Modelling and Design of Transport Networks with Electric Vehicles

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ABSTRACT

Carbon-based emissions and greenhouse gases (GHG) are critical global issues, where transport sector is a significant contributor to GHG emissions in most countries. The automobile transport is the principal CO2 emitter. From the energy safety point of view, the transport sector as a whole is also exceedingly dependent on fossil oil and would probably be affected by any changes in energy resources. Therefore, changes to energy structure are in urgent need to reduce emissions and oil-dependence.

Battery Electric Vehicles (BEV) are deemed to be a solution as a type of new alternative fuel vehicles (AFV) to reduce GHG emissions, noise pollution and reliance on fossil. There are a number of studies on the BEV market potential, BEV characteristic, BEV charging facility location problem (CFLP) and BEV routing problem. Given the current scarce deployment of charging facilities and driving range limit, it is crucial for BEV drivers to choose the best route to fulfil their trips while satisfying the charging needs. Although there is an increase in current research in BEV CFLP and equilibrium network modelling with BEV, there are several unresolved methodological issues as well as the practical concerns of such models. This research aims at developing BEV charging facility location models while investigating stochastic equilibrium network models considering a mixed BEV and gasoline vehicle (GV) flows. This research aims to examine the interaction between BEV charging facility location and BEV equilibrium flows and the effects of BEV range limits on stochastic traffic assignment.

In order to accomplish this broader research goal, the study has defined four main research objectives: i) investigate the impact of BEV driving distance limit on BEV drivers' route choice behaviour and equilibrium BEV flow ii) explore the effects of a flow-dependent energy consumption rate and fixed battery capacity on equilibrium BEV flow iii) explore the applicable BEV charging facility location model that maximize the charging facility coverage with a limited financial budget, and iv) explore the location and configuration of battery swapping station (BSS) in the application of battery electric buses (BEBs). Each component is the focus of a thesis chapter where detailed research context, methodologies and the key findings are presented.

Before investigating the first objective, the key problem is to find the unique and distinctive behaviour of BEVs that distinguish it from GV. The driving range limit and scarcity of charging facilities are the main concerns at current stage. Public charging facilities are not considered in this model. The focus of the first objective is to investigate a general stochastic traffic assignment problem (STAP) model with mixed BEV and GV flows considering path distance constraints. It was found that a modified method of successive averages (MSA) can be applied to solve the model for both multinomial logit and multinomial probit loading. A feasibility check procedure is essential to ensure the feasibility of this problem owing to the fixed travel demand between each O-D pair. The BEV drivers would prefer physically short paths when their driving distance limit is low. The GV drivers would use those longer links with less BEVs to reduce their travel time.

The second objective is to investigate the effect of a flow-dependent link energy consumption on stochastic traffic assignment of BEVs. This task employed a more realistic assumption that BEVs' energy consumption rate is based on not only distance but also travel time which makes it a path-based flow-dependent general stochastic traffic assignment problem. A key contribution of this task is to enrich the STAP family with path-based constraints. A solution framework was proposed to solve this type of model.

The third task involved identifying the potential location of charging facilities to maximize the exposure to BEV drivers. The results suggest that the equilibrium traffic flows are affected by charging speed, range limit, and charging facilities' utility and that BEV drivers incline to choose the route with charging stations and less charging time.

The final objective investigates a new way of refuelling for BEBs: battery swapping. Four fundamental questions are answered: How many BSSs should there be? Where should they be? Which BEBs should they serve? How big should they be? Results shows that the battery capacity would affect the number of BSS to locate and the local charging system configuration mainly depends on the charger and battery costs.

In summary, this thesis provides a number of original contribution to knowledge in the field of transport network modelling by addressing important methodological issues as well as the consideration of practical application.

DECLARATION

This thesis contains no material which has been accepted for the award of any other degree or diploma at any university or equivalent institution and that, to the best of my knowledge and belief, this thesis contains no material previously published or written by another person, except where due reference is made in the text of the thesis.

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Date: 15/05/2018

LIST OF PUBLICATION

The following publications have arisen from the research reported in this thesis:

Journal Papers (Published)

- *Jing, Wentao, Yadan Yan, Inhi Kim, and Majid Sarvi. "Electric vehicles: A review of network modelling and future research needs." Advances in Mechanical Engineering 8, no. 1 (2016): 1687814015627981.*
- *Jing, Wentao, Inhi Kim, Mohsen Ramezani, and Zhiyuan Liu. "Stochastic traffic assignment of mixed electric vehicle and gasoline vehicle flow with path distance constraints." Transportation Research Procedia 21 (2017): 65-78.*
- *Jing, Wentao, Kun An, Mohsen Ramezani, and Inhi Kim. "Location Design of Electric Vehicle Charging Facilities: A Path-Distance Constrained Stochastic User Equilibrium Approach." Journal of Advanced Transportation 2017 (2017).*
- *Jing W, Ramezani M, An K, et al. Congestion patterns of electric vehicles with limited battery capacity[J]. PloS one, 2018, 13(3): e0194354-e0194354*

Journal Papers (Under Review)

- *Jing, Wentao, Inhi Kim, and Kun An (under review). "The capacitated battery swapping facility location problem with local charging system serving electric bus fleet." Submitted to Journal of Transport Geography*

Conference Papers (Under Review)

- *Jing, Wentao, Inhi Kim, and Kun An (under review). "The uncapacitated battery swapping facility location problem with localized charging system serving electric bus fleet." Submitted to International Symposium of Transport Simulation (ISTS'18) and the International Workshop on Traffic Data Collection and its Standardization (IWTDCS'18)*

THESIS INCLUDING PUBLISHED WORKS DECLARATION

I hereby declare that this thesis contains no material which has been accepted for the award of any other degree or diploma at any university or equivalent institution and that to the best of my knowledge and belief, this thesis contains no material previously published or written by another person, except where due reference is made in the text of the thesis.

This thesis includes three original papers published in peer reviewed journals. The core theme of the thesis is the modelling and design of transport network with electric vehicles. The ideas, development and writing up of all the papers in the thesis were the principal responsibility of myself, the candidate, working within the Institute of Transport Studies, Department of Civil Engineering under the supervision of Dr Inhi Kim and Dr Kun An. The inclusion of co-authors reflects the fact that the work came from active collaboration between researchers and acknowledges input into team-based research.

In the case of chapters 4 to 6, my contribution to the work involved the following:

Thesis Chapter	Publication title	Status	Nature and extent (%) of student's contribution	Co-author name(s) Nature and % of Co-author's contribution	Co-author(s) Monash student Y/N
4	Stochastic traffic assignment of mixed electric vehicle and gasoline vehicle flow with path distance constraints.	*published*	70%, design of the work, drafting the article	1) Inhi Kim, conception of the work 10% 2) Mohsen Ramezani, critical revision of the article 10% 3) Zhiyuan Liu, critical revision of the article 10%	No No No
5	Congestion patterns of electric vehicles with limited battery capacity	*published*	65%, design of the work, writing the first draft	1) Mohsen Ramezani, critical revision of the article, conception of the work, 15% 2) Kun An, critical revision of the article 10% 3) Inhi Kim, critical revision and final approval of the version to be published 10%	No No No
6	Location Design of Electric Vehicle Charging Facilities: A Path-Distance Constrained Stochastic User Equilibrium Approach.	*published*	65%, design of the work, writing the first draft	1) Kun An, critical revision of the article, 15% 2) Mohsen Ramezani, conception of the work 10% 3) Inhi Kim, critical revision and final approval of the version to be published 10%	No No No

I have not renumbered sections of submitted or published papers in order to generate a consistent presentation within the thesis.

Student signature: [redacted] **Date: 15/05/2018**

The undersigned hereby certify that the above declaration correctly reflects the nature and extent of the student and co-authors' contributions to this work. In instances where I am not the responsible author I have consulted with the responsible author to agree on the respective contributions of the authors

Main supervisor signature: [redacted] **Date: 15/05/2018**

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LIST OF ABBREVIATIONS

AC	Alternating Current
BEV	Battery Electric Vehicle
BEB	Battery Electric Bus
BPR	Bureau of Public Roads
BSS	Battery Swapping Station
CFLP	Charging Facility Location Problem
DC	Direct Current
DUE	Deterministic User Equilibrium
EB	Electric Bus
EV	Electric Vehicle
FLP	Facility Location Problem
GHG	Greenhouse Gases
GV	Gasoline Vehicle
GP	Gradient Projection
IIA	Independent and Irrelevant Alternatives
MIP	Mixed Integer Programming
MNL	Multinomial Logit
MNP	Multinomial Probit
MP	Mathematical Programming
MSA	Method of Successive Averages
NP	Non-deterministic Polynomial
PHEV	Plug-in Hybrid Electric Vehicle
SNL	Stochastic Network Loading
SO	Social Optimum
SSO	Stochastic Social Optimum
STAP	Stochastic Traffic Assignment Problem
SUE	Stochastic User Equilibrium

UE	User Equilibrium
VRP	Vehicle Routing Problem

PART I:

BACKGROUND, LITERATURE AND DATA

CHAPTER 1 INTRODUCTION

1.1 Overview

This thesis focuses on transport network modelling and designing with the emphasis on electric vehicles link flow prediction and charging facility location problem (CFLP) for both private electric vehicles and electric buses. The thesis seeks to understand the primary behaviour and characteristic of battery electric vehicles (BEV) that distinguishing BEV from gasoline vehicles (GV) and to predict the traffic network equilibrium flows when BEVs account for a given amount of total travel demand. Moreover, it develops general stochastic user equilibrium (SUE) models with flow-independent path distance constraints and flow-dependent battery capacity constraints in a traffic network, which accounts for BEV's behaviour and characteristic. The thesis further investigates the refuelling facility location problems for both private vehicles and public transport, namely BEV and electric buses (EB), in terms of their different characteristics. This introductory chapter presents the background to the thesis, followed by a description of the research aims and objectives. The last part of this chapter shows the organization of the thesis.

1.2 Background and motivations

Electric vehicle (EV) is a broad term that Any passenger vehicle with a battery component that provides the propulsion to the vehicle could be an EV. Generally, there are two types of propulsion designs to be found in passenger road vehicles today. The internal combustion engine (ICE) is the dominant design for propulsion. As we are moving towards an increased electrification of the road vehicles, an increased use of electric motors (EMs) is applied within passenger road vehicles. The different designs have been developed to foster the transition, including the hybrid electric vehicle (HEV), plugin hybrid electric vehicle (PHEV), the extended range electric vehicle (EREV), the battery electric vehicle (BEV) and the fuel cell electric vehicle (FCEV). The HEV and the PHEV use both an ICE and EM for the propulsion of the vehicle. However, an EREV uses the ICE only to charge the battery, which in turn propels the vehicle. One could argue that it is in essence a BEV, but uses additional means (i.e. a gasoline engine) to charge its batteries. The BEV makes fully use of an electric motor for its propulsion as well as the FCEV (Bakker 2011). In this thesis, BEV would be the main focal point including the electric buses (EB).

Many studies have been done over the years to recognize the value of EV: helping to solve the environment problem and easy their potential impact in transport sector, as well as governments and EV companies are encouraging the ownership of EV through economic incentives. However, at the initial stage of the market, a more wide-spread use of EV is still hindered by limited battery capacity and deployment of charging stations. And they do not mention that the capacity of these stations is typically quite limited and some of them are not open to the public (Jiang et al. 2013). On top of that, high purchase cost for EV, safety and liability concerns, long charging time, fuel cost and improvements of the competitors remain to be other barriers for EV adoption. It is obvious that the driving range limit inevitably adds a certain level of restrictions to BEV drivers' travel behaviours, at least in a long future period prior to the coverage of recharging infrastructures reaching a sufficient level.

The widespread adoption of BEVs calls for fundamental changes to the existing network flow modelling tools, which is for properly capturing changed behaviours and induced constraints in forecasting travel demands and evaluating transportation development plans. In the past, one of the most challenging aspects of traffic assignment research is the inability to adequately predict the link flows, in order to propose appropriate counter-measures or design transportation infrastructure, that will improve social resource distribution.

Researchers have attempted to investigate the equilibrium BEV flow regarding the growing BEV travel demand. Deterministic traffic assignment problem (TAP) in the context of EV is widely studied (Jiang,Xie 2014; Jiang et al. 2012; Jiang et al. 2013; Xie,Jiang 2016; Xie et al. 2017; Xu et al. 2017; Zheng et al. 2017). BEV drivers' route choice behaviour can potentially lead to more accurate assignment results by taking BEVs' driving and charging behaviour into account (Adler et al. 2014; Okan et al. 2014). Omission of drivers' imperfect knowledge of path travel cost may lead to biased estimates of the predicted BEV flows. Accounting for the travel cost perception errors improves the precision of the estimated equilibrium link flows. In other words, stochastic traffic assignment models have not been studied considering EVs and its limited driving range in the network with urban transportation planning to predict the EV flow patterns in the near future. The driving range limit and the lack of charging infrastructure are two main characteristics of EVs at the current stage. To our best knowledge, it remains unsolved about how to develop the general SUE traffic assignment model with path distance constraints as well as the corresponding solution algorithms. The main reason is that adding path distance constraints into Daganzo's model cannot yield an optimization one of the generalized SUE conditions, in spite of the successful application in modelling the generalized DUE conditions. It is believed that this thesis is the first study attempting to seek the solution to such a challenging problem.

Charging/swapping facility are regarded as an indispensable component for BEV network as gas stations for GV network. However, currently, the chicken and egg problem (Melaina 2003)—who would build and buy the BEVs if a refuelling infrastructure is not in place and who would build the refuelling infrastructure before the BEVs are built—remains the most intractable barrier. A more wide-spread use of EVs is still hindered by limited battery capacity, which allows cruising ranges of only 150 to 200 kilometres (Andreas et al. 2010) except for Tesla models. An extensive research exists on refuelling infrastructure problem, adopting different assumptions (fixed travel demand, simple distance-constrained vehicle routing, etc), various objectives (e.g. maximization of BEV service levels, minimization of total cost, etc) and different types of constraints. Most of them have not considered the equilibrium BEV flows in the transport networks. As part of the transportation electrification plan, EBs have received significant attention worldwide with the advance in battery and bus manufacturing technologies. Theoretically, EBs can travel up to 250 km. Various factors, including air conditioning, driving behavior, and battery aging issues can significantly reduce the EBs' operational range, often making EBs incapable of finishing a whole day's work without battery recharging (Li 2016). BSS are recommended as a promising strategy to eliminate barriers of long charging time and limited mileage range faced by EVs. Generally, there are two types of operation modes for BSSs in terms of the way of dealing with depleted batteries: central charging and local charging (Tan et al. 2014). To avoid the tedious battery shipping between BSSs and central charging stations and promote the development of BSSs for EBs, the optimal BSSs' location and its local charging system design should be investigated together. Overall, the deployment of charging facility should consider charging/swapping facilities not only for private BEVs but also for public EBs.

1.3 Problem statement

One of the major problems facing transportation engineers and urban planners is to predict the impact of given transportation scenarios. For a transport network with EVs, the amount of EV drivers' trip taking place at a given moment on any street in an urban area is the result of many EV drivers' decisions. EV drivers choose whether and when to take a trip, which mode of transportation to use, where to go, and which way to get there. In this thesis, the analytical part of this problem can be dealt with in two main stage. First, the scenarios are specified mathematically in the traffic assignment models as a set of inputs. which are used to predict the flow pattern resulting from such a scenario in transport planning. Second, the resulting flow pattern is used to calculate an array of charging facility location plans that characterize the scenario under study. In the second stage, the charging facility location plans use the flow pattern as a major input, especially for private BEVs.

1.3.1 Range anxiety and range limit

Many cities are planning the construction and expansion of charging infrastructures for BEVs. It is likely that BEV commuters will need to charge their vehicles at home most of the time due to the availability of public charging stations in the foreseeable future (Marrow et al. 2008). For many EVs, the current method of recharging the vehicle battery is to plug the battery into the power grid at places like the home or office(Bakker 2011; Kurani et al. 2008) , which requires an extended period of time to recharge before massive adoption of fast chargers and swapping facilities. However, it is much more costly to operate fast charging stations and it still takes much more time to recharge than a standard gasoline vehicle would take to refuel(Botsford,Szczepanek 2009). Due to the lack of standardization in batteries and its charging interfaces, BSSs are more suitable for buses and taxis rather than private vehicles (Zheng et al. 2014). These inherent problems, combined with a lack of refuelling infrastructure, are inhibiting a wide-scale adoption of EVs, especially apparent in longer trips, or inter-city trips. Range anxiety, when the driver is concerned that the vehicle will run out of charge before reaching the destination, and range limit are major hindrance for the market penetration of EVs(Yu et al. 2011; Jeeninga et al. 2002; Sovacool,Hirsh 2009; Mock et al. 2010). Thus, comparing with GVs, range limit and range anxiety need to be taken into consideration in stochastic traffic assignment models in order to effectively contribute to a more reliable BEV flow prediction and subsequent charging facility deployment. A major objective of this thesis is to investigate range limit and its impact on stochastic traffic assignment models. The range limits of BEVs are defined in two ways: flow-independent energy consumption (driving distance limit only) and flow-dependent energy consumption (energy consumption increases with the travel time). For the driving distance constraint, the usability of a path chosen by BEVs is independent of traffic flow and can be pre-determined. According to a study by Bigazzi,Clifton (2015), the traffic congestion affects the fuel economy of BEVs and BEVs may become more fuel efficient when the average speed increases. The insufficient public charging stations and its impact on STAP are also investigated.

1.3.2 General SUE models with side constraints

There has been a great interest among transport researchers in developing and using advanced choice models to represent EV drivers' route choice behaviour and their reaction and adaptation to different changes in the transport system in a sufficient accuracy. Modelling route choice behaviour, however, is one of the most challenging issues in travel demand analysis. The presence of a huge number of feasible alternative routes connecting each O-D pair in a typical transport network, as well as the fact that route characteristics, notably travel times, are dependent on users' behaviours and decisions, has made this one of the most challenging areas of transport engineering.

A probabilistic approach of network analysis has been originally developed to represent the uncertainties involved in modelling route choice behaviour including errors in perception, measurement and model specification. This class of stochastic models can potentially provide a more precise representation of behaviour through the more flexible modelling structure. The stochastic user equilibrium (SUE) model is well recognized in the literature. It relaxes the perfect information assumption of the deterministic user equilibrium model by incorporating a random error term in the route cost function to simulate travellers' imperfect perceptions of travel times. Route choice models, under this approach, have different specifications according to the modelling assumptions on the random error term. The two commonly-used random error terms are Gumbel and normal distributions, corresponding to the logit-based and probit-based route choice models, respectively (Dial 1971; Daganzo,Sheffi. 1977). In recent years, there has been a growing recognition of the advantages of path-based stochastic traffic assignment methods. It has been established that SUE models allow the adaptation of random-utility in the analysis of transport networks to address different behavioural aspects of travellers' decision. Comparing with GVs, the choice sets that EV drivers considered are considerably ambiguous (and potentially large) to the modeler in a route choice setting, but also the attributes of alternatives are subject to alteration according to decision makers' perception of EV's range, energy consumption and travel time, and hence, they could be determined through solution of a large-scale stochastic equilibrium mathematical problem instead of deterministic user equilibrium (DUE).

In addition, a considerable amount of research has formerly been conducted in DUE area with aforesaid constraints focusing on EVs' behaviours. Shortest path is commonly used in DUE to do all-or-nothing assignment, while advances in the efficiency of computer analysis have allowed modelers to generate and store path-flow variables explicitly in SUE. Having their own theoretical and computational challenges, the path set problems share a similar concern about how to produce manageable-sized and heterogeneous subsets from universal sets of alternatives which include the actual competitor paths mostly considered by EV travellers while excluding the irrelevant paths or infeasible paths which are rarely considered by EV users. This study will investigate the influence of the size of generated path sets on the outcome of SUE flow.

Side constraints are usually introduced for refining a descriptive or prescriptive traffic equilibrium assignment model. There are several diverse reasons for side constraints, such as describing the effects of traffic control policies, describing flow restrictions and improving an existing traffic equilibrium model for a given application by introducing further information about the traffic flow situation at hand. Various side constraints have been introduced to DUE models (Larsson,Patriksson 1999; Yang,Huang 2005) and the corresponding solution algorithms have been extensively investigated. In this study, we show that the side constraints regarding BEVs are range limit constraints in essence. The constrained problem is equivalent to an SUE model with travel cost functions properly adjusted to consider the range limit through the side constraints. Although the SUE principle plays a role as same as the DUE principle in describing drivers' route choice behaviour, the general SUE TAP has received little attention due to the complexity of general SUE problem. It has been pointed out that directly adding side constraints into the well-known minimization model for probit-based SUE problem does not give us an equivalent minimization model to the probit-based SUE traffic assignment with side constraints (Meng,Liu 2011). The modelling technique developed by (Meng et al. 2008) remains to be the sole model to address the general SUE TAP with link capacity constraints. To deal with BEVs' range limit, the proper SUE TAP models are explored to address the battery capacity issues and insufficient public charging stations.

1.3.3 Charging/swapping facilities

Lastly, there are three levels of EV chargers, which are categorized by voltage and power levels: Level I is 120V alternating current (AC) up to 20A (2.4kW), Level II is 240V AC up to 80A (19.2kW), and Level III (which is yet to be defined fully) will likely be 240V AC and greater at power levels of 20-250kW. Level I and Level II charging can be referred to as slow charging. A Level III connector are DC fast charger (SAE J1772 2010). Slow charging usually takes hours to refuel whereas fast charging may only need less than 20 minutes to charge a depleted battery. According to Huang et al. (2016), the charger costs is from \$1,000 to \$100,000 depending on the charging speed. One should weigh the costs, charging efficiency, battery life and other factors in choosing the charging method. BSS, which removes depleted batteries on board and replace the batteries with fully charged ones, is an alternative strategy to reduce charging time and range anxiety (Avci et al. 2014). The most outstanding feature of this strategy is that BSSs can complete the swapping process in less than 10 minutes. The depleted battery can be left overnight to get charged at a discounted electricity price. However, due to the lack of standardization in batteries and its charging interfaces, BSSs are more suitable for buses and taxi rather than private vehicles (Zheng et al. 2014). Different charging methods and charging equipment are suitable for different EV types. Charging/swapping facilities deployment for both private BEVs and public EBs are investigated in this thesis using different modelling techniques.

1.4 Research objectives

The broad aim of this thesis is to develop stochastic traffic assignment models for BEV network equilibrium analysis and charging facility location models for BEV and BEB whilst considering the BEVs' behaviour and the charging facilities characteristics. To achieve this research aim, a few specific objectives are outlined as follows:

1. Investigating the primary behaviour and characteristic of BEV that distinguish BEV from GV
2. Developing a general SUE model with flow-independent path distance constraints in a mixed BEV and GV flow network accounting for BEV's behaviour and characteristic
3. Extending the SUE model with battery capacity constraints that flow-dependent path energy consumption depends on both travel time and travel distance
4. Developing charging facility location model that maximize the coverage with limited budget
5. Developing a new model for BSS location problem with local charging system to serve BEB fleet

1.5 Contributions of this thesis

In response to four research gaps associated with the STAP models with BEVs and location problems of the battery charging/swapping facility serving BEVs and BEBs, this thesis makes eight original contributions to knowledge:

Methodological and algorithmic developments:

- New general SUE model considering flow-independent BEVs' driving distance constraints.
- New method for solving the proposed general SUE model with driving distance constraints of BEVs.
- New general SUE model with flow-dependent battery capacity constraints accounting for a more reasonable battery consumption based on both distance and travel time.
- New methodology for solving general SUE model with limited battery capacity of BEVs.
- New charging facility location model considering a SUE BEV flow pattern and charging facility deployment.
- New method for solving the proposed bi-level SUE-based BEV charging facility location problem.
- New swapping facility model considering local charging system serving BEB fleet.

- An understanding of the effects of various factors in the swapping facility system.

1.6 Organization of the thesis

Figure 1-1 presents the structure of this thesis. The thesis is divided into four parts and is made up of eight chapters.

- Part I (Chapters 1-3) covers background, literature and methodology,
- Part II (Chapters 4-5) consists of the development of stochastic traffic assignment models with BEVs,
- Part III (Chapters 6-7) discusses refuelling facility location models for both BEVs and EBs, and
- Part IV (Chapter 8) includes thesis synthesis and conclusions.

Part I of the thesis is made up of three chapters. Chapter 1 introduces the background , the research aims and the objectives and the overall structure of the thesis. Chapter 2 presents a summary review of literature on previous traffic assignment studies of BEVs and charging facility location problem. This chapter reviews the various issues in current EV charging network design studies and presents the gaps or limitations in the literature which are addressed by the thesis in the later chapters. In Chapter 3, there is a brief overview of the research methodology used this study.

Part II of the thesis focuses on the development of SUE traffic assignment model of BEVs with limited range limit using advanced techniques such as the modified MSA, Lagrangian dual, gradient projection and column generation. These models are developed because they are able to address drivers' perception errors, and path choice limitation. Chapter 4 develops SUE model with driving distance limit in a network with mixed GV and BEV flows. In Chapter 5, the issue of range limit is further defined by BEVs' battery capacity. The energy consumption is based on not only distance travelled but also time consumed. A more general SUE model is developed to address the impact of the flow-dependent battery consumption rate. Chapter 4 & 5 include published materials from paper 1 and 2 respectively.

Part III concentrates on the development of charging facility models. Chapter 6 investigates the deployment of charging facility for BEVs to maximize the coverage of BEV flows. The lower level problem is to address SUE BEV flow pattern considering the deployment plan in the upper level. Chapter 6 includes published materials from paper 3. Chapter 7 focuses on the application of charging facility location model for electric buses fleet and the understanding of the operating EBs with BSS as a likely replacement for conventional diesel bus.

Part IV, Chapter 8 is the concluding chapter of the thesis and provides a summary of key findings, implications, limitations, and directions for future research.

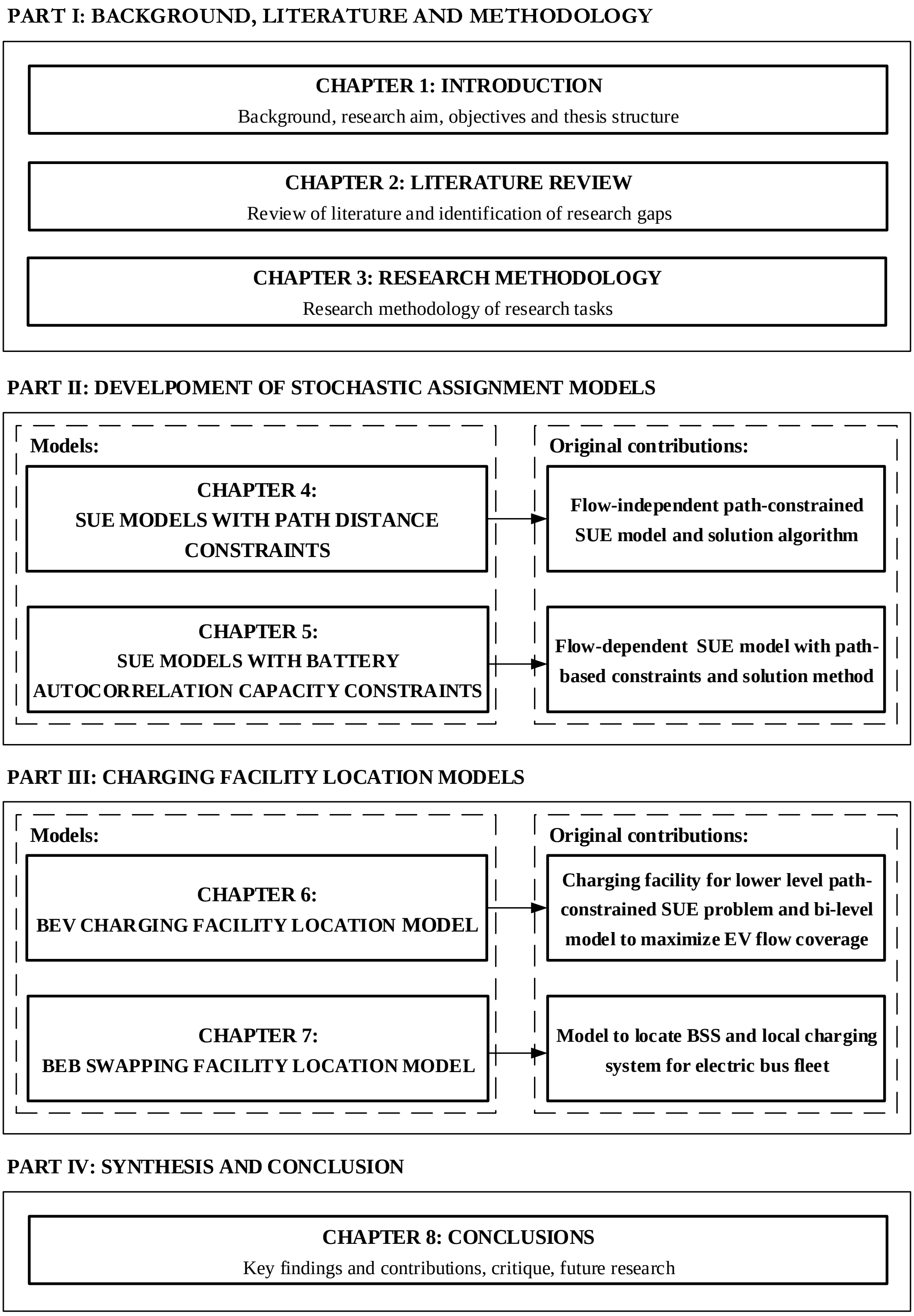

Figure 1-1: Thesis structure

CHAPTER 2 LITERATURE REVIEW

Part of the material in this chapter is from the peer-reviewed journal paper: *Jing, Wentao, Yadan Yan, Inhi Kim, and Majid Sarvi. "Electric vehicles: A review of network modelling and future research needs." Advances in Mechanical Engineering 8, no. 1 (2016): 1687814015627981.*

2.1 Introduction

The aim of this chapter is to provide a review of the existing literature on modelling and designing of transport networks with electric vehicles and their combination effects. Considering that the focus of this thesis are the EV flow prediction and EV refuelling facility deployment identified in Section 1.3, the objectives of this review are to provide the understanding of:

- EV market potential, demand & behaviour study (Section 2.2)
- Deterministic traffic assignment problem of vehicles with range limit (Section 2.3)
- Methodological issues of stochastic traffic assignment problem with side constraints (Section 2.4)
- BEV Charging stations location problem studies (Section 2.5.1)
- BEB swapping station location problem studies (Section 2.5.2)

This chapter is organised in accordance with these objectives. It concludes with a summary of gaps in knowledge identified through this literature review. Opportunities to advance knowledge in addressing these gaps are also discussed, which are then addressed in the following chapters of the thesis.

2.2 EV market potential, demand & behaviour study

Battery electric vehicles (BEVs), as one of the alternative fuel vehicles (AFVs), are believed to be a solution, for alternative fuels are addressed as a new fuel choice to reduce GHG emissions (OECD-ITF Joint Transport Research Centre 2008). However, a more wide-spread use of EVs is still hindered by limited battery capacity, which allows cruising ranges of only 150 to 200 kilometres (Andreas et al. 2010). Currently, the chicken and egg problem (Melaina 2003)—who will build and buy the AFVs if a refuelling infrastructure is not in place and who will build the refuelling infrastructure before the AFVs are built—remains the most intractable barrier.

It is obvious that the driving range limit inevitably adds a certain level of restrictions to battery electric vehicle (BEV) drivers' travel behaviours, at least in a long future period prior to the coverage of recharging infrastructures reaching a sufficient level. The widespread adoption of plug-in electric vehicles (PEVs) calls for fundamental changes to the existing network flow modelling tools for properly capturing changed behaviours and induced constraints in forecasting travel demands and evaluating transportation development plans (Jiang et al. 2013).

Smart et al analysed the BEV drivers (Nissan LEAF) drove 6.9 miles per trip and 30.3 miles per day on average, and the average number of charge times per day were 1.05 per day. Besides that, 82% of charging events were conducted at home(Smart,Schey 2012). Chargers and associated cords are categorized by voltage and power levels: Level I is 120V alternating current (AC) up to 20A (2.4kW), Level II is 240V AC up to 80A (19.2kW), and Level III (which is yet to be defined fully) will likely be 240V AC and greater at power levels of 20–250kW. The SAE J1772 standard defines a five-pin configuration that will be used for Level I and Level II charging. A Level III connector and the use of the current connector for direct current (DC) power flow are under development (SAE J1772 2010). Markel summarized the components of the PEV infrastructure, challenges and opportunities related to the design and deployment of the infrastructure and the potential benefits (Markel 2010). Dong,Lin (2014) proposed a stochastic modelling approach to characterize BEV drivers' behaviour using longitudinal travel data to account for more realistic analysis of the charging station impact on BEV feasibility. The actual range of a BEV is regarded as a Weibull-distributed variable and between-charge travel distances is represented by Poisson-gamma distribution. Hidrue et al. (2011) used a stated preference study to analyse customers' willingness to pay for EVs and their attributes, showing that driving range, fuel cost savings and charging time lead in importance and battery cost must drop significantly before EVs will find a mass market without subsidy. He et al. (2013). proposed a model that captures the interactions among availability of public charging opportunities, prices of electricity and destination and route choices of PHEVs.

2.3 Deterministic traffic assignment problem of vehicles with range limit

2.3.1 DUE of EVs with range limit

Traffic assignment is in general characterized as an uncapacitated nonlinear multi-commodity network flow problem under some given optimal or equilibrium routing principle. It has long been recognized as the last step of the traditional four-step travel demand modelling process and widely used an evaluation tool for a variety of urban and regional traffic network analyses (Xie,Waller 2012). Since Beckmann et al. (1956) first devised a set of nonlinear programming formulations for the TAPs with the first and second Wardropian principles, various types of traffic assignment models have been developed in past decades. These TAPs have been written as optimization programs, variational inequalities, complementarity systems, or fixed-point problems (Patriksson 1994; Florian,Hearn 1995). It has been shown by Beckmann et al. (1956) that the link traffic flow pattern in agreement with the UE principle could be uniquely determined by solving a convex mathematical program, if link travel times on a road network are separable/integrable, convex and monotonically increasing functions of link traffic flows. Estimating these link cost functions (or link performance functions) is a non-trivial task that involves choosing appropriate functional forms and calibrating corresponding parameters. Most link performance functions used in practice, including the well-known Bureau of Public Roads

(BPR) function, are polynomials whose degree and coefficients are specified from statistical analysis of real data (Nie et al. 2004).

In addition to Beckmann et al.'s classic formulations [i.e., deterministic user-equilibrium (DUE) and deterministic system-optimal (DSO) models], the Wardropian principles have also been extended to stochastic problems represented by optimization programs, including the stochastic user-equilibrium (SUE) model (Sheffi,Powell 1982) and the stochastic system-optimal (SSO) model (Maher et al. 2005). More general elastic-demand cases of these TAPs as optimization programs have also been proposed, such as the DUE problem with elastic demand (Beckmann et al. 1956) and the SUE problem with elastic demand (Maher 2001). Xie,Waller (2012) first presented an alternative common optimization formulation that can be used to represent each of TAPs (DUE, DSO, SUE and SSO), if a proper specification of its cost and cost variance terms is given.

It is well recognized that the standard TAP can be solved efficiently with a Frank-Wolfe type algorithm. The existing TAP models should be modified to better describe commuters' behaviour with the prevalence of BEVs. There have been many endeavors to address this problem. Among which, some studies enforce flow of a path to be zero if the path distance is greater than the driving range limit of BEVs. The classic Frank-Wolfe method with a constrained shortest path algorithm can be applied to solve this problem under DUE (Jiang et al. 2012). As an extension of static path distance constraint, stochastic range anxiety resulting in stochastic path distance constraint has been considered in networks (Xie et al. 2014; Xie et al. 2017; Wang et al. 2016). Network equilibrium problem is further addressed when modelling transportation networks that accommodated both gasoline vehicles (GVs) and BEVs (Jiang,Xie 2014; Jiang et al. 2013; Xu et al. 2017). A multi-class dynamic user equilibrium model is proposed to evaluate the performance of the mixed traffic flow network, where GVs chose paths with minimum travel time and BEVs selected paths to minimize the generalized costs including travel time, energy cost and range anxiety cost. It is also pointed out that the BEV energy consumption rate per unit distance travelled is lower at moderate speed than at higher speed resulting in an equilibrium that BEVs choose paths with lower speed to conserve battery energy (Agrawal et al. 2016). Relay/charging requirement has been taken into account in network equilibrium problems and is formulated as a nonlinear integer programming (Xie,Jiang 2016). It is found that traffic congestion would affect fuel economy of BEVs and BEVs might become more fuel-efficient as the average speed increases, particularly at local arterials (Bigazzi,Clifton 2015). Hence, another work considered recharging time based on flow-independent energy consumption in the base network equilibrium model and further extended the proposed DUE model with flow-dependent energy consumption assumption (He et al. 2014).

2.3.2 Shortest path problem with range limit

It is well recognized that the standard TAP can be solved efficiently with a Frank-Wolfe type algorithm in which the linearized sub-problem is finding shortest paths for each OD pair. The problem of finding the shortest path for an EV was originally discussed by Ichimori et al. (1981), where a vehicle has a limited battery and is allowed to stop and recharge at certain locations. Lawler (2001) sketched a polynomial algorithm for its solution. Adding refuelling stations to the shortest weight-constrained path problem (SWCPP), which is known to be NP-Complete (Desrochers,Soumis 1989; Desrosiers et al. 1984), has been discussed by Laporte,Pascoal (2011) and Smith et al. (2012), and, since the fuel and length components of the arcs are not related, the problem is still NP-hard.

Numerous works have addressed the classical vehicle routing problem (VRP) with capacity and distance constraints (Laporte et al. 1985.). Erdoğan,Miller (2012) extended the VRP to accounts for the additional challenges associated with operating a fleet of AFV considering the driving range limit as well as the limited refuelling infrastructure. In SWCPP, there are typically two independent measures such as cost and time associated with a path (Desaulniers,Villeneuve 2000; Ahuja et al. 2002). Kobayashi et al. (2011) and Siddiqi et al. (2011) further included battery recharging stations in their models and proposed heuristic techniques as solution methodologies. Ryan,Miguel (2011) introduced the so-called recharging vehicle routing problem where vehicles with limited range are allowed to recharge at customer locations mid-tour. Okan et al. (2014) introduced the minimum cost path for PHEV in a network with refuelling and battery switching stations, considering electricity and gasoline as sources of energy with different cost structures and limitations. Adler et al. (2014) proposed an EV shortest-walk problem to determine the shortest travel distance route which may include cycles for detouring to recharging batteries from origins to destinations with minimum detouring. Besides, it improved on that work by adding a limit to the number of times the vehicle can stop. Cabral et al. (2007) studied the network design problem with relays (NDPR) on an undirected graph, which generalized the shortest path problem with relays and the weight constrained shortest path problem, trying to minimize the total edge costs plus relay costs. The length between two consecutive relays would not exceed a pre-set upper bound. The problem of energy efficient routing of EV has been addressed and polynomial time algorithms have been developed in the literature by considering limited cruising range and regenerative breaking (i.e. the EV increases its level of energy when breaking) capabilities of EV which is actually a special case of the constrained shortest path problem (Andreas et al. 2010; Sachenbacher et al. 2011; Eisner et al. 2011).

Several articles addressed the minimum cost path problem of conventional vehicles (MCPP-CV) in the literature (Lin 2008a, b; Lin et al. 2007; Khuller et al. 2007; Suzuki 2008, 2009, 2012). If each arc required an amount of fuel that did not depend on the length of the arc, and the goal is to find the shortest path constrained on the amount of fuel used (and the vehicle cannot stop to refuel), then the problem is exactly the shortest weight-constrained path problem (Garey,Johnson 1979). This problem is NP-hard and has been discussed extensively in the literature (Handler,Zang 1980; Beasley,Christofides 1989; Desrochers,Soumis 1988; Xiao et al. 2005). Some new logistical problems which were relevant to the design and operations of a fleet of EV vehicles operating within a battery-exchanging infrastructure were discussed from an operations research perspective (Mirchandani et al. 2014). Table 2-2 is the summary of the relevant research.

