moving out distance. For example, if $M L$ is $0.7 \tilde{3} .0$, whereby 0.7 and 3.0 are two adjacent split nodes in current split nodes list, then 0.7 refers to the split node that is located at the point

```
Algorithm 1 Algorithm VC \(k\) NN ( \(q, k\), moving path \(S E\) )
    \(1^{s t} N N=\) contain \((q)\)
    Initial \(C S=1^{s t} N N\) 's neighbor generator.
    \(M=1\)
    Result \(=1^{\text {st }} N N\) (moving interval of query point)
    if \(M>1\) then
        for each polygon where \(S E\) goes across do
            Expand \(q\) to each border point
            Draw the line for each border point AND get piecewise function for each
            border point
            Add border to generator distance to the line
        end for
    end if
    The lowest line will the \(2^{\text {nd }} N N\). Intersect points will be split nodes. Set \(M=\)
    2
    Result \(+=2^{\text {nd }} N N\left(\right.\) MovingInterval \(\left._{1}\right), 2^{\text {nd }} N N\left(\right.\) MovingInterval \(\left._{n}\right)\)
    while \(M<K\) do
        for each intervals which separate by split node do
            \(C S=C S+M^{t h}\) neighbor generator
            for each interest point in \(C S\) do
            draw a line for this interval
            The lowest line will be the \((M+1)^{t h} N N\)
            Result \(+=(M+1)^{\text {th }}\) MovingInterval \(_{1},,(M+1)^{\text {th }}\) MovingInterval \(_{n}\)
                \(M=M+1\)
            Intersect nodes are split nodes
            end for
        end for
    end while
    if \(M=K\) then
        Terminate the algorithm
    end if
```

where query point moves out in a distance of 0.7 . The same is applied to 3.0 which is the split node location away from the current query point. The proposed VCkNN algorithm is given in Algorithm.1. Our VCkNN algorithm is explained as follows:

- Step 1: $1^{\text {st }} \mathrm{NN}$

Use the contain $(q)$ function to get the Voronoi polygon, which includes the query point. This polygon's generator will be the $1^{\text {st }}$ NN until it moves out from this polygon (according to preposition 1).

- Step 2: Split nodes

The intersections between query paths and polygon borders are split nodes (refer to lemma 1).

- Step 3: Moving Interval (ML)

Moving interval ( $M L$ ) will have segments within the Voronoi polygons and the query path is divided into several $M L \mathrm{~s}$. For each $M L$, we do the following. From the beginning point of the interval, expand the road network to every border point of this polygon and record the distance. For each border point, monitor the change of the distance. Get the piecewise function for each border point according to query point's moving out distance, and then a set of candidate interest points $(C S)$ is initialized that contains all adjacent neighbors of $1^{\text {st }} \mathrm{NN}$.

- Step 4: Candidate Interest Points ( $C S$ )

For all interest points in CS, calculate its distance to the beginning of the interval and generate the corresponding lines and functions. Every time a line is generated, put it into a chart which $x$ axis is the moving distance of the query point. The chart records all the changes of $k \mathrm{NN}$. One thing should be mentioned here is that, if one interest point has more than one border point in the current polygon, keep the one which has the shortest distance.

- Step 5: $2^{\text {nd }}$ NN and more split nodes

After finishing all interest points in $C S$, the lowest line (the one closest to $x$
axis) will be the $2^{\text {nd }} \mathrm{NN}$ and the intersections of lines will be the split nodes. These split nodes divide the current interval into multiple small ones. Then add the $2^{\text {nd }}$ NN's adjacent interest points into $C S$.

- Step 6: $k>2$

If $k>2$, then for every new interval, do the following: Remove the lowest lines from the chart in this interval. For all interest points in $C S$, calculate its distance to the beginning of interval and generate the corresponding line and functions. Every time we generate a line, put it into the chart. The lowest lines will be the next level of NN. New split nodes are the intersections on the lowest lines, and new intervals are generated by these split nodes. Update $C S$ by adding new NN's neighbor into $C S$. If the NN level is still less than $k$, do this step again until all $k N N$ s have been found.

- Step 7: Process termination conditions

Finally, after all Voronoi polygons where a query path goes across have been checked, and all split nodes have been found, the algorithm terminates.

## Walk through Example of VC $k N N$

This section describes a walk through of the VCkNN process. It not only explains the VCkNN step by step but also compares it with other works, including DAR and IE. We will list the piecewise function and draw the line in the chart to make it easy to understand. Fig 3.3 shows an example. The query is to find $\mathrm{C} k \mathrm{NN}$ along the query path, shown as a thick black line, which starts from q and ends at $p_{10}$. The borders of $V\left(P_{1}\right)$ and the paths from P 1 to the border points are also shown.

The first set of split nodes is the intersection nodes between Voronoi polygons and the moving path. In this case, SplitNodes $=b_{2}, b_{9}, b_{10}$. Refer to Lemma 3.2.1 on the split nodes. Split node b2 is the border point between Voronoi polygon $V\left(P_{1}\right)$ and $V\left(P_{2}\right)$, split node b 9 is the border point between $V\left(P_{2}\right)$ and $V\left(P_{3}\right)$, and split node $b_{10}$ is the border point between $V\left(P_{3}\right)$ and $V\left(P_{10}\right)$.


Figure 3.3: Example of VCkNN

Note the $1^{\text {st }}$ NN results are $p_{1}$ with a range of distance from 0.0 and $5.0, p_{2}$ with a range of distance from 5.0 and $10.0, p_{3}$ with a range of distance from 10.0 and 13.0 , and P10 with a range of distance 13.0 and 14.0 . In short, we can write the $1^{\text {st }}$ NN results something like this:

$$
1^{s t} \mathrm{NN}=p_{1}(0.05 .0), p_{2}(5.0 \text { 10.0 }), p_{3}(10.0 \text { 13.0 }), p_{10}(13.014 .0)
$$

All ranges of distance are the distance from the starting query point. This means that when the query point $q$ moves from 0.0 to $5.0, p_{1}$ is $1^{s t} \mathrm{NN}$, and when $q$ moves from 5.0 to $10.0, p_{2}$ will be the $1^{\text {st }} \mathrm{NN}$, and so on.

Then for $V\left(P_{1}\right), V\left(P_{2}\right), V\left(P_{3}\right)$ and $V\left(P_{10}\right)$, do the following steps. Take $V\left(P_{1}\right)$ as an example.

Firstly we need to set some initial values according to the VCkNN algorithm (1) $M=1$ as we have found the $1^{s t}$ level of $k \mathrm{NN}$, and $C S=p_{2}, p_{3}, p_{4}, p_{5}, p_{6}, p_{7}, p_{8}$, which are the adjacent nodes of $V\left(P_{1}\right)$.

Secondly, expand the query point q to every border point in this polygon. With the movements of $q$, draw a line for every border point and get the piecewise function
for each border point. Table 3.2 shows the line along the movement of query point. The first column is the moving distance from the current location of query point $q$.

Tab 3.2 Movement of each border point in $p_{1}$.

| q moving distance (km) NN | $b_{1}$ | $b_{2}$ | $b_{3}$ | $b_{4}$ | $b_{5}$ | $b_{7}$ | $b_{8}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0 | 4.0 | 5.0 | 8.0 | 5.0 | 1.0 | 3.5 | 5.5 |
| 1.0 | 3.0 | 4.0 | 7.0 | 4.0 | 2.0 | 2.5 | 4.5 |
| 1.5 | 2.5 | 3.5 | 7.5 | 4.5 | 2.5 | 2.0 | 4.0 |
| 2.0 | 2.0 | 3.0 | 8.0 | 5.0 | 3.0 | 2.5 | 4.5 |

Table 3.2: Movement of each border point in $p_{1}$

The corresponding chart (see fig. 3.4 ) and the piecewise function 3.2 are shown as follows. Note from fig. 3.4 , the line for border point $b_{2}$ goes down from 5 when the position of $q$ is 0 , to 0 when the position of $q$ is around 5 . The opposite is the line for border point $b_{5}$ where it goes up as $q$ is moving from 0 to 5 (in this case the line for $b_{5}$ increases from 1 to 6 ). These two lines explain that when $q$ moves, the distance from $q$ to $b_{2}$ will be decreasing and $b_{2}$ is getting closer to $q$. The opposite is to $b_{5}$, where q is actually moving away from it.

The rest of the border points, such as $b_{1}, b_{3}, b_{4}, b_{7}$, and $b_{8}$, are all getting closer to $q$ when $q$ moves from 0 to some points before 2, but then they all increase after that. This indicates that initially the distance from $q$ to these border points is decreasing (the border points are getting closer to $q$ ), but later it will be increased as $q$ moves away from these border points.

In term of mathematical functions, these distance movements can be expressed in a piecewise function as shown in fig 3.4. Note that the functions for $b_{2}$ and $b_{5}$ are straight functions, whereas the rest have some conditions when to increase and when to decrease.

$$
b_{1}=\left\{\begin{array}{ll}
4-x & x \in[0,2] \\
x & x \in(2,5]
\end{array} \quad b_{3}=\left\{\begin{array}{ll}
8-x & x \in[0,1] \\
x+6 & x \in(1,5]
\end{array} \quad b_{5}=x+1[0,5]\right.\right.
$$

$$
b_{8}=\left\{\begin{array}{ll}
5.5-x & x \in[0,1.5]  \tag{3.1}\\
x+2.5 & x \in(1.5,5]
\end{array} b_{4}=\left\{\begin{array}{ll}
5-x & x \in[0,1] \\
x+3 & x \in(1,5]
\end{array} \quad b_{7}=\left\{\begin{array}{cc}
3.5-x & x \in[0,1.5] \\
x+0.5 & x \in(1.5,5]
\end{array}\right.\right.\right.
$$

Thirdly, for each interest point, add its distance to the corresponding border into table 3.2 and do the chart again (as shown in fig.3.5). Suppose their distances to the borders are as follows:

$$
\left\{\begin{array}{l}
\operatorname{Distn}\left(b_{1}, P_{2}\right)=2.2  \tag{3.2}\\
\operatorname{Distn}\left(b_{2}, P_{2}\right)=3.2 \\
\operatorname{Distn}\left(b_{3}, P_{2}\right)=7.8, \operatorname{Distn}\left(b_{3}, P_{3}\right)=7.8, \operatorname{Distn}\left(b_{3}, P_{4}\right)=7.8 \\
\operatorname{Distn}\left(b_{4}, P_{5}\right)=4.8 \\
\operatorname{Distn}\left(b_{5}, P_{6}\right)=2.8 \\
\operatorname{Distn}\left(b_{7}, P_{7}\right)=2.3 \\
\operatorname{Distn}\left(b_{8}, P_{8}\right)=4.3
\end{array}\right.
$$

Note that $p_{2}$ is adjacent to $b_{1}, b_{2}$, and $b_{3}$. Also note that the adjacent polygons of $b_{3}$ are $p_{2}, p_{3}$, and $p_{4}$. Others indicate that $b_{4}$ adjacent with $p_{5}$, b5 with $p_{6}, \mathrm{~b} 7$ with $p_{7}$, and finally b8 with $p_{8}$. Fig 3.5 shows how each interest point, $p_{2}$ to $p_{8}$ are adjacent with the corresponding border points. For example, the top line in fig 3.5 indicates the distance from $q$ to $p_{4}, p_{3}$ and $p_{2}$ (all through $b_{3}$ ). The line, as explained previously, shows that initially $p_{4}, p_{3}$ and $p_{2}$ are getting closer to q but are then getting farther.

In fig 3.6, we only focus on the bottom lines. The intersections between all bottom lines are new split nodes. The first intersection is between line $p_{6}$ and line $p_{7}$ at $q=1.0$ (This is pointed by the first vertical dotted line). The second intersection is between line $p_{7}$ and line $p_{2}$ at $q=1.7$ (pointed by the second vertical dotted line). The third intersection is between line $p_{2}$ (through border point $b_{2}$ ) and line $p_{2}$ (through border point $b_{1}$ ). And finally the last intersection for this interval is the lowest point of line $p_{2}$ (through border point $b_{2}$ ). Hence, we have four new split nodes for this interval.


Figure 3.4: $p_{1}$ border

|  |
| :---: |

Figure 3.6: $2^{\text {nd }} \mathrm{NN}$

Figure 3.5: Each $p$


Figure 3.7: $3^{r d} \mathrm{NN}$
$2^{\text {nd }} \mathrm{NN}$ for this interval are: $p_{6}, p_{7}, p_{2}$ (through $b_{1}$ ), and $p_{2}$ again (but through $b_{2}$ ). In summary, $2^{\text {nd }} \mathrm{NN}$ for the $0.0-5.0$ interval are:
$2^{\text {nd }} \mathrm{NN}$ for $0.0-5.0$ interval $=\left\{p_{6}(0.0-1.0), p_{7}(1.0-1.7), p_{2}(1.7-3.0), p_{2}(3.0-5.0)\right\}$
In other words:

- When $q$ moves from 0.0 to $1.0,2^{\text {nd }} \mathrm{NN}=p_{6}$
- When $q$ moves from 1.0 to $1.7,2^{\text {nd }} \mathrm{NN}=p_{7}$
- When $q$ moves from 1.7 to $3.0,2^{\text {nd }} \mathrm{NN}=p_{2}\left(\right.$ through $\left.b_{1}\right)$
- When $q$ moves from 3.0 to $5.0,2^{\text {nd }} \mathrm{NN}=p_{2}\left(\right.$ through $\left.b_{2}\right)$

Fourthly, after we get $2^{\text {nd }}$ NN, we update $C S$ for every interval of the new split nodes, that is interval $0.0-1.0$, interval $1.0-1.7$, and interval 1.7-5.0. There is no need to split interval 1.7-5.0 into two intervals of 1.7-3.0 and 3.0-5.0, since the $2^{\text {nd }} \mathrm{NN}$ for this interval is the same, that is $p_{2}$.

- For interval 0.0-1.0: $C S=\left\{p_{2}, p_{3}, p_{4}, p_{5}, p_{7}, p_{8}, p_{15}, p_{16}, p_{17}, p_{18}\right\}$, and $2^{\text {nd }} \mathrm{NN}$ $=\left\{p_{6}(0.0-1.0)\right\}$. This means that when $q$ moves from 0.0 to $1.0, p_{6}$ is $2^{\text {nd }} \mathrm{NN}$.
- For interval 1.0-1.7: $C S=\left\{p_{2}, p_{3}, p_{4}, p_{5}, p_{6}, p_{8}, p_{18}\right\}$, and $2^{n d} \mathrm{NN}=\left\{p_{7}(1.01 .7)\right\}$. This means that when $q$ moves from 1.0 to $1.7, p_{7}$ is $2^{n d} \mathrm{NN}$.
- For interval 1.7-5.0: $\mathrm{CS}=\left\{p_{3}, p_{4}, p_{5}, p_{6}, p_{7}, p_{8}, p_{9}, p_{10}\right\}$, and $2^{\text {nd }} \mathrm{NN}=$ $\left\{p_{2}(1.75 .0)\right\}$. This means that when $q$ moves from 1.7 to $5.0, p_{2}$ is $2^{\text {nd }} \mathrm{NN}$.

Fifthly, if $k>2$, for every interval listed above, we need to process further. Note that the process is done iteratively from a larger interval to a smaller interval, until the smallest interval cannot further be divided. To illustrate our example, we take the 1.0-1.7 interval. This process can be thought like using a magnifying glass on the $1.0-1.7$ interval of the previous process (in fig 3.6), and the result is shown in fig 3.7. We need to update the line in fig 3.7 for all interest points in $C S$.

Fig 3.7 shows the 1.0-1.7 interval, where the lines are updated for all interest points in $C S . C S$ for 1.0-1.7 interval is $C S=\left\{p_{2}, p_{3}, p_{4}, p_{5}, p_{6}, p_{8}, p_{18}\right\}$, and the $2^{\text {nd }} \mathrm{NN}$ for this interval is $p_{7}$. The adjacent nodes of $p_{7}$ are $p_{6}, p_{18}$, and $p_{8}$.

Suppose the distances between $b_{7}$ and these adjacent nodes are:

$$
\left\{\begin{array}{l}
\operatorname{Distn}\left(b_{7}, P_{6}\right)=3  \tag{3.3}\\
\operatorname{Distn}\left(b_{7}, P_{18}\right)=5 \\
\operatorname{Distn}\left(b_{7}, P_{8}\right)=4
\end{array}\right.
$$

Note that we only need to get the distance between border point $b_{7}$ and all adjacent polygons of the $2^{\text {nd }} \mathrm{NN}$ which is $p_{7}$, because border point $b_{7}$ is the border between $p_{7}$ and $p_{1}$ (the Voronoi polygon of the query point).

After calculating the above three distances, which represent three lines on the chart, we draw the three lines on the chart again. The split nodes are found at the interactions of the bottom lines (refer to figure 13). As a result the 1.0-1.7 intervals is now divided into two smaller intervals: $1.0-1.2$ and $1.2-1.7$.

For interval 1.0-1.2:CS $=\left\{p_{2}, p_{3}, p_{4}, p_{5}, p_{8}, p_{15}, p_{16}, p_{17}, p_{18}\right\}$, and $3^{r d} \mathrm{NN}=\left\{p_{6}\right.$ $(1.0-1.2)\}$. This means that when q moves from 1.0 to $1.2, p_{6}$ is $3^{r d} \mathrm{NN}$.

And For interval 1.2-1.7: $C S=\left\{p_{3}, p_{4}, p_{5}, p_{8}, p_{9}, p_{18}\right\}$, and $3^{r d} \mathrm{NN}=\left\{p_{2}\right.$ $(1.2-1.7)\}$. This means that when $q$ moves from 1.2 to $1.7, p_{2}$ is $3^{r d} \mathrm{NN}$.

In summary, $3^{r d} \mathrm{NN}$ for the $1.0-1.7$ interval are: $3^{r d} \mathrm{NN}$ for $1.0-1.7$ interval $=$ $\left\{p_{6}(1.0-1.2), p_{2}(1.2-1.7)\right\}$

We need to do the same thing for the other two intervals of the 2nd NN, which are $0.0-1.0$, and $1.7-5.0$. This is repeated until the desired $k$ is achieved.

Finally, after the algorithm finishes, we can see clearly where the split nodes are and also every point on query path; in other words, we can tell the $k \mathrm{NN}$ results straightaway, without processing $k$ NN on every single split node like DAR and IE.

If we just look at the $1.0-1.2$ interval, for an example sake, if the query is $3^{r d}$ NN, then the $3^{r d}$ NN for this interval is $p_{1}, p_{7}$, and $p_{6}$. $p_{1}$ will remain 1 stNN until distance 5.0 ( $p_{1}$ actually starts becoming the $1^{s t} \mathrm{NN}$ from distance 0.0 ), and $p_{7}$ will remain $2^{\text {nd }}$ NN until distance 1.7 . Finally, $p_{6}$ is only the $3^{r d}$ NN for this interval only (e.g. 1.0-1.2 interval).

### 3.3 Approach 2: Voronoi based Multiple kNN Search

In this section, we present our proposed algorithms for the three kinds of multiple-object-type $k N N$ queries. The first two proposed query processing (M_NN and iM_NN) use two approaches, namely: using one NVD for each object type, and using one NVD for all object types, whereas the last proposed query processing for PM_NN uses one NVD for all object types model.

Before proposing approaches for $k \mathrm{NN}$ queries for multiple object types, we firstly introduce the taxonomy of these three queries with examples:

1. Multiple-object-types Nearest Neighbor (M_NN) query is to find nearest neighbors for multiple object types. It is common in mobile navigation. Around the query point, there are $k$ different types of interest points. For each object type, to find the nearest neighbor among the same object type is the objective of the query.

Example 3.3.1. Suppose a group of colleagues wants to have dinner together, and around their company there are hundreds of restaurants. They prefer French, Italian and Chinese food. As a result, they want to know the nearest French, Italian and Chinese restaurant respectively first and then make the final decision.
2. Incremental Multiple-object-types Nearest Neighbors (iM_NN) query is to find optimum/shortest path to pass multiple object types in the pre-defined sequence. This query can be used when the sequence of passed interest points is critical for the user.

Example 3.3.2. Suppose a person falls ill at home suddenly, the family wants to tell their driver the following path. Firstly, obviously they want to go to the nearest hospital because the sickness is acute. Secondly, they need to go to the nearest medical checkup clinic. After that, they will go to find the nearest

GP office to check the checkup result and finally go to the nearest pharmacy according GP's prescription.
3. Optimum Path Multiple-object-type Nearest Neighbors (PM_NN) query is to find optimum/shortest path to pass multiple object types in random sequence. Although it seems similar with $2^{\text {nd }}$ query, it is a novel issue actually because the interest points can be random passed.

Example 3.3.3. Suppose a secretary has plan to do the following things: go to post office to post a letter, go to bank to deposit a cheque, go to shop to buy some print chapter and go to dry cleaner to deliver a piece of clothes. So she wants to get the best routine which not only covers all places but also makes her travelling path shortest.

In summary, they are novel queries as there is no approach touching the query about $k \mathrm{NN}$ of multiple object types and they are reality-oriented and practical. Now let using the following 3 sections to proposed approaches to these quires in sequence.

## Multiple-object-types Nearest Neighbor(M_NN)

In this section, we propose two ways to solve M_NN query: (i) For each object type, generate a NVD and find the nearest neighbor. (ii) Generate one NVD for all objects then filter them while searching the target result. NVD for each object type:

A straight approach is firstly generating NVD for each object type. For each type, find its nearest neighbor for query point using its NVD. The result comes out directly when all nearest neighbors of each type have been found. There is no reason to doubt its correctness. But concerning its efficiency, it becomes infeasible because if there are too many different kinds of object, loading different NVD $s$ will consume most of the processing time. Consequently in this section, an alternative way is proposed for the query: one NVD for all objects.


Figure 3.8: Example 3.3.1- One NVD for each object type

Based on example 3.3.1, Fig 3.8(a), 3.8(b) and 3.8(c) represent NVD $s$ of French, Italian and Chinese restaurants respectively. As a result, the nearest French $\left(p_{1}\right)$, Italian $\left(p_{7}\right)$ and Chinese ( $p_{12}$ ) restaurant can be told directly.

One NVD for all object types: Generate just one NVD for all objects which not only includes the types that the user concerns but also includes the objects of other types. It will definitely improve the performance both in time and storage aspects. The algorithm performs as follows.

Firstly, generate NVD considering all objects as polygon generators. Then "contain" function is invoked to get the generator whose polygon covers query point. This generator is the first nearest neighbor of its type.

Secondly, do an expansion within this polygon and record the distance from the query point to all border points. Calculate the distances from the query point to


Figure 3.9: Example 3.3.1- One NVD for all objects
all adjacent polygon generators. As all border points to generators' distances are pre-computed, this process can be finished transitorily.

Thirdly, the generators will be put in a queue sorting by their distance to the query point. From the shortest one, if by now a query result for its type have not found, it will be recorded as query result for this type; otherwise, just discard it. Then add its adjacent generators into the list and sort again. Do this step iteratively until all object types' nearest neighbors have been found.

Finally, we get a result list which is for each object type there is an interest point nearest to query point among others in this type.

The algorithm can be expressed in Algorithm 2.

The following example fully illustrates how the algorithm works. The scenario is based on example 3.3.1 as well. In this case, the query should retrieve 3 restaurants because the user only concerns 3 types of restaurants, French, Italian and Chinese. The processing steps are as follows:

- Generate NVD as in Fig. 3.9. White triangle, black dot and black triangle indicate French, Italian and Chinese restaurants respectively. Use contain() function to locate $p_{1}$ which is the $1^{s t} \mathrm{NN}$ of $q$.
- As Type $\left(p_{1}\right)=$ French, initial $R L=\left\{\left(\right.\right.$ French, $\left.p_{1}\right),($ Italian, $\emptyset),($ Chinese, $\left.\emptyset)\right\}$ Initial $N P=\left\{p_{2}, p_{3}, p_{4}, p_{5}, p_{6}, p_{7}\right\}$ by adding all $p_{1}$ 's adjacent into $N P$.
- Expand $q$ within $p_{1}$ 's polygon and record all distance from $q$ to border points. Calculate the distance from $q$ to each $p$ in $N P$ and sort them in ascending order. Update $N P$, suppose $N P=\left\{\left(p_{5}, 5\right),\left(p_{7}, 7\right),\left(p_{2}, 9\right),\left(p_{3}, 11\right),\left(p_{6}, 16\right)\right.$, $\left.\left(p_{4}, 18\right)\right\}$
- Pop out $p_{5}$. As type $\left(p_{5}\right)=$ French \& in $R L$, French already has value $p_{1}$, ignore $p_{5}$. Add $p_{5}$ 's adjacent neighbors into $N P$ and update $N P$. Suppose the distance is: $N P=\left\{\left(p_{7}, 7\right),\left(p_{2}, 9\right),\left(p_{3}, 11\right),\left(p_{8}, 14\right),\left(p_{6}, 16\right),\left(p_{4}, 18\right),\left(p_{9}\right.\right.$, 19), $\left.\left(p_{10}, 22\right),\left(p_{11}, 28\right)\right\}$
- Then Pop out $p_{7}$. As type $\left(p_{7}\right)=$ Italian $\&$ in $R L$, Italian has null value, update $R L$ as $R L=\left\{\left(\right.\right.$ French, $\left.p_{1}\right),\left(\right.$ Italian, $\left.p_{7}\right),($ Chinese,$\left.\emptyset)\right\}$. After that, add all $p_{7}$ 's adjacent neighbors into $N P$ and update $N P$. Suppose the distance is: $N P=$ $\left\{\left(p_{2}, 9\right),\left(p_{12}, 10\right),\left(p_{3}, 11\right),\left(p_{8}, 14\right),\left(p_{6}, 16\right),\left(p_{4}, 18\right),\left(p_{9}, 19\right),\left(p_{10}, 22\right),\left(p_{11}\right.\right.$, 28) $\}$
- Then pop out $p_{2}$ and ignore it as it is Italian restaurant. Then Pop out $p_{12}$. As type $\left(p_{12}\right)=$ Chinese \& in $R L$, Chinese has null value, update $R L$ as $R L=$ $\left\{\left(\right.\right.$ French,$\left.p_{1}\right),\left(\right.$ Italian,$\left.p_{7}\right),\left(\right.$ Chinese, $\left.\left.p_{12}\right)\right\}$. Algorithm terminates.


## Incremental Multiple-object-types Nearest Neighbors (iM_NN)

The query of incremental nearest neighbors for sequential multiple object types is to find the shortest path which goes through multiple object types in pre-defined sequence. In this case, the sequence is crucial to the user and the user wants to pass these object types in a certain order as in example 3.3.2.

```
Algorithm 2 M_NN( \(k\), query point)
    Generate Voronoi diagram using all interest points
    \(R L(\) Result List \()=\left\{\left(\right.\right.\) type \(\left._{1}, \emptyset\right),\left(\right.\) type \(\left._{2}, \emptyset\right), \ldots,\left(\right.\) type \(\left.\left._{k}, \emptyset\right)\right\}\)
    \(p_{i}=1^{s t} \mathrm{NN}=\) contain \((q)\)
    type \(_{i}=\) Check_type \(\left(p_{i}\right)\)
    \(R L(\) Result List \()=\left\{\left(\right.\right.\) type \(\left._{1}, \emptyset\right),\left(\right.\) type \(\left._{2}, \emptyset\right), \ldots,\left(\right.\) type \(\left._{i}, P_{i}\right), \ldots,\left(\right.\) type \(\left.\left._{k}, \emptyset\right)\right\}\)
    Initial NP (Neighbor point) \(=\left\{p_{i}\right.\) 's adjacent generator \(\}\)
    Expand \(q\) within this polygon \& record distance from \(q\) to border point.
    Calculate distance from \(q\) to each \(p\) in \(N P \&\) sort them in ascending distance order.
    \(N P=\left\{\left(p_{1}, \operatorname{dist}\left(q, p_{1}\right)\right), \ldots, p_{i}, \operatorname{dist}\left(q, p_{i}\right)\right\}\)
    Pop out the first \(p\) in \(N P\), suppose it is \(p_{j}\)
    type \(_{j}=\) Check_type \(\left(p_{j}\right)\)
    if in \(R L\), type \(_{j}\) has null values then
        update \(R L\) as \(\left(\right.\) type \(\left._{j}, p_{j}\right)\)
    else
        ignore \(p_{j}\)
    end if
    if all type has values in \(R L\) then
        terminate algorithm
    else
        Add \(p_{j}\) 's adjacent neighbor into \(N P\) \& go to step 8
    end if
```

From the example, we can tell that the sequence of object types is crucial, in other words, the routine should begin at home then pass the hospital, the medical checkup clinic, the GP office and end at one pharmacy. It is not hard to see that the approach performs in the following steps: when the path reaches the interest point, it is treated as the new query point. Then continue to search the nearest neighbor of the next type until all object types have been found. There are two ways in which we can solve this query, naming as one NVD for each object type and one NVD for all objects.

One NVD for each object type: This method firstly generates NVD for the $1^{\text {st }}$ object type and finds the nearest one of this type. Then it generates NVD for the $2^{\text {nd }}$ object type and finds the nearest one of this type considering $1^{\text {st }} \mathrm{NN}$ as the query point. Continue to do so for the following object types until nearest neighbors for all types have been found.

Fig. 3.10 shows the processing steps based on example 3.3.2. The result is automatically shown in the figures: Shortest path starts at q, firstly goes to hospital $p_{2}$,


Figure 3.10: Example 3.3.2- One NVD for each object type
then heads to checkup clinic $p_{3}$, after that, towards to GP office $p_{8}$, finally arrives pharmacy $p_{12}$ for medicine.

One NVD for each object type is actually dividing this query into multiple 1_NN queries. There is no reason to doubt its correctness. But concerning its efficiency, it becomes infeasible because if there are too many different kinds of interest points, loading different NVD $s$ will consume most of the processing time. One NVD for all object types: This method generates just one NVD for all object types, including not only the type of user concerns but also other object types. It saves time and storage. The following steps illustrate how it works.

Firstly, one NVD is generated considering all objects as polygon generators. Then invoke the "contain" function to get the first nearest neighbor. Check whether
it is the $1^{\text {st }}$ type the user wants. If yes, go to the $2^{\text {nd }}$ step; otherwise check the adjacent neighbors of this interest point until find the nearest neighbor of 1st type.

```
Algorithm 3 iM_ \(k \mathrm{NN}(k\), query point)
    Generate Voronoi diagram using all interest points within given types
    \(R L(\) Result List \()=\left(\right.\) type \(\left._{1}, \emptyset\right),\left(\right.\) type \(\left._{2}, \emptyset\right), \ldots,\left(\right.\) type \(\left._{k}, \emptyset\right)\)
    Initial \(M=1\)
    \(p_{i}=1^{s t} \mathrm{NN}=\operatorname{contain}(q)\)
    type \(_{i}=\) Check_type \(\left(p_{i}\right)\)
    if type \(_{i}=\) type \(_{m}\) then
        update type \({ }_{m}\) 's values as \(p_{i} \& M=M+1\)
    else
        Expand \(q\) within this polygon\&record distance from \(q\) to border
        Initial \(N P\) (Neighbor point) \(=p_{i}\) 's adjacent generator
        Calculate distance from \(q\) to each \(p\) in \(N P\) \& sort them in ascending order.
        \(\mathrm{NP}=\left\{\left(p_{1}, \operatorname{dist}\left(q,\left(p_{1}\right)\right)\right), \ldots,\left(p_{1}, \operatorname{dist}\left(q,\left(p_{1}\right)\right)\right)\right\}\)
        Pop out the first \(p\) in \(N P\), suppose it is \(p_{j}\). type \(_{j}=\) Check_type \(\left(p_{j}\right)\)
        if type \(_{j}=\) type \(_{m}\) then
            update type \(_{m}\) 's values as \(p_{j} \& M=M+1\)
        else
            update \(N P\) by adding \(p_{j}\) 's adjacent neighbor into \(N P \&\) go to step 11
        end if
    end if
    while \(M \leq k\) do
        Suppose \(p_{m-1}=\) type \(_{m-1}\) 's values in \(R L\)
        Initial \(N P(\) Neighbor point \()=p_{m-1}\) 's adjacent generator
        Calculate distance from \(p_{m-1}\) to each \(p\) in \(N P\) and sort them in ascending order.
        \(\mathrm{NP}=\left\{\left(p_{1}, \operatorname{dist}\left(q, p_{1}\right)\right), \ldots,\left(p_{i}, \operatorname{dist}\left(q, p_{i}\right)\right)\right\}\)
        Pop out the first \(p\) in \(N P\), suppose it is \(p_{n}\). Type \(e_{n}=\) Check_type \(\left(p_{n}\right)\)
        if Type \(_{n}=\) type \(_{m}\) then
            update type \(_{m}\) 's values as \(p_{n} \& M=M+1\)
        else
            update \(N P\) by adding \(p_{n}\) 's neighbor into \(N P \&\) go to step 22
        end if
    end while
    return \(N P\)
```

Secondly, consider the $1^{\text {st }} \mathrm{NN}$ as query point; find its nearest neighbor of $2^{\text {nd }}$ type using the pre-computed distance to its adjacent neighbors.

Thirdly, do these operations iteratively until all object types have been found.
Finally, a shortest path comes out which begins at the query point, passes multiple object types in user defined sequence until it reaches the last object. That is the optimum path of this kind of query.

The algorithm can be expressed in Algorithm 3 .

A case study based on example 3.3 .2 fully illustrates the approach. In this case, the user concerns 4 object types $(k=4)$ because they want to pass the hospital, checkup clinic, GP office and pharmacy one by one. The processing steps are as follows:

- Generate NVD as Fig. 3.11. White triangle, black dot, black triangle and white dot indicate hospital, checkup clinic, GP office and pharmacy respectively.
- Initial $R L=\{($ hospital, $\emptyset)$, (checkup clinic, $\emptyset)$, (GP office, $\emptyset)$, (pharmacy, $\emptyset)\}$ $\& M=1$
- Use contain() function to locate $p_{1}$ which is the $1^{s t} \mathrm{NN}$ of $q$.
- Expand $q$ within $p_{1}$ 's polygon and record all distances from $q$ to borders.
- As type $\left(p_{1}\right)=$ pharmacy $\neq$ type $_{m}$, Initial $N P=\left\{p_{2}, p_{3}, p_{4}, p_{5}, p_{6}, p_{7}, p_{8}\right\}$ by adding all $p_{1}$ 's adjacent into $N P$.
- Calculate the distance from $q$ to each $p$ in $N P$ and sort them in ascending order. Update $N P$ as $N P=\left\{\left(p_{2}, 2\right),\left(p_{3}, 4\right),\left(p_{4}, 6\right),\left(p_{8}, 8\right),\left(p_{7}, 9\right),\left(p_{5}, 12\right),\left(p_{6}, 16\right)\right\}$
- Pop out $p_{2}$. As Type $\left(p_{2}\right)=$ hospital $=$ type $_{m}$, update $R L$ as $R L=$ (hospital, $\left.p_{2}\right),($ checkup clinic, $\emptyset),($ GP office, $\emptyset),($ pharmacy,$\emptyset) . M=2$
- As $M<k$, type ${ }_{m-1}$ 's value is $p_{2}$. Initial $N P=\left\{p_{1}, p_{3}, p_{4}, p_{9}\right\}$ by adding all $p_{2}$ 's adjacent into $N P$.
- Calculate the distance from $p_{2}$ to each $p$ in $N P$ and sort them in ascending distance to $p_{2}$. Update $N P$, suppose $N P=\left\{\left(p_{1}, 3\right),\left(p_{3}, 6\right),\left(p_{9}, 8\right),\left(p_{4}, 9\right)\right\}$
- Then pop out $p_{1}$. As type $\left(p_{1}\right)=$ pharmacy $\neq$ type $_{m}=$ checkup clinic, ignore $p_{1}$. Add $p_{1}$ 's adjacent neighbors into $N P$ and update $N P$. Suppose the distance is $N P=\left\{\left(p_{3}, 6\right),\left(p_{9}, 8\right),\left(p_{4}, 9\right),\left(p_{8}, 13\right),\left(p_{7}, 15\right),\left(p_{5}, 18\right),\left(p_{6}, 21\right)\right\}$
- Pop out $p_{3}$. As type $\left(p_{3}\right)=$ checkup clinic $=$ type $_{m}$, update $R L$ as $R L=$ $\left\{\left(\right.\right.$ hospital, $\left.p_{2}\right),\left(\right.$ checkup clinic, $\left.p_{3}\right),($ GP office, $\emptyset),($ pharmacy, $\left.\emptyset)\right\} . M=3$


Figure 3.11: Example 3.3.2- One NVD for all objects

- As $M<k$, type ${ }_{m-1}$ 's value is $p_{3}$. Initial NP $=\left\{\left(p_{8}, 12\right),\left(p_{1}, 14\right),\left(p_{2}, 17\right),\left(p_{9}\right.\right.$, $\left.23),\left(p_{12}, 25\right),\left(p_{13}, 27\right),\left(p_{11}, 30\right),\left(p_{10}, 31\right)\right\}$.
- Then pop out $p_{8}$. As type $\left(p_{8}\right)=$ GP office $=$ type $_{m}$, update $R L$ as $R L=$ $\left\{\left(\right.\right.$ hospital, $\left.p_{2}\right),\left(\right.$ checkup clinic, $\left.p_{3}\right),\left(\right.$ GP office, $\left.p_{8}\right),($ pharmacy,$\left.\emptyset)\right\} . M=4$
- As $M=k$, type ${ }_{m-1}$ 's value is $p_{8}$. Initial $N P=\left\{\left(p_{12}, 15\right),\left(p_{1}, 16\right),\left(p_{7}, 17\right)\right.$, $\left.\left(p_{14}, 26\right)\right\}$.
- Then pop out $p_{12}$. As type $\left(p_{12}\right)=$ pharmacy $=$ type $_{m}$, update $R L$ as $R L=$ $\left\{\left(\right.\right.$ hospital,$\left.p_{2}\right),\left(\right.$ checkup clinic,$\left.p_{3}\right),\left(\right.$ GP office,$\left.p_{8}\right),\left(\right.$ pharmacy,$\left.\left.p_{12}\right)\right\} . M=5$
- As $M>k$, algorithm terminates.

Results are: The optimum path firstly goes to hospital $p_{2}$, then heads to checkup clinic $p_{3}$, GP office $p_{8}$ and finally arrives at pharmacy $p_{12}$ for medicine.

## Optimum Path Multiple-object-type Nearest Neighbors (PM_NN)

Optimum path for multiple object types' query is similar with the $2_{n d}$ query except that object types can be passed in any sequence. In this query, the length of whole
path is the criterion of assessment. As multiple 1_NN cannot guarantee the final path is the shortest one, this approach is different with IM_NN approach in the last section. More details can be told based on example 3.3.3.

In example 3.3.3, the sequence of interest points is unimportant because posting letter, depositing cheque and so on are independent tasks and it does not matter which task the user does first. In addition, the objective of this query is to make the whole path short not to find any nearest object. There may be an instance that after choosing the nearest post office, the path to other place will become farther. Maybe choosing the second or even third nearest post office is better. In addition, how to arrange the sequence of interest points is another issue needed to be solved. The following steps illustrate the process of the approach.

Firstly, generate NVD considering all objects as polygon generators. Then invoke "contain" function to get the nearest generator $P$. Check its type and record $P$ as the first object type that the user will visit. For all P's adjacent neighbors, sort them in the ascending sequence of their distance to $P$. Check their types one by one, if the path has not visited that object type, record it as the next $P$. From now on, start from this $P$, do the same operation as the first $P$ until all types have been found and the path is completed. The obove operation cannot guarantee this path is shortest but it did set a boundary for the query ( $d_{\max }$ ) which means once expansion is over this boundary, it should be terminated.

Secondly, every object whose distance to $q$ is smaller than $d_{\text {max }}$ can be treated as potential first interest point. Sort them in a queue by their distance to $q$.

Thirdly, for each interest point in the queue, pop it out, find its closest neighbor whose type has not been covered and then from that neighbor do the same things until all types of interest points have been covered. If in the process of the expansion, the distance is over the boundary, terminate it directly. If the path is completed, compare its path length with the boundary and update the boundary if it is smaller.

Terminate the algorithm when there is no interest point in the queue. The optimum path shows how the user can pass multiple object types in random sequence.

```
Algorithm 4 PM_NN( \(k\), query point)
    Initial \(T S=\left\{\right.\) type \(_{1}, \ldots\), type \(\left._{k}\right\} R=\left\{\right.\) dist \(\left._{q}, \emptyset_{1}, \emptyset_{2}, \ldots, \emptyset_{k}\right\}, d_{\max }=\infty, R L=\emptyset, S=\emptyset\)
    \(p_{1}=1^{\text {st }} \mathrm{NN}=\operatorname{contain}(q), t_{p 1}=\) Check_type \(\left(p_{1}\right)\)
    Suppose type \(_{i}=t_{p 1}\), remove it from \(T S\)
    \(R=\left\{\right.\) dist \(\left._{q}, p_{1}, \emptyset_{2}, \ldots, \emptyset_{k}\right\}\)
    Initial \(N P(\) Neighbor point \()=p_{1}\) 's adjacent generator
    Calculate distance from \(p_{1}\) to each \(P\) in \(N P\) in ascending order.
    \(\mathrm{NP}=\left\{\left(p_{1}, \operatorname{dist}\left(q,\left(p_{1}\right)\right)\right), \ldots,\left(p_{i}, \operatorname{dist}\left(q,\left(p_{i}\right)\right)\right)\right\}\)
    Pop out the first \(P\) in \(N P\), suppose it is \(p_{j}\)
    if \(t_{p j}=\) Check_type \(\left(p_{j}\right)\) is in \(T S\) then
        update \(t_{p j}\) in \(R \&\) remove \(t_{p j}\) from \(T S\)
        if \(T S\) is not \(\emptyset\) then
            add \(p_{j}\) 's neighbor into \(N P\) \& go to step 7
        else
            if dist \(_{q}<d_{\text {max }}\) then
                update \(d_{\max }=\) dist \(_{q} \& R L=R\)
            else
                ignore it
            end if
        end if
    else
        add \(p_{j}\) 's adjacent neighbor into \(N P\) \& go to step 7
    end if
    Expand \(q\) within this polygon \& record distance from \(q\) to border point
    Update \(S=\left\{\right.\) all objects (dist) to \(q<d_{\max }\) sort in ascending distance order \(\}\)
    for each \(P\) in \(S\) do
        Pop out the first \(P \&\) Initial \(T S=\left\{\right.\) type \(_{1}\), type \(_{2}, \ldots\), type \(\left._{k}\right\}\)
        \(t=\) Check_type \((P)\)
        \(R=\left\{\right.\) dist \(\left._{q}, P, \emptyset_{2}, \ldots, \emptyset_{k}\right\}\)
        Initial \(N P\) (Neighbor point \()=P\) 's adjacent generator
        Calculate distance from \(P\) to each \(p_{i}\) in \(N P \&\) Wipe out \(P\) whose \(\operatorname{dist}(q, P)>d_{\max }\)
        \(\mathrm{NP}=\left\{\left(p_{1}, \operatorname{dist}\left(q,\left(p_{1}\right)\right)\right), \ldots,\left(p_{i}, \operatorname{dist}\left(q,\left(p_{i}\right)\right)\right)\right\}\)
        if \(N P \neq \emptyset\) then
            Pop out the first \(p\) in \(N P\), suppose it is \(p_{j}\)
        else
            go to step 23
        end if
        \(t_{p j}=\) Check_type \(\left(p_{j}\right)\)
        if \(t_{p j}\) is in \(T S\) then
            update \(t_{p j}\) in \(R \&\) remove \(t_{p j}\) from \(T S\)
            if \(T S\) is not \(\emptyset\) then
                add \(p_{j}\) 's neighbor into \(N P \&\) go to step 23
            else
                if dist \(_{q}<d_{\text {max }}\) then
                    update \(d_{\max }=\operatorname{dist}_{q} \& R L=R \&\) wipe off \(p\) in \(S \operatorname{dist}(P)>d_{\max }\)
                end if
                go to step 29
            end if
        end if
        add \(p_{j}\) 's adjacent neighbor into \(N P \&\) go to step 29
    end for
```



Figure 3.12: Example 3.3.3- One NVD for all objects
The algorithm can be express in Algorithm 4 .
To clarify the algorithm, a case study will fully illustrate how it works.

- Generate NVD as in Fig. 3.12. White triangle, black dot, black triangle and white dot indicate post office, bank, shop and dry cleaner respectively.
- Initial $d_{\max }=\infty, T S=\{$ post office, bank, shop, dry cleaner $\}, R=\left\{\right.$ dist $_{q}, \emptyset_{1}$, $\left.\emptyset_{2}, \ldots, \emptyset_{k}\right\}, R L=\emptyset$
- Use contain() function to locate $p_{1}$ which is the $1_{s t} \mathrm{NN}$ of $q$.
- As Type $\left(p_{1}\right)=$ dry cleaner, update $T S=\{$ post office, bank, shop $\}$ by removing it from $T S \& R=\left\{1, P_{1}, \emptyset_{2}, \ldots, \emptyset_{k}\right\} / /$ suppose $p_{1}$ to $p_{q}$ 's distance is 1
- From $p_{1}$, find nearest neighbor whose type in TS. Suppose $p_{4}$. As Type $\left(p_{3}\right)$ $=$ bank, update $T S=\{$ post office, shop $\} \& R=\left\{5, P_{1}, P_{4}, \ldots, \emptyset_{k}\right\} / /$ suppose $p_{4}$ to $p_{1}$ 's distance is 4
- Do the same to $p_{4}$ as the step above until $T S=\emptyset$. Suppose $R=\left\{15, p_{1}, p_{4}\right.$, $\left.p_{11}, p_{53}\right\}$ Update $d_{\max }=15$
- Expand $q$ within $p_{1}$ 's polygon and record all distances from $q$ to borders.
- Initial $S=\left\{p_{4}, p_{3}, p_{6}, p_{5}, p_{7}, p_{2} \ldots p_{n}\right\} / /$ whose distance to $q$ within $d_{\max }$
- Pop out $p_{4} \& u p d a t e ~ T S=\{$ post office,dry cleaner,shop $\} \& R=\left\{3, p_{4}, \emptyset_{2}, \ldots, \emptyset_{k}\right\}$. Search $p_{4}$ 's nearest neighbor whose type in $T S$ and do the same for the rest interest points iteratively until $T S=\emptyset$. Suppose $R=\left\{12, p_{4}, p_{3}, p_{12}, p_{11}\right\}$, then update $d_{\max }=12$. Wipe out $p$ in $S$ dist $_{q}>12$.
- Do the same operation to every $P$ in $S$ until $S$ is empty. In the process, once the distance is over $d_{\text {max }}$, terminate expansion for this path.

The result comes out finally $R=\left\{10, p_{3}, p_{12}, p_{13}, p_{14}\right\}$. The user firstly goes to post office $p_{3}$, then heads to dry cleaner $p_{12}$, after that, towards bank $p_{13}$ and finally arrives at shop $p_{14}$ and the length of the final path is 10 .

### 3.4 Performance Evaluation

In this section, we evaluate these two methods using different data and environment settings.

### 3.4.1 Voronoi based Continuous $k N N$

Melbourne city map and Geelong map in Victoria, Australia, are chosen in the experimentations from the "whereis" website Cor to represent high-density and low-density scenarios of interest points. All interest points, network links and intersect nodes are real-world data. We analyze the behavior of our approach in the aspects such as segment division in different path or point of interest density by DAR/IE and VCkNN and runtime with various lengths of path and the values of $k$.

## Segment division

Firstly, we aim at finding the differences in the number of segments divided along the path in different path densities. The Melbourne city map is used to indicate
high path density, in other words, more network intersections along the path (2.1 intersection/km). Correspondingly, the Geelong city map is used to indicate low path density ( 1 intersection $/ \mathrm{km}$ ). Interest points are distributed at $10.93 / \mathrm{km} 2$ on two different maps. From fig 3.13, we can draw several conclusions:

- Segment increases show a nearly linear trend;
- In the $\mathrm{VC} k \mathrm{NN}$ algorithm, paths are divided into the same segment no matter whether the path density is high or low;
- DAR/IE algorithm divides into more segments in high path density than in low path density;
- VCkNN always generates less segments than DAR/IE no matter the path density.

Secondly, we aim at finding the differences of number of segments divided along the path in different point of interest point density. Restaurants in the Melbourne city map indicate a high point of interest density ( $23 / \mathrm{km} 2$ ), whereas petrol stations indicate a low point of interest density $(1.8 / \mathrm{km} 2)$. Path density is about 1.2 intersection/km. From fig. 3.14, several conclusions lists below:

- In the VCkNN algorithm, more segments occur if the objects of interest are distributed in high density then in low density;
- DAR/IE algorithms remain the same no matter the points of interest are in low or high density;
- VCkNN always generates less segment than DAR/IE no matter the density of objects of interest.


## Run time

Firstly, we report our experimentation results on runtime of different density of points of interest. We use 20 points of interest to represent a low-density sample


Figure 3.13: Segment in different path Figure 3.14: Segment in different POI density density
and 100 interest points to represent a high density sample. Also we test 20 different query positions to get the average runtime based on $k$ from 1 to 7 .

For the runtime factor, we can easily tell that if $k$ increases, runtime increases sharply, and in a high-density scenario it is even more time consuming. Figure 16 shows the trend of these two scenarios.

From fig 3.15, we can also conclude that the runtime increases sharply after $k>5$. This is because too many operations on small intervals and too many operations and checkings need to be executed for the candidate interest points. The high density will do more looping and the runtime consequently goes up.

Secondly, we aim at finding the differences of runtime between shorter and longer query paths. We put 50 interest points on each map to compare the runtime. We choose 20 query paths (all equal to 20 km ) to get the average runtime in the Melbourne city map based on $k=3$. For these 20 moving paths, we record the runtime every time when query point moves 1 km and after query point moves 5 km . As the shorter distance query point moves out, the less time is consumed as less polygon is checked and less expansion is involved.

Fig 3.13 presents the average runtime and it can tell that the line is nearly linear which means that every part of the query path is generally independent. With the increase of query path's length, the runtime will definitely increase.


Figure 3.15: Runtime in high and low Figure 3.16: Runtime in different density of interest points query path lengths

## Split nodes number between different point of interest density

In this section, we use the same experimental conditions as the previous ones to compare the split nodes number in different point of interest density. It is known that normally for the same map, if the interest point density is low, the split nodes will be less than the high density one, because there is less chance that another interest point will be found.

From fig 3.17, because the density decides the polygon average area, the same query path will go across fewer polygons in the low density map than in the high density map. So when $k=1$, the low density performance is better than the high density performance. While we cannot conclude that for the same $k$ and query path, the low density one has less split nodes than that of high density. At the same time, we can draw a conclusion that for the same map and same query path, split nodes will increase or decrease with $k$ but the increasing amount is not constant.

### 3.4.2 Voronoi based Multiple types $k \mathbf{N N}$

In the experimentations, Melbourne city map and Frankston map in Australia are chosen from the "whereis" website wM06. In these maps, shops and restaurants represent a high-density scenario of interest points, on the other hand, hospitals and shopping centers represent low-density scenario of interest points. All interest points


Figure 3.17: Split nodes in high and low density of interest points
are real-world data. The performance of our approaches is analyzed in runtime aspect in different diversity of interest point or in different interest points' density.