2.4 Stochastic traffic assignment problem with side constraints& path-based Algorithm

2.4.1 Models and solution algorithms for the general SUE problem

The SUE model is well known in the literature. It relaxes the perfect information assumption of the DUE model. This assumption is unrealistic even if the users have a long-term experience about the network conditions, due to the daily variations of travel times and the diversity from users' sense of time. A well-known breakthrough on this issue was made by (Daganzo,Sheffi. 1977),where a random error term was incorporated in the route cost function to simulate travellers' imperfect perceptions of travel times . Travellers' perceived travel times equal to the actual travel time plus a multivariate random variable. The route choice models, under this approach, can have different specifications according to the modelling assumptions on the random error term. The two commonly used random error terms are Gumbel and normal distributions, corresponding to the logit-based and probit-based route choice models, respectively (Dial 1971; Daganzo,Sheffi. 1977). The travellers would choose the route with minimal perceived travel time. In this work, a conceptual framework of general SUE problem as well as stochastic network loading (SNL) is provided. Regarding the equivalent mathematical model for the general SUE problem, (Daganzo 1982) provided an unconstrained convex optimization model which requires calculating the inverse travel time functions and are computationally demanding. (Sheffi,Powell 1982) therefore transformed this model and developed an convergent solution algorithm to solve the proposed model (MSA) (Powell,Sheffi 1982) which is much easier in terms of computation. Convergence of the MSA type algorithms are usually proven by virtue of the Blum's theory (Daganzo 1983; Cantarella 1997). Another efficient solution algorithm called Stochastic Assignment Method (SAM) was developed by Maher,Hughes (1997) which adopts the Clark's approximation to calculate the objective function.

The logit-based route choice model has a closed-form probability expression, and the equivalent mathematical programming (MP) formulation can be formulated with an entropy-type model for the logit-based SUE problem (Fisk 1980). The choice probability of logit-based SUE merely depends on the cost difference between two paths. The logit-based SUE has an inherent defect which is known as Independent and Irrelevant Alternatives (IIA) property despite of its computational advantages. That depends on the differences of travel time only and is insensitive to network tepology (Sheffi 1985). Probit-based SUE takes into account the correlation of the travel costs on different paths, thus overcomes the IIA problem. Therefore, probit-based model has better representativeness to the practical conditions and it is a superior representativeness of the SUE problems. However, despite these robust characteristics, no closed form can be provided for the choice probability of probit-based problem, thus it is approximately solved by two types of methods: analytical approximation methods and Monte Carlo simulation-based methods. In this study, Monte Carlo simulation would be used for probit-based SNL.

Compared with logit-based SUE, the research for probit case is quite limited in recent years(Liu,Meng 2013; Meng,Liu 2011; Meng,Liu 2012).

The aforementioned models and algorithms for general SUE problem are effective for both logit-based and probit-based SUE problem. In this thesis, models of general SUE problem are of my interests and general SUE models with path-based constraints have been developed to fill the research gaps.

2.4.2 Path-based algorithms for solving SUE problem

As claimed above, various solution algorithms have been proposed to solve either logit-based or probit-based SUE problems. Early algorithms developed to solve the logit-based SUE problem were link-based [e.g.(Maher 1998)], These link-based algorithms do not require path storage and often use Dial's STOCH algorithm or Bell's alternative as the stochastic loading step (Bell 1995a; Dial 1971). This algorithm only covers those "reasonable" routes which take the drivers farther from the origins and closer to the destinations. Path-based algorithms require explicit path storage to directly compute the logit route choice probabilities. Olof et al. (1996) developed a path-based algorithm based on the disaggregated simplicial decomposition algorithm to solve the MNL SUE problem. Bekhor,Toledo (2005). compared path-based algorithms for the MNL SUE problem, and showed that the disaggregated simplicial decomposition algorithm is superior to the path-based MSA algorithm.

Xu et al. (2012) investigated different strategies for determination of step size of the path-based algorithms developed to solve the C-logit SUE models based on an adaptation of the GP method. Three strategies were investigated: (a) predetermined step size(Nagurney,Zhang 1996), (b) Armijo line search (Larry 1966; Bertsekas 1976), and (c) self-adaptive line search. The predetermined step size circumvents the difficulty in line search but also brings an inferior sub-linear convergent speed. The solution procedure of the general GP algorithm to solve the C-logit SUE problem is provided. The self-adaptive step size strategy was originally proposed by He et al. (2002) for the Goldstein–Levitin–Polyak projection algorithm. Recently, Chen et al. (2012). adopted this strategy in the GP algorithm to solve the non-additive traffic equilibrium problem. The main idea of this strategy is to determine a suitable step size automatically from the information derived from previous iterations. This strategy is reminiscent of Bertsekas's generalized Armijo rule. However, it is more practical and robust since the step size sequence is allowed to be non-monotone. A particular strategy for step size determination is not specified. It is found the GP algorithm with the self-adaptive step size strategy performs better than other step size determination strategies. The MSA strategy has a fast convergence in the early iterations. However, it cannot achieve an accurate solution within an acceptable computational budget because of the sub-linear convergence rate. The Armijo strategy is a widely used inexact line search strategy. However, it always starts from a fixed initial step size, which is nontrivial to choose without a priori knowledge. The quality of initial step size thus strongly affects the algorithmic performance. In contrast, the self-adaptive step size strategy adjusts the next initial and, consequently, the next acceptable step sizes according to the previous iterative information. This treatment permits the initial and acceptable step size sequences to be non-monotone (i.e., to decrease as well as increase). This mechanism makes the algorithm insensitive to the initial step size setting, thereby guaranteeing the robustness and efficiency of the algorithm (Xu et al. 2012).

Thus, different strategies can be embedded. To have a fair comparison of different step size strategies, a working route set is used. This set could be obtained from a route choice set generation algorithm (Bekhor et al. 2006). Behaviourally, it had the advantage of explicitly identifying those routes that were most likely to be used and also allowed greater flexibility to include route-specific attributes that might not be obtainable directly from the link attributes(Cascetta et al. 1997; Bekhor et al. 2006).A column generation procedure could also be readily embedded in the GP algorithm (Chen,Jayakrishnan 1998). He et al developed a class of projection and contraction method (He 1997). Chen et al. (2001) considered solving the non-additive traffic equilibrium problem, which is formulated as a nonlinear complementarity problem (NCP) and solved by a self-adaptive projection and contraction method. Among the path-based algorithms for the traffic equilibrium problem with additive path costs, much of the recent attention has been focused on the disaggregate simplicial decomposition (DSD) algorithm, which was proposed by Larsson and Patriksson (Larsson,Patriksson 1992), and the GP algorithm (Chen et al. 2001). A comparison work between these two path-based algorithm could be found (Chen,Lee 1999).

Meng et al. (2007) found that adding link capacity constraints into Daganzo's model (Daganzo 1982), undoubtedly, would lead to a linearly constrained minimization problem. Nevertheless, any optimal solution of the induced minimization model did not fulfil the generalized SUE conditions. This indicated that the typical technique used in modelling the generalized DUE conditions isnot available for the generalized SUE conditions except the logit-based generalized SUE conditions formulated by Bell (Bell 1995b).Then a general SUE TAP with link capacity constraints is proposed. It first proposed a novel linearly constrained minimization model, inspired by Maher et al. (2005) who formulated a SSO that related to SUE in the same way as the SO related to the UE, in terms of path flow. As the objective function of the proposed model involved path-specific delay functions without explicit mathematical expressions, its Lagrangian dual formulation is analysed. On the basis of the Lagrangian dual model, a convergent Lagrangian dual method with a predetermined step size sequence was developed (Meng et al. 2008) . Meng,Liu (2011) extended Meng's model for the side-constrained probit-based SUE problem with elastic demand to investigate availability of trial-and-error method for the effective toll pattern of cordon-based congestion pricing scheme

This study aims to use two general SUE models based on models developed by Sheffi (1985) and Meng et al. (2008) respectively for solving the general SUE problem of EVs with range limits. The MSA method is modified to eliminate the paths exceeding driving distance limit in the first model by adding a path processing step and modifying SNL procedure.

2.5 Charging stations location problem studies

2.5.1 Location problem of charging facility for private BEVs

Although many cities are planning the construction and expansion of charging infrastructures for BEVs, it is likely that in the foreseeable future BEV commuters will need to charge their vehicles at home most of the time (Marrow et al. 2008). For many electric vehicles, such as the Nissan LEAF or Chevrolet VOLT, the current method of recharging the vehicle battery is to plug the battery into the power grid at places like the home or office (Kurani et al. 2008). The battery requires an extended period of time to recharge, this method has an implicit assumption that vehicle will be used only for driving short distances. EV companies are trying to overcome this limited range requirement with fast charging stations; locations where a vehicle can be charged in only a few minutes to near full capacity. Besides being much more costly to operate rapid recharge stations, the vehicles still take more time to recharge than a standard gasoline vehicle would take to refuel (Botsford,Szczepanek 2009). These inherent problems, combined with a lack of refuelling infrastructure, are inhibiting a wide-scale adoption of electric vehicles. These problems are especially apparent in longer trips, or inter-city trips. Range anxiety, when the driver is concerned that the vehicle will run out of charge before reaching the destination, is a major hindrance for the market penetration of EVs (Mock et al. 2010). Hybrid vehicles, vehicles which have both an electric motor and a gasoline engine, have been successful since they overcome the range anxiety of their owners by also running on gasoline. However, since hybrids still require gasoline, these vehicles do not fully mitigate the environmental consequences (Bradley,Frank 2009).

Another refuelling infrastructure design is to have quick BSSs. These stations will remove a pallet of batteries that are nearly depleted from a vehicle and replace the battery pallet with one that has already been charged (Shemer 2012). This method of refuelling has the advantage that it is reasonably quick. The unfortunate downside is that all of the vehicles serviced by the battery exchange station are required to use the identical pallets and batteries which is unrealistic before battery and charger standardization. It is assumed here that the developers of these battery pallets will coalesce around a single common standard, as has been the case for other car parts such as tires, wipers, etc. Battery exchange stations have been tried out by taxi vehicles in Tokyo in 2010(Schultz 2010). Denmark is investigating the possibility of having sufficient battery exchange locations so that the country relies on none, or very few, gasoline powered vehicles (Mahony 2011).

Of course, there is a complementary location problem (not addressed here) where we wish to locate “refuelling” stations (battery recharging, battery exchanging and, other alternative refuelling options can all addressed similarly) in a region where there are currently none. The problem of optimally locating such refuelling stations has been investigated by several researchers using the flow refuelling location model (Kuby,Seow 2005; Kuby,Lim 2007; Upchurch et al. 2009). Frade et al. (2010). formulated a maximum covering model to locate a certain number of charging stations to maximize the demand covered within a given distance. A conceptual optimization model was proposed by Nie,Ghamami (2013) to analyse travel by EVs along a long corridor whose objective is to select the battery size and charging capacity (in terms of both the charging power at each station and the number of stations needed along the corridor) to meet a given level of service in such a way that the total social cost is minimized. Wang,Lin (2009).and Wang,Wang (2010) used set cover concept to propose refuelling-station- location model based on vehicle-routing logics considering both intercity and intra-city travel. The flow refuelling location model is reformulated and a flexible mixed-integer linear programming model is presented, which is able to obtain an optimal solution much faster than the previous set cover version. Besides, The model also could be solved in the maximum cover form MirHassani,Ebrazi (2013). Xi et al. (2013). developed a simulation–optimization model that determines where to locate EV chargers to maximize their use by privately owned EVs. Dong et al. (2014). studied EV charging station location problems and analysed the impact of public charging station deployment on increasing electric miles travelled. Wang et al. (2013). developed global optimization methods for discrete network design problem which can be applied in EV network design when formulated as a bi-level programming model, where the upper level aims to minimize the total cost and the lower level is a traditional UE problem.

2.5.2 Location problem of charging facility for public EBs

As part of transportation electrification plan, battery electric buses (BEBs) have received significant attention worldwide with the development and advance in battery technology and bus manufacturing recently. The governments aim to reduce the proportion of diesel-powered buses that were dominant in bus transportation and transition to alternative fuel buses, such as natural gas, hydrogen, electric battery, etc. It is estimated that more than 45% nitrogen oxides and 75% of particulate matter are generated by heavy-duty diesel trucks and buses (Elkins et al. 2003). In contrast, EBs have a unique advantage: zero emissions.

The use of BEBs has been reported in many countries all over the world. Several cities in United States introduced BEBs in transit service prior to mid-2000s. In 2012, Uruguay signed a deal for 500-heavy-duty BEBs and Tel Aviv, Israel, ordered 700 BEBs. In 2013, Shenzhen, China, ordered 1000 heavy-duty BEBs. The large-scale BEB adoptions are largely motivated by government incentives, such as the TIGER program in the United States, the Green Bus Fund Program in the UK, the Electric Mobility Program in German and the Ten Cities and Thousand Vehicles Program in China (SUTP 2015).

BEBs are characterized by fixed running routes, fixed depots, near-identical battery capacity. However, configuring an overall BEB system is challenging; this would include possible battery recharging and swapping concepts, choice of battery technology, battery sizing, positioning and dimensioning of charging and swapping stations (Leou,Hung 2017). Comparing to conventional diesel-powered buses, BEBs still suffer from long charging time, limited mileage range, and insufficient charging infrastructure regardless of its regenerative braking attribute.

Typically, three charging methods are available at current stage, namely slow charging, fast charging and battery swapping. Slow charging usually takes hours to refuel and reduces the utilization of BEBs, while fast charging reduces battery life (Sarker et al. 2013). It is pointed out that EV's orderly charging is the prerequisite for realizing its environmental benefits especially in countries with fossil-dominated power and the disordered charging will cause load fluctuations, and increase generation costs (Rao et al. 2015).

The deployment of BSSs, which removes the depleted batteries on the BEBs and replace the batteries with fully charged ones, is an alternative strategy to eliminate these barriers (Avci et al. 2014). The most outstanding feature of this strategy is that BSSs can complete the swapping process in less than 10 min, while another advantage is that charging the depleted battery can be left for the night at a discounted electricity price. Since BSSs achieve a unified management of batteries, it contributes to the effective maintenance of batteries and is beneficial to extend the batteries' lifetime. However, due to the lack of standardization of batteries and interfaces, the BSSs are more suitable for fleets of buses and taxies (Zheng et al. 2014). Based on these contexts, China is leading the way on deploying the BSSs. In April 2015, Ziv Av Engineering signed the deal with China's Bustil to design 7000 BSSs for BEBs in Nanjing city (Elis 2015). In 2016, new energy automaker BAIC BJEV has built 50 EV charging and swapping stations to address the needs of at least 6000 EV taxis (PRNewswire 2016). So far 1300 BSSs have been constructed and 12000 more are planned through 2020 in many pilot cities of China (Liang et al. 2017).

Theoretically BEBs can travel up to 250 km, various factors influence the operational range in the real-world operations. It is shown that the air conditioning, driving behavior and battery aging issue can largely (more than 30%) reduce the BEBs' operational range, thus making BEBs often incapable of finishing a whole day's work without battery recharging (Li 2016). Moreover, BSSs require large capital investment to purchase additional batteries that are necessary to swap with ones near depletion. The land-use is another issue, including the parking space for bus awaiting and the space for installing local chargers on spot for local charging mode (Li 2016). Therefore, the location of BSSs and the choice of charger types become an inevitable issue when designing battery swapping system due to their charging speed and financial cost.

Generally there are two types of operation mode for BSSs in terms of the strategy of charging the depleted batteries, namely central charging and local charging. In the central charging mode, EVs swap their batteries in BSSs, and the empty batteries are sent to the central charging station. After the empty batteries are fully charged, they will be delivered back to the BSSs. Another mode is local charging system which excludes the empty battery depot and charges the depleted batteries in BSSs themselves.

To date, some studies have been done regarding the optimal planning, operation and location of BSSs. The relevant research may be classified into three types. In the first type, the optimal location of BSSs and the interaction between BSSs and power grid are the primary concern. A p-median based model is applied to solve the BSS location problem with central charging system (Xiang,Zhang 2017). The optimal configuration of central charging station and its location were studied (Xu et al. 2013). A bi-level optimal configuration model to plan the capacity and location of BSS is proposed to maximize net profit of BSS while minimizing the operation cost of distribution company (Liu et al. 2016). The second type primarily focuses on the operation of both BSSs and BEBs. An single-depot optimization model for BEB scheduling with BSS is proposed to minimize the total operation cost (Li 2013) and more operation features of BEBs were taken into consideration to minimizing the capital investment in another single BEB depot scheduling model (Zhu,Chen 2013). Another study focused on schedule the battery charging in the BSS so that every BEB arrives to find a full battery for swapping (You et al. 2016). Articles in the third type mainly focus on the operation details of BSS including optimal power capacity (Leou,Hung 2017), charging scheduling (You et al. 2017) and meeting total swapping demand (Xiong et al. 2012). A simulation-based BSS load demand model is presented considering the stochastic charging characteristics of BSS and BEB arrival pattern (Dai et al. 2014). A central charging strategy and scheduling of BEBs for BSS is designed to minimize charging cost based on optimal charging priority and charging location electricity price (Kang et al. 2016). To promote development of BSSs for BEBs, the optimal BSSs' location and its local charging system design should be researched first and the major factors affecting the capital investment for the stakeholder should be fully considered in the process of planning. Moreover, transport cost between BEB transit depot and BSSs would also be one of the major concerns because of range limit and the energy waste during the detour to swap the depleted battery.

2.6 Knowledge Gaps

On reviewing the literature to date regarding the transport network model and charging facility location model development of electric vehicles scheme, clear gaps in the knowledge are identified.

(1) There have been few researches on the stochastic traffic assignment of electric vehicle considering the driving range limit and insufficient public charging infrastructure at the current stage.

(2) The flow-dependent energy consumption rate of EVs which increases with the traffic congestion level has not been considered in general SUE model.

(3) Charging facility location model for private BEVs have not taken SUE flow pattern into consideration which incorporates an upper-level of EV charging facility network design and a lower-level of stochastic traffic assignment of EVs.

(4) Swapping facility location model with local charging system for public EBs has not been studied.

2.7 Summary

In this chapter, a review of the relevant literature is undertaken focusing on the key characteristics of BEVs, the development of traffic assignment model with BEVs, the development of stochastic traffic assignment model with side constraints, the overall charging facility location problem (CFLP) of BEVs and the BSS location problem serving EBs. The review has identified important gaps in the existing knowledge. These gaps are summarised in Table 2-3 and the research to be undertaken to address these gaps is outlined.

Driving distance limit has been extensively used as a side constraint in equilibrium network modelling (Xie,Jiang 2016; Wang et al. 2016; Jiang,Xie 2014; Jiang et al. 2012; Jiang et al. 2013) . In order to explore the impact of driving distance limit on equilibrium EV flow pattern, previous equilibrium models focus on DUE model which do not consider the stochasticity of travellers' perception error on travel time. To address this gap, this thesis develops the new general SUE model for predicting EV flow pattern, using modified MSA algorithms and modified probit-loading algorithm (see Chapter 4).

Given the impacts of travel speed on the battery energy consumption (Bigazzi,Clifton 2015; Agrawal et al. 2016), the effects of combining flow-dependent energy consumption on BEVs' route choice behaviour should be explored with battery capacity constraint. In existing general SUE models with side constraints, only link capacity constraints have been considered (Meng et al. 2008) and battery capacity constraints have only been studied in DUE models. This thesis will close this gap by proposing new general SUE model of EVs as well as its solution methodology (see Chapter 5).

Most charging facility location models for private BEVs do not consider the traffic congestion and equilibrium BEV flow patterns (Upchurch et al. 2009; Capar,Kuby 2012; Kuby,Seow 2005). The BEV flow may change resulting from the change of the BEV charging facility locations considering their route choice behaviour. Furthermore, the change of flow patterns may affect the utilization rate of the deployed charging facilities. One focus of this thesis is the effects of combining existing SUE models and classical facility location model into a bi-level model. Yet, it is prudent to explore if a bi-level model, involving stochastic traffic assignment and available public charging facility, can have a fair performance. Litter is understood about the location design of BEV charging facility accounting for a SUE equilibrium flow pattern. This thesis will help to fill this gap by developing a new bi-level charging facility location model for private BEVs (see Chapter 6).

Although previous studies have investigated CFLP for public EBs (Riemann et al. 2015), little attention has been paid to battery swapping facility planning for EBs with local charging system considering its own characteristics, such as fixed routes and given demand (Xiang,Zhang 2017; Zhu,Chen 2013; Sarker et al. 2013). This thesis will help to fill this gap by exploring the optimal location and configuration of BSSs as well as local charging system for EBs (see Chapter 7).

This research project aims to address the knowledge gaps identified above and attempts to obtain a deeper knowledge in assessing public charging/swapping facility deployment in urban areas. In the next chapter, the research methodology will be presented in detail and the detail of data used in this research will be also provided.

The following chapter of this thesis presents the research methodology to address the identified gaps in knowledge, based on the abovementioned research opportunities.

Table 2-1: Summary of Charging Station Location Studies

Author	Hodgson (1990)	Bapna et al. (2002)	Kuby and Seow (2005),Kuby and Lim (2007),Upchurch et al. (2009)	Wang and Lin (2009)	Wang and Wang (2010)	Frade et al. (2010)	Capar and Kuby (2012)	Mak et al. (2012)	Xi et al. (2013)	MirHassani and Ebrazi (2013)	Nie and Ghamami (2013)	Dong et al. (2014)
Facility type												
Battery charging station				√		√			√		√	√
Battery swapping station								√			√	
Alternative fuel station		√	√	√	√		√			√		
Gas station	√	√										
The number of the facility												
Fixed	√		√			√	√				√	
Variable				√	√			√	√	√		√
Model type												
Interger programming		√										√
Mixed interger programming			√	√	√		√	√		√		
Linear programming	√								√			
Objective function												
Maximize coverage		√				√	√					
Minimize cost		√		√	√			√		√	√	
Maximize flow coverage	√		√		√				√	√		

Note:‘ √ ’ indicates the use of such a factor or technique.

Table 2-2: Summary of Traffic Assignment Problem and Vehicle Routing Problem Related to EV

	Ichimori et al. (1981)	Desrosiers et al. (1984); Desrochers and Soumis (1989)	Cabral et al. (2007)	Andreas et al. (2010); Eisner et al. (2011);Sachenbacher et al. (2011)	Kobayashi et al. (2011)	Siddiqi et al. (2011)	Ryan and Miguel (2011)	Laporte and Pascoal (2011); Smith et al. (2012)	Jiang et al., (2012)	Erdoğan and Miller (2012)	Jiang et al. (2013)	Okan et al. (2014)	Adler et al. (2014)
Constraints													
Shortest path problem	√	√	√	√	√							√	
Relay requirement	√		√	√	√	√	√			√		√	√
Distance-constrained			√						√		√		√
Fuel-constrained				√	√		√			√			
Time-constrained						√							
Computational complexity													
Polynomial solvable	√			√	√				√				√
NP-complete		√				√	√					√	
NP-hard			√					√		√			
Objective function													
Minimize distance	√					√				√			√
Minimize flow cost		√											
Minimize total cost			√	√			√				√	√	

Note:‘ √ ’ indicates the use of such a factor or technique

CHAPTER 3 RESEARCH METHODOLOGY

3.1 Introduction

The previous chapter provides a literature review of TAP of EVs and their charging facility location problem. The review identifies research gaps and discusses opportunities to advance knowledge in addressing these gaps.

This chapter describes the overall research approach to address the identified research gaps and achieve the objectives outlined in Section 1.4, including the application of analytical methods based on classic facility location problem theory and STAP Models.

3.2 Overall research approach

The overall research approach includes five key research components which were designed to achieve the five research objectives. The first research component focuses on EV drivers' behaviour and characteristics developments, which were then applied for understanding the key constraints to be considered in STAP models in the subsequent components of the thesis. The second and third components investigate STAP models with the consideration of range limit constraints to predict BEV flow patterns in the future. The last two research components focus on deploying charging/swapping facilities to maximize their utility. Linkages between research gaps, research objectives, research components, and thesis chapters are shown in Table 3-1. The following sections present a brief description of each research component.

3.2.1 Research component 1: Characteristics and behaviours of EVs affecting equilibrium network modelling

An attempt is made to investigate the various aspects of EVs that explicitly address the problems which come with EVs' development as well as network modelling of EVs. Starting with the concept of EVs, it discussed both the EVs market studies and special characteristics of EVs as well as its charging infrastructures. From network modelling point of view, it is, therefore, important to take their special characteristics into account when predicting EVs route choice behaviour and designing charging infrastructure networks accordingly. It is found that a number of factors contribute to BEV network modelling. For instance, range limit, range anxiety, charging time, charging cost, availability of charging infrastructure, etc. The commonly used factors were examined in the five remaining research components. EVs' driving distance limit and scarcely available public charging stations were considered in the development of general SUE models of transport network with mixed GVs and EVs (research component 2). A dynamic EVs' energy consumption rate is considered to depend on both travel time and travel distance. Limited battery capacity and lack of public charging facility were taken into consideration in another general SUE model (research component 3). More factors were adopted in the formulation of bi-level charging facility location problem for private BEVs (research component 4) and evaluation of the proposed battery swapping facility location with local charging system serving EB fleet (research component 5). The summary of several research directions is given to address the emerging of BEVs in the field of network modelling in Chapter 2.

3.2.2 Research component 2: Addressing stochastic traffic assignment of mixed EV and GV flow with path distance constraints

The methodological issues of a general SUE model of mixed EV and GV flow with path distance constraint and how to solve this model is investigated in this component of the thesis. Directly adding side constraints into a SUE model cannot generate a SUE flow pattern. Incorporating the path distance constraints into the general STAP needed a mathematical proof. The BEV range limit is defined by the path distance it can travel without charging. A classical minimization model is used with a modified MSA method to address the SUE problem. Solution properties of equivalence and uniqueness were provided. Path feasibility check is employed to address the path distance issue whenever generating a path in K-shortest path algorithm or shortest path algorithm. The results suggested that range limit would have a great impact on EV users' route choice, especially for those with short range limit. When the range limit became large enough, EV behaves similarly to GV. This component of research were then adopted and further extended in research component 4 by incorporating the available public charging facility into the general SUE model. The detailed formulation and evaluation are provided in Chapter 4.

3.2.3 Research component 3: Addressing stochastic traffic assignment of EV with battery capacity constraints

This component of the thesis investigated a more complicated STAP in transportation networks with BEVs owing to the fact that BEV energy consumption depends on not only the path distance but also the travel time. The main objective is to theoretically understand how a flow-dependent path-based constraint can be incorporated into a general SUE model. Battery capacity constraint is a flow-dependent one, while path distance constraint is flow-independent. The flow-independent driving distance constraint in research component 2 can be processed in the route choice procedures, while the flow-dependent battery energy consumption depends on not only distance but also traffic flow (travel time). A mathematical programming model is proposed for the flow-dependent path-based SUE traffic assignment. A convergent Lagrangian dual method is employed to transform the original problem into a concave maximization problem and a customized gradient projection algorithm is developed to solve it. A column generation procedure is adopted to generate the path set. The solution framework, Lagrangian dual-gradient projection-stochastic network loading, can be applied to solve path-based SUE problem. Further details of this research component are given in Chapter 5.

3.2.4 Research component 4: Location of EV charging facilities: A path distance constrained SUE approach

This component of the thesis investigated a way of locating charging facilities in the network since no public charging facilities have been considered in the previous SUE components. A bi-level model is adopted with maximum covering objective in the upper level and STAP with path-distance constraints in the lower level. Public charging facilities were taken into consideration in the trip chain in the lower level STAP to accomplish this component of research. An important concept of sub-path is used to identify the scenarios of charging need. A key application of this concept is to calculate the generalized path travel cost composed of path travel time, charging time and equivalent travel time reduction (the utility of charging facilities on attracting BEV drivers). Comparing to research component 2&3, the SUE approach is extended to consider public charging facilities in the network. It is demonstrated that the driving distance limits, charging speed and utility of charging facilities affect the equilibrium network flow and charging facility location. It is also found that the BEVs with shorter driving distance and risk-neutral attitude would probably have a larger value of charging facilities utility, because charging facilities helped to ease their range anxiety. While for those with larger batteries, they would behave more like GV users. A potential drawback of this method of defining flow coverage is that it may lead to the location of charging facilities on several adjacent links of some high-volume freeways. Further details of this research component are provided in Chapter 6.

3.2.5 Research component 5: Battery swapping station location serving BEB fleet

The objective of last component is to develop location models for BSS serving BEB fleet. The service capability of BSS is restricted by the number of installed swapping robots. Depleted batteries will be charged at BSSs which adopts a local charging system equipped with a number of batteries and chargers in various types. This study intends to answer four fundamental questions: How many BSSs should be installed? Where should they be? Which EBs should be assigned to each BSS? What is the service capability of the BSSs? A mixed-integer linear program is formulated to represent this problem, which is then solved by a GUROBI solver implemented on Python interface. The test on a real network of the southeast region of Melbourne in Australia verifies the feasibility of the proposed model and investigates the effects of BSS locations and configurations.

As a base model with simple assumptions, future work should consider more realistic scheduling of electricity-price-based battery charging and BEB operation to increase the utilization rate of batteries. Furthermore, comparing with local charging system, models of central charging system should be considered to make an economic comparison to identify a favorable charging mode. Additionally, the battery capacity and the charging power of BEB used for public transportation are several times greater than those of electric cars, which can result in high energy consumption and negatively impact on power distribution networks. Therefore a BSS deployed at a given region should be considered as capacitated before power grid upgrade to accommodate more local charging demand. This component of research has the potential to develop many new models for future works. Detailed model formulation and results are given in Chapter 7.

3.3 Conclusion

This chapter has presented the overall research approach composed via five research components, in line with the five research objectives. The first research component focuses on identifying key characteristics and main behaviour that distinguish EVs from GVs, as well as the factors affecting charging/swapping facility location design. The second and third research components focus on the SUE model and solution methodological developments, which are then applied for understanding the equilibrium network flow in the fourth research components when charging facility becomes available in the network. The last research component investigates location design of swapping facility serving public EBs. Limitations of the research approach associated with each research component are discussed in each corresponding thesis chapter.

The next chapter of this thesis presents the model that developed to formulate SUE model of mixed GV and EV flow with flow-independent path distance constraints.

Table 3-1: Summary of research gaps and opportunities to advance knowledge

Research component.	Research topic	Research gaps	Research opportunities
1	Factors affecting BEVs drivers' charging and route choice behaviour (Chapter 2)	The factors distinguishing BEV from GV have not been thoroughly studied in transport network modelling	summarizing the existing researches and identifying key factors affecting potential future research directions
2&3	SUE models of transport network with electric vehicles (Part II)	Existing traffic assignment models tend to ignore the stochasticity of travel time perception. There is a need for stochastic traffic assignment model of a transport network with BEVs whose driving distance is limited. (see section 2.6)	Developing a general SUE model and solution algorithms to predict BEV flow pattern when considering the maximum distance BEVs can travel. (see chapter 4)
		No studies have considered a SUE model of BEVS with battery capacity constraints where BEVs' range limit is restricted by both travel distance and travel time. (see section 2.6)	Proposing a general SUE model and solution algorithm for BEVs with limited battery capacity that restricts BEVs' travel time and travel distance. (see Chapter 5)
4&5	Charging/swapping facility location models of BEVs and BEBs (Part III)	There is no bi-level charging facility location model dedicated to considering a SUE BEV flow pattern in the lower level problem and maximize BEV flow coverage in the upper level (see section 2.6)	Proposing a new bi-level model for deploying the charging facility considering a SUE link flow pattern and availability of charging facility (see Chapter 6)
		Battery swapping is designed to be more suitable for electric buses. Moreover, there is no swapping facility location model dedicated to swapping facility location with local charging system serving EB fleet (see section 2.6)	Proposing a new BSS facility location model for BEBs considering BEBs' characteristics (see Chapter 7)

PART II:

DEVELOPMENT OF STOCHASTIC TRAFFIC ASSIGNMENT MODELS

CHAPTER 4 STOCHASTIC TRAFFIC ASSIGNMENT MODEL WITH PATH DISTANCE CONSTRAINTS

4.1 Introduction

The previous chapter describes the overall research approach, comprising five research components aligned to five research objectives. The first research component focuses on identifying key characteristics and main behaviour that distinguish EVs from GVs. The second and third research components focus on the SUE model and solution methodological developments, which are then applied for understanding the equilibrium network flow in the fourth research components when charging facility becomes available in the network. The last research component investigates location design of swapping facility serving public EBs.

This chapter presents the results of research component 2, which focuses on model and methodological developments of SUE. The methodological issues of a general SUE model of mixed EV and GV flow with path distance constraint and its solution algorithm remain unsolved. However, directly adding side constraints into a SUE model cannot generate a SUE flow pattern like the way in DUE models with side constraints. Incorporating the path distance constraints into the general STAP needed a mathematical proof. The BEV range limit is defined by the path distance it can travel without charging.

The aim of this chapter is to therefore propose new model to formulate this problem. A minimization model for path-constrained SUE is first proposed as an extension of path-constrained deterministic user equilibrium (DUE) TAP, which also extends the existing general SUE models with link-based constraints to path-based constraints. The resulting SUE model and solution algorithm can be used for other conditions with similar path-based constraints. The research gap and objective associated with this research component is described in Table 4-1.

Table 4-1: Research gap and objective associated with research component 2

Research topic	Research gaps	Research opportunities
SUE models of transport network with electric vehicles (Part II)	Existing traffic assignment models tend to ignore the stochasticity of travel time perception. There is a need for stochastic traffic assignment model of a transport network with BEVs whose driving distance is limited. (see section 2.5.2)	Developing a general SUE model and solution algorithms to predict BEV flow pattern when considering the maximum distance BEVs can travel. (see chapter 4)

This chapter includes the following research paper:

Jing, Wentao, Inhi Kim, Mohsen Ramezani, and Zhiyuan Liu. "Stochastic traffic assignment of mixed electric vehicle and gasoline vehicle flow with path distance constraints." Transportation Research Procedia 21 (2017): 65-78.

4.2 Paper 1: Stochastic traffic assignment of mixed electric vehicle and gasoline vehicle flow with path distance constraints

The following paper details the formulation of a general SUE model of mixed EV and GV flow with path distance constraint. It begins by discussing the shortcomings of existing methods of DUE model to predict the EV flow pattern. It also reviews network equilibrium models for EV schemes and solution algorithms for general SUE models. Incorporating the path distance constraints into the general STAP needed a mathematical proof. The BEV range limit was defined by the path distance it can travel without charging. A classical minimization model is proposed with a modified MSA method to address the SUE problem. Solution properties of equivalence and uniqueness are provided. Path feasibility check was employed to address the path distance issue whenever generating a path in K-shortest path algorithm or shortest path algorithm. Finally, the results suggested that range limit would have a great impact on EV users' route choice, especially for those with short range limit. When the range limit became large enough, EV behaves similarly to GV. This component of research is then adopted and further extended in Chapter 6 by incorporating the available public charging facility into the general SUE model.

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Stochastic traffic assignment of mixed electric vehicle and gasoline vehicle flow with path distance constraints

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Abstract

This paper addresses a general stochastic user equilibrium (SUE) traffic assignment problem (TAP) for transport networks with electric vehicles (EV), where EV paths are restricted by the EV driving range limits. A minimization model for path-constrained SUE is first proposed as an extension of path-constrained deterministic user equilibrium (DUE) TAP, which also extends the existing general SUE models with link-based constraints to path-based constraints. The resulting SUE model and solution algorithm can be used for other conditions with similar path-based constraints. The equilibrium conditions reveal that any path cost in the network is the sum of corresponding link costs and a path specific out-of-range penalty term, while path out-of-range term should equal to zero to ensure feasible flows. We develop a modified method of successive averages (MSA) with a predetermined step size sequence where both multinomial logit and multinomial probit based loading procedure are applied to solve the TAP. The suggested methods incorporate K-shortest paths algorithm to generate the path set on a need basis. Finally, two numerical examples are presented to verify the proposed model and solution algorithms.

Keywords: Traffic assignment; stochastic user equilibrium; path distance constraints; K-shortest path algorithm; Multinomial Logit; Multinomial Probit

1. Introduction

Carbon-based emissions and greenhouse gases (GHG) are critical global issues as addressed by the Kyoto Protocol in 1998 (U.S. Envirionmental Protection Agency, 2006). The transport sector is a significant contributor to

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GHG emissions in most countries, comprising 23% (worldwide) of CO2 emissions from fossil fuel combustion in 2005 while automobile transport is the principal CO2 production source. From the energy safety point of view, the transport sector as a whole is 98% dependent on fossil oil which is also exceedingly affected by changes in energy resources (OECD-ITF Joint Transport Research Centre, 2008). So changes to the current energy structure in transport sector are in urgent need.

Alternative fuels are addressed as a new fuel choice to reduce GHG emissions and electric vehicles (EV)are believed to be a sustainable solution (OECD-ITF Joint Transport Research Centre, 2008). Governments and automotive companies have recognized the value of these vehicles in helping the environment and are encouraging the ownership of EV through economic incentives (Hacker et al., 2009). It is mentioned that one million plug-in hybrid and electric vehicles will be on the road by 2015 in United States to reduce greenhouse gas emission and dependence on oil (Saber and Venayagamoorthy, 2009). According to Electric Drive Transportation Association, the plug-in electric vehicles (PEV) in US has exceeded 190,000 between January of 2011 and Mach of 2014 (Ghamami et al., 2014).

Although many cities are planning construction and expansion of charging infrastructures for EV, it is likely that in the foreseeable future EV commuters will need to charge their vehicles at home most of the time (Morrow et al., 2008). It is obvious that the driving range limit inevitably adds a certain level of restrictions to EV drivers' travel behaviors, at least in a long future period prior to the coverage of recharging infrastructures reaching a sufficient level (Jiang et al., 2013). EV companies are trying to overcome this limited range requirement with fast charging stations, where a vehicle can be charged in only a few minutes to near full capacity. Besides being much more costly to operate rapid recharge stations, the vehicles still take more time to recharge than a standard gasoline vehicle would take to refuel (Botsford and Szczepanek, 2009).

However, the widespread adoption of PEV calls for fundamental changes to the existing network flow modelling tools for properly capturing changed behaviors and induced constraints in forecasting travel demands and evaluating transportation development plans (Jiang et al., 2013).

In order to take into consideration of driving range limit and insufficient charging facility status in traffic assignment, Jiang et al. (2012) proposed an approach to restrict flow of a path to zero if the path distance is greater than the driving range limit of EV. They employed a path travel time function that is the sum of the corresponding link cost such as the Bureau Public Road (BPR) function and showed the Lagrangian multiplier of its optimal solution stands for the unit out-of-range travel distance cost. Classic Frank-Wolfe algorithm with a constrained shortest path algorithm as its subroutine can be applied to solve this problem.

The deterministic user equilibrium (DUE) condition characterizes route choice behavior where users have perfect traffic network information and always choose the shortest path accurately. A convex minimization model for DUE conditions can be built by adding path distance constraints into the Beckmann's conventional DUE model. A more realistic and general situation is that travel times are random variables or travel times are perceived by travellers in imperfect, stochastic manner. Although the stochastic user equilibrium (SUE) principle plays a more realistic role than DUE principle in addressing road user's route choice behavior, the SUE traffic assignment problem with path-distance constraints has received little attention. To be consistent with the generalized DUE with path distance constraints, the SUE traffic assignment model with generalized path travel times are referred to as generalized SUE traffic assignment with path distance constraints. A milestone in formulating SUE conditions is Daganzo's unconstrained minimization model (Daganzo, 1982) of conventional SUE conditions, which can lead to a convergent algorithm for solving the general SUE traffic assignment problem. However, adding side constraints (e.g. link capacity constraints) into Daganzo's model cannot yield solution fulfilling generalized SUE conditions.