For the nearest neighbor for multiple object types, the processing time is increasing with the number of object types (M). In Fig. 3.18(a), the dash line indicates the performance of one NVD for each object type approach and the solid line indicates the performance of one NVD for all objects approach. In this case, we use different types of shops as candidate types and the average density is $5 / \mathrm{km}^{2}$. From Fig. 3.18(a), we can easily tell that one NVD for each object type performs better than one NVD for all objects if objects types are small, especially smaller than 4 . Otherwise, one NVD for all objects is a better choice because it saves time for generating NVD. We can also tell that with the increasing object types, the processing time increases sharply because more polygon expansions will be invoked and more NVD $s$ should be generated.

For incremental nearest neighbors for sequential multiple object types query, the processing time is increasing with the number of object types $(M)$. Here a definition is introduced: density relative rate ( DRR ). DRR is ratio of the highest density to lowest density of all object types. As a result, DRR is not smaller than 1. The closer to 1 DRR is, the more evenly objects distribute. For example, if the user concerns 4 object types and their densities are $5.5 / \mathrm{km}^{2}, 3.6 / \mathrm{km}^{2}, 2.5 / \mathrm{km}^{2}$ and $1.1 / \mathrm{km}^{2}$ respectively. So this scenario's $\operatorname{DRR}$ is $5.5 / \mathrm{km}^{2}$ (highest) $1.1 / \mathrm{km}^{2}$ (lowest) $=5$. In

Fig. 3.18(b), the first two bars indicate the processing time of one NVD for each object type approach and the last two bars indicate the processing time of one NVD for all objects approach. The first and third bars are operating in low DRR scenario $(\mathrm{DRR}=1)$ and the second and forth bars are in high DRR scenario $(\mathrm{DRR}=10)$.

Fig. 3.18(b) lllustrates that processing time will increase if DRR increases. In addition, the higher DRR is, the closer two approaches (one NVD for each \& one NVD for all) performs. In addition, generally, one NVD for all interest points performs better than one NVD for each object types because generating and loading NVD are time consuming tasks.

The processing time for optimum path for multiple object types query, the processing time is increasing with the object types $(M)$. In Fig. 3.18(c), the dash line indicates the performance of the query when density relative rate $(\mathrm{DRR})=1$ and the solid line indicates the performance of the query when density relative rate (DRR) $=5$.

From Fig. 3.18(c), with the increasing object types, the processing time increases sharply because more polygon expansions will be invoked. Moreover, DRR is another critical factor for the performance of the approach. The processing time increases more sharply if DRR increases from 1 to 5 .

### 3.5 Summary

In this chapter, firstly we present a novel approach of Voronoi-based continuous k nearest neighbor search based on network distance, which we call VCkNN. The basis of VCkNN is using network expansion within each polygon and a drawing line for every border point. VCkNN gives users the split nodes as $k$ increases and there is no need to perform $k N N$ processing for any node on the path. In addition, VCkNN does not consider the segment between every intersection. This feature improves the performance because finding split nodes segment by segment is not efficient, especially if there are too many intersections on the query path. We have performed several experiments to measure the performance of VCkNN in different

(a) M_NN

(c) PM_NN

(b) iM_NN

Figure 3.18: Processing Time Comparison
network conditions. In general, our algorithm performs better if the density is low, especially in segment division mechanism. If the number of interest objects is smaller than 5 , the performance is acceptable no matter how complex the road condition is. However, as expected, if $k$ is greater than 5 , the runtime increase sharply. Also the runtime is related to the length of the query path and the polygon it goes across. On average, high density of interest points and more crossing polygons will let the runtime and expansion step increase. If $k$ is large, the runtime will increase sharply. When comparing VCkNN with other approaches, we can conclude that the advantage of VCkNN becomes obvious if the interest points are highly density distributed.

Secondly, inspired by novel $k$ NN search involving multiple object types, we discussed another 3 set of queries using the Voronoi Diagram. The first query (nearest neighbor for multiple object types) provides a solution if the user wants to get 1_NN for each category of interest points. The second query (incremental nearest neighbors for sequential multiple object types) helps users to find the shortest path to pass through multiple object types in pre-defined sequence. The last query (optimum path for multiple object types) provides an optimum path for users if they want to pass multiple object types without any sequential constrain. These approaches investigate novel $k \mathrm{NN}$ in multiple object types using a network Voronoi Diagram which enriches the content of our mobile navigation system and gives more benefits to mobile users.

To sum up, both approaches can solve their corresponding queries efficiently when the Voronoi Diagram is utilized compared to Network Expansion.

## Chapter 4

## Route and Path related $k \mathbf{N N}$ Queries ${ }^{11}$

### 4.1 Introduction

Traditional query in spatial databases are range search PZMT03, JT05 and $k$ nearest neighbor search ( $k \mathrm{NN}$ ) [?, RKV95, Saf05]. Range search is to find all interest objects within a predefined range, while $k \mathrm{NN}$ is to find $k$ interest objects which are closest to a query point. Both range and $k \mathrm{NN}$ searches provide users the candidate set of interest points and allow users to choose any one in the set because they have been previously filtered by user's conditions. From the description, we call tell the traditional range search and $k$ nearest neighbor search are retrieving discrete points. Motivated by this, we propose 3 approaches in this chapter which brings path into the input or/and output of spatial queries.

Firstly, a possible query that a user may invoke is as follows: A market researcher may want to do a survey on restaurants and the sample size should be 10. The question is to find the shortest path for the user to visit all the 10 restaurants one by one. Range search cannot be used as there is no fixed range. $k \mathrm{NN}$ search cannot be used either, as after we visit the first interest point, the user may not want to

[^1]

Figure 4.1: Result comparisons
return to query point and go to the second one. In this case, the user wants to continue to go to the second location from the first, and so on. This is a typical path based $k$ nearest neighbor query ( $p k \mathrm{NN}$ ). The difference between range search, traditional $k \mathrm{NN}$, and our proposed $p k \mathrm{NN}$ is highlighted in Fig.4.1 in section 4.2,
$p k \mathrm{NN}$ is described as given a set of candidate interest objects, a query point and the number of objects $k$, find the shortest path which starts from the query point and goes through $k$ interest objects. By following this path, a user can visit all $k$ interest objects one by one, and furthermore, this path has the shortest distance among all other possible paths.

Secondly, another query comes into our minds that is called path branch point $(P B P)$ and which is discussed in section 4.3, $P B P$ can be defined as: given a set of candidate interest objects and a pre-defined path which starts at $S$ and end at $E$, find a path which starts at $S$, via an interest point $p$ and ends at $E$. This path should overlap with the pre-defined path as much as possible with acceptable distance increment. This is a novel query which is motivated by users' common requirements because most users have ad hoc paths in their daily travel and they can tolerate a longer driving distance to some extent if they can drive on a familiar path. In this
proposed approach, an Adjust Score is calculated for each path which is determined by overlapping distance and increased distance cost. The following example explains the query.

Fig 4.6 is an example of a path branch points query. The pre-defined path is marked as a red line in Fig 4.6 which starts at $S$ and ends at $E$. Our aim is to find another path which starts from $S$, via one $p$ and ends at $E$. This path should overlap the pre-defined path as much as possible under the condition that the driving distance increment is acceptable. As an example, take two paths which go through $p_{1}$ and $p_{2}$ respectively. One of the possible paths via $p_{1}$ ( Path $_{1}$ ) is $S \rightarrow p_{1} \rightarrow n_{2} \rightarrow n_{6}$ $\rightarrow E$ and one of the possible paths via $p_{2}\left(\right.$ Path $\left._{2}\right)$ is $S \rightarrow n_{2} \rightarrow n_{10} \rightarrow p_{2} \rightarrow E$. How to determine which path is more suitable (optimal) to the user's requirement is the main target of this chapter.

This section makes three main contributions. First, the path branch point $(P B P)$ query problem is defined. Second, we incorporate $k \mathrm{NN}$ search query and route search algorithms to process our $P B P$ query using the network distance metric and three factors, Distance Cost $(D C)$, Overlap Factor $(O F)$ and Adjust Score $(A S)$ are introduced and defined. By using these three factors, we can scale whether the path is the one the user wanted or not. For the third contribution, we evaluate this approach using experiments under different interest point distributions. Our experiments verified the applicability of the proposed approach to solve the queries, which involve finding the optimal path branch points.

Thirdly, as route search has been extended to include locations to be visited along the planned route KSSD08, YS05, HJ04, ZXTS08, ZXR ${ }^{+}$, TBPM05, KSS09, KSSD08, Zha08. The aim was to find the shortest distance, and sometimes the most reliable route, that covers all user-defined locations or places. Although this is certainly useful, it is often impractical, due to a couple of reasons: (i) each location or place, which are normally a spatial business entity (e.g. bank, dry cleaner, supermarket) has the opening hours - this implies that when this place is visited, it must be during their business hours; and (ii) the traveling time from one location to another needs
to be considered, as in many cases, traveling time is more useful than the distance alone. Hence, in order to make route planning over visiting locations, one must take into account these two constraints. In section 4.4, we refer to these constraints as Time Constraints. Therefore, our chapter focuses on route search over multiple locations taking into consideration time constraints.

It is therefore imperative to assume that the route or path that arrives on the location outside the operation hour is considered as an invalid path. This problem exists in daily life, whereby we sometime have to choose a longer path to go back and forth places just to meet the business hours of one location before its closing time. Hence, we need to draw time constraints into our proposed methods.

Route search over multiple locations is often assumed to be the problem of $k \mathrm{NN}$ or continuous $k \mathrm{NN}$ in spatial and mobile databases [?]. There is a huge distinction between $k \mathrm{NN}$ and route search. $k \mathrm{NN}$ finds spatial objects that are closest located to the query point, without considering the path that needs to be established as the user has to visit each objects in the query result. Because of this, existing work on $k \mathrm{NN}$ is inapplicable to solve route search problems. Our previous work on incremental $k \mathrm{NN}$ (called ikNN) ZXTS08 attempts to solve route search problem whereby it could find the shortest distance to cover $k$ number of homogenous type of locations - however, it does not consider time constraints, nor heterogenous multiple types of locations.

In section 4.4, we focus on two problems of route search over multiple heterogenous locations: one for fixed locations, and the other for flexible locations. Fixed locations refer to predetermined locations by the user, such as Citibank on a specific location, Pharmore pharmacy on a specific location, etc. In this case, not only a specific business entity is specified, such as Citibank and not any bank, or Pharmore pharmacy and not any pharmacy, but also the specific location, such as Citibank on 180 High Street, or Pharmore pharmacy on 25 Cure Road, etc. Hence, a Route Search over Fixed Locations (our proposed algorithm is then called RFix) finds
the most efficient route to visit the user-defined fixed locations in a non-predefined order.

Flexible locations, on other hand, refer to predetermined location types, are not the exact location itself. For example, if user wants to visit a pharmacy, which can be the pharmacy anywhere; or to visit Citibank, but can be in any branch. So, a route search over flexible locations for example is to find the most efficient route to visit Citibank, a pharmacy, etc, in a non-predefined order. Our proposed algorithm for Route Search over Flexible Locations is abbreviated as RFlex. Both RFix and RFlex use the travel time network to estimate the travel time between any two locations, as well as using the time constraints imposed by not only the operating hours of each location, but also the traveling time itself.

To sum up, chapter 4 is the second main chapter of this thesis, which includes 3 approaches of path/route based $k$ Nearest neighbor search. More specific descriptions are:

- Section 4.2 proposes a query that is called path based $k$ nearest neighbor search. It aims at providing a path that visits $k$ objects and the length of the path that is the shortest.
- Section 4.3 explains a query which is called path branch point route search. By given the query path and an object type, path branch point route search retrieves the optimal path that balances the overlap ratio of query path and the length of result path.
- Section 4.4 describes a novel route research which adds time constraint into the search. In addition, a user may define the objects visiting sequence as sequential or random.

Let us begin the main part of these three approaches with the performance evaluation followed after.


Figure 4.2: An example of road networks

### 4.2 Approach 1: Path based $k N N$ Search Queries

Nowadays, most queries in mobile databases take the road networks into consideration because Euclidean distance, in most situations, cannot reflect the real connections between interest objects. The driving distance, or time cost, are determined by the shortest distance in road connection between objects. As a result, firstly, we define the method that how a weighted map is constructed. Also in this section, we introduce the data structures which is going to be used in the approach.

### 4.2.1 Definition of road network elements

Fig. 4.2 is an example of road networks, in which query point $q$, road network intersections $n_{1}-n_{7}$ (white points), and interest points $p_{1}-p_{8}$ (black points) are vertices and the solid lines connecting these vertices are edges. The number on each edge represents the shortest distance, in other word, the weight of the edge.

Definition 4.2.1. (Expansion) Expansion is the traversal from $a$ vertex $v_{i}$ to all of its adjacent vertices.

In Fig 2.1, from $q$, three $q$ 's adjacent vertices $n_{1}, n_{2}$ and $p_{5}$ will be expanded.

### 4.2.2 Data structure

In the approach processing progress, couple of data structures are going to be used, such as $p k$ NN tuple, Result Set, Expanded Set, Boundary Set and distance. The following definitions defines them in details.

Definition 4.2.2. ( $p k \boldsymbol{N} \boldsymbol{N}$ tuples) $t=\left(v, d_{\text {net }}(v, q), V P\right)$ where $v \in V$, visited point set $V P=\left(p_{1}, p_{2}, \ldots, p_{m}\right)$ where $m \leq k$. Each tuple represents a path which starts from $q$ to $v$.

In Fig 2.1, $t_{1}=\left(n_{3}, 4,\left\{p_{5}\right\}\right)$ and $t_{2}=\left(n_{3}, 4,\{\emptyset\}\right)$ are two paths. $t_{1}$ starts from $q$ to $n_{3}$ via $p_{5}$, in other word, $q \rightarrow p_{5} \rightarrow n_{3}$ while $t_{2}$ is the path via null object of interest which is $q \rightarrow n_{2} \rightarrow n_{3}$.

Definition 4.2.3. (Result Set $R S$ ) $R S=\{t \mid t . v$ is a current expansion node $\bigcap$ $\left.t . d_{\text {net }}(v, q) \leqslant d_{\max }\right\}$. RS holds the pkNN tuples after expansion and sorted by $t . d_{n e t}(v, q)$.

Definition 4.2.4. (Expanded Set $E S$ ) $E S=\{t \mid t$ is the expanded $p k N N$ tuple $\bigcap$ $\left.t . d_{n e t}(v, q) \leqslant d_{\max }\right\}$. ES is designed to hold all expanded $p k N N$ tuples to do further pruning.

See example shown in Fig 2.1, after expanded from $q, R S=\left\{\left(n_{2}, 1,\{\emptyset\}\right),\left(n_{1}, 2\right.\right.$, $\left.\{\emptyset)\},\left(p_{5}, 3,\left\{p_{5}\right\}\right)\right\}$. And the $p k \mathrm{NN}$ tuple which has been expanded is moved into $E S$. As result, $E S=\{(q, 0,\{\emptyset\})\}$.

Definition 4.2.5. (Boundary Set and distance) Boundary distance $d_{\text {max }}=\min$ $\left(t . d_{\text {net }}(v, q)\right)$ where $\forall t . V P=\left(p_{1}, p_{2}, \ldots, p_{k}\right)$. Boundary Set $B S=\left\{t \mid t_{i} . V P=\left(p_{1}, p_{2}\right.\right.$,
$\left.\left.\ldots, p_{k}\right) \bigcap t \cdot d_{\text {net }}(v, q)=d_{\text {max }}\right\}$. BS stores all candidate shortest path based on current $d_{\text {max }}$.

Suppose the query of example in Fig 2.1 is to find $2 k \mathrm{NN}$, in the query processing, $d_{\max }$ is 8 because one complete path is found, which is $q \rightarrow p_{5} \rightarrow p_{4}$ and the the path length is 8 . As a result, $B S=\left\{p_{4}, 8,\left\{p_{5}, p_{4}\right\}\right\}$. Although at last, another shorter path is found as optimal path, at this stage, $B S$ holds the candidate result path.

### 4.2.3 Proposed Method

In this subsection, the $p k \mathrm{NN}$ approach will introduce basic expansion, pruning conditions, accelerated approach and special issues in turn.

## Basic Expansions

The approach of $p k \mathrm{NN}$ performs network expansion, which is similar with INE (Incremental Network Expansion). The expansion starts from query point $q$ to all adjacent vertex and store the $p k \mathrm{NN}$ tuples into $R S$. Every time pop out one $p k \mathrm{NN}$ tuple to do further expansion until boundary set $(B S)$ is found. In the processing progress, using pruning conditions to prune some redundant $p k \mathrm{NN}$ tuples to speed up the expansion. Then keep updating the boundary distance ( $d_{\max }$ ) until $R S$ has been cleared. In INE, the visited nodes will not be expanded during the expansion, while in $p k \mathrm{NN}$, all adjacent nodes are expanded to no matter whether the nodes have been visited or not.

Specifically, $p k \mathrm{NN}$ first initials $E S$ and $R S$ as $\{(q, 0,\{\emptyset\})\}$. Secondly, as $q$ is on the top of the $R S$, we pop it out and retrieve all adjacent nodes of $q\left(n_{1}, n_{2}\right.$ and $\left.p_{5}\right)$, expand to each of them and put their $p k \mathrm{NN}$ tuples into $R S$. As a result, $R S=\left\{\left(n_{2}\right.\right.$, $\left.1,\{\emptyset\}),\left(n_{1}, 2,\{\emptyset)\right\},\left(p_{5}, 3,\left\{p_{5}\right\}\right)\right\} .\left(p_{5}, 3,\left\{p_{5}\right\}\right)$ tuple means the path starts from $q$, ends at $p_{5}$ with distance 3 via interest objects $p_{5}$. Then following the former steps, pop ( $n_{2}, 1,\{\emptyset\}$ ) out, add it into $E S$ and do expansion to all $n_{2}$ 's adjacent
nodes. Update boundary set $B S$ and $d_{\text {max }}$ until at least one completed route has been found. Keep updating $B S$ and $d_{\max }$ until $R S$ is empty.

## Lemma (Pruning conditions)

As we stated before, in every expansion, all adjacent nodes are expanded which will cause a lot of redundant $p k$ NN tuples because we allow go-and-back expansion. As a result, pruning conditions can speed up the algorithm by reducing useless $p k \mathrm{NN}$ tuples.

Lemma 4.2.1. Given $t_{x}, t_{y} \in R S$, if $t_{x} \cdot v_{x}=t_{y} \cdot v_{y}, t_{x} . V P=t_{y} . V P$ and $t_{x} \cdot d_{n e t}\left(v_{x}, q\right)$ $\leq t_{y} \cdot d_{n e t}\left(v_{x}, q\right)$, then $t_{y}$ needs to be pruned. Summarized as prune path with vain distance.

Proof. Given an $p k \mathrm{NN}$ query, all candidate interest points are in Set POI, suppose $t_{x}, t_{y} \in R S, t_{x} \cdot v_{x}=t_{y} \cdot v_{y}, t_{x} \cdot V P=t_{y} \cdot V P, t_{x} \cdot d_{n e t}\left(v_{x}, q\right) \leq t_{y} \cdot d_{n e t}\left(v_{x}, q\right)$, we should prove that $t_{y}$ can not lead the optimal path under this assumption.

As $t_{x} \cdot v_{x}=t_{y} \cdot v_{y}, t_{x} \cdot V P=t_{y} . V P=\left\{p_{1}, \ldots, p_{m}\right\}$, then this becomes another $\mathrm{i} k^{\prime} \mathrm{NN}$ query where $k^{\prime}=k-m$ and $P O I^{\prime}=P O I-\left\{p_{1}, \ldots, p_{m}\right\}$. Suppose $P a t h^{\prime}$ is the optimal path of $\mathrm{i} k^{\prime} \mathrm{NN}$ which starts from $t_{x} \cdot v_{x}$.

Under this condition $t_{x} \cdot d_{\text {net }}\left(v_{x}, q\right) \leq t_{y} \cdot d_{\text {net }}\left(v_{x}, q\right)$, then $t_{x} \cdot d_{\text {net }}\left(v_{x}, q\right)+$ Path $\leq$ $t_{y} \cdot d_{n e t}\left(v_{x}, q\right)+$ Path $^{\prime} . p k \mathrm{NN}$ is to find the shortest path which starts from $q$ and goes through $k$ interest points, given $t_{x} \cdot d_{\text {net }}\left(v_{x}, q\right)+P a t h^{\prime}$ and $t_{y} \cdot d_{n e t}\left(v_{x}, q\right)+P a t h^{\prime}$ are two candidate $p k$ NN path, we should prune the longer path which is $t_{y} \cdot d_{n e t}\left(v_{x}, q\right)+P a t h^{\prime}$. So $t_{y}$ needs to be pruned.

Example 4.2.1. As $t_{3}=\left(p_{4}, 8,\left\{p_{5}, p_{4}\right\}\right)$ and $t_{4}=\left(p_{4}, 10,\left\{p_{5}, p_{4}\right\}\right)$, because $t_{3} \cdot v=p_{4}$ $=t_{4} \cdot v, t_{1} \cdot V P=\left\{p_{5}, p_{4}\right\}=t_{2} \cdot V P$ and $t_{3} \cdot d_{\text {net }}(v, q)=8 \leq t_{2} \cdot d_{\text {net }}(v, q)=10$, so $t_{4}$ is pruned. In other words, $t_{3}$ and $t_{4}$ represent two paths with some start and end node as well as the visited nodes are identical (includes both interest nodes name and visiting sequence). The path with longer distance is pruned because it costs vain driving distance.

```
Algorithm 5 PrunCond1_Lemma1( \(R S\) )
    for \(i=0, i<\operatorname{Size}(R S), i++\) do
        for \(j=0, j<\operatorname{Size}(R S), j++\) do
            if \(d_{\text {net }}\left(v_{i}, q\right) \leq d_{\text {net }}\left(v_{i}, q\right) \| v_{i} . V P=v_{j} . V P\) then
                \(R S=R S-t_{j}\)
            end if
        end for
    end for
    return \(R S\)
```

Lemma 4.2.2. Given $t_{x}, t_{y} \in R S$, if $t_{x} \cdot v_{x}=t_{y} \cdot v_{y}, t_{x} \cdot V P \supset t_{y} \cdot V P$ and $t_{x} \cdot d_{n e t}\left(v_{x}, q\right)$ $\leq t_{y} \cdot d_{n e t}\left(v_{y}, q\right)$, then $t_{y}$ needs to be pruned. Summarized as prune path with less efficiency.

Proof. Given an $p k$ NN query, all candidate interest points are in Set POI, suppose if $t_{x} \cdot v_{x}=t_{y} \cdot v_{y}, t_{x} \cdot d_{n e t}\left(v_{x}, q\right) \leq t_{y} \cdot d_{n e t}\left(v_{y}, q\right)$ and $t_{x} \cdot V P \supset t_{y} \cdot V P$, we should prove that $t_{y}$ can not lead the optimal path under this assumption.

Given the conditions $t_{x} \cdot v_{x}=t_{y} \cdot v_{y}, t_{x} . V P \supset t_{y} . V P$, suppose $t_{x} . V P=\left\{p_{1}, \ldots, p_{n}, \ldots, p_{m}\right\}$, $t_{y} . V P=\left\{p_{1}, \ldots, p_{n}\right\}$. Then from $t_{x} \cdot v_{x}$, how to find optimal path becomes another $\mathrm{i} k^{\prime} \mathrm{NN}$ query where $k^{\prime}=k-m$ and $P O I^{\prime}=P O I-\left\{p_{1}, \ldots, p_{m}\right\}$, and from $t_{y} \cdot v_{y}$, how to find optimal path becomes another $\mathrm{i} k^{\prime \prime} \mathrm{NN}$ query where $k^{\prime \prime}=k-n$ and $P O I^{\prime \prime}=P O I-$ $\left\{p_{1}, \ldots, p_{n}\right\}$.

Suppose Path" is the optimal path of $p k^{\prime \prime} \mathrm{NN}$ which starts from $t_{x / y} \cdot v_{x / y}$, so Path $_{t_{y}}+$ Path" $^{\prime \prime}$ is one candidate $p k \mathrm{NN}$ path.

If Path" contains some interest points say $0 \leq i \leq m-n$ of $t_{x} \cdot V P-t_{y} . V P$, then Path $_{t_{x}}+$ Path" contains $m+k-n-i>k$ interest points. As $t_{x} \cdot d_{n e t}\left(v_{x}, q\right) \leq t_{y} \cdot d_{n e t}\left(v_{y}, q\right)$, so Path $_{t_{x}}+$ Path ${ }^{\prime \prime} \leq$ Path $_{t_{y}}+$ Path ${ }^{\prime \prime}$. Suppose Path ${ }_{s u b}$ is the sub-path of Path $_{t_{x}}+$ Path $^{\prime \prime}$ which goes through $k$ interest points, so Path sub $\leq$ Path $_{t_{x}}+$ Path ${ }^{\prime \prime} \leq$ Path $_{t_{y}}+$ Path $^{\prime \prime}$. As a result, $t_{y}$ needs to be pruned.

Example 4.2.2. Suppose $t_{1}=\left(n_{3}, 4,\left\{p_{1}, p_{5}\right\}\right)$ and $t_{2}=\left(n_{3}, 4,\left\{p_{1}\right\}\right)$, because $t_{1} \cdot v=$ $n_{3}=t_{2} \cdot v, t_{1} \cdot d_{\text {net }}(v, q)=4 \leq t_{2} \cdot d_{n e t}(v, q)=4$ and $t_{1} \cdot V P=\left\{p_{1}, p_{5}\right\} \supset t_{2} \cdot V P=\left\{p_{1}\right\}$, as a result, $t_{2}$ is pruned. To sum up, $t_{1}$ and $t_{2}$ represent two paths with some start and end node. $t_{1}$ visits more interest objects than $t_{2}$, in other word, $t_{1}$ visits more objects
after visiting all $t_{2} . V P$ in same sequence. Moreover, $t_{1}$ 's distance is not longer than $t_{2}$ 's distance. We can conclude that $t_{2}$ is a path with less efficiency, so prune it.

```
Algorithm 6 PrunCond2_Lemma2( \(R S\) )
    for \(i=0, i<\operatorname{Size}(R S), i++\) do
        for \(j=0, j<\operatorname{Size}(R S), j++\) do
            if \(d_{\text {net }}\left(v_{i}, q\right) \leq d_{\text {net }}\left(v_{i}, q\right) \| v_{i} . V P \supset v_{j} . V P\) then
                \(R S=R S-t_{j}\)
            end if
        end for
    end for
    return \(R S\)
```

Lemma 4.2.3. Given $t_{x} \in R S$, if $t_{x} \cdot d_{\text {net }}\left(v_{x}, q\right)>d_{\max }$, then $t$ should to be pruned. Summarized as shrink $R S$ by Boundary Distance ( $d_{\max }$ ).

Proof. Suppose Path is the full path which contains $t_{x}$, then length ${ }_{\text {Path }}>t_{x} \cdot d_{n e t}\left(v_{x}, q\right)$. Because $t_{x} \cdot d_{\text {net }}\left(v_{x}, q\right)>d_{\text {max }}$, then length ${ }_{\text {Path }}>d_{\text {max }}$, in other word, Path is not the optimal path of $p k \mathrm{NN}$. So $t_{x}$ is pruned.

Example 4.2.3. If $d_{\text {max }}=7$, there is a tuple $t_{5}=\left(n_{1}, 8,\left\{p_{3}\right\}\right)$ in RS. It is pruned because $t_{5} . d_{\text {net }}\left(n_{1}, q\right)=8>d_{\text {max }}$. This lemma is summarized as following: the $p k N N$ tuples in $R S$ represent uncompleted path but with longer distance than the completed path has been found. So there is no chance this tuple can be the optimal path, this tuple is pruned. As a result, $R S$ shrink using $d_{\text {max }}$.

```
Algorithm 7 PrunCond3_Lemma3 \(\left(R S, d_{\max }\right)\)
    for \(i=0, i<\operatorname{Size}(R S), i++\) do
        if \(d_{\text {net }}\left(v_{i}, q\right)>d_{\text {max }}\) then
            \(R S=R S-t_{j}\)
        end if
    end for
    return \(R S\)
```

Lemma 4.2.4. Tuples in ES can prune pkNN tuples in RS using Lemma 1 and 2.

Proof. Def. 4.2.4 tells $R S$ is to keep the expansion history. Suppose $t_{1} \in R S, t_{2} \in E S$, $t_{1}$ and $t_{2}$ fit Lemma 1 and $2, t_{1}$ can be pruned. The reason are the same as proof of Lemma 1 and 2.

Example 4.2.4. As stated before, after first expansion from $q$, tuple $t_{i}=(q, 0,\{\emptyset\})$ is in ES. Then after couple of expansions, there is tuple $t_{j}=(q, 2,\{\emptyset\})$ in $R S . t_{i}$ and $t_{j}$ fit Lemma 1, so $t_{j}$ in $R S$ is pruned which accelerates the approach as well. This is main reason we use ES to keep expansion history.

```
Algorithm 8 PrunCond4_Lemma4( \(R S, E S\) )
    for \(i=0, i<\operatorname{Size}(E S), i++\) do
        for \(j=0, j<\operatorname{Size}(R S), j++\) do
            if \(d_{\text {net }}\left(v_{i}, q\right) \leq d_{n e t}\left(v_{i}, q\right) \| v_{i} . V P=v_{j} . V P\) then
                \(R S=R S-t_{j}\)
            end if
            if \(d_{\text {net }}\left(v_{i}, q\right) \leq d_{n e t}\left(v_{i}, q\right) \| v_{i} . V P \supset v_{j} . V P\) then
                \(R S=R S-t_{j}\)
            end if
        end for
    end for
    return \(R S\)
```

Most redundant $p k \mathrm{NN}$ tuples can be pruned by lemma 1-4 which accelerates the processing time of $p k \mathrm{NN}$ approach. There is no order between four pruning conditions. For each tuple in $R S$, if it fits one pruning condition, just prune it. $R S$ keeps the tuples which are unfit for all four pruning conditions. Tuples which fits one pruning conditions commonly exist after each expansion. As a result, the performance can be accelerated by using four pruning lemmas after each expansion. The following accelerated approach algorithm (Algorithm 9) is produced based on these pruning conditions.

## Special Issues

After addressing the approach, there are three important issues need to be clarified in this section.

```
Algorithm 9 pkNN \((q, k)\)
    Initial \(P=\{p \mid\) all interest objects \(\}, N=\{n \mid\) all intersection nodes \(\}, V=q \cup P \cup N\)
    Initial \(R S=\{(q, 0,\{\emptyset\})\}, E S=\emptyset, B S=\emptyset, d_{\text {max }}=\infty\)
    Initial \(p k\) NN tuple structure \(t=\left(v, d_{\text {net }}(v, q), V P \subset P\right.\), when \(\operatorname{Size}(V P)=k\), the path is
    complete.
    Do\{
    De-queue the top \(t\) in \(R S\) and \(E S=E S \cup t\)
    Find all \(t\) 's adjacent nodes in \(V\)
    for Each \(t\) 's adjacent node \(n d\) do
        if \(n d \in P\) and \(n d \notin t . V P\) then
            if Sizet. \(V P<k-1\) then
                \(R S=R S \cup\left(n d, d_{n e t}(v, q)+d_{n e t}(v, n d),\{t . V P \cup n d\}\right)\)
            else
            if \(d_{\text {net }}(v, q)+d_{\text {net }}(v, n d)<d_{\text {max }}\) then
                \(d_{\text {max }}=d_{\text {net }}(v, q)+d_{\text {net }}(v, n d)\)
                \(B S=\left\{\left(n d, d_{\text {net }}(v, q)+d_{\text {net }}(v, n d),\{t . V P \cup n d\}\right)\right\}\)
            else
                    if \(d_{\text {net }}(v, q)+d_{\text {net }}(v, n d)=d_{\text {max }}\) then
                    \(B S=B S \cup\left(n d, d_{n e t}(v, q)+d_{n e t}(v, n d),\{t . V P \cup n d\}\right)\)
                    end if
            end if
        end if
        else
            \(R S=R S \cup\left(n d, d_{n e t}(v, q)+d_{n e t}(v, n d),\{t . V P\}\right)\)
        end if
    end for
    PrunCond1_Lemma1 \((R S)\)
    PrunCond2_Lemma2 \((R S)\)
    PrunCond3_Lemma3 \(\left(R S, d_{\max }\right)\)
    PrunCond4_Lemma4( \(R S, E S\) )
    \} While ( \(R S \neq \emptyset\) )
```



Figure 4.3: An example of Local minima scenario

Local minima: One main question from the $p k N N$ queries is that whether we can use multiple 1_NN, instead of $p k$ NN to answer the same query? For example, if we want to find $p 2 \mathrm{NN}$, can we use $1 \_\mathrm{NN}$ first in query point to find nearest $p_{i}$ and then use $1 \_$NN on $p_{i}$ to find nearest $p_{j}$ ? The answer is NO. See example in Fig. 4.3 , using multiple 1_NN, we will get a path $q \rightarrow p_{5} \rightarrow p_{4}$ because $p_{5}$ is closest to $q$ and $p_{4}$ is closest to $p_{5}$. Whereas using $p 2 \mathrm{NN}$, the path would be $q \rightarrow p_{2} \rightarrow p_{1}$. And actually distance $\operatorname{dist}\left(q \rightarrow p_{2} \rightarrow p_{1}\right)=7$, which is shorter than $\operatorname{dist}\left(q \rightarrow p_{5} \rightarrow p_{4}\right)=8$. This simple example shows that it is impossible to use multiple 1_NN technique to answer $p k \mathrm{NN}$ queries.

Looping scenario: In $p k \mathrm{NN}$, we do a full expansion, which means we expand to all adjacent neighbor nodes. It is different from INE and Dijistra's algorithm because will full expansion, we allow go-and-back path. As a result, a looping scenario is well performed in $p k \mathrm{NN}$. But lemma 1 and 2 can prune the redundant looping paths.

See example in Fig 4.4. Starting from $q$, after few expansion to $n_{1}, n_{2}, p_{4}, n_{3}, n_{4}$ in turn, the path ends at $n_{4}$. Then the next expansion will go to $q$ and this is a typical looping scenario. This looping tuple can be pruned by lemma 1 because the second round will cost longer distance than the first round.


Figure 4.4: An example of looping scenario


Figure 4.5: An example of U-Turn scenario

U-Turn scenario: As we expand the path to all adjacent nodes, we allow the user to go back anytime. Fig 4.5 shows that this condition should be allowed in the algorithm; otherwise the answer of the query can be wrong.

Suppose the query is $2 p \mathrm{NN}$. After expansion from $q$, we got the following tuples in $R S=\left\{\left(p_{1}, 1,\left\{p_{1}\right\}\right),\left(n_{1}, 1,\{\emptyset\}\right)\right\}$. When we expand $p_{1}$, if we do not allow U-Turn, we will not add $q$ into $R S$, so we cannot find $n_{1}$, then $p_{2}$ in future, so the result will be $q \rightarrow p_{1} \rightarrow p_{4}$. Actually, the shortest path should be $q \rightarrow p_{1} \rightarrow p_{2}$ because $\operatorname{dist}\left(q \rightarrow p_{1} \rightarrow p_{2}\right)=5$ is smaller than $\operatorname{dist}\left(q \rightarrow p_{1} \rightarrow p_{4}\right)=6$.

Now we can summarize $p k \mathrm{NN}$ algorithm in general, our $p k \mathrm{NN}$ approach essence is full expansion with four pruning conditions to improve the performance evaluation, in other word, accelerate the query processing, at the same, three issues (local
minima, looping and U-turn scenarios) are been emphasized to ensure the correctness of the $p k \mathrm{NN}$ result.

### 4.3 Approach 2: Path Branch Point based $k$ NN Search

### 4.3.1 Preliminaries

Euclidean distance, which has been widely used in most queries in mobile database, cannot reflect the real connections between interest points. The measure which is investigated in this approach is network distance. The network distance depends on the underlying road network which links the interest points, while Euclidean distance reflects the relative positions of interest points. So at the beginning of this section, the road network and network distance are defined.

In addition, the two factors $D C$ (distance cost) and $O F$ (overlap factor) are introduced. By using $D C$ and $O F$, a formula can distinguish one path from the others.

Finally, the data structure is also illustrated to clarify the algorithm in section 4.3.2.

## Definitions

In the query of our approach, one of the given condition is pre-defined path. Def. 4.3.1 defines the notation of pre-defined path.

Definition 4.3.1. (Pre-defined Path) is one condition in the query, which starts at point $S$ and ends at point E. Pre-defined path can be notated as Path pre.

Definition 4.3.2. (Overlap Segment) If both of Path $_{a}$ and Path $h_{b}$ contain segment $s_{i}, s_{i}$ is defined as Path $_{a}$ and Path ${ }_{b}$ 's overlap segment, marked as $O S_{a, b}$.

Definition 4.3.3. (Separation) and (Regression Point) If $n_{i} n_{j}$ is one of the overlap segment $O S_{a, b}$ of $\mathrm{Path}_{a}$ and $\mathrm{Path}_{b}$ and according to the direction of Path $h_{\text {pre }}$,
$n_{i}$ is antecedent point while $n_{j}$ is the succedent point, $n_{i}$ and $n_{j}$ are defined as a pair of Separation Point $\left(S P_{a, b}\right)$ and Regression Point $\left(R P_{a, b}\right)$ respectively.

## Factors and Formula

Definition 4.3.4. Overlap Factor $(O F)$ of path $n_{n}$ is the percentage of $O S_{n, p r e}$ out of $\operatorname{dist}\left(\right.$ Path $\left._{n}\right)$, if path $h_{n}$ starts from $S$, via $p_{i}$ and ends at $E$, $p_{i} \in P$.

$$
\begin{equation*}
O F_{\text {path }}^{n}, ~=\frac{\sum_{i=1}^{n} \operatorname{dist}\left(O S_{n, p r e}\right)}{\operatorname{dist}\left(\text { Path }_{n}\right)} \tag{4.1}
\end{equation*}
$$

From Def. 4.3.4 and equation 4.1, we can see that the $O F$ is the factor of overlap distance divide Path ${ }_{n}$. For example, if the overlap part of Path $h_{n}$ and Path pre is 7 km and Path $_{n}$ is 11 km , we can calculate that the $O F$ of $P a t h_{n}$ is $\frac{7}{11}$.

Definition 4.3.5. Distance $\boldsymbol{C o s t}(D C)$ of path $h_{m}$ is the percentage of dist $\left(\right.$ Path $\left._{\text {pre }}\right)$ compared to dist $\left.(\text { Path })_{m}\right)$, if path $h_{m}$ starts from $S$, via $p_{i}$ and ends at $E$, $p_{i} \in P$.

$$
\begin{equation*}
D C_{p a t h_{m}}=\frac{\operatorname{dist}\left(\text { Path }_{\text {pre }}\right)}{\operatorname{dist}\left(\text { Path }_{m}\right)} \tag{4.2}
\end{equation*}
$$

From Def. 4.3.5 and equation (4.2), we can see that is the $D C$ is the percentage of $\operatorname{dist}\left(\right.$ Path $\left.h_{\text {pre }}\right)$ compared to $\operatorname{dist}\left(\right.$ Path $\left._{m}\right)$. For example, if the Path $_{\text {pre }}$ is 10 km , while $P a t h_{m}$ is 11 km , the $D C$ is $\frac{10}{11}$.

Definition 4.3.6. Adjust Score $(A S)$ is determined by the result of Overlap Decrement (OF) multiple Distance Cost (DC). The closer to 1, the better the path is.

$$
\begin{equation*}
A S_{\text {path }_{i}}=O F_{\text {path }}^{i} \text { } * D C_{\text {path } h_{i}} \tag{4.3}
\end{equation*}
$$

After clarifying the Defs. 4.3.54.3.6 and equations 4.24.4, we can summarize as follows: factor $D C$ examines the distance cost of path, factor $O F$ represents the overlap, while $O F$ multiplied by $D C$, which is factor $A S$ examines whether the


Figure 4.6: An example of path branch points query
balance between $O F$ and $D C$ is optimal. The closer to 1 , the better the path is. To continue the example above, the $A S$ is $\frac{7}{11} * \frac{10}{11}=\frac{70}{121} \approx 0.579$.

Definition 4.3.7. (Path branch points) Given Path $h_{\text {pre }}$ and $p_{i} \in P$, a Path branch point query finds the optimal Path $h_{a}$ (with largest $A S$ ) and the pair of $S P_{a, p r e}$ and $R P_{a, p r e}$. Path $_{a}$ should start at $S$, via $p_{i}$ and end at $E$.

Example 4.3.1. Fig 4.6 is an example of a path branch points query. The predefined path is marked as a red line in Fig.4.6 starting at $S$ and ending at E. Taking $p_{1}$ and $p_{2}$ as an example, one possible path through $p_{1}$ (Path ${ }_{1}$ ) is $S \rightarrow p_{1} \rightarrow n_{2} \rightarrow n_{6}$ $\rightarrow E$ and one possible path through $p_{2}\left(\right.$ Path $\left._{2}\right)$ is $S \rightarrow n_{2} \rightarrow n_{10} \rightarrow p_{2} \rightarrow E$. The factors of $\mathrm{Path}_{1}$ and Path $h_{2}$ are calculated as follows.

$$
\begin{aligned}
& O F_{\text {Path }_{1}}=\frac{1+5}{3+2+1+5}=\frac{6}{11} \\
& D C_{\text {Path }_{1}}=\frac{3+1+5}{3+2+1+5}=\frac{9}{11} \\
& A S_{\text {Path }_{1}}=O F_{\text {Path }_{1}} * D C_{\text {Path }_{1}}=\frac{6}{11} * \frac{9}{11}=\frac{54}{121} \approx 0.446 . \\
& O F_{\text {Path }_{2}}=\frac{3}{3+2+3+2}=0.3 \\
& D C_{\text {Path }_{2}}=\frac{3+1+5}{3+2+3+2}=0.9 \\
& A S_{\text {Path }_{2}}=\text { OF }_{\text {Path }_{1}} * D C_{\text {Path }_{1}}=0.3 * 0.9=0.27 .
\end{aligned}
$$

From this example, we call see that Path $h_{1}$ is better than Path $h_{2}$ even Path ${ }_{1}$ 's distance is longer than Path $h_{2}$. As Path $h_{1}$ has more overlap with the pre-defined path, which means the user can drive further on the familiar path, Path $h_{1}$ is determined to be a better path than Path $h_{2}$ which fits our motivation as well.

### 4.3.2 Proposed Approach

In this part, the path branch points approach $(P B P)$ will be introduced, including lemmas, pruning conditions, algorithms and the process will be outlined.

## $P B P$ query

In this part, the query of our proposed path branch points $(P B P)$ approach is described.

In daily life, most people have a preferred route if the start point and destination are given. For example, when traveling from the office to home, a person generally takes the same route each day, which is usually the shortest or fastest. Users are disinclined to change the route, even when they need to visit another destination along the route. Most users agree that if the driving distance is not too much greater, they prefer to keep as much as possible to the same route. How to balance the increment of the driving path with the overlapping percentage of the pre-defined path is the motivation of this chapter.

Given a pre-defined path which starts at $S$ and ends at $E$, a user decides to visit one specific type of interest point along the path. Choosing the best path which not only intersects with the user's pre-defined path, but also overlaps with it as much as possible, providing the increase in the driving distance is acceptable, is a general description of the $P B P$ query. In fig 4.6, our approach is to determine which is the better path from $S \rightarrow p_{1} \rightarrow n_{2} \rightarrow n_{6} \rightarrow E$ and $S \rightarrow n_{2} \rightarrow n_{10} \rightarrow p_{2} \rightarrow E$, and to finally find the optimal path of all possible paths.

## Lemmas

Section 4.3.1 has already clarified the factors which determine the optimal path. In this section, lemmas are illustrated based on an in depth analysis of $O F, D C$ and $A S$ factors.

Lemma 4.3.1. If dist $\left(\right.$ Path $\left._{a}\right)=\min \{\forall d i s t($ Path $) —$ Path starts at $S$ and ends at $E\}$ and dist $\left(\right.$ Path $\left._{b}\right)=\min \{\forall d i s t($ Path $) —$ Path starts at $S$, via $P$ and ends at $\left.E, p_{i} \in P\right\}, \operatorname{dist}\left(\right.$ Path $\left._{a}\right) \leq \operatorname{dist}\left(\right.$ Path $\left._{b}\right)$, in other words, $\forall$ path $_{b}, 0 \leq D C_{\text {path }} \leq 1$.

Proof. Lemma 4.3.1 can be proven by contradiction.
Suppose $\operatorname{dist}\left(\right.$ Path $\left._{a}\right)>\operatorname{dist}\left(\right.$ Path $\left._{b}\right)$, which means $\operatorname{Path} h_{b}$ is shorter than Path ${ }_{a}$. Because $P a t h_{b}$ starts at $S$ and ends at $E$, which satisfies the condition of $P a t h_{a}$ as well, and $\operatorname{dist}\left(\right.$ Path $\left._{a}\right)>\operatorname{dist}\left(\right.$ Path $\left._{b}\right)$, Path $_{a} \neq \min \forall \operatorname{dist}($ Path $—$ Path starts at $S$ and ends at $E$. This is against the given condition.

As a result, we can conclude that $\operatorname{dist}\left(\operatorname{Path}_{a}\right) \leq \operatorname{dist}\left(\operatorname{Path}_{b}\right)$ under given conditions.

$$
\text { So } 0 \leq D C_{\text {path }}=\frac{\operatorname{dist}\left(\text { Path }_{\text {pre }}\right)}{\operatorname{dist}\left(\text { Path }_{m}\right)} \leq 1 \text {. }
$$

Lemma 4.3.2. $\forall$ path $_{c}-$ path $_{c}$ starts at $S$, via $P$ and ends at $E, p_{i} \in P, a$ conclusion can be drawn: $0 \leq O F_{\text {path }_{c}} \leq 1$.

Proof. Because $O F_{\text {path }_{n}}=\frac{\sum_{i=1}^{n} \operatorname{dist}\left(O S_{n, p r e}\right)}{\operatorname{dist}\left(\text { Path }_{n}\right)}$, according to Lemma 4.3.1. $\operatorname{dist}\left(\right.$ Path $\left._{\text {pre }}\right)$ $\leq \operatorname{dist}\left(P a t h_{n}\right)$.

With reference to Def. 4.3.2, we can conclude $\sum_{i=1}^{n} \operatorname{dist}\left(O S_{n, p r e}\right) \leq \operatorname{dist}\left(\right.$ Path $\left._{\text {pre }}\right)$.

$$
\begin{aligned}
& \quad \text { As } \sum_{i=1}^{n} \operatorname{dist}\left(O S_{n, p r e}\right) \leq \operatorname{dist}\left(\text { Path }_{\text {pre }}\right) \leq \operatorname{dist}\left(\text { Path }_{n}\right), O F_{\text {path }}^{n}
\end{aligned}=\frac{\sum_{i=1}^{n} \operatorname{dist}\left(O S_{n, p r e}\right)}{\operatorname{dist}\left(\text { Path }_{n}\right)}
$$

Lemma 4.3.3. Given dist $\left(\right.$ Path $\left._{\text {pre }}\right)=\min (\forall d i s t($ Path $) —$ Path at from $S$ and ends at $E)$, then $0 \leq A S \leq 1$.


Figure 4.7: An example of Lemma 4.3.4
Proof. According to Lemma 4.3.1 and 4.3.2, $0 \leq O F_{\text {path }_{i}} \leq 1$ and $0 \leq D C_{\text {path }} \leq 1$, as a result, $0 \leq A S_{\text {path }_{i}}=O F^{*} D C \leq 1$.

By deep analysis of Lemma 4.3.1, 4.3.2 and 4.3.3, we can conclude that the closer $A S$ to 1 , the more optimal the path.

Lemma 4.3.4. If there is any interest point on the Path pre, the optimal path is Path pre with no branch point.

Proof. If there is any interest point on the Path pre, we can conclude that $\operatorname{dist}\left(\operatorname{Path}_{k}\right)$ $=\operatorname{dist}\left(\right.$ Path $\left._{\text {pre }}\right)$ as well as $\sum_{i=1}^{n} \operatorname{dist}\left(O S_{k, p r e}\right)=\operatorname{dist}\left(\right.$ Path $\left._{\text {pre }}\right)$.

Consequently $O F_{\text {path }_{k}}=\frac{\sum_{i=1}^{n} \operatorname{dist}\left(O S_{k, p r e}\right)}{\operatorname{dist}\left(\text { Path }_{k}\right)}=1$ and $D C_{\text {path }_{k}}=\frac{\operatorname{dist}\left(\text { Path }_{\text {pre }}\right)}{\operatorname{dist}\left(\text { Path }_{k}\right)}=1$. So $A S_{\text {path }}=D C^{*} O F=1$.

This is the optimal path because it has the largest $A S$ value.
Example 4.3.2. Fig 4.7 is an example of Lemma 4.3.4. In Fig 4.7, there is an interest point $p_{3}$ located on the pre-defined path. According to Lemma 4, it is the optimal path with no branch point. We can demonstrate this using the following calculations.

Suppose path $h_{i}$ is $S \rightarrow P_{3} \rightarrow n_{2} \rightarrow n_{6} \rightarrow E . O F_{\text {path }}^{i} 10 ~=\frac{12}{12}=1$ and $D C_{\text {path }}^{k}=\frac{12}{12}=1$. So $A S_{\text {pathi }}=D C^{*} O F=1$.


Figure 4.8: An example of Lemma 4.3.5
path $_{i}$ is the optimal path as it already reaches the top boundary of $A S$. It also fits our motivation because this path will allow the user to travel an a familiar route for as long as possible (in this case, the whole path is familiar to the user) with an acceptable increase in the driving distance (no distance increase in this case). It is clear that this is the optimal path.

Lemma 4.3.5. If $\forall P a t h_{j}-\sum_{i=1}^{n} \operatorname{dist}\left(O S_{j, p r e}\right)=0, A S_{p_{\text {path }}^{j}}=0$ and should be discarded.

Proof. If $\sum_{i=1}^{n} \operatorname{dist}\left(O S_{j, p r e}\right)=0$
obviously $O F_{\text {path }}^{k}$ $=\frac{\sum_{i=1}^{n} \operatorname{dist}\left(O S_{j, p r e}\right)}{\operatorname{dist}\left(\text { Path }_{k}\right)}=0$,
so $A S_{\text {path }}^{j}$ $=D C^{*} O F=0$.
This is the worst path because it has the smallest $A S$ value.

Example 4.3.3. Fig 4.8 is an example of Lemma 4.3.5. In Fig 4.8, the red path is Path pre and the blue path ( Path $_{\text {blue }}$ ) is the candidate path which starts at $S$, via $p_{2}$ and ends at E. According to Lemma 4.3.4, it is the worst path because there is no overlap between Path pre and Path blue . We can demonstrate this using the following calculations.

$$
\text { As } \sum_{i=1}^{n} \operatorname{dist}\left(O S_{b l u e, p r e}\right)=0
$$

$$
\text { obviously } O F_{\text {path }_{\text {blue }}}=\frac{\sum_{i=1}^{n} \operatorname{dist}\left(O S_{\text {blue }, \text { pre }}\right)}{\operatorname{dist}\left(\text { Path }_{\text {blue }}\right)}=0
$$

$$
\text { so } A S_{\text {path } h_{\text {blue }}}=D C^{*} O F=0 \text {. }
$$

From lemma 4.3.5, we can see that although path $h_{b l u e}$ has no driving distance increment $\left(\operatorname{dist}\left(\right.\right.$ path $\left._{\text {blue }}\right)=\operatorname{dist}\left(\right.$ path $\left.\left._{\text {pre }}\right)\right)$, it is still considered as the worst path because the entire path is not familiar to the user. The user may want to drive further provided most of the path is familiar to him.