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1.1. Literature Review

It is well known that the standard TAP under DUE can be solved efficiently with a Frank-Wolfe type algorithm whose linearized sub-problem finds shortest paths for each OD pair at each iteration. The problem of finding the shortest path for an EV was originally discussed by Ichimori et al. (1981)， where a vehicle has a limited battery and is allowed to stop and recharge at certain locations. Lawler (2001) developed a polynomial algorithm for its solution. Adler et al. (2014) proposed an EV shortest-walk problem to determine the shortest travel distance route which may include cycles for detouring to recharging batteries from origins to destinations with minimum detouring. Kobayashi et al. (2011) and Siddiqi et al. (2011) included battery recharging stations in their shortest weight-constrained path problem models, which is known to be NP-Complete (Desrosiers et al., 1984; Desrochers and Soumis, 1989), and proposed heuristic techniques as solution methodologies. There has been some recent consideration of the effect of EV on traffic assignment and DUE. (Jiang et al., 2012) studied the effect of restricted the EV path distances and assumes charging events only occur at OD nodes, which corresponds to the real circumstance of insufficient charging facilities.

As a rational extension to the DUE, the stochastic user equilibrium (SUE) principle can be adopted to formulate the TAP. Meng et al. (2007) found that adding link capacity constraints into Daganzo's model would lead to a linearly constrained minimization problem. Nevertheless, any optimal solution of the induced minimization model did not fulfil the generalized SUE conditions. This indicates that the typical technique used in modelling the generalized DUE conditions was not available for the generalized SUE conditions except the logit-based generalized SUE conditions formulated by Bell (1995b). Meng et al. (2007) proposed a general stochastic user equilibrium (SUE) traffic assignment problem with link capacity constraints, inspired by Maher et al. (2005), who proposed a formulation for stochastic social optimum (SSO) with the objective of minimizing the total perceived travel time and found that the solution to SSO can be achieved by solving a SUE problem using the marginal cost function. Meng et al. (2007) found that SUE flow pattern can be generated by solving a SSO problem applying a modified link travel time function.

Previous studies on general SUE traffic assignment problem mainly tackled link-based constraints [see, e.g., (Meng et al., 2007; Meng and Liu, 2011; Meng et al., 2014)]. However, for EV users, the route choice is restricted by their driving range limit, which imposes path distance constraints to the general SUE model.

Early algorithms developed to solve the unconstrained logit-based SUE problem were link-based [e.g. Maher (1998)]. These link-based algorithms do not require path storage and often use Dial's STOCH algorithm or Bell's alternative as the stochastic loading step (Dial, 1971; Bell, 1995a). Path-based algorithms require explicit path storage to directly compute the logit route choice probabilities. Olof et al. (1996) developed a path-based algorithm based on the disaggregated simplicial decomposition algorithm to solve the multinomial logit (MNL) SUE problem. Bekhor and Toledo (2005) compared path-based algorithms for the MNL SUE problem, and showed that the disaggregated simplicial decomposition algorithm is superior to the path-based method of successive averages (MSA) algorithm. Among the path-based algorithms for the traffic equilibrium problem with additive path costs, much of the recent attention has been focused on the disaggregate simplicial decomposition (DSD) algorithm, which was proposed by Larsson and Patriksson (1992), and the gradient projection (GP) algorithm. A comparison work between these two path-based algorithm could be found in Chen and Lee (1999).

Xu et al. (2012) investigated different strategies for determination of step size of the path-based algorithms developed to solve the C-logit SUE models based on an adaptation of the GP method. Three strategies were investigated: (a) predetermined step size (Nagurney and Zhang, 1996), (b) Armijo line search (Larry, 1966; Bertsekas, 1976), and (c) self-adaptive line search (He et al., 2002; Chen et al., 2012). To have a fair comparison of different step size strategies, Bekhor et al. (2006). Used a working route set, obtained from a route choice set generation algorithm, such as labelling, link penalty, link elimination and simulation, in path-based problem.

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Behaviorally, it had the advantage of explicitly identifying those routes that were most likely to be used and also allowed greater flexibility to include route-specific attributes that might not be obtainable directly from the link attributes (Cascetta et al., 1997; Bekhor et al., 2006). A column generation procedure could also be readily embedded in the GP algorithm (Chen and Jayakrishnan, 1998).

To the best of our knowledge, it is still an open question to find an exact solution method for solving the general SUE problem with path distance constraints on a transport network with EV.

1.2. Objectives and Contributions

Since SUE relaxes the perfect information assumption of the DUE model by incorporating a random error term in the path cost function to model travelers' imperfect perceptions of travel times, it would be more realistic to apply general SUE model to assign EV flows to the transport network. It is interesting to compare the assignment results between stochastic models and deterministic one. Hence, the results from proposed model are compared with that from DUE with path distance constraints. The path-based approach requires path generation/enumeration, where the size of a path set between a single O-D pair can be extremely large. Thus predetermined path set is applied in this path-based problem.

Meng et al. (2007) proposed a solution framework for general SUE problem with link-based constraints. For the general SUE traffic assignment problem addressed in this study, we investigate the properties of the path distance constraints and the solution method framework. Furthermore, K-shortest paths algorithm is applied to avoid path enumeration.

To sum up, the contributions of this study are twofold. First, a holistic methodology is proposed for general SUE traffic assignment model with path distance constraints on EV scheme, in which the classic unconstrained SUE model can be used to incorporate path distance constraints by modifying MSA algorithm and finding the distance-constrained K-shortest paths in stochastic network loading process. It is assumed that the EV route choices are restricted by the distance EV can travel with a single charge. Second, comparison results between SUE and DUE are provided. The major part of this paper is a discussion of the modelling and solution methods for the SUE traffic assignment problem with path distance constraints.

The remainder of this paper is organized in the following order. In Sections 2 & 3, we elaborate the problem formulation, and analyse its solution properties. Section 4 presents the solution algorithms for both Logit-based and Probit-based stochastic network loading models, while Section 5 presents the numerical results from applying the algorithm procedure for a small network and Sioux Falls network as well as the comparison work between SUE TAP with path distance constraints and DUE TAP with path distance constraints. In the end, Section 6 provides a few concluding remarks.

2. Notation, assumptions and problem description

Consider a strongly connected network, denoted by $G=(N,A)$, where N and A are sets of nodes and links, respectively. (r,s) stands for certain ordered pairs of nodes, $r \in R$ and $s \in S$,where node r is an origin and node s is a destination. $R \subset N$ and $S \subset N$ are sets of origins and destinations, respectively. There are non-negative travel demand q_n^{rs} of n-th vehicle type between (r,s). $\mathbf{q}=\left(q_n^{rs},n\right)^{\mathrm{T}}, \forall (r,s)$ is a column vector for all the travel demands. Let K_{rs} be the set of paths connecting O-D pair (r,s), f_{kn}^{rs} be traffic flow of n-th vehicle type on

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path $k \in K_{rs}$, $\mathbf{f}^{rs} = \left(f_{kn}^{rs}, \forall n\right)^{\mathrm{T}}, k \in K_{rs}$ be a column vector of all these path flows between OD pair (r,s), and $\mathbf{f} = \left(\mathbf{f}^{rs}\right)^{\mathrm{T}}, \forall(r,s)$ be a column vector of all the path flows over the entire network. Let v_a denote traffic flow on link $a \in A$ and $\mathbf{v} = \left(v_a\right)^{\mathrm{T}}, a \in A$ is a column vector of all the link flows. The path flows and link flows should comply with fundamental flow conservation equations:

$$v_a = \sum_n \sum_{(r,s)} \sum_k f_{kn}^{rs} \delta_{a,k}^{rs}, \forall a \in A \tag{1}$$

$$\sum_k f_{kn}^{rs} = q_n^{rs}, \forall (r,s), n \tag{2}$$

$$f_{kn}^{rs} \geq 0, \forall (r,s), n, k \in K_{rs} \tag{3}$$

where $\delta_{a,k}^{rs}$ =1 if path $k \in K_{rs}$ between O-D pair (r,s) traverses link $a \in A$, and 0 otherwise.

Let $t_a\left(v_a\right)$ denote the separable travel time function of link $a \in A$, which is assumed to be a positive, strictly increasing, convex and continuously differentiable function of the traffic flow on the link. All the link travel time functions are grouped into a column vector $\mathbf{t}(\mathbf{v}) = \left(t_a\left(v_a\right)\right)^{\mathrm{T}}, a \in A$. Travel time on path $k \in K_{rs}$ between O-D pair (r,s) can be considered as a function of all the path flows, denoted by $c_k^{rs}(\mathbf{f})$ with the expression

$$c_k^{rs}(\mathbf{f}) = \sum_a t_a(v_a) \delta_{a,k}^{rs} \tag{4}$$

Given any positive feasible path flow pattern $\mathbf{f}$, $\mathbf{f}$ satisfies the conventional SUE conditions associated with the path travel time functions, $\mathbf{c}^{rs}(f) = \left(c_k^{rs}\right)^{\mathrm{T}}, , \forall (r,s), n, k \in K_{rs}$ is a column vector of all these path travel time between OD pair (r,s) ,namely

$$f_{kn}^{rs} = q_n^{rs} \cdot P_{kn}^{rs}(\mathbf{c}^{rs}(f)) \tag{5}$$

where P_{kn}^{rs} is the probability that vehicle type n choose path k between O-D pair (r,s).

2.1. Path distance constraints and insufficient charging facility

Based on EV's market potential, it is expected that in the future gasoline vehicles (GV) and EV will coexist in the automobile market. For this reason, the proposed model includes multiple classes of vehicles, namely GV and EV, which distinguish from each other in terms of driving distance range and travel cost composition. To derive the theoretical properties of the problem, we consider a set of assumptions regarding demand heterogeneity and travel behavior.

First without loss of generality, it is assumed that the demand population is only comprised of GV and EV. Plug-in hybrid electric vehicle (PHEV) are not explicitly considered since they can be simply treated as an in-between class of GV and EV in terms of the technological and economic features (i.e., driving range limit and travel cost composition), or a special type of GV with lower operating costs. Readily multiple types of EV with different

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driving range limits and operating costs can be incorporated into the model.

Second, we assume the total travel demand between each O-D pair for every vehicle type is deterministically known a-priori. SUE concept is devised for route choice procedure, in which each traveler chooses a route that minimizes his/her perceived travel cost while no one can reduce his/her perceived travel cost by unilaterally switching to an alternative route. For an individual GV traveler, stochastic user equilibrium simply implies a conventional stochastic traffic assignment problem of searching for perceived minimum cost (travel time); whereas for an EV traveler, it poses a path distance-constrained perceived minimum cost problem. In this paper, we scrutinize the integrated effect of different vehicle types with various path constraints.

Third, without loss of generality, we assume that both GV and EV travelers use a common form of systematic travel cost function for determining their travel choices. The link travel time functions are assumed to be separable between different network links and identical for different vehicle classes, implying the travel time on a particular link only depends on its own traffic flow. These functions are assumed to be positive, monotonically increasing, and strictly convex.

In our network equilibrium analysis, it is implicitly assumed that all EV are fully charged at their origins. The possible availability of commercial battery-charging or battery-swapping stations emerging in urban areas can be considered in the future when charging infrastructures achieve a certain level of coverage. EV users would choose a path whose distance l_k^{rs} is less than or equal to the driving range limit of the vehicle type, denoted by D_n. Hence, any feasible path flow pattern should satisfy the path distance constraints:

$$f_{kn}^{rs}(D_n - l_k^{rs}) \geq 0, \forall (r,s), n, k \in K_{rs} \tag{6}$$

which means that if the flow of that class of EV users going through this path is positive, the path distance is smaller than or equal to the driving range of a given class of EV; otherwise, the trip flow should equal to zero.

3. Mathematical model

Due to the complexity of probit-based SUE problem, directly adding side constraints into that general SUE model does not give us an equivalent minimization model to the probit-based SUE traffic assignment with side constraints (Meng and Liu, 2011). However, we can still add the path distance constraint into this minimization model developed by Sheffi (1985) as follows, only if predetermining path set to ensure distances of all the used paths are less than the range limit for each O-D pair.

$$\min_v Z(\mathbf{v}) = -\sum_n \sum_{rs} q_n^{rs} S^{rs}[c^{\mathbf{rs}}(\mathbf{v})] + \sum_a v_a t_a(v_a) - \sum_a \int_0^{v_a} t_a(\omega) d\omega \tag{7}$$

s.t.: (1)(2)(3)(6)

Compared to Sheffi's model which can be solved as an unconstrained minimization problem and still yield a solution that satisfies the flow conservation constraints (1)(2)(3), the extra path distance constraints are the constraints that needs a careful consideration. To prove the equivalence between the solution of the problem given in Eq.(7) and the SUE equations, the first-order derivative of this problem have to coincide with the SUE conditions.

$$L(\mathbf{x}, \boldsymbol{\mu}) = -\sum_n \sum_{rs} q_n^{rs} S^{rs}[c^{\mathbf{rs}}(\mathbf{x})] + \sum_a x_a t_a(x_a) - \sum_a \int_0^{x_a} t_a(\omega) d\omega - \sum_n \sum_{rs} \sum_k \mu_{kn}^{rs} \cdot f_{kn}^{rs} \cdot (D_n - l_k^{rs}) \tag{8}$$

The first-order derivative require that

$$\nabla L(\mathbf{x}, \boldsymbol{\mu}) = 0 \tag{9}$$

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The gradient is taken with respect to link flow vector $\mathbf{x}$, and the derivatives of first three summation terms of Eq. (7) can be calculated as

$$\frac{\partial L(\mathbf{x},\boldsymbol{\mu})}{\partial x_b} = (-\sum_n \sum_{rs} \sum_{k \in K_{rs}} q_n^{rs} P_{kn}^{rs} \delta_{b,k}^{rs} + x_b)\frac{dt_b}{dx_b} - \sum_n \sum_{rs} \sum_{k \in K_{rs}} \mu_{kn}^{rs} \cdot (D_n - l_k^{rs})\delta_{b,k}^{rs} \qquad (10)$$

Note that the extra path distance constraints could be infeasible. For some OD pairs, if none of those paths connecting them satisfies the range limit of a certain class of vehicles, the travel demand between them of this class of vehicles cannot be assigned to the network and the problem has no feasible solution. However, those infeasible OD pairs can be found easily by checking distance of the shortest path from the start, and if the distance of shortest path is physically longer than range limit of a certain class of vehicles, then there would be no feasible path for the SUE TAP between this OD pair for this class of vehicles.

If it is feasible, which means there exists at least one path between each OD pair that is within the range limit of a certain class of vehicles. Then the term $\mu_{kn}^{rs} \cdot (D_n - l_k^{rs})$, which is path out-of-range cost incurred when the path length exceeds the distance limit of that class of vehicles should equal to zero. μ_{kn}^{rs} is a proxy of equivalent travel time value of the out-of-range cost per unit distance. Therefore, by ensuring the distance of each chosen path less than range limit, the derivative of the SUE objective function with respect to a link-flow variable becomes

$$\frac{\partial L(\mathbf{x},\boldsymbol{\mu})}{\partial x_b} = (-\sum_n \sum_{rs} \sum_{k \in K_{rs}} q_n^{rs} P_{kn}^{rs} \delta_{b,k}^{rs} + x_b)\frac{dt_b}{dx_b}, \forall b \qquad (11)$$

Assume that link performance functions are strictly increasing, the gradient becomes zero if and only if

$$x_b = \sum_n \sum_{rs} \sum_{k \in K_{rs}} q_n^{rs} P_{kn}^{rs} \delta_{b,k}^{rs}, \forall b \qquad (12)$$

The above equation expresses the SUE link flows when it has any feasible solution, namely whenever we assign travel demand to paths between each OD pair, path distance should be less than the range limit of that class of vehicles.

Following the same procedure of demonstrating uniqueness in page 319, Chap. 12, Sheffi (1985), it is obvious that the Hessian matrix of the SUE objective function is positive definite, because the second derivative of $\sum_n \sum_{rs} \sum_k \mu_{kn}^{rs} \cdot f_{kn}^{rs} \cdot (D_n - l_k^{rs})$ with respect to path flow equals to zero. Therefore the model possesses two vital propositions as follows.

Proposition 1: Any local feasible minimum $\mathbf{x}^*$ of this model satisfies the generalized SUE conditions, and the Lagrangian multipliers associated with path distance constraint (6) are path out-of-range costs.

Proposition 2: The SUE link flow pattern induced by any local minimum solution of the linear constrained minimization model is unique.

4. Solution method

4.1. Modified MSA algorithm

It was shown that the MSA algorithm can still be applied to the path distance constrained stochastic traffic assignment problem (STAP) with a direction-finding step different from that of classic STAP and feasibility check step.

For multinomial logit model (MNL), the steps are as follows:

Step 0: Feasibility check. For each OD pair, find the shortest path according to physical distance. If the distance of this path is longer than the range limit of a certain type of vehicle and the corresponding travel demand is positive, then there is no feasible path for this type of vehicle between this OD pair. Record this OD pair and infeasible vehicle type to Set A. If Set A is empty, go to the next step; if not, stop and display Set A.

Step 1: Initialization. Set $x_a(0)=0$, $t_a = t_a[x_a(0)]$. For each OD pair, find K shortest path for each class of vehicles in terms of free flow travel time. If the path distance is greater than the range limit of this class of vehicles, set the path travel time to infinite. Calculate the probability of choose each path, record them as initial path set and perform stochastic network loading to assign all the demand of each class of vehicles between this OD pair to the corresponding K shortest paths. This yields $\mathbf{x_a}(1)$. Set iteration counter $n=1$.

Step 2: Update. Calculate a new link cost in terms of $t_a = t_a[\mathbf{x_a}(1)], \forall a$.

Step 3: Direction finding. Follow the same procedure described in step 1 to find K shortest path for each class of vehicles based on the current set of link travel times, $\{t_a^n\}$. If all the K paths between an OD pair exceed range limit of this type of vehicle, use initial path set in step 1 and perform stochastic network loading. This yields an auxiliary link flow pattern $\{y_a^n\}$.

Step 4: Step size. $\{\alpha_n\}$ is a predetermined step size sequence satisfying the three conditions:

$$0<\alpha_n<1 \text{ and } \lim_{n\to\infty}\alpha_n = 0$$

$$\sum_{n=1}^{\infty}\alpha_n = +\infty$$

$$\sum_{n=1}^{\infty}\alpha_n^2 < \infty$$

There are a few step size sequences fulfilling the above conditions; for example

$$\alpha_n = \frac{\rho}{n}, n = 1,2,\ldots,\infty$$

where parameter $0<\rho\le 1$

Step 5: Move. Find the new flow pattern by setting $\mathbf{x_a^{n+1}} = \mathbf{x_a^n} + (1/n)(\mathbf{y_a^n} - \mathbf{x_a^n})$.

Step 6: Convergence test. Let

$$\bar{x}_a^{\,n} = \frac{1}{m}(x_a^n + x_a^{n-1} + \cdots + x_a^{n-m+1})$$

If the convergence criterion

$$\sqrt{\sum_a (\bar{x}_a^{\,n+1} - \bar{x}_a^{\,n})^2} \,/ \sum_a \bar{x}_a^{\,n} \le \kappa$$

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is met, stop and $\{\mathbf{x}_{\mathbf{a}}^{\mathbf{n+1}}\}$ is the set of equilibrium link flows; otherwise, set $n = n+1$ and go to step 2.

4.2. Modified probit-based loading algorithm

For multinomial probit model (MNP), the steps are

Step 0: Feasibility check. This step is the same as that of logit model.

Step 1: Sampling. Set iteration counter $n = 1$. Sample T_a^n from $T_a^n \sim N(t_a, \beta t_a)$ for each link a.

Step 2: All-or-nothing assignment. For each OD pair, find the distance-constrained shortest path based on perceived link travel time T_a^n. Calculate the path distance of shortest path for this class of vehicles, if the path distance exceeds range limit, calculate second shortest one, etc. If all the K path distances are greater than range limit, use the initial shortest path generated in Step 0. Assign the travel demand of each class of vehicles between this OD pair to the distance-constrained shortest path based on perceived link travel time T_a^n. This yields the set of link flow $\mathbf{X}_a^n$.

Step 3: Flow averaging. Let $\mathbf{x}_a^n = [(n-1)\mathbf{x}_a^{n-1} + \mathbf{x}_a^n] / n$.

Step 4: Stopping test. Let $\boldsymbol{\sigma}_a^n = \sqrt{\frac{1}{n(n-1)}\sum_{m=1}^{n}[\mathbf{X}_a^m - \mathbf{x}_a^n]^2}$, if $\max_a \boldsymbol{\sigma}_a^n / \mathbf{x}_a^n \le \kappa$,stop; otherwise, set $n = n+1$ and go to step 1.

5. Numerical example

Two numerical examples are adopted in this section to assess the proposed methodology.

5.1. Small network example

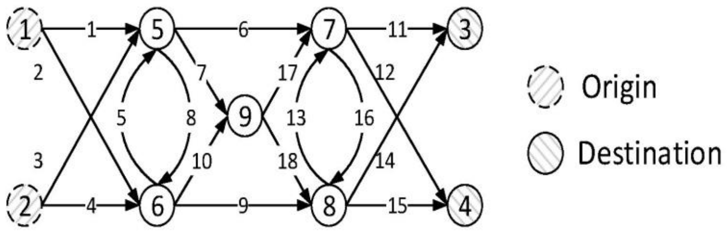

Fig. 1. Small network

The first example consists of 9 nodes, 18 links, and 4 OD pairs: (1,3), (1,4), (2,3), and (2,4), as shown in Figure 1. The free-flow travel time is used as a proxy for the link length for each link. Travel time on each link is defined by the following BPR (Bureau of Public Road) type function

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$$t_a(v_a) = t_a^0\left(1+0.15\times\left(\frac{v_a}{H_a}\right)^\rho\right), a\in A \quad (13)$$

where t_a^0 is the free flow travel time, H_a is the link a capacity and ρ is a prescribed parameter. OD demands are assumed to be the same for both GV and EV (Given in Table 1). Free-flow travel time and link capacity are indicated in Table 2.

Table 1. OD demand of small network example.

Origin \ Destination	3	4
1	10	20
2	30	40

We use this example to evaluate performance of proposed algorithms for solving both logit-based and probit-based SUE TAP with path distance constraints. The link flow patterns under two different cases, MNL and MNP with two classes of users (EV and GV), are estimated and compared in Table 2. EV and GV range limits are set as 20 and 100, respectively. K is set to be 8.

Table 2. The comparison of equilibrium EV and GV flows on the small network.

Link #	Link length	Link Capacity	MNL flow			MNP flow		
			EV, D=20	GV, D=100	EV+GV	EV, D=20	GV, D=100	EV+GV
1	5	40	16	16	32	12	14	25
2	6	30	14	14	28	19	16	35
3	3	50	70	24	94	61	35	96
4	9	80	0	46	46	9	35	44
5	9	30	0	0	0	0	21	21
6	2	60	86	40	126	81	20	101
7	8	30	0	1	2	10	29	39
8	4	30	0	0	1	19	22	40
9	6	90	14	59	72	7	22	29
10	7	30	0	1	1	2	29	31
11	3	30	35	20	55	29	20	49
12	6	30	42	18	60	24	28	52
13	2	30	9	4	16	42	26	68
14	8	30	5	20	25	12	20	32
15	6	30	18	42	60	36	32	67
16	4	30	0	1	2	2	22	23
17	4	40	1	1	3	12	33	45
18	8	30	0	0	0	0	26	26

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It is observed that the SUE assignment with different path distance limit resulted in different equilibrium link flows. For some links, e.g. link 1 & 2, both GV and EV obtained similar assignment results, while for the other links, it can be seen that the more EV use a certain link, the less GV choose that link, because the EV's path choices are restricted by its driving range and EV user prefer paths of short distance. When EV users crowded into those links with short distance, they became over-saturated, thus increasing corresponding link travel time, and GV user would rather use those unsaturated links to reduce their travel time to obtain equilibrium.

Comparing MNL with MNP, it is clear that there are 7 links close to zero flow in MNL while all the links have been assigned some flow in MNP. This result might come from Independence of Irrelevant Alternatives (IIA) property of MNL, which makes MNP more realistic even if it suffers from its low efficiency.

5.2. Sioux falls network example

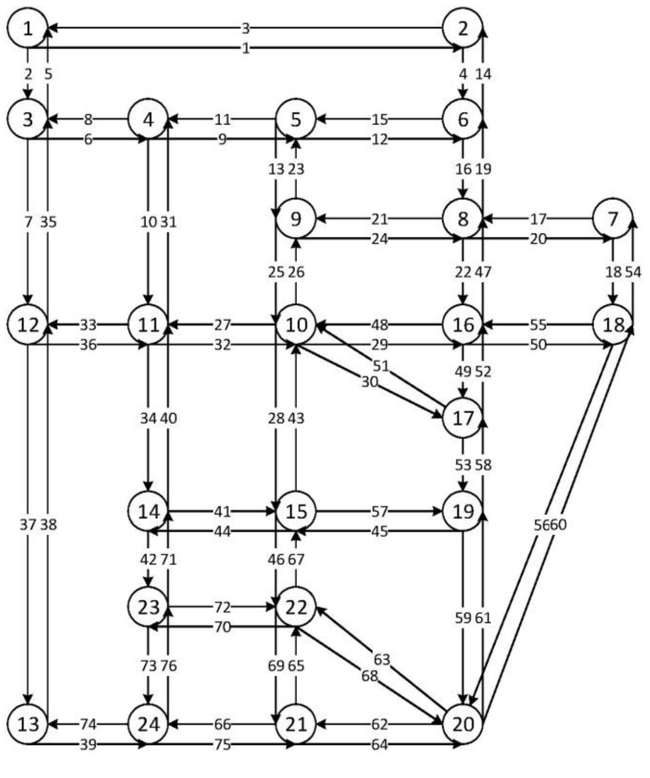

Fig. 2. Sioux Falls network

As shown in Figure 2, Sioux Falls network has a total of 24 nodes and 76 links. The travel demand table used in the application are from Suwansirikul et al. (1987).The link flow patterns under the same scenarios are compared. Without loss of generality, for the first three cases, only one class of vehicles are considered. Based on free-flow travel time, the range limit for EV is set to $\beta * \max_{(r,s)}\{\min_{k \in K^{rs}} l_k^{rs}\}$, where in this example, $\max_{(r,s)}\{\min_{k \in K^{rs}} l_k^{rs}\}$ is the

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maximum of all shortest path between each O-D pair and $\beta \geq 1$ is a parameter to ensure there is at least one feasible path connecting each OD pair. The K is set to 3 in K-shortest path algorithm.

A similar OD travel demand is applied to all the four experiments, including UE, MNL, MNP & MNL with MCU (Multiclass users). Two classes of users, i.e. EV and GV users, are involved in MNL with MCU, where the travel demand of each class is half of original travel demand, which means, the EV market share is 50%. For GV users, the range limit is set to $\beta = 10$.

The results illustrate that the equilibrium flows change significantly on a number of links. A few example links were selected randomly and their flow variations were observed in terms of different range limits (Figure 3).

As can be seen from Figure 2-6 that when β begins to increase, the network flows on these links behave in a different manner; UE flow patterns change little as β increases because the base range limit ($\beta = 1$) is the maximum distance of shortest paths among all OD pairs while UE usually requires shortest path (All-or-nothing Assignment, for example). Comparing with MNP, flow patterns resulting from MNL model change dramatically. As β continuously increases, the flow rates change mildly and finally converge to values without range limit constraints. However, these changes may not be necessarily monotone.

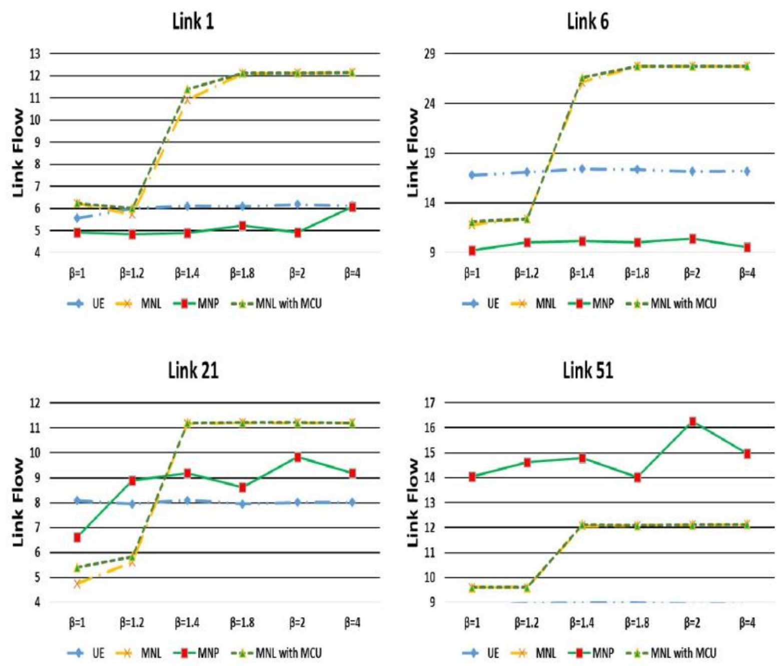

Fig. 3. Equilibrium flow pattern (a) link 1 (b) link 6 (c) link 21 (d) link 51.

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5.3. Sensitivity analysis

We conduct a sensitivity test with respect to the total demand in Sioux Falls network, multiplied its value by a constant factor and performed SUE assignment to observe the effect of the congestion level on algorithm performance. Since the Sioux Falls matrix is quite congested (Bekhor and Toledo, 2005), the factor ranges from 0.1 to 1.5, in intervals of 0.1. The number of iterations required by MNL and MNP to reach given convergence rate is presented in Figure 5. For all levels of demand, the modified MSA algorithm of MNL model requires less than 50 iterations to reach 0.01% precision level especially when demand level is low. However, comparing with MNL, the convergence rate of MNP model is quite slow, which requires more than 200 iterations to achieve 0.1 precision.

The performance of MNL and MNP for the Sioux Falls network is quite different in terms of convergence rate. For efficiency, the MNL model outperforms MNP. However, as is known to all, MNL model suffers from IIA property, and MNP might be better when distributed computing approaches are applied.

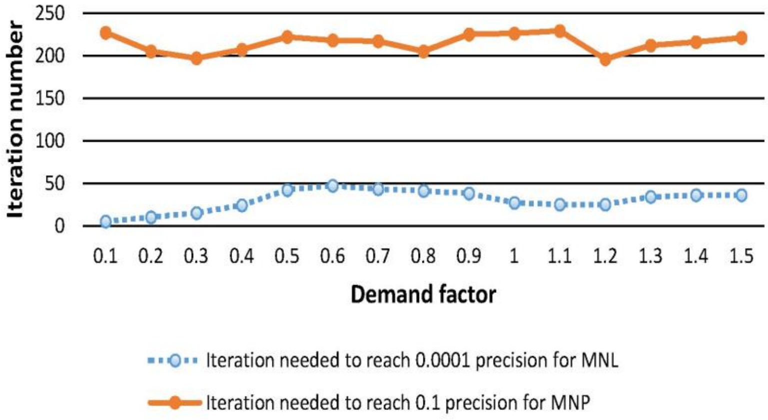

Fig. 4. Sensitivity of the algorithm performance to the level of demand.

6. Conclusions

This paper worked on the traffic assignment models with path distance constraints, where new SUE TAP is formulated, solved and numerically analysed. SUE models, which include perception error of travel time, are considered more rational than UE model. Multiclass users in SUE model represents a simplified case of current traffic networks that carry both EV and GV. More classes of users with various range limit can also be taken into consideration. The vehicles' range limit is determined based on its travel distance only, while rationally the range limit should be related to both travel distance and travel time.

This paper shows that at the equilibrium point the selected paths to assign the travel demand are different from that of basic SUE TAP. The distance of each path must be less than the range limit of that class of vehicles. The well-known and widely used MSA procedure and probit-based network loading method are adopted and modified to solve this problem, following the idea of putting the path distance constraints into the path selection rules of stochastic network loading procedure. The direction finding step for MNL, involves finding K feasible paths to load the travel demand between each OD pair, while it requires finding feasible shortest path for MNP in all-or-nothing assignment step. The proposed algorithm is easy to understand and implement. The application of the algorithms in

Sioux Falls network justifies the applicability of the solution procedures to general network with path-based constraints. The numerical analysis results show the impact of range limit on network equilibrium flows.

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4.3 Conclusion

The paper included in this chapter contributes to knowledge by developing new SUE traffic assignment model with path distance constraints. It can be seen as an extension of DUE model with the same constraints, which include perception error of travel time, are considered more rational than UE model. Multiclass users in SUE model represents a simplified case of current traffic networks that carry both EV and GV. More classes of users with various range limit can also be taken into consideration. For most cases, one realistic assumption is that the vehicle will have full battery level for each path, because drivers are basically rational to choose path which they are able to travel through without running out of battery. Stochastic battery levels or driving ranges are not necessary to be considered as well. Overall, the new model shows that at the equilibrium point the selected paths to assign the travel demand are different from that of basic SUE TAP. The distance of each path must be less than the range limit of that class of vehicles. The well-known and widely used MSA procedure and probit-based network loading method are adopted and modified to solve this problem, following the idea of putting the path distance constraints into the path selection rules of stochastic network loading procedure. The direction finding step for MNL, involves finding K feasible paths to load the travel demand between each OD pair, while it requires finding feasible shortest path for MNP in all-or-nothing assignment step. The proposed SUE model with driving distance is therefore adopted to achieve reliable outputs from tasks related to SUE EV flow patterns in remaining research components.

- Driving distance constraints and lack of public charging facilities are identified as EV's key characteristics in research component 1, Chapter 2.
- Driving distance constraints and public charging facilities are considered in research component 4, Chapter 6.
- Driving distance constraints are extended to a more general case-battery capacity constraint in general SUE model in research component 3, Chapter 5.
- Driving distance constraints are used to calculate the swapping demand for EBs in research component 5, Chapter 7.

The next chapter of the thesis presents new general SUE model of EVs with limited battery capacity where the vehicles' range limit is determined by both travel distance and travel time, which corresponds to research component 3.

CHAPTER 5 STOCHASTIC TRAFFIC ASSIGNMENT OF ELECTRIC VEHICLES WITH FLOW-DEPENDENT BATTERY CAPACITY CONSTRAINTS

5.1 Introduction

The previous chapter describes new model to predict the EV flow pattern in a mixed GV and EV flow network under general SUE principle, corresponding to research component 2. The proposed driving distance constraints and the availability of public charging stations will be adopted or extended to achieve reliable outputs in the remaining chapters involving SUE models with distance limits.

This chapter presents the results from research component 3. Previous research has shown that the EV energy consumption rate does not only depend on driving distance but also travel speed. Given the impacts of travel speed on the battery energy consumption (Bigazzi,Clifton 2015; Agrawal et al. 2016), the effects of combining flow-dependent energy consumption on BEVs' route choice behaviour should be explored with battery capacity constraint. In existing general SUE models with side constraints, only link capacity constraints have been considered (Meng et al. 2008) and battery capacity constraints have only been studied in DUE models. This chapter therefore aims to develop new general SUE models for EVs which considers flow-dependent battery energy consumption. The research gap and objective associated with this research component are provided in Table 5-1.

Table 5-1: Research gap and opportunities associated with research component 3

Research topic	Research gaps	Research opportunities
SUE models of transport network with electric vehicles (Part II)	No studies have considered a SUE model of BEVS with battery capacity constraints where BEVs' range limit is restricted by both travel distance and travel time. (see section 2.5.3)	Proposing a general SUE model and solution algorithm for BEVs with limited battery capacity that restricts BEVs' travel time and travel distance. (see Chapter 5)

This chapter begins with a description of DUE models with range limit constraints, including driving distance limit and battery capacity constraints. General SUE model with side constraints, such as link capacity constraints are then presented. Algorithms for solving the proposed model are discussed. This chapter continues by validating the proposed general SUE models of EVs with limited battery capacity using Lagrangian dual and gradient projection algorithms, followed by a conclusion.

This paper is included in this chapter: *Jing W, Ramezani M, An K, et al. Congestion patterns of electric vehicles with limited battery capacity[J]. PloS one, 2018, 13(3): e0194354-e0194354.*

5.2 Paper 2: Congestion patterns of electric vehicles with limited battery capacity

The following paper details a more complicated STAP in transportation networks with BEVs owing to the fact that BEV energy consumption depends on not only the path distance but also the travel time. The main objective was to theoretically understand how a flow-dependent path-based constraint can be incorporated into a general SUE model. It begins by discussing the battery capacity constraint was a flow-dependent one, while path distance constraint was flow-independent. The flow-independent driving distance constraint in research component 2 can be processed in the route choice procedures, while the flow-dependent battery energy consumption depends on not only distance but also traffic flow (travel time). A mathematical programming model was proposed for the flow-dependent path-based SUE traffic assignment. A convergent Lagrangian dual method was employed to transform the original problem into a concave maximization problem and a customized gradient projection algorithm was developed to solve it. A column generation procedure was adopted to generate the path set. The solution framework, Lagrangian dual-gradient projection-stochastic network loading, can be applied to solve path-based SUE problem. Finally, two numerical examples are presented to demonstrate the applicability of the proposed model and the solution algorithm.

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RESEARCH ARTICLE

Congestion patterns of electric vehicles with limited battery capacity

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Abstract

The path choice behavior of battery electric vehicle (BEV) drivers is influenced by the lack of public charging stations, limited battery capacity, range anxiety and long battery charging time. This paper investigates the congestion/flow pattern captured by stochastic user equilibrium (SUE) traffic assignment problem in transportation networks with BEVs, where the BEV paths are restricted by their battery capacities. The BEV energy consumption is assumed to be a linear function of path length and path travel time, which addresses both path distance limit problem and road congestion effect. A mathematical programming model is proposed for the path-based SUE traffic assignment where the path cost is the sum of the corresponding link costs and a path specific out-of-energy penalty. We then apply the convergent Lagrangian dual method to transform the original problem into a concave maximization problem and develop a customized gradient projection algorithm to solve it. A column generation procedure is incorporated to generate the path set. Finally, two numerical examples are presented to demonstrate the applicability of the proposed model and the solution algorithm.

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Introduction

Battery electric vehicles (BEVs) have received much attention in the past few years due to their advantages in reducing greenhouse gas emissions, noise pollution, reliance on fossil oil and improving the efficiency of electricity grid by vehicle-to-grid technology [1]. Similarly, most of the autonomous vehicles currently under development appear to be electric powered. When they are matured enough to be wide-spread in car manufacturing market, electrified autonomous vehicles are likely to change travel behavior and traffic patterns. Governments and automotive manufacturers have recognized the value of these vehicles in helping the environment and are encouraging BEV ownership through economic incentives and public charging station deployment [2]. Currently, however, BEV users still suffer from the inconvenience of limited driving range, long charging time and insufficient public charging stations [3]. In addition, range anxiety of BEV drivers will inevitably add a certain level of restrictions to BEV drivers' path choices, at least for a long future period prior to the coverage of recharging infrastructures reaching a sufficient level [4], especially for those BEVs with limited battery capacities.

BEV companies are trying to overcome this limited range requirement by implementing fast charging stations, where a vehicle can be charged in minutes rather than hours to full capacity [5]. However, operating fast charging stations is costly and fast charging reduces the life of a battery due to the irreversible damages to charging cells [6]. Despite the development of fast charging techniques, BEVs still take more time to recharge than the time needed for a standard gasoline vehicle to refuel. Hence, BEV commuters are more likely to charge their vehicles at home rather than at stations [7].

Nevertheless, insufficient charging stations and limited driving range for BEVs make traffic assignment problem (TAP) more challenging due to the incorporation of path distance constraints and battery capacity constraints. The existing TAP models should be modified to better describe commuters' behavior with the prevalence of BEVs. There have been many endeavors to address this problem. Among which, some studies enforce flow of a path to be zero if the path distance is greater than the driving range limit of BEVs. The classic Frank-Wolfe method with a constrained shortest path algorithm can be applied to solve this problem under deterministic user equilibrium (DUE) [8]. As an extension of static path distance constraint, stochastic range anxiety resulting in stochastic path distance constraint has been considered in networks [9–11]. Network equilibrium problem was further addressed when modeling transportation networks that accommodated both gasoline vehicles (GVs) and BEVs [4, 12, 13]. A multi-class dynamic user equilibrium model was proposed to evaluate the performance of the mixed traffic flow network, where GVs chose paths with minimum travel time and BEVs selected paths to minimize the generalized costs including travel time, energy cost and range anxiety cost. It was also pointed out that the BEV energy consumption rate per unit distance traveled is lower at moderate speed than at higher speed resulting in an equilibrium that BEVs choose paths with lower speed to conserve battery energy [14]. Relay/charging requirement has been taken into account in network equilibrium problems and was formulated as a nonlinear integer programming [15]. It was found that traffic congestion would affect fuel economy of BEVs and BEVs might become more fuel-efficient as the average speed increases, particularly at local arterials [16]. Hence, another work considered recharging time based on flow-independent energy consumption in the base network equilibrium model and further extended the proposed DUE model with flow-dependent energy consumption assumption [3].