## Implementing the $P B P$ query

In this section, the steps involved in implementing the $P B P$ query are described.
Firstly, initial Boundary $A S=0$, ResultList $=\emptyset$, array Intersections $=[\emptyset]$, DistBoun $=\infty$, ResultList $=\emptyset$.

Secondly, find all interest points along Path pre. If any, terminate the algorithm by returning these interest points and Path pre. According to Def.3, this is the optimal path of this query.

Thirdly, consider $S$ and $E$ as group $k \mathrm{NN}$ and find all group NN results then group them into a set CandidateSet sorted by the sum distance to $S$ and $E$. Whenever BoundaryAS is updated, update the DistBoun using the following formula:

$$
\text { DistBoun }=\sqrt[2]{\frac{\text { dist }^{\text {Bath } \text { Prere }^{2}}}{\text { BoundaryAS }}}
$$

Then pop out the first $p_{i}$ from CandidateSet. Find the shortest path path $h_{l}$ from $S$ via $p_{i}$ to $E$. Let $n_{x}$ be the separation point of path $h_{l}$ and pathpre $\left(s p_{l, p r e}\right)$ and $n_{y}$ the regression point of path $l_{l}$ and path pre $\left(r p_{l, p r e}\right)$. Calculate $A S_{p_{\text {path }}^{l}}$. If $A S_{\text {path }}$ $>$ BoundaryAS, set BoundayAS $=A S_{\text {path }}^{l}$, and replace ResultList as path $h_{l}, n_{x}$, $n_{y}$. If $A S_{\text {path }}^{l}=$ Bounday $A S$, add path $h_{l}, n_{x}, n_{y}$ into ResultList. If $A S_{\text {path }}<$ Boundary $A S$, put all intersections between $n_{x}$ and $n_{y}$ into Intersections including $n_{x}$ and $n_{y}$ in the visiting sequence. For each pair of $n_{i}$ and $n_{j}$ in Intersections and $n_{i}$ is forehead of $n_{j}$, form a path $h_{i}$ as $S \rightarrow n_{i} \rightarrow p_{i} \rightarrow n_{j} \rightarrow E$. If $\operatorname{dist}\left(\right.$ path $\left._{i}\right) \leq$ DistBoun, calculate $A S_{\text {path }_{i}}$ and follow the comparison above.

```
Algorithm 10 PBP \(\left(\right.\) Path \(\left._{\text {pre }}, P\right)\)
    Initial BoundaryAS=0
    Initial ResultList=Ø
    Initial array Intersections=[ \(=\) ]
    Initial DistBoun= \(\infty\)
    Initial ResultList \(=\emptyset\)
    OnPath \(=p-p_{i}\) on Path \(_{\text {pre }}\)
    if Size \((\text { OnPath })_{\dot{¿}} 0\) then
        Return OnPath
    end if
    CandidateSet \(=\mathrm{G} k \mathrm{NN}(S, E)\) sorted by \(\operatorname{dist}\left(S, p_{i}\right)+\operatorname{dist}\left(p_{i}, E\right)\).
    Pop out first \(p_{i}\) in CandidateSet
    Find the shortest path path from \(S\) via \(p_{i}\) to \(E\)
    if \(\operatorname{dist}\left(\right.\) path \(\left._{l}\right)\);DistBoun then
    \(n_{x}=s p_{l, p r e}\) and \(n_{y}=r p_{l, p r e}\)
        Calculate \(A S_{\text {path }}^{l}\)
        if \(A S_{\text {path }}^{l} \dot{\mathrm{~B}}\) Boundary \(A S\) then
            Set BoundayAS \(=A S_{\text {path }_{l}}\), ResultList \(=\left\{\right.\) path \(\left._{l}, n_{x}, n_{y}\right\}\)
            Update DistBoun \(=\sqrt[2]{\frac{\text { dist }\left(\text { Path }_{\text {pre }}\right)^{2}}{\text { BoundaryAS }}}\)
        end if
        if \(A S_{\text {path }}^{l}{ }^{=}\)Boundary AS then
            ResultList \(=\) ResultList \(\left\{\right.\) path \(\left._{l}, n_{x}, n_{y}\right\}\)
        end if
        if \(A S_{\text {path }{ }_{l} \text { Boundary } A S \text { then }}\)
            Intersections \(=\left\{n-n_{x}\right.\), intersections between \(n_{x}\) and \(\left.n_{y}, n_{y}\right\}\) in visiting sequence
        for Each pair of \(n_{i}, n_{j}, i_{i} j\) do
            Form a path \(h_{i}\) as \(S \rightarrow n_{i} \rightarrow p_{i} \rightarrow n_{j} \rightarrow E\)
            if \(\operatorname{dist}\left(\right.\) path \(\left._{i}\right)<\) DistBoun then
                Calculate \(A S_{\text {path }}\)
                if \(A S_{\text {path }}^{i}\) ¿Boundary \(A S\) then
                    Set Bounday \(A S=A S_{\text {path }_{i}}\), ResultList \(=\left\{\right.\) path \(\left._{i}, n_{i}, n_{j}\right\}\)
                    Update DistBoun \(=\sqrt[2]{\frac{\text { dist }^{\left(\text {Path }_{\text {pre }}\right)^{2}}}{\text { BoundaryAS }}}\)
                end if
                if \(A S_{\text {path }_{i}}=\) Boundary \(A S\) then
                    ResultList=ResultList \(\left\{\right.\) path \(\left._{l}, n_{x}, n_{y}\right\}\)
                end if
                Break
            end if
        end for
        end if
        Break
    end if
    if Size(CandidateSet) ¿0 then
        Go to Line 11
    end if
    Return ResultList
```

Next, continue pop out $p_{i}$ from CandidateSet and follow the previous steps until the sum distance to $S$ and $E$ in CandidateSet is larger than DistBoun. Lastly, the optimal path is in ResultList. Algorithm 10 is produced to find the path branch points.

### 4.4 Approach 3: Time Constraint Route Search

### 4.4.1 Preliminaries

Some variations of route search have been investigated in earlier works KSSD08, YS05, HJ04, ZXTS08, TBPM05, KSSD08, Zha08, EL05, PG98. In YS05, HJ04, they try to find the route with smallest deviation to visit a new point when a user travels a pre-defined route. ZXTS08,TBPM05] have single type of interest points and no time constraint involved. Considering the inaccuracy and incomplete issues, some works assigned scores or probabilities to each locations and the result path should pass the locations with high probabilities KSS09, KSSD08. In our query, the criteria is the path with shortest travel time which is different from shortest path ZXT $^{+} 09 \mathrm{~b}$. In addition, in some route search, the path can go through multiple locations of the same type KSSD08, KSS09 while our path only visits one location for each type. Moreover, even if a lot of different constraints have been studied, there is no paper which draws time constraint into Route Search query. Although our proposed methods are significantly different from existing works EL05, they are still worth to be reviewed as they become our motivation.

## Route Search Query

Route search query was proposed by Yaron Kanza et. al in 2008 KSSD08. There are three semantics covered in this chapter, such as given start point, end point and all types of user interests, the first semantic is to return the shortest route that goes via all relevant entities. A second semantic is to find the most-profitable route, which is the route having the highest accumulative relevance and the length of the
route is within the given limit. A third semantic is to compute the most-reliable route, which goes through as much higher relevant entities as possible and the length of the route is within the given limit.

Route search query uses greedy insertion from both start point and end point to find the final path. In route search, similar as our methods, multiple location types are concerned and the final path must pass one location of each type in any order.

## Outstanding Problems

- Route search query chooses multiple criteria such as shortest distance, highest relevance or more relevant entities with higher relevance. While we choose the shortest travel time because we draw time constraint into the query and shortest distance cannot guarantee the shortest travel time.
- Route search query has a pre-defined path length limit while our methods are indifferent to travel distance.
- Route search query pre-defined the start and end point while our methods only give the start point.
- Route search query does not concern arriving time for any interest point while the optimum path of our methods should meet the time constraint of each type.


## Path Based $k$ NN

Path Based $k$ nearest neighbor ( $\mathrm{P} k \mathrm{NN}$ ) [ZXTS08] is given a set of candidate interest points to find the shortest path which starts at query point and goes through $k$ interest points. For example, find the shortest path which goes through 3 restaurants from $q$. It is a novel $k N N$ because the result is the shortest path and the interest points are visited one by one. While it is not a Route Search query neither because all candidate interest points are single type, while our Route Search query involves
multiple types which needs access to different data structures such as multiple Network Voronoi Diagram [ZXT ${ }^{+}$09b, KS04, OBSC00] and the result path must cover every type.
$\mathrm{P} k N N$ uses network expansion as Incremental Network expansion (INE). In the process of network expansion, $\mathrm{P} k \mathrm{NN}$ records all expansion branches until one path is full of $k$ interest points. The path is set as the boundary. Continue to do the expansion, once there is a path shorter than the boundary; shrink it until all possible branches are expanded out of boundary. $\mathrm{P} k \mathrm{NN}$ is similar with our proposed methods, such as both methods have a boundary identifier $\left(D / T_{\max }\right)$ to shrink the expansion scope and numbers of points to be visited are given.

## Outstanding Problems

Single vs. Multiple types: $\mathrm{P} k \mathrm{NN}$ considers all interest points as single type. In reality, the user may want to process the $k$ nearest neighbor search in a multiple type objects environment.

Time Constraint: PkNN does not concern arriving time constraint for any interest point while the optimum path of our methods should meet the time constraint.

## Additional Specifications

Before we move on to the proposed methods, more specifications are described and compared with other existing works:

- Why cannot kNN solve Route Search query?
$k N N$ cannot guarantee all locations to be fully covered, since the distance from query point to all interest points is the only criteria, and not the complete path.
- What are the differences between incremental kNN and Route Search query? In path based $k \mathrm{NN}$, all interest points are considered as single type while in route search query, there are multiple types of interest points and the optimal path should go through all types of interest points.
- Can multiple 1_NN find all interest points?

If we use 1_NN to find the nearest neighbor until all types have been found, there is high possibility that the final path is not the most optimum one. The first nearest interest point may lead to a further distance to other interest points, as a result, it does not yield an optimum path. This is a common local minimal problem in scheduling.

- Why does the shortest time need to be used instead of the shortest distance? Since the problem of answering Route Search queries is a generalization of the traveling salesman problem, it is unlikely to have an efficient solution, i.e., there is no polynomial-time algorithm that solves the problem (unless $\mathrm{P}=\mathrm{NP}$ ) [KSSD08]. Hence, as a solution, this chapter incorporates time constraint in order to prune as many expansion branches as possible and makes the query more realistic. If we use time constraint to prune the expansion branch, choosing traveling time as criteria is straightforward and can be adjusted to different travel time period.


### 4.4.2 Proposed Methods

In this section, our proposed Route Search for fixed locations (RFix) and flexible locations (RFlex) are described. In each method, the query is given first followed by detailed explanations of the key issues, then after listing the algorithm, an example will illustrate the processing steps. As travel time is chosen as criteria, travel time network needs to be introduced first.

In Travel time network KZWW05, the measurement between nodes is the travel time 4.9(b) instead of network distance 4.9(a). This is often more desirable because under certain conditions travel time is more meaningful than network distance, such as whether the path arriving at locations within their operating hours depends on the travel time, not travel distance. In this chapter, we use the average travel speed in routine profile to estimate the approximate travel time. Fig. 4.4.2 gives an


Figure 4.9: Road network vs. travel time network
example of a travel time network. We assume that the average travel time for each road segment is read from the traffic profile.

## Route Search for fixed locations (RFix)

When the query is a route search for fixed locations, this query can be categorized as Route Search for fixed locations (RFix). Example 4.4.1 illustrates a RFix query. It can be expressed as follows: Start at $q$ at $4: 30 \mathrm{pm}$, find the optimum path whose travel time is shortest and this path should visit $A, B, C$ and $D$ between 9:00am5:00pm, 9:00am-5:30pm, 4:30pm-5:40pm and 6:00pm-6:30pm respectively. Now we can treat $q, A, B, C$ and $D$ as locations and invoke our proposed method RFix to find an optimum path. The pruning conditions are explained as follows:

Example 4.4.1. Secretary will leave her office at 4:30pm. She has a plan to do:
$\diamond$ Fetch a suit from dry cleaner $A$ and $A$ 's trading time is 9:00am-5:00pm.
$\diamond$ Fetch a contract from Company B and B's open hours are 9:00am-5:30pm.
$\diamond$ Send a report to manager's apartment $C$ and he is at home 4:30pm-5:40pm.
$\diamond$ Pick up her son from kindergarten $D$ and $D$ 's pick up period is 6:00pm-6:30pm.

## RFix Definition

The RFix query can be formally defined like this:

Definition 4.4.1. RFix is a route search query consisting of:
Input: Type Set $T=\left\{t_{1}, t_{2}, \ldots, t_{n}\right\}$, Locations set $P=\left\{p_{1}, p_{2}, \ldots, p_{n}\right\}, \forall p_{i} \in t_{i}$.
Output: A Path l which goes through all ps in $P$ and distancel is the shortest.
Pruning Conditions Since the problem of answering Route Search queries is a generalization of the traveling salesman problem, it is unlikely to have an efficient solution, hence an efficient pruning method is crucial. With the prune conditions, the candidate permutation is greatly reduced and that is the basis of our solutions. Two pruning conditions are discussed in this section. Firstly, definition for Invalid Path and Valid Path are introduced here.

Definition 4.4.2. A path is invalid when at least one location's arriving time followed by this path is out of its operating hours; otherwise if it visits all user defined location types, it is a valid path.

$$
\left.\begin{array}{l}
\exists P_{i}  \tag{4.4}\\
\operatorname{Path}\left(q \rightarrow P_{1} \rightarrow \ldots \rightarrow P_{i} \rightarrow \ldots \rightarrow P_{j}\right) \\
T\left(q \rightarrow P_{1} \rightarrow \ldots \rightarrow P_{i}\right) \notin \text { OperatingHour }\left(P_{i}\right)
\end{array}\right\} \Rightarrow \operatorname{Invalid}\left(q \rightarrow P_{1} \rightarrow \ldots \rightarrow P_{i} \rightarrow \ldots \rightarrow P_{j}\right)
$$

Pruning condition 1: non-reversible visiting sequence. If $q \rightarrow p_{i} \rightarrow p_{j}$ is a valid path while $q \rightarrow p_{j} \rightarrow p_{i}$ is an invalid path as in Equation (1), ( $p_{i}, p_{j}$ ) have nonreversible visiting sequence $\left(q \rightarrow p_{i} \rightarrow p_{j}\right)$. Hence, any solution visiting $p_{j}$ before $p_{i}$ should be pruned.

$$
\left.\begin{array}{l}
\operatorname{valid}\left(q \rightarrow P_{i} \rightarrow P_{j}\right)  \tag{4.5}\\
\operatorname{Invalid}\left(q \rightarrow P_{j} \rightarrow P_{i}\right) \\
\operatorname{Type}\left(P_{i}\right), \text { Type }\left(P_{j}\right) \subseteq \text { LocationTypelist }
\end{array}\right\} \Leftrightarrow P_{i \nless} \rightarrow P_{j}
$$

Proposition 2. Given a query point $q$ and two locations $p_{1}$ and $p_{2}$

$$
\left.\begin{array}{l}
\operatorname{StartTime}>\operatorname{OpenTime}\left(P_{1}\right)  \tag{4.6}\\
\operatorname{StartTime}>\operatorname{OpenTime}\left(P_{2}\right) \\
\operatorname{CloseTime}\left(P_{1}\right)<\operatorname{CloseTime}\left(P_{2}\right) \\
\operatorname{Invalid}\left(q \rightarrow P_{1} \rightarrow P_{2}\right)
\end{array}\right\} \nRightarrow \operatorname{Invalid}\left(q \rightarrow P_{2} \rightarrow P_{1}\right)
$$

Proof. If CloseTime $\left(p_{1}\right)<\operatorname{CloseTime}\left(p_{2}\right)$ which means $p_{1}$ closes earlier than $p_{2}$, it is possible that $\left(p_{1}, p_{2}\right)$ have a non-reversible visiting sequence $q \rightarrow p_{1} \rightarrow p_{2}$ because if we visit $p_{2}$ first, when we arrive $p_{1}$, it is already closed. It is self-evidence.

If $\operatorname{CloseTime}\left(p_{1}\right)<\operatorname{CloseTime}\left(p_{2}\right)$ and $q \rightarrow P_{1} \rightarrow P_{2}$ is invalid, we will prove that we can not conclude $q \rightarrow P_{2} \rightarrow P_{1}$ is invalid by contrapositive.

$$
\left.\begin{array}{l}
T\left(q, P_{1}\right)+T\left(P_{1}, P_{2}\right) \approx 2 * T\left(q, P_{1}\right)>\operatorname{Close} T\left(P_{2}\right) \Rightarrow \operatorname{Invalid}\left(q \rightarrow P_{1} \rightarrow P_{2}\right)  \tag{4.7}\\
T\left(q, P_{2}\right)+T\left(P_{2}, P_{1}\right) \approx T\left(q, P_{1}\right)<\operatorname{Close} T\left(P_{1}\right) \Rightarrow \operatorname{Valid}\left(q \rightarrow P_{2} \rightarrow P_{1}\right)
\end{array}\right\} \Leftrightarrow \exists P_{2 \nleftarrow}^{\rightarrow} P_{1}
$$

Pruning condition 2: Invalid sub-paths make the entire path invalid. Any permutation containing invalid sub-path should be pruned. See Proposition 2.

## Proposition 3.

$$
\left.\begin{array}{l}
\operatorname{Invalid}\left(q \rightarrow P_{i} \rightarrow P_{j}\right)  \tag{4.8}\\
\text { StartTime }>\operatorname{Max}\left(\text { OpenTime }\left(P_{i}\right), \text { OpenTime }\left(P_{j}\right)\right)
\end{array}\right\} \Rightarrow \operatorname{Invalid~}\left(q \rightarrow \ldots \rightarrow P_{i} \rightarrow \ldots \rightarrow P_{j}\right)
$$

Proof.

$$
\begin{align*}
& T\left(q \rightarrow P_{i} \rightarrow P_{j}\right)>\operatorname{CloseTime}\left(P_{j}\right)  \tag{4.9}\\
& T\left(q \rightarrow \ldots \rightarrow P_{i} \rightarrow P_{k} \rightarrow P_{j}\right)>T\left(q \rightarrow P_{i} \rightarrow P_{j}\right) \\
& \left.\begin{array}{r}
P_{k} \neq P_{i} \neq P_{j}
\end{array}\right\} \\
& \Rightarrow T\left(q \rightarrow \ldots \rightarrow P_{i} \rightarrow \ldots \rightarrow P_{j}\right)>\operatorname{CloseTime}\left(P_{j}\right) \\
& \quad \Rightarrow \operatorname{Invalid}\left(q \rightarrow \ldots \rightarrow P_{i} \rightarrow \ldots \rightarrow P_{j}\right)
\end{align*}
$$

The RFix method is processed in the following steps and the algorithm is shown in Algorithm 11 .

```
Algorithm 11 RFix(q,StartTime,LocationSet,LocationOperatingHour)
    Load routine traffic speed and calculate travel time for all segments
    Initial EntitySet \(=q+\) LocationSet
    For any two in EntitySet, calculate its travel time
    First=All locations whose travel time to \(q\) within earliest CloseTime
    : For any two locations \(p_{i}\) and \(p_{j}\) in LocationSet, check \(q \rightarrow p_{i} \rightarrow p_{j}\) is valid or not. If
    it is invalid, put \(q \rightarrow p_{i} \rightarrow p_{j}\) in PruneList
    : CandidatePath \(=\) Permutations of LocationSet whose first point in First and contain
    no subpath in PruneList
    Initial Total_TimeCost \(=\infty\)
    for each candidate path in CandidatePath do
        TimeCost=sum up all travel time and once TimeCost>Total_TimeCost, terminate
        this loop
10: if at some step, TimeCost is out of \(p_{i}\) 's LocationOperatingHour then
11: ignore this path and prune all path from CandidatePath whose has it as sub
        path
            TimeCost \(=\infty\)
        end if
        if Total_TimeCost \(>\) TimeCost then
            Total_TimeCost \(=\) TimeCost
        end if
    end for
    Final Total_TimeCost is travel time cost and its path is the optimum path
```



Figure 4.10: RFix Example

| Traversal Permu. | Action \& Reason | CandidatePath |
| :---: | :---: | :---: |
| $q \rightarrow B \ldots$ | Pruned ( $B$ not in First) | None |
| $q \rightarrow C \ldots$ | Pruned (C not in First) | None |
| $q \rightarrow A \rightarrow B \rightarrow C \rightarrow D$ |  | None |
| $q \rightarrow A \rightarrow B \rightarrow D \rightarrow C$ |  | None |
| $q \rightarrow A \rightarrow D \rightarrow B \rightarrow C$ | Pruned | None |
| $q \rightarrow D \rightarrow A \rightarrow B \rightarrow C$ | Invalid subpath | - |
| $q \rightarrow D \rightarrow B \rightarrow A \rightarrow C$ | $(q \rightarrow B \rightarrow C)$ | None |
| $q \rightarrow D \rightarrow B \rightarrow C \rightarrow A$ |  | None |
| $q \rightarrow A \rightarrow C \rightarrow D \rightarrow B$ |  | None |
| $q \rightarrow A \rightarrow D \rightarrow C \rightarrow B$ | Pruned | None |
| $q \rightarrow D \rightarrow A \rightarrow C \rightarrow B$ | Invalid subpath | None |
| $q \rightarrow D \rightarrow C \rightarrow A \rightarrow B$ | $(q \rightarrow D \rightarrow B)$ | None |
| $q \rightarrow D \rightarrow C \rightarrow B \rightarrow A$ |  | None |
| $q \rightarrow A \rightarrow C \rightarrow B \rightarrow D$ | Unpruned | $q \rightarrow A \rightarrow C \rightarrow B \rightarrow D$ |

Table 4.1: RFix Filter Process

Firstly, depending on current time period, retrieve the traffic speed and calculate the time cost between any two locations in the set which includes the query point and all fixed locations. The process is similar to Dijkstra algorithm if the weight between entities are travel time cost.

Secondly, find all locations in First whose travel time to $q$ is within the earliest close time, meaning that if the path goes to the other points first, the path already misses the location with the earliest close time. After checking its arriving time is in the operating hours, put it into First. As a result, First holds all possible locations which can be the first visited.

Thirdly, for any two locations $\left(p_{i}\right.$ and $\left.p_{j}\right)$, calculate whether $q \rightarrow P_{i} \rightarrow P_{j}$ match $i, j$ 's operating hours or not. If not, record $q \rightarrow P_{i} \rightarrow P_{j}$ into PruneList. Then
generate all permutations of visiting sequence whose first visiting node is in First and do not include any sub path in PruneList.

Fourthly, for each candidate path, sum up its cost time and compare the time with time constraint. Once it exceeds the time constraint, ignore it and filter the other path who has the same sub path. E.g. if $q \rightarrow p_{1} \rightarrow p_{2} \rightarrow p_{3}$ fails to match time constraint and when query starts, $p_{1}, p_{2}$ and $p_{3}$ have opened already, $q \rightarrow p_{1} \rightarrow p_{5} \rightarrow p_{2} \rightarrow p_{6} \rightarrow p_{3}$ should be pruned as well.

Finally, compare the time cost of the paths left and choose the optimum one.
A Case Study To clarify the algorithm, a case study (see in Fig. 4.10) is presented. This case study is based on Example 4.4.1. Table 4.1 shows the RFix filter process. Firstly, $q \rightarrow B \rightarrow A$ will make $A$ over its close time, so $B$ is not in the First list. The same goes for $C$.

Secondly, according to Proposition 2, $q \rightarrow C_{4: 55 p m} \rightarrow B_{5: 19 p m}$ is valid and $q \rightarrow$ $B_{5: 18 p m} \rightarrow C_{5: 42 p m}$ cannot meet $C$ close time (5:40pm), so $B_{\nrightarrow}^{\rightarrow} C$. Add $q \rightarrow B \rightarrow C$ into pruneList.

Thirdly, the same as second step, add $q \rightarrow D \rightarrow B$ into PruneList.
Finally, generate the permutation whose first node is $A$ or $D$ (in First) and does not contain sub path in PruneList. In this case, only one path lists. After checking this path satisfy all time constraints, it is the result of this query $\left(q \rightarrow A_{4: 35 p m} \rightarrow\right.$ $\left.C_{5: 03 p m} \rightarrow B_{5: 27 p m} \rightarrow D_{6: 22 p m}\right)$.

## Route Search for flexible locations (RFlex)

When the user has pre-defined the location types whereby any locations of that type can be visited, this query can be categorized as Route Search for flexible locations, see example 4.4.2. As there are no fixed locations, we should distill the location types first according to the query specification. Then the query can be summarized as finding an optimum path which goes through these types within the time constraint.

Example 4.4.2. Secretary will leave her office at 4:30pm. She has a plan to do:
$\diamond$ Deposit a cheque in any bank and all banks' trading hour is 10:00am-5:00pm.
$\diamond$ Buy a printer in any shop and all shops' operating hours is 11:00am-5:30pm.
$\diamond$ Post a letter in any post office and posts' trading hour is 10:00am-5:40pm.
$\diamond$ Buy some medicine in any pharmacy and all pharmacies' trading hour is 10:00am6:30pm.

RFlex Definition The RFlex query can be formally defined like this:
Definition 4.4.3. RFlex is a route search query consisting of:
Input: Type Set $T=\left\{t_{1}, t_{2}, \ldots, t_{n}\right\}$, Locations set $P=\left\{p_{1}, p_{2}, \ldots, p_{m}\right\}, m>n$. $\operatorname{Type}\left(p_{i}, \ldots, p_{j}\right)=t_{k} \in T$

Output: A Path l which goes through ps and ps cover all types in T. Also distance ${ }_{l}$ is the shortest.

Pruning Conditions Traditional Route Search query is an NP complete problem and the focus of this section is how to use time constraint to prune most of expansion branches. Basically, our pruning conditions prune the path which leads to unreachable point or which is out of time.

Proposition 4. (Pruning condition 3): If P satisfies Equation 7, P_NN holds Pś nearest NN of all types in unvisited_type, $P$ leads to a unreachable point. $P$ will be pruned out candidate_next set. See Algorithm 12.


$$
\begin{equation*}
\Rightarrow \text { Unreachable(P) } \tag{4.10}
\end{equation*}
$$

Proof.

$$
\left.\begin{array}{l}
\operatorname{CloseTime}\left(\text { Type }_{i}\right)<\operatorname{TimeCost}\left(P, P_{\_} N N\left(P_{k}\right) \leq \operatorname{TimeCost}\left(P, \forall P_{i}\right)\right.  \tag{4.11}\\
P_{i} \& P_{k} \in \text { Type }_{m} \in \text { unvisited_type } \\
\Rightarrow \operatorname{TimeCost}\left(P, \forall P_{i}\right)>\operatorname{CloseTime}\left(\text { Type }_{i}\right) \Rightarrow \operatorname{Unreachable}(P)
\end{array}\right\}
$$

Example of Pruning condition 3: Fig. 4.11 is a case study of Example 4.4.2. Suppose $p_{1}$ is the candidate_next point, according to Proposition 3, whether it

```
Algorithm 12 Untouch(candidate_next,visited_type,T,TimeCost)
    for each \(p\) in candidate_next do
        p_type \(=\) get_Location_type ()
        visited_type \(=\) visited_type \(+P_{t} y p e\)
        unvisited_type \(=\) unvisited_type \(-P_{t}\) ype
        Initial boolean=0
        Find \(p\) ś nearest NN of all types in unvisited_type and put in \(p_{-} N N\)
        p_TimeCost=get_POI_TimeCost()
        TimeCost=TimeCost + p_TimeCost
        for each \(N N\) in \(p_{-} N N\) do
            \(N N_{-}\)type \(=\)get_POI_type \((N N)\)
            NN_TimeCost=get_Location_TimeCost \((N N)\)
            if NN_TimeCost + TimeCost \(>\) NN_type's close time then
                boolean=1
                Return
            end if
        end for
        if boolean=1 then
            Delete this \(p\) from candidate_next
        end if
    end for
```

should be pruned or not depends on the data in Table 4.2. $p_{i}$ is the nearest neighbor of $p_{1}$ in Type $_{i}$. As $p_{1}$ 's type is $T_{1}$, then unvisited_type $=T_{2}, T_{3}, T_{4}, T_{5}$. Find $p_{1}$ 's NN for each type in unvisited_type (Column 1 in Table 2). We can easily tell that $p_{2}$ is out of OperatingHours of Type ${ }_{2}$, in other words, $p_{1}$ leads unvisited type $T_{2}$ unreachable. Consequently $p_{1}$ cannot be the candidate_next point.


Figure 4.11: Pruning Cond. 3

| $p_{1}$ NN | Type | Oper.Time | Arriving Time |
| :---: | :---: | :---: | :---: |
| $p_{2}$ | $T_{2}$ | $5: 00 \mathrm{pm}$ | Start $T+15+65=5: 20 \mathrm{pm}$ |
| $p_{3}$ | $T_{3}$ | $5: 30 \mathrm{pm}$ | Start $T+15+45=5: 00 \mathrm{pm}$ |
| $p_{4}$ | $T_{4}$ | $5: 40 \mathrm{pm}$ | Start $T+15+25=4: 40 \mathrm{pm}$ |
| $p_{5}$ | $T_{5}$ | $6: 30 \mathrm{pm}$ | Start $T+15+20=4: 35 \mathrm{pm}$ |

Table 4.2: Proposition 3 Demo for RFlex

Proposition 5. (Pruning condition 4): Locations whose types are in unvisited_Type within TimeCon (see equation 9) can be in candidate_next. Equation 10 which collects the candidate_next set can prune lots of interest points.

StartTime $>\max _{\forall i \in n}\left(\right.$ OperatingHours $\left.\left(P_{i}\right)\right) \Rightarrow$ TimeCon $=\max ($ CloseTime $($ unvisited_Type $))$

$$
\text { candidate_next }(P)=\left\{\begin{array}{l}
\forall P_{i} \\
\operatorname{TimeCost}\left(P, P_{i}\right)<\text { TimeCon } \\
\text { Type }\left(P_{i}\right) \in \text { unvisited_Type }
\end{array}\right.
$$

Proof. Suppose when we start the query, all locations are open, TimeCon should be set as the earliest close time in unvisited_Type as if the earliest close time type has not been visited, the locations which are going to be visited must be finished ahead of the earliest close time, otherwise when expanding to the earliest close time type, the arriving time is already out of the time constraint.

```
Algorithm 13 CandidateNext(p,visited_type,T,TimeCost)
    unvisited_type \(=T-\) visited_type
    TimeCon \(=\) Earliest CloseTime of unvisited_type
3: Candidate_next = all points whose type in unvisited_type and travel time within TimeCon
```

Example of Pruning condition 4:Referring to Fig. 4.12 and Table 3, suppose $p_{3}$ of $T_{3}$ is the current expansion point. In Table 4.2, each line represents one scenario as the unvisited_Type is different. Take the first line as an example, if the only visited type is $T_{3}$, then the unvisited_Type $=\left\{T_{1}, T_{2}, T_{4}, T_{5}\right\}$ and TimeCon $=T_{1}$ 's


Figure 4.12: Pruning Cond. 4
close time $=4: 30 \mathrm{pm}$. Only $p_{1}$ can be the candidate_next point because all of the other points are out of TimeCon according to Proposition 4. Algorithm 13 shows the process of how to find candidate_next points which satisfies pruning conditions 4.

| Visit_Type | Unvisit_Type | TimeCon | Cand_Next |
| :---: | :---: | :---: | :---: |
| $T_{3}$ | $T_{1}, T_{2}, T_{4}, T_{5}$ | $5: 00 \mathrm{pm}$ | $p_{1}$ |
| $T_{1}, T_{3}$ | $T_{2}, T_{4}, T_{5}$ | $5: 30 \mathrm{pm}$ | $p_{4}, p_{5}, p_{7}$ |
| $T_{2}, T_{3}$ | $T_{1}, T_{4}, T_{5}$ | $5: 00 \mathrm{pm}$ | $p_{1}$ |
| $T_{1}, T_{2}, T_{3}$ | $T_{4}, T_{5}$ | $5: 40 \mathrm{pm}$ | $p_{5}, p_{7}$ |

Table 4.3: Proposition 4 Demo for RFlex

With these two pruning conditions, most expansions branches have been pruned. Although the execution time could potentially be exponential in the worst case because the pruning strategy is only heuristic, the pruning conditions do improve the performance significantly.

The detailed steps are shown as follows.
Firstly, initialize $T_{\max }$ as $\infty$ and it is the boundary identifier which will hold the final result. Initialize the visited_type as $\emptyset$ which is the collection of the location types that have been visited. Also, initialize $T$ as all location types of user interest.

```
Algorithm 14 RFlex(q,StartTime,T,LocationTtable)
    if \(q=\) start point then
        Load routine traffic speed in current period
        Initial \(T_{\max }=\infty\)
        visited_type \(=\emptyset\)
        unvisited_type \(=T\)
        Static TotalTimeCost=0
        TimeCost=0
    end if
    CandidateNext(q,visited_type,T,TimeCost)
    Untouch(candidate_next,visited_type,T,p_TimeCost,TimeCost)
    for each \(p\) in candidate_next do
        TotalTimeCost \(=\) TotalTimeCost + TimeCost
        StartTime \(=\) StartTime + TimeCost
        if visited_type \(=T\) then
            if TimeCost \(\leq T_{\max }\) then
            Update \(T_{\max }=\) TotalTimeCost and record its path tree
            end if
            Break
        else
            RFlex \(\left(p, S t a r t T i m e, T-v i s i t e d \_t y p e, L o c a t i o n T t a b l e\right) ~\)
        end if
    end for
    Final \(T_{\text {max }}\) is Time Cost and its path is optimum path
```

Secondly, according to the current time period, load routine traffic speed and get all interest points around $q$ whose travel time to $q$ is within TimeCon. Collect them in a set call candidate_next and prune these points using pruning condition 3.

Thirdly, for any point in candidate_next, do the following. Remove any $p$ in candidate_next, add $p$ 's type into visited_type. Then repeat the second step until all types have been visited. Once this path's cost time is shorter than $T_{\text {max }}$, replace $T_{\text {max }}$ with its cost time.

Finally, $T_{\max }$ is the shortest travel time and its path is the optimum path.
The algorithm of RFlex is shown in Algorithm 14

### 4.5 Performance Evaluation

In this section, we evaluate these three methods using different data and environment setting.

### 4.5.1 Path based $k N N$ search

In the experimentations, different simulation data (stored as several tables) are chosen to represents high (45 interest points), medium (20 interest points) and low density of interest points (10 interest points) respectively. In addition, a netlike map is created to represent more looping map (136 links), a general map with couples of loops (127 links) and an emanative map with few loops (101 links) are chosen to represent the medium and less looping maps in our performance evaluation.

Moreover, we examined $p k \mathrm{NN}$ for different values of $k$. All interest points, network links and intersect nodes are simulated data. The experiments were performed on a Mac with Intel Core 2 Duo processors, 2GB of RAM, and MS Access as our database. We analyze the behavior of our approach in the aspects such as expansion steps and run time with different densities of interest points and the values of $k$. The looping in the map has also been evaluated in expansion steps and run time


Figure 4.13: Expansion steps for different loops in maps


Figure 4.14: Runtime for different loops in maps
using different values of $k$. We also compare the performance of our approach in expansion steps and run time with pruning and without pruning.

## Looping map

In this section, we aim at finding the differences of expansion steps and runtime if less or more loops are involved in the searching map. A simulate map like Melbourne city is chosen as more looping map because it is a netlike map and in the expansions, more looping scenarios accrue there. A simulate map like Malvern suburb is chosen as normal which consists of couples of loops. A simulate map like Peninsula map is chosen as less looping map because it is emanative map. Also 50 different query positions which were generated randomly are tested to get the average expansion steps and runtime based on $k$ which from 1 to 8 . Less looping maps will give us a better performance because less expansion will be acted and runtime will be less.

Fig 4.13 and Fig 4.14 show the trend of the expansion steps and runtime if $k$ increases and gives the comparison among less looping map, medium looping map and more looping map.

From Fig 4.13 and Fig 4.14, we can draw a conclusion that the expansion steps and runtime go up with the increasing value of $k$. This can be easily understood as more $k$ requires more computations. At the same time, we found there is no


Figure 4.15: Expansion steps of different POI densities


Figure 4.16: Runtime of different POI densities
significant difference in expansion steps and runtime under different conditions of loops. In other words, the performance of $p k N N$ performs at the same scale in a netlike map or an emanative map. This may surprise us because according to common sense, more loops mean more redundant cost. This outcome is due to 4 pruning conditions proposed by us.

## Density of interest points

In this section, we aim at finding the difference of expansion steps and runtime with various densities of interest points. The performance is evaluated based on the different interest points distributions, such as high density distributed objects (42 interest points), medium density distributed objects (20 interest points), and low density distributed objects ( 9 interest points). In addition, 50 different query positions are tested to get the average performance result based on different values of $k$. It is known that for the same map, if the interest point density is low, the runtime will increase because more time will be a cost to do expansion. Also if $k$ increases, the runtime will increase because more time will be spend on trying to find more interest points. Fig 4.15 and Fig 4.16 demonstrate the trend of each scenario.

From Fig 4.15 and Fig 4.16, we can conclude that with the increment of $k, p k$ NN performs exponential growth in expansion steps as well as runtime. Another factor


Figure 4.17: Expansion steps with or without Pruning conditions


Figure 4.18: Runtime with or without Pruning conditions
which effects expansion steps and runtime is the density of interest points. The higher the density is, the lower cost in expansion and runtime, in other words, the better performance. The performance result coincides with our prediction because the more $k$ required, the more operations are in the approach. Also the lower density distributed of the interest points, the more cost to find the next node on the route.

## Pruning Conditions

The pruning conditions are the bright spots of our $p k N N$ approach. Instead of computing all possible permutations, full expansion is inducted. In addition, in the process of expansion, the expansion history is stored into a list in order to optimize the expansion as well as accelerate the processing efficiency. Fig 4.17 and Fig. 4.18 show the expansion steps and runtime improvements respectively between algorithms without and with pruning conditions.

Fig 4.17 and Fig 4.18 illustrate how pruning conditions improves the performance. With the increasing values of $k$, the expansion steps and runtime go up in linear growth instead of exponential growth. With the pruning condition, it is possible to implement $p k$ NN with larger values of $k$. To sum up, pruning conditions not only improve the performance, but also enhance the feasibility of our $p k \mathrm{NN}$ algorithm.


Figure 4.19: Operation time of different POI densities


Figure 4.20: Memory size of different POI densities

### 4.5.2 Path Branch Point based $k$ NN Search Queries

In these experiments, Sydney city map, Canberra city map and Hobart city map from the Whereis website (www. whereis.com) were chosen to represent high, medium and low density of interest points. All interest points, network links and intersect nodes are real-world data. We analyze the behavior of our approach in aspects such as operation time, memory size and $A S$ values with different densities of interest points and the length of $P a t h_{p r e}$.

## Interest Point distribution density

In this section, we aim to find the differences in runtime, memory size and $A S$ values between low, medium and high density interest points. We use restaurants in Sydney to represent a high density sample, parks in Canberra represent a medium density sample and hospitals in Hobart to represent a low density sample. Also we test 20 different query positions to obtain the average runtime, memory size and $A S$ values based on the distance of the pre-defined path from 10 to 80 .

From Fig 4.19, the conclusion can be drawn that the run time will increase if the density decreases. This is because the lower the density, the less chance that the interest point is close to the path. As a result, lower density will cause more


Figure 4.21: $A S$ values of different POI densities


Figure 4.22: Factor change based on different overlap increment-AS all negative
comparisons which will cause the delay of the operation. Also, we can see that there is no fixed relation between run time and the length of the pre-defined path.

From Fig 4.20, the conclusion can be drawn that the memory size will increase when the length of the pre-defined path increases, at the same time, the density of interest points and memory size seem to be unrelated according to our experiments. This is because the shorter the pre-defined path, the shorter the boundary distance. In other word, there is only a small chance that this will be the optimal path. At the same time, most interest points are pruned by boundary distance, which saves memory size.

From Fig 4.21, the conclusion can be drawn that the $A S$ value is closer to 1 when either the density of interest points or the length of the pre-defined path increases. Both these two factors increase the possibility that there are some interest points on the path or around the path. In other words, with an increase in the density of the interest points and the length of the pre-defined path, the greater the possibility that the path is more suitable to the user.

## Factor Increment/decrement

In this section, we aim to find the factor's increment or decrement when the path becomes longer in order to have more overlapping distance. From Figs $4.22,4.23$ and


Figure 4.23: Factor change based on different overlap increment-AS partial negative and partial positive


Figure 4.24: Factor change based on different overlap increment-AS all positive
4.24 , we can tell that the $D C$ becomes increasingly smaller because the increment is always negative, while the $O F$ becomes increasingly larger because the increment is already negative. After the calculation, we can see that there are three possible results: the $A S$ increases, the $A S$ decreases or the $A S$ increases then decreases.

Fig 4.22 shows an example where the $A S$ decreases. In this case, the shortest path is the one which has chance to be optimal, after the increment in the driving distance, although it may take overlap in return, but the overlap increment cannot increase the $A S$.

Fig 4.23 shows an example where the $A S$ increases then decreases. The turning point is where the optimal path is and this shows that the shortest path is not always the optimal path.

Fig 4.24 shows an example where the $A S$ increases. In this case, there is a chance that by increasing the driving distance to increase the section of the path that overlaps, the path becomes better than the shortest path.

### 4.5.3 Time Constraint Route Search over Multiple Locations

We used network and interest points data in Los Angeles in our experiments. We extracted 8 different types of interest points to simulate different location types, including 15 parks, 29 coffee lounges, 31 bank branches, 54 hotels, 78 post offices, 158 pharmacies, 283 shops and 597 restaurants and all interest points are normally distributed. In our experiments, we varied the following parameters: the number of location types, the congestion level (speed), density of interest points and the average time interval between locations to observe their effects on average processing time, memory as well as their improvement compared with the exhaustive traversal of all permutation approach.

## Experimental Results of RFix

Since number of locations highly influences our method's performance, we test the processing time (Fig. 4.25(a)) and memory (Fig. 4.25(b) for 2 to 8 locations based on 3 different traffic status (low, medium and high congestion). From Fig. 4.25(a) and Fig. 4.25(b), we can easily tell that with the increasing number of locations, the processing time and memory are directly proportional to the number of locations. In addition, when the congestion level increases from low to high, the speed decreases at the same time and it causes a slight increase of the processing time and memory. When it is extremely congested, the processing time and memory will cost approximately $8 \%$ to $14 \%$ more than the lower level. While adding one more location into query list, there will be exponential growth in processing time and logarithmic growth in memory size required, referring to Fig. 4.25(c) and Fig. 4.25(d) when location number is greater 8 , the performance scale increases sharply.

Without using our method, Route Search for fixed locations query can be solved by traversing all permutations at the cost of processing time and memory size. To improve its performance, we have proposed two pruning conditions in this chapter. Fig. 4.26(a) and Fig. 4.26(b) give the performance comparison in processing time


Figure 4.25: Time and memory comparison between different number of locations and traffic status in RFix and Time and memory incremental ratio when adding more locations


Figure 4.26: Proc. time and memory comparison between RFix and traversal methods


Figure 4.27: $P D T$ is optimum (RFix) ratio
and memory between with and without pruning conditions. Fig. 4.26(c) highlights the advantages of our pruning conditions.

To sum up, with the increasing number of locations, our methods with pruning conditions outperform the traversal methods in both processing time and memory aspects especially when the location number is greater than 3 .

In section 4.4, we include time constraint into Route Search query, normally the user will follow rules: earliest close, first visit. In other words, the path is sorted by the close time sequence and this path is abbreviated as $P D T . P D T$ ratio is the possibility that $P D T$ is the optimum path (Equation 11). In this section, we analyze factors that will affect the visiting path, see Fig. 4.27(a) and Fig. 4.27(b).

$$
\begin{equation*}
\text { PDT Ratio }=\frac{\text { number of times }(P D T=\text { optimum path })}{n \text { times experiments }} \tag{4.14}
\end{equation*}
$$

Before analyzing our experiment results, first we define a factor called Travel distance span in average Time interval to objects Distribution region (TTD). Object Distribution Region $(O D R)$ is the size of the region that user can arrive within the last location close time. TTD represents the coverage percentage of $O D R$ in average time interval between locations.

$$
\begin{gather*}
O D R=\pi\left(\left(\max \text { CloseTime }_{i}-\text { StartTime }\right)+\bar{t}\right)^{2}  \tag{4.15}\\
T T D=\frac{\left(\sum_{i=1}^{n}\left(\text { CloseTime }_{n}-\text { CloseTime }_{n-1}\right) / n\right) * \bar{t}}{O D R} \tag{4.16}
\end{gather*}
$$

Fig. $4.27(\mathrm{a})$ shows that low travel speed will lead to a high possibility that an optimum path is $P D T$ until the average interval increases to 2 hours or more, while high travel speed leads more possibility that optimum path is different from PDT when the average interval is greater than 0.4 hour. This result is in conformity with common sense as if the average time interval between locations is small and speed is relatively slow, visiting locations along the close time sequence has a higher possibility to meet the time constraint because if we visit the later close time location, there will be a high possibility that we cannot catch the earlier location, and vice versa. Fig. 4.27(b) illustrates that if TTD increases, which means its coverage percentage
in average time interval increases, the possibility that $P D T$ is the optimum path drops. The percentage remains stable until $T T D$ increases to around 1.

## Experimental Results of RFlex

For Route Search for flexible locations query, the average time interval between locations is a factor which affects the visiting sequence. People generally believe that if the average time interval between locations is large, the performance falls badly. While Fig. 4.29(a) and Fig. 4.29(b) prove that this conjecture is not correct because our RFlex goes down to get the first path and this path is set as boundary. As a result, the processing time is nearly linear and the memory decreases a little with the increasing average time interval between locations. As a result, our approach performs well even if there is no time constraint when the number of locations remains constant.

The processing time and memory will steeply increase with the increasing number of locations and this is already proven in RFix. In this section, we compare the differences between high density distributed objects (e.g. restaurant, density $=$ 0.09375 ) and low density distributed objects (e.g. parks, density $=0.0024$ ). In our experiments we test the processing time and memory for locations numbers from 2 to 7 refer to Fig. 4.28(a) and Fig. 4.28(b) for low and high densities of locations. Fig. 4.28(c) and Fig. 4.28(d) show that if Route Search query involves more low density objects, with the increasing number of locations, the processing time and memory do increase, but much slower than the high density objects.

### 4.6 Conclusion

This chapter discusses the route and path related $k N N$ queries. The motivation is to bring route/path into the input or output of spatial queries.

In this chapter, a novel approach called path based $k$ nearest neighbor based on network distance on road network is been introduced firstly. The basis of $p k \mathrm{NN}$ is network expansion. The proposed approach, $p k \mathrm{NN}$, gives users correct paths, even


Figure 4.28: Proc. time and memory comparison for different object densities in RFlex and Proc. time and memory incremental ratio when adding more locations


Figure 4.29: RFlex in different time intervals
when the route is complex like that in real world. We have also taken care of complex circumstances involving local minimum, loops, and U-Turns. We performed several experiments to measure the performance of $p k \mathrm{NN}$ in various network conditions. In general, our algorithms performs well if the density is high and the number of interest objects is smaller than 7. However, as expected, if the density of the interest objects is low and number of interest objects is large, the performance of $p k \mathrm{NN}$ will degrade sharply. On average, if $k$ is given, lower density of interest points will let the runtime and expansion step increase. If $k$ is large, the runtime and expansion step will increase sharply. Furthermore, there is no significant increase if more loops are involved on the underlying map because pruning conditions are proposed in this chapter. With pruning conditions, even the expansion steps and runtime will increase with the increasing values of $k$, it makes the increase in linear growth instead of exponential growth.

Secondly, we defined the path branch path query and proposed an approach which can scale the path into fit or unfit user requirement categories. We aim at using a mathematical formula to quantify the percentage of fitness to the user's requirement. In this chapter, we examine several special scenarios such as an interest point on the pre-defined path, no overlap with the pre-defined path, and a scenario where distance cost is negligible. We performed several experiments to measure the performance of $P B P$ in different interest point distributions. In general, our algorithms perform well if the density is high. However, as expected, if the density of the interest objects is low and the pre-defined path is too short, there will be a high possibility that the path is not optimal because it is likely that there is no interest point around the path and driving path to it will not have any overlap with the pre-defined path. The more interest points around the path, the longer the pre-defined path, hence the more optimal the path.

The third part of this chapter proposes novel Route Search methods with time constraint involving multiple object types. Route Search for fixed locations provides a solution to users if they want to find the shortest travel time path for multiple
location types and the locations of these types are fixed. Route Search for flexible locations helps users to find the shortest travel time path if the locations of these types are flexible. Both queries do not concern visiting sequence of objects subject to the final path as long as they arrive at each location within its operating hours. In our method, the network Voronoi Diagram is used to find the candidate next visiting point within certain time range and it enriches the content of our mobile navigation system and gives more benefit to mobile users as well. We performed several experiments to measure the performance of RFix and RFlex in different network conditions and object distributions. In general, our algorithm performs better if the number of locations is small. If the number of locations is smaller than 7 , the performance is acceptable no matter how complex the road condition is and how objects are distributed. However, as expected, if the number of locations is greater than 7 , the processing time and memory increase sharply. In addition, if the average location close time interval is large, our optimum path has a high possibility that it is not $P D T$ which means discarding $P D T$ and using our methods can give users a better path choice. Lastly when comparing RFix with the traditional traversal permutation method, it performs better and the advantage becomes obvious when the number of locations increases up to 4 .

To sum up, all approaches bring route into input or/and output of spatial queries which highly enrich the type of spatial queries contents. These approaches have been proven that they can solve their corresponding queries efficiently.

## Chapter 5

## Conclusion

### 5.1 Contributions

In this thesis, we presented efficient techniques to deal with Voronoi Diagram based $k$ Nearest Neighbor Search queries and route based $k$ nearest neighbor queries search under different settings. Chapter 3 presents some variants of $k$ nearest neighbor search respectively in section 3.2 , section 3.3. Chapter 4 presents the novel category of $k$ nearest neighbor search which is based on route/path. Three different approaches are discuss respectively in section 4.2, section 4.3, section 4.4. At last, section 5 summarizes this thesis as well as pointing out the future work.