However, a more realistic and general situation is that travel time is a random variable and is perceived by travelers in an imperfect, stochastic manner. For example, travel time varies due to stochastic traffic flow conditions. Moreover, battery energy consumption rate is demonstrated to be not only distance-dependent but also time-dependent because it is pointed out that heating and air-conditioning systems of BEVs may consume a substantial amount of energy of the total battery capacity and reduce the BEV's range limit [17]. Therefore it can be estimated well if link flow volume can be predicted more precisely. Although the stochastic user equilibrium (SUE) principle plays a more realistic role than DUE principle in describing road user's path choice behavior, the general SUE TAP considering both multinomial logit (MNL) and multinomial probit (MNP) loading with driving range limit constraints has received little attention because of its complexity. As a rational extension of general SUE with flow-independent path distance constraints [18, 19], the SUE traffic assignment model considering flow-dependent link energy consumption is referred to as general SUE traffic assignment with battery capacity constraints.

Previously proposed side constraints for the TAP are basically imposed on traffic flows through nodes, links, paths or O-D pairs and may be grouped into two major categories, i.e. link-based and path-based constraints [9]. The first category is generally referred to as the stochastic TAP with link capacity [see e.g., [20–22]]. A milestone is Meng's linearly constrained minimization model, which is a general SUE traffic assignment problem with link capacity constraints. This work was inspired by the stochastic social optimum (SSO) traffic assignment with the objective of minimizing the total perceived travel time by Maher, Stewart [23]. Maher, Stewart [23] found that the solution of SSO could be achieved by solving a SUE problem using the marginal cost function. Meng, Lam [22] demonstrated that SUE flow pattern could be generated by solving an SSO problem applying a modified link travel time function. Early developed algorithms to solve the unconstrained logit-based SUE problem were link-based, e.g. [24]. These link-based algorithms do not require path storage and often use Dial's STOCH algorithm or Bell's alternative as the stochastic loading step [25, 26].

The second category, the stochastic TAP with path-based constraints, has received much less attention. Only deterministic TAP with flow-independent path-based constraints under BEV scheme has been considered in these studies [3, 4, 8, 12, 15, 19, 27]. In general, they assumed that BEV users could charge only at trip origins and destinations, that the distance of any feasible trip must not exceed the given distance limit [8, 9, 12]. However, for general SUE TAP, no research has taken flow-dependent path energy consumption constraints into the general SUE TAP model.

The main challenge of using a path-based algorithm in the past is the memory requirement. This restriction has been relaxed considerably in recent years due to rapid advances in the computing resources. Different from link-based algorithms above, path-based algorithms require explicit path storage to directly compute the logit path choice probabilities. Olof, Jan [28] developed a path-based algorithm based on the disaggregated simplicial decomposition (DSD) algorithm to solve the MNL SUE problem. Among the path-based algorithms for the traffic equilibrium problem with additive path costs, much attention has been paid to the DSD algorithm and the gradient projection (GP) algorithm [29]. GP algorithm has been shown as a successful path-based algorithm for solving traditional traffic equilibrium problem with additive and non-additive path costs due to its global convergence and simple implementation [30]. A comparison work of evaluating the performance and robustness of these two path-based algorithms can be found in Chen and Lee [31]. Furthermore, to investigate the impact of step size scheme, different step size strategies of the path-based algorithms developed to solve the C-logit SUE models based on an adaptation of the GP method were investigated in [32]. Another inevitable problem of the path-based problem is the way of generating paths. A possible alternative path set can be obtained from a path choice set generation algorithm [33]. Behaviorally, this has an advantage of explicitly identifying paths which are most likely to be used and also allows a greater flexibility to include path-specific attributes that might not be obtainable directly from the link attributes [33, 34].

This paper is concerned with a general SUE TAP with battery capacity constraints, as an extension and generalization of the previous DUE TAP version with driving distance constraints or battery capacity constraints. To the best of our knowledge, it remains to be an open question to find an exact solution method for solving the general SUE problem with path-based constraints, incorporating column generation to avoid path enumeration on a transport network with BEV.

Meng, Lam [22] proposed a solution framework combining Lagrangian dual (LD) method with GP algorithm for general SUE problem with link capacity constraints. However, due to the existence of the implicit path-specific battery out-of-energy function, it remains uninvestigated if this framework can also be applied for path-based constraints. Hence, for the general SUE TAP, we also adopt the solution method framework of combining LD with GP and prove its applicability. Specifically, the path set in this paper is generated prior to the assignment using column generation procedure which has been embedded in the GP algorithm [35] to avoid path enumeration. The GP algorithm iteratively updates the Lagrangian multiplier corresponding to each path, until the optimal solution is obtained.

To sum up, the contributions of this study are threefold. Firstly, to enrich the general SUE family with side constraints (link-based and path-based) and make consistence with side-constrained general DUE condition, it is believed that this is the first paper studying a general SUE model with path-based constraints. Secondly, a holistic methodology is proposed for general SUE traffic assignment model with battery capacity constraints on BEV scheme, in which the path choice is restricted by the battery capacity with a single charge. Thirdly, a Lagrangian dual based exact solution method incorporating column generation is developed for solving this path-constrained general SUE model.

The remainder of this paper is organized as follows. In Sections 2, we elaborate the problem formulation and analyze its solution properties. Section 3 presents an LD reformulation and details its algorithmic implementations by incorporating a convergent GP subroutine and column generation procedure. Section 4 presents the numerical results of applying the algorithm procedure to two case studies. Section 5 provides the concluding remarks.

Notation, problem description and model formulation

Let us assume the transport network is modeled as a connected graph, denoted by $G = (N,A)$, where N and A are sets of nodes and links, respectively. (r,s) stands for certain ordered pairs of nodes, $r \in R$ and $s \in S$,where node r is an origin and node s is a destination. $R \subset N$ and $S \subset N$ are sets of origins and destinations, respectively. There are non-negative travel demands q^{rs} between (r,s). $\mathbf{q} = (q^{rs})^{\mathrm{T}}, \forall (r,s)$ is a column vector for all the travel demands. Let K_{rs} be the set of paths connecting O-D pair (r,s), f_k^{rs} be traffic flow on path $k \in K_{rs}$, $\mathbf{f}^{rs} = (f_k^{rs})^{\mathrm{T}}, k \in K_{rs}$ be a column vector of all these path flows between O-D pair (r,s), and $\mathbf{f} = (\mathbf{f}^{rs})^{\mathrm{T}}, \forall (r,s)$ be a column vector of all the path flows over the entire network. Let v_a denote traffic flow on link $a \in A$ and $\mathbf{v} = (v_a)^{\mathrm{T}}, a \in A$ is a column vector of all link flows. The path flows and link flows should comply with fundamental flow conservation equations:

$$v_a = \sum_{(r,s)} \sum_k f_k^{rs} \delta_{a,k}^{rs}, \forall a \in A \tag{1}$$

$$q^{rs} = \sum_k f_k^{rs}, \forall (r,s) \tag{2}$$

$$f_k^{rs} \geq 0, \forall (r,s), k \in K_{rs}, \tag{3}$$

where $\delta_{a,k}^{rs} = 1$ if path $k \in K_{rs}$ between O-D pair (r,s) traverses link $a \in A$, and 0 otherwise.

Let $t_a(v_a)$ denote the separable travel time function of link $a \in A$ that is assumed to be a positive, strictly increasing, convex and continuously differentiable function of the traffic flow on the link. All the link travel time functions are grouped into a column vector.
$\mathbf{t}(\mathbf{v}) = (t_a(v_a))^{\mathrm{T}}, a \in A$. Travel time on path $k \in K_{rs}$ between O-D pair (r,s) can be considered as a function of all the path flows, denoted by $c_k^{rs}(\mathbf{f})$ with the expression

$$c_k^{rs}(\mathbf{f}) = \sum_a t_a(v_a) \delta_{a,k}^{rs} \tag{4}$$

To generate a SUE flow pattern by solving a SSO problem, a modified link travel time function $\bar{t}_a(v_a)$ corresponding to link travel time function $t_a(v_a)$,$a \in A$ is defined to be positive, strictly increasing and continuously differentiable [22].

$$\bar{t}_a(v_a) = \begin{cases} \dfrac{\int_0^{v_a} t_a(x)\mathrm{d}x}{v_a}, v_a > 0 \\ t_a(0), v_a = 0 \end{cases} \quad (5)$$

With modified link travel time functions $\{\bar{t}_a(v_a), a \in A\}$, the corresponding path modified travel time can be expressed as

$$\bar{c}_k^{rs}(\mathbf{f}) = \sum_{a\in A} \bar{t}_a(v_a)\delta_{ak}^{rs}, \forall(r,s), k \in K_{rs} \quad (6)$$

Let $\bar{c}^{rs}(\mathbf{f}) = (\bar{c}_k^{rs}(\mathbf{f}))^{\mathrm{T}}, k \in K_{rs}$ be a column vector of all the modified path travel times between O-D pair (r,s). In terms of any positive feasible path flow pattern $\mathbf{f}$, there are continuously differentiable path-specific dummy functions, $\bar{d}^{rs}(\mathbf{f}^{rs}) \in \Re^{|K_{rs}|}, \forall(r,s)$, so that the conventional SUE conditions associated with the path travel time functions can be satisfied by path flow pattern $\mathbf{f}$, $\{(\bar{c}^{rs}(\mathbf{f}) + \bar{d}^{rs}(\mathbf{f}^{rs})\}, \forall(r,s)$, namely

$$f_k^{rs} = q^{rs} \cdot P_k^{rs}((\bar{c}^{rs}(\mathbf{f}) + \bar{d}^{rs}(\mathbf{f}^{rs})) \quad (7)$$

where $P_k^{rs}((\bar{c}^{rs}(\mathbf{f}) + \bar{d}^{rs}(\mathbf{f}^{rs}))$ is referred to as the probability of choosing a given path that has the minimum perceived generalized path cost and $P_k^{rs}((\bar{c}^{rs}(\mathbf{f}) + \bar{d}^{rs}(\mathbf{f}^{rs})) = Pr(U_k^{rs} < U_l^{rs}, \forall l \in K_{rs})$. The perceived generalized path cost of any path $k \in K_{rs}$ connecting O-D pair (r,s), U_k^{rs} is random variable, where $U_k^{rs} = \bar{c}_k^{rs} + \bar{d}_k^{rs} + \varepsilon_k^{rs}$. ε_k^{rs} is the random perception error of the path cost and $\bar{d}_k^{rs}$ represents an additional additive cost variable across all links associated with path $k \in K_{rs}$ to fulfill Eq (7) which is the SUE condition. At the optimum, the additional path-specific cost term $\bar{d}_k^{rs*}(f^{rs}) = \sum_a v_a(\partial\bar{t}_a/\partial v_a)\delta_{a,k}^{rs}$, thus $\bar{c}_k^{rs} + \bar{d}_k^{rs} = \sum_{a\in A}(\bar{t}_a + v_a(\partial\bar{t}_a/\partial v_a))\delta_{a,k}^{rs}$ representing the induced marginal system cost if a new traveler is added into the system traversing on path $k \in K_{rs}$ connecting O-D pair (r,s) [36] or the so-called marginal social cost [23]. The analytical expressions of $\bar{d}^{rs}(\mathbf{f}^{rs})$ is presented under logit-based SUE conditions in [23, 36]. The path-specific cost $\bar{d}_k^{rs}(f^{rs})$ can be expressed as,

$$\bar{d}_k^{rs} = -\frac{1}{\rho}\ln[\frac{f_k^{rs}}{q^{rs}}\sum_k \exp(-\rho(\bar{c}_k^{rs} + \bar{d}_k^{rs}))] - \bar{c}_k^{rs}, \forall(r,s), k \in K_{rs}. \quad (8)$$

where ρ is the scale parameter of the logit model.

To simplify the traffic network modeling, only BEV (as an alternative traffic mode) is considered and a set of assumptions regarding demand heterogeneity and travel behaviors are considered. First, it is assumed that the demand population is only comprised of a single class of BEV. Certainly, if needed, multiple types of BEV with different battery capacities, initial battery charging state (fully charged or not), range anxiety level (a safety margin that BEV drivers would like to reserve before battery depletes) and energy consumption functions can be readily incorporated into the model without changing the problem's nature and model's structure [12].

Second, we assume a given fixed travel demand and SUE principle. In other words, stochasticity and elasticity of travel demand are not considered regardless of its stochastic nature [37, 38]. Each BEV traveler chooses a path that minimizes his/her perceived travel cost and no one can reduce his/her perceived cost by unilaterally switching to an alternative path. The travel cost consists of two parts: path energy consumption and possible battery out-of-energy cost.

When BEV runs out of battery before reaching destination, battery out-of-energy cost occurs, e.g. a roadside assistance cost. Furthermore, without loss of generality, we assume that BEV travelers use a common form of systematic travel cost for determining their travel choices.

In our network equilibrium analysis, we implicitly assume that all BEV are fully charged at their origins (e.g. home garages), and there is no battery-charging or battery-swapping stations in the network. In most transportation networks, it may take a number of years to deploy sufficient electricity-recharging infrastructures for achieving a certain level of coverage. Consequently, BEV users would choose the path whose energy consumption is less than or equal to the battery capacity, denoted by D. Although it is difficult to foresee how future developments in battery and vehicular technologies may enhance the fuel economy of BEVs at various traffic conditions, the link energy consumption in this paper is assumed to increase with the increasing energy consumption of heating and air-conditioning system over time, which is a linear function of link length and modified link travel time, namely, $e_a(v_a) = \alpha l_a + \beta \bar{t}_a(v_a), \forall a \in A$ [3]. The authors, however, do not claim the applicability and suitability of the defined energy consumption function for accurate quantification of link energy consumption. One must consider the relationship between energy consumption and travel time (speed). Note that $\bar{t}_a(v_a)$ is the modified link travel time function. In practice, each path $k \in K_{rs}$ would have a path energy cost and EV drivers have perception error on this cost. Any feasible path flow pattern should satisfy the battery capacity constraints:

$$f_k^{rs}(D - \alpha l_k^{rs} - \beta \bar{c}_k^{rs}) \geq 0, \forall (r,s), k \in K_{rs} \tag{9}$$

With the above battery capacity constraints, the generalized path travel cost is defined by

$$\hat{c}_k^{rs} = \alpha l_k^{rs} + \beta \bar{c}_k^{rs} \tag{10}$$

which means that if the energy consumption is smaller than or equal to the battery capacity, the flow of BEV users going through path k is nonnegative,; otherwise, the trip flow should be equal to zero.

$$\begin{cases} f_k^{rs} \geq 0, \text{if } \alpha l_k^{rs} + \beta \bar{c}_k^{rs} \leq D \\ f_k^{rs} = 0, \text{if } \alpha l_k^{rs} + \beta \bar{c}_k^{rs} > D \end{cases}, \forall (r,s), k \in K_{rs} \tag{11}$$

Remark 1. If we set $\alpha = 0$, the problem would turn into a travel time-constrained SUE TAP; if setting $\beta = 0$, it becomes a SUE TAP with driving distance constraints. Therefore, flow-dependent battery capacity constraint is a generalization of BEV's driving distance constraint.

Model formulation

This section introduces the general SUE traffic assignment model in terms of path flows with battery capacity constraints as follows:

$$\min Z(f) = \sum_r \sum_s q^{rs} S^{rs}(\bar{\mathbf{c}}^{\mathbf{rs}}(\mathbf{f}) + \bar{\mathbf{d}}^{\mathbf{rs}}(\mathbf{f}^{\mathbf{rs}})) - \sum_r \sum_s \sum_k \bar{d}_k^{rs}(f^{rs}) \cdot f_k^{rs} \tag{12}$$

s.t. (1), (2), (3), (9)

where $S^{rs}(\bar{\mathbf{c}}^{\mathbf{rs}}(\mathbf{f}) + \bar{\mathbf{d}}^{\mathbf{rs}}(\mathbf{f}^{\mathbf{rs}})) = E[\min\{\bar{\mathbf{c}}^{\mathbf{rs}}(\mathbf{f}) + \bar{\mathbf{d}}^{\mathbf{rs}}(\mathbf{f}^{\mathbf{rs}})\}]$ is the satisfaction function, i.e., the expected value of the minimum perceived travel time for travelers between O-D pair (r,s). The satisfaction function is a continuously differentiable, concave function [39]. Compared to the Meng's model [22] for general SUE with link capacity constraints, the difference lies in the constraints of model (12). This model also possesses two vital propositions as follows.

Proposition 1. Any local minimum $\mathbf{f}^*$ of the minimization model (12)- satisfies the generalized SUE conditions, and the optimal Lagrangian multipliers associated with battery capacity constraint (9) are battery out-of-energy costs.

Proposition 2. The SUE link flow pattern induced by any local minimum solution of the minimization model (12)- is unique.

The mathematical proof of proposition 1 can be accessed in the appendix file "S1 Text", while proposition 2 can be proved by following exactly the same procedure as in Meng, Lam [22] and substituting μ_a in their work with φ_a in Eq (17).

Problem feasibility

The extra battery capacity constraint in the above model could result in problem infeasibility. If the energy consumption of all the paths connecting an O-D pair exceeds the battery capacity, the travel demand between this O-D pair cannot be assigned to the network without causing additional battery out-of-energy cost. The infeasible O-D pairs can be detected by comparing minimum energy cost path with battery capacity under free flow scenarios.

Solution method

Lagrange Dual (LD) method. The objective function (12) includes an inexplicit path-specific dummy functions, $\bar{d}^{rs}(\mathbf{f}^{rs})$, therefore the original problem cannot be solved directly. Nevertheless, LD formulation of the original model can be established to examine if the proposed algorithms can successfully solve the proposed problem. The solution equivalence between the original problem and the LD problem can be realized if the dual problem is maximized with respect to the Lagrangian multipliers according to the dual theorem.

In order to get the optimal Lagrangian multipliers with respect to the battery capacity constraint in the minimization model, the LD maximization is defined as

$$\max_{\mu \geq 0} L(\boldsymbol{\mu}) \tag{13}$$

$$L(\boldsymbol{\mu}) = \min_{\mathbf{f} \in \Omega} [Z(\mathbf{f}) + \sum_r \sum_s \sum_k \mu_k^{rs} f_k^{rs} (\alpha l_k^{rs} + \beta \tilde{c}_k^{rs} - D)] \tag{14}$$

where $\boldsymbol{\mu} = (\cdots, \mu_k^{rs}, \cdots) \in R^{|K_{rs}|}$, where μ_k^{rs} is the Lagrangian multiplier associated with battery capacity constraints (9), where $|K_{rs}|$ denotes the number of elements in set K_{rs}. Ω is the set of all the feasible path flows without consideration of battery capacity constraints, i.e. $\Omega = (\mathbf{f}|\mathbf{f}$ satisfies Eqs (1), (2) and (3)). $\boldsymbol{\mu}$ acts as a role to convert the battery capacity constraints (9) into the objective function (12). Moreover, $L(\boldsymbol{\mu})$ is a concave function with respect to non-negative Lagrangian parameter μ_k^{rs}.

Following the same procedure in the proof of Proposition 1, it can be demonstrated that any local minimum of above concave function $L(\boldsymbol{\mu})$ fulfills the conventional SUE conditions (see Eq 6 in Meng, Lam [22]) in terms of the generalized path travel cost function. The well-defined generalized path travel time function is

$$\tilde{c}_k^{rs} = c_k^{rs} + \mu_k^{rs}(\alpha l_k^{rs} + \beta c_k^{rs} - D), \forall (r,s), k \in K_{rs} \tag{15}$$

where $\mu_k^{rs}(\alpha l_k^{rs} + \beta c_k^{rs} - D)$ is called the battery out-of-energy cost incurred when the battery energy needed to travel through a given path exceeds the battery capacity of the BEV. The generalized path travel cost and it should satisfy the following conditions:

$$\begin{cases} \mu_k^{rs} = 0, \text{if } \alpha l_k^{rs} + \beta \bar{c}_k^{rs} \leq D \\ \mu_k^{rs} \geq 0, \text{if } \alpha l_k^{rs} + \beta \bar{c}_k^{rs} > D \end{cases} \tag{16}$$

Hence, travel time experienced by a driver on a path consists of two parts: normal travel cost and additional cost incurred when energy needed exceeds the battery capacity. The accumulation of the battery out-of-energy cost on a link a is defined as:

$$\sum_r \sum_s \sum_k \mu_k^{rs}(\alpha l_k^{rs} + \beta \bar{c}_k^{rs} - D)\delta_{ak}^{rs} = \sum_r \sum_s \sum_k \lambda_k^{rs}\delta_{ak}^{rs} = \sum_a \varphi_a \delta_{ak}^{rs} \tag{17}$$

where φ_a accounts all the paths going through it. According to the generalized path travel cost, the generalized SUE conditions that take battery capacity constraints into consideration can be defined as follows.

$$f_k^{rs} = q^{rs} \cdot P_k^{rs}(\bar{c}^{rs}(f) + \mu_k^{rs}(\alpha l_k^{rs} + \beta \bar{c}_k^{rs} - D)), \forall (r,s), k \in K_{rs} \tag{18}$$

$$f_k^{rs}(D - \alpha l_k^{rs} - \beta \bar{c}_k^{rs}) \geq 0, \forall (r,s), k \in K_{rs} \tag{19}$$

$$\mu_k^{rs} \geq 0, \forall (r,s), k \in K_{rs} \tag{20}$$

$$\mu_k^{rs} \cdot f_k^{rs}(D - \alpha l_k^{rs} - \beta \bar{c}_k^{rs}) = 0, \forall (r,s), k \in K_{rs} \tag{21}$$

The generalized link travel time functions can be defined as:

$$\tilde{t}_a(v_a) = \bar{t}_a(v_a) + \varphi_a, a \in A \tag{22}$$

For any given $\boldsymbol{\mu} \geq 0$, let $v(\boldsymbol{\mu})$ be the link flow pattern induced from a local minimum of the minimization problem shown in right-hand side of (14). Following the similar proof in Meng, Lam [22], $v(\boldsymbol{\mu})$ is a unique SUE link flow pattern for networks with the modified path travel time functions and Lagrangian dual formulation (13) is a continuously differentiable concave maximization model. The uniqueness of the optimal link flow solution implies that the gradient of $L(\boldsymbol{\mu})$ is:

$$\nabla L(\boldsymbol{\mu}) = (\cdots, f_k^{rs}(\boldsymbol{\mu})(\alpha l_k^{rs} + \beta \bar{c}_k^{rs} - D), \cdots)_{|K_{rs}|} \tag{23}$$

Applying the Karush-Kuhn-Tucker (KKT) conditions to (13) can lead to proposition 3 that

Proposition 3. Assume that $\boldsymbol{\mu}^*$ is an optimal solution of the LD maximization model (13). $v(\boldsymbol{\mu}^*)$ is the SUE link flow pattern with battery capacity constraint.

Hence, the LD formulation (13) can be efficiently solved by a global convergent GP method with iterative solution updating scheme:

$$\boldsymbol{\mu}^{(n+1)} = P_{R_+^{|A|}}[\boldsymbol{\mu}^{(n)} + \alpha_n \nabla L(\boldsymbol{\mu}^{(n)})] \tag{24}$$

where n is the number of iterations; $P_{R_+^{|A|}}[\boldsymbol{\mu}^{(n)} + \alpha_n \nabla L(\boldsymbol{\mu}^{(n)})]$ is the projection of vector $\boldsymbol{\mu}^{(n)} + \alpha_n \nabla L(\boldsymbol{\mu}^{(n)})$ onto the $|A|$-dimensional non-negative orthant, i.e., $R_+^{|A|}$; and the projection operation $P_{R_+^{|A|}}[\cdot]$ is defined by

$$P_{R_+}[\mathbf{y}] = \arg \min_{\mathbf{x} \in R_+} \sum_{a \in A} (x_a - y_a)^2. \tag{25}$$

Furthermore, $\{\alpha_n\}$ is step size sequence and given at any point $\boldsymbol{\mu}^{(n)} \in \mathbf{Q}$, where $\mathbf{Q}$ is the feasible set, denoted by

$$\boldsymbol{\mu}^{(n)}(\alpha_n) = P_{\mathbf{Q}}[\boldsymbol{\mu}^{(n)} + \alpha_n \nabla L(\boldsymbol{\mu}^{(n)})], \quad \alpha_n \geq 0 \tag{26}$$

The unique projection of the vector $[\boldsymbol{\mu}^{(n)} + \alpha_n \nabla L(\boldsymbol{\mu}^{(n)})]$ on $\mathbf{Q}$ where $\alpha_n \geq 0$ is a nonnegative scalar parameter. Since the feasible set of $\boldsymbol{\mu}$ is the whole nonnegative orthant, the Lagrangian multiplier updating formula shown in (24) can be rewritten in the following way:

$$\mu_l^{ij(n+1)} = \max\{0, \mu_l^{ij(n)} + \alpha_n f_k^{rs}(\boldsymbol{\mu}^{(n)})(\alpha l_k^{rs} + \beta \bar{c}_k^{rs} - D)\}, \forall (i,j), l \in K_{ij} \tag{27}$$

It has been proved that without the requirement of the Lipschitz condition, every limit point of the sequence $\{\boldsymbol{\mu}^{(n)}\}$ generated by the GP algorithm is a stationary point, as well as a solution point. For step size, a predetermined step size which has simple structure and is commonly used by [Meng and Liu [20], Meng, Lam [22]] is applied in this model instead of the generalized Armijo rule, which belongs to the inexact line search strategies and is for constrained minimization problems. The reason lies in that at each step, the gradient information of the objective function and objective function evaluations are required to determine an appropriate step size to improve the solution when using Armijo rule.

Proposition 3 confirms that solving the SUE link flow pattern with battery capacity constraint can be obtained by solving LD maximization model (13). Although the LD function $L(\boldsymbol{\mu})$ does not possess an explicit expression, its gradient for any $\boldsymbol{\mu} \geq 0$ can be evaluated by implementing a conventional SUE traffic assignment procedure without consideration of battery capacity constraint. Difficulties in calculating the LD function value and applying Armijo rule render us to employ a GP method with a predetermined step size sequence for solving the continuously differentiable maximization problem (13), which is stated as follows.

Stage 0: Feasibility Check. For each O-D pair, find the minimum energy consumption path according to link length and free flow travel time. If the path energy consumed is greater than the BEV battery capacity and the corresponding travel demand is positive, then there is no feasible path between this OD pair without causing additional energy out-of-battery cost. Record this OD pair and infeasible vehicle type to Set A. If Set A is empty, go to the next step; if not, stop.

Stage 1. Initialization. Set $v_a(0) = 0$, $\bar{t}_a = t_a[v_a(0)]$, iteration counter $n = 1$ and define the path set $K_{rs} = \emptyset$

1. Solve the acyclic K shortest path problem in terms of path energy cost by Yen's algorithm [40] to generate an initial path set $\bar{k}_{rs}(n)$, $K_{rs} = \bar{k}_{rs}(n) \cup K_{rs}$ and initialize its corresponding multiplier $\mu_k^{rs(1)} = 0, \forall k \in K$
2. Perform stochastic network loading to assign the travel demand to the paths generated based on $\bar{t}_a(0)$, $\sum f^{rs}_{\bar{k}_{rs}(1)}(1) = q_{rs}$. Logit loading results in K paths while probit loading generates one shortest path between each O-D pair.
3. Assign path flows to links $v_a(n) = \sum_{rs} \sum_{k \in K_{rs}} f_k^{rs}(n) \delta_{ak}^{rs}$

Stage 2. Column Generation. Increment iteration counter $n = n+1$

4. Update link travel time $\bar{t}_a(n) = [v_a(n-1)]$ based on $\bar{t}_a(v_a) = \begin{cases} \dfrac{\int_0^{v_a} t_a(x)\mathrm{d}x}{v_a}, v_a > 0 \\ t_a(0), v_a = 0 \end{cases}$ and link energy consumption $e_a(n) = \alpha l_a + \beta \bar{t}_a(n-1), \forall a$
5. Solve the K minimum path energy cost problem to generate new paths $\bar{k}_{rs}(n)$ and initialize the corresponding Lagrangian multiplier $\mu_k^{rs(n)}$ of the newly generated paths.

8.1. Update path set $K_{rs}(n) = \bar{k}_{rs}(n) \cup K_{rs}(n-1)$, if $\bar{k}_{rs}(n) \notin K_{rs}(n-1)$; otherwise use current path set $K_{rs}(n)$ in stochastic network loading procedure.

Stage 3. Equilibration. Compute the generalized path travel cost $C_k^{rs(n)} = \bar{c}_k^{rs} + \mu_k^{rs}(\beta \bar{c}_k^{rs} + \alpha l_k^{rs} - D)\ \forall k \in K_{rs}(n)$

6. Perform stochastic network loading procedure (logit or probit) to generate new path flow patterns $f_k^{rs}(\boldsymbol{\mu}^{(n)})$ in terms of the current path set $K_{rs}(n)$

7. Obtain the set of link flows according to link/path incidence relationship $V_a(\boldsymbol{\mu}^{(n)}) = \sum_r \sum_s \sum_k f_k^{rs} \delta_{a,k}^{rs}, \forall a \in A$

8. Average flow. Let $v_a(\boldsymbol{\mu}^{(n)}) = [(n-1)v_a(\boldsymbol{\mu}^{(n-1)}) + V_a(\boldsymbol{\mu}^{(n)})]/n, \forall a \in A$

9. Check stopping criterions of both flow change rate and Lagrangian multiplier change rate. (Flow change rate can be referred to page 301 for probit loading and page 327 for logit loading respectively in Sheffi [39].) If the following criterion hold, then terminate.

$$\max\{|\mu_k^{rs(n)} - \max[0, \mu_k^{rs(n)} + \alpha_n f_k^{rs}(\boldsymbol{\mu}^{(n)})(\alpha l_k^{rs} + \beta c_k^{rs} - D)]|\} \le \varepsilon,\ \forall \mu_k^{rs}$$

10. Otherwise, update Lagrangian multipliers according to the following equation:

$$\mu_k^{rs(n+1)} = \max\{0, \mu_k^{rs(n)} + \alpha_n f_k^{rs}(\mu_k^{rs(n)})(\alpha l_k^{rs} + \beta c_k^{rs} - D)\}$$

Note that $\{\alpha_n\}$ is a predetermined step size sequen ce satisfying the three conditions:

$$0 < \alpha_n < 1 \text{ and } \lim_{n\to\infty} \alpha_n = 0; \sum_{n=1}^{\infty} \alpha_n = +\infty; \sum_{n=1}^{\infty} \alpha_n^2 < \infty$$

Numerical examples

This section presents 2 numerical case studies to assess the performance and properties of the proposed method.

The first example, which is also adopted by Nie, Zhang [41] and Meng and Liu [20], consists of 9 nodes, 18 links, and 4 O-D pairs: (1,3), (1,4), (2,3), and (2,4), as shown in Fig 1. The free-flow travel time is used as a proxy for the link length for each link. Travel time on each link is defined by the following BPR (Bureau of Public Road) type function

$$t_a(v_a) = t_a^0\left(1 + 0.15 \times \left(\frac{v_a}{H_a}\right)^4\right), a \in A \tag{28}$$

where t_a^0 is the free flow travel time and H_a is link a capacity. OD demands, free-flow travel time and link capacity are the same as that in Meng, Lam [22].

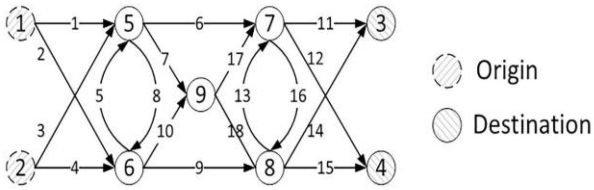

Fig 1. Small network schematic with 2 origins, 2 destinations, 9 nodes, and 18 links.

https://doi.org/10.1371/journal.pone.0194354.g001

Table 1. Path sets and corresponding optimal Lagrangian multipliers for MNL.

O-D pair	Path generated and its Lagrangian multiplier						
(1,3)	[1,5,7,3] **0**	[1,6,5,7,3] **0**	[1,5,7,8,3] **0.63**	[1,6,8,7,3] **4.23**	[1,5,9,7,3] **5.24**	[1,6,9,7,3] **5.24**	[1,6,8,3] **4.61**
(1,4)	[1,5,7,4] **0**	[1,5,7,8,4] **0**	[1,6,8,4] **7.07**	[1,6,5,7,4] **8.34**	[1,6,5,7,8,4] **10.83**	[1,6,8,7,4] **10.79**	[] /
(2,3)	[2,5,7,3] **0**	[2,5,7,8,3] **0**	[2,6,5,7,3] **10**	[2,5,9,7,3] **9.95**	[2,6,8,7,3] **12.92**	[2,6,8,3] **12.45**	[] /
(2,4)	[2,5,7,4] **0**	[2,5,7,8,4] **0**	[2,6,8,4] **18.54**	[2,5,9,7,4] **17.17**	[2,6,5,7,4] **17.13**	[2,5,9,7,8,4] **14.79**	[] /

https://doi.org/10.1371/journal.pone.0194354.t001

We use this example to evaluate the performance of proposed algorithms for solving both logit-based and probit-based SUE TAP with battery capacity constraints (further details are provided in S1 Matlab Code). The generated paths and their corresponding Lagrangian multipliers at the equilibrium under MNL are shown in Table 1. EV range limit is set to 4, and K, α, β are 6, 0.174 and 0.116 respectively. The convergence criterion of both flow change rate and Lagrangian multiplier change rate is 0.01. The step size sequence $\{\alpha_n\}$ and the initial multiplier $\mu_k^{(0)}$ are $1/n$ and 0, respectively. The non-zero multipliers indicate that the energy consumptions of traveling on these paths exceed the BEV battery capacity at the equilibrium, while zero multipliers (e.g. for paths 1-5-7-3 and 1-6-5-7-3) denote paths within the battery capacity, which will not trigger the out-of-energy cost. The number of paths generated in the column generation procedure is related to the value of K. Fig 2 shows the convergence performance of the solution method under MNL loading, where the equilibrium is reached after 130 iterations. Note that, in Fig 2 y-axis is in logarithm unit. The Euclidean distance equals to $\max_{\mu} |\mu_k^{rs(n+1)} - \mu_k^{rs(n)}|$.

Furthermore, we perform a thorough sensitivity analysis with respect to travel demand, battery capacity and the logit parameter. The high demand is double of the medium demand in the first numerical example. Table 2 demonstrates that after a certain level of battery capacity (e.g. battery capacity equals to 6 and 7), there is no influence on the equilibrium link flows, whereas in extreme cases (e.g. battery capacity = 2) where battery capacity is too small to travel through any path between an O-D pair, every corresponding Lagrangian multiplier would be positive and each EV user would experience the battery out-of-energy cost. In addition, higher travel demand may impose congestion on the network and thereby increase the energy consumption rate for the same path comparing to the original demand because of the increasing path travel time. Table 2 shows that the equilibrium link flow pattern is affected by travel demand and battery capacity. For example, link flows of 9, 11, 12, and 13 in the fifth and eighth column have much difference with each other because the network becomes congested and link travel time goes up when travel demand is high and increasing path travel time results in more energy consumption and more paths infeasible on which BEV will run out of energy and incorporate additional out-of-battery cost.

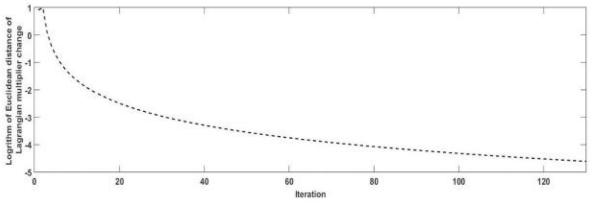

Fig 2. Convergence performance of MNL loading for the small-sized case study.

https://doi.org/10.1371/journal.pone.0194354.g002

Table 2. Equilibrium link flow for different scenarios of the travel demand and battery capacity under MNL network loading.

Link No.	Link capacity	Medium demand				High demand	
		capacity = 2	capacity = 4	capacity = 6	capacity = 7	capacity = 4	capacity = 6
1	40	21.36	23.85	14.46	14.06	14.38	20.39
2	30	8.64	6.15	15.54	15.94	45.62	39.61
3	50	68.44	68.31	52.20	47.71	39.22	66.20
4	80	1.56	1.69	17.80	22.29	100.78	73.80
5	30	0.00	0.00	0.00	4.40	14.14	2.97
6	60	92.62	94.58	71.22	61.67	21.55	81.34
7	30	3.00	1.47	11.27	14.35	40.18	6.47
8	30	5.81	3.89	15.83	18.65	22.27	4.18
9	90	2.46	3.53	16.47	19.38	92.92	102.82
10	30	1.92	0.42	1.04	4.60	45.36	9.37
11	30	39.03	35.65	30.11	27.71	35.82	10.39
12	30	60.00	37.49	31.70	30.17	2.21	0.00
13	30	0.62	24.43	26.81	29.32	44.48	83.50
14	30	0.97	4.35	9.89	12.29	44.18	69.61
15	30	0.00	22.51	28.30	29.83	117.79	120.00
16	30	2.11	1.10	5.08	6.58	11.47	1.14
17	40	4.92	1.89	12.31	18.95	49.49	11.40
18	30	0.00	0.00	0.00	0.00	36.04	4.44

https://doi.org/10.1371/journal.pone.0194354.t002

Table 3 shows path usage status, revealing the number of total generated paths and the proportion of feasible paths without additional out-of-battery cost corresponding to the scenarios in Table 2. When the battery capacity is extremely small and travel demand is medium, e.g. medium demand, capacity = 2, all the generated paths exceed range limit and every path user would experience a battery out-of-energy cost. However, while the battery capacity increase to 4, comparing two scenarios of different demand, all paths in the highly congested network are still out of range limit, because the energy consumption increase sharply as the path travel time increases. For the medium demand case, there are at least 2 paths within the range limit for each O-D pair.

For MNP, K is set to be 1 because the all-or-nothing assignment is applied in MNP loading and only the shortest path is used to load the demand. Sample size of drawing perceived travel time in Monte Carlo simulation is 200. Fig 3 shows the convergence performance under MNP loading where there is a fast trend during the first 10 iterations while equilibrium is reached at iteration 100. The equilibrium path sets and their corresponding Lagrangian multipliers are listed in Table 4. It is observed that the number of paths generated is affected by travel demand as well. Comparing to MNL, fewer paths are used between each O-D pair because the all-or-

Table 3. Path status under different travel demand and battery capacity for MNL.

O-D pair	The number of paths within range limit V.S. total paths generated					
	Medium demand				High demand	
	capacity = 2	capacity = 4	capacity = 6	capacity = 7	capacity = 4	capacity = 6
(1,3)	0/7	1/7	7/7	8/8	0/19	3/12
(1,4)	0/10	0/6	5/6	8/8	0/24	3/16
(2,3)	0/6	1/6	4/6	7/10	0/21	2/15
(2,4)	0/10	1/6	2/6	7/8	0/24	1/16

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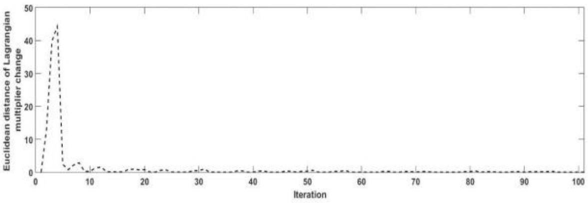

Fig 3. Convergence performance under MNP loading for the small-sized case study.

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nothing assignment is used in probit loading step to assign the travel demand to the shortest path. When the demand is low and the network is not congested, only several paths would be calculated in column generation step as the shortest path are stored in path sets. A sensitivity analysis is conducted with respect to probit parameter in Table 5. As we can see, the effect of changing probit parameter values is not that obvious in terms of the link flow volume. In MNL, different K values in K-shortest path algorithm used for column generation step would lead to different path size. It is well known that MNL model suffers from independence of irrelevant alternative (IIA) property [39], which is the reason why larger K value is used in MNL model.