- Contribution 1: Chapter 3 does the optimization by utilizing Voronoi Diagram to merge the road segments into polygons in order to replace the Network Expansion. Two approaches are proposed in the chapter, summarized as follows:

In section 3.2, we proposed an alternative approach for Continuous $k$ Nearest Neighbor query processing, which is based on Network Voronoi Diagram (we call our proposed method VCkNN, for Voronoi C $k N N$ ). This approach avoids the weakness of existing work GR03, GR99 and improves the performance by utilizing the Voronoi diagram. VCkNN ignores intersections on the query path; instead, it uses Voronoi polygons to subdivide the path. Our proposed

VCkNN approach is based on the attributes of the Voronoi diagram itself and using a piecewise continuous function to express the distance change of each border point. Our experiment verified the applicability of VCkNN approach to solve $\mathrm{C} k \mathrm{NN}$ queries and demonstrated that it outperforms existing algorithms. Section 3.3 presents new approaches on three different queries involving multiple object types using a network Voronoi Diagram, including: a) query to find nearest neighbor for multiple types of interest point (or 1NN for each object type), b) query to give the shortest path to cover multiple-object-types in a pre-defined sequence, and c) query to find an optimum path for multiple object types that gives the shortest path that covers the required interest objects in a random sequence. In these queries, more than one object type is considered and the query result is highly related with the object types. Every object belongs to one of the category and there is no overlap between categories. That is the basic property of multiple-object-type query. Our experiment verified the applicability of our approach to solve $k$ nearest neighbor over a multiple type of objects.

- Contribution 2: Chapter 4 opens up new route search queries which are able to bring path into the input or/and output of spatial queries. The tradition spatial queries use discrete points as input and output. So this chapter is mainly doing exploring a new area. Three approaches are proposed in the chapter, summarized as follows:

Section 4.2 investigates a novel route based $k$ nearest neighbor query which is called Path based $k$ NN Search Query. Path bases $k$ nearest neighbor search is to find the shortest path which goes through $k$ objects. In general, the overall distance of the path becomes the selection criteria. We propose an efficient algorithm and present several pruning conditions to do optimization that significantly reduces the overall computation cost and processing time. The efficiency of our proposed approach is demonstrated using real data sets and simulated data sets.

Section 4.3 brings a novel query which is called path branch point(PBP). PBP can be defined as: given a set of candidate interest objects and a pre-defined path which starts at $S$ and end at $E$, find a path which starts at $S$, via an interest point $P$ and ends at $E$. This path should overlap with the pre-defined path as much as possible with acceptable distance increment. This is a novel query which is motivated by users' common requirements because most users have ad hoc paths in their daily travel and they can tolerate a longer driving distance to some extent if they can drive on a familiar path when they want to visit a certain type of objects on the way. In this proposed approach, an Adjust Score is calculated for each path which is determined by overlapping distance and increased distance cost.

Section 4.4 introduces time constraint into route search over multiple locations. Each spatial business entity has its own valid time which implies the time constraint of the route. Moreover, instead of finding the shortest path, the aim is to find the path with shortest time cost. Meanwhile, all spatial entities are visited within their valid time frame. The query definitions are clearly stated at first, followed by two types of query scenarios: route search over objects with flexible locations and route search over objects with fixed locations. Extensive experiments demonstrate the efficiency of our proposed algorithms.

To sum up, the main contribution of the thesis can be summarized as the following two aspects in general: 1) optimization using Voronoi Diagram 2) exploring new area in spatial query search which brings route into it.

### 5.2 Open Problems and Future Work

Spatial query processing has been studied over decades, but we still see that possible extensions can be made in the future.

Firstly, a novel route based $k$ nearest neighbor query is proposed which is called Path based $k$ NN Search Query. Our approach has been proven that it can give the user precise results. But when the $k$ increases over 10 , the efficiency of this approach drops dramatically. One open problem leaves to us is how to improve the efficiency with small preciseness sacrifice.

Secondly, we bring a novel query which is called path branch point(PBP). In this chapter, I defined a function to judge the length increment cost as well as the length overlap percentage. There might be the other proposals on defining the concepts. With the different cost modeling, the query will produce different routes as result because some users might want to sacrifice the driving distance but prefer the familiar road, while other users might lay emphasis on shortest distance and are tolerant of unfamiliar paths.

Thirdly, time constraint is introduced into route search over multiple locations. We present two different processing procedures when the user wants to visit flexible or fixed locations. Another open problem which can be investigated in future work might be the various visiting sequences assigned by users. In other words, the visiting sequence of the objects can be sequential, random or partial sequential and random. The objects location can be flexible, fixed or partial flexible and fixed.

To sum up, there are still a few open problems for us to investigate in the future work. Our algorithm performs well under some settings. We may introduce more parameters into the spatial queries. By solving these open problems, the queries can assist user's lives. The following are several possible directions for future work.

- Query processing in P2P and Ad-hoc networks: In this thesis, all algorithms and queries are based on client-server architectures. Whereas, it is interesting to adjust our proposed algorithms for P2P based networks or Adhoc networks [Muh09]. As our algorithms outperform existing works in the most of circumstances, we conjecture that Voronoi based algorithms in P2P or Ad-hoc networks will outperform existing techniques as well. We also would like to investigate whether it is possible to let each peer to handle a portion
of Voronoi diagrams, which may reduce the load and computation cost of the server seriously. Another potential problem is how to let moving objects or queries to cooperate to manage the relative positions in the P 2 P or Ad -hoc networks.
- Spatial queries in a high dimensional space: Most of the existing works of spatial query concentrate on the processing in a 2D space. The spatial query processing in a high dimensional space, e.g., 3D space, land surface, inside space of a building, has only become a target ever in the past few years STX08, XSP09. It is an interesting problem to investigate how the higher dimensional Voronoi diagram or other geometric theories would help to improve the performance of spatial query processing.
- Join up spatial query results: All the existing spatial queries are univocal. But some of the user queries might contain more than one query types, e.g. joining $k$ nearest neighbor search with range search. In the future, we may find that the joining result of two categories might represent users's special requirements.
- Incorporate some intelligent features in mobile navigation In the future, it can be expected that mobile navigation incorporates some intelligent features, such as extracting movement patterns of mobile users GT04b, and these can be adopted for mobile navigation. There have also been some successful works in incorporating Voronoi diagrams and network Voronoi diagrams in mobile navigation XZTS08] as well as the use of ontology in query expansion [WST03.WST04]. Further, there are two important issues in mobile navigation, scalability and performance. Data broadcasting has been known to be able to address the scalability issues and indexing can be used to speed up performance. Further investigations on these two issues in mobile navigation can be useful TR04, TR02 and the $k$ nearest neighbor algorithm can be broadly investigated in various area of research, such as digital ecosystems MP07, LZL08, BX11, CLLP11.


## Appendix A

## Simulation Source Code

## A. $1 \quad k$ NN Implementation

## Point.Java

```
package algorithm
```

import java.util.ArrayList;
import java.util. Collections;
import java.util.HashMap;
import java.util.List;
import java.util.Map;
public final class Algorithm \{
public Algorithm(final List<Point> all, final int maxK) \{
this. all $=$ all;
this. $\operatorname{maxK}=\operatorname{maxK}$;
this. nearest $=$ new HashMap $<$ Point, List $<$ Point $\gg()$;
this.findNearestAtMaxK ();
\}
@SuppressWarnings(\{ "null", "boxing" \})
public Map<Integer, List<Cluster>> findClusters() \{
final Map<Integer, List<Cluster>> clusters = new HashMap<Integer, List<Cluster
$\gg() ;$

```
clusters.put(0, new ArrayList<Cluster>());
for (final Point p : this. all) \{
    clusters.get(0).add(new Cluster(p));
\}
for (int \(\mathrm{i}=1 ; \mathrm{i}<=\) this.maxK; ++i ) \(\{\)
    final Map<Point, List<Point>> pairs = this.findPairs(i);
    final List \(<\) Cluster \(>\) previousClusters \(=\) clusters.get \((\mathrm{i}-1)\);
    for (final Cluster c : previousClusters) \{
        if \((\mathbf{c}\). isMerged ()\()\{\)
            continue;
        \}
        final Cluster mergedCluster \(=\) new Cluster();
        final List \(<\) Cluster \(>\) mergingCluster \(=\) new ArrayList \(<\) Cluster \(>()\);
        mergedCluster.getPoints().addAll(c.getPoints());
        c.setMerged(true);
        int \(\mathrm{j}=0\);
        double mergingMinStableFactor \(=\) c.getStableFactor () ;
        while (mergedCluster.getPoints().size ()\(>j)\{\)
            final Point \(\mathrm{p}=\) mergedCluster.getPoints().get(j);
            if (pairs.containsKey(p)) \{
                for (final Point p2 : pairs.get(p)) \{
                if (!c.getPoints().contains(p2)) \{
                Cluster p2Cluster = null;
                for (final Cluster whereIsP2 : previousClusters) \{
                    if (whereIsP2.getPoints().contains(p2)) \{
                        p 2 Cluster \(=\) whereIsP2;
                    break;
                        \}
        \}
                for (final Point p2Point : p2Cluster.getPoints()) \{
                        if (!mergedCluster.getPoints().contains(p2Point)) \{
                        mergedCluster.getPoints().add(p2Point);
                \}
        \}
```

```
                    mergingCluster.add(p2Cluster);
                    p2Cluster.setMerged(true);
                    if (p2Cluster.getStableFactor() < mergingMinStableFactor) {
                                    mergingMinStableFactor = p2Cluster.getStableFactor();
                    }
            }
            }
    }
    ++j;
}
if (! clusters .containsKey(i)) {
    clusters.put(i, new ArrayList<Cluster>());
}
int connections = 0;
for (final Point p : mergedCluster.getPoints()) {
    if (pairs.containsKey(p)) {
        connections += pairs.get(p).size();
        System.out.println(p + "\t" + pairs.get(p).size());
    }
}
System.out.println(mergedCluster.getPoints().size() + " merged points");
final double newStableFactor = connections
    / Math.pow(mergedCluster.getPoints().size(), 2);
System.out.println(newStableFactor + " is newStableFactor");
System.out.println(mergingMinStableFactor
    + " is mergingMinStableFactor");
if (newStableFactor > mergingMinStableFactor
    || Math.abs(newStableFactor - mergingMinStableFactor) < 0.00001) {
    clusters .get(i).add(mergedCluster);
    mergedCluster.setStableFactor(newStableFactor);
    System.out.println("!!! MERGED");
} else {
    clusters .get(i).add(c);
    clusters .get(i).addAll(mergingCluster);
```

91 92

```
                    System.out.println("!!! ORIGINAL CLUSTERS RETAINED");
```

                    System.out.println("!!! ORIGINAL CLUSTERS RETAINED");
            }
            }
        }
        }
        for (final Cluster c : clusters .get(i)) {
        for (final Cluster c : clusters .get(i)) {
            c.setMerged(false);
            c.setMerged(false);
    }
    }
    }
    }
    return clusters;
    return clusters;
    }
}
public Map<Point, List<Point>> findPairs(final int k) {
public Map<Point, List<Point>> findPairs(final int k) {
final Map<Point, List<Point>> pairs = new HashMap<Point, List<Point }>>>()\mathrm{ ;
final Map<Point, List<Point>> pairs = new HashMap<Point, List<Point }>>>()\mathrm{ ;
for (final Point p1 : this.all) {
for (final Point p1 : this.all) {
for (final Point p2: this.nearestTo(p1, k)) {
for (final Point p2: this.nearestTo(p1, k)) {
if (this.nearestTo(p2, k).contains(p1) \&\& p1 != p2) {
if (this.nearestTo(p2, k).contains(p1) \&\& p1 != p2) {
if (! pairs.containsKey(p1)) {
if (! pairs.containsKey(p1)) {
pairs.put(p1, new ArrayList<Point>());
pairs.put(p1, new ArrayList<Point>());
}
}
pairs.get(p1).add(p2);
pairs.get(p1).add(p2);
}
}
}
}
}
}
return pairs;
return pairs;
}
}
@SuppressWarnings("boxing")
@SuppressWarnings("boxing")
private void findNearestAtMaxK() {
private void findNearestAtMaxK() {
for (final Point p : this.all) {
for (final Point p : this.all) {
final Map<Double, Point> distances = new HashMap<Double, Point>();
final Map<Double, Point> distances = new HashMap<Double, Point>();
for (final Point p2 : this. all ) {
for (final Point p2 : this. all ) {
if (p== p2) {
if (p== p2) {
continue;
continue;
}
}
distances.put(p.distanceTo(p2), p2);
distances.put(p.distanceTo(p2), p2);
}

```
        }
```

\}

$$
\text { final List }<\text { Double }>\text { sortedDistances }=\text { new ArrayList }<\text { Double }>(
$$ distances.size () ); sortedDistances.addAll(distances.keySet()); Collections.sort(sortedDistances); final List $<$ Point $>$ nearestPoints $=$ new ArrayList $<$ Point $>(100)$; for (final Double d: sortedDistances.subList(0, this.maxK)) \{ nearestPoints.add(distances.get(d));

            \}
            this.nearest.put(p, nearestPoints);
        \}
    \}
    private List<Point> nearestTo(final Point p, final int k) \{
        return this.nearest.get (p).subList ( \(0, \mathrm{k}\) );
    \}
    private final List \(<\) Point \(>\) all;
    private final int maxK;
    private final Map<Point, List<Point>> nearest;
    
## A. 2 kNN Demo Code

## Point.Java

```
package demo;
import java.awt.BorderLayout;
import java.awt.Color;
import java.awt.Graphics;
import java.awt.Graphics2D;
import java.awt.RenderingHints;
import java.awt.event.ActionEvent;
```


import java.awt.event.ActionListener; import java.awt.event.MouseAdapter; import java.awt.event.MouseEvent; import java.util.ArrayList; import java.util.HashMap; import java.util.List; import java.util.Map; import java.util.Random;
import javax.swing.JButton;
import javax.swing.JFrame;
import javax.swing.JLabel;
import javax.swing.JOptionPane;
import javax.swing.JPanel;
import javax.swing.JTextField;
import javax.swing.WindowConstants;
import algorithm.Algorithm;
import algorithm.Cluster;
import algorithm.Point;
public final class DemoFrame extends JFrame \{
public DemoFrame() \{
this. setTitle ("Knn Demo");
this. setSize (1024, 600);
this.setDefaultCloseOperation(WindowConstants.DISPOSE_ON_CLOSE);
this.setLayout(new BorderLayout());
this.canvas $=$ new $\operatorname{Canvas}()$;
this.controlPanel $=$ new ControlPanel(this.canvas);
this.getContentPane().add(this.controlPanel, BorderLayout.NORTH);
this.getContentPane().add(this.canvas, BorderLayout.CENTER);
\}
private final Canvas canvas;
private final ControlPanel controlPanel;
public static void main(final String[] args) \{
new DemoFrame().setVisible(true);
\}
private static final long serialVersionUID $=-2297653670935822145 \mathrm{~L}$;
\}
@SuppressWarnings("serial")
final class Canvas extends JPanel \{
public Canvas() \{
this.addMouseListener(new MouseAdapter() \{
@Override
public final void mousePressed(final MouseEvent e) \{
for (final Point p : Data.points) \{
if $($ p.distanceTo(new Point $(e . g e t X()$, e.getY ()$))<10)\{$
double sum $=0$;
double previous $=0$;
double giniIndex $=0$;
for (int $\mathrm{i}=1 ; \mathrm{i}<=$ Data.clusterK; ++i ) \{
final Algorithm $\mathrm{a}=$ new Algorithm(Data.points, i ;
final Map<Point, List<Point>> pairs $=$ a.findPairs(i);
if (pairs.containsKey $(\mathrm{p})$ ) \{
sum $+=$ pairs.get $(\mathrm{p}) \cdot \operatorname{size}()+$ previous;
JOptionPane.showMessageDialog(null, " (" + (int)p.getX()+
" , "+(int)p.get $Y()+$ ") " +"When $K=$ " $+\mathrm{i}+$ ", the MKNN number is "+ pairs.get
p). $\operatorname{size}())$;
previous=pairs.get(p). size ();
\}
else
JOptionPane.showMessageDialog(null, " (" + (int)p.getX()+
" , "+(int)p.get Y()$+$ ") " +"When $\mathrm{K}=\mathrm{"}+\mathrm{i}+\mathrm{"}$, the MKNN number is 0 ");
\}

```
                    giniIndex=1-sum/(Data.clusterK * Data.clusterK);
                    JOptionPane.showMessageDialog(null, "GINI-index: " + giniIndex);
                    break;
            }
            }
        }
    });
}
@SuppressWarnings("boxing")
@Override
public final void paintComponent(final Graphics g) {
    ((Graphics2D) g).setRenderingHint(RenderingHints.KEY_ANTIALIASING,
        RenderingHints.VALUE_ANTIALIAS_ON);
    g.setColor(new Color(0, 3, 97));
    g. fillRect (0, 0, this.getWidth(), this.getHeight());
    g.setColor(Color.white);
    for (final Point p : Data.pairs.keySet()) {
        for (final Point to : Data.pairs.get(p)) {
            g.drawLine((int) p.getX(), (int) p.getY(), (int) to.getX(),
                (int) to.getY());
        }
    }
    g.setColor(Color.yellow);
    for (final Point p : Data.points) {
        g. fillOval ((int) p.getX() - 2, (int) p.getY() - 2, 5, 5);
        g.drawString(p.toString(), (int) p.getX(),(int) p.getY());
    }
    if (Data.showClusters) {
        final Random r = new Random();
        if (Data.clusters.containsKey(Data.clusterK)) {
        for (final Cluster c : Data.clusters.get(Data.clusterK)) {
            g.setColor(new Color(r.nextFloat(), r.nextFloat(), r.nextFloat()));
```

```
                    for (final Point p : c.getPoints()) {
```

                    for (final Point p : c.getPoints()) {
                        g. fillOval ((int) p.getX() - 5, (int) p.getY() - 5, 10, 10);
                        g. fillOval ((int) p.getX() - 5, (int) p.getY() - 5, 10, 10);
                }
                }
            }
            }
            }
            }
        }
        }
    }
    }
    }
}
@SuppressWarnings("serial")
@SuppressWarnings("serial")
final class ControlPanel extends JPanel {
final class ControlPanel extends JPanel {
public ControlPanel(final Canvas canvas) {
public ControlPanel(final Canvas canvas) {
this.numberTextField.setText("15");
this.numberTextField.setText("15");
this.nearDistanceTextField.setText("50");
this.nearDistanceTextField.setText("50");
this.nearPercentageTextField.setText("20");
this.nearPercentageTextField.setText("20");
this.kTextField.setText("3");
this.kTextField.setText("3");
// this.clusterKTextField.setText("1");
// this.clusterKTextField.setText("1");
this.add(this.numberLabel);
this.add(this.numberLabel);
this.add(this.numberTextField);
this.add(this.numberTextField);
this.add(this.kLabel);
this.add(this.kLabel);
this.add(this.kTextField);
this.add(this.kTextField);
// this.add(this.clusterKLabel);
// this.add(this.clusterKLabel);
// this.add(this.clusterKTextField);
// this.add(this.clusterKTextField);
this.add(this.nearDistanceLabel);
this.add(this.nearDistanceLabel);
this.add(this.nearDistanceTextField);
this.add(this.nearDistanceTextField);
this.add(this.nearPercentageLabel);
this.add(this.nearPercentageLabel);
this.add(this.nearPercentageTextField);
this.add(this.nearPercentageTextField);
this.add(this.viewClusterButton);
this.add(this.viewClusterButton);
this.add(this.randomButton);
this.add(this.randomButton);
this.add(this.runButton);
this.add(this.runButton);
this.add(this.clearButton);
this.add(this.clearButton);
this.randomButton.addActionListener(new ActionListener() {
this.randomButton.addActionListener(new ActionListener() {
@SuppressWarnings({ "boxing", "synthetic-access" })
@SuppressWarnings({ "boxing", "synthetic-access" })
@Override

```
        @Override
```

```
    public final void actionPerformed(final ActionEvent e) \{
```

    public final void actionPerformed(final ActionEvent e) \{
        ControlPanel.clear () ;
        ControlPanel.clear () ;
        final Random \(\mathrm{r}=\) new Random();
        final Random \(\mathrm{r}=\) new Random();
        final double width \(=\) canvas.getWidth(), height \(=\) canvas.getHeight () ;
        final double width \(=\) canvas.getWidth(), height \(=\) canvas.getHeight () ;
        final int nearDistance \(=\) Integer
        final int nearDistance \(=\) Integer
            .valueOf(ControlPanel.this.nearDistanceTextField.getText()),
            .valueOf(ControlPanel.this.nearDistanceTextField.getText()),
    nearPercentage $=$ Integer
nearPercentage $=$ Integer
.valueOf(ControlPanel.this.nearPercentageTextField.getText());
.valueOf(ControlPanel.this.nearPercentageTextField.getText());
Point previous $=$ null;
Point previous $=$ null;
for (int $\mathrm{i}=0$; $\mathrm{i}<$ Integer.valueOf(ControlPanel.this.numberTextField
for (int $\mathrm{i}=0$; $\mathrm{i}<$ Integer.valueOf(ControlPanel.this.numberTextField
. $\operatorname{get} \operatorname{Text}()) ;++\mathrm{i})\{$
. $\operatorname{get} \operatorname{Text}()) ;++\mathrm{i})\{$
if (previous $==$ null) $\{$
if (previous $==$ null) $\{$
previous $=$ new Point(r.nextDouble() $*$ width, r.nextDouble()
previous $=$ new Point(r.nextDouble() $*$ width, r.nextDouble()
* height);
* height);
\} else \{
\} else \{
double $\mathrm{x}, \mathrm{y}$;
double $\mathrm{x}, \mathrm{y}$;
do \{
do \{
double d;
double d;
final double angle $=$ r.nextDouble() $* 2 *$ Math.PI;
final double angle $=$ r.nextDouble() $* 2 *$ Math.PI;
if (r.nextInt(101) $<=$ nearPercentage) $\{$
if (r.nextInt(101) $<=$ nearPercentage) $\{$
$\mathrm{d}=$ r.nextDouble() $*$ nearDistance;
$\mathrm{d}=$ r.nextDouble() $*$ nearDistance;
\} else \{
\} else \{
$\mathrm{d}=$ r.nextDouble( $) *$ width;
$\mathrm{d}=$ r.nextDouble( $) *$ width;
\}
\}
$\mathrm{x}=$ previous.getX( $)+\mathrm{d} *$ Math.cos(angle);
$\mathrm{x}=$ previous.getX( $)+\mathrm{d} *$ Math.cos(angle);
$\mathrm{y}=$ previous.get Y()$+\mathrm{d} *$ Math.sin(angle);
$\mathrm{y}=$ previous.get Y()$+\mathrm{d} *$ Math.sin(angle);
$\}$ while ( $\mathrm{x}<0 \| \mathrm{x}>$ width $-1\|\mathrm{y}<0\| \mathrm{y}>$ height -1 );
$\}$ while ( $\mathrm{x}<0 \| \mathrm{x}>$ width $-1\|\mathrm{y}<0\| \mathrm{y}>$ height -1 );
final Point $\mathrm{p}=$ new $\operatorname{Point}(\mathrm{x}, \mathrm{y})$;
final Point $\mathrm{p}=$ new $\operatorname{Point}(\mathrm{x}, \mathrm{y})$;
previous $=\mathrm{p}$;
previous $=\mathrm{p}$;
\}
\}
Data.points.add(previous);
Data.points.add(previous);
\}
\}
canvas.repaint ();
canvas.repaint ();
\}
\}
\});

```
\});
```

```
this.runButton.addActionListener(new ActionListener() {
    @SuppressWarnings({ "boxing", "synthetic-access" })
    @Override
    public final void actionPerformed(final ActionEvent e) {
        final Algorithm a = new Algorithm(Data.points, Integer
                .valueOf(ControlPanel.this.kTextField.getText()));
        Data.pairs = a.findPairs(Integer.valueOf(ControlPanel.this.kTextField
                .getText()));
        Data.clusterK = Integer.valueOf(ControlPanel.this.kTextField.getText());
        Data.showClusters = false;
        canvas.repaint();
    }
});
this.viewClusterButton.addActionListener(new ActionListener() {
    @SuppressWarnings({ "boxing", "synthetic-access" })
    @Override
    public final void actionPerformed(final ActionEvent e) {
        Data.clusterK = Integer.valueOf(ControlPanel.this.kTextField.getText());
        final Algorithm a = new Algorithm(Data.points, Integer
            .valueOf(ControlPanel.this.kTextField.getText()));
        Data.pairs = a.findPairs(Integer.valueOf(ControlPanel.this.kTextField
            .getText()));
        Data.clusters = a.findClusters();
        Data.showClusters = true;
        canvas.repaint();
    }
});
this.clearButton.addActionListener(new ActionListener() {
    @SuppressWarnings("synthetic-access")
    @Override
        public final void actionPerformed(final ActionEvent e) {
        ControlPanel.clear();
        canvas.repaint();
    }
});
```

```
N
```

```
    }
```

    }
    private final JLabel numberLabel = new JLabel("Number of points: "),
    private final JLabel numberLabel = new JLabel("Number of points: "),
        kLabel = new JLabel("K:"), clusterKLabel = new JLabel("Cluster K:"),
        kLabel = new JLabel("K:"), clusterKLabel = new JLabel("Cluster K:"),
        nearPercentageLabel = new JLabel("Near%: "),
        nearPercentageLabel = new JLabel("Near%: "),
        nearDistanceLabel = new JLabel("Near distance: ");
        nearDistanceLabel = new JLabel("Near distance: ");
    private final JTextField numberTextField = new JTextField(4),
    private final JTextField numberTextField = new JTextField(4),
        kTextField = new JTextField(2), nearPercentageTextField = new JTextField(
        kTextField = new JTextField(2), nearPercentageTextField = new JTextField(
            2), nearDistanceTextField = new JTextField(3),
            2), nearDistanceTextField = new JTextField(3),
        clusterKTextField = new JTextField(2);
        clusterKTextField = new JTextField(2);
    private final JButton randomButton = new JButton("Random"),
    private final JButton randomButton = new JButton("Random"),
        runButton = new JButton("Draw the link"), clearButton = new JButton("
        runButton = new JButton("Draw the link"), clearButton = new JButton("
        Clear"),
        Clear"),
        viewClusterButton = new JButton("Show Clusters");
        viewClusterButton = new JButton("Show Clusters");
    private static void clear() {
    private static void clear() {
        Data.points.clear ();
        Data.points.clear ();
        Data.pairs.clear ();
        Data.pairs.clear ();
        Data.clusters.clear ();
        Data.clusters.clear ();
    }
    }
    }
final class Data {
final class Data {
static {
static {
Data.points = new ArrayList<Point>();
Data.points = new ArrayList<Point>();
Data.pairs = new HashMap<Point, List<Point>>();
Data.pairs = new HashMap<Point, List<Point>>();
Data.clusters = new HashMap<Integer, List<Cluster>>();
Data.clusters = new HashMap<Integer, List<Cluster>>();
}
}
public static int clusterK;
public static int clusterK;
public static Map<Integer, List<Cluster>> clusters;
public static Map<Integer, List<Cluster>> clusters;
public static Map<Point, List<Point>> pairs;
public static Map<Point, List<Point>> pairs;
public static List<Point> points;
public static List<Point> points;
public static boolean showClusters;
public static boolean showClusters;
}

```

\section*{A. 3 Path \(k N N\) Query Search Simulation}
using System;
using System.Collections.Generic;
using System.ComponentModel;
using System.Data;
using System.Data.Odbc;
using System.Drawing;
using System.Text;
using System.Windows.Forms;
using System.Collections;
namespace IKNN
\{
    public partial class IKNNMain : Form
    \{
        private DataTable dtRoutine;
        private DataTable dtPassedRoutine;
        private DataTable dtSegment \(=\) new DataTable();
        private DateTime StartTime;
        private int intDmax;
        private struct Query
        \{
            public string strSP;
        public int intSPDistance;
        public string strEP;
        public int intEPDistance;
        public int intKpoints;
        \}
        public IKNNMain()
        \{
```

    InitializeComponent();
    LoadSegmentData();
    }
private void btnRun_Click(object sender, EventArgs e)
{
dtPassedRoutine = new DataTable();
dtRoutine = new DataTable();
intDmax = int.MaxValue;
Query myQ = new Query();
myQ.strSP = txtA.Text.Split(' , ')[0];
myQ.intSPDistance = Convert.ToInt32(txtA.Text.Split(' , ')[1]);
myQ.strEP = txtB.Text.Split(', ')[0];
myQ.intEPDistance = Convert.ToInt32(txtB.Text.Split(' , ')[1]);
myQ.intKpoints = Convert.ToInt32(txtK.Text);
DataColumn dcNode = new DataColumn("Node", Type.GetType("System.String"
));
dtRoutine.Columns.Add(dcNode);
DataColumn dcDistance = new DataColumn("Distance", Type.GetType("System.
Int32"));
dtRoutine.Columns.Add(dcDistance);
for(int i = 0; i < myQ.intKpoints; i++)
{
DataColumn dcWaypoint = new DataColumn("WP" + i, Type.GetType("
System.String"));
dtRoutine.Columns.Add(dcWaypoint);
}
DataRow myQueryRowA = dtRoutine.NewRow();
myQueryRowA["Node"] = myQ.strSP;

```
    myQueryRowA["Distance"] = myQ.intSPDistance;
    dtRoutine.Rows.Add(myQueryRowA);
    DataRow myQueryRowB \(=\) dtRoutine. \(\operatorname{NewRow}()\);
    myQueryRowB["Node"] = myQ.strEP;
    myQueryRowB["Distance"] \(=\) myQ.intEPDistance;
    dtRoutine.Rows.Add(myQueryRowB);
    int intDminRow \(=-1 ;\)
    int \(\operatorname{intK}=1\);
    StartTime \(=\) DateTime.Now;
    intDminRow \(=\) FindMinDistanceRow ()\(;\)
    GetLinkedSegments(intDminRow, myQ.intKpoints);
    while (intDminRow \(==-1 \| \operatorname{intDmax}>\) Convert.ToInt32(dtRoutine.Rows[
FindMinDistanceRow()]["Distance"]) || intDmax == int.MaxValue)
    \(\{\)
        intDminRow \(=\) FindMinDistanceRow ()\(;\)
        GetLinkedSegments(intDminRow, myQ.intKpoints);
        intK++;
    \}
    lstOutput.Items.Add(intK + " Time: " + DateTime.Now.Subtract(StartTime));
    //for (int \(i=0 ; i<d t\) Routine.Rows.Count \(; i++\) )
    //\{
    // lstOutput.Items.Add(dtRoutine.Rows[i]["Node"] + "" + dtRoutine.Rows[i]["
Distance"] + dtRoutine.Rows[i]["WP0"]);
    //\}
```

        dtRoutine.Clear();
    ```
        dtRoutine.Clear();
        dtPassedRoutine.Clear();
        dtPassedRoutine.Clear();
        dtRoutine.Dispose();
        dtRoutine.Dispose();
        dtPassedRoutine.Dispose();
        dtPassedRoutine.Dispose();
}
private void GetLinkedSegments(int intMinRowId, int intKpoints)
{
        for (int i = 0; i < dtSegment.Rows.Count; i++)
        {
            DataRow myRow = dtRoutine.NewRow();
        if (dtSegment.Rows[i]["StartPoint"].Equals(dtRoutine.Rows[intMinRowId]["
Node"]))
            {
            if (!isPassedRoutine(intMinRowId, intKpoints, i))
            {
                    myRow["Node"] = dtSegment.Rows[i]["EndPoint"];
                    UpdateWayPoints(myRow, intMinRowId, intKpoints, i);
            }
        }
        else if (dtSegment.Rows[i]["EndPoint"].Equals(dtRoutine.Rows[intMinRowId][
"Node"]))
            {
            if (!isPassedRoutine(intMinRowId, intKpoints, i))
            {
                    myRow["Node"] = dtSegment.Rows[i]["StartPoint"];
                    UpdateWayPoints(myRow, intMinRowId, intKpoints, i);
            }
        }
    }
    //lstOutput.Items.Add(dtRoutine.Rows[intMinRowId]["Node"].ToString() + " "+
    // dtRoutine.Rows[intMinRowId]["Distance"].ToString() + ""+
```

```
            // (dtRoutine.Rows[intMinRowId]["WP0"].Equals(DBNull.Value) ? "Null":
dtRoutine.Rows(intMinRowId]["WP0"].ToString()));
    if (dtPassedRoutine.Rows.Count \(<=0\) )
        \(\mathrm{dtPassedRoutine}=\mathrm{dtRoutine}\). Clone ()\(;\)
    DataRow myPassedRow \(=\) dtPassedRoutine.NewRow();
    for (int \(\mathrm{i}=0 ; \mathrm{i}<\mathrm{dtRoutine}\).Columns.Count; \(\mathrm{i}++\) )
    \{
        myPassedRow[i] \(=\mathrm{dtRoutine.Rows[intMinRowId][i];}\)
    \}
    dtPassedRoutine.Rows.Add(myPassedRow);
    dtRoutine.Rows.RemoveAt(intMinRowId);
    FilterDuplicateRoutines(intKpoints);
\}
private void UpdateWayPoints(DataRow myRow, int intMinRowId, int intKpoints, int
        i)
\{
            myRow["Distance"] = Convert.ToInt32(dtRoutine.Rows[intMinRowId]["Distance"
]) + Convert.ToInt32(dtSegment.Rows[i]["Length"]);
for (int \(\mathrm{j}=0 ; \mathrm{j}<\) intKpoints; \(\mathrm{j}++\) )
    \{
        myRow["WP" +j ] \(=\) dtRoutine.Rows[intMinRowId]["WP" +j\(]\);
    \}
        int intTmp;
        if (!int.TryParse(myRow["Node"].ToString(), out intTmp))
        \{
        for (int \(\mathrm{k}=0 ; \mathrm{k}<\operatorname{intKpoints;~} \mathrm{k}++\) )
        \{
            if (myRow["WP" +k ].Equals(DBNull.Value))
```

```
\({ }^{227}\)
```

```
            {
```

            {
                isNotSame = true;
                isNotSame = true;
            break;
            break;
            }
            }
            }
            }
            if (!isNotSame)
            if (!isNotSame)
            {
            {
            if (Convert.ToInt32(dtPassedRoutine.Rows[m]["Distance"])}<
            if (Convert.ToInt32(dtPassedRoutine.Rows[m]["Distance"])}<
    Convert.ToInt32(dtRoutine.Rows[intMinRowId]["Distance"]) + Convert.ToInt32(
Convert.ToInt32(dtRoutine.Rows[intMinRowId]["Distance"]) + Convert.ToInt32(
dtSegment.Rows[i]["Length"]))
dtSegment.Rows[i]["Length"]))
{
{
return true;
return true;
}
}
}
}
}
}
}
}
}
}
return false;
return false;
}
}
private void FilterDuplicateRoutines(int intKpoints)
private void FilterDuplicateRoutines(int intKpoints)
{
{
for (int i = 0; i < dtRoutine.Rows.Count; i++)
for (int i = 0; i < dtRoutine.Rows.Count; i++)
{
{
for (int j = i + 1; j < dtRoutine.Rows.Count; j++)
for (int j = i + 1; j < dtRoutine.Rows.Count; j++)
{
{
if (dtRoutine.Rows[i]["Node"].Equals(dtRoutine.Rows[j]["Node"]))
if (dtRoutine.Rows[i]["Node"].Equals(dtRoutine.Rows[j]["Node"]))
{
{
bool isNotSame = false;
bool isNotSame = false;
for (int k = 0; k < intKpoints; k++)
for (int k = 0; k < intKpoints; k++)
{
{
if (!dtRoutine.Rows[i]["WP" + k].Equals(dtRoutine.Rows[j]["WP" +
if (!dtRoutine.Rows[i]["WP" + k].Equals(dtRoutine.Rows[j]["WP" +
k]))

```
k]))
```

```
            isNotSame \(=\) true;
```

            isNotSame \(=\) true;
            break;
            break;
        \}
        \}
            \}
            \}
        if (!isNotSame)
        if (!isNotSame)
        \{
        \{
            if (Convert.ToInt32(dtRoutine.Rows[i]["Distance"]) \(>=\) Convert.
            if (Convert.ToInt32(dtRoutine.Rows[i]["Distance"]) \(>=\) Convert.
    ToInt32(dtRoutine.Rows[j]["Distance"]))
ToInt32(dtRoutine.Rows[j]["Distance"]))
$\{$
$\{$
dtRoutine.Rows.RemoveAt(i);
dtRoutine.Rows.RemoveAt(i);
$\mathrm{i}=\mathrm{i}>0$ ? $\mathrm{i}--: \mathrm{i} ;$
$\mathrm{i}=\mathrm{i}>0$ ? $\mathrm{i}--: \mathrm{i} ;$
$j=\mathrm{i}+1 ;$
$j=\mathrm{i}+1 ;$
\}
\}
else if (Convert.ToInt32(dtRoutine.Rows[i]["Distance"]) $<$
else if (Convert.ToInt32(dtRoutine.Rows[i]["Distance"]) $<$
Convert.ToInt32(dtRoutine.Rows[j]["Distance"]))
Convert.ToInt32(dtRoutine.Rows[j]["Distance"]))
\{
\{
dtRoutine.Rows.RemoveAt(j);
dtRoutine.Rows.RemoveAt(j);
\}
\}
\}
\}
else
else
\{
\{
for (int l $=0 ; 1<\operatorname{intKpoints} ; 1++$ )
for (int l $=0 ; 1<\operatorname{intKpoints} ; 1++$ )
\{
\{
if (!dtRoutine.Rows[i]["WP" +l$]$.Equals(dtRoutine.Rows[j]["WP
if (!dtRoutine.Rows[i]["WP" +l$]$.Equals(dtRoutine.Rows[j]["WP
$"+1])$
$"+1])$
\{
\{
if (dtRoutine.Rows[i]["WP" +1$]$.Equals(DBNull.Value))
if (dtRoutine.Rows[i]["WP" +1$]$.Equals(DBNull.Value))
\{
\{
if (Convert.ToInt32(dtRoutine.Rows[i]["Distance"])
if (Convert.ToInt32(dtRoutine.Rows[i]["Distance"])
$>=$ Convert.ToInt32(dtRoutine.Rows[j]["Distance"]))
$>=$ Convert.ToInt32(dtRoutine.Rows[j]["Distance"]))
\{
\{
dtRoutine.Rows.RemoveAt(i);
dtRoutine.Rows.RemoveAt(i);
$\mathrm{i}=\mathrm{i}>0$ ? $\mathrm{i}--: \mathrm{i} ;$
$\mathrm{i}=\mathrm{i}>0$ ? $\mathrm{i}--: \mathrm{i} ;$
$\mathrm{j}=\mathrm{i}+1 ;$

```
                                    \(\mathrm{j}=\mathrm{i}+1 ;\)
```

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## A. 4 Time Constraint Route Search Simulation

1
package undesignated;
import classes. Global;
import classes.Point;
import classes.QueryPoint;
import java. util .*;
import java. util . ArrayList;
public class Main \{
public static void main(String[] args) \{
// for (int $p=0 ; p<10 ; p++$ ) \{
generateRandomPoints();
createQueryPoint();
printPoints(Global.pointList);
ArrayList<Object> candidateNext=candidateNext(Global.queryPoint);
ArrayList<Integer $>$ visitedType $=$ new ArrayList $<$ Integer $>()$;
for (int $\mathrm{i}=0 ; \mathrm{i}<$ Global.typeList.size () ; $\mathrm{i}++$ ) \{
Global.TC.add(0.00);
\}
run(candidateNext,visitedType);
\}
public static void printObjects(ArrayList<Object> aObjects)\{
ArrayList<Point> pointSet=(ArrayList<Point>)aObjects.get(1);
printPoints(pointSet);
\}
public static void run(ArrayList<Object> candidateNext, ArrayList<Integer>
visitedType) $\{$
Global.TC.set(Global.level-1, Global.timeCost);
ArrayList $<$ Point $>$ candidateNext1 $=($ ArrayList $<$ Point $>$ )candidateNext.get(1);
for $($ int $\mathrm{i}=0 ; \mathrm{i}<$ candidateNext1.size ()$; \mathrm{i}++)\{$
if $($ Global. level $==1)\{$
if (getTravelTime(getDistance(candidateNext1.get(i),(QueryPoint)
candidateNext.get(3)))+ Global.timeCost<Global.Tmax)\{
Global.timeCost+=getTravelTime(getDistance(candidateNext1.get(i),(
QueryPoint)candidateNext.get(3)));\}
else \{
if $(\mathrm{i}==$ candidateNext1.size ()$-1)\{$

```
                    Global. level --;
            if (Global. level \(==0)\{\)
            System.exit(0);
            \}
            else\{
            Global.timeCost \(=\) Global.TC.get(Global.level-1);
            visitedType.remove(visitedType.size()-1);
            break;
            \}
            \}
            else continue;
            \}
            \}
            else\{
            if (getTravelTime(getDistance((Point)candidateNext.get(3), candidateNext1.get(
i)))+Global.timeCost<Global.Tmax) \(\{\)
            Global.timeCost+=getTravelTime(getDistance((Point)candidateNext.get(3),
candidateNext1.get(i)));\}
        else \{
            System.out.println("*****TimeCost is too large go back to super
            if \((\mathrm{i}==\) candidateNext1.size ()\(-1)\{\)
            Global. level--;
            if \((\) Global. level \(==0)\{\)
            System.exit(0);
            \}
            else\{
            Global.timeCost= Global.TC.get(Global.level-1);
            visitedType.remove(visitedType.size()-1);
            break;
            \}\}
            else continue;
    \}
    \}
```

```
visitedType.add(candidateNext1.get(i).getType());
Global. level + + ;
for(int \(\mathrm{k}=0 ; \mathrm{k}<\) visitedType.size ()\(; \mathrm{k}++)\{\)
System.out.println("****Visited Type**"+visitedType.get(k)+" ");
\}
if \((\) visitedType.size ()\(==\) Global.typeList.size ()\()\{\)
if (Global.timeCost<=Global.Tmax) Global.Tmax=Global.timeCost;
System.out.println("Tmax: "+Global.Tmax);
visitedType.remove(visitedType.size()-1);
Global. level --;
if \((\) Global. level \(==1)\)
Global.timeCost \(=0\);
else
Global.timeCost-=getTravelTime(getDistance((Point)candidateNext.get(3),
candidateNext1.get(i)));
if \((\mathrm{i}==\) candidateNext1.size ()\(-1)\{\)
Global. level --;
if \((\) Global. level \(==0)\{\)
System.out.println("The algorithm finish");
System.exit(0);
\}
else\{
Global.timeCost \(=\) Global.TC.get(Global.level -1 );
visitedType.remove(visitedType.size()-1);
break;
\}\}
else continue;
\}
ArrayList<Object> CN=new ArrayList<Object>();
CN=candidateNext(candidateNext1.get(i), visitedType, Global.timeCost, Global. level);
ArrayList<Point \(>\) temp \(=(\) ArrayList \(<\) Point \(>)\) CN.get(1);
```

```
        System.out.println("Global.level: "+Global.level);
        System.out.println("Global.typeList.size(): "+Global.typeList.size());
        if (temp.size() \(!=0\) )
        \{
            run(CN,visitedType);
            \}
        else
        \{
            Global. level--;
            Global.timeCost \(=\) Global.TC.get(Global.level -1 );
            visitedType.remove(visitedType.size()-1);
        \}
        if \((\mathrm{i}==\) candidateNext1.size ()\(-1)\{\)
            Global. level--;
            if \((\) Global. level \(==0)\{\)
            System.exit(0);
            \}
            else\{
            Global.timeCost \(=\) Global.TC.get(Global.level -1 );
            visitedType.remove(visitedType.size()-1);
            break; \(\}\)
            \}
        \}
\}
public static void generateRandomPoints ()\(\{\)
            int noPoints, noTypes \(=0\);
            Random \(\mathrm{r}=\) new Random();
            Global. \(\mathrm{mx}=8\);//input.nextInt();
            Global.my=8;//input.nextInt();
            noTypes=10;//input.nextInt();
            for (int \(\mathrm{i}=0 ; \mathrm{i}<\) noTypes \(; \mathrm{i}++)\{\)
        int hour \(=0\);
```

```
            double \(\min =0\);
```

            double \(\min =0\);
            hour \(=5\); //input.nextInt();
            hour \(=5\); //input.nextInt();
            \(\min =0.0+(\) double \() \mathrm{i} * 30.0 ; / /\) input.nextDouble();
            \(\min =0.0+(\) double \() \mathrm{i} * 30.0 ; / /\) input.nextDouble();
            Global.typeList.add(hour \(+\min / 60\) );
            Global.typeList.add(hour \(+\min / 60\) );
            noPoints=6;//input.nextInt();
            noPoints=6;//input.nextInt();
            for (int \(j=0 ; j<\) noPoints \(; j++)\{\)
            for (int \(j=0 ; j<\) noPoints \(; j++)\{\)
            int \(\mathrm{x}, \mathrm{y}=0\);
            int \(\mathrm{x}, \mathrm{y}=0\);
            \(\mathrm{x}=\mathrm{r}\).nextInt(Global.mx \()+1\);
            \(\mathrm{x}=\mathrm{r}\).nextInt(Global.mx \()+1\);
            \(\mathrm{y}=\mathrm{r}\). nextInt(Global.my) +1 ;
            \(\mathrm{y}=\mathrm{r}\). nextInt(Global.my) +1 ;
            Global.pointList.add(new Point(x,y,i));
            Global.pointList.add(new Point(x,y,i));
            \(\}\)
            \(\}\)
    \}
    \}
    \}
\}
public static void createQueryPoint()\{
public static void createQueryPoint()\{
int qx, qy, hour $=0$;
int qx, qy, hour $=0$;
double $\min =0$;
double $\min =0$;
$\mathrm{qx}=4 ; / /$ input.nextInt();
$\mathrm{qx}=4 ; / /$ input.nextInt();
$\mathrm{qy}=4 ; / /$ input.nextInt();
$\mathrm{qy}=4 ; / /$ input.nextInt();
hour $=4 ; / /$ input.nextInt();
hour $=4 ; / /$ input.nextInt();
$\min =30.0 ; / /$ input.nextDouble();
$\min =30.0 ; / /$ input.nextDouble();
Global.queryPoint=new QueryPoint(qx, qy, hour + min $/ 60$ );
Global.queryPoint=new QueryPoint(qx, qy, hour + min $/ 60$ );
Global.speed $=30.00 ; / /$ input.nextDouble();
Global.speed $=30.00 ; / /$ input.nextDouble();
\}
\}
public static double getDistance(Point x, Point y) \{
public static double getDistance(Point x, Point y) \{
return Math.sqrt(Math.pow(x.getX()-y.getX(),2) + Math.pow((x.getY()-y.getY()),2))
return Math.sqrt(Math.pow(x.getX()-y.getX(),2) + Math.pow((x.getY()-y.getY()),2))
\}
\}
public static double getDistance(Point x, QueryPoint y)\{
public static double getDistance(Point x, QueryPoint y)\{
return Math.sqrt(Math.pow(x.getX()-y.getX(),2) + Math.pow((x.getY()-y.getY()),2))
return Math.sqrt(Math.pow(x.getX()-y.getX(),2) + Math.pow((x.getY()-y.getY()),2))
\}

```
    \}
```

public static double getTravelTime(double distance)\{
return distance/Global.speed;
\}
public static void printPoints(ArrayList<Point> aSet)\{
for $($ int $\mathrm{i}=0 ; \mathrm{i}<\operatorname{aSet} . \operatorname{size}() ; \mathrm{i}++)\{$
aSet.get(i). printPoint();
\}
\}
public static ArrayList<Object> candidateNext(Point p, ArrayList<Integer> visitedType,
double timeCost, int level) \{
ArrayList<Integer> T=new ArrayList<Integer>();
for(int $\mathrm{i}=0 ; \mathrm{i}<$ Global.typeList.size(); $\mathrm{i}++$ ) T.add(i);
ArrayList<Point $>$ result=new ArrayList $<$ Point $>()$;
ArrayList<Integer> unvisitedType=new ArrayList<Integer>();
for (int $\mathrm{i}=0 ; \mathrm{i}<\mathrm{T} . \operatorname{size}() ; \mathrm{i}++)\{$
if $($ visitedType.indexOf(T.get $(\mathrm{i}))==-1)\{$ unvisitedType.add(T.get(i)) $;\}$
\}
double timeCon=100;
for(int $\mathrm{i}=0 ; \mathrm{i}<$ unvisitedType.size(); $\mathrm{i}++)\{$
double time=Global.typeList.get(unvisitedType.get(i));
if (time<timeCon)timeCon=time;
\}
for(int $\mathrm{i}=0 ; \mathrm{i}<$ unvisitedType.size (); $\mathrm{i}++)\{$
for $($ int $\mathrm{j}=0 ; \mathrm{j}<$ Global.pointList.size ()$; \mathrm{j}++)\{$
if (Global.pointList.get( j ) $\cdot \operatorname{get}$ Type ()$==$ unvisitedType.get(i)) \{
double distance=getDistance(Global.pointList.get(j),p);
if (getTravelTime(distance) $<=($ timeCon-timeCost-Global.queryPoint.
getQueryTime())\&\&touch(Global.pointList.get(j), unvisitedType, distance, timeCost))\{
if $($ getTravelTime(distance) $<=($ timeCon-timeCost-Global.queryPoint.
getQueryTime()))\{
result.add(Global.pointList.get(j));

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```
            }
```

            }
            }
            }
        }
        }
    }
    }
    ArrayList<Object> r=new ArrayList<Object>();
    ArrayList<Object> r=new ArrayList<Object>();
    r.add(level);
    r.add(level);
    r.add(result);
    r.add(result);
    r.add(timeCost);
    r.add(timeCost);
    r.add(p);
    r.add(p);
    printObjects(r);
    printObjects(r);
    return r;
    return r;
    }
public static boolean touch(Point p, ArrayList<Integer> unvisitedType, double
public static boolean touch(Point p, ArrayList<Integer> unvisitedType, double
pTimeCost, double timeCost){
pTimeCost, double timeCost){
for(int i=0; i<unvisitedType.size(); i++){
for(int i=0; i<unvisitedType.size(); i++){
double dmax=Math.sqrt(Global.mx` 2+Global.my }2)         double dmax=Math.sqrt(Global.mx` 2+Global.my }2)
for(int j=0; j<Global.pointList.size(); j++){
for(int j=0; j<Global.pointList.size(); j++){
if(Global.pointList.get(j).getType()==unvisitedType.get(i)){
if(Global.pointList.get(j).getType()==unvisitedType.get(i)){
double distance=getDistance(Global.pointList.get(j),p);
double distance=getDistance(Global.pointList.get(j),p);
if (distance<dmax)dmax=distance;
if (distance<dmax)dmax=distance;
}
}
}
}
if (getTravelTime(dmax)}>\mathrm{ (Global.typeList.get(unvisitedType.get(i))-timeCost-
if (getTravelTime(dmax)}>\mathrm{ (Global.typeList.get(unvisitedType.get(i))-timeCost-
pTimeCost-Global.queryPoint.getQueryTime())){return false;}
pTimeCost-Global.queryPoint.getQueryTime())){return false;}
}
}
return true;
return true;
}
public static ArrayList<Object> candidateNext(QueryPoint p){
public static ArrayList<Object> candidateNext(QueryPoint p){
ArrayList<Integer> unvisitedType=new ArrayList<Integer>();
ArrayList<Integer> unvisitedType=new ArrayList<Integer>();
for(int i=0; i<Global.typeList.size(); i ++) unvisitedType.add(i);
for(int i=0; i<Global.typeList.size(); i ++) unvisitedType.add(i);
ArrayList<Point> result=new ArrayList<Point>();

```
    ArrayList<Point> result=new ArrayList<Point>();
```


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