For the second case study, a variation of the Sioux Falls network (see Fig 4) is adopted which has been chosen as a benchmark network in numerous traffic assignment studies [Suwansirikul, Friesz [42]]. One particular reason for presenting the Sioux Falls network example here is to highlight the effect of parameter setting on computational cost. This network consists of 24 nodes, 76 links, and 576 O-D pairs. For computational experiments, the number of iterations (ITR) and the total computational cost (TCC) were compared for MNL and MNP under different battery capacities (BC), stochastic parameter values, and K values. The weight value of link energy consumption function, namely α, β, and convergence criteria used here are the same as the first example.

Table 6 **and** Table 7 list the computational cost with different parameters under logit and probit-based loading. Assuming the travel cost coefficient of the logit model, referred to as logit parameter, is 0.2, it can be seen from Table 6 that K value has a great impact on computational cost. By looking into ITR before convergence and comparing the first two scenarios, bigger K value would decrease the ITR needed while increasing the TCC. Clearly, most computational cost is spent on calculating the K shortest paths at column generation and logit loading steps for each iteration. Therefore, a more efficient K-shortest path algorithm would improve TCC. According to these two tables, it can also be observed that smaller battery capacity, bigger stochastic parameters and larger battery capacity, lead to slow convergence speed. Intuitively, the larger the BC is, the more paths can be selected in the path set. More time is

Table 4. Path sets for MNP and its Lagrangian multiplier.

O-D pair	Medium demand		High demand		
(1,3)	[1,5,7,3] **0**	[] /	[1,5,7,3] **0**	[1,6,8,3] **0**	[] /
(1,4)	[1,5,7,4] **0**	[1,5,7,8,4] **0.35**	[1,5,7,4] **0**	[1,6,8,4] **0**	[] /
(2,3)	[2,5,7,3] **0**	[] **0**	[2,5,7,3] **0**	[2,6,8,3] **0**	[] /
(2,4)	[2,5,7,4] **0**	[2,5,7,8,4] **0.26**	[2,5,7,4] **0**	[2,6,8,4] **0**	[2,5,7,8,4] **2.63**

https://doi.org/10.1371/journal.pone.0194354.t004

Table 5. Equilibrium link flow for different scenarios of the travel demand and probit parameter under MNP network loading.

Link No.	Medium demand		High demand	
	Parameter = 0.2	Parameter = 1.2	Parameter = 0.2	Parameter = 1.2
1	30.00	29.80	29.84	30.53
2	0.00	0.20	30.16	29.47
3	70.00	70.00	111.77	108.60
4	0.00	0.00	28.23	31.40
5	0.00	0.00	0.00	0.00
6	100.00	99.80	141.61	139.12
7	0.00	0.00	0.00	0.00
8	0.00	0.00	0.00	0.00
9	0.00	0.20	58.39	60.88
10	0.00	0.00	0.00	0.00
11	40.00	40.00	72.58	71.05
12	45.74	49.11	61.94	63.16
13	14.26	10.69	7.10	4.91
14	0.00	0.00	7.42	8.95
15	14.26	10.89	58.06	56.84
16	0.00	0.00	0.00	0.00
17	0.00	0.00	0.00	0.00
18	0.00	0.00	0.00	0.00

https://doi.org/10.1371/journal.pone.0194354.t005

needed to generate the paths, calculate the path choice probability, and assign the flows. When BC is large enough and travel demand is fixed, BEV can actually travel to every destination with no concern about running out of energy, thus making it a conventional SUE with no additional battery capacity constraints. In reality, BEV may not fully charged under some circumstances, e.g. power grid failure, multiple trips. Therefore, multi-class users with different battery capacities can be further taken into consideration without changing the problem's structure.

Moreover, the bigger value of the stochastic parameter, the larger is the random perception error on both travel time and energy cost. From the results, it is found that it took less time and less iterations for probit-based network loading to converge than that of logit-based network loading. This result is because the all-or-nothing assignment is used in probit-based loading. Only the shortest path is generated between each O-D pair at each iteration. When BC is relatively large, all the paths energy consumption would be within the capacity level and Lagrangian multipliers are equal to zero.

Conclusions

This paper works on the stochastic traffic assignment models with battery capacity constraints, where new path-constrained stochastic user equilibrium (SUE) traffic assignment problem is formulated, solved and numerically analyzed. The method considers a flow-depend energy consumption assumption for battery electric vehicles (BEV), which is a generalization of flow-independent driving distance constraint. The BEV's range limit is determined based on both its travel distance and travel time that is a function of traffic congestion. Flow-dependent constraint inevitably calls for fundamental changes to the existing network flow modeling tools for properly capturing traffic patterns and evaluating traffic assignment results. It is proved that the solution method framework, LD-GP-stochastic network loading, could be applied not only in link-based problems but also in path-based problems. In this path-based SUE problem, the

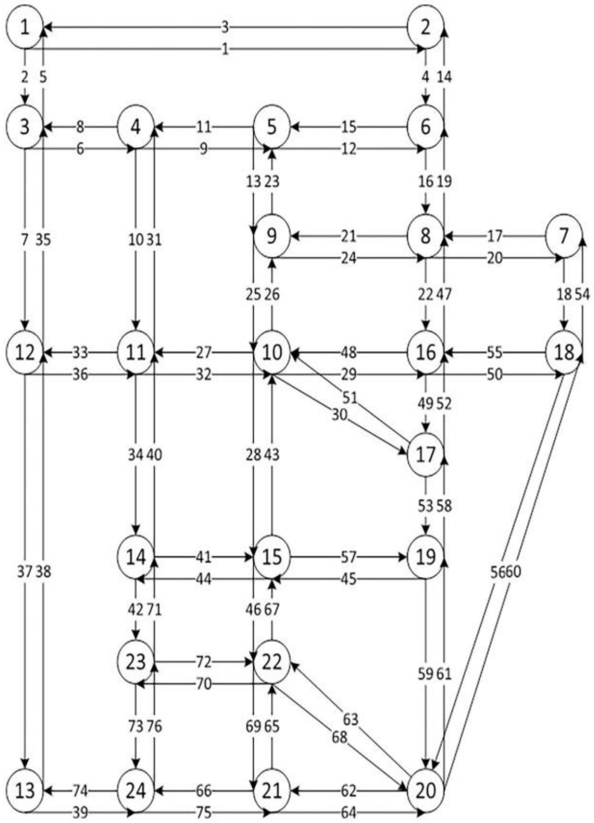

Fig 4. Sioux falls network with 24 nodes and 76 links.

https://doi.org/10.1371/journal.pone.0194354.g004

column generation procedure is applied to the path choice set generation which turns out to works well with GP and stochastic network loading and provides basic insights of solving path-constrained SUE problem to avoid path enumeration. The application of the algorithms in the small network justifies the applicability of the solution procedures to general network with path-based constraints. The numerical analysis results show the impact of battery capacity, travel demand and stochastic parameters on network equilibrium flow and computational cost.

Table 6. Computational cost with different parameter settings for MNL.

	K = 3,logit parameter = 0.2				K = 6,logit parameter = 0.2			
BC	0.05	0.2	0.6	1	0.05	0.2	0.6	1
ITR	36	26	8	4	28	21	11	6
TCC(s)	136.21	102.06	27.98	12.62	285.61	209.10	106.75	54.21
	K = 6,logit parameter = 0.4				K = 6,logit parameter = 1			
BC	0.05	0.2	0.6	1	0.05	0.2	0.6	1
ITR	20	13	5	3	12	6	3	2
TCC(s)	204.58	131.88	44.81	23.10	124.51	56.61	23.09	13.59

https://doi.org/10.1371/journal.pone.0194354.t006

Table 7. Computational cost with different parameter settings for MNP.

	K = 1,probit parameter = 0.2				K = 1,probit parameter = 0.4			
BC	0.05	0.2	0.6	1	0.05	0.2	0.6	1
ITR	8	6	6	3	6	6	6	3
TTC(s)	6.40	5.07	5.21	2.63	4.89	4.90	4.63	2.55
	K = 1,probit parameter = 1				K = 1,probit parameter = 2			
BC	0.05	0.2	0.6	1	0.05	0.2	0.6	1
ITR	6	3	3	3	6	3	3	3
TTC(s)	4.91	3.12	2.67	2.60	4.55	2.86	2.46	2.56

https://doi.org/10.1371/journal.pone.0194354.t007

As a pure mathematical modeling tool to characterize BEVs' travel behavior in the network with some ideal socioeconomic assumptions, we expect that the modeling technique and solution methods demonstrated in this work would potentially trigger the interest of investigating other types of stochastic traffic assignment problems with path-based constraints in logit-type or weibit route choice models. The model itself can also be applied for more accurate quantification of network flows, travel demand and battery capacity levels. As a modeling platform for more practical and realistic model, the proposed model should be enhanced to accommodate mixed traffic flows of different types of vehicles such as BEVs, hybrid vehicles and conventional gasoline vehicles as well as the availability of charging infrastructure. Our future study will investigate the possibility of incorporating charging time, range anxiety level and value of time in model extensions. Based on the SUE models proposed in this paper, we will also investigate how to optimally locate charging stations in the network in terms of different objectives.

Supporting information

S1 Matlab Code. Details of the numerical test of the first example including code and network attributes.
(ZIP)

S1 Text. Proof of Proposition 1.
(DOCX)

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S1 Text. Proof of Proposition 1

Consider the Lagrangian function of the minimization model (12) with constraints (1)(2)(3)(9) below.

$$L(\mathbf{f},\boldsymbol{\mu}) = \sum_r \sum_s q^{rs} S^{rs}(\overline{\mathbf{c}}^{\mathbf{rs}}(\mathbf{f}) + \overline{\mathbf{d}}^{\mathbf{rs}}(\mathbf{f}^{\mathbf{rs}})) - \sum_r \sum_s \sum_k \overline{d}_k^{rs}(f^{rs}) \cdot f_k^{rs} - \sum_r \sum_s \sum_k \mu_k^{rs} f_k^{rs} \cdot (D - \alpha l_k^{rs} - \beta \overline{c}_k^{rs})$$
$$+ \sum_r \sum_s \pi^{rs} (q^{rs} - \sum_k f_k^{rs}) \tag{29}$$

where $\boldsymbol{\mu}$ is the Lagrangian multiplier with respect to the battery capacity constraints, and row vector $\boldsymbol{\mu} = (\ldots, \mu_k^{rs}, \ldots) \in R^{|K_{rs}|}$.

Keep in mind that the vector $\overline{\mathbf{d}}^{\mathbf{rs}}(\mathbf{f}^{\mathbf{rs}})$ is a function of path flows between O-D pair (r,s) only. Take partial derivative of Lagrangian function above with respect to path flow f_l^{ij} on a designated path $l \in K_{ij}$ between O-D pair (i, j) can yield

$$\frac{\partial L(\mathbf{f},\boldsymbol{\mu})}{\partial f_l^{ij}} = \sum_r \sum_s q^{rs} (\sum_{k \in K_{rs}} P_k^{rs} \cdot \frac{\partial \overline{c}_k^{rs}(\mathbf{f})}{\partial f_l^{ij}}) + q^{ij} \sum_{k \in K_{ij}} (P_k^{ij} \cdot \frac{\partial \overline{d}_k^{ij}(\mathbf{f}^{\mathbf{ij}})}{\partial f_l^{ij}}) - \overline{d}_l^{ij}(\mathbf{f}^{\mathbf{ij}}) -$$
$$\sum_{k \in K_{ij}} f_l^{ij} \frac{\partial \overline{d}_k^{ij}(\mathbf{f}^{\mathbf{rs}})}{\partial f_l^{ij}} + [\mu_l^{ij} \cdot (\alpha l_l^{ij} + \beta \overline{c}_l^{ij} - D) + \sum_r \sum_s \sum_{k \in K_{rs}} \mu_k^{rs} f_k^{rs} \beta \frac{\partial \overline{c}_k^{rs}(\mathbf{f}^{\mathbf{rs}})}{\partial f_l^{ij}}] - \pi^{ij} \tag{30}$$

The first term of right-hand side (RHS) of the equation above can be rewritten by

$$\sum_r \sum_s q^{rs} (\sum_{k \in K_{rs}} P_k^{rs} \cdot \frac{\partial \overline{c}_k^{rs}(\mathbf{f})}{\partial f_l^{ij}}) = \sum_r \sum_s q^{rs} \sum_{k \in K_{rs}} P_k^{rs} \sum_{a \in A} \frac{d\overline{t}_a(v_a)}{dv_a} \cdot \delta_{ak}^{rs} \cdot \delta_{al}^{ij}$$
$$= \sum_r \sum_s f_k^{rs} \sum_{k \in K_{rs}} \delta_{ak}^{rs} \sum_{a \in A} \frac{d\overline{t}_a(v_a)}{dv_a} \cdot \delta_{al}^{ij} = \sum_{a \in A} v_a \cdot \frac{d\overline{t}_a(v_a)}{dv_a} \cdot \delta_{al}^{ij} \tag{31}$$

The second and fourth terms can be canceled out. The fifth term equals to

$$\mu_l^{ij} \cdot (\alpha l_k^{rs} + \beta \overline{c}_k^{rs} - D) + \sum_r \sum_s \sum_{k \in K_{rs}} \mu_k^{rs} f_k^{rs} \beta \frac{\partial \overline{c}_k^{rs}(\mathbf{f}^{\mathbf{rs}})}{\partial f_l^{ij}} = \mu_l^{ij} \cdot (\alpha l_l^{ij} + \beta \overline{c}_l^{ij} - D) + \beta \mu_l^{ij} \sum_a v_a \frac{d\overline{t}_a(v_a)}{dv_a} \delta_{al}^{ij}$$

Hence, the equation can be simplified as

$$RHS = (1+\beta\mu_l^{ij})c_l^{ij} - \overline{c_l}^{ij} - \mu_l^{ij}\cdot(D-\alpha l_k^{rs}) - \overline{d_l}^{ij}(\mathbf{f}^{ij}) \tag{32}$$

Since f^* is a local minimum, according to KKT conditions, there are optimal Lagrangian multiplier μ^* such that

$$\frac{\partial L(\mathbf{f}^*, \boldsymbol{\mu}^*)}{\partial f_l^{ij}} = 0, l \in K_{ij}, i \in R, j \in S \tag{33}$$

$$f_l^{*ij}(D - l_l^{ij}) \geq 0, \forall i, j, l \tag{34}$$

$$\mu_l^{*ij} \geq 0, \forall i, j, l \tag{35}$$

$$\mu_l^{*ij}\cdot f_l^{*ij}(D - l_l^{ij}) = 0, \forall i, j, l \tag{36}$$

$$\overline{d_l}^{ij}(\mathbf{f}^{*\mathbf{rs}}) = (1+\beta\mu_l^{*ij})c_l^{*ij} - \overline{c_k}^{*rs} - \mu_l^{*ij}\cdot(D-\alpha l_k^{rs}) \tag{37}$$

$$\begin{aligned}\overline{d}^{ij}(\mathbf{f}^{*\mathbf{rs}}) &= \sum_{l\in K_{ij}}\sum_{m}\overline{d}_{lm}^{ij}(\mathbf{f}^{*\mathbf{rs}}) = \sum_{l\in K_{ij}}[(1+\beta\mu_l^{*ij})c_l^{*ij} - \overline{c_l}^{*ij} - \mu_l^{*ij}\cdot(D-\alpha l_l^{ij})] \\ &= c^{*ij} - \overline{c}^{*ij} + \sum_{l\in K_{ij}}\mu_l^{*ij}(\beta c_l^{*ij} + \alpha l_l^{ij} - D)\end{aligned} \tag{38}$$

Let $\lambda_l^{*ij} = \mu_l^{*ij}(\beta c_l^{*ij} + \alpha l_l^{ij} - D)$, $\boldsymbol{\lambda}^{*\mathbf{ij}} = (\ldots, \lambda_l^{*ij}, \ldots) \in R^{|K_{ij}|}$

Thus

$$\overline{d}^{ij}(\mathbf{f}^{*\mathbf{rs}}) = c^{*ij} - \overline{c}^{*ij} + \boldsymbol{\lambda}^{*\mathbf{ij}} \tag{39}$$

where $\lambda_l^{*ij} = \mu_l^{*ij}(\beta c_l^{*ij} + \alpha l_l^{ij} - D)$ is the battery out-of-energy cost incurred when the energy needed to travel through a given path exceeds the battery capacity of the EV. The Lagrangian multiplier μ_l^{*ij} stands for an equivalent travel cost value of the unit energy.

$$f_l^{*ij} = q^{ij} \cdot P_l^{ij}(c^{ij}(\mathbf{f}^*) + \boldsymbol{\lambda}^{*\mathbf{ij}}), \forall (i, j), l \in K_{ij} \tag{40}$$

Eq. (40), (34)-(36) state that $\mathbf{f}^*$ fulfills the generalized SUE conditions and that $\{\mu_k^{rs}\}$ is the relevant SUE battery out-of-energy cost pattern.

5.3 Discussion

The paper included in this chapter contributes to knowledge by developing new stochastic traffic assignment models of BEVs with limited battery capacity, where new path-constrained SUE traffic assignment problem is formulated, solved and numerically analyzed.

As a pure mathematical modeling tool to characterize BEVs' travel behavior in the network with some ideal socioeconomic assumptions, the modeling technique and solution methods demonstrated in this work are expected to trigger the interest of investigating other types of stochastic traffic assignment problems with path-based constraints in other logit-type or weibit route choice models. The model itself can also be applied for more accurate quantification of network flows, travel demand and battery capacity levels.

As a modeling platform for more practical and realistic model, the proposed model should be enhanced to accommodate mixed traffic flows of different types of vehicles such as BEVs, hybrid vehicles and conventional gasoline vehicles (GV) as well as the availability of charging infrastructure. The driving range of BEV is subject to the battery capacity and electricity consumption rates, while the drivers usually keep the battery full for any path they choose without worrying about so called range anxiety concern among most driving population. Future study should focus on the possibility of incorporating charging time, range anxiety level and value of time in model extensions.

5.4 Conclusions

In this chapter, the research adopts a battery capacity constraint as an extension and generalization of driving distance constraints by incorporating travel time into the range limit consideration. The method considers a flow-dependent energy consumption assumption for BEV, which is a generalization of flow-independent driving distance constraint. The BEV's range limit is determined based on both its travel distance and travel time that is a function of traffic congestion. Flow-dependent constraint inevitably calls for fundamental changes to the existing network flow modeling tools for properly capturing traffic patterns and evaluating traffic assignment results. It is proved that the solution method framework, LD-GP-stochastic network loading, could be applied not only in link-based problems but also in path-based problems. In this path-based SUE problem, the column generation procedure is applied to the path choice set generation which turns out to works well with GP and stochastic network loading and provides basic insights of solving path-constrained SUE problem to avoid path enumeration. The application of the algorithms in the small network justifies the applicability of the solution procedures to general network with path-based constraints. The numerical analysis results show the impact of battery capacity, travel demand and stochastic parameters on network equilibrium flow and computational cost.

This chapter brings Part II to an end, which has focused on the development of general SUE models of EVs with range limits as well as their solution algorithms. The first chapter of Part II, chapter 4 started with the general SUE model of mixed GV and EV flow using driving distance constraints. Modified MSA method and modified probit-based loading method are applied to solve the proposed model. And Chapter 5 have further investigated EV's SUE flow pattern in a more realistic and more general way. The model formulation and some propositions are discussed and the solution method for general SUE model with path-based constraints are first addressed.

The next part of the thesis, Part III, looks at charging facility location model for both private BEVs and public electric buses. There are two chapters, one devoted to battery charging facility location model for private BEVs in Chapter 6 and the other to battery swapping facility location model for public EBs in Chapter 7.

PART III:

CHARGING FACILITY LOCATION MODELS

CHAPTER 6 LOCATION DESIGN OF CHARGING FACILITY FOR PRIVATE ELECTRIC VEHICLES

6.1 Introduction

In accordance with research objective 4, the aim of this chapter is to develop a bi-level charging facility location model for BEVs. In the upper level, the objective is to maximize coverage of BEV flows in the network by locating a given number of charging stations on road segments considering budget constraints. In the lower level, BEV drivers follow the SUE principle with path distance constraint as we have addressed in Part II. Moreover, the availability of public charging stations, battery charging time have been considered in the lower level problem as an extension of SUE model in Part II. This is a key contribution to knowledge as no studies have investigated SUE models with driving distance limit and battery charging. An investigation of how range limit and location of charging facilities affect drivers' path choice behavior and equilibrium flows of BEVs in a transportation network is yet to be explored. This research also investigates the method of the deploying a given number of public charging facility to maximize the coverage of BEVs public charging facilities on a network with mixed conventional GVs and BEVs. The chapter addresses a research gap identified in the literature review: No bi-level charging facility location model dedicated to considering a SUE BEV flow pattern in the lower level problem and maximize the BEV flow coverage in the upper level. This is in accordance with research objective 4 to develop a new bi-level model to locate a given number of public charging facility to maximize their exposure to the BEV users in order to eliminate their range anxieties. Table 6-1 details the research component, research gaps and research opportunities.

Table 6-1: Research gap, opportunity and objective associated with research component 4

Research topic	Research gaps	Research opportunities
Charging/swapping facility location models of BEVs and BEBs (Part III)	There is no bi-level charging facility location model dedicated to considering a SUE BEV flow pattern in the lower level problem and maximize BEV flow coverage in the upper level (see section 2.5.4)	Proposing a new bi-level model for deploying the charging facility considering a SUE link flow pattern and availability of charging facility (see Chapter 6)

The following paper is included in this chapter:

Jing, Wentao, Kun An, Mohsen Ramezani, and Inhi Kim. "Location Design of Electric Vehicle Charging Facilities: A Path-Distance Constrained Stochastic User Equilibrium Approach." Journal of Advanced Transportation 2017 (2017).

An equilibrium-based heuristic algorithm is developed to obtain the solution of this program. Finally, two numerical tests are presented to demonstrate applicability of the proposed model and feasibility and effectiveness of the solution algorithm. The results demonstrate that the equilibrium traffic flows are affected by charging speed, range limit, and charging facilities' utility and that BEV drivers incline to choose the route with charging stations and less charging time.

6.2 Paper 3: Location Design of Electric Vehicle Charging Facilities: A Path-Distance Constrained Stochastic User Equilibrium Approach

The location design problem of charging facilities can be modelled as a Leader-Follower Stackelberg game where the decision makers are the leaders who decide the facility deployment and the BEV users are the followers who can choose their paths freely. Most of previous studies focused on DUE problems with BEVs. However, the driving distance limit, to the best of our knowledge, has not been considered in stochastic network equilibrium models, especially in the mixed flow transport network. Moreover, to tackle the range anxiety problem with a limited budget, the charging facilities should be accessible to as many EVs as possible. Deploying the pubic charging facilities on the links where most BEV drivers use is an efficient way to increase the utilization and perception of the public charging facilities, which promotes BEV acceptance and relieve range anxiety. Given the high cost of building public charging stations and financial constraints, it is essential to optimize the location of facilities in a network that provide the maximum exposure and utilization by BEV drivers. Since various factors influence BEV drivers' charging decision, such as stochasticity of range anxiety, initial battery energy state, battery energy consumption ratio and battery capacity, considering those factors in the model is of great importance.

The following paper details a bi-level charging facility location model for deploying the public charging facility for private BEVs. A maximal flow-covering (MFC) model is proposed to maximize BEV flow coverage by locating a fixed number of charging facilities in the bi-level, equilibrium-optimization framework. Coverage is achieved when the charging facilities is located on the BEV route. Secondly, the effects of driving distance limit constraints, charging facility availability, charging facility utility and traffic congestion are accommodated in BEVs' route choice behaviour. The equilibrium BEV flow pattern is determined endogenously by the general SUE traffic assignment model with driving distance limit constraints, in which the mutual interactions between the location of charging facilities and resultant equilibrium BEV link flow patterns are modelled. Finally a heuristic algorithm is proposed to solve the mixed-integer nonlinear program.

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Research Article

Location Design of Electric Vehicle Charging Facilities: A Path-Distance Constrained Stochastic User Equilibrium Approach

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Location of public charging stations, range limit, and long battery-charging time inevitably affect drivers' path choice behavior and equilibrium flows of battery electric vehicles (BEVs) in a transportation network. This study investigates the effect of the location of BEVs public charging facilities on a network with mixed conventional gasoline vehicles (GVs) and BEVs. These two types of vehicles are distinguished from each other in terms of travel cost composition and distance limit. A bilevel model is developed to address this problem. In the upper level, the objective is to maximize coverage of BEV flows by locating a given number of charging stations on road segments considering budget constraints. A mixed-integer nonlinear program is proposed to formulate this model. A simple equilibrium-based heuristic algorithm is developed to obtain the solution. Finally, two numerical tests are presented to demonstrate applicability of the proposed model and feasibility and effectiveness of the solution algorithm. The results demonstrate that the equilibrium traffic flows are affected by charging speed, range limit, and charging facilities' utility and that BEV drivers incline to choose the route with charging stations and less charging time.

1. Introduction

Carbon-based emissions and greenhouse gases are critical global issues, where transport sector is a significant contributor. A cost-effective strategy for reducing emissions is efficient use of alternative fuels. Cities, businesses, and governments have recognized electric vehicles (EVs) as an indispensable part of smart and sustainable city frameworks [1], because, comparing to conventional internal combustion engines, EVs are more energy efficient [2]. Moreover, battery electric vehicles (BEVs), as a type of alternative fuel vehicles, have been developed as a promising solution for reducing local air pollution at the point of operation [3], greenhouse gas emissions [4], dependency on fossil oil, and improving energy safety. Furthermore, EVs can be utilized to store energy from renewable resources, such as wind, wave power, and solar cells, to smoothen out the daily power fluctuation in low peak periods [5] with the development of vehicle-to-grid (V2G) technology [6–8]. For consumers, the monetary savings of switching to a BEV can be significant due to cheaper electricity cost comparing with gasoline [9]. However, the early BEV users still suffer from the inconvenience of limited driving range, long charging time, and insufficient public charging stations [1, 10].

Currently the driving range of EVs can vary greatly between 60 km and 400 km by model and manufacturer, while most of them have ranges between 100 km and 160 km [11]. The EVs can be recharged using plug-in charging or battery-swapping facilities. The plug-in charging is categorized by voltage and power levels, leading to different charging times. Slow charging usually takes hours to charge while fast charging can achieve 50% charge in 10–15 minutes [11]. Range anxiety, when the driver is concerned that the vehicle will run out of battery before reaching the destination, is a major hindrance for the market penetration of EVs [12] and will inevitably add a certain level of restrictions to BEV drivers' path choices, at least in a long future period prior to the massive coverage of recharging infrastructures [13].

Governments and automotive manufacturers have recognized the environmental value of EVs and, therefore, are encouraging BEV ownership through economic incentives and more public charging station deployment [14].

Explicitly incorporating the range limit into facility location problem (FLP) can be traced back to flow refueling facility location problems (FRFLP) which utilized optimization models to determine a set of locations to serve the refueling demand in a network subject to a financial budget. One branch of FRFLP sought to maximize demand coverage by locating a fixed number of refueling facilities, which was referred to as the maximal covering location problem (MCLP). This problem has been typically formulated as flow refueling location model (FRLM) [15–18], which served demand along their shortest paths rather than demand at their end points to maximize the coverage of these flows. Typically, they used modifications of flow-capturing or flow interception location models (FILM) [19, 20], which were path-based version of MCLP. In FILM, for each O-D pair, the shortest path between the O-D pair is considered as covered if it passes through at least one node that contains a refueling facility. The developed FRLM models have been compared empirically for specific scenarios in order to choose one location model over another [21]. Furthermore, in another attempt, a flow-based refueling-station-location model was proposed based on a set covering concept and vehicle-routing logics considering both intercity and intracity travel [22, 23]. The above model was reformulated and a flexible mixed-integer linear programming model was presented, which was able to obtain an optimal solution much faster than the previous set cover version. Moreover, the model also could be solved in the maximum cover form [24].

Along another track, a large variety of other approaches have been proposed to address the locations of EV public charging infrastructures. Huang et al. [11] proposed a geometric segmentation method to find the optimal location for both slow and fast charging stations. Sweda and Klabjan [25] developed an agent-based decision support system and a variant maximal covering location problem for EV charging infrastructure deployment. Asamer et al. [26], by using 800 electric taxis' operational data in the city of Vienna, Austria, proposed a two-phase decision support system. Nie and Ghamami [3] presented a conceptual optimization model to analyze travel by EV along a long corridor whose objective was to select the battery size and charging capacity (in terms of both the charging power at each station and the number of stations needed along the corridor) to meet a given level of service. They further proposed a fixed charge facility location model with charging capacity constraints, considering drivers' preference for familiar parking lots [27]. Chen et al. [28] investigated the optimal deployment of charging stations and lanes along a long traffic corridor to serve the charging need of EVs and examined the competitiveness of charging lanes over charging stations. Xi et al. [29] developed a simulation-optimization model that determined where to locate EV charging stations to maximize their use by privately owned EVs. Jung et al. [30] reported a simulation-optimization location model including an upper level multiple-server allocation model with queueing delay and a lower level dispatch simulation and provided a solution algorithm that featured itinerary-interception, stochastic demand, and queueing delay. Dong et al. [31] analyzed the impact of public charging station deployment on increasing electric miles traveled. By considering transportation and power networks and maximizing the social welfare, He et al. [32] developed an equilibrium-based modeling framework for locating plug-in charging facilities. Riemann et al. [33] incorporated stochastic user equilibrium (SUE) into a FCLM and aimed to capturing the maximum EV path flow on a network. A global optimal solution was applied to solve the proposed model. Wu and Sioshansi [34] proposed a stochastic flow-capturing model to optimize the location of fast charging stations, addressing the uncertainty of BEV flows. Zhu et al. [35] proposed a model that simultaneously handled the problem of where to locate the charging stations and how many chargers should be established in each charging station to minimize the total cost.

The location design problem of charging facilities can be modeled as a Leader-Follower Stackelberg game where the decision makers are the leaders who decide the facility deployment and the BEV users are the followers who can choose their paths freely. Most of the previous studies focused on user equilibrium (UE) problems with BEVs. Among these studies, Jiang et al. [13] first introduced a path-constrained deterministic traffic assignment problem and further extended this work by considering trip chain and range anxiety analysis [36–39]. Zheng et al. [40] presented a bilevel model to locate charging facility and minimize all users cost in the upper level and to find path-constrained equilibrium BEV flows in the lower level. Jing et al. [41] provided a comprehensive review for the equilibrium network modeling. However, the driving distance limit, to the best of our knowledge, has not been considered in stochastic network equilibrium models, especially in the mixed flow transport network. Moreover, to tackle the range anxiety problem with a limited budget, the charging facilities should be accessible to as many EVs as possible [11]. It can be an efficient way to deploy the public charging facilities on the links where most BEV drivers use to increase the utilization and perception of the public charging facilities, which promotes BEV acceptance and relieve range anxiety [31]. Given the high cost of building public charging stations and financial constraints, it is essential to optimize the location of facilities in a network that provides the maximum exposure and utilization by BEV drivers. Since various factors influence BEV drivers' charging decision, such as stochasticity of range anxiety, initial battery energy state, battery energy consumption ratio, and battery capacity, considering those factors in the model is of great importance.

In this study, we present a novel bilevel public charging infrastructure location model that maximizes the total captured BEV link flows, considering BEV range limits and SUE principle to capture BEV drivers' route choice behavior in a network with mixed BEV and gasoline vehicles (GVs). The objective of the upper level of the model is to cover the maximum BEV link flows in a network by deploying a given number of charging facilities. In other words, the model aims to maximize the number of BEVs who can access the charging facilities along their routes. In the lower level, the stochastic traffic assignment on the network is the primary factor that determines the location of charging facility deployment. In general, a network equilibrium problem with multiple vehicle/mode classes cannot be written as a convex mathematical programming model, due to the existence of the asymmetric Jacobi matrix caused by different impacts on travel cost from different vehicle/mode classes [42]. The approaches to deal with the asymmetric Jacobian elements can be attributed to Jiang and Xie [43] and Ryu et al. [44]. It should be noted that relaxing the asymmetric restriction inevitably degrades the realism of traffic assignment model. However, in our model, the general compositions of path travel cost functions of the two vehicle classes, that is, GVs and BEVs, are similar. The only differences between these two types of vehicles lie in two flow-independent terms, namely, charging facility utility and charging time and thus their flow-time impacts on each other are symmetric (i.e., the impact of GVs on the travel times of BEVs is the same as the impact of BEVs on the travel times of GVs).

Modeling a traffic network with realistic refueling behaviors may require accommodating different routing objectives (e.g., minimization of travel time, charging time, and/or fuel consumption), different refueling services (e.g., battery-charging service or battery-swapping service), and different types of vehicles (e.g., GVs and BEVs) [38]. All these factors result in different path travel cost perception and route choice behaviors. It is evident that BEV drivers have inherent differences in travel behavior from GV drivers and specifically range limit, charging speed, and charging stations locations have significant influence on BEV drivers' decision-making process [45].

This paper focuses on several factors to explicitly capture BEV drivers' behavior with the stochastic traffic assignment. However, we understand the limitations of the stochastic traffic assignment in the lower level for accurately capturing realistic situations. It is believed that the results from this paper can provide some guidelines for locating BEV charging facility and basic insights of BEV drivers' behavior. Despite all the realistic situations, most data, such as demand, initial battery state of charge, and actual range limit, are difficult to obtain and this method and objective are easy to implement especially at the early stage of expanding EV market share. First, BEVs' range limit is considered as travel distance such that any path whose distance is greater than its range limit (referred to as infeasible paths) would not be chosen if the existing charging facility could not help finish the trip. Second, availability of charging facility would affect the route choice in a way that those infeasible paths may become feasible after recharging at the charging facilities on the path. Furthermore, the utility theory is applied to charging facility; that is, BEV drivers are more likely to choose the path with charging facilities over others without charging facilities even if they have equal path travel time. In addition, traffic congestion effects on travel time are also taken into consideration in BEV drivers' route choice behavior but not in the range limit constraint. Lastly, under the principle of perceived individual cost minimization, the path cost structure in the lower level model consists of flow-dependent path travel time, charging time, and utility of charging facilities (equivalent to given amount of travel time reduction). Specifically, the lower level model can be stated as follows: in a traffic network with fixed GV and BEV travel demand between each O-D pair and a set of charging facilities at known locations, the problem is to find such a traffic flow pattern that each trip maker chooses a path along which his or her least perceived cost is minimized and the vehicle can be charged before running out of energy before arriving at the destination. Meanwhile, no one can improve his/her perceived travel cost by unilaterally changing a path. Given the sufficient coverage of gasoline stations and GVs' large fuel capacity, GVs' route choice is not affected by any other costs incurred by refueling requirement, except for travel time.

The contributions of this study are threefold. Firstly, a maximal flow-covering (MFC) model, that is, a modification of classic MCLP, is proposed to maximize BEV flow coverage by locating a fixed number of charging facilities in the bilevel, equilibrium-optimization framework. Coverage is achieved when the charging facilities are located on the BEV route. Secondly, the effects of driving distance limit constraints, charging facility availability, charging facility utility, and traffic congestion are accommodated in BEVs' route choice behavior. The equilibrium BEV flow pattern is determined endogenously by the general SUE traffic assignment model with driving distance limit constraints, in which the mutual interactions between the location of charging facilities and resultant equilibrium BEV link flow patterns are modeled. Finally a heuristic algorithm is proposed to solve the mixed-integer nonlinear program.

The remainder of this paper is organized as follows. In Sections 2 and 3, we elaborate the problem definition and formulation. Section 4 presents the solution methodology and details its algorithmic implementations, while Section 5 describes the numerical results from applying the algorithmic procedure for a small network and Sioux Falls network. In the end, we conclude the article and point out some future research directions in Section 6.

2. Problem Description, Assumptions, and Notation

BEVs rely entirely on electricity as a single power source and are designed to be charged at the charging facilities. BEVs' electricity consumption is typically proportional to the driving distance, resulting in a driving range limit because of the battery capacity. On the basis of current battery technology, charging a BEV still takes more time than refueling a GV's fuel tank. The distance limit, the charging time, and the location of the charging facilities inevitably change BEV drivers' route choice behavior in a stochastic manner where BEV drivers may have imperfect information regarding their travel cost over the entire mixed flow (i.e., BEVs and GVs) traffic network. The massive adoption of BEVs requires a certain level of coverage of the charging facility. Given the financial budget and high cost of installing public chargers, it is a sound approach to maximize the passing BEV population on the links where charging facilities are deployed.

This paper considers a strongly connected transportation network with both BEVs and GVs demands, denoted by

$G = (N, A)$, where N is the set of nodes and A is the set of links. $R \subseteq N$ and $S \subseteq N$ denote the sets of origins and destinations, respectively. The objective of this proposed bilevel model is to locate a given number of BEV charging facilities for covering maximum BEV flows on the mixed traffic flow network. All the candidate charging facility locations are grouped into a set of pseudonodes in the middle of the links denoted by Z. GVs and BEVs are distinguished by their driving distance limits, travel cost composition, and the availability of refueling facilities.

Without loss of generality, the following assumptions are made:

(A1) The technological characteristics of BEVs and demographic features of BEV drivers are homogeneous in the network, and so are GVs and GV drivers. Only one type of BEV with identical driving distance limit and battery consumption rate is considered.

(A2) Every vehicle is fully charged at its origin.

(A3) The variation of BEV drivers' range anxiety level and risk-taking behaviors are ignored.

(A4) A charging facility is deployed on the midpoint of the link in the network.

(A5) The facilities have unlimited charging capacity. Hence, an EV can get charged without delay after its new arrival. En route charging time at the public charging facilities is linear related to the remaining distance to reach the destination.

(A6) The BEV link flow is covered if a charging facility exists on this link.

(A7) The deployment of a charging facility on a route/path would increase the "attractiveness" or "utility" of this route. The utility of a charging facility is considered as a fixed value and converted into travel time reduction.

(A8) Travel demand of both GV and BEV between each O-D pair is fixed. That is, elastic and stochastic demands are not considered in this model.

See the Notations for variables and parameters used throughout this paper, where subscripts g and e indicate variables or parameters associated with GV and BEV, respectively.

3. Model Formulation

In this section, we formulate the bilevel optimization model for the charging facility location problem. Bilevel problems split the decisions of the system planner (leader, i.e., infrastructure developer in this paper) and system users (followers, i.e., drivers) into two levels so that the subproblems are solvable and an iterative approach can be used to achieve an equilibrium state. The upper level aims at determining the locations of charging facilities to increase an objective to maximize the covered BEVs flows assuming BEVs flows remain unchanged. The lower level subproblem is characterized as BEV drivers' route choice behavior with a generalized travel cost structure. SUE conditions with mixed BEVs and GVs assuming fixed locations of charging facilities from the upper level subproblem are analysed.

3.1. Preliminaries. A feasible path for GVs between a given O-D pair may be infeasible for BEVs because of the limited driving distance range and absence of a charging facility. Hence, a feasible path used by GVs can be decomposed into several parts for BEVs according to whether a charging action should be taken by BEV drivers at each charging station. To model BEVs paths, three notions, namely, subpath, pure subpath, and feasible subpath, proposed by Xie and Jiang [38], are introduced in the formulation of the lower level stochastic assignment problem and three charging action based scenarios are analyzed as follows.

Subpath. A part of path k connecting O-D pair (r, s) is a subpath if charging stations are located at the head and tail nodes/pseudonodes of this part. A subpath consists of a number of consecutive links and half links since we assume charging stations locate in the middle of the links. We denote k^{ij}, $i, j \in Z$, as a subpath of path k, where charging station $i(j)$ is the head (tail) node of this subpath. $l_k^{rs,ij}$ is the length of the subpath.

Pure Subpath. Subpath k^{ij} is a pure subpath if there are no other charging facilities on this subpath except i and j.

Feasible Subpath. Subpath k^{ij} is feasible on path k of O-D pair (r, s), if its length is no greater than BEV driving distance limit; that is, $l_k^{rs,ij} \le D_e$.

The concept of subpaths allows us to better illustrate the BEV drivers' path travel cost structure and add the driving distance constraint.

The generalized path travel cost is composed of three parts: path travel time, path charging time, and equivalent travel time reduction (the utility of charging facilities on attracting BEV drivers). Without loss of generality, we consider 3 scenarios based on the relationship between the driving distance limit D_e and subpath distances. For a given path k shown in Figure 1, path travel time and equivalent travel time reduction are fixed and can be represented by a consistent form: $c_k^{rs} + t_{u,k}^{rs}$, where $t_{u,k}^{rs} = 2 \cdot t_u^o$. Note that t_u^o is a nonpositive value.

Scenario 1. There is no need for charging. When $l_k^{rs} \le D_e$, the BEV driver can reach the destination without en route charging. The generalized path travel cost is $\overline{c}_{ke}^{rs} = c_k^{rs} + 2 \cdot t_u^o$.

Scenario 2. If any pure subpath distance exceeds the driving distance limit D_e, this path becomes infeasible to BEV drivers. In other words, if path k cannot be decomposed into a set of feasible subpaths, path k is not feasible. In this case, the generalized path travel cost becomes extremely large and the probability of choosing this path is zero.

Scenario 3. Charging is needed to reach the destination. If the path distance is larger than the distance limit (i.e., $l_k^{rs,rs} \ge D_e$) and the distances of its all pure subpaths are less than D_e, the BEVs need to charge at least once. BEVs would charge as little as possible to reduce the path travel time. The minimum charging time is $t_{c,k}^{rs} = \varepsilon \cdot (l_k^{rs,rs} - D_e)$. The generalized path travel cost is $\overline{c}_{ke}^{rs} = c_k^{rs} + 2 \cdot t_u^o + t_{c,k}^{rs}$.

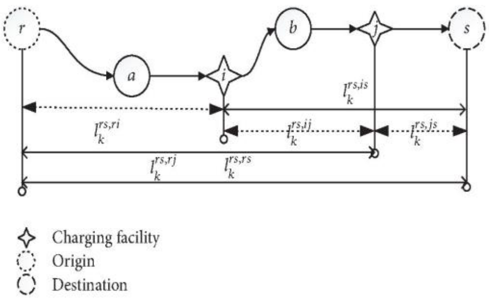


FIGURE 1: Illustration of subpaths definitions. We consider a path k of O-D pair (r, s), along which nodes i and j are located in the middle of links ab, bs, respectively. There exist 3 pure subpaths denoted by dotted lines, namely, k^{ri}, k^{ij}, and k^{js}, and another 3 subpaths k^{rj}, k^{rs}, and k^{is} by solid lines on path k. These subpaths are feasible if their distance is less than the BEV driving distance limit D_e.

For GVs, the generalized path travel cost is $\bar{c}_{kg}^{rs} = c_k^{rs}$. Hence, BEV drivers are more likely to choose path k than GV users under Scenario 1 due to the utility (attractiveness) of the charging facilities on this path, while only GV drivers would choose this path under Scenario 2 because of the infeasible pure subpath. A trade-off between charging time and charging facility utility should be made to identify the generalized travel cost difference of BEVs and GVs under Scenario 3. For example, charging time of fast charging or battery swapping may be shorter than the equivalent travel time reduction converted from the charging facility utility, and thus more BEVs would be assigned to this path even if they may need several charging instances on this path. If multiple charging stations are available on a path, BEV drivers will go through the following process to decide whether charging should be conducted at a station. Let us consider Scenario 3 only where each pure subpath is feasible for BEVs to reach the destination without running out of energy. When arrived at a charging station, BEVs would not charge at the current charging station if they can reach the next one without charging.

3.2. Bilevel Model Formulation. Given the key concepts and terms above, we define the upper level problem as

$$\max_{\mathbf{x}} \quad F[\mathbf{x}, \mathbf{v}(\mathbf{x})]$$
$$\text{Subject to} \quad E[\mathbf{x}, \mathbf{v}(\mathbf{x})] \leq 0, \tag{1}$$

where $\mathbf{v}(\mathbf{x})$ is implicitly determined in the lower level problem

$$\min_{\mathbf{v}} \quad f[\mathbf{x}, \mathbf{v}]$$
$$\text{Subject to} \quad e[\mathbf{x}, \mathbf{v}] \leq 0, \tag{2}$$

where F and E are the objective function and constraints of the upper level problem while f and e are those of the lower level distance-constrained SUE model. F models the total covered BEV link flows and E guarantees the number of charging facilities to be equal to the given design value. $\mathbf{x}$ and $\mathbf{v}$ are decision variables for upper and lower level problems; that is, $\mathbf{x}$ and $\mathbf{v}$ denote charging facility locations and BEV link flow pattern, respectively. Subsequent sections detail the mathematical properties of both upper and lower level subproblems.

Furthermore, in the lower level distance-constrained SUE problem in mixed traffic flow networks, the link performance functions are assumed to be a BPR (Bureau of Public Road) type function as follows:

$$t_a\left(v_{a,g}, v_{a,e}\right) = t_a^0\left(1 + 0.15 \times \left(\frac{v_{a,g} + v_{a,e}}{H_a}\right)^4\right), \quad a \in A. \tag{3}$$

As $t_{c,k}^{rs}$ and t_u^0 are flow-independent, we can easily obtain the Jacobi matrix for the lower level problem, with its elements given for GVs and BEVs, respectively, as follows:

$$\frac{\partial \bar{c}_{kg}^{rs}}{\partial v_{a,e}} = \frac{\partial \bar{c}_{ke}^{rs}}{\partial v_{a,g}} = 0.6 \sum_{a \in A} t_a^0 \delta_{a,k}^{rs} \frac{\left(v_{a,g} + v_{a,e}\right)^3}{H_a^4}. \tag{4}$$

This proves that the Jacobi matrix is symmetric so that the lower level model can be established as a convex mathematical problem.

3.2.1. Upper Level Formulation. The upper level problem aims to maximize the total covered BEV link flows with the deployment of a given number of charging facilities, where the network coverage is defined as the total sum of BEV link flows on only links with charging facility. That is,

$$\max \quad \sum_a v_{a,e} x_a \tag{5}$$

$$\text{Subject to} \quad \sum_{a \in A} x_a = p. \tag{6}$$

Equation (6) is the budget constraint and can be relaxed as locating the maximum number of p facilities in the network as shown in constraint (7). Consider

$$0 \leq \sum_{a \in A} x_a \leq p. \tag{7}$$

3.2.2. Lower Level Problem. The lower level problem is to obtain the equilibrium BEV flow under SUE routing principle in a congested mixed traffic network considering charging facility locations. The network is assumed to be connected; that is, there is at least one path connecting each O-D pair. We formulate the flow conservation and nonnegativity constraints in the mixed traffic network as follows:

$$\begin{aligned}
v_a &= \sum_r \sum_s \sum_k f_{kg}^{rs} \delta_{a,k}^{rs} + \sum_r \sum_s \sum_k f_{ke}^{rs} \delta_{a,k}^{rs}, \quad \forall a \in A \\
q_g^{rs} &= \sum_k f_{kg}^{rs}, \quad \forall (r,s) \\
q_e^{rs} &= \sum_k f_{ke}^{rs}, \quad \forall (r,s) \\
f_{kg}^{rs} &\geq 0, \quad \forall (r,s), \; k \in K_g^{rs} \\
f_{ke}^{rs} &\geq 0, \quad \forall (r,s), \; k \in K_e^{rs}.
\end{aligned} \tag{8}$$

The link travel cost functions are assumed to be separable between different network links, and they are positive, monotonically increasing, and strictly convex as well. The travel cost for GV drivers includes travel time only, whereas BEV drivers travel cost consists of travel time, charging time, and charging facilities' utility. The perceived path cost is equal to the generalized path travel cost plus a random error term.

$$\begin{aligned} C_{kg}^{rs} &= \bar{c}_{kg}^{rs} + \xi_{kg}^{rs}, \quad k \in K_g^{rs} \\ C_{ke}^{rs} &= \bar{c}_{ke}^{rs} + \xi_{ke}^{rs}, \quad k \in K_e^{rs}. \end{aligned} \tag{9}$$

Under SUE, for each O-D pair, GV and BEV flows are distributed on those paths that experience a minimum perceived travel cost and no user can improve its perceived travel cost by unilaterally changing its path. The probability that path k is chosen (by both GV and BEV drivers) can be expressed as

$$P_k^{rs}\left(C_k^{rs}\right) = \Pr\left[C_k^{rs} \le C_r^{rs},\ \forall r \in K^{rs},\ r \ne k\right]. \tag{10}$$

Thus, the SUE path flows are the solution of the following equations:

$$f_{kg}^{rs} = q_g^{rs} P_{kg}^{rs}\left(C_{kg}^{rs}\right), \quad \forall k \in K_g^{rs},\ \forall (r,s) \tag{11}$$

$$f_{ke}^{rs} = q_e^{rs} P_{ke}^{rs}\left(C_{ke}^{rs}\right), \quad \forall k \in K_e^{rs},\ \forall (r,s). \tag{12}$$

It has been proved that adding side constraints directly into the general SUE model does not generate the probit-based SUE traffic assignment with side constraints [46]. Jing et al. [47] proposed a solution framework by properly selecting the path set for each O-D pair to ensure the distances of all the used paths are within the BEV range limit with no charging facilities in the network. We extend that SUE model with path-distance constraints to include public charging facilities.

$$\begin{aligned} \min_{v_a} \quad & Z(\mathbf{v}) \\ &= -\sum_{rs} q_g^{rs} S_g^{rs}\left[c^{\mathbf{rs}}(\mathbf{v})\right] - \sum_{rs} q_e^{rs} S_e^{rs}\left[c^{\mathbf{rs}}(\mathbf{v})\right] \\ &\quad + \sum_a v_a t_a(v_a) - \sum_a \int_0^{v_a} t_a(\omega)\, d\omega \end{aligned} \tag{13}$$

$$\begin{aligned} \text{Subject to} \quad & f_{ke}^{rs}\left(D_e - l_k^{rs,ij}\right) \ge 0, \\ & \forall (r,s),\ k \in K_e^{rs},\ (i,j) \in Z_k^{rs}. \end{aligned} \tag{14}$$

The objective function (13) of the lower level problem is the classical unconstrained minimization model proposed by Sheffi [48], whose solution is equivalent to SUE conditions satisfying network constraints (8). The novelty of this problem lies in the introduction of subpaths in path selection procedure in constraints (14). It is easy to decide whether a charging action should be taken when arriving at a charging station to make sure BEVs can reach the next charging station or destination; namely, the subpath distance $l_k^{rs,ij}$, $(i,j) \in Z_k^{rs}$, of path $k \in K_e^{rs}$ is less than D_e. Supposing that there are Z_L charging stations deployed along a path for BEVs, only less than 2^{Z_L} charging decision should be made and $c_{Z_L+2}^2$ subpaths exist when going through this path. Therefore, by comparing the driving distance limit D_e with subpath distance $l_k^{rs,ij}$, the set of feasible subpaths generated from finite paths between each O-D pair can be predetermined. The generation of feasible subpaths is illustrated in Figure 1 which is similar to the way of predetermining battery-swapping action based feasible paths in Xu et al. [49]. First we prove the equivalence between the solution of the proposed model (see (13)) and SUE solution. The Lagrangian function can be written as

$$\begin{aligned} L(\mathbf{v},\boldsymbol{\mu}) &= -\sum_r\sum_s q_g^{rs} S_g^{rs}\left[c^{\mathbf{rs}}(\mathbf{v})\right] - \sum_r\sum_s q_e^{rs} S_e^{rs}\left[c^{\mathbf{rs}}(\mathbf{v})\right] \\ &\quad + \sum_a x_a t_a(x_a) - \sum_a \int_0^{x_a} t_a(\omega)\, d\omega \\ &\quad - \sum_r\sum_s\sum_k \mu_{ke}^{rs,ij} \cdot f_{ke}^{rs} \cdot \left(D_e - l_k^{rs,ij}\right), \end{aligned} \tag{15}$$

where $\mu_{ke}^{rs,ij}$ is the Lagrangian multiplier corresponding to path/subpath-distance constraint (14). $\mu_{ke}^{rs,ij} f_{ke}^{rs} \cdot (D_e - l_k^{rs,ij})$ can be perceived as the path out-of-range cost incurred when the path/subpath distance exceeds the driving distance limit of the BEV and it should fulfill the following conditions:

$$\begin{aligned} \mu_{ke}^{rs,ij} &= 0, \quad \text{if } l_k^{rs,ij} \le D_e \\ \mu_{ke}^{rs,ij} &\ge 0, \quad \text{if } l_k^{rs,ij} > D_e. \end{aligned} \tag{16}$$

If the flow of BEV drivers going through this path is positive, the path/subpath distance is smaller than or equal to the driving distance limit; otherwise, the trip flow is zero. $\mu_{ke}^{rs,ij}$ is the unit path/subpath out-of-range cost.

The first-order derivative of (13) must satisfy the SUE conditions. Let

$$\nabla L(\mathbf{v},\boldsymbol{\mu}) = 0. \tag{17}$$

The gradient with respect to link flow vector is

$$\begin{aligned} &\frac{\partial L(\mathbf{v},\boldsymbol{\mu})}{\partial v_b} \\ &= \left(-\sum_r\sum_s\sum_{k\in K_g^{rs}} q_g^{rs} P_{kg}^{rs}\delta_{b,k}^{rs} - \sum_r\sum_s\sum_{k\in K_e^{rs}} q_e^{rs} P_{ke}^{rs}\delta_{b,k}^{rs} + v_b\right)\frac{dt_b}{dv_b} \\ &\quad - \sum_r\sum_s\sum_{k\in K_e^{rs}} \mu_{ke}^{rs,ij} \cdot \left(D_e - l_k^{rs,ij}\right)\delta_{b,k}^{rs}. \end{aligned} \tag{18}$$

Note that the extra path/subpath-distance constraints could be infeasible if the distance of any selected subpath exceeds the BEVs' driving distance limit. If all the selected paths and their subpaths are within driving distance limit, the subpath out-of-range cost $\mu_{ke}^{rs,ij} \cdot (D_e - l_k^{rs,ij})$ should be equal to zero. The derivative of the SUE objective function becomes

$$\begin{aligned} &\frac{\partial L(\mathbf{v},\boldsymbol{\mu})}{\partial v_b} \\ &= \left(-\sum_r\sum_s\sum_{k\in K_g^{rs}} q_g^{rs} P_{kg}^{rs}\delta_{b,k}^{rs} - \sum_r\sum_s\sum_{k\in K_e^{rs}} q_e^{rs} P_{ke}^{rs}\delta_{b,k}^{rs} + v_b\right)\frac{dt_b}{dv_b}. \end{aligned} \tag{19}$$

The gradient equals zero if and only if

$$v_b = \sum_r \sum_s \sum_{k \in K_e^{rs}} q_e^{rs} P_{ke}^{rs} \delta_{b,k}^{rs} + \sum_r \sum_s \sum_{k \in K_g^{rs}} q_g^{rs} P_{kg}^{rs} \delta_{b,k}^{rs}, \quad \forall b \in A. \tag{20}$$

Equation (20) expresses the SUE link flows consisting of BEV and GV flows and the feasible solution can be ensured by properly selecting paths. Then we can prove the Hessian matrix of the SUE objective function is positive definite, because the second derivative of $\sum_r \sum_s \sum_{k \in K_e^{rs}} \mu_{ke}^{rs,ij} \cdot f_{ke}^{rs} \cdot (D_e - l_k^{rs,ij})$ with respect to path flow equals zero. This proves that the resulting SUE link flow pattern is unique.

4. Solution Method

The bilevel programming problem is NP-hard. Thus, we propose an equilibrium-based heuristic to iteratively solve the lower level SUE problem and the upper level problem. The interaction between the upper and lower levels, shown in Figure 2, captures the effect of charging facility location on the routing behavior of BEV drivers, which further determines the BEV and GV flow patterns. Initially, we assume no charging facilities in the network. The lower level problem is a stochastic traffic assignment of mixed GV and BEV flows under path-distance constraints. After the first run of the lower level problem, we can obtain the initial BEV link flow pattern. The upper level problem then finds the best p charging facility locations to maximize the total covered BEV flow. The obtained charging facility locations will be compared with the previous location solutions. If there is no change in charging facility location, the procedure ends with the current solution; otherwise, the lower level SUE assignment is repeated with updated charging facility locations.

The detailed procedure is as follows. Note that a Multinomial Logit choice model is used in the lower level SUE TAP.

Step 1. Set upper level iteration counter $z = 1$. Input initial charging facility location, namely, no charging facility in the network. Relax BEVs' distance constraints and perform conventional SUE assignment to identify the corresponding SUE link flow pattern.

Step 2. Increase the upper level iteration counter by 1. Sort all the links in ascending order of their BEV flows and find the top p of them. Locate the charging facilities (uncapacitated) in the middle of the p links.

Step 3. Perform SUE assignment with charging facilities in the network from Step 2. The detailed steps are listed below.

Step 3.1 (subpath feasibility check). Set $x_a(0) = 0$ and $t_a = t_a[x_a(0)]$. For each O-D pair, find K shortest paths for both GVs and BEVs in terms of free-flow travel time and record them as initial path set. For each path of BEVs, identify the path distance, the number of charging facilities on this path, the location of charging facilities, and pure subpath distances. If any pure subpath distance is greater than the BEVs' driving distance limit, set its corresponding path travel time to infinity and this path becomes infeasible. If all the K paths are infeasible, record this O-D pair to Set A. If Set A is empty which means there exists at least one feasible path between each O-D pair, go to the next step; otherwise, stop.

Step 3.2 (initialization). Calculate the generalized BEV path travel cost $\bar{c}_{ke}^{rs}$ and the probability of choosing each path to get the auxiliary link flow pattern. Perform stochastic network loading to assign the entire demand of each class of vehicles between each O-D pair to the corresponding K shortest paths. This yields $\mathbf{v}_{a,g}(1)$ and $\mathbf{v}_{a,e}(1)$. Set iteration counter $n = 1$.

Step 3.3 (update). Calculate a new link cost in terms of $t_a = t_a[\mathbf{v}_\mathbf{a}(1)]$, $\forall a$.

Step 3.4 (direction finding). Follow the same procedure described in Step 3.1 to find K shortest path for each class of vehicles based on the current set of link travel times, $\{t_a^n\}$. If all the pure subpaths of the generated K paths between an O-D pair exceed the range limit, use initial path set in Step 3.1 and perform stochastic network loading. This yields an auxiliary link flow pattern $\{y_{a,g}\}$, $\{y_{a,e}\}$.

Step 3.5 (step size). A predetermined step size sequence $\{\alpha_n\}$ is used: $\alpha_n = 1/n$, $n = 1, 2, \ldots, \infty$.

Step 3.6 (move). Find the new flow pattern by setting

$$\begin{aligned} \mathbf{v}_a^{n+1} &= \mathbf{v}_a^n + \left(\frac{1}{n}\right)\left(\mathbf{y}_a^n - \mathbf{v}_a^n\right) \\ \mathbf{v}_{a,g}^{n+1} &= \mathbf{v}_{a,g}^n + \left(\frac{1}{n}\right)\left(\mathbf{y}_{a,g}^n - \mathbf{v}_{a,g}^n\right) \\ \mathbf{v}_{a,e}^{n+1} &= \mathbf{v}_{a,e}^n + \left(\frac{1}{n}\right)\left(\mathbf{y}_{a,e}^n - \mathbf{v}_{a,e}^n\right). \end{aligned} \tag{21}$$

Step 3.7 (convergence test). Let

$$\bar{v}_a^n = \frac{1}{m}\left(v_a^n + v_a^{n-1} + \cdots + v_a^{n-m+1}\right). \tag{22}$$

If the convergence criterion

$$\frac{\sqrt{\sum_a \left(\bar{v}_a^{n+1} - \bar{v}_a^n\right)^2}}{\sum_a \bar{v}_a^n} \leq \kappa \tag{23}$$

is met, stop and $\{\mathbf{v}_a^{n+1}\}$, $\{\mathbf{v}_{a,e}^{n+1}\}$ are the sets of equilibrium link flows and BEV link flows, respectively; otherwise, set $n = n+1$ and go to Step 3.3.

Step 4. Repeat Step 2 and update the current charging facility location. Compare the current location with previous location status at Step 2. If the locations do not change, stop and record the current charging facility location; otherwise, go to Step 3.

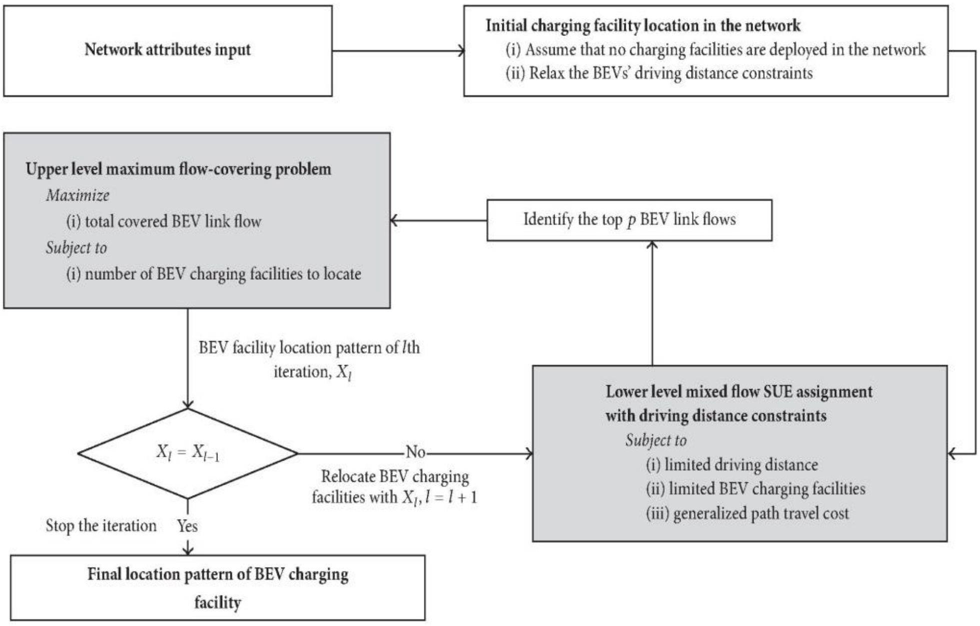


FIGURE 2: Framework of the bilevel proposed method for the equilibrium-optimization-based BEV charging facility location problem.

TABLE 1: O-D demand of Nguyen-Dupuis network.

O-D	BEV	GV
(1, 2)	200	200
(1, 3)	400	400
(4, 2)	300	300
(4, 3)	100	100

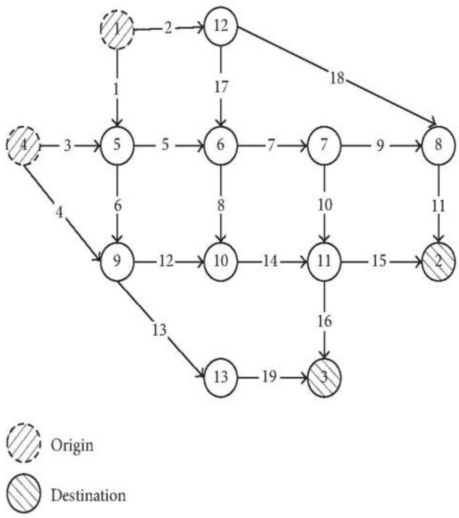


FIGURE 3: The Nguyen-Dupuis network with 2 origins, 2 destinations, 13 nodes, 19 links, and 25 paths between the 4 O-D pairs.

5. Numerical Analysis

This section presents the numerical results of the model and solution algorithm applied to two network case studies. The analysis aims at assessing the impacts of charging facility utility, charging speed, and driving distance limit on the optimal placement of charging facility locations.

The first numerical example is the Nguyen-Dupuis network; see, for example, [49]. The network consists of 13 nodes, 19 links, and 4 O-D pairs: (1, 2), (1, 3), (4, 2), and (4, 3), as shown in Figure 3. The network supply and O-D demands information are from Nguyen and Dupuis [50]. The O-D demand is assumed to be the same for both GVs and BEVs; that is, BEV market penetration rate is 50% (given in Table 1) to facilitate the equilibrium flow comparison between BEV and GV. The free-flow travel time is used as a proxy for the link length for each link. Due to the small size of the Nguyen-Dupuis network, the enumerated path sets information is obtained from Jiang and Xie [43] in Table 2.

We use this case study to evaluate the performance of the proposed algorithm for solving the bilevel model where lower level problem is logit-based SUE assignment with driving distance constraints. The following parameter values are considered. We do not claim the suitability of the defined parameters for accurate quantification of network performance. To avoid the dominant role of t_u^0 in the path cost, a relatively small proportion of charging facility is deployed in this 19-link network: $p = 3$. The BEV driving distance limit is set to 20, the scale parameters of the logit model for route choice of

Table 2: Path compositions and lengths in the Nguyen-Dupuis network example.

O-D	Path number	Path composition	Length
(1, 2)	1	1-5-6-7-8-2	29
	2	1-5-6-7-11-2	33
	3	1-5-6-10-11-2	38
	4	1-5-9-10-11-2	41
	5	1-12-6-7-8-2	35
	6	1-12-6-7-11-2	39
	7	1-12-6-10-11-2	44
	8	1-12-8-2	32
(1, 3)	9	1-5-6-7-11-3	32
	10	1-5-6-10-11-3	37
	11	1-5-9-10-11-3	40
	12	1-5-9-13-3	36
	13	1-12-6-7-11-3	38
	14	1-12-6-10-11-3	43
(4, 2)	15	4-5-6-7-8-2	31
	16	4-5-6-7-11-2	35
	17	4-5-6-10-11-2	40
	18	4-5-9-10-11-2	43
	19	4-9-10-11-2	37
(4, 3)	20	4-5-6-7-11-3	34
	21	4-5-6-10-11-3	39
	22	4-5-9-10-11-3	42
	23	4-5-9-13-3	38
	24	4-9-10-11-3	36
	25	4-9-13-3	32

GV and BEV are $\gamma_g = \gamma_e = 0.1$, the charging speed is $\varepsilon = 1$, the utility of a charging facility on path is $t_u^0 = -2$, and K in the K shortest paths is set to be 5. In addition, the link capacity and free-flow travel time (link length) are given in Table 3 with the equilibrium BEV link flow at each upper level iteration.

The relationship between charging facility location pattern in the upper level and BEV link flows in the lower level is first examined. Table 3 lists the charging facility locations and the corresponding BEV link flows in each iteration. At the first iteration, we assume no charging facility is available in the network and relax the driving distance constraints. The results clearly show the overall BEV link flow pattern in the first iteration is quite different from those in the others, especially after the first iteration when charging facilities are located in the network. In the first iteration, every enumerated path is feasible for BEV drivers since the driving distance constraint is relaxed. As for the other iterations, some paths become infeasible due to the lack of charging facilities. For example, only path 18 between O-D pair $(4, 2)$ is feasible in the last iteration because two charging stations are deployed on links 6 and 14 so that each pure subpath distance is smaller than the range limit.

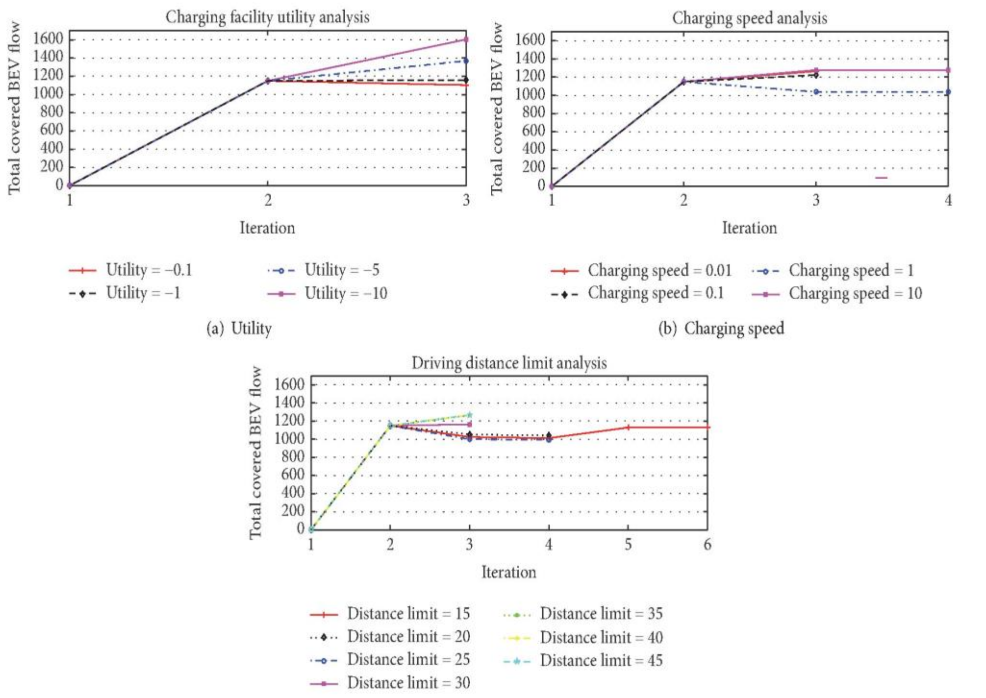
The total covered flows by locating 3 charging facilities in this example are "0, 1054.3, 1048.5, and 1048.5" during the four iterations. The amount of total covered BEV flows in the third iteration may decrease comparing to the second iteration because the BEV flow covered in the second iteration is actually generated by using the charging facility locations in the first iteration. Therefore, when new locations are generated, the BEV link flow changes accordingly until the last two iterations that produce the same facility locations. The potential drawback of this modified definition of maximum covering flow is that if a route contains multiple links with charging stations (e.g., paths 4 and 11), a trip by a driver is counted multiple times even though BEV drivers may not charge or only charge once during the trip. As a result, this method could locate charging facilities on several adjacent links of some high-volume freeways, while in practice fast charging facilities are usually deployed with long intradistances along the freeways.

A sensitivity analysis is conducted with respect to the charging facility utility, charging speed, and BEV driving distance limit. The results are illustrated in Figure 4, where only one parameter is changed in each scenario. In scenario (a), we set the charging speed as $\varepsilon = 0.1$ which can be regarded as relatively fast charging and we conduct tests on different level of charging facility utility. The utility value can be perceived as the risk-taking level of BEV drivers. A smaller utility value indicates that BEV drivers are willing to take more risks. As the equivalent travel time reduction value (i.e., utility) goes up, the total covered BEVs flows increases, because BEVs drivers are more likely to choose feasible lengthy paths with fast charging facilities instead of paths with less travel time. If we consider multiple classes of BEV drivers with different driving distance limits, the BEVs with shorter driving distance and risk-neutral attitude would probably have a larger value of charging facilities utility, because charging facilities help to ease their range anxiety, while, for those with larger batteries, they would behave more like GV users. In general, large travel time reduction value should apply to fast charging method, small battery capacities, and risk-taking BEV drivers.

We then examine the impacts of charging speed, that is, ε, in scenario (b), where a smaller value represents a faster charging speed, with charging time estimated as $t_{c,k}^{rs} = \varepsilon \cdot (l_k^{rs,rs} - D_e)$. This parameter translates to different charging methods (i.e., slow charging, fast charging, or battery-swapping technology) that lead to different charging facility location patterns. Given a charging facility location pattern (e.g., $\{1, 5, 7\}$), charging speed affects the total travel cost on a feasible path. As a result, the probability of choosing each path changes if there exist at least two feasible paths between each O-D pair. With $\varepsilon = 0.01$, the charging facilities are deployed on link $\{1, 5, 7\}$ and the feasible paths are paths 9 and 13 between O-D pair $(1, 3)$, whereas, with $\varepsilon = 10$, the charging facilities are located on $\{6, 12, 14\}$. Only path 11 is feasible between O-D pair $(1, 3)$, and all the BEV drivers will be assigned to this path if no other paths are feasible. In this case, charging speed does not affect the path choice probability. Fast charging attracts more BEV flows compared to slow charging when at least one another path with no charging need is available to BEV users, because the charging speed would have the influence on the total travel cost and path choice probability only if BEV drivers take charging

TABLE 3: The charging facility locations and BEV flows over iterations.

Link number		Link length	Link capacity	Upper level iteration 1		2		3		4	
				Location	BEV flow	Location	BEV flow	Location	BEV flow	Location	BEV flow
(1, 5)	1	7	500	/	349.8	√	316.2	√	335.5	√	335.5
(1, 12)	2	9	500	/	250.2	/	283.8	/	264.5	/	264.5
(4, 5)	3	9	500	/	257.1	/	188.0	/	194.5	/	194.5
(4, 9)	4	12	400	/	142.9	/	212.0	/	205.5	/	205.5
(5, 6)	5	3	500	/	395.4	√	202.5	/	228.1	/	228.1
(5, 9)	6	9	500	/	211.5	/	301.6	√	301.8	√	301.8
(6, 7)	7	5	500	/	404.4	√	172.5	/	196.5	/	196.5
(6, 10)	8	13	500	/	159.0	/	175.7	/	158.1	/	158.1
(7, 8)	9	5	500	/	161.4	/	75.4	/	87.8	/	87.8
(7, 11)	10	9	500	/	243.0	/	97.2	/	108.7	/	108.7
(8, 2)	11	9	500	/	243.5	/	213.5	/	225.9	/	225.9
(9, 10)	12	10	500	/	164.7	/	260.8	/	253.0	/	253.0
(9, 13)	13	9	400	/	189.7	/	252.9	/	254.3	/	254.3
(10, 11)	14	6	500	/	323.8	/	436.5	√	411.2	√	411.2
(11, 2)	15	9	500	/	256.5	/	286.5	/	274.1	/	274.1
(11, 3)	16	8	500	/	310.3	/	247.1	/	245.7	/	245.7
(12, 6)	17	7	500	/	168.1	/	145.7	/	126.5	/	126.5
(12, 8)	18	14	400	/	82.1	/	138.1	/	138.0	/	138.0
(13, 3)	19	11	500	/	189.7	/	252.9	/	254.3	/	254.3



(a) Utility

(b) Charging speed

(c) Driving distance limit

FIGURE 4: Sensitivity analysis for various input parameters (a) charging facility utility; (b) charging speed; and (c) driving distance limit on the total covered BEV flows.

action with these facilities. The generalized travel cost on paths with charging actions would be too high when charging speed is extremely slow (e.g., $\varepsilon = 10$) and charging time takes over path travel time. BEVs would probably choose saturated paths with high travel time. However, as can be seen from the results, the total covered BEV flow is not strictly increasing with the increase of charging speed and it is also influenced by the feasible path set between O-D pairs.

In scenario (c), the lower bound of distance limit is set to 15 to make sure there exists at least one feasible path between each O-D pair. In addition, given that all paths are enumerated in Table 2, the distance limit 45 is the path length upper bound in the network without imposing the distance limit. The charging facility locations for distance limits 15, 20, and 25 are $\{5, 7, 14\}$, $\{1, 6, 14\}$, and $\{1, 5, 14\}$, respectively, and the total covered BEV flows are 1131.6, 1040.9, and 995.5. Additionally, the charging facilities are all located on $\{1, 5, 7\}$ for distance limits 30, 35, 40, and 45, covering, respectively, 1164.7, 1265.4, 1267.5, and 1267.6 BEV flows. It is observed that as the distance limit increases, the total covered BEV flow decreases at first, while after the distance limit reaches a certain value, the total covered BEV flows increase till it reaches a stable value. The driving distance limit affects the number of feasible subpaths and charging time. As the driving distance limit increases, more paths are eligible to carry flows and a larger K value should be used to generate more feasible paths during the assignment process. However, as indicated in [38], the change in the number of feasible paths does not always increase with the distance limit, since each subpath of the generated K shortest paths would be feasible when the distance limit is large enough.

From the three sensitivity analysis scenarios, it is observed that the proposed model can satisfy the stopping criteria after 3 or 4 iterations for this small network. Although there is no significant difference in the total covered BEV flows, the charging facility locations vary for each scenario. It is noteworthy that the deployment of charging facilities changes BEV path flow patterns while the aggregated covered BEV link flows do not change significantly. Therefore, the strategy of locating charging facilities is still focusing on those BEV saturated links to increase the exposure of charging facilities to BEV flows. Taking realistic situation into consideration, when budget is limited, the number of charging facilities can be flexible by adjusting its size and configuration. It would be better to scatter more small size charging facilities than large ones to increase the exposure to BEV drivers. The charging speed affects the BEVs perceived travel cost only when they need charging. Thus fast charging station or chargers should be deployed along freeways or highways to reduce the charging time of long-distance trips while slow chargers can be deployed along urban roads to eliminate range anxiety and to increase exposure. Under some circumstances, charging station equipped both slow and fast chargers may enable more flexible charging operation. We also found that the BEVs are restricted to some relatively short paths especially when distance limit is low; however, the equilibrium mechanism will assign more GVs to relatively long paths since the GV drivers still try to minimize their perceived travel time.

The second numerical experiment is done on the Sioux Falls network shown in Figure 5, which has been chosen as a benchmark network in numerous traffic assignment studies. We adopt a variation of this network presented in Suwansirikul et al. [51]. The exact network attributes and travel demands are also used in our study. For simplicity, the free-flow travel time is used as proxy for link length and BEV penetration rate is assumed to be 50%. Sioux Falls network consists of 24 nodes, 76 links, and 576 O-D pairs. The number of charging facilities is $p = 8$. This example is to evaluate the computational performance of the proposed solution algorithm. For computational experiments, the number of iterations (ITR) and the total computational cost (TCC) were compared under different parameter settings.

Table 4 lists the computational cost under different parameter settings. Assuming the logit scaling parameter be 0.1, it can be seen from Scenario 1 that the computational cost generally increases as the driving distance limit increases. The underlying reason might be that many paths become feasible in the K paths generated, thus requiring the related path/subpath choice probability calculation and assignment. From the first two scenarios, clearly K value has an impact on the computational cost, because bigger K value would increase the computational time in the K shortest path algorithm as well as the stochastic network loading procedure in the lower level problem. Comparing Scenario 2 with Scenarios 3 and 4, respectively, the results demonstrate that charging speed and charging facilities' utility affect computational time marginally. Finally, we can observe that K value has the most impact on increasing computational time and the number of iterations needed for the upper level problem.

6. Conclusions and Future Work

This paper formulates, solves, and evaluates the problem of potential location of public charging facilities for BEV in a network with mixed GVs and BEVs. The path travel cost of BEVs is modeled by considering path travel time, charging time, driving distance limit, and charging facilities' utility, where driving distance limit restricts the path choice. A bilevel model has been proposed to address the issue of coexisting equilibrium GV-BEV flows. A mix-integer nonlinear program is constructed based on MSA to maximize the total BEV flow coverage on high-BEV-traffic paths. The key part of this formulation is the lower level path-distance constrained stochastic traffic assignment. The solution equivalency is proved to satisfy SUE condition as well as the uniqueness of link flow pattern. Moreover, a modified MSA method with K shortest path algorithm and generalized BEV path travel cost are applied to solve the charging facility location problem. In the numerical analysis, we also demonstrated how the driving distance limits, charging speed, and utility of charging facilities affect the equilibrium network flow and charging facility location.

We expect that the strategy of locating charging facilities and the modeling technique presented in this work would potentially trigger the interest of incorporating other types of BEV-specific constraints in the lower level problem, such

TABLE 4: Computational cost with different parameter settings for MNL.

	Scenario 1: $K = 3$, $\varepsilon = 1$, $t_u^0 = 0.001$				Scenario 2: $K = 5$, $\varepsilon = 1$, $t_u^0 = 0.01$			
D_e	0.25	0.4	0.6	0.8	0.25	0.4	0.6	0.8
ITR	5	3	3	3	4	4	3	3
TCC(s)	117.5	168.74	166.56	173.15	197.33	474.45	328.17	329.46
	Scenario 3: $K = 5$, $\varepsilon = 0.1$, $t_u^0 = 0.01$				Scenario 4: $K = 5$, $\varepsilon = 1$, $t_u^0 = 0.001$			
D_e	0.25	0.4	0.6	0.8	0.25	0.4	0.6	0.8
ITR	4	4	3	3	4	4	3	4
TCC(s)	199.18	477.28	328.06	326.61	198.15	475.62	327.09	474.98

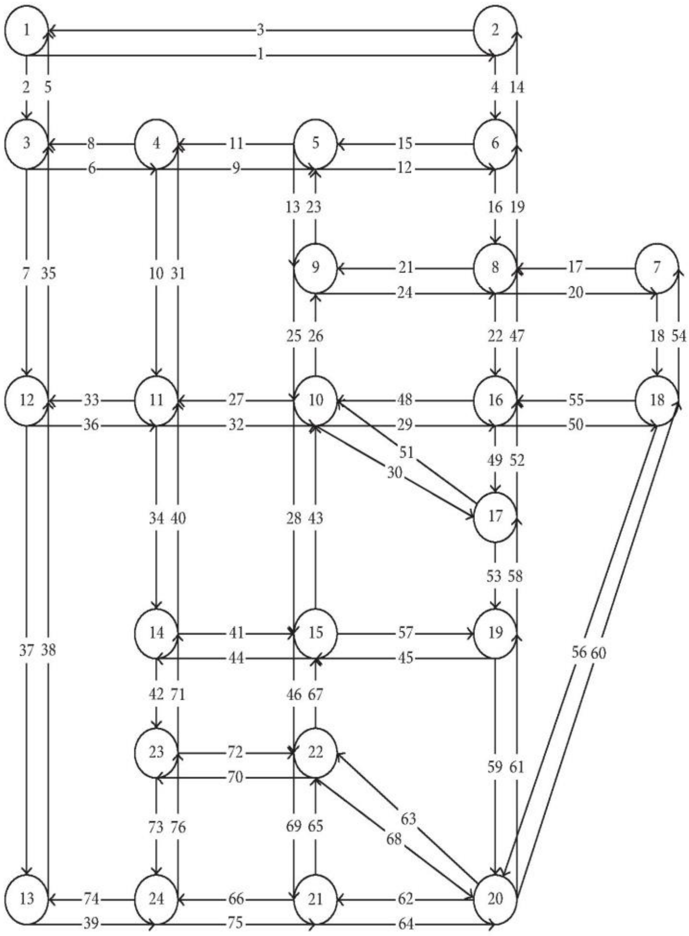


FIGURE 5: Sioux Falls network with 24 nodes and 76 links.

as flow-dependent battery capacity constraints and time-dependent battery-charging price. As for the upper level problem, some other approaches, such as FILM and FRLM, locating charging facilities to maximize passing BEV flows without double counting, can be explored to better serve the BEV travel demand. The model uses a number of assumptions to simplify the problem and make it tractable, which will be relaxed in the future work to deal with more complicating and realistic issues.

Notations

K_g^{rs}, K_e^{rs}: Set of paths connecting O-D pair (r, s) of GV and BEV, respectively

Z_k^{rs}: Set of pseudonodes of charging stations on path $k \in K_e^{rs}$ connecting O-D pair (r, s)
$l_k^{rs,ij}$: Length of subpath k^{ij} in path k, $(i, j) \in Z_k^{rs}$
l_a: Length of link a, $a \in A$
v_a: Traffic flow on link $a \in A$, which is the summation of GV link flow $v_{a,g}$ and BEV link flow $v_{a,e}$that is, $v_a = v_{a,g} + v_{a,e}$
$\mathbf{v}$: A column vector of all the link flows; $\mathbf{v} = (v_a)^T$, $a \in A$
x_a: Binary variable, equaling 1 if there is a charging facility at location $z \in Z$ on link a 0 otherwise
$\mathbf{x}$: A column vector of all the location variables; $\mathbf{x} = (x_a)^T$, $a \in A$
$t_a(v_a)$: Link travel time on link a
$\delta_{a,k}^{rs}$: Link path incidence: $\delta_{a,k}^{rs} = 1$ if path $k \in K_g^{rs}$, K_e^{rs} between O-D pair (r, s) traverses link $a$and 0 otherwise
l_k^{rs}: $l_k^{rs} = \sum_a l_a \delta_{a,k}^{rs}$, length of path k between O-D pair (r, s)
D_e: Driving distance limit of BEV
f_{kg}^{rs}, f_{ke}^{rs}: Traffic flow of GV and BEV on path $k \in K_g^{rs}, K_e^{rs}$
$c_k^{rs}(\mathbf{f})$: Path k travel time between O-D pair (r, s), $k \in K_e^{rs}, K_g^{rs}$; $c_k^{rs}(\mathbf{f}) = \sum_a t_a(v_a)\delta_{a,k}^{rs}$
$t_{u,k}^{rs}$: Total travel time reduction on path $k \in K_e^{rs}$
t_u^0: The utility of one charging facility on the path, equivalent to a constant nonpositive travel time reduction value
$\bar{c}_{kg}^{rs}$, $\bar{c}_{ke}^{rs}$: Generalized travel cost of GV or BEV on a given path $k \in K_g^{rs}, K_e^{rs}$
ε: Battery-charging speed, min/km
$t_{c,k}^{rs}$: Charging time needed on a given path $k \in K_e^{rs}$ between O-D pair (r, s)
t_a^0: Free-flow travel time on link a
H_a: Capacity of link a
p: The number of charging facilities to be located
q_g^{rs}, q_e^{rs}: GV and BEV travel demand between O-D pair (r, s)
P_{kg}^{rs}, P_{ke}^{rs}: The probability that GV or BEV choose path k between O-D pair (r, s)
γ_g, γ_e: Scale parameter of the logit model for route choice of GV and BEV, respectively
S_g^{rs}, S_e^{rs}: The satisfaction function: the expected value of the minimum perceived travel time for GV and BEV travelers between O-D pair (r, s) respectively
ξ_{kg}^{rs}, ξ_{ke}^{rs}: Random error term of perceiving generalized GV and BEV path k cost between O-D pair (r, s).

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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6.3 Conclusion

BEVs rely entirely on electricity as a single power source and are designed to be charged at the charging facilities. BEVs' electricity consumption is typically proportional to the driving distance, resulting in a driving range limit because of the battery capacity. On the basis of current battery technology, charging a BEV still takes more time than refuelling a GV's fuel tank. The distance limit, the charging time and the location of the charging facilities inevitably change BEV drivers' route choice behaviour in a stochastic manner where BEV drivers may have imperfect information regarding their travel cost over the entire mixed flow (i.e. BEVs and GVs) traffic network. The massive adoption of BEVs requires a certain level of coverage of the charging facility. Given the financial budget and high cost of installing public chargers, it is a sound approach to maximize the passing BEV population on the links where charging facilities are deployed.

This paper formulates, solves and evaluates the problem of potential location of public charging facilities for BEV in a network with mixed GVs and BEVs. The path travel cost of BEVs are modelled by considering path travel time, charging time, driving distance limit and charging facilities' utility, where driving distance limit restricts the path choice. A bi-level model has been proposed to address the issue of co-existing equilibrium GV-BEV flows. A mix-integer non-linear program is constructed based on MSA to maximize the total BEV flow coverage on high-BEV-traffic paths. The key part of this formulation is the lower level path-distance constrained stochastic traffic assignment. The solution equivalency is proved to satisfy SUE condition as well as the uniqueness of link flow pattern. Moreover, a modified MSA method with K shortest path algorithm and generalized BEV path travel cost are applied to solve the charging facility location problem. In the numerical analysis, we also demonstrated how the driving distance limits, charging speed and utility of charging facilities affect the equilibrium network flow and charging facility location.

Although this study provides insights into the definition of charging facilities' coverage, we still expect the strategy of locating charging facilities and the modelling technique presented in this work would potentially trigger the interest of incorporating other types of BEV-specific constraints in the lower level problem, such as flow-dependent battery capacity constraints, time-dependent battery price, etc. As for the upper level problem, some other approaches, such as FILM and FRLM, locating charging facilities to maximize passing BEV flows without double counting, can be explored to better serve the BEV travel demand. The model uses a number of assumptions to simplify the problem and make it tractable, which will be relaxed in the future work to deal with more complicating and realistic issues.

This chapter focused on deploying public charging facilities for private BEVs based on the SUE flow patterns of BEVs. The SUE models used in the lower-level problem is an extension of those in Part II. The next chapter, which is the last chapter of Part III, is dedicated to another charging facility, battery swapping facility for public EBs, which is an important part of transportation electrification process.

CHAPTER 7 BATTERY SWAPPING FACILITY LOCATION MODEL OF PUBLIC ELECTRIC BUSES

7.1 Introduction

In accordance with research objective 5, the aim of this chapter is to specifically design the refueling facility network for electric buses (EBs) that are known to follow fixed routes and timetable during their operation. This chapter considers EBs' own characteristics which are different from BEVs and addresses the research gap identified in the Literature Review: No studies have explored the BSS location model with local charging system serving EB fleet. Table 7-1 details the research objective, research component, research gap and research opportunity associated with this chapter.

Table 7-1: Research gap, opportunity and objective associated with research component 5

Research topic	Research gaps	Research opportunities
Charging/swapping facility location models of BEVs and BEBs (Part III)	Battery swapping is designed to be more suitable for electric buses. Moreover, there is no swapping facility location model dedicated to swapping facility location with local charging system serving EB fleet (see, section 2.5.5)	Proposing a new BSS facility location model for BEBs considering EBs' characteristics (see Chapter 7)

The models and findings of the research in this chapter have been submitted in the form of one journal paper as follows:

Jing, Wentao, Inhi Kim and Kun An. (Under review) " The capacitated battery swapping facility location problem with local charging system serving electric bus fleet." Submitted to Journal of Transport Geography.

As battery charging stations for private BEVs in chapter 6, battery swapping technology has been considered more suitable for EBs. In this chapter, a BSS location problem is considered for battery EBs with local charging system. The depleted batteries will be charged at BSSs with local charging system in terms of its charger type and the quantity of both chargers and batteries. This chapter starts with an introduction of EBs' adoption and its refueling system around the globe. The intention of this chapter is to answer four fundamental questions: How many BSSs should be optimal? Where should they be? Which EBs should be assigned to them? How big should they be in terms of service capability?

The service capability of the BSS is restricted by the number of swapping robots in each BSS. Understanding the key cost factors in BSS system that serves EBs will assist city planners or bus service operators in making better EB-related decisions at the planning level. This is particular important in public transit, as there is limited research on BSS location problem studies and BSS can largely improve the utilization rate of EBs by reducing the refueling time.

7.2 Background

Along with increasing environment and energy concerns, electric vehicles (EVs) are regarded as a promising solution to alleviate the global energy crisis and reduce greenhouse gas emissions. As part of the transportation electrification plan, battery electric buses (EBs) have received significant attention worldwide with the advance in battery and bus manufacturing technologies. In public transport, diesel-powered buses are still dominant, which accounts for more than 45% nitrogen oxides and 75% of particulate matter emissions(Elkins et al. 2003). In contrast, EBs have a unique advantage: zero emissions. Governments thus have created various incentives to switch to alternative fuel buses, powered by natural gas, hydrogen, or electric batteries. For example, the TIGER program in the United States, the Green Bus Fund Program in the UK, the Electric Mobility Program in German and the Ten Cities and Thousand Vehicles Program in China, all aimed to promote green transport(SUTP 2015). Motivated by these government incentives, EBs are being extensively used in many metropolitans. Several cities in the United States, such as Santa Barbara, Chattanooga, Berkeley, and Denver, introduced EBs in transit service prior to mid-2000s. In 2012, Uruguay signed a deal for 500 heavy-duty EBs and Tel Aviv in Israel ordered 700 EBs. In 2013, Shenzhen, China, ordered 1000 heavy-duty EBs.

EBs usually have fixed running routes, fixed depots, and near-identical battery capacity. However, configuring a public transport system using EBs is challenging; this includes possible battery recharging or swapping strategy decisions, battery sizing, and charging station sitting and sizing problems(Leou,Hung 2017). Comparing to conventional diesel-powered buses, EBs still suffer from long charging time, limited mileage range, and insufficient charging infrastructures problem regardless of its regenerative braking attribute of recovering energy from the braking process. Theoretically, EBs can travel up to 250 km. Various factors, including air conditioning, driving behavior, and battery aging issues can significantly reduce the EBs' operational range, often making EBs incapable of finishing a whole day's work without battery recharging (Li 2016).

Three charging methods are available, namely slow charging, fast charging, and battery swapping. Slow charging usually takes hours to refuel a bus and thus reduces the utilization rate of EBs, whereas fast charging may only need ¼ of the time but is to the detriment of battery life (Sarker et al. 2013). According to Huang et al. (2016), a charger costs from $1,000 to $100,000 depending on the charging speed. One has to weigh the costs, charging efficiency, battery life and other factors in choosing the charging method. It is pointed out that well-aligned charging strategies with evolutionary electric vehicle adoption are the prerequisite for realizing its environmental benefits especially in countries with fossil-dominated power. Otherwise, the disordered charging will cause load fluctuations and increase generation costs (Rao et al. 2015).

The deployment of battery swapping stations (BSSs), which remove depleted batteries on EBs and replace the batteries with fully charged ones, is an alternative strategy to eliminate most of these barriers (Avci et al. 2014). The most outstanding feature of this strategy is that BSSs can complete the swapping process in less than 10 minutes. The depleted battery can be left overnight to get charged at a discounted electricity price. Such battery management method allows effective battery maintenance and is beneficial to extend the batteries' lifetime. However, due to lack of standardization in batteries and its charging interfaces, BSSs are more suitable for buses and taxis rather than private vehicles (Zheng et al. 2014). Many countries are keen to explore the application possibilities of BSS systems. In April 2015, Ziv Av Engineering signed a deal with China's Bustil to design 7000 BSSs for EBs in Nanjing city (Elis 2015). So far 1,300 BSSs have been constructed and additional 12,000 are planned through 2020 in many pilot cities of China (Liang et al. 2017).

Generally, there are two types of operation modes for BSSs: central charging and local charging (Tan et al. 2014). In the central charging mode, EVs swap their batteries in BSSs, and the empty batteries are sent to the central charging station. After empty batteries are fully charged, they will be delivered back to BSSs. The other mode utilizes a local charging system which charges depleted batteries in local BSSs (Mak et al. 2012). While avoiding the tedious battery shipping, the local charging method calls for careful land-use planning to reserve sufficient spaces for bus awaiting/parking and for local charger installation(Li 2016). Moreover, BSSs require large capital investment in purchasing additional batteries to be swapped with ones near depletion. The location of BSSs and the choice of charger types become an inevitable issue when designing a battery swapping system to balance the tradeoff between their charging speed and costs.

Many efforts have been devoted to optimizing the planning and operation for BSSs. The existing research can be classified into three categories. In the first category, the optimal location of BSSs and the interaction between BSSs and the power grid are the primary concerns. Xiang,Zhang (2017) developed a p-median based model to solve the BSS location problem with a central charging system. Xu et al. (2013) studied the optimal configuration of a central charging station and its location. Liu et al. (2016) proposed a bi-level model to plan the capacity and location of BSSs to maximize the net profit of BSSs in the upper level while minimizing the operational costs of the distribution company in the lower level. The second category primarily focuses on the operation of both BSSs and EBs. Li (2013) proposed a single-depot optimization model for EB scheduling to minimize the total operating costs with battery swapping constraints at BSSs. Zhu,Chen (2013) looked into the minimum number of standby batteries to ensure non-stop bus operations and studied the required power supply to meet the charging demand of EBs. You et al. (2016) focused on scheduling the battery charging in BSSs so that every EB could find a fully charged battery for swapping. The third category addresses the operation details of BSSs including optimal power capacity (Leou,Hung 2017), charging scheduling (You et al. 2017) and swapping demand analysis (Xiong et al. 2012). A simulation-based model was utilized to estimate the uncontrolled energy consumption of BSSs considering random EB arrivals and disorderly charging behavior (Dai et al. 2014). Based on a central charging system, EBs are scheduled for charging at BSSs to minimize the charging costs considering electricity price fluctuations and EB charging priority (Kang et al. 2016). Regarding local charging systems, Zhu et al. (2016) proposed a mathematical model to simultaneously determine the charging station location and the number of chargers to install in each station.

However, no study has investigated the optimization of swapping station location, charger number, charger type and electric bus assignment in the BSS planning problem. In addition, to swap the depleted battery, an EB may travel a long distance to the assigned BSS. More electricity energy should be reserved to sustain the trip to the BSS. This further reduces the number of trips that an EB can serve on a bus route and adds to the difficulties of BSS assignment. To promote the development of BSSs for EBs, the optimal BSSs' location and its local charging system design should be investigated together. Transport costs between EB transit depots and BSSs is another major factor to capture the energy waste during the detour to swap the depleted battery, which will be considered in this study.

In this paper, we propose an optimization framework for locating capacitated BSSs incorporated with local charging systems. Comparing with the previous studies, the main contributions of this paper can be summarized as follows. First, to our best knowledge, this is the first study investigating the deployment of BSSs with different types of local charging infrastructures (including batteries, chargers and swapping robots) while taking the tradeoff between BSS installation costs and transportation costs from EBs to BSSs into account. Second, the proposed model can provide insights for city planners and bus operators of deploying battery swapping and charging systems. The optimal number of batteries, chargers and swapping robots and the type of chargers initially purchased at BSSs can also be decided through the proposed model to satisfy the swapping and charging demand of EBs. Third, a case study of the southeast region of Melbourne network verifies the effectiveness of the proposed model and provides cost analysis if EBs serve the current bus routes and demand. The approach proposed in this paper may be used by city planners, power grid companies, and transit service providers to plan the battery charging and swapping infrastructures, estimate how many chargers and what type of chargers to install to fulfill the potential demand while minimizing the total capital investment.

This paper is organized as follows. The next section introduces the assumptions and problem settings. Section 7.4 discusses the mechanism of capacitated BSSs with local charging systems and its mathematical formulation. In Section 7.5, we use a case study to demonstrate the effectiveness of the proposed model. Finally, Section 7.6 is devoted to the conclusions and future research.

7.3 Basic consideration

Facility location decisions are strategic in nature. BSSs and their local charging system will remain in place for many years. While the conditions and policies under which BSSs will operate in the future are not clear yet, the tactical and operational decisions of BSSs can be adjusted to some extent subject to the pre-determined long-term decisions of the BSS location. In particular, the swapping demand of EBs could change with weather and road conditions. With the advance of charger and battery technologies, their service capability is expected to improve at a lower cost. The battery charging scheduling methods that the operator may adopt are also uncertain. Thus, it is often unwarranted to insist on the strict satisfaction of battery quantity constraints. Since the aim of this paper is to explore a location model regarding the newly emerging battery charging and swapping system for EBs, we make some simplifications to model the BSSs deployment with local battery charging systems serving EB depots. Assumptions of the modeling framework are summarized as follows:

1. All EBs use a uniform type of batteries.
2. The demand for EB battery swapping services is evenly distributed during the T time slots and the charging scheduling optimization is not considered here.
3. EBs go to BSSs for swapping from the transit depot/bus terminals only, but not from intermediate stops.
4. Every EB can get a fully charged battery immediately when they arrive at a BSS.

7.4 Model formulation

Facility The electric bus operating company or power grid company is considered as the major investor to build the BSS network. They decide the location and configuration of BSSs in terms of charger type, charger quantity, swapping robot quantity and battery inventory. The objective aims to minimize the total investment of BSSs network while fulfilling the EB charging demand. Therefore, this paper intends to minimize the total system investment, including fixed swapping facility costs, total transportation costs, battery purchase costs, and installation costs of chargers and swapping robots.

The following table provides the notation of variables and parameters used throughout this paper.

Variables	Description
X_j	binary variable, equals 1 if a BSS is installed at candidate site $j \in J$, 0 otherwise
Y_{ij}	assignment variable, fraction of demand at transit depot $i \in I$ that is assigned to the BSS at candidate site $j \in J$
N_{ja}	number of chargers of type $a \in A$ installed at station $j \in J$
R_j	number of swapping robots at station $j \in J$
Z_j^t	number of fully-charged batteries stored at candidate site $j \in J$ in time slot $t \in T$
W_{ja}^t	number of depleted batteries assigned to chargers of type $a \in A$ at station $j \in J$ in time slot $t \in T$,$W_{ja}^0 = 0$

H_j^t	swapping demand arrived at BSS station $j \in J$ in time slot $t \in T$

Parameters

γ	coefficient, monetary costs of transporting a depleted battery between the bus depot and the BSS for one km
h_i	swapping demand at transit depot $i \in I$
d_{ij}	distance between transit depot i and candidate BSS site $j \in J$
f_j	fixed costs of constructing a BSS facility at candidate site $j \in J$, converted to the depreciation costs per day
c_a	costs of installing a type $a \in A$ charger, including land costs, transformer and power line costs, converted to the depreciation costs per day
c_b	costs of purchasing a battery, converted to the depreciation costs per day
c_c	costs of installing a swapping robot, converted to the depreciation costs per day
s_a	service capability of a type $a \in A$ charger in a typical service period (No. of batteries fully charged per day)
s_c	service capability of a swapping robot in a typical service period (No. of batteries swapped per day)

Objective function:

$$\min_{\mathbf{X,Y,N,R,Z,W,H}} \sum_{j \in J} f_j X_j + \gamma \sum_{i \in I} \sum_{j \in J} h_i d_{ij} Y_{ij} + \sum_{j \in J} \sum_{a \in A} c_a N_{ja} + \sum_{j \in J} c_b Z_j^0 + \sum_{j \in J} c_c R_j \tag{1}$$

s.t

$$\sum_{j \in J} Y_{ij} = 1, \ \forall i \in I \tag{2}$$

$$Y_{ij} \le X_j, \ \forall i \in I, j \in J \tag{3}$$

$$X_j = \{0,1\}, \ \forall j \in J \tag{4}$$

$$0 \le Y_{ij} \le 1, \ \forall i \in I, j \in J \tag{5}$$

$$Z_j^{t+1} = Z_j^t + \sum_{a \in A} W_{ja}^{\max\{t+1-\left\lceil \frac{T}{s_a} \right\rceil, 0\}} - H_j^t, \ \forall t \in T, j \in J \tag{6}$$

$$H_j^t \le Z_j^t, \ \forall t \in T, j \in J \tag{7}$$

$$H_j^t = \sum_{a \in A} W_{ja}^t = \sum_{i \in I} h_i Y_{ij} / \mathrm{T}, \forall t \in T, j \in J \tag{8}$$

$$\sum_{r=t}^{t+\left\lceil \frac{T}{s_a} \right\rceil - 1} W_{ja}^r \le N_{ja} \left\lceil \frac{s_a}{T} \right\rceil, \ \forall t \in T, j \in J, a \in A \tag{9}$$

$$\sum_{i \in I} h_i Y_{ij} \le \sum_{a \in A} N_{ja} s_a, \forall j \in J \tag{10}$$

$$\sum_{a \in A} N_{ja} \le M * X_j, \forall j \in J \tag{11}$$

$$R_j \le M * X_j, \forall j \in J \tag{12}$$

$$H_j^t \le R_j s_c / T, \forall t \in T, j \in J \tag{13}$$

The objective function (1) is to minimize the costs for building battery swapping stations, transportation costs between demand point (electric bus depot) and battery swapping stations, charger costs, battery costs and swapping robot costs. The fixed BSS construction investment and transportation costs, which are the first and the second terms, take the form of the classical fixed-charge facility location problem(An,Ouyang 2016; An et al. 2017). Chargers, swapping robots, and batteries are the major capital investment considered in configuring BSSs. Constraint (2) requires that all demand at depot i should be assigned to BSSs, while constraint (3) ensures EBs can only be assigned to open battery swapping facilities. Constraint (4) and (5) are the integrality and non-negativity constraints for location variables $\mathbf{X}$ and assignment variables $\mathbf{Y}$. The continuous variable Y_{ij} takes a nonnegative value in [0,1], indicating that the demand at each EB transit depot $i \in I$ may be assigned to multiple BSSs. Constraint (6) is the fully-charged battery quantity conservation constraint at time slot $t+1$. The second term on the right-hand side $\sum_{a \in A} W_{ja}^{\max\{t+1-t_a,0\}}$ calculates the number of batteries that have completed charging during the time period from t to $t+1$. Variable H_j^t is the swapping demand arrived at BSS j, i.e. the number of buses with depleted batteries. Constraint (7) ensures every EB can get a fully-charged battery immediately when they arrive for swapping services. Constraint (8) captures the assumption that the battery swapping demand is evenly distributed over T time slots during the BSS operation period by carefully arranging the swapping schedules. This assumption can ensure full utilization of the charging facilities and batteries. Constraint (9) can be explained in two ways according to the charger type. Slow chargers can usually charge 2 batteries per day, i.e. $s_a = 2$. If we consider a time duration of one hour, there are $T = 24$ time slots, making the term $\left\lceil \frac{s_a}{T} \right\rceil$ equal to 1. It states that the number of slow chargers should be greater than the number of depleted batteries assigned to them over the $\left\lceil \frac{T}{s_a} \right\rceil - 1$ time slots, so that the depleted batteries can start the charging process once they are swapped out at the BSS. For fast chargers, $\left\lceil \frac{s_a}{T} \right\rceil$ is basically greater than 1 and it explains that the number of depleted batteries assigned to them should be less than its charging capability at any time slot $t \in T$. Constraint (10) indicates that the total demand assigned to site j should not exceed its charging capability at this site. Constraints (11) and (12) indicate that chargers and swapping robots are only deployed at open BSSs where M is a large number. The swapping robot capacity constraint (13) guarantees that the service capability of swapping robots at station j in time slot t satisfies the swapping demand.

7.5 Case study

We consider a pilot battery swapping program for EBs as a replacement of the existing diesel bus services. It serves a given number of bus routes with a limited number of EB fleet. In an urban area, the number of BSSs in a given region is typically orders of magnitude less than the number of EB transit depots. Tens of transit depots usually accommodate over hundreds of buses in service. We assume bus depots and large bus terminals can be candidate sites for BSSs.

7.5.1 Data preparation

We take the east region of Melbourne city for example. The study region centers around the city of Monash consisting of eight local suburbs, namely Monash, Knox, Glen Eira, Stonnington, Whitehorse, Maroondah, Kingston and Greater Dandenong. Bus routes information and timetables are available at Public Transport Victoria for the eight local councils (Victoria 2017a). Bus route length, origins and destinations are extracted from an open dataset of PTV bus route metro using ArcGIS software (Victoria 2017b). There are 109 bus routes operating in this area. The swapping demand of a bus route is assumed to occur at the two terminals, each terminal taking half demand generated. On-route battery swapping is not considered in this model. The swapping demand of a bus route is calculated based on the total daily vehicle-mile traveled (route length multiply by bus service frequency) divided by the range limit of EBs. The demand h_i at terminal i is the sum of the half demand for all routes using terminal i. We rounded up the swapping demand to integers for calculation convenience. The demand set I contains 81 demand generation points composed by origins and destinations of these bus routes. Note that several bus routes could share the same terminal. We select the 81 terminals in I and the 38 existing bus depots under operation in Melbourne as the candidate sites of BSSs, namely, J has 119 elements (see Fig.7-1). The distance d_{ij} between demand point $i \in I$ and candidate BSS site $j \in J$ is calculated by Google Maps Distance Matrix API during off-peak hours. The fixed cost f_j is calculated based on the median land price per sqm (available from https://www.microburbs.com.au/). The land value, candidate site index, and the name of each candidate site can be found in the appendix. We implement the proposed model in Gurobi MIP solver to find the optimal BSS location and its charging system configuration.

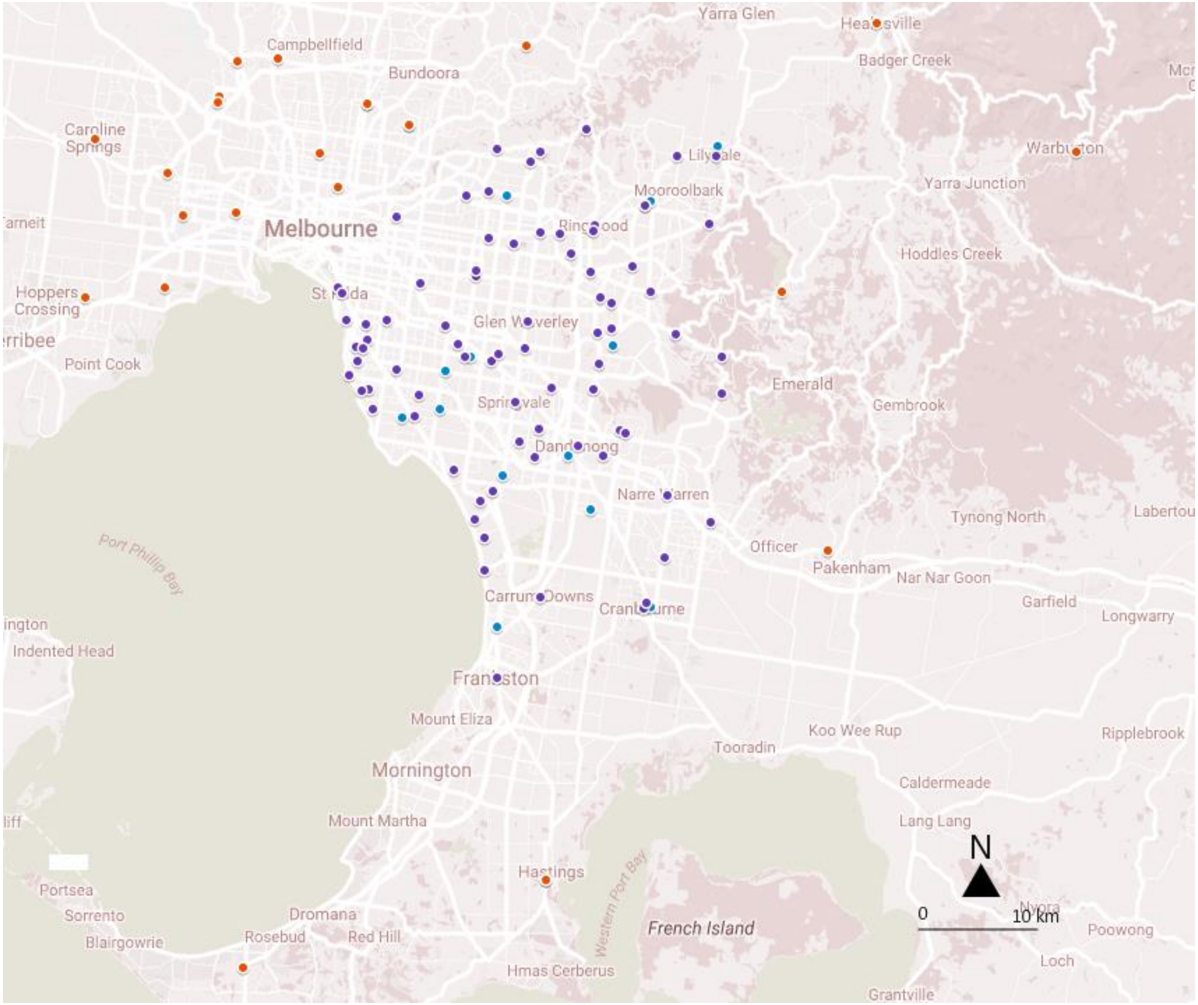

Figure 7-1:East region of Melbourne with 81 demand points and 119 candidate sites

Note: Purple marks represent demand points. Blue and orange marks represent existing bus depots within and out of the studied region respectively.

The parameters used in this case study are listed in Table 7-2. A single EB type with a range limit of 250 km is used. In this study, we take EB manufactured by Sunwin China for example, to which battery swapping applies. This manufacturer (Sunwin) only published part technical specifications of EBs. Therefore, we have to refer to the parameters from several similar manufacturers. Hopefully, these key parameters in the operational stage are all within a reasonable range for a macro level cost analysis. We assume slow charging and fast charging are applicable to the local charging system. The costs of chargers, batteries, and swapping robots are only referenced values used in Melbourne. The operation time is divided into 12 time slots, i.e. $T = 12$.

Table 7-2: Basic model input

EB range limit	250 km	slow charging time	8 h

EB battery capacity	324 kWh	fast charging time	2 h
charging power	60 kw	swapping time	15 min
operation time	24 h	battery life	3 years
slow charger price	$ 25,250	vehicle life	8 years
fast charger price	$757,500	swapping robot life	10 years
vehicle price	$1,320,000	charger life	10 years
land size	1,000 $\mathbf{m^2}$	land use period	100 years
electricity price	$0.16/km	swapping robot price	$100,000
battery price	$192/kwh		

Sources. EB range limit, battery capacity, charging power, charger price, vehicle price, charging capability, and swapping time are from Li (2016); EB electricity price is from Eudy et al. (2016) and Lajunen (2014); land size is 1/6 that of a real case in Qingdao, China with 3 lanes, 6 swapping robot from Li (2016); bus operation time of Melbourne metro bus differs from route to route. 24 hours is considered in this case including night buses; swapping robot price, charger life and swapping robot life are estimated; Battery life and costs are from Wikipedia (2017); Land purchase price is from Microburbs (2017). Note: all $ here is AUD

7.5.2 Numerical result

We consider a pilot battery swapping program for EBs as a replacement of the existing diesel bus services. It serves a given number of bus routes with a limited number of EB fleet. In an urban area, the number of BSSs in a given region is typically orders of magnitude less than the number of EB transit depots. Tens of transit depots usually accommodate over hundreds of buses in service. We assume bus depots and large bus terminals can be candidate sites for BSSs.

In the following section, we investigate two base scenarios with different construction cost settings and further conduct sensitivity analysis on the second base scenario. In scenario A, we assume all BSSs have the same land size and fixed land value. In scenario B, varying land size and annual land value appreciation are taken into consideration. The land size increases with the number of swapping robots deployed in the BSS and land value increases 3% per year.

Scenario A: common BSS land size and fixed land value

In scenario A, we utilize the basic parameters in Table 7-2 considering a fixed land size of 1,000 m^2 for each BSS and fixed land price. The optimal BSS location is shown in Fig.7-2 with green marks. The constructed 8 BSSs are scattered in the studied area, which are located at Box Hill, Clayton, Croydon Station, Dandenong, Frankston, Gardenvale, Mordialloc, and Upper Ferntree Gully. The detailed results are listed in Table 7-3 and Fig.7-2, including the selected BSS sites, the configuration of each BSS and the swapping demand assigned to the BSSs.

The total costs reach $15,037, including five cost components- land costs of $278, transportation costs of $507, charger costs of $1,522, battery costs of $12,511, and swapping robot costs of $219. Owing to its short lifespan and high price, batteries are the primary cost component in this case, accounting for over 80% of the total cost. The company has to maintain 16 to 32 additional batteries at each BSS (see

Table 7-3) to ensure seamless battery swapping with no delay. The charger costs rank the second. The quantity of slow chargers is of the same as that of the batteries stored in the BSS. Slow chargers are deployed exclusively in the BSSs, while no fast chargers are used. The fast charger is 30 times more expensive, while only providing 4 times charging capability than slow chargers. It is not cost-effective to use fast chargers to charge EBs with a large battery capacity at the current stage, although fast charging can provide more flexibility in managing the battery charging schedule. The other three costs are of the same order of magnitude. Since the annual appreciation of land value is not considered here, the daily land costs f_j are quite low due to the long service period (100 years). The final location of BSSs results from the tradeoff between BSS construction costs and battery transportation costs. As can be seen from the results, the maximum number of battery swapping robots installed is 2. Generally, one lane at BSS can install one or two swapping robots. This result justifies the use of uniform land size at BSSs.

Furthermore, the assignment of the swapping demand is shown in Fig.7-2 Most of the demand assignment variables equal to 1 with six exceptions (demand points No. 22, 40, 42, 52, 72 and 80). Their values are as follows: $Y_{22,64} = 0.71$, $Y_{22,76} = 0.29$; $Y_{40,59} = 0.5$, $Y_{40,77} = 0.5$; $Y_{42,77} = 0.8$, $Y_{42,94} = 0.2$; $Y_{52,46} = 0.68$, $Y_{52,63} = 0.32$; $Y_{72,64} = 0.14$, $Y_{72,113} = 0.86$; $Y_{80,59} = 0.44$, $Y_{80,64} = 0.56$. It shows that assigning all demand at a bus terminal to one BSS may not be cost effective but can greatly simplify the operations of bus drivers. This phenomenon can be seen in the overlapping area of the dashed line, showing that these demand points in the boundary area are assigned to two BSSs. The installed BSSs are approximately evenly distributed in the study region so that an EB does not have to travel too long to get refueled.

Table 7-3: Optimization results in scenario A

location(No.)	land value ($)	swap robot	battery	slow charger	fast charger	swapping demand
Box Hill (46)	2180	1	32	32	0	96
Clayton (59)	1500	1	32	32	0	96
Croydon Station (63)	850	1	32	32	0	96
Dandenong (64)	870	1	28	28	0	84
Frankston (76)	730	1	16	16	0	48
Gardenvale (77)	2070	1	32	32	0	96
Mordialloc (94)	1410	1	28	28	0	84
Upper Ferntree Gully (113)	520	1	20	20	0	60
Sum	/	**8**	**220**	**220**	**0**	**660**

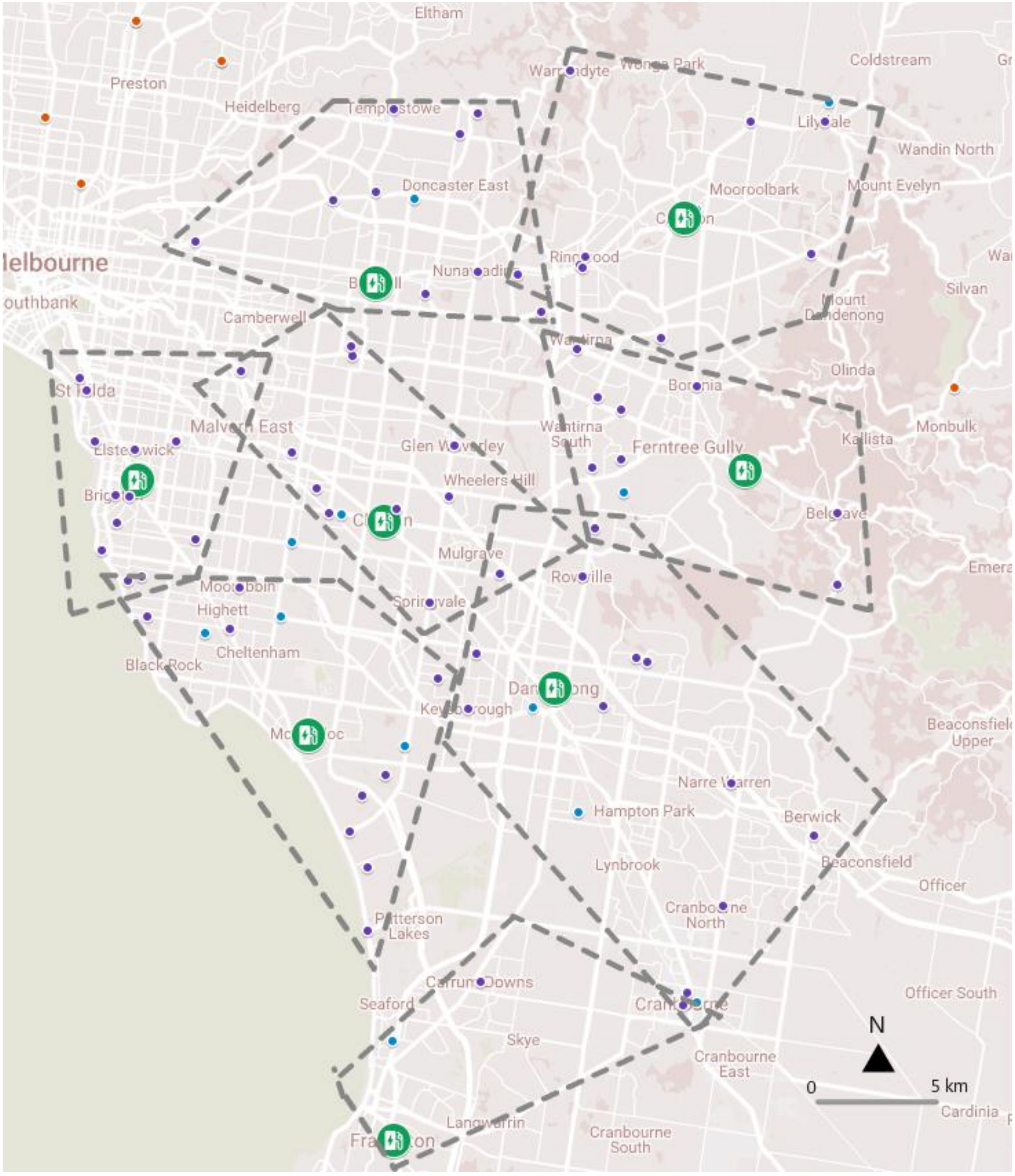

Figure 7-2: Optimal location of BSSs and swapping demand assignment in scenario A

Note: Green marks represent the optimal BSS location. Purple marks represent swapping demand points. Blue and orange marks represent the existing bus depots within and out of the studied region respectively.

Scenario B: varying BSS land size and with 3% annual land value appreciation

This section further investigates the impact of land costs by considering 3% annual land value appreciation and land use size. In scenario A, we can see that land costs only account for 2% of the total cost when we consider a fixed land use size (1000 $\mathbf{m}^2$) and constant land value. With 3% annual land value appreciation, the land costs increase $1.03^{99} = 18.66$ times at the 100th year. The average cost would be $\frac{\sum_{n=0}^{99}(1+3\%)^n}{100} = 6.06$ times of the current land costs, making it a more significant cost term in the objective function. We further consider a variable land size to avoid unreasonable configuration of BSSs. The impacts of the BSS's land size are investigated by adding one extra term $\sum_{j \in J} f_j N_{jc}$ in the objective function (1). We assume the BSS's land size linearly increases with the number of swapping robots. Otherwise, it would be unimaginable to install multiple swapping robots with hundreds of slow chargers in a 1000 $\mathbf{m^2}$ site.

The total cost rises to $16,804, 12% higher than that in scenario A. The results show that only three BSSs are deployed--in Frankston Depot, Noble Park and Boronia (see Fig.7-3 and Table 7-4). These are not the cheapest areas but are relatively closer to the region center. It indicates that the planner intends to construct BSSs in regions with lower construction costs while maintaining reasonable transportation costs. The number of batteries and chargers in the whole system is the same as that in the previous case. In addition, the total number of swapping robots is 7, 1 less than the previous case. This is because that the service capability of swapping robot is abundant in the previous case. For example, the robot in Frankston services 84 swapping demand only while its capacity is 96. When assigning these swapping demands to one BSS, it can help to make full use of the swapping robots. Comparing with the previous case, the main differences lie in the land costs and transportation costs. The Land costs increase from $278 to $1,364, while the transportation costs increase from $507 to $1,214. The transportation costs have greatly increased because EBs need to travel to rural areas to swap the depleted batteries. Under some circumstances, e.g. from Kew to Boronia, the distance is more than 60 km for a round trip of a battery swapping service. It consumes 24% of total battery energy for such a detour to the BSS. If the planner wants to decrease the time or costs EBs spent on the way to BSSs, they should deploy BSSs closer to demand points with higher land values, rather than rural areas far from demand points.

Table 7-4: Model output with 3% annual land value appreciation and land size

location (No.)	land value	swap robot	battery	slow charger	fast charger	swapping demand
Ventura Bus Lines-Frankston Depot (15)	730	1	28	28	0	84
Noble Park (96)	950	3	96	96	0	288
Boronia (115)	740	3	96	96	0	288
Sum	**/**	**7**	**220**	**220**	**0**	**660**

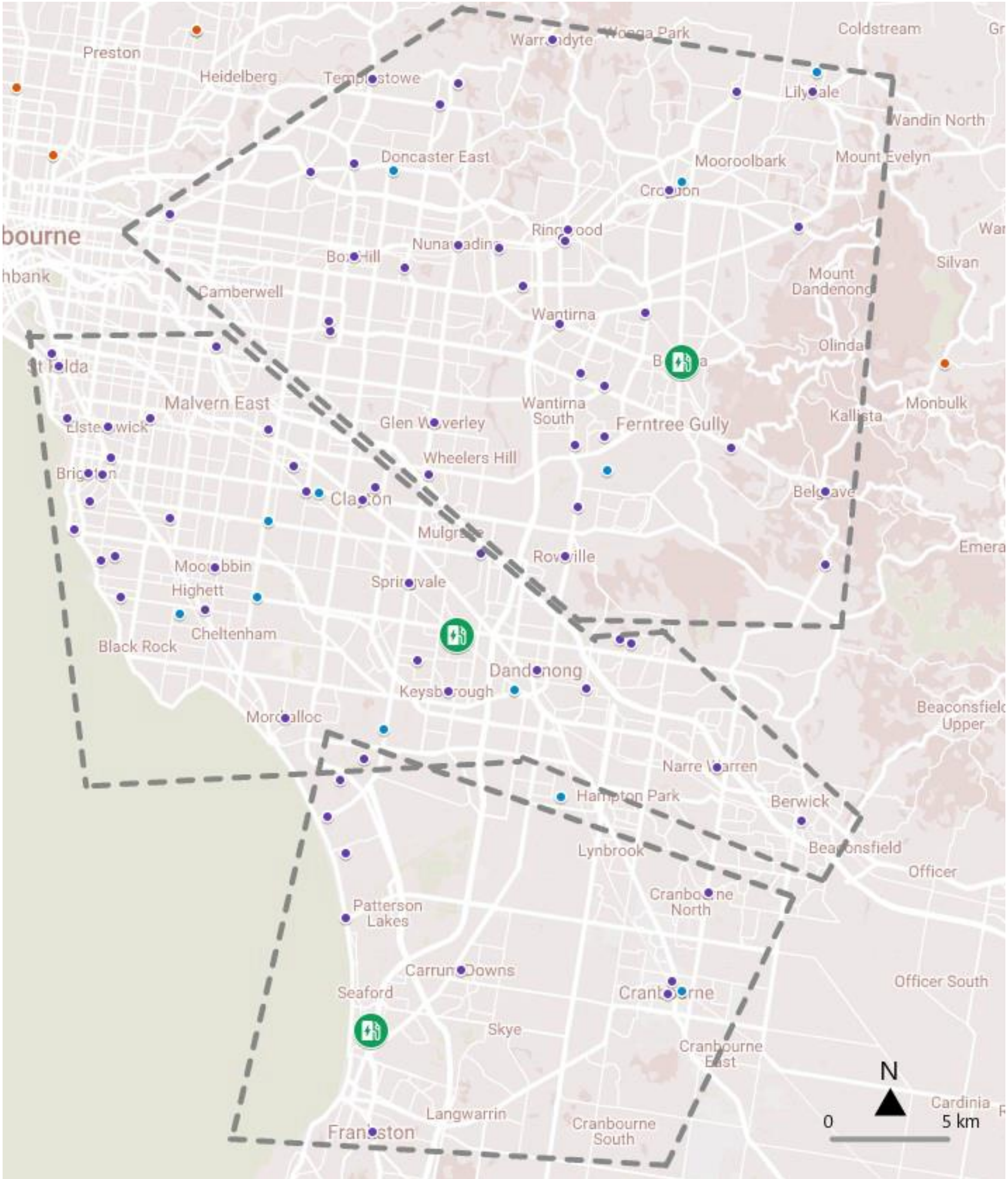

Figure 7-3: Optimal location of BSS and swapping demand assignment of scenario B

Note: The notations are the same as those in **Fig.7-2**

Sensitivity analysis on unit transportation cost in Scenario B

This section looks into the impacts of unit transportation cost on the number of constructed BSSs. Table 7-5 provides the results on locations of the deployed BSSs under various unit transportation costs, γ. The amount of BSSs deployed increases with the unit transportation cost. γ =1 indicates that only energy cost as in the base case is considered. A larger unit transportation cost means that the EB operators values more than electricity price. Travel time and actual operation range might be other concerns, especially when EBs detour more than 60 km to swap a depleted battery out. It takes more than one hour and consumes 20% of the battery for the refueling service. As a result, planners could assign more weight on transportation costs. If the electricity price dramatically drops to 10%-20% of the current price, only one or two BSSs are deployed in the network. When transportation costs remain within a given range, e.g., 30%-100% and 400%-700%, the number of deployed BSSs does not change but in different locations. In the last scenario with 1500% unit transportation cost, 215 batteries, 3 fast chargers, and 212 slow chargers are used in the system and 2 BSSs utilize both fast chargers and slow chargers. Comparing this configuration with other scenarios in Table 7-5, we can see that the adoption of fast chargers helps to reduce the number of batteries needed in the system.

Table 7-5: The number of BSSs deployed considering different transport costs

Transport Cost	The number of BSSs	BSS location No.
0.1	1	14
0.2	2	14,26
0.3	3	14,17,69
0.4	3	17,69,113
0.5	3	17,69,113
1	3	15,96,115
1.5	4	0,15,69,115
2	4	15,69,93,115
3	5	15,69,93,115,116
4	8	59,62,69,72,76,94,98,113
5	8	44,59,62,69,76,77,94,113
6	8	44,59,62,69,76,77,94,113
7	8	44,59,62,69,76,77,94,113
8	10	44,58,59,69,76,77,94,113,115,116
9	11	46,58,59,60,64,76,77,94,113,115,116
10	11	46,58,59,60,64,76,77,94,113,115,116

Sensitivity analysis on fast charger price in Scenario B

A further analysis is conducted on how fast charger price would affect the choice of chargers and battery inventory. The other parameters are fixed as in scenario B. We only reduce the price of fast chargers to simulate the advance in charging technology. In scenario B, the fast charger costs $757,500 and can only fully charge 12 batteries in 24 hours, while the slow charger can charge 3 batteries and costs $25,250. On average, it costs $8,416 and $63,125 to fully charge a depleted battery by the slow and fast charger respectively. Slow mode seems to be an economical choice. In terms of charging capability, the price of the fast charger has to drop to 8,416 /63,125 =13.3% of the current price to be more attractive if battery inventory impact is not considered. However, fast charging can increase the utilization rate of batteries. In scenario B, the swapping demand arrives every two hours and fast charging also takes two hours to fully charge a depleted battery. Fast charging can greatly reduce the number of batteries required at each BSS. The local charging system needs either 220 slow chargers and 220 batteries or 55 fast chargers and 55 batteries. From Table 7-6, we can see that, when fast charger price drops by around 4.5%, the local charging system can switch from slow charging to fast charging because the rise of fast charger costs would be less than the decrease of the battery costs. Therefore, deploying local charging system at BSS is a tradeoff between charger costs and battery costs.

Table 7-6: The charger selection based on the reduction of fast chargers' price

fast charger price (AUD)	battery quantity	fast charger quantity	slow charger quantity
757,500 (0.0% reduction)	220	0	220
723,759 (4.5% reduction)	220	0	220
723,758 (4.5% reduction)	55	55	0
684,750 (10.0% reduction)	55	55	0
606,000 (20.0% reduction)	55	55	0

Several other factors, such as a relationship between charger and land requirement, will probably help produce a better cost evaluation when data is available. Battery capacity can affect the BSS location decision by influencing the total swapping demand. When battery capacity is relatively limited or EB companies are making a strategic long-term planning, they value more on reducing the energy loss due to the detour between BSSs and EB depots. Bus arrival patterns or swapping service scheduling may also have an influence on the choice of charger types and battery quantity, which will be left for future studies.

7.6 Discussions and conclusions

This chapter investigates the BSS location models serving public EBs. The objective was to answer five key questions that the EB company concerns most are answered simultaneously: 1) the location of BSSs; 2) the assignment of EB swapping demand, i.e. the demands in existing EB depots should be assigned to which BSS 3) the charger selection in terms of type and quantity; 4) the number of batteries needed in each BSS; 5) the number of battery swapping robots installed.

Heavy capital investment on electric bus refueling infrastructures calls for prudent planning of the system. Battery swapping stations encapsulated with local battery charging provide a promising solution to refuel EBs with minimum delay. This paper establishes a novel and compact mixed integer program for the BSS location problem with distinct charging system configurations to minimize the sum of the construction and operation costs, which is then solved by a GUROBI solver implemented on Python interface. The test on a real network of the southeast region of Melbourne in Australia verifies the feasibility of the proposed model and investigates the effects of BSS locations and configurations. Results show that more BSSs would be built in candidate sites closer to demand points when annual appreciation of land value and varying land size are not considered. Otherwise, BSSs would be deployed in low land value sites. The decision would be a tradeoff between land costs and transportation costs. The total number of swapping robots, batteries and chargers mainly depend on technical parameters and the swapping demand arrival pattern. Fast chargers are unattractive at this stage unless they become cheaper or faster with technology advances.

The case study shows the validity of the proposed model and provides insights on the BSS planning. The study of the local charging system configuration problem can be the foundation for designing the combination of chargers and battery inventory to satisfy EB swapping demand. In the expanded model, we have further investigated the BSS location problem by considering not only land value appreciation but also land size based on the number of swapping robots installed. It is found that charger price and unit transportation costs are important factors affecting the answers to the five key questions proposed.

For this pioneering research, several simple assumptions are made to ensure its tractability. Future work should relax these assumptions to address a more realistic problem. For example, the scheduling of electricity price-based battery charging and EB operation will be considered to increase the utilization rate of batteries. A more accurate BSS land use model should be adopted considering the availability of land. Comparing with the local charging system, models of the central charging system should be created to make an economic comparison to identify a favorable charging mode. Additionally, the battery capacity and the charging power of EB used for public transportation are several times greater than that of electric cars, which can result in high energy consumption and negative impact on power distribution networks. Thus, a BSS deployed in a given region should be considered as capacitated before power grid upgrade to accommodate more local charging demand.

From Chapter 6, it was found that public charging facility are more appropriate for locating on those road segments with a large amount of private BEV flows as the objective is to maximize their exposure to the BEV users. However, for public EBs, they follow fix routes and timetables, they do not refuel on route to avoid delay. It can be easy to calculate their refueling demand based on their operation details. It was also found that battery swapping technology is more suitable for EBs to increase EBs' utilization rate and a local charging system which charges depleted batteries in local BSSs helps avoiding the tedious battery shipping. The local charging method calls for careful land-use planning to reserve sufficient spaces for bus awaiting/parking and for local charger installation. Besides, one has to weigh the costs, charging efficiency, battery life and other factors in choosing the charging method. It is pointed out that well-aligned charging strategies with evolutionary EV adoption are the prerequisite for realizing its environmental benefits especially in countries with fossil-dominated power. Otherwise, the disordered charging will cause load fluctuations and increase generation costs, subsequently losing the meaning of transportation electrification.

The next and final part, Part IV, of this thesis presents the conclusions of this research, including a summary of key findings and directions for future research.

PART IV:

SYNTHESIS AND CONCLUSIONS

CHAPTER 8 CONCLUSION AND RECOMMENDATIONS

8.1 Introduction

This thesis has explored the general stochastic user equilibrium (SUE) models with side constraints such as flow-independent driving distance constraints and flow-dependent battery capacity constraints and charging facility location models for both private BEVs and public EBs based on their own characteristics and refueling demand pattern. The research presented in previous chapters has generated a number of original contributions to knowledge. This chapter concludes the thesis through a summary of key findings, a summary of contributions to knowledge, and a discussion of implications for practice. A critique of the research is then presented, followed by a discussion of future research directions.

8.2 Summary of key findings

The major contributions of the thesis commenced in chapter 2 where an attempt was made to investigate the various factors affecting route choice behavior of EV users as well as the factors that need to be considered when deploying any type of battery charging/swapping facilities. Starting with the concept of EVs, it discusses both the EVs market studies and those special characteristics of EVs and its charging infrastructures that distinguish EVs from GVs. From network modelling and design point of view, it is, therefore, important to take their special characteristics into account when predicting EVs flow patterns and designing charging infrastructure networks accordingly. For instance, driving distance limit, availability of public charging infrastructures, long charging time, battery swapping time, location of charging facilities, battery energy consumption rate, and the attraction level of different charging facility are likely to affect EV users' route choice behavior and the equilibrium network flow pattern. For example, if a Nissan Leaf user is taking a trip with remaining battery energy that can only travel 20 km, he would not choose any route longer than 20 km unless there are public charging stations en-route. The charger costs vary with charging speed, which will definitely affect the installation choice considering their cost-benefit effect. Hence, this research summarizes the existing barriers for EVs and selects some factors that matter most as the key constraints in the following general SUE models and refueling facility location models for both private BEVs and public EBs.

Chapter 4 investigated the methodological issues of general SUE model of mixed EV and GV flow with path distance constraint and how to solve this model was investigated in this component of the thesis. Directly adding side constraints into a SUE model cannot generate a SUE flow pattern. Incorporating the path distance constraints into the general STAP needed a mathematical proof. The BEV range limit was defined by the path distance it can travel without charging. A classical minimization model was used with a modified MSA method to address the SUE problem. Solution properties of equivalence and uniqueness are provided. It can be seen as an extension of DUE model with the same constraints, which include perception error of travel time, are considered more rational than UE model. Multiclass users in

SUE model represents a simplified case of current traffic networks that carry both EV and GV. More classes of users with various range limit can also be taken into consideration. Overall, the new model shows that at the equilibrium point the selected paths to assign the travel demand are different from that of basic SUE TAP. The distance of each path must be less than the range limit of that class of vehicles. The well-known and widely used MSA procedure and probit-based network loading method are adopted and modified to solve this problem, following the idea of putting the path distance constraints into the path selection rules of stochastic network loading procedure. The direction finding step for MNL, involves finding K feasible paths to load the travel demand between each OD pair, while it requires finding feasible shortest path for MNP in all-or-nothing assignment step. Path feasibility check was employed to address the path distance issue whenever generating a path in K-shortest path algorithm or shortest path algorithm. The results suggested that range limit would have a great impact on EV users' route choice, especially for those with short range limit. When the range limit became large enough, EV behaves similarly to GV. This component of research was then adopted and further extended in Chapter 6 by incorporating the available public charging facility into the general SUE model.

In Chapter 5, this thesis detailed a more complicated STAP in transportation networks with BEVs owing to the fact that BEV energy consumption depends on not only the path distance but also the travel time. The main objective was to theoretically understand how a flow-dependent path-based constraint can be incorporated into a general SUE model. It begins by discussing the battery capacity constraint was a flow-dependent one, while path distance constraint was flow-independent. The flow-independent driving distance constraint in Chapter 4 can be processed in the route choice procedures, while the flow-dependent battery energy consumption depends on not only distance but also traffic flow (travel time). A mathematical programming model was proposed for the flow-dependent path-based SUE traffic assignment. A convergent Lagrangian dual method was employed to transform the original problem into a concave maximization problem and a customized gradient projection algorithm was developed to solve it. A column generation procedure was adopted to generate the path set. Three solution propositions are provided regarding equivalence and uniqueness of the solution. The solution framework, Lagrangian dual-gradient projection-stochastic network loading, can be applied to solve path-based SUE problem. The path set generated and their corresponding Lagrangian multiplier are demonstrated. It was shown that the network becomes congested and link travel time goes up when travel demand is high and increasing path travel time results in more energy consumption and more paths infeasible on which BEV will run out of energy and incorporate additional out-of-battery cost. The numerical analysis results show the impact of battery capacity, travel demand and stochastic parameters on network equilibrium flow and computational cost.

The focus of Chapter 6 was to investigate a way of locating charging facilities in the network since no public charging facilities have been considered in the previous SUE components. A bi-level model was adopted with maximum covering objective in the upper level and STAP with path-distance constraints in the lower level. Public charging facilities were taken into consideration in the trip chain in the lower level STAP to accomplish this component of research. An important concept of sub-path was used to identify the scenarios of charging need. A key application of this concept was to calculate the generalized path travel cost composed of path travel time, charging time and equivalent travel time reduction (the utility of charging facilities on attracting BEV drivers). Comparing to research component 2&3, the SUE approach was extended to consider public charging facilities in the network. It was demonstrated that the driving distance limits, charging speed and utility of charging facilities affect the equilibrium network flow and charging facility location. It was also found that the BEVs with shorter driving distance and risk-neutral attitude would probably have a larger value of charging facilities utility, because charging facilities helped to ease their range anxiety. While for those with larger batteries, they would behave more like GV users. A potential drawback of this method of defining flow coverage is that it may lead to the location of charging facilities on several adjacent links of some high-volume freeways. Further details of this research component are provided in Chapter 6

Chapter 7 which is the last contributory chapter had two main objectives. The first objective was to understand the location problem of BSSs serving public EBs considering the fixed construction cost and transportation cost between EB depots and BSSs. The second objective was to briefly investigate its local charging system configuration including charger quantity, charger type and battery inventory so that the depleted batteries can charge at the BSS itself and do not need any battery distribution centre for centralize charging. A mixed-integer linear program is formulated to represent this problem, which is then solved by a GUROBI solver implemented on Python interface. The test on a real network of the southeast region of Melbourne in Australia verifies the feasibility of the proposed model and investigates the effects of BSS locations and configurations. Results show that more BSSs would be built in candidate sites closer to demand points when annual appreciation of land value and varying land size are not considered. Otherwise, BSSs would be deployed in low land value sites. The decision would be a trade-off between land costs and transportation costs. The total number of swapping robots, batteries and chargers mainly depend on technical parameters and the swapping demand arrival pattern. Fast chargers are unattractive at this stage unless they become cheaper or faster with technology advances. Several other factors, such as a relationship between charger and land requirement, will probably help produce a better cost evaluation when data is available. Battery capacity can affect the BSS location decision by influencing the total swapping demand. When battery capacity is relatively limited or EB companies are making a strategic long-term planning, they value more on reducing the energy loss due to the detour between BSSs and EB depots. Bus arrival patterns or swapping service scheduling may also have an influence on the choice of charger types and battery quantity, which will be left for future studies.

8.3 Contributions to knowledge

The thesis has provided four key contributions in the areas of the STAP models with BEVs and location problems of the battery charging/swapping facility serving BEVs and EBs. Focus The contributions of this thesis are dedicated to the methodological developments and summarized as follows:

- <u>New general SUE model considering flow-independent BEVs' driving distance constraints and new method for solving the proposed general SUE model with driving distance constraints of BEVs (Chapter 4):</u> It has long been recognized as the last step of the traditional four-step travel demand modelling process and widely used an evaluation tool for a variety of urban and regional traffic network analyses (Xie,Waller 2012). Although there have been a number of research projects in recent years in traffic assignment of EVs, most of these studies have focused on DUE models along with various constraints (Xie et al. 2014; Xie et al. 2017; Wang et al. 2016; Jiang,Xie 2014; Jiang et al. 2013; Xu et al. 2017). Very little research has payed attention to SUE models especially general SUE model including both logit and probit stochastic loading. This thesis demonstrated a holistic methodology is proposed for general SUE traffic assignment model with path distance constraints on EV scheme, in which the classic unconstrained SUE model can be used to incorporate path distance constraints by modifying MSA algorithm and finding the distance-constrained K-shortest paths in stochastic network loading process. It is assumed that the EV route choices are restricted by the distance EV can travel with a single charge.

- <u>New general SUE model with flow-dependent battery capacity constraints accounting for a more reasonable battery consumption based on both distance and travel time and a new methodology for solving general SUE model with limited battery capacity of BEVs (Chapter 5):</u> Few studies have addressed stochastic traffic assignment models for EVs (Jing et al. 2017). It has been pointed out that directly adding side constraints into the well-known minimization model for probit-based SUE problem does not give us an equivalent minimization model to the probit-based SUE traffic assignment with side constraints (Meng,Liu 2011). This thesis is the first to study a general SUE model with path-based constraints and enrich the general SUE family with side constraints (link-based and path-based) and make consistence with side-constrained general DUE condition. A holistic methodology is proposed for general SUE traffic assignment model with battery capacity constraints on BEV scheme, in which the path choice is restricted by the battery capacity with a single charge. A Lagrangian dual based exact solution method incorporating column generation is developed for solving this path-constrained general SUE model.

- <u>New charging facility location model considering a SUE BEV flow pattern and charging facility deployment and a new method for solving the proposed bi-level SUE-based BEV charging facility location problem (Chapter 6):</u> The literature on CFLP has focused on a larger variety of charging facility models for private BEVs. Few studies attempted to apply the bi-level model for public charging facility deployment. The location design problem of charging facilities can be modelled as a Leader-Follower

Stackelberg game where the decision makers are the leaders who decide the facility deployment and the BEV users are the followers who can choose their paths freely. Most of previous studies focused on DUE problems with BEVs. However, the driving distance limit, to the best of our knowledge, has not been considered in stochastic network equilibrium models, especially in the mixed flow transport network. Moreover, to tackle the range anxiety problem with a limited budget, the charging facilities should be accessible to as many EVs as possible. Deploying the pubic charging facilities on the links where most BEV drivers use is an efficient way to increase the utilization and perception of the public charging facilities, which promotes BEV acceptance and relieve range anxiety. This thesis used a maximal flow-covering (MFC) model, i.e., a modification of classic MCLP, is proposed to maximize BEV flow coverage by locating a fixed number of charging facilities in the bi-level, equilibrium-optimization framework. Coverage is achieved when the charging facilities is located on the BEV route. The effects of driving distance limit constraints, charging facility availability, charging facility utility and traffic congestion are accommodated in the lower-level general SUE problem where the equilibrium BEV flow pattern is determined endogenously by the general SUE traffic assignment model with driving distance limit constraints, in which the mutual interactions between the location of charging facilities and resultant equilibrium BEV link flow patterns are modeled.

- New swapping facility model considering local charging system serving BEB fleet and an understanding of the effects of various factors in the swapping facility system (Chapter 7): It has been pointed out that BEBs are characterized by fixed running routes, fixed depots, near-identical battery capacity. However, configuring an overall BEB system is challenging; this would include possible battery recharging and swapping concepts, choice of battery technology, battery sizing, positioning and dimensioning of charging and swapping stations (Leou,Hung 2017). Existing BSSs research have concentrated on the interaction between BSSs and power grid and the operation of both BSSs and BEBs. This research proposed an optimization framework for locating capacitated BSSs incorporated with local charging systems. It was the first study investigating the deployment of BSSs with different types of local charging infrastructures (including batteries, chargers and swapping robots) while taking into account the tradeoff between BSS installation costs and transportation costs from EBs to BSSs. The optimal number of batteries, chargers and swapping robots and the type of chargers initially purchased at BSSs can also be decided through the proposed model to satisfy the swapping and charging demand of EBs.

8.4 Implications for practice

Based on the findings presented in this thesis, it is possible to discuss their implications for practice. The stochastic traffic assignment models and charging facility location models in this thesis is intended to help city planners, traffic and transportation engineers, EV companies and policy makers to make informed decisions. The thesis has identified a number of factors including EV demographic, land use, traffic and charging infrastructure variables that have the potential to influence EV flows patterns in the traffic network. The planning-level factors such as the selection of charger type, charger location, battery quantity matter in practice. Even if battery charging might speed up to minutes and battery cost might decreases greatly with the advance of technology, EVs, as an alternative vehicle, may still behave differently from GVs because of fuel cost, demographic, vehicle type and other factors affecting the EVs' performance. A driver with less income may still choose a cheap EV with small battery and slow charging only instead of high-end EV with super charger and large battery. Slow charging might be more attractive than fast charging if its price is lower. The choice of battery capacity and the charging speed would always be a trade-off between time and cost. High-end EVs could behave more like GVs and more class users should be taken into consideration at that time.

First, Chapter 4 and 5 of the thesis the problem of finding the equilibrium flow pattern over a given urban transportation network is known as traffic assignment. The amount of travel taking place at a given moment on any street in an urban area is the result of many EV users' decisions. In this research, EVs are considered as part of the transportation network with their own travel behaviors. The travel time on each of the paths connecting the origins and destinations is a function of the total traffic flow due to congestion. EV drivers may consider more than travel time because of the range anxiety and availability of public charging facility. The analytical approach described in this research can help to predict the EV flow pattern in order to calculate an array of measures. These may include the following:1) Level of service measures such as travel time and travel cost control 2) Operating characteristics such as revenues, profits, toll pricing setting 3) Flow by-products such as pollution reduction 4) Welfare measures such as equity and priority of EVs. The flow patterns could also help optimize the location choice of charging facility.

Chapter 6 provides EV companies or urban planners with critical application of public charging facility location model for maximize chargers' exposure to the BEV users. A number of factors were identified as being critical in determining EVs' route choice behavior. To increase EV charging facilities' utility given the restricted budget, both researchers and practitioners should be interested in how EV charging facility location can affect flow patterns and this is presented in chapter 6. The thesis provides a theoretical/methodological basis for evaluating the utility/exposure of deployed charging facilities. An improved and reliable tool can be used by planners and engineers at a planning level.

Chapter 7 emphasizes to both researchers and EV companies the importance of addressing swapping facility for EBs. No study has investigated the optimization of swapping station location, charger number, charger type and electric bus assignment in the BSS planning problem. The optimization framework in this chapter for locating capacitated BSSs incorporated with local charging systems can give practitioners a general view given the inputs and costs they have in hand. The outputs can provide insights for city planners and bus operators of deploying battery swapping and charging systems. The optimal number of batteries, chargers and swapping robots and the type of chargers initially purchased at BSSs can also be decided through the proposed model to satisfy the swapping and charging demand of EBs. To our best knowledge, this is the first study investigating the deployment of BSSs with different types of local charging infrastructures (including batteries, chargers and swapping robots) while taking into account the tradeoff between BSS installation costs and transportation costs from EBs to BSSs. The case study of the southeast region of Melbourne network verifies the effectiveness of the proposed model and provides cost analysis if EBs serve the current bus routes and demand. The approach proposed in this paper may be used by city planners, power grid companies, and transit service providers to plan the battery charging and swapping infrastructures, estimate how many chargers and what type of chargers to install to fulfill the potential demand while minimizing the total capital investment. This will save the government and EV companies time, money and funds which would be useful to support other EV programs.

8.5 Critique

While the thesis has provided a number of original contributions to knowledge, there are opportunities to improve it. Some specific improvements could be:

- In chapter 4, it is noted that only one type of driving distance of EVs and no public charging facility was considered. Note that the underlying SUE model focused on driving distance constraints only. The vehicles' range limit is determined based on its travel distance only, while rationally the range limit should be related to both travel distance and travel time. Elastic demand, link capacity and dynamic battery consumption were all omitted in this model.

- The general SUE models of EVs with battery capacity constraints reported in Chapter 5 ignored the other vehicle types in the network as a sole extension of static driving distance constraints. Similarly all the other considerations such as elastic demand, link capacity, availability of charging stations were omitted as well. These limitations generally affect the general traffic flow prediction in real conditions. The findings presented in this thesis may be improved if multi-class EV users were considered.

- The bi-level charging facility location model in Chapter 6 used a simple way of defining EV coverage by exposing charging facilities to EV drivers instead of using them. The solution algorithm is a heuristic to find a local optimum instead of a global one. Still the EV travel demand is assumed to be known as a prior. Another limitation is that dynamic energy consumption was not considered.

- The BSS location model serving electric buses in Chapter 7 used an assumption that the demand for EB battery swapping services was evenly distributed during the time slots. Another limitation was that the charging scheduling optimization was not considered. Local charging station may have some disadvantages over another mode of operating BSS with central charging system which was not discussed. For the evaluation case study, some technical features were not available for current available EBs and their compatible chargers. This may affect the precision of the presented results.

8.6 Future research directions

Given the limitations discussed in the previous section, a number of areas for future research can be identified.

The driving range limit and the lack of charging infrastructure are two main characteristics of EVs at the current stage. There is a need to extend the general SUE with driving distance limit considering elastic demand. There have been few researches on the stochastic or dynamic traffic assignment of electric vehicle considering elastic demand.

As a pure mathematical modeling tool to characterize BEVs' travel behavior in the network with some ideal socioeconomic assumptions, we expect that the modeling technique and solution methods demonstrated in chapter 5 would potentially trigger the interest of investigating other types of stochastic traffic assignment problems with path-based constraints in logit-type or weibit route choice models. The model itself can also be applied for more accurate quantification of network flows, travel demand and battery capacity levels. As a modeling platform for more practical and realistic model, the proposed model should be enhanced to accommodate mixed traffic flows of different types of vehicles such as BEVs, hybrid vehicles and conventional gasoline vehicles as well as the availability of charging infrastructure. Our future study will investigate the possibility of incorporating charging time, range anxiety level and value of time in model extensions. Based on the SUE models proposed in this paper, we will also investigate how to optimally locate charging stations in the network in terms of different objectives.

Another key avenue for future research regarding the bi-level model involves incorporating other types of BEV-specific constraints in the lower level problem, such as flow-dependent battery capacity constraints, time-dependent battery charging price, etc. As for the upper level problem, some other approaches, such as FILM and FRLM, locating charging facilities to maximize passing BEV flows without double counting, can be explored to better serve the BEV travel demand. The model uses a number of assumptions to simplify the problem and make it tractable, which will be relaxed in the future work to deal with more complicating and realistic issues.

There is also a need for studying BSSs with central charging system where batteries can be treated as goods and transported between central charging facilities and EB depots. There have been few studies found on the operating mode of the battery swapping station which incorporates logistic management into the battery pack transportation. Local charging mode may have a lot of disadvantages. First, it is hard to accurately predict the demand of battery swapping service or the EB arriving pattern (e.g. More EV users may swap during peak hours or public holidays) at each station, thus making it a hard choice to decide the number of charger and battery inventory each station needs. It is a waste of money and resource if the chargers are over-built. If the number of the chargers is less than we need, it means the EV drivers may have to wait for hours to get a full energy battery which will discourage the user and further influence the market penetration. Second, DC fast charger needs a power of around 100kw per charger for DC fast charging or level 3 charging. Building one charger at a station is already a great burden for local electricity power grid, not to mention it usually needs more than that. So it will make few locations available for building new battery swapping stations or rebuilding the existing gas station restricted by the power grid and the safety issue. Last but not least, fast charging does damage to the battery itself and reduces the battery life. By contrast, building a battery distribution center can help solve all the problems above to ensure the acceptable level of service by proper operation of logistic management and inventory information system. Also the existing gas station can be reconstructed by just adding a battery swapping facility and a warehouse for battery storage. A battery distribution center can give more flexibility of battery use with regard to spatial and temporal distribution of the demand by adjusting the battery shipment scheme, thus reducing the number of battery needed in the system by leveraging the battery transportation cost and battery manufacturing cost. Therefore, it is of great value to do this research towards developing a new operating mode for battery swapping station, especially along the corridor between cities for the optimal design of future battery swapping systems which would help in improving the level of service and attracting more drivers to the EBs.

In summary, it is worth to highlight that this thesis has established two STAP models for predicting BEV flows and two charging/swapping facility location models for deploying the facilities to minimize the cost or maximize their utility. It also identified opportunities to enhance the modelling tool for EV schemes for both private BEVs and public EBs. Firstly the methods developed in this research can be used to explore the congestion effects of the upcoming decades when EV market share increases. Secondly while the methods adopted in this research are considered to be robust, it is acknowledged that they come with their own limitations. Further research can address these limitations to build on the knowledge gained from this thesis.

APPENDIX A – CHAPTER 7

No.	BSS Candidate Site	Land Value per Sqm	Bus Depot	demand
0	Ventura Bus Lines - Oakleigh South	1450	Aspendale Gardens	2
1	Ventura Bus Lines - Dandenong Depot	870	Bayswater	9
2	Ventura Bus Lines - Knoxfield Depot	830	Belgrave	8
3	Ventura Moorabbin Transit	1560	Belgrave South	3
4	Ventura Bus Lines - Lilydale Depot	670	Bentleigh	2
5	Ventura Bus Lines - Croydon Depot	830	Berwick Station	8
6	CDC Melbourne - Oakleigh Depot	1640	Blackburn	13
7	Kingstons Tours	1030	Boronia	7
8	Heatherton Bus Depot - Transdev Melbourne	1520	Box Hill	46
9	Transdev - Doncaster Depot	1820	Brandon Park SC	2
10	Transdev - Keysborough Depot	1310	Brighton	6
11	Cardinia Transit	770	Brighton Beach	7
12	broadmeadows bus service	600	Burwood	1
13	Ventura Bus Lines - Heidelberg West Depot	910	Caroline Springs	7
14	Ventura Bus Lines - Monbulk Depot	300	Carrum	6
15	Ventura Bus Lines- Frankston Depot	730	Carrum Downs	2
16	Ventura Bus Lines- Hastings Depot	560	Casey Central SC	3
17	Dysons Bus Service	290	Caulfield	7
18	Cranbourne Transit	660	Chadstone SC	26
19	East West Bus Company	1040	Chelsea	15
20	CDC Melbourne - Altona Depot	1280	Chirnside Park SC	18
21	CDC Melbourne - Sunshine Depot	850	Clayton	2
22	CDC Melbourne - Werribee Depot	650	Cranbourne	7
23	Ventura Bus-Ivanhoe Bus Company	910	Cranbourne Station	2
24	Ventura Bus-Portsea Passenger Service	670	Croydon	14
25	Ventura Bus-SEAFORD	880	Croydon Station	1
26	Kastoria Bus Lines	300	Dandenong	42
27	Martyrs Bus Service	270	Deakin University	4
28	McKenzie's Tourist Services	340	Deep Creek	1
29	Moonee Valley Bus Lines	810	Doncaster Park & Ride	1
30	Moreland Bus Lines	2760	Doncaster Shoppingtown	3
31	Panorama Coaches	900	Doveton	1
32	Reservoir Bus Company	1040	Eastland SC	5
33	Ryan Bros Bus Service	810	Edithvale	2
34	Sita Bus Lines	1540	Elsternwick	9
35	Sunbury Bus Service	580	Elwood	4
36	Transdev - Fitzroy North Depot	5290	Endeavour Hills	3
37	transdev sunshine west	810	Fountain Gate	1
38	Aspendale Gardens	1250	Frankston	44

39	Bayswater	820	Gardenvale	18
40	Belgrave	430	Glen Iris	2
41	Belgrave South	520	Glen Waverley	18
42	Bentleigh	2180	Hampton	5
43	Berwick Station	910	Hampton Station	8
44	Blackburn	1610	Huntingdale Station	5
45	Boronia	740	Kew	4
46	Box Hill	2180	Keysborough	5
47	Brandon Park SC	1370	Knox City	2
48	Brighton	3030	Knox City SC	15
49	Brighton Beach	3030	Knoxfield	1
50	Burwood	1860	Lilydale	4
51	Caroline Springs	1060	Middle Brighton	11
52	Carrum	1000	Mitcham	22
53	Carrum Downs	700	Monash University	15
54	Casey Central SC	850	Montrose	1
55	Caulfield	2300	Moorabbin	8
56	Chadstone SC	2130	Mordialloc	32
57	Chelsea	1519	Mossgiel Park	2
58	Chirnside Park SC	710	Noble Park	1
59	Clayton	1500	North Brighton	1
60	Cranbourne	660	Nunawading	4
61	Cranbourne Station	680	Oakleigh	16
62	Croydon	830	Ringwood	18
63	Croydon Station	850	Ringwood Station	4
64	Dandenong	870	Rowville	5
65	Deakin University	1860	Sandringham	3
66	Deep Creek	1330	Scoresby	1
67	Doncaster Park & Ride	1820	Southland SC	12
68	Doncaster Shoppingtown	1820	Springvale	2
69	Doveton	610	Springvale South	1
70	Eastland SC	1120	St Kilda Station	8
71	Edithvale	1160	St. Kilda	3
72	Elsternwick	2000	Stud Park SC	7
73	Elwood	3300	Templestowe	6
74	Endeavour Hills	760	The Pines SC	2
75	Fountain Gate	730	Upper Ferntree Gully	16
76	Frankston	730	Vermont East	1
77	Gardenvale	2070	Wantirna / Boronia	1
78	Glen Iris	2690	Warrandyte	15
79	Glen Waverley	1720	Waterways	2
80	Hampton	2650	Waverley Gardens SC	9
81	Hampton Station	2670		
82	Huntingdale Station	1640		
83	Kew	3460		
84	Keysborough	1310		

85	Knox City	950
86	Knox City SC	1200
87	Knoxfield	830
88	Lilydale	670
89	Middle Brighton	3030
90	Mitcham	1330
91	Monash University	1500
92	Montrose	590
93	Moorabbin	1520
94	Mordialloc	1410
95	Mossgiel Park	760
96	Noble Park	950
97	North Brighton	3030
98	Nunawading	1330
99	Oakleigh	1640
100	Ringwood	1120
101	Ringwood Station	1140
102	Rowville	1040
103	Sandringham	2320
104	Scoresby	920
105	Southland SC	1560
106	Springvale	1130
107	Springvale South	1030
108	St Kilda Station	4000
109	St. Kilda	4000
110	Stud Park SC	1060
111	Templestowe	1430
112	The Pines SC	1590
113	Upper Ferntree Gully	520
114	Vermont East	1460
115	Wantirna / Boronia	740
116	Warrandyte	640
117	Waterways	1520
118	Waverley Gardens SC	1090

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